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# TOWARD A HOLISTIC LOAD MODEL FOR STRUCTURES IN BROKEN ICE

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## ABSTRACT
Over the past decade we have seen an increase in marine operations in Arctic waters. Despite research and work on several offshore standards and ship rules, the ice loads on fixed and floating structures are not fully understood. We are still a long way from being able to formulate standards and rules strictly from theory.

If physical ice management is involved, where icebreakers reduce floe sizes and break ridges upstream of the floating structure, we are thus given a possibility to define/design our structure's working ice environment. Different from level ice and ice ridges, the design codes do not provide standard procedures for calculating actions on offshore structures from broken ice fields. Engineers still have to utilize available full-scale data, to use empirical formulae and to perform physical and numerical modelling in order to give answers to practical problems. Within this context, there is a strong interest to develop 'predictive' tools that will allow new structures to be optimized so as to minimize ice loadings and to evaluate operational performance prior to final design verification in an ice test basin.

This paper presents several semi-analytical solutions that are useful to model interaction between floe ice and structures. Our ambition is to support the development of multi-body numerical simulators that incorporate rigid-body dynamics, hydrodynamics and ice mechanics in a three-dimensional space. Furthermore, as an extension to a previously developed map of competing failure modes of ice floes, we delineate a new map that includes ice crushing depth distribution for the dominant ice failure modes. This new map is based on observations of ice failure in contact with floating ship-shaped structures in level ice and in low ice concentrations. Localized crushing (as the major bridge between initial contact and other possible failure modes), bending, radial cracking, splitting failure modes and a possibility for rotation of an ice floe of a finite size are considered.

## 1. INTRODUCTION
There is a continuing interest to study ice loads on various types of man-made structures located within or transiting through Arctic waters. The interaction between a structure and sea ice is a complex process that depends strongly on the ice conditions, the hull geometry, the relative velocity between the ice and the structure and the hydrodynamic aspects of interaction. For example, the interaction process between ship and level ice is usually divided into several phases such as breaking (crushing, bending, splitting), rotation, sliding, clearing; see e.g., Enkvist et al. (1979), Kotras et al. (1983) and Valanto (2001). When ship advances in

level ice, the ice breaking phase begins with a localized crushing of the free ice edge at the contact zone. The crushing force increases as the ship advances, and the contact area increases. This causes the ice sheet to deflect and the bending stresses to build up until the ice sheet fails. Flexural failure occurs some distance from the crushing region. This distance (or the breaking length) depends on the ice thickness and the ship speed, among other factors.

The complexity of this interaction process necessitates formulation of an idealized yet realistic model for the interaction. For example, the Unified Requirements of the International Association of Classification Societies for Polar Ships (IACS UR 2011) specify a particular design scenario as the design basis for local plating, i.e., an oblique collision with an infinitely large ice floe, where ice crushing and bending failure models are considered (IACS, 2011). This is just a theoretical idealization; the actual ice conditions may include discontinuous and inhomogeneous ice features, such as ice ridges, leads or discrete ice floes of various shapes and sizes, which form a broken ice field.

For fixed or floating offshore installations in Arctic waters, the design scenario may be different from the original ice conditions because icebreakers may reduce floe size of the drifting ice upstream the floating platform. Broken ice-structure interactions are not well studied compared with those in level ice conditions. Different from level ice and ice ridges, the design codes do not provide standard procedures for calculating actions on offshore structures from broken ice fields. Engineers still have to utilize available full-scale data, to use empirical formulae and to perform physical and numerical modelling in order to give answers to practical problems. Within this context, there is a strong interest to develop 'predictive' tools that will allow new structures to be optimized so as to minimize ice loadings and to evaluate operational performance prior to final design verification in an ice test basin.

Numerically, time-domain modelling is inevitable due to considerable nonlinearities in the ice-structure interaction process. Some of the existing frameworks for simulating interaction between floating structures and sea ice (e.g., the model by Alawneh, 2014; Lubbad and Løset, 2011; Metrikin, 2014 and Septseault et al., 2014) allow for supplementing the multi-body solver (or the governing equations of motion) with analytical closed form solutions to represent icebreaking processes.

This paper presents several solutions that are useful to model the interaction between floe ice and a structure. The term '*floe ice*' is used here to describe any fragmented ice field whether it is naturally broken, e.g., by gravity waves, or artificially broken as in the case of ice management operations. Our ambition is to support the development of multi-body numerical simulators that incorporate rigid-body dynamics, hydrodynamics and ice mechanics in a three-dimensional space.

The present study combines a currently existing contact model for ice crushing (Daley, 1999) with recent developments in the mechanics of ice loads in terms of finite sized ice floe's fracturing calculations (Lu, 2014). As an extension to a previously developed map of competing failure modes of ice floes, we delineate a new map that includes ice crushing depth distribution for the dominant ice failure modes. It is recognized that the fracture energy serves an important role within the defined failure modes, and the crushing depth increases with increasing mass of the ice floe.

## 2. MODEL DESCRIPTION

Consider an ice floe that is moving at speed $V$ and impacting a stationary ship type structure (Figure 1). The speed is fast enough to impart brittle ice behaviour. The collision occurs at point '$O$' and, in the frictionless case, results in a normal force $F_n$ along the collision line $O\eta$; see a side view in Figure 1. The ice edge has in-plane front angle $\varphi$. The hull is assumed to be rigid; only the ice is deforming. Localized crushing, bending, radial (or circumferential) cracking, splitting failure modes and a possibility for rotation of an ice floe of a finite size are considered (Figure 2). This consideration is based on the field observations of failure patterns and failure modes during the KV Svalbard's transit and during interaction of IB/RV Oden with finite size ice floes in the Greenland Sea (Lu, 2014).

For a given failure mode and hull-ice contact conditions, theoretical formulations are used to estimate the forces required to fail an ice floe. In these formulations, dynamic response from the ice floe and the fluid beneath the ice floe is neglected. The presented model is an extension of a model for calculating the finite-sized ice floe's failure loads proposed by Lu (2014), and it includes a direct load calculation due to ice crushing. It should be emphasized that the 'localized crushing' of the free ice edge always takes place at the ice-structure contact region, irrespective of the eventual failure pattern.

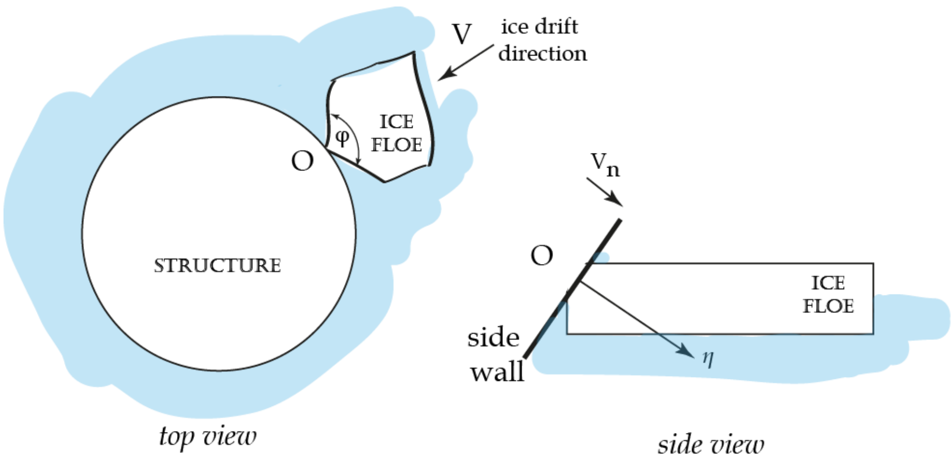


Figure 1. Illustration of a scenario of impact with an ice floe.

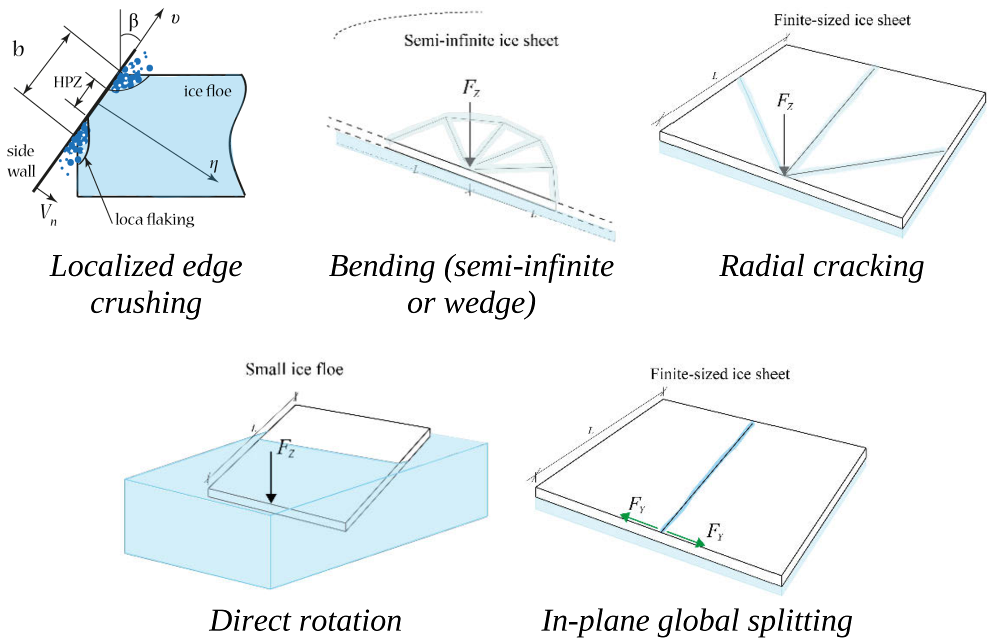


Figure 2. Idealised load models describing different failure scenarios of a nearly square-shaped ice floe.

In order to consider the different failure modes of the ice floe, another coordinate system *X*, *Y* and *Z* is introduced (Figure 3), with origin at the center of contact area and fixed with the ice floe.

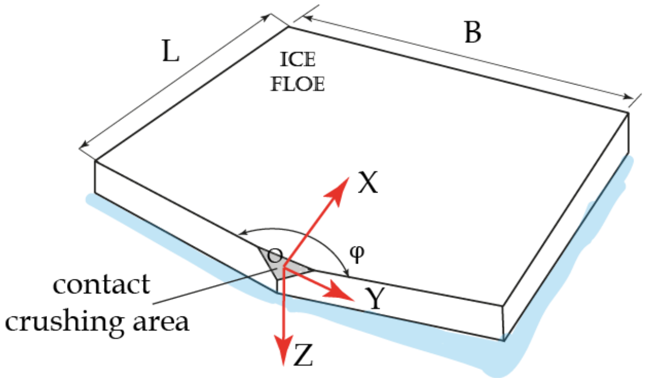


Figure 3. Coordinate system for ice floe failure calculations.

The vertical force component $Fz$ produces a potential out-of-plane flexural failure mode, radial cracking and a direct rotation of the floe; a pair of horizontal forces $F_Y$ produces a potential in-plane splitting failure; and the in-plane force component $F_X$ increases the compression within the floe. The force components applied on the ice floe are related to the normal contact force $F_n$ according to Eq. (1). The force components corresponding to different failure modes (i.e., local crushing, bending, in-plane splitting or radial cracking) are separately evaluated, including a scenario of direct rotation of an ice floe without fracture. The ice loads due to localized crushing are calculated in accordance with the IACS UR approach (see Daley, 1999 and Daley, 2000), whereas ice loads caused by bending, radial (or circumferential) cracking, rotation or in-plane splitting of the ice floe are determined by the methods proposed by Lu et al. (2015a,b,c) for an ice floe of a finite size. The minimum of these load components determines the failure mode, the maximum contact force and also the maximum crushing depth for a given impact scenario.

$$\begin{aligned} F_X &= F_n \cdot \cos\beta \\ F_Y &= 0.5F_X = 0.5F_n \cdot \cos\beta \\ F_Z &= F_n \cdot \sin\beta \end{aligned} \tag{1}$$

*2.1 Ice loads due to localized ice edge crushing*

This section presents the idealized *initial* interaction process between a structure's hull and an ice edge. The main equations are presented for ice-load calculations due to local crushing of an ice edge with a front opening angle $\varphi$. An analytical solution was proposed by Daley (1999 and 2000). The notation and formulations given in Daley (2000) are used here with minor changes. The crushing force depends on the geometry of the ice floe and the depth of penetration into the ice ($\delta$). The ice force ($F_n$) is characterized by an average pressure ($p_{cr}$) that is uniformly distributed over the nominal contact area ($S$). The force is calculated by integrating the ice crushing pressure over the nominal contact area. The average pressure depends on the size of the nominal contact area and is determined using a Sanderson-type '*process pressure-area relationship*' (Sanderson, 1988):

$$p_{cr}(\eta) = P_0 \left( \frac{S(\eta)}{S_0} \right)^{ex}, \tag{2}$$

where $S_0$ is the reference contact area ($S_0$=1.0 m$^2$), the leading coefficient $P_0$ and the exponent $ex$ are constants. $P_0$ can be interpreted as the ice pressure that occurs when the area is 1.0 m$^2$. In turn, $p_{cr}$ can be interpreted as the average pressure that accounts for softening behaviour of ice, direct hull-ice contact (or high-pressure zones) and contact with the crushed ice, where the crushed ice can extend from a high-pressure zone to the edge of the nominal contact area. The average contact pressure decreases with the nominal contact area (i.e., $ex$<0 in Eq. (2)). The crushing force is calculated as in the following:

$$F_{cr} = F_n(\delta) = \int_0^{\eta=\delta} p_{cr}(\eta) \cdot S(\eta) d\eta = P_0 \left( \frac{\delta^2 \tan 0.5\varphi}{\cos^2 \beta \cdot \sin \beta} \right)^{1+ex} . \tag{3}$$

Here $\delta$ is the ice crushing depth in the direction $\eta$, $P_0$ and $ex$ are the leading coefficient and the exponent of Sanderson's process pressure-area relationship, respectively; $\beta$ is the frame angle and $\varphi$ is the ice edge front opening angle.

*2.2 Ice loads due to bending of a semi-infinite ice floe*

According to the description of a semi-infinite ice floe's failure process (Kerr, 1976) and Nevel's (1972) simplification of his previous works (Nevel, 1958; Nevel, 1961; Nevel, 1965), the following Eq. (4) was proposed to calculate the out-of-plane bending failure of a semi-infinite ice floe (Lu et al., 2015c). The calculation model is illustrated in Figure 4.

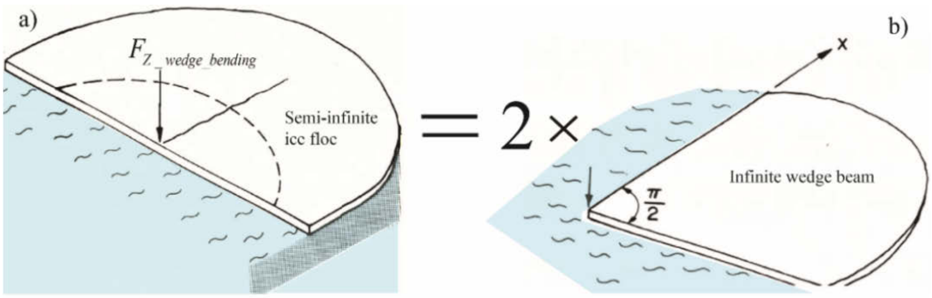


Figure 4. Failure of a semi-infinite ice floe.

$$F_Z = \frac{2}{3}\sigma_f t^2 [1.05 + 2.0(\frac{R}{\ell}) + 0.50(\frac{R}{\ell})^3] \tag{4}$$

in which,

| | |
|---|---|
| $\sigma_f$ | the flexural strength of sea ice; |
| $t$ | thickness of the considered ice floe ($t \leq l$/l0); |
| $R$ | half width the contact area; and |
| $\ell$ | The characteristic length of a floating ice plate. It can be calculated with Eq. (5). |

$$\ell = \sqrt[4]{\frac{D}{k}} \tag{5}$$

in which,

| | |
|---|---|
| $D$ | flexural rigidity of a sea ice plate with $D$=$E^3t$/(12(1–$v^2$)), in which, $E$ and $v$ are Young's modulus and Poisson ratio, respectively; and |
| $k$ | elastic foundation modulus. For the fluid base $k$=$\rho_w$ $g$, where $\rho_w$ and $g$ are the fluid |

density and the gravitational acceleration, respectively.

The force calculated by Eq. (4) can be interpreted as a critical vertical force to initiate circumferential cracks in a semi-infinite ice floe.

*2.3 Ice loads due to rotation*
Rather small ice floes ($L \leq l$) are treated as a short beam with its flexural deflections neglected (Hetényi, 1946). When the vertical deflection at the loading end is below the water line, the required rotation force $F_z$ is assumed to decrease. Potential ventilation effect (Lu et al., 2012; Valanto, 2001), which increases the rotation force, is disregarded. The formulation has been derived for floes with typical size $L \leq l$ as in Eq. (6). Detailed calculation model can be found in Figure 5.

$$F_Z = (1 - \frac{\rho_i}{\rho_w})\frac{t}{4}\rho_w gBL \tag{6}$$

Here,

| | |
|---|---|
| $\rho_i$ and $\rho_w$ | are the ice and fluid density, respectively; |
| $g$ | is gravitational acceleration; |
| $B$ | is the width of the considered ice floe; and |
| $L$ | is length of the floe. |

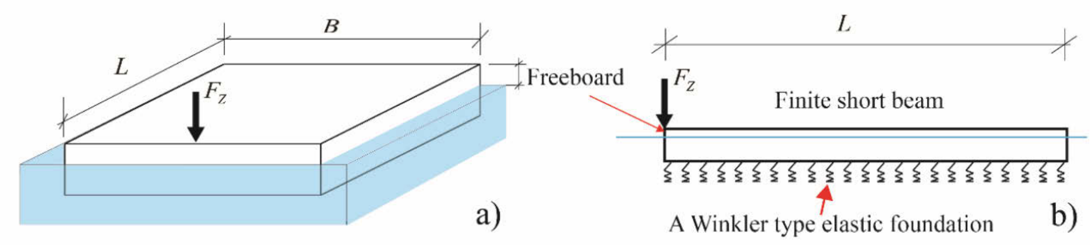


Figure 5. Model for the direct rotation of a small ice floe.

*2.4 Ice loads due to radial (or circumferential) crack*
Radial crack initiation and propagation in a finite sized ice floe was studied by Lu et al. (2015b and 2015c). They found that a square ice floe fails by directly forming a radial crack under the vertical force *Fz* if the floe size *L* is in a range $L \leq 2l$. Within such floe size range, the initiated crack can 'feel' the boundary and thus propagate through the whole floe. A rectangular ice floe of arbitrary size (*B* – width and *L* – length) can fail by directly forming circumferential cracks when $B \geq L$; or it can fail by a directly forming radial crack when $B < L$. A conservative displacement controlled criteria was utilised to solve this engineering problem analytically. The mathematical solution that is based on the *symplectic mechanics method* (Li et al., 2013) was adopted for this purpose. This method was proved advantageous and useful in describing the direct radial cracking (see Figure 2) and direct circumferential cracking of a finite sized ice floe. Its formulation is rather complicated and details can be found in the literature (Li et al., 2013) with a specific application in Lu et al. (2015c).

In summary, this analytical solution takes care of rectangular ice floes whose lengths are $l < L \leq 2l$. An ice floe is assumed to fail once its maximum displacement exceeds the freeboard of an ice floe. Superposition of results that are based on three different boundary conditions is utilised to achieve the final solution of a floating ice floe under a vertical concentrated edge load with free boundaries. The critical force reads:

$$F_z = \frac{D}{WL^2}\left(1 - \frac{\rho_i}{\rho_w}\right)t\,, \tag{7}$$

where $W$ is the normalized deflection of the ice floe in the z-direction. A brief description of the derivation of $W$ can be found in Lu et al. (2015c) whereas the detailed mathematical formulation and derivations can be found in the original work by Li et al. (2013).

*2.5 Ice loads due to in-plane splitting failure*

This section presents an idealized model of ice splitting failure that utilizes fracture mechanics approach to calculate the driving force behind crack propagation. The potential dynamic effects (e.g., stress wave propagation) within the ice floe have been neglected. The floe ice – structure contact was simplified by considering only a pair of equally opposed force components $F_Y$. Splitting failure has been addressed as an in-plane Mode-I fracture scenario (Lu et al., 2015a) for which 'one parameter' Linear Elastic Fracture Mechanics was combined with a weight function method (Bueckner, 1970; Rice, 1972). For symmetrically loaded ice floe with a pre-existing crack of length $A$, the critical force $F_Y$ required to propagate the splitting crack can be calculated by Eq. (8).

$$F_Y(A) = \frac{tK_{IC}\sqrt{L}}{H(A,0,k_{AR})} \tag{8}$$

in which,

| | |
|---|---|
| $A$ | is the length of the initial crack, where $A$=0 denotes a crack free body; |
| $F_Y(A)$ | is the splitting load with a normalized crack length $A/L$ |
| $t$ | is the ice floe thickness; |
| $K_{IC} = \sqrt{G_f E}$ | is the fracture toughness of sea ice, $G_f$ is the fracture energy; |
| $H(A,0,k_{AR})$ | is a weight function of the given symmetrically cracked ice floe with the splitting force acting at the crack mouth. |
| $k_{AR}$ | is a factor that takes into account ice floe's width-to-thickness ratio. |

Specifically, for an edge-cracked square ice floe and a circular ice floe, the maximum splitting force that is required to propagate a splitting crack of length $A$ can be calculated as in Eq. (9) (Lu et al., 2015a). Similar results have also been obtained by other authors before (Bhat, 1988; Bhat et al., 1991; Dempsey et al., 1993).

$$\begin{aligned} &F_Y = 0.19tK_{IC}\sqrt{L} \quad \text{for a square ice floe with critical crack length } A=0.145L. \\ &F_Y = 0.17tK_{IC}\sqrt{L} \quad \text{for a circular ice floe with critical crack length } A=0.165L. \end{aligned} \tag{9}$$

*2.6 Summary*

A closed form solution that accounts for aforementioned ice failure modes is difficult to obtain. In lieu of this, the normal force ($F_n$) may be determined as follows:

$$F_n = \min \begin{cases} P_0 \left( \dfrac{\delta^2 \tan 0.5\varphi}{\cos^2 \beta \cdot \sin \beta} \right)^{1+ex} & \text{crushing} \\ k_{\varphi b} \dfrac{1}{\sin \beta} \left( \dfrac{2}{3} \sigma_f t^2 (1.05 + 2.0R/l + 0.50R/l) \right) & \text{bending} \\ k_{\varphi r} \dfrac{1}{\sin \beta} (1 - \rho_i / \rho_w) \dfrac{t}{4} \rho_w gBL & \text{rotation} \\ k_{\varphi rc} \dfrac{1}{\sin \beta} \dfrac{D}{WL^2} \left( 1 - \dfrac{\rho_i}{\rho_w} \right) t & \text{radial } \textit{(circumferential)} \text{ craking} \\ k_{\varphi s} \dfrac{2}{\cos \beta} \dfrac{tK_{IC} \sqrt{L}}{H(A, 0, k_{AR})} & \text{in - plane global splitting} \end{cases} \tag{10}$$

Here $k_{\varphi b}$, $k_{\varphi r}$, $k_{\varphi rc}$ and $k_{\varphi s}$ are factors that account for ice front opening angle $\varphi$. The minimum of the load components in Eq. (10) determines the failure mode, the maximum contact force and also the maximum crushing depth for a given impact scenario.

## 3. CALCULATION EXAMPLE

Through theoretical analysis and numerical simulations Lu (2014) delineated *a map of competing failure modes of ice floes*, which for given contact forces on the ice floe determines whether or not failure will take place and in which mode. As an extension of Lu's work, the load model in Section 2 is used to construct a new map that includes the ice crushing depth distribution for dominant failure modes of the ice floe.

In this calculation example, a relatively open and broken ice field with minimal confinement is assumed. Within this ice environment, we consider impact of an ice floe with a sloping front structure. It is recognized that in higher ice concentrations, floe-floe interactions generate somewhat different boundary conditions. However, we do not quantify the effects of these herein. The size $L$ of the ice floe is considered as a variable. Other important variables are listed in the Table 1.

Table 1. Summary of input data.

| | |
|---|---|
| Frame angle, $\beta$ | 45° |
| Ice thickness, $t$ | 3.0 m |
| Yong's modulus, $E$ | 5 GPa (in accordance with ISO A.8.2.8.9) |
| Poisson ratio, $v$ | 0.3 |
| Water density, $\rho_w$ | 1025 kg/m$^3$ |
| Ice density, $\rho_i$ | 900 kg/m$^3$ |
| Flexural strength, $\sigma_f$ | 650 kPa |
| Leading coefficient in Sanderson's pressure-area relationship, $P_0$ | 6.0 MPa·m$^{0.2}$ |
| Exponent in Sanderson's pressure-area relationship, $ex$ | –0.1 |
| Ice floe front opening angle[(a)], $\varphi$ | 150° (as in the IACS UR scenario for local design of plating ) |
| Half width of the contact area, $R$ | 0.1$l$ ($l$ is the characteristic length) |
| $k_{\varphi b}$, $k_{\varphi r}$, $k_{\varphi rc}$ and $k_{\varphi s}$ | 1.0 (for simplicity) |
| Width of the ice floe, $B$ | $L$ (floe aspect ratio1.0) |
| Fracture energy, $G_f$ | 1.0 N/m and 15 N/m |
| Critical crack length, $A$ | 0.145$L$ |

[(a)] Popov et al. (1967) calculated the magnitude of $\varphi$ by assuming the dimensions of the segments that were broken off by the icebreaker. Their calculations indicated that $\varphi$ can vary over a wide range, from 45° to 145°; average values between 90° and 100° were recommended for calculations. In the IACS UR approach, $\varphi$ =150° is used.

Figures 6 and 7 demonstrate results of the calculations for two different values of ice fracture energy.

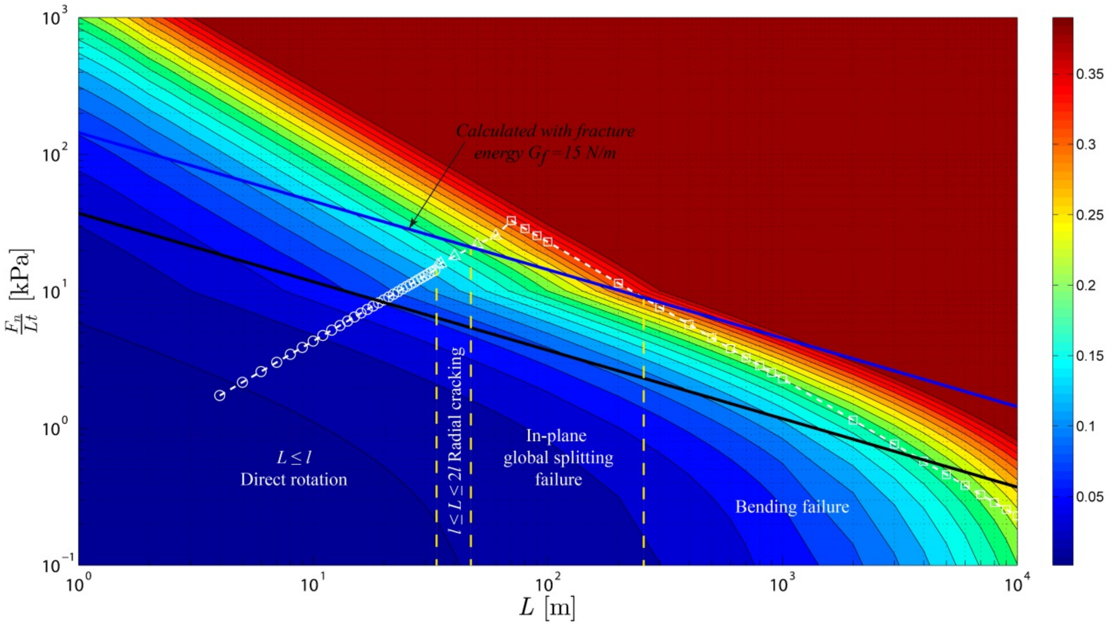


Figure 6. Sloping front structure (45$^{\circ}$): Understanding of possible ways in which a nearly square ice floe of varying sizes and the thickness of 3.0 m may fail when the value of fracture energy is based on *field measurements* (Dempsey et al., 1999); the resulting loads are limited by limit-stress conditions; contour plot of crushing depth (units: m).

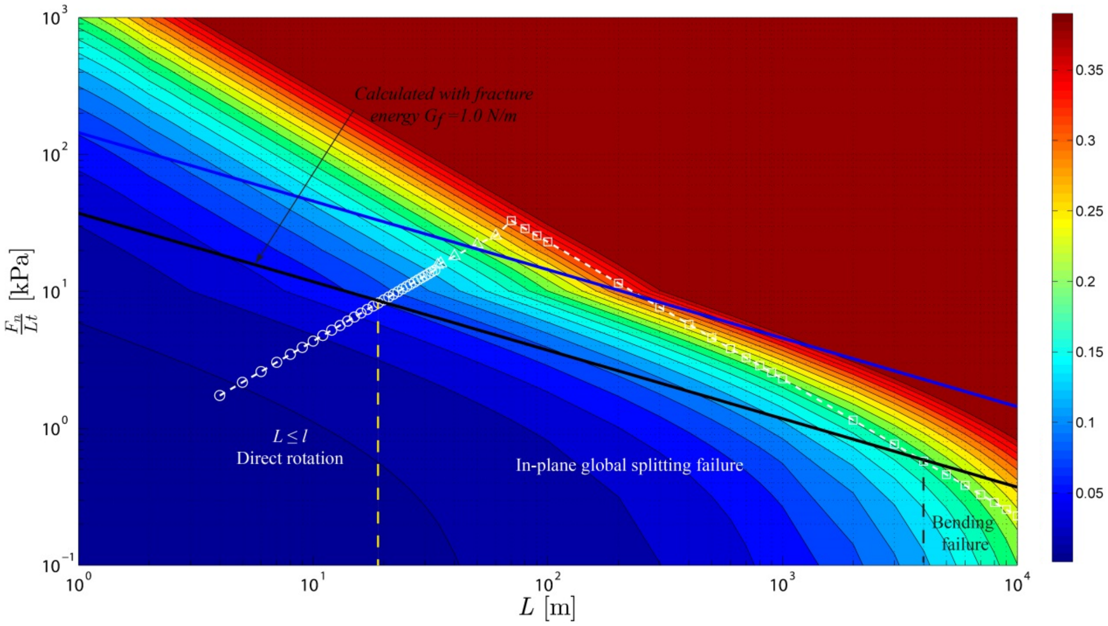


Figure 7. Sloping front structure (45$^{\circ}$): Understanding of possible ways in which a nearly square ice floe of varying sizes and the thickness of 3.0 m may fail when the value of fracture energy is based on *laboratory measurements* (Schulson and Duval, 2009); the resulting loads are limited by limit-stress conditions; contour plot of crushing depth (units: m).

To interpret Figures 6 and 7, the dominant failure scenarios are highlighted over the considered floe sizes (up to 10 km). The ice crushing depth distribution, which corresponds to different failure scenarios, is also presented in the figures.

## 4. DISCUSSION

The calculation example above applies to open and broken ice fields with minimal confinement. The ice failure modes 'compete' with each other. As the ice floe with $B=L \leq l$ impacts the sloping front structure, a complex stress field forms within the contact area and the ice edge undergoes local crushing followed by direct floe rotation (Figures 6 and 7). When conditions permit (i.e., a relatively small ice floe $l < B=L \leq 2l$ and a relatively low ice concentration), the radial cracking (Figure 6) and in-plane global splitting failure (Figures 6 and 7) tend to dominate the bending failure mode. The results of the calculations in Figures 6 and 7 demonstrate that small ice floes with $L \leq l$ will be tilted with no significant ice edge crushing (i.e., the crushing depth is approximately 0.05 – 0.10 m).

When the sloping-front structure interacts with a relatively large ice floe (e.g., level ice $L>>2l$), or when an ice floe's lateral boundary confinement is significant, the local edge crushing is expected to followed by bending failure. This behaviour is similar to the ice-failure-mode scenario of IACS UR for local design of plating.

The crushing depth increases with increasing floe size (mass). The calculations also showed that the dominant failure mode, the corresponding load and the maximum crushing depth are significantly influenced by the value of the fracture energy. For 3-m thick ice floe, if the value of $G_f$ =1 N/m is used (Figure 7), the splitting failure dominates over other failure modes whereas radial cracking mode is absent. With decreasing fracture energy a larger ice floe is required to be considered as level ice (reference is made to the intercession between bending failure and in-plane global splitting in Figures 6 and 7). In order to have a better understanding of this fracture energy number for sea ice, a test campaign is now under planning at the Centre for Sustainable Arctic Marine and Coastal Technology (SAMCoT). A pilot *in-situ* test has been carried out (Lu et al., 2015d) in 2015. More results shall be reported in separate papers in this regard.

## 5. CONCLUDING REMARKS

The presented solutions are based on observations of ice failure in contact with floating ship-shaped structures in level ice and in low ice concentrations. The load model is an extension of existing models for calculating the finite-sized ice floe's failure loads, and it includes a direct load calculation due to ice crushing. The ice breaking begins with a localized crushing of the free ice edge at the contact zone. The crushing force increases with increasing contact area until another failure mode corresponding to the lowest estimated load occurs at the ice–structure interface.

To demonstrate model performance, a calculation example has been presented where a 3-m thick ice floe interacts with the 45$^\circ$ sloping front structure. The ice failure modes are analytically quantified by studying each possible failure scenario and the corresponding load components in a decoupled manner. The minimum of these load components determines the failure mode, the maximum contact force and also the maximum crushing depth for a given impact scenario. Through the theoretical analysis, a new map of competing failure modes was delineated for a nearly square the ice floe of various sizes.

The model, presented in this paper, allows for a direct calculation of the critical normal force and identification of the corresponding ice failure mode and the crushing depth for ice floes of different sizes.

- The fracture energy plays an important role within the defined failure modes. The general trend is that the smaller the fracture energy, the greater is the chance for ice to fail in global splitting. With a greater value of fracture energy (15 N/m), a smaller size ice floe is required to be considered as level ice at the initial contact with sloping structure.
- The crushing depth increases with increasing size of the ice floe and is greatest for the level ice-structure interaction scenario. The crushing depth is the smallest for a direct rotation of small ice floes. This result verifies assumption made earlier about the tilting of an ice floe with no significant material failure.

The presented solutions can account for the geometry of ice floes that makes them well suited for the framework of multi-body simulations, which are a very useful tool not only to optimize the structural and moorings design but also to enhance marine operations such as ice management and dynamic positioning operations. The calculation of floe ice actions is also important for the design of free-going vessels and possibly for route optimization.

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