

CFD based study of Steep Irregular Waves for Extreme Wave Spectra

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ABSTRACT

Offshore structures are exposed to irregular sea states. It consists of breaking and non-breaking waves. They experience extreme wave loads perpetually after being installed in the open ocean. Thus, the study of steep waves is an important factor in the design of offshore structures. In the present study, a numerical investigation is performed to study steep irregular waves in deep water. The irregular waves are generated using the Torsethaugen spectrum which is a double-peaked spectrum defined for a locally fully developed sea. The Torsethaugen spectrum takes both the sea and swell waves into account. Thus, the generated waves can be very steep. The numerical investigation of such steep waves is quite challenging due to their high wave steepness and wave-wave interaction. The present investigation is performed using the open-source computational fluid dynamics (CFD) model. The wave generation and propagation of steep irregular waves in the numerical model is validated by comparing the numerical wave spectrum with the experimental input wave spectrum. The numerical results are in a good agreement with experimental results. The changes in the spectral wave density during the wave propagation are studied. Further, the double-hinged flap wavemaker is also tested and validated by comparing the numerical and experimental free surface elevation over time. The time and the frequency domain analysis is also performed to investigate the changes in the free surface horizontal velocity. Complex flow features during the wave propagation are well captured by the CFD model.

KEYWORDS: Irregular waves, CFD, double-peaked spectrum, double-hinged flap wavemaker.

INTRODUCTION

Offshore wind turbines are exposed to the extreme irregular sea states. Extreme waves exert extreme hydrodynamic loads on the substructures. Thus, the study of such irregular waves is very important in the design of offshore wind turbines. Sev-

eral experimental and field investigations have been performed in the past to study extreme waves. Such spectra exhibit two peaks, due the presence of swell and wind waves. Ochi and Hubble (1976) carried out a statistical analysis of 800 measured wave spectra at the North Atlantic Ocean. They derived a six-parameter double-peaked spectrum. The spectrum is composed of two parts: one which primarily includes the low frequency wave components and second which contains the high frequency wave components. Each part of the wave spectrum is represented by three parameters. The six-parameter spectrum represents almost all stages of the sea condition associated with a storm. Guedes and Nolasco (1992) analysed wave data from the North Atlantic and the North Sea and proposed a four-parameter double-peaked spectrum. This double-peaked spectrum was formulated by superimposing individual spectral components of the JONSWAP type single-peaked spectrum. Torsethaugen (1996) used a similar approach of combining two individual JONSWAP spectra for different frequency ranges, but instead of averaging he used other parameters of the JONSWAP spectrum. Violante-Carvalho et al. (2004) studied the influence of swell waves on wind waves by using buoy data measurements in deep water in the South Atlantic sea. Other researchers have also made efforts in this direction to study the double-peaked spectra (Masson (1993) ; Dobson et al. (1989)). Pákozdi et al. (2015) performed laboratory experiments with breaking irregular waves using the Toresethaugen spectrum to measure the global impact loads on offshore structures . Their study highlighted the importance of double-peaked spectra for a better representation of extreme sea states. The real state is composed of the sea and swell. Most of the widely used spectra like JONSWAP and PM spectra do not consider both sea states. The Torsethaugen spectrum gives an opportunity to study the real state more closely (Torsethaugen, 1996).

Computational Fluid Dynamics (CFD) can be used as an effective tool to study such double-peaked spectra. CFD has been used previously by many researchers to numerically study the breaking and non-breaking waves. Alagan Chella et al. (2017,

2016); Alagan Chella et al. (2015); Kamath et al. (2016) studied breaking waves and breaking wave forces on a vertical slender cylinder over an impermeable sloping seabed and they observed a good match with experiments. Bihs et al. (2016b) investigated the interaction of breaking waves with tandem cylinders under different impact scenarios. Bredmose and Jacobsen (2010) investigated breaking wave impacts on offshore wind turbine foundations for the focused wave groups using CFD. They compared the numerical and theoretical free surface and wave forces in time-domain by using the linear reconstruction of waves. Östman et al. (2015) performed CFD investigations with irregular waves using the Torsethaugen spectrum. They compared their numerical results with experimental data (Pákozdi et al., 2015). They found a reasonable match between CFD and experiments. However, the numerical extreme wave crest heights were lower and wave phases were not correct in comparison with experiments. Also, the wave energy content for the higher frequencies was not accurately captured in comparison with the experiments.

In the present paper, an attempt has been made to numerically model the extreme waves. The experiments were performed for 1:60 scaled $H_s=20\text{m}$ and $T_p=20.1\text{s}$ (Pákozdi et al., 2015). Most of the previous CFD studies with irregular waves are limited to the single-peaked spectra and non-extreme wave heights. The present paper used two different wave generation methods. To the best of the authors knowledge, this is the first study where a double-peaked wave spectrum can be used as an input to generate the irregular waves. The goal of the present study is to numerically investigate the irregular wave generation and propagation for the steep irregular waves generated using the double-peaked spectrum: Torsethaugen spectrum, by using the open-source computational fluid dynamics (CFD) model REEF3D (Bihs et al., 2016a). The wave generation and propagation is tested by comparing the numerical wave spectrum with the experimental wave spectrum in a numerical wave tank without any structures. The numerical results are in a good agreement with the experimental results. Next, the double-hinged flap wavemaker theory in the numerical model is validated by performing comparison of the numerical wave free surface elevation in time-domain with the experimental data (Pákozdi et al., 2015). The reason of modelling the double-hinged wavemaker is to simulate the waves as much as possible close to the experimental conditions. Usually, the experimental tests are more economically expensive and time-consuming. The numerical double-hinged flapmaker can be used as the preliminary test before running the experiments for all cases. This could be beneficial both time and money wise. Further, the changes in the horizontal velocities at the free surface during wave propagation are also investigated in the time and frequency domain.

NUMERICAL MODEL

The present numerical model is based on the governing equations of fluid dynamics: the continuity equation and the Reynolds Averaged Navier-Stokes equations (RANS) with the assumption of an incompressible fluid given as:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[(\nu + \nu_t) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + g_i \quad (2)$$

where, u is the velocity averaged over time t , ρ is the fluid density, p is the pressure, ν is the kinematic viscosity, ν_t is the eddy viscosity, i and j denote the indices in x and y direction, respectively and g_i is the acceleration due to gravity.

The numerical model uses the fifth-order finite difference Weighted Essentially Non-Oscillatory (WENO) scheme in multi-space dimensions for the spatial discretization (Jiang and Peng, 2000). The third order TVD Runge Kutta scheme is used for the time discretization (Shu and Oscher, 1988). An adaptive time stepping scheme is used in the numerical model (Griebel et al., 1998). The present study uses the $k - \omega$ model (Wilcox, 1994) along with the Reynolds Averaged Navier Stokes (RANS) equation. The level set method is used to capture the free surface (Osher and Sethian, 1988). Detailed information about the numerical model can be obtained in Bihs et al. (2016a). In the numerical model, the irregular waves are generated by the superpositioning of the linear regular waves components (Aggarwal et al. (2016a,b)). The second-order irregular wave theory is used here (Schaffer, 1996). The present study uses the experimental spectrum as an input for the numerical model which is generated using the Torsethaugen spectrum (Torsethaugen, 1996). The input values to the spectrum are the significant wave height H_s and the peak period T_p .

The double-hinged flap wave-maker is also tested and validated in the present work. The schematic sketch of the double-hinged flap wave-maker is shown in Fig. 1. The paddle motion is directed positive towards the wall and negative towards the water. Angles for both hinges with respect to time are given as input to the numerical model (β for H1 and $\beta + \gamma$ for H2). The angles are converted to distance vector $X(z)$ using:

$$sign = -\frac{\theta}{|\theta|}; X(z) = sign * |\sin\theta| \quad (3)$$

where, θ is the angle at hinge measured from the vertical (β for H1 and $\beta + \gamma$ for H2).

The velocity, $U(z, t)$ which is the inflow boundary condition is calculated as:

$$U(z, t) = \frac{\delta X(z)}{\delta t} \quad (4)$$

where $X(z)$ varies as a function of depth (normalised with respect to the flap length). The values are zero at the hinge and maximum at the tip of the flap.

SETUP OF THE NUMERICAL WAVE TANK

The numerical tests are conducted in a two-dimensional numerical wave tank (NWT) as shown in Fig. 2. The numerical model is validated by comparing the numerical results with the experimental data (Pákozdi et al., 2015). The NWT is 56 m long and 15 m high with a water depth of 10 m is used in the simulations. Five wave gauges and three velocity probes are placed along the length of NWT to study the changes in the wave surface elevation and free surface wave velocity.

RESULTS

Grid Refinement Study With Wave Spectrum Input

The grid refinement study is conducted for the wave spectra in the NWT under steep irregular waves. The numerical tests are

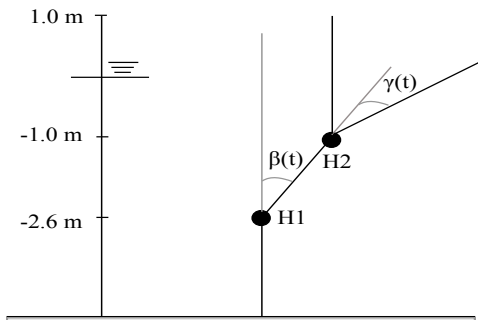


Fig. 1: A schematic sketch of the double-hinged flap wavemaker

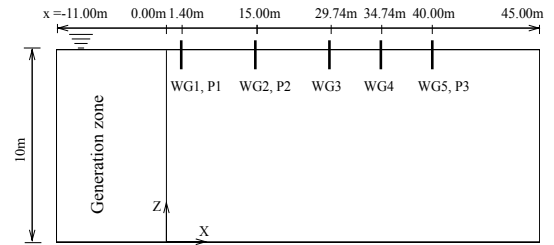


Fig. 2: Setup of the numerical wave tank (side view)

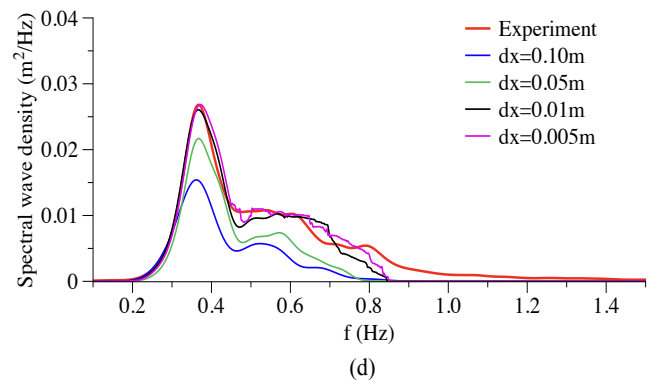
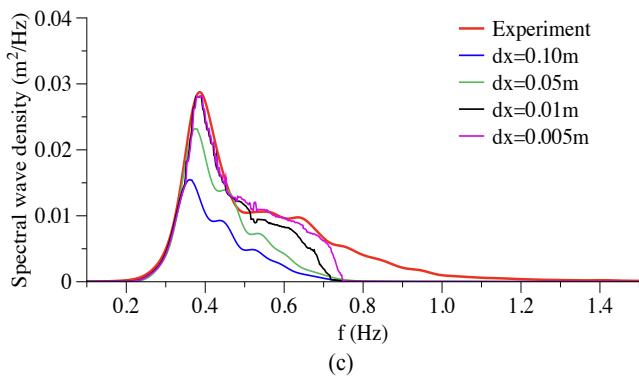
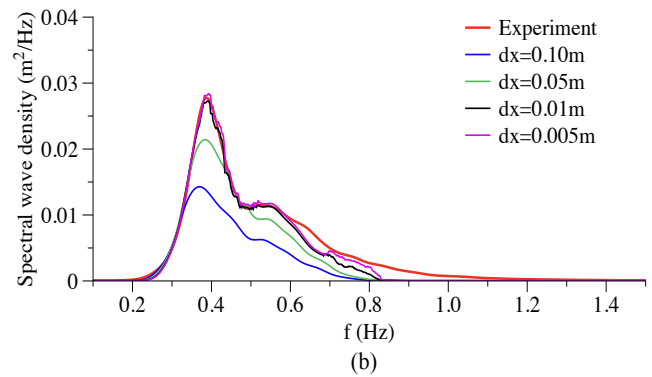
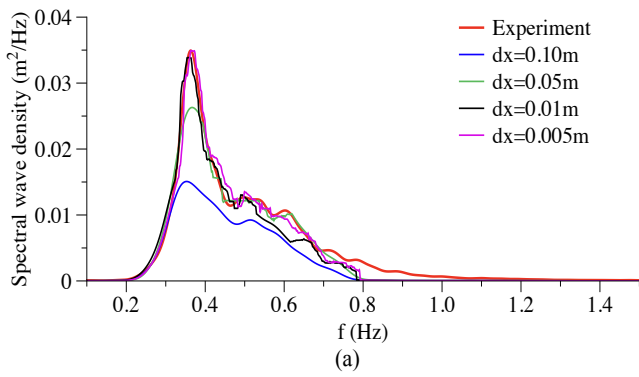


Fig. 3: Comparison of the numerical results with different grid sizes and the experimental data (Pákozdi et al., 2015) for the wave spectra at (a) WG1 (b) WG2 (c) WG3 (d) WG4

performed for the significant wave height $H_s = 0.345$ m and the peak period $T_p = 2.6$ s with four different grid sizes, $dx = 0.10$ m, 0.05 m, 0.01 m and 0.005 m for the grid refinement study. Fig. 3 presents the comparison of numerical and experimental spectral wave density over the frequency for different grid sizes at WG1, WG2, WG3 and WG4. For the wave gauge located next to the wave generation (WG1), the numerical results with $dx = 0.10$ m, $dx = 0.05$ m are not captured properly in comparison with the experimental results in the high frequency range, also the numerical peak spectral wave density is lower than the experimental peak spectral wave density by 57.14 % and 25.71 % at grid sizes $dx = 0.10$ m and 0.05 m, respectively. The results with $dx = 0.01$ m show a good match with the experimental ranges in most of the frequency range. The difference between the experimental and numerical peak spectral wave density reduces to 3.92 % (Fig. 3 (a)). However, the numerical wave spectrum is relatively narrow compared to the experimental case, and some difference is observed between experimental and numerical spectra in higher frequencies (0.81 Hz to 1.00 Hz). The results with $dx = 0.005$ m show a slight improvement and the difference between the experimental and numerical peak spectral wave density reduces to 2.22 %. However, the improvement in results is not very significant at $dx = 0.005$ m. The numerical results are converged at $dx = 0.01$ m. A similar behaviour is observed for the comparison between the other wave gauges (Figs. 3 (b), 3(c) and 3 (d)). For WG3, for $dx = 0.01$ m and 0.005 m, the numerical spectrum is narrower (0.2 Hz to 0.78 Hz) as compared to the experimental spectrum (0.2 Hz to 1.1 Hz). The spectral peaks and spectral wave densities at all wave gauges are well captured by the numerical model in most of the frequency range when compared with the experimental spectrum. A decrease in the peak value of spectral wave density is noticed in both numerical and experimental spectra during the wave propagation. The secondary peaks are observed at all wave gauge locations in both the experimental and numerical results. The peak spectral wave density for WG4 is lower than the wave gauge located next to the generation zone (WG1). This is due to wave-wave interactions leading to some steep waves. These steep waves are very close to breaking, and thus, they lose some energy during this process resulting in lower peak spectral wave density at the wave gauge located close to the end of the wave tank (WG4). The non-linear energy transfer takes place, and the energy is transferred to higher harmonics via this transfer. Therefore, the primary peak is reduced and some secondary peaks are observed in the higher frequencies. This means that the contribution of shorter waves become significant when higher-order wave-wave interaction takes place. The numerical spectrum can not capture the waves in highest frequency range (0.9 to 1.2 Hz), it could be because the numerical results employ the second-order irregular wave theory but there could be even higher-order components in the irregular wave train due to steepening of the wave crest and wave-wave interactions.

Grid Refinement Study For Double-Hinged Flap Wavemaker

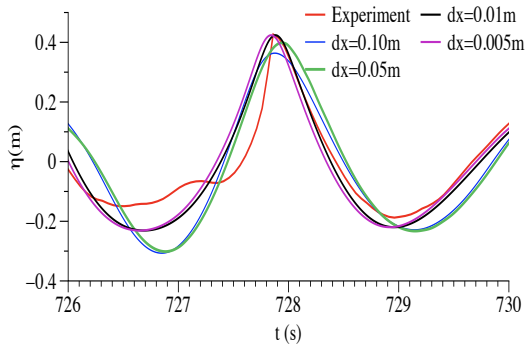
In this part of the paper, the steep irregular wave generation using the double-hinged flap wavemaker theory is tested and validated. The input angle signals for the upper and lower flaps from the experiments are used as inputs for the double-hinged flap wavemaker in the numerical model. The numerical tests are performed for the significant wave height $H_s = 0.345$ m and T_p

$= 2.6$ s with four different grid sizes, $dx = 0.10$ m, 0.05 m, 0.01 m and 0.005 m for the grid refinement study. In the CFD simulations, the waves are numerically generated using 43 s of wave flap signal. Two extreme wave events are generated here using the signals from the experiments. Fig. 4 presents the comparison of numerical and experimental wave free surface elevation (m) over time (s) for three different grid sizes around the steepest wave crest for extreme events 1 and 2. For the extreme event 1, a difference of 14.82 % and 6.5 % is observed between the experimental and numerical crest for grid sizes $dx = 0.10$ m and 0.05 m, respectively. The difference between the experimental and numerical crest height for the extreme event 1 reduces to 0.04 % and 0.02 % for grid sizes $dx = 0.01$ m and 0.005 m, respectively. There is almost no observed phase difference between the experimental and numerical extreme event at fine grids $dx = 0.10$ m and 0.005 m around the steep wave crest (Fig. 4 (a)). A similar behaviour is observed for extreme event 2. The difference between the numerical and experimental wave crest height for the extreme event is 36.4 % and 26.56 % for the grid sizes $dx = 0.10$ m and 0.05 m, respectively. This difference reduces to 9.81 % and 8.97 % at grid sizes $dx = 0.01$ m and 0.005 m, respectively. The phase information is also captured reasonably well (Fig. 4(b)). The improvement in the results is not very significant with $dx = 0.005$ m compared to $dx = 0.01$ m. The numerical results are converged at $dx = 0.01$ m.

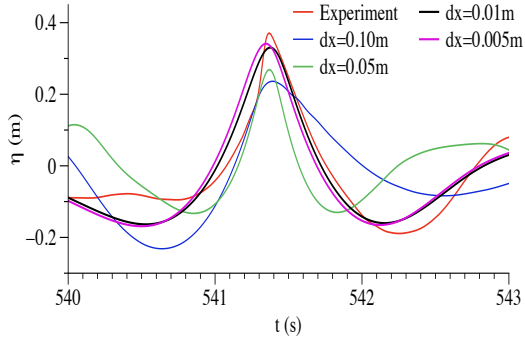
Fig. 5 presents the comparison of experimental and numerical ($dx = 0.01$ m) results for the wave free surface elevation over a longer time-series. For extreme event 1, the numerical results are in a good agreement with the experimental results for most of the time-series. The wave crests peaks for the steep waves are captured in the numerical model with good accuracy, when compared with the experimental data (Fig. 5(a)). For extreme event 2, a behaviour similar to extreme event 1 is observed. The wave propagation and peaks of the wave crests are in good agreement with the experimental data. However, it is noticed that the numerical peak for steepest wave in this case is slightly lower than the experiments. Also, for some parts of time-series, the wave troughs are slightly overpredicted by the numerical model in comparison with the experiments and wave phases are slightly disagree with experiments. It is possible that the energy transfer between wave crests is simply sufficient to increase the local wave steepness after the extreme event point where another instabilities, associated with maximum phase velocities lead to phase differences for the preceding wave crests.

CHANGES IN THE FREE SURFACE HORIZONTAL VELOCITY

A transformation of the free surface wave velocity is observed during the wave propagation. In this section of the paper, the variation of the horizontal component of particle velocity (at the free surface) during the wave propagation in the numerical wave tank is investigated. Fig. 6 presents the variation of numerical velocity (m/s) over time (s) at different velocity probe locations. It is observed that for event 1, the velocity computed at the probe located at $x = 15$ m (P2) have higher value of peak velocities as compared with the velocity measurements at the probe located next to wave generation (P1). This might be due to the generation of steeper waves (with larger wave height and larger velocity) during wave propagation. The peak velocities measured at the velocity probe located at $x = 40$ m (P3) shows lower crest values

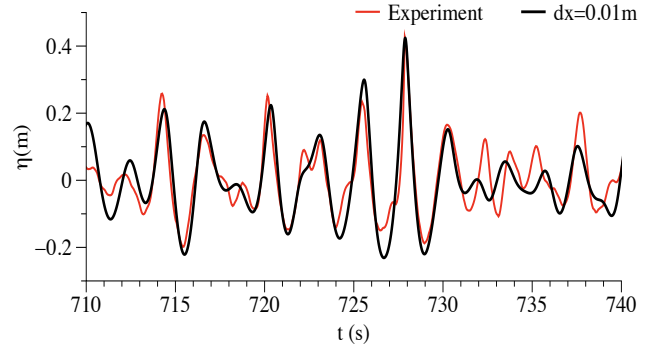


(a)

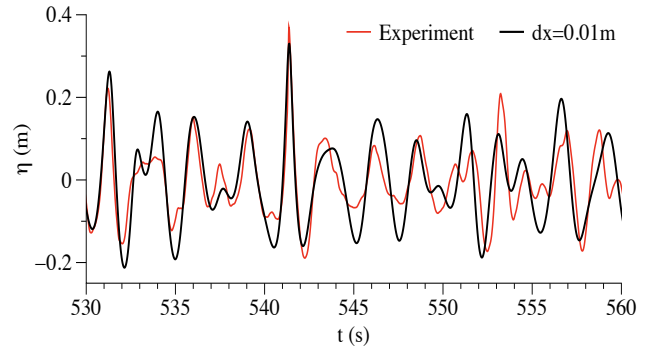


(b)

Fig. 4: Comparison of numerical and experimental wave free surface elevation for different grid sizes during the (a) extreme event 1 (b) extreme event 2

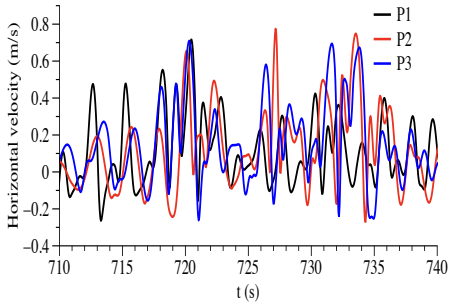


(a)

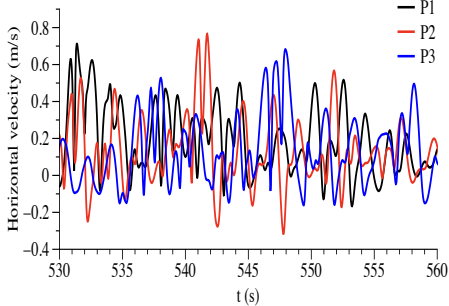


(b)

Fig. 5: Comparison of numerical and experimental wave free surface elevation for $dx = 0.01$ m for longer time series during the (a) extreme event 1 (b) extreme event 2

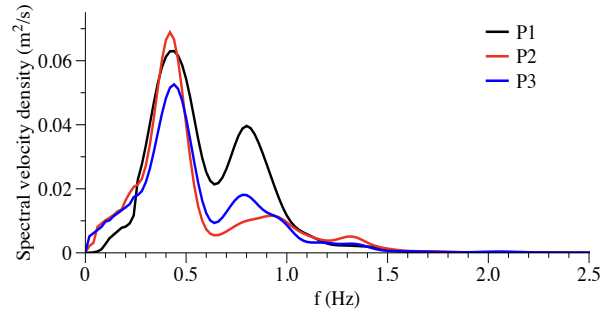


(a)

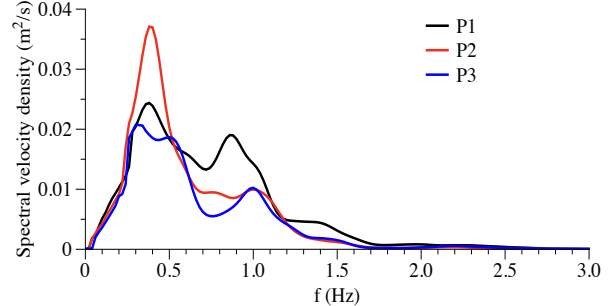


(b)

Fig. 6: Numerical horizontal velocity with respect to time during the (a) extreme event 1 (b) extreme event 2



(a)



(b)

Fig. 7: Numerical velocity spectral density over frequency during the (a) extreme event 1 (b) extreme event 2

as compared with the velocity measured at the probe located at $x = 15\text{m}$ (P2) (Fig. 6(a)). A similar behaviour is observed for the extreme event 2 (Fig. 6(b)).

Further, the frequency domain analysis is performed to study the changes in the horizontal component of velocity (at the free surface). Fig. 7 presents the velocity spectral density (m^2/s) over frequency (Hz) at different velocity probes along the numerical wave tank. It is observed that two spectral peaks are observed in the frequency spectrum. For extreme event 1, the primary peak is observed at $f = 0.42$ Hz with a value of $0.063 m^2/s$, while the secondary peak occurs at $f = 0.80$ Hz with a value of $0.0396 m^2/s$ for the velocity probe located next to wave generation (P1). The value of the primary peak is increased by 8.7 % and the value of secondary peak is decreased by 62 %, also a shift of 0.02 Hz is observed in the location of secondary peak occurrence for the probe located at $x = 15\text{m}$ (P2). A decrease in the values of both primary and secondary peaks is observed for the velocity probe located at $x = 40\text{m}$ (P3). Also, more than one secondary peak is noticed (Fig. 7 (a)). A similar behaviour is observed for the extreme event 2, the increase of 54.23 % is noticed in the primary spectral peak for the probe located at $x = 15\text{m}$ (P2) as compared with the primary spectral peak at the velocity probe located next to wave generation (P1). This increase for extreme event 2 is higher compared to the extreme event 1, but the spectral peak values in general for event 1 are larger than event 2. This might be due to higher steepest wave crest during event 1 (Fig. 5). This again corresponds to the energy-transfer and energy loss due wave breaking and wave-wave interaction to higher frequencies range, which leads to the decrease in the spectral velocity peaks. The primary spectral at P2 is much higher than P1 and P3 for event 2 than event 1. However, the spectral peak value for event 2 at P2 is lower than that for event 1. The reason might be higher number of wave-wave interactions have taken place for event 2 than event 1. As event 1 has already occurred, it is possible that some of the waves which lead to event 1 of relatively larger wave heights and velocities are present in the irregular wave train, which might have possibly contributed towards the higher primary spectral peak for event 2.

FREE SURFACE CHANGES

Fig. 8 presents the simulated free surface changes with velocity magnitude (m/s) variation during the propagation of irregular steep waves in the numerical wave tank (NWT) at different time-steps. Fig. 8 (a) shows a two-dimensional view of the irregular wave propagation along the whole domain of NWT at $t = 38.27$ s. The waves with different velocities in the irregular free surface are observed. Fig. 8 (b) show a zoomed view around the steep wave, it is noticed that the velocity of the wave crest starts to get larger as compared to the other waves. In the next time step at $t = 38.49$ s, the steep wave (extreme event 1) with very high velocity and elevation is observed (Fig. 8 (c)). After the wave crest reaches its maximum height and attains the maximum crest velocity as shown in Fig. 8 (c), the velocities and the wave heights start to decrease again (Figs. 8 (d) and 8 (e)). A uniform inflow similar to conditions for a bore propagating onto a weak current, leads to an internal-flow pattern, with turbulence spreading downward into the incident flow. An inflow that has the fully developed profile of a steady turbulent flow gives rise to separation under the front of the wave and a substantially different flow field, though without any significant change in the

surface profile.

CONCLUSIONS

The CFD based numerical model is used to model irregular steep waves for the extreme wave spectrum. The numerical model is validated with measured data for simulating steep irregular waves using the double-peaked Torsethaugen spectrum. A grid refinement study is performed in order to study the effect of the grid size on the numerical results for the wave spectral density. A good match is observed between the experimental and numerical results. The wave spectrum shows two peaks for all wave gauges. The value of the primary peak is slightly lower for the wave gauge located close to the end of wave tank as compared to the wave gauge located next to the wave generation zone due to the loss of energy during wave propagation. Further, the numerical double-hinged flap wavemaker is validated by comparing the wave free surface elevation time series with the experimental data. This is performed for two extreme events as measured in the experiments. A grid refinement study is also conducted to study the effect of grid sizes on the wave free surface elevation. It is noticed that numerical values for the steepest wave crest are in a good agreement with the experimental peak crest values. In the next section, investigations are performed to study the changes in the horizontal velocity at the free surface. The following conclusions can be drawn from the study:

- For the steep irregular waves propagating in deep water, as the wave propagates, they undergo wave-wave interaction which increases non-linearity in the wave spectrum. The energy from the primary peak is transferred towards higher frequencies and contribution of secondary peaks and the shorter waves increases.
- The numerical model represents well the wave crest peaks and wave phases until the extreme event. After the extreme events, the extreme wave crest with large wave height interact with preceding wave crests and further increases wave non-linearity. This might be reason that wave phases are not represented well after extreme wave event occurs.
- The horizontal wave velocities and the spectral velocity densities are affected by the wave spectral density. The wave heights for extreme waves increases which leads to increase in the primary spectral peak for second wave probe. Some of the waves close to the beach, either break or are close to breaking which leads to dissipation of wave energy and thus reduction in spectral velocity density peaks.
- The effect of turbulence increases with increasing wave non-linearity due to non-linear energy transfer and dissipation. This is important factor and is recommended to take into account for the design of offshore and coastal structures.

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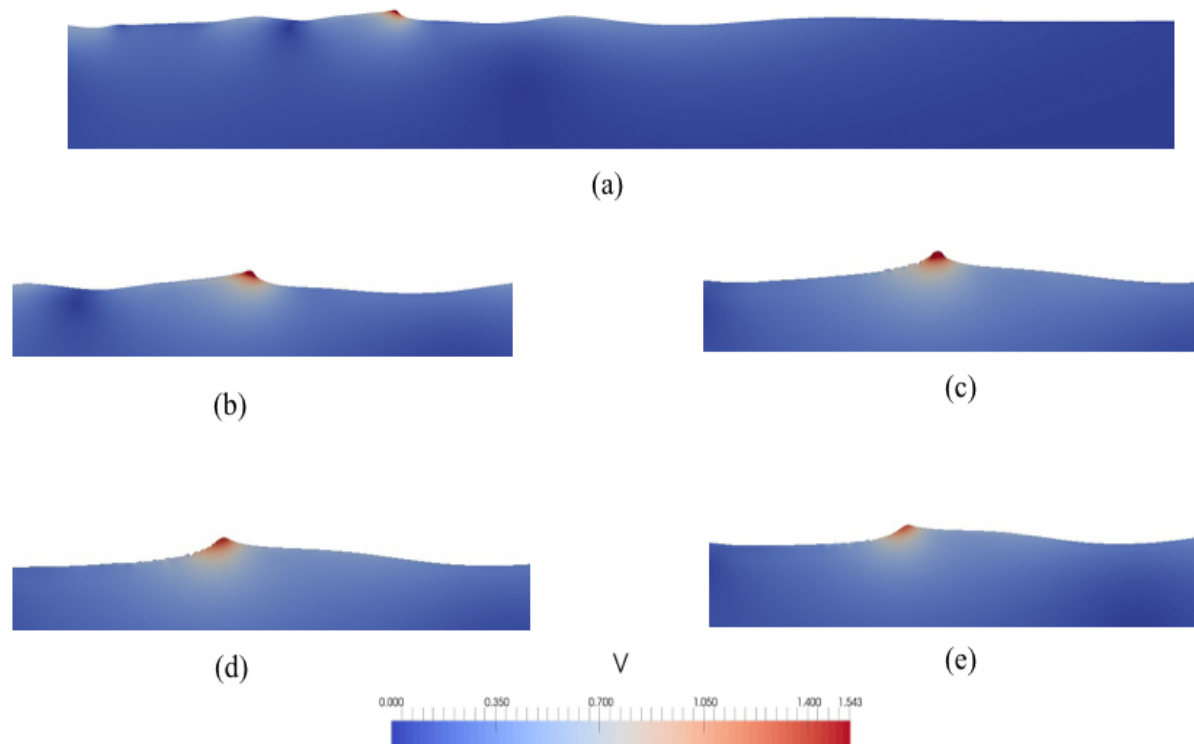


Fig. 8: Simulated free surface changes with velocity magnitude (m/s) variation during the propagation of the irregular steep wave along the wave tank a) $t=38.27$ s (b) Zoomed-in-view at $t=38.27$ s (c) Zoomed-in-view at $t=38.49$ s (d) Zoomed-in-view at $t=38.69$ s (e) Zoomed-in-view at $t=38.81$ s

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