Journal of Energy and Power Engineering 6 (2012) 570-579

Covariance Estimation Using High-Frequency Data: An Analysis of Nord Pool Electricity Forward Data

Gudbrand Lien[1], Erik Haugom[1, 2], Sjur Westgaard[2] and Per Bjarte Solibakke[3]

1. Faculty of Economics and Organization Science, Lillehammer University College, Lillehammer NO-2624, Norway

2. Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Trondheim NO-7491, Norway

3. Department of Business Administration and Economics, Molde University College, Molde NO-6402, Norway

Received: March 18, 2011 / Accepted: August 05, 2011 / Published: April 30, 2012.

Abstract: The modeling of volatility and correlation is important in order to calculate hedge ratios, value at risk estimates, CAPM (Capital Asset Pricing Model betas), derivate pricing and risk management in general. Recent access to intra-daily high-frequency data for two of the most liquid contracts at the Nord Pool exchange has made it possible to apply new and promising methods for analyzing volatility and correlation. The concepts of realized volatility and realized correlation are applied, and this study statistically describes the distribution (both distributional properties and temporal dependencies) of electricity forward data from 2005 to 2009. The main findings show that the logarithmic realized volatility is approximately normally distributed, while realized correlation seems not to be. Further, realized volatility and realized correlation have a long-memory feature. There also seems to be a high correlation between realized correlation and volatilities and positive relations between trading volume and realized volatility and between trading volume and realized correlation. These results are to a large extent consistent with earlier studies of stylized facts of other financial and commodity markets.

Key words: Realized volatility and correlation, high-frequency data, distribution properties, temporal dependence, Nord Pool forward data.

1. Introduction

Balance risk and expected returns within a portfolio approach constitute some of the key concepts in modern finance. Accordingly, the estimation and forecasting of both volatility and correlation is arguably among the most important pursuits in empirical asset pricing, asset allocation and risk management. Examples of the crucial role that volatility and correlation estimates and applications play in finance include the calculation of hedge ratios, the calculation of portfolio value at risk estimates, the calculation of CAPM (Capital Asset Pricing Model) betas, option and derivate pricing, and volatility transmission between assets and markets.

Corresponding author: Gudbrand Lien, professor, research fields: energy and agricultural economics. E-mail: gudbrand.lien@hil.no.

Many classical models in financial economics assume constant volatilities and correlations, even if the dynamic properties of volatilities and correlations have been widely accepted. The research on dynamic properties of volatilities and correlation has typically been based on the estimation of parametric univariate and multivariate ARCH (autoregressive conditional heteroscedasticity) (and its generalizations) and stochastic volatility models. One drawback with these models (and parametric models in general) is that they depend on specific distributional assumptions, which reduce the robustness of the empirical findings [1].

The recognition of the limitations of the traditional volatility and correlation models has led to a different approach. The development of computer technology over the past decades and the increased availability of high-frequency financial data have opened up a new field of research within this framework [2]. The idea has been used to historically relevant and reliable high-frequency data in order to improve the modeling and forecasting of outcome variability and correlation. The availability of high-frequency data has made it possible to construct i.e. "Afterward", "after the event", post realized daily volatilities and correlations, through the summation of squares and cross-products of intraday observations, respectively. This approach has the advantage of allowing for the characterization of the distributional features of the volatilities and correlation without an attempt to fit multivariate conditional and stochastic volatility models.

Andersen and Bollerslev's [3] seminal paper shows that realized volatility computed from high-frequency intraday returns is effectively a model-free volatility measure. On the basis of mainly the same ideas and procedures as for univariate realized volatility, both Andersen, [4] and Barndorff-Nielsen [5] have spelled out the concept of realized covariances and correlations. With this framework, volatility and correlation can simply be treated as observables (and not latent or modeled), which has opened up several opportunities. For example, conventional statistical techniques can be used to characterize i.e. "Afterward", "after the event", post realized volatilities and correlations measures' distributional properties. Further, these observed measures can be modeled and dynamically forecasted with simple standard regression techniques.

Some studies have also given some indication that nonparametric models based on high-frequency data provide superior out-of-sample forecasts of volatility compared to parametric volatility models based on daily data (e.g. Refs. [6-8]).

Until about the late 1980s only sparse data were available, and the research consisted of formulating adequate models and verifying that the models reproduced the data patterns and gave reasonable predictions for the future, with an emphasis on the methodology. In a situation with sparse data, it was important to gather as much as possible information from what little there was, and so it was natural for the researcher to take great pains to make sure that the methodology was correct. Of course, it is equally important for studies to employ the correct methodology and to implement certain analytical improvements. However, owing to the availability of high-frequency data, the research community has begun to focus on in-depth, model-free (or models with less strict assumptions) analyses in order to discover the fundamental statistical properties of the data. These studies aim to document the statistical characteristics of the financial returns or the "stylized facts", as it is often named in the econometric and finance literature, where the understanding of the market is of interest in itself and can give useful information for the specification and estimation of predictive models. Notable examples of studies within the financial field of stylized facts are those of individual stocks and stock indices [9-12], bonds [11], currencies [2, 4, 11, 12] and agricultural commodities [12]. Some of these studies, in addition to variance and volatility analyses, also focus on the stylized facts of realized covariances and correlations between assets investigated [4, 9, 11]. To our knowledge, Ref. [13] has been the first to examine the distributions of realized energy-futures volatilities and correlation in their study of the NYMEX light crude oil and natural-gas futures contracts.

Ulrich [14] and Chan [15] have analyzed high-frequency US and Australian electricity spot-market data. Haugom, et al. [16] have studied univariate realized volatility on the basis of high-frequency electricity-futures data from the Nord Pool market. However, to our knowledge, nobody has analyzed realized covariance based on high-frequency financial electricity data.

The main findings obtained in these earlier studies mentioned above show that the logarithmic realized volatility and correlation are approximately normally distributed. Further, realized volatility and correlation have a long-memory feature, which can be modeled by fractionally integrated processes, and there seems to be a high correlation between realized correlation and volatilities.

Recently, high-frequency, tick-by-tick data have also become available for the two most liquid electricity forward contracts (year and quarter contracts) at the central Nord Pool data source. This is a quickly growing derivative market, and more knowledge and tools for risk management and trading applications are needed. This study builds on the build framework employed in Refs. [4, 9] on stylized volatility and correlation facts, and high-frequency data is applied in the electricity market to analyze the stylized volatility and correlation facts of these electricity forward contracts. As far as it is aware, this is the first study to undertake this endeavor. As electricity is distinct in several important ways from other commodities (e.g., non-storability, uncertainty in load and generation, inelastic demand, oligopolistic generation), it is interesting to compare whether the use of high-frequency data reveals that the behavior of the financial electricity market differs significantly from traditional financial markets. The data covers the period of June 2005 to May 2009.

The next section includes a brief review of the concepts of realized volatility and correlation, followed by a description of the data sets. The main results of the analysis are presented in Section 4. The last section offers some implications for future research and concluding comments.

2. The Concepts of Realized Volatility and Correlation

The main focus in this study is covariance and correlation between series. A *N*-dimensional log-price process $p(s)$ over the period $[t-1,t]$ is considered. Assumed that the *N*-dimensional or multivariate log-price process is governed by a diffusion process, which can be formulated as follows:

$$dp(s) = \mu(s)ds + \Omega(s)dw(s) \quad (1)$$

where the drift, $\mu(s)$, is a *N*-dimensional vector process, the instantaneous volatility, $\Omega(s)$, is a $(N \times N)$ matrix such that $\sum(s) = \Omega(s)\Omega'(s)$ is the covariance matrix process of the continuous sample path component and $W(t)$ is a vector of N independent Brownian motions. Assuming that the returns do not allow arbitrage and they have a finite instantaneous means, the multivariate log-price process in Eq. (1) belongs to the class of semi-martingales. Because the price process in Eq. (1) is a semi-martingale, it has a well-defined quadratic variance/covariance process. [1] Then, the quadratic covariance, $QCov_t$ in Eq. (2) below is the theoretical covariance of the price process in Eq. (1).

$$QCov_t = \int_{t-1}^{t} \sum(s)ds \quad (2)$$

Assumed that the value of this price process is observed in equally spaced intervals, Δ, in the period $[t-1,t]$. The log price is observed every Δ units of time, where Δ is small, and $M \equiv [1/\Delta]$, as equally spaced intraday returns. Then, the *i*-th intraday return of day *t* is:

$$r_{t,i} = p(t+i\Delta) - p(t+(i-1)\Delta), i = 1,\cdots,M \quad (3)$$

Practical implementation of the quadratic covariance measure in Eq. (2) confronts the reality that no market provides continuous-time arbitrage-free trading environment. However, the quadric covariation may be approximated directly from high-frequency intraday returns by the realized covariance measure, $(RCov_t)$:

$$RCov_t = \sum_{i=1}^{M} r_{t,i}\, r'_{t,i} \quad (4)$$

where $r_{t,i}$ is now defined as an $(N \times 1)$ vector. Note, the realized variance, $RVar_t$ are simply the diagonal elements of the matrix $RCov_t$. When $M \to \infty$ in Eq. (4) converges to the theoretical quadratic covariance measure in Eq. (2). Also, Eq. (4) is a consistent estimator for Eq. (2).

In this study the distributional properties and temporal dependencies for the following daily measures are investigated:

- Realized variance, $RVar_t$;
- Root of $RVar_t$, or realized volatility measure,

[1] Semi-martingales allows for a decomposition of the price process into a predictable finite variation process and a local martingale including an infinite variation component. More specific details about the theory of quadratic variation and semi-martingale assumptions are described in for example Refs. [4, 6].

henceforth named RV_t;

- Logarithm of the root of $RVar_t$, that is, realized logarithmic volatility measure, denoted $lnRV_t$;
- Realized covariance measure, $RCov_{t,ab}$. a and b refer to reported realized covariance between assets or variables a and b, at day t. It is here referred to the element $[a, b]$ of the matrix, not the whole matrix. The same applies for the measure is mentioned below;
- Realized correlation,

$$RCorr_{t,ab} = \frac{RCov_{t,ab}}{\sqrt{\left(RVar_{t,a} \times RVar_{t,b}\right)}}$$

All estimates in this study were calculated with the OxMetrics package called RE@LIZED [17].

3. Data

The Nord Pool data source includes transactional prices and trading volume (per contract) in MW (megawatt). The data encompass forward prices for two financial contracts: (1) one-quarter-ahead prices traded the last quarter before maturity; (2) one-year-ahead prices traded the last year before maturity. The financial trading at Nord Pool takes place between 08:00 to 15:30. The average time between tick-observations for the two contracts is 2 minutes and 53 seconds for quarter series and 6 minutes and 17 seconds for year series. The average number of unique ticker observations over the business day is therefore approximately 178 and 73 for the quarter and year contract series, respectively. The analysis concept requires a strategy about sampling schemes, where in this study equally spaced price and return interval (calendar time sampling) is applied.[2] It will always be a balance between the accuracy of the continuous-record asymptotics underlying the construction of the realized volatility and correlation concept on the one hand, and the influences from market microstructure noise on the other [6]. As it is implausible to push the continuous record asymptotics beyond an average (or median) level of trade duration, equally spaced 30-minute intervals were constructed from the raw data, using closest tick interpolation [2].[3] The contract series consist then of approximately 16000 contract price interval observations from June 1, 2005 up to May 29, 2009.

In the sample used in this study, there are 22% and 35% of the samples consisting of zero-returns, for the quarter and year time series, respectively. Zero-returns, caused by flat prices (consecutively sampled prices in calendar time with the same value) and no-trading (no observation at sampling points), are a potential bias source in estimation of realized volatility and correlation. Intraday seasons in the trading pattern can, among others, be a reason for flat-prices. For example, the volatility may be larger at the opening and closing than during the lunch time. It will also typically be an increase in volatility in the electricity market at the time of publication of the spot price. At Nord Pool for example, the spot price, which is being decided in an auction market, is announced at approximately 13:00 every day for the 24 hours the following day (00:00-24:00).

30-minutes returns were calculated from the equally spaced price data. A few observations that did not match in date were removed from the sample. In order to avoid problems with large jumps in returns between contracts, the returns at 08:00 for the first trading day of the new contract are defined as missing for both data-sets. Some earlier studies have argued for dropping overnight returns in the volatility and correlation measures because these non-trading overnight hours may differ from the volatility and correlation during trading hours and consequently introduce more noise than useful information (e.g. Ref. [19]). The contract-price data used in this study show very small or, more typically, no changes during the night, between 15:30 and 08:00, which further supports dropping these observations. Hence, in this study the daily realized

[2] An overview and discussion of different sampling schemes, both in univariate and multivariate applications, are discussed in e.g. Ref. [18].

[3] In the highly liquid Deutschemark/Dollar and Yen/Dollar spot exchange rate markets equally-spaced 30-minutes return strikes are also used.

volatilities and correlation measures were calculated based on 15 intraday prices, i.e. $M = 15$. The sample used in this study consists of 995 daily-return data and daily-realized variability measures.

4. Results

4.1 The Distribution of Daily Volatility and Correlation

The summary statistics in Table 1 show that all realized variances, realized volatilities and realized covariance distributions for both quarter and year contracts are significantly right-skewed and have significantly excessive kurtosis that exceeds the normal value of zero; the result is that the normal distribution is a poor approximation (based on a number of normality tests). The normality tests used are the Jarque-Bera test, the Anderson-Darling test and the Shapiro-Wilk test.[4]

The realized logarithmic volatility (lnRV) seems to be approximately normal distributed, especially the quarter contract (see the two uppermost panels in Table 1 and the two uppermost rows of panels in Fig. 1).

Compared to the corresponding variance measure, the realized correlation ($RCorr$) is more symmetric, with less skewness and kurtosis, but non-normally distributed (see the lowermost panel in Table 1 and the lowermost rows of panels in Fig. 1). Note, as expected, the strong positive realized correlation, on average, between the year and quarter contracts. However, these correlations also display high variation, ranging from -0.67 to 0.99. A highly negative correlation is unexpected, due to the fact that both contracts analyzed have the same underlying market. However, a closer look at the results show that only 3 (out of 995) daily-realized correlation estimates were less than -0.5 and only 36 estimates less than zero.[5]

The electricity forward-contract distributional property evidence on realized variance, covariance, volatility, logarithmic volatility and correlation is to a large extent consistent with earlier studies of individual stocks and stock indexes (e.g. Refs. [9, 11]), bonds (e.g. Ref. [11]), currencies (e.g. Refs. [2, 4, 11]) and oil and gas (e.g. Ref. [13]).

Table 1 Summary statistics of realized variance, volatility, covariance and correlation.

Variance and volatility	Quarterly		
	RVar	RV	lnRV
Mean	0.00076	2.359%	-3.90
Median	0.00039	1.980%	-3.92
Min.	0.00001	0.348%	-5.66
Max.	0.01987	14.096%	-1.96
Std.Dev.	0.00119	1.429%	0.56
Skew.	6.84 **	2.01 **	0.06
Kurt.	81.72 **	7.66 **	-0.10
JB	284580 **	3105 **	1.06
AD	+Inf. **	32.11 **	0.30
SW	0.513	0.851 **	0.999
	Yearly		
	RVar	RV	lnRV
Mean	0.00043	1.724%	-4.24
Median	0.00018	1.359%	-4.30
Min.	0.00000	0.209%	-6.17
Max.	0.01852	13.608%	-1.99
Std.Dev.	0.00082	1.140%	0.58
Skew.	12.12 **	2.47 **	0.18 *
Kurt.	240.95 **	13.95 **	-0.16
JB	2431220 **	9073 **	6.39 *
AD	+Inf **	+Inf **	2.60 **
SW	0.390 **	0.816 **	0.994 **
Covariance and correlation			
	RCov	RCorr	
Mean	0.00037	0.598	
Median	0.00016	0.669	
Min.	-0.00034	-0.677	
Max.	0.01819	0.986	
Std.Dev.	0.00079	0.292	
Skew.	12.53 **	-0.955 **	
Kurt.	257 **	0.715 **	
JB	2765775 **	172 **	
AD	+Inf. **	20.52 **	
SW	0.384 **	0.923 **	

RVar = realized variance, RV = realized volatility, lnRV = logarithmic realized volatility, RCov = realized covariance, RCorr = realized correlation. 5% significant level is marked by *, 1 % by **. JB = Jarque-Bera test, AD = Anderson-Darling test, SW = Shapiro-Wilk test.

[4] For data with long memory, the Jarque-Bera test over-rejects normality and is thus not recommended [11]. Owing to this, a number of normality tests were used.

[5] Inspection of the extreme negative correlation estimates association to zero-returns in the quarterly and yearly contracts did not show any systematic pattern, compared to the non-extreme correlation estimates association to zero-returns.

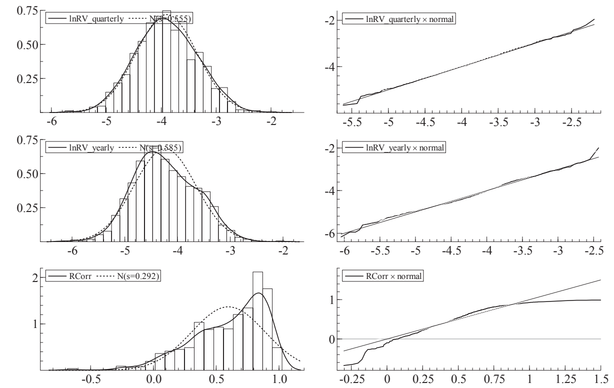

Fig. 1 Density distribution (solid line), histogram (bars) and normal reference distribution (dots) (to the left) and QQ-plot (to the right) of quarter contract, lnRV_quarterly, for year contract, lnRV_yearly, and for realized correlation between quarter and year contracts, RCorr.

4.2 Temporal Dependence

In this section the time-series properties of the realized variance, volatility, covariance, and correlation of the quarter and year forward contract data are explored. Table 2 reports a number of test statistics about the temporal-dependency properties for the aforementioned variables. The lines denoted Q20 summarize the value of the standard Box-Pierce test for joint significance of the first 20 autocorrelations. The null hypothesis of no autocorrelation is clearly rejected for all realized variance, volatility, covariance and correlation measures analyzed. The right panels of Fig. 2 depict the autocorrelations for logarithmic realized volatility for quarter and year contracts. The figure illustrates that autocorrelations are systematically above the conventional Bartlett 95% confidence error bounds (at least for more than 100 lags), which confirm the autocorrelation tests from the Box-Pierce tests. Moreover, the realized correlation ($RCorr$) measure in Fig. 3 shows evidence of autocorrelation in the data, but weaker than for the individual variance variables.

Fig. 2 shows that the autocorrelations for the volatility variable starts around 0.5 and decays very slowly, which suggests a long-memory pattern. The correlation measures are less autocorrelated and have fewer significant lags than the volatility measures (Dropped c.f. Fig. 3). In any case, the autocorrelation

Table 2 Temporal dependence of realized variance, volatility, covariance and correlation.

Variance and volatility	Quarterly		
	RVar	RV	lnRV
Q20	509.24 **	1911.18 **	3115.86**
ADF	-6.18 **	-6.27 **	-4.84 **
KPSS	1.373 **	4.142 **	3.055 **
AR	9	4	10
d_{RH}	0.221 **	0.310 **	0.353 **
	Yearly		
	RVar	RV	lnRV
Q20	669.03 **	3072.44 **	3883.26**
ADF	-9.34 **	-5.35 **	-4.52 **
KPSS	2.407 **	3.617 **	3.297 **
AR	4	6	9
d_{RH}	0.296 **	0.357 **	0.343 **
Covariance and correlation			
	RCov	RCorr	
Q20	344.85 **	347.00 **	
ADF	-9.81 **	-5.65 **	
KPSS	2.173 **	1.402 **	
AR	4	10	
d_{RH}	0.234 **	0.168 **	

RVar = realized variance, RV = realized volatility, lnRV = logarithmic realized volatility, RCov = realized covariance, RCorr = realized correlation. The Q20 is the Box-Pierce statistics, where the null hypothesis rejects zero autocorrelation from lag 1 up to 20. ADF is the augmented Dickey-Fuller test (where the null-hypothesis is non-stationarity) with an intercept and no time trend. The test by Kwiatkowski, Phillips, Schmidt, and Shin [20], named the KPSS test, where the null-hypothesis is conventional stationarity, also has no time trend. Number of lags in the ADF and KPSS tests are determined by the AIC (Akaike's Information Criteria), and are given in the rows denoted AR. d_{RH} is the long-memory test in Ref. [21]. A level of significance of 5% is marked by *, 1% by **.

functions seem to decay at a slow hyperbolic rate, in contrast to the geometric decay rate associated with the conventional stationary $I(0)$ process, or alternatively to an infinite persistence pattern resulting from a non-stationary unit-root $I(1)$ process. The hyperbolic-decay process is a fractionally integrated process with a fractional order ranging from 0 to 1. When the fractional order is between 0 and 0.5, the process is mean-reverting stationary [12]. To test the decay rate both the unit-root ADF test [22] and the KPSS

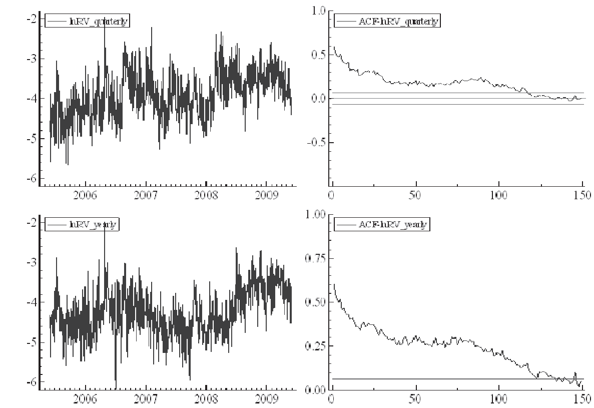

Fig. 2 Plots of logarithm of realized volatility and ACF (autocorrelation function) up to 150 lags for the quarter and year electricity forward contracts.

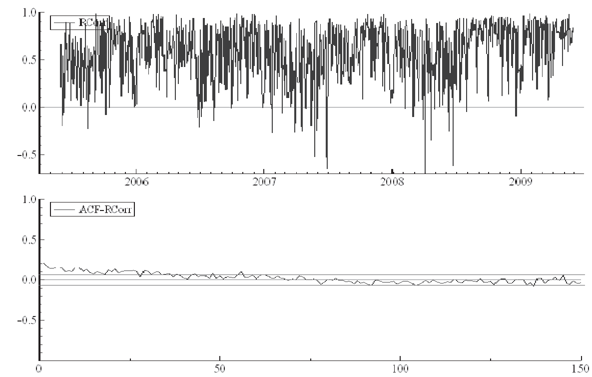

Fig. 3 Plots of realized correlation between the quarter and year electricity forward contracts, in addition to the ACF (autocorrelation function) up to 150 lags.

test [20] were applied. Further, the fractional integrated estimate, d_{RH}, with the method proposed by Robinson and Henry [21] was estimated. For all measures analyzed (in Table 2), the null hypothesis of non-stationarity, or unit root (via ADF test), is rejected. Conventional stationarity, examined with the KPSS method, is also rejected for all contracts and measures investigated. These two tests provide initial evidence for long memory and hyperbolic decay of the autocorrelation function. The estimated d_{RH} values for the variance/volatility and covariance/correlation measures all have $0 < d_{RH} < 0.5$, showing that these measures are fractionally integrated and mean-reverting stationary. Also it is noted that the realized volatility measures are in general more persistent (i.e., higher d_{RH} values) than the realized covariance/correlation measures.

The temporal characteristics of realized variance/volatilities and covariance/correlations for the electricity forward data are consistent with the findings from Ref. [4] for exchange rates, Ref. [9] for stocks and Ref. [11] for various futures contracts. In contrast to the results in this study, Ref. [13] did not find that realized correlation between futures contract prices for crude oil and natural gas exhibit any long-memory patterns.

Fig. 3 illustrates that the realized correlation (*RCorr*) is very volatile. In many periods the realized correlation is close to 1, but in some periods close to 0 and sometimes clearly negative.

Table 3 documents to what extent the volatility measures linearly move together, and how volatilities measures and correlation measures are correlated, which is frequently termed the volatility-in-correlation effect ([9], p. 66). Table 3 indicates a positive association at 0.27 and 0.40 between realized correlation (*RCorr*) and logarithmic realized volatility (*lnRV*) for quarter and year contracts, respectively. These results are consistent with the findings by, for example, Ref. [4] on currency or Ref. [9] on stocks, imply that correlations between year and quarter forward contracts on electricity are, on average, higher when the volatility in the year and quarter contracts is high, compared to when the volatility in these contracts are in a low-volatility state. This again suggests that models based on constant correlation, such as mean-variance efficiency analysis, are misguided. The year and quarter forward contracts become more synchronized when more volatile.

Correlation between realized volatility for quarter and year contracts are 0.67. The tendency of realized volatility to vary in tandem was also found for currencies [4] and for stocks [9].

As also shown in Table 3, the total trading volume change (from day to day) are a positive correlation with both realized volatility and realized correlation. This result is consistent with the findings of Giot, et al. [23] who state that much of the empirical literature documents

Table 3 Pearson's correlation coefficients between realized correlation, the corresponding logarithmic realized volatility, time-to-delivery (in days) and change in trading volume (from day to day).

Variables	InRV yearly	InRV quarterly	RCorr
InRV yearly		0.670	0.397
Time-to-delivery yearly	-0.027		-0.105
Volume change yearly	0.139		0.122
InRV quarterly			0.270
Time-to-delivery quarterly		-0.235	-0.062
Volume change quarterly		0.131	0.109

Values in bold are different from 0 with 5% significance level. Non-relevant correlation coefficients are not reported.

are a positive contemporaneous relation between volume and volatility in financial markets. Moreover, the multivariate analysis for stock returns in Ref. [24] also reports evidence of contemporaneous relationship in volatility.

According to Samuelson [25], the volatility of futures price returns should increase as time-to-maturity decrease. For the year contracts, the realized volatility measure shows no statistically clear evidence of the Samuelson effect. However, the quarter-contract series show evidence of increasing realized volatility as time-to-delivery decreases (significant at the 5% level). Table 3 also shows that decreasing time-to-delivery is positively related to increasing realized correlation.

For modeling and forecasting realized volatilities and correlation, future studies will do well to take the stylized facts into account that are described in this section.

5. Implications and Conclusions

The main findings obtained in this study of Nord Pool electricity forward data show that the logarithmic realized volatility are approximately normally distributed, while realized correlation seems not to be. Further, realized volatility has a long-memory feature, and there is a high correlation between realized correlation and volatilities. These results are to a large extent consistent with early, similar studies of stylized facts of other financial and commodity markets. In contrast to the study of crude oil and natural gas in Ref. [13], the result in this study also find a long-memory pattern in realized correlation.

The results from this study have implications for future research and modeling. Electricity volatility modeling and forecasting based on high-frequency data can draw from similar studies of other financial and commodity markets. Models based on constant correlation seem inappropriate. Realized correlation is, as expected, almost always positive, but very volatile and so the hedge position with these quarter and year electricity forward contracts points to high risk.

Realized volatilities and probably also realized correlation should account for the long-memory feature in modeling. Since the logarithmic realized volatility is approximately normal distributed a fractionally integrated processes (ARFIMA model) with normal distributed error terms may be appropriate. Realized correlation seems not to be normally distributed, and an ARFIMA model with non-normal distributed error terms should describe this process well. The strong correlation between realized volatilities and correlation demands a multivariate GARCH/VARFIMA framework (e.g. [26]). For modeling of realized volatility and correlation dynamic, quantile regression models and state space regression models could be useful approaches and should also be investigated.

Future research could also combine realized correlation with similar measures from multivariate conditional volatility models, multivariate stochastic volatility models and implied correlation from option-market data. Further, realized correlation analyses between various energy markets and between energy and other markets would be fruitful areas for future research.

This study has the ignored possibility for discontinues jumps and co-jumps in the price process. To account for jumps and co-jumps the multivariate diffusion in Eq. (1) would be replaced by a multivariate jump-diffusion equation. Further, the quadratic covariance specification in Eq. (2) would be replaced by two variation components, one accounting for the continuous covariance components and one accounting for the jump/co-jump component of the discontinuous part. It is an extensive ongoing research on how to estimate this model in a reliable way.[6] One challenge for future research in that respect is how to empirically estimate the continuous covariance matrix, that are robust to co-jumps. While in the univariate case there exists formal tests for significant jumps, based on the authors' knowledge, it don't exist any formal test that can distinguish significant differences between $RCov_t$ and continuous covariance matrix for the multivariate case. In other words, for the multivariate case it is not, at this point, possible to separate out significant co-jumps. More research is also needed to document the eventually improved forecasting abilities for realized variance and correlation by using decomposed continuous and jump/co-jump variables in modeling.

Another challenge is the ways to account for market microstructure noise in the estimation of realized covariance and correlation. The realized covariance and correlation estimates may be affected by microstructure noise.[7] In this study the traditional realized covariance and correlations by Andersen, et al. [4] and Barndorff-Nielsen, et al. [5] have been used. Some recent alternative realized covariance and correlations estimators that deal with non-synchronous trading are discussed in for example Refs. [33, 34]. Schulz [35] proposes a robust approach in the case of flat prices and zero-returns in the univariate framework. Ideas considering how to deal with zero-returns in a multivariate setting are welcome. These extensions are left for further research and will be dealt with in a separate article.

[6] Several alternative integrated covariance estimators exist. Examples are the Bipower Covariation estimator [27], the nearest-neighbor truncation estimator [28], the range-based covariance estimator [29], the Gaussian rank covariance estimator [30], and the realized outlyingness weighted quadratic covariation estimator [31].

[7] Ref. [32] discusses potential microstructure noise problems in detail.

Acknowledgments

The authors gratefully acknowledge an anonymous referee for useful comments and suggestions. This paper is a part of the project "ELCARBONRISK-Modeling and Forecasting Risk in Electricity, Carbon and Related Energy Markets (oil, gas, coal)", financed by the Research Council of Norway, Eidsiva Energi and Tafjord Kraftproduksjon AS.

References

[1] T.G. Andersen, T. Bollerslev, P.F. Christoffersen, F.X. Diebold, Volatility and correlation forecasting, in: G. Elliott, C.W.J. Granger, A. Timmermann (Eds.), Handbook of Economic Forecasting, Vol. 1, North-Holland, Amsterdam, 2006, pp. 777-878.

[2] M.M. Dacorogna, R. Gencay, U. Muller, R.B. Olsen, O.V. Pictet, An Introduction to High-Frequency Finance, Academic Press, San Diego, CA, 2001.

[3] T.G. Andersen, T. Bollerslev, Answering the sceptics: Yes, standard volatility models do provide accurate forecasts, International Economic Review 39 (4) (1998) 885-905.

[4] T.G. Andersen, T. Bollerslev, F.X. Diebold, P. Labys, Distribution of realized exchange rate volatility, Journal of the American Statistical Association 96 (453) (2001) 42-55.

[5] O.E. Barndorff-Nielsen, N. Shephard, Econometric analysis of realized covariation: High frequency based covariance, regression, and correlation in financial economics, Econometrica 72 (3) (2004) 885-925.

[6] T.G. Andersen, T. Bollerslev, F.X. Diebold, P. Labys, Modeling and forecasting realized volatility, Econometrica 71 (2) (2003) 579-625.

[7] M. Martens, J. Zein, Predicting financial volatility: High-frequency time-series forecasts vis-à-vis implied volatility, The Journal of Futures Markets 24 (11) (2004) 1005-1028.

[8] F.É. Racicot, R. Théoret, A. Coën, Forecasting irregularly spaced UHF financial data: Realized volatility vs UHF-GARCH models, International Advances in Economic Research 14 (1) (2008) 112-124.

[9] T.G. Andersen, T. Bollerslev, F.X. Diebold, H. Ebens, Distribution of realized stock rate volatility, Journal of Financial Economics 61 (1) (2001) 43-76.

[10] N.M.P.C. Areal, S.J. Taylor, The realized volatility of FTSE-100 futures prices, Journal of Futures Markets 22 (7) (2002) 627-648.

[11] D.D. Thomakos, T. Wang, Realized volatility in the futures markets, Journal of Empirical Finance 10 (3) (2003)

321-353.

[12] Z. Chen, R.T. Daigler, A.M. Parhizgari, Persistence of volatility in futures markets, Journal of Futures Markets 26 (6) (2006) 571-594.

[13] T. Wang, J. Wu, J. Yang, Realized volatility and correlation in energy futures markets, Journal of Futures Markets 28 (10) (2008) 993-1011.

[14] C.J. Ulrich, Realized volatility and price spikes in electricity markets: The importance of observation frequency, 2009, Social Science Research Network Web site, http://ssrn.com/abstract=1342586.

[15] K.F. Chan, P. Gray, B. Campen, A new approach to characterizing and forecasting electricity price volatility, International Journal of Forecasting 24 (4) (2008) 728-743.

[16] E. Haugom, S. Westgaard, P.E. Solibakke, G. Lien, Realized volatility and the influence of market measures on predictability: Analysis of Nord Pool forward electricity data, Energy Economics 33 (6) (2011) 1206-1215. (forthcoming)

[17] S. Laurent, Estimating and Forecasting ARCH Models Using G@RCH™ 6, Timberlake Consultants Press, UK, 2009.

[18] M. McAleer, M.C. Medeiros, Realized volatility: A review, Econometric Reviews 27 (1-3) (2008) 10-45.

[19] M. Martens, Measuring and forecasting S&P 500 index-futures volatility using high-frequency data, Journal of Futures Markets 22 (6) (2002) 497-518.

[20] D. Kwiatkowski, P.C.B. Phillips, P. Schmidt, Y. Shin, Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?, Journal of Econometrics 54 (1-3) (1992) 159-178.

[21] P.M. Robinson, M. Henry, Long and short memory conditional heteroskedasticity in estimating the memory parameter of levels, Econometric Theory 15 (3) (1999) 299-336.

[22] D.A. Dickey, W.A. Fuller, Distributions of the estimators for autoregressive time series with unit root, Journal of the American Statistical Association 74 (366) (1979) 427-431.

[23] P. Giot, S. Laurent, M. Petitjean, Trading activity, realized volatility and jumps, Journal of Empirical Finance 17 (1) (2010) 168-175.

[24] A.R. Gallant, P.E. Rossi, G. Tauchen, Stock prices and volume, Review of Financial Studies 5 (2) (1992) 199-242.

[25] P.A. Samuelson, Proof that properly anticipated prices fluctuate randomly, Industrial Management Review 6 (2) (1965) 41-49.

[26] R. Chiriac, V. Voev, Modeling and forecasting multivariate realized volatility, Journal of Applied Econometrics 26 (6) (2011) 922-947. (forthcoming)

[27] O.E. Barndorff-Nielsen, N. Shephard, Measuring the impact of jumps in multivariate price processes using bipower covariation, Discussion Paper, Nuffield College, Oxford University, 2004.

[28] T.G Andersen, D. Dobrev, E. Schaumburg, Jump-robust volatility estimation using nearest neighbor truncation, CREATES Research Paper 2009-52, IDEAS Web site, http://ideas.repec.org/p/aah/create/2009-52.html.

[29] K. Bannouh, D. Dijk, M. Martens, Range-based covariance estimation using high-frequency data: The realized co-range, Journal of Financial Econometrics 7 (4) (2009) 341-372.

[30] K. Boudt, J. Cornelissen, J.C. Croux, Jump Robust Daily Covariance Estimation by Disentangling Variance and Correlation Components, North-Holland Pub. Co., 2011. (forthcoming)

[31] K. Boudt, C. Croux, S. Laurent, Outlyingness weighted covariation, Journal of Financial Econometrics 9 (4) (2011) 657-684. (forthcoming)

[32] J.Y. Campbell, A.W. Lo, A.C. MacKinlay, The Econometrics of Financial Markets, Princeton University Press, New Jersey, 1997.

[33] J.E. Griffin, R.C.A. Oamen, Covariance measurement in the presence of non-synchronous trading and market microstructure noise, Journal of Econometrics 160 (1) (2011) 58-68.

[34] L. Zhang, Estimating covariance: Epps effect, microstructure noise, Journal of Econometrics 160 (1) (2011) 33-47.

[35] F.C. Schulz, Robust estimation of integrated variance and quarticity under flat price and no trading bias, Journal of Energy Markets 4 (1) (2011).