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An Exploratory Study of a Wet Bulk Platform Supply Vessel using Operations Research

Single-Vessel Scheduling with Bulk Loads
and Mud-Cleaning Process Plant

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Preface

This Master Thesis is my final work in my Master of Science in Marine Technology at the Norwegian University of Science and Technology. My specialisation is within Marine Design and Logistics, and this thesis concerns topics within Operations Research. The Master Thesis is a continuation of the project thesis handed in by Martin Otteraaen and me in December 2016.

The thesis is exploring the use of operations research to assess a new way of wet bulk management, and model a route for transporting wet bulk to offshore installations in the North Sea. Statoil ASA presented me with the problem in the autumn of 2016, and has made real data available along with other important information. For this I would like to thank them.

I would also like to thank my supervisor Professor Bjørn Egil Asbjørnslett for helpful guidance throughout my process of writing the Master Thesis. Further I would like to express my gratitude to Professor Kjetil Fagerholt, who made time to help me develop my mathematical model, and to Inge Norstad at SINTEF Ocean for helping me understand the MIP-solver Xpress-IVE. An additional gratitude goes to Svein Hermann Nilsen, Sigbjørn Sangesland and Bjørn Christensen for answering my questions in their fields of work.

Trondheim, July 14, 2017



Rebekka Resell

Summary

In order to deal with an increased demand for wet bulk on the offshore installations on the Norwegian continental shelf, a new system for handling wet bulk logistics is investigated in this thesis. The reason for the increased demand is because the production rate on an installation must be maintained as the field matures, which is done by injecting produced water into the field. Additionally, operators experience that large and unexpected cargo demands are hard to implement on pre-scheduled platform supply vessels (PSV), forcing them to use expensive, spot chartered PSVs. This thesis covers this problem in the North Sea, with offshore installations connected to Base Mongstad.

The new PSV introduced in this thesis is dedicated to wet bulk operations, and has a process plant on board to clean oil-based drilling mud (OBM). There are several, equally sized tanks on the PSV that are not dedicated to carry a certain type of product, unlike ordinary PSVs. This tank philosophy makes the vessel more flexible and only limits the amount of a certain cargo it can carry to the vessel's capacity. The wet bulk supply problem is characterised by several cargoes that need to be delivered to different offshore installations from the onshore base, and some cargoes that must be transported back from the installations to the offshore base. A maritime pickup and delivery model is thought to best describe the problem, and a suiting mathematical model is thus build.

A study of state of the art shows that there is performed little or no research on scheduling a PSV solely after the wet bulk demands. Therefore this thesis contributes to insight into how optimisation can be used to schedule a PSV with the aim to avoid delays. The objective function in the mathematical model is ambiguous as it minimises delay and maximises the number of available tanks at the same time. However, it is the delays at each cargo and the flexibility of the vessel in number of available tanks that are of interest. The mathematical model is implemented and run in the commercial solver FICO[®] Xpress Optimization Suite.

Different cases divided into two main sections, demand today and increased demand, are developed to investigate the capacity of the wet bulk PSV in a case study. Relevant data is identified and collected primarily from actors in the Norwegian petroleum industry to best suit the thesis problem.

Results from the case study indicate that the introduced wet bulk PSV has capacity to handle large amounts of cargo, which implies that it can stay offshore for a long time. The results also show that there is no difference in letting the wet bulk PSV and an ordinary PSV handle small, frequent cargoes. It is therefore proposed that the wet bulk PSV should primarily handle large cargo demands and treatment of OBM. Treating the OBM offshore is the cause of higher delays in the results, but the delays are also explained by poor input in form of time of demand. The OBM process plant allows the PSV to stay offshore for more than a week, which makes the vessel obviate two to three returns back to base. By staying offshore for such amounts of time it can operate as a bank, holding and delivering some products that might be unexpectedly wanted at an installation. Such a vessel can save an operator money in terms of less PSVs chartered from the spot market and less fuel consumption connected to less returns to base, but also more time gained for the vessel to be available for service.

Sammen drag

For å kunne håndtere en økning i våtbulkbehov ved offshore-installasjoner på norsk sokkel undersøkes det i denne oppgaven en ny måte å behandle våtbulklogistikk på. Bakgrunnen for det økte behovet er at produksjonsraten på en installasjon må opprettholdes når petroleumsfeltet modnes, noe som gjøres ved å injisere produsert vann i feltet. I tillegg opplever operatører at store og uventede produktbehov er vanskelige å implementere i en PSVs forhåndsplanlagte rute. Dette fører til at operatørene må bruke dyre PSVer hentet fra spot-markedet. Denne oppgaven omhandler dette problemet knyttet til offshore-installasjoner i Nordsjøen som blir betjent fra basen på Mongstad.

Den nye PSVen som blir introdusert i denne oppgaven er dedikert til våtbulkoperasjoner, og har et prosessanlegg om bord som renser oljebasert borevæske (OBM). Lasttankene på våtbulk-PSVen er like i størrelsen og er ikke dedikert til å inneholde en spesiell type last, i motsetning til vanlige PSVer. Denne måten å behandle tankene på gjør skipet mer fleksibelt, og begrenser lastkapasiteten til en type last til skipets totale lastkapasitet. Forsyning av våtbulk karakteriseres av at mange produkter fra basen skal leveres til ulike offshore-installasjoner, mens noen produkter skal hentes inn fra installasjonene, og leveres tilbake til basen. En maritim "pickup and delivery"-modell er introdusert som det som best beskriver problemet, og det er bygget en passende matematisk modell.

En studie av relevant litteratur tilsier at det er gjort lite, eller ingen, forskning på området der en PSV er rutet kun på bakgrunn av våtbulkbehov. Denne oppgaven bidrar derfor med innsikt i hvordan optimering kan brukes til å rute en PSV med et mål om å unngå forsinkede våtbulk leveranser. Målfunksjonen i den matematiske modellen er tvetydig ettersom den minimerer forsinkelser og maksimerer antallet ledige tanker samtidig. Men siden det er forsinkelsene for hver enkelt leveranse, og fleksibiliteten til skipet demonstrert ved antall ledige tanker, som er av interesse, går dette relativt fint. Det fordrer derimot at man må tolke resultatene. Den matematiske modellen er implementert og kjørt i en kommersiell programvare kalt FICO[®] Xpress Optimization

Suite.

Forskjellige caser er satt opp for å utforske kapasiteten til våtbulk-PSVen. Disse er hovedsakelig delt opp i to ulike kategorier, behovsbildet i dag og et bilde der behovene har økt. Relevant data er funnet og hentet først og fremst fra aktører i den norske petroleumsindustrien for å passe best til oppgaven.

Resultater fra casestudiene indikerer at våtbulk-PSVen har kapasitet nok til å behandle store mengder last, som antyder at den kan være offshore over en lengre tidsperiode. Resultatene viser også at det ikke er noen forskjell i å la våtbulk-PSVen og en vanlig PSV håndtere små behov som oppstår hyppig. Det foreslås derfor at våtbulk-PSVen først og fremst håndterer store lastbehov og OBM. En av ulempene ved å behandle OBM om bord på PSVen er at det blir store forsinkelser på de oppsatte lastleveransene, selv om disse også kan forklares av dårlige input-verdier for behovstider. Prosessanlegget tillater våtbulk-PSVen å være offshore mer enn en uke i strekk, noe som gjør at skipet unngår to til tre turer tilbake til basen. Ved å være offshore over slike tidsperioder kan skipet operere som en bank som oppbevarer og leverer produkter det kan oppstå uventet behov for. Et slikt skip kan spare en operatør for penger i form av færre PSVer som leies inn fra spot-markedet og mindre drivstofforbruk forbundet med færre turer til basen. Våtbulk-PSVen vil også være mer tilgjengelig til å betjene installasjoner når den bruker en mindre andel av tid på å returnere til basen.

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Abbreviations

1-M-1	-	One-to-many-to-one
B	-	Breadth
D	-	Draft
FSO	-	Floating Storage and Offloading
FPSO	-	Floating Production Storage and Offloading
H_s	-	Significant wave height
LOA	-	Length overall
m-VRPPD	-	Multi-Vessel Routing Problem with Pickups and Deliveries
MGO	-	Marine Gas Oil
MIP	-	Mixed Integer Problem
MIR	-	Maritime Inventory Routing
NTNU	-	Norwegian University of Science and Technology
OBM	-	Oil Based Mud
OSV	-	Offshore Supply Vessel
PDP	-	Pickup and Delivery Problem
PSV	-	Platform Supply Vessel
SBM	-	Synthetic Based Mud
SP	-	Set Partitioning
TP	-	Thesis Problem
TSPPD	-	Travelling Salesman Problem with Pickups and Deliveries
VRP	-	Vehicle Routing Problem
WBM	-	Water Based Mud

Chapter 1

Introduction

The petroleum production in Norway is located offshore on the Norwegian continental shelf. Many of the petroleum fields in production are getting mature, and some has already been for a while. As a result, the operators experience an increase of wet bulk is necessary to uphold the petroleum production per time unit. The production rate is maintained by injecting produced water into the reservoir through injection wells. The wet bulk needed to uphold the production is mainly water and various types of chemicals (Sangesland, 2016).

For all petroleum fields there exist a production strategy concerning drilling of both new production wells, and injection wells (Nilsen, 2017). Drilling wells offshore require the use of drilling mud. Due to limited storage capacities on the offshore installations, it is important to make the logistics suit the drilling operation's progress. Further, most drilling mud consists of oil which can not be discharged to the sea, thus requiring the drilling mud to be transported back onshore for cleaning. Produced water which comes up together with oil and/or gas from a wellbore, is principally re-injected in dedicated injection wells offshore and does not require cleaning (Nilsen, 2017). The produced water has two origins, it exists naturally in the rock sediments in a reservoir, and it is injected in an ageing oil reservoir for production purposes.

According to the Norwegian Petroleum Directorate (2012) the produced water-oil ratio has increased since 2004. In 2011, 161 million standard cubic metres of produced water were produced on Norwegian petroleum fields, compared to 97.5 million standard cubic metres of oil. The ratio is expected to increase further with the reservoirs' increasing age.

Everything requested at an offshore installation must be transported somehow, and

the most common mean of cargo transportation are platform supply vessels (PSV). Routing and scheduling of PSVs to offshore installations is an area where a lot of research has been conducted. Fagerholt and Lindstad (2000) studies a way to find the optimal fleet size and mix to provide a given number of installation visits in a time period. The routing of a given fleet of PSVs is addressed by Sopot and Gribkovskaia (2014), who applies a neighbourhood heuristic to solve the problem. More recently, the uncertainty which characterises marine operations is studied by adding stochastic aspects, as done by Albjerk et al. (2016). However, most papers concerning scheduling of PSVs consider deck space as the limiting factor, and that the wet bulk demand is met by a sufficient amount of visits. The problem arises when an installation needs unexpected- or extra large orders (Vik and Gullberg, 2016). Such orders are hard to fit into the pre-scheduled PSV routes and are often assigned to high-cost spot chartered vessels.

PSVs today have cargo tanks that are dedicated to hold a certain type of product, for example fuel tanks, mud tanks and special product tanks. An average PSV, represented by Far Scotsman from Farstad shipping, has a mud tank capacity of $1270m^3$ (Farstad Shipping, 2017), while an average mud delivery in a well displacement is $500-1000m^3$ (Vik and Gullberg, 2016). If the capacity is limited to only one tank the vessel may only handle one type of mud at a time, and if the capacity is divided into several tanks it might handle several cargoes but of small amounts. The PSV's design limits the vessel to only handle one large or a few small mud deliveries on each route, which is often not enough to serve the offshore installation in a satisfying manner both with regards to amount and time.

The reason for looking into wet bulk logistics now might be the trend of making all operations cost effective. With a decline in the oil price, the oil companies' margins has been lowered or vanished, and new solutions are necessary to maintain the profit. Earlier the money was often more available, and the logistics were conducted in an easy way. In many cases this means with more vessels than necessary, which implies a higher cost than necessary. Statoil ASA is the leading oil company on the Norwegian continental shelf, and has initiated the following study on their wet bulk logistics from Base Mongstad.

The proposed new system consists of a PSV with extended wet bulk capacity in order to focus on holding and transporting wet bulk to offshore installations in the North Sea. Further, a process plant which cleans oil-based drilling mud is placed on board the vessel in order to change the logistics chain.

The aim of the thesis is to investigate a new way of managing wet bulk logistics in the offshore petroleum industry on the Norwegian continental shelf. How a new type of vessel can be operated when highlighting each wet bulk delivery and changing the wet bulk logistics chain. The thesis will figure out if the proposed system is suited to handle large and unexpected demands in addition to an increase in wet bulk demand and.

Objectives of the thesis are to describe the wet bulk supply system as it is today before presenting the new system. State of the art on the subject will help build a mathematical model within operations research in FICO[®] Xpress Optimization Suite suited to the presented problem. The mathematical model will analyse different scenarios to serve as a basis for the discussion on how the new system should be operated and where it is of most use.

This thesis investigates only the wet bulk logistics from the onshore base to the offshore installations, and vice versa. All cargoes that are required offshore are assumed to be available on the base when the PSV is there. The operations on the base are not of interest and thus not addressed in this thesis, but some of the functions the base holds are relevant for the paper. Data acquired for the thesis is to the greatest extent tied to the Norwegian petroleum business which is characterised with many Norwegian actors. Finally, the weight and stability requirements for the PSV is neglected in the thesis. The thesis is also limited not to assess fuel consumption and the restrictions it adds to the vessel. Note that in this thesis, all facilities that drill for, produce and store oil and gas are referred to as offshore installations.

Upcoming chapters

Chapter 2 describes the wet bulk logistics system as it is today to support the problem description in chapter 3, which elaborates the characteristics of the thesis problem. In chapter 4 state of the art on offshore supply logistics and pickup and delivery problems are presented. Chapter 5 presents the model developed to suit the problem in this thesis. Chapter 6 describes how input data is acquired and treated to fit the presented model. In chapter 7 different cases for a case study are presented. Chapter 8 includes the results from the case studies, before they are discussed in chapter 9. Chapter 10 presents the thesis conclusion and further work.

Chapter 2

System Description

This chapter presents relevant information to support the following problem description. At first some content on how offshore installations on the Norwegian continental shelf are serviced by platform supply vessels is presented. Followed by some necessary information on important aspects for the thesis problem, such as drilling mud and offshore drilling operations.

2.1 Platform Supply Vessels

Platform supply vessels, or PSVs, are like lorries on the sea. They transport whatever the offshore installations need of equipment, provisions, return loads, and other necessities. These necessities can be divided into deck cargo and wet bulk cargo, and the vessels are specially designed for this purpose. The design mainly consist of a large deck space to carry containers with equipment and dry bulk, and is complemented with several tanks below deck dedicated to carry different liquid products. To ensure flexibility on the PSV, large deck space and many tanks dedicated to different liquid products has been prioritised. The length of the vessels ranges primarily between 30 and 100 metres.

In areas with many offshore installations and a lot of marine traffic, the PSVs can be assigned to tasks such as fire fighting and collecting oil spill, and as stand by vessel to an installation. Norway has many ship owners in the supply segment, and Norwegian shipowners owns most of the supply vessels in use on the Norwegian continental shelf.

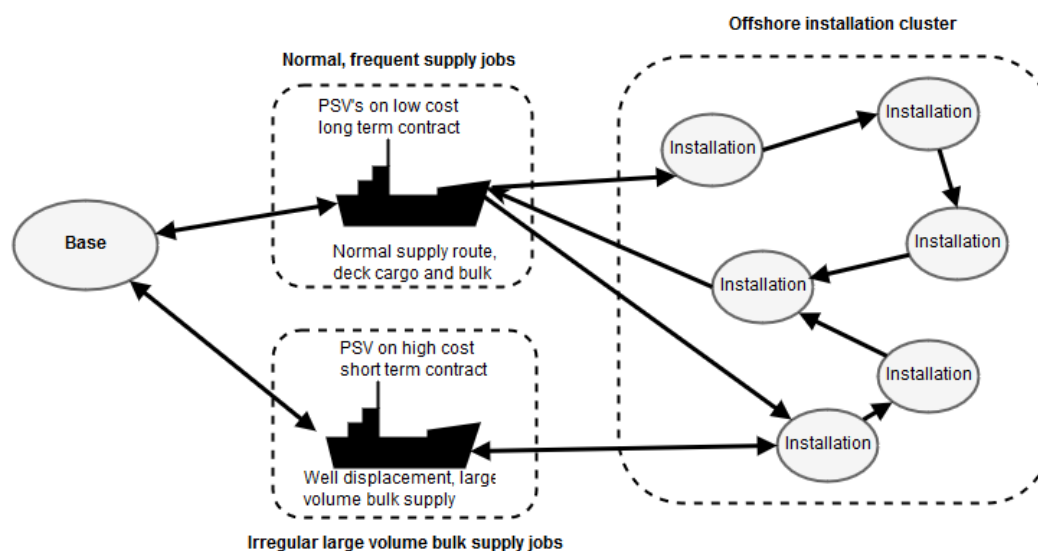


Figure 2.1: The current offshore supply logistics. Showing how PSVs on low cost, long term contracts are routed to several installations in accordance with frequent demands. In contrast to PSVs on high cost, short term contracts which are hired for large or unexpected demands to one or few installations. Adapted from Project Thesis by Resell and Otteraaen (2016)

2.2 Supplying Offshore Installations in Norway

Along the Norwegian coast line there are several onshore bases that each supply a cluster of offshore installations. To service them, PSVs are hired on contracts of various time periods. Most installation demands are easy to predict, and are delivered with PSVs on pre-scheduled routes set to intervals of 1-3 months. However, there are some necessities that are large and less frequent, or that occur unexpectedly, which are difficult to put on a pre-scheduled vessel (Resell and Otteraaen, 2016). From a wet bulk point of view, such necessities are drilling mud, brine and other chemicals needed for petroleum production.

The logistics of offshore supply vessels today are shown in Figure 2.1. The main activity is performed by PSVs chartered on low cost, long term contracts, and are the ones referred to as pre-scheduled. When they leave the base they are assigned to serve several installations for efficiency and good vessel utilisation. Some activity is performed by PSVs that are chartered on high cost, short term contracts. These vessels are often chartered because of a certain cargo that is either too large for a pre-scheduled PSV, or is needed at a certain time/as soon as possible. When these PSVs are chartered, they get assigned to other demands considered useful with regards to time or capacity. Such shipments are costly as the PSVs are chartered from the spot market, but at the same time vital to uphold an installation's operation. In the case

of drilling mud, these irregular shipments are connected to mud demand in general, and well displacements specifically. A well displacement is an operation where the entire mud volume in an oil well is changed. Brine and other chemicals are usually scheduled on low cost PSVs and follows the routing of the vessel it is assigned to.

2.3 Drilling Mud

Drilling mud is used during drilling operations. The purpose of its use is to bring up cuttings, and to lubricate and cool the drill bit. Drilling mud also prevents the borehole from collapsing, and keeps the pressure under control to prevent an uncontrolled blowout (NOG, 2015). As described in Resell and Otteraaen (2016), the mud is circulated between the topside and the borehole during an offshore drilling operation, and at a certain point it gets too contaminated to proceed. At this point it must be cleaned in order to be reused. The mud consists of several expensive products which makes it desirable to reuse it when it is possible. There are few offshore installations that has a process plant on board to perform the cleaning. Thus there is a need for transportation of clean drilling mud to the offshore installations, and removal of the dirty drilling mud for cleaning onshore.

There are three main bases of drilling mud, water-based mud (WBM), synthetic-based mud (SBM) and oil-based mud (OBM). SBM is made by using synthetic oil and has the same properties as mud from hydrocarbon oil. Therefore, talk about OBM will include both SBM and OBM from this stage in the thesis. WBM is the cheapest mud base and is the one that is most used. However, the OBM has some advantages which makes it worth the higher cost. For use in wells with high temperature and high pressure, and for minimising the damage in the well formation, OBM is a better alternative than WBM. Additionally, OBM has better lubricating characteristics, is resistant to contaminants, has faster penetration rate, and is effective against all types of corrosion (Abduo et al., 2016). OBM is the preferred mud base for drilling operations in the North Sea (Vik and Gullberg, 2016).

Another reason for changing drilling mud, apart from contamination, is due to a change in required mud property. It can be related to switching from WBM to OBM, or the necessity of an OBM with higher viscosity or density. OBM consists of water - usually brine, bentonite, barite, and various emulsifiers and detergents. Bentonite and barite is applied for viscosity and weight, respectively, and the emulsifiers and detergents for lubricity (Britannica ACADEMIC, 2017). The composition of the drilling

mud is different dependent on what well it is applied to, and in what section of the well drilling it is to be used. The final mixing of the mud is done on the offshore installations with a drilling mud engineer supervising it (Vik and Gullberg, 2016).

Recycling of drilling mud is a practise that has increased over the years on the Norwegian continental shelf. This comprises of collecting dirty drilling mud and taking it to a process plant to remove the content which makes it dirty. The cleaning process leads to a loss in drilling mud volume, but recycling also lowers the demand for procuring expensive chemicals (Vik and Gullberg, 2016). On most supply bases onshore there are established "banks" that stores recycled drilling mud for later reuse (Norwegian Petroleum Directorate, 2011).

2.4 Offshore Drilling Operations

As described in Resell and Otteraaen (2016), a drilling operation leads to losses of drilling mud. There are residuals of mud in the removed cuttings, and some mud is lost to cracks in the well formation. Based on data from Statoil, approximately 20 % of the gross weight of OBM sent out to the installations is returned to base. Returned percentage by volume is unknown as mud has varying specific gravity, but in this thesis it is assumed to be 20 % as well.

In Table 2.1 a time line of drilling mud demand is estimated with the help of information from Vik and Gullberg (2016). The estimate is made and taken from the project thesis by Resell and Otteraaen (2016). The table shows the demand for drilling mud of a single well throughout its drilling operation. Note that the number of well displacements and time line can vary significantly between wells. Required amount of mud is depending on the size of the well, but mainly the displacement volume is in the range of $500-1000m^3$. The reason for the varying number of well displacements is the nature of the drilling operation. A drilling engineer does not know how the progress will precede before he has initiated the operation, which causes the uncertain time for mud demand (Vik and Gullberg, 2016).

In the beginning an offshore well must be drilled using WBM, until a certain amount of meters is reached. Environmental legislations states that WBM is the only drilling mud that can be discharged in the ocean (Ministry of Climate and Environment, 2017). Drilling on the sea bottom and in the top bottom layer leads to losses of drilling mud to the sea, which restricts the drilling operation to WBM only. The first displacement is due 12 days after commenced drilling operation, and a large delivery of OBM is

required. The WBM that was used to drill the first section can be discharged to the sea and makes room for the clean OBM. After 24 days the first OBM displacement is due. Either the OBM is dirty, or another mud composition is required. The old mud is collected and new mud delivered to the installation for it to continue its drilling operation. The same ship operation is required after approximately 36 days, and after about 50 days the drilling operation is completed. Completion of a drilling operation leaves used drilling mud on the installation, which needs to be transported back to base. It seems like a well displacement is needed every 12 days.

Table 2.1: Estimated time line of drilling mud demand when drilling an offshore well (Vik and Gullberg, 2016). Adapted from project thesis by Resell and Otteraaen (2016)

Day	Well activity	Ship Operation
0	Start drilling offshore well, WBM is used	Delivery of large drilling mud load.
~ 12	First displacement. Change from WBM to OBM.	Delivery of large drilling mud load.
~ 24	Second displacement. Change to different OBM.	Delivery of large drilling mud load, return of used mud.
~ 36	Third displacement. Change to different OBM.	Delivery of large drilling mud load, return of used mud.
~ 50	Drilling completed	Return of used mud.

Concluding Remarks

To sum up the two previous sections, 2.3 and 2.4, Table 2.2 presents relevant mud data.

Table 2.2: Summation of mud data. Adopted from project thesis by Resell and Otteraaen (2016)

What	Description	Comment
Mud types	Oil based mud and water based mud.	Due to different properties, oil based mud is favoured but more expensive.
No. deliveries per well	~ 4	Highly fluctuating number. Depends on the well.
Displacement volume	$500-1000m^3$.	Depends on the size of the well.
Losses	Losing mud volume in the operation.	Remains in removed cuttings, escapes in cracks in the well formation, 10-20% volume loss in cleaning process.

Chapter 3

Problem Description

The problem treated in this thesis will be elaborated and described in all its aspects in this chapter. Section 3.1 presents the different aspects which defines the problem, and in Section 3.3 features which makes the problem stochastic are listed. In section 3.4, a summarised and simplified version to be investigated further is presented.

3.1 What is the Problem and its Main Characteristics

The matter to be discussed in this thesis is a new way of managing wet bulk, with a new type of PSV, between Base Mongstad and offshore installations in the North Sea. A PSV with extended wet bulk capacity exist on the market today, but there are none, or very few, PSVs with a process plant to clean OBM on board. The new system the PSV is to be a part of is characterised by several aspects; wet bulk properties, demand and storage on the installations, capacity and tank allocation on the vessel and an OBM cleaning process plant. Elaborations on said topics are following in this section. Design features on the PSV are presented in section 7.

Base Mongstad is located north of Bergen, Norway, and handles most supply services to the offshore installations in the northern part of the North Sea. Approximate locations of the base and some installations are visualised in Figure 3.1. The offshore oil fields these installations are connected to are well developed and have been operating for between 17 (Oseberg Sør) and 37 years (Statfjord A) (Statoil ASA, 2017).

The North Sea is known for harsh weather conditions which in many cases impacts

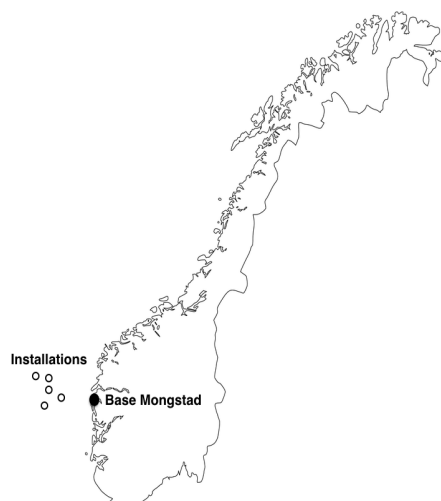


Figure 3.1: Illustration of locations for onshore base and offshore installations

the operability of the vessels working in the area. Especially the winter months are tough, with large waves and regular storms. Such conditions cause a decrease in a vessel's cruising speed, and might subsequently lead to delays if the weather has not been taken into account when scheduling. The unloading- and loading operations of the cargo will also be affected by harsh weather conditions. If the significant wave height, H_s , surpasses 3.5-4.5 m (Larsen, 2016), the operation must wait until a suitable weather window comes. Note that wind conditions will also have an impact on the vessel's operability, not only the H_s alone. The limits introduced by Larsen (2016) in his lecture are related to oil offloading offshore. But as wet bulk is a generalisation of oil, it is assumed that the same criteria is applicable to wet bulk loading as well. A delay of loading operations might affect the rest of the vessel's schedule, unless this is accounted for or the time between demands is large.

Wet Bulk in General, Drilling Mud Specifically

There are several suppliers providing drilling mud and other liquid products to the market today. They are responsible for providing what the installations need, and are usually contracted on a series of drilling operations (Nilsen, 2017). It is assumed that the different suppliers are not willing to let their product be mixed with other competitors' products, due to quality specifications in the contracts. Even though the mud is finalised at the installation, it is to be expected that different suppliers do not want to mix products. Such an assumption will highly inflict and limit the vessel's operability. It is however important to follow, as the market today requires certainty in the delivered cargo.

Drilling mud needs to be cleaned in order to be reused. The normal practise for the vessels connected to Base Mongstad, is to deliver the dirty mud to the base where a process plant cleans it. If the cleaning process could be moved onto the vessel, the amount of trips between base and field will decrease and related costs will be reduced. It is also fair to assume that implementing the cleaning process into the operator's scope of work will lessen its cleaning expenses.

Another aspect concerning the drilling mud is that recycled mud should not be used for drilling exploration wells, or other wells where collecting data is a priority. An exploration well is drilled to gather information regarding the content in the soil and its whereabouts. Some small particles can not be removed from the drilling mud in the recycling process, and will thus contaminate tests that are extracted from exploration wells. This must be considered when storing the mud on board and assigning mud to installations, but is not considered in this thesis. As mentioned in 2.3, there are established "banks" for recycled drilling mud on most supply bases. If there is enough capacity on the dedicated vessel, it might operate as a forward stock with recycled drilling mud from the supplier/s that has the most contracts in the area.

Demand and Storage

Talking with Statoil, a large part of their problem is the uncertainty in the demand of wet bulk and drilling mud in particular. The uncertainty is connected to the volatile progress of the drilling operation. Due to the uncertainty, and the amounts needed in a well displacement, these tasks are usually given to the PSVs on the spot market. Thus resulting in higher costs than if it was placed on a pre-routed vessel.

There is a lack of storage space on the installations, especially on the elder ones (Vik and Gullberg, 2016) which most installations of interest are. As these small storage capacities are different for each installation, they are assumed to be negligible. This implies that when a well displacement is needed, all mud in the installation's drilling system is collected and then replaced with new mud. Implementing this assumption leads to handling large amounts of cargo, as the systems often contain between 500 m^3 and 1000 m^3 of drilling mud, as listed in table 2.2. Another consequence of this assumption is that the drilling operation is thought to stop during the well displacement. Delays in this operation thus leads to induced costs in form of longer rent of drilling rigs and personnel.

The problem mentioned above is commonly solved by having a PSV as a storage vessel beside the offshore installation. This option is not considered in this thesis.

Tank Allocation and Capacity

Drilling mud and other types of wet bulk are in general defined by two variables, the supplier and its condition. *Condition* refers to whether the product is clean or dirty. As the drilling mud is properly mixed on the installations to suit its purpose of use, clean drilling mud is assumed similar in composition.

As mentioned in section *Demand and Storage*, most offshore installations lack storage capacity. This results in an assumption that an installation requiring a well displacement must load its dirty mud onto the vessel before receiving the clean mud. The assumption might restrict the vessel's operability, as it implies that the vessel must have capacity to hold both the clean and the dirty load at the same time. Especially the cases without process plant on board are affected by this assumption.

An example of cargo handling and tank allocation

Following is a description of the tank allocation for an imagined route handling drilling mud only. There are four nodes to visit, but only the three first nodes are shown in detail. There is an incident of well displacement in node 1, one cargo of clean drilling mud to deliver in node 2, and one cargo of dirty drilling mud to collect in node 3. Figure 3.2 shows illustrations of a set of tanks, and clean- and dirty drilling mud used in the following example. In figures 3.3 and 3.4 there are numbers inside the depicted drilling mud which represents suppliers, different numbers means different suppliers.

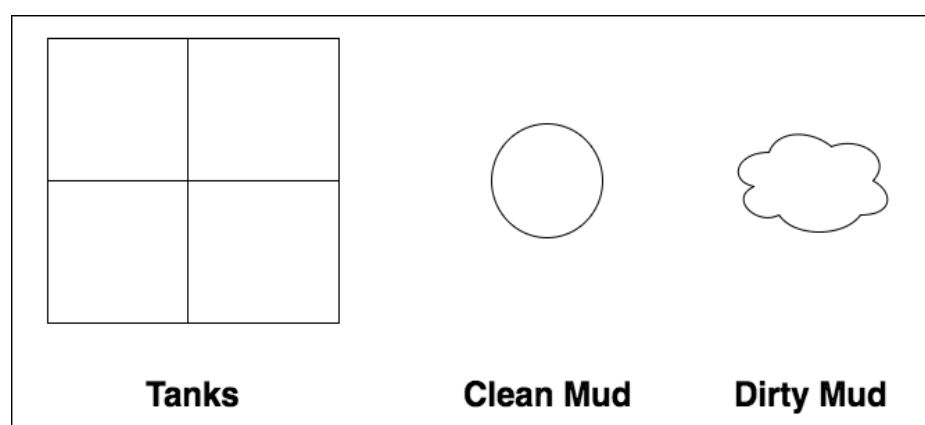


Figure 3.2: Description of figures depicting tanks, clean drilling mud and dirty drilling mud, used when describing tank capacities and allocation

As Figure 3.3 shows, the initial tank allocation consists of two tanks occupied with clean drilling mud from supplier 1, one tank with clean drilling mud from supplier 2,

in addition to one empty tank. In node 1 it is requested a well displacement with mud from supplier 1, which is handled as shown in Figure 3.3. First the dirty drilling mud is collected and placed in the empty tank, then the clean drilling mud is loaded onto the installation. According to the assumption made earlier, this well displacement would not have been feasible if the fourth tank was occupied with, for example, clean drilling mud.

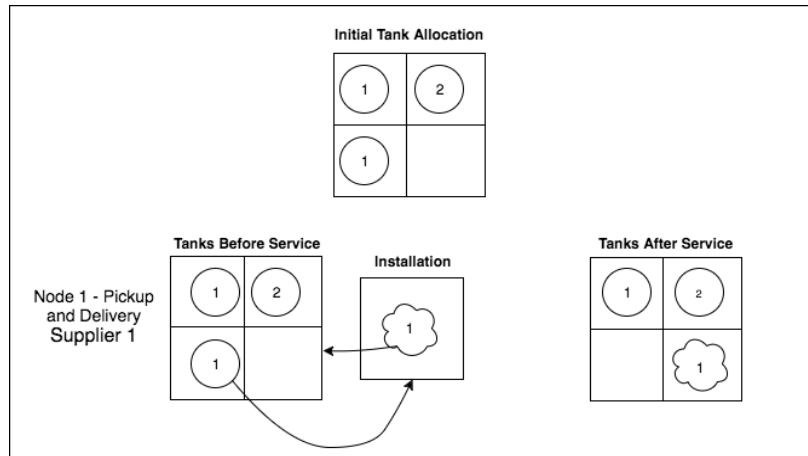


Figure 3.3: Initial tank allocation and the allocation during a mud switch at an installation

Figure 3.4 shows tank allocation when servicing node 2 and 3 in the imagined route. Node 2 requires a delivery of clean drilling mud from supplier 1, which is easily handed over. The vessel now has two empty tanks, the upper and lower left corner. However, the upper left corner may have some leftover residuals from the drilling mud that it delivered in node 1, which might affect the further operation. Node 3 is an installation that requires the collection of a large amount of dirty drilling mud from supplier 2. The amount surpasses the vessel's tank capacity, and it is therefore necessary to allocate two cargo tanks to allow service of the installation.

After servicing these three nodes, the vessel with one tank of clean drilling mud and three tanks with dirty drilling mud is considered fully loaded. To fully utilise the vessels tank capacity, the vessel should deliver its load of clean drilling mud to another installation, and pick up a load of dirty drilling mud before returning to base.

In the case of an unforeseen event at an installation, the vessel might be unable to service its route as scheduled. In these cases it will be an advantage if the vessel has the capacity to service the installation later on the scheduled route. A specific example of this is the last situation in Figure 3.4. With all tanks occupied, the possible vessel operations are strictly limited to delivering the clean drilling mud that is left, or to return back to base. If the vessel is equipped with one more tank which is empty, the

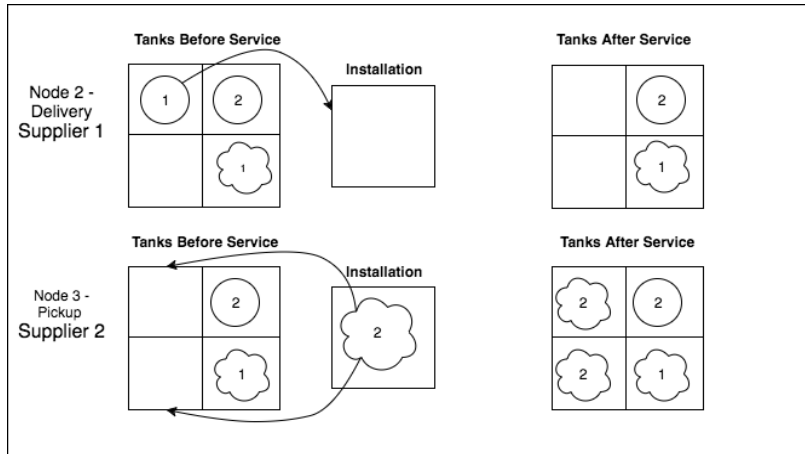


Figure 3.4: Tank allocations of clean and dirty drilling mud when servicing a delivery cargo, node 2, and a pickup of a large cargo, node 3

service possibilities increases to include well displacements and pickups as well. By allowing one tank to stay empty the vessel may be better suited to service installations in another sequence than scheduled.

3.2 Process Plant Cleaning OBM

On Base Mongstad there are process plants which cleans the dirty OBM that PSVs deliver. By installing such a process plant on a PSV, the OBM can be cleaned without being transported back to base and thus save money and time. In this thesis the process plant acts as a black box which has a few inputs and outputs due to missing information on its size and capabilities.

3.3 Why is this Problem Stochastic?

The problem described above is stochastic due to several features.

As described under section 3.1, rough weather conditions will limit offshore marine operations. The part which makes the weather situation difficult is that it is hard to predict, especially long time ahead. Consequences of the weather stopping loading or offloading operations are delays, which are costly. Therefore it is unlikely that a deterministic optimisation model will provide realistic solutions for the problem, creating a large gap between model and reality.

Another part of the stochastic problem is related to the demand. As described in

section 3.1 *Demand and Storage*, the time for when the demand occurs is highly fluctuating.

Predicting the progress of the drilling operation is difficult. Results from exploration wells in the area can provide good indications, but a finite solution is only obtained by drilling the actual well.

Whether cargoes can be mixed in the same tank or not is of great importance, since there is a limited number of tanks on board. Skipping an installation with the purpose of serving it later might make the problem infeasible to solve, due to lack of space both before or after servicing the next.

For the problem to be modelled as realistic as possible, all aspects mentioned above should be accounted for in an optimisation model. This is however difficult, as the future is impossible to foresee and can only be predicted with background in known changes and statistics.

3.4 Closing Remarks on Problem Description

The new wet bulk management is characterised by several properties. A PSV with extended wet bulk capacity and a process plant for cleaning OBM is introduced to the current PSV fleet, and needs to be investigated and scheduled. Scheduling the vessel will contribute to corroborate the new system and introduce a mathematical model that is applicable to the PSV. The mathematical model must be built in accordance with weather, geography, wet bulk properties, demand trends, vessel capacity and the process plant. In order to look at the cost aspect on the vessel without looking at it directly, it is possible to minimise waiting times, which in itself represents costs.

The weather is difficult to assess in a deterministic model, and in the mathematical model it is important to distinguish between different products and product suppliers. Additionally it is of significance that the PSV picks up a cargo before a delivery is made at the same installation. To meet the listed requirements, a maritime pickup and delivery model seems to fit the problem best.

Chapter 4

Literature Review

This chapter presents state of the art within offshore supply routing and scheduling. To include the cargo aspect, literature on maritime cargo routing is included as well as some pickup and delivery problems in other business segments.

The process of acquiring knowledge started with the Oria-portal, which is provided by the university library at NTNU. All collections the university library possesses, both printed and electronic, are available for students and employees to use. This includes a large number of scientific journals within in a vast variety of fields, where state of the art literature is published. Searching for relevant papers, and looking at papers that cites important methodology, has contributed to the content of the following literature review. The problem studied in this thesis is referred to as the Thesis Problem (TP) in this chapter.

The chapter's structure consists of an introduction to shipping industry characteristics, before a brief explanation of pickup and delivery problems and heuristics. Following are literature on maritime routing and scheduling in general and offshore supply problems specifically, ending with concluding remarks.

4.1 Characteristics of the Shipping Industry

There are three general modes in shipping, *industrial*, *tramp* and *liner* (Fagerholt, 2016). Vessels performing liner shipping operates like a bus line, where cargo owners must assign cargo to departures. Tramp shipping can be compared to taxi business, where the vessels follow cargo that needs transportation, and wants to maximise profit.

Industrial shipping is when the owner of the cargo also controls the vessels that performs the transportation. The objective is thus to minimise the costs related to the cargo transport as all cargoes must be serviced. Industrial shipping is the applicable mode for the TP, as the operator of an installation in most cases are responsible of transporting demanded cargo back and forth. Shipping companies experience three levels of planning problems: *strategic*, *tactical* and *operational*. Determining a company's fleet size and mix is a strategic planning problem, whilst cargo routing and scheduling are tactical and operational planning problems (Christiansen et al., 2004). Ronen (1993) defines *routing* as assigning a sequence of ports to a vessel, and *scheduling* as assigning times to the different events on a vessel's route. The different methods are often connected with the time aspect of the planning. The scheduling problems usually have shorter planning horizons due to uncertainties in marine operations (Ronen, 1993).

The different modes are connected to each other in an important interface. Determining the fleet size and mix defines the configuration of vessels that are available for routing, scheduling and deployment. Then again, determining the optimal fleet demands a knowledge of the ports' requests and representative routes (Christiansen et al., 2004).

4.2 Pickup and Delivery Problems (PDP)

According to Berbeglia et al. (2007), pickup and delivery problems are conveniently classified into a three-field scheme of *structure*, *visits* and *vehicles*. The structure refers to the number of origins and destinations, which for the TP is one-to-many-to-one (1-M-1). Gribkovskaia et al. (2008) states that scheduling a single vehicle to pickup and delivery cargoes back and forth between offshore installations and an onshore base is characterised as a 1-M-1-problem. This is because all deliveries originate at a depot, and all pickups are sent back to the same depot. Visits refer to how the pickups and deliveries are performed in the nodes. For the TP, P/D is the yielding description because every cargo is represented by a node which represents either a pickup or a delivery demand. The vehicles refer to the number of vehicles that are available in the problem solving, which in this case is one vessel.

PDPs with time windows are computationally hard problems and are well researched with several papers studying the area. Dumas et al. (1991) create an exact model that solves a general problem with instances where the capacity constraints are restrictive.

The health industry encounters challenges in material logistics between different depots and consumers, and is studied by Liu et al. (2013). The time windows assure that both pickup and delivery demands are treated simultaneously, as the consumers only can be visited once. This differs from the TP, where an installation might be visited several times as each node represents a cargo.

An aspect which characterises most logistic operations, and PDPs, is uncertainty. In recent years the amount of studies on this area has risen. Uncertainty in operational research is often handled by introducing stochastic processes to the uncertain problem elements. Arnesen et al. (2017) investigates stochastic in-port routing for chemical tankers, which has several similarities with the TP. "The problem of routing and scheduling a ship in port is, in principal similar to the *Traveling Salesman Problem with Pickups and Deliveries* (TSPPD), which is also known as the *Single-Vehicle One-to-Many-to-One Pickup and Delivery Problem* (1-M-1-PDP)" (Arnesen et al., 2017). The 1-M-1-PDP and the multi-product nature of the problem are similar to the TP, but the time window and draft limit aspects differ. The stochastic single-vehicle PDP is also studied by Swihart and Papastavrou (1999) for onshore traffic, with the objective to minimise the expected time in system for the demands.

4.3 Heuristics

Heuristics are often used to solve difficult optimisation problems. A heuristic solution method generates a good solution, but gives no guarantee on its quality. Different methods are designed for specific classes of optimisation problems, and can be based on very simple rules. Choosing heuristics as a solution method is favoured when exact methods take too long time, or the exact method requires a lot of computer memory, amongst others. All theory in this section is found in Lundgren et al. (2012).

There are several types of heuristics, following are some which are common in use.

- **Constructive Heuristics** - Builds a feasible solution successively. Often used to find an initial feasible solution. Example: Nearest-neighbour and sweep heuristic.
- **Local Search Methods** - Improves a feasible solution iteratively. Provides a local optimum.
- **Metaheuristics** - Prevents the local search method from getting stuck in a local optimum. Allows a temporary worse solution to be able to reach better

solutions in other areas. Example: Tabu search.

Amongst the papers reviewed in Christiansen et al. (2004), 40% of the ship scheduling problems were solved by using set partitioning (SP) heuristic. SP-problems are solved in two steps. The first step is to construct feasible routes for the vessels that are available. The routes are created according to the constraints for time windows, load capacities and other problem specific restrictions. This is done either by a constructive heuristic or by an exact mathematical model. Feasible routes are then input to a model, which picks out the ones that together best fulfil the objective function. The reason for its popularity within the ship routing and scheduling segment is that the SP model often can be solved to optimality with the use of standard optimisation solvers, and that feasible routes are easy to construct either by optimisation or heuristics (Fagerholt and Lindstad, 2000; Christiansen et al., 2004). An SP solution method is flexible in the sense that it is possible to adjust the way of generating routes in accordance with the intended use of the solution. If the use requires the optimal solution, *all* feasible routes must be constructed and put into the model. Finding the optimal solution gets more time consuming as the problem size increases, and a heuristic might be a better choice for route generation. However, due to ship scheduling problems often being of limited size and well constrained in comparison to other vehicle scheduling problems, the optimal solution is often found solving the SP-problem (Fagerholt and Lindstad, 2000). If an adequate solution is sufficient, or a problem is large, a heuristic can generate many feasible routes and send them to the model. Using heuristics is less time consuming, but there is no way of checking the solution's quality (Christiansen et al., 2004).

4.4 Maritime Routing and Scheduling

According to Al-Khayyal and Hwang (2007), maritime routing and scheduling problems for bulk products can be divided into two groups; *cargo routing* and *inventory routing* problems. Cargo routing assigns a vessel to a specific cargo, and is thus constrained by the cargo size, its loading ports, and often time windows. In general, vessels are able to transport multiple cargoes simultaneously. However, for large bulk commodities, one cargo is often a full shipload. For minor commodities, such as chemicals, vessels can transport several products on the same voyage. Inventory routing complies to the stock constraints in different ports. The decision is then what amount of cargo that should be transported to stay within the stock limitations at all times. The TP falls under the cargo routing problem.

An early study on the transportation of crude oil by Brown et al. (1987) initiated a series of complementary studies on maritime routing and scheduling of bulk products. Brown et al. (1987) introduces an elastic set partitioning model to determine which cargoes to be served by a controlled fleet, and which cargoes to assign to a spot chartered vessel. The proposed mixed integer problem (MIP)-model is solved to optimality in short time for cargoes set to be full shiploads. Bausch (1998) changed the cargoes such that they consist of up to five products, and includes the possibility of optional back hauls. Optional back hauls are not considered in the TP, but several products lie in the nature of the problem presented. A vessel schedule in Bausch (1998) consists of assignments covering two to three weeks, according to the term of short-time scheduling. The aim of the study is to establish a user friendly interface in Microsoft Excel to support the dispatcher. Another expansion of the study by Brown et al. (1987) is done by Sherali (1999). Sherali (1999) creates an arc flow model applicable to the smaller instances. For the larger problems a rolling horizon solution algorithm is applied, which sequentially fixes integer variables until a feasible solution of good quality is obtained.

For maritime wet bulk transportation, the amount of different products and the number of compartments on the assigned vessel are important factors when constructing feasible schedules. This yields especially for chemical tankers. Thus it is important to decide which load to allocate to a given tank. This stowage problem is investigated by Hvattum et al. (2009) and takes into consideration aspects as vessel roll and trim to secure a feasible sailing condition, as well as a feasible route. In a dry cargo perspective, the compartment allocation is of equal importance as for the wet bulk. This is treated by Fagerholt and Christiansen (2000) who introduce flexible cargo holds. Determining the size of the cargo holds and cargo allocation simultaneously will directly influence the vessel's operability. These aspects of the problem is not considered in the TP, where it is assumed that a feasible sailing solution always can be obtained. A significant difference between wet and dry bulk transportation is that wet bulk must be transported in tanks, not open compartments. Thus making flexible cargo holds difficult and not relevant for the TP.

In a paper by Foss et al. (2016) a multi-product maritime routing problem is studied without dedicating compartments to certain liquid products. Constructed as a maritime inventory routing (MIR)-problem, vessels in a fleet can carry several liquid products, but only one type of product in each compartment. Washing of tanks between the holding of different products is neglected as the washing time is less than loading/unloading time in port. However, it is an important aspect to consider

when handling different products in the same tanks, as some products will badly contaminate other products. The same logic is applied to the TP with non-dedicated tanks, only occupied tanks. Washing of tanks is also neglected in the TP, but assumed performed when necessary between voyages. A similar problem is addressed by Christiansen et al. (2016) with fuel supply vessels supplying customer ships anchored outside a port. A feature equal to the TP is that if different customers order the same type of fuel, it can be allocated to the same compartment.

Fagerholt et al. (2010) addresses the problem of strategic planning in maritime bulk shipping by combining optimisation and simulation methods. An optimisation model solving the short-term routing and scheduling problem acts as input to a simulation model. The simulation model provides the optimal fleet size and mix for the schedules based on scenarios constructed by Monte Carlo-simulations. Fagerholt et al. (2010) states that a combination of optimisation and simulation will handle weaknesses with the respective methods, stochastic elements and routing aspects, better than by just using one. The paper by Norlund et al. (2015) brings the combined solution method into the offshore supply business, and provides an algorithm that creates cost-efficient and "green" vessel schedules. The TP has several elements which in reality are stochastic, but as they are chosen to be treated as deterministic simulation is not considered necessary.

4.5 Routing and Scheduling of Offshore Supply Vessels (OSV)

Fagerholt and Lindstad (2000) brought the SP solution method into a new shipping area, the offshore supply business. They look at a model which can help decide the optimal fleet size to service offshore installations in the Norwegian Sea, based on a required number of visits at the installations. As the authors state that the bulk capacities on the PSVs exceed the installations demand, the deck capacity is set as the binding constraint. This is in opposition to the TP, where the wet bulk cargo is treated making the tank capacities the binding constraint. The aim of the vehicle routing problem (VRP) modeled by Fagerholt and Lindstad (2000) is to provide a fleet that can handle different scenarios of available service hours on the installations. Said paper is one of the first papers treating routing of offshore supply vessels.

Several papers have included the amount of cargo into the routing decision, and can be characterised as pickup and delivery problems. Fernández Cuesta et al. (2017)

presents two models which routes vessels according to different cargoes that is needed at offshore installations. Routes with a weekly regularity are however determined a priori, and the vessels only visit the installations with a pickup or a delivery demand on the route. A *Vessel Routing Problem with Selective Pickups and Deliveries* (VRPSPD) is introduced to allow a vessel to revisit an installation - if it means the vessel will have better utilisation, and the need for a voyage chartered vessel becomes unnecessary. Sometimes this is not possible and an extra vessel must be assigned, the introduced *Multi-Vessel Routing Problem with Pickups and Deliveries* (m-VRPPD) assigns the normal and extra vessels in parallel to fully exploit their utilisation. An adaptive large neighbourhood search heuristic is used to solve the models. Sopot and Gribkovskaia (2014) are first out with opening for a revisit of installations, which makes it more likely to build feasible schedules to the PDP. The large neighbourhood search heuristic is the chosen method for solving the problem by both Sopot and Gribkovskaia (2014) and Shyshou et al. (2012). Shyshou et al. (2012) provides optimal or near-optimal solutions to small instances of the periodic routing problem of PSVs. Said problems differ from the TP as they handle routing of several vessels. But especially Fernández Cuesta et al. (2017) proposes interesting aspects on how to handle insufficient fleet capacity.

As mentioned in section 4.2 the number of papers trying to handle the uncertainty which characterise the shipping industry has increased. Stochastic models are studied, as well as an introduction of disruption management to the maritime routing business. Albjerk et al. (2016) investigates the latter subject. The problem is to determine the further voyage for OSVs after a disruption has occurred. Creating both an arc flow- and a path flow PDP-model, all deck cargoes in the short time period should be serviced either by a vessel in the fleet or by a vessel from the spot market. A dynamic programming algorithm is used to generate the paths that minimises costs and maintains a sufficient service level to the offshore installations. The study by Albjerk et al. (2016) is highly relevant for an implementation to the wet bulk aspect, even though the TP does not consider uncertainty.

Aas et al. (2007) presents a MIP-model that takes into account pickup and delivery demands, vessel capacity and free storage capacity on the installations, but does not include the stochastic perspective. A single vessel is scheduled according to said constraints with the possibility of visiting an installation twice. With deck space as the limiting factor, which stands in contrast to the TP. The objective of the model in Aas et al. (2007) is to design a minimum length route for the single vessel. A tabu search heuristic is applied to the problem presented by Aas et al. (2007) in Gribkovskaia et al. (2008). In the paper by Seixas et al. (2016) the stowage of cargoes

on PSVs servicing offshore installations is investigated using a local search heuristic on fixed route schedules. Again, the deck capacity is the limiting factor and wet bulk cargo not handled specifically.

4.6 Closing Remarks on Literature

It has come to my attention that most features with the thesis problem; no tank allocation, single vehicle offshore scheduling and scheduling with regards to wet bulk demand, are relatively well researched. However, to my best knowledge, no research has been conducted that unifies them. Most papers on PDP-scheduling of single vessels has deck cargo space as the limiting factor. The papers that consider routing of PSVs in general are the ones that most often refer to a wet bulk demand. The preferred solution method for these problems is versions of VRP-models, set up after a demanded amount of visits for a given time horizon. Thus assuming that a given number of visits will suffice to handle an installations wet bulk demand.

Chapter 5

Method

This chapter contains the thoughts behind the making of the mathematical model solving the problem presented in chapter 3 *Problem Description*, and its explanation. Modelling assumptions are presented in section 5.1, and in section 5.2 and 5.3 a Pickup and Delivery Problem is presented.

5.1 Model Assumptions

This section covers different considerations that are made when interfacing the real problem with a mathematical model. The considerations are divided into three parts, General, Cargo and Tank considerations.

General considerations

The mathematical model studied in this thesis must be linear. This is because of the commercial MIP-solver to be used, Xpress-IVE, only handles linear expressions. If an expression is nonlinear there are two, or several, variables multiplied with each other.

In most cases there is only one supplier of drilling mud engaged in a drilling operation on an installation. It is therefore an assumption that there is only one type of drilling mud that is handled at each installation. If a mud switch needs service, it will appear in the model as two different cargoes but with the same location. In other words, there is no additional sailing time when servicing these cargoes.

The model does not open for servicing cargoes before the time of demand or multiple visits at the same installation on a single route.

In a large sized problem the model might end up running for an interminable time in order to search the entire solution space. It is therefore decided that the model should stop running after 10800 seconds and present the best solution that is found at that time.

Cargo considerations

As clean drilling mud in most cases is mixed properly at the installation, it is assumed that mud destined to different locations might be mixed in the vessel tanks as long as the supplier and condition is the same. This assumption implies that the cargo tanks on the vessel can be characterised with the same indices as the cargo. Further, it binds the tanks to contain products of a certain type from the time it is filled until it is empty. However, according to Norwegian Petroleum Directorate (2011), recycled drilling mud should not be used in exploration wells due to contamination of test results. Thus exploration wells needs new, clean drilling mud. To keep track of the clean drilling mud types it is necessary to label the different batches, which is not done in the following model. Further, with the introduction of the process plant this is an important feature to have control over.

When the vessel picks up drilling mud from the installations it is always considered dirty, and in practice it will be in most cases. However, unforeseen events might lead to a need for pickup of clean mud, but that is not considered in this model.

As mentioned in section 3.1 under *Tank Allocation and Capacity*, it is assumed that the vessel must pick up dirty drilling mud before loading clean drilling mud onto an installation. This requires the model to contain restrictions regarding sequencing of the two operations.

Tank considerations

The tanks on board the vessel are assumed to be equally sized. Stability is not considered, as it is expected that ballast tanks always can make the vessel have a feasible stability situation.

Tank allocation and cargo tracking is not considered in the following model. A simplification is made to keep track of *how much* load there is of each cargo type on board

the vessel, but not *which* cargoes that are present and their *placement*.

Regarding occupation of tanks, it is assumed that an empty tank which is not washed contains very small amounts of leftover residuals from the product that occupied it earlier. This is mostly relevant after emptying a tank with a clean product and wanting to fill it with a dirty one, which is a situation with high probability of occurring. As all dirty product is going to be recycled afterwards, and the leftover residuals are few, the recycled product is practically only from the supplier of the dirty product. This assumption is applied in this thesis as it is necessary in order to avoid tracking each cargo throughout the route. All tanks are thus considered washed before the routing begins.

5.2 A Maritime Pickup and Delivery Problem

The problem is defined on a graph $G = (N, A)$, where N is the set of all nodes, and A is the set of all arcs in the network. The cargoes are modelled as nodes and indexed by i and j , and the number of cargoes is n . Origin and destination nodes $o = 0$, and $d = n+1$, acts as Base Mongstad, and thus have equal properties. $N = \{0,1,\dots,n+1\}$ is the set of nodes, while $N^C = \{1,2,\dots,n\}$ is the set of cargoes. Cargoes are divided into delivery nodes and pickup nodes. $N^D \subset N^C$ is the set of delivery nodes, and $N^P \subset N^C$ is the set of pickup nodes. Delivery cargoes are cargoes which the vessel *delivers to* the node, while pickup cargoes are cargoes which the vessel *picks up from* a node.

Let k be the number of different suppliers and products, and $S = \{1,2,\dots,k\}$ be the set of all different products indexed by s . From this point, the number of different products and suppliers are referred to as products only. $C = \{\text{clean, dirty}\}, \{1,2\}$, is the set of cargo conditions, indexed by c .

The parameter Q_{is} is the amount of cargo in cubic meter of product s to be serviced in node i , and let $L_{0s1} = \sum_{i \in N^D} Q_{is}$ for all $s \in S$, be the initial amount of load on board the vessel for each type of product. The cargo quantity is summed over the delivery nodes, as they represent the amount of cargo that is to be delivered from the base to the installations. All cargo quantities are given as positive numbers regardless of being on a delivery- or a pickup node.

V^{CAP} is the parameter for the vessel's load capacity, parameter H is the number of tanks on the vessel, while H^{CAP} is the equal sized tanks' capacity.

The time the ship uses from the start of service in node i until the start of service in

node j is set by parameter T_{ij} :

$$T_{ij} = U_i + E_i + S_{ij} \quad (5.1)$$

where E_i is the time when leaving node i and S_{ij} is the sailing time from i to j . U_i is the time it takes to load or unload cargo j . Let $U_i = \frac{\sum_{s \in S} Q_{is}}{R}$ where parameter R is the loading and unloading rate at the installation. This can be done as it is assumed there is only one supplier of drilling mud, and only one product to deliver per route, at an offshore installation. Thus the sum over all quantities from different suppliers at an installation is equal to the amount of cargo demanded from the contracted product.

Waiting time before entering a node is included in the sailing time S_{ij} . As two cargoes may be situated at the same installation (one delivery- and one pickup load), there might be an issue with letting E_i be the same for both cargoes, when the time actually is passing by. This is solved by letting T_{ij} only contain a negative entering equal to E_i .

All cargoes have a time for when it is needed, also referred to as occurrence of demand, D_i . The time parameter refers to a point of time after the vessel leaves Base Mongstad, assuming the vessel leaves in time 0.

The vessel's capacity is denoted by V^{CAP} , and is the product of the number of tanks on the vessel H and the tanks' size H^{CAP} .

There are six variables. The time variable t_i is the time *before* service is commenced in node i . Load variable l_{isc} is the load, in volume, on the vessel in condition c of product s *after* servicing node i . h_{isc} is an integer variable that states how many tanks that are occupied by load in condition c of product s after service in node i . The arc flow variable x_{ij} is 1 if the vessel sails directly from node i to node j , and 0 otherwise. y_i is a time variable which represents the positive time gap between the time for demand at installation i , and the time which the vessel commences service the installation. Finally, w_i is a variable containing the amount of empty tanks available after service of each cargo.

Objective function

$$\text{minimise} \quad \sum_{i \in N^C} y_i - \sum_{i \in N^C} w_i \quad (5.2)$$

y_i represents the time gap between the point of time for the demand of cargo i , and the

time which the vessel starts servicing the cargo. This time gap represents costs and no added value, and is to be minimised. Subtracting the w_i variable favours solutions where the amount of available tanks are at a maximum.

Flow constraints

$$\sum_{j \in N \setminus \{d\}} x_{oj} = 1 \quad (5.3)$$

$$\sum_{i \in N^C} x_{ij} - \sum_{i \in N^C} x_{ji} = 0, \quad j \in N^C \quad (5.4)$$

$$\sum_{i \in N \setminus \{o\}} x_{id} = 1 \quad (5.5)$$

$$\sum_{j \in N} x_{ij} = 1, \quad i \in N \quad (5.6)$$

Constraints (5.3) let only one arc go out from the base/origin node, while constraints (5.5) ensure only one arc going in to base/destination node. Constraints (5.4) let a cargo that has an in-going arc also have an outgoing to make sure the flow is maintained. Constraints (5.6) ensure that all cargoes are serviced along the route.

Time constraints

$$x_{ij}(t_i + T_{ij} - t_j) \leq 0 \quad (i, j) \in A \quad (5.7)$$

$$t_i + T_{ij} - t_j - M(1 - x_{ij}) \leq 0, \quad (i, j) \in A \quad (5.8)$$

Constraints (5.7) ensure that the time at the start of service in node j is larger than or equal to the time from the node it came from, node i . This is ensured by adding T_{ij} as discussed above. Constraints (5.8) show the linearised version, where M is a big number coefficient used to make the constraint binding when x_{ij} equals 1, and non-binding otherwise. M is set to be a large value surpassing the intended time horizon of the problem.

$$t_i - t_0 \geq 0, \quad i \in N \setminus o \quad (5.9)$$

$$t_{n+1} - t_i \geq 0, \quad i \in N \setminus d \quad (5.10)$$

Constraints (5.9) confirm that the vessel must visit the base/start node before visiting all the other nodes. The constraints containing the start node are excluded as it will be 0 in all cases, thus creating a model with one less constraint. Constraints (5.10) make sure that the base/end node is the last node to be visited on the route. Adding these specific constraints ensures that the vessel performs a round trip. The constraints containing the end node is excluded because it will always be equal to 0, and one less constraint shortens solution time.

$$t_i - D_i - y_i = 0, \quad i \in N^C \quad (5.11)$$

Constraints (5.11) make variable y_i take the value of time between the demand for cargo i occurs, D_i , and the vessel starts servicing the cargo, t_i .

Load constraints

The load constraints are in general divided into two sets, one set treating the clean products, and one set treating dirty products. Therefore are all load constraints in pairs with differences in cargo condition $\{1,2\}$, and the cargo to which it is applicable $\{\text{delivery, pickup}\}$.

$$x_{ij}(l_{is1} - Q_{js} - l_{js1}) \leq 0, \quad i \in N \setminus \{d\}, j \in N^D, s \in S \quad (5.12)$$

$$l_{is1} - Q_{js} - l_{js1} - B1(1 - x_{ij}) \leq 0, \quad i \in N \setminus \{d\}, j \in N^D, s \in S \quad (5.13)$$

Constraints (5.12) update the load variable with matching product as cargo i , given that the node is a delivery node - hence the 1 for clean cargo condition. Constraints (5.13) are the linear version of constraints (5.12), and require a large number $B1$ to ensure that the constraints only are applied when the vessel sails directly from node i to node j . The value of parameter $B1$ is set to the total amount of all delivery cargoes on the scheduled route.

$$x_{ij}(l_{is2} - Q_{js} - l_{js2}) \leq 0, \quad i \in N \setminus \{d\}, j \in N^P, s \in S \quad (5.14)$$

$$l_{is2} - Q_{js} - l_{js2} - B2(1 - x_{ij}) \leq 0, \quad i \in N \setminus \{d\}, j \in N^P, s \in S \quad (5.15)$$

Constraints (5.14) and (5.15) are equivalent to constraints (5.12) and (5.13), except that they apply to pickup nodes and dirty products, instead of delivery nodes. The parameter $B2$ is set to the total amount of all pickup cargoes to ensure that the constraints only are applied when the vessel sails directly from node i to node j .

$$x_{ij}(l_{is1} - l_{js1}) = 0, \quad i \in N \setminus \{d\}, j \in N^P, s \in S \quad (5.16)$$

linearised to

$$l_{is1} - l_{js1} + V^{CAP} x_{ij} \leq V^{CAP} x_{ij}, \quad i \in N \setminus \{d\}, j \in N^P, s \in S \quad (5.16a)$$

$$l_{is1} - l_{js1} - V^{CAP} x_{ij} \geq -V^{CAP} x_{ij}, \quad i \in N \setminus \{d\}, j \in N^P, s \in S \quad (5.16b)$$

While constraints (5.13) and (5.15) take care of the load variables that are changing when servicing cargo i , constraints (5.16) make sure the other load variables remain the same. These constraints must be added, otherwise loads will disappear in nodes they are not called upon, and the load situation at all times will solely consist of the load being treated. The constraints are set to maintain the amount of clean drilling mud when handling dirty drilling mud in pickup nodes. Constraints (5.16) is not linear, and must be divided into two linear constraints (5.16a) and (5.16b) to handle the equality sign. Constraints (5.16a) state that the sum of clean drilling mud before and after picking up dirty drilling mud, plus the vessel's load capacity, must be *less than* or equal to the vessel's load capacity when the vessel sails directly from node i to node j . And constraints (5.16b) state that the sum of clean drilling mud before and after picking up dirty drilling mud, minus the vessel's load capacity, must be *greater than* or equal to the vessel's load capacity when the vessel sails directly from node i to node j . With this "less than - greater than" binding the load maintains the same value from node to node when it is not treated in constraints (5.13) or (5.15).

$$x_{ij}(l_{is2} - l_{js2}) = 0, \quad i \in N \setminus \{d\}, j \in N^D, s \in S \quad (5.17)$$

linearised to

$$l_{is2} - l_{js2} + V^{CAP} x_{ij} \leq V^{CAP} x_{ij}, \quad i \in N \setminus \{d\}, j \in N^D, s \in S \quad (5.17a)$$

$$l_{is2} - l_{js2} - V^{CAP} x_{ij} \geq -V^{CAP} x_{ij}, \quad i \in N \setminus \{d\}, j \in N^D, s \in S \quad (5.17b)$$

Constraints (5.17) are the equivalent to constraints (5.16), except for maintaining

dirty drilling mud in delivery nodes instead of clean drilling mud in pickup nodes. The constraints are divided into two linear constraints (5.17a) and (5.17b) to handle the equality sign in the same way as constraints (5.16).

Tank constraints

$$h_{isc} - \frac{l_{isc}}{HCAP} \geq 0, \quad i \in N, s \in S, c \in C \quad (5.18)$$

Constraints (5.18) let h_{isc} take an integer value higher than the load of product s in condition c in node i divided by the tank capacity. The constraints decide how many tanks that must be dedicated to each type of cargo.

$$\sum_{s \in S} \sum_{c \in C} h_{isc} \leq H, \quad i \in N \quad (5.19)$$

Constraints (5.19) limit the sum of the cargo holding tanks, h_{isc} , to a value lower than or equal to the total number of tanks.

$$w_i + \sum_{s \in S} \sum_{c \in C} h_{isc} = H, \quad i \in N \quad (5.20)$$

Constraint (5.20) ensure that the w_i variable takes the value of the available tanks on board the vessel.

Sequence constraints

$$x_{ij} = 1, \quad S_{ij} = 0, i \in N_P, j \in N_D \quad (5.21)$$

$$x_{ij} = 0, \quad S_{ij} = 0, i \in N_D, j \in N_P \quad (5.22)$$

Constraints (5.21) state that a pickup cargo must be serviced before a delivery cargo if the sailing time between two cargoes is 0. No sailing time between two cargoes implies that they are located at the same installation. These constraints are in accordance with the assumption of no storage space on the installations, and forces the vessel to have enough space for both cargoes on board when performing a well displacement. By adding (5.22), the assumption is acknowledged once again, and the delivery can not be made before the pickup.

Gap constraint

$$t_i - D_i = y_i, \quad i \in N^C \quad (5.23)$$

Constraints (5.23) let y_i take the gap value between the actual time of service of a cargo, and the time the service is demanded for the cargo.

Variable constraints

$$x_{ij} \in \{0, 1\}, \quad (i, j) \in A \quad (5.24)$$

Constraints (5.24) make the arc flow variable x_{ij} binary.

$$t_i \geq 0, \quad i \in N \quad (5.25)$$

Constraints (5.25) let the time variable t_i take a non-negative value.

$$l_{isc} \geq 0, \quad i \in N, s \in S, c \in C \quad (5.26)$$

Constraints (5.26) let the load variable l_{isc} take a non-negative value.

$$h_{isc} \in \{0, 1, 2, \dots, H\}, \quad i \in N, s \in S, c \in C \quad (5.27)$$

Constraints (5.27) let the tank variable h_{isc} take an integer value between 0 and H.

$$w_i \in \{0, 1, 2, \dots, H\}, \quad i \in N^C \quad (5.28)$$

Constraints (5.28) let the tank variable w_i take an integer value between 0 and H.

$$y_i \geq 0, \quad i \in N^C \quad (5.29)$$

Constraints (5.29) let the time variable y_i take a non-negative value.

In Figure 5.1 the model description is simplified and summarised. The required input to the model is listed along with the different types of constraints which are applied to give the listed output.

The code build for implementing the mathematical model without process plant into the commercial solver FICO[®] Xpress Optimization Suite is located in Appendix C.

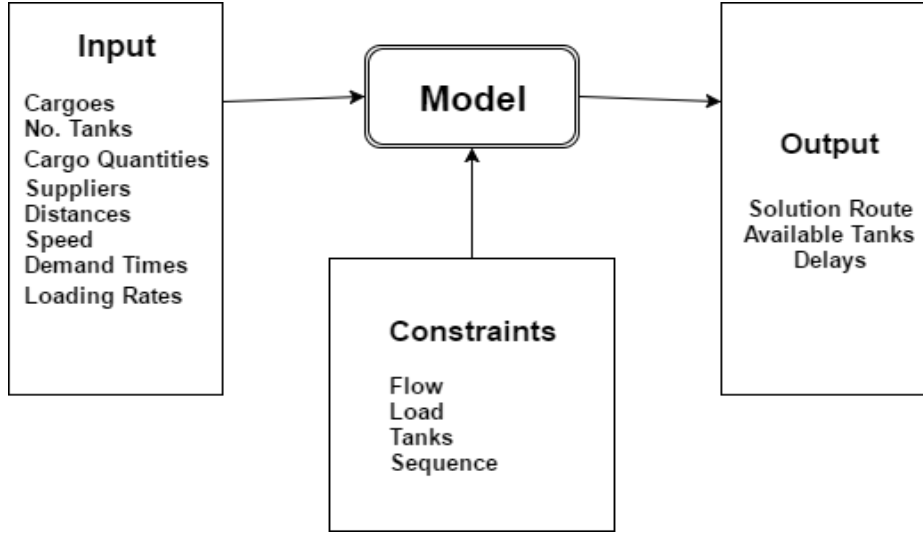


Figure 5.1: Simplified model overview with required input, applied constraints and output

5.3 The Mathematical Formulation

The problem is defined on an undirected graph $G = (N, A)$, where $N = \{0, 1, \dots, n+1\}$ with indices i and j is the set of nodes, and $A = (i, j)$ is the set of all feasible arcs in the network. $N^C = \{1, 2, \dots, n\}$ are the cargoes that are to be served in the network, which are divided into pickup cargoes $N^P \subset N$ and delivery cargoes $N^D \subset N$. Delivery cargoes are cargoes that the vessel *delivers* to a node, while pickup cargoes are cargoes that the vessel *picks up* from a node. Origin- and destination nodes $o=0$ and $d=n+1$ represents the same physical location. The cargoes consist of different products with different suppliers, which are represented by $S = \{0, 1, \dots, k\}$ indexed by s , and have two conditions $C = \{\text{clean}, \text{dirty}\}$, or $\{1, 2\}$, indexed by c .

Every cargo is described by Q_{is} which represents the amount of cargo of product s , and its node i . The initial load condition on the vessel is set as the sum of all delivery cargoes, thus let $L_{0s1} = \sum_{i \in N^P} Q_{is}$. The time the demand occurs at a node is denoted by D_i . Between all nodes there are sailing times S_{ij} . And the loading/unloading time for cargo i is determined by a rate R , which gives $U_i = \frac{\sum_{s \in S} Q_{is}}{R}$, where U_i is the loading/unloading time of cargo i . Thus the time between two nodes is defined by letting $T_{ij} = U_i + E_i + S_{ij}$, where E_i is the time when leaving node i . The vessel's capacity is denoted by V^{CAP} . There are H number of tanks on the vessel of equal size, which is set to H^{CAP} .

The model is routing a vessel with binary variable x_{ij} equal to 1 if the vessel sails directly from node i to j . The time variable t_i determines the time before service in node $i \in N \setminus d$. Load variable l_{isc} keeps track of the load of product s in condition c

after service in node i . h_{isc} is a variable that counts the number of tanks which are occupied in node $i \in N$ of cargo from supplier s in condition c , and w_i is a variable that counts the number of available tanks after each cargo service. y_i is a time variable with the time between the demand occurs at node $i \in N^C$, and the time the vessel starts servicing it. M , $B1$ and $B2$ are parameters used to linearise time- and load constraints.

The problem can then be formulated as follows:

Sets and indices

Set	Description	Index	Range
N	Set of nodes	i, j	$\{0, 1, \dots, n+1\}$
N^C	Set of cargoes	i, j	$\{1, 2, \dots, n\}$
N^P	Set of pickup cargoes	i, j	$N^P \subset N^C$
N^D	Set of delivery cargoes	i, j	$N^D \subset N^C$
A	Feasible arcs between nodes	i, j	$i \in N, j \in N$
S	Set of products	s	$\{1, 2, \dots, k\}$
C	Set of cargo conditions	c	$\{\text{clean, dirty}\} \{1, 2\}$
$\{o, d\}$	Origin and destination nodes		$\{0, n+1\}$

Parameters

Q_{is}	Cargo [m^3] in node i of product s .
S_{ij}	Sailing time from node i to j .
U_i	Loading time of cargo i .
T_{ij}	Time between nodes i and j .
R	Loading rate [m^3/h].
D_i	Time cargo i is demanded.
L_{0s1}	$\sum_{i \in N^P} Q_{is}$ for all s in S .
V^{CAP}	Total vessel capacity.
H	Number of tanks on vessel.
H^{CAP}	Tank capacity.
M	Big M used to linearise time constraint.
$B1$	Big B1 used to linearise load constraint for cargo condition 1.
$B2$	Big B2 used to linearise load constraint for cargo condition 2.

Variables

x_{ij}	1 if the vessel sails from node i to j , 0 otherwise.
t_i	Time before service starts in node i .
l_{isc}	Load, in volume, on the vessel of product s in condition c after servicing node i .
h_{isc}	Number of occupied tanks of product s in cargo condition c after servicing node i .
w_i	Number of available tanks after service of cargo i .
y_i	Time between the time for demand of cargo i , and the vessel starts servicing it.

Objective function

$$\text{minimise } z = \sum_{i \in N^C} y_i - \sum_{i \in N^C} w_i \quad (5.30)$$

Subject to the following constraints.

$$\sum_{j \in N \setminus \{d\}} x_{oj} = 1 \quad (5.31)$$

$$\sum_{i \in N^C} x_{ij} - \sum_{i \in N^C} x_{ji} = 0, \quad j \in N^C \quad (5.32)$$

$$\sum_{i \in N \setminus \{o\}} x_{id} = 1 \quad (5.33)$$

$$\sum_{j \in N} x_{ij} = 1, \quad i \in N \quad (5.34)$$

$$x_{ij}(t_i + T_{ij}^S - t_j) \leq 0, \quad (i, j) \in A \quad (5.35)$$

$$t_i - t_0 \geq 0, \quad i \in N \quad (5.36)$$

$$t_N - t_i \geq 0, \quad i \in N \quad (5.37)$$

$$t_i - D_i - y_i = 0, \quad i \in N^C \quad (5.38)$$

$$x_{ij}(l_{is1} - Q_{js} - l_{js1}) = 0, \quad i \in N \setminus \{d\}, j \in N^D, s \in S \quad (5.39)$$

$$x_{ij}(l_{is2} - Q_{js} - l_{js2}) = 0, \quad i \in N \setminus \{d\}, j \in N^P, s \in S \quad (5.40)$$

$$x_{ij}(l_{is1} - l_{js1}) = 0, \quad i \in N \setminus \{d\}, j \in N^P, s \in S \quad (5.41)$$

$$x_{ij}(l_{is2} - l_{js2}) = 0, \quad i \in N \setminus \{d\}, j \in N^D, s \in S \quad (5.42)$$

$$h_{isc} - \frac{l_{isc}}{H^{CAP}} \geq 0, \quad i \in N, s \in S, c \in C \quad (5.43)$$

$$\sum_{s \in S} \sum_{c \in C} h_{isc} \leq H, \quad i \in N \quad (5.44)$$

$$w_i + \sum_{s \in S} \sum_{c \in C} h_{isc} = H, \quad i \in N \quad (5.45)$$

$$x_{ij} = 1, \quad S_{ij} = 0, i \in N_P, j \in N_D \quad (5.46)$$

$$x_{ij} = 0, \quad S_{ij} = 0, i \in N_D, j \in N_P \quad (5.47)$$

$$(5.48)$$

$$x_{ij} \in \{0, 1\}, \quad (i, j) \in A \quad (5.49)$$

$$h_{isc} \in \{0, 1, \dots, H\}, \quad i \in N^C, s \in S, c \in C \quad (5.50)$$

$$w_i \in \{0, 1, \dots, H\} \quad i \in N^C \quad (5.51)$$

$$t_i \geq 0, \quad i \in N \quad (5.52)$$

$$l_{isc} \geq 0, \quad i \in N, s \in S, c \in C \quad (5.53)$$

$$y_i \geq 0, \quad i \in N^C \quad (5.54)$$

The objective function (5.30) minimises the amount of time between the demand occurs until the vessel starts servicing it, while favouring a large amount of available of tanks. This gap time is considered to represent only costs, and adds no value. Constraints (5.31) - (5.34) describe the flow in the network. The vessel has to leave the base (5.31) and return to it (5.33), there must be both an in-going and an out-going arc from each node (5.32), and all nodes must be visited once (5.34). Constraints (5.35) define the time in node j as larger than the time in node i plus the time between the nodes, with a permission to wait before servicing node j . Constraints (5.36) ensure that origin node is serviced before all other nodes, and constraints (5.37) ensure that destination node is serviced after all other nodes. Constraints (5.38) let y_i take the time value between the demand of cargo i occurs, until the cargo is serviced. Constraints (5.39) and (5.40) keep track of the load condition for clean products in delivery nodes, and dirty products in pickup nodes, respectively. Constraints (5.41) and (5.42) keep track of the load condition for clean products in pickup nodes, and dirty products in delivery nodes, respectively. Constraints (5.43) check how many tanks that are occupied in a certain load condition, and constraints (5.44) ensure that the number of occupied tanks does not exceed the total number of tanks on board. Constraints (5.45) let w_i take the value of available tanks on board the vessel after servicing cargo i . Constraints (5.46) and (5.47) make the vessel service the pickup cargo before the delivery cargo if two cargoes i and j are located at the same geographical location ($S_{ij} = 0$). Constraints (5.49) are binary constraints for the routing variable, constraints (5.50) and (5.51) force the occupied and available tank variables, respectively, to take a positive integer value, while constraints (5.52) - (5.54) force non-negativity on the remaining variables.

5.3.1 Mathematical Formulation With Process Plant

There are some differences between the mathematical formulations with and without the process plant cleaning OBM. The main differences are in the parameters, and which parameters that are used in different constraints. These differences are elaborated in this section.

The process plant is introduced by increasing the loading time for an OBM pickup cargo. This increased loading time represents the time the process plant uses to clean the pickup cargo. Introducing the process plant thus leads to three significant differences between the two mathematical models.

For the cases without the process plant there is only one parameter, Q_{is} , containing the cargo quantity for pickup and delivery cargoes, respectively. This parameter is then used for both calculating loading times and controlling the amount of load on board the vessel. For the case with the process plant, there are two parameters containing two different amounts of cargoes depending of whether it is used to calculate loading time or for controlling load on board the vessel. The differences are not very perceptible in the mathematical model, but more so in the pre-processing of parameters done in the commercial solver.

Table 5.1 presents the differences in the two mathematical formulations. The full mathematical formulation including the process plant is located in Appendix B.

Table 5.1: Differences between mathematical formulations with and without OBM cleaning process plant

Process Plant	Parameter/ Constraint	Description
Without	Q_{is}	Cargo quantity for pickup and delivery nodes
With	Q_{is}^{PLT}	Cargo quantity for pickup node used for calculating loading time
	Q_{is}^{PLO}	Cargo quantity for pickup node used for controlling load on board vessel
	Q_{is}^{DLT}	Cargo quantity for delivery node used for calculation loading time
	Q_{is}^{DLO}	Cargo quantity for delivery node used for controlling load on board vessel
Without	$x_{ij}(t_i + T_{ij}^S - t_j) \leq 0$	Calculating loading time using Q_{js} is done in pre-processing and is part of T_{ij}^S
With	$x_{ij}(t_i + T_{ij}^S - t_j) \leq 0$	Calculating loading time using Q_{js}^{PLT} and Q_{js}^{DLT} is done in pre-processing and is part of T_{ij}^S
Without	$x_{ij}(l_{isc} \pm Q_{js} - l_{jsc}) = 0$	Controlling load on board vessel
With	$x_{ij}(l_{is2} + Q_{js}^{PLO} - l_{js2}) = 0$	Controlling pickup loads on board vessel
	$x_{ij}(l_{is1} - Q_{js}^{DLO} - l_{js1}) = 0$	Controlling delivery loads on board vessel
Without	R	Loading rate is equal for all cargoes
With	R_i	Loading rate depends on the pickup cargo

Chapter 6

Input Data

The following chapter concerns the input data, what it is, how it is of use and how it is made possible to use.

Most of the input data is based on data sheets (Statoil ASA, 2016) provided by Statoil. The data received is from their systems, and needs therefore some processing before it can be used.

6.1 Which Offshore Installations are of Interest?

The offshore installations of interest are the ones listed in Table 6.1.

Table 6.1: The offshore installations of interest in the North Sea.

Statfjord A	Statfjord B	Statfjord C
Gullfaks A	Gullfaks B	Gullfaks C
Oseberg B	Oseberg C	Oseberg Øst
Oseberg Sør	Stena Don	COSL Innovator MNG
Songa Dee MNG	COSL Promotor MNG	Songa Equinox MNG

In Table 6.1, counting from left to right, the ten first installations are permanent platforms, while the five last ones are mobile rigs (Kartverket, 2017). It is assumed that these installations are the ones Statoil services from Base Mongstad, as they are marked as relevant amongst a list of all their installations in the provided data sheets.

6.2 Calculating Distances between Installations from Coordinates

As the location data for the installations are available by their Cartesian coordinates, they need some processing to be able to deduce distances between them.

The Haversine formula, Equation (6.1), gives the distance between two coordinates, with the longitude and latitude coordinates and the Earth's radius as the only input. The function "CalculateDistance.m" made in Matlab performs this calculation. The Matlab codes are located in Appendix D.1.

Further, a script made in Matlab, "DistanceMatrix.m" calls on function "CalculateDistance.m" to perform this calculation between all the relevant installations. The latitude and longitude for the installations are manually inserted to two vectors, as the number of installations is relatively small. A double for-loop going through the vectors makes sure that every installation is paired up with the other ones, and calculates the distance between them in metres using the "CalculateDistance.m" function. The complete distance matrix is constructed in the for-loop as well, which makes it easy to call out the desired distances to put into Xpress.

$$d = 2r \arcsin \left(\sqrt{\sin^2\left(\frac{\varphi_2 - \varphi_1}{2}\right) + \cos(\varphi_1) \cos(\varphi_2) \sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)} \right) \quad (6.1)$$

It should be noted that these distances are in direct airline, and does not take into account any land obstacles, islands and shears that must be passed by. This is, most likely, only a problem when going to and from Base Mongstad, and not between the installations. The calculated distances between installations used in the following case study are located in Appendix D.2

6.3 Defining the Loading Rate

To calculate the amount of time a loading/offloading operation lasts on an installations, two parameters must be known - the quantity of the cargo and the rate at which it is loaded. In Table 2.2 taken from Resell and Otteraaen (2016), a mean lay time of three hours during an offshore loading operation is presented, fortified in information from Statoil ASA (2016). Table 2.2 also presents the average amount of drilling mud that is handled to be between 500 m^3 and 1000 m^3 . The mean value of 500 and 1000

is 750, which is the thought behind setting the loading rate as $\frac{750m^3}{3h}$, or $250 m^3/h$. However, this rate varies a lot between the installations and the trend is that the older installations have lower rates than the newer ones (Vik and Gullberg, 2016). This is not considered in this thesis.

6.4 Finding Vessel Data

For the case study, vessel data is needed. Currently there is at least one PSV with extended wet bulk capacity in the Norwegian fleet, *Far Solitaire* from Farstad Shipping. The properties of *Far Solitaire* give the foundation for the extended wet bulk capacity PSV's data in the case study. Comparable data for a vessel from the current PSV fleet is thus collected from the average PSV in Farstad Shipping's fleet. The average PSV is assumed to be the design type with most occurrences, and corresponds to the design of *Far Scotsman*. The process of finding the average PSV is located in Appendix E.

The vessel data is collected at Farstad Shipping (2017), and presented in Table 6.2. Length overall (LOA), breadth (B), draft (D) and sizes of tanks designated to contain the cargo listed are considered interesting. *Far Solitaire* is a larger vessel than *Far Scotsman* in all physical aspects.

Table 6.2: Interesting vessel data for *Far Scotsman* and *Far Solitaire*

Data \ Vessel	Far Scotsman	Far Solitaire
LOA	81,7 m	91,6 m
B	18 m	22 m
D	6,5 m	7,2 m
Drill Water	1915 m ³	2447 m ³
Mud	1270 m ³	1316 m ³
Brine	1270 m ³	1559 m ³
Fuel Oil	917 m ³	1146 m ³
Pot Water	730 m ³	739 m ³
Base Oil	319 m ³	403 m ³
Methanol	100 m ³	403 m ³
Total Tank Volume	6521 m³	8013 m³
Average Tank Size	931,6 m³	1144,7 m³

6.5 Finding Wet Bulk Demand

In order to build a mathematical model rooted in reality, the demand for wet bulk must be mapped. Statoil ASA has provided two large data sheets with information

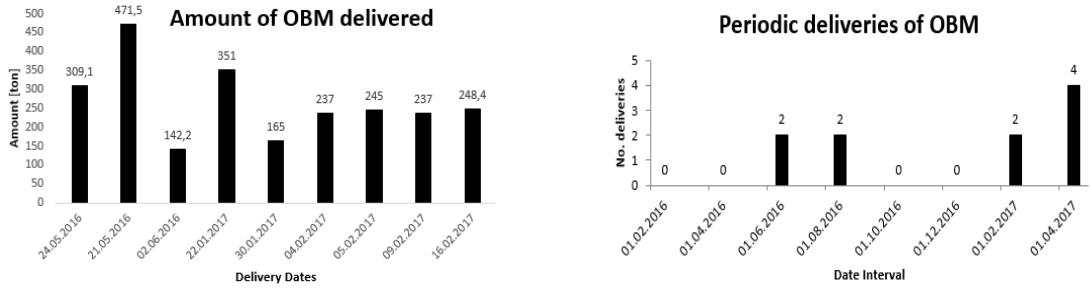
regarding vessels and amounts of transported cargoes. Excerpts from the sheets are found in Appendix F.1 and F.2. As the sheets consist of information regarding 17000 deliveries in a 13-month period of time, it is decided to extract only some of the information and make it general. Statfjord A is an installation of interest, and thus a good starting point. All wet bulk transportation back and forth between Statfjord A is gathered and processed in Excel. Both data sheets are used to find the date and amount of the delivered cargo. For some of the deliveries matching information is missing, and is therefore not included. This is thought to have little significance for the further study.

The wet bulk transported between base and installations from the data sheets are stated in ton. The mathematical model is build to handle cargoes in cubic metres, so the ton must be converted. In Section 6.5.1, the densities are determined for the products of interest, in order to convert the cargoes from ton to volume.

Figures 6.1 and 6.2 present the deliveries of OBM and wet bulk, respectively, to Statfjord A from January 2016 to mid-February 2017. The amounts of delivered OBM in ton are listed in Figure 6.1a. The data sheets contain little information on transport of OBM away from the installation, and is therefore not considered. In the further study it is assumed that all delivered OBM, minus the experienced losses, must be transported back to base. Together with Figure 6.1b it is evident that the deliveries are periodic. From the Figures it might seem like Statfjord A had drilling operations going on in late-May to June 2016 and late-January to February 2017. The total amount of OBM delivered during these periods are 922 ton and 1483 ton, respectively. Such amounts may represent 1-3 well displacements.

Figure 6.2 contains information regarding general wet bulk that is transported to Statfjord A in said period. The products of interest are various special products, slop, marine gas oil (MGO), base oil and brine, as shown in Figure 6.2a. Black bars represent amounts transported to the installation, and grey bars are amounts transported from the installation to Base Mongstad. There is a significant amount of ton MGO and brine delivered to Statfjord A. Looking at Figure 6.2b, there are wet bulk deliveries to Statfjord A all year. However, there is no correlation between periods of many wet bulk deliveries and the periods for delivery of OBM, as seen in figures 6.2b and 6.1b. Figures showing amount of transported special products, MGO and brine, and their respective delivery periods, are located in Appendix G.

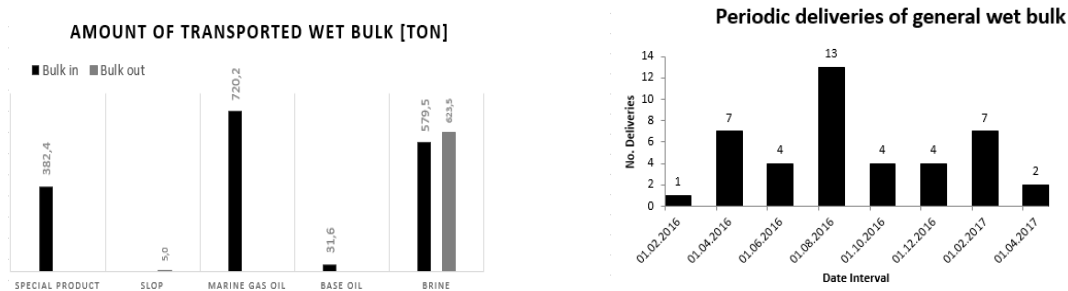
Analysing Statfjord A does not yield any strong correlations between deliveries of wet bulk. A small exception are the deliveries of brine, which are all concentrated in June



(a) Dates for, and amount of, OBM delivered.

(b) Number of deliveries in different time periods.

Figure 6.1: Delivered OBM to Statfjord A over a 13-month period.



(a) Amount of transported wet bulk, both deliveries and pickups, not included OBM.

(b) Number of wet bulk deliveries, not included OBM.

Figure 6.2: Delivered wet bulk to Statfjord A over a 13-month period.

2016 in close proximity to the delivered OBM. However, the times for delivery are all after the last OBM delivery in that period, which makes it hard to say anything of its correlation. Since no brine is delivered in the other period of OBM deliveries in February, it is assumed that said correlation is accidental.

Concluding remarks on the wet bulk demand is therefore as following. OBM is delivered in large amounts in short periods of time. A trend regarding deliveries of special products is many small deliveries spread throughout the period, as seen in Appendix G. For MGO there are few but large deliveries, approximately one every two months as seen in Appendix G.3. Brine is only delivered two times throughout the period, and then collected soon after as seen in Appendix G.4. The reason for this is unknown and it is therefore hard to say anything about the delivery pattern for brine.

6.5.1 Determining Densities

The different product deliveries are listed in ton, but for the mathematical model and case study they must be given in cubic metres. Converting from mass to volume is given by $Volume = \frac{Mass}{Density}$. Thus the densities for the products must be determined

in order to find the volume.

Even though there are large variations in density within the same product, the densities listed in Table 6.3 are the ones applied in this thesis. Special products are chemicals such as methanol, which has a density on 800 kg/m^3 , MGO is slightly heavier with 860 kg/m^3 . The densities of slop and OBM vary a lot, but are set to 1000 kg/m^3 and 1500 kg/m^3 , respectively. Slop may contain both oil, water and particles which makes a total density hard to determine.

Table 6.3: Wet bulk densities

Product	Density
Special product	800 kg/m^3
OBM	1500 kg/m^3
MGO	860 kg/m^3
Slop	1000 kg/m^3

6.6 Finding Vessel Schedules

Looking at two deliveries of OBM, and the voyages of the vessels behind the delivery, form a basis for determining the extend of cargoes to be serviced on a single voyage. The information for one of the voyages is located in Appendix F.1 and F.2, with voyage number 94052. One voyage contains a large OBM delivery to Statfjord A on 471,5 ton, and is illustrated in Figure 6.3 where the "B" acts as Base Mongstad. The voyage consists of five installation visits to Statfjord C, Statfjord A, Kvitebjørn, Safe Scandinavia and Oseberg Sør. Wet bulk deliveries were made at Statfjord A and C, while the reason for servicing the last three installations is unknown. As the data sheets does not contain information regarding deck loads, it is assumed that deck cargo was serviced on these installations. The exact wet bulk deliveries are shown in Table 6.4.



Figure 6.3: Voyage 1 with installations visited by a PSV.

Table 6.4: Wet bulk deliveries on voyage 1

Installation	Delivery	Ton
Statfjord A	OBM	471,5
Statfjord C	Water	400
Statfjord C	MGO	118,5

The second voyage is of a smaller OBM delivery to Statfjord A of 237 ton. The vessel behind the delivery services in total 13 installations on its voyage, and the ones related to wet bulk are illustrated in Figure 6.4. Gullfaks A, Statfjord B and Statfjord C are supplied with special products, while at Songa Equinox the vessel collects slop. The remaining installations that are visited are assumed not to be related to wet bulk supply. The exact wet bulk deliveries are shown in Table 6.5.



Figure 6.4: Voyage 2 with installations visited by a PSV for wet bulk services.

Table 6.5: Wet bulk deliveries on voyage 2

Installation	Delivery	Ton
Gullfaks A	Special Product	66
Songa Equinox	Slop	260
Statfjord A	OBM	237
Statfjord B	Special Product	110
Statfjord C	Special Product	50

Comparing the two voyages, the first one is significantly shorter than voyage number two. Five installation visits on the first voyage compared to 13 visits on the second one implies a large span in the number of assigned visits to a voyage. Another observation is the difference in amount of cargo on the vessels on the two voyages. The first voyage transports two large wet bulk cargoes and one smaller, at a total weight of ~ 1000 ton. The second voyage transports two medium sized cargoes and three smaller ones, at a total weight of 723 ton. As mentioned, the weight of the cargoes delivered to the other installations is unknown, but from the two said voyages it can be stated that short voyages implies larger cargo deliveries. This assumption seems reasonable, as few visits with small cargoes to deliver in most cases yields low vessel utility.

The conclusion of the quick overview of voyages is that large cargo deliveries are on short voyages. On long voyages there are several small deliveries, and in the case of voyage 2, several of the same product which can be transported in the same cargo tank. This forms a good basis for developing test cases for the mathematical model and problem described.

Chapter 7

Case Study

This chapter presents the cases that are due for analysis with the mathematical model. The cases are introduced in section 7.1, and the specific data for the respective cases are presented in tables in section 7.2.

7.1 Introducing Cases for Analysis

There are several cases which the mathematical model has been used to find correlations and solutions on scenarios the vessel might experience. They are mainly divided into two sections, one treating the number of cargoes serviced by PSVs today, and one where the number of cargoes are increased to a large extend. The cases will only treat wet bulk cargoes, and therefore exclude all other operations a PSV might conduct.

7.1.1 Several Visits, Small Cargo Sizes

The seemingly most occurring voyages for a PSV are those with several visits of small cargoes. Such a voyage therefore represents an important and relevant case to study. Specific data for the case is introduced in Table 7.1. In total eight visits are scheduled, with four different types of products; special product, MGO, OBM and slop. There are two different suppliers of special products, which implies keeping them separated in different cargo tanks. There are also two different suppliers of OBM and MGO which the same separation criteria applies to. The sizes of the products are varying, but in the range 75-250 m^3 , where six are delivery cargoes and two pickup cargoes.

The case is tested with two different vessels, one PSV of average size, and one with extended wet bulk capacity, to see which one fits best. The times for demand are scattered at an interval of 24 hours.

7.1.2 Few Visits, Large Cargo Sizes

A voyage representing the large sized problem cargoes is also an interesting case to study. This voyage only visits four different installations, and handles two well displacements and two MGO deliveries. Specific data for the case is presented in Table 7.2. The sizes of the deliveries range from $175 m^3$ to $450 m^3$, and there are in total four delivery cargoes and two pickup cargoes.

This case is also tested with two different vessels, one PSV of average size, and one with extended wet bulk capacity. And the demand times are also scattered at an interval of 24 hours. Additionally, another feature is tested up against this case, which is adding the OBM cleaning process plant.

With OBM Cleaning Process Plant

When well displacements are due, it might be beneficial to have a process plant cleaning the OBM on board the vessel. This case is only studied with Far Solitaire, the PSV with extended wet bulk capacity. Specific data for the case is equal to the data in the case with few visits and large cargo sizes, which is presented in Table 7.2.

The process plant function is achieved by setting the unloading rate very low, which forces the vessel to stay at an installation for a longer period of time. This time serves as the time it takes to clean the OBM, as it is assumed that the vessel must stay by the installation in order to load back OBM that is cleaned (Vik and Gullberg, 2016). The loading rate at which the OBM is loaded back onto the installation stays normal, at $250 m^3/h$. Approximately 20 % of the unloaded dirty OBM from an installation is left on the vessel. This 20 % is the dirty leftovers from the cleaning process which must be transported back to base for disposal (Resell and Otteraaen, 2016).

7.1.3 Many Visits, Various Cargo Sizes

Since the vessel mainly services wet bulk cargoes, the vessel potential should be fully exploited. To test the vessel's potential, several cargoes of different sizes are added to the route along with a longer time horizon. The route set up contains 17 cargoes

to service, which include two large well displacements of OBM, medium OBM, small MGO and various sized special product deliveries and slop pickups. As presented in Section 6.6 the regular number of cargoes to serve on a voyage is up to 13. In this case the number is increased to 17 in order to investigate how the vessel can deal with a rise in wet bulk demand. The specific case data is presented in Table 7.3. In this case, all installations are visited once due to the mathematical model limit of allowing only one visit per installation per route. The demand times in this case are scattered at an interval of 168 hours, equal to one week.

Allowing Mixing of Products with Different Suppliers

To best investigate the advantages of mixing products with different suppliers, it is allowed in the case with many different cargoes of various sizes. In accordance with the statement of the OBM being mixed properly at the installation where it is used, mixing clean OBM from different producers is not assumed as a problem. The same assumption is made on the dirty OBM, which is going back to base for treatment and disposal. MGO is assumed to have equal properties but come from different suppliers. It is therefore opened for mixing the MGO as well. Slop and special products are treated in the same manner as in the previous case. The specific case data is equal to the case without mixing except for the amount of suppliers, and is also presented in Table 7.3.

7.2 Input Data for Case Study

The times for when the demand is needed at an installation are set randomly, but with root in real, manageable points. Setting the times randomly does not exclude any solutions as there are no time windows in the model. However, since the objective function is related to the demand times the random setting can be of great significance for the solution schedule. The other data needed for the respective cases are presented in tables 7.1, 7.2 and 7.3.

There are two vessels used in the study, and in the mathematical model the tanks on the vessel are thought to be equally sized. The vessels presented in Section 6.4 have different sized cargo tanks which must be changed to be applicable for the model. As none of the cases transports pot water, the pot water tanks are removed from the total tank capacity of the initial vessels. This is also done due to assumptions of strict regulations regarding what can be stored in tanks that carry pot water.

Table 7.1: Visits and services for case with several visits and small cargo sizes

Cargo No.	Installation	Product (supplier no.)	Product Size	Pickup/ Delivery	Demand Time
1	Oseberg B	Special Product (1)	108 m^3	Delivery	8 hours
2	Statfjord A	Special Product (1)	78 m^3	Delivery	4 hours
3	Gullfaks C	OBM (2)	83 m^3	Delivery	9 hours
4	Songa Equinox	MGO (3)	116 m^3	Delivery	13 hours
5	COSL Innovator	Special Product (4)	150 m^3	Delivery	6 hours
6	Oseberg Sør	MGO (5)	163 m^3	Delivery	17 hours
7	Statfjord C	Slop (6)	250 m^3	Pickup	24 hours
8	Stena Don	OBM (7)	130 m^3	Pickup	6 hours

Table 7.2: Visits and services for case with few visits and large cargo sizes, and for the case with process plant cleaning OBM

Cargo No.	Installation	Product (supplier no.)	Product Size	Pickup/ Delivery	Demand Time
1	Gullfaks B	OBM (1)	287 m^3	Pickup	4 hours
2	Gullfaks B	OBM (1)	320 m^3	Delivery	4 hours
3	Oseberg Øst	OBM (2)	400 m^3	Pickup	19 hours
4	Oseberg Øst	OBM (2)	454 m^3	Delivery	19 hours
5	Statfjord A	MGO (3)	239 m^3	Delivery	9 hours
6	Songa Dee	MGO (4)	177 m^3	Delivery	12 hours

Table 7.3: Visits and services for case with many visits and various cargo sizes, ordinary and mixed products

Cargo No.	Installation	Product (supplier no. ord/mix)	Product Size	Pickup/ Delivery	Demand Time
1	Statfjord C	Slop (1/1)	250 m^3	Pickup	58 hours
2	Oseberg B	Slop (1/1)	120 m^3	Pickup	18 hours
3	Songa Dee	Slop (1/1)	363 m^3	Pickup	5 hours
4	Gullfaks A	Slop (1/1)	180 m^3	Pickup	156 hours
5	Songa Equinox	OBM (2/2)	630 m^3	Pickup	10 hours
6	Songa Equinox	OBM (2/2)	650 m^3	Delivery	10 hours
7	COSL Innovator	OBM (3/2)	450 m^3	Pickup	69 hours
8	COSL Innovator	OBM (3/2)	490 m^3	Delivery	69 hours
9	Oseberg Sør	Special Product (4/3)	120 m^3	Delivery	123 hours
10	Statfjord A	Special Product (4/3)	100 m^3	Delivery	43 hours
11	Statfjord B	Special Product (5/4)	80 m^3	Delivery	65 hours
12	Gullfaks C	Special Product (5/4)	160 m^3	Delivery	90 hours
13	Gullfaks B	MGO (6/5)	230 m^3	Delivery	76 hours
14	COSL Promoter	MGO (6/5)	170 m^3	Delivery	57 hours
15	Oseberg Øst	MGO (7/5)	200 m^3	Delivery	35 hours
16	Oseberg C	OBM (2/2)	400 m^3	Delivery	27 hours
17	Stena Don	OBM (3/2)	380 m^3	Delivery	139 hours

Table 7.4: Vessel data for the case study

Data \ Vessel	Far Scotsman	Far Solitaire
Total Tank Volume	5791 m^3	7274 m^3
Tank Sizes	360 m^3	455 m^3
No Tanks	16	16
Service Speed	10 kts	10 kts

The number of tanks is difficult to determine. Many tanks implies a smaller tank size and will force a large cargo to spread over several tanks. Few, large tanks will lessen the possibility of carrying many different products, but at the same time allow a large cargo to be stored in the same tank. The number of tanks on both vessels is set to 16, and serves as a compromise between few and many tanks. Relevant vessel data is presented in Table 7.4, and consists of total tank volume, number of tanks on the vessels, their tank sizes and service speed. The service speed is set to 10 knots for both vessels.

Chapter 8

Results

This chapter presents the results from the case study divided into sections to fit the cases presented in Chapter 7.

The mathematical model presented in Chapter 5 is implemented using Xpress Mosel Version 3.8.0 as the modelling language, and is run using Xpress-IVE Version 1.24.06 64 bit with Xpress Optimizer Version 27.01.02. The computer used runs Windows 8 64-bit Operating System with AMD Radeon R5 Graphics A8-6410 APU @ 2.00 GHz and has installed memory of 4 GB. The code with which the mathematical model without process plant is implemented in the solver is located in Appendix C.

8.1 Several Visits, Small Cargo Sizes

In this case, the results are the same for both the ordinary PSV, Far Scotsman, and for the PSV with extended wet bulk capacity, Far Solitaire. The cases are solved to optimality after approximately 12 minutes, and the respective output files from Xpress are located in Appendix H.1. Figure 8.1 depicts the route the vessels are scheduled to, while Table 8.1 shows the delays of service for the cargoes and the amount of available tanks on the vessel after service of a cargo.

The vessels starts by visiting Gullfaks C and services the OBM delivery which is required there on time. Thereafter they visit Statfjord A for a special product delivery before picking up an OBM cargo at Stena Don. Both cargoes are serviced with a delay on 7.64 hours, and 6.06 hours, respectively. The vessel then continues with servicing Songa Equinox' MGO delivery on time, before delayed deliveries of special products and MGO are performed for COSL Innovator, Oseberg B and Oseberg Sør,

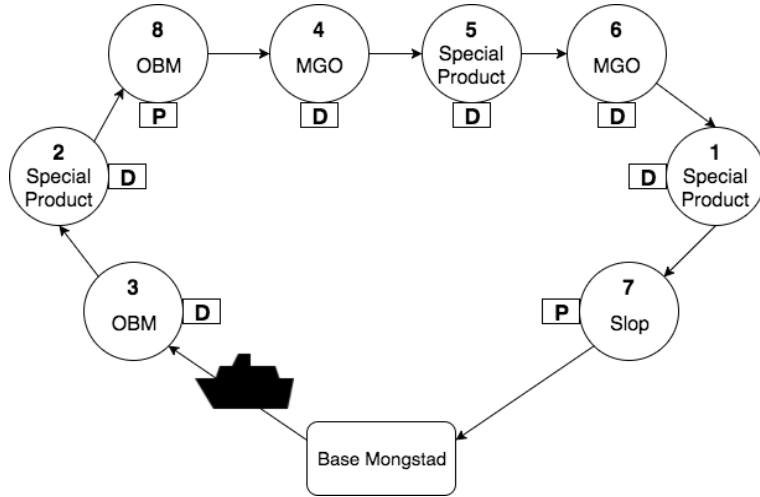


Figure 8.1: Route for case 1 for both Far Scotsman and Far Solitaire

Table 8.1: Service delays and available tanks for the solution routes of Far Scotsman and Far Solitaire, for case of several visits and small cargo sizes

Cargo No.	Installation	Product	Service Delay	Available Tanks
3	Gullfaks C	OBM	0 hours	12
2	Statfjord A	Special Product	7.64 hours	12
8	Stena Don	OBM	6.06 hours	11
4	Songa Equinox	MGO	0 hours	12
5	COSL Innovator	Special Product	8.15 hours	13
6	Oseberg Sør	MGO	4.43 hours	14
1	Oseberg B	Special Product	15.2 hours	15
7	Statfjord C	Slop	0.66 hours	14

respectively. The slop pickup at Statfjord C is serviced only 0.66 hours late and is the last cargo on the route. Along the route there are many tanks available, ranging between 11 and 15 after servicing the different cargoes.

8.2 Few Visits, Large Cargo Sizes

In this case the results in available tanks differ for the two vessels, which can be seen in Table 8.2 deduced from the output files in Appendix H.2. Both cases are solved to optimality in under one second. The routes scheduled to the two vessels are equal, and is illustrated in Figure 8.2.

The vessels starts by servicing the OBM well displacement at Gullfaks B on time. There is a service delay of 1.15 hours on the delivery cargo, which is equal to the time it takes to load the pickup cargo from the installation onto the vessel. Further, the vessels deliver MGO to installations Statfjord A and Songa Dee on time and 0.66 hours after demanded time, respectively. Lastly, the vessels service the OBM well

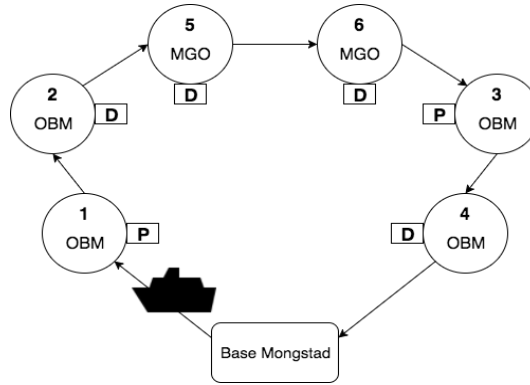


Figure 8.2: Route for case 2 for both Far Scotsman and Far Solitaire

Table 8.2: Service delays and available tanks for the solution routes of Far Scotsman and Far Solitaire, for case of few visits and large cargo sizes

Cargo No.	Installation	Product	Service Delay		Available Tanks	
			Far Scotsman	Far Solitaire	Far Scotsman	Far Solitaire
1	Gullfaks B	OBM	0 hours	0 hours	10	11
2	Gullfaks B	OBM	1.15 hours	1.15 hours	11	12
5	Statfjord A	MGO	0 hours	0 hours	12	13
6	Songa Dee	MGO	0.66 hours	0.66 hours	13	14
3	Oseberg Øst	OBM	0 hours	0 hours	11	13
4	Oseberg Øst	OBM	1.69 hours	1.69 hours	13	14

displacement at Oseberg Øst on time. Remark the differences in number of available tanks between the two vessels. Far Solitaire has one, or several, more available tanks in all stages of the route.

8.2.1 With Process Plant Cleaning OBM

The solution route for the case with OBM cleaning process plant is the same as for the case with few visits and large cargo sizes, which is illustrated in Figure 8.2. Delays and the amount of tanks available differs from the solution, and is presented in Table 8.3.

Table 8.3: Service delays and available tanks for the solution route of Far Solitaire, for case with OBM cleaning process plant

Cargo No.	Installation	Product	Service Delay	Available Tanks
1	Gullfaks B	OBM	0 hours	11
2	Gullfaks B	OBM	9.57 hours	12
5	Statfjord A	MGO	8.03 hours	13
6	Songa Dee	MGO	8.68 hours	14
3	Oseberg Øst	OBM	8.12 hours	13
4	Oseberg Øst	OBM	21.45 hours	14

The only thing that is different between case 2 and this case is the introduction of the process plant, which makes the service delays increase by several hours.

8.3 Many Visits, Various Cargo Sizes

Solution route, service delays and available tanks for the case with many visits and various cargo sizes are presented in Table 8.4. The commercial solver is set to stop running after 10800 seconds, which affects the running of both sub-cases. The solver located ten and nine integer solutions in the two sub-cases, respectively.

Table 8.4: Service delays and available tanks for the solution route of Far Solitaire, for case with many visits and various cargo sizes

Cargo No.	Installation	Product	Service Delay	Available Tanks
5	Songa Equinox	OBM	0 hours	7
6	Songa Equinox	OBM	21 hours	8
15	Oseberg Øst	MGO	3.55 hours	9
2	Oseberg B	Slop	23.87 hours	8
16	Oseberg C	OBM	16.71 hours	9
10	Statfjord A	Special Product	11.07 hours	9
3	Songa Dee	Slop	52.17 hours	8
14	COSL Promoter	MGO	9.63 hours	8
1	Statfjord C	Slop	19.81 hours	8
11	Statfjord B	Special Product	14.90 hours	8
13	Gullfaks B	MGO	6.25 hours	8
7	COSL Innovator	OBM	22.60 hours	8
8	COSL Innovator	OBM	37.60 hours	9
12	Gullfaks C	Special Product	26.86 hours	10
17	Stena Don	OBM	0 hours	11
9	Oseberg Sør	Special Product	24.87 hours	12
4	Gullfaks A	Slop	1.72 hours	11

The vessel starts by servicing the OBM well displacement at Songa Equinox on time before servicing three installations on the Oseberg field with a significant delay of about 20 hours at Oseberg B and C. MGO is delivered, slop collected, and OBM delivered at Oseberg Øst, B and C respectively. The vessel then delivers special product to Statfjord A 11 hours late, before collecting slop at Songa Dee even more delayed. COSL Promoter is then receiving MGO before slop is collected at Statfjord C and special product is delivered to Statfjord B, these services are also delayed. Following is a MGO delivery ay Gullfaks B before the well displacement is performed at COSL Innovator 22 hours after demanded time. Then the vessel visits Gullfaks C for a delayed special product delivery and an on time delivery of OBM at Stena Don. Finally, the vessel delivers special product at Oseberg Sør, and collects slop at

Gullfaks A before travelling back to base. The number of available tanks is never less than seven, and at most 12.

Allowing Mixing of Products

When allowing mixing of equal products of different suppliers the solution route changes. The three first cargoes to be served are equal, but the other installations are served in a different sequence which is presented in Table 8.5. Apart from the difference in the route, there are in general more available tanks throughout the voyage.

Table 8.5: Service delays and available tanks for the solution route of Far Solitaire, for case with many visits and various cargo sizes allowing mixing of products

Cargo No.	Installation	Product	Service Delay	Available Tanks
5	Songa Equinox	OBM	0 hours	8
6	Songa Equinox	OBM	21 hours	9
15	Oseberg Øst	MGO	3.55 hours	10
16	Oseberg C	OBM	13.79 hours	10
10	Statfjord A	Special Product	8.15 hours	10
11	Statfjord B	Special Product	0 hours	10
14	COSL Promoter	MGO	18.72 hours	10
7	COSL Innovator	OBM	8.09 hours	10
8	COSL Innovator	OBM	23.09 hours	11
1	Statfjord C	Slop	46.55 hours	10
13	Gullfaks B	MGO	31.43 hours	11
12	Gullfaks C	Special Product	18.73 hours	12
3	Songa Dee	Slop	105.88 hours	11
2	Oseberg B	Slop	101.72 hours	10
9	Oseberg Sør	Spacial Product	0 hours	12
17	Stena Don	OBM	0 hours	13
4	Gullfaks A	Slop	0 hours	12

Chapter 9

Discussion

This chapter discusses the approach to the problem and the results obtained.

Results

For the case with several visits and small cargo sizes taken from the current market size the results with the two different vessels are the same. All cargoes are smaller than the tank sizes of both PSVs, which makes the number of available tanks equal, and there is no difference in time. For the case with large cargoes, the results are different in the amount of available tanks. This is because some of the large cargoes fits in one cargo tank on Far Solitaire, which they do not do on Far Scotsman. When there are more available tanks on the vessel, it is more flexible in terms of adding more tasks and cargoes to its route. There is also a reduction in the amount of tanks that needs to be cleaned when the route is finished.

Common for all well displacements in all the cases is that the demand times for both the pickup OBM and the delivery OBM are set to be equal. It is impossible for the model to serve both cargoes at the same time, and the pickup cargo will always be picked up before the delivery. This will in the model induce a negligible delay in the service time on the delivery cargo equal to the time it takes to unload the pickup cargo. It can be avoided by setting the demand time of the delivery cargo equal to said loading time, but is not done in this thesis.

Introducing the process plant initially leads to many service delays on the case with few and large cargo sizes, but the visiting sequence stays the same. The increased service delays are directly connected to the elongation of loading times for the well

displacements, acting as the cleaning process. As the process plant cleans OBM that already is at the installation before delivering the same OBM back, the vessel does not need to bring all the clean OBM that is demanded from the base. Therefore the vessel has more available cargo space initially, but it still needs to be able to store the OBM when it is cleaned. Since the process plant acts as a black box for now, it is not clear whether the OBM is stored in tanks solely connected to the plant or in the vessel's cargo tanks. Installing the process plant will most likely demand some space from the cargo tanks. This has not been considered in the thesis, and the number of tanks have stayed the same for analyses with and without the process plant. The size of the process plant is unknown, which makes it hard to tell how much space it will require.

Having many cargoes of different sizes scheduled on the same route is done to investigate the flexibility and capacity of the vessel. From the results the vessel is more than capable of transporting the assigned cargoes, as it has seven or more available tanks at all times. However, the amount of available tanks might not be the real number in practice. Rules regarding products that are not allowed to be stored in the same tank without the tank being cleaned in between might reduce the number slightly. The extend of the issue is not possible to measure without tracking the different cargoes, which is not done in this thesis.

Throughout the results with many cargoes and the process plant there are many and long service delays. They arise from two causes. The first one is the objective function which will be elaborated later, and the initial setting of demand times has a part in it. These demand times are set at random with no correlation between cargoes or installations, which might cause an illogical routing. To get a good route, the demand time should be set in accordance with the real demands. This way, the model can be used to figure out if there are any cargoes that should be excluded from the route. For example cargoes causing large delays, or eventually makes the problem infeasible, should be serviced by another vessel in the fleet if the cost of including it is larger than excluding it.

From the results it might seem like an operator will benefit from having the same suppliers of different products on installations in the same area, as for those in the northern part of the North Sea. The reduction in the amount of available tanks when allowing a mix of equal products proves this. The PSV with extended wet bulk capacity and process plant will thus have capacity to stay offshore for a long period of time acting as a bank for wet bulk cargoes that are needed. In addition it can be scheduled around as an ordinary PSV. Especially large deliveries and OBM services

should be dedicated to such a vessel when it operates as a bank. The wet bulk PSV can get more flexible if it does not commit to the small frequent cargo demands, which can be serviced with an ordinary PSV. If it is free of these tasks it is more available for sudden demands, but then again it is costly to have the wet bulk PSV offshore with no tasks. Assigning tasks to such a vessel should therefore be done wisely.

The vessel seems to be capable of handling large amounts of cargoes that allows it to stay offshore for more than a week, for which the demand times in the case with many cargoes are divided into. If the vessel stays offshore for more than a week, it has obviated two to three returns to the onshore base which a scheduled PSV today does.

With a decrease in amount of base visits in a PSV route, there are several positive outcomes. There are costs to spare on less fuel consumption from fewer sailing legs and less chemicals delivered to the base which needs to be bought back. Sailing back to base also takes time, which now is more time that the vessel is available to service installations.

Finally, a small error is detected in the code. The PSVs are meant to sail in 10 knots, but are mistakenly set to sail at 10 km/h. This error is present in all the results, and leads to an approximate doubling of the sailing time between two sites. It does impact the results in matter of service delays, and might also change the sequence of services. The error was addressed late in the process, but is chosen only to highlight instead of running new analyses. Either way, the aim of the thesis is to build a mathematical model as a tool to explore the operation possibilities a wet bulk PSV has, which makes the specific numbers less important compared to the resulting trends and model functionality.

Problem Description and Limitations

Normally, PSVs have specified tanks for specific bulk products which only carry these types of products. In this thesis, cargo tanks are considered the same way as tanks on a chemical tanker. They are not dedicated to a specific product, and they are washed in between the scheduled routes when necessary. There are both advantages and disadvantages to this approach, where the greatest advantage is a larger degree of flexibility in cargo services without the specified tanks. The vessel is then capable of transporting more cargo than what specified tanks allows, which is valuable if there are many deliveries of the same product due at the same time. A downside to non-

specified tanks might be a rise in tanks that needs to be cleaned, which implies higher cleaning costs. It will also demand more of the crew to keep control on which tank contains what, and what available tanks have contained earlier to avoid pollution of products.

The aspect of fuel consumption and fuel tanks is not assessed in this thesis. It is to expect that the process plant will consume fuel in addition to the vessel itself, and the stored amount of fuel on the vessel will limit the time it can spend offshore. A return to base can be avoided by letting another PSV serve the wet bulk PSV with fuel, and thereby extend the time offshore.

Regarding the on board process plant there are some aspects that must be considered. Today there are none or very few vessels holding other functions than loading and offloading cargoes the offshore installations need that approach installations. There are some tankers that collect gas and/or oil, but that is mainly done from a loading buoy near the installation or from an offshore floating storage and offloading and/or production unit (FSO/FPSO). Approaching an offshore installation with a system running on board that is not connected to loading or offloading implies an increased risk for something going wrong, as more instruments must be under control. This increased risk might turn out unacceptable for the government, and must therefore be meticulously mapped out. In addition, there might be some difficulties related to regulations considering waste product treatment and discharge which can cause issues for an offshore process plant. As there are none of these types of vessels operating on the Norwegian continental shelf at the moment, the regulations that will apply to its operations must be carefully investigated.

A difficult area to beat in the existent system is that the unexpected and large cargo demands most often are delivered on time, or just a few hours delayed. This feature makes the system contain few delays at the expense of spot chartered PSVs. An important aspect to investigate further is thus the comparison of expenses between the old system and the flexibility of the new with the offshore process plant.

Input Data

The problem addressed is large, and it is therefore hard to consider all the factors that contribute to the problem. This is especially hard when determining what data to use as input for the mathematical model. From the large data sheet that was provided from Statoil, it is decided to use the historical data from Statfjord A as an indicator

for the amount and frequency of cargo transport on the Norwegian continental shelf. This is most likely not applicable for all installations as they differ in both production rate and storage space, but it is easy to exchange these numbers with numbers from a real case scenario. However, the differences are not considered big enough to question the results.

Mathematical Model

The objective function in the mathematical model consists of two different variables that are normally not "allowed" to be mixed, the delay in service, y_i , and the number of available tanks, w_i . y_i is as mentioned chosen because it represents indirect costs in the system. Since the number of available tanks on the vessel is a depiction of the vessel's flexibility and possibilities, it is included in the objective function to ensure that a cargo is not stored in more tanks than necessary. It is necessary to include it in the objective function because the constraint that delegates tanks to a cargo has no upper limit, only a lower. The constraint thus makes sure that enough tanks are at disposal for the cargo, but it might end up assigning more tanks to the cargo than necessary.

As w_i is in the objective function, one hour delay is weighted equal to one available tank at any point along the route in the case study. This might not be the best decision, but is easy to weight differently in future research. If one hour delay is more valuable than one available tank, the solution route will include less hours of delay and less available tanks. It will also most likely differ in the visiting sequence because the cargo sequence is less important than the installation sequence. As the results from the case study with many cargoes of various cargo sizes show, the amount of available tanks is never below seven and is probably driving the results more than the delay times. This makes sense, as for example nine available tanks will "neutralise" nine hours of delay with this objective function. However, if the number of available tanks is irrelevant, and the best route without delays is more sought for, the term including w_i can be removed from the objective function along with constraint (5.20).

The goal of the objective function is to find the solution with the lowest overall delay. Therefore it does not consider whether there are small delays all over the route, or if there is one cargo that is significantly delayed while the others are served on time. If there for some cargoes are more important to be served on time, they can be emphasised in the objective function by multiplying with a scalar. This way a delay

at the emphasised cargo will gravely impact the objective function. Such modifications could be done on, for example, well displacement operations, as they are highly time sensitive unlike frequent delivered cargoes such as MGO and pot water.

A way to avoid these ambiguities in the objective function is to introduce sequencing variables and constraints for each cargo tank. Implementing such features opens for having tanks of different sizes, making the model able to guide a user into which tanks to load with which cargo in addition to the visiting sequence. If sequencing variables and constraints are added to the model it will increase a lot in size, and make it harder to solve to optimality.

Sadly, the model is not very user friendly when it comes to creating the input files to the commercial solver. A thorough work is required in order to convert a matrix into one line and make sure the correct information is located in its correct place. It is probably easier to create a script to write the input files, but is not done in this thesis.

Stochastic vs. Deterministic Mathematical Model

With an aim to make a mathematical model to use in the planning of every single route, the stochastic aspect should be implemented to account for uncertainties met when scheduling a PSV. These uncertainties are mainly related to the weather restricting the vessel from maintaining a normal sailing time, or from loading cargoes between PSV and installation. Additionally, the occurrence of certain demands is fluctuating, which also should be included in a stochastic model. As this thesis has its focus on exploring the possibilities of a new type of PSV rather than the actual scheduling, the stochastic aspect is excluded.

Introducing simulation is the best way to apply the stochastic perspective to the short-term scheduling problem. A simulation model can be used to verify the strength of an optimal route from an optimisation model. According to Fagerholt et al. (2010) these methods will together most likely provide a more robust answer to the problem, compared to an optimisation model alone. No matter if it is deterministic or stochastic.

Chapter 10

Conclusion

The maritime pickup and delivery problem introduced in this thesis describes the problem in a satisfying manner. The mathematical model's objective function is ambiguous due to the desired effect of available tanks in the solution. This will impact the solution route, but is easy to modify if the visiting sequence is the subject of interest. Output from the mathematical model is highly dependent on the inserted, random, demand times, making the results look bad from the objective function's point of view. However, the results show that the maritime pickup and delivery model works as it is supposed to, and that it will provide correct results with realistic demand times as input.

The results from the case study show that the PSV with extended wet bulk capacity and OBM cleaning process plant can handle large amounts of cargo, and cargoes of different types. As there are multiple tanks available at all times on the scheduled route, the PSV is able to carry even more cargo than what is assigned in this thesis. Since the assigned cargo in the thesis surpasses the amount of cargo delivered in the market today, the PSV proves to be able to handle an increase in wet bulk demand. The amount of cargo the vessel is capable of handling allows it to stay offshore for more than a week at a time. By staying offshore for such amounts of time it can operate as a bank holding and delivering some products that might be unexpectedly wanted at an installation. Such a vessel can save an operator money in terms of less PSVs chartered from the spot market and less fuel consumption connected to less returns to base, but also more time gained for the vessel to be available for service. However, the full extent and feasibility of the process plant is not elaborated in this thesis, which makes it difficult to determine the full effect on the supply system.

10.1 Further Work

Further work should include more data on the process plant to find out how much it will help and affect the operation of the vessel.

Regarding the mathematical model, the objective function can be weighted in accordance with a cost-benefit analysis determining how much an available tank is worth compared to an hour delay. And in order to get a better understanding of the system and find a feasible solution considering all system aspects, a simulation model can verify the robustness of the optimisation model.

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Appendix A

Problem Description



NTNU Trondheim
Norwegian University of Science and Technology
Department of Marine Technology

MASTER THESIS IN MARINE TECHNOLOGY

SPRING 2017

For stud.techn.

Rebekka Resell

An Exploratory Study of a Wet Bulk Platform Supply Vessel using Operations Research Single-Vessel Scheduling with Bulk Loads and Mud-Cleaning Process Plant

Background

According to the Norwegian Petroleum Directorate, the produced water-oil ratio has increased since 2004. In 2011, 161 million standard cubic metres of produced water were produced on Norwegian petroleum fields, compared to 97.5 million standard cubic metres of oil. An increased demand for injection of produced water in order to maintain the production rate on the offshore installations is causing this increasing ratio. This is a fact especially for mature petroleum fields, which is the situation for many of the fields on the Norwegian continental shelf. Maintaining the production rate causes an increased demand for wet bulk on the installations.

The increase in wet bulk demand on the Norwegian continental shelf has caused operators to look at their offshore supply logistics in order to make it suit the situation better. Hopefully it will also reduce costs related to the transport and treatment of wet bulk.

Objective

The objective of this thesis is to investigate how a new type of platform supply vessel can be operated when highlighting wet bulk deliveries and changing the wet bulk logistics chain by adding a process plant that cleans drilling mud offshore. The use of optimisation as a decision support tool is chosen to provide insight in the offshore supply business.

Tasks

The candidate shall/is recommended to cover the following tasks in the project thesis:

- a. Present and describe important aspects with the real problem and the proposed new solution.
- b. Review and present relevant literature.
- c. Develop a mathematical model which describes real problem and the proposed new system.
- d. Identify and collect relevant necessary data for a computational study.
- e. Present different case scenarios to implement in a computational study.
- f. Implement data and solve the mathematical model in Xpress IVE.
- g. Present the results deduced from the computational study.
- h. Discuss the possibilities of the new wet bulk logistics system with background in the provided results.



General

In the thesis the candidate shall present his personal contribution to the resolution of a problem within the scope of the thesis work.

Theories and conclusions should be based on a relevant methodological foundation that through mathematical derivations and/or logical reasoning identify the various steps in the deduction.

The candidate should utilize the existing possibilities for obtaining relevant literature.

The thesis should be organized in a rational manner to give a clear statement of assumptions, data, results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Telegraphic language should be avoided.

The thesis shall contain the following elements: A text defining the scope, preface, list of contents, summary, main body of thesis, conclusions with recommendations for further work, list of symbols and acronyms, reference and (optional) appendices. All figures, tables and equations shall be numerated.

The supervisor may require that the candidate, in an early stage of the work, present a written plan for the completion of the work.

The original contribution of the candidate and material taken from other sources shall be clearly defined. Work from other sources shall be properly referenced using an acknowledged referencing system.

Deliverable

- The thesis shall be submitted in two (2) copies:
- Signed by the candidate
- The text defining the scope included
- In bound volume(s)
- Drawings and/or computer prints that cannot be bound should be organized in a separate folder.
- The bound volume shall be accompanied by a CD or DVD containing the written thesis in Word or PDF format. In case computer programs have been made as part of the thesis work, the source code shall be included. In case of experimental work, the experimental results shall be included in a suitable electronic format.

Supervision:

Main supervisor: Bjørn Egil Asbjørnslett

Deadline: 28.07.2017

Appendix B

Mathematical Formulation with Process Plant

Sets and indices

Set	Description	Index	Range
N	Set of nodes	i, j	$\{0, 1, \dots, n+1\}$
N^C	Set of cargoes	i, j	$\{1, 2, \dots, n\}$
N^P	Set of pickup cargoes	i, j	$N^P \subset N^C$
N^D	Set of delivery cargoes	i, j	$N^D \subset N^C$
A	Feasible arcs between nodes	i, j	$i \in N, j \in N$
S	Set of products	s	$\{1, 2, \dots, k\}$
C	Set of cargo conditions	c	$\{\text{clean, dirty}\} \{1, 2\}$
$\{o, d\}$	Origin and destination nodes		$\{0, n+1\}$

Parameters

Q_{is}^{PLT}	Cargo [m^3] in pickup node i of product s used for calculating loading time.
Q_{is}^{PLO}	Cargo [m^3] in pickup node i of product s used for controlling load on vessel.
Q_{is}^{DLT}	Cargo [m^3] in delivery node i of product s used for calculating loading time.
Q_{is}^{DLO}	Cargo [m^3] in delivery node i of product s used for controlling load on vessel.
S_{ij}	Sailing time from node i to j .
U_i	Loading time of cargo i .
T_{ij}	Time between nodes i and j .
R_i	Loading rate [m^3/h] for cargo i .
D_i	Time cargo i is demanded.
L_{0s1}	$\sum_{i \in N^P} Q_{is}^{PLO}$ for all s in S .
V^{CAP}	Total vessel capacity.
H	Number of tanks on vessel.
H^{CAP}	Tank capacity.
M	Big M used to linearise time constraint.
$B1$	Big B1 used to linearise load constraint for cargo condition 1.
$B2$	Big B2 used to linearise load constraint for cargo condition 2.

Variables

x_{ij}	1 if the vessel sails from node i to j , 0 otherwise.
t_i	Time before service starts in node i .
l_{isc}	Load, in volume, on the vessel of product s in condition c after servicing node i .
h_{isc}	Number of occupied tanks of product s in cargo condition c after servicing node i .
w_i	Number of available tanks after service of cargo i .
y_i	Time between the time for demand of cargo i , and the vessel starts servicing it.

Objective function

$$\text{minimise } z = \sum_{i \in N^C} y_i - \sum_{i \in N^C} w_i \quad (\text{B.1})$$

Subject to the following constraints.

$$\sum_{j \in N \setminus \{d\}} x_{oj} = 1 \quad (\text{B.2})$$

$$\sum_{i \in N^C} x_{ij} - \sum_{i \in N^C} x_{ji} = 0, \quad j \in N^C \quad (\text{B.3})$$

$$\sum_{i \in N \setminus \{o\}} x_{id} = 1 \quad (\text{B.4})$$

$$\sum_{j \in N} x_{ij} = 1, \quad i \in N \quad (\text{B.5})$$

$$x_{ij}(t_i + T_{ij}^S - t_j) \leq 0, \quad (i, j) \in A \quad (\text{B.6})$$

$$t_i - t_0 \geq 0, \quad i \in N \quad (\text{B.7})$$

$$t_N - t_i \geq 0, \quad i \in N \quad (\text{B.8})$$

$$t_i - D_i - y_i = 0, \quad i \in N^C \quad (\text{B.9})$$

$$x_{ij}(l_{is1} - Q_{js}^{DLO} - l_{js1}) = 0, \quad i \in N \setminus \{d\}, j \in N^D, s \in S \quad (\text{B.10})$$

$$x_{ij}(l_{is2} - Q_{js}^{PLO} - l_{js2}) = 0, \quad i \in N \setminus \{d\}, j \in N^P, s \in S \quad (\text{B.11})$$

$$x_{ij}(l_{is1} - l_{js1}) = 0, \quad i \in N \setminus \{d\}, j \in N^P, s \in S \quad (\text{B.12})$$

$$x_{ij}(l_{is2} - l_{js2}) = 0, \quad i \in N \setminus \{d\}, j \in N^D, s \in S \quad (\text{B.13})$$

$$h_{isc} - \frac{l_{isc}}{HCAP} \geq 0, \quad i \in N, s \in S, c \in C \quad (\text{B.14})$$

$$\sum_{s \in S} \sum_{c \in C} h_{isc} \leq H, \quad i \in N \quad (\text{B.15})$$

$$w_i + \sum_{s \in S} \sum_{c \in C} h_{isc} = H, \quad i \in N \quad (\text{B.16})$$

$$x_{ij} = 1, \quad S_{ij} = 0, i \in N_P, j \in N_D \quad (\text{B.17})$$

$$x_{ij} = 0, \quad S_{ij} = 0, i \in N_D, j \in N_P \quad (\text{B.18})$$

$$(\text{B.19})$$

$$x_{ij} \in \{0, 1\}, \quad (i, j) \in A \quad (\text{B.20})$$

$$h_{isc} \in \{0, 1, \dots, H\}, \quad i \in N^C, s \in S, c \in C \quad (\text{B.21})$$

$$w_i \in \{0, 1, \dots, H\} \quad i \in N^C \quad (\text{B.22})$$

$$t_i \geq 0, \quad i \in N \quad (\text{B.23})$$

$$l_{isc} \geq 0, \quad i \in N, s \in S, c \in C \quad (\text{B.24})$$

$$y_i \geq 0, \quad i \in N^C \quad (\text{B.25})$$

Appendix C

Mathematical Model Implemented in Xpress

!!!!!!!Created by Rebekka Resell, spring 2017

```
model MaritimePickupDelivery

uses "mmsxprs";!gain access to the Xpress-Optimizer solver

options explterm
options noimplicit

uses "mmsxprs", "mmmodbc","mmsystem";

parameters
  Datafile = "Case2.txt";
  SolutionTime = 10800;
end-parameters

! Enable message printing by the Optimizer
setparam("xprs_verbose", true);
! Turn off the automatic cut generation
setparam("xprs_cutstrategy", 0);
! Turn off the automatic heuristics
setparam("xprs_heurstrategy", 0);

!*****
!   Declaring sets
!*****

declarations
  nCargoes:          integer;          !number of nodes/cargoes
  nTanks:            integer;          !number of tanks on the vessel
  Nodes:             set of integer;
  PickupNodes:      set of integer;
  DeliveryNodes:    set of integer;
  Suppliers:         set of integer;   ![1 2 3 4] hypothetical
  CargoConditions:  set of integer;   ![clean, dirty], [1,2]
  CargoNodes:       set of integer;
  solutionTime1:    real;
end-declarations

initializations from Datafile
  nCargoes;
  nTanks;
  Nodes;
  PickupNodes;
  DeliveryNodes;
  Suppliers;
  CargoConditions;
  CargoNodes;
end-initializations

finalize(Nodes);
finalize(PickupNodes);
finalize(DeliveryNodes);
finalize(Suppliers);
finalize(CargoConditions);
finalize(CargoNodes);

!*****
!   Declaring parameters
!*****

declarations
  VesselCapacity:    real;
  TankCapacity:      real; !Assuming equally sized tanks
  TotalDeliveryLoad: dynamic array(Suppliers)          of real;
  TotalPickupLoad:   dynamic array(Suppliers)          of real;
```

```

CargoQuantityDel:      dynamic array(DeliveryNodes, Suppliers)      of real;
CargoQuantityPick:    dynamic array(PickupNodes, Suppliers)        of real;
CQPick:                dynamic array(PickupNodes)                  of real;
CQDel:                 dynamic array(DeliveryNodes)                of real;
VesselSpeedkmh:       real;
SailingDistanceskm:   dynamic array(Nodes, Nodes)                  of real;
SailingTime:           dynamic array(Nodes, Nodes)                  of real;
DemandTime:           dynamic array(CargoNodes)                    of real;
LoadingRate:           real;
LoadingTimeDel:        dynamic array(DeliveryNodes)                of real;
LoadingTimePick:       dynamic array(PickupNodes)                   of real;

A:                     dynamic array(Nodes, Nodes)                  of boolean;
BigM:                  integer;!                                   of real;
BigB1:                 real;
BigB2:                 real;
teller:                integer;
index:                 integer;
!a:                    real;   !Acts as temporary variable
!b:                    real;   !Acts as a temporary variable
end-declarations

initializations from Datafile
  TankCapacity;
  CargoQuantityDel;
  CargoQuantityPick;
  VesselSpeedkmh;
  SailingDistanceskm;
  DemandTime;
  LoadingRate;
  BigM;
  A;
end-initializations

!*****
!Preprocessing of Parameters
!*****

!Vessel Capacity
VesselCapacity := TankCapacity*nTanks;

!Total Delivery Load
forall(ss in Suppliers) do
  TotalDeliveryLoad(ss) := sum(ii in DeliveryNodes) CargoQuantityDel(ii,ss);
end-do

!Total Pickup Load
forall(ss in Suppliers) do
  TotalPickupLoad(ss) := sum(ii in PickupNodes) CargoQuantityPick(ii,ss);
end-do

!Big B1 and Big B2
BigB1 := sum(ss in Suppliers) TotalDeliveryLoad(ss);
BigB2 := sum(ss in Suppliers) TotalPickupLoad(ss);

!Time of unloading delivery cargo i
forall(ii in DeliveryNodes) do
  CQDel(ii) := sum(ss in Suppliers) CargoQuantityDel(ii,ss); !Bound in the assumption
of only ONE supplier of cargo to an installation
  LoadingTimeDel(ii) := CQDel(ii)/LoadingRate;
end-do

!Time of loading pickup cargo i
forall(ii in PickupNodes) do
  CQPick(ii) := sum(ss in Suppliers) CargoQuantityPick(ii,ss);

```

```

LoadingTimePick(ii) := CQPick(ii)/LoadingRate;
end-do

!*****
!Calculating sailing times
!*****

!Calculate sailing time
forall(ii in Nodes, jj in Nodes | ii<>nCargoes+1 and jj<>0) do
SailingTime(ii,jj) := SailingDistanceskm(ii,jj)/VesselSpeedkmh;
end-do

!*****
! Create variables
!*****

declarations
    x: dynamic array(Nodes, Nodes)                of mpvar;
    t: dynamic array(Nodes)                        of mpvar;
    l: dynamic array(Nodes, Suppliers, CargoConditions) of mpvar;
    h: dynamic array(Nodes, Suppliers, CargoConditions) of mpvar;
    y: dynamic array(CargoNodes)                  of mpvar;
    w: dynamic array(Nodes)                        of mpvar;
!of mpvar; !!***
end-declarations

!x-variable, binary - 1 if vessel v sails directly from node i to node j.
!Except when nodes are equal, from end-base, and to start-base
forall(ii in Nodes, jj in Nodes | ii<>nCargoes+1 and jj<>0) do
    if (A(ii,jj)) then
        create(x(ii,jj));
        x(ii,jj) is_binary;
    end-if
end-do

!time-variable, continuous
forall(ii in Nodes) do
    create(t(ii));
end-do

!load-variable, continuous
forall(ii in Nodes, ss in Suppliers, cc in CargoConditions | ii<>nCargoes+1) do
    create(l(ii,ss,cc));
end-do

!occupied tanks-variable, integer
forall(ii in Nodes, ss in Suppliers, cc in CargoConditions) do
    create(h(ii,ss,cc));
    h(ii,ss,cc) is_integer;
end-do

!gap-variable, continuous
forall(ii in CargoNodes) do
    create(y(ii));
end-do

!available tanks-variable, integer
forall(ii in CargoNodes) do
    create(w(ii));
    w(ii) is_integer;
end-do

!*****
! Declare Constraints
!*****

```

```

declarations
    Gap: linctr;
    AllNodesVisited: dynamic array(Nodes) of linctr;
    FlowFromOrigin: linctr;
    FlowConservation1: dynamic array(Nodes) of linctr;
    FlowConservation2: dynamic array(Nodes) of linctr;
    FlowToDestination: linctr;
    TimeConstraint1: dynamic array(DeliveryNodes, Nodes) of linctr;
    TimeConstraint2: dynamic array(PickupNodes, Nodes) of linctr;
    StartNodeFirst: dynamic array(Nodes) of linctr;
    EndNodeLast: dynamic array(Nodes) of linctr;
    TimeWindowMin: dynamic array(Nodes) of linctr;
    TimeWindowMax: dynamic array(Nodes) of linctr;
    LoadConstraintDel: dynamic array(Nodes, DeliveryNodes, Suppliers) of
linctr;
    LoadConstraintDel1: dynamic array(Nodes, DeliveryNodes, Suppliers) of
linctr;
    LoadConstraintDel2: dynamic array(Nodes, DeliveryNodes, Suppliers) of
linctr;
    LoadConstraintPick: dynamic array(Nodes, PickupNodes, Suppliers) of linctr;
    LoadConstraintPick1: dynamic array(Nodes, PickupNodes, Suppliers) of linctr;
    LoadConstraintPick2: dynamic array(Nodes, PickupNodes, Suppliers) of linctr;
    LoadOrigin: dynamic array(Suppliers, CargoConditions) of linctr;
    OccTanks1: dynamic array(Nodes, Suppliers, CargoConditions) of
linctr;
    NoTanks: dynamic array(Nodes) of linctr;
!!***
    LimOccupiedTanks: dynamic array(Nodes) of linctr;
    GapConstraint: dynamic array(CargoNodes) of linctr;
end-declarations

!*****
! Objective Function
!*****

Gap :=
    sum(ii in CargoNodes)y(ii)
    -
    sum(ii in CargoNodes)w(ii) ;

!*****
! Flow Constraints
!*****

!All nodes/cargoes are visited/handled only once
forall (ii in Nodes | ii<>0 and ii<>(nCargoes+1)) do
    AllNodesVisited(ii) := sum(jj in Nodes) x(ii,jj)= 1;
end-do

!Only one arc from origin
FlowFromOrigin := sum(jj in Nodes) x(0,jj) =1;

!Only one visit per node, has to be an arc out from node i
forall (ii in Nodes | ii<>0 and ii<>(nCargoes+1)) do
    FlowConservation1(ii) := sum(jj in Nodes) x(ii,jj) = 1;
end-do
!Only one visit per node, has to be an arc to node j
forall (jj in Nodes | jj<>0 and jj<>(nCargoes+1)) do
    FlowConservation2(jj) := sum(ii in Nodes) x(ii,jj) = 1;
end-do

!Only one arc to destination
FlowToDestination := sum(ii in Nodes) x(ii,(nCargoes+1))=1;

!*****
! Time constraints
!*****

```



```

!Visits start node (node 0) before all other nodes
forall (ii in Nodes | ii<>0) do
    StartNodeFirst(ii) := t(ii) - t(0) >= 0;
end-do

!Time for arrival in j is to be bigger than the time for arrival in i + time spent
in i
forall (ii in DeliveryNodes, jj in Nodes | ii<>jj and ii<>nCargoes+1 and jj<>0) do
    TimeConstraint1(ii,jj) := t(ii) + LoadingTimeDel(ii) + SailingTime(ii,jj) -
t(jj) - BigM*(1-x(ii,jj)) <= 0;
end-do

!Time for arrival in j is to be bigger than the time for arrival in i + time spent
in i
forall (ii in PickupNodes, jj in Nodes | ii<>jj and ii<>nCargoes+1 and jj<>0) do
    TimeConstraint2(ii,jj) := t(ii) + LoadingTimePick(ii) + SailingTime(ii,jj) -
t(jj) - BigM*(1-x(ii,jj)) <= 0;
end-do

!Visits end node (node nCargoes+1) after all other nodes
forall (ii in Nodes | ii<>(nCargoes+1)) do
    EndNodeLast(ii) := t(nCargoes+1) - t(ii) >= 0;
end-do

!*****
! Load constraints
!*****

!forall(ii in Nodes, ss in Suppliers | ii=0) do
forall(ss in Suppliers) do
    LoadOrigin(ss,1) := l(0,ss,1) = sum(jj in DeliveryNodes)
CargoQuantityDel(jj,ss);
end-do

forall(ii in Nodes, jj in DeliveryNodes, ss in Suppliers | ii<>jj and
ii<>nCargoes+1) do
    LoadConstraintDel(ii,jj,ss) := l(ii,ss,1) - CargoQuantityDel(jj,ss) -
l(jj,ss,1) - BigB1*(1-x(ii,jj)) <= 0;
    LoadConstraintDel1(ii,jj,ss) := l(ii,ss,2) - l(jj,ss,2) +
VesselCapacity*x(ii,jj) <= VesselCapacity;
    LoadConstraintDel2(ii,jj,ss) := l(ii,ss,2) - l(jj,ss,2) -
VesselCapacity*x(ii,jj) >= -VesselCapacity;
end-do

forall(ii in Nodes, jj in PickupNodes, ss in Suppliers | ii<>jj and ii<>nCargoes+1)
do
    LoadConstraintPick(ii,jj,ss) := l(ii,ss,2) + CargoQuantityPick(jj,ss) -
l(jj,ss,2) - BigB2*(1-x(ii,jj)) <= 0;
    LoadConstraintPick1(ii,jj,ss) := l(ii,ss,1) - l(jj,ss,1) +
VesselCapacity*x(ii,jj) <= VesselCapacity;
    LoadConstraintPick2(ii,jj,ss) := l(ii,ss,1) - l(jj,ss,1) -
VesselCapacity*x(ii,jj) >= -VesselCapacity;
end-do

!*****
!Tank constraints
!*****

!Finding number of occupied tanks
forall(ii in Nodes, ss in Suppliers, cc in CargoConditions) do
    OccTanks1(ii,ss,cc) := h(ii,ss,cc) - (l(ii,ss,cc)/TankCapacity) >= 0;
end-do

```

```

!No more occupied tanks than there are tanks on the vessel
forall(ii in Nodes) do
    LimOccupiedTanks(ii) := sum(ss in Suppliers, cc in CargoConditions) h(ii,ss,cc)
<= nTanks;
end-do

!Ensure correct no. of tanks are in use
forall(ii in Nodes) do
NoTanks(ii) := w(ii) + sum(ss in Suppliers, cc in CargoConditions) h(ii,ss,cc) =
nTanks; !!***
end-do

!*****
!Sequence constraints
!*****

!Forcing the vessel to service the pickup cargo before the delivery cargo at the
same installation
forall(ii in PickupNodes, jj in DeliveryNodes | SailingTime(ii,jj) = 0) do
    x(ii,jj) = 1;
end-do

forall(ii in DeliveryNodes, jj in PickupNodes | SailingTime(ii,jj) = 0) do
    x(ii,jj) = 0;
end-do

!*****
!Gap constraint
!*****

forall(ii in CargoNodes) do
    GapConstraint(ii) := t(ii) - DemandTime(ii) = y(ii);
end-do

!*****
setparam('xprs_maxtime', SolutionTime);

solutionTime1:=gettime;
minimize(Gap);
!solutionTime2:=gettime;

declarations
    FileName:    string;
end-declarations

FileName := 'Output_' + Datafile + '.txt';

fopen(FileName, F_OUTPUT);

writeln("-----");
----");

if(TankCapacity=360) then
    writeln("Total delay is ",getsol(Gap), " hours for Far Scotsman");
else
    writeln("Total delay is ",getsol(Gap), " hours for Far Solitaire");
end-if

writeln("-----");
----");

index:=0;
teller:=1;

while (teller < nCargoes+1) do
forall(jj in Nodes) do

```

```

if(getsol(x(index,jj)) > 0.5) then
  if (index < nCargoes+1 and jj<>nCargoes+1) then
    write("The ship sails directly from ",index, " to ",jj);
    index:=jj;
    if(getsol(y(index)) > 0.5) then
      writeln("    The vessel starts service ", getsol(y(index)), " hours after
the demand occurs in node ",index);
      teller:=teller+1;
    else
      writeln("    The vessel services node ",index," on time");
      teller:=teller+1;
    end-if
  else
    writeln("The ship sails directly from ",index, " to ",jj);
    teller:=teller+1;
    writeln("-----");
    -----");
  end-if
end-if
end-do
end-do

teller:=0;
index:=0;
while (teller < nCargoes) do
forall(jj in CargoNodes) do
  if(getsol(x(index,jj)) > 0.5) then
    writeln("The vessel has ", getsol(w(jj)), " available tanks in node ", jj);
    index:=jj;
    teller:=teller+1;
  end-if
end-do
end-do

writeln("-----");
----");
fclose(F_OUTPUT);

end-model

```

Appendix D

Making Distance Matrices

D.1 Matlab Codes

```
1 %Make a Distance Matrix
2
3 %Pre-allocate the distance matrix
4 DistMatrix = zeros(16);
5
6 %Make vectors containing the latitude (LA) and longitude
7 %(LO) coordinates, with the corresponding names of the installations
8 %as a string vector.
9
10 LA = [60.79 61.25 61.2 61.29 61.17 61.2 61.2 61.07 60.89 60.87 60.6 ...
11       60.38 60.7 60.48 60.84 60.78];
12 LO = [5.063 1.85 1.82 1.9 2.18 2.2 2.27 2.19 3.65 3.69 2.77 2.79 2.93...
13       2.82 3.58 3.55];
14 Name ={'Base Mongstad', 'Statfjord A', 'Statfjord B', 'Statfjord C',...
15       'Gullfaks A', 'Gullfaks B', 'Gullfaks C', 'Gullfaks - Songa Dee',...
16       'Troll - Stena Don', 'Troll - Songa Equinox', 'Oseberg C',...
17       'Oseberg S r', 'Oseberg st ', 'Oseberg B', 'Troll COSL Innovator '
18       ,...
19       'Troll COSL Promoter'};
20 %Double for-loop to allocate the distances between the installations
21 %according to the sequence given in the "Name"-vector.
22
23 for i = 1:16
24     for j = 1:16
25         %Set the distance to 0 on the upper left-lower right diagonal.
```

```

26     if i == j
27         DistMatrix(i,j) = 0;
28     else
29         %Call the function "CalculateDistance" to calculate the distance
30         %and allocate it accordingly.
31         DistMatrix(i,j) = CalculateDistance(LA(i),LA(j),LO(i),LO(j));
32     end
33 end
34 end

```

```

1 %Use the Haversine formula to calculate distances between installations
2
3 function [Distance] = CalculateDistance(Lat1,Lat2,Lon1,Lon2)
4
5 R =6360752; %Radius of the Earth[m] (6356 at poles, 6378 at equator)
6
7 %Convert latitude and longitude from degrees to radians
8 Lat1_r = Lat1*(pi/180);
9 Lat2_r = Lat2*(pi/180);
10 Lon1_r = Lon1*(pi/180);
11 Lon2_r = Lon2*(pi/180);
12
13 A = sqrt(sin((Lat2_r-Lat1_r)/2).^2+(cos(Lat1_r)*cos(Lat2_r)*...
14     sin((Lon2_r-Lon1_r)/2).^2));
15 %Haversine formula
16 Distance = 2*R * asin(A);
17
18 end

```

D.2 Distance Matrices

Table D.1: Distance matrix [m] for case with large cargo sizes

	Base Mongstad	Gullfaks B	Oseberg Øst	Statfjord A	Songa Dee
Base Mongstad	-	$1.67 \cdot 10^5$	$1.16 \cdot 10^5$	$1.80 \cdot 10^5$	$1.58 \cdot 10^5$
Gullfaks B	$1.67 \cdot 10^5$	-	$6.80 \cdot 10^4$	$1.95 \cdot 10^4$	$1.44 \cdot 10^4$
Oseberg Øst	$1.16 \cdot 10^5$	$6.80 \cdot 10^4$	-	$8.43 \cdot 10^4$	$5.73 \cdot 10^4$
Statfjord A	$1.80 \cdot 10^5$	$1.95 \cdot 10^4$	$8.43 \cdot 10^4$	-	$2.70 \cdot 10^4$
Songa Dee	$1.58 \cdot 10^5$	$1.44 \cdot 10^4$	$5.73 \cdot 10^4$	$2.70 \cdot 10^4$	-

Table D.2: Distance matrix [m] for case with small cargo sizes

	Base Mongstad	Oseberg B	Gullfaks C	Songa Equinox	COSL Innovator	Oseberg Sor	Statford C	Statford A	Stena Don
Base Mongstad	-	$1.27 \cdot 10^5$	$1.57 \cdot 10^6$	$7.48 \cdot 10^4$	$8.05 \cdot 10^4$	$1.32 \cdot 10^5$	$1.79 \cdot 10^5$	$1.80 \cdot 10^5$	$7.72 \cdot 10^4$
Oseberg B	$1.27 \cdot 10^5$	-	$8.53 \cdot 10^4$	$6.41 \cdot 10^4$	$5.75 \cdot 10^4$	$1.12 \cdot 10^4$	$1.03 \cdot 10^5$	$1.00 \cdot 10^5$	$6.41 \cdot 10^4$
Gullfaks C	$1.57 \cdot 10^6$	$8.53 \cdot 10^4$	-	$8.47 \cdot 10^4$	$8.10 \cdot 10^4$	$9.53 \cdot 10^4$	$2.21 \cdot 10^4$	$2.31 \cdot 10^4$	$8.18 \cdot 10^4$
Songa Equinox	$7.48 \cdot 10^4$	$6.41 \cdot 10^4$	$8.47 \cdot 10^4$	-	$6.82 \cdot 10^3$	$7.32 \cdot 10^4$	$1.07 \cdot 10^5$	$1.08 \cdot 10^5$	$3.10 \cdot 10^3$
COSL Innovator	$8.05 \cdot 10^4$	$5.75 \cdot 10^4$	$8.10 \cdot 10^4$	$6.82 \cdot 10^3$	-	$6.68 \cdot 10^4$	$1.03 \cdot 10^5$	$1.04 \cdot 10^5$	$6.72 \cdot 10^3$
Oseberg Sor	$1.32 \cdot 10^5$	$1.12 \cdot 10^4$	$9.53 \cdot 10^4$	$7.32 \cdot 10^4$	$6.68 \cdot 10^4$	-	$1.12 \cdot 10^5$	$1.09 \cdot 10^5$	$7.35 \cdot 10^4$
Statford C	$1.79 \cdot 10^5$	$1.03 \cdot 10^5$	$2.21 \cdot 10^4$	$1.07 \cdot 10^4$	$1.03 \cdot 10^4$	$1.12 \cdot 10^5$	-	$5.18 \cdot 10^5$	$1.04 \cdot 10^5$
Statford A	$1.80 \cdot 10^5$	$1.00 \cdot 10^5$	$2.31 \cdot 10^4$	$1.08 \cdot 10^5$	$1.04 \cdot 10^5$	$1.09 \cdot 10^5$	$5.18 \cdot 10^5$	-	$1.05 \cdot 10^5$
Stena Don	$7.72 \cdot 10^4$	$6.41 \cdot 10^4$	$8.18 \cdot 10^4$	$3.10 \cdot 10^3$	$6.72 \cdot 10^3$	$7.35 \cdot 10^4$	$1.04 \cdot 10^5$	$1.05 \cdot 10^5$	-

Appendix E

Finding Average PSV

Note that this work was done prior to the merging of the companies Farstad, Solstad and Deep Sea Supply.

Skip	LOA [m]	B [m]	Draft [m]	Drill Water/WB [m^3]	Mud [m^3]	Brine [m^3]	Fuel Oil [m^3]	Pot Water [m^3]	Base Oil [m^3]	Methanol [m^3]	Deck Area [m^2]	Type	Year Build
Far Solitaire	91,6	22	7,2	2447	1316	1559	1146	739	403	403	1022 (54,4 x 18,8)	UT 654 WP	2012
Far Sygna	94,7	21	7,1	2806	1005	787	1334	828	152	151	1179	VAR D 1 07	2014
Far Sun	94,7	21	7,1	2751	1003	785	1331	813	152	151	1170	VAR D 1 07	2014
Far Starling	81,7	18	6,5	1915	1270	1270	917	730	319	100	810	PSV 08 CD	2013
Far Spica	81,7	18	6,5	1915	1270	1270	917	730	319	100	810	PSV 08 CD	2013
Far Sitella	81,7	18	6,5	1915	1270	1270	917	730	319	100	810	PSV 08 CD	2013
Far Skimmer	81,7	18	6,5	1897	1270	1270	917	730	319	100	810	PSV 08 CD	2012
Far Scotsman	81,7	18	6,5	1915	1270	1270	917	730	319	100	810	PSV 08 CD	2012
Far Server	78,6	17,6	6,5	1257	975	975	877	823	155	178	800	Hayyard 832 CD	2010
Far Swan	73,4	16,6	6,5	714	1072	859	214	775	214	179	704	VS 470 MK II	2006
Far Serenade	93,9	21	7,3	2522	1137	388	1260	1158	276	218	1002	UT 751 CD	2009
Far Searcher	93,9	21	6,6	1457	911	404	1319	989	240	206	1091	UT 751 E	2008
Far Seeker	93,9	21	6,6	1457	911	404	1319	989	240	206	1091	UT 751 E	2008
Far Spirit	73,4	16,6	6,5	716	1072	856	1048	776	214	191	725	VS 470 MK II	2007
Far Symphony	86,2	19	6,7	550	837	804	1678	736	305	163	950	P 105	2003
Far Splendour	74,3	16	6,3	1281	785	417	749	502	190	211	692	P 106	2003
Lady Melinda	71,0	16,0	5,8	946	775	387	1047	572	0	169	567	UT 755	2003
Far Star	84,6	18,6	6,3	1063	695	306	1502	1683	260	0	815	UT 745	1999
Far Supplier	82,9	19,0	6,3	894	680	352	1245	1017	221	108	896	VS 483	1999
Far Strider	82,9	19,0	5,9	1057	0	0	3086	973	0	0	902	VS 483	1999
Far Supporter	83,8	18,8	6,2	832	532	740	1668	1735	196	0	902	UT 750	1996
Far Service	83,8	18,8	6,2	894	503	377	1518	1399	198	0	965	UT 745	1995
Average	83,5476190476191	18,6190476190476	6,49523809523809	1464,47619047619	962,15	759,55	1227,61904761905	924,666666666667	242,526315789474	154,764705882353	881	-	2006,47619047619
Average PSV	83,5	18,6	6,5	1465	962	760	1228	925	243	155	881	-	2007
Most Common Type - Far Scotsman	81,7	18	6,5	1915	1270	1270	917	730	319	100	810	PSV 08 CD	2012

Appendix F

Excerpts from Statoil Data Sheets

F.1 Cargo Information

Voyage number	Installation ID	Installation ID2	Utilisation Group	BULK OUT (TON)	BULK IN (TON)	# LIFTS
94052	GFA	GULLFAKS A	BLK18	66,0		1
94052	SEQ	SONGA EQUINOX	BLK17		260,0	1
94052	STA	STATFJORD A	BLK11	237,0		2
94052	STB	STATFJORD B	BLK18	110,0		1
94052	STC	STATFJORD C	BLK18	50,0		1
94161	KVB	KVITEBJØRN	BLK13	70,0		1
94161	STA	STATFJORD A	BLK11	245,0		2
94161	STA	STATFJORD A	BLK16	172,0		1

F.2 Vessel Information

Departure port	Voyage Type	Voyage Nr.	Boat name	Estimated date departure port	Actual date departure port	Actual time departure port	Legnumber	Destination on port	Deviation	Estimated date of arrival at destination	Estimated time of arrival at destination	Estimated date of arrival at destination	Estimated time of arrival at destination	Actual date of departure at destination	Actual time of departure at destination	Actual date of arrival at destination	Actual time of arrival at destination
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	2	FBB		03.02.2017	16:57:12	03.02.2017	13:57:12	03.02.2017	06:40:00	02.02.2017	16:15:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	3	UHV		03.02.2017	00:00:00	03.02.2017	00:00:00	03.02.2017	08:01:00	03.02.2017	08:00:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	4	FBB	D58	03.02.2017	16:57:12	03.02.2017	13:57:12	03.02.2017	11:30:00	03.02.2017	09:00:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	6	STA	D51	04.02.2017	09:31:26	04.02.2017	03:31:26	04.02.2017	05:40:00	04.02.2017	00:00:01
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	7	STC	D51	04.02.2017	16:00:55	04.02.2017	10:00:55	04.02.2017	10:10:00	04.02.2017	06:05:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	8	STB	D60	05.02.2017	00:46:27	04.02.2017	16:46:27	04.02.2017	18:00:00	04.02.2017	10:50:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	9	STC	D60	04.02.2017	16:00:55	04.02.2017	10:00:55	04.02.2017	21:45:00	04.02.2017	18:35:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	10	STB	D51	05.02.2017	00:46:27	04.02.2017	16:46:27	05.02.2017	10:05:00	04.02.2017	23:20:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	11	GFA	D11	05.02.2017	00:00:00	05.02.2017	00:00:00	05.02.2017	13:35:00	05.02.2017	11:25:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	12	KVB		05.02.2017	05:54:22	05.02.2017	02:54:22	05.02.2017	17:05:00	05.02.2017	14:40:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	13	VMO		05.02.2017	00:00:00	05.02.2017	00:00:00	05.02.2017	18:35:00	05.02.2017	17:40:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	14	SEQ	D57	05.02.2017	00:00:00	06.02.2017	00:00:00	06.02.2017	04:45:00	05.02.2017	22:10:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	16	SLØ		05.02.2017	14:34:17	05.02.2017	13:04:17	06.02.2017	13:50:00	06.02.2017	09:00:00
FMO	10	94052	Viking Energy	03.02.2017	10:54:55	00:00:00	17	FMO		05.02.2017	00:00:00	05.02.2017	15:08:45	06.02.2017	00:00:00	06.02.2017	14:40:00
FMO	10	94161	Juanta	03.02.2017	18:30:00	00:00:00	4	OSS	D57	04.02.2017	04:13:09	04.02.2017	01:13:09	04.02.2017	09:50:00	04.02.2017	04:30:00
FMO	10	94161	Juanta	03.02.2017	18:30:00	00:00:00	5	KVB		04.02.2017	10:17:57	04.02.2017	08:17:57	04.02.2017	15:15:00	04.02.2017	14:10:00
FMO	10	94161	Juanta	03.02.2017	18:30:00	00:00:00	6	STC		04.02.2017	13:30:25	04.02.2017	12:30:25	04.02.2017	20:30:00	04.02.2017	17:45:00
FMO	10	94161	Juanta	03.02.2017	18:30:00	00:00:00	7	STA		04.02.2017	23:16:23	04.02.2017	19:46:23	05.02.2017	07:35:00	04.02.2017	20:50:00
FMO	10	94161	Juanta	03.02.2017	18:30:00	00:00:00	8	STB		04.02.2017	19:15:57	04.02.2017	14:15:57	05.02.2017	13:25:00	05.02.2017	08:15:00
FMO	10	94161	Juanta	03.02.2017	18:30:00	00:00:00	9	OSC		04.02.2017	00:00:00	05.02.2017	00:00:00	05.02.2017	20:25:00	05.02.2017	18:25:00
FMO	10	94161	Juanta	03.02.2017	18:30:00	00:00:00	11	FMO		05.02.2017	00:00:00	05.02.2017	10:22:36	06.02.2017	00:00:00	06.02.2017	06:00:00

Appendix G

Wet Bulk Transport at Statfjord A



Figure G.1: Dates for, and amounts of special product delivered.

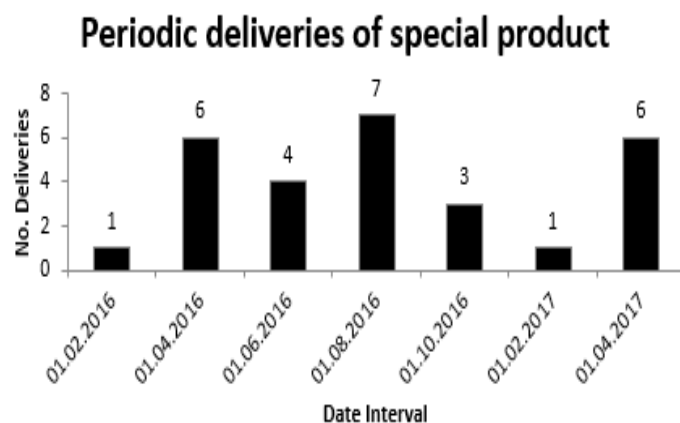
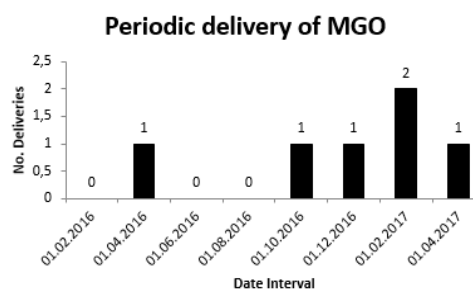
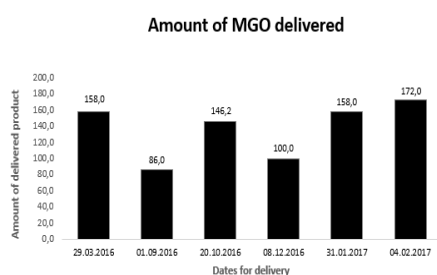


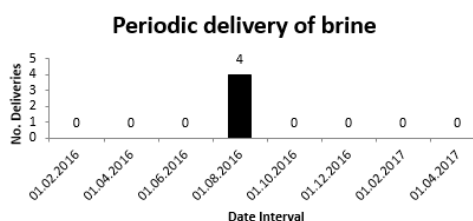
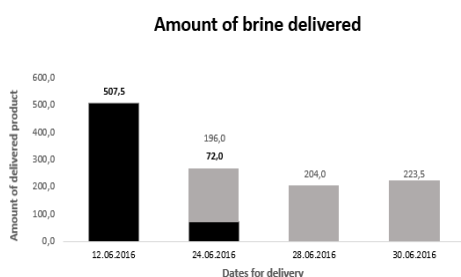
Figure G.2: Number of deliveries in different time periods.



(a) Dates for, and amount of, MGO delivered.

(b) Number of deliveries in different time periods.

Figure G.3: Delivered MGO to Statfjord A over a 13-month period.



(a) Dates for, and amount of, brine delivered (black) and picked up (grey).

(b) Number of deliveries in different time periods.

Figure G.4: Delivered brine to Statfjord A over a 13-month period.

Appendix H

Some Output Files from Xpress

H.1 Results Several Visits with Small Cargo Sizes

```
-----  
Total delay is -60.869 hours for Far Scotsman  
-----  
The ship sails directly from 0 to 3    The vessel services node 3 on time  
The ship sails directly from 3 to 2    The vessel starts service 7.642 hours after the demand occurs in node 2  
The ship sails directly from 2 to 8    The vessel starts service 6.059 hours after the demand occurs in node 8  
The ship sails directly from 8 to 4    The vessel services node 4 on time  
The ship sails directly from 4 to 5    The vessel starts service 8.146 hours after the demand occurs in node 5  
The ship sails directly from 5 to 6    The vessel starts service 4.426 hours after the demand occurs in node 6  
The ship sails directly from 6 to 1    The vessel starts service 15.198 hours after the demand occurs in node 1  
The ship sails directly from 1 to 7    The vessel starts service 0.66 hours after the demand occurs in node 7  
The ship sails directly from 7 to 9  
-----  
The vessel has 12 available tanks in node 3  
The vessel has 12 available tanks in node 2  
The vessel has 11 available tanks in node 8  
The vessel has 12 available tanks in node 4  
The vessel has 13 available tanks in node 5  
The vessel has 14 available tanks in node 6  
The vessel has 15 available tanks in node 1  
The vessel has 14 available tanks in node 7  
-----
```

Output file from Xpress for case 1 with Far Scotsman

```
-----  
Total delay is -60.869 hours for Far Solitaire  
-----  
The ship sails directly from 0 to 3    The vessel services node 3 on time  
The ship sails directly from 3 to 2    The vessel starts service 7.642 hours after the demand occurs in node 2  
The ship sails directly from 2 to 8    The vessel starts service 6.059 hours after the demand occurs in node 8  
The ship sails directly from 8 to 4    The vessel services node 4 on time  
The ship sails directly from 4 to 5    The vessel starts service 8.146 hours after the demand occurs in node 5  
The ship sails directly from 5 to 6    The vessel starts service 4.426 hours after the demand occurs in node 6  
The ship sails directly from 6 to 1    The vessel starts service 15.198 hours after the demand occurs in node 1  
The ship sails directly from 1 to 7    The vessel starts service 0.66 hours after the demand occurs in node 7  
The ship sails directly from 7 to 9  
-----  
The vessel has 12 available tanks in node 3  
The vessel has 12 available tanks in node 2  
The vessel has 11 available tanks in node 8  
The vessel has 12 available tanks in node 4  
The vessel has 13 available tanks in node 5  
The vessel has 14 available tanks in node 6  
The vessel has 15 available tanks in node 1  
The vessel has 14 available tanks in node 7  
-----
```

Figure H.1: Output file from Xpress for case 1 with Far Solitaire

H.2 Results Few Visits with Large Cargo Sizes

```

-----
Total delay is -60.869 hours for Far Scotsman
-----
The ship sails directly from 0 to 3   The vessel services node 3 on time
The ship sails directly from 3 to 2   The vessel starts service 7.642 hours after the demand occurs in node 2
The ship sails directly from 2 to 8   The vessel starts service 6.059 hours after the demand occurs in node 8
The ship sails directly from 8 to 4   The vessel services node 4 on time
The ship sails directly from 4 to 5   The vessel starts service 8.146 hours after the demand occurs in node 5
The ship sails directly from 5 to 6   The vessel starts service 4.426 hours after the demand occurs in node 6
The ship sails directly from 6 to 1   The vessel starts service 15.198 hours after the demand occurs in node 1
The ship sails directly from 1 to 7   The vessel starts service 0.66 hours after the demand occurs in node 7
The ship sails directly from 7 to 9
-----
The vessel has 12 available tanks in node 3
The vessel has 12 available tanks in node 2
The vessel has 11 available tanks in node 8
The vessel has 12 available tanks in node 4
The vessel has 13 available tanks in node 5
The vessel has 14 available tanks in node 6
The vessel has 15 available tanks in node 1
The vessel has 14 available tanks in node 7
-----

```

Figure H.2: Output file from Xpress for case 2 with Far Scotsman

```

-----
Total delay is -60.869 hours for Far Solitaire
-----
The ship sails directly from 0 to 3   The vessel services node 3 on time
The ship sails directly from 3 to 2   The vessel starts service 7.642 hours after the demand occurs in node 2
The ship sails directly from 2 to 8   The vessel starts service 6.059 hours after the demand occurs in node 8
The ship sails directly from 8 to 4   The vessel services node 4 on time
The ship sails directly from 4 to 5   The vessel starts service 8.146 hours after the demand occurs in node 5
The ship sails directly from 5 to 6   The vessel starts service 4.426 hours after the demand occurs in node 6
The ship sails directly from 6 to 1   The vessel starts service 15.198 hours after the demand occurs in node 1
The ship sails directly from 1 to 7   The vessel starts service 0.66 hours after the demand occurs in node 7
The ship sails directly from 7 to 9
-----
The vessel has 12 available tanks in node 3
The vessel has 12 available tanks in node 2
The vessel has 11 available tanks in node 8
The vessel has 12 available tanks in node 4
The vessel has 13 available tanks in node 5
The vessel has 14 available tanks in node 6
The vessel has 15 available tanks in node 1
The vessel has 14 available tanks in node 7
-----

```

Figure H.3: Output file from Xpress for case 2 with Far Solitaire

H.3 Results with Process Plant

```

-----
Total delay is -60.869 hours for Far Solitaire
-----
The ship sails directly from 0 to 3   The vessel services node 3 on time
The ship sails directly from 3 to 2   The vessel starts service 7.642 hours after the demand occurs in node 2
The ship sails directly from 2 to 8   The vessel starts service 6.059 hours after the demand occurs in node 8
The ship sails directly from 8 to 4   The vessel services node 4 on time
The ship sails directly from 4 to 5   The vessel starts service 8.146 hours after the demand occurs in node 5
The ship sails directly from 5 to 6   The vessel starts service 4.426 hours after the demand occurs in node 6
The ship sails directly from 6 to 1   The vessel starts service 15.198 hours after the demand occurs in node 1
The ship sails directly from 1 to 7   The vessel starts service 0.66 hours after the demand occurs in node 7
The ship sails directly from 7 to 9
-----
The vessel has 12 available tanks in node 3
The vessel has 12 available tanks in node 2
The vessel has 11 available tanks in node 8
The vessel has 12 available tanks in node 4
The vessel has 13 available tanks in node 5
The vessel has 14 available tanks in node 6
The vessel has 15 available tanks in node 1
The vessel has 14 available tanks in node 7
-----

```

Figure H.4: Output file from Xpress for process plant case