A physically-based constitutive model applied to AA6082 aluminium alloy at large strains, high strain rates and elevated temperatures

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Abstract

This paper presents a physically-based constitutive model applied to an AA6082 aluminium alloy subjected to large strains, high strain rates and elevated temperatures. The model accounts for thermo-elasticity, thermo-viscoplasticity, and strain-rate and temperature dependent work-hardening, using the dislocation density as an internal variable without relating it to a detailed characterization of the microstructure evolution. The parameter identification is based on previously reported experimental results from quasi-static and dynamic tensile tests performed at temperatures from 20 to 350 °C on a cast and homogenised AA6082 aluminium alloy. The equivalent stress-strain curves were determined beyond necking by monitoring the cylindrical specimens with a digital camera and applying the Bridgman correction to account for the triaxial stress state. The experimental results revealed a distinct increase of the work hardening at high strain rates, indicating a change of strengthening mechanism. A new formulation for the dynamic recovery provided reasonable predictions over the considered range of strains, strain rates and temperatures. The constitutive model was implemented into finite element software and simulations of the quasi-static and dynamic tensile tests were made to assess the accuracy of the Bridgman correction used to determine the flow stress in the post-necking range. The simulations reproduced the complex behaviour observed in the tests and validated the applied parameter identification procedures for the entire range of strains, strain rates and temperatures.

*Keywords: Constitutive modelling; Quasi-static and dynamic response; Coupled strain-rate and temperature sensitivity; Finite element analysis; Large plastic strains*

# Introduction

Due to the combination of good mechanical properties and low density, the use of aluminium alloys is constantly increasing, especially in automotive applications and fields where fuel consumption is of major concern. AA6xxx alloys are often used for extruded profiles and rolled sheets or plates. Finite element (FE) analysis of thermo-mechanical forming processes requires knowledge about the behaviour of these alloys for a wide range of strains, strain rates and temperatures. Application of FE codes can decrease design costs, but calls for reliable constitutive models. The availability of such models is quite limited when it comes to cover the coupled effects of strain, strain rate and temperature on the flow stress up to large deformation levels, and development of improved constitutive models is thus required.

Plastic deformation by dislocation glide in fcc materials is assisted by thermally activated processes to overcome obstacles for mobile dislocations like solute atoms and forest dislocations [[1](#_ENREF_1)]. Accordingly, aluminium alloys exhibit temperature and strain-rate sensitivity (SRS) that should be accounted for in constitutive models. [Vilamosa et al. [2]](#_ENREF_2) investigated three cast and homogenised AA6xxx class alloys, and showed that they exhibit a pronounced temperature sensitivity of both yielding and work hardening. The alloys displayed a slight positive SRS at room temperature. Due to the dynamic recovery of dislocations, which is highly strain-rate and temperature sensitive [[3](#_ENREF_3)], the alloys were found to exhibit a much higher SRS with increasing temperature beyond 200 °C. The SRS was also shown to increase with increasing plastic strain, which, according to [Romhanji et al. [4]](#_ENREF_4), is related to the increased density of dislocations in cell walls. Metals and alloys often display a significant increase of the flow stress for strain rates higher than 103  [[5-8](#_ENREF_5)]. This sudden change of behaviour is by several authors attributed to a change of the strengthening mechanism involved. The flow stress is governed by thermally activated obstacles-controlled dislocation motion at low and medium strain rates, while dissipative dislocation drag occurs at sufficiently high strain rates [[8](#_ENREF_8), [9](#_ENREF_9)]. This modification of the physical mechanism results in an increase of the SRS which is often assumed to be temperature dependent [[10](#_ENREF_10)]. However, several works argue that dissipative dislocation drag occurs only at higher strain rates than 104  and that the strengthening behaviour from 103  up to 104  is due to a changed mechanism for the evolution of the microstructure [[11-13](#_ENREF_11)].

In FE codes the most employed constitutive models are classified as phenomenological models and are based on experimental observation, e.g. the Johnson-Cook model [[14](#_ENREF_14)]. Such models have few parameters and are often expressed as algebraic relations that are easy to implement numerically. However, physically-based models can be more appropriate for describing the behaviour of materials over a wide range of strain rates and temperatures or for capturing transient effects caused by changes of strain rate or temperature [[15](#_ENREF_15), [16](#_ENREF_16)]. Physically-based material models relate the flow stress to the distribution of stored dislocations and other dislocation obstacles by assuming a profile for the thermal activation energy required for dislocations to overcome these obstacles. A challenge is to combine the influence of different types of pinning points on the flow stress, because they have different thermal activation properties. Furthermore, the dislocation density increases during straining and its evolution has to be modelled to keep track of the increased number of dislocation induced pinning points. A simple evolution rule for the dislocation density was proposed by [Bergström [17]](#_ENREF_17), while more complex evolution rules for the microstructure, accounting for the different stages of work hardening, have also been suggested, e.g. [Nes [18]](#_ENREF_18). Based on various simplifications and assumptions physically-based models are frequently expressed mathematically in a similar manner as the phenomenological models, i.e., the flow stress is defined as a function of strain, strain rate and temperature, but with a sound physical basis [[1](#_ENREF_1), [19](#_ENREF_19)]. The constitutive model of [Voyiadjis and Abed [1]](#_ENREF_1) was applied by [Vilamosa et al. [2]](#_ENREF_2) for the three AA6xxx alloys involved in their experimental study. The Voyiadjis-Abed model provided a rather good prediction of the flow stress as function of strain, strain rate and temperature considering the restricted number of material parameters. However, it did not capture the increase of the SRS beyond 200 °C and the sudden increase in work hardening for strain rates higher than 1000 .

The thermo-mechanical response of materials is often investigated by series of tension or compression tests at different strain rates and temperatures. Low and medium strain-rate testing can be performed by use of universal testing machines, while high strain-rate testing usually is carried out with a Hopkinson bar apparatus [[20](#_ENREF_20)]. Different approaches such as heat guns [[21](#_ENREF_21)] and ovens [[22](#_ENREF_22)] can be applied to heat the specimen depending on the test rig involved. [Vilamosa et al. [23]](#_ENREF_23) showed that a fast induction heater was a practical solution for high strain-rate testing performed in a split-Hopkinson tension bar (SHTB), since the heating of the bars is limited and the stress wave propagation remains undisturbed. This set-up allows applying image based measuring techniques which provide information on the local deformation of the sample up to large strains (see [Vilamosa et al. [2]](#_ENREF_2), [Vilamosa et al. [23]](#_ENREF_23) for more details).

The aim of the present paper is to develop a physically-based constitutive model for AA6xxx alloys based on the experimental data presented by [Vilamosa et al. [2]](#_ENREF_2). The constitutive model allows for thermo-elasticity, thermo-viscoplasticity, and strain-rate and temperature dependent work-hardening. The dislocation density is adopted as the single internal variable of the flow stress model, yet its evolution is not correlated to a detailed characterization of the microstructure. Two AA6060 alloys and one AA6082 alloy were investigated by [Vilamosa et al. [2]](#_ENREF_2), while the constitutive model is only calibrated for the AA6082 material in the present study. This alloy exhibited the strongest strain-rate and temperature coupling effects on the flow stress. Concerning the AA6060 alloys, calibration of the model and some numerical results are provided in [Vilamosa et al. [24]](#_ENREF_24).

The paper is organized as follows. A brief summary of the experimental study by [Vilamosa et al. [2]](#_ENREF_2) is given in Section 2. The physically-based model for AA6xxx alloys is described in Section 3 together with the numerical implementation into the FE code LS-DYNA [[25](#_ENREF_25)]. Section 4 presents the parameter identification strategy and gives a detailed evaluation of the model against the experimental data. Results from FE simulations of the tensile tests are presented in Section 5 and used for verification of the calibration procedure and further validation of the constitutive model. Section 6 and 7 contain a discussion of certain aspects of the proposed constitutive model and the main conclusions, respectively.

# Experimental

The chemical composition of the AA6082 aluminium alloy is provided in Table 1. The material was delivered as cast and homogenised billets by Hydro Aluminium and was stored in room temperature for six months before the experimental tests.

Table 1: Chemical composition of the aluminium alloy AA6082 (weight %).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Mg | Si | Fe | Cu | Mn | Zn | Ti | Cr | Al |
| 0.80 | 0.76 | 0.20 | 0.24 | 0.56 | - | 0.01 | 0.16 | balance |

The thermo-mechanical response of the material was reported by [Vilamosa et al. [2]](#_ENREF_2) based on series of tensile tests at different nominal strain rates  (0.01 , 0.33 , 350 , 750 ) and temperatures  (20 °C, 200 °C, 250 °C, 300 °C, 350 °C). Cylindrical samples with 3 mm diameter and 5 mm parallel gauge length were used. Low and intermediate nominal strain-rate tests were achieved in a universal tensile machine, while high strain-rate tests at 350  and 750  were performed with a SHTB system (see also Section 5.2). The samples were heated with an induction heating system from MSI Automation. The temperature was measured by a pyrometer from LumaSense Technologies connected in a closed loop with the heating system.

In all tests, the local strain and strain rate were computed from an edge detection technique based on frames of the sample recorded with a digital camera during testing [[23](#_ENREF_23)]. The applied local strain measurement technique allows for investigation of the material response at large strains beyond necking. It also facilitates determination of the local logarithmic strain rate  in the neck. The Cauchy stress  was calculated from the force and the minimum cross-section area of the sample and was corrected according to Bridgman’s method to take into account that the stress state after necking is no longer uniaxial [[26](#_ENREF_26)]. Using this method, the flow stress  was calculated as



with



where  is the current radius of the minimum cross-section of the specimen and  is the current radius of curvature of the neck. More details about the test rigs, local strain measurements, heating of specimen, temperature measurements and stress correction beyond necking are provided in [Vilamosa et al. [2]](#_ENREF_2), [Vilamosa et al. [23]](#_ENREF_23).

Some of the main results from the experimental study by [Vilamosa et al. [2]](#_ENREF_2) are compiled in Figure 1. Small oscillations were observed in the tests, in particular the dynamic ones, and the curves shown in the figure represent a fit of the experimental data sets to a smooth mathematical function. Flow stress versus plastic strain curves at low and intermediate strain rates over the entire temperature range are shown in Figure 1 a), while Figure 1 b) compares results at low and high strain rates in a similar way. The different curves are plotted up to the maximum plastic strain level measured for each test. At room temperature, low positive SRS is observed for both yielding and work hardening. On the other hand, the SRS of the flow stress displays a marked increase beyond 200°C.

|  |  |
| --- | --- |
|  |  |
| a) | b) |
| Figure 1: Representative flow stress versus plastic strain curves from room temperature up to 350°C at a) nominal strain rates of 0.01 s–1 (round symbol) and 0.33 s–1 (diamond symbol) and b) nominal strain rates of 0.01 s–1 (round symbol) and 750 s–1 (triangle symbol). | |

# Constitutive model

A thermoelastic-thermoviscoplastic model is developed using a physically-based formulation where the flow stress is expressed as a function of dislocation density, strain rate and temperature. The corotational stress approach is adopted to allow for large deformations and to fulfil the principle of material-frame indifference. A hypoelastic formulation of plasticity is used and the elastic strains are assumed to remain small. Since the considered material was cast and homogenised, the thermo-elastic moduli tensors and the yield function are assumed to be isotropic.

## **Corotational hypoelastic plasticity**

The corotational Cauchy stress tensor  and corotational rate-of-deformation tensor  are defined as



where  is the Cauchy stress tensor,  is the rate-of-deformation tensor, and  is the rotation tensor, defined by the polar decomposition of the deformation gradient tensor  as ,  being the right stretch tensor. The corotational rate-of-deformation tensor is decomposed into elastic and plastic parts



The isotropic linear thermo-hypoelastic relation is defined as



The isotropic thermo-elastic moduli tensors  and  are given by



where  is the 2nd order identity tensor and  is the 4th order symmetric identity tensor. The linear thermo-hypoelastic relation is then expressed as



It is here assumed that Young’s modulus  varies with the temperature , while Poisson’s ratio  and the thermal expansion coefficient  are constant. In reality, both  and  are temperature dependent. The temperature sensitivity of the Young’s modulus is described empirically as [[27](#_ENREF_27)]



where  is Young’s modulus at 0 K,  is a material constant and  is the melting temperature of the material.

The plastic part of the corotational rate-of-deformation tensor  is obtained from the associated flow rule as



where  is the equivalent plastic strain rate. The equivalent stress  is defined by the Hershey yield function [[28](#_ENREF_28), [29](#_ENREF_29)]



where  are the eigenvalues of , and the exponent , which determines the shape of the yield surface, is usually set to 8 for fcc materials [[30](#_ENREF_30)]. The dynamic yield criterion is defined as



where  is the work-hardening variable. The flow stress incorporates the yield stress, the work-hardening and the viscous overstress. In the current study, either isothermal conditions, , or adiabatic conditions are assumed, and in the latter case the rate of temperature increase is estimated as



where  is the Taylor-Quinney coefficient [[31](#_ENREF_31)] that represents the proportion of plastic work converted into heat,  is the material density and  is the specific heat capacity of the material. Due to lack of experimental data, it is assumed that both  and  are independent of temperature.

## **Flow stress and work-hardening**

Following [Bergström [17]](#_ENREF_17), the flow stress  is split into three parts



The athermal yield stress  depends on temperature only through the shear modulus and is here used to capture the strengthening effects from particles (i.e., primary particles, dispersoids and precipitates). The viscous stress  depends explicitly on strain rate and temperature for a given concentration of elements in solid solution. With a change in either the strain rate or the temperature, or both, there will be an instantaneous change of the viscous stress. The work-hardening variable  is a function of the dislocation density , as defined by the Taylor equation, and depends indirectly on strain rate and temperature through the evolution rule for the dislocation density and on temperature via the shear modulus.

The temperature sensitivity of the athermal stress is assumed to be the same as the temperature sensitivity of the shear modulus



where  and  are respectively the yield stress and shear modulus at 0 K. The temperature sensitivity of the shear modulus is defined by



Recall that the Poisson ratio  is assumed here to be independent of temperature.

Using an Arrhenius type of expression for the strain rate with the activation energy profile suggested by [Kocks et al. [32]](#_ENREF_32) and the regularization proposed by [Holmedal [33]](#_ENREF_33) to avoid instabilities at low strain rates, the viscous stress is defined as



with



In these equations,  is a dimensionless activation energy at 0 K,  is the modulus of the Burgers vector,  is Boltzmann’s constant,  is a reference strain rate,  and  define the shape of the obstacle profile, and  is a material parameter. This formulation was originally developed for pure metals, but it is assumed here that by modifying the profile parameters, it can be used also for alloys.

The work-hardening  is related to the dislocation density  through the Taylor equation [[34](#_ENREF_34)]



where  is a scaling factor set to 1.0 andis the Taylor factor for a tensile test in the reference direction, which depends on the texture of the material. According to the Taylor model,  equals 3.07 for materials with random texture. A special case of the general formalism by [Kocks and Mecking [35]](#_ENREF_35) is the particular formulation of their master curve proposed by [Bouaziz [36]](#_ENREF_36)



where the constant  describes storage of dislocations due to interaction with forest dislocations and  is a characteristic length scale representing the capture distance for dynamic recovery. This evolution equation for the dislocation density is adopted here since it is assumed to give improved results for large strains. The reader is referred to [Bouaziz [36]](#_ENREF_36) for a physical interpretation of the evolution equation.

The characteristic length scale  is assumed here in the form



where is a reference capture length for dynamic recovery. The parameter  accounts for the fact that dynamic recovery is a thermally activated process that can be related to the Zener-Hollomon parameter, see [Bergström and Hallén [3]](#_ENREF_3). The functional form of  is slightly modified to account for the wider range of strain rates investigated in the present study. Thus, the parameter  is defined as



where  represents the main contribution to the dynamic recovery at low strain rates and  accounts for the combined effect of strain rate and temperature on dynamic recovery at low and medium strain rate. This formulation is valid up to relatively high nominal strain rates, say 103 . At higher strain rates from 103 to 104 , a large increase of the flow stress in fcc materials is usually observed and linked to a change in the strain-rate sensitivity of the microstructure [[11](#_ENREF_11), [13](#_ENREF_13)]. The factor  is introduced to account for reduced dynamic recovery at very high strain rates and expressed as



where  is a material parameter and  is a reference strain rate. In their original model, [Bergström and Hallén [3]](#_ENREF_3) assumed a constant recovery term  for low strain rates at room temperature or colder conditions. It is more realistic to expect an increase of the flow stress for decreasing temperatures below room temperature for aluminium alloys [[37](#_ENREF_37)], and  is therefore expressed as



where  is a material constant, and  is the dynamic recovery at low strain rate and 0 K. Inspired by the Zener-Hollomon equation, [Bergström and Hallén [3]](#_ENREF_3) proposed to express the strain-rate and temperature sensitivity of the dynamic recovery at low and medium strain rates as



where  is an activation energy,  is the gas constant which is equal to 8.31 J mol-1 K-1, and  and  are two material constants. The parameter  is used as a fitting parameter in the current model and should not differ much from 1/3 according to [Bergström and Hallén [3]](#_ENREF_3).

Combination of Equations and leads to the following evolution rule for the work-hardening



with



Note that the initial work hardening rate  is a function of temperature via the temperature dependence of the shear modulus, while  depends on both strain rate and temperature. As pointed out by [Bouaziz [36]](#_ENREF_36), a first-order expansion of Equation when  gives the well-known Voce hardening law, where  would be the saturation value of  at a given strain rate and temperature. However, in the Bouaziz model there is no saturation stress and  plays the role of a characteristic stress.

## **Stress update algorithm**

The constitutive model was implemented as a user material model in the finite element code using a semi-implicit formulation [[38](#_ENREF_38)]. Assume that all variables at time  are known, and that the incremental strain tensor  is given as



where  is the time increment and  is the rate-of-deformation tensor at the mid-step . An elastic trial state is first calculated to check whether the time step from  to  is purely elastic or if plastic flow occurs, i.e.,



where  is the temperature increment from the previous time step. The time step is elastic if



and then



If , a semi-implicit return map is used to reestablish consistency







subject to the constraint



It is noted that  is the only unknown variable in the semi-implicit return map, and a secant method is used to solve the resulting nonlinear equation in . In the case of high strain rates leading to adiabatic heating, the temperature is updated explicitly at the end of the time step according to Equation as



If isothermal conditions prevail,  is set to zero and the temperature remains equal to the ambient temperature. Since the update scheme is semi-implicit, small time steps are required for stability and accuracy. In order to ensure a robust stress update, a sub-stepping scheme is adopted in the subsequent simulations.

# Parameter identification and model evaluation

This section firstly presents the different steps used to identify the parameters of the proposed constitutive model. The test results presented by [Vilamosa et al. [2]](#_ENREF_2) for the aluminium alloy AA6082 at nominal strain rates of 0.01, 0.33 and 750  and temperatures from 20 to 350 °C were employed for this purpose. The experimental database is herein augmented with some tests serving to explore the temperature dependence of Young’s modulus. The constitutive model is thereafter assessed by comparing it with the experimental results for the entire range of strain, strain rate and temperature covered in the test programme.

## Calibration of the model constants

The constitutive model is calibrated in three steps. Firstly, the temperature sensitivity of the Young’s modulus is determined. Secondly, the athermal stress  and the strain-rate and temperature dependent viscous stress  are fitted using the experimental yield stresses as a function of strain rate and temperature. Finally, the parameters governing the evolution of the work-hardening variable  (or equivalently the dislocation density) are identified employing the experimentally-obtained flow stress versus plastic strain curves up to large strains for all combinations of strain rate and temperature.

The temperature sensitivity of Young’s modulus was estimated from a series of tensile tests previously performed for an AA6060 alloy which has a similar thermo-mechanical response as the AA6082 alloy considered here (see [[2](#_ENREF_2)]). In these tests, the sample was loaded and unloaded ten times within the elastic domain of the alloy (i.e., the stress never reached the yield stress). Using a chamber limiting the maximum temperature to 430 °C, tests were conducted at seven different temperatures and one sample was tested per temperature. The strain was measured with a high-precision extensometer, while the applied force was measured by the load cell of the testing machine. Young’s modulus was determined as the average slope of the intermediate part of the resulting stress-strain curves. The test results together with the fit of ,  obtained with Equation are presented in Figure 2. The test results are presented as the average Young’s modulus at a given temperature while the error bars denote the range of the measured values from the ten loading/unloading cycles. The parameters governing the temperature sensitivity of Young’s modulus are presented in Table 2. It is assumed here that these results are also valid for the AA6082 alloy. Table 2 also compiles the other thermoelastic properties of the material.

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|  |
| Figure 2: Temperature sensitivity of Young’s modulus for an AA6060 alloy. The bars denote the range of the measured values. |

Table 2: Thermoelastic parameters applied for the AA6082 alloy

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| [MPa] |  | [K] |  | [] |
| 72 000 | 2.295 | 915 | 0.33 | 2.3⋅10-5 |

In the high-rate tests performed in the SHTB, the temperature of the test sample increases with straining due to adiabatic heating. It was not possible to measure this temperature increase due to the short duration of the tests. Instead, the temperature was estimated as a function of plastic strain for the high strain-rate tests, using Equation with ,  and . Isothermal conditions were assumed for the tests at low and intermediate strain rates.

To calibrate the model parameters, a representative experimental stress-strain curve was selected for each combination of temperature and nominal strain rate, see Figure 1. The parameters defining the athermal stress  and the viscous stress , see Equations and , were first estimated using the measured initial yield strengths. This temporary set of parameters was subsequently used as a starting point for including the work hardening  in the calibration procedure. To this end, each representative stress-strain curve was discretized into 100 equidistant points along the strain axis, and the experimental flow stress was expressed as



where  is a discrete value of equivalent plastic strain,  is the local equivalent strain rate and  is the local temperature. The local values of strain rate and temperature refer to the values at the minimum cross section of the tensile specimen. In the case of adiabatic heating, the temperature evolves with plastic straining and is estimated from Equation . The selected strain range was from onset of yielding until the maximum value of the locally measured strain , i.e., . The model parameters presented in Table 3 were optimised using [Matlab [39]](#_ENREF_39) and a least squares method involving the measured and predicted flow stresses. Constants taken from the literature and used in the Taylor equation are presented in Table 4.

Table 3: Thermo-viscoplastic parameters applied for the AA6082 alloy

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Yield stress and viscous stress | | | | | | | | | | | | |
| [MPa] | |  | | |  | |  | | [] | |  | |
| 20 | | 2.482⋅10-3 | | | 1 | | 1 | | 1.842⋅108 | | 1.047 | |
| Evolution of dislocation density | | | | | | | | | | | | |
|  | [m] | |  |  | |  | | [] |  | [] | |  |
| 0.0488 | 5.618⋅10-8 | | 4.621 | 3.256 | | 2.348⋅106 | | 1.995⋅105 | 0.3600 | 5000 | | 0.95 |

Table 4: Taylor equation parameters

|  |  |  |
| --- | --- | --- |
| Taylor equation | | |
|  |  | [m] |
| 1.0 | 3.07 | 2.858⋅10-10 |

## Evaluation of the calibrated model

The strain-rate and temperature dependence of the flow stress calculated with the calibrated constitutive model is compared with experimental data at plastic strains of 0.01, 0.05 and 0.6 in Figures 3, 4 and 5, respectively. The flow stress is plotted as function of strain rate for given nominal temperature in sub-figures a) and as a function of temperature for given nominal strain rate in sub-figures b). The calculated flow stresses at these three strain levels are then compared to all the experimental data used in the calibration procedure. In addition, experimental results from tests at nominal strain rate 350  are included in the figures. It appears that the constitutive model captures the complex strain-rate and temperature sensitivity with good overall accuracy.

|  |  |
| --- | --- |
|  |  |
| a) | b) |
| Figure 3: Comparison of flow stress at plastic strain equal to 0.01 from tests (discrete symbols) and constitutive model (isolines) as function of a) strain rate and b) temperature. The isolines in a) represent the model at prescribed temperatures while the isolines in b) are computed at the four levels of nominal strain rate. | |

|  |  |
| --- | --- |
|  |  |
| a) | b) |
| Figure 4: Comparison of flow stress at plastic strain equal to 0.05 from tests (discrete symbols) and constitutive model (isolines) as function of a) strain rate and b) temperature. The isolines in a) represent the model at prescribed temperatures while the isolines in b) are computed at the four levels of nominal strain rate. | |

|  |  |
| --- | --- |
|  |  |
| a) | b) |
| Figure 5: Comparison of flow stress at plastic strain equal to 0.6 from tests (discrete symbols) and constitutive model (lines) as function of a) strain rate and b) temperature. The isolines in a) represent the model at prescribed temperatures and the shaded area addresses the change of flow stress between isothermal and adiabatic conditions. The solid isolines in b) are computed at the four levels of nominal strain rate. The dashed lines represent the increase of strain rate within the neck. | |

At the low strain level in Figure 3, corresponding approximately to the yield stress, the constitutive model compares well with the test data for temperatures higher than 200 °C and slightly overestimates the flow stress at room temperature. At the moderate plastic strain level of 0.05 in Figure 4, the flow stress is in good agreement with the test data over the entire strain rate and temperature range under consideration. A significant increase of the local strain rate in the neck was observed at the largest strain level of 0.6 shown in Figure 5. This increase was earlier reported to be temperature insensitive [[2](#_ENREF_2)]. At this large plastic strain in the neck, the correction for increased temperature due to adiabatic heating calculated with Equation is represented by the areas under the isolines in Figure 5 a), ranging from adiabatic to isothermal conditions. The adiabatic heating has a significant influence on the flow stress and should be properly corrected for when interpreting the tests at large strains and high strain rates. It is evident from Figure 5 a) that the way the microstructural SRS is modified in the model is capable of capturing the sudden increase of flow stress at the highest strain rates. The solid lines in Figure 5 b) correspond to the flow stress variation with temperature at a plastic strain of 0.6 and for the prescribed nominal strain rates. The dashed lines represent the flow stress when the local increase of strain rate in the neck is taken into account. A slight increase in the flow stress is observed for low and medium nominal strain rates, while the influence is considerable at the highest strain rates. The differences between the calculated and measured flow stresses at this largest strain are mostly of the same magnitude as the experimental scatter.

An alternative way of evaluating the calibrated model is to use scatter plots between the calculated and experimentally obtained flow stresses at different strains, strain rates and temperatures. Figure 6 shows such scatter plots for low and high strain rates. It is clearly seen that the correlation is generally good, but apparently somewhat better at low strain rates and large strain levels.

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| --- | --- |
|  |  |
| a) | b) |
| Figure 6: Scatter plots of calculated versus experimentally determined flow stresses at different plastic strains and temperatures: a) nominal strain rate of 0.01 , b) nominal strain rate of 750 . | |

# Numerical simulations

The use of experimental data acquired in the post-necking regime of the tensile test and under adiabatic conditions at high strain rates introduces some additional assumptions in the evaluation of the test data. Firstly, the Bridgman correction is used to estimate the flow stress after necking, and, secondly, the temperature increase by adiabatic heating is not measured but estimated by assuming that a certain percentage of the plastic work is dissipated. Detailed simulations of all the tensile tests were therefore performed to check if these assumptions hold. The constitutive model was implemented as a user material model in LS-DYNA [[25](#_ENREF_25)] and then applied in numerical simulations of the tensile tests using the identified set of parameters. The simulations of the high strain-rate tests included the entire SHTB test setup, while only the sample was modelled for the tests performed at lower strain rates. Since the cast and homogenised AA6082 aluminium alloy material was found to be isotropic [[2](#_ENREF_2)], axisymmetric models were used to reduce the computational time. The constitutive model describes the material behaviour up to large strains, which calls for fine spatial discretization to obtain accurate predictions. A mesh sensitivity study was therefore performed before the final simulations were run and the results compared with the experimental data. All simulations were carried out with the explicit solver of LS-DYNA, and mass scaling was used for the low and intermediate strain rates. In the latter cases, it was checked that the kinetic energy remained a small fraction of the internal energy throughout the deformation process.

## Simulations of the quasi-static tensile tests

Simulations of the tensile tests at low and medium nominal strain rates were performed using 2D axisymmetric elements with selective reduced integration, and accounting for the symmetry in the longitudinal direction of the specimen. A coarse mesh (Mesh 1), an intermediate mesh (Mesh 2) and a fine mesh (Mesh 3) were considered. The different meshes are presented in Figure 7 for the initial state and after deformation at 350 °C and a nominal strain rate of 0.33 s-1 to a plastic strain of 1.75 in the neck. The reason for selecting a tensile test at elevated temperature for the mesh sensitivity study is that necking is more pronounced at hot conditions. The coarse, intermediate and fine meshes have in turn 6, 12 and 24 elements over the radius and 282, 924 and 2316 elements in total. The numbers given refer to the finely meshed part within the gauge area of the sample. The velocity was applied at the nodes of the upper edge of the sample, while the displacement component in the longitudinal direction of the bottom nodes was set to zero. The cross-head velocities applied on the upper edge were ramped smoothly up to 0.025 mm/s and 0.825 mm/s to reach nominal strain rates of respectively 0.01  and 0.33 . The cross-section diameter of each sample was measured before the experimental tests, and the geometry of all FE models were scaled so that the areas of the gauge section corresponded to the dimension of each test sample. A mass scaling factor of 106 was applied in these simulations.

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| --- | --- | --- |
|  |  |  |
| a) | b) | c) |
| Figure 7: Initial finite element meshes and corresponding deformed meshes at a logarithmic strain level of 1.75 at 350°C and 0.33 : a) coarse mesh (Mesh 1) , b) intermediate mesh (Mesh 2) and c) fine mesh (Mesh 3). | | |

The tensile stress-strain curves were computed from the simulations in a similar way as from the experimental tests. Assuming constant volume, the logarithmic strain was derived from the initial radius  and current radius  of the smallest cross section as



The Cauchy stress was obtained by dividing the force measured in the upper section by the smallest cross-section area



The radius of curvature of the neck  was used in the Bridgman correction of the Cauchy stress to determine the flow stress after necking, cf. Equation . It is interesting to compare the evolution of  as obtained from simulations and experiments, since this gives a mutual assessment of the experimental method and the simulation of the necking process. To this end,  was computed from the deformed mesh in the simulations in the same manner as from the digital pictures taken during the tests ([Vilamosa et al. [23]](#_ENREF_23)). In both cases  was determined using the least squares method and Chebyshev polynomials to represent the shape of the contour of the necked section.

The tensile stress-strain curve obtained experimentally for a tensile test at nominal strain rate 0.33  and temperature 350 °C is presented in Figure 8a) together with the corresponding simulations using the three meshes. The simulations are all seen to be in good agreement with the test result until a strain of about unity. The coarsest mesh (Mesh 1) gives a dramatic increase of the stress at larger strains and is therefore discarded. The same artificial stress increase is observed for the intermediate mesh (Mesh 2), but at a later stage.

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| a) | b) |
| Figure 8: FE simulations of an experimental test performed at nominal strain rate of 0.33  and temperature of 350°C: a) Cauchy stress versus logarithmic strain curves and b) Bridgman correction factor  versus logarithmic strain. Symbols are plotted based on every three frames in a). | |

In order to evaluate the prediction of the necking process, the Bridgman correction factor  as defined in Equation is used, since it depends on both the radius of curvature of the neck  and the radius of the smallest cross section . Figure 8b) shows the evolution of  with logarithmic strain for the experiment and the simulations with the three meshes. The two finer meshes give rather consistent results, slightly underestimating  at small strains and overestimating its value at large strains. The coarsest mesh does not provide reliable results at strains beyond unity. It should be noted here that  might be underestimated at an early stage in the tests because the coil that was used to heat the sample partly covered the gauge section ([Vilamosa et al. [23]](#_ENREF_23)).

Based on these results, it was decided to use the intermediate mesh in the subsequent simulations, as it gives similar results as the fine mesh up to large strains (about 1.5) and at the same time demands reasonable computation times. The resulting stress-strain curves are compared with the experimental results in Figure 9 for the different temperatures. In general, the simulations are capable of reproducing the global behaviour of the AA6082 aluminium alloy with rather good agreement over the entire range of strains, strain rates and temperatures. Figure 10a) presents scatter plots of the simulated versus the experimentally obtained Cauchy stress at different levels of plastic strain and temperature with strain rate equal to 0.01 . It is found that the correlation between simulations and tests is good and similar to the correlation seen in Figure 6a), thus validating the use of Bridgman’s analysis to correct the flow stress in the post-necking region for this material.

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| a) | b) |
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| c) | d) |
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| e) | |
| Figure 9: Cauchy stress versus logarithmic strain curves from test and simulation at nominal strain rate of 0.01  and 0.33  at a) 20°C, b) 200°C, c) 250°C, d) 300°C and e) 350°C. Symbols are plotted every six frames at plastic strain rate of 0.01  and every three frames at plastic strain rate of 0.33 . | |

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| a) | b) |
| Figure 10: Scatter plots of simulated versus experimentally determined Cauchy stress at different logarithmic strains and temperatures: a) nominal strain rate of 0.01 , b) nominal strain rate of 750 . | |

## SHTB simulations

As outlined in Section 2, a split-Hopkinson tension bar (SHTB) was applied to investigate the mechanical response of the AA6082 material at high strain rates and temperatures. A high-speed camera monitored the tests, facilitating determination of the flow stress up to large strains. The SHTB set-up is briefly repeated here, while more details about the test rig and the data processing are provided by [Vilamosa et al. [23]](#_ENREF_23). The SHTB consisted of an 8140 mm long input bar and a 7100 mm long transmission bar, see Figure 11 and the threaded test specimen was attached between these two bars. The input bar was clamped at a location 2060 mm from the sample and stretched at its free extremity. The clamp was then broken to release a stress wave which propagates through the input bar AC. At point C, a part of the wave was reflected back into the input bar and a part was transmitted into the sample CD and further into the transmission bar DE. The local strain in the neck was determined from digital pictures obtained with a high-speed camera. The force in the bars, and hence also the sample, was determined by the strain  measured in strain gauge station ➁ in the transmission bar, i.e.,



where  and  respectively represent the cross-section area and the Young’s modulus of the bar. Applying the photos from the high-speed camera, the logarithmic strain and the Cauchy stress were calculated with Equations and .

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| Figure 11: Sketch of SHTB apparatus with camera (measures in mm). |

An axisymmetric FE model of this set-up was used to assess the agreement between the simulations, using the calibrated constitutive model and the test results. The entire rig, including the two bars and the sample, was included in the FE model to allow calculating the force from Equation and the logarithmic strain from Equation in the same way as in the experimental tests. [Chen et al. [40]](#_ENREF_40) applied a 3D FE model of the same SHTB apparatus to investigate the behaviour of different anisotropic aluminium alloys at room temperature and showed that the strain wave propagation through the bars can be well represented by LS-DYNA [[25](#_ENREF_25)]. In the study presented herein, the bars contain a total of 16752 elements and were discretized using 6 elements over the radius. The interaction between the sample and the two bars could be modelled either by merging coincident nodes between the sample and the two bars [[40](#_ENREF_40)] or by discretizing the threads of both the sample and the bars and using contact between the threaded parts [[41](#_ENREF_41)]. An accurate modelling of the threads might improve the prediction of plastic strains outside the gauge part of the sample. On the other hand, representing the threads necessitates a very fine mesh which drastically increases the computational time. The first option, which gave good results in the study of [Chen et al. [40]](#_ENREF_40), was therefore selected in this study. As shown in Figure 12, the mesh of the bars was adapted to the mesh of the sample in a neighbouring region to the merged common nodes. The three different meshes of the sample depicted in Figure 7 were applied also in these simulations.

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| Figure 12: Finite element mesh of the specimen (mesh 2) and the two split-Hopkinson bars. |

The FE calculations were run using both the implicit and explicit solvers of LS-DYNA. The implicit solver was employed for the pre-stretching of the input bar. The clamping of the input bar was simulated by preventing the longitudinal displacements of the nodes located 2060 mm from the centre of the sample. Thereafter, the input bar was pre-stretched by applying a displacement at the free end so that the resultant tensile force was equal to the pre-stretching force in the experimental test. Finally, the solver was switched from the implicit to the explicit one, the clamp at B was removed, and the released stress wave propagated towards the specimen. As in the experimental test, the stress wave was partly reflected at C and partly transmitted further to bar DE.

To allow a better comparison with the experiments, in which fracture of the samples always occurred, the failure criterion proposed by [Cockcroft and Latham [42]](#_ENREF_42) was applied in the simulations



where  is the Cockcroft-Latham integral,  is the maximum principal stress, and  is the failure parameter. Using inverse modelling and the test performed at room temperature and nominal strain rate 750 , the failure parameter was estimated to = 250 MPa. This value was used in all simulations.

Also for the dynamic simulations, a mesh sensitivity study was performed at 350 °C. The tensile stress-strain curves from the simulations with the three meshes were in good agreement with the test results up to a logarithmic strain level of 0.75, see Figure 13 a). The stress-strain curve from the experimental test exhibited some minor oscillations due to noise in the unfiltered measurements. The coarse mesh diverged again from the expected behaviour at large strains and was discarded, while the intermediate and fine meshes gave consistent results. The measurement of the radius of curvature in the simulation was performed in the same way as for low strain rates. The Bridgman correction factor  is plotted as a function of the logarithmic strain for the test and the three simulations in Figure 13 b). The intermediate and fine meshes give comparable results, consistently overestimating the experimental value. However, the discrepancy is rather small within the whole strain range. The reason for this overestimation of  is that necking occurred earlier in the simulations than in the test. As the intermediate and fine meshes gave similar results, the intermediate mesh was used in the rest of the study.

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| a) | b) |
| Figure 13: FE simulations of a test performed at nominal strain rate of 750  and temperature of 350°C: a) Cauchy stress versus logarithmic strain curves and b) Bridgman correction factor  versus logarithmic strain. Symbols are plotted every three frames in a). | |

Simulations of the SHTB tests were performed at temperatures from 20 °C to 350 °C and nominal strain rate equal to 750 . Both the global response of the system and the local behaviour of the sample were evaluated, using respectively the signals of the strain gauges and the stress-strain curves. Figure 14 shows strain versus time at the positions of strain gauges ➀ and ➁, see Figure 11, obtained from both tests (strain gauge data) and FE simulations. It appears that the FE model with sample and bars represents the overall behaviour of the SHTB tests with good agreement for all temperatures. The simple failure model gave accurate results up to 300 °C, but the failure strain was underestimated for the highest temperature. Due to the stress wave propagation, the simulated strains in the bars exhibit oscillations of Pochhammer-Cree type. It was found by [Chen et al. [40]](#_ENREF_40) that these oscillations are not affected by the mesh size used to discretize the sample. The experimental and simulated local responses of the specimen, as represented by the computed Cauchy stress versus logarithmic strain curves in Figure 15, are in good agreement for all the temperatures considered. The stress-strain curves labelled FE were determined with Equations and , i.e., in the same way as in the experimental tests.

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| a) | b) |
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| c) | d) |
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| e) | |
| Figure 14: Strain gauge measurements in the bars from test and simulation at nominal strain rate 750  and temperature a) 20°C, b) 200°C, c) 250°C, d) 300°C and e) 350°C. | |
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| a) | b) |
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| e) | |
| Figure 15: Cauchy stress versus logarithmic strain curves from test and simulation at nominal strain rate 750  and temperature a) 20°C, b) 200°C, c) 250°C, d) 300°C and e) 350°C. Symbols are plotted every three frames. | |

To further display the correlation between the dynamic simulations and tests, scatter plots of simulated versus experimentally obtained Cauchy stress for different levels of strain and temperature with strain rate 750  are plotted in Figure 10 b). The correlation is good and similar to the correlation found in Figure 6 b), showing that Bridgman’s analysis is accurate also for the dynamic tensile tests for this material.

# Discussion

The constitutive model for the flow stress was formulated in a similar manner as the mechanical threshold stress (MTS) model, see [Follansbee and Kocks [11]](#_ENREF_11) and [Kocks and Mecking [35]](#_ENREF_35). The adopted work hardening formulation is based on the evolution law for the dislocation density proposed by [Bouaziz [36]](#_ENREF_36), which is shown here to describe the work hardening of the aluminium alloy to large levels of strain. The dynamic recovery was assumed to depend on strain rate and temperature, using a formulation proposed by [Bergström and Hallén [3]](#_ENREF_3), but modified to account for the marked increase of the work hardening observed at high strain rates, e.g. higher than 1000 s-1. In the present case, the suggested phenomenological extension of the dynamic recovery model gives good results for local strain rates up to 5000 . [Follansbee and Kocks [11]](#_ENREF_11) claim that this sudden increase of the flow stress is caused by a decrease of the average slip length of the dislocations. A consequence of that would be a significant increase of the initial work hardening, which they also reported in their study. However, no such increase of the initial work hardening with increasing strain rate was visible from the data by [Vilamosa et al. [2]](#_ENREF_2). The increase of dislocation density at high strain rates is therefore instead believed to be due to a decrease of the dynamic recovery. A decreased dynamic recovery at high strain rates would have to be caused either by more pinning points for the dislocations or by modified strength of the existing ones. One might speculate what kind of pinning points could be responsible for this. A possible explanation could be debris left behind due to dragging of jogs on screw dislocations, i.e., dipoles of height equal to one atomic distance that assumingly would have enough time to vanish by rearrangement of the lattice at low and medium strain rates. At high strain rates there might be little time for this, and they might accumulate and act as additional pinning points. However, a detailed study of this, and other potential mechanisms, is beyond the scope of this paper.

Experimentally obtained flow stress values at three distinct plastic strains were used to assess the agreement between the constitutive model and the test data, see Figure 3 to Figure 5. These strain levels were selected to obtain flow stress data in stage II, III and IV of the work hardening. The low plastic strain level of 0.01 corresponds to the beginning of stage II, which is dominated by athermal storage of dislocations. The transition to stage III, at which dynamic recovery becomes important, occurs about a plastic strain level of 0.05 at room temperature and earlier with increasing temperature. At the highest plastic strain level of 0.6, the work hardening is in stage IV for all tests considered, i.e., stress saturation occurs at higher temperatures.

The dislocation evolution equation proposed by [Bouaziz [36]](#_ENREF_36) was found to represent the experimentally observed behaviour of the AA6082 alloy with good agreement. In particular, the Bouaziz equation gives markedly better description of the work hardening at large strains than the Kocks-Mecking equation, since there is no saturation of the work hardening. In contrast to the Kocks-Mecking equation, the dislocation density evolution is always non-negative according to the Bouaziz equation; i.e., the rate of dynamic recovery can never balance the rate of storage of dislocations. Hence, the transient reduction of the work hardening that often occurs subsequently to an instantaneous increase of temperature or a jump down in strain rate cannot be captured by this formulation. In this study, the work hardening at large strains was deemed more important than such transient responses and the Bouaziz equation was therefore adopted.

# Conclusions

A physically-based constitutive model was suggested to reproduce the test results obtained by [Vilamosa et al. [2]](#_ENREF_2) for the AA6082 alloy, which exhibits a complex and highly coupled strain-rate and temperature sensitivity of the yield stress and work hardening. The yield strength was described with an athermal stress accounting for strengthening effects from particles and a viscous stress derived from an activation energy profile function described by [Kocks et al. [32]](#_ENREF_32). The work hardening was formulated with a dislocation evolution law originally proposed by [Bouaziz [36]](#_ENREF_36). The dynamic recovery of dislocations was developed within the Bergström framework, adopting a phenomenological formulation to represent high strain-rate strengthening. The model parameters were calibrated to the earlier reported quasi-static and dynamic tensile tests. The Bridgman analysis was used to account for the triaxial stress field after necking of the cylindrical tensile specimen, and the temperature increase due to adiabatic heating was estimated for the dynamic tests in the calibration process.

Applying the constitutive model, finite element simulations of the quasi-static and dynamic tests at different strain rates and temperatures were performed and compared with the experimental data. In the dynamic tests, the entire split-Hopkinson tension bar including the test specimen was modelled. The simulations gave in general consistent agreement with test results in terms of stress-strain curves and diffuse necking. The obtained results show also that the use of the local measurement technique is mandatory to investigate the work hardening at large strains and high temperatures.

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