

## INVESTIGATION OF PARAMETRIC RESONANCE IN ROLL FOR CONTAINER CARRIER SHIPS

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### ABSTRACT

Parametric roll resonance is of concern for container and fishing vessels, especially in head-sea waves. Here this phenomenon is investigated with a numerical method based on potential-flow theory with viscous corrections for the roll damping. The seakeeping problem is handled by considering a strip theory and assuming a 5-DOF system. Nonlinearities are accounted for in the Froude-Krylov and hydrostatic loads. The solver has been validated against experiments on a C11 class container carrier ship in terms of parametric resonance occurrence and features for different ship forward speeds and headings, wavelengths, wave amplitudes and wave headings. The overall agreement is good but there are some discrepancies. For instance, the simulations show capsizing in some cases while it does not happen in the experiments. The results from present method can be used to generate 2D and 3D polar diagrams identifying the zones with parametric roll occurrence, and are very handy for masters aboard ships. This type of information is valuable at design stage and can be used aboard vessels for a safer voyage.

### INTRODUCTION

William Froude [9] was to our knowledge the first to observe parametric roll resonance of ships. He reported about a ship that may capsize especially when the oscillation frequency in heave and pitch is almost twice the natural frequency in roll. Thereafter Grim [14] and Kerwin [20] were the pioneers that investigated ship rolling in waves. Dunwoody [4] and [5]

studied this phenomenon in astern seas and some model test studies in the same conditions leading to capsize were carried out by Hamamoto et al. [15] and Umeda et al. [32]. Hua et al. [17] represented the GM variation in an irregular seaway using the Volterra series.

After the China disaster of an APL Container carrier ship and other similar incidents, the attention of hydrodynamic researchers turned to parametric rolling more than ever [31]. Another consequence of dramatic events is that a lot of maritime regulating organizations and classification societies like IMO, ABS and ITTC started to provide recommendations to designers and masters to avoid wave-induced parametric rolling [25,1,19].

Belenky et al. [2] investigated the parametric roll of C11 class container ships using LAMP code and they also investigated the ergodicity qualities and distribution of heave, pitch and roll motions in head sea and the effect of U-tube anti-rolling tank on parametric rolling.

Neves [23] proposed a 3 DOF nonlinear model in which he considered the coupled restoring moments up to 2nd order using Taylor expansion, and then Neves and Rodríguez [24] updated the previous model by a 3 DOF model using a Taylor series expansion up to the 3<sup>rd</sup> order and strip theory for calculating hydrodynamic loads [16]. Bulian [3] made a comprehensive analysis of this phenomenon in both regular and irregular waves and proposed a 1.5 DOF model. This means that the roll motion is modelled dynamically using a single degree of freedom while the coupling with heave and pitch is

taken into account by means of hydrostatic calculations without considering dynamic effects.

Spyrou [29] provided an analytical formulation of rate of growth of parametric roll based on probabilistic analysis by using the concept of wave groups.

France et al [11] analyzed and investigated the motion of the post-Panamax C11 class container ship through a series of experiments and a numerical analysis and also studied the parametric roll effect on container securing and lashing system. Greco et al [12] examined numerically and experimentally the parametric rolling of a fishing vessel in head sea with possibility of bottom slamming and water on deck by using a 3D domain decomposition strategy.

Parametric roll can happen in both regular and irregular waves. Vessels with non-vertical hull sides at water line are more vulnerable to this instability. Examples are fishing vessels, container carriers and passenger vessels. One of the vessels that has been investigated is the C11 class post-Panamax container ship. A set of model experiments have been performed at the CEHIPAR basin (Canal de Experiencias Hidrodinámicas de El Pardo) in Spain and some other basins. Wave amplitude, wave frequency, wave heading, ship loading and ship forward speed were varied in the experiments and the most dangerous operational conditions were identified.

From a massive research carried out in the last decade, it can be concluded that parametric roll resonance occurs when the following requirements are met:

1. The natural period of roll is equal to approximately twice the wave encounter period
2. The wave length is of the order of the ship length (between 0.8 and 2 times)
3. The wave height exceeds a critical level
4. The roll damping is low

Here, a numerical method based on linear potential-flow strip theory and nonlinear Froude-Krylov and hydrostatic loads, modelling the vessel as a 5-DOF system and including viscous roll damping, is used to study parametric roll.

There should be a balance between simplicity (which leads to reasonable and practical computational time) and comprehensiveness of the simulation method. In some literature totally nonlinear codes (like FREDYN and LAMP) have been used, which need very long CPU time for simulations. This makes them sometimes impractical because, for instance, to produce a 2D polar diagram one might need couple of hundreds simulations and it might take very long time for that. In some literature [3,16,23,24] they used a 3 DOF or 1DOF or 1.5 DOF which might not cover the whole phenomenon physics. So something in between would be desirable. We believe that the present method is fast and robust and is precise enough at the same time.

Present method is described in the next section, then it is validated against available experiments for a C11 class container ship model and used to carry on additional studies on this vessel. Finally the main conclusions are drawn.

## MODELLING FORMULATION

There are some susceptibility criteria, which are based on Mathieu instability formulation and could be used at initial design stage. Besides there are some severity criteria as well which can be used if the susceptibility criteria indicate danger of parametric resonance [1, 19].

For investigating susceptibility criteria of parametric rolling, we can consider the 1-DOF roll equation of motion with zero excitation moment as:

$$\ddot{\eta}_4 + 2\delta^*\dot{\eta}_4 + \frac{W \cdot GM(t)}{I_{44} + A_{44}}\eta_4 = 0 \quad (1)$$

where  $\eta_4$  is the roll motion,  $\delta^*$  is the linearized damping coefficient,  $W$  is the ship weight,  $GM$  is the metacentric height,  $I_{44}$  is the transverse moment of inertia and  $A_{44}$  is the added moment of inertia in roll.

In wave, due to the wave-body interaction and resulting heave and pitch motions,  $GM$  is in general time dependent due to variations of the water-plane area and of the vertical position of the center of buoyancy relative to the center of mass. For instance, depending also on the fore-aft ship geometry, a wave trough mid-ship tends to increase the  $GM$  while the opposite occurs with a wave crest mid-ship (see example in fig. 1).

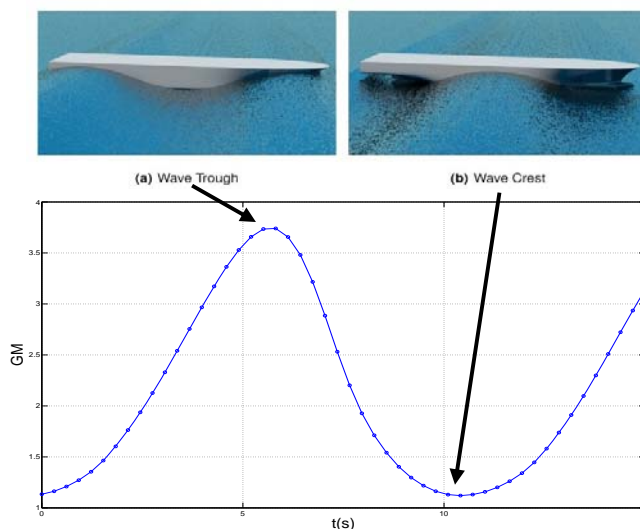


Figure1: Top:Change of waterplane area in waves [33]. Bottom: GM variation of C11 class container ship in waves (Head sea, wave amplitude=3m, ship forward speed=8kn, wave period=12.95s)

So, we look for unstable conditions of the ship assuming a periodic variation of  $GM$  with the excitation frequency.

We can consider the roll equation of motion as a damped Mathieu equation with known unstable solution zones. A stability diagram, which includes some damping values, can be found in [7, 21].

Now we can say that if the combination of parameters falls in a stable zone, then the roll motion will be stable, i.e. any disturbance will die out with time. If the Mathieu solution is in an unstable zone then the roll motion will be unstable.

This provides on-off information of parametric occurrence. To complement these predictions, some procedures and simplified formulas were proposed for predicting the roll amplitude in case of parametric resonance, as discussed in the ITTC report [19].

In our approach, when this simplified analysis indicates a possible instability, a more general numerical investigation is carried out for a quantitative estimation of the ship behavior.

The equations of motion are considered for a ship with constant forward velocity.

We considered two coordinate systems. One is a right-handed inertial frame moving with the steady forward ship velocity and fixed with respect to the mean oscillatory ship position. The z axis is vertically upward through the center of gravity, x is pointing in the direction of forward motion, y is pointing to the starboard and the origin is in the plane of undisturbed free surface. Let us assume  $\eta_1$ ,  $\eta_2$  and  $\eta_3$  as surge, sway and heave displacements in x, y and z directions, respectively.  $\eta_4$ ,  $\eta_5$  and  $\eta_6$  are roll, pitch and yaw rotations of the vessel around x, y and z axes, respectively.

The other coordinate system is a body-fixed coordinate system, i.e. translating and rotating with the body. Moreover no approximation is used in the matrices transforming point coordinates and forces and moments between these two frame references.

Since an objective of our studies is to produce 2D and 3D polar diagrams showing dangerous combinations of wave heading, wave amplitude and ship speed in regard of parametric rolling, a simplified computational method needs to be developed. The computational method has the linear frequency-domain potential-flow strip theory by Salvesen, Tuck and Faltinsen [26] as a basis. Because of the assumption of ship slenderness in strip theory, surge is neglected.

A generalization is that the Froude-Krylov and hydrostatic loads are estimated as nonlinear loads. More in detail, assuming known the regular incident wave profile, based on linear theory, and the rigid body motions, we can find the instantaneous submerged part of the body at each cross-section along the ship hull. The pressure is approximated as  $p = -\rho g z - \rho \partial \varphi_0 / \partial t + p_a$  where  $\rho$ , g,  $\varphi_0$ , t and  $p_a$  are mass density of water, acceleration of gravity, incident wave potential, time and atmospheric pressure, respectively. When  $z > 0$ ,  $\varphi_0$  is assumed constant consistent with linear theory and it is based on linear wave potential formulation for  $z < 0$ . It should be noted that, with the approximation, the free surface dynamic condition is correctly satisfied at the wave crest whereas there is a higher order-error at the wave trough [6].

Then, integrating the pressures along the wetted portion of each ship cross-section, 2D forces and moments in any section are estimated. Finally their integration along ship length, provides the nonlinear 3D Froude-Krylov and restoring forces and moments at any time instant. Besides, the weight of the ship must also be properly considered. The forces and moments are obtained in body-fixed coordinate system and have to be decomposed in inertial frame.

In the damping part, we added linear equivalent viscous roll damping to radiation roll damping also. We used Ikeda semi-empirical formulation for calculating viscous roll damping [18]. After estimating all the terms we can integrate the equations of motion in time by numerical integration using Runge-Kutta fourth order method.

Examples on error sources in the computational methods are:

- The linear potential-flow problem ought to be formulated in time domain in terms of convolution integrals. However, studies for zero speed showed a small influence.
- The linear potential-flow problem ought to be solved as a 3D problem with correct interactions between steady and unsteady flow. However, heave and pitch studies do often not show a dominant effect relative to strip theory. Even though a more rational linear 3D potential flow method was used as in Shao and Faltinsen [27] with consequent more complexity, we are not guaranteed better parametric roll predictions due to the facts that viscous roll damping and other nonlinearities matter.
- We do not consider the effect of ship-generated waves because the considered strip theory cannot predict the diffraction waves. However, according to the weak-scatterer assumption sometimes used [13], the elevation of ship-generated waves is secondary relative to the incident waves.
- Higher-order wave effects in the incident waves are neglected. The latter effect on the incident wave amplitude can first be estimated by considering second order theory. For the largest wave steepness which should be studied later in the text, the relative error is 0.075. Furthermore more results presented by [8] shows that higher than second order effects are negligible.
- Since nonlinear hydrodynamic effects may matter for the large roll angles in connection with parametric roll, nonlinear free surface and body-boundary effects ought to be examined. The latter can obviously be studied within state-of-the-art in CFD including viscous effects. However, computational time for our purposes prohibit the use of CFD.

## MODEL STUDIES AND RESULTS:

A Series of experiments for parametric rolling on a container ship model have been done during Hydralab III project in CEHIPAR (Canal de Experiencias Hidrodinámicas del Pardo, Spain) [28], which could be used for validation of our numerical method. The full-scale particulars for post-Panamax C11 containership are as follows:

Table 1: C11 Containership particulars

Length between perpendiculars	262m
Height at main deck	24.40m
Breadth	40m
Displacement at design waterline	76056 ton
Block Coefficient	0.66
Draft	12.34 m
Transverse Metacentric Height	1.97 m
Natural roll period	22.78 s
Maximum Speed	20 knots

Body plan of this ship and a 3D view of the vessel are provided in figure 2.

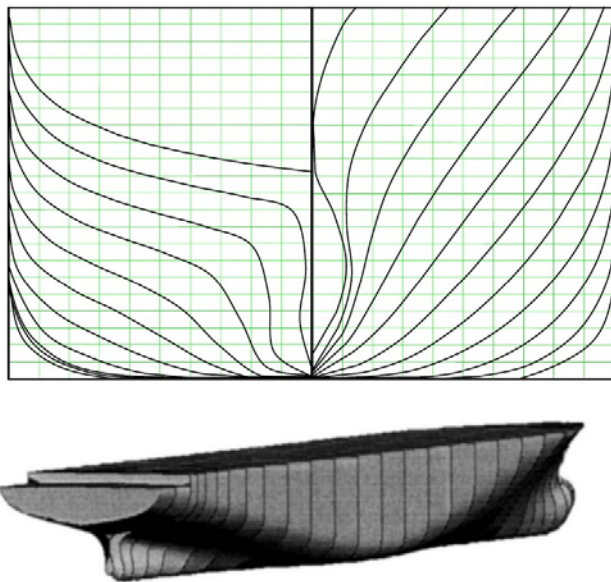


Figure2: Body plan and 3D view of C11 class containership

The experiments were performed on a model with scale of 1:65 in some scenarios which are discussed in the following. A tuning was set for parameters so that parametric rolling could occur. Based on conditions that were discussed in the previous section, the frequency of excitation should be almost twice the roll frequency and, at the same time, the wave height should be higher than a threshold value and the damping should be lower than a limit. Besides, the running time should be long enough so that the rising phase of the roll due to its instability can lead to steady-state conditions with the roll oscillating at its natural frequency.

In the experiments examined in the following, the wave length varies from 0.8 to 1.4 times of ship length and the wave amplitude ranges between 3 and 5 m. The ship speeds in these experiments are 8 knots in head sea and 0 knot in following sea.

Figure 3 provides an example of parametric-roll occurrence from the model tests in [28] in terms of the time evolution of the wave elevation and of heave, roll and pitch for a given

incident wave amplitude  $\zeta_a = 3m$ , wave period  $T_w = 12.95s$  and ship forward speed  $U=8kn$ . These values correspond to an incident wave steepness  $k\zeta_a = 0.072$ , a calm water roll natural frequency to excitation frequency ratio  $\omega_{An}/\omega_e = 0.472$  and  $\lambda/L_{pp} = 1$ .

Here  $\lambda$  and  $L_{pp}$  mean the incident wavelength and the length between perpendiculars, respectively.

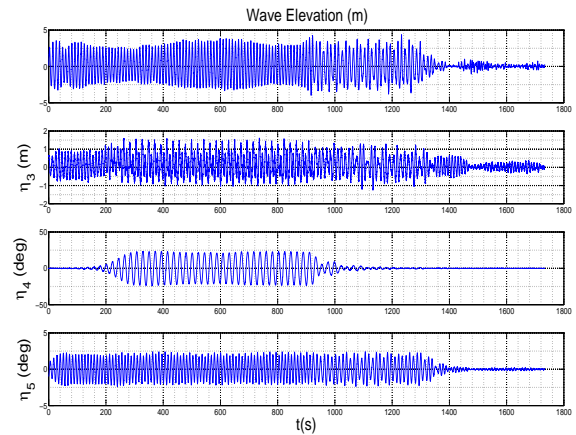


Figure3: Development of parametric rolling in experiments in regular waves (Head sea,  $\zeta_a = 3m$ ,  $U=8kn$ ,  $T_w = 12.95 s$ ) [28]

In this scenario, the maximum roll amplitude measured in experiments is 23.2 degrees. For the same scenario, the numerical simulation was carried out and the results are given in figure 4 including all motions modeled by the developed solver.

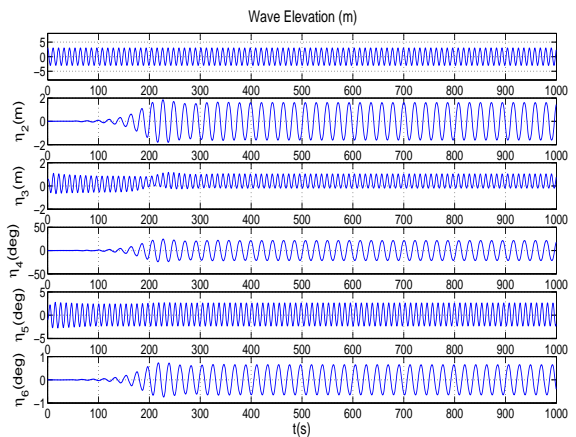


Figure4: Development of parametric rolling in simulations in regular waves (Head sea,  $\zeta_a = 3m$ ,  $U=8kn$ ,  $T_w = 12.95 s$ )

The maximum numerical roll amplitude is 21.3 degrees. In the next experiment examined, all the parameters are the same except the wave amplitude which is changed to 4m

( $k\zeta_a = 0.096$ ). We can see the effect of increasing the wave amplitude in figure 5.

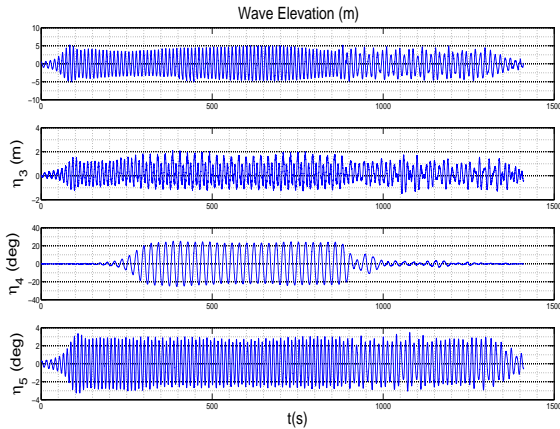


Figure5: Development of parametric rolling in experiments in regular waves (Head sea,  $\zeta_a = 4m$ ,  $U=8kn$ ,  $T_w = 12.95 s$ ) [28]

The maximum roll amplitude changes from 21.3 degrees for  $\zeta_a = 3m$  to 25 degrees for  $\zeta_a = 4m$ . The same scenario with numerical simulations is shown in figure 6.

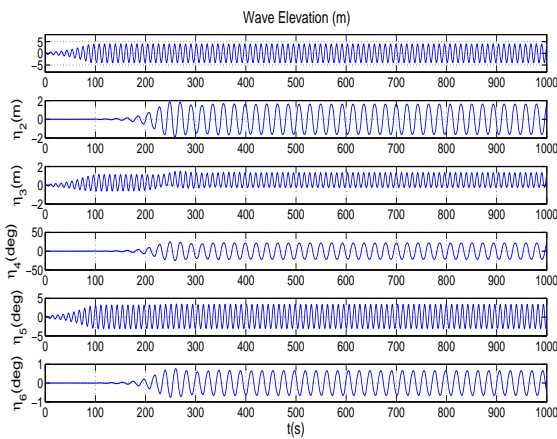


Figure6: Development of parametric rolling in simulations in regular waves (Head sea,  $\zeta_a = 4m$ ,  $U=8kn$ ,  $T_w = 12.95 s$ )

The simulated results show 21.3 degrees as the maximum roll amplitude, which is in good agreement with experiments but it does not show an increase when comparing with previous scenario.

Now we want to see how the ship would behave in another combination of parameters. For this scenario, the experiment was done for an incident wave amplitude  $\zeta_a = 4m$ , wave period  $T_w = 14.19s$  and ship forward speed  $U=8kn$ . These values correspond to an incident wave steepness  $k\zeta_a = 0.08$ , a calm water roll natural frequency to excitation frequency ratio  $\omega_{4n}/\omega_e = 0.526$  and wavelength-to-ship length ratio  $\lambda/L = 1.2$  and the related measurements are shown in the figure 7.

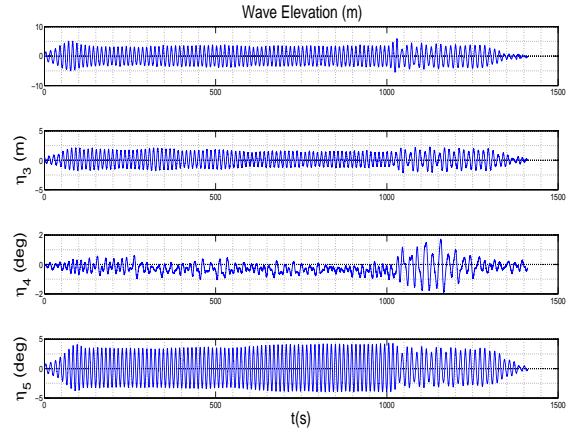


Figure7: Development of parametric rolling in experiments in regular waves (Head sea,  $\zeta_a = 4m$ ,  $U=8kn$ ,  $T_w = 14.19 s$ ) [28]

As it can be seen no parametric rolling occurred because of change of frequency of encounter conditions relative to roll natural frequency and the initial disturbance died out in time. The corresponding numerical results are provided in figure 8.

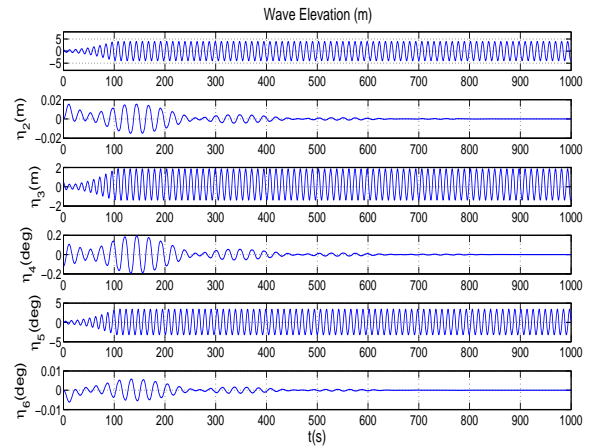


Figure8: Development of parametric rolling in simulations in regular waves (Head sea,  $\zeta_a = 4m$ ,  $U=8kn$ ,  $T_w = 14.19 s$ )

Consistently with the experiments, also in this case there is no parametric resonance in roll.

There are some other scenarios in the experiments and numerical simulations and the overall results can be seen in the table 2.

Table2: Occurrence of parametric roll (PR) in terms of roll amplitude from experiments in [28] and present method in head-sea regular waves.

Test No	$\frac{\lambda}{L_{pp}}$	$\zeta_a$ (m)	$T_w$ (s)	$U$ (knots)	Roll angle experiment (deg.)	Roll angle simulation (deg.)	Roll angle simulation [33] (deg.)
5	0.8	3	11.59	8.0	31.0	38.5	/
6	1	3	12.95	8.0	23.2	21.3	17.58
7	1.2	3	14.19	8.0	2.0	No PR	/
8	1.4	3	15.33	8.0	0.5	No PR	/
9	0.8	4	11.59	8.0	32.8	40	/
10	1	4	12.95	8.0	25.0	21.3	45.04
11	1.2	4	14.19	8.0	1.7	No PR	/
12	1.4	4	15.33	8.0	0.8	No PR	/
13	0.8	5	11.59	8.0	35.7	40.5	11.57
14	1	5	12.95	8.0	27.3	20.4	/
15	1.2	5	14.19	8.0	1.3	No PR	/
16	1.4	5	15.33	8.0	0.6	No PR	/

The simulation results for the occurrence of parametric roll and for the values of steady-state roll amplitude are in good agreement with the experimental data.

For the conditions in which the parametric rolling occurred and the roll maximum amplitude are almost the same in experiments and simulations.

As it can be seen in the experiments, for  $T_w = 11.59$  and  $12.95$ s the roll amplitude increases by increasing wave amplitude. For  $T_w = 11.59$ s it goes from 31 degrees for  $\zeta_a = 3$ m to 32 degrees for  $\zeta_a = 4$ m and 35.7 degrees for  $\zeta_a = 5$ m. This last value represents the largest amplitude observed in the head-sea tests.

In six scenarios, there is no parametric resonance both in the model tests and in the simulations. More in detail, this occurs in tests with  $T_w = 14.19$  and  $15.33$ s ( $\omega_{4n}/\omega_e = .525$  and  $0.575$ ) at all incident-wave steepnesses examined.

The most severe cases did not occur for  $\lambda/L_{pp} = 1$  but for 0.8 and the maximum roll amplitude for  $\zeta_a = 5$ m reached 35.7 degrees in experiments and 40.5 degree in simulations. These values are very high and dangerous for ships.

Turk [31] compared his results with the same experiments for tests No. 6, 10 and 13 by using an apparent similar method as ours. However there were not given any details on e.g. how the nonlinear Froude-Krylov and restoring loads were calculated. We cannot explain why, but our results agree reasonably well with the experiments while the results by [31] do not.

Figures 9 and 10 examine the initial transient phase leading to the build-up of the parametric resonance from one experimental case and the corresponding numerical simulation. The roll time history is synchronized with the evolution of the incident-wave elevation and of the heave and pitch motions.

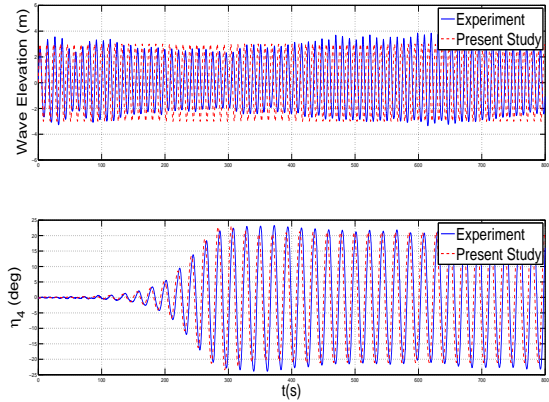


Figure9: Comparison of development of parametric rolling in experiments [28] and simulations in regular waves (Head sea,  $\zeta_a = 3$ m,  $U=8$ kn,  $T_w = 12.95$  s)

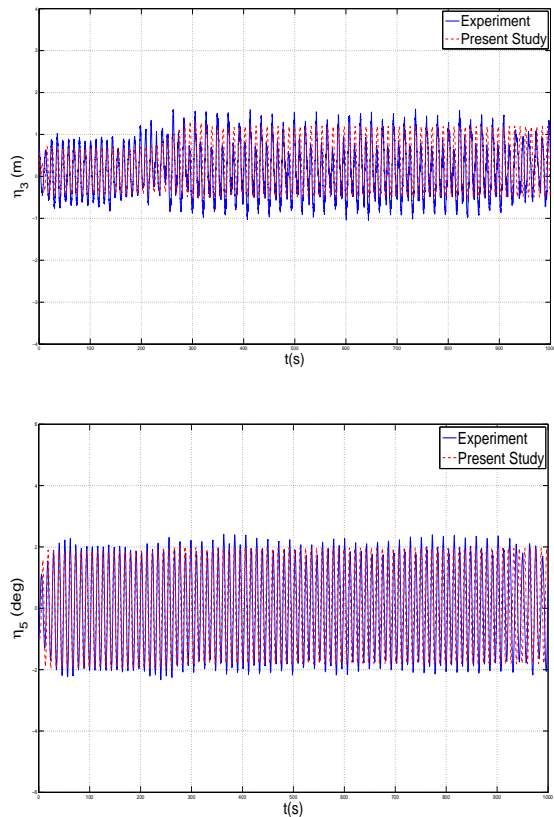


Figure10: Comparison of heave and pitch motions in experiments [28] and simulations in regular waves (Head sea,  $\zeta_a = 3$ m,  $U=8$ kn,  $T_w = 12.95$  s)

There is good agreement between the two results. The small differences could be due to a non-perfectly regular behavior of the incident waves in the experiments (see top plot of figure 9).

However, we have no possibilities to assess experimental error sources.

As it was told before, one of the objective of this model is to identify dangerous zones for parametric rolling by producing polar diagrams. The polar diagrams for selected wave periods and wave amplitudes are shown in the figures 11 to 13. The toolbar in the right of the figures represents the steady state roll amplitude in degrees. The heading intervals are considered as 15 degrees and the interval in speed is 2 knots. For each polar diagram 117 simulations were performed for a time duration of 2000 seconds. With these polar diagrams one can identify which speed and heading combinations, within the chosen values, can be dangerous regarding parametric rolling.

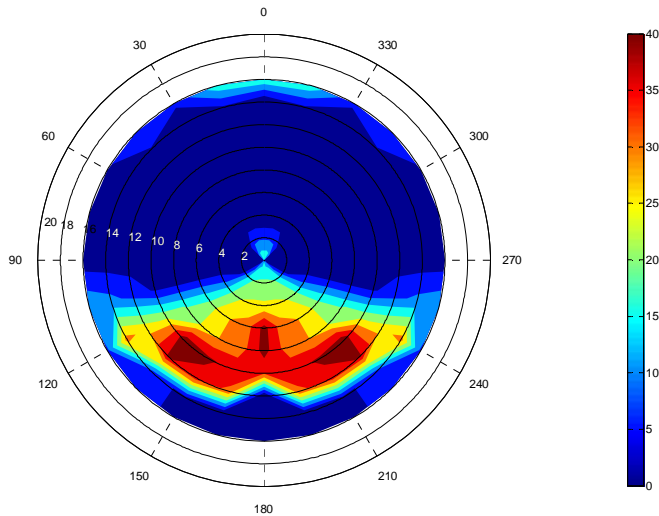


Figure11: Numerical polar plot for roll (degree) in regular waves versus heading and forward speed in knots  
( $GM=1.97m, \zeta_a = 3m, T_w = 11.59 s$ )

The first polar plot is in figure 11 and is for  $\zeta_a = 3m$  and  $T_w = 11.59s$ . As it can be seen, we have a weak amplification at head sea (180 degrees heading) with no forward speed but it becomes higher with increasing forward speed, it will have the maximum amplification at around 8 knots and starts to reduce after that vanishing at around 12 knots. In bow sea, the maximum roll amplitude is in between 10 and 12 knots and vanishes at around 14 knots. In beam sea there is no parametric rolling observed. In following sea we have small amplification at zero forward speed, then it vanishes at around 2 knots and it seems it starts again at 16 knots.

The next polar diagram in figure 12 is for same wave period but with amplitude of 4m.

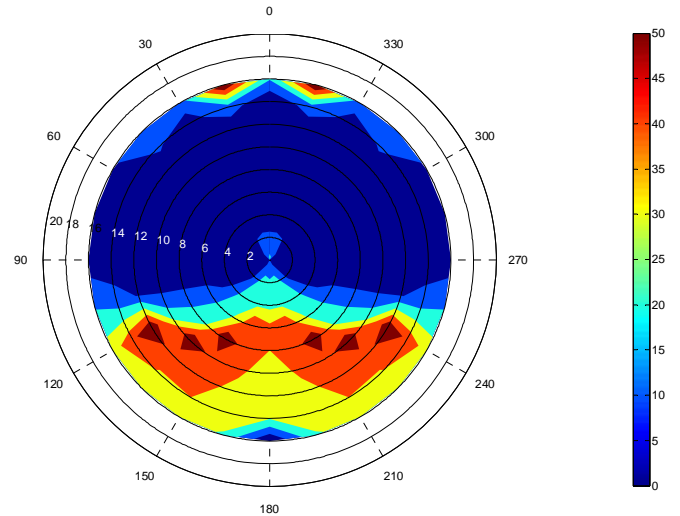


Figure12: Numerical polar plot for roll (degree) in regular waves versus heading and forward speed in knots  
( $GM=1.97m, \zeta_a = 4m, T_w = 11.59 s$ )

As it can be seen, by increasing the wave amplitude, the danger zone in head sea widens and the speed at which the parametric rolling vanishes increases. The parametric rolling zone in following sea seems also to start at lower speeds. We can also see some danger zones at heading of 15 degrees in speed of 16 knots while we do not see a very high danger in the same area for wave amplitude of 3m. The highest roll amplitude observed is also greater in the larger wave-amplitude case and so the capsizing risk is higher.

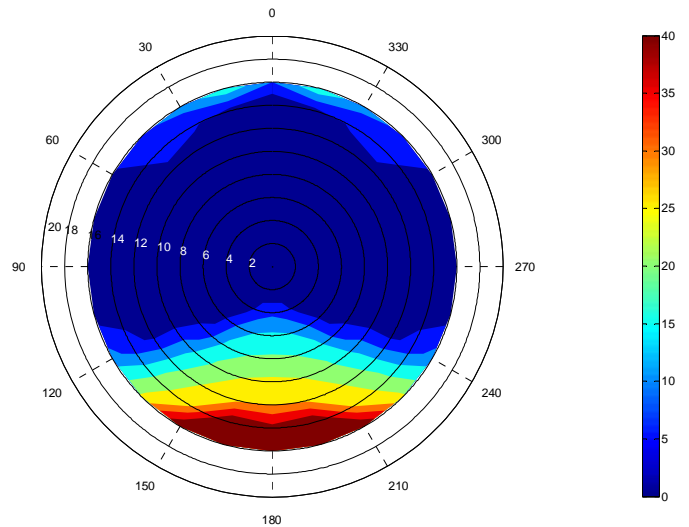


Figure13: Numerical polar plot for roll (degree) in regular waves versus heading and forward speed in knots  
( $GM=1.97m, \zeta_a = 3m, T_w = 12.95 s$ )

The last polar diagram is shown in figure 13 for a higher wave period than figure 11. We see that at zero forward speed there is no parametric rolling in any heading while we have some small amplification for shorter incident-wave period. The situation in

following sea with higher speeds are almost the same in the two cases. In head sea, it is clear that the strong amplification starts at higher speeds than in figure 11. For instance at speed of 10 knots the roll amplitude is around 35 degree for  $T_w = 11.59$  while in the same condition the amplitude is around 25 degrees for  $T_w = 12.95$  s.

From these polar plots, the specified danger zones should be considered and as it can be seen, there are more dangerous zones in head and bow sea than in following and beam sea, at least in the studied situations and in the operational conditions. The ship masters should avoid these situations by ship handling tactics based on forward speed and heading or a combination of them. As it can be seen from the results, reducing the forward speed always does not bring out of parametric rolling and sometimes increasing the speed might be more useful in this regard.

The damping plays an important role in the occurrence of parametric roll. Here the basic damping is calculated at the encounter frequency due to the fact that we do not use a convolution integral formulation. The quadratic viscous damping is small relative to the wave radiation damping. For example, the total damping ratio at roll amplitude of 20 degrees for  $T_w = 12.95$  s, head sea and  $U = 8$  kn is around 8% and for  $T_w = 11.59$  s is around 9.6%. Unluckily free-decay tests are not available from the experiments in [28] to assess the roll damping used in our simulations. Therefore, a sensitivity analysis has been performed on the influence of damping on the parametric roll. We added a fraction of critical damping to the system to find the threshold damping that can prevent parametric resonance. The results are shown in figure 14.

Head Sea,  $U=8$ kn

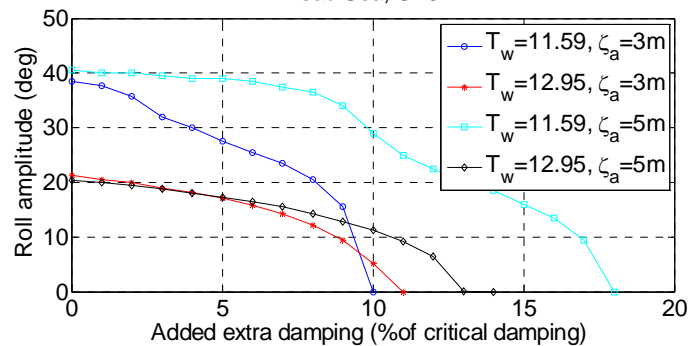


Figure14: The effect of added damping on parametric roll occurrence and roll amplitude

It is clear that the needed damping for higher wave amplitude is larger. For the studied incident waves, with additional damping between 10% and 18% of the critical damping we can avoid parametric resonance in roll. These results suggest that the use of other damping devices (like bilge keels or anti rolling tank) might avoid the parametric roll.

We also did a sensitivity test on the effect of freeboard on parametric roll. By increasing and decreasing the freeboard by 5%, we did not see much difference in rolling amplitude. In fact the difference was less than 1 degree.

A set of experiments were also done for  $GM=0.99$ m in following sea waves and without forward speed. The results from the model tests and the simulations are shown in table 3.

Table3: Occurrence of parametric roll in terms of roll amplitude from experiments in [31] and present method in following-sea regular waves and  $GM=0.99$ m. (Cap=Ship Capsize)

Test No	$\frac{\lambda}{L_{pp}}$	$\zeta_a$ (m)	$T_w$ (s)	$U$ (knots)	Roll angle experiment (deg.)	Roll angle simulation (deg.)
309	0.8	3	11.59	0.0	35.79	Cap
310	1	3	12.95	0.0	35.79	Cap
311	1.2	3	14.19	0.0	31.53	29.6
312	1.4	3	15.33	0.0	25.96	22.5
313	0.8	4	11.59	0.0	35.74	Cap
314	1	4	12.95	0.0	38.20	Cap
315	1.2	4	14.19	0.0	34.07	Cap
316	1.4	4	15.33	0.0	26.35	24.5

From the comparison, the numerical predictions are a bit conservative in terms of roll amplitude. In three conditions, the agreement is good but in other conditions, which lead to high roll amplitudes, the simulation shows capsizing while the experiments did not record such critical event. A possible experimental error source is wave reflection from tank walls, which are more important than for forward speed.

### CONCLUSION:

A robust and computationally efficient numerical method for investigating ship parametric-resonance in roll is presented and applied to a C11 class container-carrier ship. The numerical method uses linear frequency-domain potential-flow strip theory, nonlinear Froude-Krylov and hydrostatic loads. Empirical viscous roll damping is included. The solver has been validated against model experiments and showed good agreement with a tendency of the developed solver to give conservative results in some examined scenarios.

The conditions in which the parametric rolling occurred and the roll maximum amplitude are almost the same in experiments and simulations in both amplitude and frequency. Furthermore, we speculate about the influence of wave reflection from tank walls at zero forward speed in following waves.

A sensitivity study on the influence of damping and freeboard on the parametric roll was performed numerically. By adding damping in the range between 10% and 18% of the critical damping, we could avoid parametric resonance in roll for the examined incident waves. The freeboard seems to have a small effect on parametric rolling.

Further work with the numerical method is to obtain critical conditions in regard of parametric rolling so they can be starting points for model tests and for further studies. These conditions are highly valuable for calibrating the experimental setup and instruments.

Besides, it could also be used for producing 2D and 3D polar diagrams, which show dangerous combinations of wave heading, wave amplitude and ship speed in regard of parametric rolling. These diagrams are highly valuable and useful for



shipmasters aboard ships to avoid such dangerous zones. Using those diagrams the ship masters will be aware of the risk of parametric rolling and will take precautionary actions.

3D polar diagrams are made of 2D polar diagrams and in a third dimension the wave amplitude will change. In this way, we can have a 3D view of parametric rolling based on wave amplitude, wave period and forward speed at the same time. Based on the authors' experience the 2D polar diagrams are more illustrative, therefore in the paper three samples of 2D polar diagrams are shown.

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