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PIPE NETWORKS: COUPLING CONSTANTS IN A JUNCTION FOR THE ISENTROPIC EULER EQUATIONS

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Abstract

The modelling of junctions in pipe networks with subsonic flows is discussed, where pipes are described by one-dimensional, single-phase isentropic flow models. We first study the Riemann problem in a pipe to understand what information is needed to couple two pipes in a flat junction. Using this insight, we generalise the Riemann problem to an arbitrary number of pipes meeting together at a junction. Three coupling strategies found in the literature are presented, where only one leads to physically sound solutions for all the selected test cases. The theoretical derivation is performed in previously published literature.

The junction is considered to be a point with no volume. The three coupling strategies are, first, to impose all the pipe sections to be at the same pressure at the junction. The second is to impose equal momentum fluxes at the inlet of all the pipes coupled to the junction. The third is to impose all the pipe sections to reach the junction at a unique stagnation enthalpy, that is, equal for all of them. Only the latter satisfies the second law of thermodynamics, expressed through an entropy condition, in all the test cases run in the study. For the two former coupling strategies, test cases where the entropy condition is violated could be found and are presented.

The different coupling strategies are implemented in a numerical model. The one-dimensional models for the pipe sections are solved using a Roe scheme. We illustrate with numerical cases that we can find initial conditions for which the entropy condition is violated for the two first coupling strategies, while the third verifies it in all the cases.

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1. Introduction

Junctions between one-dimensional pipe models have a number of applications. A possible one is the simulation of heat exchangers involving parallel channels, like the shell-tube or plate-fin heat exchangers used in gas processing. Offshore especially, the reduction of size and weight of the equipment is an important

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factor for cost reduction. However, compact heat exchangers may be subject to fluid-dynamic instabilities, possibly leading to early aging or accidents.

In the present article, the pipe sections are modelled by one-dimensional models. The fluid is modelled by a single-phase isentropic model. However, junctions are three-dimensional objects that need to be carefully handled numerically. The first obvious condition is that mass should be conserved across the junction. A second coupling condition is necessary. In previous works [1, 2, 3, 4], three modelling strategies have been proposed for the second condition. The first one is to impose that all the pipes join at the same pressure. The second one is to impose equal momentum fluxes at the inlet of all the pipes coupled to the junction. The third one is to impose that all the pipes join at the same stagnation enthalpy [5], bearing similarities with the Bernoulli law. It was proved in [5] that only the latter coupling condition leads to a model that satisfies the second law of thermodynamics, in shape of an entropy condition. This can be illustrated, as is done in the present article, by running the model using the different coupling conditions, and checking whether the entropy condition is fulfilled.

To solve such a model, finite-volume methods for conservation laws can be used [6]. It consists in cutting the pipe in cells, and averaging the solution in each cell. Thus, the real solution is approximated by a piecewise continuous solution, having discontinuities at the interfaces. Thus, a discontinuous problem has to be solved at each interface, even when the real solution is smooth. Such a discontinuous problem is called a Riemann problem. Thus, deriving the coupling conditions for discontinuous problems at junctions, which is the theme of the present paper, is necessary to obtain numerical methods for pipe networks.

In Section 2, the isentropic flow model is presented together with the thermodynamic equation of state. Then, in Section 3, the Riemann problem is studied to understand the principle of coupling pipes. In Section 4, the Riemann problem is generalised to an arbitrary number of pipes. This is where the coupling conditions previously mentioned are involved. In section 5, two numerical test cases are presented to illustrate why the two first coupling conditions are physically wrong, while the third one yields physically sound solutions when entropy condition is used as criterion. Finally, Section 6 concludes the paper.

Nomenclature

A	Cross-sectional area
$\boldsymbol{A}$	Jacobian matrix
C_p	Heat capacity at constant pressure
C_v	Heat capacity at constant volume
$\boldsymbol{F}$	Vector of fluxes
$\mathcal{H}$	Coupling quantity
$\boldsymbol{R}$	Eigenvectors
$\boldsymbol{U}$	Vector of conserved variables
a	Speed of sound
h	Specific enthalpy
k	Factor in the isentropic equation of state
p	Pressure
q	Parameter vector in the Roe scheme derivation
v	Velocity
γ	Heat capacity ratio
λ	Eigenvalues
ρ	Density
σ	Entropy production rate

2. The model

2.1. Fluid dynamic model

In problems like the modelling of heat exchangers with parallel channels, one needs to model networks of pipelines. Several pipes in parallel will be coupled at junctions. To have computationally tractable models, the pipes may be modelled with one-dimensional fluid-dynamical models of the form

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}(\boldsymbol{U})}{\partial x} = 0, \tag{1}$$

where $\boldsymbol{U}$ is the vector of the conserved variables, mass, momentum and total energy, and $\boldsymbol{F}$ is the vector of the fluxes of these variables. When coupling together the equation systems for two or more pipes at a junction, care must be taken.

2.2. Equation of state

The isentropic Euler equations are derived from the Euler equations for compressible gas dynamics, by assuming that the flow is isentropic. The resulting system is of dimension two. The equations can be written in the form of Equation (1), where

$$\boldsymbol{U} = \begin{pmatrix} \rho \\ \rho v \end{pmatrix} \tag{2}$$

and

$$\boldsymbol{F}(\boldsymbol{U}) = \begin{pmatrix} \rho v \\ \rho v^2 + p(\rho) \end{pmatrix} \tag{3}$$

where ρ is the density of the gas, p its pressure and v its velocity. The fluid equation of state enters *via* the term $p(\rho)$, defined as

$$p(\rho) = k\rho^{\gamma}. \tag{4}$$

with $\gamma = C_p/C_v > 1$.

2.3. Some useful mathematical transformations

The model (1) can be rewritten in quasilinear form as

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{A}(\boldsymbol{U})\frac{\partial \boldsymbol{U}}{\partial x} = 0, \tag{5}$$

where the Jacobian $\boldsymbol{A}$ is equal to

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 \\ a^2 - v^2 & 2v \end{pmatrix} \tag{6}$$

where a is the speed of sound in the fluid. The eigenvalues of $\boldsymbol{A}$ are

$$\begin{aligned} \lambda_1 &= v - a, \\ \lambda_2 &= v + a, \end{aligned} \tag{7}$$

and the associated right eigenvectors

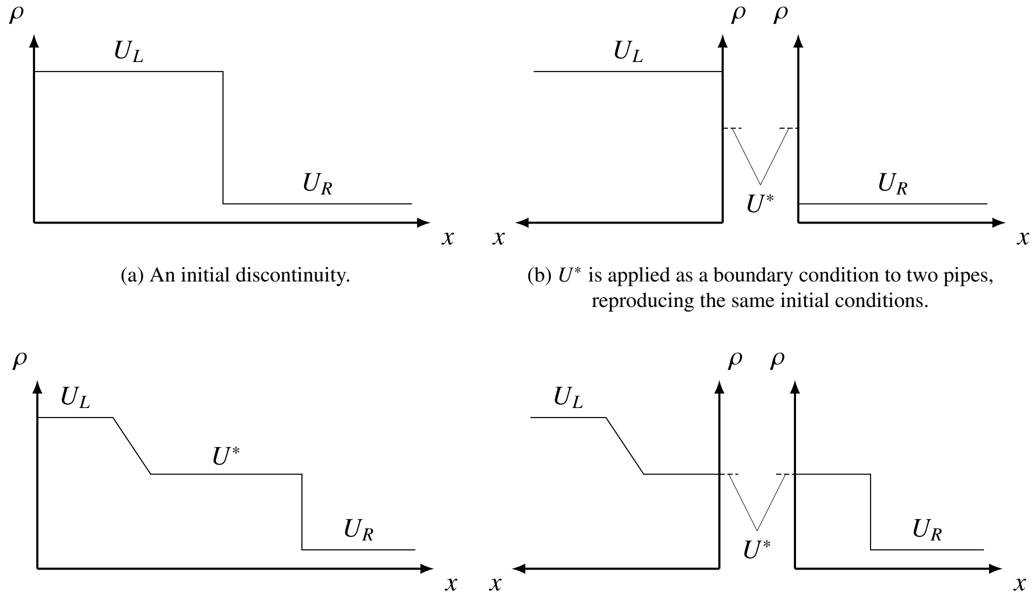
$$\boldsymbol{R}_1 = \begin{pmatrix} 1 \\ v - a \end{pmatrix}, \boldsymbol{R}_2 = \begin{pmatrix} 1 \\ v + a \end{pmatrix}. \tag{8}$$

3. Riemann problem and coupling of two pipes

The study of the Riemann problem gives useful insight in the structure of equations of the type of (1). A Riemann problem consists of two states separated by a discontinuity at time $t = 0$ s. It can be written as

$$\boldsymbol{U}(x, 0) = \begin{cases} \boldsymbol{U}_L & \text{if } x \leq 0, \\ \boldsymbol{U}_R & \text{if } x > 0. \end{cases} \tag{9}$$

When time starts to evolve, two waves in general will propagate from the initial discontinuity (cf. Figures 1a–1c). Between the two states $\boldsymbol{U}_L$ and $\boldsymbol{U}_R$, a third state appears, $\boldsymbol{U}^*$. Its particularity is that, at the location of the initial discontinuity, the solution will always remain equal to the $\boldsymbol{U}^*$-state. From the eigenstructure of the model (7)–(8), it is possible to derive the relations between the $\boldsymbol{U}^*$-state and each of the initial states.



(a) An initial discontinuity.

(b) U^* is applied as a boundary condition to two pipes, reproducing the same initial conditions.

(c) Two waves propagate in opposite directions, separated by the U^*-state.

(d) The same waves propagate. In their local references, they are all propagating with positive x-velocities.

Fig. 1: Comparison of, on the one hand, a pipe with an initial discontinuity, and on the other hand, two half-pipes, each containing one of the two initial states. By applying the U^*-state as boundary condition where the pipe is cut in half, an identical behaviour is recovered. The density corresponding to the state U is plotted.

3.1. Wave equations

The equations are symmetrical in the space variable x, that is when x is transformed into $-x$. Thus, the waves propagating to the left (with negative speeds) are symmetrical to the waves propagating to the right (with positive speeds). In the present work, we concentrate on the waves of positive speed since they will be needed in the following. There are two types of possible waves, depending on whether the density of the right-hand side is higher than the density of the left-hand side. We will from now on study waves which have the $\boldsymbol{U}^*$-state as left-hand side, and $\overline{\boldsymbol{U}}$ as right-hand side.

3.1.1. Rarefaction waves

Rarefaction waves are continuous pieces of curves, which happen when $0 < \rho^* \leq \bar{\rho}$. In such a case, the solution between the two states is given by

$$v^*\left(\rho^*; \bar{\rho}, \bar{v}\right)_{\mathrm{R2}} = \bar{v} + \frac{2\sqrt{\gamma k}}{\gamma - 1}\left(\rho^{*\,\frac{\gamma-1}{2}} - \bar{\rho}^{\,\frac{\gamma-1}{2}}\right) \tag{10}$$

which can be inverted in

$$\rho^*\left(v^*; \bar{\rho}, \bar{v}\right)_{\mathrm{R2}} = \left(\frac{\gamma - 1}{2\sqrt{\gamma k}}\left(v^* - \bar{v}\right) + \bar{\rho}^{\,\frac{\gamma-1}{2}}\right)^{\frac{2}{\gamma-1}} \tag{11}$$

3.1.2. Shocks

Shocks are discontinuities, separating the left and right-hand sides. They happen when $\rho^* > \bar{\rho}$, and the left and right-hand sides are related through

$$v^*\left(\rho^*; \bar{\rho}, \bar{v}\right)_{\mathrm{S2}} = \bar{v} + \sqrt{\frac{k\left(\rho^* - \bar{\rho}\right)\left(\rho^{*\gamma} - \bar{\rho}^{\gamma}\right)}{\bar{\rho}\rho^*}} \tag{12}$$

To invert it, the following expression has to be solved numerically for ρ^*

$$I(\rho^*) = \frac{(v^* - \bar{v})^2}{k} + \bar{\rho}^{\gamma-1} - \frac{\rho^{*\gamma}}{\bar{\rho}} + \rho^{*\gamma-1} - \frac{\bar{\rho}^{\gamma}}{\rho^*} = 0 \tag{13}$$

This is done using a Newton algorithm, the derivative of the expression with respect to ρ^* being

$$\frac{\partial I}{\partial \rho^*} = -\frac{\gamma \rho^{*\gamma-1}}{\bar{\rho}} + (\gamma - 1)\rho^{*\gamma-2} + \frac{\bar{\rho}^{\gamma}}{\rho^{*2}} \tag{14}$$

As starting point, we use the density evaluated at $\gamma = 1$

$$\frac{(v^* - \bar{v})^2}{k} + 2 - \frac{\rho^*}{\bar{\rho}} - \frac{\bar{\rho}}{\rho^*} = 0, \tag{15}$$

which gives

$$\rho^* = \bar{\rho}\left(C + \sqrt{C^2 - 1}\right) \tag{16}$$

where

$$C = \frac{(v^* - \bar{v})^2}{2k} + 1 \tag{17}$$

3.2. The $\boldsymbol{U}^*$-state as the coupling information

Now, we mentioned that the waves are symmetrical with respect to the space variable x. If the pipe is cut in half at the location of the initial discontinuity, and the left side of the x-axis is reversed, we have only waves of positive velocities propagating from the initial discontinuity, on both sides. In addition, if the pipes are completely separated, we need a boundary condition in place of the initial discontinuity. Using the $\boldsymbol{U}^*$-state, we can see that we keep the same wave relations between $\boldsymbol{U}^*$ and $\boldsymbol{U}_R$ on the one hand, and between $\boldsymbol{U}^*$ and $\boldsymbol{U}_L$ on the other hand, though in the latter case, the $x \leftrightarrow -x$ symmetry has been used. This is illustrated in Figures 1b–1d. Thus we have two independent pipes coupled through, and only through, the $\boldsymbol{U}^*$-state, behaving exactly as if it was one initial discontinuity in one pipe.

We are thus able to couple two pipes together with a flat junction. The objective of the present work was to couple more than two pipes, in a three-dimensional arrangement. The question is whether it is possible to find such a coupling state.

4. The generalised Riemann problem

In a network of pipes, each pipe section, denoted by k, will be described by the system (1). Junctions are the meeting point of two or more sections (See Figure 2). Following the idea of the previous section, where two pipes where coupled using the $\boldsymbol{U}^*$-state as boundary condition, we try in this section to find the proper boundary condition to apply to the pipe sections to describe the junction. To this end, we define the generalisation of the Riemann problem (9) at the junction of N pipe sections. Each pipe section is defined on a domain $[0, x)$ and is initialised with a uniform state. It can be written as

$$\text{For } k \in (1, \cdots, N), \begin{cases} \dfrac{\partial \boldsymbol{U}_k}{\partial t} + \dfrac{\partial \boldsymbol{F}(\boldsymbol{U}_k)}{\partial x} = 0, \\ \boldsymbol{U}_k(x, 0) = \begin{cases} \boldsymbol{U}_k^* & \text{if } x = 0, \\ \overline{\boldsymbol{U}}_k & \text{if } x > 0. \end{cases} \end{cases} \tag{18}$$

where the state $\boldsymbol{U}_k^*$ serves as a boundary condition at the junction for the k^{th} pipe

$$\boldsymbol{U}_k^*\left(\overline{\boldsymbol{U}}_1, \cdots, \overline{\boldsymbol{U}}_N\right) = \lim_{x \to 0^+} \boldsymbol{U}_k(x, t). \tag{19}$$

$\overline{\boldsymbol{U}}_k$ is the initial condition of pipe section k.

4.1. The $\boldsymbol{U}_k^*$-states as the coupling information

In the case with two pipe sections, a single $\boldsymbol{U}^*$-state was necessary to couple the two half pipes. Now, each pipe section k has its own boundary state $\boldsymbol{U}_k^*$, all being in general different from each other. However, each $\boldsymbol{U}_k^*$ is in general dependent on the initial conditions of all the N segments. This is where the coupling of the pipes happens. To summarise, solving the junction means to find the $\boldsymbol{U}_k^*$-states such that

- all the $\boldsymbol{U}_k^*$-states together fulfil a set of junction coupling conditions,
- and each $\boldsymbol{U}_k^*$-state in the k^{th} pipe section is related to the initial state $\overline{\boldsymbol{U}}_k$ by a wave of positive speed, considering the definition of the x-axis in Figure 2, either (11) or (13).

4.2. Coupling conditions

To fully determine the coupling boundary conditions $\boldsymbol{U}_k^*$, two coupling conditions need to be defined for the isentropic Euler equations (cf. [3, 7]). In the following, we will only discuss subsonic regimes. The first coupling condition expresses the conservation of mass at the junction, that is, all the mass from the in-flowing pipes leaves through the out-flowing pipes. This is expressed by

$$\sum_{k=1}^{N} A_k \rho_k^* v_k^* = 0, \tag{20}$$

where A_k is the cross-section of the k^{th} pipe.

The second coupling condition concerns the momentum. Momentum differs from mass in that it is a vector quantity. In a one-dimensional model, linear momentum is conserved in the same way as mass. Junctions, however, are three-dimensional objects, and linear momentum cannot be used. To handle the momentum coupling condition, we have to return to a scalar quantity, named $\mathcal{H}$. The condition is written as

$$\forall k \in \{1, \cdots, N\}, \quad \mathcal{H}\left(\rho_k^*, v_k^*\right) = \widetilde{\mathcal{H}}, \tag{21}$$

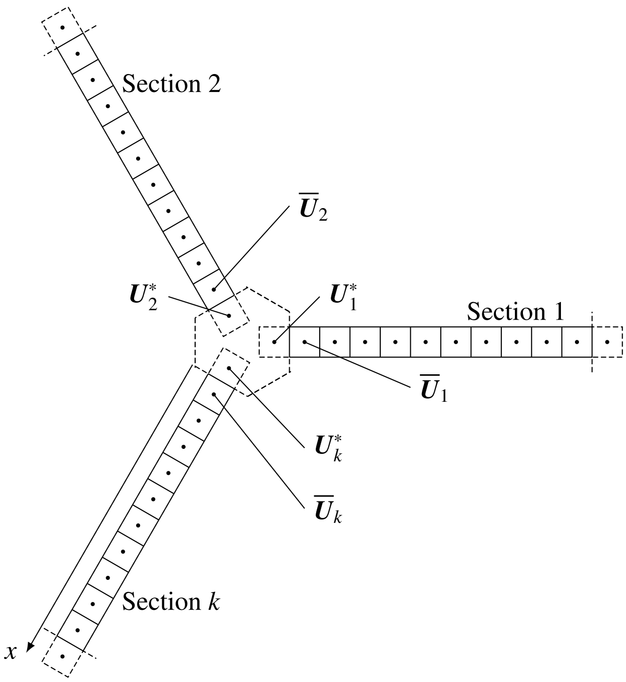


Fig. 2: Junction of k pipe sections. The junction, actually a point with no volume, contains the ghost cells that are used as boundary conditions for the pipe sections. The ghost cells are coupled together by the coupling condition. The x-axis always starts at the junction and point outwards for all pipe sections.

which means that there exists a scalar constant $\widetilde{\mathcal{H}}$ such that, for all pipe sections, the quantity $\mathcal{H}\left(\rho_k^*, v_k^*\right)$, function of $\boldsymbol{U}_k^*$, is equal to that constant. $\widetilde{\mathcal{H}}$ is called the coupling constant. Recall that $\boldsymbol{U}_k^*$ is the boundary state of pipe section k at the junction, and that it is related to the initial state $\overline{\boldsymbol{U}}_k$ through the relevant wave equation (11) or (13). It was shown for the isentropic Euler equations that the coupling constant $\widetilde{\mathcal{H}}$ is unique given a set of initial conditions in the pipe sections [5].

Now, the quantity $\mathcal{H}\left(\rho_k^*, v_k^*\right)$ has to be defined. It has been proposed [5] that all the pipes should be at the same pressure at the junction, which is expressed using the equation of state (4) as

$$\mathcal{H}\left(\rho_k^*, v_k^*\right) = p = k\rho_k^{*\gamma}. \tag{22}$$

Then, the $\boldsymbol{U}_k^*$-states of all the pipes in (18) share the same pressure. Another proposition was to set all $\boldsymbol{U}_k^*$-states to have equal momentum fluxes (the second component of $\boldsymbol{F}(\boldsymbol{U})$ in (3)), which results in

$$\mathcal{H}\left(\rho_k^*, v_k^*\right) = \rho_k^* v_k^{*2} + p = \rho_k^* v_k^{*2} + k\rho_k^{*\gamma}. \tag{23}$$

However, it was shown by Reigstad [5] that none of these options were satisfying the entropy condition at the junction for all the sets of initial conditions. A third choice was shown to fulfill the entropy condition for all subsonic flows [5], in which the coupling quantity is the Bernoulli invariant $h + 1/2v^2$, also known as the stagnation enthalpy. The coupling quantity can then be written

$$\mathcal{H}\left(\rho_k^*, v_k^*\right) = h_k^* + \frac{1}{2}v_k^{*2} = \frac{k\gamma}{\gamma-1}\rho_k^{*\gamma-1} + \frac{1}{2}v_k^{*2}. \tag{24}$$

The conservation of mass at the junction together with the Bernoulli invariant for the momentum coupling were proved to satisfy the entropy condition at a junction for any number of connected pipe sections with arbitrary cross-sectional areas, as long as the velocities are subsonic [5].

4.3. Algorithm to evaluate the value of the coupling quantity

The problem to solve is thus to find $\widetilde{\mathcal{H}}$ and the k different $\boldsymbol{U}_k^*$-states, which verify Equations (20) and (21), knowing that each $\boldsymbol{U}_k^*$-state has to be related to the initial conditions in the pipe section k by the wave relations

- of a rarefaction wave, if $0 < \rho_k^* \leq \bar{\rho}_k$ (Equation (11)),
- of a shock, if $\rho_k^* > \bar{\rho}_k$ (Equation (13)).

Since Equations (11) and (13) give relations between ρ_k^* and v_k^*, we are able to write the problem to be solved as

$$\sum_{k=1}^{N} A_k \rho_k^*(v_k^*) v_k^* = 0, \tag{25}$$

$$\forall k \in \{1, \cdots, N\}, \quad \mathcal{H}\left(v_k^*\right) = \widetilde{\mathcal{H}}. \tag{26}$$

where the v_k^* and $\widetilde{\mathcal{H}}$ are the unknowns.

In [8], Reigstad presented a numerical algorithm to solve a similar system for the isothermal Euler equations. We use here the same solution strategy as in [8], where the Mach number is replaced by the velocity. A Newton-Raphson algorithm is used to solve (26), using the fact that $d\mathcal{H}_k^*/dv_k^*$ is known from the derivation in [5].

5. Numerical results

Two test cases are run with the numerical algorithm to compare the different momentum coupling quantities (22), (23) and (24). The first one shows whether the entropy function is satisfied at a junction. The second one illustrates how the wrong coupling quantities lead to an intuitively wrong physical behaviour, while the stagnation-enthalpy-based coupling quantity gives a physical solution.

5.1. *Roe scheme*

The system is solved using a Roe scheme (See for example [6]). The Roe-averaged Jacobian is

$$\tilde{\boldsymbol{A}} = \begin{pmatrix} 0 & 1 \\ \widetilde{a^2} - \tilde{v}^2 & 2\tilde{v} \end{pmatrix} \tag{27}$$

where the Roe-averaged variables are

$$\tilde{v} = \frac{\tilde{q}_2}{\tilde{q}_1} = \frac{\sqrt{\rho_L} v_L + \sqrt{\rho_R} v_R}{\sqrt{\rho_L} + \sqrt{\rho_R}} \tag{28}$$

and

$$\widetilde{a^2} = \frac{p_R - p_L}{\rho_R - \rho_L} \tag{29}$$

5.2. *An isolated junction*

The first test case consists in an isolated junction, from which pipes are emerging and are extending to infinity, using extrapolation boundary conditions (Figure 3a). With this case, we evaluate whether the entropy condition [5]

$$\sigma_{\mathrm{J}} = \sum_{k=1}^{N} A_k \rho_k^* v_k^* \left(h_k^* + \frac{1}{2} v_k^{*2} \right) \leq 0 \tag{30}$$

is satisfied or not. Remark that the model being isentropic, the entropy condition is actually a condition on energy. It can be seen as the rate of energy production. The initial conditions are summarised in Table 1. The velocity v_3 is either 0 m/s or 50 m/s. The results are shown in Table 2. We can see that for the equal pressure and momentum flux coupling conditions, the entropy condition is violated in either one or the other case. For the stagnation enthalpy coupling condition, the entropy condition is very close to zero, which is why it is written as ≈ 0 J/s in the table. The entropy condition is considered satisfied here, the residual being caused by loss of numerical accuracy in the solution of the coupling-constant problem.

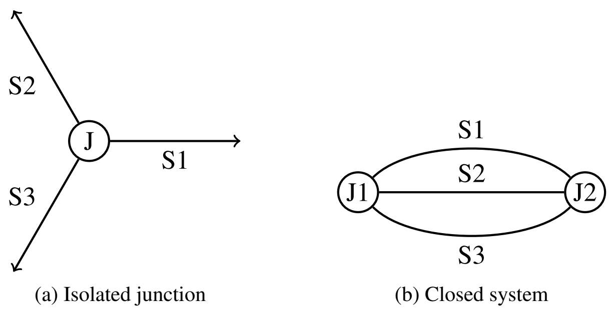


Fig. 3: Illustration of the two test cases.

Table 1: Initial conditions for the isolated junction test case

	Pressure (bar)	Velocity (m/s)
Section 1	1	0
Section 2	1.5	0
Section 3	1.4	v_3

Table 2: Value of the entropy condition at steady state

	Equal pressure	Equal momentum flux	Stagnation enthalpy
$v_3 = 0\,\mathrm{m/s}$	$1.1 \times 10^5\,\mathrm{J/s}$	$-8.2 \times 10^4\,\mathrm{J/s}$	$\approx 0\,\mathrm{J/s}$
$v_3 = 50\,\mathrm{m/s}$	$-6.6 \times 10^4\,\mathrm{J/s}$	$9.8 \times 10^4\,\mathrm{J/s}$	$\approx 0\,\mathrm{J/s}$

5.3. A closed system

To illustrate the concept of energy production at the junction when the wrong coupling condition is used, we study a closed system of three pipes joined at both their ends (see Figure 3b). The system being closed, the sum of the energy over the whole system should not increase. It may however decrease in the presence of shock, since in an isentropic model, they will dissipate energy. In a non-isentropic model, shocks would generate entropy instead. The initial conditions in the pipe sections are summarised in Table 3. Figure 4 shows the evolution of the energy in the system, relative to the initial energy. We can see again that the equal pressure and momentum flux coupling conditions show unphysical behaviour by producing energy in the system. With the stagnation enthalpy coupling condition, the amount of energy in the system is going uniformly down, as expected since shocks are present.

Table 3: Initial conditions for the closed system test case

	Pressure (bar)	Velocity (m/s)
Section 1	1	0
Section 2	1.5	0
Section 3	1.4	0

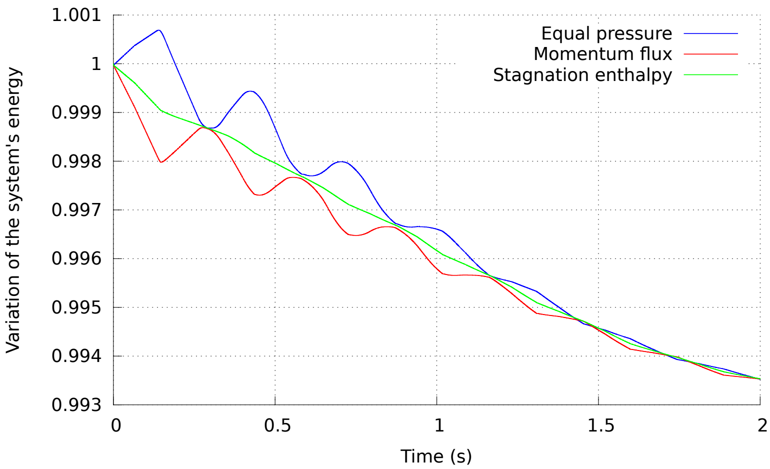


Fig. 4: Evolution of the total energy content of the system

6. Conclusions

The numerical algorithm for the junction flow for the isentropic Euler equations has been implemented. In addition to the conservation of mass at the junction, a coupling condition has to be defined regarding the momentum. Momentum being a vector quantity, a scalar quantity has to be defined on which the condition can be applied. Earlier, three coupling quantities have been proposed, and the one involving the stagnation enthalpy was proved to be the only one yielding physically sound solutions in general. In the present paper, we have illustrated why the two other, equal pressure at the junction and equal momentum flux, were not physically sound. They violate the entropy condition.

The physical insights gained may now be used as a starting point to derive coupling conditions for other models, including multiphase flow models. Two points will need to be addressed. The first one is to handle an energy transport equation, as the one appearing in the Euler equations. Since it is a scalar equation, the coupling should be similar to the one for the mass equation. The second point to address is when two momentum equations are present in multiphase flow models, one for each phase. In particular, phase change through the junction due to the change of pressure should be handled, in a similar way as phase change should be handled through choke valves.

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