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# Predicting credit spreads in the Norwegian Corporate Bonds Market

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# Preface

This master thesis concludes my Master of Science degree with specialization within Investment, Finance and Financial Management at the Department of Industrial Economics at the Norwegian University of Science and Technology, NTNU.

The motivation for writing this thesis arose from my interest in the capital markets, and a specific desire to explore and increase my knowledge of the dynamics of Debt Capital Markets. The Norwegian corporate bonds market is, compared to e.g. the US market, relatively unexplored in academia, which further increased my motivation to study the Norwegian market and uncover possible differences between the markets.

This report has been prepared using ShareLaTeX, the online version of LaTeX. Figures have been prepared in both Microsoft Excel and R Studio, depending on the nature of the plots. Statistical analysis has in general been performed in R Studio, with some simple tests performed in Excel. The implementation of a bond pricing model has been performed in R Studio. The data used in this thesis are collected from several sources, including Titlon/Oslo Børs, Stamdata and Proff Forvalt.

I would like to thank my supervisor, Associate Professor Einar Belsom, for his feedback and commitment in studying the Norwegian corporate bonds market and for providing access to Proff Forvalt. I would also like to express deep gratitude to Per-Marius Pettersen and his colleagues at Stamdata for granting me access to their comprehensive databases on both bond data and Default and Recovery rates and answering my questions regarding the Norwegian corporate bonds market.

Trondheim, June 8, 2017

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Harald Eskerud



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# Abstract

Using a dataset comprising 62 fixed coupon bonds in the period 2000 to 2015 I implement and analyze a structural model for bond pricing in order to develop an understanding of the pricing of bonds in the Norwegian corporate bonds market. The model estimates the part of the credit spread related to compensation for default risk, which consists of the probability of default and the expected loss given default.

The results indicate that the base case implementation does not yield accurate estimates for bond prices and credit spreads in the Norwegian market. The model overestimates compensation for default risk for most bond price observations, particularly for bonds with low coupons and observations with short remaining time to maturity. The analysis of the results indicate that the model's accuracy is sensitive to the specification of recovery rate, which in the base case is assumed to be the same for all firms and static throughout the time period analyzed. Furthermore, the default barrier estimation of total liabilities causes the model to overestimate the probability of default in the short run.

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# Sammendrag

Ved å bruke et datasett bestående av 62 obligasjoner med fast rente utstedt i tidsrommet 2000 til 2015 implementerer og analyserer jeg en strukturell modell for obligasjonsprising for å utvikle en forståelse av prising i det norske obligasjonsmarkedet. Modellen estimerer den delen av kredittmarginen som utgjør kompensasjon for forventet tap, som består av sannsynlighet for mislighold og forventet tap gitt mislighold.

Resultatene indikerer at grunnimplementeringen ikke gir nøyaktige estimater for priser og kredittmarginer. Modellen overpredikerer kompensasjon for forventet tap for de fleste prisobservasjonene, særlig for obligasjonene med lav kuponrente og for prisobservasjoner med kort gjenværende løpetid. Analysen av resultatene indikerer at modellens nøyaktighet er sensitiv til spesifikasjonen av forventet tap gitt mislighold, som i grunnimplementeringen antas lik for alle selskap over tid. Videre kan unøyaktigheten for observasjonene med kort gjenværende løpetid forklares ved spesifikasjonen av grensen for mislighold, som for alle observasjoner implementeres til å være summen av bokført gjeld.

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# Chapter 1

## Introduction

The Norwegian corporate bonds market offers great opportunities for firms seeking debt financing. Since the turn of the millennium the Norwegian corporate bonds market has developed from a small market dominated by banks and the public sector to become a viable source of debt financing for both Norwegian and foreign firms. The Norwegian High Yield market in particular has developed to become one of the largest and most important markets worldwide for the Oil and Gas, Shipping and Offshore sectors (Nordic Trustee, 2015). Despite its position as an important High Yield market and a growing but relatively small Investment Grade market, the literature on the Norwegian corporate bonds market is quite limited.

Pricing of bonds is essential to both investors and issuers. The literature on bond pricing theory has two main stands, namely the structural model approach pioneered by Merton (1974) and the reduced form approach pioneered by Jarrow and Turnbull (1992). Both types of models show varying degrees of empirical accuracy. This can to some extent be explained by the fact that investors are not only compensated for default risk (as assumed in structural models) but other factors such as liquidity and differences in taxation (see, e.g. Elton et.al., 2001; Sæbø, 2015).

My main objective is to predict credit spreads and assess the predictive accuracy of a bond pricing model in the Norwegian corporate bonds market. My dataset consists of 62 different fixed coupon bonds issued in the years 2000 to 2015, all in Norwegian kroner. The aim is to identify pricing mechanisms in the Norwegian market and also to identify possible differences between the Norwegian market and foreign markets, especially the U.S. market. Due to the nature of my available bond price data and the fundamental properties of structural models I choose to implement a structural model of credit risk to predict credit spreads. I implement an extended version of Merton's original structural model for credit pricing as described by Eom et.al. (2004), which models a coupon bond as a portfolio of zero coupon bonds and prices each cash flow using the zero coupon version of Merton's model.

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The literature on empirical assessment of structural models of credit risk is largely focused on the US market. The empirical testing of structural models' ability to predict prices and spreads in the Norwegian market is, to the best of my knowledge, rather limited. This is caused, partly, by the lack of reliable bond price data and also by the rather recent development of the Norwegian bond market. The approach in this thesis is similar to that of and inspired by Eom et.al. (2004) and Knappskog and Ytterdal (2015), but differs from the first by studying the Norwegian market rather than the US and the second by studying bond prices for both Investment Grade and High Yield rather than only the latter at issue.

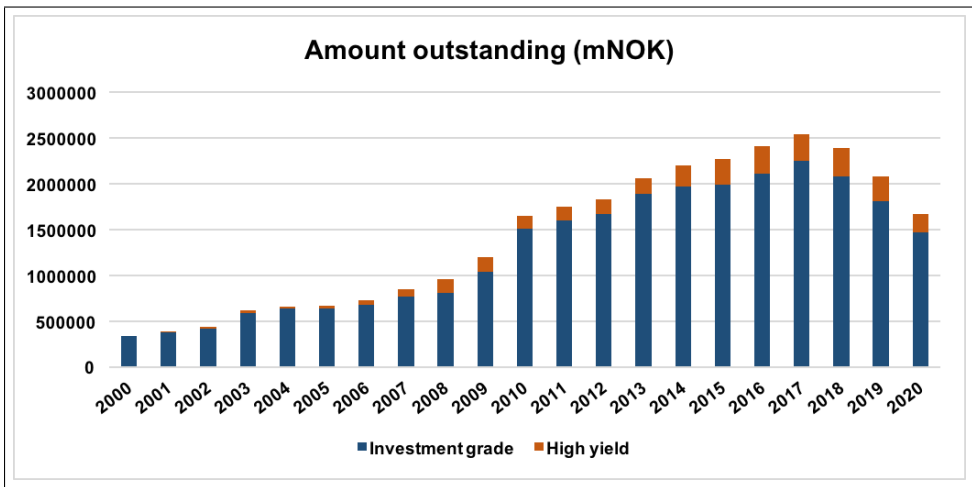
My main finding is that the model tends to overpredict credit spreads, especially for low coupon bonds associated with low risk. The overprediction of spreads can be split in two main sub-problems: overestimating the likelihood of default and overestimating the expected loss given default, resulting in a too low expected recovery rate in the event of default. I find that the market likely prices, to quite large extent, more individuality with respect to expected recovery rates than indicated by the results of Eom et.al. (2004), who apply a single recovery rate across their entire sample. Additionally, my results indicate that the estimates of the model parameters are more accurate for observations with longer remaining time to maturity, as the model underprices bonds more with short remaining time to maturity than long remaining time to maturity. The default barrier estimate with short remaining time until maturity is a feasible source of the short-run overprediction of default.

The remainder of this thesis is organized as follows. First I give a brief presentation of some of the key characteristics of the Norwegian Bonds market. I then present bond pricing theory with special focus on structural models in Section 3, before I present the data in Section 4. In Section 5 present the methodology, with a presentation of the proposed structural model and the estimation of the input parameters. I then present and analyze the model accuracy in Section 6, before I conclude and give recommendations for future research in Section 7.



# The Norwegian Bonds Market

Since the turn of the millennium the Norwegian bonds market has seen significant developments for both Investment Grade and High Yield bonds. The market has developed from a small regional market to a viable source of financing, as illustrated in Figure 2.1. The High Yield market has been of particular interest in recent years, mainly due to the ease of issuing such bonds compared to in other markets (Nordic Trustee, 2015). Despite the relative size of the High Yield market the Norwegian corporate bonds market is dominated by Investment Grade issues, as illustrated in Figure 2.1.



**Figure 2.1:** Development of outstanding amount in Norwegian bonds market. Source: Stamdata

The most prominent sectors are Bank and Finance, followed by Government and Public Sector issues. This reflects the size and importance of the public sector in the Norwe-

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gian economy. The remaining part of the market, commonly referred to as the "corporate bonds" market, is dominated by issuers in capital intensive industries such as Offshore, Shipping and Oil and Gas, which are all important Norwegian industries (PwC, 2016).

One of the most important characteristics of the Norwegian corporate bonds market is the relative ease of both issuing and listing bonds for secondary market trading. The ease of issuing bonds has been driven, among other factors, by shadow ratings. Unlike in e.g. the US market the Norwegian market has no requirements of ratings from official rating agencies such as S&P and Moody's for listing of bonds. Instead, Norwegian investment banks have historically provided shadow ratings on issuers/bonds as a part of the issuing process service, which reduces the transaction cost. The shadow ratings use a rating scale which is intended to correspond to the scales defined by Moody's and S&P. Investment grade issues are issues with ratings over BBB, while high yield issues are rated BBB or lower. Shadow ratings are regarded as an important success factor for the Norwegian bonds market (Nordic Trustee, 2015; Finans Norge, 2017). As official ratings are costly, they tend to only be available to large companies. Therefore, shadow ratings facilitate in opening the Debt Capital Markets to smaller firms. It should be noted that the practice of providing shadow ratings has come under scrutiny, and as of 2017 the Norwegian brokerages have been forced to stop providing shadow ratings. This is because the European Securities and Markets Authority argue that the use of an official rating scale in an unofficial rating is misleading (Schwartzkopff, 2016) and the use of the scale should be reserved for official rating agencies.

While issuers are not required to list their bonds many investors, which are usually institutional funds, have mandates of only investing in listed bonds (PwC, 2016). The Norwegian corporate bonds market offers issuers two different marketplaces for listing, namely Oslo Børs and Nordic ABM (Alternative Bonds Market). Nordic ABM opened in 2005 to serve as an alternative to Oslo Børs. The main differences between the market places are related to listing requirements. To list bonds on the Oslo Børs marketplace financial reporting has to follow IFRS standards and prepare a EEA approved prospectus (usually prepared by investment banks), in addition to other requirements. At Nordic ABM the listing process is less demanding than for Oslo Børs, and offers greater freedom with respect to accounting standards. For instance, no prospectus is required for listing on ABM (Oslo Børs, 2015). Additionally, Oslo Børs offers great flexibility with respect to issuing currency, which allows issuers to hedge exchange rate risk. The ease of both issuing and listing bonds are regarded as the most important factors for the development of the Norwegian Corporate Bonds market (Nordic Trustee, 2015).

It should also be noted that bonds need not be traded centrally. In fact, according to Rakkestad et.al (2013), most trading of bonds takes place off market. All off market transactions are required by Oslo Børs to be reported immediately or at the end of the trading day (Rakkestad et.al., 2013).

## Bond Pricing Theory and Credit Models

There are two main classes of credit risk models, namely structural and reduced form models (see Jarrow and Protter, 2004; Wang, 2009). In addition, a third class consists of purely statistical models. In this section I present a review of literature and theory related to the different model classes. Structural models of credit risk originate with the pioneering work of Black-Scholes (1973) and Merton (1974). The models are based on contingent claims analysis (CCA) as they utilize option pricing theory to value debt. As structural models require market input parameters, the empirical testing of structural models is mainly focused on debt of publicly traded firms (Eom et.al., 2004). Reduced form models model a firm's time to default as a stochastic process with price parameters that are fitted to past bond price data. Reduced form models are classified as such as they model default as a statistical event without providing an economic explanation as to why default occurs (Arora, Bohn and Zhu, 2005).

I implement an extended version of Merton's basic credit model, and motivate this by several factors. The first is the relative intuitiveness of structural models in general compared to reduced form models. As Wang (2009) argues, the structural approach led by Merton has the appealing feature of connecting credit risk to underlying structural variables, rather than simply a statistical explanation. Furthermore, the properties of available bond price data makes it difficult to implement a reduced form model. More specifically in the case of the Extended Merton Model it treats a coupon bond as a portfolio of zero coupon bonds. This approach leads to a relatively straightforward formula, which is explained in detail in Section 5.1. The Extended Merton Model is also implemented by Knappskog and Ytterdal (2015). Due to the model's solid theoretical foundation, implementation in other studies and relative intuitiveness I find the Extended Merton Model to be an appropriate model choice.

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## 3.1 Structural Models

In this section I present the most basic structural model of credit risk, namely the basic Merton model, and its underlying assumptions. The basic Merton model is the cornerstone of all structural models of credit risk. I then present an overview of the developments of structural models and advantages and disadvantages.

### 3.1.1 Basic Merton Model

Merton (1974) argues that the value of a bond essentially depends on the required rate of return on risk-free debt, the various provisions and restrictions contained in the debt indenture and the probability of default. It follows that for two otherwise identical bonds the difference in yield to maturity arises from a different probability of default. The Basic Merton Model is based on the following assumptions (Sundaresan, 2013):

1. There are no transaction costs, taxes or problems with indivisibility of assets
2. There is a sufficient number of investors with comparable wealth levels such that each investor believes he can buy and sell as much of an asset as he wants at the market price
3. An exchange market exists for lending and borrowing at the same rate of interest.
4. Short sales of all assets, with full use of the proceeds, are allowed.
5. Trading takes place continuously in time
6. The Modigliani-Miller theorem that firm value is invariant to capital structure holds.
7. The term structure is flat and known with certainty, so that the price of a riskless discount bond with unit face value at time  $T$  in the future is  $P(t, T) = e^{-r(T-t)}$ , where  $r$  is the riskless rate of interest, the same for all time.
8. The dynamics for the value of the firm,  $V_t$ , follows a geometric Brownian motion under the physical measure  $dV_t = \mu V_t dt + \sigma_V V_t dW_t$ ,  $V_0 > 0$ , where  $\mu$  is the mean rate of return on the assets,  $\sigma_V$  is the volatility of the return of the firm's assets and  $dW_t$  is the standard Wiener process.

Several of Merton's assumptions seem rather unrealistic. It is, for instance, unrealistic to assume trading takes place continuously in time. Merton (1974) notes, however, that the first four assumptions can easily be relaxed. The assumption of a flat term structure is also unrealistic; however it is not a serious drawback and its generalization is proven in Merton (1974). According to Sundaresan (2013) the critical assumptions are assumptions 5, 6 and 8.

A payout ratio,  $\delta$ , can be incorporated into the dynamics of asset value by making a simple modification to Merton's 8th assumption. This leads to the following stochastic process for firm value:

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$$dV_t = [\mu - \delta]V_t dt + \sigma_V V_t dW_t, V_0 > 0 \quad (3.1)$$

The first part of Equation 3.1 describes the expected drift of the firm's assets, while the second part is used to model unpredictable events. The asset volatility can be interpreted as a scale parameter, which makes the impact of the unpredicted events on firm value larger or smaller depending on the magnitude of volatility (Poon and Granger, 2003). In the following, the dynamics of firm value is assumed to follow Equation 3.1.

Suppose that at time  $t$  a firm has assets  $V_t$ , financed by equity  $E_t$  and *zero-coupon* debt  $D_t$  with face value  $B$  and maturity  $T$ . The value of the firm's assets  $V_t$  is obtained from the balance sheet relation

$$V_t = E_t + D_t \quad (3.2)$$

In the basic Merton model default can only occur at maturity  $T$ , and if default occurs creditors take over the firm without incurring any distress costs and realize an amount  $V_T$ . If the asset value  $V_T$  is greater than  $B$  the debt is paid in full, and the remainder,  $(V_T - B)^+$ , is distributed among shareholders. On the other hand, if  $V_T < B$  shareholders will receive nothing, but are not required to inject additional funds to pay the debt (by limited liability). From the above observations it is seen that at maturity  $T$  equity holders receive payoff equal to

$$E = \max[V_T - B, 0] \quad (3.3)$$

while the payoff to creditors is equal to

$$D = \min[V_T, B] = B - (B - V_T)^+, \quad (3.4)$$

The key insight from Equations 3.3 and 3.4 is that shareholders are long a call option written on the firms assets. On the other hand, the creditors are short a put option written on the assets of the borrowing firm. The strike price of the option is  $B$ , i.e. the face value of debt. The value of equity and debt can be calculated using option valuation as developed by Black-Scholes (1973):

$$E(V_t, t) = \text{Call}_{BS}(V_t, B, r, T - t, \sigma) \quad (3.5)$$

$$D(V_t, t) = \text{BP}(t, T) - \text{Put}_{BS}(V_t, B, r, T - t, \sigma) \quad (3.6)$$

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Equation 3.6 can be viewed as a hedged position for creditors. By buying a put option written on the firm's assets creditors can hedge the default risk. This way, they are guaranteed a payoff of  $B$ . By rearranging the terms of equation 3.6 the hedged position can be expressed as

$$D(V_t, t) + Put_{BS}(V_t, B, r, T - t, \sigma) = BP(t, T) \quad (3.7)$$

From Equations 3.6 and 3.7 the spread in Merton's model between risky debt and otherwise identical risk-free debt is simply the value of the put option written on the firm's assets. Using Black-Scholes option valuation framework, the value of equity can be shown to be equal to

$$E_t = V_t N(d_1) - B e^{-rT} N(d_2), \quad (3.8)$$

while the value of debt is equal to

$$D(t, T) = V_t e^{-\delta(T-t)} N(-d_1) + BP(t, T) N(d_2), \quad (3.9)$$

where  $N(\cdot)$  is the cumulative standard normal distribution function and

$$d_1 = \frac{\ln(\frac{V_t}{B}) + (-\delta + \frac{1}{2}\sigma_V^2)t}{\sigma_V \sqrt{t}}, d_2 = d_1 - \sigma_V \sqrt{t} \quad (3.10)$$

### 3.1.2 Development of structural models

The Basic Merton Model has its obvious shortcomings. One of the most critical is related to the firm value process, assumed to follow the dynamics of Equation 3.1, which cannot be easily observed. In addition to the limitations previously mentioned it is incorrect to assume that firm value is tradeable. Furthermore, default can only occur at maturity, which is hardly realistic, and costs of financial distress are not incorporated. The subsequent development of structural models therefore attempts to address these obvious shortcomings.

The first and simplest so-called *first passage model*, presented by Black-Cox (1976), allows for default before maturity, as they incorporate safety covenants in the model. In Merton's model default can only be exogenous (determined by the value process), while Black-Cox incorporate endogenous default, i.e. that the default is incurred as the results of management decisions, often to maximize equity value. In the Black-Cox model the debt is

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assumed perpetual (i.e. no principal payment) with a constant coupon rate (Leland, 2006). Leland (1994) and Leland and Toft (1996) address the shortcomings of earlier structural models by incorporating four additional aspects, namely costs of financial distress, payout ratios, taxes and a finite average maturity of debt.

The Kealhofer, McQuown and Vasicek (KMV) Model is one of the more successful structural models. From a purely theoretical view the Basic Merton and KMV Models are very similar; the KMV model relies heavily on empirical testing with a very large proprietary database (Cirillo, 2014). The KMV model overcomes two of the weaknesses of the Merton model, namely Gaussianity and default before maturity, by incorporating a different value of the default barrier which better reflects the firm's capital structure.

In their study Eom et.al. (2004) test and analyze the performance of five different structural models. The first is the Extended Merton Model, which as mentioned models coupon bonds as a portfolio of zero coupon bonds where each cash flow is priced using the zero coupon version of the Merton model. The model incorporates costs of financial distress by including a recovery rate in the event of default. In addition to the Extended Merton model Eom et.al. (2004) test the Geske (1977), Longstaff-Schwartz (1995), Leland-Toft (1996) and Collin-Dufresne and Goldstein (2001) models. The models differ in a number of important features, including the specification of default boundary, recovery rate and coupon.

The Geske (1977) model treats each coupon payment as a compound option (i.e. an option on an option), in which if the equity holders decide to pay the coupon the firm stays alive, otherwise default occurs. In the Leland-Toft (1996) the firm continuously issues a constant amount of debt that pays continuous coupon. The Longstaff-Schwartz (1995) model allows for stochastic interest rates, while Collin-Dufresne and Goldstein (2001) model extends the Longstaff-Schwartz model by incorporating a stationary leverage ratio. Eom et.al. (2004) conclude that the five models are unable to accurately predict spreads. The Extended Merton Model predicts too low spreads on average, as does the Geske model, while the remaining three predict spreads that are too large on average.

## **3.2 Reduced Form Models**

Reduced form models originate from the work of Jarrow and Turnbull (1995). The model class does not assume that default is directly based on a firm's cash flow or value, but instead estimate a jump rate to default empirically (Leland, 2006). An advantage of reduced form models is that they can allow for other sources of risk premia to compensate investors for other sources of risk than default risk (Campbell and Taksler, 2003). Such types of risk include liquidity and systematic credit risk.

In their study Jarrow and Protter (2004) compare structural and reduced form models. They argue that the difference between the model types lie essentially in the information set assumed known by the modeller. In structural models the authors argue that the modeller has complete knowledge of the firm's assets and liabilities, which implies that the modeller has the same information available as a firm manager. In other words, structural

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models essentially assume that the modeller has more knowledge than he can actually have. On the other hand, reduced form models in essence assume that the modeller only has market information available - that is, incomplete knowledge of the firm's condition. The knowledge available leads to (in most cases) predictable default times in structural models, while in reduced form models default time is inaccessible. Consequently, Jarrow and Protter (2004) argue that reduced form models should be preferred for pricing and hedging.

### **3.3 Other Sources of Risk Compensation**

According to Eom et.al. (2004) much of the development in structural models literature is motivated by the perception that Merton's model is unable to generate sufficiently high yield spreads as to what is observed in the market. Structural models in general predict credit spreads as if investors were only compensated for default risk. The empirical literature finds, however, that only a small part of the credit spread can be contributed to compensation for default risk (see, e.g. Elton et.al., 2001; Amato and Remolona, 2003; Sæbø, 2015). This is commonly referred to as the credit spread puzzle.

Sæbø (2015) argues that the so-called puzzle arises because investors are in fact not risk-neutral but risk-averse and therefore demand compensation for other types of risk in addition to default risk. Therefore, quite extensive literature has emerged on other explanatory factors for credit spreads than compensation for default risk. The most common factors, according to Sæbø (2015), are differences in taxation between government and corporate bonds, liquidity between the bond types and e.g. Fama-French-factors. In their study Elton et.al.(2001) find that differences in taxation is an important explanatory variable to the credit spread in the U.S. market. This is, however, not a possible explanatory factor in the credit spreads for Norwegian corporate bonds as government and corporate bonds are taxed equally (Sæbø, 2015).



# Chapter 4

## Data

The dataset used in this thesis is obtained from several sources. In this chapter I present the various sources of data on bond issues, bond prices, financial and market data and recovery rates. First, I discuss the assumptions and methods that have been used to identify the final sample. I then present descriptive statistics on the final bond sample, before I present the Default and Recovery data sample.

### 4.1 Bond Prices

Bond price data are retrieved from Titlon (NTNU, 2017). Titlon is a financial database created in cooperation between seven Norwegian higher education institutions and Oslo Stock Exchange. The database contains data on securities that have been traded on Oslo Stock Exchange from 1980 until 2016. Types of securities include equities, bonds, warrants, options and funds. The Titlon bond price data includes open prices, high and low intraday prices, last traded price, official volume and unofficial volume. All prices are quoted as a percentage of par value. For a full list of participating institutions and other info about Titlon please visit <https://titlon.uit.no/>.

An issue with the Titlon data is that prices are only quoted on dates with an official transaction volume. For many bonds there is a long period between transactions, which could indicate illiquidity. I have made the assumption that the bond price remains constant between the Titlon recorded trade dates. A check with bond pricing data obtained from Thomson-Reuters suggests that this is reasonable.

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## 4.2 Bond Sample Construction

Data on bond characteristics such as maturity, coupon rate, industrial classification and bond rating are retrieved from Stamdata. Stamdata is a subsidiary of Nordic Trustee, which is the leading provider of bonds data in the Nordics. The initial Titlon bond price sample consists of 7243 debt securities, issued by 838 different issuers. Key characteristics for the debt securities, such as redemption type, coupon, maturity and convertibility, are found in the Stamdata bonds database by matching the ISIN code of each security.

### Preliminary Bond Sample Overview

In the following I present an overview of the characteristics of the securities in the initial sample. 1496 securities were not found in Stamdata and are consequently left out. Table 4.1 presents the industrial classification of the initial sample. It is clear that bonds issued by banks make up the largest industry group in the sample, followed by utilities and finance.

Industry	N	% of total
Agriculture	1	0.02 %
Auto	1	0.02 %
Bank	2682	46.67 %
Consumer services	112	1.95 %
Convenience Goods	80	1.39 %
Finance	577	10.04 %
Government	113	1.97 %
Health Care	3	0.05 %
Industry	133	2.31 %
Insurance	31	0.54 %
Media	16	0.28 %
Oil and gas E&P	61	1.06 %
Oil and gas services	183	3.18 %
Public Sector	337	5.86 %
Pulp, paper and forestry	12	0.21 %
Real Estate	350	6.09 %
Seafood	21	0.37 %
Shipping	99	1.72 %
Telecom/IT	56	0.97 %
Transportation	95	1.65 %
Utilities	784	13.64 %
<b>Total</b>	<b>5747</b>	<b>100.00 %</b>

**Table 4.1:** Initial sample industrial classifications.

Table 4.2 presents the rating of the debt securities in the initial sample. Most bonds are rated as investment grade.

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Rating Group	N	% of sample
Investment grade	5056	88%
High Yield	691	12%

**Table 4.2:** Initial sample bond ratings.

Table 4.3 presents the initial sample with respect to coupon type. Floating rate notes constitute the largest part of the sample, followed by fixed coupon issues. "Others" refers to Adjust, CPI, Linked Notes and Zero-coupon bonds.

Coupon type	N	% of sample
Fixed	2085	36.3%
Floating Rate Notes	3171	55.2%
Other*	491	8.5%

**Table 4.3:** Overview of initial sample coupon types.

## Screening of bonds

To avoid biased results it is important that the bond sample is relatively homogeneous with respect to a number of properties. For instance, the spread on a convertible bond will on average be lower than for an otherwise similar plain vanilla bond, as the convertible bond contains an upside in the embedded call option. One model would not be valid to analyze and compare credit spreads for the different bond types. The various steps in the sample construction are described in the following.

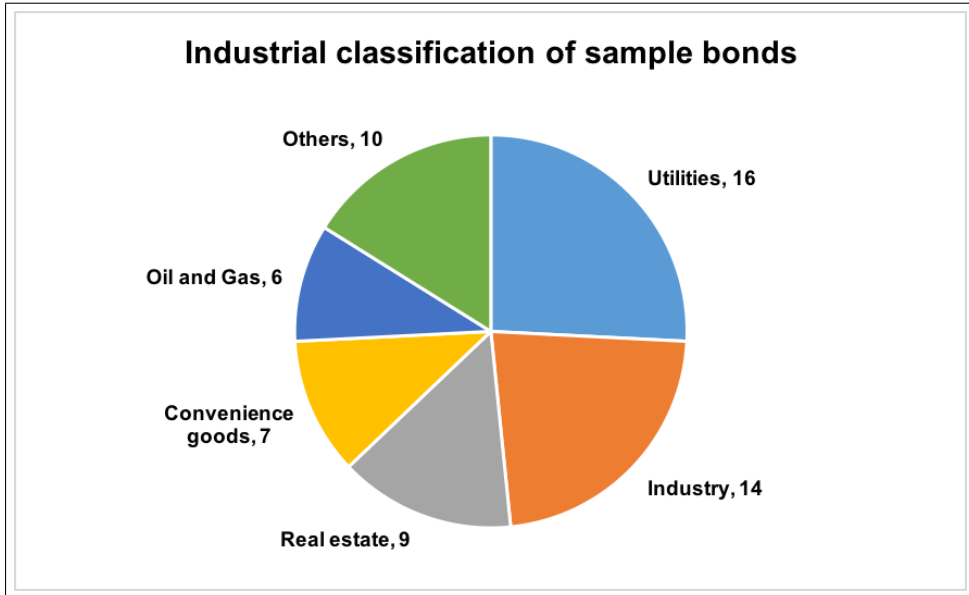
The first step in the sample construction was, as mentioned, to exclude debt securities securities not found in Stamdata 2000-2016, in order to ensure that the classification of the bonds is consistent. This excludes 1496 securities. Excluding non-bullet bonds and other debt securities such as convertibles leaves out 1703 securities. To ensure comparable leverage ratios across firms bonds issued by banks, finance and insurance companies are excluded, leaving out 2791 bonds. Excluding bonds issued by governments and the public sector leaves out another 316 securities.

As is described in detail in Section 5, the proposed bond pricing model requires market and financial data. Therefore, all bonds issued by non-public firms are excluded. Following Warga (1991), I exclude bonds with maturity of less than one year, as they are highly unlikely to trade. Note that such debt securities should have been excluded earlier in the screening, as it is the norm to classify such securities as notes. At the time of writing, financial data for the ASAs are only available until the end of 2015. Therefore, all bonds issued after 1.1.2016 are excluded. Only including fixed coupon bonds leaves 62 bonds to be analyzed.

## 4.2.1 Final Bond Sample Overview

In the following section I provide an overview of the final screened bond sample. A full list of bonds is presented in Appendix B.

Figure 4.1 illustrates the industrial classification of the bonds in the sample. Bonds issued by utility companies (mainly Hafslund) account for the most bonds in the sample, followed by industry and real estate.



**Figure 4.1:** Industrial classification of final bonds sample.

Table 4.4 presents how the sample bonds are rated. Most bonds are investment grade, as is the case with the initial sample.

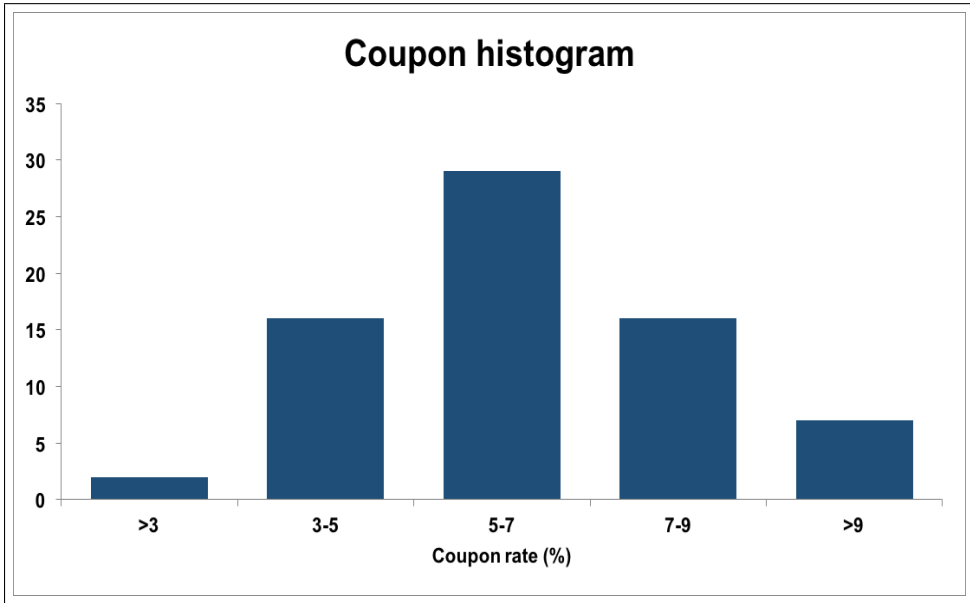
Rating	N
Investment grade	44
High yield	18

**Table 4.4:** Ratings of final bonds sample.

Table 4.5 and Figure 4.2 describe the distribution of coupon rates of the final sample. The average coupon rate lies in the investment grade range.

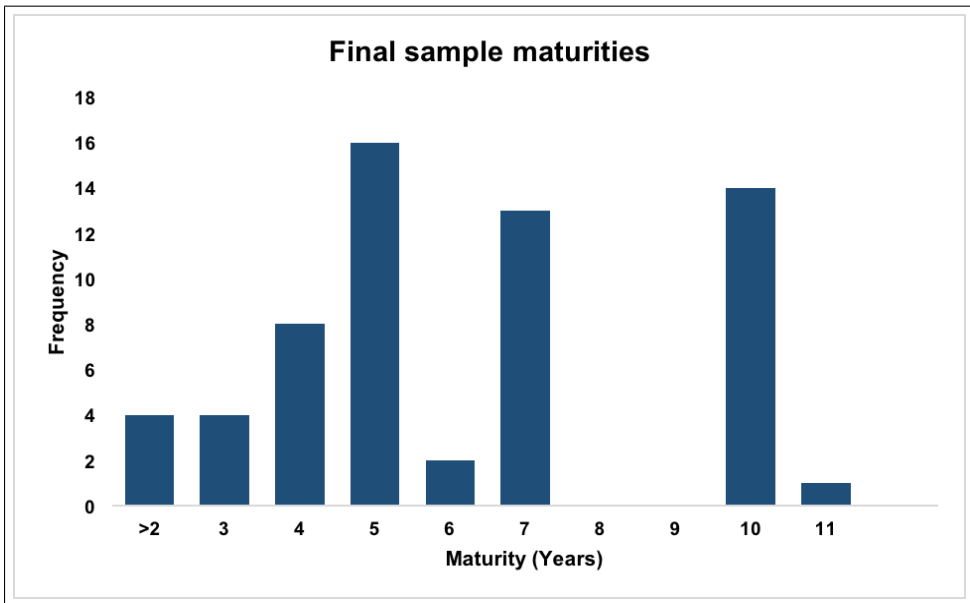
Coupon type	N	Mean	Standard deviation	Max	Min
Fixed	62	6.52%	2.51%	15.50%	2.51%

**Table 4.5:** Descriptive statistics for coupons of final bonds sample



**Figure 4.2:** Histogram of coupon rates for final bonds sample

Descriptive statistics for the maturities of the bonds in the final sample are illustrated and presented in Figure 4.3 and Table 4.6. Most bonds in the final sample have a maturity of around 5 years, which is slightly less than the average maturity of 6.2 years.



**Figure 4.3:** Final sample maturities.

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Mean	Standard deviation	Max	Min
6.2	2.6	11	1.07

**Table 4.6:** Descriptive statistics for final bond sample maturities.

## 4.2.2 Financial and market data

Financial and market data are obtained from several data sources to obtain a comprehensive set of input variables for my models. Financial data, including debt book values and interest expenses, are obtained from Proff Forvalt, which is a leading Norwegian provider of accounting data. Financial market data are obtained from Titlon (NTNU, 2017). This data includes market capitalization, number of shares outstanding, dividends and share repurchases.

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## 4.3 Default and recovery rate data

The expected recovery rate, or 1-expected loss given default, is an important input parameter in the extended Merton model. My estimate for the parameter is based on data from Stamdata's Default and Recovery database on default events from 2006 to 2017. The database contains data on recovery rates for 298 debt issues. In the following I first present an overview of Stamdata's methodology for defining credit events and calculating recovery rates, followed by a presentation of the Default and Recovery data.

### 4.3.1 Stamdata methodology

#### Definition of default

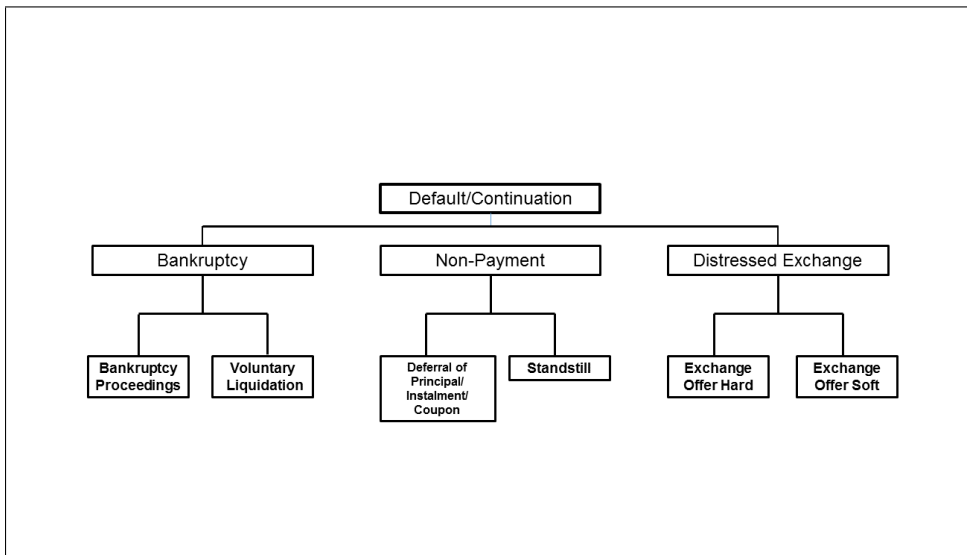
Stamdata may declare a debt instrument to be in default if one of the following events occurs (Stamdata, 2017):

1. The issuer or obligor fails to fulfil the contractually-agreed principal or interest payment, unless in the opinion of Stamdata it is likely that (i) such payment will be made in full within the short-term, and (ii) it is not due to illiquidity, insolvency or willingness or ability to pay
2. If for issuer or obligor any step is taken in relation to insolvency proceedings or dissolution that will cause the suspension of payments under the contractually-agreed debt obligation
3. Any debt exchange or restructuring where the following apply:
  - The issuer or obligor arrange for a tender or exchange offer inferior to the terms of the original contractual agreement, or;
  - The tender or exchange offer is in the opinion of Stamdata recognized to be meaning that the event took lace to avoid a future bankruptcy or payment default.

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## Types of credit events

Figure 4.4 presents an overview of Stamdata's credit event classifications. The different classifications are described in detail below.



**Figure 4.4:** Overview of default event classifications in the Stamdata Default and Recovery database.

According to Stamdata (2017) an event is recognized as "bankruptcy" if any of the following statements apply:

- The issuer of the reference debt instrument faces insolvency or bankruptcy proceedings;
- The issuer voluntarily liquidates the assets of the company to avoid bankruptcy with bondholders' approval;
- The issuer of the reference debt instrument is subject to the appointment of an administrator, liquidator, conservator, receiver, trustee, and custodian or similar for all/majority of its assets.

"Non-payments" occur when the issuer of the reference debt instrument fails to make a coupon payment and/or principal on any payment date (or any given grace period). Standstill agreements where the issuer and bondholders agree to suspend the loan agreement for a given time period in order to negotiate a new deal will also count as a default (Stamdata, 2017).

A "distressed exchange" refers to a restructuring event, where a debt instrument is restructured at diminished economic value to investors in order to relieve the issuer of financial pressures and to avoid non-payment or bankruptcy events (Stamdata, 2017). In such incidents the new offer is below the contractually bound claim. It is however often superior to



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what investors could anticipate in a bankruptcy settlement. "Exchange Offer Hard" refers to default events where the issuer exchanges all or parts of the debt instrument for a new security of inferior value or a reduction of in the amount of principal at maturity. "Exchange Offer Soft" refers to default events that arise as a result of deterioration of the creditworthiness or financial condition of the issuer. Such results will typically be extended maturities or decreases in coupons.

There may be multiple credit events in a bond. To avoid skewed statistics Stamdata define an additional classification called "Continuation". According to Stamdata (2017) the purpose of the classification is to indicate whether an event is deemed to be a new default rather than a continuation of an event that did not have a resolving exit.

### **Estimation of Recovery Rates**

In their recovery rate estimation Stamdata include credit events defined as bankruptcy and distressed exchange (see Figure 4.4), while events in the Non-Payment group are excluded. Stamdata estimate three main types of recoveries, namely ultimate recoveries, interim recoveries and insufficient data. Ultimate recoveries refer to the actual value an investor receives by holding the bond until settlement. The ultimate recovery is estimated using one of or a combination of the following three methods:

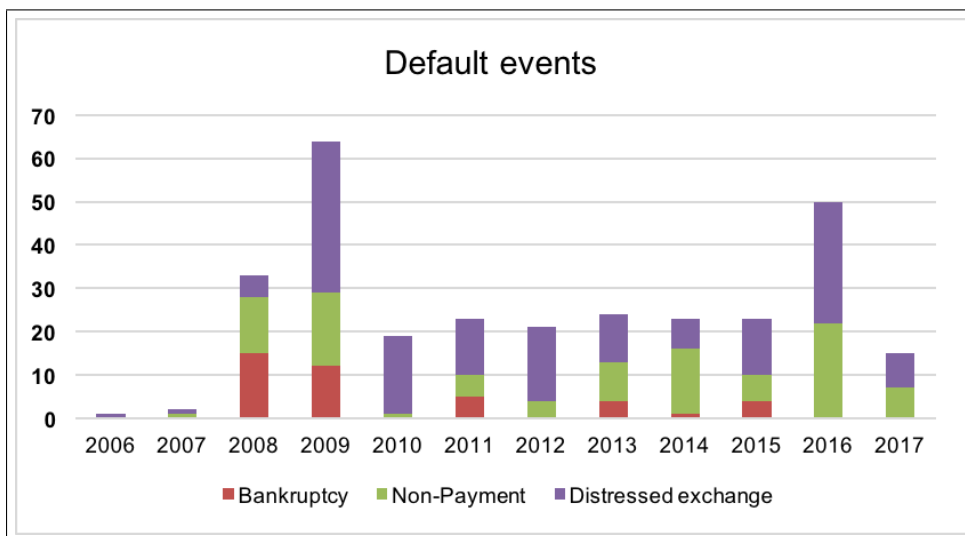
- Trading price method: Trading prices (market value) of the prepetition ("original") instrument upon emergence of the credit event.
- Settlement method: Earliest available trading prices (market value) of the new instrument received in exchange for the prepetition instrument
- Liquidity method: Value of cash or instrument settlement from the liquidations of issuer's assets in exchange for the prepetition instrument, or other cash distributions made to debt holders.

If the ultimate recovery value may not be determined for some time an interim recovery rate is estimated. Lastly, in the event of absence of available liquid market value or any forms of cash settlement Stamdata set the recovery value at 100%. According to Stamdata (2017) such events are usually linked to older defaults where secondary market prices have been difficult to obtain. As mentioned bonds may experience several credit events and consequently multiple recovery rates. Stamdata estimate the *final recovery* to measure the overall recovery an investor will receive when holding the bond from disbursement date to maturity/solution.

## **4.3.2 The Default and Recovery Database**

Stamdata's proprietary Default and Recovery database contains detailed information on credit events for 298 different issues. Figure 4.5 illustrates the default events sample with respect to credit event date and type of credit event. 2009 saw the highest number of default events, followed by 2016. 2008 saw the highest number of bankruptcies followed by 2009,

which reflects the impact of the financial crisis. Distressed exchanges make up the highest number of default events in most years, apart from 2008 and 2014. The number of default events is fairly stable in the years after the financial crisis, before a sharp increase in 2016. Note that default events until April 2017 are included, which helps explain the low number of default events in 2017.

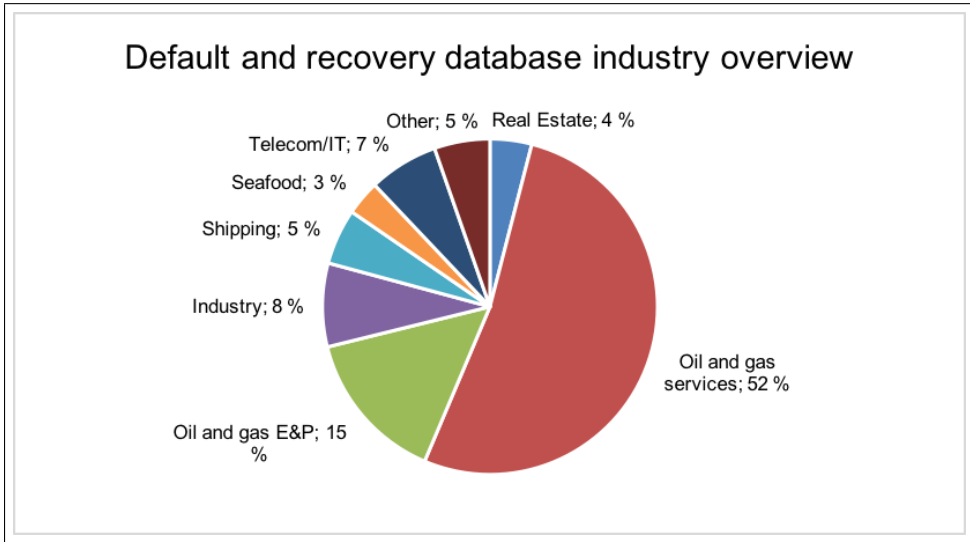


**Figure 4.5:** Overview of credit events with respect to year of event and type of default event.

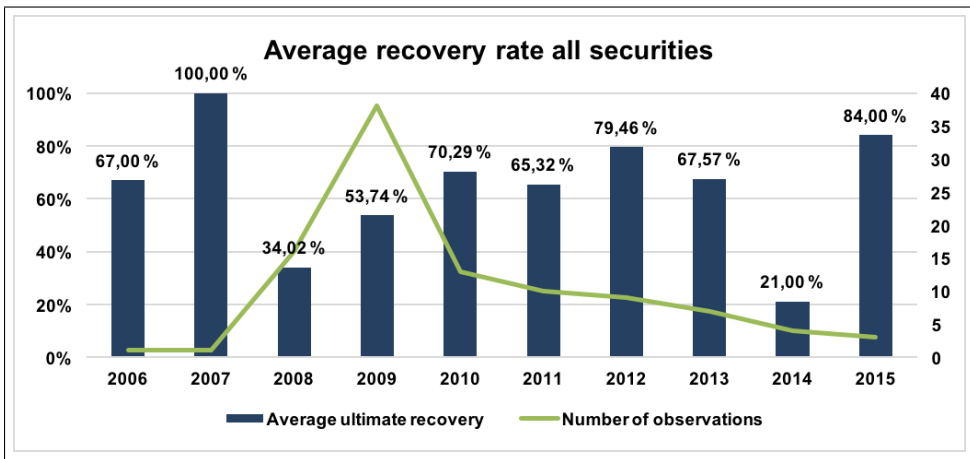
Table 4.7 presents an overview of default events for different issue types. From Table 4.7 it is clear that most default events involve distressed exchanges for bonds. The vast majority of default events in the Default and Recovery database are related to firms in the Oil and Gas industries, as illustrated in Figure 4.6. This is not surprising, considering the nature of Norwegian business. Neither is it surprising given the capital intensity of the industry.

Issue type	Bankruptcy	Non-Payment	Distressed Exchange
Bonds	28	85	117
Convertibles	12	14	38
CDs	1	1	2

**Table 4.7:** Overview of default events for different issue types.



**Figure 4.6:** Overview of default event sample with respect to year of event and type of default event.



**Figure 4.7:** Development of average final recovery rate for all issues in Default and Recovery database.

The Default and Recovery database contains data on final recovery rates for 102 credit events from 2006 to 2015. Figure 4.7 illustrates the development in average final recovery for each year and the number of events per year. The very high recovery rates of 2006 and 2007 should be interpreted with care due to the very low number of observations. The low average ultimate recovery rate for credit events in 2008 should also be interpreted with care, as these events occurred during the financial crisis. The figure indicates that the historical recovery rates are higher in times of financial stability.

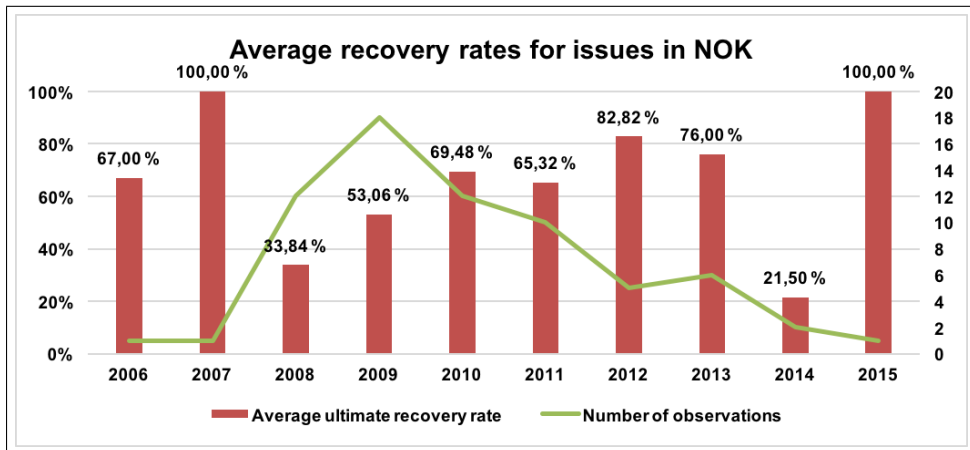
### 4.3.3 Final Recovery Data for Securities Issued in Norwegian kroner

As the focus of this thesis is on bonds issued in Norwegian kroner I exclude all bonds issued in other currencies from the Default and Recovery rate database. This leaves 68 securities, which are classified as bonds, convertibles and CDs (certificate of deposit) as per Table 4.8.

Issue type	N	Average final recovery
Bonds	39	56.77%
Convertibles	28	63.32%
CDs	1	41%

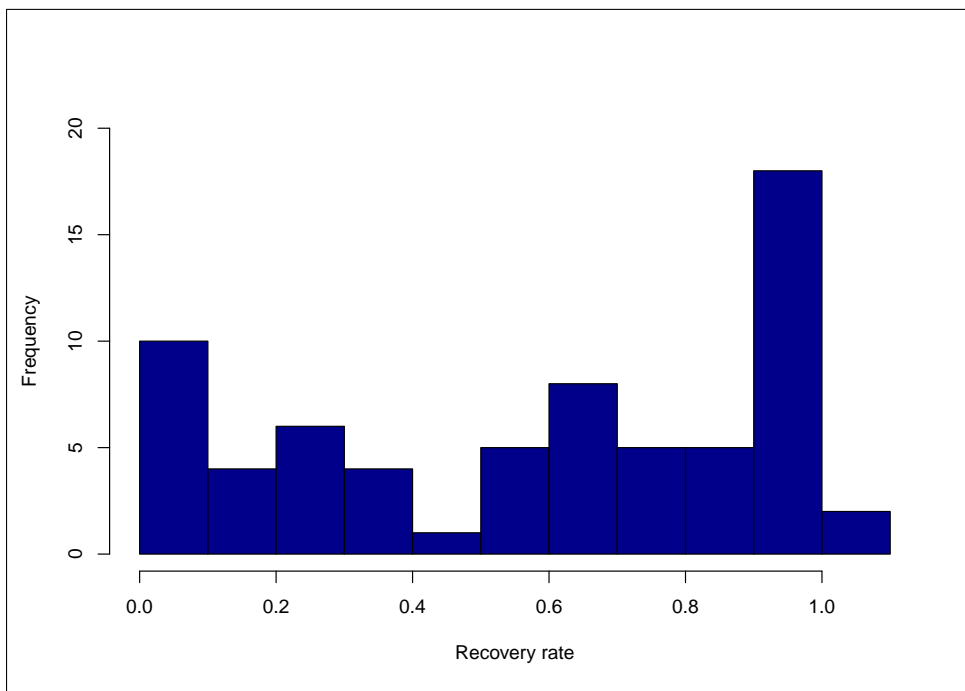
**Table 4.8:** Average final recovery rates for issue type and issues in Norwegian kroner.

Figure 4.8 illustrates the time-varying properties of the average final recovery rate for issues in Norwegian kroner. It shows a similar pattern as the final recovery rate for all bonds in the sample, with the highest number of events in 2009.



**Figure 4.8:** Development of average final recovery rate for issues in NOK

Figure 4.9 and Table 4.9 describe the distribution of final recovery rates for issues in Norwegian kroner. The recovery rate fluctuates widely from its mean, and it is clear that the distribution of recovery rates is not Gaussian. The maximum recovery rate seems slightly peculiar, but is explained by the fact that investors in the end actually gained an upside through the settlement process.



**Figure 4.9:** Distribution of final recovery rates for issues in NOK

Mean	Standard deviation	Max	Min
0.5924	0.3572	1.06	0

**Table 4.9:** Descriptive statistics for average final recovery rate for issues in NOK

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# Methodology

In this section I present the proposed credit pricing model: The Extended Merton model, as described by Eom et.al. (2004). I begin by presenting and explaining the model, followed by a presentation of the estimation of the different model parameters.

## 5.1 Bond Pricing Model

The extended Merton model, as described by Eom et.al. (2004), allows for coupon payments by modelling the coupon bond as a portfolio of zero coupon bonds as well as allowing default before maturity. Each of the zero coupon bonds can be priced with the zero coupon version of Merton's model. The model considers a defaultable bond with maturity  $T$  with *unit face value* and semiannual coupon payments at an annual rate  $c$ . For simplicity, Eom et.al. (2004) assume that  $2T$  is an integer, an assumption followed in this thesis. In practical terms this implies that if a bond has maturity of e.g. 10.1 years the maturity is rounded to 10. It should be noted that most bonds have an actual integer maturity. Furthermore, I assume that coupon payments are made with six month intervals. This assumption is likely to introduce noise to the estimates, it is however not likely to be a source of systematic error. The asset value is assumed to follow the process defined by Equation 3.1. From each point in time the default barrier is assumed constant,  $K_t = K \forall t \in [0, T]$ , throughout the lifetime of the bond. Default is triggered if the asset value falls below  $K$  on a coupon date.

Let  $w$  be the expected recovery rate in the event of default,  $D(0, T_i) = e^{-rT_i}$  be the value at time 0 of a risk-free zero-coupon bond maturing at time  $T_i$ ,  $E^Q$  be the expected value under the *risk neutral measure* and  $I$  be the indicator function. Then  $P(0, T_i)$ , the price of a bond  $i$  with maturity  $T_i$ , is calculated by the formula

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I

$$+D(0,T)E^Q\left(\underbrace{\left(1 + \frac{c}{2}\right)I_{V_T \geq K} + \min\left[w\left(1 + \frac{c}{2}\right), V_T\right]I_{V_T < K}}_{\text{II}}\right) \quad (5.1)$$

The first part of Equation 5.1, I, is the expected value of a coupon payment made before principal. If default does not occur on coupon date  $i$ , the creditor will receive a cash flow equal to the expected value of the coupon payment,  $E^Q I_{V_{T_i} \geq K} \frac{c}{2}$ . This cash flow is discounted by  $D(0, T_i)$ . On the other hand, if default does occur on coupon date  $i$  the creditor will receive a cash flow equal to the expected *recovery* value of the coupon payment,  $E^Q(I_{V_{T_i} < K} \min[\frac{wc}{2}, V_t])$ . Parts I and II express essentially the same, with the only difference being II including the principal payment of 1 (as it is assumed the bond has unit face value).

Let  $\psi$  be the expected *recovery* value of the cash flow on a coupon date. On a coupon date  $\psi = \frac{wc}{2}$ , while on the last payment date  $\psi = w(1 + \frac{c}{2})$ . If on a cash flow payment date the asset value  $V_t$  is greater than the default barrier  $K$  the promised payment/cash flow is paid in full. If  $V_t$  falls below  $K$ , but is higher than  $\psi$  the payment is limited to  $\psi$ . Lastly, if  $V_t$  falls below  $K$  and  $\psi$  on a payment date the payment is equal to  $V_t$ .

Following the assumptions postulated by Merton (see Section 3.1.1) it can be shown that

$$E^Q I_{V_{T_i} \geq K} = N(d_2(K, t)), \quad (5.2)$$

which is the risk neutral probability of not default on the coupon date. Furthermore, it can be shown that (see Eom et.al. 2004)

$$E^Q(I_{V_{T_i} < K} \min[\psi, V_t]) = V_0 D(0, t)^{-1} e^{-\delta t} N(-d_1(\psi, t)) + \psi [N(d_2(\psi, t)) - N(d_2(K, t))], \quad (5.3)$$

where  $\psi$  is the expected recovery value of the cash flow.  $E^Q(I_{V_{T_i} < K})$  is the probability of default.  $N(\cdot)$  represents the cumulative normal distribution function and

$$d_1(x, t) = \frac{\ln\left(\frac{V_0}{x D(0, t)}\right) + (-\delta + \frac{1}{2}\sigma_V^2)t}{\sigma_V \sqrt{t}}, \quad d_2(x, t) = d_1(x, t) - \sigma_V \sqrt{t} \quad (5.4)$$

By combining equations 5.1, 5.2, 5.3 and 5.4 the price of the bond can be calculated under Merton's assumptions.



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## Calculation of Credit Spreads

The credit spread refers to the difference in yield to maturity between a corporate bond and a risk-free bond with the same maturity. Mathematically it can be expressed as

$$s_{i,t} = r_{i,t} - r_{f,t} \quad (5.5)$$

where  $s_{i,t}$  is the spread for bond  $i$  at time  $t$ ,  $r_{i,t}$  is the yield to maturity for bond  $i$  at time  $t$  and  $r_{f,t}$  is the spot rate. The yield to maturity for the coupon bonds are calculated using the `jrvFinance` package in R.

## Implementation

As with any other financial instrument its present value is equal to the expected value of future cash flows. To estimate bond prices at different points in time (effectively the coupon dates), the *remainder* of the bond's cash flows is modelled as a new portfolio of zero coupon bonds. At each point in time, new information is available. Therefore, the estimates for the parameters in Table 5.1 are updated at payment day  $t$ . Thus, by modelling the remainder of the bond as a new zero coupon bond with new updated input parameters I can estimate bond prices at various points in time. I estimate prices and spreads at six month intervals, from issue until only the last payment remains. Based on the literature review of structural models and other sources of risk premium it is expected that the Extended Merton Model produces credit spreads that are too low, on average. This is equivalent to the model overpricing bonds.

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## 5.2 Bond Pricing Model parameters

The previous section describes the proposed structural model and the required input parameters: the term structure of risk-free debt, the coupon rate, maturity, default barrier, recovery rate, payout ratio, asset value and asset volatility. Table 5.1 presents an overview of the model parameters and how they are estimated. In the following I go into detail on how the parameters are estimated.

Parameter group	Description	Estimated as
<b>Bond features</b>	Coupon $c$	Given
	Default barrier $K$	Book value of total liabilities
	Maturity $T$	Given
	Recovery rate $w$	Average final recovery
<b>Firm characteristics</b>	Asset Value $V_0$	Book value of debt + market value of equity
	Asset volatility $\sigma_V^2$	See Section 5.2.2
	Payout ratio $\delta$	See Section 5.2.2
<b>Interest rates</b>	Risk-free rate $r_f$	Nelson-Siegel Model

**Table 5.1:** Parameters in the Extended Merton Model.

### 5.2.1 Bond features

#### Default barrier

The default barrier,  $K$ , is defined as the level that asset value must reach in order for default to occur. The literature utilizes different approaches to modeling the default barrier. A common choice for the default barrier is to use one that is less than that of total liabilities because all debt is unlikely to be due within the estimation period. One such approach is a simplification of the one utilized in the KMV model (Crosbie and Bohn, 2003), where

$$K = D_{shortterm} + kD_{Longterm}, \quad (5.6)$$

with  $k = 0.5$ , i.e. the sum of short term debt and half of long term debt. Afik, Arad

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and Galil (2012) test the sensitivity of the accuracy of the Merton model to changes in the default barrier specification. They find that the Merton model is only slightly sensitive to the default barrier specification.

Eom et.al. (2004) argue that in most structural models, including the extended Merton model, equity holders earn the residual value of the firm once all debt is paid off. Equity residual values accrue when the par value of the bond is paid off only if the firm has no other debt. As most firms have several sources of debt, it would be erroneous to model the default barrier as the face value of the bond. This would likely result in severely overpricing bonds, as it would be very unlikely for the asset value diffusion to cross the default barrier. I find Eom et.al. (2004)'s approach the most reasonable and intuitive and choose to follow their approach by modelling the default barrier as the total book value of liabilities at the end of the year of the observation.

### Recovery rate

The expected recovery rate,  $w$ , is a key input parameter in the model. Simply put, it measures of how much the market expects to be recovered of a payment in the event of default. Eom et.al. (2004) use Moody's data on the recovery rate. In their study on the Nordic High Yield bond market Knappskog and Ytterdal (2015) attempt to estimate firm-specific recovery rates by specifying a multivariate OLS regression, with equity ratio, receivables, long term debt, intangibles, profitability and distance to default as explanatory variables, in addition to seniority and security and industry dummy variables. They are, however, unable to obtain significant results. As my available Default and Recovery data is similar to the data analyzed by Knappskog and Ytterdal (2015) I choose, based on their findings, to estimate the  $w$  as the average *final* recovery rate for debt securities issued in Norwegian kroner. This approach is in the lines of Eom et.al.(2004), who implement a single recovery rate for all firms throughout the entire time period analyzed.

As can be seen from Table 4.8 the average final recovery is different for bonds and convertibles. Since I only model bond prices it interesting to test whether convertibles are likely to have a higher recovery rate than bonds, and if only recovery rates for bonds should be included. I perform a Welch t-test with  $\alpha = 0.05$  to test whether the different debt instruments have equal/different mean recovery rates. The results from the test are presented in Table 5.2.

$\alpha = 0.05$	Bonds	Convertibles
Mean	56.77%	63.32%
Variance	12.656	13.4689
N	39	28
df	57	
t Stat	-1.45886	
P(T≤t) two-tail	0.150092	
t Critical	2.002465	

**Table 5.2:** Welch t-test for recovery rates

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The null hypothesis is not rejected. Therefore, I include both bonds and convertibles in my estimation of  $w$ . The average final recovery rate for all issues in Norwegian kroner is  $\bar{w} = 59.24\%$ , and this value is implemented in the bond pricing model. This value is slightly higher than the value implemented by Eom et.al. (2004), who estimate  $w = 51.31\%$ .

## 5.2.2 Firm characteristics

### Asset value and asset volatility

As discussed in Section 3.1.2 a general drawback of structural models is that the key input parameters asset value and volatility are unobservable and need to be estimated. This is a serious disadvantage, as the asset volatility is regarded as the most important input parameter in derivatives pricing (Poon and Granger, 2003).

There are several approaches to estimating the asset volatility. One possibility is to simply adjust the equity volatility by multiplying by the equity ratio. This would, however, be erroneous, as one would assume debt has zero volatility and is risk-free. A more realistic approach to estimate asset volatility (Acharya and Schaefer, 2009) is to calculate a weighted sum of equity and debt volatility. A challenge with this approach is the value of debt. While equity value is easily defined as the observed market capitalization, there is no such straightforward approach to determine debt value. They argue that using the book value of debt is usually adequate in such an adjustment. Another possible approach, as implemented by Knappskog and Ytterdal (2015), is to estimate both asset volatility and value by solving two functions derived from the Black-Scholes-Merton framework simultaneously. As described in Section 3.1.1, equity value can be expressed as

$$E_t = V_t N(d_1) - B e^{-rT} N(d_2) \quad (5.7)$$

where  $d_1(x, t) = \frac{\ln(\frac{V_0}{x D(0, t)}) + (-\delta + \frac{1}{2}\sigma_V^2)t}{\sigma_V \sqrt{t}}$ ,  $d_2(x, t) = d_1(x, t) - \sigma_V \sqrt{t}$ . Furthermore, it can also be shown that

$$\sigma_{tE} = \frac{V_t}{E_t} N(d_1) \sigma_{tV} \quad (5.8)$$

Estimates for the asset value and asset volatility can thus be obtained by solving the system of equations (numerically). The required input parameters are equity value,  $E_t$ , volatility of equity returns,  $\sigma_E$ , and an estimate for  $B$  (face value of debt).  $E_t$  is estimated as the observed market capitalization on the date of bond price observation.

There are several approaches to estimate and forecast the volatility of equity returns. Examples of "simple" methods include Moving Average and Exponential Weighted Moving

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Average. More sophisticated methods include ARCH and GARCH modeling. Knappskog and Ytterdal (2015) estimate equity standard deviation from five years of monthly data, while Eom et.al. (2004) use different estimation periods, ranging from 30 to 150 trading days prior to the bond price observation, fitting GARCH(1,1) models. In addition to the different time frames before the bond price observation they also include a 150 day forward looking estimate, under the assumption that some of the future path of equity price volatility can be anticipated.

For the ease of implementation I choose to implement a simple approach by calculating a 150 day moving average for equity volatility and use this as an input parameter in Equation 5.8. I estimate asset value in the lines of Eom et.al (2004) as the book value of debt plus the market capitalization of equity. Through this estimation only Equation 5.8 is required to estimate asset volatility. The nature of Equation 5.8 dictates that it must be solved numerically. These calculations are performed using Excel's iterative equation solver function.

### **Payout ratio**

The inclusion of the payout ratio is to reduce the drift of asset value. Eom et.al. (2004) estimate the firm's payout ratio as the weighted average of the coupon payment  $c$  and the share repurchase-adjusted dividend yield. In this paper the payout ratio is estimated by Equation 5.9, where  $Div_E$  represents dividends paid to equity holders,  $rep$  represents share repurchases and  $int$  represents interest paid to equity and debt holders, respectively.

$$\delta = \frac{Div_E + rep + int}{TotalAssets}, \quad (5.9)$$

Total assets is the book value of total assets. The time of observation is the year prior to the observation date. This is because dividends for a year are normally paid out the following year.

At each point in time the payout ratio is assumed to remain constant throughout the lifetime of the bond, even if the firm experiences solvency issues in the future. Knappskog and Ytterdal (2015) address this possibility by setting dividends and share repurchases to zero for issuers with estimated asset values less than two standard deviations from debt value. I choose to not perform this adjustment, because actual data on dividends and share repurchases should reflect if a firm believes it is likely to experience solvency issues in the (near) future.

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## Interest rates

Eom et.al (2004) note that any term structure model is allowed in their extended Merton model, and use the Nelson-Siegel (1987) and Vasicek (1977) models. I implement the Nelson-Siegel (1987) model to estimate the yields on missing dates. The general Nelson-Siegel function takes the form

$$y_t(m) = \beta_{t0} + \beta_1 \frac{1 - e^{-m/\lambda}}{m/\lambda} + \beta_2 \left( \frac{1 - e^{-m/\lambda}}{m/\lambda} - e^{-m/\lambda} \right), \quad (5.10)$$

where  $m$  is maturity,  $y_t(m)$  is the yield curve at time  $t$  for maturity  $m$  and  $\lambda$  is a decay factor. The method of estimating the parameters ( $\beta$ s) involves reducing the sum of the residual of the actual observations and model estimates. I have used the `xts` and `YieldCurve` packages in R to estimate the parameter values and estimate the yield curve values for missing maturities.

I have chosen to use Norwegian government bonds for as a proxy for the risk-free rate. While not completely risk-free, Norwegian government bonds are in general considered virtually risk-free. From 2003 to 2015 I have gathered the rates for maturities 3 months, 6 months, 9 months, 1 year, 3 years, 5 years and 10 years, respectively, from Norges Bank. All rates obtained from Norges Bank represent the monthly average for the respective maturities. I have gathered interest rates from two different datasets, as interest rates for short term government securities were not available before January 2003. From 2000 to 2003 I have gathered NIBOR rates for 3, 6, 9 and 12 months. While not technically a government security NIBOR rates should be good proxies for risk-free rates.

## Results and Analysis

An issue with the bond price time series for many bonds is that there is no observation at the time of issue. One way to address this issue is to assume that the bond trades at par at issue. This approach is implemented by Knappskog and Ytterdal (2015). This does, however, leave room for errors, as it is not certain that the bond trades as par at issue. For instance, government bonds are often auctioned and traded at a discount at issue. To ensure consistency in the analysis of the model accuracy I therefore choose to exclude all prices and spreads observations from before the first recorded transaction. Furthermore, I exclude the observations for the bonds that were exercised early. Following these exclusions there are 491 estimates of bond prices and spreads. In the following I first present the model estimates for bond prices and spreads. I then analyze the performance of the model.

### 6.1 Extended Merton Model Results

Table 6.1 presents the model results compared to observed bond prices. On average the model underprices bonds, as indicated by the lower mean. The maximum predicted bond price is very close to the maximum observed bond price, while the minimum model price is relatively much higher than the minimum observed price.

Bond prices	Mean	St. Dev.	Max	Min
Observed	102.539	8.548	121.14	61.25
Model	98.88	7.74	121.10	73.93

**Table 6.1:** Observed versus predicted bond prices

Table 6.2 presents model output compared to observed values with respect to credit spreads.

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The model overpredicts credit spreads by 167 basis points, on average. The model output has similar standard deviation to the observed spreads, while the output differs to large extent with respect to maximum and minimum credit spread. The minimum observed spread is rather peculiar, and could indicate that there has been a calculation error. On inspection I find that the value is reasonable, as the observed market price for the specific bond is very high with very short remaining maturity, meaning the bond coupon would not compensate an investor sufficiently to achieve a positive return. It should also be noted that for the specific bond the observed price is constant for the last three price observations, which indicates that it has been a long time since the last transaction of the bond.

Credit spreads	Mean	St. Dev.	Max	Min
Observed	2.50%	4.30%	29.50%	-11.72%
Model	4.17%	4.73%	45.48%	0.074%

**Table 6.2:** Observed versus predicted credit spreads

The results presented above both indicate that the model tends to underprice bonds, i.e. overestimate credit spreads. This is a rather unexpected result based on the findings of Eom et.al. (2004) on the Extended Merton Model and the findings of Sæbø (2015), who finds that only 21.5% of the credit spread in the Norwegian market is compensation for default risk. The model has a rather low  $R^2 = 0.1479$ , which implies that the model is able to explain (only) 14.79% of the variance in observed credit spreads. This means 85.21% of the variance in credit spreads is unexplained by the model.

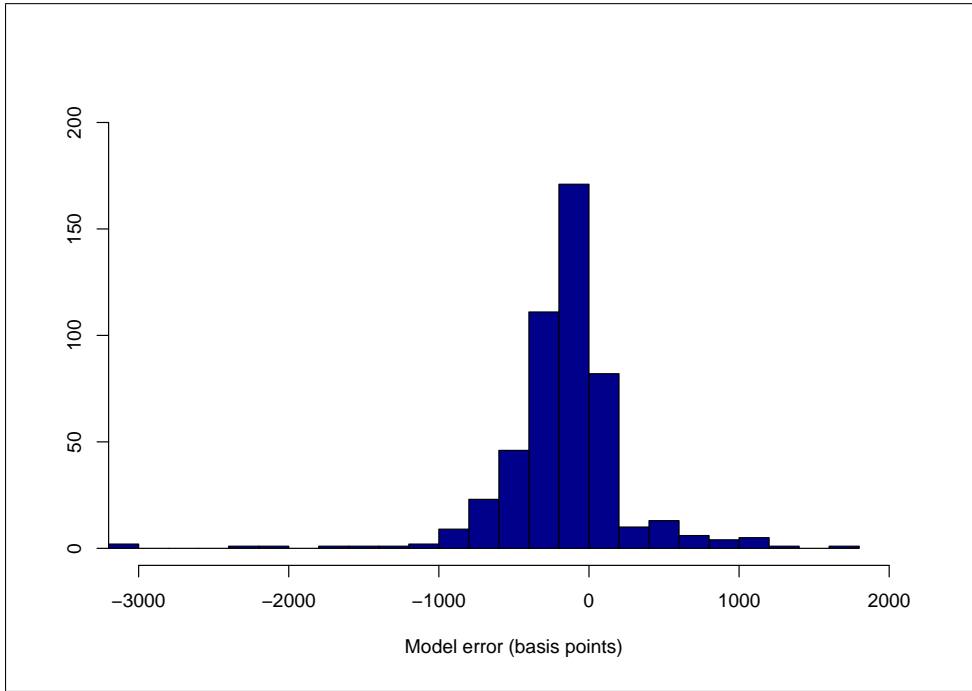
### 6.1.1 Model Performance

There are several possible ways to measure the model accuracy. One such measure is model mispricing. Let  $\epsilon_{i,t}$  be the model mispricing in credit spreads for bond  $i$  at time  $t$ ,  $s_{act,i,t}$  be the observed credit spread for bond  $i$  at time  $t$  and let  $s_{mod,i,t}$  be the model estimate for the credit spread for bond  $i$  at time  $t$ . Then

$$\epsilon_{i,t} = s_{act,i,t} - s_{mod,i,t}, \quad (6.1)$$

The model mispricing metric on credit spreads has the advantage of indicating whether the model overpredicts or underpredicts spreads. This is important, because predicted default risk can at most be as large as the spread observed in the market. The expected sign of the model mispricing is positive, given the other potential and likely sources of risk compensation discussed in Section 3.3. Figure 6.1 illustrates the model performance measured by model mispricing, as defined by Equation 6.1. Most spreads are relatively close to the mean, with some extreme observations of both overprediction and underprediction of spreads. More importantly, the model estimates, on average, higher levels of compensation default risk than can be observed in the market, as shown in Table 6.3.





**Figure 6.1:** Distribution of model mispricing, measured by basis points

	Mean	Standard deviation	Max	Min
$\epsilon_{i,t}$	-167	429	1770	-3179

**Table 6.3:** Model mispricing, measured by basis points

The analysis of model errors, as defined above, provides insight to the relation between the model input variables and model over and underprediction of spreads. It is also important to analyze the relations between the model input parameters and the magnitude of errors, which is simply the absolute value of the model mispricing metric defined above,

$$|\epsilon_{i,t}| = |s_{act,i,t} - s_{mod,i,t}| \quad (6.2)$$

Table 6.4 presents the model performance measured by magnitude of errors. The minimum magnitude of error is very small, by predicting a credit spread only 0.22 basis points from the observed credit spread. The value seems very low, as it allows virtually zero room for other sources of risk premium.

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	Mean	Standard deviation	Max	Min
$ \epsilon_{abs,i,t} $	292.28	355.79	3179.28	0.22

**Table 6.4:** Magnitude of model errors, measured by basis points

## 6.2 Analysis of Errors

It is evident that the model yields fairly inaccurate estimates of credit spreads in general, with a tendency toward overpredicting credit spreads. In the following sections I analyze the relations between model input parameters and the two different metrics of model accuracy to uncover sources of inaccuracy. First, I perform simple linear regression analysis of the different error metrics on the different input variables to assess the relation between the input variables and model inaccuracy. I then assess what part of the mispricing can be explained by overpredicting the probability of default, before I assess the sensitivity of the model inaccuracy to changes in the expected recovery rate.

### 6.2.1 Regression Analysis

To assess the relation between the different error metrics and variables I perform a set of simple linear regressions. The results of the regressions are presented in Tables 6.5, 6.6, 6.7 and 6.8. Plots of errors versus the model input variables (and some additional) are presented in Appendix C. In the following I present findings on the errors with respect to the various input parameters.

<i>Dependent variable:</i>					
Model mispricing ( $\epsilon_{i,t}$ )					
Coupon	2,796*** (881.18)				
Remaining maturity		53.60*** (7.48)			
Equity volatility ( $\sigma_E$ )			220.70*** (77.18)		
Asset volatility ( $\sigma_V$ )				925.85*** (120.06)	
Market capitalization ( $E_t$ )					$9.03 \cdot 10^{-7}$ ** ( $3.71 \cdot 10^{-7}$ )
Intercept	-347.92*** (60.25)	-378.11*** (34.79)	-249.17*** (34.66)	-332.25*** (28.22)	-195.96*** (22.71)
Observations	491	491	491	491	491
R <sup>2</sup>	0.020	0.095	0.016	0.108	0.012
Adjusted R <sup>2</sup>	0.018	0.093	0.014	0.107	0.010
Residual Std. Error (df = 489)	425.423	408.849	426.232	405.812	427.194
F Statistic (df = 1; 489)	10.067***	51.351***	8.176***	59.469***	5.937**

Note: Standard errors in parenthesis

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

**Table 6.5:** Regression results for model mispricing, I

<i>Dependent variable:</i>					
Model mispricing ( $\epsilon_{i,t}$ )					
Asset value ( $V_0$ )	5.04 · 10 <sup>-7</sup> *				
	(2.86 · 10 <sup>-7</sup> )				
Default barrier ( $K$ )	- 3.58 · 10 <sup>-7</sup>				
	(9.55 · 10 <sup>-7</sup> )				
Payout ratio ( $\delta$ )	445.02				
	(444.60)				
Quasi-market leverage ratio $\gamma_{i,t}$	-621.05***				
	(81.04)				
Time until year end	-71.28				
	(67.38)				
Intercept	-195.607***	-157.789***	-189.024***	187.769***	-133.706***
	(25.367)	(30.758)	(29.522)	(49.754)	(36.738)
Observations	491	491	491	491	491
R <sup>2</sup>	0.006	0.0003	0.002	0.107	0.002
Adjusted R <sup>2</sup>	0.004	-0.002	0.00000	0.105	0.0002
Residual Std. Error (df = 489)	428.428	429.718	429.340	406.084	429.289
F Statistic (df = 1; 489)	3.092*	0.140	1.002	58.735***	1.119

Note: Standard errors in parenthesis

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

**Table 6.6:** Regression results for model mispricing, II

<i>Dependent variable:</i>					
Magnitude of error ( $ s_{act,i,t} - s_{mod,i,t} $ )					
Coupon	2,053.5*** (731.8)				
Remaining maturity		-59.51*** (5.9)			
Equity volatility ( $\sigma_E$ )			559.03*** (59.33)		
Asset volatility ( $\sigma_V$ )				208.14** (104.95)	
Market capitalization ( $E_t$ )					$-1.863 \cdot 10^{-6}$ *** ( $2.973 \cdot 10^{-7}$ )
Intercept	159.199*** (50.035)	526.966*** (27.603)	83.452*** (26.643)	255.066*** (24.664)	352.559*** (18.213)
Observations	491	491	491	491	491
R <sup>2</sup>	0.016	0.171	0.154	0.008	0.074
Adjusted R <sup>2</sup>	0.014	0.169	0.152	0.006	0.072
Residual Std. Error (df = 489)	353.324	324.361	327.655	354.734	342.669
F Statistic (df = 1; 489)	7.873***	100.570***	88.775***	3.934**	39.256***

*Note: Standard errors in parenthesis*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

**Table 6.7:** Regressions on magnitude of errors, I

	<i>Dependent variable:</i>				
	Magnitude of error ( $ s_{act,i,t} - s_{mod,i,t} $ )				
Asset value ( $V_t$ )	$-1.50 \cdot 10^{-7}$ *** ( $2.28 \cdot 10^{-7}$ )				
Default Barrier ( $K$ )		$-4.45 \cdot 10^{-6}$ *** ( $7.66 \cdot 10^{-7}$ )			
Payout ratio ( $\delta$ )			1,495*** (362.570)		
Quasi-market leverage ratio ( $\gamma_{i,t}$ )				459.78*** (67.964)	
Time until year end					-71.91 (55.805)
Intercept	378.726*** (20.205)	403.477*** (24.656)	217.384*** (24.075)	29.835 (41.728)	325.590*** (30.428)
Observations	491	491	491	491	491
R <sup>2</sup>	0.082	0.065	0.034	0.086	0.003
Adjusted R <sup>2</sup>	0.080	0.063	0.032	0.084	0.001
Residual Std. Error (df = 489)	341.241	344.465	350.123	340.577	355.554
F Statistic (df = 1; 489)	43.684***	33.761***	17.000***	45.765***	1.660

Note: Standard errors in parenthesis

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

**Table 6.8:** Regressions on magnitude of errors, II

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## Coupon

The plots in Figure A1 and results in Tables 6.5 and 6.7 provide interesting insight in how the errors are related to coupons. Bonds with low coupons tend to be underpriced by the model, as illustrated in Figure A1 and indicated by the negative intercept. The positive coefficient indicates that there is a tendency of less underpricing with increasing coupons, with a p-value 0.001604, and that bonds with lower coupons are more underpriced than bonds with higher coupons. It should also be noted that the magnitude of the error increases ( $p < 0.01$ ) with increasing coupon. This observation seems fairly reasonable, as high coupons are associated with high yield issues with greater uncertainty than investment grade issues. What is more interesting is that bonds with low coupons are consistently underpriced.

## Remaining Time to Maturity

Figure A2 illustrates the different error metrics versus remaining time to maturity. The sign and significance level of the coefficient ( $p < 0.01$ ) reported in Table 6.7 indicate that the magnitude of the error is larger for observations with short remaining time to maturity. The result reported in Table 6.5 indicates that bonds with short remaining maturity tend to be underpriced more than bonds with longer remaining maturity. This implies that the model tends to overestimate the compensation for expected default more in the short run than in the long run. This is an interesting observation, because it indicates that the model is better at pricing bond prices when the bond has long remaining time to maturity than short.

## Equity Volatility

Equity volatility is an important parameter to assess because it is vital in estimating asset volatility. The different error metrics are plotted against estimated equity volatility in Figure A3. The results from Table 6.7 indicate that the magnitude of errors tend to increase with increasing (150 day) equity volatility ( $p < 0.01$ ). This makes sense, as a greater degree of uncertainty about the firm's future makes it more difficult to accurately forecast the pricing parameters. Furthermore, the model tends to overpredict spreads for firms with low equity volatility, as reported in Table 6.5. All else equal, this would indicate that the estimated volatility is higher than the forecast volatility used in pricing the bonds.

## Asset Volatility

Figure A4 plots the different accuracy measures against estimated asset volatility. The plots are similar to those observed for equity volatility, as are the results from the simple linear regressions reported in Tables 6.5 and 6.7. The model tends to underprice bond price observations with low estimated asset volatility more than bond price observations with high estimated asset volatility. The magnitude of errors increases with increasing

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estimated asset volatility, which makes sense as noted in the analysis of errors with respect to estimated equity volatility.

### **Market Capitalization**

Another important metric is the market capitalization, which constitutes an important part of the estimate for asset value. The different error metrics are plotted against market capitalization in Figure A5. The magnitude of errors tends to decrease with increasing market capitalization, all else equal. This indicates that the model is better at estimating compensation for expected default for larger than for small firms. Furthermore, the model tends to underprice bonds issued by firms with low market capitalization more than for bonds issued by firms with higher market capitalization ( $p < 0.01$ ). These observations make sense, as smaller firms (measured by market capitalization) are associated with greater uncertainty than larger companies. It should be noted that the largest observed errors, both with respect to underpricing and overpricing, are for bonds issued by firms with very low market capitalization. Firms that have experienced a dramatic drop in market capitalization are in particular associated with uncertain futures. Some of the most extreme pricing errors are for firms that have recently experienced dramatic drops in market capitalization, such as Norske Skogindustrier during the financial crisis.

### **Asset Value**

The different error metrics are plotted against estimated asset value in Figure A6. The plots and regression results for asset value are, as expected, similar to those obtained for market capitalization. The magnitude of errors tends to decrease ( $p < 0.01$ ) with increasing estimated asset value, which, as discussed in the previous paragraph, makes sense. The model tends to overestimate credit spreads more for firms with smaller estimated asset value, indicated by the negative constant and positive slope of the coefficient ( $p < 0.01$ ).

### **Default Barrier**

Figure A7 illustrates the different error metrics plotted against the default barrier, which is estimated as the book value of total liabilities at year end for each observation. The regression result from Table 6.6 indicates that there is no significant relation between the level of default barrier level and whether the model underprices or overprices bonds (p-value 0.7082). The regression for the magnitude of errors on the other hand yields significant results. The magnitude of errors tends to decrease with higher values of default barrier. High values of total liabilities are commonly related to larger firms, which as discussed earlier are associated with less uncertainty than smaller firms.



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## Payout Ratio

The plots in Figure A8 illustrate the different error metrics versus payout ratio. The linear regression for model mispricing yields no significant results (p-value 0.317) and offers little insight to whether the model under or overprices bonds price observations with different estimated payout ratios. The regression on the magnitude of error does, however, yield significant results. The magnitude of error increases with estimated increasing payout ratio ( $p < 0.01$ ). This could cause the drift term of Equation 3.1 to become inaccurate. A high payout ratio offsets the positive drift from expected return on assets, which could lead to too little positive drift and consequently a too high probability of default.

## Leverage Ratio

It is interesting to assess the relation between model inaccuracy and leverage ratios. The leverage ratio could be viewed as an indicator of a firm's distance to default. If the leverage ratio is low this indicates that the firm must experience a severe fall in asset value for the asset value to drop below the default barrier. The opposite holds for high levels of leverage ratio. Let  $D_{i,t}^{book}$  be the book value of debt for the issuing firm  $i$  at time  $t$ . Furthermore, let  $E_{i,t}^{mkt}$  be the market capitalization for the issuing firm  $i$  at time  $t$ . The quasi-market leverage ratio,  $\gamma_{i,t}$ , is defined as

$$\gamma_{i,t} = \frac{D_{i,t}^{book}}{E_{i,t}^{mkt} + D_{i,t}^{book}} \quad (6.3)$$

As can be seen in Figure A9 the model tends underprice bonds more with increasing leverage ratio ( $p < 0.01$ ), while there are extreme observations of both over and underpricing of bonds in the high leverage environment. Figure A9 and the results from the simple simple linear regression indicate that the magnitude of the model mispricing increases with increasing quasi-market leverage ratio. The plot and result from Table 6.8 provide indications for increasing overprediction of spreads with increasing quasi-market leverage, all else equal. The bonds with the largest errors belong in the high-leverage environment. Such companies include Norske Skogindustrier and Norwegian Property.

## Time Until Year End

A difference from this study to Eom et.al. (2004) is that I estimate prices and spreads for bonds at different times of year, while Eom et.al. (2004) test observations on the last trading day of December only. This is done in order to match the price observation with year-end financial data. In my implementation I essentially assume that the book value of total liabilities at year end is known well before year end, which is unrealistic. Intuitively, it seems reasonable that the model accuracy would increase if data from the most recently available financial reports (i.e. quarterly reports) were used, in order to better match the time of observation for the different input parameters. The errors for

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observations close to year end are therefore expected to be smaller than for observations early in the year. The results of the simple linear regression indicate however that there is no significant relationship between the model error and the payout ratio (p-value 0.2906) nor the magnitude of the errors (p-value 0.1982). The time until year end does likely introduce some noise to the sample.

## 6.2.2 Overestimating the Probability of Default

The analysis conducted in the previous section indicates; especially for bond price observations for bonds with low coupon, short remaining time to maturity and high estimated quasi-market leverage ratio, that the model fails to predict accurate price and spread estimates and consistently overpredict spreads. It seems the market is more certain that the firms will meet their remaining (short term) obligations than the model is able to predict. The question is why the model fails to capture these pricing mechanisms.

If it is correct that Merton's critical assumptions hold (See Section 3.1) and that the bonds can be correctly priced using Equation 5.1 a combination of the following factors would lead to overpredicting the probability of default:

1. The estimated asset volatility is too high. This would lead the fluctuations following the Standard Wiener Process to become too large, leading to too high probability of default
2. The default barrier is too high. This causes the "distance to default" to be too short, leading to too high probability of default
3. The estimated asset value is too low. This causes the "distance to default" to be too short, leading to too high probability of default

In addition to the three possibilities listed above there may be issues related to the payout ratio, leading to too little positive drift. This does not, however, seem a likely source of systematic error, because the underpricing issue does not seem to be more evident for high values than for low values of estimated payout ratio.

A possible explanation is that the estimated asset volatility is consistently too high. Assuming Merton's assumptions hold the market could be more certain the firm will meet its obligation than what estimated asset volatility would indicate. The properties of the firm's assets could be an explanation, as could the ownership structure. However, the model accuracy actually increases with longer term observations. It should be easier to forecast short-term volatility than long term. Therefore, it is rather surprising that the model accuracy actually increases with longer-term observations as illustrated in Figure A2. The asset volatility estimate likely introduces noise to the results, but does not seem like the most viable source of systematic underpricing for short-term observations.

The arguably most feasible explanation is related to the default barrier specification. The default barrier is estimated the same way for all price observations, i.e. the book value of total liabilities. The model does not explicitly incorporate capital and debt structure complexity. The firm may have several sources of debt with very different properties such

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as coupon and maturity. In the model implementation it is assumed that the all of the debt is due within the specified time period, which most certainly is not always the case. It seems reasonable that the market would put greater emphasize on whether the firm will meet its short term obligations rather than long term. Thus, implementing the firm's total liabilities as the default barrier could lead to a specification that in many instances is unreasonably high, leading to a too high probability of default. It would possibly be more realistic to implement the simplified KMV specification (see Equation 5.6), or simply just using short term liabilities for these observations.

It is interesting to assess how much of the error can be explained by the input parameters and additional variables analyzed above. I therefore run a multiple linear regression with the model mispricing as the dependent variable and the different model input parameters as explanatory variables. The correlations between some of the variables is very high, as indicated in Table A3. Some of the pairwise correlations are hardly surprising, e.g. asset value and market capitalization. To avoid multicollinearity issues I exclude equity volatility, default barrier, asset value and market capitalization. This is because these are important components of other variables, namely asset volatility and quasi-market leverage ratio, and their impact is to some extent incorporated through the other variables. The results are presented in Table 6.9.

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**Table 6.9:** Multiple linear regression

	<i>Dependent variable:</i>
	Model mispricing
Coupon	2,888*** (811.50)
Remaining time to maturity	60.51*** (6.91)
Asset volatility	692.19*** (148.05)
Leverage ratio	-421.49*** (84.20)
Payout ratio	-968.52** (446.58)
Time until year end	-95.36* (57.68)
Intercept	-382.99*** (88.940)
Observations	491
R <sup>2</sup>	0.281
Adjusted R <sup>2</sup>	0.272
Residual Std. Error	366.412 (df = 484)
F Statistic	31.460*** (df = 6; 484)

*Note:* Standard errors in parenthesis \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

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The results from Table 6.9 indicate that the included variables explain 28.1% of the model mispricing. The significance and signs of the coefficients for coupon, remaining time to maturity, asset volatility and leverage ratio are the same as is the simple linear regressions. The estimates for the coefficients are slightly different from the simple linear regressions. Holding all the other variables fixed the change in error with increasing coupon is slightly larger than when only including coupon in the regression. The sign of the coefficient that the model will underprice less with increasing coupon. The same is observed for remaining time to maturity and asset volatility.

An interesting observation is made for the significance of the coefficients of time until year end and payout ratio. Holding all else in the model fixed, the sign of the coefficient of time until year end indicates that the model mispricing becomes more negative with increasing time until year end. In other words, the model underprices bonds more with longer time until year end than with short. Accounting for the effect of the other variables this result is in line with expectations, as more accurate results are obtained for observations closer to year end. The significance of the variable is however fairly low (p-value 0.09892).

The sign of the coefficient of payout ratio indicates that the model underprices bonds more with increasing estimated payout ratio. This can be related to Equation 3.1, and could imply that for many bonds the estimated payout ratio is actually too high. With higher payout ratios the model underprices bonds significantly (p-value 0.03059) more than with lower payout ratios, holding the other variables fixed. It is therefore possible that higher payout ratios cause the asset diffusion process to become less accurate than with lower payout ratios.

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### 6.2.3 Recovery Rates Sensitivity Analysis

The analysis so far addresses if the model overpredicts the probability of default. The analysis and discussion in the previous section indicate that the model produces more accurate results for observations with low coupon and long remaining time until maturity. Despite the increased accuracy for long-term observations the model tends to underprice bonds, as illustrated in Figure A2. Throughout my implementation and analysis of model inaccuracy I have used a single static measure based on the average final recovery rate for all firms. The results from the analysis above indicate that the use of a single recovery rate across all firms might be a major source of error. It should be noted that in their implementation Eom et.al. (2004) use a single recovery rate for all firms and still end up with overpricing of bonds in general. Therefore, it is rather surprising that the model tends to overpredict spreads. This could indicate that there are different pricing mechanisms related to the expected recovery in the Norwegian corporate bonds market compared to the sample of US bonds analyzed by Eom et.al. (2004).

As Figure 4.9 illustrates the final recovery rate fluctuates widely from its mean, with relatively large standard deviation. Furthermore, the average final recovery rate changes from year to year, as illustrated in Figure 4.8. It is possible that the actual market expected recovery rate for most issues actually lies far from the mean used in the implementation. For investment grade bonds in particular it seems reasonable that the expected recovery rate is higher than  $\bar{w}$ . In the following, I assess the sensitivity of the model performance to changes in recovery rate using different measures of model performance. I begin with the mean absolute deviation (MAD), followed by the changes in the errors measured by model mispricing. Still following the assumption that the recovery rate is the same for all firms I only assess increases in the recovery rate. This is because I find it likely that the recovery rate in many cases should be higher than  $\bar{w} = 59.24\%$ , especially for investment grade firms in general, and in times of financial stability, as the observed average final recovery rate is higher than during the crisis.

#### Sensitivity of Mean Absolute Deviation (MAD) to Recovery Rate

The mean absolute deviation is defined as

$$MAD = \sum \frac{|s_{act,i,t} - s_{mod,i,t}|}{n}, \quad (6.4)$$

which is simply the average of the magnitude of errors defined earlier. By using increments of 5% I measure the change in MAD and assess the change of the model performance. The results of the tests are presented in Table 6.10. From Table 6.10 it is evident that the performance of the model is sensitive to changes in the recovery rate. The MAD decreases for the first three steps, before it increases with increments over 15%. This indicates that for most bonds the model output becomes more accurate with a 15% higher  $w$  value, all else equal. There are, however, still some instances of extreme mispricing, as indicated in Table 6.10. Furthermore, the minimum error still seems very low.

	Mean absolute error	Standard deviation	Max	Min
Base case	292.28	355.79	3179.28	0.22
$\bar{w} + \sigma_{rec}$	323.89	415.69	2841.91	1.59
$\bar{w} + 5\%$	258.59	320.64	2641.45	0.82
$\bar{w} + 10\%$	234.72	304.01	2134.19	0.24
$\bar{w} + 15\%$	227.11	302.75	2204.24	0.29
$\bar{w} + 20\%$	230.30	319.16	2335.90	0.32
$\bar{w} + 25\%$	235.76	326.14	2461.86	0.14

**Table 6.10:** Model accuracy measured by MAD with different values for recovery rate

### Sensitivity of Model Mispricing to Changes in Recovery Rate

A drawback of using the mean absolute deviation as a measure of the model performance sensitivity to the recovery rate is that it does not provide any information as to whether the model over or underpredicts spreads. As this is a serious issue in the base case implementation I also assess the sensitivity in the model mispricing to the recovery rate. The results reported in Table 6.11 show that the average model mispricing is highly sensitive to changes in the recovery rate. With increasing  $w$  the average model mispricing is shifted toward positive values. This is as expected, given the relationship between bond price and recovery rate (see Equation 5.1). All else equal, an increasing recovery rate shifts the sign of the error to more positive values, yielding results that are more in line accurate for a number of observations. This indicates that, all else equal, the expected recovery rate is higher than  $\bar{w}$  for many bonds. The minimum model mispricing (greatest overprediction of credit spread) is highly sensitive to increases in recovery rate. On further inspection I find that the minimum at different recovery rates are for high yield bonds, and for a single high yield bond for all  $w$ s except the maximum. There are still relatively many occurrences of overpredicting spreads after adjusting the recovery rate, but many of these observations have an observed negative yield to maturity.

	Average model mispricing	Standard deviation	Max	Min
Base case	-166.73	429.34	1769.86	-3179.28
$\bar{w} + \sigma_{rec}$	301.67	432.11	2841.91	-1254.86
$\bar{w} + 5\%$	-94.37	401.12	1921.81	-2641.45
$\bar{w} + 10\%$	-24.53	383.44	2066.40	-2134.19
$\bar{w} + 15\%$	43.03	376.15	2204.24	-1787.99
$\bar{w} + 20\%$	108.45	378.45	2335.90	-1656.81
$\bar{w} + 25\%$	171.88	389.02	2461.86	-1527.26

**Table 6.11:** Model accuracy measured by average model mispricing with different values for recovery rate

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## **Firm Specific and Time-Varying Recovery Rates**

The analysis and results above provide interesting insight on the model performance's sensitivity to different recovery rates. It is, however, still based on the assumption that the recovery rate is the same for all firms. Increasing the recovery rate seems to reduce the problems related to overestimating spreads for most issues, but causes unrealistically low compensation for expected default for some issues. This indicates that it is likely that the market prices individual or at least industry specific recovery rates for firms, and that this varies over time.

## **Realism of Merton's Assumptions**

Throughout the analysis it has been assumed that Merton's fundamental assumptions for the structural model hold. There are, however, clear indications that this is not the case, in particular the assumption of perfectly liquid markets. There are 49 observations of negative actual yields to maturity, most of which observed with short remaining maturity. For many such observations there is a large time gap between transactions (as indicated by Titlon data on official trading volumes) and a constant bond price for long periods of time. This clearly violates Merton's assumption of perfectly liquid markets. In addition to indicating illiquidity the lack of transactions might imply that the observed market price does not reflect a reasonable and tradeable price for the bond. This is in particular true for the instances of observed negative yield to maturity, which makes it unlikely that the bond would actually trade at the observed price. These large time gaps might potentially have an impact on the performance of the model, as they are treated equally to the cases of overpredicting default compensation when the observed yield to maturity is positive.



## Conclusion

The purpose of this thesis was to predict credit spreads on fixed coupon bonds issued in Norwegian kroner in the Norwegian corporate bonds market. I have implemented a structural model of credit pricing and used bond price data from 2000 to 2015 to assess the model's accuracy. Following the approach of Eom et.al. (2004), with a single recovery rate for all firms, I find that the base case implementation of the model tends to overestimate credit spreads, especially for low-coupon bonds with short remaining maturity. After analyzing the model input parameters I find that model accuracy is very sensitive to higher recovery rates. The results indicate that the market prices more individuality in recovery rates than was done in the base case model implementation.

There are several potential and interesting ways to go forward with the research conducted in this thesis. Due to the relatively small sample analyzed in this thesis it is difficult to draw any solid conclusions about the pricing mechanisms in the Norwegian corporate bonds market as a whole. Using a larger sample of fixed coupon bonds would increase the robustness of my results. Additionally, it would be interesting to use other methods of volatility forecasting. My approach is rather simplistic and could be inaccurate. It is possible that the market incorporates more sophisticated techniques such as GARCH for volatility forecasting rather than simply using a moving average measure with time window of 150 days. This is particularly interesting, as the volatility forecast is the most important input variable in general contingent claims analysis. Additionally, it would be interesting to use a different estimation for the default barrier in the short run, possibly one that only takes short term liabilities into account or one specified by Equation 5.6 to ensure a more reasonable estimate for the default barrier.

The most apparent drawback of this study is the use of a single recovery rate across all industries and time. This study indicates that such a recovery rate does not yield reasonable estimates for bonds in the Norwegian corporate bonds market. It would therefore be highly interesting to develop a model for industry specific and time-varying recovery rates and then assess the model's performance. It should also be noted that using the final recovery

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rate might be problematic in itself. In many cases the time from credit event to final settlement is very long. This implies that the final recovery rate cannot be known with certainty by investors before the actual settlement. The market's expected future recovery rates may at a point in time be completely different from the final recovery rate, due to uncertainty about the firm's future. Using the final recovery rate essentially implies that in the model investors have more knowledge about the firm's future than they actually have. It would therefore be interesting to use a different metric for the recovery rate.

# Appendix

## A Screening of bonds

<b>Exclusion criterion</b>	<b>N excluded per criterion</b>	<b>N remaining in sample</b>
Initial Titlon Sample		7243
- Securities not in Stamdata 2000-2016	1496	5747
- Other security types	1703	4044
- Issued by banks and financials	2791	1253
- Issued by government and public sector	316	937
- Less than 1 year, non-ASA, non-fixed coupon	875	62
<b>Final Sample</b>		<b>62</b>

**Table A1:** Screening of bond sample

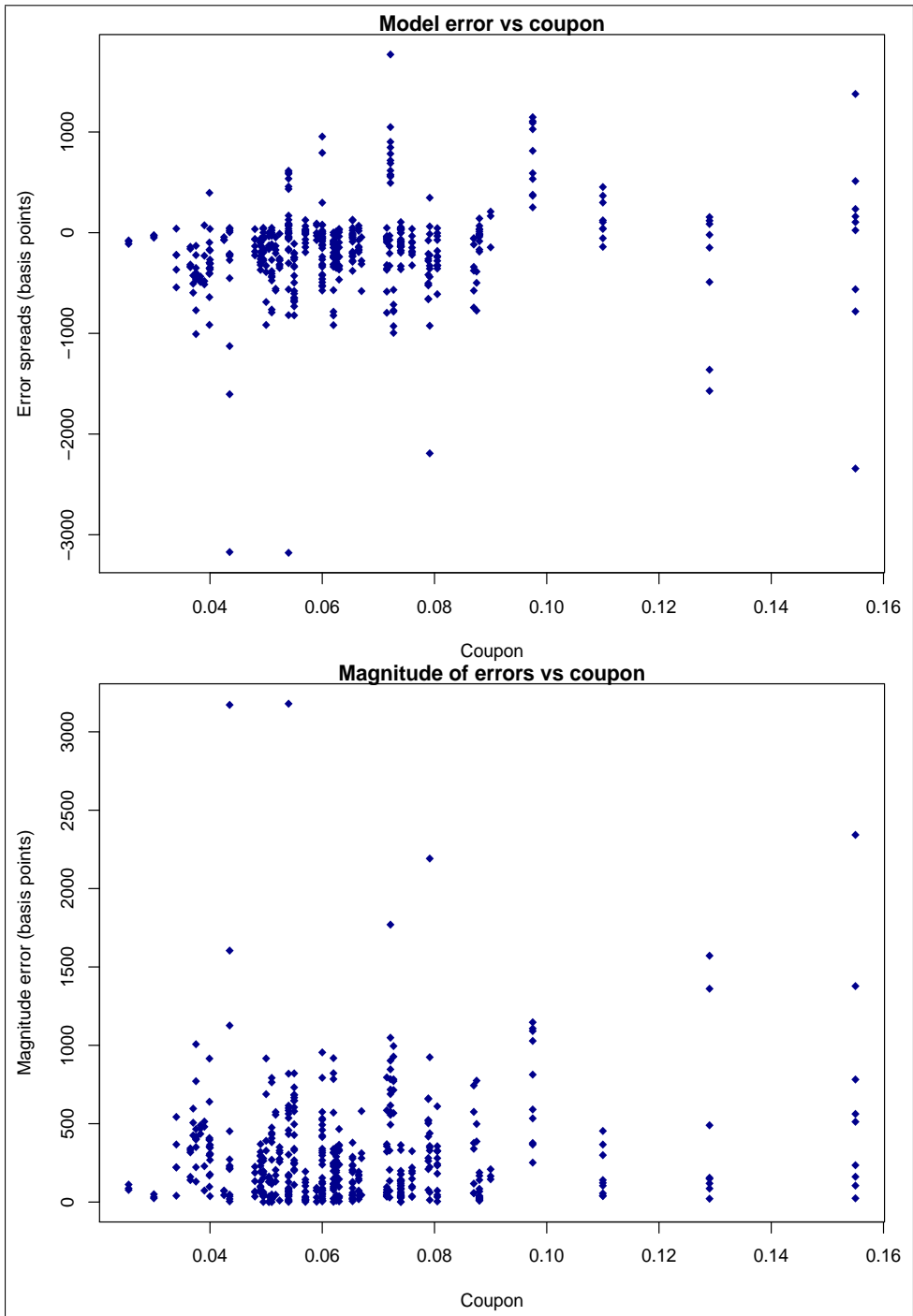
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## B List of bonds in final sample

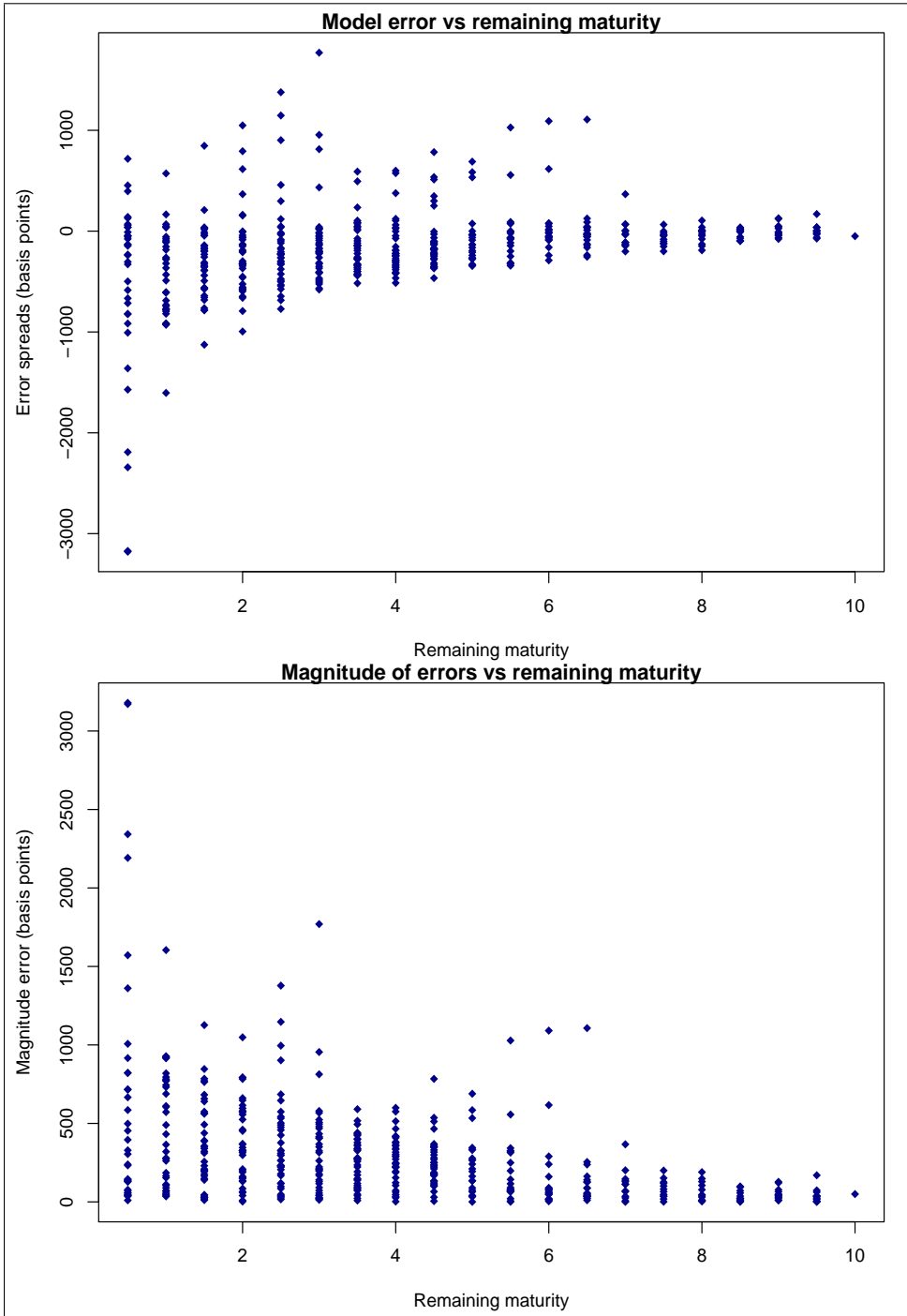
Issuer	Issue Date	Issuer	Issue date
Norske Skogind. ASA	27/07/2000	Hafslund ASA	12.03.2009
Kværner ASA	15/12/2000	Yara Inter. ASA	26/03/2009
Marine Harvest ASA	20/12/2000	Yara Inter. ASA	30/03/2009
Hafslund ASA	05/10/2001	Orkla ASA	22/04/2009
Hafslund ASA	20/03/2002	Akastor ASA	26/06/2009
Orkla ASA	19/06/2002	Norske Skogind. ASA	30/06/2009
Telenor ASA	18/10/2002	REC Silicon ASA	16/09/2009
Orkla ASA	13/11/2002	Norwegian Ener. Co. ASA	20/11/2009
Telenor ASA	18/11/2002	Olav Thon Eiendoms. ASA	27/11/2009
Orkla ASA	12/02/2003	Aker ASA	23/11/2010
Hafslund ASA	16/05/2003	REC Silicon ASA	03/05/2011
Kongsberg Gruppen ASA	10/10/2003	Hafslund ASA	14/02/2012
Hafslund ASA	09/01/2004	Schibsted ASA	01/03/2012
DNO ASA	01/06/2004	Olav Thon Eiendoms. ASA	30/03/2012
Norske Skogind. ASA	26/10/2004	Borgestad ASA	22/06/2012
Norske Skogind. ASA	26/10/2004	Norsk Hydro ASA	05/07/2012
Olav Thon Eiendoms. ASA	01/12/2004	Arendals Fosse. ASA	06/07/2012
Aker ASA	02/03/2005	Kongsberg Gruppen ASA	11/09/2012
Sevan Marine ASA	31/03/2005	Norwegian Ener. Co. ASA	30/10/2012
Olav Thon Eiendoms. ASA	15/06/2005	Schibsted ASA	13/12/2012
DNO ASA	12/10/2005	Hafslund ASA	25/01/2013
Telenor ASA	10/07/2006	Hafslund ASA	01/02/2013
Hafslund ASA	21/11/2006	Hafslund ASA	06/05/2013
Akastor ASA	01/12/2006	Olav Thon Eiendoms. ASA	24/05/2013
Norwegian Property ASA	22/03/2007	Olav Thon Eiendoms. ASA	31/10/2013
Orkla ASA	27/04/2007	Orkla ASA	22/11/2013
Hafslund ASA	12/06/2007	Norwegian Property ASA	27/11/2013
Orkla ASA	06/06/2008	Norwegian Ener. Co. ASA	09/12/2013
Hafslund ASA	12/11/2008	Hafslund ASA	24/01/2014
Hafslund ASA	21/01/2009	Yara Inter. ASA	18/12/2014
Hafslund ASA	04/02/2009	Yara Inter. ASA	18/12/2014

**Table A2:** Final bonds sample

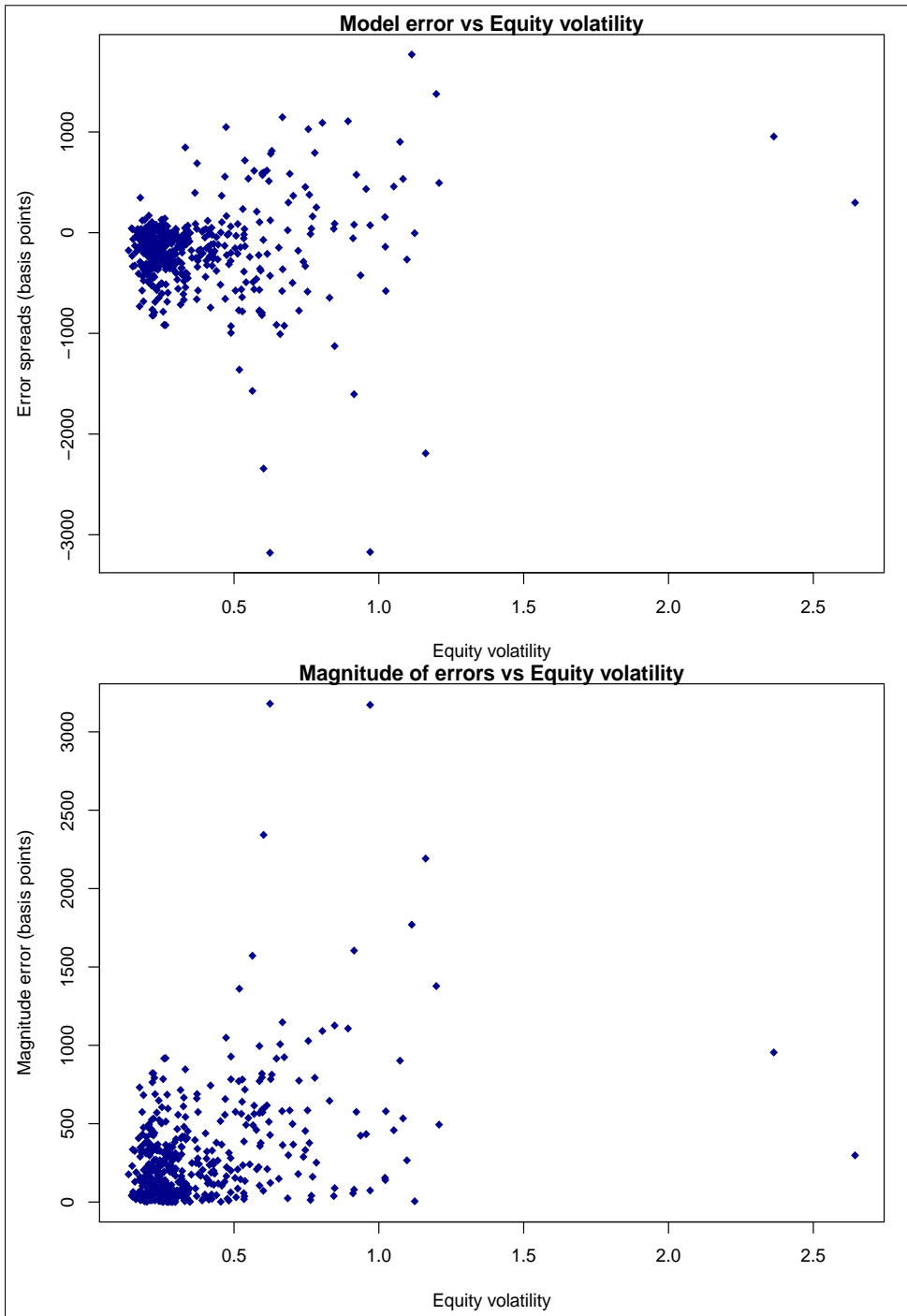
## C Plots of errors



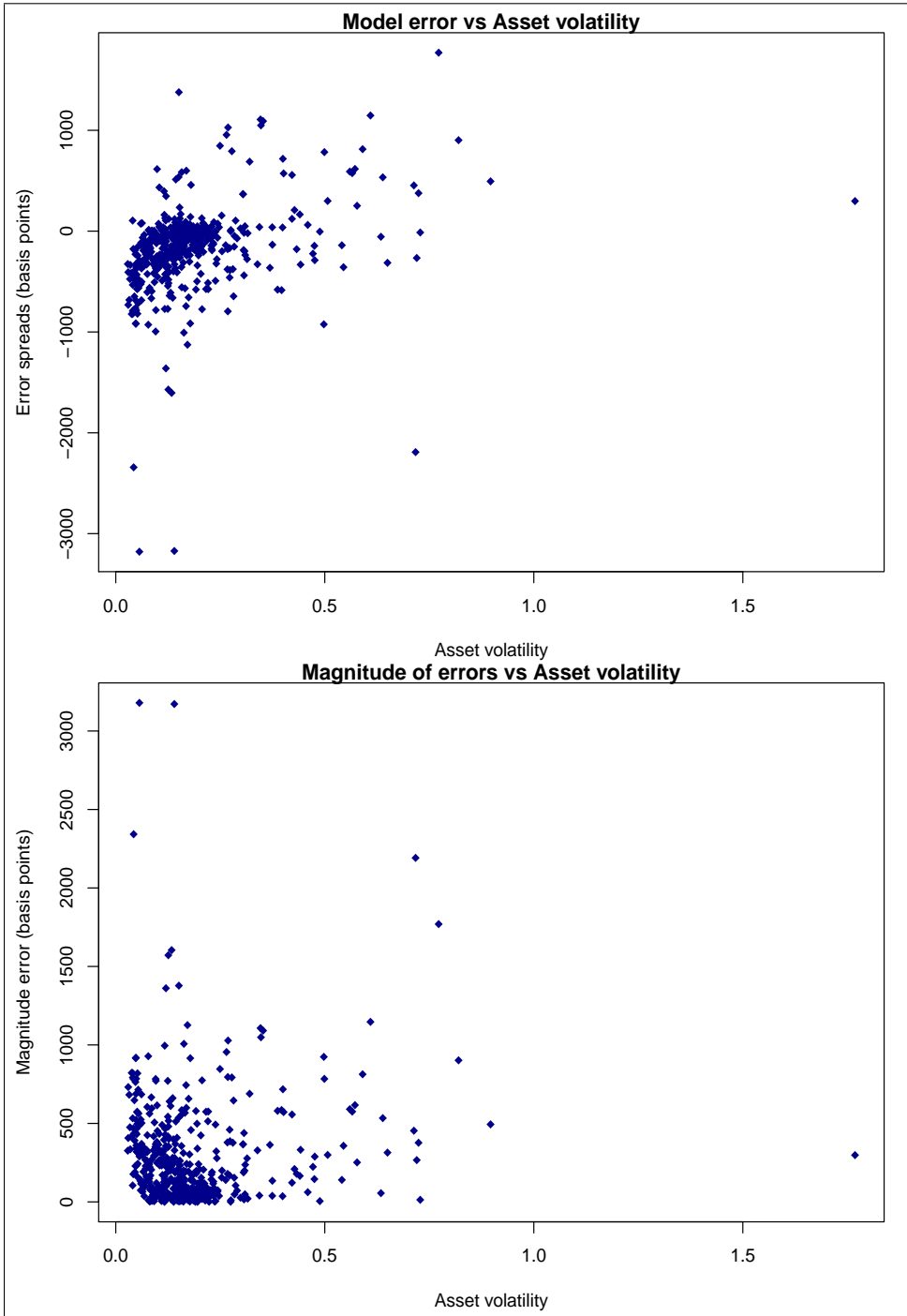
**Figure A1:** Errors versus coupon



**Figure A2:** Errors versus remaining time to maturity

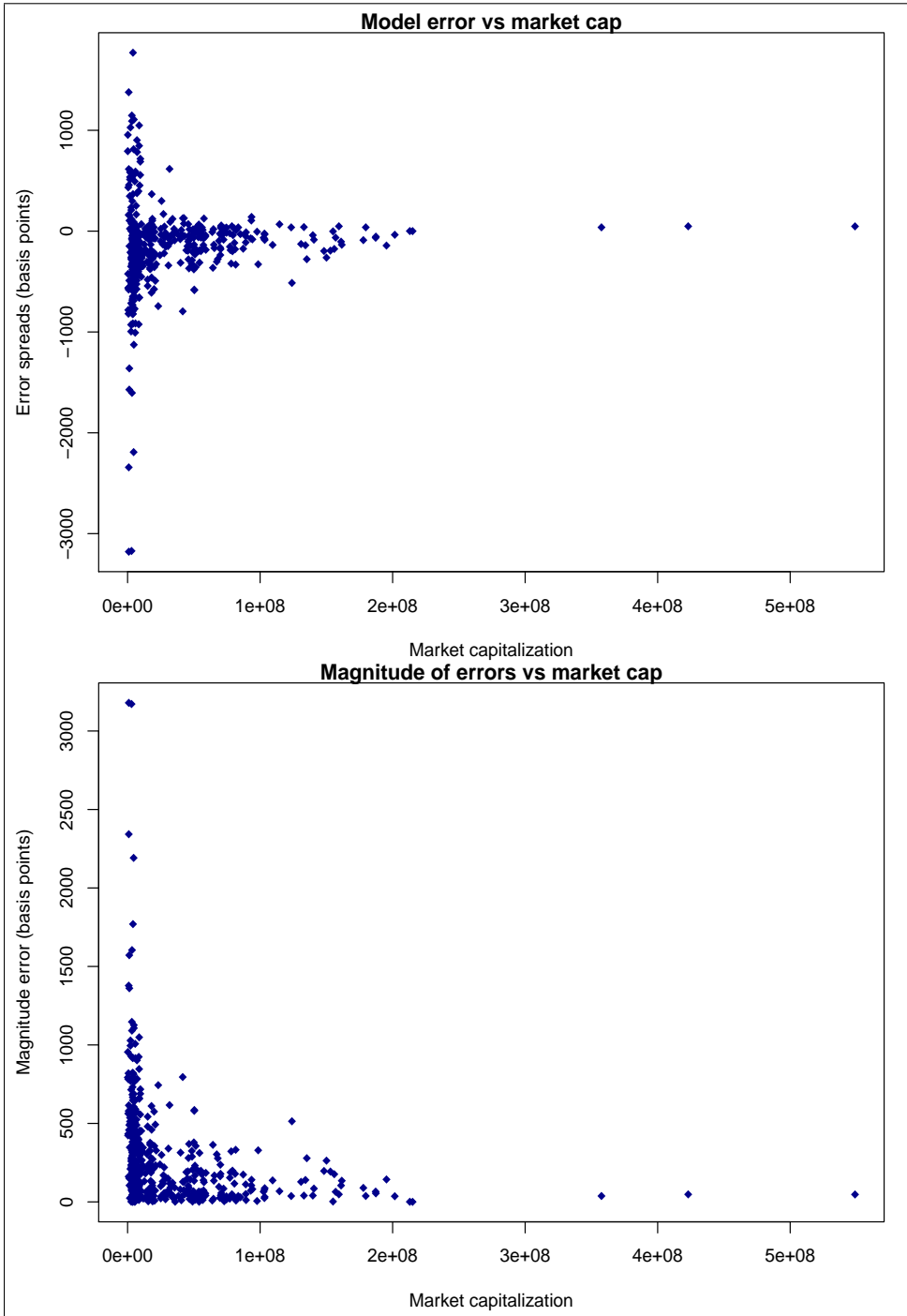


**Figure A3:** Errors versus 150 day equity volatility

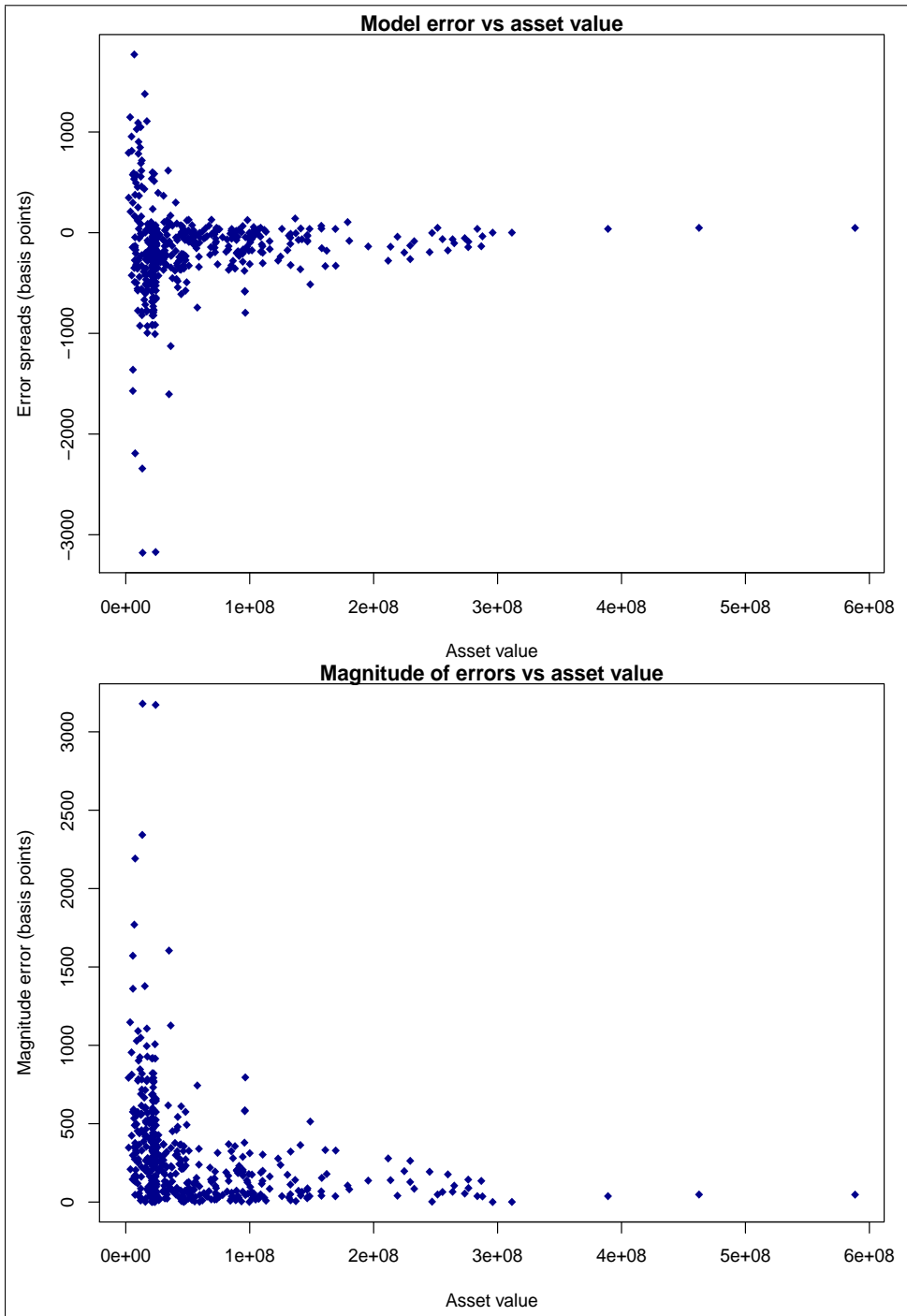


**Figure A4:** Errors versus 150 day asset volatility

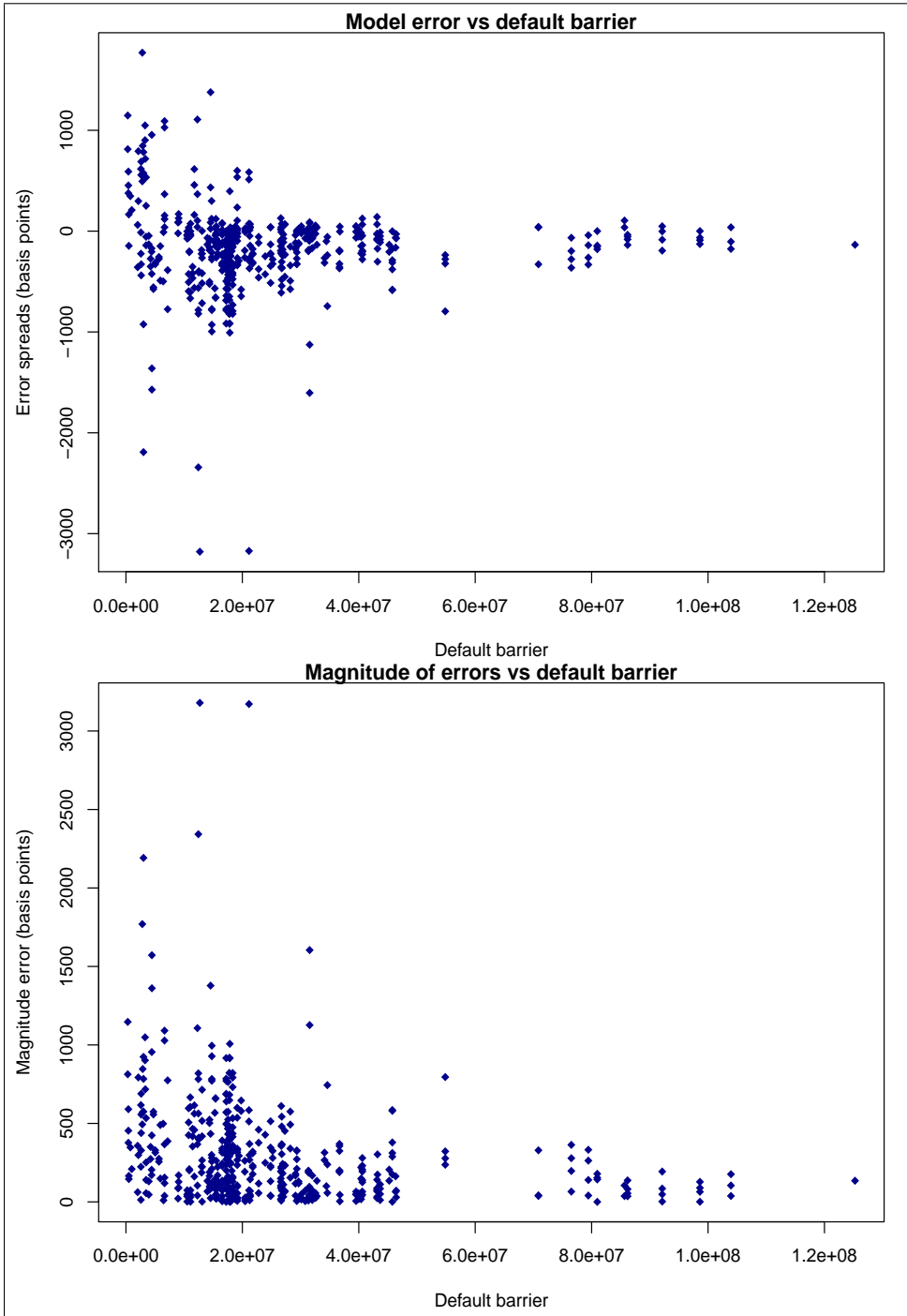




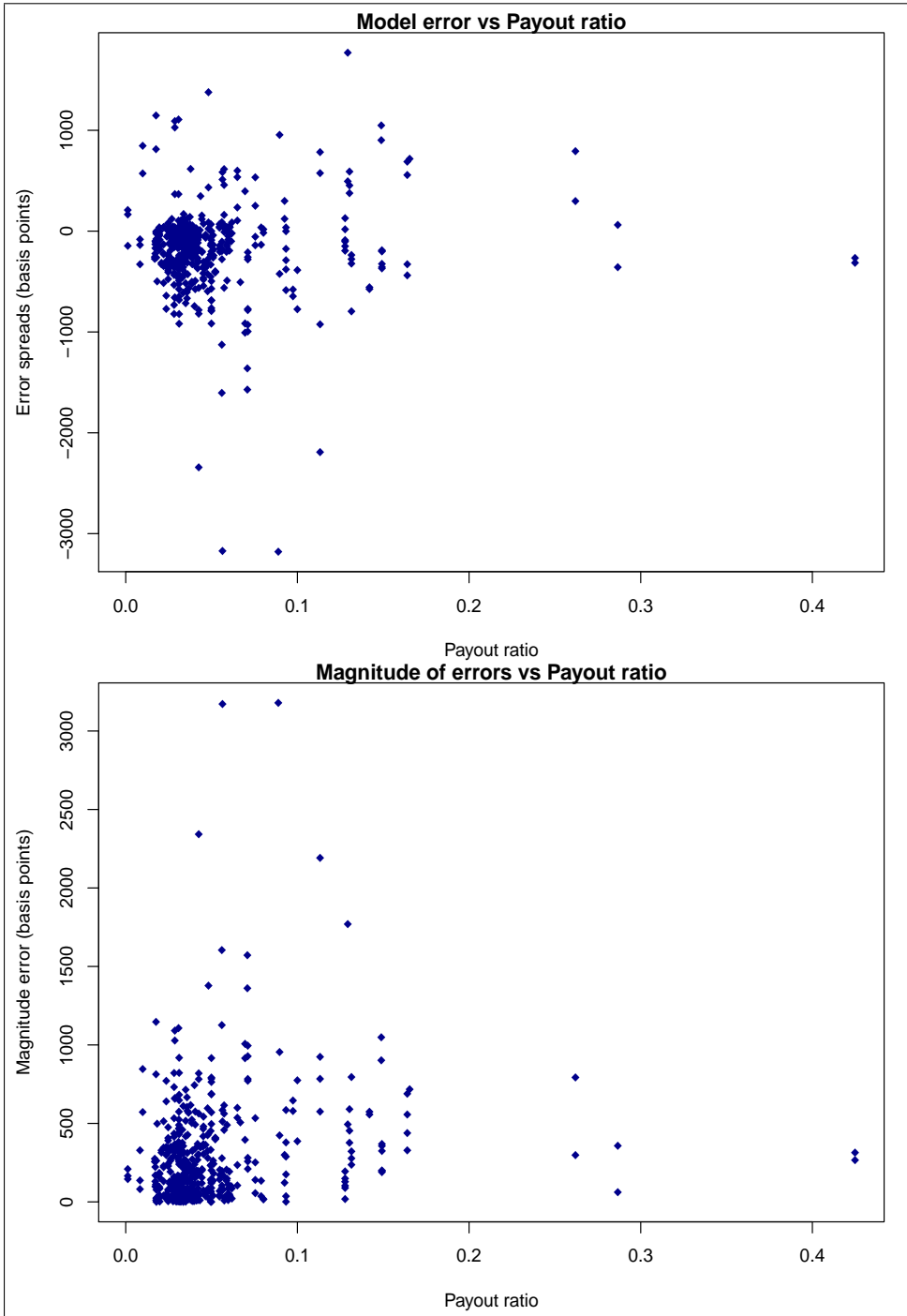
**Figure A5:** Errors versus market capitalization



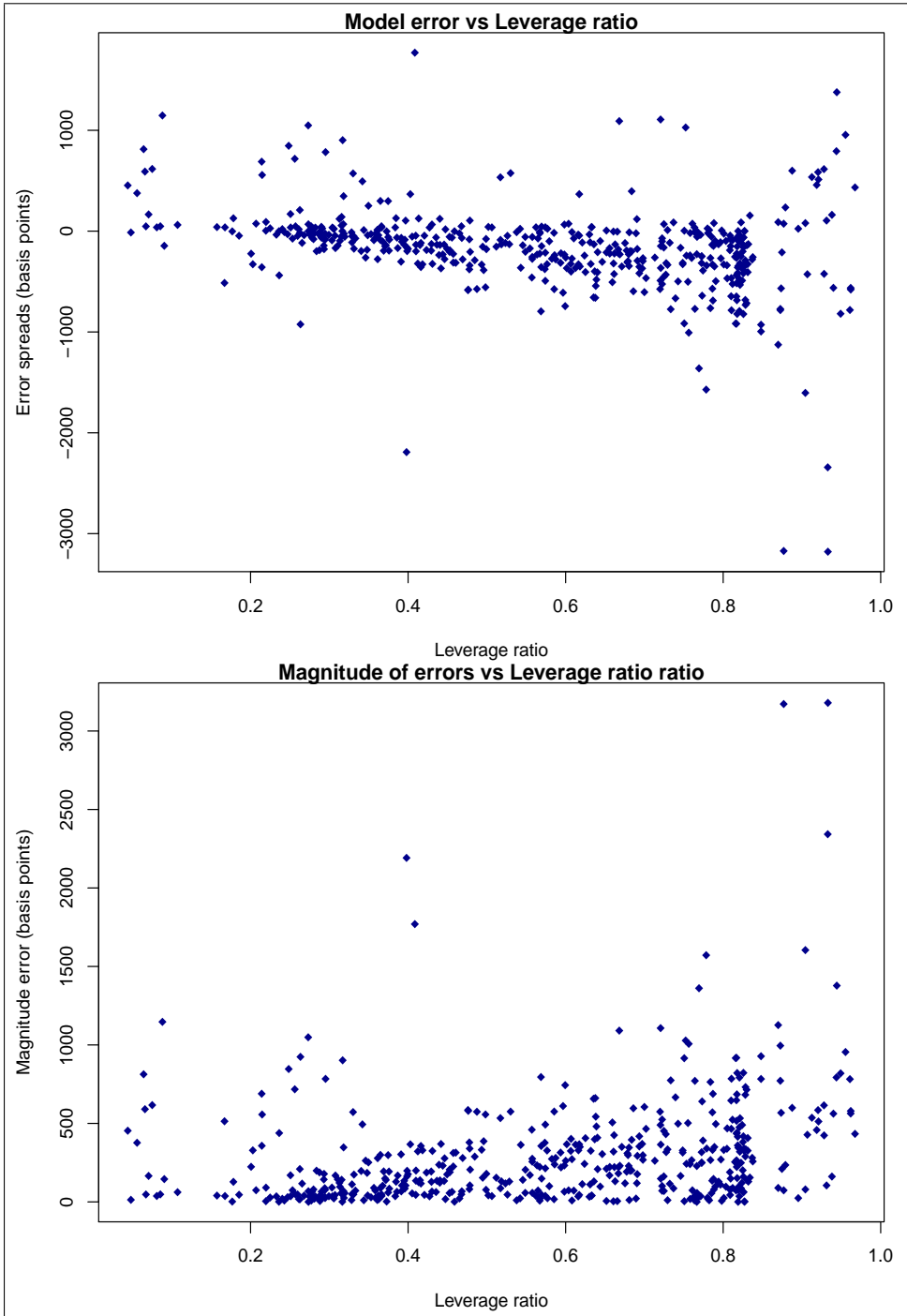
**Figure A6:** Errors versus asset value



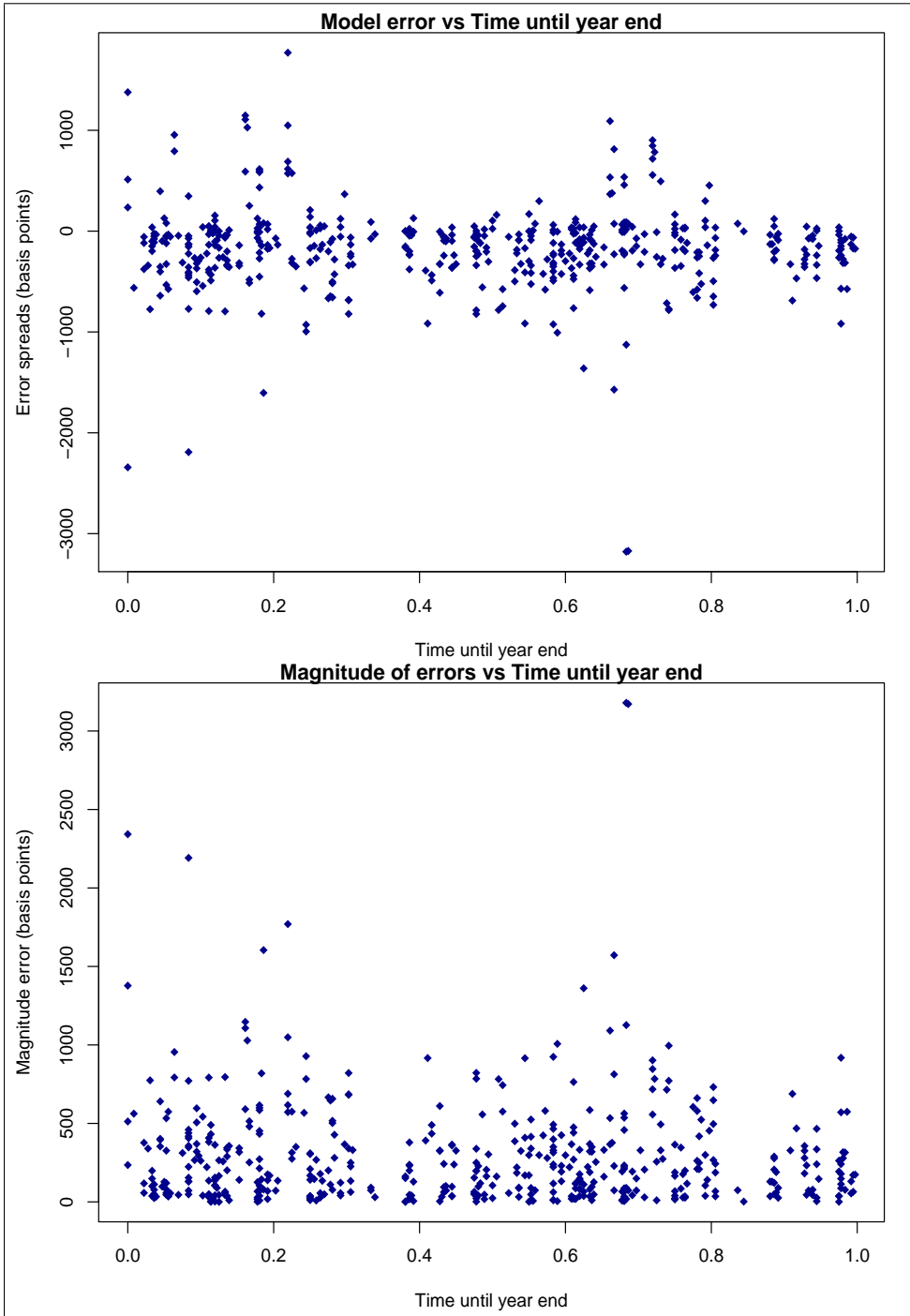
**Figure A7: Errors versus default barrier**



**Figure A8:** Errors versus Payout ratio



**Figure A9:** Errors versus quasi-market leverage ratio



**Figure A10:** Errors versus Time until year end

## D Correlation between parameters

	Coupon	rem.mat	$\sigma_E$	$\sigma_V$	$E$	$V_t$	$K$	$\delta$	$\gamma_{i,t}$	year_end
Coupon	1	-0.217	0.338	0.271	-0.020	-0.040	-0.083	0.119	-0.059	-0.044
rem.mat	-0.217	1	-0.175	-0.040	0.004	0.019	0.054	-0.097	0.017	0.064
$\sigma_E$	0.338	-0.175	1	0.702	-0.180	-0.198	-0.197	0.374	0.099	-0.084
$\sigma_V$	0.271	-0.040	0.702	1	0.075	0.037	-0.071	0.508	-0.477	-0.019
$E$	-0.020	0.004	-0.180	0.075	1	0.976	0.683	-0.040	-0.560	0.043
$V_t$	-0.040	0.019	-0.198	0.037	0.976	1	0.827	-0.048	-0.507	0.051
$K$	-0.083	0.054	-0.197	-0.071	0.683	0.827	1	-0.059	-0.252	0.059
$\delta$	0.119	-0.097	0.374	0.508	-0.040	-0.048	-0.059	1	-0.147	-0.035
$\gamma_{i,t}$	-0.059	0.017	0.099	-0.477	-0.560	-0.507	-0.252	-0.147	1	-0.007
year_end	-0.044	0.064	-0.084	-0.019	0.043	0.051	0.059	-0.035	-0.007	1

**Table A3:** Correlations

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