

The Dutch Disease revisited: absorption constraint and learning by doing

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Abstract This paper revisits the Dutch disease by analyzing the general equilibrium effects of a resource shock on a dependent economy, both in a static and dynamic setting. The novel aspect of this study is to incorporate in one coherent framework two distinct features of the Dutch disease literature that have previously been analyzed in isolation from each other: capital accumulation with absorption constraint, and productivity growth induced by learning-by-doing. The result of long run exchange rate appreciation is maintained in line with part of the Dutch Disease literature. In addition, a permanent change in the employment shares occurs after the resource windfall, in favor of the non-traded sector and away from the traded sector growth engine of the economy. In other words, in the long run both of the classic symptoms of the Dutch Disease remain in place.

Key words Dutch Disease, Exchange rate, Dual Economy, Resource wealth.

1 Introduction

The Dutch disease is an economic phenomenon reflecting changes in the structure of employment and production of an economy, in the wake of a favorable resource income shock. This paper aims at revisiting the macroeconomic mechanisms behind this phenomenon by figuring out the general equilibrium effects of a resource shock on a dependent economy, both in a static and dynamic setting. The novel aspect of this study is to incorporate in one coherent framework two distinct features of the Dutch disease economic literature that have previously been analyzed in isolation from each other: capital accumulation with absorption constraint on one side, productivity growth induced by learning-by-doing on the other side. First,

let us briefly present the economic literature on the Dutch Disease, in order to thoroughly motivate the development of the framework of the current study. The discovery of natural resources and the changes that its related resource income cause for small open economies has been an issue of interest for macroeconomists and policy-makers throughout the last decades. As regards the theoretical literature, the standard one-sector production models turned out to be insufficient to provide useful insights. The innovation brought in by the model of the dependent economy, pioneered by Salter (1959), was precisely that different sectors of the economy could be affected by the resource income to varying degrees. A couple of decades later Corden and Neary (1982) came out with their pioneering work on the Dutch Disease and became the obligatory reference point for any further study on the issue. Since then, the essence of the hypothesis of the Dutch Disease has been that an unexpected resource income induces in the long-run an appreciation of the real exchange rate and a decline of the employment level in the manufacturing traded sector, with possible detrimental consequences on income *levels* if the traded sector happens to be the productive engine of the economy. However, the question about income *growth rates* remained unanswered. The theoretical literature has therefore subsequently developed with the scope of suggesting that a *dynamic* version of the Dutch-Disease model can generate a negative correlation between resource abundance and the pace of income growth. The general argument carried out is that, among the different sectors that operate in the economy, some are relatively more growth-enhancing than others. Imagine that, in the context of a multi-sectoral model, the growth-enhancing sector of the economy is represented by manufacturing: a resource boom crowding out production inputs from manufacturing has negative consequences not only for the level of real income, but also for the subsequent growth rate of the economy, because the negative shock implies reduced accumulation of technological progress. This is the argument carried out by Van Wijnbergen (1984), Sachs and Warner (1985), Krugman (1987) and Matsuyama (1992) – amongst others – who argue that de-industrialization effects reduce income growth by weakening technological progress externalities. More specifically, Sachs and Warner (1995) extended the work of Matsuyama (1992) by constructing an overlapping generations model of endogenous growth in which the key assumption about technical progress is that the accumulation of knowledge is generated exclusively in the traded sector of the economy as a by-product of the employment level. The theoretical analysis of a three-sector resource-rich small open economy in Alberola and Benigno (2017) has recently confirmed the result that, if productivity gains are exclusively concentrated in the sector of tradable goods, then resource boom has detrimental effect on the traded sector and slows down productivity growth. Notice that the assumption of learning by doing taking place only in one sector simplifies the analytical tractability of the model, but bears some costs. In reality, the stock of knowledge raises the productivity level of employed workers in all sectors of the economy, in other words there are perfect spillovers from the traded to

the non-traded sector. In this line, Torvik (2001) introduced the possibility that *both* traded and non-traded sectors were contributing to learning, with additional spillovers between the sectors themselves. Due to this structure of the model, Torvik (2001) shows that it is actually relative productivity that drives factor allocation and real exchange rate dynamics. The unconventional result coming out of his work is that a foreign exchange gift might determine an exchange rate depreciation in the long-run. Another important innovation for the dynamic Dutch disease literature has been lately put forward by the models in van der Ploeg (2011, 2012), van der Ploeg and Venables (2013). These models challenge the common belief that the temporary loss of learning associated with shrinking manufacturing sector constitutes the main factor of risk, focusing instead the attention on another possible cause of *disease*. If capital goods are produced solely by the non-traded sector, which in turn needs domestically produced capital goods to function (to mention the typical example, teachers are needed to educate more teachers, roads to produce roads etc.), then the economy might fail to absorb the boom in demand after a resource boom (*absorption constraint*). In other words, van der Ploeg and Venables (2013)'s assumption determines a sluggish adjustment of the production sector of the economy to natural resource windfall, resulting in a real exchange rate appreciation in the short run. In the long run, investment in non-traded goods permits gradual expansion of capital and *reversal* of the initial real exchange rate appreciation. The issue of the absorption constraint and the related exchange rate appreciation has been recently empirically highlighted by Harding et al. (2016), which have estimated for a large group of resource-rich economies that resource booms substantially appreciate the real exchange over the first decade following the resource discovery, via the prices of non-tradable goods. Will the mechanism of the absorption constraint and its effect on the exchange rate play a decisive role in the current model as well? In conclusion of this excursus on the Dutch Disease economic literature, notice that the recent surveys by Vaz (2017) and van der Ploeg and Poelhekke (2016) can be consulted for additional details. The scope of this paper can now be properly introduced. The two distinct aspects of learning externalities á la Sachs and Warner (1995) (which corresponds to a slightly simplified version of the more thorough approach by Torvik, 2001) on one hand, and absorption constraint due to domestically produced capital goods on the other hand (as in van der Ploeg and Venables, 2013), have yet not been analyzed by the economic literature. This paper aims at filling this gap in the literature of the Dutch Disease by precisely providing such a complete framework. The research question then becomes that of investigating whether, both in a static and dynamic setting, the classic symptoms of the Dutch Disease from the background literature remain in place or not. In other words, will the structural allocation of labor still respond to the resource boom in the way it does in the classic framework of the Dutch Disease (namely with a drop in the share of labor in the manufacturing sector)? How will the labour share in the traded and non-traded sectors interact with the assumption of domestically produced

capital goods? The complete analysis of the impact of the resource boom on the relative prices and on the real exchange rate in a dual small open economy is precisely the contribution of this study. Next section introduces the modeling framework of the economy, whilst section 3 presents the static results. After that, section 4 and 5 will present respectively the dynamic model and the dutch disease dynamic mechanisms. Section 6 will summarize the results and conclude.

2 The model

Consider the supply side of a resource-rich dependent economy in which only two goods are produced, tradables and non-tradables. Assume that all markets clear instantaneously and that firms operate under perfect competition. Time is continuous and indexed by $t = [0, \dots, \infty)$. No population growth is considered and there is balanced trade. The assumption of balanced trade excludes assets accumulation and it may result from imperfect capital markets or policy controls. To put some structure on the analysis, I assume that the production technology for both goods is a linearly homogeneous Cobb-Douglas production function. Sector N produces a non-tradable good by means of labor and domestic capital. Sector T produces a tradable good by means of labor. Labor is inelastically supplied by households and is fully mobile between the two sectors. Normalizing labor supply to unity, sector N employs a share η_t whereas sector T employs a share $(1 - \eta_t)$ of workers. The production functions read:

$$X_{Nt} = K_{Nt}^\alpha (A_t \eta_t)^{1-\alpha}, \quad (1)$$

$$X_{Tt} = A_t(1 - \eta_t), \quad (2)$$

where X_{jt} is the physical output at time t of sector $j = N, T$, A_t represents labor-augmenting technical progress, K_N is domestic (non-tradable) capital, $\alpha \in (0, 1)$ is the production elasticity of capital. This production structure captures in a simple way the features of an economy with a labour-intensive traded sector (i.e. agriculture) and with a non-traded sector that is constrained by the domestic production of capital goods.

Since our dependent economy is small relative to international markets, the price of tradable goods P_{Tt} is exogenously fixed at the world level. Hence, we can assume that it is constant over time and set it at unity. The real exchange rate of our economy is therefore given by P_t :

$$P_t = \frac{P_{Nt}}{P_{Tt}} = \frac{P_{Nt}}{1}. \quad (3)$$

This implies that the value of total production is given by $X_t = P_t X_{Nt} + X_{Tt}$. As in Sachs and Warner (1995), productivity growth is driven by

learning being external to firms in the traded sector (i.e. $A_t = A_{Tt}$) with perfect spillovers to non-traded sector¹:

$$\frac{\dot{A}_t}{A_t} = \gamma(1 - \eta_t), \quad (4)$$

where $\gamma > 0$ is an exogenous parameter which captures the marginal impact on technological progress of additional labor units in the traded sector. The implication of this formulation in which learning is only generated in the traded sector, is that a decreased size of the sector will determine a lower growth of productivity. A particular aspect of this formulation is that productivity growth is clearly bounded in its domain. In other words, in corner solutions there will be either null productivity growth in case the non-traded sector fully absorbs the labour force at $\eta_t = 1$, otherwise productivity growth will be bounded from above at $\frac{\dot{A}_t}{A_t} = \gamma$ as long as the labour force is entirely employed in the traded sector at $\eta_t = 0$.

Capital goods K_{Nt} are produced uniquely by the non-traded sector and can not be imported. This assumption resembles closely the absorption constraint formulated in van der Ploeg and Venables (2013). In other words, the non-traded sector produces a homogeneous good X_{Nt} which can be either directly consumed or invested for further production (i.e. $X_{Nt} = C_{Nt} + I_{Nt}$), whilst the traded sector produces a homogeneous good X_{Tt} which is directly consumed ($X_{Tt} = C_{Tt}$). Capital accumulation is defined as follows:

$$\dot{K}_{Nt} = I_{Nt} = \varphi \left(\frac{P_t \cdot (\partial X_N / \partial K_N)}{P_t} \right) \cdot K_{Nt}, \quad \varphi(1) = 0, \varphi' > 0 \quad (5)$$

$$\dot{K}_{Nt} = \varphi \left(\alpha \left(\frac{A_t \eta_t}{K_{Nt}} \right)^{1-\alpha} \right) \cdot K_{Nt}, \quad (6)$$

where depreciation of capital is absent, investment demand I_{Nt} (i.e. demand of capital goods) is a function $\varphi(\cdot)$ of the ratio between the value of an additional unit of capital and the cost of acquiring it. Due to the assumption that non-traded capital goods are sold on the non-traded goods market at equal price regardless of whether they are purchased for consumption or investment reasons, the cost of acquiring one additional unit of capital will therefore be equal to P_t . In other words, the higher the ratio between the value and the cost of additional capital units, the stronger the incentive of additional investment expenditures. The assumption $\varphi(1) = 0$ also implies that, if this ratio is as low as unity or lower, the incentives will be absent and no additional investment will take place.

To close the model, the aggregate resource constraint of the dependent economy is given by the equality between aggregate income from production and aggregate demand:

$$\mathfrak{R}_t + P_t X_{Nt} + X_{Tt} = P_t(C_{Nt} + I_{Nt}) + C_{Tt}, \quad (7)$$

¹ This formulation of technical progress implies balanced growth by definition since relative productivity is exogenous to the model.

where $\mathfrak{R}_t = A_t R_t$ is the flow of foreign exchange R_t measured in traded sector productivity units. The assumption of balanced trade implies that on the right-hand side of this constraint it appears no assets accumulation, in other words null current account.

In conclusion, a note about the demand side. Households choose at each point in time how to allocate consumption expenditures between traded and non-traded consumption goods. The representative household is endowed with a Cobb-Douglas utility function, hence the standard result that a constant fraction of the aggregate consumption expenditure C_t is respectively spent on traded and non-traded consumption goods (details in Appendix A1):

$$C_{Nt} = \left(\frac{1 - \delta}{P_t} \right) C_t, \quad C_{Tt} = \delta C_t. \quad (8)$$

We can already draw some observations from this structure of the model. The return on savings is determined by the return to investment in capital goods, which are the only assets that can be accumulated in the model. Therefore, a higher return on savings will determine on aggregate higher savings and lower consumption. Aggregate consumption is thus considered as a residual after demand of capital goods has been formulated. The model therefore implies that the representative household decides uniquely on the composition of consumption expenditure, setting the shares of his/her income to spend on each of the available goods. In other words, the supply side of the economy with firms profit maximization and factor markets equilibria suffices in driving the dynamics and determining the results of the model.

3 Static mechanisms and equilibrium

This section develops the model in its static version. To simplify notation I will therefore skip the time index for all the variables. The productivity A and the non-traded capital K_N are given at each point in time in the current static setting.

I start by deriving the relation between the exchange rate P and the employment level of the non-traded sector η implied by the static equilibrium. On the supply side of the economy, competitive firms demand labour which is supplied inelastically and assumed to be instantaneously mobile across sectors. The amounts of labor units demanded for each sector is the result of profit maximization for the representative sector firm, subject to the respective technologies taking all prices as given. The first order conditions of profit maximization with respect to labor inputs for the two sectors are:

$$P(1 - \alpha) \frac{K_N^\alpha (A\eta)^{1-\alpha}}{\eta} = \bar{w}, \quad (9)$$

$$A = \bar{w}, \quad (10)$$

in which, due to the traded sector using only labour and having constant returns, the wage in terms of tradables is fixed. By merging the first-order

conditions (9,10) we obtain the labour market equilibrium (one equation in two unknowns P, η):

$$P = \frac{1}{1-\alpha} \left(\frac{A\eta}{K_N} \right)^\alpha. \quad (11)$$

Taking the derivative of this equation we find the response of P to a change in η arising from the labor market equilibrium:

$$\frac{\partial P}{\partial \eta} = \frac{\alpha}{1-\alpha} \left(\frac{A}{K_N} \right)^\alpha \eta^{\alpha-1} > 0. \quad (12)$$

This result shows that a higher non-traded employment level η would obviously decrease the marginal productivity of labour in the non-traded sector, causing the exchange rate P to increase in order for the equilibrium to be re-established. I label this upward sloping relation as the *LL* curve.

The labor market equilibrium is however only one side of our dependent economy. Let us therefore derive the other curve of the static diagram. Substituting the consumer demands of consumption goods C_T and C_N given in (8) and the supply given by the production functions X_N and X_T (1,2) into the aggregate resource constraint of the economy (7) gives the following expression for the goods market equilibrium (again one equation in two unknowns P, η):

$$P = \frac{\left(\frac{1-\delta}{\delta}\right) [AR + A(1-\eta)]}{K_N^\alpha (A\eta)^{1-\alpha} - \varphi \left(\alpha \left(\frac{A\eta}{K_N} \right)^{1-\alpha} \right) K_N}. \quad (13)$$

Taking the derivative of this equation and rearranging by exploiting the labor market equilibrium in (11), I find the response of P to a change in η arising from the goods market equilibrium:

$$\frac{\partial P}{\partial \eta} = \frac{A \left[\alpha \varphi'(\cdot) - \frac{1}{\delta} \right]}{K_N^\alpha (A\eta)^{1-\alpha} - \varphi \left(\alpha \left(\frac{A\eta}{K_N} \right)^{1-\alpha} \right) K_N}. \quad (14)$$

Let us give a closer look at this result. The denominator of this derivative is positive by definition since it is equal to $X_N - I_N = C_N > 0$. The numerator is instead composed of investment and production responses to changes in η which are pulling in different directions. In order to highlight the mechanism at work behind these counteracting responses, let us imagine for a while that investment demand of non-traded goods I_N were absent. An increase in the non-traded employment level η will then increase production of non-traded goods and correspondingly decrease production of traded goods. This will result in excess supply of non-traded goods which calls for a decreased exchange rate P in order to restore equilibrium. In other words, P has to fall in order to shift back demand from traded to non-traded goods so that the market will be back to balance.

However, the model does include investment demand and postulates that this demand does react as well to a change in the non-traded employment

level η . Let us in turn observe this effect in isolation. Other things being equal (recall that A and K_N are given at each point in time), higher η will determine higher investment demand via higher marginal productivity of capital which translates into excess demand of non-traded goods, which in turn would require higher exchange rate P in order to restore equilibrium. The following positive derivative of the investment demand I_N with respect to the non-traded employment level shows the analytical side of this mechanism:

$$\frac{\partial I_N}{\partial \eta} = \varphi'(\cdot)(1 - \alpha)\alpha \left(\frac{K_N}{\eta}\right)^\alpha A^{1-\alpha} > 0. \quad (15)$$

Hence we have observed how production and investment effects are pushing the exchange rate P towards opposite directions. I thereby conclude that as long as higher non-traded employment η causes supply of non-traded goods to increase more (less) than their demand, the net result will be excess supply (demand) and a lower (higher) exchange rate P for the equilibrium to be restored. From here onwards I will assume that the production effects are sufficiently large to prevail over the investment effect. The necessary analytical condition for this assumption to be verified can be obtained by setting a negative sign to the numerator of (14):

$$\left[\alpha\varphi'(\cdot) - \frac{1}{\delta}\right] < 0, \quad \Rightarrow \varphi'(\cdot) < \frac{1}{\alpha\delta}. \quad (16)$$

This result tells us that the marginal response of the incentive to invest must be limited by the upper threshold $\frac{1}{\alpha\delta}$ in order for the production effects to prevail over the investment effect. In other words, as long as this condition holds we conclude that $\frac{\partial P}{\partial \eta} < 0$ and label this downward sloping relation as the NN curve. This result implies that in this framework the factor reallocation or shift of labor from the traded to the non-traded sector is accompanied by a decrease in the relative price of non-traded goods.

3.1 Static Dutch Disease

Some comparative statics results of the model can be now sorted out. To begin with, let us point out that in the static model a resource windfall R directly influences only the goods market equilibrium, by causing a shift exclusively for the NN curve. Higher resource windfall R determines higher consumption demand of non-traded goods which means that, for any given level of employment η , the relative price of non-traded goods P will increase. Thus, from the goods market equilibrium equation (13) I derive the positive response of P to a change in R for a given η :

$$\frac{\partial P}{\partial R} = \frac{A(1 - \delta)/\delta}{K_N^\alpha (A\eta)^{1-\alpha} - \varphi\left(\alpha\left(\frac{A\eta}{K_N}\right)^{1-\alpha}\right) K_N} > 0. \quad (17)$$

This derivative is unambiguously positive, confirming that a resource windfall implies an upwards shift of the NN curve and an appreciation of the exchange rate. A short note on the collateral effect of the resource windfall on the non-traded employment level η . Analytically, the response of η to a change in R (for a given P) is given by

$$\frac{\partial \eta}{\partial R} = \frac{A(1-\delta)}{1-\alpha\delta\varphi'(\cdot)}, \quad (18)$$

in which I used (11) to simplify the expression. As long as the necessary condition (16) for the NN curve to be downwards sloping is met, the denominator of this derivative is positive. I will refer to this result several times in the derivation of the dynamic model.

In conclusion, the new static equilibrium will thus be characterized by the common symptoms of the Dutch Disease: exchange rate appreciation and a larger share of employment in the non-traded sector as in fig.1²:

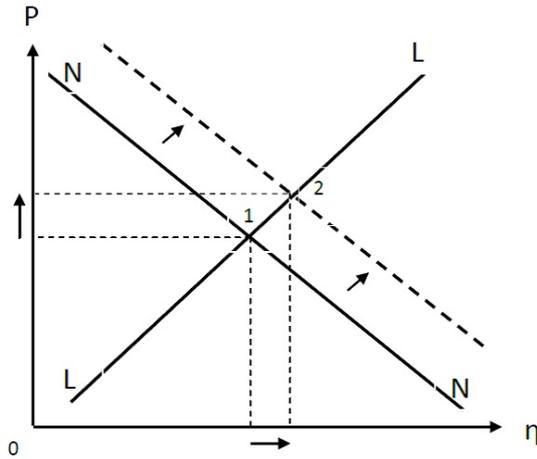


Figure 1 - Static effects of resource windfall

Resource income induces therefore a static factor reallocation pulling labor away from the growth engine of the model, the traded sector. In order to shed light on the mechanisms at work in the dynamic version of the model which will be developed in the next sections, let us proceed with analyzing the static behavior of the state variables of the model.

² It is relevant to point out that a downward sloping NN curve (due to the assumption of weak reaction of the investment) is *not* necessary for the resource windfall R to determine positive changes in P and η . Even an upward sloping NN curve (although less steep than the LL) would ensure the presence of the two classic symptoms of the Dutch Disease.

3.2 State variables shocks

At first, let us analyze how productivity A respectively affects the exchange rate P and the non-traded employment η in the current static setting. As initial remark, let us notice that productivity A influences both the labour market and the goods market equilibrium. From the labor market equilibrium equation (11) we observe that higher productivity A induces a relatively higher increase in the marginal productivity of labour of the traded with respect to the non-traded sector, thus calling for a higher P to restore factor price equality, i.e. graphically the LL curve of fig.1 shifts up to the left.

The other side of the story comes from the goods market equilibrium (13). Noting that the exchange rate is defined in (3) as $P = P_N$ due to the price of traded goods being constant, I infer that the (partial equilibrium) response of P to a change in productivity A will only depend on the net effect between excess supply and demand of non-traded goods. This implies that the effect of higher productivity A on the exchange rate P will be the net result of two separate effects pulling in different directions. On one side, higher productivity A directly induces higher non-traded production X_N thereby determining excess supply of non-traded goods. On the other side, higher productivity A translates as well into higher consumption (via the resource windfall) and investment demand I_N , determining a counterbalancing excess demand effect of non-traded goods. In other words, the direction of the shift of the NN curve and in particular the overall sign of the effect of productivity A on the exchange rate P depends on which of these two effects is prevailing over the other.

Let us now investigate the static effect of higher productivity A on non-traded sector employment η , since this relation plays a key role for the dynamic model of the next section. Rather than merging partial equilibrium effects, let us notice that by combining the labour market and the goods market equilibrium (11,13) and rearranging we obtain an equation in only one endogenous variable (i.e. the non-traded employment level η , since P cancels out whilst productivity A and capital stock K_N are given at each point in time in the static equilibrium):

$$\eta - \left(\frac{K_N}{A}\right)^{1-\alpha} \eta^\alpha \cdot \varphi\left(\alpha \left(\frac{A\eta}{K_N}\right)^{1-\alpha}\right) = \frac{(1-\alpha)(1-\delta)}{\delta} [R + (1-\eta)]. \quad (19)$$

This equation allows me to derive the general equilibrium response of η to a change in A :

$$\frac{\partial \eta}{\partial A} = \frac{\frac{(1-\alpha)\eta}{A} \left[\varphi' \alpha - \varphi(\cdot) \left(\frac{K_N}{A\eta}\right)^{1-\alpha} \right]}{1 + \frac{(1-\alpha)(1-\delta)}{\delta} - \alpha \left[\varphi(\cdot) \left(\frac{K_N}{A\eta}\right)^{1-\alpha} + \varphi'(1-\alpha) \right]}. \quad (20)$$

This derivative is crucial for the stability of the model. A negative sign will imply that sustained productivity growth will lead to the corner solution in which non-traded sector collapses. A positive response of the non-traded employment level η to a jump in productivity would instead be a convenient result since it would mean that, higher productivity A determines lower traded sector employment (the main source of productivity growth) and in turn a slowdown in productivity growth, thereby avoiding explosive productivity dynamics (precise analytical conditions for a positive sign are given in A2 in Appendix).

3.2.1 The role of the capital stock Let us now focus on the other factor of production which will play an important role in the dynamic analysis, namely the capital stock K_N . I begin by observing that a change in the amount of capital K_N will influence both curves of the static equilibrium. From the labor market equilibrium (11) I initially derive the partial equilibrium response of P to a change in K_N (for a given η):

$$\frac{\partial P}{\partial K_N} = \frac{\alpha}{\alpha - 1} (A\eta)^\alpha K_N^{-\alpha-1} < 0. \quad (21)$$

This negative derivative implies that higher capital level K_N would induce higher marginal productivity of labour in the non-traded sector which in turn requires a lower P to restore equilibrium. In other words, the LL curve shifts down to the right. The (partial equilibrium) response of P to a change in K_N (for a given η) coming from the goods market (13) is instead given by:

$$\frac{\partial P}{\partial K_N} = \frac{P \left[\frac{\varphi(\cdot)}{\alpha} + \varphi'(\cdot)(\alpha - 1) \left(\frac{A\eta}{K_N} \right)^{1-\alpha} - \left(\frac{A\eta}{K_N} \right)^{1-\alpha} \right]}{K_N^\alpha (A\eta)^{1-\alpha} - \varphi \left(\alpha \left(\frac{A\eta}{K_N} \right)^{1-\alpha} \right) K_N}. \quad (22)$$

Two separate effects are at work, one on production and one on investment. On one side, higher capital level K_N increases production X_N thus creating excess supply and requiring a depreciated P in the new static equilibrium. On the other side things are slightly more cumbersome since, by recalling the investment demand formulation in (6), we observe that higher capital level K_N has opposite effects on this demand. On one hand, higher capital level decreases its marginal productivity thereby diminishing investment demand, however on the other hand higher capital level enters directly the investment demand function and therefore increases it. In order to simplify I assume the decrease in marginal productivity to be stronger³

³ This happens to be the case as long as $\frac{\partial I_N}{\partial K_N} < 0$ which is analytically verified as long as $\varphi'(\cdot) > \frac{\varphi(\cdot)}{\alpha(1-\alpha)} \left(\frac{K_N}{A\eta} \right)^{1-\alpha}$. It is important to notice that this assumption does not necessarily need to hold for the total effect of capital stock on the exchange rate to be negative, since in any case the excess supply from production is likely to overcome the demand generated by the positive investment effect.

thereby concluding that higher capital level diminishes investment demand and contributes further to the excess supply of non-traded goods. Therefore the production and investment effects jointly imply a lower P and the NN curve shifts down to the left. As shown analytically by these results and in the figure below, I conclude that an increase in the non-traded capital stock unambiguously implies a depreciation effect for the exchange rate P . The economic intuition behind this result is that the increase in the stock of capital goods (by assumption home-grown capital) directly reduces the bottleneck effects faced by the booming resource economy, thereby determining a reversal of the eventual initial appreciation of the exchange rate caused by a resource windfall.

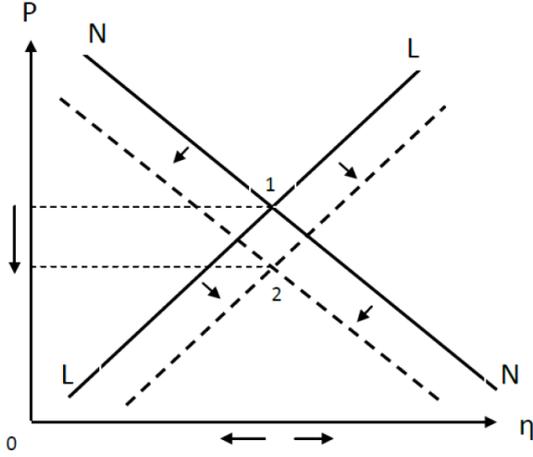


Figure 2 - Static effects of a capital boost

Fig.2 shows as well that the net effect of higher K_N on the employment level η appears ambiguous and is depending on the magnitude of the shifts of the two curves of the static diagram. There are two counteracting effects at work. As regards the labor market in (11), higher K_N implies higher marginal productivity of labour in the non-traded sector which requires higher non-traded employment η to restore equality. From the equilibrium in the goods market in (13) instead we observe that the production and investment effects of higher capital K_N call for a lower η in equilibrium. In order to analytically compare these counteracting effects, I use again (19) to derive the general equilibrium response of η to a change in K_N :

$$\frac{\partial \eta}{\partial K_N} = \frac{\frac{(1-\alpha)\eta}{K_N} \left[\varphi(\cdot) \left(\frac{K_N}{A\eta} \right)^{1-\alpha} - \varphi' \alpha \right]}{1 + \frac{(1-\alpha)(1-\delta)}{\delta} - \alpha \left[\varphi(\cdot) \left(\frac{K_N}{A\eta} \right)^{1-\alpha} + \varphi'(1-\alpha) \right]}. \quad (23)$$

The sign of this derivative is going to play as well a decisive role in the stability analysis of the dynamic model, therefore it deserves more attention.

The subsection A2 of the Appendix gives the precise analytical conditions for an overall negative sign of this derivative. I now proceed with the analysis of the dynamic model.

4 The dynamic model

The model consists of a system of two differential equations, one for the productivity and one for the capital stock. As regards the employment level η_t , a result of the static model was to highlight that η_t is at each point in time a function of productivity, capital stock and of the potential resource windfall $\eta_t = \eta(A_t, K_{Nt}, R_t)$. In other words, the employment level is a function only of state variables and of an exogenous variable, and will therefore be endogenously determined in the following dynamic model:

$$\dot{A}_t = A_t \cdot \gamma [1 - \eta_t(A_t, K_{Nt}, R_t)], \quad (24)$$

$$\dot{K}_{Nt} = \varphi \left(\alpha \left(\frac{A_t \eta_t(A_t, K_{Nt}, R_t)}{K_{Nt}} \right)^{1-\alpha} \right) \cdot K_{Nt}. \quad (25)$$

4.1 Consistency with static equilibrium

A first step is to investigate whether this system could display a dynamic equilibrium at which both productivity and the capital stock grow at a common rate g_t , i.e. a steady-state dynamic equilibrium with endogenous growth:

$$\frac{\dot{A}_t}{A_t} = \frac{\dot{K}_{Nt}}{K_{Nt}} = g_t. \quad (26)$$

In order to prove the existence of this equilibrium I start by verifying whether it is consistent with the static model, in other words verifying whether the dynamic equilibrium implies that the static market equilibrium equations be constant over time. Let us rewrite the static labour market equilibrium (11) as follows:

$$P_t = \frac{1}{1-\alpha} \left(\frac{A_t \eta_t}{K_{Nt}} \right)^\alpha. \quad (27)$$

This equilibrium equation will be constant over time as long as the ratio between productivity and capital stock $\frac{A_t}{K_{Nt}}$ stays constant (the employment level is also constant at its dynamic equilibrium level), which is indeed what the dynamic equilibrium with common growth rate (26) implies. As regards the goods market equilibrium (13), it can be rearranged as follows:

$$P_t = \frac{\left(\frac{1-\delta}{\delta} \right) [R_t + (1-\eta_t)]}{\left(\frac{K_{Nt}}{A_t} \right)^\alpha \eta_t^{1-\alpha} - \varphi \left(\alpha \left(\frac{A_t \eta_t}{K_{Nt}} \right)^{1-\alpha} \right) \frac{K_{Nt}}{A_t}}. \quad (28)$$

Again as above, the ratio between productivity and capital stock $\left(\frac{K_{Nt}}{A_t}\right)$ appearing now three times in the denominator of this market equilibrium equation will be constant under the dynamic equilibrium condition. This equilibrium equation will then be constant over time as well. In other words, I have shown that the dynamic steady-state with growth is consistent with the market equilibrium equations defining the static equilibrium.

4.2 Local stability and phase diagram

As anticipated in Section 2 when I described the possible corner solutions for productivity dynamics, I show here for the sake of completeness that the long-run equilibrium given in (26) is not the only possible long-run equilibrium of our dependent economy. By finding the isoclines for which $\dot{A}_t = 0$ we obtain:

$$\dot{A}_t = \gamma A_t - \gamma A_t \eta_t(A_t, K_{Nt}) = 0, \quad \Rightarrow \quad \bar{\eta} = 1, \quad (29)$$

$$\dot{A}_t > 0 \quad 0 \leq \eta_t(A_t, K_{Nt}) < 1. \quad (30)$$

As regards the other isocline $\dot{K}_N = 0$ we have:

$$\dot{K}_{Nt} = \varphi \left(\alpha \left(\frac{A_t \eta_t(A_t, K_{Nt}, R_t)}{K_{Nt}} \right)^{1-\alpha} \right) \cdot K_{Nt} = 0. \quad (31)$$

By definition of the investment demand function in (6) we remember that $\varphi(1) = 0$, therefore implying that:

$$\alpha \left(\frac{A_t \eta_t(A_t, K_{Nt})}{K_{Nt}} \right)^{1-\alpha} = 1, \quad \Rightarrow \quad K_N^* = \alpha^{\frac{1}{1-\alpha}} A \eta. \quad (32)$$

This long-run equilibrium point $[A^*, K_N^*, \bar{\eta} = 1]$ can be denoted as the "*fully closed economy*" equilibrium and turns out to be quite unrealistic since the entire labour force of the economy is employed in the non-traded sector. Productivity growth becomes null since the engine of growth given by the traded sector employment, is null as well. Note importantly that, in case a potential resource windfall shock leads the system to this equilibrium with null growth rates of productivity and capital, the effect of such a windfall can indeed be considered as a *disease* for the economy in the long-run (as in Sachs and Warner (1995)). Note that there is an additional solution, namely the "*fully open economy*" equilibrium in which all the labor force is employed in the traded sector and the non-traded sector collapses. Both these corner solutions will be left aside from now on.

Let us then proceed with the analysis of the equilibrium with endogenous growth. I rewrite the dynamic system (24,25) in terms of growth rates, as

follows:

$$\frac{\dot{A}_t}{A_t} = \gamma [1 - \eta_t(A_t, K_{Nt}, R_t)] = g_t(R_t), \quad (33)$$

$$\frac{\dot{K}_{Nt}}{K_{Nt}} = \varphi \left(\alpha [\Phi_t(R_t) \cdot \eta_t(A_t, K_{Nt}, R_t)]^{1-\alpha} \right) = g_t(R_t), \quad (34)$$

in which I redefined for future convenience and without loss of generality the state variables ratio between the productivity level and the non-traded capital stock as $\frac{A_t}{K_{Nt}} = \Phi_t(R_t)$.

The proof of stability is provided in the subsection A3 of the Appendix. The stability result (for any given g_t) is important inasmuch it allows in the next section to disentangle the transitional effect from a dynamic equilibrium to another, after the economy is subject to a resource boom. In other words, a locally stable dynamic equilibrium with endogenous growth indicates that the constant growth of productivity and capital stock keeps the economy on a balanced long-run growth path.

Let us once again exploit the results from the static model in order to compute the linear approximation of the isoclines for the non-linear dynamic system, as follows:

$$\left[\frac{\partial A_t}{\partial K_{Nt}} \right]_{\frac{\dot{A}_t}{A_t} = g_t} = - \frac{(\partial \eta_t / \partial K_{Nt})}{(\partial \eta_t / \partial A_t)} > 0, \quad (35)$$

$$\left[\frac{\partial A_t}{\partial K_{Nt}} \right]_{\frac{\dot{K}_{Nt}}{K_{Nt}} = g_t} = - \frac{\left(A_t \frac{\partial \eta_t}{\partial K_{Nt}} - \frac{A_t \eta_t}{K_{Nt}} \right)}{\left(\eta_t + A_t \frac{\partial \eta_t}{\partial A_t} \right)} > 0. \quad (36)$$

The dynamic steady-state equilibrium with endogenous growth can thus be graphically represented as follows⁴:

⁴ Looking exclusively at the derivation in (35, 36), I cannot infer which of the two isoclines has higher slope. In case (35) has higher slope than (36), the dynamic equilibrium would be a saddle point. In the opposite case, the dynamic equilibrium is locally stable. The results obtained from the stability analysis in A3 of the Appendix with negative trace and non-negative determinant allows me to disregard the former case and to plot the dynamic system as it appears in Fig.3.

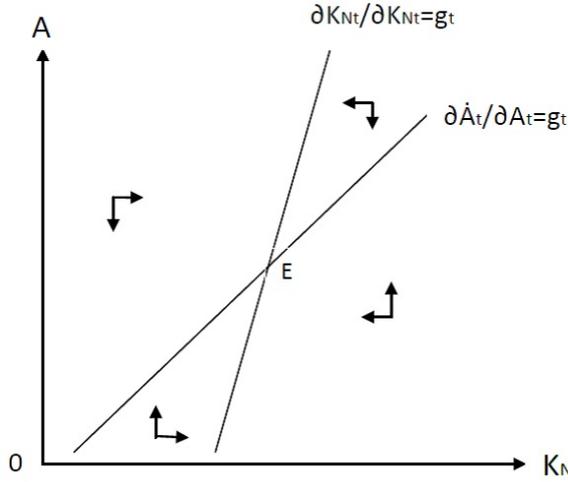


Figure 3 - The initial dynamic steady-state equilibrium

Importantly, the stability of the dynamic steady-state for any given g_t does not imply that the system will remain constantly at E . A resource windfall will shock the dynamic system and cause the transition to a new stable dynamic equilibrium, at which the state variables will display a different growth rate.

5 Dutch disease dynamics

The target of this section is to investigate the transitional and comparative dynamics effects of a permanent increase in the flow of the foreign exchange gift, and to determine the long-run outcomes for the exchange rate and the employment levels of our dependent economy.

5.1 The resource shock

As soon as the resource windfall hits the economy, the dynamic model is thrown out of the steady-state equilibrium with endogenous growth depicted in fig.3 at the point E . Let us observe how the isoelines of fig.3 react in turn to the exogenous shock. At first, let us compute the response of the productivity level A_t to the resource windfall implied by the productivity dynamics equation (33), for any given level of the capital stock:

$$\left[\frac{\partial A_t}{\partial R_t} \right]_{K_N} = - \frac{(\partial \eta_t / \partial R_t)}{(\partial \eta_t / \partial A_t)} > 0. \quad (37)$$

The isoeline related to the capital stock dynamic equation (34) will as well shift in response to the resource shock, precisely the response of the

capital stock level to the resource windfall for any given level of productivity will be given by:

$$\left[\frac{\partial K_{Nt}}{\partial R_t} \right]_{\bar{A}} = - \frac{\partial \left(\dot{K}_{Nt}/K_{Nt} \right) / \partial R_t}{\partial \left(\dot{K}_{Nt}/K_{Nt} \right) / \partial K_{Nt}} = - \frac{\varphi'(\cdot) \alpha (1 - \alpha) \left(\frac{\bar{A}}{K_{Nt}} \right)^{1-\alpha} \left(\frac{\partial \eta_t}{\partial R_t} \right)}{\eta_t^\alpha \left(\bar{A} \frac{\partial \eta_t}{\partial K_{Nt}} - \frac{\bar{A} \eta_t}{K_{Nt}} \right)} > 0, \quad (38)$$

where I have exploited the findings from the static model $(\partial \eta / \partial R) > 0$ and $\left(A_t \frac{\partial \eta_t}{\partial K_{Nt}} - \frac{A_t \eta_t}{K_{Nt}} \right) < 0$. These results allow me to state that the dynamic system after the resource shock will transit from the stable dynamic equilibrium E to the new stable dynamic equilibrium F as depicted in fig.4:

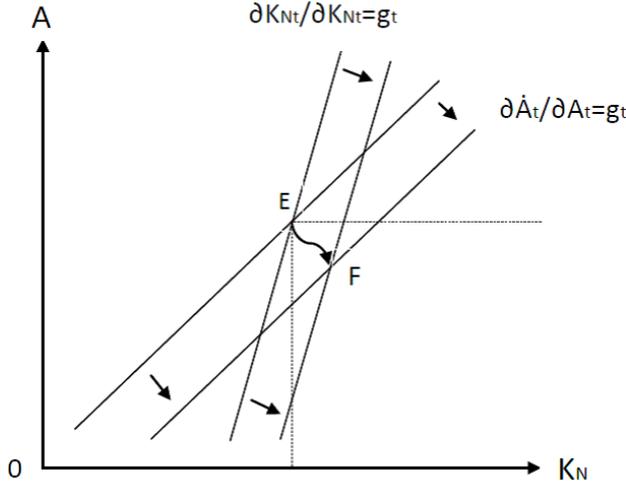


Figure 4 - The dynamic effects of a resource windfall

A note on the new dynamic steady-state equilibrium F . As it comes out from a close graphical investigation of the different possible shifts in the isoclines of fig.4, it can be observed that the new stable dynamic equilibrium point F might imply, depending on the respective magnitudes of the shifts, both a new higher (lower) level for productivity and a new higher (lower) level for the capital stock. For example, a strong downwards shift of the isocline $\left[\frac{\dot{K}_{Nt}}{K_{Nt}} = g_t \right]$ together with a weak response of the isocline $\left[\frac{\dot{A}_t}{A_t} = g_t \right]$ would determine that in the new dynamic equilibrium both productivity level A_t and capital stock K_{Nt} are higher. As opposed to that, a weak downwards shift of the isocline $\left[\frac{\dot{K}_{Nt}}{K_{Nt}} = g_t \right]$ coupled with a strong response of the isocline $\left[\frac{\dot{A}_t}{A_t} = g_t \right]$ would determine that in the new dynamic equilibrium both productivity level A_t and capital stock K_{Nt} are lower than before the resource shock.

The decisive result I can however infer from the transition to the new dynamic equilibrium is that the *ratio* Φ_t of the productivity level with respect to the non-traded capital stock level has in any case decreased in F with respect to its level under the initial equilibrium E (the analytical proof behind this statement is shown in A4 of the Appendix). In other words, the dynamic effect of a resource shock works in the sense of reducing the "gap" between labor productivity and the amount of domestic capital goods accumulated in the economy.

The result $[\partial\Phi_t/\partial R_t] < 0$ of a reduced ratio is not after all unexpected, given the structure of the model. Productivity growth slows down due to a decline on impact of the traded employment share $(1 - \eta_t)$ caused by the resource windfall as in (18). On the other hand, the positive impact response of the speed of accumulation of non-traded capital $\left[\partial \left(\dot{K}_{Nt}/K_{Nt} \right) / \partial R_t\right] > 0$ shows that the resource windfall has the immediate effect of stimulating the production of non-traded goods and thereby relaxing the bottlenecks limitation of the economy.

5.2 *Dynamic responses of labor allocation and exchange rate*

This section aims at providing an answer to the research question of the present paper by investigating how the employment levels and the exchange rate react to the resource windfall in a fully dynamic setting. As opposed to the static results derived in section 3.1, in which we could abstract from the intermediate effects of resources on the state variables (A_t and K_{Nt} were given at each point in time), these effects have to be taken into account here. Speaking in terms of the diagram of fig.1, the effect of resources will now determine a shift for both the LL and the NN curve. Recall for instance that the result from the static model in (18) and fig.1, implied that a new static equilibrium with increased foreign exchange gift induces a factor re-allocation towards the non-traded sector of the economy. By fully incorporating the dynamic effects of resource on the state variables and in turn on equilibrium employment, I will now try to evaluate whether in the long-run the non-traded employment level will revert towards its initial equilibrium level or it will instead be permanently altered. As regards the exchange rate, the conventional result both in the static section of the current paper and in the wide literature on the Dutch disease is that the foreign exchange gift causes a real appreciation as a response to higher demand for non-traded goods in the economy. This result has not always been supported by the empirical evidence creating a "puzzle" about the relation between resources and relative prices of goods in a dependent economy. In addition, Torvik (2001) and van Wijnbergen (1984) have developed dynamic models showing precisely that, by adding endogenous relative productivity as an additional determinant of the real exchange rate, the conventional appreciation result is turned around.

Let us start by recalling that in the initial dynamic equilibrium (for example at point E in fig.3) we have that $\left[\frac{\dot{A}_t}{A_t} = g_t = \frac{\dot{K}_{Nt}}{K_{Nt}}\right]$, thus the dynamic system of two differential equations can be merged as to get a single equation that could be solved to find the equilibrium value (for example at E) of the non-traded sector employment level:

$$\gamma [1 - \eta_E^*(R_t)] = \varphi \left(\alpha [\Phi_t(R_t) \cdot \eta_E^*(R_t)]^{1-\alpha} \right). \quad (39)$$

However, rather than focusing on solving for this stable (although not constant to changes in R_t) equilibrium value for the labor allocation of the economy, I proceed by totally differentiating this equation in order to observe the general equilibrium dynamic response of the non-traded employment level to the resource windfall:

$$\frac{\partial \eta^*}{\partial R_t} = - \frac{\varphi'(\cdot) \alpha (1 - \alpha) (\Phi_t)^{-\alpha} \eta_t^{1-\alpha}}{2 \left[\gamma + \varphi'(\cdot) \alpha (1 - \alpha) (\Phi_t)^{1-\alpha} \eta_t^{-\alpha} \right]} \left(\frac{\partial \Phi_t}{\partial R_t} \right) > 0, \quad (40)$$

in which I used the previous section's crucial result $[\partial \Phi_t / \partial R_t] < 0$ to determine the overall positive sign of the derivative. This derivative is fully dynamic in the sense that includes all the endogenous variation by incorporating the intermediate effects of resource shock on the ratio of the state variables. The result of positive sign shows that the resource windfall alters permanently the equilibrium level of the labor allocation, confirming also in the present dynamic setting the factor reallocation of labor from the traded to the non-traded sector of the economy. This finding differs from the results in Sachs and Warner (1995) and Torvik (2001) in which the non-traded employment level would instead revert towards its long-run steady-state equilibrium after a temporary increase. In their models, the resource windfall does not have any permanent effect on the labour allocation of the economy and therefore it does not induce detrimental growth consequences in the long-run.

Let us incorporate this result in the same diagram as in fig.1. As anticipated above, the resource windfall will determine a shift for both the LL and the NN curve. At first, by re-arranging the static labour market equilibrium (11) as $\Phi_t(R_t) = \eta_t(R_t) \cdot [P_t(1 - \alpha)]^{-1/\alpha}$ and computing the effect of resource windfall on the exchange rate P_t for a given η_t , gives:

$$\left[\frac{\partial P_t}{\partial R_t} \right]_{\bar{\eta}} = - \frac{(\partial \Phi_t / \partial R_t)}{(\partial \Phi_t / \partial P_t)} = \frac{\alpha \bar{\eta} \cdot (\partial \Phi_t / \partial R_t)}{[P^{1-\alpha} (1 - \alpha)]^{1/\alpha}} > 0, \quad (41)$$

which implies that the LL curve shifts up to the left. The previous general equilibrium result of (40) together with this shift of the LL curve, already allows us not only to infer that the NN curve will shift up to the right as for the static model, but also that the magnitude of the shift of this curve will be at least enough as to ensure that the new dynamic equilibrium implies indeed a higher non-traded employment level for the economy. The

immediate implication as regards the exchange rate is that its dynamic general equilibrium response to the resource windfall is inevitably that of a permanent appreciation⁵. The static diagram of fig.1 can be therefore revisited in the present dynamic setting as such:

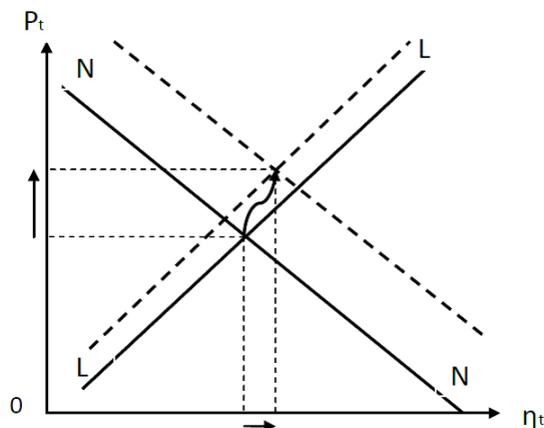


Figure 5 - The dynamic effects of resource windfall

A note on the several mechanisms at work behind this fig.5 and more in general. In the present model, the result of exchange rate appreciation obtained both in the static and dynamic setting, can be explained by the combination of increased demand for non-traded goods coupled with a non-traded sector absorption constraint. Subsequently, this increased demand of non-traded goods boosts a capital accumulation process that allows a gradual relaxation of the initial absorption constraint. As opposed to van der Ploeg and Venables (2013), this process of accumulation of non-traded capital goods is accompanied here by a parallel learning process of productivity growth. On one side, higher productivity contributes as well to increased production. On the other side, productivity growth increases the flow of the foreign exchange gift and thereby boosts aggregate demand. All in all, capital accumulation and productivity growth keep the economy on a stable balanced growth path and do not induce in the longer run a reversal of the initial exchange rate appreciation as in van der Ploeg and Venables (2013).

As regards the complementary effect of resource windfall on the labor allocation of the economy, the current model predicts a factor reallocation towards the non-traded sector, both in the static and in the dynamic setting.

⁵ The general equilibrium derivation of $[\partial P_t / \partial R_t]$ in the present dynamic setting gives a derivative with undefined overall sign, due to production and investment effects pointing in different directions. The analytical formulation is of course available on request. I have carefully verified that, by imposing for instance an overall positive sign for $[\partial P_t / \partial R_t]$, no inconsistency is found with respect to each and one of the analytical conditions assumed to hold for the derivative of the investment function, as given in (16,42).

In turn, higher non-traded equilibrium employment has on one side the effect of slowing down productivity growth. On the other side, it relaxes the absorption constraint by increasing supply of non-traded goods and thereby slowing down the pace of capital accumulation. All in all, the factor reallocation of labor away from the traded sector and growth engine of the economy indicates that resource booms can have detrimental growth consequences in the longer run.

6 Concluding remarks

This study revisited the macroeconomic mechanisms behind the Dutch Disease phenomenon by working out the general equilibrium effects of a resource boom both in a static and dynamic setting. The intention behind this research was to provide new theoretical insights by merging two distinct features of the Dutch disease economic literature in a coherent and simplified framework: capital accumulation as in van der Ploeg and Venables (2013) on one side, productivity growth induced by learning-by-doing as in Sachs and Warner (1995) on the other. The theoretical results obtained in this work differ from the previous papers on which the current model builds. More precisely, the current model followed van der Ploeg and Venables (2013) in assuming a capital stock absorption constraint which to a large extent induced a short-run appreciation of the exchange rate after the resource boom. However, in van der Ploeg and Venables (2013) the subsequent gradual increase in the capital stock "cools down" the economy allowing the initial exchange rate appreciation to be reverted in the long-run, whilst the additional assumption of learning-by-doing employed by the current model allows to maintain the conventional long-run appreciation result in line with a large part of the Dutch Disease literature. In addition, as regards the complementary effect of resources on the labor allocation, the current model predicts a factor reallocation effect towards the non-traded sector, both in the static and in the dynamic setting. In other words, the crowding out of labor away from the traded sector and growth engine of the economy indicates that resource booms can indeed have detrimental growth consequences in the longer run. This result differs from the dynamic models of Sachs and Warner (1995) and Torvik (2001) in which the non-traded employment level would instead revert towards its long-run steady-state equilibrium after a temporary increase. In conclusion, the present paper has shown both statically and dynamically that the set of classic symptoms of the Dutch Disease (real exchange rate appreciation and crowding out of the manufacturing sector) remain in place.

A Appendix

[A1] The representative household endowed with Cobb-Douglas utility function maximizes the static utility $u(C_N, C_T) = C_T^\delta C_N^{1-\delta}$ subject to the static version of the aggregate income constraint given in (7):

$$\begin{aligned} & \max_{(C_N, C_T)} C_T^\delta C_N^{1-\delta}, \\ \text{s.t.} \quad & PC_N + C_T = AR + X - PI_N. \end{aligned}$$

Setting the Lagrangian Γ and computing the first order conditions, the solution to this static problem is:

$$\begin{aligned} \Gamma &= \delta \log C_T + (1 - \delta) \log C_N - \lambda(PC_N + C_T), \\ [C_T] \quad & \frac{\delta}{C_T} = \lambda \quad [C_N] \quad \frac{1 - \delta}{C_N} = P\lambda, \\ & C_N = \left(\frac{1 - \delta}{P} \right) C, \quad C_T = \delta C. \end{aligned}$$

[A2] Let us give a closer look at the overall sign of the derivatives in (20,23):

$$\begin{aligned} \frac{\partial \eta}{\partial A} &= \frac{\frac{(1-\alpha)\eta}{A} \left[\varphi' \alpha - \varphi \left(\frac{K_N}{A\eta} \right)^{1-\alpha} \right]}{1 + \frac{(1-\alpha)(1-\delta)}{\delta} - \alpha \left[\varphi \left(\frac{K_N}{A\eta} \right)^{1-\alpha} + \varphi'(1-\alpha) \right]}, \\ \frac{\partial \eta}{\partial K_N} &= \frac{\frac{(1-\alpha)\eta}{K_N} \left[\varphi \left(\frac{K_N}{A\eta} \right)^{1-\alpha} - \varphi' \alpha \right]}{1 + \frac{(1-\alpha)(1-\delta)}{\delta} - \alpha \left[\varphi \left(\frac{K_N}{A\eta} \right)^{1-\alpha} + \varphi'(1-\alpha) \right]}. \end{aligned}$$

The common denominator of both derivatives is always positive since:

$$\begin{aligned} & \varphi \left(\frac{K_N}{A\eta} \right)^{1-\alpha} + \varphi'(1-\alpha) < 0, \\ & \varphi' > - \frac{\varphi \left(\frac{K_N}{A\eta} \right)^{1-\alpha}}{(1-\alpha)}, \end{aligned}$$

which is always true since by definition $\varphi' > 0$. As regards the numerator, we observe that as long as the following condition holds:

$$\varphi' > \frac{\varphi \left(\frac{K_N}{A\eta} \right)^{1-\alpha}}{\alpha}, \quad (42)$$

we can determine the overall signs of both derivatives and conclude that $\frac{\partial \eta}{\partial A} > 0$ and $\frac{\partial \eta}{\partial K_N} < 0$. Notice that this condition is *not* inconsistent with the condition given in (16).

[A3] Dynamic stability analysis. At first, by totally differentiating (33,34) and exploiting the convenient result that $(\partial g_t / \partial R_t) = 0$, the dynamic system can be rewritten as:

$$\begin{aligned} -\frac{\partial \eta_t}{\partial A_t} - \frac{\partial \eta_t}{\partial K_{Nt}} &= 0, \\ \left[\eta_t + A_t \frac{\partial \eta_t}{\partial A_t} \right] + \left[A_t \frac{\partial \eta_t}{\partial K_{Nt}} - \frac{A_t \eta_t}{K_{Nt}} \right] &= 0. \end{aligned}$$

Let us then insert these derivatives into the Jacobian J and evaluate it at the dynamic steady-state (26) (for any given g_t):

$$J = \begin{vmatrix} -\frac{\partial \eta_t}{\partial A_t} & -\frac{\partial \eta_t}{\partial K_{Nt}} \\ \eta_t + A_t \frac{\partial \eta_t}{\partial A_t} & A_t \frac{\partial \eta_t}{\partial K_{Nt}} - \frac{A_t \eta_t}{K_{Nt}} \end{vmatrix}$$

By recalling from the static model and from the section A2 of the Appendix that $\frac{\partial \eta}{\partial A} > 0$ and $\frac{\partial \eta}{\partial K_N} < 0$ we can immediately evaluate that:

$$tr(J) = -\frac{\partial \eta_t}{\partial A_t} + \left[A_t \frac{\partial \eta_t}{\partial K_{Nt}} - \frac{A_t \eta_t}{K_{Nt}} \right] < 0.$$

The trace is unambiguously negative. The determinant is instead given by:

$$\begin{aligned} \det(J) &= -\frac{\partial \eta_t}{\partial A_t} \left[A_t \frac{\partial \eta_t}{\partial K_{Nt}} - \frac{A_t \eta_t}{K_{Nt}} \right] + \frac{\partial \eta_t}{\partial K_N} \left[\eta_t + A_t \frac{\partial \eta_t}{\partial A_t} \right], \\ \det(J) &= \eta_t \left[\frac{A_t}{K_{Nt}} \frac{\partial \eta_t}{\partial A_t} + \frac{\partial \eta_t}{\partial K_N} \right]. \end{aligned}$$

Let us now substitute for the analytical expression of the two derivatives (20,23). Redefine for convenience the positive common denominator as $\Psi = 1 + \frac{(1-\alpha)(1-\delta)}{\delta} - \alpha \left[\varphi(\cdot) \left(\frac{K_N}{A\eta} \right)^{1-\alpha} + \varphi'(\cdot)(1-\alpha) \right]$ and rewrite $\det(J)$ as:

$$\begin{aligned} \det(J) &= \frac{\eta_t}{\Psi} \left\{ \frac{A_t}{K_{Nt}} \frac{(1-\alpha)\eta_t}{A_t} \left[\varphi'(\cdot)\alpha - \varphi(\cdot) \left(\frac{K_{Nt}}{A_t \eta_t} \right)^{1-\alpha} \right] + \right. \\ &\quad \left. \frac{(1-\alpha)\eta_t}{K_{Nt}} \left[\varphi(\cdot) \left(\frac{K_{Nt}}{A_t \eta_t} \right)^{1-\alpha} - \varphi'(\cdot)\alpha \right] \right\}, \\ \det(J) &= \frac{(1-\alpha)\eta_t^2}{\Psi K_{Nt}} \left\{ \left[\varphi'(\cdot)\alpha - \varphi(\cdot) \left(\frac{K_{Nt}}{A_t \eta_t} \right)^{1-\alpha} \right] + \left[\varphi(\cdot) \left(\frac{K_{Nt}}{A_t \eta_t} \right)^{1-\alpha} - \varphi'(\cdot)\alpha \right] \right\} = 0. \end{aligned}$$

[A4] Let us analyze closely the change in the *ratio* of the productivity level with respect to the non-traded capital stock level Φ_t , after the resource shock. As it can be seen from fig.4 in the paper and the fig.6 below, the information at our disposal is that the new dynamic equilibrium F will in any case lay in the area down to the right of the two initial isoclines. The border of this area is marked by thicker isoclines:

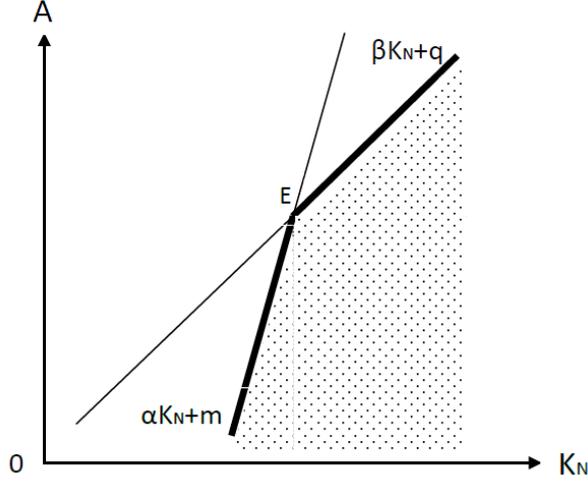


Figure 6

Let us redefine for convenience, in the most general case, the two isoclines as such (with $A > 0, K_N > 0$):

$$\begin{aligned} A &= \alpha K_N + m, & \alpha > 0, \\ A &= \beta K_N + q, & \beta > 0, \\ \alpha &> \beta, & q > m. \end{aligned}$$

This allows me to start computing the ratio at the initial dynamic equilibrium in E :

$$\Phi_E = \frac{\alpha K_N + m}{K_N} = \frac{\beta K_N + q}{K_N}.$$

In order to cover all the possible outcomes for the new ratio between productivity and the capital level, let us consider the two following "corner solutions", in which all the possible new equilibria lay to the right of (but infinitely close to) the thicker parts of the isoclines:

$$\begin{aligned} \Phi'_F(A', K'_N) &= (A' = \alpha K'_N + m; \quad K'_N = K_N + \varepsilon), \\ \Phi''_F(A'', K''_N) &= (A'' = \beta K''_N + q - \varepsilon; \quad K''_N = K_N), \end{aligned}$$

where $\varepsilon > 0$ is infinitely small. It is easy to show that, for both of these cases:

$$\begin{aligned} \Phi'_F &= \frac{\alpha K_N + m}{K_N + \varepsilon} = \frac{\beta K_N + q}{K_N \left(1 + \frac{\varepsilon}{K_N}\right)} = \Phi_E \left(\frac{1}{1 + \frac{\varepsilon}{K_N}} \right) < \Phi_E, \\ \Phi''_F &= \frac{\beta K_N + q - \varepsilon}{K_N} = \frac{(\beta K_N + q) \left(1 - \frac{\varepsilon}{\beta K_N + q}\right)}{K_N} = \Phi_E \left(1 - \frac{\varepsilon}{\beta K_N + q} \right) < \Phi_E, \end{aligned}$$

which completes the proof. The ratio between productivity and capital stock in the new dynamic equilibrium has in any case decreased after the resource shock, with respect to the initial dynamic equilibrium [$\Phi_F < \Phi_E$].

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