

Volume 33, Issue 3

Uniform price auctions with profit maximizing seller

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Abstract

We study multiunit uniform price auctions where the seller is allowed to decrease the quantity supplied in order to maximize his profit. We show that he never chooses to do so in equilibrium. However, the existence of this option eliminates such equilibria where objects for sale are sold for too low a price. Our model explains the size of underpricing in Treasury auctions and provides guidance for the design of uniform price auctions.

I would like to thank Milan Bašta, Gernot Doppelhofer, Espen Henriksen, Lukáš Laffèrs, Michal Zdėnek and anonymous referee for helpful comments and the PURELEC project for the financial support.

Citation: Peter Molnár, (2013) "Uniform price auctions with profit maximizing seller", *Economics Bulletin*, Vol. 33 No. 3 pp. 1840-1846.

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Submitted: August 18, 2012. **Published:** July 15, 2013.

1 Introduction

A multiunit auction is an auction in which several identical items are sold. Multiunit auctions are widely used particularly in financial and commodity markets, including markets for electricity, emission permits, timber from national forests, FCC sale of spectrum rights, initial public offerings (IPOs) and Treasury auctions. In terms of traded volume, the most important multiunit auctions are Treasury auctions. The units sold in multiunit auction can be sold either each at the same price (a uniform price auction) or at prices offered by the bidders (a discriminatory price auction).

It was argued by Friedman (1960) that uniform price auction should result in higher revenue, because it attracts more bidders.¹ On the other hand, as pointed out by Wilson (1979) and extended by Back and Zender (1993), uniform price auction allows for so-called underpricing equilibria, where the object is sold for only a fraction of its true value. This underpricing can be arbitrarily large and exists even when supply is uncertain. However, these equilibria are not observed in the real world. This puzzle is addressed in Back and Zender (2001) and Kremer and Nyborg (2004). Both show that some of the underpricing equilibria are eliminated: in Back and Zender (2001) due to the revenue maximization of the seller and in Kremer and Nyborg (2004) due to the discreteness in prices and quantities. However, both these explanations still allow for much larger underpricing than is observed, as documented e.g. in Keloharju et al. (2005).

This paper studies possible equilibrium outcomes in uniform price auctions when the seller decides how much to sell in order to maximize his profit. Our results show that possible underpricing becomes much smaller than Back and Zender (2001) suggest. This is much more in line with real world observations. The model of Back and Zender (2001) is a special case of the situation of the price of objects sold outside the auction set to zero.

Another paper investigating a profit maximizing behavior of the seller in the uniform price auction is McAdams (2007). He shows that when the seller has continuously increasing marginal costs, adjustable supply eliminates all underpricing equilibria. However, there is a big difference between his and our model. In our model the seller sells only a limited quantity of the items, whereas McAdams (2007) implicitly assumes unlimited supply. In his model, the seller increases the supply as long as the price is above his marginal cost. The reasoning of McAdams (2007) does not apply when supply is limited, which is the most typical case in real world.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 discusses the results and Section 4 concludes.

2 The model

The model is based on Back and Zender (2001). There is a single seller and n bidders. The seller is offering \bar{Q} units of a perfectly divisible good for sale, but he might reduce this quantity after observing the bids. The value of the object v is common knowledge for the

¹Less informed (or less sophisticated) bidders can lose more in discriminatory price auctions because they pay what they bid. On the other hand, the total price paid in uniform price auction is less sensitive to one's own demand schedule, because there is only one clearing price.

bidders. Bidders maximize their expected profit. Each bidder i submits a non-increasing left-continuous demand function $q_i = [0, \infty) \rightarrow [0, \bar{Q}]$. The auction is a standard uniform price auction. The stop-out price s is the maximum price at which demand equals (or exceeds) the supply Q

$$s(Q) = \max \left\{ p \mid \sum_{i=1}^n q_i(p) \geq Q \right\}. \quad (1)$$

All bids above or at the stop-out price are accepted. If demand exceeds the supply at the stop-out price², the seller satisfies all the bids above the stop-out price and the bids at the stop-out price are satisfied only partially, proportional to their size. This (pro rata rationing) rule is most commonly used in the real world. This means that the quantity received by bidder i is

$$\theta_i(Q) = q_i(s(Q)) - \Delta q_i(p) \frac{\sum_{i=1}^n q_i(s(Q)) - Q}{\sum_{i=1}^n \Delta q_i(s(Q))}, \quad (2)$$

where $\Delta q_i(p) = q_i(p) - \lim_{p' \searrow p} q_i(p')$ is the bidder i 's marginal demand and $\sum_{i=1}^n \Delta q_i(s(Q))$ is the total marginal demand.

The seller has an option to sell the goods outside the auction for the price p_{min} ³. After observing the bids, the seller chooses the quantity $Q \leq \bar{Q}$ for sale to maximize his profit $\sum_{i=1}^n (s(Q) - p_{min}) \theta_i(Q)$.

As described above, the whole game consists of two stages. At first stage, bidders choose their strategies (submit their demand functions). At second stage, seller observes the bids and chooses his strategy (quantity Q). If seller does not observe the bids, the game reduces to one stage. By equilibrium we mean symmetric pure strategy equilibrium as usually defined. We say that the seller is not acting strategically if he determines the quantity for sale before observing the bids, whether it is a pre-specified quantity \bar{Q} known to the bidders or a random quantity $Q \leq \bar{Q}$ not known to the bidders. Underpricing is defined as the difference between the value of the object v and the price p bidders pay in the auction. Before showing our finding in Theorem 3, we summarize existing literature in Theorem 1 and Theorem 2.

Theorem 1 (Back and Zender 1993)

In the uniform price auction there exist equilibria with arbitrary large underpricing if the seller is not acting strategically.

Intuitively, the underpricing equilibria exist because the demand schedule in those equilibria is very steep. If the bidder wants a higher quantity, the bidder must bid higher for the marginal unit sold in the auction. However, since this is a uniform price auction, increased price for the marginal unit translates into increased price for all the units. If the demand curve is very steep, the loss caused by increased price (the bidder would have to pay higher price for all the units he buys) would overweight the gain from obtaining additional unit(s).

²This can happen when there is a discontinuity at the stopout price in the demand curve of at least one of the bidders.

³ p_{min} can be equivalently interpreted as the per unit cost of producing the good.

Theorem 2 (Back and Zender 2001)

Assume that the seller chooses quantity $Q \leq \bar{Q}$ after observing the bids to maximize revenue. Only those equilibria remain where the stop-out price is at least $(n-1)v/n$. In equilibrium $Q = \bar{Q}$.

Intuitively, many of the underpricing equilibria are eliminated, because the demand schedule cannot be arbitrarily steep. If it is very steep, the seller would sell a smaller quantity but at much higher price, resulting in higher revenue. Let us consider the following example. Seller can sell 10 items at price 10 per item. However, if demand curve is very steep (e.g. decrease in number of items sold from 10 to 9 leads to increase in price per piece from 10 to 12), seller prefers to sell less items at higher price (9 items at price 12), since this choice gives him higher revenue (108 instead of 100). This happens when demand curve is very steep. If demand curve would not be steep enough (e.g. decrease in number of items sold from 10 to 9 would lead to increase in the from 10 to only 11), revenue maximizing seller would prefer to sell the whole quantity.

Theorem 3

Assume that the seller chooses quantity $Q \leq \bar{Q}$ after observing the bids to maximize his profit. Only those equilibria remain where the stop-out price is at least $\frac{n-1}{n}v + \frac{1}{n}p_{\min}$. In equilibrium $Q = \bar{Q}$.

Intuition is similar to that in Theorem 2. The difference is that profit is more sensitive to the price than revenue. Let us consider the previous example, but remember that seller now maximizes his profit. Assume that production cost of every item is 5. In this case, even moderately steep demand curve (e.g. such that decrease in quantity from 10 to 9 leads to increase in price from 10 to 11) would result in decrease in quantity sold. Selling 9 items instead of 10 brings smaller revenue ($9 \times 11 = 99$ instead of $10 \times 10 = 100$), but higher profit ($9 \times (11 - 5) = 54$ instead of $10 \times (10 - 5) = 50$). Therefore, profit maximization eliminates several of the outcomes which would be plausible under revenue maximization.

Proof.

Let us denote equilibrium values of variables with asterisk. We are interested in underpricing equilibria ($p^* < v$). The formula for maximal possible underpricing is derived considering two equilibrium conditions: the seller cannot increase his profit by selling less (and increasing price) and no bidder can increase his profit by offering a higher price to get a larger quantity. The equilibrium condition for the seller is $\forall Q \leq \bar{Q} : (p^* - p_{\min}) Q^* \geq (s(Q) - p_{\min}) Q$, which can be rewritten as:

$$\frac{p^* - p_{\min}}{Q} \geq \frac{s(Q) - p^*}{Q^* - Q}. \quad (3)$$

When every bidder uses equilibrium strategy q^* , the bidder's profit is $(v - p^*)\bar{Q}/n$. When the bidder deviates and offers a higher price $p^* + \epsilon$, the bidder's profit becomes $(v - p^* - \epsilon)(\bar{Q} - (n-1)q^*(p^* + \epsilon))$. The equilibrium condition for the bidder is therefore

$$(v - p^*)\bar{Q}/n \geq (v - p^* - \epsilon)(\bar{Q} - (n-1)q^*(p^* + \epsilon)). \quad (4)$$

Let us denote $Q \equiv nq^*(p^* + \epsilon)$. Then by definition of the stop-out price $s(Q) = p^* + \epsilon$.

Condition (4) can be therefore written as

$$\frac{(v - p^*)\bar{Q}}{n} \geq (v - s(Q)) \left(\bar{Q} - \frac{(n-1)}{n}Q \right) \quad (5)$$

and rewritten as

$$\frac{s(Q) - p^*}{\bar{Q} - Q} \geq \frac{v - p^*}{\frac{n}{n-1}\bar{Q} - Q}. \quad (6)$$

Before finishing the proof, we need to show two additional facts. First, let us show that $Q^* = \bar{Q}$ in equilibrium. If $Q^* < \bar{Q}$, the seller would prefer to sell the whole quantity if he can do this by decreasing the price only slightly. In other words, $(p^* - p_{\min})Q^* \leq (p^* - \epsilon - p_{\min})\bar{Q}$ holds for ϵ sufficiently small. Any bidder of course prefers a higher allocation at a lower price. Therefore $Q^* < \bar{Q}$ cannot hold in equilibrium.

Second, let us show that aggregate demand $Q \equiv nq^*(p)$ converges to \bar{Q} as the price converges to p^* . If it would converge to some smaller number Q' , one of the bidders would offer only a slightly higher price to get all the additional quantity $\bar{Q} - Q'$, because a higher allocation at a (only marginally) smaller price would increase his profit. The seller of course prefers to sell more at a higher price.

Now we can finish the proof. Conditions (3) and (6) together with $Q^* = \bar{Q}$ imply

$$\frac{p^* - p_{\min}}{Q} \geq \frac{v - p^*}{\frac{n}{n-1}\bar{Q} - Q}, \quad (7)$$

which can be rewritten as

$$p^* \geq \left(\frac{n-1}{n}v + \frac{1}{n}p_{\min} \right) \frac{Q}{\bar{Q}}. \quad (8)$$

Since Q converges to \bar{Q} as price converges to p^* , the limit for equilibrium underpricing is $\frac{n-1}{n}v + \frac{1}{n}p_{\min}$. \square

3 Discussion

Here we discuss how well our model describes the real world. Since the most important (and the most studied) uniform price auctions are Treasury auctions, we focus on them.

Keloharju, Nyborg, Rydqvist (2005) find an underpricing of 0.041% in Finnish Treasury Auctions. Nyborg and Sundaresan (1996) find that yield markup in US Treasury auctions varies depending on the way how it is measured, but typically stays in the range 0.01% – 0.1%. Such a small underpricing is unexpected as the standard model of uniform price auctions tells us that an arbitrary large underpricing is possible. This puzzle is addressed by Back and Zender (2001) and Kremer and Nyborg (2004). For 10 bidders, which is a reasonable assumption for Treasury auctions, the model of Back and Zender (2001) allows for underpricing as large as 10%; underpricing possible in the model of Kremer and Nyborg (2004) is (said in their own words) “bounded above by approximately 1% of face value. Empirically this would be considered a “large” underpricing...”. Therefore none of the existing models is able to explain this phenomenon sufficiently. Now we discuss whether our model fits the real world better.

To find the largest possible underpricing allowed by our model we must estimate the price which the seller could sell the bonds for outside the auction (the discount). Given the high quality of the Treasury as the borrower, it seems reasonable to assume that the bond could be easily sold at the discount 1% (or even much less) of the true value of the bond outside the auction. In this case, from Theorem 3, it follows that the highest possible underpricing in the auction with 10 bidders is 0.1%.

Our model fits the real world not only in terms of correct predictions, but is based on sound theoretical considerations. If there is a cost associated with goods sold, revenue maximization is not optimal and it can easily result in the goods being sold under the production cost. There is an obvious cost associated with the Treasury bonds: they must be repaid later. Therefore, the Treasury need not care only about raising as much revenue as possible, but at the same time about paying back as little as possible in the future. An alternative reasoning for the behavior of the Treasury is that it (potentially) has alternative ways of borrowing money and would therefore not accept too low a price. Both these considerations are consistent with the profit maximization, but not with revenue maximization. Moreover, there is a direct empirical evidence against revenue maximization. Keloharju, Nyborg and Rydqvist (2005) test revenue maximization in Finnish Treasury Auctions and find that "...revenue maximizing level is not picked as the stop-out price, something which holds true in each and every auction in our sample. Thus, the Treasury does not follow the strategy studied by Back and Zender (2001)...". On the contrary, profit maximization seems to be good explanation of the behavior of the Treasuries around the world.

4 Conclusion

The underpricing (sale for too low price) observed in multiunit uniform price auctions is much smaller than what could be explained by existent literature. Our model fills in this gap. It shows that if the profit maximizing seller has an option to reduce the quantity sold, then the equilibrium underpricing must be indeed small. The best example of this are Treasury auctions. Treasuries around the world often have an option of reducing the quantity sold after receiving the bids and some of them, e.g. Mexico, Italy, Finland (see Umlauf (1993), Scalia (1997) and Keloharju, Nyborg, Rydquist (2005)) sometimes utilize this option. Even if the seller does not formally have this option, as is the case of US Treasury, bidders understand that the seller would change the rules if they (the bidders) play equilibria with large underpricing. Thus, the possible strategic behavior of the seller is the main reason for the non-existence of equilibria with large underpricings.

This result has important practical implications. To avoid large underpricings in uniform price auctions the seller should have an option to act strategically (to make final decision about quantity sold only after he receives the bids). Having this option does not mean that he will need to utilize it. In equilibrium, buyers understand this, adjust their bidding behavior and seller does not need to utilize this option (but still benefits from having it).

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