HVDC Meshed Multi-Terminal Networks for Offshore Wind Farms: Dynamic Model, Load Flow and Equilibrium

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Abstract

This paper presents sufficient conditions for the existence of an equilibrium point of multi-terminal HVDC (MT-HVDC) network for offshore applications. Newton's method is used both as a computational tool and as the basis for proving the existence and uniqueness of the equilibrium. A simple but generalized dynamic model of the MT-HVDC grid for offshore wind farm applications is presented which is non-linear due to the constant power loads with droop regulation. The classic Kantorovitch theorem is used to define the requirements for the existence of the equilibrium and the super-convergence of Newton's method starting from voltages close to 1pu. Finding this equilibrium is equivalent to the power flow in power systems applications. Computational results corroborate the requirements for equilibrium as well as the convergence of the algorithm in a realistic MT-HVDC grid.

Index Terms

Offshore wind energy, Multiterminal HVDC systems, HVDC, transient stability, equilibrium point, dynamic systems.

I. Introduction

Nowadays, wind energy is considered a competitive alternative to conventional power plants and a promising technology for reducing the causes of global warming. However, large-scale expansion of wind farms is limited by factors such as the land use and visual impact of wind turbines. These limitations can be overcome by the installation of offshore wind farms and the use of multi-terminal high-voltage direct current transmission (MT-HVDC).

In these type of grids, offshore wind farms are usually integrated through a centralized AC/DC converter that maintains constant voltage at the offshore ac grid. From the dc point of view, the centralized converter can be modeled as a constant power injection which makes the problem intrinsically non-linear. Any dynamical model must consider this non-linearity but a trade-off between simplicity and accuracy is required.

In this context, it is challenging to determine the exact conditions for the existence of the equilibrium since non-linear grids could have several equilibrium points; in some extreme cases, the equilibrium may not even exist. Finding the equilibrium point entails the classic power flow problem for power systems applications. Numerical methods such as the Newton Rapshon algorithm are used to find this equilibrium but additional analysis is required in order to guarantee convergence and uniqueness of the solution.

Existence of the equilibrium in grids with constant power terminals has been studied for dc grids with particular conditions, specially in microgrid applications. In [1] the equilibrium was ensured for systems with constant power terminals where certain inequality is satisfied. In [2], stability and power flow of ad hoc dc microgrids were investigated considering a worst-case scenario. The large-signal stability for microgrids was considered in these contributions, which can be used as starting point for the study of MT-HVDC where not only constant power loads but constant power generation and droop controls are considered.

Stability of MT-HVDC has been also studied from the small signal point of view (*e.g.* [3]), including the effect of droop controllers (*e.g.* [4]), and from the more practical transient stability perspective (*e.g.* [5]) based on extensive numerical simulations. On the other hand, power flow in MT-HVDC grids has been studied for long time (see for example [6]) but the subject is still an active research problem as can be inferred from [7] and [8], to name a few. However, in all these cases, the existence of the equilibrium is taken for granted without a formal demonstration. In addition, convergence of the algorithm is studied by numerical simulations without a generalized approach. To the best knowledge of the authors, exact conditions for the existence of the equilibrium and convergence of Newton's method in MT-HVDC grids have not been presented before. This paper addresses these subjects by a general modeling of the MT-HVDC grid and the use of the Kantorovitch theorem. The existence of the equilibrium is demonstrated as well as conditions for convergence. The general demonstration of the existence and uniqueness is complemented by numerical simulations that allows to find the equilibrium itself. This numerical study leads to a simple power flow algorithm.

The remaining of this paper is organized as follows: In section II a general model for MT-HVDC grids is presented. Moreover, in section III the conditions for the existence of the equilibrium and super-convergence of Newton's method are demonstrated. Simulations result are presented in section IV followed by conclusions and references.

II. Modeling MT-HVDC grids

Consider an MT-HVDC represented by terminals $\mathcal{N} = \{1, 2, ..., N\}$. Each terminal has a converter with either constant voltage control or constant power with voltage droop control represented by non-empty disjoint sets $\mathcal{V}$ and $\mathcal{P}$ such that $\mathcal{N} = \{\mathcal{V}, \mathcal{P}\}$. HVDC lines are characterized by a set of edges $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ each of them represented by a π-model. Edge and nodal variables are related by the edge-to-node oriented incidence matrix $A = [A_{\mathcal{V}}, A_{\mathcal{P}}] \in \mathbb{R}^{\mathcal{E}\times\mathcal{N}}$ as follows

$$V_{\mathcal{E}} = A_{\mathcal{V}} \cdot V_{\mathcal{V}} + A_{\mathcal{P}} \cdot V_{\mathcal{P}}, \tag{1}$$

$$I_{\mathcal{V}} = A_{\mathcal{V}}^T \cdot I_{\mathcal{E}}, \tag{2}$$

$$I_{\mathcal{P}} = A_{\mathcal{P}}^T \cdot I_{\mathcal{E}}, \tag{3}$$

where $V_{\mathcal{V}} \in \mathbb{R}^{\mathcal{V}}$ is the voltage in the constant voltage terminals, a value given by the tertiary control (i.e it is an input of the control), $A_{\mathcal{P}} \in \mathbb{R}^{\mathcal{E}\times\mathcal{P}}$, $A_{\mathcal{V}} \in \mathbb{R}^{\mathcal{E}\times\mathcal{V}}$ and $I_{\mathcal{E}} \in \mathbb{R}^{\mathcal{E}}$ are the current in each HVDC line, whose dynamics are given by

$$V_{\mathcal{E}} = L \cdot \frac{d}{dt} I_{\mathcal{E}} + R \cdot I_{\mathcal{E}}, \tag{4}$$

where $L, R \in \mathbb{R}^{\mathcal{E}\times\mathcal{E}}$ are diagonal matrices that represent inductive and resistive effects on the π-model. Both, L and R must be non-singular.

The dynamics of each power terminals are obtained by using Kirchhoff laws as follows:

$$I_{\mathcal{P}} + C_{\mathcal{P}} \cdot \frac{d}{dt} V_{\mathcal{P}} = G(V_{\mathcal{P}}), \tag{5}$$

where $C_{\mathcal{P}}$ is a non-singular diagonal matrix that includes the capacitance of each power terminal and the capacitive effect of the connected line segments. Capacitance of constant voltage terminals is neglected since any element in parallel to a voltage source is redundant. Furthermore, $G(V_{\mathcal{P}})$ is a vector function that represents the current on the power terminals as function of its voltage $V_{\mathcal{P}}$ and the drop control $K_{\mathcal{P}}$, *i.e.*,

$$G(V_{\mathcal{P}}) = diag(V_{\mathcal{P}})^{-1} \cdot (S + K_{\mathcal{P}} \cdot V_{\mathcal{P}}), \tag{6}$$

with

$$S = P_{ref} - K_{\mathcal{P}} V_{ref} \tag{7}$$

and P_{ref} and V_{ref} the references given by the tertiary control.

The dynamical system takes finally the following structure:

$$\frac{d}{dt} I_{\mathcal{E}} = -L^{-1} R I_{\mathcal{E}} + L^{-1} A_{\mathcal{V}} \cdot V_{\mathcal{V}} + L^{-1} A_{\mathcal{P}} \cdot V_{\mathcal{P}}, \tag{8}$$

$$\frac{d}{dt} V_{\mathcal{P}} = C^{-1} G(V_{\mathcal{P}}) - C^{-1} A_{\mathcal{P}}^T \cdot I_{\mathcal{E}}, \tag{9}$$

where it is assumed that all values are given in per unit.

III. Operation point

The fist step to analyze the dynamics of the MT-HVDC grid is to determine the operating point. In this regard, the following lemma is presented first:

Lemma 1 (operation point)**.** *An MTDC-HVDC network at its operating point, with constant input $\mathcal{V}_{\mathcal{P}} \in \mathbb{R}^{\mathcal{P}}$, admits the following representation:*

$$R \cdot I_{\mathcal{E}} + A_{\mathcal{V}} \cdot V_{\mathcal{V}} + A_{\mathcal{P}} \cdot V_{\mathcal{P}} = 0 \tag{10}$$

$$G(V_{\mathcal{P}}) - A_{\mathcal{P}}^T \cdot I_{\mathcal{E}} = 0 \tag{11}$$

Proof. since it is the equilibrium point, make zero the derivatives of (8) and (9) and simplify the resulting equations. □

Recall that $V_{\mathcal{V}}$ are the voltages in the voltage-controlled terminals at the operation point. This vector is known and constant. Notice that the operation point does not depend on the inductances or capacitances of the grid, then, finding the equilibrium point entails the same problem as the power flow. Notice also that R is invertible if the graph is connected. Therefore, the resulting system can be represented as $F(V_{\mathcal{P}}) = 0$ with

$$F(V_{\mathcal{P}}) = G(V_{\mathcal{P}}) - A_{\mathcal{P}}^T R^{-1} A_{\mathcal{V}} V_{\mathcal{V}} - A_{\mathcal{P}}^T R^{-1} A_{\mathcal{P}} \cdot V_{\mathcal{P}} = 0 \tag{12}$$

This equation can be rewritten in terms of the Y_{bus} matrix and a constant vector J defined as follows:

$$Y_{\mathcal{P}\mathcal{P}} = A_{\mathcal{P}}^T R^{-1} A_{\mathcal{P}} \tag{13}$$

$$Y_{\mathcal{P}\mathcal{V}} = A_{\mathcal{P}}^T R^{-1} A_{\mathcal{V}} \tag{14}$$

$$J = Y_{\mathcal{P}\mathcal{V}} V_{\mathcal{V}} \tag{15}$$

Then the system of non-linear equations yields:

$$F(V_{\mathcal{P}}) = G(V_{\mathcal{P}}) - Y_{\mathcal{P}\mathcal{P}} \cdot V_{\mathcal{P}} - J = 0 \tag{16}$$

The objective is not only to solve numerically this problem (i.e load flow) but also to analyze the existence of the solution which is equivalent to the existence of the equilibrium point. A Newton's method is proposed as presented in the next section.

IV. Newton's method

The resulting power flow problem is solved by using Newton's method. This method is used as a computational tool but also as the basis for proving the existence of the equilibrium. Our program is the following: first the basic Kantorovitch's theorem is presented which guarantee convergence of Newton's method and uniqueness of the solution, next we demonstrate the equilibrium point in the MT-HVDC grid fulfills all the conditions required by that theorem, finally, we present some stronger results about numerical convergence.

Theorem 1. *(Kantorovitch's theorem) Let x_0 be a point in $\mathbb{R}^n$, U an open neighborhood of x_0 in $\mathbb{R}^n$ and $F : U \to \mathbb{R}^n$ a differentiable mapping, with its derivative $DF(x_0)$ invertible. Define*

$$h_0 = -[DF(x_0)]^{-1} \cdot F(x_0) \tag{17}$$

$$a_0 = h_0 + x_0 \tag{18}$$

$$U_0 = \{x : \ \|x - x_0\| \leq \|h_0\|\} \tag{19}$$

If the derivative $[DF(x_0)]$ satisfies the Lipchitz condition

$$\|DF(x) - DF(y)\| \leq M \cdot \|x - y\| \tag{20}$$

for all points $x, y \in U_0$, and if the inequality

$$\|F(x_0)\| \cdot \left\|[DF(x_0)]^{-1}\right\|^2 \cdot M \leq \frac{1}{2} \tag{21}$$

*is satisfied, the equation $F(x) = 0$ has a **unique** solution in U_0,and Newton's method with initial guess x_0 converges to it. In addition, if the left side of Equation 21 is strictly $< 1/2$ then we can assure super-convergence of Newton's method.*

Proof. See chapter 2 of [9] □

Remark: The theorem guarantees that the solution is unique and hence it is a formal proof of the existence of the equilibrium of the original non-linear dynamical system in a closed ball given by U_0.

In order to use the theorem above, we consider the following assumptions for the MT-HVDC grid

- A1 The graph is connected and consequently $Y_{\mathcal{P}\mathcal{P}}$ is non-singular.
- A2 Nodal currents are lower than short circuit currents (i.e the system is not in short circuit).
- A3 The system is under non-critical conditions (i.e injected power in each terminal is bellow or equal its nominal).
- A4 The system is represented in per-unit.

These conditions are easily satisfied by any practical MT-HVDC grid. Hence, the main result be general for any feasible steady state operation.

Now consider the following proposition:

Proposition 1. *The jacobian matrix (derivative) of F in (16) satisfies the Lipchitz condition under the assumptions (A1-A4) for all points v_k in an open ball $\mathcal{B} = \{v : |v - 1| < \delta < 1\}$*

Proof. The derivative of F is given by

$$DF(V_{\mathcal{P}}) = -diag(S) \cdot diag(V_{\mathcal{P}}^{-2}) - Y_{\mathcal{P}\mathcal{P}} \tag{22}$$

then select two different points $V_{\mathcal{P}}, U_{\mathcal{P}} \in \mathcal{B}$ and calculate

$$\|DF(V_{\mathcal{P}} - U_{\mathcal{P}})\| \leq \|S\| \left\|diag(V_{\mathcal{P}}^{-2}) - diag(U_{\mathcal{P}}^{-2})\right\| \tag{23}$$

A submutiplicative norm is assumed in order to be able to use the CauchySchwarz inequality. In this case the norm is induced by the uniform norm as follows:

$$\|M\| = \sup_{\|x\| \neq 0} \frac{\|Ax\|}{\|x\|}. \tag{24}$$

Let us also define $\alpha = \|P_{ref} + K_{\mathcal{P}} V_{ref}\| \geq \|S\|$, then

$$\|DF(V_{\mathcal{P}} - U_{\mathcal{P}})\| \leq \alpha \cdot \max_i \left\{ \frac{1}{v_i^2} - \frac{1}{u_i^2} \right\} \tag{25}$$

$$\leq \alpha \cdot \frac{2}{(1-\delta)^3} \cdot \|V_{\mathcal{P}} - U_{\mathcal{P}}\| \tag{26}$$

then it satisfies the Lipchitz condition with

$$M = \alpha \cdot \frac{2}{(1-\delta)^3}. \tag{27}$$

□

Now consider the following result:

Proposition 2. *Under de assumptions (A1-A4), $DF(V_{\mathcal{P}})$ is invertible and its inverse is bounded*

Proof. Define $DF(V_{\mathcal{P}}) = DF_0$ for $V_{\mathcal{P}} = 1$ (i.e flat start) and consider its inverse

$$(-diag(S) - Y_{\mathcal{P}\mathcal{P}}) \cdot DF_0^{-1} = I \tag{28}$$

where I is the identity. Now multiply by $W = Y_{PP}^{-1}$ (this inverse exists due to assumption A1)

$$-W \cdot diag(S) \cdot DF_0^{-1} - DF_0^{-1} = W \tag{29}$$

Taking the norm

$$\begin{aligned} \|W\| &= \left\|-W \cdot diag(S) \cdot DF_0^{-1} - DF_0^{-1}\right\| \\ &\geq \left\|DF_0^{-1}\right\| - \|R\| \cdot \|S\| \cdot \left\|DF_0^{-1}\right\| \end{aligned} \tag{30}$$

and consequently

$$\left\|DF_0^{-1}\right\| \leq \frac{\rho}{1 - \alpha\rho} \tag{31}$$

where $\rho = \|W\|$.

□

Notice that W is diagonal dominant and the terms of the diagonal represent the Thevenin equivalent of each node. Therefore $1/\rho$ is the maximum short-circuit current in the system. This term must be higher than the operational current which is given by α for flat start conditions. This behavior agrees with assumption A2.

Now, the main result is presented:

Theorem 2. *An MT-HVDC network admits a unique operation point and this point can be calculated by Newton's-method with super-convergency iff*

$$\gamma = \left(\frac{2\alpha}{(1-\delta)^3}\right)\left(\frac{\rho}{1-\rho\alpha}\right)^2 \|F\| < \frac{1}{2} \tag{32}$$

with

$$\|F\| \leq \alpha + \|Y_{\mathcal{PP}}\| \tag{33}$$

Proof. The proof of this proposition is obtained by directly invoking Theorem 1 and propositions 1 and 2. □

Remark The value $\rho/(1-\rho\alpha)$ is very small for almost all practical MT-HVDC grids that fulfills assumptions A1 to A4. Condition of Proposition 1 can be evaluated before calculating Newton's method.

V. Stability Analysis

Lemma 2 (Structural stability)**.** *If the Jacobian matrix $DF(x)$ is negative definite then the dynamical system is stable.*

Proof. The linearized form of the dynamic system is

$$\begin{pmatrix} \Delta\dot{V}_{\mathcal{P}} \\ \Delta\dot{I}_{\mathcal{E}} \end{pmatrix} = B \left(\begin{array}{c|c} DG(x_0) & -A_{\mathcal{P}}^T \\ \hline A_{\mathcal{P}} & R \end{array}\right) \begin{pmatrix} \Delta V_{\mathcal{P}} \\ \Delta I_{\mathcal{E}} \end{pmatrix} \tag{34}$$

with B

$$B = \left(\begin{array}{c|c} C^{-1} & 0 \\ \hline 0 & L^{-1} \end{array}\right) \tag{35}$$

It is easy to demonstrate that the system with reduced rows is

$$\Delta\dot{V}_{\mathcal{P}} - (C^{-1}A_{\mathcal{P}}^T R^{-1} L)\Delta\dot{I}_{\mathcal{E}} = C^{-1}\cdot DF_0 \Delta V_{\mathcal{P}} \tag{36}$$

where $DF_0 = DG(x_0) - Y_{\mathcal{PP}}$ and $DG(x_0) = -diag(S)\dot{d}iag(V_{\mathcal{P}}^{-2})$. The matrix C^{-1} is positive definite. and if DF_0 is positive or negative semi-definite, then $C^{-1}DF_0$ is positive or negative semi-definite. This does not mean that the eigenvalues are the same. In fact they are different, but if for one cases they are negative for the other too. Therefore, it is sufficient to analyze the load flow Jacobian.

□

Proposition 3. *Under all considerations of Theorem 2, if all constant power terminals are generating then the equilibrium is stable.*

Proof. The proof is strainghtforward if $S \geq 0$, thus; the Jacobian (DF_0) is negative semi-definite, because $Y_{\mathcal{PP}}$ is positive definite, which is due to (A1) and $diag(V_{\mathcal{P}}^{-2})$. Moreover, the sum of two semi-definite matrices is a semi-definite matrix. □

Proposition 4. *Under all considerations of Theorem 2, the equilibrium is stable if*

$$-diag(S_{\mathcal{P}^-})\cdot diag(V_{\mathcal{P}^-}^{-2}) - Y_{\mathcal{PP}^-} \prec 0 \tag{37}$$

Proof. For this case it is necessary to split the power constants on two terms. $\mathcal{P}^+$ for the generation nodes and $\mathcal{P}^-$ for the nodes that are consuming, and in consecuence, the system is

$$\begin{aligned} -&\begin{pmatrix} diag(S_{\mathcal{P}^+}\cdot V_{\mathcal{P}^+}^{-2}) & 0 \\ 0 & diag(S_{\mathcal{P}^-}\cdot V_{\mathcal{P}^-}^{-2}) \end{pmatrix} \\ &\qquad - \begin{pmatrix} Y_{\mathcal{PP}^+} & Y_{\mathcal{PP}^\pm} \\ Y_{\mathcal{PP}^\pm}^T & Y_{\mathcal{PP}^-} \end{pmatrix} \prec 0 \end{aligned} \tag{38}$$

The nodes generating do not affect the problem, using the proposition 3; it was demonstrated that the subsystem is negative semi-definite. Therefore, with the Schur complement it is sufficient to analyze (37), which completes the proof.

□

VI. Computational results

Two set of simulations were performed in order to corroborate the theoretical results presented in the previous sections. First, a three-node MT-HVDC grid is tested. This grid allows graphical analysis since the resulting space is $\mathbb{R}^2$. After that, the methodology is tested in a modified version of the CIGRE MT-HVDC grid.

A. Simple MT-HVDC grid

The MT-HVDC grid depicted in Fig 1 is used to corroborate the analysis presented in the sections above. A first order approximation of the frequency dependent series impedance of the cable yielding $Z_s(\omega) = 0.0151 + j\omega 0.75 \times 10^{-3}$ Ω/km. Two offshore wind parks were considered whose droop gains are of 0.05 pu and reference voltages of 1.0pu. Distances of the cables are depicted in the figure as well as the generated power in each offshore wind farm. All numerical results are available in [10] The system is represented in per unit with a $P_{base} = 1000MW$ and $V_{base(dc)} = 400kV$. The matrix Y_{bus} for this case yields:

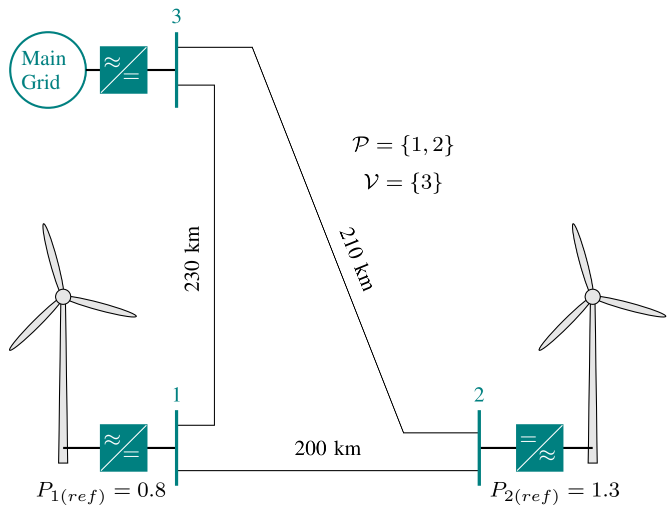

Fig. 1. Example of a simple MT-HVDC grid with two offshore wind farms

$$Y_{bus} = \begin{pmatrix} 99.0498 & -52.9801 & -46.0697 \\ -52.9801 & 103.4374 & -50.4573 \\ -46.0697 & -50.4573 & 96.5269 \end{pmatrix} \tag{39}$$

As $V_{\mathcal{P}} \in \mathbb{R}^2$ the ball U_0 around the initial point $V_{\mathcal{P}} = (v_2, v_3)^T = (1.00, 1.00)^T$ can be represented graphically as shown in Fig 2. We selected $\delta = 0.5$ to perform the numerical evaluations. At this point we can evaluate the expression 27, 31 and 32:

$$M = 38.2240 \tag{40}$$
$$\left\| DF_0^{-1} \right\| = 0.0186 \tag{41}$$
$$\gamma = 0.0319 < 1/2 \tag{42}$$

Since $\gamma < 1/2$ then super-convergence of Newton's method is assured initializing from $(1.00, 1.00)^T$. In addition, there is a unique equilibrium point inside the ball marked by U_0 in Fig 2.

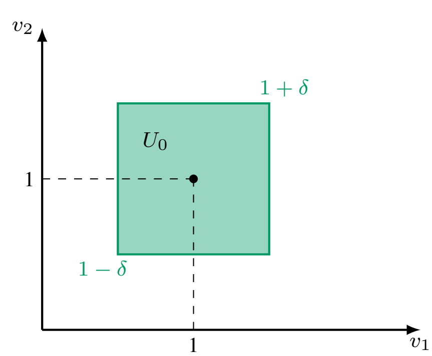

Fig. 2. Space of solutions in which it is guaranteed there is an equilibrium for the simple MT-HVDC grid depicted in the previous figure

Newton's method was evaluated in this system by executing only three iterations. Results are shown in Fig 3 were super-convergence is evident.

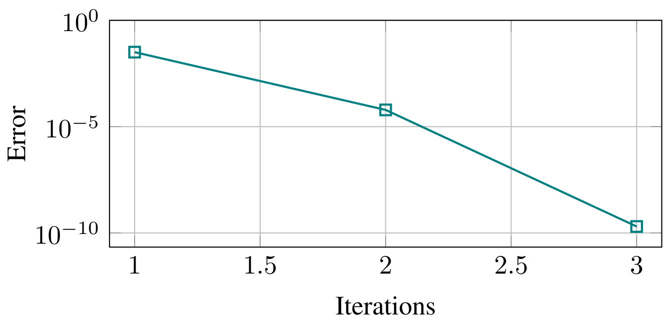

Fig. 3. Super-convergence of Newton's method applied to a simple MT-HVDC grid from an initial point $V_{\mathcal{P}} = (1.00, 1.00)^T$

B. CIGRE MT-HVDC grid

A second simulation was performed in modified version of the CIGRE MT-HVDC grid depicted in Fig 4.

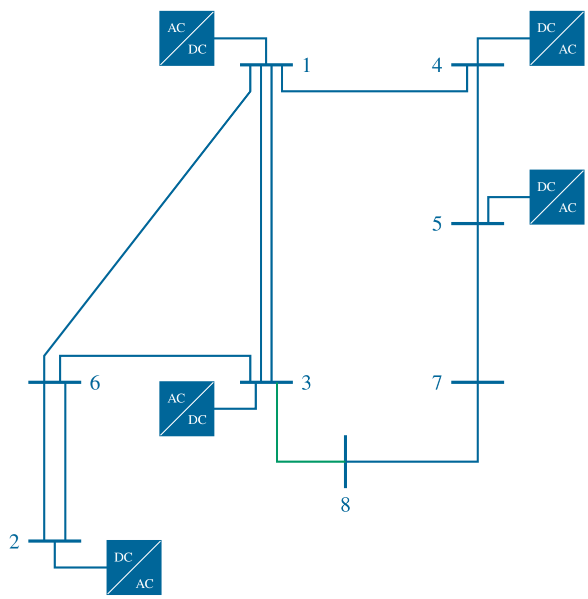

Fig. 4. Reduced CIGRE bipolar DC grid test system

The Parameters of the system were taken from [11] and the recommendations from [12], [13]. For the sake of completeness, they are presented in Tables I and II. Terminal 1 is the slack node ($v_1 = 1$) while the available voltage in the other terminals are between 0.9 and 1.01 pu. The maximum capacity of DC transmission lines were defined in 1.0pu. Nodes 6, 7 and 8 were eliminated by a simple Kron reduction.

TABLE I
PARAMETERS OF THE HVDC LINES

From	To	$r_{km}(\Omega)$
2	6	3.42/2
1	6	5.70
6	3	2.28
1	3	4.56/2
1	4	1.90
4	5	1.90
5	7	2.85
8	7	1.90

TABLE II
PARAMETERS OF THE HVDC CONVERTERS

Node	$P(MW)$	$V_{nom}(kV)$	$K(\%)$
1	236	400	5
2	300	400	4
3	400	400	5
4	-500	400	6
5	-400	400	3

Under flat starting initialization (i.e $v = 1$pu) the parameter $\gamma = 0.005856$ and hence existence of the equilibrium point is guaranteed inside an U_0 with $\delta = 0.5$. In addition, superconvergence is achieved as can be inferred from fig 5.

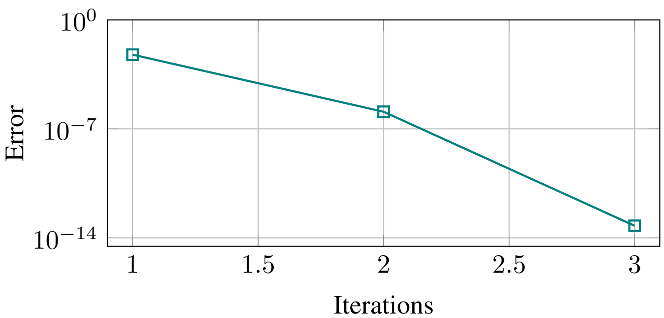

Fig. 5. Superconvergence of Newton's method applied to a reduced CIGRE bipolar DC grid test system from an initial point $v_k = 1$

VII. STABILITY ANALYSIS

The eigenvalues of the Jacobian DF_0 for the case of a simple MT-HVDC grid are $\lambda_{DF} = (-158.26, -66.08)$ and the Jacobian for the CIGRE MT-HVDC grid are $\lambda_{DF} = (-219.92, -129.44, -33.75, -31.68)$. This demonstrates the stability of the systems.

VIII. CONCLUSIONS

A simple but accurate methodical modeling of multi-terminal high-voltage direct current grids has been demonstrated. The model presented is in a general form, where any network configuration can be studied.

A procedure to obtain the existence of the equilibrium in a meshed HVDC network has been showned, based on a direct application of Kantorovitch theorem. Numerical results corroborated the theoretical analysis.

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