Modelling the Plastic Anisotropy of Aluminium Alloy 3103 Sheets by Polycrystal Plasticity Models

# Abstract

The plastic anisotropy of AA3103 sheets in the cold rolled condition (H18 temper) and in the fully annealed condition (O temper) was studied experimentally and numerically in this work. The microstructure and texture of the two materials were characterized and the anisotropic plastic behaviour was measured by in-plane uniaxial tension tests along every 15° from the rolling direction (RD) to the transverse direction (TD) of the sheet. Five polycrystal plasticity models, namely the full-constraint Taylor model, the Alamel model, the Alamel Type III model, the visco-plastic self-consistent crystal plasticity model (VPSC) and the crystal plasticity finite element method (CPFEM), were employed to predict the plastic anisotropy in the plane of the sheet. Experimentally observed grain shapes have been taken into consideration. In addition, a multi-level modelling method was employed where the advanced yield function Yld2004-18p is fitted to stress points provided by CPFEM simulations along 89 strain-paths, and the plastic anisotropy is then produced by the yield function. Based on comparisons between the experimental and the predicated results, the multi-level fitting method is considered as the most accurate way of describing the plastic anisotropy. The Alamel Type III and Alamel models are also recommended as accurate and time-efficient models for predicting the plastic anisotropy of the AA3103 sheets in H18 and O tempers.

# Introduction

Due to the thermo-mechanical history, sheet metals commonly feature a crystallographic orientation distribution which is termed crystallographic texture – or texture for short. The significance of texture lies in the plastic anisotropy, i.e. direction-dependent strengths and flow patterns, which are mainly attributed to the texture. In a forming process, the plastic anisotropy of the material greatly influences the final shapes and dimensions of the products. Hence, proper description of the plastic anisotropy is of importance for the design and optimization of forming processes. Crystal plasticity (CP) models and anisotropic yield functions are two principal approaches to describe the plasticity and its anisotropy of engineering metallic alloys [[1](#_ENREF_1" \o "Grytten, 2008 #2)]. Due to higher computational efficiency than the CP models, anisotropic yield functions are widely applied in finite element method (FEM) simulations of forming processes.

In the last decade, many advanced yield functions have been proposed that are capable of accurately describing the plastic anisotropy of aluminium alloys [[2-5](#_ENREF_2" \o "Barlat, 2005 #18)]. These yield functions are generally calibrated by mechanical tests, e.g. uniaxial tensile tests and biaxial tension/compression tests. However, most of the stress space is left unexplored when fitting the parameters of the yield function due to severe experimental limitations, and biaxial tension/compression testing is not universally accessible. To overcome these limitations, many researchers have employed CP models to facilitate the fitting of anisotropic yield functions, i.e. a hierarchical multi-level modelling. This can be done by conducting all or part of the required experimental tests virtually using CP models [[2](#_ENREF_2" \o "Barlat, 2005 #18), [6](#_ENREF_6" \o "Inal, 2010 #494), [7](#_ENREF_7" \o "Gawad, 2013 #519)]. Alternatively, the yield functions can also be calibrated against the stress points at yielding provided by CP models in the stress space [[1](#_ENREF_1" \o "Grytten, 2008 #2), [8-11](#_ENREF_8" \o "Saai, 2013 #65)]. As pointed out by Zhang et al. [[12](#_ENREF_12" \o "Zhang, 2014 #524)], the success of the hierarchical multi-level modelling critically depends on the accuracy of the lower-scale CP models in representing the plastic anisotropy. Hence, quantitative evaluations of the performance of CP models for predicting plastic anisotropy are of great value for the research on and applications of the hierarchical multi-level modelling.

Over the past few decades, materials scientists have become more aware of the important role played by the distribution and connectivity of different grain boundary types in governing various mechanical and functional properties of materials [[13](#_ENREF_13" \o "Patala, 2012 #3)]. It has been shown that the grain boundary can be of importance to texture and microstructure evolution during deformation [[14](#_ENREF_14" \o "Chang, 2010 #525), [15](#_ENREF_15" \o "Xie, 2013 #500)]. The plastic anisotropy predicted by CP models is also influenced by the grain morphology [[15](#_ENREF_15" \o "Xie, 2013 #500)]. Hence, the grain morphology should be taken into consideration when conducting CP calculations, and thus make the predictions more physically reasonable.

In this work, the plastic anisotropy of AA3103 sheets, in a cold-rolling state (H18 temper, denoted AA3103-H18 in the following) and in a fully annealed state (denoted AA3103-O in the following), were investigated by experiments and by various CP models. Uniaxial tensile tests at every 15° from the rolling direction (RD) to the transverse direction (TD) were carried out to get the directional normalized yield stresses and Lankford coefficients (*r*-values). Five different crystal plasticity models, namely the full-constraint (FC) Taylor model [[16-18](#_ENREF_16" \o "Taylor, 1938 #102)], the Alamel model [[19](#_ENREF_19" \o "Van Houtte, 2005 #458)] and the variant with Type III relaxation (Alamel Type III) [[20](#_ENREF_20" \o "Mánik, 2013 #449)], the visco-plastic self-consistent (VPSC) model [[21](#_ENREF_21" \o "Lebensohn, 1993 #447)] and the crystal plasticity finite element method (CPFEM) [[22](#_ENREF_22" \o "Zhang, 2014 #523)], were employed to simulate the uniaxial tension tests. Besides the texture, the grain shape of the material has also been taken into consideration in the CP models that allows relaxed constraints. Due to the non-equiaxed grain structures of the AA3103 sheets (especially AA3103-H18), the in-plane uniaxial tensile tests along directions other than the RD and TD are difficult to perform by the CPFEM while keeping a realistic grain structure in the representative volume element (RVE). Instead, a hierarchy multi-level modelling method is used, i.e. fitting the advanced yield function Yld2004-18p [[2](#_ENREF_2" \o "Barlat, 2005 #18)] by CPFEM stress points for different in-plane loadings. The numerical results are compared with the experiments to evaluate the performance of different CP models.

In Section 2 of this paper, the procedures for microstructure characterization and uniaxial tensile testing are described along with the experimental results. The five CP models and the virtual uniaxial tensile tests are described in Section 3. The methods adopted for considering the grain morphology in the different CP models are described in the same section. Section 4 deals with the hierarchical multi-level modelling of plastic anisotropy, including the Yld2004-18 yield function and CPFEM calculations. All the numerical results are presented and discussed in Section 5, while the main conclusions of the study are provided in Section 6.

# Experimental methods and results

The AA3103-H18 and AA3103-O sheets were provided by the R&D Center of Hydro Aluminium in Bonn, Germany. Starting from a hot rolled strip of a 4.1mm thickness, the AA3103-H18 sheets were produced by cold rolling to a final thickness of 1.2 mm, i.e. with a 71% thickness reduction. The AA3103-O was then prepared through a simulated batch anneal of the AA3103-H18 sheets with a holding time of 2 hours at a peak metal temperature of 350 ºC. The chemical composition is shown in Table 1.

The microstructure at both RD- ND and TD-ND sections (ND being the normal direction) of the two variants of the AA3103 sheets were measured by electron back scatter diffraction (EBSD) in a Zeiss Ultra/Supra 55 field emission scanning electron microscope (FESEM) equipped with TSL software. For the AA3103-O material, the scanned area for RD-ND and TD-ND sections were 1.0×1.6 mm2 and 0.8×1.2 mm2, respectively, with a step size of 2 µm. For the AA3103-H18 material, the scanned area for both sections was 0.3×0.3 mm2 with a step size of 0.5 µm. The orientation distribution functions (ODF) were then generated using all scanning points by the series expansion method with  and . In order to get EBSD diffraction patterns with a higher quality, the EBSD samples of the AA3103-H18 material were soft-annealed at 250 ºC for 10 minutes in a furnace before the measurement. Textures of the AA3103-H18 samples before and after the rapid soft-anneal at 250 ºC were also measured by X-ray diffractions, and confirmed that there was little influence introduced by that soft-annealed treatment on the texture. The constitutive particles were characterized by back-scattered electrons (BSE) in the FESEM while the area fraction and size of constitutive particles were analyzed by quantitative metallography.

Uniaxial tensile tests at every 15° from the RD to the TD (a total of seven directions) were conducted to obtain the anisotropy in strength and plastic flow of the two AA3103 materials. The tensile direction with respect to RD is represented with the angle , i.e.,  for the RD and  for the TD. The uniaxial tensile samples had a uniform section with a gauge length of 50 mm and a width of 12.5 mm. The experiments were carried out with a load speed of 2 mm per minute until fracture. At least two samples were tested in one single direction. Extensometers were attached in the longitudinal and in the transverse direction to record the respective deformations. The *r*-value was then determined from the recorded deformations employing the plastic incompressibility condition. Elastic deformations were calculated with longitudinal true stress, Young’s modulus and Poisson’s ratio and were deducted from the total deformations when calculating the *r*-value [[23](#_ENREF_23" \o "An, 2013 #522)]. The *r*-values of the AA3103-H18 material were calculated in the uniform plastic deformation region with logarithmic strain between ~0.5% and 2.0%. For the AA3103-O sheet, the *r*-values were calculated for logarithmic strains between 3% and 15%. The yield stresses were determined at 0.2% plastic strain.

Examples of the typical microstructure of the AA3103-H18 and AA3103-O sheets are shown in Fig. 1. The RD-ND section of AA3103-H18 shows a typical cold rolling structure, i.e. fibrous grain shapes, whereas the pancake grain shape can be found at the TD-ND section. Due to the large deformation, the grain size and aspect ratio along three orthotropic directions of the AA3103-H18 sheet are difficult to measure accurately. Instead, they can be estimated according to the thickness reduction and the grain morphology before the cold rolling, i.e. a partially recrystallized grain structure after the hot-rolling. The AA3103-O sheet shows a fully recrystallized grain structure where smaller grain size along ND can be observed. Grain sizes along the RD, TD and ND directions are measured using the line intercept method. The aspect ratios along three orthotropic directions of the sheet were then determined based on the measurements. The grain sizes and grain aspect ratios of the two AA3103 sheets are compiled in Table 2.

The BSE micrographs in Fig. 2 further reveal that the AA3103 alloys contain a large fraction of micron-sized second-phase particles. The size and area fraction was analyzed by quantitative metallography and are summarized in Table 2.

Crystallographic textures of the two AA3103 sheets are illustrated in Fig. 3 by means of ODF. The β-fiber which consists of the copper, S and brass components is clearly shown in the ODF of the AA3103-H18 material. For the AA3103-O material, the main components are the cube and P orientations together with the Goss orientation at a lower intensity. The AA3103 alloy in both tempers exhibit weak textures, which is attributed to the high fraction of second-phase particles [[24](#_ENREF_24" \o "Engler, 2005 #527)].

In Fig. 4, experimental stress-strain curves of the two AA3103 sheets are shown. Due to the pre-cold-rolling, the A3103-H18 sheet has short uniform elongations before necking, only ~1.5%, while fracture occurred at ~3% elongation. In contrast, the AA3103-O sheets can withstand tension up to 50% elongation along the RD and around 30% along the TD before fracture. The yield stress along the RD for the AA3013-H18 and the AA3103-O sheets are 201.6 MPa and 39.8 MPa, respectively. The yield stress along the RD was taken as a reference to normalize the yield stress along the other directions. The anisotropy in strength and plastic flow is plotted in Fig. 5. The AA3103-H18 sheet shows weak strength anisotropy. The maximum deviation from the reference value occurs at  and is only about 3%. The strength anisotropy of the AA3103-O sheet is different and significantly stronger, and the maximum deviation from the reference value occurs at the TD () and is about 8%. The directional variation of the *r*-value of AA3103-H18 sheet is strong with a minimum at the RD of about 0.5 and a maximum at  of about 1.5. The AA3103-O sheet exhibits less and somewhat different variation of the *r*-value. The maximum value of about 0.9 occurs at , while the minimum value, about 0.4, is found in the TD. Within the investigated strain ranges, only minor changes in the anisotropy in strength and plastic flow of the AA3103 materials were found.

Table 1 Chemical composition of the AA3103 alloy (in wt%).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Si | Fe | Cu | Mn | Mg | Cr | Zn | Ti | Al |
| 0.063 | 0.53 | 0.0027 | 1.03 | 0.01 | 0.0006 | 0.0054 | 0.006 | Rest |

E:\Doucuments\10.2013\AA3103- work for Paper III\7-Paper writing\Figures\FIG 1 - EBSD MAPS-2.tif

Fig. 1 EBSD microstructure of (a, b) the AA3103-H18 material and of (c, d) the AA3103-O material.

Table 2 Measured grain size, aspect ratios, particle size (mean diameter of an equivalent circle) and area fraction of the AA3103 sheets.

|  |  |  |
| --- | --- | --- |
|  | AA3103-H18 | AA3103-O |
| Grain size *Φ* | — | *Φ*RD: 38.4 ± 4.3 µm |
| — | *Φ*TD: 30.8 ± 3.1 µm |
| *Φ*ND: ~5 µm | *Φ*ND: 15.0 ± 1.7 µm |
| Aspect ratio (*Φ*RD: *Φ*TD: *Φ*ND) | 5 : 1 : 0.2 | 2.5 : 2 : 1 |
| Particle size | 2.6 ± 0.14 µm | 2.6 ± 0.15 µm |
| Area fraction | 3.5 ± 0.3 % | 3.2 ± 0.29 % |

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Fig. 2 BSE images of the constitutive particles of the AA3103 sheets (a) in the H18 temper and (b) in the O temper.



(a) (b)

Fig. 3 ODFs of (a) the AA3103-H18 and (b) the AA3103-O sheets.



(a) (b)

Fig. 4 Stress-strain curves from uniaxial tensile tests along seven in-plane directions of the AA3103 sheets: (a) temper H18 and (b) temper O.

(a) (b)

Fig. 5 Plastic anisotropy of the AA3103 sheets based on tensile tests at seven in-plane directions with respect to the RD: (a) normalized yield stress and (b) *r*-values.

# Crystal plasticity modelling of plastic anisotropy

The five CP models employed in this work will be briefly introduced in Section 3.1, while the virtual uniaxial tension tests performed with the Taylor-type and VPSC models are described in Section 3.2. Section 3.3 deals with the CPFEM simulations.

# Crystal plasticity models

The FC-Taylor model assumes that each grain experiences the same deformation as the aggregate, and the deformation is accommodated by at least five slip systems according to the principle of maximum plastic work or the complementary minimum principle, which follows from the yield criteria of the slip systems. Based on the active slip systems, the stress state is found in one of the 56 vertices of the yield surface of each FCC crystal. The stress state of the aggregate is defined as the volume average over all grains.

In the Alamel-type models, a pair is assembled by two grains and a common grain boundary. Local interactions between the two grains in a pair are considered by means of relaxations of constraints on the shear components of the prescribed deformation. Stress tensors in each crystal of one pair are calculated from the yield criteria of the slip systems and the equilibrium conditions of the shear stress components [[12](#_ENREF_12" \o "Zhang, 2014 #524)]. The Taylor-type models studied here, i.e. the FC-Taylor and Alamel-type models, are rate-independent.

The VPSC model regards each grain of the polycrystal as an ellipsoidal inclusion embedded in a homogeneous effective medium whose mechanical response corresponds to the volumetric average of all grains. Grains in the CPFEM are represented by single or multiple elements and both stress equilibrium and strain compatibility are fulfilled.

The rate-dependent VPSC and CPFEM models assume that all slip systems are active and that the slip activity is determined via a power-law type equation



where  is a reference shearing rate,  is the instantaneous strain-rate sensitivity and  represents the slip resistance which evolves during the plastic deformation according to the hardening law.  is the resolved shear stress and is calculated from the current stress-states and crystallographic orientation. The plastic power per unit volume is defined as



The hardening law employed by the CPFEM in this work assumes that the critical resolved shear stress , which is initially equal to , evolves through



where  is the instantaneous strain hardening matrix;  and  are indices referring to slip systems; and  is the number of slip systems. In this work, is described phenomenologically by a Voce-type law:



where  is the initial hardening rate, while  and  describe the asymptotic hardening. The accumulated plastic shear strain  is defined by



The parameter  represents latent hardening, while  is the Kronecker delta function.

# Set-up of Taylor-type and VPSC calculations

Grain orientations are the necessary input for all of the CP models motioned above. A total of 2500 orientations were randomly selected from the EBSD scanning points, i.e. an aggregate of 2500 grains for both the AA3103-H18 and AA3103-O materials. The quality of the selected orientations in representing the measured texture is evaluated by the normalized difference texture index [[19](#_ENREF_19" \o "Van Houtte, 2005 #458)], defined as



where  is the ODF. Values of  for the grain selections of AA3103-H18 and AA3103-O are 0.39% and 1.13%, respectively, which indicates a rather good representation of the experimental ODF for both materials.

For the AA3103-H18 sheet, these 2500 orientations were randomly assembled into 3750 Alamel pairs in a manner that each grain was reused three times. These pairs were then considered by the Alamel and the Alamel Type III models. The cold-rolled AA3103-H18 sheet shows extensively elongated grain shapes, as illustrated in Fig. 1 and by the grain aspect ratio compiled in Table 2. To reflect such rolling grain structure in the Alamel-type models, grain boundaries in all Alamel pairs were assumed parallel with the RD-TD plane, which reduced the Alamel-type models into Lamel-type models [[25](#_ENREF_25" \o "Van Houtte, 1999 #456)], i.e. the Lamel model and the Lamel Type III model.

For the AA3103-O material, the same number of Alamel pairs was considered as its H18 counterpart. The grain morphology in terms of the grain aspect ratio was taken into consideration following the next two steps. First, an equiaxed grain structure was defined using a microstructure file consisting of 1875 random orientations [[19](#_ENREF_19" \o "Van Houtte, 2005 #458)]. Then, a gradient tensor  of which the three diagonal components take values of the grain aspect ratio of the AA3103-O material shown in Table 2 was applied to distort the plane normal of the equiaxed grain boundaries. By such an hypothetical deformation, a new grain boundary coordinate is obtained and the volume fraction can be estimated [[15](#_ENREF_15" \o "Xie, 2013 #500)]. The updated grain boundaries were then randomly assigned to these 3750 Alamel pairs.

The VPSC code reads the grain aspect ratio to define the initial lengths of the three ellipsoid axes. The aspect ratios shown in Table 2 for AA3103-H18 and AA3103-O were then input into the VPSC code for their respective calculations. The instantaneous strain-rate sensitivity was set as 0.01 to represent the low strain-rate sensitivity of the materials under study. Several formulations exist for the interaction equation that linearly relates stress and strain rates in the grain with the overall stress and strain rates of the effective medium. The intermediate approximation with one adjustable parameter  was used [[26](#_ENREF_26" \o "Carlos, 1999 #528)]. The approximation approach gives a response of the polycrystal which lies in-between the stiff secant and the compliant tangent approaches.

For these Taylor-type models and the VPSC model, virtual in-plane uniaxial tensile tests were carried out using the initial texture, along every 3° from the RD to the TD. The global strain-rate components are applied and iteratively adjusted until the average stress of the aggregate is uniaxial along the tensile axis. Only the strain-rate components are iterated for the FC-Taylor and Alamel-type models when determining *r*-values [[27](#_ENREF_27" \o "Bunge, 1970 #10), [28](#_ENREF_28" \o "Li, 2005 #492)], whereas both strain-rate and stress components enter into the iteration procedure in the VPSC model.

# Direct CPFEM simulations of tensile tests

For the CPFEM simulation of the AA3103-H18 material, Voronoi tessellation [[29](#_ENREF_29" \o "Quey, 2011 #106)] was first utilized to generate 2500 grains of random structure in a cuboid of size 0.8 mm, 4 mm and 4.8 mm along the RD, TD and ND, respectively. About 26 grains were resolved along the ND while only about 4 grains along the RD. This equiaxed tessellation was then scaled according to the grain aspect ratio shown in Table 2 to obtain a final RVE of 4.0×4.0×0.96 mm3. The RVE was finally meshed with 89×89×21 = 166341 cubic solid elements with one integration point, i.e., under-integrated elements with hourglass control. In average, each grain is represented by 67 elements and the maximum lengths along the RD, TD and ND in most grains follows the experimental grain aspect ratio, see Fig. 6 (a).

The RVE of AA3103-O material was made in a similar manner, but the final RVE size is 2×2×1 mm3. The RVE was meshed into 131072 elements. With such a fine mesh, each grain in the RVE is represented by about 52 elements in average and holds the general experimental grain shape, see Fig. 6 (b).

Particles were not represented in the CPFEM mesh. Periodic boundary conditions were applied to the nodes located on the faces of the RVE in order to ensure periodicity in displacements and minimize constraint effects [[8](#_ENREF_8" \o "Saai, 2013 #65)].

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(a) (b)

Fig. 6 The RVE used in the CPFEM simulations for (a) the AA3103-H18 and (b) the AA3103-O sheets; the *x*, *y* and *z* axes coincide with the RD, TD and ND, respectively.

The strain-rate sensitivity takes the same value as used in the VPSC calculations, i.e. *m* = 0.01; the latent hardening parameter *q* is set to 1.4 as often used for FCC metals [[30](#_ENREF_30" \o "Kalidindi, 1992 #100), [31](#_ENREF_31" \o "Dumoulin, 2012 #285)]. The elastic constants *c*11,*c*11 and*c*44 were set to 106 GPa, 60.4 GPa and 28.2 GPa, respectively.The calibration of other material parameters in Eqns. and were carried out with the design optimization tool LS-OPT [[32](#_ENREF_32" \o "Stander, 2012 #501)]. The calibration was made against the experimental uniaxial tensile stress-strain curves along the RD. Since the initial plastic anisotropy is the main concern in this work, the calibration was made in a relatively small deformation range, i.e. up to 2.25% elongation for AA3103-H18 and up to 9.5% elongation for AA3103-O. The calibrated values of the coefficients are compiled in Table 3.

Uniaxial tension tests were simulated for both materials by CPFEM at the seven experimentally investigated directions. Tension along the RD was simulated by stretching along the RD with a constant speed. The final elongations for the AA3103-H18 and the AA3103-O materials were 2.25% and 9.5%, respectively. Yield stresses were determined at a constant plastic work per unit volume, namely 0.36 MPa for AA3103-H18 and 0.07 MPa for AA3103-O, corresponding to 0.2% plastic strain along the RD for the materials. The *r*-value was determined from 0.5% plastic strain to the final deformation for AA3103-H18 and from 3% plastic strain to the final deformation for AA3103-O. For uniaxial tension at other directions than RD and TD, a technically convenient ‘texture rotation’ method [[6](#_ENREF_6" \o "Inal, 2010 #494), [31](#_ENREF_31" \o "Dumoulin, 2012 #285)] was applied, i.e. fixing the RVE and boundary conditions used for uniaxial tension along the RD while the texture was rotated through decreasing the first Euler angle by the tensile angle  . Uniaxial tension along the TD was simulated by stretching the RVE along this direction.

Table 3 Model parameters used in the simulations.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| AA3103-H18 | 0.01 | 1.4 | 0.001 | 66.52 | 3000 | 9.49 | 10.98 |
| AA3103-O | 0.01 | 1.4 | 0.0021 | 12.02 | 467.64 | 15.59 | 27.76 |

# Multi-level modelling by Yld2004-18p and CPFEM

A two-scale modelling scheme was used here to describe the plastic anisotropy. The lower scale (meso-scale) model is the grain aggregate consisting of 2500 grains. Plane stress states are assumed, i.e., all stress components in the ND are assumed to be zero. Stress points at yielding are provided by CPFEM for a number of prescribed in-plane deformations. The macroscopic yield surface is described by the advanced yield function Yld2004-18p which is fitted to these stress points at yielding. The generation of the plane stress states at yielding by CPFEM follows the method proposed by Saai et al [[8](#_ENREF_8" \o "Saai, 2013 #65)]. For the sake of completeness, the method is briefly described in the following.

The RVE has four master nodes, shown by full black circles in Fig. 7 and numbered 0, 1, 2 and 3. The axes *x*, *y* and *z* coincide with the RD, TD and ND of the rolled sheet, respectively. Master node 0 is always fixed. A constant velocity  is applied to the master node 1 along the RD, while master node 2 is given a constant velocity  along the TD for normal loading and/or a constant velocity  in the RD for shear loading. Master node 3 is free to move along the ND to adapt the incompressibility of the deformation.

The stress states on the yield surface are generated by controlling the three velocities of master nodes 1 and 2. Three sets of deformation are prescribed, namely







where  is a constant reference velocity, the variable  varies between -1 and 1 with increment , and the variable  varies between 0 and 3 with increment . The deformation according to Eq. corresponds to pure in-plane shear. For each deformation condition, the CPFEM simulation will be made until the volume-weighted average of  among all integration points, denoted , reaches a critical value . The Cauchy stress tensor of the RVE is defined as a volume-weighted average of the stress tensors of all integration points. Since all deformations are prescribed in the sheet plane, the stress components ,  and  are negligibly small compared to the three in-plane stress components, i.e., the averaged stress tensor  corresponds to a state of plane stress.



***x***

2



0

***y***

***z***

1

3

Fig. 7 Boundary conditions applied to the RVE; grey marks at nodes show the constraints of translation motions along axes.

In this study, the increments  and  were set to 0.2 and 1.0, respectively. To obtain stress states near initial yielding of each material, the parameter  was taken as the plastic work per unit volume at 0.2% plastic strain calculated with the experimental stress-strain curves for tensions along the RD. The value of  was found to be 0.36 MPa for AA3103-H18 and 0.07 MPa for AA3103-O. With the described settings, a total of 89 stress points at yielding were obtained for each material. The stress component  obtained at  for uniaxial tension along the RD was considered as the reference yield stress of the material.

The analytical yield function, denoted Yld2004-18p, was proposed by Barlat et al [[2](#_ENREF_2" \o "Barlat, 2005 #18)] as



where  is the equivalent stress;  is the exponent of the yield function;  and , , are the principal values of the tensors  and , where  is the deviatoric stress tensor; and  are fourth-order tensor containing all the 18 parameters of the yield function. The exponent is usually set to 8 for FCC materials. The yield function is then fitted to the stress points at yielding provided by the CPFEM. Among the 18 parameters of Yld2004-18p, there are four related to the through-thickness stress components. Since only plane stress states are considered here, these four parameters were set to unity, which is the value for an isotropic material. The other 14 parameters were identified by fitting to the 89 stress points at yielding computed by CPFEM. It is noted that these 89 stress points were normalized by the yield stress along the RD before entering into the fitting procedure.

More details about the yield function and associated parameter identification procedures can be found in the literature [[1](#_ENREF_1" \o "Grytten, 2008 #2), [2](#_ENREF_2" \o "Barlat, 2005 #18), [12](#_ENREF_12" \o "Zhang, 2014 #524), [33](#_ENREF_33" \o "Yoon, 2006 #502)].

# Numerical results and discussion

Stress-strain curves for uniaxial tension in the RD predicted by CPFEM with the parameters compiled in Table 3 are compared to the experimental data in Fig. 8. For both materials, the predicted curves show perfect match with the experiments, thus validating the parameter calibration.

The  yield loci derived from the fitted Yld2004-18p for both materials are plotted in Fig. 9, where also the CPFEM stress points at yielding in biaxial tension, i.e., loading with *λ* = 0 in Eq. and , are shown. Fig. 9 shows good agreement between the yield loci and the CPFEM stress points. Since all 89 stress points were equally weighted in the fitting procedure, the same accuracy of the Yld2004-18p yield surface in representing the CPFEM calculations is expected for other plane stress states.

(a) (b)



Fig. 8 Comparisons between the experimental stress-strain curves and the CPFEM predictions with calibrated material coefficients.

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(a) (b)

Fig. 9 Yld2004-18p yield loci for (a) the AA3103-H18 and (b) the AA33103-O material, including the stress points at yielding in biaxial tension from the CPFEM calculations.

The predicted anisotropy of normalized yield stress and *r*-value by all the CP models described above is shown in Fig. 10 and Fig. 11 for AA3103-H18 and AA3103-O, respectively. The average of duplicate experimental tests is added these figures to evaluate the quality of predictions.

As shown in Fig. 10 for AA3103-H18, all CP models and the fitted Yld2004-18p yield surface predict weak anisotropy in strength but strong anisotropy in plastic flow. The trends of the experimental data are well captured by all CP models and the yield surface, while there are significant quantitative differences. The FC-Taylor model gives the most accurate prediction of the anisotropy in the normalized yield stress, and gives results within 2% of the experimental values. The other CP models consistently predict lower normalized yield stresses. Note that for all CP models the normalized yield stress equals unity in the RD owing to the normalization. The Lamel and Lamel-type III model give similar predictions but the latter model performs slightly better. The predictions obtained with the VPSC model, CPFEM and the Yld2004-18p yield function are similar, but less accurate. Concerning the *r*-value at the RD, the Lamel-type models, CPFEM and the Yld2004-18p yield function give reasonable predictions, while the FC-Taylor and VPSC models are 25% higher and 50% lower than the experimental data, respectively. At , the FC-Taylor and Lamel Type III models give about 30% higher *r*-value than the experiments, whereas the predictions with the Lamel and VPSC models, CPFEM and the Yld2004-18p yield function are close to the experiments. At the TD, only the prediction with the VPSC model agrees well with the experiment, while the other predictions are lower than the experimental results. In summary, the CPFEM-fitted Yld2004-18p yield surface gives better description of the variation of the *r*-value than the CP models, whereas the Lamel Type III model works slightly better than other models when considering both the variation of the normalized yield stress and the *r*-value.

Simulations have also been done by Alamel-type models where the grain morphology was represented by fictitiously deforming a random grain structure according to the measured grain aspect ratio of the AA3103-H18 sheet. The results are very similar to the Lamel-type calculations, confirming that the assumption made with respect to the grain boundary in the Lamel-type models is reasonable for heavily cold rolled materials.



(a)



(b)

Fig. 10 Plastic anisotropy of the AA3103-H18 material predicted by different CP models and the CPFEM-fitted Yld2004-18p yield surface: (a) normalized yield stress and (b) *r*-value.

The predicted anisotropy in normalized yield stress and *r*-value for AA3103-O is presented in Fig. 11. All models show similar variations of both the normalized yield stress and the *r*-value, but there are some quantitative differences. The low strength exhibited in the experiments for  is not captured by any model. The predictions obtained with CPFEM, either with direct calculations or the fitted yield surface, are closest to the experiments, only 3% and 6% higher values of the normalized yield stress than in the experiments at  equal to 45° and 90°, respectively. The predictions of the normalized yield stress by the Alamel, Alamel Type III and VPSC models are nearly identical and slightly higher than the CPFEM results, while the FC-Taylor model gives the least accurate predictions. The variation of *r*-value predicted by all CP models and the Yld2004-18p yield function agrees well with the experimental results. The predictions obtained with the Alamel model, CPFEM and the Yld2004-18p yield surface are closest to the experimental *r*-values. In summary, the Alamel model, CPFEM and the Yld2004-18p yield surface perform slightly better than other models in predicting the in-plane plastic anisotropy of the AA3103-O in uniaxial tension.

Even the simple FC-Taylor model was found to give reasonable predictions of the plastic anisotropy of the AA3103-H18 and AA3103-O sheets based solely on the texture. This indicates that the plastic anisotropy exhibited by the two materials is mainly due to their respective textures. The predictions of the Alamel-type models, the VPSC model and CPFEM are generally better than those of the FC-Taylor model. In these advanced CP models, the strain constraints imposed in the FC-Taylor model are relaxed to various extents, rendering these models more physically reasonable. The grain morphology can be considered in these advanced CP models in addition to texture. Despite their difference in handling the grain morphology, the performance of these CP models in predicting plastic anisotropy and its evolution is believed to be improved, even though not significantly [[15](#_ENREF_15" \o "Xie, 2013 #500)].



(a)



(b)

Fig. 11 Plastic anisotropy of the AA3103-O material predicted by different CP models and the CPFEM-fitted Yld2004-18p yield surface: (a) normalized yield stress and (b) *r*-value.

As mentioned above, the ‘texture rotation’ method has been employed in the CPFEM simulations of uniaxial tension tests in directions between the RD and TD. However, it should be noted that the grain morphologies have not been properly represented in these simulations. The ‘texture rotation’ method is physically meaningful for equiaxed grain structures. For elongated grain structures as studied here, however, the grain boundary configurations change with the tensile direction. As an example, the number density of grain boundaries along the RD differs significantly from that along the TD of the AA3103-H18 material, see Fig. 6. As a consequence, different RVEs should be prepared for uniaxial tension along different directions. Unfortunately, preparing such RVEs for directions other than the three orthotropic axes is rather complex and inconvenient. Alternatively, the hierarchical multi-level modelling method described in Section 4 can be used, where uniaxial tension is no longer the only desired deformation mode. Instead, a number of virtual deformation modes are prescribed to obtain stress states on the yield surface. The flexible yield function Yld2004-18p is then calibrated against these stress points at yielding and used to represent the yield surface of the material. In these CPFEM simulations, the effect of the grain shape has been considered implicitly, since all the deformations are prescribed with respect to the same RVE. Another distinct advantage of this multi-level method is that the plastic anisotropy is known for any stress direction once the fitting is done. The yield loci shown in Fig. 9 are such examples. Furthermore, the virtually fitted yield surface can be used in large-scale FE simulations of forming process.

The current materials feature large fractions of constitutive particles which were ignored in all the simulations made in this work. During the cold rolling, back-stresses can be introduced due to these non-shearable particles [[34](#_ENREF_34" \o "Zhao, 2013 #529)]. The back-stress may contribute to the weak strength anisotropy observed in the AA3103-H18 material. For the fully annealed material, the back-stress should have completely vanished during the recrystallization, and there should then be little influence of particles on the observed strength anisotropy. Other micro-structural features besides the texture and grain morphology, such as aligned dislocation structures [[35-37](#_ENREF_35" \o "Holmedal, 2008 #530)] and grain size [[38](#_ENREF_38" \o "Delannay, 2012 #77)], may also be the sources of mechanical anisotropy. Taking particles and other micro-structural information into consideration by CP models is beyond the scope of this paper, while it is suggested for future research work.

# Conclusion

The anisotropic plastic behaviour of AA3103 sheets in cold-rolled and fully recrystallized states has been investigated experimentally by uniaxial tensile tests and virtually by five different CP models and a hierarchical multi-level modelling method. The experimentally observed grain shapes have been taken consideration in the computations. For both materials, all five CP models give reasonable predictions when compared to the experiments, but the Alamel-type models, CPFEM and the multi-level fitting method perform slightly better than other CP models. Since high-resolution CPFEM calculations are prohibitively time-expensive, the Alamel-type models are good candidates for predicting the plastic anisotropy of AA3103 sheets.

In the multi-level fitting method, the yield surface of the material was represented by the Yld2004-18p yield function and calibrated to stress points at yielding obtained with CPFEM. This method makes it possible to consider the real grain shapes even for non-equiaxed grain structure in the CPFEM simulations. Due to theoretical advantages of CPFEM, the multi-level fitting method used here is considered to be the most accurate method for predicting the plastic anisotropy of different materials.

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