Optimization Approach to Target Costing under Uncertainty with Application to ICT-Service

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Abstract

Target costing is a modern approach applied during product development that defines cost targets for products and its components. These cost targets are driven by customer requirements and achievable revenues. The intention of this paper is the integration of target costing with modern concepts of modeling uncertainty and management of risk based on optimization. Contrary to the traditional focus of target costing on cost targets this paper prefers a strategy for achieving a target profit. Moreover, in this paper target costing is understood as a continuous process with incremental changes of cost drivers, product and component design as well as product prices. Therefore, the change in costs and profit with respect to aforementioned control parameters is modeled by linear approximations. Hence, improved decisions concerning design and prices are derived by linear programming models. In practice, information concerning product and component costs, demand or customer preferences are not given with certainty. Therefore, we apply a stochastic programming approach to manage the risk inherent in the target costing process. After a general presentation we apply our approach to the provision of an information and communication technology service where the level of uncertainty is considerable.

Keywords: Cost Management, Optimization, Risk Management, Product Design, Target Costing.

1 Introduction

Target costing has been a successful methodological framework that aims at setting product costs during the product development and design stage such that a sufficient profit margin can be achieved once the product enters the production. This methodology departs from a competitive market price of the product and defines the target costs of the product by subtracting the required profit margin. If the target costs are different from what is presently achievable with the available product design and production processes then the product design and production processes need to be restructured and improved accordingly. Target costing is principally carried out with a strong focus on customer needs and requirements. Therefore target costing is often integrated with Quality Function Deployment (QFD) that translates customer requirements into engineering and production characteristics (see for example Hauser and Clausing, 1988). One of the main principles applied in target costing and QFD is that the costs of a component or engineering characteristic should be proportional to its contribution to the customer's requirements.

During the last decade, considerable literature has been dedicated to the study of different aspects of target costing including economic and organizational perspectives as well as the implementation and adoption in different industrial branches (Cooper and Slagmulder, 1997; Ansari and Bell, 1997; Ellram, 2002 and 2006; Everaert et al., 2006; Ibusuki and Kaminski, 2007; Yazdifar and Askarany, 2012).

The contribution of this paper arises from the following two issues. Conventional target costing mostly departs from a defined product with given customer requirements and achievable price. However, during the product design stage engineers often have the opportunity to change the design in order to fulfill different customer requirements which allows for different pricing. Therefore this paper extends the target costing framework to allow for alternative customer requirements and prices. The second issue is related to uncertainties present in the early stages of the product's life cycle. In order to implement the target costing methodology and to set the correct cost targets already in the development stage, it is necessary to have substantial information about the acceptance of the product or service in the market, the relation between engineering characteristics and customer requirements, achievable market prices and potential cost reductions in the future. In the case of innovative industries this information can be affected by substantial uncertainty. In such situations there will be the risk that the cost management techniques will not guarantee the targeted costs or profit. Therefore the application of target costing in such industries should be supplemented with appropriate risk management methods. Indeed, the importance of environmental uncertainty for the adoption of target costing is recognized in the literature, although with conflicting findings: Dekker and Smidt (2003) argue that the adoption of this management tool is highest among firms in an unpredictable environment while Ax et al. (2008) reason that its adoption is negatively correlated with the environmental uncertainty. Particularly the latter study drives our suspicion that firms in an unpredictable environment reluctantly use a deterministic target costing approach without explicit considerations of uncertainty and possibilities for controlling risk.

We address these issues by associating target costing with an appropriate optimization model. Since target costing can be understood as a continuous process with incremental changes of cost drivers, product and component design as well as product prices we model the change of costs and profit with respect to aforementioned control parameters by linear approximations. Hence, design and price decisions towards the goal of profitability are represented by a linear programming model.

In order to incorporate uncertainties concerning product and component costs as well as demand and customer preferences, we apply quantitative decision support models for risk management based on the recent advances in optimization under uncertainty that proved to be successful in other domains like finance and investment analysis (see Birge and Louveaux, 1997; Gaivoronski and Pflug, 2005; Kall and Wallace, 1994; Zenios, 2007). Furthermore, this paper draws from modern portfolio theory of finance (Markowitz, 1991). More precisely, the target costing process is guided by the optimal choice of product design and price changes in order to achieve the expected cost reduction or profit improvement on one hand and the reduction of risk of ex-post non-profitability on the other hand.

Our reference application is the development of an ICT (information and communication technology) service. The environmental uncertainty in this domain is substantial due to a high pace of innovation, relatively short lifetimes of services, changing usage patterns, and the entrance of new actors. There is also an increasing need for quantitative decision support tools due to the development of electronic marketplaces.

The rest of the paper is organized as follows. Section 2 sets the stage for further discussion by describing the main stages of target costing and identifying the key quantitative relationships that are required for the development of optimization based target costing strategies and risk management tools. Section 3 introduces the optimization framework for incremental target costing in case of deterministic data. The risk management framework is integrated with the target costing in Section 4. How an integrated approach of target costing and risk management can be adopted in the ICT environment is shown in Section 5 where a numerical example is considered. Section 6 concludes the paper.

2 Quantitative Formulation of the Traditional Target Costing Model

This section sets the stage for the development and integration of risk management tools in the context of target costing under uncertainty. It contains a description of the main components of the traditional target costing model and its phases as described in the literature. This will serve as departing point for the incremental target costing and profit model introduced in Section 3 and the risk management approach developed in Section 4.

At first suppose a firm that attempts to maximize the profit G from producing and selling a single product to the customers. The demand for this product depends on the price p and the product quality that is composed of N product attributes (customer requirements) perceived by the customer. These attributes are quantified by the vector $\mathbf{z} = (z_1, z_2, \dots, z_N)$. The dependence of the demand on the price p and the product attributes $\mathbf{z}$ is expressed by the demand function $f_D(\mathbf{z}, p)$. The firm controls several variables $\mathbf{v} = (v_1, v_2, \dots, v_K)$ that govern the design of the product and its components as well as the costs of the product. Let the dependence of the product attributes be expressed by an attribute function $f_Z(\mathbf{v})$, $f_Z: \mathbb{R}^K \to \mathbb{R}^N$. This is often a complex and multilayered system. Approaches referred to as quality function deployment (Hauser and Clausing, 1988; Zengin and Ada, 2010) and function deployment analysis (Prasad, 1994) attempt to operationalize this complexity. The total costs are expressed by the cost function $f_C(\mathbf{v}, d)$ with $f_C: \mathbb{R}^{K+1} \to \mathbb{R}^1$. The total costs depend on the control variables $\mathbf{v}$ and the production volume. For simplicity we assume that the production volume is equivalent to the demand d. Note that the demand itself depends on the price and the product quality as described above. Finally, the maximization of the profit can be stated by the following optimization model:

$$\underset{\mathbf{v},p}{\text{maximize}} \qquad G = f_D(\mathbf{z}, p) \cdot p - f_C(\mathbf{v}, d) \tag{1}$$

$$\text{where } \mathbf{z} \equiv f_Z(\mathbf{v}) \tag{2}$$

$$\text{Subject to} \qquad p \geq 0, \mathbf{v} \geq \mathbf{0} \tag{3}$$

In this formulation any restrictions concerning production or sales capacities, technical constraints or financial budgets are neglected but can be easily added if needed. An overview on the components of the target costing model for the purpose of this paper is given in Figure 1.

There are a couple of models related to the optimization of product design (Prasad, 1993; Moskowitz and Kim, 1997; Fung et al., 2002; Chen and Weng, 2003; Lai et al., 2004; Chen and Weng, 2006; Chen and Ko, 2009; Delice and Güngör, 2009). The objective functions of all of these models aim at the maximization of customer utility in one or another way. Some of these studies explicitly treat the effect of the design on the product's cost by budgetary constraints (Fung et al., 2002; Weng and Cheng, 2003; Lai et al., 2004; Chen et al., 2005; Chen and Ko, 2009; Delice and Güngör, 2009). The only work that considers costs in the objective function is Chen and Weng (2006) who apply a goal programming approach with the three objectives: maximization of customer satisfaction, minimization of costs and minimization of technical difficulty. The uncertainty inherent in the relationships between customer requirements and product requirements or relationships between costs and product design are treated in Fung et al., 2002; Chen and Weng (2003 and 2006), Chen and Ko, (2009) in form of fuzzyness. In contrast, in our paper the optimization of the product design is guided by the objective of profit maximization. Uncertainty in the parameters of the functions involved (see Figure 1) will be quantified by means of a probabilistic framework. For differences of a fuzzy versus probabilistic representation of data see for example Gaines (1978). Finally yet importantly, the work of Hoque et al. (2005) deserves attention. They represent a compromising framework for conflicting weights concerning customer, engineering, and cost requirements as stated by the merchandizing, development and production departments. This is an interesting and complementary approach to the framework represented in our study.

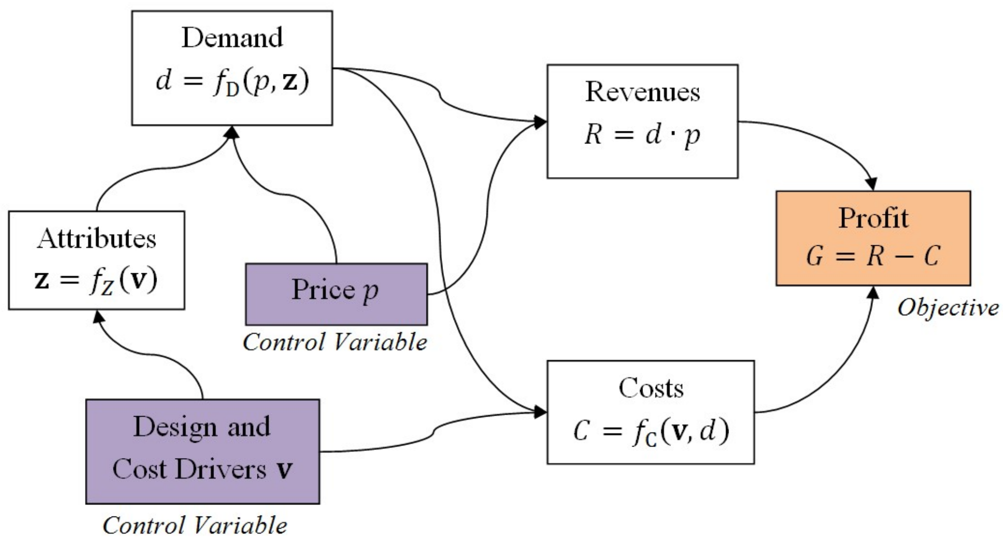

Figure 1: Components of the Target Costing Model

Knowing the specifications of the demand function $f_D(\mathbf{z}, p)$, cost function $f_C(\mathbf{v}, d)$ and attribute function $f_Z(\mathbf{v})$ one will attempt to find the optimal price p^* and design/cost drivers $\mathbf{v}^*$. Depending on the properties of this optimization problem one can actually obtain these optimal decisions, otherwise, one will try to find an improved or near optimal solution. However, practically the straight forward implementation of such an optimization model is not possible because there exists limited or imprecise knowledge on the demand function f_D, cost function f_C, and attribute function f_Z. For the design stage of new services and products cost management practitioners and researchers have therefore developed the concept of target costing (See Feil et al. (2004) or Burrows and Chenhall (2012) for a historical perspective on target costing). In what follows we apply the notation from above and give an overview on the target costing phases similarly to Cooper and Slagmulder (1997), Ellram (2006), Everaert et al. (2006) or Ax et al. (2008).

Phase 1 – Identifying the product and service attributes: Target costing starts with the identification of the customer needs and the selection of product attributes that are required to meet these needs. This phase will yield the description of a service by a vector of quantified attributes $\mathbf{z} = (z_1, \ldots, z_N)$. These attributes define the service features that are important from the point of view of consumers, like functionality and quality, and they are measured in appropriate units. This is a crucial step because target costing is a market-driven approach and its results will be used throughout the whole target costing process. Different means are used to extract this information like market assessments, customer surveys, focus groups, product prototype tests, and organized interviews with key customers. Based on market research the target values $\hat{\mathbf{z}} = (\hat{z}_1, \ldots, \hat{z}_N)$ can be defined. A number of factors are considered here, not all of them possible to formalize like the position of the product in the firm's product matrix, the dependence of the sales volume on these attributes, the product's quality and functionality compared to competitive offerings, the characteristics of anticipated customers, or the firm's strategy as well as competitors' strategies. In section 3 we will consider the product attributes as dependent on the price and the product specifications.

Phase 2 – Establishing the target price: In this phase the target selling price $\hat{p}$ is established. In addition to above mentioned factors the following aspects may play an important role in this phase: firm's long-term sales and profit objectives, the perceived value of the product to the customers, the desired market share of the product and the dependence of the sales volume on this price. Like the product requirements, the price becomes a control variable in Section 3.

Phase 3 – Establishing target demand and revenues: Marked research now focuses on the projection of the demand $\hat{d}$ for the service with attributes $\hat{\mathbf{z}}$ and unit price $\hat{p}$. Having an estimate for the target demand, we can also calculate the value for the revenues $\hat{R}$. In our analysis we suppose an undifferentiated price per unit such that the revenues become: $\hat{R} = \hat{p} \cdot \hat{d}$. Again, we want to accentuate that Section 3 considers price and product specifications as control variables on which the satisfaction of customer, demand and revenues will depend.

Phase 4 – Determining the target profit margin and the target profit: In this phase the target profit rate $\hat{\mu}$ needs to be determined. This decision will depend on different benchmarks like the profit levels for similar or preceding products, the relative strength of competitive offerings or the firm's long-term profit plan. The target profit is then determined as $\hat{G} = \hat{\mu} \cdot \hat{C}$.

Phase 5 – Determining the product target costs: The target costs of the product are obtained by taking the difference between the target revenues $\hat{R}$ and profit $\hat{G}$, i. e. $\hat{C} = \hat{R} - \hat{G}$. The target costs can also be determined through the target revenues $\hat{R}$ and the profit margin $\hat{\mu}$ as follows: $\hat{C} = \frac{1}{1+\hat{\mu}} \cdot \hat{R} = \frac{1}{1+\hat{\mu}} \cdot \hat{p} \cdot \hat{d}$.

Phase 6 – Decomposition of target costs to components, materials and processes: This stage aims at the establishment of target costs for the product components, production processes, materials and external suppliers. The decomposition can be obtained by different methods, the most common one being the function-oriented method and the component-allocation method (Everaert et al. 2006). In this paper we suppose that the product or service is composed of K cost and design drivers. These are for example product components, production activities, materials, etc. The vector $\mathbf{v} = \{v_k\}_{k=1,\dots,K}$ contains a quantification of the quality or extent of these cost and design drivers. As mentioned above, an attribute function $\mathbf{z} = f_Z(\mathbf{v})$ connects these drivers with the attributes of the product. The total costs of the product are dependent on the quality of these drivers and the demand, which is expressed by some cost function: $f_C(\mathbf{v},d)$. With this specification the problem of target costing becomes the following. Find the optimal product attributes and cost drivers $\mathbf{v}$ such that the gap between the achievable costs $f_C(\mathbf{v},\hat{d})$ and the target costs $\frac{f_D(\hat{\mathbf{z}},\hat{p})\cdot\hat{p}}{1-\hat{\mu}}$ will be reduced:

$$\underset{\mathbf{v}}{\text{minimize}} \qquad f_C(\mathbf{v},\hat{d}) - \frac{f_D(\hat{\mathbf{z}},\hat{p})\cdot\hat{p}}{1-\hat{\mu}} \tag{4}$$

$$\text{Subject to} \qquad \hat{\mathbf{z}} \equiv f_Z(\mathbf{v}) \tag{5}$$

$$p \geq 0, \mathbf{v} \geq \mathbf{0} \tag{6}$$

This problem is equivalent to solely minimizing costs since the target profit and target profit margin are fixed. Furthermore, this problem is also equivalent to problem (1) to (3) for fixed customer requirements $\hat{\mathbf{z}}$ and price $\hat{p}$. Since we will allow for changes in the price and satisfaction of customers we will later regress to problem (1) to (3) instead of (4) to (6).

Phase 7 – Continuous improvements: Studies like the one by Ellram (2006) show that firms may introduce a product even if the target costs have not been achieved. In this case, efforts aim at a continuous improvement throughout the whole life cycle of the product. This process is often referred to as Kaizen costing (Modarress et al., 2005).

3 Incremental Target Costing

The process outlined in the previous section does not completely describe the engineering practice. More particularly, after stage 6 has been performed the target values of the product attributes (customer requirements) and target price can be adjusted in order to define the realistic cost reduction targets. This is due to the fact that the development of a new service or product will be subject to uncertainties concerning the cause-effect relationships between product design, costs, satisfaction of customer requirements and demand shown in Figure 1. For this reason we consider target costing as an iterative process as indicated in Figure 2. This process starts from some initial design of the product or service and goes several times through the target costing phases described in the previous section. Each loop through these steps can bring some adjustment to the cost targets and cost decomposition.

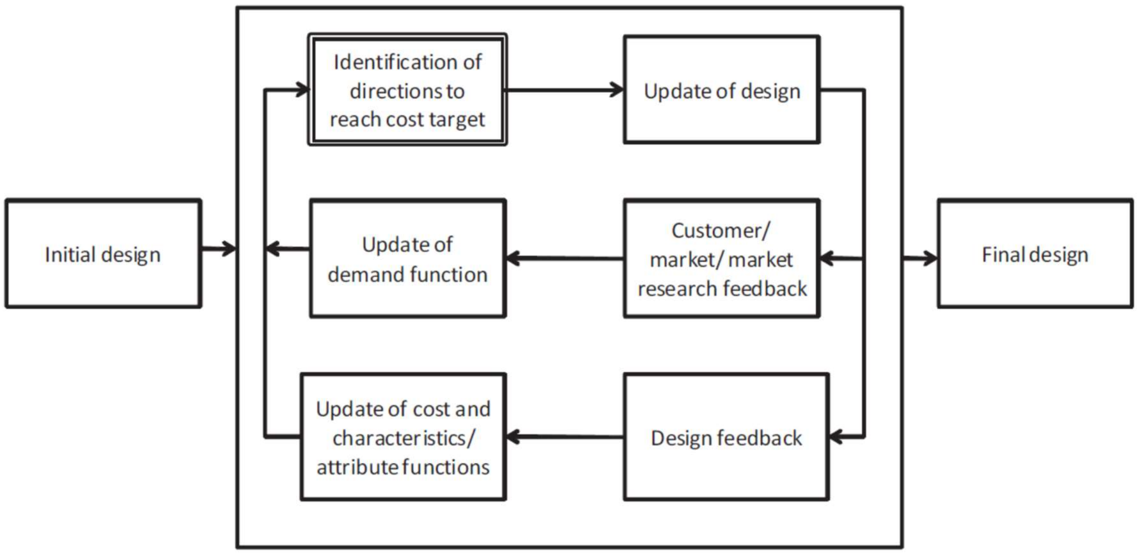

Figure 2: Incremental Target Costing

In this section we shall be concerned with the upper left box on this figure. More particularly, we develop a model that will help the service or product designer to identify directions that will bring him closer to the cost target while maintaining the necessary level of customer's response. Due to uncertainty inherent in this process, there is always the possibility that cost reductions will bring about undesirable effects on revenues and demand. In fact the objective of the service designer will be to find the acceptable trade-off between provision costs and the resulting revenues. Our purpose is to facilitate the identification of this trade-off by providing the designer with a set of the most effective directions of reaching the target costs and an estimate of accompanying risks for each direction. Having this information, the designer will choose which direction to pursue.

For this we depart from model (1) to (3). Since, the incremental target costing process does not aim at the optimization of the product design in one immediate step like indicated by this model, but should be interpreted as an iterative process with relatively minor updates in price and design at each iteration, we will formally describe the changes in costs, revenues and profit with respect to a change in the control variables p and $\mathbf{v}$ as linear approximations. This will indicate what information is needed for an improvement towards the profit goal. The target costing stages will be extended as follows:

In phase 1 (*identifying the product and service attributes*) we suppose that we start from some initial design $\mathbf{z}_0$. The change in the satisfaction of the product requirements $i = 1, \dots, N$ is defined as:

$$\Delta z_i = \frac{\partial z_i}{\partial v_1} \cdot \Delta v_1 + \cdots + \frac{\partial z_i}{\partial v_K} \cdot \Delta v_K \quad \text{for all} \quad i = 1, \dots, N \tag{7}$$

In phase 2 (*establishing the target price*) we suppose that we have some initial price estimate p_0. In phase 3 (*establishing of target demand and revenues*) marked research defines the effect of changes in product attributes and price (Δz_i and Δp) on the demand:

$$\Delta d = \frac{\partial d}{\partial z_1} \cdot \Delta z_1 + \cdots + \frac{\partial d}{\partial z_N} \cdot \Delta z_N + \frac{\partial d}{\partial p} \cdot \Delta p \tag{8}$$

By substituting all Δz_i in (8) by (7) we obtain the change in the demand dependent on a change in the component design and price:

$$\Delta d = \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{\partial d}{\partial z_i} \cdot \frac{\partial z_i}{\partial v_k} \cdot \Delta v_k + \frac{\partial d}{\partial p} \cdot \Delta p \tag{9}$$

The effect on the revenues can now be approximated by:

$$\Delta R = \frac{\partial R}{\partial d} \cdot \Delta d + \frac{\partial R}{\partial p} \cdot \Delta p \tag{10}$$

where Δd is defined by (9). We again suppose an undifferentiated price per unit such that the underlying revenue function is $R = p \cdot f_{\mathrm{D}}(p, \mathbf{z})$.

Concerning phase 5 (*determining the product target costs*), information concerning the change in the costs need to be retrieved:

$$\Delta K_T = \frac{\partial K_T}{\partial v_1} \cdot \Delta v_1 + \cdots + \frac{\partial K_T}{\partial v_K} \cdot \Delta v_K + \frac{\partial K_T}{\partial d} \cdot \Delta d \tag{11}$$

where Δd is defined by (9).

Finally, the change in the profit is $\Delta G = \Delta R - \Delta K_T$ with ΔR defined by (10) and ΔK_T by (11). Jointly, we obtain:

$$\begin{aligned} \Delta G = \left(\frac{\partial R}{\partial d} - \frac{\partial K_T}{\partial d}\right) \cdot \sum_{k=1}^{K} \left(\left(\sum_{i=1}^{N} \frac{\partial d}{\partial z_i} \cdot \frac{\partial z_i}{\partial v_k} \right) - \frac{\partial K_T}{\partial v_k} \right) \cdot \Delta v_k \\ + \left(\left(\frac{\partial R}{\partial d} - \frac{\partial K_T}{\partial d} \right) \cdot \frac{\partial d}{\partial p} + \frac{\partial R}{\partial p} \right) \cdot \Delta p \end{aligned} \tag{12}$$

We, however, keep in mind that the derivatives used in the aforementioned equations cannot be derived from the "actual" functions $f_D(\mathbf{z}, p)$, $f_C(\mathbf{v}, d)$ and $f_Z(\mathbf{v})$ for the reason of limited or imprecise knowledge. Instead, these derivatives will be estimates based on knowledge from market research or engineering experience. Let us now state the optimization model that aims at an incremental improvement of the profit by changing price and cost/design drivers:

$$\underset{\Delta\mathbf{v},\Delta p}{\text{maximize}} \qquad \Delta G \text{ as defined in (12)} \tag{13}$$

$$\text{subject to:} \qquad |\Delta p| \leq \mathscr{p}, \qquad \|\Delta\mathbf{v}\| \leq \boldsymbol{\mathscr{v}} \tag{14}$$

Here $|\Delta p|$ denotes the absolute change in the price and $\|\Delta\mathbf{v}\|$ describes the absolute change (expressed by the Euclidian norm) in the vector of changes in the engineering characteristics; $\mathscr{p}$ and $\mathscr{v}$ denote the maximum allowable changes in the price and the engineering characteristics respectively.

In the practical implementation of such a model there will be substantial uncertainty concerning the acceptance of the product or service in the market, the relation between engineering characteristics and customer requirements, the achievable market price and potential cost reductions in the future. The next section will therefore allow for stochastic parameters in the proposed model.

4 Incremental Target Costing under Uncertainty and Risk Management

It is inevitable that there is a great deal of uncertainty and imprecision in the parameters of the optimization problems associated with incremental target costing as presented in the previous section. In what follows we therefore include uncertainty in the data and the possibility of controlling risk into our target costing problem (13) and (14). We adopt here a probabilistic description of uncertainty with ω indicating a random event that governs a particular outcome of the stochastic parameters of the target costing model. The demand function, the attribute and the cost function will be denoted by $d = f_{\mathrm{D}}(p, \mathbf{z}, \omega)$, $\mathbf{z} = f_{\mathrm{Z}}(\mathbf{v}, \omega)$ and $f_{\mathrm{C}}(\mathbf{v}, d, \omega)$ respectively. Again the revenues are supposed to be based on an undifferentiated unit price: $R = p \cdot f_{\mathrm{D}}(p, \mathbf{z}, \omega)$. The expected revenues are $\bar{R} = p \cdot \mathbb{E}_\omega[f_{\mathrm{D}}(p, \mathbf{z}, \omega)]$ where $\mathbb{E}_\omega$ is the mathematical expectation with respect to ω. The stochastic profit $G(\omega)$ and the expected profit $\bar{G}$ are formally given by:

$$G(\omega) = p \cdot f_{\mathrm{D}}(p, \mathbf{z}, \omega) - f_{\mathrm{C}}(\mathbf{v}, d, \omega)$$

$$\bar{G} = p \cdot \mathbb{E}_\omega[f_{\mathrm{D}}(p, \mathbf{z}, \omega)] - \mathbb{E}_\omega[f_{\mathrm{C}}(\mathbf{v}, d, \omega)]$$

There is a risk that the actual profit will differ from this estimate because it depends on the realization of the uncertain parameters ω. We shall measure the variation or deviation from the expectation by some risk measure $\mathbb{R}$. The most traditional and widely used measure is the standard deviation or variance. Modern risk management, however, has developed different other risk measures that represent a better alternative in specific risk management cases (see Artzner, et al. (1999), Uryasev and Rockafellar (1999), Jorion (2001), Gaivoronski and Pflug (2005)). In our approach we apply the expected shortfall below some benchmark or target profit π:

$$\mathbb{R} = -\mathbb{E}_{\omega}[\min\{G(\mathbf{v}, p, \omega) - \pi, 0\}]$$

One possible candidate for the target π can be the break-even (profit of zero). Through an appropriate selection of the control parameters (decision variables) p (price) and $\mathbf{v}$ (design and cost drivers) the risk of a deviation from these targets can be controlled. We shall now discuss how price and cost/design drivers can be chosen in order to obtain the desired trade-off between achieving targets and reducing the risk of failure. Let us therefore reformulate the problem (1) to (3) such that the risk exposure will be limited. More particularly, we constrain the hazard of having a high expected deviation from the benchmark π (for example high expected loss):

$$\underset{\mathbf{v},p}{\text{maximize}} \quad \mathbb{E}_{\omega}[G(\mathbf{v}, p, \omega)] \tag{15}$$

$$\text{subject to} \quad -\mathbb{E}_{\omega}[\min\{G(\mathbf{v}, p, \omega) - \pi, 0\}] \leq \hat{\sigma} \tag{16}$$

$$p \geq 0, \mathbf{v} \geq \mathbf{0} \tag{17}$$

Here $\hat{\sigma}$ is some pre-specified level of risk.

In the present form the objective function and the constraints are nonlinear and contain the expectation operator. This operator itself represents a challenge because it may be hard to compute. The problem becomes simpler if we assume that random parameters have finite support defined by a finite number of scenarios. These scenarios can be derived from the market research that shall yield demand projections. Suppose that we have S scenarios of uncertainty and each scenario is described by the value $s = 1, \ldots, S$ and its probability ρ_s.

Then the problem (15) to (17) assumes the following form:

$$\underset{\mathbf{v},p}{\text{maximize}} \quad \sum_{s=1}^{S} \rho_s \cdot G(\mathbf{v},p,s) \tag{18}$$

$$\text{where } \mathbf{z}(s) = f_Z(\mathbf{v},s) \quad \text{for all } s = 1,\dots,S \tag{19}$$

$$\text{subject to} \quad \sum_{s=1}^{S} \rho_s \cdot (-\min\{0,\ G(\mathbf{v},p,s)\}) \leq \hat{\sigma} \tag{20}$$

$$p \geq 0, \mathbf{v} \geq \mathbf{0}$$

This problem can be further reformulated by introducing a composite objective function that will be the linear combination with weight λ between the performance measure (18) and the risk value from the left hand side of expression (20) above. The problem becomes:

$$\underset{\mathbf{v},p}{\text{maximize}} \quad \lambda \cdot \sum_{s=1}^{S} \rho_s \cdot G(\mathbf{v},p,s) - (1-\lambda) \cdot \sum_{s=1}^{S} \rho_s \cdot (-\min\{0,\ G(\mathbf{v},p,s)\}) \tag{21}$$

$$\text{subject to} \quad p \geq 0, \mathbf{v} \geq \mathbf{0} \tag{22}$$

In fact, one can prove that under mild technical assumptions any solution of problem (21) and (22) for any fixed $\lambda \in (0,1]$ is also a solution of problem (18) to (20) for some specific $\hat{\sigma}$ which depends on this solution. This relationship between the performance maximization problem under risk constraints of type (18) to (20) and the optimization of integrated performance/risk measure of type (21) and (22) holds for a wide variety of risk and performance measures. It is well known fact in the theory of financial portfolio management, where this transformation is widely used, see for example Zenios (2007: chapter 3). We provide a mathematical proof of this fact in Appendix for completeness of exposition.

Let us reformulate this problem further by introducing auxiliary variables $u = (u_1, \ldots, u_S)$:

$$\underset{\mathbf{v},p}{\text{maximize}} \qquad \lambda \cdot \sum_{s=1}^{S} \rho_s \cdot G(\mathbf{v}, p, s) - (1-\lambda) \cdot \sum_{s=1}^{S} \rho_s \cdot u_s \tag{23}$$

$$\text{subject to} \qquad u_s + G(\mathbf{v}, p, s) \geq 0 \tag{24}$$

$$u_s \geq 0, p \geq 0, \mathbf{v} \geq \mathbf{0} \tag{25}$$

Now we have to replace $G(\mathbf{v}, p, \omega) \approx G_0(\omega) + \Delta G(\omega)$ by the linear approximation from (12):

$$\underset{\Delta\mathbf{v},\Delta p}{\text{maximize}} \qquad \lambda \cdot \sum_{s=1}^{S} \rho_s \cdot \Delta G(s) + \lambda \cdot \sum_{s=1}^{S} \rho_s \cdot G_0(s) - (1-\lambda) \cdot \sum_{s=1}^{S} \rho_s \cdot u_s \tag{26}$$

$$\text{subject to} \qquad \begin{aligned} & u_s + \Delta G(s) \geq -G_0(s) \\ & u_s \geq 0 \end{aligned} \tag{27}$$

$$\begin{aligned} & -\boldsymbol{\mathscr{v}} \leq \Delta\mathbf{v} \leq \boldsymbol{\mathscr{v}} \\ & -\mathscr{p} \leq \Delta p \leq \mathscr{p} \end{aligned} \tag{28}$$

Note that $\lambda \cdot \sum_{s=1}^{S} \rho_s \cdot G_0(\omega)$ is a constant that can be neglected in the objective function. This is a linear programming problem which can be solved by standard software. Let us now discuss how the solution to this problem can assist the service designer to select a desirable trade-off between moving towards the targets defined above and accompanying risks like reducing the appeal to customers and decreasing the resulting revenue. This can be achieved by the following two-stage process.

1. Problem (26) to (28) can be solved for different values of the weighing factor λ within a range from 0 to 1. Each such solution will consist of the following:
   - the portfolio of cost reduction activities $\Delta\mathbf{v}$ that will indicate which components and which characteristics should be altered and at which rate; Possibly, this portfolio will include also the recommendation for the price change Δp,
   - the expected performance of this portfolio measured as improved expected profit,
   - the change in the risk associated with these activities measured as the expected negative deviation from some benchmark.

   By changing the value of λ from 0 to 1 one will obtain the dependence of performance on risk as shown in Figure 3 in Section 5. Drawing on the analogy with the portfolio theory from finance and investment science one can refer to the curve in this figure as the efficient frontier (Markowitz, 1991). Each point on this frontier corresponds to a portfolio of activities that a rational designer should prefer to any other portfolio of activities that generates a risk/performance point below or to the right of this frontier.

2. Having the efficient frontier as in Figure 3 the designer can decide on his upper level of risk tolerance $\hat{\sigma}$ and obtain the corresponding level of performance, i.e. cost reduction or profit improvement together with the portfolio of activities to undertake in the current iteration of incremental cost targeting.

5 Target Costing with Controlled Risk for an Advanced Mobile Data Service

In this section we consider the application of the incremental target costing framework under risk discussed in the previous sections for the design of a specific advanced mobile data service, namely a mobile tourist information service (MTIS). This service is a location based service that offers specific information about travel destinations (news and information in form of weather reports, emergency news, health support news, traffic conditions, general news bulletins and other useful tourist information). The description of the MITS is identical with a real case analyzed in Telenor Norway, one of the world's largest mobile telecommunications companies at this time. The data concerning customer preferences and costs are deteriorated for this paper. This, however, will not derogate the applicability of our approach, and the conclusions remain valid. For the MTIS seven ($i = 1, \ldots, 7$) service functions or attributes have been specified:

1. delivering information
2. availability
3. news precision
4. easy usage
5. attractiveness
6. service security
7. easy installation

These attributes are quantified by $z_i,\ i = 1, \ldots, N$. One way to do this is to assign a number on the scale from 0 to 1, where 0 corresponds to "very bad" while 1 corresponds to "stellar". For some of the attributes other measures are possible. For example, availability and the corresponding z_2 can be measured as the percentage of calls to the service that go through, easy installation (z_7) can be measured in installation time, etc. Service attributes should reflect market and customer requirements, which are identified through market research.

In the context of the incremental target costing we assume that some current reference service design can be identified for obtaining the values $\mathbf{z}_0$ of the aforementioned service attributes and the reference price p_0. Table 1 shows the results of this assessment where the values range from 0 (miserable) to 1 (excellent).

Table 1: Initial Assessment of the Quality of the Service Attributes

	(1) $i = 1$	(2) $i = 2$	(3) $i = 3$	(4) $i = 4$	(5) $i = 5$	(6) $i = 6$	(7) $i = 7$	Price p
	0.4	0.45	0.28	0.51	0.41	0.58	0.47	0.3

With these values for quality realization and initial price a reference demand d_0 of 1,100,000 access times was projected. Hence, the expected revenues can be determined as $R = p \cdot d = 0.3 \cdot 1{,}100{,}000 = 330{,}000$. These figures are partly based on conjoint analysis (Green and Srinivasan, 1990) and interactions with a selected number of key customers. This also provides the estimates and distribution of the sensitivity of the demand with respect to the quality of the service ($\frac{\partial d}{\partial z_i}, i = 1, \dots, N$) and the price ($\frac{\partial d}{\partial p}$). Table 2 summarizes these results in form of expected values and standard deviations.

Table 2: Sensitivity of the Demand on the Attributes of the Product and the Price

	(1) $\frac{\partial \bar{d}}{\partial z_1}$	(2) $\frac{\partial \bar{d}}{\partial z_2}$	(3) $\frac{\partial \bar{d}}{\partial z_3}$	(4) $\frac{\partial \bar{d}}{\partial z_4}$	(5) $\frac{\partial \bar{d}}{\partial z_5}$	(6) $\frac{\partial \bar{d}}{\partial z_6}$	(7) $\frac{\partial \bar{d}}{\partial z_7}$	Price $\frac{\partial \bar{d}}{\partial p}$
Expectation	510,000	320,000	450,000	310,000	180,000	240,000	100,000	−4,780,000
Standard Deviation	77,000	45,000	64,000	44,000	24,000	37,000	16,000	650,000

The design of a telecommunication service is often based on a multilayered architecture, i. e. the enablers of one architecture layer use service enablers from the layers below (see for example the OSI reference model for data communication). The term *enabler* corresponds to a *product component*. For the MTIS a simplified one-layer architecture with the following enablers was presumed:

A: The network enabler (NE) provides the physical transfer of data between users of services, for example GPRS and UMTS networks.
B: The context enabler (CxE) provides the service context, e.g. information on location, time and presence as well as service metadata.
C: The content enabler (CtE) delivers multimedia information.
D: The service composition enabler (SCE) coordinates various service platform mechanisms (e.g. discovery, brokering and mediation) and composes various enablers to the end user service.

E: The A4C enabler (A4C) provides authentication, authorization, accounting, auditing and charging services.

F: The identity management enabler (IdM) is used for the retrieval and management of the user's identity information and the user's profile.

The quality of these enablers is furthermore described by physical attributes like processing power, software failure rate, memory and transmission characteristics of the information to be transmitted, etc. An estimate of the costs of the service enablers, as required for the reference design, is given in Table 3.

Table 3: Total Costs Decomposed by Service Enablers

$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	Total Costs
100,000	40,000	21,000	50,000	20,000	55,000	286,000

Hence, the profit from service provision can be computed as $G = R - C = 330{,}000 - 286{,}000 = 44{,}000$.

In order to process the target costing optimization problem (26) to (28) the impact of cost/design drivers on the attributes of the product ($\frac{\partial z_i}{\partial v_k}$, $i = 1, \ldots N$ and $k = 1, \ldots, K$) need to be determined. Table 4 summarizes this information. Table 5 shows the estimated sensitivity of the costs on the cost/design drivers ($\frac{\partial C}{\partial v_k}$, $k = 1, \ldots, K$) and the demand ($\frac{\partial C}{\partial d}$). These estimates are based on an assessment by telecommunication engineers and partly drawn from experiences with preceding and similar products. The uncertainty in this evaluation is again indicated by the standard deviation.

The uncertainty in the data is represented by means of 50 scenarios with equal probability $\rho_s = 1/50$. These scenarios have been generated from a uniform distribution. The underlying dispersion of the data was retrieved from the uncertainty uncovered by customer interactions and conjoint analysis as well as the stated data from engineers and management. For each scenario the sensitivity of the profit with respect to the cost/design drivers and the price was computed according to (12). Finally the following bounds on the changes in the cost/design drivers and the price were chosen: $\boldsymbol{\mathscr{v}} = \{0.1\}_{k=1,\ldots,K}$ and $\mathscr{p} = 0.1$.

Table 4: Impact of Cost/Design Drivers on the Attributes of the Product

	(1) $i = 1$	(2) $i = 2$	(3) $i = 3$	(4) $i = 4$	(5) $i = 5$	(6) $i = 6$	(7) $i = 7$
A: $= 1$	25 %	20 %	10 %	0 %	10 %	10 %	10 %
B: $= 2$	10 %	20 %	20 %	0 %	10 %	10 %	0 %
C: $= 3$	25 %	25 %	30 %	30 %	20 %	10 %	30 %
D: $= 4$	20 %	15 %	20 %	40 %	30 %	10 %	40 %
E: $= 5$	10 %	10 %	10 %	15 %	10 %	30 %	5 %
F: $= 6$	10 %	10 %	10 %	15 %	20 %	30 %	15 %

Table 5: Sensitivity of the Costs with Respect to the Cost/Design Drivers and the Demand

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	Demand
	$\frac{\partial C}{\partial v_1}$	$\frac{\partial C}{\partial v_2}$	$\frac{\partial C}{\partial v_3}$	$\frac{\partial C}{\partial v_4}$	$\frac{\partial C}{\partial v_5}$	$\frac{\partial C}{\partial v_6}$	$\frac{\partial K_T}{\partial d}$
Expectation	66,500	56,300	121,000	110,000	60,800	74,170	0.07
Standard Deviation	11,500	9,700	20,000	18,900	11,900	12,000	0.0133

With these data the target costing problem with controlled risk as described by (26) to (28) can be solved by standard optimization software. In Table 6 the optimal decisions, i. e. the changes in the cost/design drivers and the change in the price are given for different values of λ. This table also shows the change of the expected profit $\Delta\bar{G}$ (the higher the better) and the change in the risk $\Delta\mathbb{R}$ (the lower the better). Figure 3 illustrates the trade-off between expected profit and risk graphically. The origin in this figure is the departing point with no changes in design and price. For a lambda-value $\lambda = 0$ we have the maximum reduction of risk. However, the reduction of risk also implies a heavy reduction of the expected profit. For $\lambda = 1.0$ we obtain the maximum profit increase which comes with a substantial increase in the risk. For a lambda-value $\lambda \approx 0.5$ risk can be reduced without having a reduction in the expected profit. Respectively, at $\lambda \approx 0.62$ the expected profit can be increased without incurring additional risk. Depending on the decision maker's (product designer's) attitude towards risk and expected profit the focus can now be directed on the appropriate adjustment of service components and price.

Table 6: Target Costing Results

λ	Δv_1	Δv_2	Δv_3	Δv_4	Δv_5	Δv_6	Δp	$\Delta\mathbb{R}$	$\Delta\bar{G}$
0	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.09152	-1863	-1481
0.3	-0.1	-0.1	-0.00113	-0.1	0.1	-0.1	-0.0614	-1564	-536
0.5	-0.1	-0.1	0.1	-0.1	0.1	-0.1	-0.04011	-1158	-23
0.6	-0.1	-0.1	0.1	-0.1	0.1	-0.1	0.00539	-248	771
0.7	-0.1	-0.1	0.1	-0.1	0.1	-0.1	0.0551	1420	1639
0.9	0.1	-0.1	0.1	-0.1	0.1	-0.1	0.1	3988	2522
1.0	0.1	0.1	0.1	-0.1	0.1	-0.1	0.1	4209	2533

Table 6 highlights these changes for different trade-offs between risk and expected profit. Entries Δv_k highlighted with gray indicate a necessary reduction of the costs, which needs to be achieved by appropriate measures like reducing the complexity of components, simplifying production processes or negotiating lower procurement prices. Accordingly, entries Δp highlighted with gray indicate a necessary price reduction. Lower prices will have a positive effect on the demand and the costs per unit of the service components (enablers) and the whole service (economies of scale). Customers may also be willing to accept a lower quality at lower prices. Non-colorized entries Δv_k indicate that the costs of the corresponding components are relatively low compared to their contribution to the customer's utility. Therefore these components are allowed to carry more costs. Engineers should therefore focus on potential improvements of the design and quality. Accordingly, non-colorized entries Δp propose an increase in the price. However, design and cost adjustments cannot be seen isolated from price adjustments. If, for example, the strategy represented by $\lambda = 0.7$ is chosen, then the moderate price increase is only justified if the design of components C and E is improved while the costs of the other components are reduced.

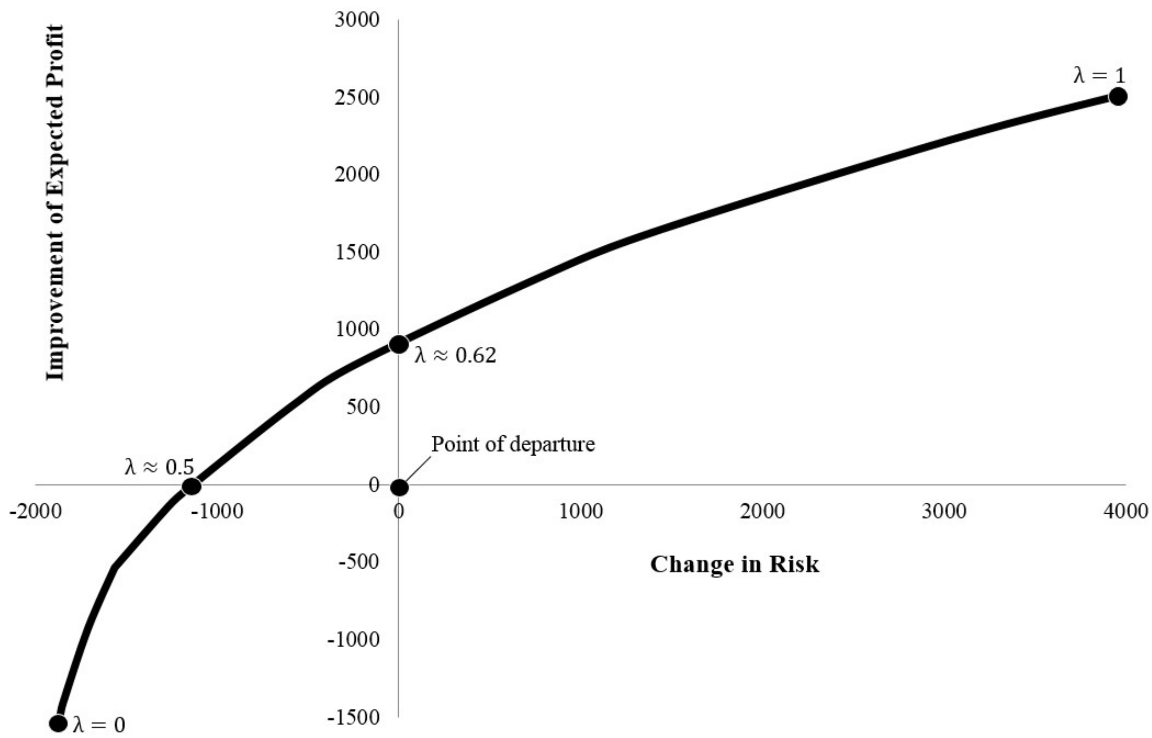

Figure 3: Trade-off between the Improvement of Expected Profit and the Change in Risk

6 Conclusion

In this paper we have extended the traditional target costing approach with concepts from optimization and risk management. More particularly, we have developed a model that helps decision makers in product design to appropriately adjust product components and price. These adjustments are driven by a trade-off between financial improvement on one hand and risk reduction on the other hand. Although we have applied our approach to a case in the ICT industry, our framework can be applied to any other branch of the service and manufacturing industry where fearful competition and substantial uncertainties are immanent. We believe that the treatment of uncertainty and risk in target costing as proposed by our approach is necessary for the successful application of target costing in such industries.

However, there are a few issues for further research. During the analysis of the practical case we have recognized that ICT services may not be provided in a single variant or stand alone. Services can vary with respect to the number and kind of features that they obtain for different customer segments. Furthermore ICT services can be bundled and sold to customers in an integrated form, for example cloud office services that are coupled with cloud storage services. This is obviously the case for various services and products in other industries as well. Nowadays, products and services are offered in customized versions in order to address different market segments. Furthermore, different products might use the same production processes and technologies. Target costing should therefore be further developed for the support of service and product portfolios rather than standalone services or products. A step in this direction is done by Kee and Matherly (2013) who use numerical examples for studying the effect of product and production interdependencies on target costing decisions.

We also recognized that products and services can have a changing price and cost structure over time. For example, services need to be offered for free in order to gain a critical mass. We therefore encourage research on joining target costing with multi-period concepts such as life-cycle costing and tools from investment analysis.

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Appendix

Let us consider two optimization problems:

Problem 1: $\max_{x \in X} F(x)$

$s.t.\ G(x) \leq \sigma$

Problem 2: $\max_{x \in X} [\lambda F(x) - (1-\lambda) G(x)]$

Proposition: Suppose that $X \subset R^n$ is a compact set and functions $F(x)$ and $G(x)$ are continuous on X. Then for any $\lambda' \in (0,1]$ we have that any solution x' of problem 2 for $\lambda = \lambda'$ is a solution of problem 1 for $\sigma = G(x')$.

Proof: Due to compactness of X and continuity of $F(x)$ and $G(x)$ the solutions of Problems 1 and 2 exist for any λ and $\sigma \geq \min_{x \in X} G(x)$.
Suppose now that x' is an arbitrary solution of Problem 2 for some $\lambda' \in (0,1]$. Let $\hat{x}$ be an arbitrary solution of Problem 1 for $\sigma = G(x')$. Observe that x' is feasible for Problem 1 with $\sigma = G(x')$ and therefore $F(\hat{x}) \geq F(x')$. Suppose that

$$F(\hat{x}) > F(x') \tag{A.1}$$

Since $\hat{x}$ is feasible for problem 1 with $\sigma = G(x')$ then

$$G(\hat{x}) \leq G(x') \tag{A.2}$$

Let us substitute $\hat{x}$ into the objective function of problem 2. Due to (A.1) and (A.2) and $\lambda' \in (0,1]$ we have

$$\lambda' F(\hat{x}) - (1-\lambda') G(\hat{x}) \geq \lambda' F(\hat{x}) - (1-\lambda') G(x') > \lambda' F(x') - (1-\lambda') G(x')$$

Thus, $\hat{x}$ yields a larger value for the objective of problem 2 for $\lambda = \lambda'$ than x' and is feasible for problem 2. This contradicts with our assumption that x' is a solution of Problem 2 for $\lambda = \lambda'$. Therefore (A.1) can not hold and, consequently,

$$F(\hat{x}) = F(x').$$

Since $\hat{x}$ is a solution of problem 1 this means that also x' is a solution of problem 1. This completes the proof.