

BACKING OUT EXPECTATIONS FROM HYDROPOWER RELEASE TIME SERIES

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Abstract

When planning production, hydro power reservoir managers need to form expectations for electricity prices in the future. When forming expectations, an electricity forward market is a useful tool for predicting how the underlying spot price will move. In this paper we develop a structural estimation model for a single agent hydro power producer in Norway. With this model, we analyze how primitives in the price process, related to the forward price, can be inferred from empirical data from actual production time series. By analyzing trends and patterns in observed time series we have approximately parameterized the state space transition. Central here is the connection we model between inflow and price, to capture dry- and wet year dynamics in the two. To demonstrate the model it has been applied to a specific hydro power plant in Norway. From the results we make a preliminary analysis of to what extent the producer uses forward information when planning production. The results indicate that this specific producer is inclined to take the forward price into account when planning, and that a forward price with 6 months to maturity is favored. An important byproduct of our model is the ability to calculate water values from the outputs.

Introduction

Managers of reservoir hydropower plants need to form expectations regarding future electricity prices when balancing immediate and future rewards from releasing water and generating electricity. Although surveys on forecasting methods employed by Sanda et al. (2013) shed light on how this happens, we analyse how primitives of this expectation process can be inferred from empirical data from actual production time series. Our main research goal is to apply structural estimation theory of Markov Decision Processes, developed by Rust (1987), to a hydropower planning problem. The model aims to work as a descriptive rather than a normative tool for the hydropower industry. In order to demonstrate the structural estimation model we apply it to a single-case hydropower plant. An example of insight gained from the demonstration, is to what extent a planning agent put emphasis on forward information when planning production. Another example is empirical insight in what expectations the hydro power producers use for future water values.

We employ structural estimation of a dynamic decision process as originated by Rust (1987). The idea is that if we observe a set of states and actions taken by an agent, we can work backwards to infer the objective function of that agent, by maximizing the likelihood of matching the observed data. By maximizing the likelihood function, the analyst can obtain an understanding of parameters hidden in the economic model. In order to estimate the structural parameters in a stochastic dynamic programming problem, Rust used an algorithm he called the Nested Fixed Point (NFXP) algorithm. This algorithm has two parts, an outer loop that searches for the structural parameters with the maximum likelihood value, and an inner loop that solves the stochastic dynamic programming model given a value for the structural parameter. According to Su and Judd (2012) the NFXP algorithm is computationally demanding, because it iterates over all structural parameter values and then solves the underlying stochastic dynamic programming (SDP) model with high accuracy for each structural parameter value.

Hydropower planning problems are suitable to be treated as a stochastic dynamic problem, where a decision today change the reservoir levels and thereby affect future production opportunities. Here the dynamic properties of the planning problem can be accounted for together with multiple stochastic variables, such as inflow, spot prices, and forward prices. For hydropower plants operating in a well-functioning market, price can be treated as

a stochastic variable in a stochastic dynamic programming problem. This has been explored by Wolfgang et al. (2009), among others. For our case, we treat both inflow and price as stochastic input state variables in a stochastic dynamic programming problem.

The model

First, structural estimation is presented as a method to empirically measure primitives of commodity storage operations, such as the formation of price expectations for hydropower schedulers.

The overall model use a combination of sub-models to analyze the decision-making done for a hydro power plant. In order to obtain the parametric approach for conditional expectations we model the state variables as following Markovian time series. This enables us to use the time series in a parametric approach for the structural estimation problem.

We employ a weekly resolution. Local inflow is assumed to follow an autoregressive process that consists of a deterministic seasonal term plus a residual AR(1) term. Parameters are fitted based on a time series of inflow 2000-2016 from a Norwegian hydro power plant. Inflow is one of five exogenous state variables; local reservoir level is a sixth, endogenous state variable. The other four exogenous variables are accumulated local inflow deviation, national reservoir deviation (i.e., deviation from its 10 year normal, capturing dry and wet year dynamics), spot price and forward price. We let accumulated local inflow affect the national reservoir deviation, and the national reservoir deviation affect the spot- and forward price. This way, there will be a negative relation between local inflow and price; if there is little local inflow over a time period, then it is likely that this drought is affecting neighboring reservoirs and might be a sign of a national drought. This will drive prices up, and conversely.

Accumulated local inflow is modelled as an exponentially weighted moving average. Accumulated local inflow deviation, which is the state variable, is the accumulation's relative deviation from its 10 year normal. All details can be found in Boger and Vestbøstad (2016).

National reservoir deviation is an important state variable to market analysts. This information is publicly available, published weekly by the regulator, and we measure it as its relative deviation from the 10 year normal. We estimate an autoregressive process of order 1 for this variable (ARX(1)), with the accumulated local inflow deviation as an exogenous explanatory variable.

Regarding forward prices, we select a point on the forward curve that is either two, six or twelve months ahead. We model the weekly transition in this point as a sum of a deterministic seasonal term and a residual term. The residual term is modelled as an ARX(1) with the national reservoir deviation as an exogenous explanatory variable.

The spot price is captured via a sum of three terms: a deterministic seasonal component $f^P(t)$, a base process Y_t^P , and a correction term coming from the forward price F_t :

$$P_t = f^P(t) + Y_t^P + \zeta (F_t - f^P(t) - Y_t^P) \quad (1)$$

The base process Y_t^P is modelled as a sum of an AR(1) process and a term that is linear in the national reservoir deviation. The parameters of the base process and the seasonal component for the spot price is estimated assuming that there is no correction from the forward price as in (1). The "forward expectation" parameter ζ is our primary interest, and we will let our data and behavioral model determine its magnitude. If it is zero, then the reservoir manager behaves as if forward prices do not matter for price expectations. If it is one, then forward prices determine all of the price expectations.

Below is a time series plot of weekly logarithm of prices, in the form of the base process, Y_t^P , together with the national reservoir deviation. In wet years, the reservoirs are higher than normal, and prices are lower than normal, and vice versa.

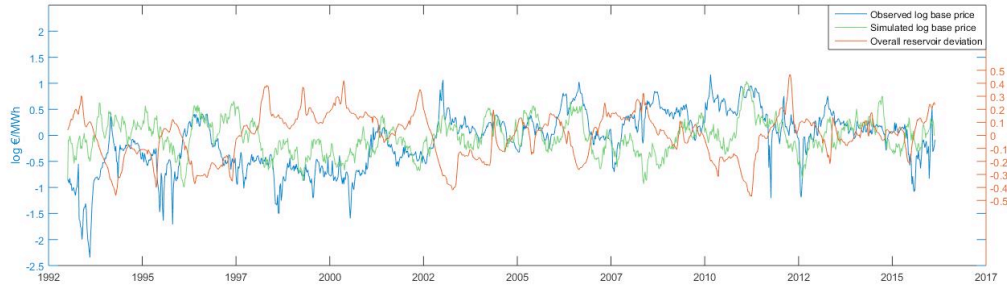


Fig. 1. Time series of log base price Y^P , observed (i.e. estimated) and simulated, and the national reservoir deviation. A negative relation between the prices and the resource state can be readily observed.

We assume that the reservoir manager for which we have data is a price taker. For the Nordic electricity market, this is a reasonable approximation. Nevertheless, we remark that since hydro power has a dominating share in the generation mix, future work should examine if forward and spot prices can be regarded as exogenous to individual reservoir decisions as we do here, or whether it is necessary to take the simultaneity of storage decisions and present and future demand and supply into account.

The parameters of the submodels above can in principle be determined not (only) by observed time series of inflow, prices and national reservoir levels, but also by the observed release decisions of the reservoir manager. That is, it could be possible to try to imply the expectations that our reservoir manager has regarding e.g. inflow dynamics. Instead, we follow Rust (1987) and determine these parameters (except the forward expectation parameter ζ) from time series only, and not release decisions.

The overall set of equations for the state variables is a system of AR(1) equations. Included in this is the reservoir balance equation, stating that the reservoir level this week equals the level last week, plus inflow, minus release and spill.

Reservoir managers are assumed to time releases from the reservoir as a trade-off between the reward associated with immediate generation of electricity versus saving the water for future release. The immediate reward is modelled as the spot price times generation release.

In order to achieve stationarity for the Markov decision process, we follow Foss and Høst (2011), who noted that since the seasonal part stays the same for different years, it will be sufficient to let the problem be conditional upon week of the year. If we set time of the year as a state variable, the problem is reduced from a non-stationary to an approximate stationary problem.

Following Rust (1987), the reservoir manager has more information than the outside researchers, and this is captured by adding a payoff shock to the immediate reward. This shock is assumed observed by the decision maker, but is hidden to the researchers. The total value of choosing a release level consists of the immediate reward, the payoff shock, and discounted expected future rewards. This gives rise to a Bellmann-like contraction that we discretize and represent as nonlinear constraints in our estimation problem, following Su and Judd (2012). The objective function in the estimation problem is maximization of the log likelihood of observing releases given our behavioral model. Payoff shocks are assumed to follow a Gumbel distribution, which allows the value function and the choice probability to be represented in closed form.

In order to discretize the Bellmann contraction, we discretize the state space as well as the random errors, assumed Gaussian, that affect inflow, national reservoir deviation, spot price and forward price.

Results

In order to implement the main optimization problem we use AMPL (A Mathematical Programming Language), which is an algebraic modelling language used to solve complex mathematical optimization problems. In combination with AMPL we use the open source solver IPOPT (version 3.10.1), used for large-scale nonlinear optimization. This version of IPOPT is limited to a total memory usage of about 2 Gb, which affect the size of the total state space. This forces us to use the minimum number of discrete levels for all of the state variables. Due to the large number of variables and constraint, the solve time is consequently long. The solver time increase because the forward expectation parameter ζ enters the model in the state transition equation. In order to ensure linearity, we need to treat it as a parameter in stead of a variable in the implementation in AMPL.

Therefore, the model has to be solved for each individual value of ζ , then to manually search for the ζ with the highest resulting log likelihood value. Solving the model for one value of ζ takes approximately 1 minute. To find the parameters for the different state variable processes, and for general data processing and analysing, we use MATLAB.

If the structural estimation model is consistent with reality, we expect to observe some of the same dynamics for water values, for different weeks and reservoir levels, as given from standard industry tools. The water value is the expected marginal value of the energy stored in the reservoir. In our model we can output the expected value function for different reservoir levels throughout the year. By finding the derivative of the expected value with respect to the reservoir level, i.e. divide the change in expected value by the change in reservoir level, we can find the water values. This is a valuable finding, and an important by-product of our model. For an external agent, this is a cost effective and fast way for finding the marginal cost of production. In Figure 2 (a) we have plotted water values based on the value function from our model. They are plotted for each reservoir level and each week of the year. Figure 2 (b) displays a comparable plot of water values from the an industry tool.

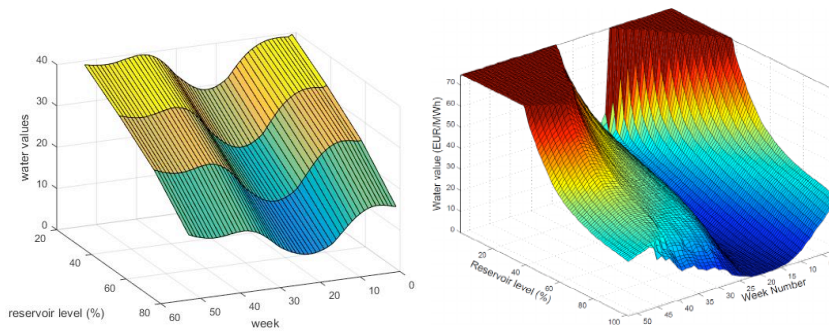


Fig. 2 (a)(left). Water values from our model. (b)(right) Water values for a reservoir in South of Norway (Gebrekiros et al. (2013))

For a given reservoir level, the water values should be low in the spring, when there is a lot of inflow to the reservoir, since getting one more unit of water gives little extra value. However, in the winter, when the inflow levels are low, getting one more unit of water is very valuable. In Figure 2 we can clearly see similarities in the two plots, in the dynamics through the year, with lower water values in the spring and higher values in the winter. The minimum is around week 20 in both figures. After week 20 the values increase quickly to a maximum around week 40, before they decrease slightly through the winter until they drop down towards the minimum in the spring. This dynamic is consistent with our expectations.

The water values for our model should develop in a similar manner as it does for the industry tool at the extremes. When the reservoir is full, the water values should be very close to 0, i.e. an extra unit of water is almost worthless. At the other extreme, when the reservoir is close to empty and little inflow is coming in, an extra unit of water is valuable. This often occurs during winters. In Figure 2 we see that the values have an inclination to be low when the reservoir is full and slightly higher when the reservoir is close to empty. We do however not see the same extreme instances as shown in the industry tool. Because of the limitations in memory we need to discretize the reservoir level quite coarsely, as mentioned in 4.1.6, which might leave us unable to explore what happens at the outer extremes. To explore this, our model might need a denser discretization grid.

Another trait we see in Figure 2 (b) is that the graph for the water values is convex, i.e. the water values increase more than linear when reservoir level decreases. This is not consistent with the water values from our model, in Figure 2 (a). Here the graph for the water values is close to linear. This could be an indication of problems in the model. Another reason could be that the results in Figure 2 (a) actually reflect how the reservoir has been operated.

One of the aims of our model was to estimate to what degree the price is adjusted towards the forward price, or if the forward price is used directly as expected price. The value of the forward expectation parameter, ζ , which gives the best fit to observed data, the one with the highest log likelihood, can provide such an indication.

If our model is consistent with reality, we expect that ζ is in the range between 0 and 1. This is because we do not expect the producer to scale the price forecasts down if $\zeta < 0$ or up if $\zeta > 1$. Such values would mean the producer uses a price larger than the forward price or smaller than the forecasted spot price when planning. If our model were approximately in the desired value range, a shift between what forecast is emphasized would be expected. We can see from Figure 3 that a value of $0 < \zeta < 1$ approximately holds.

We would expect ζ to reflect the views of Näsäkkälä and Keppo (2008) and Fleten et al. (2002), namely that the forward price is important in forming expectations, i.e. a ζ near 1.

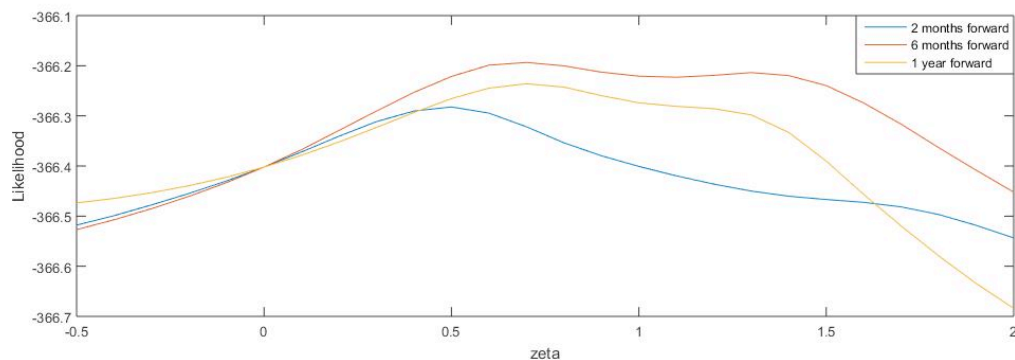


Fig. 3. Likelihood for different ζ 's, forward with 2 months- (blue), 6 months- (red) and 1 year (yellow) to maturity.

Even though the global maximum is a little smaller, the likelihood plot is weighted heavily towards a higher ζ . That is a further indication that the producers use forward price information when planning for the future, since the likelihood for the different forward contracts is generally higher around $\zeta = 1$ than at $\zeta = 0$.

Conclusions

Our results indicate that forward prices are an important factor in forming price expectations. Further, it seems possible to form an empirical structural model that describes the behavior of reservoir managers.

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