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### GEAR CONTACT FATIGUE RELIABILITY ANALYSIS FOR WIND TURBINES UNDER STOCHASTIC DYNAMIC CONDITIONS CONSIDERING INSPECTION AND REPAIR

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#### ABSTRACT

Reliability is one of the most important features of the wind turbine gearbox, especially in offshore wind turbines (OWTs). This paper describes a general way to perform gear contact fatigue reliability analysis for wind turbines considering inspection and repair. A simplified predictive surface pitting model for estimating gear fatigue lives is applied to establish the 'so-called' limit stated functions. The National Renewable Energy Laboratory (NREL)'s 750kW land-based wind turbine is used to perform time domain simulations considering different wind speeds that the turbine will experience, whose occurrence frequencies are described by a generalized gamma distribution. The time series of the torques in the main shaft are obtained from the global dynamic response analysis of the wind turbine. The time series of the gear contact forces are obtained from the dynamic analysis of the gearbox through multi-body simulation. The 2-parameter Weibull distribution is used to fit the long-term probability distribution of the gear tooth contact pressures. The reliability analysis is based on fracture mechanics (FM) analysis of crack growth. Finally the sensitivity of the reliability index and failure probability on initial crack size, critical crack size, detectable crack size, crack size after repair, material property and environmental loads is estimated considering the effect of inspection.

#### NOMENCLATURE

OWTs/OWT, Offshore Wind Turbines/Offshore Wind Turbine  
NREL, National Renewable Energy Laboratory  
FM, Fracture Mechanics  
IMMR, Inspection, Monitoring, Maintenance and Repair  
OM, Operation and Maintenance  
FORM, First Order Reliability Method  
SORM, Second Order Reliability Method  
MPP, Most Probable failure Point  
NDT, Nondestructive Testing  
POD, Probability Of Detection  
CM, Condition Monitoring  
AE, Acoustic Emission  
C.O.V., Coefficient Of Variation  
VEC, Virtual Crack Extension Method

#### INTRODUCTION

As one of the most expensive components of wind turbine systems, gearboxes have experienced higher-than-expected failure rates in the wind energy industry from its inception [1]. The nature of the failures has been investigated by researchers, e.g. De Vries [2], Musial et al. [3], Spinato et al. [4], etc. and some firm conclusions have been made. It is also believed that

the gearbox failures observed in the earlier 500 kW to 1000 kW sizes 5 to 10 years ago still exist in many of the larger 1 to 2 MW gearboxes being built today with the same architecture [3]. The cost of gearbox replacement and the down time due to these failures increases significantly with the increase in the sizes of wind turbines. The main failure modes of gears include tooth contact fatigue (pitting), tooth bending fatigue (tooth end cracking), wear (adhesion and fretting corrosion), scuffing, grinding cracks, core separation, etc. [5]. The tooth contact fatigue and bending fatigue are two identified prominent modes of fatigue damage, which are considered for design in international standards such as ISO 6336 [6]. Gear tooth pitting may be initiated on the surface (surface pitting) due to defects such as dents or scratches, or alternatively by near-surface plastic deformation in the region of the maximum cyclic shear stress caused by cyclic rolling-sliding contact (subsurface pitting). Presently ISO 6336 [6] provides different deterministic methods for the fatigue design of surface durability (pitting) and tooth bending strength under constant load and variable load with respect to spur and helical gears. However, the effects of the uncertainties of the parameters used in these methods are not explicitly considered. More reasonable probabilistic design methods are desired to refine the design and hence increase its long-term reliability. This method can also be used to manage inspection, monitoring, maintenance and repair (IMMR) to make optimal decisions regarding safety and life cycle costs of structures [7]. In the wind energy industry, this method has been applied to perform reliability-based design of wind-turbine rotor blades [8][9]. For the mechanical components of wind turbines, e.g. gears, bearings and shafts in the drive train, application has been very limited up to now.

Sørensen [10] described a risk-based life cycle approach for optimal planning of operation and maintenance (OM) of offshore wind turbines, which could be used for gears, generators, fatigue cracks, corrosion, etc.

Recently Dong et al. [11] presented a simplified predictive subsurface pitting model for estimating gear service lives and validated it by comparing with the published experimental evidence, which can be used to perform long-term time domain based gear contact fatigue analysis of a wind turbine under stochastic dynamic conditions. Furthermore, Dong et al. [12] developed a reliability-based gear contact fatigue analysis approach for wind turbines using this model.

The main purpose of this study is to establish and apply a reliability model of gear contact fatigue problem with respect to surface pitting for wind turbine drive train under stochastic dynamic conditions, considering the effect of inspection. The main benefit of this method is that uncertainty of design parameters could be considered explicitly and hence the effect of uncertainty on project costs could be estimated [13]. In addition, it is also useful for rational and optimal planning of operation (services, inspections, etc.) and maintenance (including repair and exchange) for offshore wind turbines.

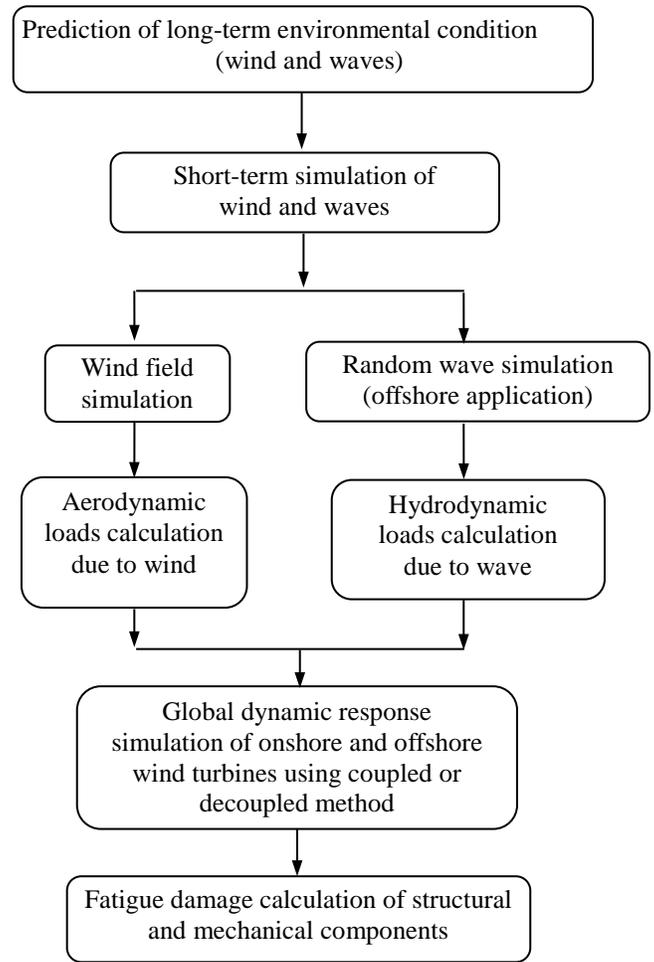


Figure 1: Schematic of the basic procedures in an integrated fatigue analysis based on time domain simulation

## PROBABILISTIC FATIGUE ANALYSIS

Reliability-based analysis was first evoked to investigate the fundamental problems of structural safety of a member under random variable load by Freudenthal in 1947 [14]. Classical reliability theory was then developed by other influential publications, such as Johnson [15], Pugsley [16] and Ferry-Borghes & Castanheta [17]. Significant developments of reliability methodology for the design of structures in general [18][19][20] and on Bayesian updating techniques in particular [21][22][23][24] have taken place since the 1980s. Reliability analyses have been extensively used in structural design of offshore structures [7], airplanes and nuclear plants. They are concerned with the rational treatment of uncertainties in structural engineering design, e.g. loads and their effects, materials and resistances, and inspection planning as well as the associated problems of rational decision making. Very limited work has been devoted to their applications to the design of mechanical components, e.g. gears and bearings.

## Limit state function

The failure criterion for fatigue limit state, based on the fracture mechanics approach, may basically be stated by

$$g(\mathbf{X}) = a_c(\mathbf{X}_1) - a_N(\mathbf{X}_2) \quad (1)$$

where  $a_c$  represents the critical crack size;  $a_N$  represents the crack size after  $N$  cycles;  $N$  represents the cycle numbers;  $\mathbf{X}_1$  and  $\mathbf{X}_2$  represent a vector of stochastic parameters respectively (stress, crack length, fatigue strength, etc.);  $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2]$ .

## Uncertainty modeling

A reasonable assessment of the uncertainties in the variables of the failure function is very important for reliability analysis at the design stage as well as in service to aid decision-making. Recently Dong et al. [25] performed a comprehensive study on the fatigue reliability of the jacket support structure for offshore wind turbines based on time domain simulation, where the major uncertainties in the whole analysis procedure are identified and analyzed. Base on the work of Veers [13], material fatigue properties can be used to approximate all the independent uncertainty from component to component. In practice, identical material specimens tested at the same stress level can have lifetimes that differ by a factor of ten or more [13]. A typical value for the standard deviation of the cycles-to-failure could be taken as 60% of the mean value [26]. Each of the other input quantities for fatigue reliability analysis has some value that varies quite little from component to component, but the exact value of the quantity is simply not known, which could be used to approximate all the common uncertainty from component to component (perfectly correlated). Figure 1 shows the basic procedures in an integrated fatigue analysis of onshore and offshore wind turbines based on time domain simulations, where uncertainties exist in each step.

For a single component, the main uncertainties associated with time domain simulation are model uncertainty and statistical uncertainty. Model uncertainty can be typically modeled as follows:

$$\chi = \frac{F_{true}}{F_{est.}}$$

where  $\chi$  represents the model uncertainty associated with a certain physical variable  $F$  (e.g. the aerodynamic loads due to wind and the hydrodynamic loads due to waves).  $F_{true}$  represents the real value of this variable, and  $F_{est.}$  represents the estimated value of this variable using approximate methods. The model uncertainties and the statistical uncertainties mentioned in the following sections of this paper are all modeled by a log-normal distribution.

## Failure probability calculation

It is assumed that probabilistic models for the assessment of the variables  $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2]$  could be described by time independent joint probability density function  $f_{\mathbf{X}}(x)$ ,  $P_f$  can be calculated by the integral:

$$P_f = \int_{g(\mathbf{X}) < 0} f_{\mathbf{X}}(x) dx \quad (2)$$

There are several methods which can be used to calculate the probability of failure  $P_f$ , e.g. FORM, SORM, Monte Carlo Simulation, etc. If the FORM and SORM methods are used, an important issue is to transform the variables  $\mathbf{X}$  into a space of variables  $\mathbf{U}$  which are independent standard normal variables. If the variables in  $\mathbf{X}$  are independent with each other, the transformation could be performed directly. If the variables in  $\mathbf{X}$  are correlated, several methods could be used to perform the transformation, e.g. the Rosenblatt transformation [27][28] and the Nataf transformation [29][30]. In the  $\mathbf{U}$  space, instead of the term of probability of failure  $P_f$ , the equivalent term of reliability index  $\beta$  could be obtained as shown in Eq. (3), which is often referred to in the design standards and relevant documentation.

$$\beta = -\Phi_U^{-1}(P_f) \quad (3)$$

where  $\Phi_U^{-1}(\cdot)$  represents the inverse transformation of the standard normal probability distribution in the  $\mathbf{U}$  space.  $\beta$  can be interpreted as the geometrical distance defined from the origin of the  $\mathbf{U}$  space to the Most Probable failure Point (MPP) on the failure surface. More details can be found in [19][31]. Presently several computer codes could be used to perform the reliability analysis, e.g. Proban [32], FAROW [33].

## Inspection and repair

Inspection, monitoring, maintenance and repair (IMMR) are important issues of ensuring safety in relation to crack growth and other deterioration phenomena for structures and mechanical components. Cracks may be detected by using various methods for inspection or monitoring and repairing the structures or mechanical components when needed. The following typical observations could be obtained from inspection and monitoring:

$$\begin{aligned} a \geq a_D & \quad \text{crack of size } a \text{ detected, where} \\ & \quad \text{size exceeds or equals detection limit, } a_D \\ a < a_D & \quad \text{crack not detected} \end{aligned}$$

Based on Eq. (1), the relative inspection event margin at time  $t$  after  $N$  cycles could be defined as:

$$H(t) = a_D(t) - a_N(t) \quad (4)$$

$H$  is negative when a crack is detected and positive otherwise. Furthermore, if the safety margin for fatigue failure before time  $t$  is defined as follows:

$$M(t) = a_c(t) - a_N(t) \quad (5)$$

and  $T_1$  and  $T_2$  are defined as four different inspection times ( $T_1 < T_2$ ), the failure probability  $P_f(t)$  for  $T_1$  and  $T_2$  could be calculated as follows:

For  $T_0 \leq t \leq T_1$ :

$$P_f(t) = P[M(t) \leq 0]$$

For  $T_1 \leq t \leq T_2$ :

$$P_f(t) = P_f(T_1) + P[M(T_1) > 0 \cap H > 0 \cap M^0(t) \leq 0] + P[M(T_1) > 0 \cap H \leq 0 \cap M^1(t) \leq 0] \quad (6)$$

$M^0$  represents no repair is performed after first inspection,  $M^1$  represents repair is performed after first inspection.  $T_0$  is the initiation period. Similar equations can be established for other inspection times. More details can be found in [34].

In general maintenance activities can be divided in corrective and preventive (time-tabled or conditioned) maintenance. Conditioned maintenance using observations from, e.g. inspections and condition monitoring, should optimally be based on risk and pre-posterior Bayesian decision theory. More details can be found in [10].

In the offshore oil and gas industry, reliability-based management of inspection, monitoring, maintenance and repair (IMMR) to make optimal decisions regarding safety and life cycle costs of offshore structures [7] has been commonly applied and performs well [35].

In the offshore wind industry, presently the application of reliability methodology is still limited, especially for the mechanical components.

In addition, it should be emphasized that the probability of detecting a crack depends on the crack size, the inspection method and the inspection team. For offshore structures, visual inspection and various Nondestructive Testing (NDT) methods have been commonly used. The Probability Of Detection (POD) curve is usually used to measure the inspection reliability. For the mechanical components of wind turbines, e.g. gears, bearings, various condition monitoring (CM) techniques, e.g. Acoustic Emission (AE), vibration, offline (or kidney loop) real-time lubricant CM, offline oil sample analysis, inline (or main loop) real-time lubricant CM and electric signature-based techniques, have been applied to diagnose the possible damage. However, each technique has its strengths and limitations. Vibration and AE-based techniques can diagnose the abnormal behavior and the damage location of the test gearbox using some analysis in the frequency domain, but they cannot provide information on lubricant condition and identify possible root causes for such damage. The real-time oil CM technique can be used to

identify possible damage to gearbox components, and the offline oil sample analysis could be used to identify possible sources of wear particles and support root causes analysis. But they cannot diagnose the location of the test gearbox where the damage occurred. The electric signature-based technique appeared less effective than the other methods. It is suggested that various techniques should be integrated when conducting wind turbine drivetrain CM [36].

## CASE STUDY

### Surface pitting model

Surface pitting may be initiated at defects such as dents or scratches on the surface of gear teeth. The prediction of crack propagation requires a proper estimate of the crack propagation rate, which can be estimated from the loading conditions and fatigue properties by using an empirical growth law. Figure 2 [37] shows the orientation of the initial crack and contact loading. Based on the Paris-Erdogans equation, the following linear elastic fracture mechanics model could be used to describe crack propagation rate:

$$\frac{da}{dN} = C \cdot (\Delta K)^m \quad (7)$$

where  $a$  is the half-length of the crack,  $N$  is the number of loading cycles,  $C$  and  $m$  are material constants, and  $\Delta K$  is the stress intensity factor range.

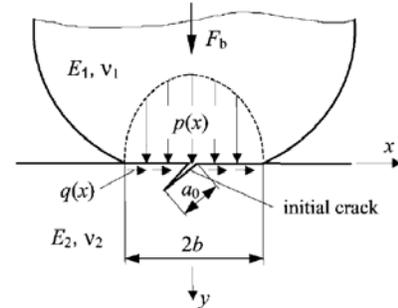


Figure 2. Orientation of the initial crack and contact loading

The driving force for crack propagation in rolling contacts depends on the maximum shear stress [37]. In 1991, Blake and Cheng [38] developed a simplified model to calculate the mode II stress intensity factors, which is given as follows:

$$\Delta K_{II} = C_k \cdot [Y \cdot \Delta \tau_{max} \cdot \sqrt{\pi a}] \quad (8)$$

Based on the work of Blake and Draper [39], a geometry correction factor  $C_D$  is incorporated in Eq. (8):

$$\Delta K_{II} = C_D \cdot C_k \cdot [Y \cdot \Delta \tau_{max} \cdot \sqrt{\pi a}] \quad (9)$$

$$C_D = [-1.497 - 0.383 \cdot \ln(\mu)] \cdot \left(\frac{a}{b}\right) + [0.945 + 0.072 \cdot \ln(\mu)]$$

$$C_k = F \cdot e^{G(a/b)}$$

$$F = 0.72557 + 1.5377 \cdot \mu - 1.61666 \cdot \mu^2$$

$$G = -0.16191 - 0.9862 \cdot \mu + 1.49067 \cdot \mu^2$$

where  $Y$  is the geometry factor.  $b$  is the half contact length.  $\mu$  is the coefficient of friction.  $\Delta\tau_{max}$  is the maximum shear stress range in the entire stress field, which could be given as follows [11]:

$$\Delta\tau_{max} = \Delta p_{max} \cdot (0.5318 \cdot \mu^2 - 0.0142 \cdot \mu + 0.3007)$$

$\Delta p_{max}$  is the maximum contact pressure range, which is calculated from time domain simulations. Therefore the following mode II stress intensity factor expression could be obtained:

$$\Delta K_{II} = C_D \cdot C_k \cdot Y \cdot \Delta p_{max} \cdot (0.5318 \cdot \mu^2 - 0.0142 \cdot \mu + 0.3007) \cdot \sqrt{\pi a} = G_{2a}(\mu, a, b, Y) \cdot \Delta p_{max} \quad (10)$$

where  $G_{2a}(\mu, a, b, Y)$  could be called the geometry function.

In Eq. (10), the random nature of the maximum contact pressure range  $\Delta p_{max}$  leads to a random cyclic stress intensity factor range  $\Delta K_{II}$  because of their linear relationship. This fundamental variability of  $\Delta K_{II}$  may however be taken into account by using the average crack growth rate obtained by weighting the crack propagation rates for given  $\Delta K_{II}$  values with their probability of occurrence.

Using Eq. (1) and Eq. (10), the following expression can be obtained:

$$\frac{da}{dN} = \int_0^\infty \frac{da}{dN}(\Delta k_{II}) \cdot f_{\Delta k_{II}}(\Delta k_{II}) \cdot d(\Delta k_{II})$$

$$= C \cdot G_{2a}^m(\mu, a, b, Y) \cdot \int_0^\infty \Delta p_{max}^m \cdot f_{\Delta p_{max}}(\Delta p_{max}) \cdot d\Delta p_{max} \quad (11)$$

### Limit state function

Using Eq. (11), the following expression could be obtained:

$$\int_{a_0}^{a_c} \frac{1}{G_{2a}^m(\mu, a, b, Y)} da$$

$$= C \cdot N_p \cdot \int_0^\infty \Delta p_{max}^m \cdot f_{\Delta p_{max}}(\Delta p_{max}) \cdot d\Delta p_{max} \quad (12)$$

Based on Eq. (12), the following contact fatigue limit state of gears with respect to surface pitting could be obtained:

$$g = \int_{a_0}^{a_c} \frac{1}{G_{2a}^m(\mu, a, b, Y)} da$$

$$- C \cdot N_p \cdot \int_0^\infty \Delta p_{max}^m \cdot f_{\Delta p_{max}}(\Delta p_{max}) \cdot d\Delta p_{max} \quad (13)$$

### Wind turbine model

The basic description of the wind turbine is given in Table 1. More details about this wind turbine can be found in [40]. The drive train configuration of the wind turbine is given in Figure 3. More details can be found in [41]. The gearbox model is given in Figure 4. More details can be found in [42].

Table 1. General description of the wind turbine

Type	3 Blade upwind
Power rating	750 kW
Rotor diameter	48.2 m
Rated rotor speed	22/15 rpm
Power regulation	Stall
Tower	Welded tubular Steel
Nominal hub height	55 m
Cut-in wind speed	3 m/s
Rated wind speed	16 m/s
Cut-out wind speed	25 m/s
Design wind class	IEC Class II
Design life	20 years

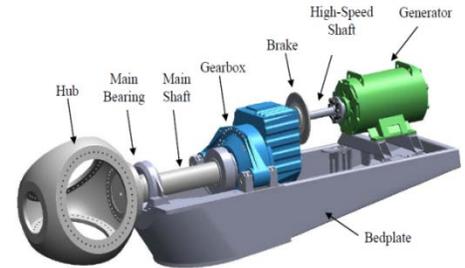


Figure 3. Drive train configuration of the NREL 750kW wind turbine [41]

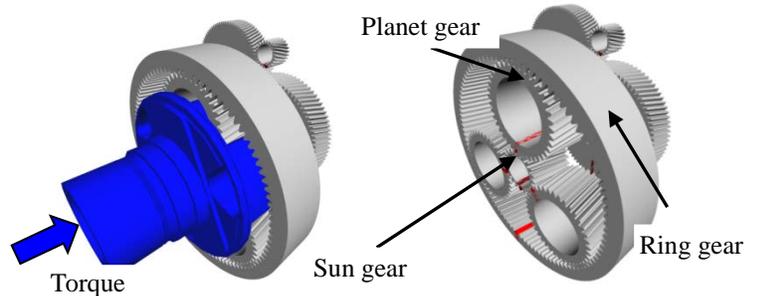


Figure 4. Gearbox model primitive

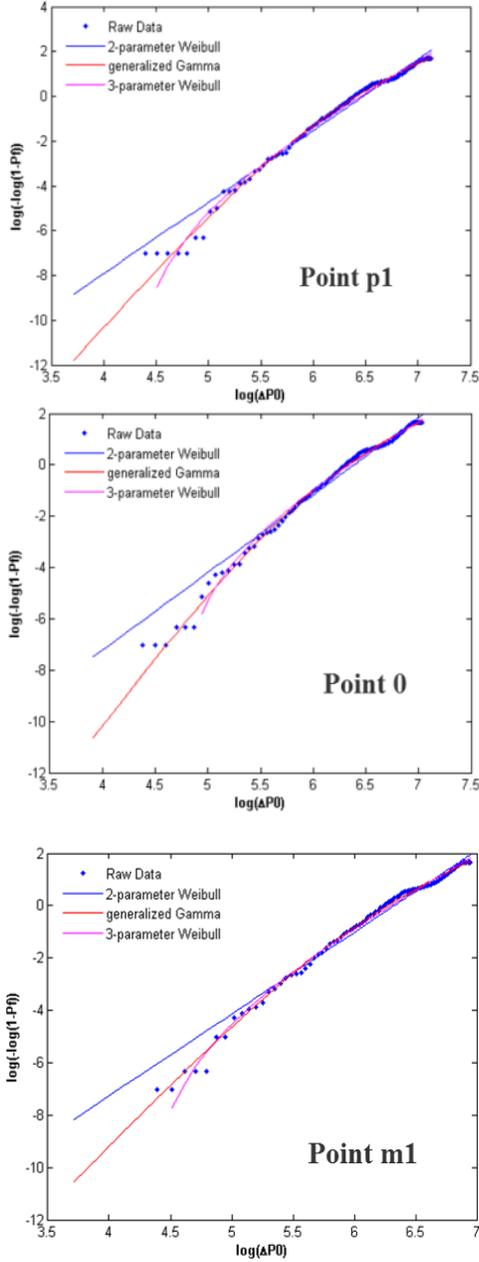


Figure 5. Long-term fitting results of  $\Delta p_{max}$  generated in the representative tooth of the planet gear

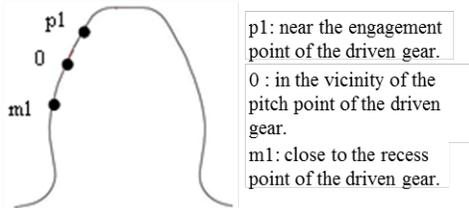


Figure 6. Positions on the profile of the characteristic points under consideration (driven gear-sun gear)

## Time domain simulation

The analysis proceeds in two steps. First, global aero-servo-elastic simulations are performed using the FAST code [43]. The time series of the main shaft torque are obtained and used as inputs in a multi-body gearbox model in SIMPACK [44]. SIMPACK is a multi-purpose multi-body code with special features available to model gearboxes.

## Probability distribution of gear tooth contact pressures

In this study the normal operation condition of the NREL 750 kW land based wind turbine is mainly considered, which is defined as the design load case 1.2 (DLC 1.2) in IEC-61400-1 [45]. Eleven different wind speeds (4 m/s-24 m/s) are used, and 20 10-minutes simulations are performed for each wind speed. The Generalized Gamma (GG) function is used to describe the probability distribution of mean wind speed. Based on the work of Dong, et al. [11], the 2-parameter Weibull distribution, the generalized gamma distribution and the 3-parameter Weibull distribution could be used to fit the long-term distribution of  $\Delta p_{max}$ . But the 2-parameter Weibull distribution is much simpler than the generalized gamma and the 3-parameter Weibull distributions. Figure 5 shows an example of the long-term fitting results of  $\Delta p_{max}$  at three different contact locations of the representative tooth of the planet gear. Figure 6 shows the relative three contact points on the profile of the gear tooth considered in this study. More details could be found in [11].

## Gear contact fatigue reliability analysis

### Simplified limit state functions

In Eq. (13), one important issue is to identify the long-term probability distribution of  $\Delta p_{max}$ . In this study, if the 2-parameter Weibull distribution is used, the following gear contact fatigue limit state could be obtained:

$$g = \int_{a_0}^{a_c} \frac{1}{G_{2a}^m(\mu, a, b, Y)} da - C \cdot v_0 \cdot T \cdot A^m \cdot \Gamma\left(1 + \frac{m}{B}\right) \quad (14)$$

where  $v_0$  represents the number of loading cycles per year,  $T$  ( $T = 1, 2, 3, \dots, 20$ ) represents the service year of the gear.  $\Gamma(\ )$  represents the gamma function.  $A$  represents the scale parameter of the Weibull distribution, and  $B$  represents the dimensionless shape parameter of the Weibull distribution. The values of  $A$  and  $B$  are estimated based on time domain simulations.

Table 2. Probabilistic data for gear contact fatigue analysis.

Variable		Distribution	Mean $\mu$	Std. Dev $\sigma$	
$\ln C$		Normal	- 42.499	0.850	
$\mu$		Log-normal	0.050	0.0025	
$m$		Fixed	5.42		
$Y$		Fixed	1.12		
$a_o$ (mm)		Exponential	0.010	0.010	
$a_R$ (mm)		Exponential	0.010	0.010	
$a_D$ (mm)		Exponential	0.040	0.040	
$\chi_{load}$		Log-normal	1.0	0.10	
$\chi_{dyn}$		Log-normal	1.0	0.05	
$\chi_{gear}$		Log-normal	1.0	0.20	
$v_o$	Sun gear	Log-normal	$5.658 \times 10^7$	$5.658 \times 10^6$	
	Planet gear	Log-normal	$1.014 \times 10^7$	$1.014 \times 10^6$	
$\ln A$ (unit of $A$ : MPa)	Sun gear	p1	Normal	6.146	0.0615
		0	Normal	6.199	0.0620
		m1	Normal	6.263	0.0626
	Planet gear	p1	Normal	6.273	0.0627
		0	Normal	6.169	0.0617
		m1	Normal	6.123	0.0612
$1/B$	Sun gear	p1	Normal	0.328	0.0164
		0	Normal	0.318	0.0159
		m1	Normal	0.296	0.0148
	Planet gear	p1	Normal	0.310	0.0155
		0	Normal	0.351	0.0175
		m1	Normal	0.339	0.0169
$a_c$ (mm)	Sun gear	p1	Log-normal	0.300	0.03
		0	Log-normal	0.300	0.03
		m1	Log-normal	0.300	0.03
	Planet gear	p1	Log-normal	0.300	0.03
		0	Log-normal	0.300	0.03
		m1	Log-normal	0.300	0.03
$b$ (mm)	Sun gear	p1	Log-normal	0.230	0.02
		0	Log-normal	0.235	0.02
		m1	Log-normal	0.190	0.02
	Planet gear	p1	Log-normal	0.193	0.02
		0	Log-normal	0.231	0.02
		m1	Log-normal	0.226	0.02

### Uncertainty treatment

A rational treatment of the uncertainties of the parameters used in Eq. (14) is one of the most important issues for reliability analysis. Here three parameters  $\chi_{load}$ ,  $\chi_{dyn}$  and  $\chi_{gear}$  are introduced to consider these model uncertainties,  $\chi_{load}$  is used to consider the model uncertainty due to aerodynamic loads calculation, as the blade element momentum (BEM) method is used in this study [25].  $\chi_{dyn}$  is

used to consider the model uncertainty due to global dynamic response analysis of the NREL 750 kW land-based wind turbine, as the decoupled analysis method is used [11].  $\chi_{gear}$  is used to consider the model uncertainty due to gear contact force calculation, as a simple rigid body gearbox model is used, and only the torque loads in the main shaft in normal operation conditions are considered [11]. It is further assumed that  $\chi_{load}$ ,  $\chi_{dyn}$  and  $\chi_{gear}$  are all multiplicative model uncertainties to  $\Delta p_{max}$ , then the following revised gear contact fatigue limit state could be obtained, which are used to perform the reliability calculation in this study:

$$g = \int_{a_o}^{a_c} \frac{1}{G_{2a}^m(\mu, a, b, Y)} da - C \cdot v_o \cdot T \cdot \chi_{load}^m \cdot \chi_{dyn}^m \cdot \chi_{gear}^m \cdot A^m \cdot \Gamma\left(1 + \frac{m}{B}\right) \quad (15)$$

The probability distribution type, mean value and C.O.V. of the parameters used in Eq. (15) are given in Table 2. Some of them are taken from [12] and [38], e.g.  $\ln C$ ,  $m$ ,  $v_o$  and  $\ln A$ . others are assumed, e.g.  $a_o$ ,  $a_R$ ,  $a_D$  and  $a_c$ . The probability distribution types are selected based on experiences. However the standard deviations of all the parameters are just estimated without verification, more accurate values should be pursued in future work.

In addition, in order to minimize the statistical uncertainty effects due to time domain simulation, 20 simulations are performed for each wind speed. More details can be found in [42].

### Strategy for inspection and repair

The design life of the gears used in this study is 20 years. Pitting is a fatigue phenomenon and is characterized by a gradual deterioration of the contacting surfaces. It originates from small, surface or subsurface initial cracks, which grow under repeated contact loading. Figure 7 shows a typical example of the pitting of gear teeth flanks [46]. As operation continues, pitting extends over a majority of the tooth surfaces and continue to form and enlarge as they break into each other. Eventually the tooth shape has been destroyed, and the gears become noisy and rough running [47]. In addition, the rough surface due to pitting on the tooth surface will cause ideal stress concentration areas from which a bending fatigue crack can originate and cause a tooth breakage failure. In order to prevent further deterioration of the failed component due to pitting, it is assumed that repair of the gear should be performed if crack with a certain length or pitting with a certain size in length or depth on the tooth surface were found during inspection. The crack surface often has a frosted or gray appearance. Generally, costs can be reduced if certain parts of a gear set are not damaged and can be reused. It makes no sense to scrap perfectly good parts of a gear set when it has been determined that they have no defects. Furthermore, Repair can restore the gears to their original condition and

minimize downtime. In general, the following repair methods are usually used:

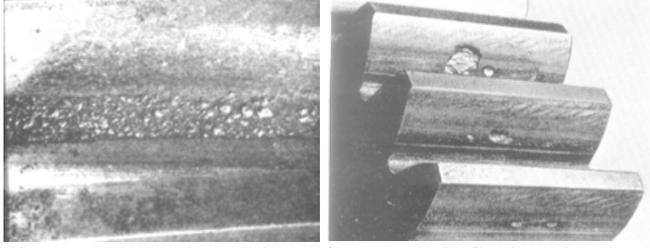


Figure 7. (a) Micropitting of gear teeth flanks (the depth usually does not exceed 20 μm [37]) (b) progressive pitting of gear teeth flanks

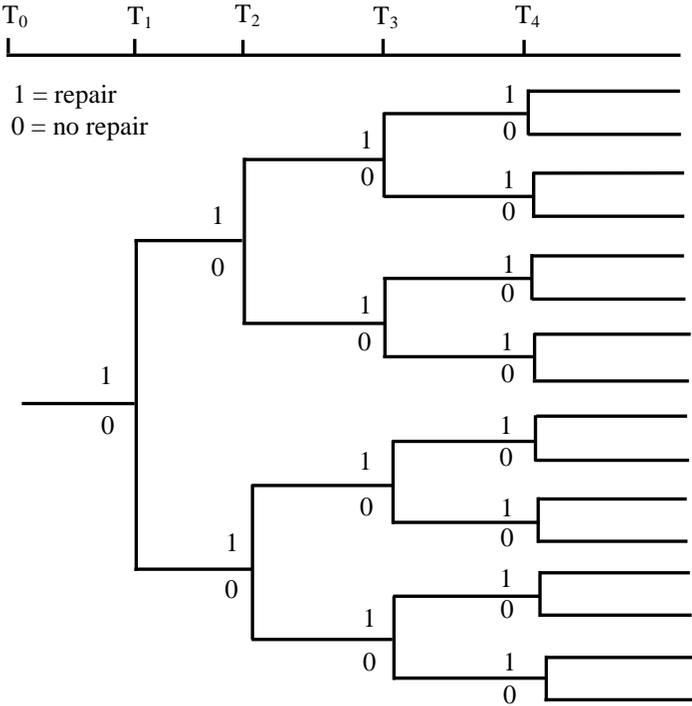


Figure 8. Inspection scheme

**Gear Grinding** Grinding is usually used to improve the finish and accuracy of the gear. When the tooth profile has only minor wear, such as from slight scoring or pitting, a quick and relatively inexpensive repair method is to grind the teeth in order to remove the damage. Typically, a gear set can be ground for one-fourth to one-third the cost of a new set. In addition, grinding the teeth of an existing gear set cost much less time than that of replacing a new gear set.

**Recutting Gear Teeth** Recutting could be used when gear teeth are worn so badly that grinding is not feasible. If the low speed gear has been found to have teeth no cracks, the diameter of the gear can be turned down and the teeth recut. As the gear is always larger than the pinion, time and expense can be saved by remachining and recutting it. A gear set repaired by this method will cost about half that of a new set.

**Rebanding the Gear** Rebanding can be the solution when the teeth in the gear are broken, severely damaged, or have cracks, if the gear is reasonably large in diameter, say over 20 inches. Rebanding is a repair method in which the gear is turned down sufficiently below the roots of the original teeth. Then a band of new steel, with the proper materials specifications, is shrunk and dowelled to the gear center. Cost savings of this method are typically from 25 to 40 percent of a complete gear set.

Here only one repair strategy is considered; all detected damaged gear teeth in terms of pitting are repaired by grinding, which is usually regarded as the most accurate method of producing gear teeth. It should be noted that a damaged tooth must be checked carefully for cracks before it can be decided if grinding the tooth is an acceptable repair method. Magnetic particle inspection methods are usually employed for this purpose. More details can be found in [47].

It is further assumed that inspections basically are performed every fourth year, i.e. after  $T_1 = 4$ ,  $T_2 = 8$ ,  $T_3 = 12$ ,  $T_4 = 16$  years. At each inspection, crack with a certain length or pitting with a certain size in length or depth may either be missed or detected and then repaired. Thus 16 different repair courses are possible. The event tree is illustrated in Figure 8, in which 0 denotes no detection and 1 denotes detection and repair.

#### Safety and event margins

Based on Eq. (15), the safety margin for fatigue failure before time  $t$  can be approximately expressed as

$$M(t) = \int_{a_0}^{ac} \frac{1}{G_{2a}^m(\mu, a, b, Y)} da - C \cdot v_0 \cdot (t - T_0) \cdot \chi_{load}^m \cdot \chi_{dyn}^m \cdot \chi_{gear}^m \cdot A^m \cdot \Gamma\left(1 + \frac{m}{B}\right) \quad (16)$$

The first inspection at time  $T_1$  leads either to crack detection or no crack detection, and the event margin is defined as

$$H = \int_{a_0}^{a_D} \frac{1}{G_{2a}^m(\mu, a, b, Y)} da - C \cdot v_0 \cdot (t - T_0) \cdot \chi_{load}^m \cdot \chi_{dyn}^m \cdot \chi_{gear}^m \cdot A^m \cdot \Gamma\left(1 + \frac{m}{B}\right) \quad (17)$$

If a crack is detected and repaired at time  $T_1$  the safety margin for failure before  $t$ , where  $T_1 \leq t \leq T_2$ , is

$$M^{x1} = \int_{a_R}^{ac} \frac{1}{G_{2a}^m(\mu, a, b, Y)} da - C \cdot v_0 \cdot (t - T_1) \cdot \chi_{load}^m \cdot \chi_{dyn}^m \cdot \chi_{gear}^m \cdot A^m \cdot \Gamma\left(1 + \frac{m}{B}\right) \quad (18)$$

The event margin for crack detection at time  $T_2$  is

$$H^1(t) = \int_{a_R}^{a_D} \frac{1}{G_{2a}^m(\mu, a, b, Y)} da - C \cdot v_0 \cdot (T_2 - T_1) \cdot \chi_{load}^m \cdot \chi_{dyn}^m \cdot \chi_{gear}^m \cdot A^m \cdot \Gamma\left(1 + \frac{m}{B}\right) \quad (19)$$

## Reliability results and sensitivity study

In this part, the reliability index  $\beta$  at the end of each service year during the designed lifetime (20 years) at three different contact points of the sun gear and the planet gear in the first transmission stage of the gearbox in the NREL's 750kW land-based wind turbine drive train is calculated by using the software Proban [32], as shown in Figure 9. The environmental condition given in Table 2, Eq. (15) and the parameters given in Tables 2 are applied.

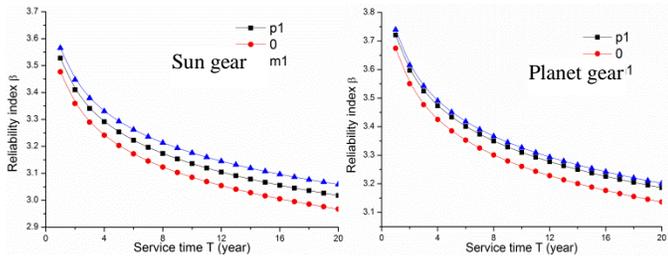


Figure 9. Reliability index  $\beta$  at 3 different contact points of the sun gear and the planet gear

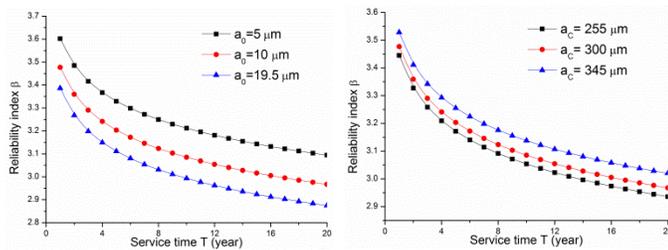


Figure 10. Reliability index  $\beta$  at the pitch contact point of the sun gear with respect to different values of  $a_0$  and  $a_c$

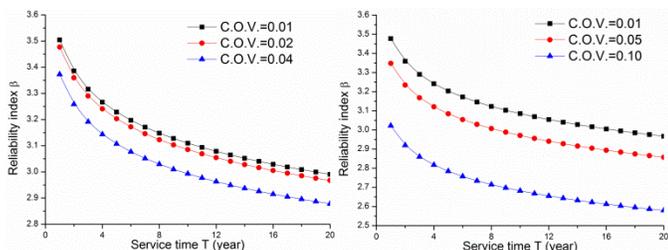


Figure 11. Reliability index  $\beta$  at the pitch contact point of the sun gear with respect to different C.O.V. of  $\ln C$  and  $\ln A$

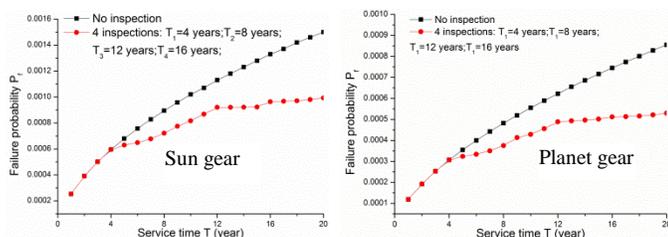


Figure 12. Failure probability  $P_f$  at the pitch contact points of the sun gear and the planet gear considering the effect of inspection

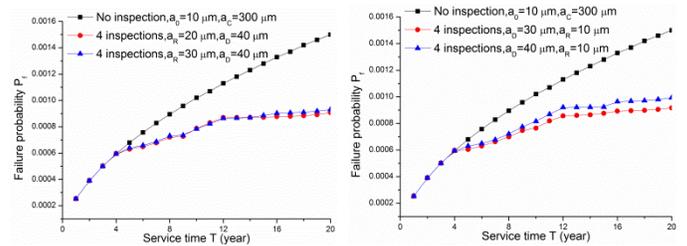


Figure 13. Failure probability  $P_f$  at the pitch contact point of the sun gear with respect to different  $a_R$  and  $a_D$  considering the effect of inspection

In the analysis reported in Figure 9, the reliability level of the sun gear is much lower than that of the planet gear. The main reason is that the same material properties are applied to the sun gear and the planet gear, and the cycle numbers of the sun gear are 3 times larger than those of the planet gear. These figures also show that the reliability levels at different contact points are different. The sensitivity studies of some parameters used in Eq. (15) are also performed, as shown in Figures 10-11. In the analysis reported in these figures, the effect of the values of  $a_0$  on the reliability index  $\beta$  seems to be larger than that of  $a_c$ . In practice, the initial crack size  $a_0$  could be determined using statistical approach based on enough samples; the critical crack size  $a_c$  can be estimated from the critical stress intensity factor  $K_{IC}$ , which could be determined from experiments [38][39] or numerical simulation methods, e.g. the virtual crack extension method (VEC) [48]. The effects of the C.O.V. values of  $\ln C$  and  $\ln A$  on the reliability index  $\beta$  are significant.  $\ln C$  is associated with the material properties; and  $\ln A$  is associated with the wind turbine models, environmental conditions and simulation techniques. A reasonable estimation of the C.O.V. values of  $\ln C$  and  $\ln A$  is an important issue for reliability-based probabilistic contact fatigue analysis of gears for onshore and offshore wind turbines. In addition, the effects of inspections on the reliability level of the sun gear and the planet gear are also investigated, as shown in Figures 12-13. In the analysis reported in these figures, inspections can reduce the failure probability  $P_f$  of the sun gear and the planet gear significantly, as shown in Figure 12. Furthermore, the effect of inspection is decreased with the initial crack size after repair  $a_R$  is increased, and is increased with the detectable crack size  $a_D$  is decreased, as shown in Figure 13. The failure probability  $P_f$  is much more sensitive to the random variable  $a_D$  than that of random variable  $a_R$ .

## CONCLUSIONS

In this study the application of gear contact fatigue (surface pitting) reliability analysis for wind turbine drive train considering inspections under stochastic dynamic conditions is presented. The main advantage of this method is that uncertainty of design parameters can be considered explicitly and hence the effect of uncertainty on project costs could be estimated, which are very useful to make a better balance

between the design and the project costs. Although the probability types and the C.O.V. values of the parameters presented in Table 2 are given without rigorous verification, some general conclusions could be still obtained and given as follows:

(I) Reliability-based probabilistic gear contact fatigue (surface pitting) analysis under stochastic dynamic conditions is available, which could be used as an alternative method or an assistant tool for the fatigue design of gears in wind turbine drive train.

(II) Eq. (17) could be used to perform the reliability-based probabilistic surface pitting analysis of gears in onshore and offshore wind turbine drive trains under stochastic dynamic conditions, which should be refined by further work.

(III) Based on the example presented in this study, the reliability levels of gear contact fatigue at different contact points are different. The effects of  $a_0$ ,  $a_C$  and the C.O.V. values of  $\ln C$  and  $\ln A$  on the reliability index  $\beta$  are significant, and a reasonable estimation of the values of them is very important, which should be performed in future work.

(IV) Inspection and repair can reduce the failure probability  $P_f$  of the sun gear and the planet gear significantly. The effect of inspection is decreased with the initial crack size after repair  $a_R$  is increased, and is increased with the detectable crack size  $a_D$  is decreased. A reasonable estimation of the values of  $a_R$  and  $a_D$  is very important, which should be performed in future work.

In this paper, only the reliability-based probabilistic surface pitting analysis of gears in wind turbine drive train under stochastic dynamic conditions is performed through an example. Other failure modes of gears in wind turbine drive train (onshore and offshore), e.g. subsurface pitting [11][12], high cycle bending fatigue, wear, scuffing [49], could be also analyzed in a similar way, where the main challenge is to obtain reasonable failure prediction models.

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