Validation of ship manoeuvring models using metamodels

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Highlights

- Metamodels are applied for validation of ship manoeuvring simulation models.
- In-service recorded data is used to identify a metamodel.
- Cross-validation demonstrates high quality of the metamodels.

Abstract

This paper describes how simplified auxiliary models—metamodels—can be used to create benchmarks for validating ship manoeuvring simulation models. A metamodel represents ship performance for a limited range of parameters, such as rudder angles and surge velocity. In contrast to traditional system identification methods, metamodels are identified from multiple trial recordings, each containing data on the ship's inherent dynamics (similar for all trials) and random disturbances such as environmental effects and slightly different loading conditions. Thus, metamodels can be used to obtain these essential data, where simple averaging is not possible. In addition, metamodels are used to represent a ship's behaviour and not to obtain physical insights into ship dynamics. The experimental trials used for the identification of metamodels can be found in in-service recorded data. After the metamodel is identified, it is used to simulate trials without substantial deviations from the ship state parameters used for the identification. Subsequently, the predictions of the metamodels are compared with the predictions of a tested manoeuvring simulation model. We present two case studies to demonstrate the application of metamodels for moderate turning motions of two ships.

Keywords: manoeuvring models; validation; metamodels; system identification; in-service data.

1. Introduction

Validating ship manoeuvring models used for training pilots and in engineering applications is important [1,2]. Despite the increasing use of simulation models, no standards or guidelines describing an objective validation method of system-based manoeuvring models are currently available. In the majority of relevant literature, manoeuvring models are typically validated for application in standard International Maritime Organization (IMO) trials [3], including turning circles and zigzag trials, for example, [4]. These trials are executed at nearly full speed and are intended to assess the emergency turning and course checking abilities of a ship. However, for other applications, such as training pilots to steer a ship in port areas, trials such as low-speed turning with the assistance of tunnel thrusters, would be more realistic. Thus, as was the conclusion of a review of the guidelines for validating aviation simulators [5], manoeuvring models should be validated for a wider range of ship-specific applications.

High-quality benchmark data are necessary for validation. Such data can be obtained through dedicated tests, such as [6]. However, such tests are cost prohibitive. An alternative approach is to use in-service recorded data and to validate a simulator against the motions recorded during real ship operation. However, in-service data are affected by numerous sources of uncertainty, such as environmental effects and difference in loading conditions. Moreover, limited data may be available; for example, for validating an emergency turning model, it is unlikely that such a record would be found in in-service recorded data. Therefore, validation against in-service data should be considered a complimentary approach to validation against the results of dedicated trials.

Data from in-service measurements tend to have larger uncertainty than do data from dedicated trials. Nevertheless, because of the long-term nature of data from in-service measurements (e.g., data from a year of operation), the uncertainty in the validation dataset may be reduced by utilizing the ‘repeated tests’ effect of the in-service data. In this paper, we use auxiliary simplified mathematical models (metamodels) for validation. Metamodels are widely used in many domains, such as systems analysis and software engineering, and typically represent the model of a model or the model of a particular phenomenon, keeping only critical features. Metamodels are identified from similar trials of a real ship and are then used as a benchmark for simulations. Thus, the identified metamodel is a manoeuvring model valid for a particular manoeuver. This approach minimizes uncertainty and can be considered as ‘smart averaging’ for that manoeuvre. The rest of this paper is organized as follows. In Section 2, we briefly describe the concept of a metamodel and its application. In Section 3, we present two case studies using the in-service data of coastal ferry Landegode and the zigzag trials of Research Vessel Gunnerus. Finally, the conclusions are presented in Section 4.

2. Metamodels and validation

System identification is widely applied in ship manoeuvring to obtain coefficients in simulation models. In [7], Abkowitz applied the extended Kalman filter to identify model coefficients and changing current from the tests of vessel Esso Osaka. Abkowitz noted that the identified coefficients could be used for validating an original mathematical model, but he did not report such a validation nor did he present instructions to perform such a validation.

The approach presented in this paper relies on system identification. Figure 1 delineates the approach in a stepwise manner. Steps 3 and 4 can be skipped for simple metamodels, but it may be essential for complex models. In Step 5, known environmental effects, such as mean drift due to current and waves identified for each individual trial, can be accounted for. The key distinctions of metamodel identification from traditional manoeuvring model identification are as follows:

- The structure of a metamodel can differ from the structure of the model that is validated. Different metamodel structures can be used in trials with different objectives.
- Several similar trials are used to identify a metamodel. Thus, the influence of random components due to environmental conditions and other disturbances is minimized by averaging.

Thus, a metamodel represents the averaged response of a ship to certain control inputs.

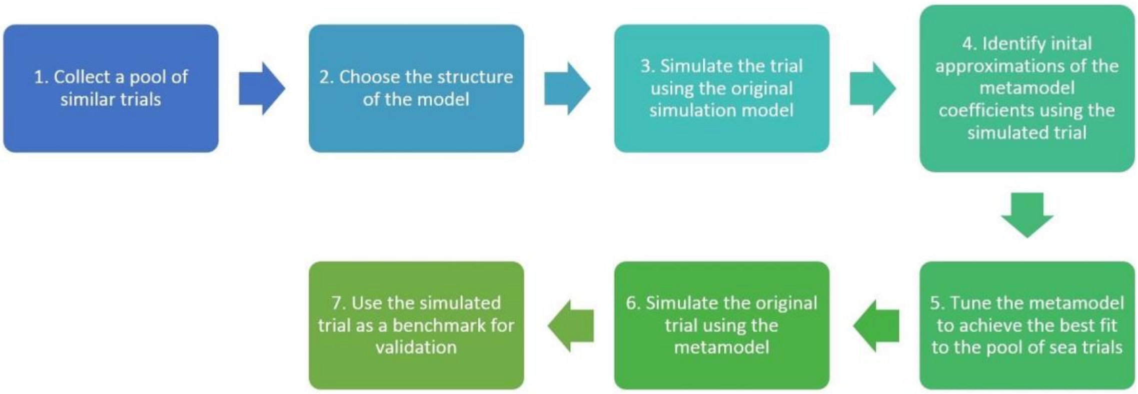

Figure 1. Validation process using metamodels.

3. Case study

3.1 Manoeuvring models

Two case studies are presented. In the first case study, metamodels are used to generate benchmark trials on the basis of in-service recorded data, and in the second, multiple repetitions of zigzag trials are used. In each case, after the metamodel is identified, we demonstrate the validation of the original manoeuvring models by using VeSim simulation models. VeSim, developed by the Norwegian Marine Technology Research Institute (MARINTEK), is an advanced six-degrees-of-freedom simulator based on unified manoeuvring and seakeeping theory [8]. A modular approach is implemented in the simulator. Because the validation of these particular models is not the primary objective of this study, the models are only briefly described in this paper.

3.2. Case study 1: ferry Landegode

In this case study, we apply metamodels to create a validation benchmark by using in-service recorded data from ferry Landegode (Figure 2). The ferry operates on routes near Bodø in Northern Norway and is one of the case vessels used in the research project ‘SimVal—Sea Trials and Model Tests for Validation of Shiphandling Simulation Models’ [9].

Figure 2. Ferry Landegode.

Table 1 shows main dimensions of the ferry. The ferry is equipped with a single-screw single-rudder propulsion system with a controllable pitch propeller and three tunnel thrusters, two in the bow and one in the stern.

Table 1. Main dimensions of ferry Landegode.

Length overall [m]	96.0
Breadth midship [m]	16.8
Draught (max) [m]	4.2

3.2.1. Searching manoeuvres in recorded data

During operation, all main parameters, such as positions, orientation, velocities, propulsion parameters, wind direction, and velocity, are recorded and stored as 1-h-long time series with short 30-s-long intervals in between. Most recorded data pertain to nearly straight motion and are not relevant for manoeuvring tests. Therefore, the first task is to identify sections of the time series representing turning motion, as follows:

Step 1. Data cleaning and preparation: All data is low-pass filtered and resampled using spline interpolation to the same sampling frequency. A simple data check is performed to exclude faulty data (e.g., data where one of the channels important for further processing shows constant zero or an unrealistic value).

Step 2. Searching for turning motion: By using the moving average (MA) algorithm with a window size of 300 samples (30 s) and a step size of 10 samples (1 s), by finding where the turning rate exceeds a threshold of 0.2°/s. All samples satisfying this condition are marked as ‘turning’. Thus, each identified section is at least 30 s long.

Step 3. Merging of the sections: Merge sections if the time interval between them is less than 90 s; this is crucial for detecting zigzag-like or course-changing motion. The resulting time series, hereafter referred to as ‘trials’, are saved as separate files. This step thus yields a relatively long time series with turning motion.

Step 4. Classification. The data used in this study have the following characteristic: the ship operates on a fixed route and follows a similar path on every trip. The position and orientation are maintained by the ship autopilot. Therefore, many of the turning trials detected in the data are nearly identical, with slightly different control inputs counteracting the changing environmental disturbances. We use the k-means algorithm to find similar trials. As a feature vector characterizing each trial, we use the positions of the beginning and the end of each trial (four features per trial). Prior to the application of the k-means algorithm, the mean values of the features are removed and the features are normalized so that the scale is similar for both latitude and longitude.

In this study, we analyse three groups of trials, from the same geographical area, representing moderate zigzag-like trials (Figure 3). The trials in Group 1 have a southeasterly direction, whereas those in Groups 2 and 3 have a northwesterly direction. Figure 4 presents the time series of surge and sway velocities, yaw rate, and rudder angles for all trials in Group 1. The velocities are similar for all repetitions, while the rudder angle has a more substantial scatter, probably because the ship autopilot is attempting to force the ship to follow a predefined route under varying environmental conditions. Thus, the direct application of these trials for validating the manoeuvring model of the vessel is difficult: each simulation with a rudder time series as an input would yield velocity time series that differ substantially from each other. Subsequently, these output time series must be compared with multiple experimental time series, which is not practicable. By contrast, by using the metamodel approach, a single relationship that represents the entire dataset can be built between the input and the output.

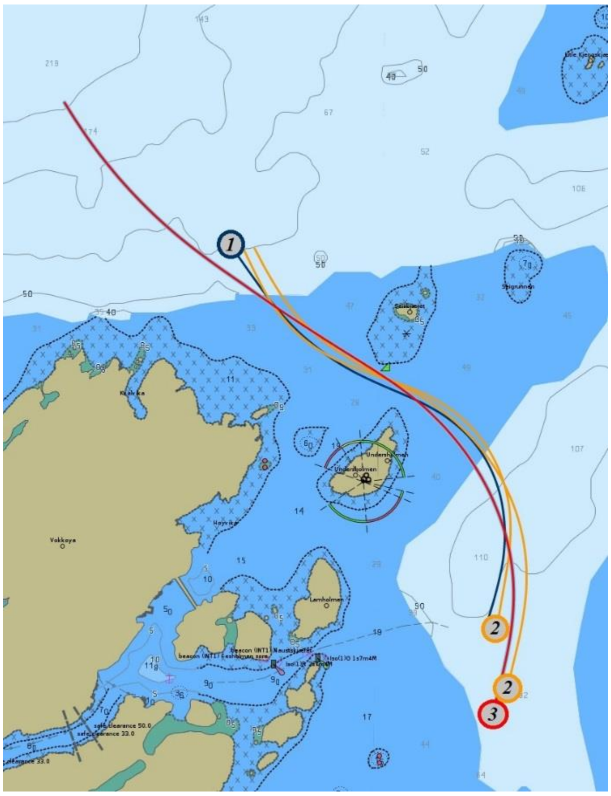

Figure 3. Illustrative tracks from each trial group used for identifying the metamodel of ferry Landegode.

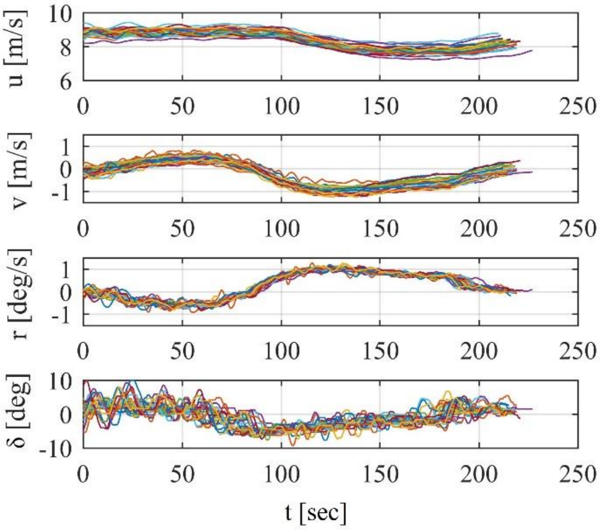

Figure 4. Surge, sway, yaw velocities, and rudder angles for trials from Group 1.

3.2.2. Metamodel identification

To build an adequate metamodel for a particular type of motion, its structure must first be chosen. Because we analyse zigzag-like trials with low rudder deflection, it is reasonable to use the nonlinear heading model proposed by Norrbin [10]:

$$\dot{r} = -a_0 - a_1 r - a_2 r^2 - a_3 r^3 + a_4 \delta \tag{1}$$

The asymmetry due to propeller rotation direction is accounted for by the coefficients a_0 and a_2. Sway velocity is not explicitly included in (1), which allows neglecting current effects under the assumption of approximately constant irrotational currents. However, wind and waves can strongly affect the yaw dynamics of the vessel. Because relative wind direction and velocity are measured by the ship's on-board equipment, these data can be used to reduce metamodel uncertainty. Moreover, excluding trials with strong wind reduces the possible effects of waves on ship dynamics. Thus, 12 trials with the minimum possible side wind are chosen from the available trials for identification. Figure 5 presents the time series of rate of turn and rudder angle for these trials.

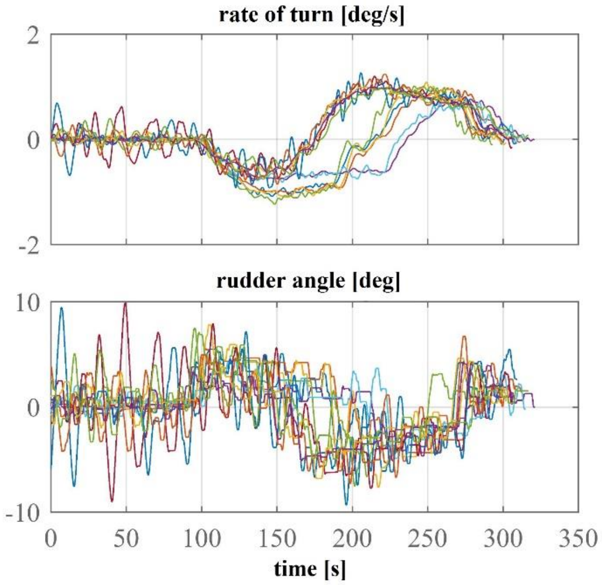

Figure 5. Trials used for metamodel identification.

To identify the metamodel, we use the nonlinear least-squares solver implemented in MatLab System Identification Toolbox (function 'nlgreyest' [11]). This function can be used to estimate the coefficients of a nonlinear grey-box model. The identified coefficient values are presented in Table 2. The positive nature of the coefficient $\boldsymbol{a_1}$ indicates that the ship is dynamically stable, which is consistent with the available information.

Table 2. Identified coefficients of the metamodel (rudder angle measured in [deg] and yaw rate measured in [deg/s]).

a_0	−0.0015
a_1	0.0201
a_2	0.0040
a_3	0.0107
a_4	−0.0090

The metamodel is cross-validated to ensure that the metamodel is identified correctly and that it accurately represents the turning motion of the ship. The metamodel is used to simulate one additional trial not used for the identification. A trial in which side wind is almost absent and rudder oscillations during approach and manoeuvre are minimal is chosen. Figure 6 presents the predictions based on the metamodel and the corresponding actual time series of rate of turn for the case trial. The following metric is used for comparison, where y is identification data and $\hat{y}$ is the output of the metamodel:

$$fit = \left(1 - \frac{\|y - \hat{y}\|}{\|y - mean(y)\|}\right) \cdot 100\% \tag{2}$$

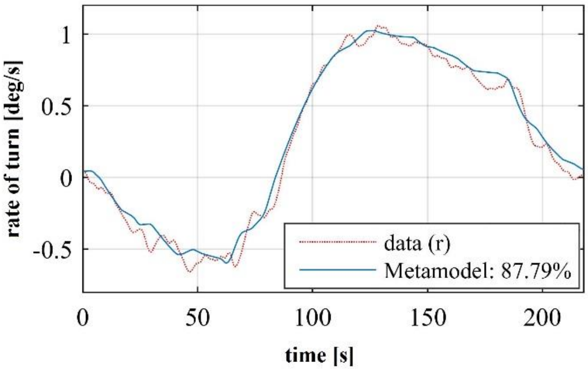

Figure 6. Metamodel cross-validation.

Thus, the metamodel accurately represents the turning dynamics of the vessel for small rudder deviations and a particular surge velocity. Note that there are several unknown sources of uncertainty in addition to wind, such as loading conditions, waves and currents, all of which can affect the result. Thus, cross-validation may show a poor fit even if the metamodel fits well. In such cases, additional trials should be used to prove the adequacy of the metamodel.

3.2.3. Validation of ship simulation model

The metamodel approach allows creating a benchmark test with reduced uncertainty relative to raw trials taken from in-service records. The next step in the analysis is to compare predictions obtained using the original ship simulation model and the metamodel for the same rudder input time series. In this case study, we use two VeSim models of ferry Landegode. Model 1 uses hull hydrodynamic coefficients identified from planar motion mechanism tests; it is therefore expected to be a high-quality model. Model 2 uses hull hydrodynamic coefficients calculated using MARINTEK's strip theory–based code with empirical corrections, called HullVisc. Ferry Landegode does not belong to the main target group of ships for HullVisc; therefore, the second model is not expected to be of high quality. The propulsion and steering module for the simulation model of the ferry was developed by Rolls-Royce Marine for the real ship. We cannot provide additional details about the models because of the proprietary and confidential nature of these models. Nevertheless, because the models are used only in the case study, this limitation does not critically affect the main objectives of this study.

We use rudder time series from the trial used for the cross-validation of the metamodel as the control input. Each simulation includes an approach segment, when the ship moves with zero rate of turn and a surge velocity of 16.5 knots (i.e., the average values for trials found in the in-service recorded data). Figure 7 presents the rate of turn as predicted by the two VeSim models and the metamodel. The fit is calculated using (2), where y and $\hat{y}$ are the results of the metamodel and VeSim simulations, respectively. For most of the simulation, the first model was close to the metamodel, but some deviation is observed in the negative rate of turn at the beginning. The first model had a relatively good fit of 67%, whereas the performance of the second model was much poorer, with a fit of only 35%. The manoeuvring simulation models should be further validated using predefined acceptance criteria or maximal allowed deviations based on the intended use. For example, for some applications, such as pilot training, the quality of Model 1 may be sufficient, but for applications demanding high precision, it may not. Note that the conclusion on the validity of the model is applicable only for a specific range of operation (turning and counterturning with moderate rudder angles of up to 5° and a moderate engine power of 30% in this case).

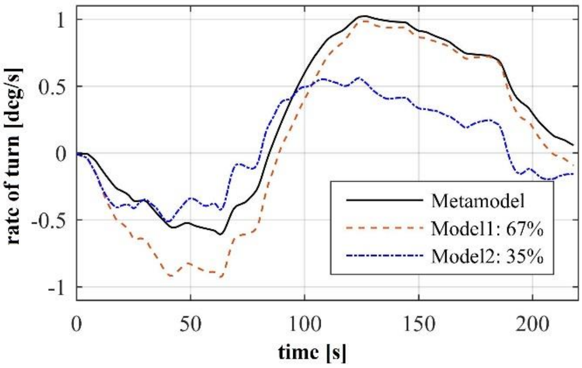

Figure 7. Validation of the simulation models using the metamodel.

3.3. Case study 2: Research Vessel Gunnerus

In this case study, we use metamodels to obtain the idealized time series of manoeuvres of Research Vessel Gunnerus (Figure 8). Gunnerus is owned by the Norwegian University of Science and Technology and is one of the case vessels used in the SimVal project. Table 3 lists the main specifications of the vessel. We use 10°/10° zigzag tests as the case trials, which is useful because for standard trials, simple characteristics, such as overshoot angles, can be defined. Thus, because data from numerous repeated 10°/10° zigzag tests are available for this ship, predictions obtained using the metamodel and the average experimentally obtained overshoot angles can be compared after accounting for experimental uncertainties.

Figure 8. Research Vessel Gunnerus.

Table 3. Main dimensions of Research Vessel Gunnerus.

Length overall [m]	31.25
Breadth midship [m]	9.60
Draught (max) [m]	2.6

Extensive uncertainty analysis of the zigzag trials is reported in [12], according to which the most influential source of uncertainty of the overshoot angles is rudder input. The ship's reaction time for changing the rudders is quick (time between the first and second rudder execute is approximately 8 seconds and the turning rate during the second rudder execute is more than 2°/s). Thus, during the trials, it was practically difficult to adjust the automatic manoeuvring control[1] or to execute the trials manually

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[1] The 10°/10° zigzag manoeuvr was programmed into the Dynamic Positioning System of the ship for automatic operation in order to minimize the delay and randomness involved in human (manual) operation.

in order to achieve the desired behaviour of the ship. In fact, none of the trials were sufficiently close to 10°/10° zigzag motion. Figure 9 shows the deviations of the heading angles from the desired value of 10° when the rudder was changed to the opposite side; both trials with starboard and port side first execute are included. Positive values indicate a delay and negative values indicate an advance; for example, at plot point (2°, 2°), in both trials, the rudder was switched when the heading change from the initial value was 12°.

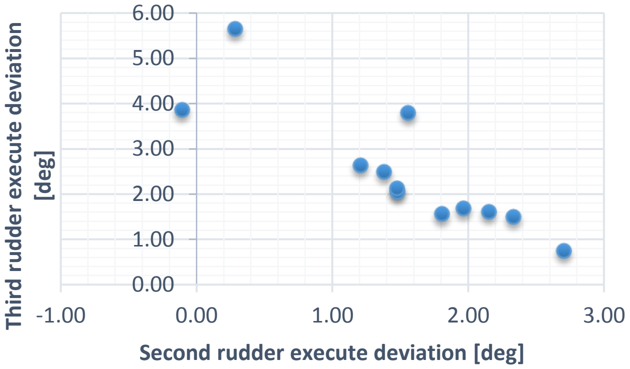

Figure 9. Deviation of the heading angle from 10° during second and third rudder execute.

Additionally, the results are affected by environmental effects, rudder angle deviations from the commanded values (~1°–2°), initial value of heading and yaw rate, and other potential sources of uncertainty. Thus, as characteristics of the trials, we use the actual overshoot angles (the maximum heading deviation after the rudders were switched to the opposite side) rather than the overshoot over 10° heading change; 95% confidence interval is estimated according to the following equation [13]:

$$\mathrm{P}\left(\bar{X} - t_{95}\frac{s_X}{\sqrt{N}} \le \mu \le \bar{X} + t_{95}\frac{s_X}{\sqrt{N}}\right) = 0.95 \tag{3}$$

where

$$s_X = \left[\frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2\right]^{1/2} \tag{4}$$

and t_{95} is a factor obtained from Student's t-distribution for number of degrees of freedom equal to $N-1$; N is the number of trials and μ is the parent population mean. Table 4 presents the results of the zigzag trials with uncertainty.

Table 4. Results of the zigzag trials. U_{95}: half-width of the confidence interval.

1st execute	Starboard		Port	
Overshoot angle	1st	2nd	1st	2nd
$\bar{X}$	7.45	5.92	5.74	7.36
$s_X / \sqrt{N}$	0.48	0.14	0.12	0.14
N	7		5	
t_{95}	2.447		2.776	
U_{95}	1.17	0.34	0.34	0.38

We divide all available zigzag trials into two groups. The first group contains six tests (four with the first execute to starboard side and two with the first execute to port side) performed on the first day of the trials. In these trials, the automatic manoeuvring control was adjusted to better compensate for the delay introduced by the processing time of the automatic system. One of the tests was executed manually. Typical wind speed on this day was 7–11 m/s. Therefore, the trials have relatively low quality and repeatability. The second group contains six tests (three each with first execute to starboard and port sides) performed on the second day of the trials. All trials were executed with the tuned control system, and the weather was calmer on this day, with a typical wind speed of 4–6 m/s. Therefore, the trials from this group have higher quality and repeatability. Figure 10 and Figure 11 show the yaw rate, heading, and rudder time series for the trials from Groups 1 and 2, respectively.

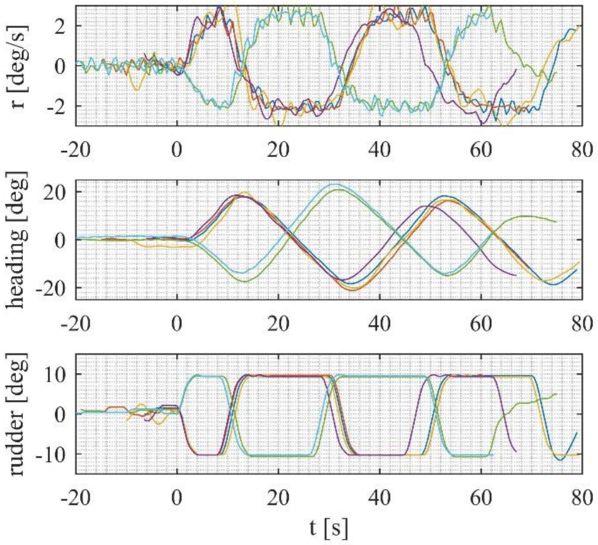

Figure 10. Low-quality zigzag trials (Group 1).

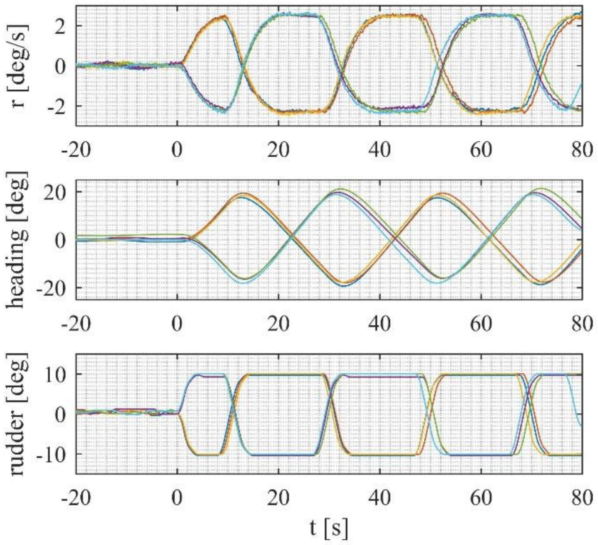

Figure 11. High-quality zigzag trials (Group 2).

We use the same metamodel structure used in the first case study—Norrbin's one-degree-of-freedom model (1). Although the ship has a twin-propeller twin-rudder system, its behaviour is asymmetrical. Therefore, the general form of the model (1) is used, where the coefficients a_0 and a_2 are nonzero. The asymmetry manifests itself as the need for providing a slight port rudder to maintain the straight course of the ship; this is likely due to the poor alignment of the rudders.

The metamodel is identified using the nonlinear least-squares solver implemented in MatLab's System Identification Toolbox [11]. Table 5 lists the identified coefficients. The rudder angle needed to achieve zero rate of turn (rudder bias) can be determined using the metamodels and was found to be 0.46° (almost the same for the first and the second metamodels). The coefficients in Table 5 are similar in magnitude, and it is difficult to conclude how important the differences are. Thus, to compare the metamodels with each other and with the experimental data, we use one of the trials from Group 2 as the benchmark trial. Figure 12 presents a comparison of yaw rate predicted by the metamodels and that measured in the benchmark experiment; the same rudder input time series is used in all three cases. The performance of the metamodels is nearly identical. Finally, we simulate an 'ideal' 10°/12° zigzag trial with the first execute to port side. The resulting values of the overshoot angles (Table 6) are within the 95% confidence intervals of the mean values of overshoot angles presented in Table 4.

Table 5. Identified coefficients of metamodels for trials from Groups 1 and 2 (rudder angle measured in [deg] and yaw rate measured in [deg/s]).

	Group 1	**Group 2**
a_0	−0.0469	−0.0480
a_1	0.4328	0.4382
a_2	−0.0009	0.0017
a_3	−0.0005	−0.0010
a_4	−0.1015	−0.1024

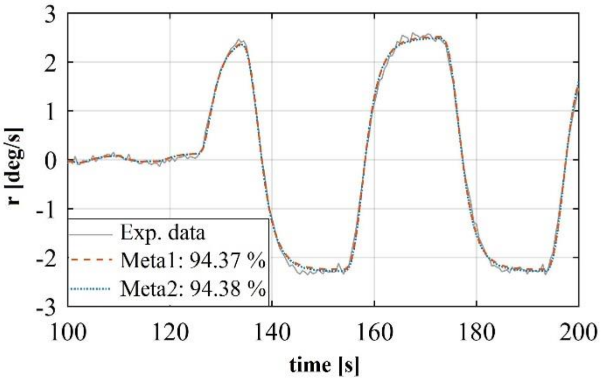

Figure 12. Comparison of the metamodels and the benchmark test.

Table 6. Results of the simulations of 10°/12° zigzag trial to port side (Model 1).

Overshoot angle [deg]	
1st	2nd
5.98	7.15

Hence, the identified metamodels accurately represent the course-checking behaviour of the ship and can be seen as an equivalent to averaging for this case. Moreover, even the metamodel identified from a few trials with relatively large uncertainty both in the input and output (Group 1) performed well. The identified metamodel can be used to simulate an idealized 10°/10° zigzag test and to obtain appropriate first and the second overshoot angles. This case study thus demonstrates another application of a metamodel for correcting standard IMO trials.

All the analysed zigzag tests were executed at approach with 50% engine power, corresponding to an approach speed of 10 knots. Additional low-speed zigzag tests were performed with approach at 20%

engine power, corresponding to an approach speed of 7.5 knots. Because multiple uncertainties affect the trials, including differences in control inputs, explaining the variation in ship performance by directly comparing the test results is a complex task, and comparing results simulated by the metamodels identified for 50% power approach and 20% power approach for the same rudder input avoids these problems. First, the 10°/10° zigzag manoeuvre at an approach speed of 10 knots is simulated using the corresponding metamodel. Second, the same rudder time series is used as input for the low-speed metamodel. The resulting time series are depicted in Figure 13. The rudder performance is clearly more efficient at high speeds (turning rate reduces by approximately 25% for 20% engine power compared with 50% engine power), which means that more time is needed to change the course at low speeds.

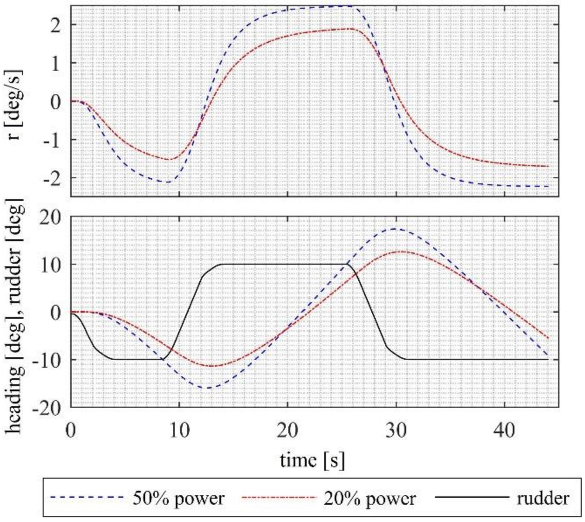

Figure 13. Yaw checking ability at 50% and 20% engine power (10 and 7.5 knots, respectively).

Finally, we validate the VeSim manoeuvring model of Gunnerus. The same rudder input time series is used in all simulations (i.e., rudder angles needed to perform 10°/10° zigzag manoeuvre using the metamodel for a 10 knots approach). However, the rudder bias is removed for simulations in VeSim, and the VeSim simulations are adjusted such that the approach speed is 10 and 7.5 knots. Figure 14 depicts the rate-of-turn time series predicted using the VeSim model and the metamodels. The VeSim model shows approximately the same reduction in the turning rate at low speed (30%), but it substantially underestimates the ship's turning performance. The validation metric, calculated using (2), is 62.3% and 64.4% at 7.5 and 10 knots, respectively. As with the previous case study, the validity of the model depends on its intended use.

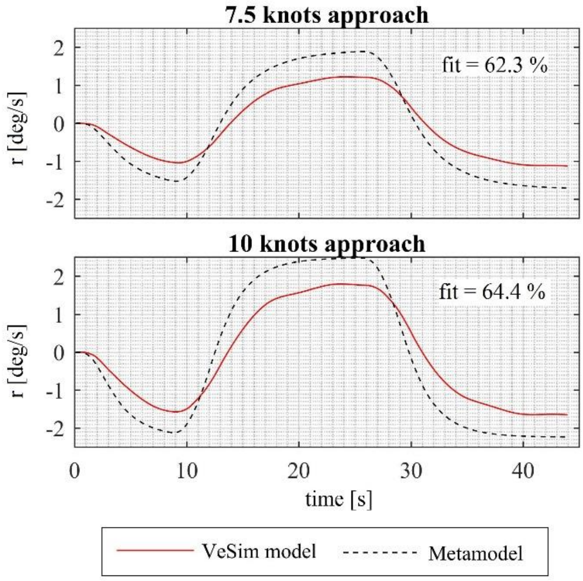

Figure 14. Validation of the VeSim manoeuvring model of Gunnerus: time series simulated using the metamodels.

4. Conclusions

We introduced the concept of applying metamodels to ship the validation of models of ship manoeuvring. Metamodels can in this context be viewed as an advanced method of averaging or filtering the undesired effects caused by various disturbances, such as environmental effects, differences in control inputs, and slight variations in loading conditions, from experimental time series. We demonstrated the main application of metamodels in the generation of validation benchmarks from in-service recorded data, where typically only wind measurements are recorded and the magnitudes of other uncertainties are unknown. The identified metamodels also has other applications, such as correction of standard IMO trials. The optimal performance of the metamodels is expected for cases with moderate random disturbances and control-input uncertainties (where simple averaging cannot be applied). Cases with strong bias factors can be excluded using objective reasoning (as we did in the first case study for strong lateral winds). Alternatively, a metamodel can be identified using all available trials, following which the trials with the worst fit can be excluded from the identification dataset; subsequently, the metamodel can be reidentified. Furthermore, both approaches can be combined. Finally, more advanced identification techniques allowing estimation bias factors, such as the identification of currents [7], may be used. Here, we demonstrated the approach for moderate turning motions of two ships by using a simple one-degree-of-freedom model. In the future, we plan to consider more complicated cases, such as three-degrees-of-freedom motion at low speed using tunnel thrusters, which is routinely used for manoeuvring in harbours.

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Conflicts of Interest

None

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