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Single Vehicle Flexible and Selective Delivery Routing Problem in Offshore Bulk Shipping

Cutting Costs and Improving Planning by Use of Optimization

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Preface

This is the thesis for my master's degree at the Norwegian University of Science and Technology (NTNU), Department of Marine Technology. It was written during the spring of 2017 in the course TMR4930.

I have worked as a logistical coordinator for Statoil ASA at their supply base in Dusavik, Stavanger in the summer of 2015 and 2016. In this thesis, I have used experience from this time. I want to thank my colleagues from the time spent there and for help with my questions on this topic.

I would like to thank representatives from Statoil ASA for providing the initial problem and historical data used in this thesis. It has been an interesting topic to write a thesis on. The problem description can be found in appendix A.1 Problem Description.

I would also like to thank my supervisor Bjørn Egil Asbjørnslett for interesting discussions, suggestions and feedback. Further, I want to thank Kjetil Fagerholt for modelling and literature suggestions.

At last, but not least, I want to express my gratitude to everyone else who have provided input, discussions and motivation throughout the process of writing this thesis.

Martin Otteraaen

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Trondheim, June 2017

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Summary

A decision support model for the Single Vehicle Flexible and Selective Delivery Routing Problem (SVFSDRP) is presented as a method to improve planning and reduce costs in offshore bulk logistics.

Reducing costs in the oil and gas industry has become the main priority with decreasing oil prices in 2014-2016. As logistics account for 14% of the cost of drilling operation, it is important to reduce costs in this area.

The routing analysis previously done in offshore shipping has deck cargo as its the focus area, instead of bulk cargo. Moreover, Statoil is considering introducing a specialized bulk vessel. The model presented in this paper would work well in combination with such a vessel.

One of the most important challenges in offshore logistics is demand uncertainty. This model may improve logistical personnel's ability to respond to this uncertainty, as the model can quickly provide a new solution to changes in demand.

The method used is a model designed through an optimization approach, the model solves the SVFSDR problem. This thesis is based on the authors work experience, research on the system and literature study on the topic. The model is developed based on earlier work stated in the literature study. Additionally, the model has novel constraints and features which are based on the authors understanding of the bulk supply logistics system. The author may be subject to misinterpreting parts of the system or factors.

When reviewing the result one should keep in mind that this is only a model; the outputs are direct results of the inputs used. The model is subject to assumptions and estimates used through this thesis. The model may be used in similar problems in bulk shipping, but it is restricted to bulk logistical problems.

Flexibility improves the solution by reducing cost, increasing transported volume and increasing revenue. However, it is at the cost of high flexibility, up to 30%, which may increase uncertainty as it will take more time before personnel at offshore installations know the exact volume they will receive. Additionally, it may be hard to achieve high flexibility on demands as it causes uncertainty in the supply chain.

The model may also give logistical personnel the ability to better respond to uncertainty in demand, as it quickly finds new solutions. The proposed model may also be used to test different designs in a bulk supply context. With regards to further work, it is recommended to make a dynamic model that includes the sequential effects of chosen route and cargo mix.

Sammendrag

Denne oppgaven skaper et grunnlag for å forbedre planlegging og redusere kostnader i offshore bulklogistikk ved bruk av et beslutningstøtteverktøy. Modellen løser «Single Vehicle Flexible and Selective Delivery Routing Problem» (SVFSDRP).

Kun halvparten av olje- og gass reservene på de norske kontinentalsokkel har blitt produsert. Når oljeprisen kollapset i 2014-2016 har det å redusere kostnader i olje- og gassindustrien har blitt en hovedpioritet. Siden logistikk står for 14% av kostnadene i boreoperasjoner, er det viktig å redusere kostnadene på dette området.

De analysene som tidligere har blitt gjort innen dette feltet har fokusert på dekklast, og ikke bulk last. I tillegg ser Statoil på mulighetene for å introdusere et spesialisert bulk fartøy. Modellen presentert i denne oppgaven ville egnet seg godt i bruk sammen med et slikt fartøy.

En av de største utfordringene i offshore logistikk er usikkerhet rundt behov. Denne modellen kan bedre logistikkpersonell sin evne til å møte usikkerhet, fordi modellen raskt kan foreslå nye løsninger dersom forandringer skulle oppstå. Modellen kan også brukes i lignende problemer innenfor bulk skipsfart.

Metoden som har blitt brukt er en optimeringstilnærming for å lage en modell som løser SVFSDR problemet. Denne oppgaven er basert på forfatteren sin arbeidserfaring, studier av systemer og litteraturstudier på emnet. Modellen er utviklet på grunnlag tidligere arbeid nevnt i literaturstudiene, samt originale elementer. Disse elementene er basert på forfatteren sin forståelse av bulkforsyningslogistikksystemet. Det tas forbehold om at forfatteren kan ha missforstått deler av systemet eller innvirkende faktorer.

Ved bruk av resultatene må det poengteres at dette kun er en modell; det man får ut er direkte resultat av det man tar inn i modellen. Modellen blir også påvirket av antagelser og estimater brukt i denne oppgaven. Modellen er begrenset til problemer innenfor bulk logistikk.

Modellen forbedrer planlegging ved raskt å gi et løsningsforslag. Fleksibilitet forbedrer løsningen ved å redusere kostnader, øke transportert volum og øke inntjening. Dette krever derimot høy fleksibilitet, opp til 30%, hvilket kan føre til økt usikkerhet, fordi offshore personell for sent får vite det eksakte volumet de blir tilsendt. Det kan også være vanskelig å oppnå tilstrekkelig fleksibilitet på behovene., fordi det øker usikkerheten i forsyningsskjeden. Å lage planer basert på usikkerhe kan være ineffektivt.

Denne modellen er et verktøy for å planlegge de nærmeste dagene, hvilket er å planlegge på er operasjonellt nivå. Modellen som blir presentert i oppgaven kan brukes i kombinasjon med en strategisk planleggingsmodell som for eksempel en Fleet Size and Mix (FSM) modell. SVFSDRP modellen kan teste forskjellige design på et operasjonellt nivå. En FSM modell kan så teste den beste måten å deployere en flåte av disse fartøyene. SVFSDRP kan også brukes for å teste nye design/fartøy i sammenheng med bulk logistikksystemet.

List of Abbreviations

NTNU – Norwegian University of Science and Technology
PSV – Platform Supply Vessel
BOP – Blowout Preventer
O&G – Oil and Gas
DP – Dynamic Positioning
TSP – Travelling Salesman Problem
VRP – Vehicle Routing Problem
SVRP – Single Vehicle Routing Problem
SVDRP – Single Vehicle Delivery Routing Problem
SVFDRP – Single Vehicle Flexible Delivery Routing Problem
SVSFDRP – Single Vehicle Selective and Flexible Delivery Routing Problem
SVSFDR – Single Vehicle Selective and Flexible Delivery Routing
SVPDPCC – Single Vehicle Pickup and Delivery Problem with Capacitated Customers
MIR – Maritime Inventory Routing
FSM – Fleet Size and Mix problem
IVP – Inventory Vehicle Problem
OBM – Oil Based Mud
WBM – Water Based Mud
SBM – Synthetic Based Mud
kNOK – Thousand (.000, -) Norwegian Kroners

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1 Introduction

This chapter will present the background, state of the art within the topic, objective, structure, scope and limitations.

1.1 Background

On the Norwegian continental shelf only 50% of the oil and gas reserves have been extracted since the Norwegian petroleum industry was established in 1969(NPD, 2016). The remaining 50% of the reserves is expected to continue to create growth and contribute significantly to the Norwegian economy in several decades. However, with the oil price significantly decreasing from $120 to $27 per barrel from 2014-2016 the Norwegian oil and gas industry have been forced to adjust their operations to a new cost level.

This has caused oil and gas field operators to reduce costs in all parts of their operation. Logistics account for 14% of the cost of drilling operations(Osmundsen et al., 2010). This thesis focus on reducing costs in offshore bulk shipping, which is part of the oil and gas upstream logistics costs. Upstream is the operational phases in production of hydrocarbons that comprises of exploration, drilling, pumping and initial transport of the oil(Orszulik, 2016). The operations in the upstream phase requires large volumes of bulk such as drilling mud, bentonite, cement, barite, brine, base oil and more. The volumes vary and whether the delivery has much flexibility in delivery amount, time and urgency of delivery can vary significantly depending on the product, the recipient and the operation it is required for(Otteraaen, 2016).

Today logistical demand offshore is served by Platform Supply Vessels (PSV). These are very versatile, one-size-fits-all vessel, with large deck space and bulk tanks under deck. However, it can lead to low vessel utilization as it is challenging to use both the deck space and bulk tanks efficiently on every round trip(Otteraaen, 2016). Due to this, the industry is considering introducing specialized bulk PSV's. Such a vessel would have increased bulk carrying capacity and reduced deck capacity.

Statoil is also interested in recycling/cleansing drilling mud on the vessel by adding a cleaning module on the vessel. This will not be a focus in this paper. A parallel thesis written by Yngve Windsland at NTNU in 2017 will focus on the vessel design, which includes this cleaning module. The operational pattern for the vessel may become drastically different with a cleansing module. The vessel may then operate as a storage vessel, laying close to a platform for several days at a time, or work as a forward storage for several platforms. In this setting a normal routing model would not be as relevant as it would be harder to make plans as they would be deviated from more often. This is therefore not a focus of the model.

1.2 State of the Art

Fagerholt and Lindstad (2000) published one of the first solutions to routing problem in the offshore supply context. After Fagerholt and Lindstad there have been published several works looking at similar problems, but with slight variations. Christiansen et al. (2013) reviews general research on ship routing and scheduling and related problems from the new millennium. Ting and Liao (2013) classifies and lists most of the relevant work done in routing in recent years until 2011.

Bjørnar et al. (2007) and Gribkovskaia et al. (2008) formulate two different mathematical models for Single Vessel Routing Problem with Pickups and Deliveries (SVRPPD) with capacity constraints on customers. However, both focus on deck cargo, and it does not allow the model to be selective on whether to pickup/deliver loads. This is an important part of the problem discussed in this thesis. Additionally, both disregard bulk commodities in their models which is the focus of this thesis.

Archetti et al. (2007), Bouly et al. (2010) and Defryn et al. (2016) propose selective vehicle routing models. Korsvik and Fagerholt (2010) discusses ship routing and scheduling with flexible cargo quantities. Hvattum et al. (2009) has extensive constraints for tank allocation problems.

The novelty in thesis is to look at bulk shipping in the offshore logistics system. Features such as flexible demand, selective delivery and tank allocation is mathematically formulated through inspiration from previous works on the topic mentioned above and original solutions by the author.

1.3 Objectives

The first objective of this paper is to create a model for the SVSFDR problem found in the offshore bulk logistic system. The model should be generic as this enables it to be easily adapted to different problems.

The second objective is to test whether having flexibility with regards to meeting the demand will cut costs, increase transported volume and increase utilization.

The third objective is for the model to be a possible decision support tool for logistical coordinators. The model should take into consideration relevant factors in offshore bulk logistics. This will require a thorough understanding of the system. Further, the limitations of the model should be clearly defined.

1.4 Structure

The remainder of this thesis is organized as follows. Section 2 contains the system description; where the offshore bulk system is analyzed and explained. Section 3 contains the problem description; with focus on the problems that are relevant to this thesis. Section 4 is a literature review where state of the art in the field is discussed and what has been done in this field is explained. The section outlines the work used to make the model and what it is based upon. Additionally, it states which issues that have not been solved from before and the novelty of this model. Section 5 describes the method used, which explains the model and how it was created. Section 6 is the computational study, which details how the computational study was done, what data is used and presents the results. Section 7 is the discussion, which contains a discussion on validity of method, computational study and the value of this work. Section 8 is conclusion and is followed by Section 9, further work.

1.5 Scope and Limitations

This thesis concerns offshore bulk logistics on the west coast of Norway. The model in this thesis is developed based on earlier work stated in the literature study, it has also been designed with original features. These features are based on the authors understanding of the bulk supply logistics system. The author is of course subject to possibly misinterpreting the system or misunderstanding factors.

One can rarely make models that include all relevant factors and variables. Because of this, there are several problems and factors not addressed in the model. This may be because they are too complex to model, or assumed to be of low significance. Further, the model has only been tested on a limited set of installations. With further testing on a larger data case it may produce different results.

This may lead to significant factors not being properly represented. Therefore, the model is based on several major assumptions. Additionally, it is only a model which means that it is highly dependent on what the inputs are. This model is far from a perfect planning tool. However, it is a decision support tool that may be used to get an initial proposal for a round-trip. Logistical coordinators may also use this model to test different cases and get insight into logistical problems. Such a routing model may also be of interest to other bulk shipping problems.

2 Offshore Bulk System

The following section describes the offshore logistics system. First the supply chain is described broadly, then upstream bulk shipping will be described in details. After upstream bulk shipping description, there will be a description of oil and gas offshore drilling operations, production and their logistical demands.

2.1 Offshore Supply Chain

The offshore supply chain is the logistical network that supplies platforms with all equipment, material and bulk needed for exploration, development, production and decommissioning of oil and gas fields. As Figure 1 shows, materials and products are transported on land between onshore bases and factories/storage sites, and from there by vessel between installations in the North Sea and onshore bases.

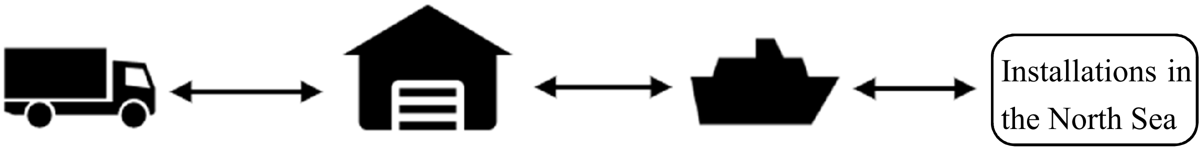


Figure 1: The upstream offshore supply chain, material and products being transported by land from production and storage sites to onshore bases and by PSV to offshore installations.

There are several bases along the Norwegian coast and each base supplies a cluster of installations. All material, equipment and bulk is delivered to these onshore bases. The deliveries may come from other onshore bases, suppliers or other storage facilities. This is shown in Figure 2, the grey stages are done by PSVs'.

Figure 2: Flow of material through upstream offshore supply chain from production/procurement to offshore installations. The grey stages are done by PSV's.

The onshore supply base acts as long term and short term/buffer storage. Most material will arrive just in time so that storage costs will be minimal and material may be checked and loaded straight on the vessel it is to be sent out with. However, some material is also stored at the base. This may be due to operational delays, where the offshore operation does not go as planned and the material is not needed as soon as possible. Material may also be stored on base as back-up. These different functions of a supply base are shown in Figure 3.

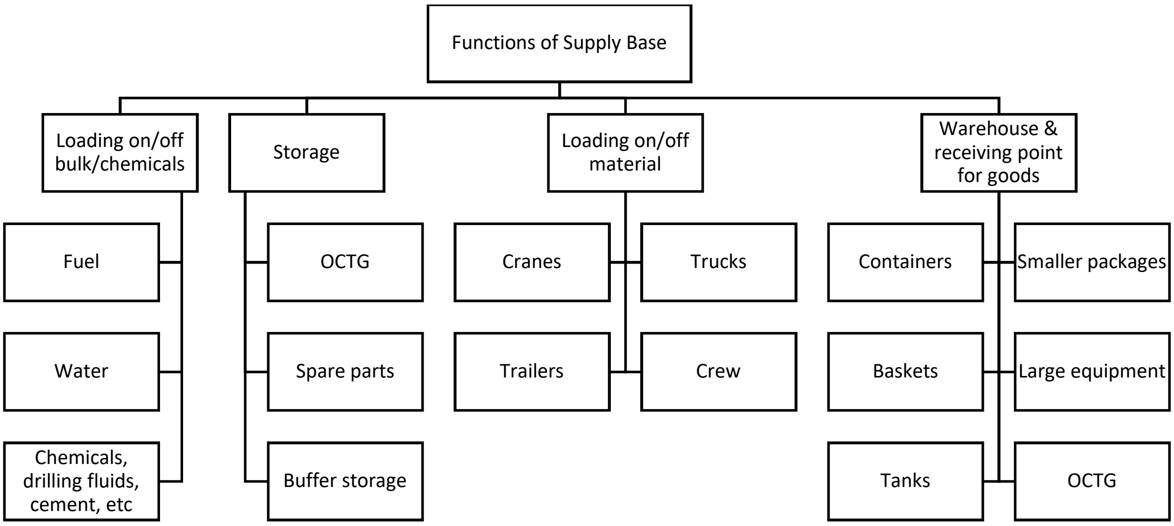


Figure 3: Functions and entities of an onshore supply base.

The supply base has several cranes, crews and trucks to lift material on/off vessels and trailers and transport material from different sites on the base. This resource can often be a limiting factor as the number of cranes or crews will limit the amount of simultaneous operations to be executed. With more ships requiring loading than available resources this will cause delays.

The base also supply and receive bulk/chemicals from/to the vessels. This is mostly done by on-base infrastructure, large tanks and pumping system, which can pump at a rate of up to 100 m^3/h (Dusavik, 2016). However, this is limited by the berth having the necessary infrastructure, which is not always the case. This will be a problem when several ships require the berth with the infrastructure for the specific bulk product. Another problem is if the supplier does not have the product in stock. It must then be delivered by truck, and this will delay the operation by much lower pumping capacity (10-20 m^3/h) and delay from set-up as the trucks only have 10-15 m^3 capacity and required volume may vary from 10-400 m^3.

Goods to be sent out is either stored at the base on the operators' area, arrive by trailer from another base/area or supplier, or arrive by sea from another port. The goods are also checked for security reasons. The checks are usually either to see if there are any undeclared/dangerous materials or that the containers/lifting equipment is in satisfactory condition.

2.2 Upstream Logistics

Logistics in the production of oil and gas is often divided broadly into upstream, midstream and downstream. Upstream find and produce the crude oil and natural gas. Midstream stores, markets and transports crude oil, natural gas, natural gas liquids and Sulphur. Downstream includes everything related to processing, selling, refining and distribution(PSAC, 2017).

Upstream logistics in the North Sea is the focus of this thesis. This includes transportation of all material, equipment and bulk, used to find and produce oil and gas. In the current transport system Statoil has a set of routes for their chartered vessels to supply the demand of their offshore installations from onshore bases.

The logistical coordinators on shore must meet the demand of the platform personnel with the available vessels and routes. Routes are set 1-3 months at a time and the offshore demands are sent to shore daily(Otteraaen, 2016). Each onshore base is designated vessels for their disposal based on the number of installations and their demand. Figure 4 describes a traditional PSV trip in the North Sea.

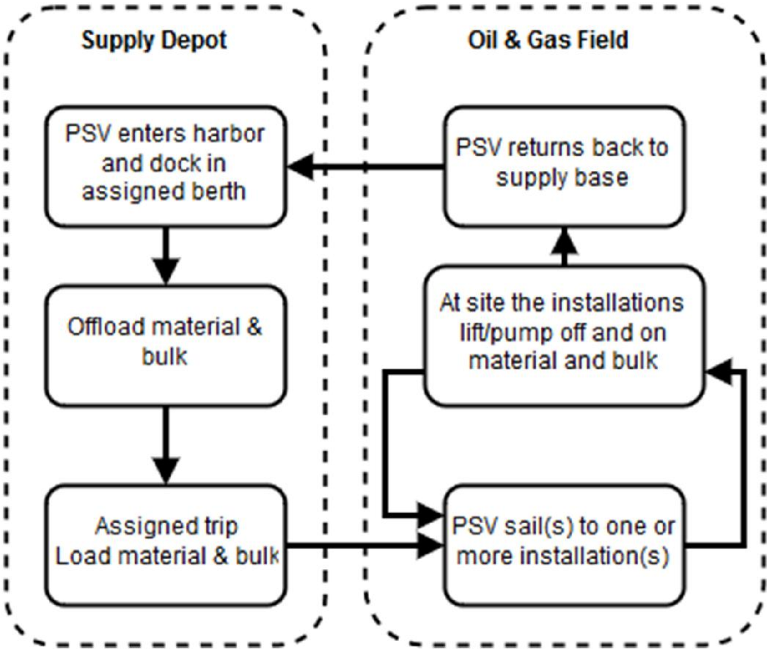


Figure 4: Standard trip for a PSV in the upstream offshore supply system, a rountrip between the onshore supply depot and the oil and gas field.

As the figure above shows, when a PSV enters a harbor it is assigned a berth depending on availability and compatibility, as some bulk products may only be handled at certain berths. The assigned crew will then start offloading material and bulk at the same time. Some bulk products require all other operations to stop due to safety concerns (toxic fumes or ignition hazard). Such products will therefore require longer time in port.

After offloading, the vessel will be assigned a trip and the corresponding cargo. A trip may on average consist of 3-6 offshore installations and will take 2-5 days. The cargo is then loaded and the vessel will start its assigned route. On the route, it delivers cargo to the installations, and receives a return load, also known as “back load”, which is sent back to the base. When the PSV has visited all its scheduled installations it returns to base and repeat the operation(Otteraaen, 2016). Offloading and loading a vessel is usually possible within an eight-hour window(Engh and Erikstad, 2015).

2.3 Routing and Planning

The routing problem today in offshore supply system is shown in Figure 5. A vessel is sent to an offshore oil and gas field to meet the installations demands.

With respect to routing the logistical coordinators on land need to figure out which installations to serve and in which order. They must consider which operations are being done and how critical their demands are. Then the coordinators need to decide how much of the demands they can fulfill. This will depend on size and quantity of the demands, vessel capacity and time constraints(Dusavik, 2016).

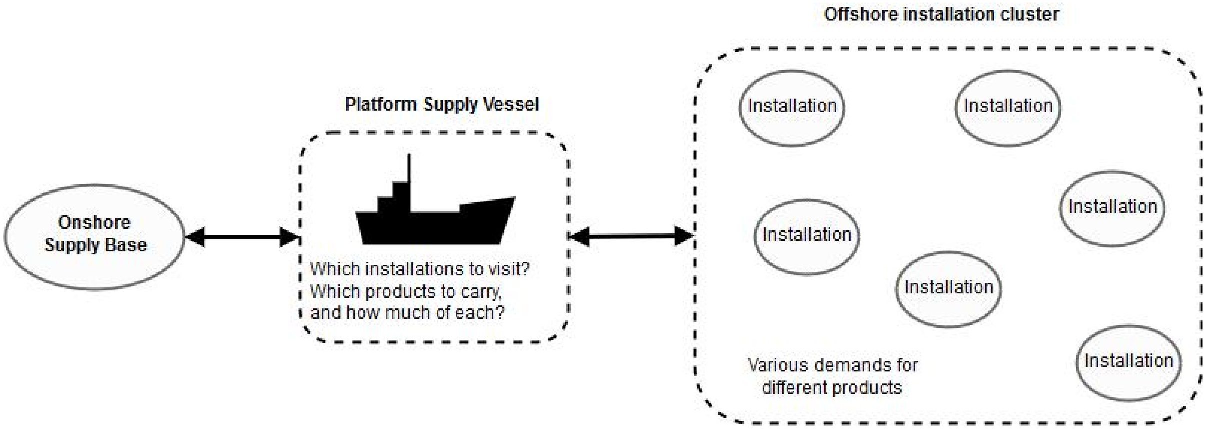


Figure 5: Routing problem today seen from onshore logistical coordinators perspective.

2.4 Offshore Operations

The following section discusses relevant offshore operations to this system.

2.4.1 Offshore Drilling

Drilling operations use drilling mud to bring up cuttings, lubricate and cool the drill bit. Drilling mud also prevents the borehole from collapsing and keeps the pressure in the well under control to prevent an uncontrolled blowout (NOG, 2015).

A standard land based drilling is presented in Figure 6. An offshore drilling rig is similar, the main architectural difference being that offshore there are conductor and riser pipes connected from the drilling platform to the sea floor. Additionally, there is a blowout preventer (BOP) at the sea surface or sea floor, depending on the platform used(NPC, 2011).

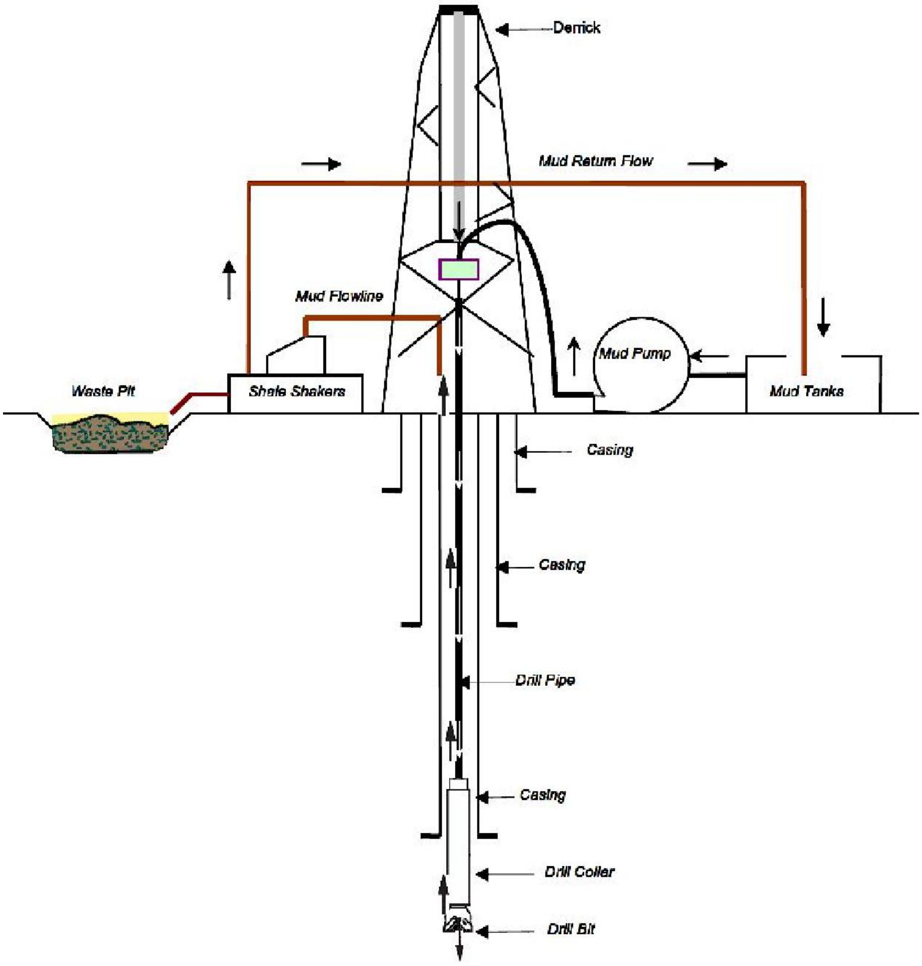


Figure 6: Schematic of a drilling rig, shows the mud circulation system and the well structure with decreasing casing size(Zoveidavianpoor and Shadizadeh, 2012).

Drilling for oil and gas offshore starts by “spudding in” which is extending a 36” drill bit on the end of the drill string to the seabed. This is then drilled from 100 to several hundred meters to prepare for 30” casing. This is the first and widest casing in Figure 6. The 30” casing is set into the drilled hole and the top of the casing is fitted with wellhead connector and the whole length is cemented into place(Gibson, 2009).

The BOP and riser (conductors) is then installed and connected to the well head (this connects the rig to the seabed). The drilling cycle may now commence. A 26” drill bit is used to drill ~300 meter into the sub strata. The drill string is then pulled out of the hole and a new narrower casing is run into the bore hole and cemented so that it is firmly held in position. This is then done several times with a narrower bit and casing each time. This may go as far as ~5000 meter well length and down to 7” casing width(Gibson, 2009).

As the drill bit rotates drilling mud is pumped down the center of the drill string and allowed to flow back up the outside, carrying debris and providing pressure to maintain well integrity. Mud volume requirements vary depending on water depth and width of hole, but at least ~320 m^3 is normally required(Gibson, 2009).

This operation requires large supply deliveries, both of deck cargo and bulk loads. The largest ones are casing, risers, conductors, drilling mud, chemicals, cement, fuel and drill water. This operation also creates a large backload demand from contaminated and used drilling mud/chemicals, drill cuttings and used material/equipment.

2.4.2 Well Displacement

Drilling mud is expensive and there is a high cost associated with the transport of it as every well requires several mud deliveries. This is due to displacement of oil and gas well which is an operation where the entire mud volume in the well is changed. Timeline of bulk demands in an "example well" is presented in Table 1.

Table 1: Timeline of bulk demand when drilling an "example well", shows how large bulk demands are needed throughout the drilling operation.

Day	Well activity	Ship operation
0	Start drilling well, WBM is used	Delivery of large drilling mud load
~ 12	First displacement. Change from WBM to OBM	Delivery of large drilling mud load
~ 24	Second displacement. Change to heavier OBM	Delivery of large drilling mud load, return of used mud
~ 36	Third displacement. Change to heavier OBM	Delivery of large drilling mud load, return of used mud
~ 50	Drilling completed	Return of used mud

Table 1 shows the time line of a well and its requirement for displacements. This is an example well, the timeline and number of displacements can vary significantly. As can be seen from the table, several deliveries of drilling mud are needed throughout the drilling of a well to do displacements. Well displacement is needed when the operator wants to switch mud base, between water (WBM), oil (OBM) or synthetic (SBM) based mud, or when they need a change in the mud properties, such as density and viscosity. From a vessel's perspective, a drilling operation begins with a large delivery of drilling mud at day 0. Approximately 12 days later, the first displacement is due, and the vessel must provide the drilling installation with oil based mud. Further, two new mud deliveries are required after two 12-day periods, where the vessel must pick up used mud and deliver new mud to the installation. After about 50 days the drilling is completed, and the mud must be retrieved by a vessel for transport to land.

Displacement is done approximately 3-5 times during the drilling of a well and is an operation that is often served by spot chartered vessels. This is because it is difficult to know exactly when the large volumes, ~500 m^3, are needed on site because it is contingent in the progress of the drilling operation. Most installations have limited storage capacity, and therefore often require the mud to be available on a vessel in standby close to the installation as the well is being displaced. This makes the delivery hard to fit into normal supply routes(Gullberg, 2016). This leads to chartering vessels on short term, high cost, contracts to do these irregular jobs, as shown in Figure 7.

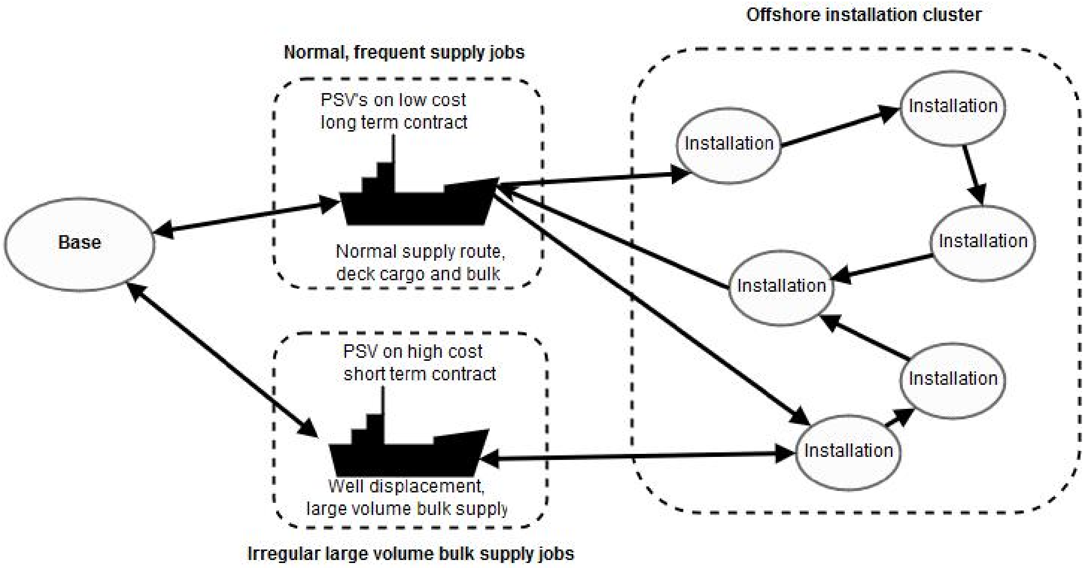


Figure 7: Current supply system when there are large irregular supply jobs, illustrates how the designated PSV's can not fulfill all demands.

On average, the larger and most common PSVs being used to supply offshore installations today have a mud capacity of around 500 m^3 of both OBM and WBM(Dusavik, 2016). This means that displacement of a single well will usually fill an entire vessel, or even require several deliveries. One aspect of this is that the PSVs cannot supply mud to other installations, as its bulk tanks are full. Since a PSV roundtrip usually stops by 3-6 installations, this will require more trips. Another aspect is that the displacement may require the vessel to stay at the installation for the entire operation, which may take several days if there are complications. This will delay the vessel on its route which will cause problems for the other installations on the roundtrip, and the vessel might not return to the supply depot early enough for its next trip.

2.5 The Cost Structure of the Bulk Supply Chain

In the current bulk supply system standard PSV's fulfill all supply jobs, it's just a matter of how many vessels one needs and how efficiently they are used. Its cost is mainly divided into, cost of the vessels, cost of operating the vessels and costs related to coordinating and organizing the vessels.

There are several drivers that make this system expensive and inefficient, such as supply jobs which cannot be done efficiently by the standard PSV's. This can be due to the volumes/quantities being too large or uncertain and irregular demand with respect to time

3 Problem Description

A challenge when creating a model is how to rank the different products and demands against each other. One approach is to analyze different offshore operations and their bulk requirements; which can accept delays and which have very little room for uncertainties in delivery quantity and time of delivery. This will require an understanding of the various offshore operations. This information must be quantified, either by assigning a relative cost (if job is not done) or revenue (for product “p” to platform “j”).

3.1 Offshore Bulk Demands in the Current System

Offshore bulk demands are driven by operational needs on the platforms. This is mainly drilling of wells and production of oil and gas. This may be fuel for the platform, different chemicals to stimulate the well during production or chemicals and products used when drilling a well.

The urgency and lead time of the demand vary significantly between different installations, operations and products. In general, the bulk demands from production are for smaller volumes, have longer lead times and can plan better, due to this the demands are often not as urgent. However, if unexpected problems occur during production it may be very urgent and costly, as some of the larger fields can have production of more than 100 million NOK in a day.

The bulk demands in drilling operations often have unexpected changes and events that are harder to plan for. Demands can therefore often be large, urgent and unexpected. This makes the demands from drilling operations the most relevant ones to look at for this system.

3.2 Factors Influencing the Model

The following section discusses the factors that influence the model.

3.2.1 Maximum Round Trip Times

In the current system, the vessels do 1-3 roundtrips each week. This is both due to space constraints, but also time constraints. If the round trips take too much time it will require planning with long lead time and it will increase uncertainties and probability of changes occurring in the logistical demands. Additionally, bulk products that need to be removed from the vessel due to cost, toxic fume production after being on the vessels tanks for too long or changing of properties by stimulating the product can limit the length of the round trips. There may also be reasons such as rented and costly equipment needing to be returned to the supplier.

3.2.2 Max Number of Tanks, Capacity and No Split Delivery Within Tanks

Platform supply vessels have different tanks that can carry the different products. Most often there are several tanks for each product. The following, in Table 2, is an example of the tank setup in a standard PSV.

Table 2: Tank setup example for a standard PSV, several dedicated tanks for different products. Dry bulk is measured in weight (MT) and wet bulk is measured in volume (m^3).

Product	Number of tanks	Est. Volume per tank
OBM	3-4	100-150 m^3
WBM	3-4	100-150 m^3
Base Oil	2	70 m^3
Barite	2	45 MT
Cement	2	45 MT
Bentonite	2	45 MT
MEG	1	115 MT
Brine	4	90 m^3
Emulsotron	2	70 m^3
Special products	4	90 m^3

As shown above, the PSV's have the highest capacity for OBM/WBM and Brine. The rest have demands in lower volumes. One important factor is that deliveries for several installations is in one tank. This means that with 4 OBM tanks a PSV can only serve 4 installations requesting OBM per trip. This can also lead to lower utilization, especially on low volume demands. However, this restriction is in place because there are different bulk suppliers and the bulk products often have different properties. Additionally, it makes offloading easier and faster as the personnel does not have to keep track of how much has been offloaded at installations or at base.

3.2.3 Penalty for Not Meeting Demand

The following flow diagram in Figure 8 describes actions and costs of not delivering, not delivering enough or delay when delivering products to a platform.

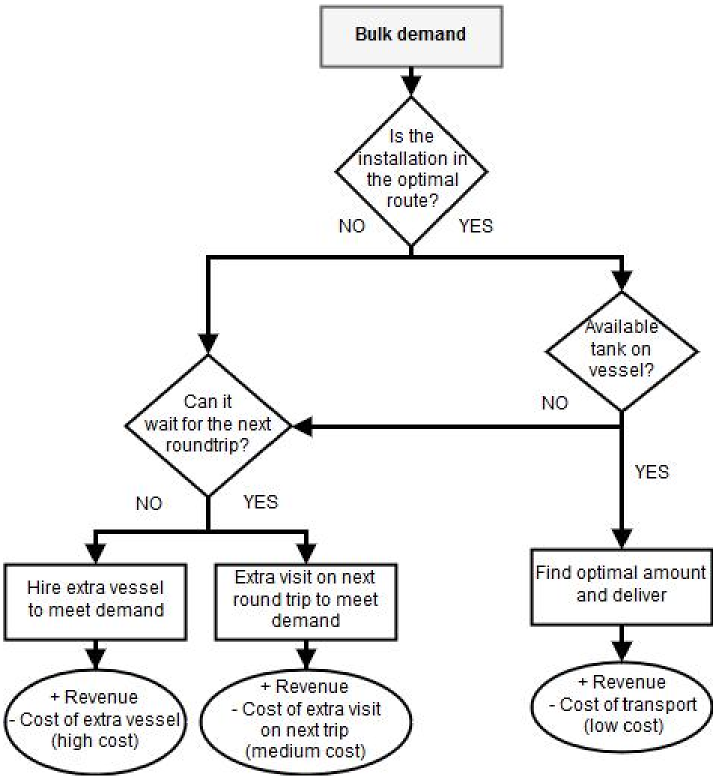


Figure 8: Bulk demand flow diagram, illustrates how the costs increase significantly if the bulk demand is urgent(Dusavik, 2016).

Cost of transport is the voyage costs for the vessel to travel to the installations, such as fuel, loading/offloading and port costs.

Cost of extra visit on next trip, is a relatively modest cost, as the cost of the entire trip is split among the installations visited, the only addition is the extra voyage costs to include a visit to the respective installation.

Cost of chartering an extra vessel is a significantly higher cost, and the day rate of a spot vessel can be estimated to kNOK 100, - (Westshore, 2016). Additionally, there would be costs related to startup, transport, loading/offloading, etc. Total cost can therefore be estimated to at least kNOK 120, - per day. If a spot vessel must be chartered for a single delivery, it will usually spend 2-3 days on a trip. This means that the cost of not delivering a product that cannot wait for the next round trip will be kNOK 240 - 460, -. Demands that cannot wait will mostly be drilling mud deliveries during time sensitive operations where a delay may cause the operation to stop. It may also be relevant for other products, if the platform has low storage capacity(Dusavik, 2016).

3.2.4 Cost of Transport

Transporting material and bulk between onshore supply bases and offshore installations have high costs. A large cost comes with the vessel used, these costs can be broken down in three main groups, capital costs, voyage costs and operational costs, as shown in Figure 9. Capital costs include everything in the construction of the vessel. Operational costs are relatively fixed costs, when the vessel is operational. The voyage costs are variable costs for each voyage.

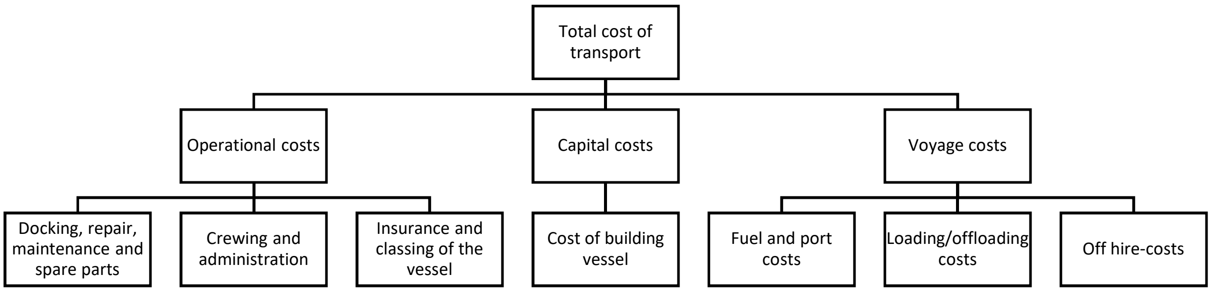


Figure 9: Cost breakdown of transport by vessel from the onshore base to offshore base(Magnussen et al., 2014). Illustrates how some of the costs are variable while some are fixed.

Comparing the current solution where standard PSV's do all supply jobs, with the proposed solution of implementing specialized bulk PSV. The hypothesis is that the proposed system would decrease operational and voyage costs, but increase capital costs. This is due to a more efficient supply system cutting post-construction costs, but a more specialized vessel will have higher building costs. However, the hypothesis is that the total cost of the proposed system would be lower than for the current system.

This increased efficiency and cost cutting will come from better use of the vessels, planning with the normal PSV's will be easier as the high volume, infrequent demands will be delivered by the new vessel.

3.2.5 Flexibility of Delivering Quantity and -Time

The various bulk demands at offshore installations can vary a lot in degree of flexibility, both with respect to when it must be delivered, but also quantity of the demand. Some demands have long lead times, and the platforms have adequate storage capacity. It is then easy to plan for and the logistical coordinators can adjust quantity and when it is delivered to fit with other deliveries. This makes it easier to have an efficient bulk demand and high vessel utilization.

However, some deliveries are critical. An example of this can be during drilling operations. It is critical to have enough mud. A requirement from the government is to always have a backup volume at the surface in case of a kick, gain or loss situation in the well. Having too much may also cause problems, as it decreases the logistical flexibility of the platform it is necessary to make changes(Dusavik, 2016).

It is often as critical to return used mud as it is to receive new mud, due to storage space on the platform. To some degree it is possible for the engineers on the platforms to send demands which define what is absolutely needed, and what would be nice to have. The minimum amount of mud to start drilling is the hull volume and the minimum amount for backup. It is also usual to add extra capacity as a secondary backup of that should become necessary (Dusavik, 2016).

Whether the platforms can store mud for extended periods depends on the type of bulk. Some may be stored for longer periods, while others need to be sent to land for treatment (Dusavik, 2016). There are also products that produce toxic or easily ignitable gasses if left on vessel for extended periods of time. This may become a problem if the product is put on the vessel and the vessel is delayed or must finish a different job or the installation that does not have room to receive the product for some unexpected reason.

3.2.6 Time at Installations

Time at installations for bulk transfer operations vary depending on three factors. First, the time it takes to get into position. As a supply vessel is closing in on an installation it will slow down and call up the installation before entering the 500-meter zone. This zone is a safety measure and no vessels are meant to enter this without permission from the installation. Within this zone the vessel will move slowly and carefully. It may therefore take at least half an hour from the vessel enters the 500-meter zone until it is close enough to the installation to start transfer operations. Leaving the 500-meter zone will also take time, although often not as much as entering.

Secondly, there will be time spent setting up for transfer. This includes safety checks, hooking up transfer cables, changing transfer cables when switching products and the offshore crew must get ready for transfer.

Thirdly, the transfer rate between vessel and installation is an important factor, especially when transferring large volumes. Normal transfer rate between land infrastructure and vessel is 100 m^3/h, but when the vessel is pumping bulk up to the installation it may have to lift it 20-50 m up in the air and this slows down the transfer rate. Therefore, a transfer rate of ~70 m^3/h is realistic (Dusavik, 2016). This may also be set at different values for different installations and vessels.

In the model, the two first factors are estimated to two hours per installation visit and denoted by the parameter T^{pos} , Time needed to get into/out of position for unloading [h]. The transfer rate is denoted by the parameter T^{rate} .

4 Literature Review

This section contains some of the literature that is most relevant to the problem discussed and method used. First the background is discussed, before relevant literature on general offshore routing models is presented. Following is a section on specific single vehicle problems. The following is a presentation of literature relevant to selective delivery and flexible delivery. The last feature discussed is no split deliveries and similar tank allocation problems. The last section highlights what parts of the problem yet remains to be solved.

Several of the authors and articles discussed in this section has been recommended to the author by the supervisor for this thesis. In addition, Google Scholar and Oria has been used to find literature. Oria is the searchable database of NTNU's library. It contains books, articles, journals, theses and more.

Aas et al. (2009) argued that as oil companies have gradually become more focused on optimizing their upstream logistics, little research regarding this has been published. Since then several works have been published, but none focusing mainly on bulk in offshore upstream logistics. Additionally, it is discussed how the demand has increased, both in variety and the total amount. In the conclusion, it is stated that one of the most important challenges is demand uncertainty.

Christiansen et al. (2013) reviews general research on ship routing and scheduling and related problems from the new millennium. The focus is mainly on vessel routing in general; however, they also discuss research done in routing of offshore supply vessels.

Fagerholt and Lindstad (2000) is one of the first to consider the real problem of efficient vehicle routing in an offshore O&G supply setting. Most of the relevant articles used in this thesis as background literature and literature reference this work in one way or another. Ting and Liao (2013) classifies and lists most of the relevant work done in routing in recent years until 2011. This has been used for background research on routing.

There are several articles investigating similar problems in offshore routing. However, an important factor about these papers is that most of them, if not all, consider deck cargo as the deciding factor, and not bulk cargo as is the case for this thesis. This is stated by Halvorsen-Weare et al. (2012), "historical data show that the deck capacity is the binding capacity resource for the supply vessels thus all demands from installations are given in m^2 deck capacity".

Bjørnar et al. (2007) formulates mathematically a solution for a Single Vessel Routing Problem with Pickups and Deliveries (SVRPPD) extended with a capacity restrictions at customers. Gribkovskaia et al. (2008) takes it a bit further and solves a problem using a single vehicle pickup and delivery problem with capacitated customers (SVPDPCC). The model can be used in to look at the system being described in this thesis. However, Gribkovskaia et al. focuses on deck cargo, and does not allow for the model to be selective on whether to pickup/deliver loads.

This is an important part of the problem being discussed in this thesis, and should therefore be included. Additionally, they disregard bulk commodities in their models, which is the focus of this thesis.

Bouly et al. (2010), and Archetti et al. (2007) proposes a selective vehicle routing model that is based on a generalization of the orienteering problem. A limited number of vehicles is to visit several customers; the vehicles have an upper time limit for the trips, there is a profit for visiting customers and a travel cost. The goal of the model is to find a collection of tours that maximize the total profit. The selectivity feature in the model can be utilized in the problem faced in this thesis.

Defryn et al. (2016) uses a selective mechanism in the model. It routes several vessels to supply the given demands, but can also remove specific demands from the solution if they are not seen as beneficial to the model. Although this is a multiple vehicle routing model, it can be adapted to a single vehicle routing problem, which makes it relevant for this thesis.

Korsvik and Fagerholt (2010) discusses ship routing and scheduling with flexible cargo quantities. This is comparable to the problems faced in offshore bulk shipping. The high priority bulk load is contract cargo while low priority cargo is spot cargo. Christiansen et al. (2013) discuss various problems in maritime inventory routing (MIR) which is very relevant to the system being proposed in this thesis. MIR problems are problems where an actor has the responsibility for both routing of the vessels and the inventory management at both ends of the transportation legs.

Hvattum et al. (2009) has extensive constraints for tank allocation problems. The author has solutions for tank allocation with respect to stability, various products and no split deliveries within tanks. These were used as inspiration, but the author developed an original approach. For further work, some of these constraints may be relevant.

The novelty of this thesis and the problem that remains to be solved is bulk shipping in the offshore logistics system. Features such as flexible demand, selective delivery and tank allocation is mathematically formulated through inspiration from previous works on the topic mentioned above and original solutions and adaptions by the author.

5 Method

This section describes the method used when creating the model by an optimization approach to solve the SVSFDRP. Figure 10 shows the standard optimization process which is an iterative approach proposed by Lundgren et al. (2010). The method used in this thesis is an adaption.

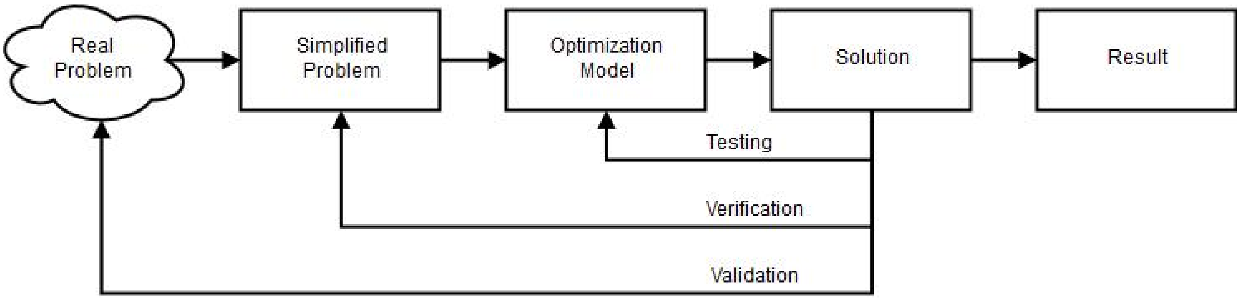


Figure 10: Steps of the optimization process(Lundgren et al., 2010).
Illustrates the iterative process of optimization and solving the problem.

The real problem is analyzed; this gives an understanding of the upstream logistics system focusing on bulk transport. This included the PSV's used in this system the infrastructure on land and the operations on the installations. The capabilities and limitations of these entities were important to understand how to logically model this problem.

As the architect of the model one must understand the problem. What is the model supposed to solve or optimize? Then one must understand the limitations of the optimization approach, where to assume variables, where to simplify the problem. If one does not limit and simplify the problem, it will quickly become too complex. Complex problems take a lot of time to solve and have many factors influencing it, making it complicated to see which factors cause specific solutions to be preferred. This is commonly referred to as a "black box" model. This process is done in section 5.1.

Once the model gets to the stage where one can run tests and get solutions, one can start iterating and improving the model. This leads to testing and adjusting the model, verifying it with the simplified problem, and validating it with the real problem. From the iterative process the final product is a model to solve the single vehicle selective and flexible delivery routing problem. This finished model is shown in section 5.2.

5.1 Creating the Model

The software used is FICO Xpress-IVE 64 which uses the programming language Mosel.

The model started out as a standard TSP model or single vehicle delivery routing problem, from there it was expanded as can be seen in the following sections.

5.1.1 Travelling Salesman Problem with Capacity

The following model routes the vessel to the different demands at the lowest cost. This is a TSP problem with capacity.

Sets (These are similar for all models)
$N - set\ of\ nodes, indexed\ by$ "i" and "j"
$P - Set\ of\ products, indexed\ by$ "p"

Objective function:

$$\min \sum_{i \in N} \sum_{j \in N} C_{ij} X_{ij} \tag{1}$$

Subject to

$$\sum_{i \in I} X_{ij} = 1, \ \ j \in J \tag{2}$$

$$\sum_{j \in J} X_{ij} = 1, \ \ i \in I \tag{3}$$

$$\sum_{j \in J} D_j \leq Q \tag{4}$$

(+subtour restrictions)

$$X_{ij} \in \{0,1\}, \ \ i,j \in N | i \neq j \tag{5}$$

$$L_j \geq 0, integer, \ \ \forall j \in J \tag{6}$$

This model is forced to visit all installations; it finds the most efficient route and checks that the demands are less than vessel capacity. The next step is adding flexibility in delivery. Flexibility is defined as acceptance for delivering more, or less, than exact quantity demanded. This means that the model can find the best cargo mix. This is done in the following model.

5.1.2 Single Vehicle Flexible Delivery Routing Problem

Optimal route and cargo mix.

Parameters
C_{ij} $- Cost\ of\ travelling\ from$ "i" to "j"
R_j $- Revenue\ of\ delivering\ at\ node$ "j", $[NOK/m^3]$
d_j^U $- Upper\ demand\ at\ node$ "j" $[m^3]$
d_j^L $- Lower\ demand\ at\ node$ "j" $[m^3]$
Q $- Capacity\ of\ vessel$ $[m^3]$

Variables
$X_{ij} -$ 1 $if\ travel\ from$ "i" to "j", 0 if else

q_i $-$ $Load\ delivered\ to\ \text{“}j\text{"}\ [m^3]$

Model

$$\max \sum_{j \in J} R_j q_j - \sum_{i \in N} \sum_{j \in N} C_{ij} X_{ij} \tag{1}$$

Subject to:

$$\sum_{i \in I} X_{ij} = 1,\ \ j \in J \tag{2}$$

$$\sum_{j \in J} X_{ij} = 1,\ \ i \in I \tag{3}$$

$$q_j \geq D^U,\ \ j \in J \tag{4}$$

$$q_j \leq D^L,\ \ j \in J \tag{5}$$

$$\sum_{j \in J} q_j \leq Q \tag{6}$$

(+subtour restrictions)

$$X_{ij} \in \{0,1\},\ \ i,j \in N | i \neq j \tag{7}$$

$$q_j \geq 0, integer,\ \ \forall j \in J \tag{8}$$

Through the objective function, $\sum_{j \in J} R_j q_j$, and upper lower limit on demand, D^U and D^L, this model found the optimal cargo mix. The next feature was the ability to skip an installation (and its demand) by taking on a penalty. Then, several products had to be included in the model.

5.1.3 SVFDRP with Additional Features

Optimal route, delivery and cargo mix.

Parameters

C_{ij} $-$ $cost\ of\ travelling\ from$ "i" to "j", $[NOK]$

R_{pj} $-$ $Revenue\ of\ delivering\ product\ \text{"}p\text{"}\ at\ node\ \text{“}j\text{"}, [NOK/m^3\ ]$

D_{pj} $-$ $Demand\ at\ node\ \text{“}j\text{"}$ for product "p", $[m^3\ ]$

T_{ij}^S $-$ $Sailing\ time\ from\ \text{"}i\text{"}\ \ to$ "j", $[h]$

T^{pos} $-$ $Time\ needed\ to\ get\ into/out\ of\ position\ for\ transfer\ [h]$

T^{rate} $-$ $Transfer\ rate\ of\ bulk\ load\ [m^3\ /h]$

Q_p^{MAX} $-$ $Capacity\ of\ vessel\ for\ product\ \text{"}p\text{"}, [m^3\ ]$

T_j $-$ $Penalty\ for\ not\ visiting\ plattform\ \text{"}j\text{"}, [NOK]$

F_p $-$ $Flexibility\ on\ product$ p, [%]

T^{MAX} $-$ $Max\ time\ for\ round\ trip, [h]$

N_p^{tanks} $-$ $Number\ of\ different\ tanks\ that\ can\ carry\ product\ \text{"}p\text{"}$

Variables

x_{ij} $-$ $1\ if\ vessel\ travels\ from\ \text{“}i\text{”}\ to\ \text{“}j\text{”}, 0\ if\ not$

q_{pj} $-$ $Amount\ of\ product\ \text{"}p\text{"}\ delivered\ to\ \text{“}j\text{"}, [m^3\ ]$

y_j $-$ $1\ if$ "j" is visited, 0 if not

t_j $-$ $Time\ at\ which\ service\ on$ "j" is started, [h]

z_{pj} $-$ $1\ if$ "p" is delivered to "j", 0 if not

Model

$$\max \sum_{p\in P}\sum_{j\in N} R_{pj}q_{pj} - \sum_{i\in N}\sum_{j\in N} C_{ij}x_{ij} - \sum_{j\in N}(1-y_j)T_j \tag{1}$$

Subject to

$$\sum_{i\in N} x_{ij} = \sum_{i\in I} x_{ji}\,, \qquad j \in N\backslash\{1\} \tag{2}$$

$$\sum_{i\in N} x_{ij} = y_j, \qquad j \in N \tag{3}$$

$$\sum_{j\in N} x_{1j} = \sum_{j\in J} x_{j1} = 1, \qquad i \in N \tag{4}$$

$$q_{pj} \geq D_{pj}(1-F_p)y_j, \qquad p \in P, j \in N \tag{5}$$

$$q_{pj} \leq D_{pj}(1+F_p)y_j, \qquad p \in P, j \in N \tag{6}$$

$$\sum_{j\in N} q_{pj} \leq Q_p^{MAX}, \qquad p \in P \tag{7}$$

$$x_{ij} \in \{0,1\}, \qquad (i,j) \in N\backslash i \neq j \tag{9}$$

$$q_{pj} \geq 0, \qquad p \in P, j \in N \tag{10}$$

$$y_j \in \{0,1\}, \qquad j \in N \tag{11}$$

As there were still sub-tours in the solutions the following was added. Sub-tour constraint with time variable. The following subtour constraints (12-14) also keep record of what time the vessel visits each node, and therefore how long time the trip takes. Constraint 12 initializes the constraint.

$$t_1 = 0 \tag{12}$$

$$t_j \geq t_i + T_{ij} + T^{pos} + \frac{\sum_{p\in P} q_{pj}}{T^{rate}} - (1-x_{ij})T^{MAX}, \qquad i \in N, j \in N\backslash\{1\} \tag{13}$$

$$t_j \leq t_i + T_{ij} + \frac{\sum_{p\in P} q_{pj}}{T^{rate}} + (1-x_{ij})T^{MAX}, \qquad i \in N, j \in N\backslash\{1\} \tag{14}$$

An additional feature is upper limit on the number of tanks, this was implemented with the following constraints.

$$\sum_{j\in N} Z_{pj} \leq N_p^{tanks}, \qquad p \in P \tag{15}$$

$$q_{pj} \leq Z_{pj}Q_p^T, \qquad p \in P, j \in N \tag{16}$$

The last feature, to ensure no split delivery within tanks was implemented through the following two constraints. This constraint may also be used when different suppliers are delivering the same product, now they will not mix within the same tank.

$$q_{pj} \leq Q_p^T n_{pj}, \qquad p \in P, j \in N \tag{17}$$

$$\sum_{j \in N} n_{pj} \leq N_p^{tanks}, \qquad p \in P \tag{18}$$

The constraint introduces a new variable, n_{pj}, for the number of tanks used per product to each destination. It also causes the capacity for each product to be tank specific, and some changes to other constraints in the model, such as total capacity. The changes are implemented in the final model.

5.2 Final Model

The setting is; one vessel and several platforms with bulk demands for varying products at varying volumes. There is some flexibility within the demands. The intention is to find a good plan that has a low cost, high volume transported and high vessel utilization.

Based on the input data, the model finds which installations to visit, optimal route, which product to deliver and at which quantities. Additionally, the model estimates the cost of this journey. It makes sure that the corresponding capacity is not exceeded for all products. There is no split delivery within tanks. Time limit is not exceeded.

5.2.1 Single Vehicle Flexible and Selective Delivery Routing Problem

Finds the optimal route, delivery and cargo mix for a single vessel and specific journey. It is an expansion of the model in the previous section.

Sets
N – set of nodes, indexed by "i" and "j"
P – Set of products, indexed by "p"

Parameters
C_{ij} – cost of travelling from "i" to "j", $[NOK]$
R_{pj} – Revenue of delivering product "p" at node "j" $[NOK/m^3]$
D_{pj} – Demand at node "j" for product "p" $[m^3]$
T^{pos} – Time needed to get into/out of position for transfer $[h]$
T^{rate} – Transfer rate of bulk load $[m^3/h]$
T_{ij}^{S} – Sailing time from "i" to "j" $[h]$
Q_p^T – Capacity per tank on vessel for product "p" $[m^3]$
C_j^{Visit} – Penalty for not visiting plattform "j" $[NOK]$
C_j^{Prod} – Penalty for not delivering product "p" $[NOK]$
F_p – Flexibility on product "p" [%]
T^{MAX} – Max time for round trip $[h]$
N_p^{tanks} – Number of tanks that can carry product "p"

Variables
x_{ij} – 1 if vessel travels from "i" to "j", 0 if not
q_{pj} – Amount of product "p" delivered to "j" $[m^3]$
y_j – 1 if "j" is visited, 0 if not
t_j – Time at which service on "j" is started [h]

z_{pj} – 1 if "p" is delivered to "j", 0 if not
n_{pj} $-$ *Number of tanks used to deliver product "p" to "j"*

Model

$$\max \sum_{p\in P}\sum_{j\in N} R_{pj}q_{pj} - \sum_{i\in N}\sum_{j\in N} C_{ij}x_{ij} - \sum_{j\in N}\left(1-y_j\right)C_j^{Visit} - \sum_{p\in P}\sum_{j\in N}\left(1-z_{pj}\right)C_j^{Product} \tag{1}$$

Subject to

$$\sum_{i\in N} x_{ij} = \sum_{i\in I} x_{ji}\,, \qquad j \in N\backslash\{1\} \tag{2}$$

$$\sum_{i\in N} x_{ij} = y_j, \qquad j \in N \tag{3}$$

$$\sum_{j\in N} x_{1j} = \sum_{j\in N} x_{j1} = 1, \qquad i \in N \tag{4}$$

$$q_{pj} \geq D_{pj}(1 - F_p)z_{pj}, \qquad p \in P, j \in N \tag{5}$$

$$q_{pj} \leq D_{pj}(1 + F_p)z_{pj}, \qquad p \in P, j \in N \tag{6}$$

$$\sum_{j\in N} q_{pj} \leq N_p^{tanks}, \qquad p \in P \tag{7}$$

$$t_j \geq t_i + T_{ij} + T^{pos} + \frac{\sum_{p\in P} q_{pj}}{T^{rate}} - (1 - x_{ij})T^{MAX}, \qquad i \in N, j \in N\backslash\{1\} \tag{8}$$

$$t_j \leq t_i + T_{ij} + T^{pos} + \frac{\sum_{p\in P} q_{pj}}{T^{rate}} + (1 - x_{ij})T^{MAX}, \qquad i \in N, j \in N\backslash\{1\} \tag{9}$$

$$t_1 = 0 \tag{10}$$

$$\sum_{j=2}^{\mathrm{N}} Z_{pj} \leq N_p^{tanks}, \qquad p \in P \tag{11}$$

$$q_{pj} \leq Z_{pj}Q_p^T N_p^{tanks}, \qquad p \in P, j \in N \tag{12}$$

$$q_{pj} \leq Q_p^T n_{pj}, \qquad p \in P, j \in N \tag{13}$$

$$\sum_{j\in N} n_{pj} \leq N_p^{tanks}, \qquad p \in P \tag{14}$$

$$\sum_{i\in N} x_{ij} \leq \sum_{p\in P} z_{pj}\,, \qquad j \in N \tag{15}$$

$$\sum_{p\in P} z_{pj} \leq y_j M, \qquad j \in N \tag{16}$$

$$t_j \leq T^{MAX}, \qquad j \in N \tag{17}$$

$$x_{ij} \in \{0,1\}, \qquad (i,j) \in N \backslash i \neq j \tag{18}$$
$$q_{pj} \geq 0, \qquad p \in P, j \in N \tag{19}$$
$$y_j \in \{0,1\}, \qquad j \in N \tag{20}$$
$$z_{pj} \in \{0,1\}, \qquad p \in P, j \in N \tag{21}$$
$$t_j \geq 0, \qquad j \in N \tag{22}$$
$$n_{pj} \geq 0 \; and \; integer, \qquad p \in P, j \in N \tag{23}$$

Equation (1) is the objective function, it maximizes the profit for the trip. This is done by adding the revenue from delivering the product and by subtracting the travel cost and the two penalties; one for not visiting a platform and the other for not delivering a product.

Equation (2) is a flow-conservation constraint, it makes sure that if a node (installation) is entered, it is also left. This makes sure that the trips are continuous and the vessel returns to the starting point. Equation (3) makes sure that if the platform is not skipped, it must be entered and (due to eq. (2)) left. Equation (4) forces the onshore base to be left and entered once. Without this constraint one could get trips not containing the base.

Equation (5) and (6) sets the upper and lower limit on the demand and makes sure loaded quantity is within the limit, or set to zero by the z_{pj} variable. Equation (7) makes sure that the loaded quantity for each product is within the vessel capacity. Equation (8) removes all subtours by ensuring that no installations are visited earlier than when the vessel visited the last node, plus the time it took to get into position, unload/load at that node and the travelling time between the nodes. With equation (9) it also keeps track of when the vessel is at each installation. Equation (10) sets the initial time to be zero for the vessel.

Equation (11) makes sure the vessel does not serve more platforms than there are tanks available, since there is no split-delivery within tanks. Equation (12) makes sure that if the product is not delivered ($z_{pj} = 0$) the quantity delivered is 0. Equation (13) and (14) are tank allocation constraints, they make sure there are no split loads within tanks and that the number of tanks is not exceeded for each product. Equation (15) makes sure the vessel does not visit node “j” if the vessel has no deliveries for it. Equation (16) forces the vessel not to visit node “j” if it has no deliveries for “j”. Equation (17) makes sure the time limit is not exceeded. Equation (18-23) forces the relevant variables to only take positive values or to be binary/integer values.

6 Computational Study

The following section describes and documents the computational study. The computational study is done through generating cases and testing them in the model.

6.1 Use Cases

The intended use for the model is to be a decision support tool for logistical coordinators. It is meant to make coordinating supply vessels to serve bulk demands easier and more efficient. The model provides a proposed solution that can be the basis for further planning.

The typical context it is to be used in is; offshore fields with several wells being drilled, both exploration and production wells. This is necessary as it is a capital-intensive system, building highly customized and specialized, large chemical vessels will be expensive. They will require enough jobs to reach high utilization. Therefore, it will be necessary to have several active drilling operations in the same geographic area so that there is a continuous demand for the vessel.

Further, the vessel will also require a different planning and logistical system. For this system to reach the highest utilization the platform personnel requesting bulk deliveries should have as much flexibility in their demand as possible. It will then be possible for coordinators on land to maximize transport volume and vessel utilization.

The system will require well-developed bulk infrastructure at onshore bases. This vessel will carry large bulk volumes, pumping this on and off the vessel will take time. If the infrastructure on land is either not present (requiring bulk to be delivered by truck) or not efficient enough, making loading/offloading a very time consuming operation, it will slow down the vessel. This will decrease its potential to improve logistics.

The model may also be used for similar logistical system transporting bulk products with some degree of flexibility on the demand.

6.2 Generating Cases

The optimization model is to be tested on various cases. The following section describes the generation of these cases. The cases are generated based on historical data from Statoil for offshore bulk deliveries in 2015. The historical data states when the various bulk products were delivered, to which platforms and at which quantities. Based on this, one can generate several cases to test the model. The following data, presented in Table 3, is needed for each case. However, some of the data may be similar in several of the cases.

Table 3: Data needed for case generation. These are the parameters that the model use as input.

Data	Explanation	Units
Travel costs	Cost of travelling between installations	$kNOK$
Travel time	Time to travel between installations	$Hours$
Bulk product demand	Demand for product to specific platforms	m^3
Flexibility	Flexibility of demand	%
Revenue for product	Revenue for delivering products to platforms	$kNOK$
Tank capacity	Capacity per tank for each product	m^3
Number of tanks	Number of tanks per product	$Integer$
Max time for a trip	Time limit before vessel needs to return to base	$Hours$
Delivery penalty	Penalty if vessel does not deliver products	$kNOK$
Visit penalty	Penalty if vessel does not visit platform	$kNOK$

Some of the data in the table above will be static parameters, such as travel costs and travel time. Others may vary slightly depending on the cases and how rigorously the cases are to be tested. Examples of this is Tank capacity, number of tanks, loading time and max time of a trip. Others will be adjusted for most cases such as bulk product demand, flexibility of demand, revenue for product and penalties.

In this computational study the input data will be changed between different cases. However, it is to be noted that whether varying the data, or setting them as constant, may have significant effect on the model. If the effects are neglectable or not relevant to the testing, it may be set to a fixed value for simplicity. This will be assumptions that make the cases easier test and the assumptions will be discussed in section 7.

With regards to the parameters, it should be noted that their specific value is often of little or no value in this thesis. The thesis will focus mainly on how they relate to other parameters, trends in the results that they are variable and that they are in the correct size order (10's, 100's or 1000's).

The different cases will be based on historical data. Mean and variance values for the volume of the demands and the frequency is estimated. With this data, several cases will be generated to test the model and hypothesis. Flexibility will be set to the intervals ±0/5/10/15% to test the effect of it. This gives a total flexibility of 0/10/20/30%.

Penalties for not delivering and not visiting will be set based on urgency and flexibility of the delivery as discussed in section 3.2.3 (Penalty for Not Meeting Demand).

The revenue is given a relatively small value. This is done so that revenue will not dominate other factors but the model will still transport as much as possible of each demand. However, revenue can be given very specific values in real cases.

6.3 Data

The following section shows the data used in the computational study. First, the historical demand from 2015 for the relevant installations and products is shown. Mean, standard deviation and weekly probability of demands are estimated based on the historical data from 2015. Historical data from 2015 was acquired from Statoil representatives(Skram, 2017). The data has been processed and is presented in Table 4 below.

Table 4: Historical demand from 2015 for relevant installations and products. This is the data used to generate the test cases.

		OBM			Bentonite		
		Volume		Probablity per week	Volume		Probability per week
	Installation	Mean	Stdev		Mean	Stdev	
GFA	Gullfaks A	206	60	15 %	89	3	21 %
GFC	Gullfaks C	168	104	37 %	122	66	29 %
OSB	Oseberg B	201	147	69 %	141	112	13 %
OSC	Oseberg C	154	129	40 %	71	14	29 %
OSS	Oseberg Sør	150	89	79 %	113	38	29 %
STB	Statfjord B	144	87	92%	94	28	40 %
STC	Statfjord C	153	103	50 %	81	18	25 %
SDO	GF Songa D	146	178	23 %	122	31	27 %

Based on the data in Table 4, several cases were generated and tested using the model

An important factor for the penalties was to set all penalties to 0 where there was no demand. This can be adjusted in future models where there is backload demand only. Further, the penalty where there is demand follows the logic explained in section 3.2.3 (Penalty for Not Meeting Demand). If OBM demand is more than 200 m^3, the penalty is 400 kNOK, if demand is more than 0 and less than or equal to 200, the penalty is 40 kNOK, and 0 kNOK if there is no demand. The penalty for not delivering Bentonite is 50 kNOK if there is a demand, or else it is 0 kNOK.

6.3.1 Computational Time

Early in the computational study, it became apparent that the model required a lot of time to find the optimal solution as the size of the problem was increased. Several cases of varying size were tested. Table 5 show the computational times.

Table 5: Computational times for problems of varying size. Nodes is the number of installations, product is the number of different products. Time constraint, etc. describes how the time constraint was set and how high the bulk demands were.

Nodes	Prod.	Time constraint, etc.	Computational time
11	2	Shorter than necessary to visit all nodes	No solution after 3000 seconds
11	2	Longer than necessary to visit all nodes	Optimal solution after 0,8s
10	2	Shorter than necessary to visit all nodes	No solution after 1600 seconds
10	1	Shorter than necessary to visit all nodes	No solution after 1600 seconds
10	1	Longer than necessary to visit all nodes	Optimal solution after 0,0s
9	1	Shorter than necessary to visit all nodes	Optimal solution after 0,0-0,1 sec
9	2	Shorter than necessary to visit all nodes, Normal bulk demand	Optimal solution after 0,0-0,1 sec
9	2	Very short time constraint, very high bulk demand from all nodes	Optimal solution after 2800 sec
9	2	Shorter than necessary to visit all nodes very bulk high demand from all nodes	No solution after 3000 sec, very large gap
9	2	Close to necessary to visit all nodes, very high bulk demand from all nodes	In some cases, a solution is found after ~500 seconds
9	2	Longer than necessary to visit all nodes, very high bulk demand from all nodes	Optimal solution after 1 sec
9	3	Shorter than necessary to visit all nodes	Optimal solution after 60-90 sec
9	3	Longer than necessary to visit all nodes	Optimal solution after 0,2 sec
9	4	Shorter than necessary to visit all nodes	Optimal solution after 60-90 sec
9	4	Longer than necessary to visit all nodes	Optimal solution after 0,2 sec
9	5	Shorter than necessary to visit all nodes	Optimal solution after 60-90 sec
9	5	Longer than necessary to visit all nodes	Optimal solution after 0,2 sec

The study was done with an Intel Core i7-4500U CPU @ 1.80GHz processor, 8 GB RAM and a Windows 64-bit Operating System. If an optimal solution was not found within reasonable time (1500 seconds), and the model was still far from finding the optimal solution, the case was stopped and assumed unsolvable within acceptable time by the current system. Whether it was close to finding the solution was based on the gap from the optimal solution. If this gap is large, or the rate at which it decreases has slowed down significantly it is assumed that the solution will not be found within acceptable time.

Number of nodes is significant to whether the model is solvable or not. If there are more than 9 nodes, the model is unsolvable if the time constraint is slightly shorter than necessary to visit all nodes. Except if the time constraint is very short (less than 20 hours), but that is unrealistic as the roundtrip only contains one node.

Figure 11 illustrate the results from a case with high demand for 2 products to 9 nodes. The variable is the time constraint for a round trip.

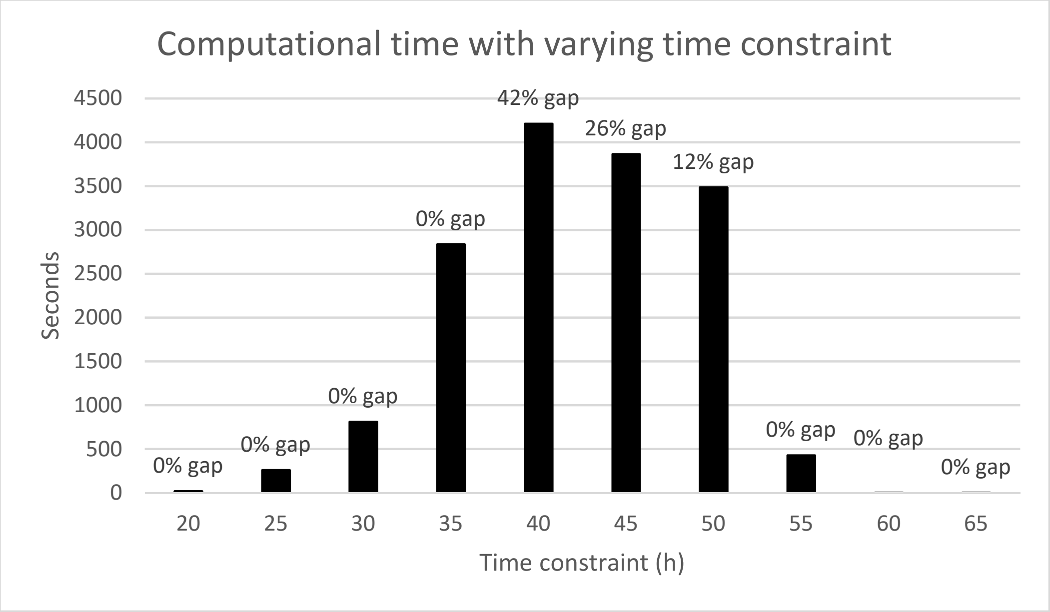


Figure 11: High demand case with 9 nodes and varying time constraint graphically represented. The gap describes how far the software was from finding the optimal solution.

With 9 nodes, whether the model is solvable or not depends mainly on the number of nodes with demand. The cases with 40, 45 and 50 hours' time constraint were still unsolved, but the gaps were relatively small (42%, 26% and 16% respectively). Therefore, most of the problems with 9 nodes are solvable within an acceptable time frame. A better computer can also be used, which would make larger problems possible, and solve the current problems faster.

Because of this, the max number of nodes was set to 9, as that was the highest number that gave reasonable computational times. As some of the installations are very close and/or connected, this was used as reasoning for which were to be clustered together. The deliveries to the removed installations were added to the closest installation. This was done to include the deliveries, even though the installations were not included. Gullfaks B (GFB) was removed, and its deliveries clustered with Gullfaks C (GFC). Statfjord A (STA) was also removed and its deliveries clustered with Statfjord B (STB).

6.3.2 Case Studies

The following section presents data from the case studies. The demand and penalty data from case 1 is presented in Table 6, the data for the rest of the cases can be found in appendix A.4.

Table 6: Demand and penalty parameters for case 1. Demand for OBM and Bentonite is given for all nodes. Visit penalty is penalty given if the respective node is not visited. Product penalty is the penalty given if the respective product is not delivered .

Case 1 | Nodes	1	2	3	4	5	6	7	8	9
Parameters	**FMO**	**GFA**	**GFC**	**OSB**	**OSC**	**OSS**	**STB**	**STC**	**SDO**
Demand OBM (m^3)	0	0	219	230	0	126	195	98	0
Demand Bento (m^3)	0	0	0	0	0	0	81	89	0
Visit penalty (kNOK)	0	0	40	40	0	40	40	40	0
Prod pen. OBM (kNOK)	0	0	400	400	0	40	40	40	0
Prod pen. Bento (kNOK)	0	0	0	0	0	0	50	50	0

Table 7 shows the parameters related to the various products in the generated cases. There are 6 OBM tanks with 150 m^3 volume and 5 Bentonite tanks with 100 m^3 volume which gives a total tank capacity of 900 m^3 and 500 m^3 respectively for OBM and Bentonite. This is a relatively high capacity compared to standard PSV's, which has OBM capacity of ~500 m^3 and Bentonite capacity of ~350 m^3.

Table 7: Product and time related parameters for case 1-5. Describes the tank set-up and capacity for OBM and Bentonite and their respective revenues. Also shows the time constraint before vessel has to be done with its trip.

Product	Tanks	Volume	Cap. (m^3)	Revenue (kNOK/m^3)
OBM	6	150	900	0,5
Bentonite	5	100	500	0,5
T^{MAX}	55	hours		

Table 8 shows the full results from case one, the full results from all cases can be found in appendix A.4.

All cases are tested with a total flexibility of 0%, 10%, 20%, 30%. As seen in Table 8 the main metrics vary when flexibility increases. The main trend is that more flexibility gives a better solution. The revenue increases and the costs decreases, thus the profit increases. Transported volume of both products increases, but Bentonite increases more than OBM. Utilization increases.

Table 8: Results from running case 1 in the model at various flexibilities. Visits describes which nodes are visited on the trip, not in which order they are visited, the visit order and full results can be found in the appendix A.4.

Case 1 | Test	1	2	3	4
Flex +/-	0 %	5 %	10 %	15 %
Comp. time (s)	0,3	0,4	0,3	0,3
Profit (kNOK)	71	79	87	96
Profit (%)	0,0 %	12,0 %	23,9 %	35,9 %
Revenue (kNOK)	843	885	927	969
Revenue (%)	0,0 %	5,0 %	10,0 %	15,0 %
Cost (kNOK)	-773	-806	-840	-874
Cost (%)	0,0 %	-4,4 %	-8,7 %	-13,1 %
OBM (m^3)	673	707	740	774
OBM (%)	0,0 %	5,0 %	10,0 %	15,0 %
Bentonite (m^3)	170	179	187	196
Bentonite (%)	0,0 %	5,0 %	10,0 %	15,0 %
Time (h)	36	34	37	35
Utilization (%)	60,2 %	63,2 %	66,2 %	69,2 %
Visits	1 3 4 6 7 8	1 3 4 6 7 8	1 3 4 6 7 8	1 3 4 6 7 8

Figure 12 shows a trend of increasing tank utilization as flexibility increases. However, the real increase is 10,8% from 0-30% flexibility which can be seen in Figure 13. The increasing trend in utilization depends on that the vessels maximum capacity is not reached with 0% flexibility. This is total utilization based on both products.

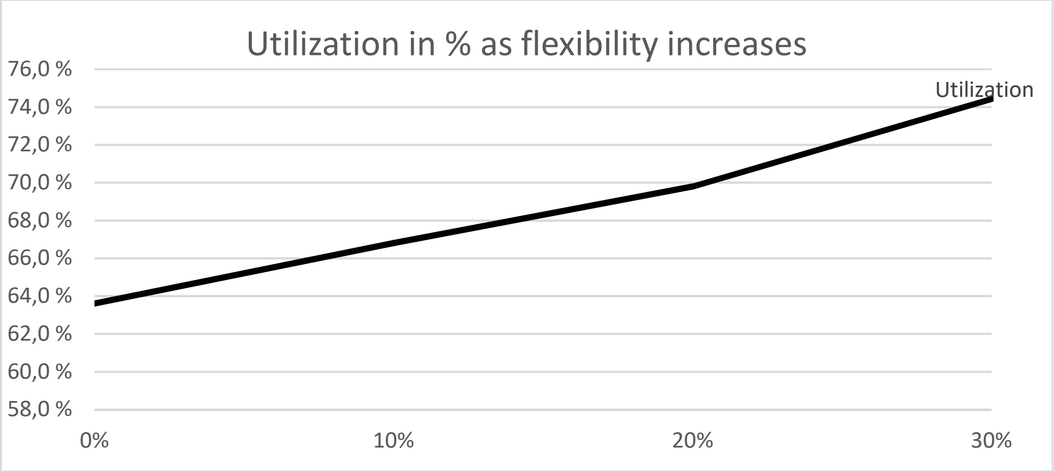


Figure 12: Average tank utilization for all products as flexibility increases. Based on the results of case 1-5.

Figure 13 shows the percentage change in OBM and Bentonite volume transported and the change in average utilization as flexibility increases. Percentage of OBM transported increases by 14% as flexibility goes from 0% to 30%. Bentonite increases at the same rate as OBM until 20% flexibility. There is a significant increase in Bentonite volume transported as flexibility increases from 20% to 30%, leading to a 25% increase of Bentonite transported at 30% flexibility.

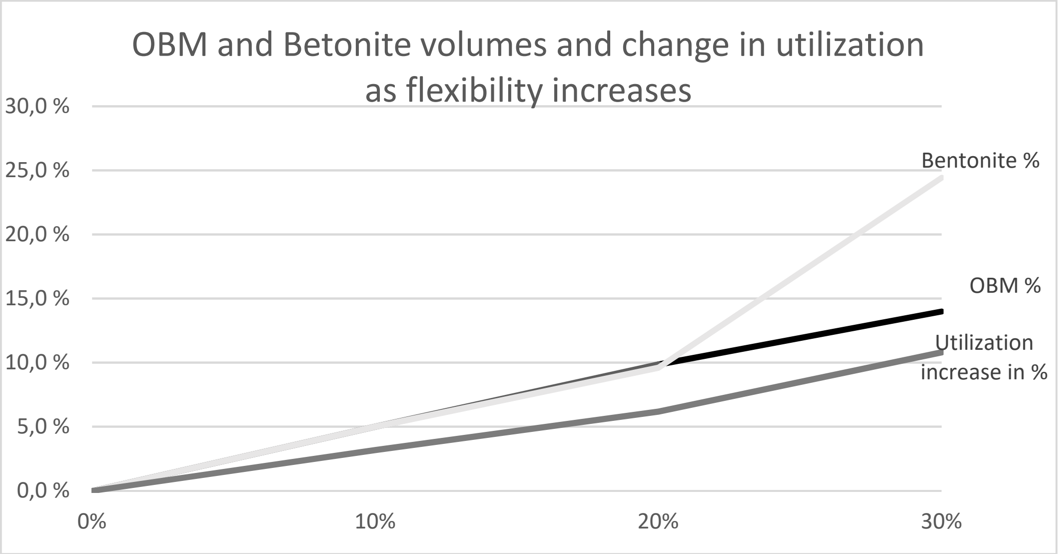


Figure 13: Average change in OBM, Betonite and total utilization as flexibility increases. Based on the results from case 1-5.

Figure 14 shows in percentage how average cost and revenue vary as flexibility increases. Cost decreases linearly to an 11% decrease with 30% flexibility. Revenue increases linearly by 10% at 20% flexibility, after that it increases at a slightly higher rate to 17% at 30% flexibility.

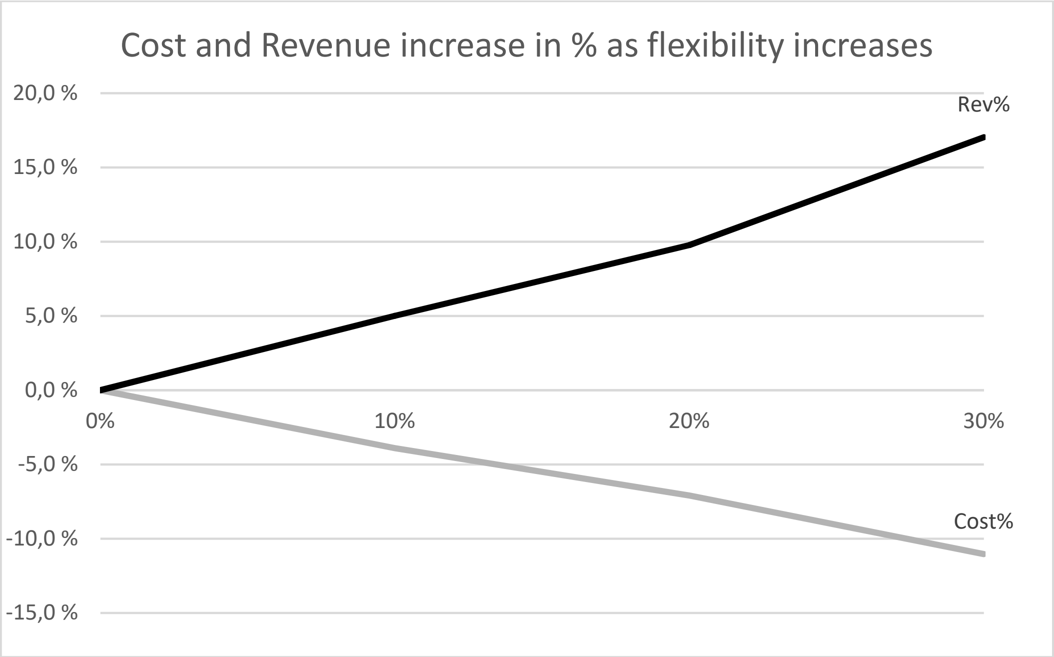


Figure 14: Average revenue, cost and profit increase in % as flexibility increases. Based on the results of case 1-5.

Figure 15 shows the average change in profit with increasing flexibility. This is an expected result as the cost decreased and revenue increased. The increase of profit, as it is presented in Figure 15 does not necessarily say that much as it is based on what input is used. However, it shows the trend of a more efficient operation as flexibility increases.

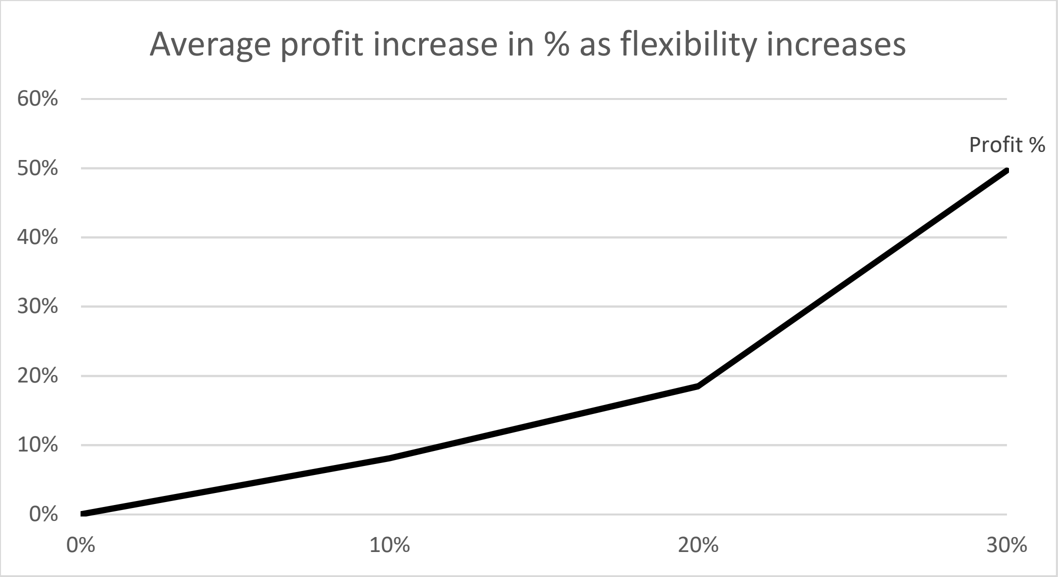


Figure 15: Average percentage change in profit with increasing flexibility. Based on the results of case 1-5.

6.3.3 High Demand Case

This section presents a case which has high demand. This case is not based on historical demand; all installations were simply given high demands (50-400 m^3). This is done to see the effects of a very high demand case on the model. The case is first tested with varying flexibility. The case is then tested with varying time constraint. In these 2 tests, all other parameters are held constant. The parameters are presented in Table 9 and Table 10, while the results are presented in Table 11 and Table 12. The two last penalties are activated if the products are not delivered.

Table 9: Demand and penalty parameters used in high demand case. Not based on historical data.

Installation	FMO	GFB	GFC	OSB	OSC	OSS	STB	STC	SDO
Demand OBM	0	250	258	150	74	180	120	385	372
Demand Bentonite	0	115	200	50	71	160	80	150	130
Penalty for no visit	0	40	40	40	40	40	40	40	40
Penalty OBM	0	400	400	40	40	40	40	400	400
Penalty Bento	0	50	50	50	50	50	50	50	50

Table 10: Tank capacity and time constraint parameters used in high demand case.

Product	Tanks	Volume (m^3)	Cap. (m^3)	Revenue (kNOK/m^3)
OBM	8	150	1200	0,5
Bentonite	8	120	960	0,5
T^{MAX}	55	hours		

Table 11: Results from high demand case with varying flexibility.

Test	1	2	3	4
Flex +/-	0 %	5 %	10 %	15 %
Comp time (s)	229,70	268,90	205,80	442,40
Profit (kNOK)	258	258	258	258
Profit %	0%	0%	0%	0%
Rev (kNOK)	1590	1590	1590	1590
Rev %	0,0 %	0,0 %	0,0 %	0,0 %
Cost (kNOK)	-1331	-1331	-1331	-1331
Costs %	0,0 %	0,0 %	0,0 %	0,0 %
OBM (m^3)	764	889	769	855
OBM %	0,0 %	16,3 %	0,6 %	11,9 %
Bento (m^3)	826	701	821	735
Bentonite %	0,0 %	-15,1 %	-0,6 %	-11,0 %
Utilization	73,6 %	73,6 %	73,6 %	73,6 %
Total time (h)	50,9	51,3	54,3	54,4
Visits	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9

The results from the high demand case with varying flexibility and constant time limit is presented in Table 11. It has a relatively high computational time, but the problems are solved within reasonable time for all flexibility levels.

Table 12 presents the results from high demand case with varying time limit. As discussed in section 6.3.1, the optimal solution was not found in test 3, 4 and 5. These tests had time limits of 40, 45, and 50, respectively. The software was run for an extended period, and the solution gaps were still relatively high. It should also be noted that the gaps were decreasing very slowly at this point.

In test 1 and 2 the model chooses the same route, but uses the extra time to transfer more bulk. This is important to see how the model prioritizes extra time. This is expected due to the penalties.

Table 12: Results from high demand case with varying time limit. Not based on historical data. Gap describes how far away the software is from finding the optimal solution.

Tmax (h)	30	35	40	45	50	55	60	65
Test	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Flex +/-	15 %	15 %	15 %	15 %	15 %	15 %	15 %	15 %
Comp time (s)	811	2838	4213	3867	3487	428	1	0,2
Profit (kNOK)	-707	-597	No solution	No solution	No solution	-330	-291	-278
Rev (kNOK)	1318	1668	0	0	0	1942	1940	2005
Cost (kNOK)	-2025	-2265	0	0	0	-2272	-2231	-2283
OBM (m^3)	855	1129	0	0	0	1177	1161	1177
OBM %	0,0 %	36,7 %	0	0	0	42,5 %	40,5 %	42,5 %
Bento. (m^3)	463	539	0	0	0	765	780	828
Bentonite %	0,0 %	32,1 %	0 %	0%	0 %	0 %	35,8 %	37,7 %
Time (h)	29	34	42% gap	26% gap	13% gap	49	58	58
Utilization	61,0 %	77,2 %	0%	0%	0%	89,9 %	89,8 %	92,8 %
Visits	1 2 3 7 8	1 2 3 7 8	0	0	0	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9

6.4 Results of Computational Study

One of the objectives in this thesis was to improve planning by making it easier and more efficient. This model gives a suggested solution to bulk supply problems. This does make it easier to plan for such cases. However, it is very dependent on having data and parameters such as travel cost, travel time, capacities, demands and possible flexibility available for each demand. It is also important for the base personnel to set appropriate revenue for delivering the demands and penalties for not delivering a product or not visiting a platform. How accurate these parameters are set will influence the degree to which logistical personnel can base their decisions on the results from the model.

Model efficiency and computational time is important for this model, when there are more than nine nodes it becomes hard to solve. However, if the time constraint is high enough for the vessel to visit all nodes, the computation is still efficient. The number of products does not have that much of an effect on the computational time, as far as the data shows.

As nine installations is a relatively high number, this does not pose as a significant problem. However, if the number of onshore supply bases were reduced and more installations were supplied from fewer bases, it might become a problem.

The model becomes more efficient when the demands are flexible. This is shown by utilization increasing by 12% and costs decreasing by 11% as flexibility goes from 0-30%. It can be discussed whether this increase in utilization and decrease in cost is worth a 30% increase in flexibility, but it will be dependent on several factors. It may also be unrealistic to expect 30% flexibility for all demands; it is more likely that each demand will have a varying flexibility from 0-30%.

With respect to total volume transported it is important that maximum bulk capacity is not reached without adding flexibility. If it is, then the added flexibility will not increase the total volume transported. However, it may choose a different mix of bulk product based on revenue/penalty parameters.

If the maximum bulk capacity is not reached with 0% flexibility, the total volume transported increases on average by 24% for Bentonite and 14% for OBM by adding 30% flexibility. This is after testing 5 cases with varying bulk demand based on historical data. Total volume transported increases, which is a more efficient use of the vessel in this case. This assumes that the increased bulk supplied is positive for the installations.

7 Discussion

The followings section is the discussion. The first subsection is how the model and results are validated, the second is the value of the model, the third discusses the problems not addressed in the model, and then the forth discusses the value of flexibility.

The model in this thesis is developed based on earlier work, as stated in the literature study. Additionally, there are several original constraints and features in the model which are based on the authors understanding and knowledge of the offshore bulk logistic system. The author may be subject to misinterpreting the system or misunderstanding factors.

This model is sensitive to the authors estimates and assumptions; this must be taken into consideration when reading the results. As it is only a model, the results will reflect the inputs used. If revenue and/or penalties are changed, the results will differ. If the objective function is adjusted the model will also change. This makes the model sensitive to the authors estimates and assumptions and should be considered with that in mind. However, we can analyze trends and study the system through this model.

One can rarely make models that include all relevant factors and variables, as that makes them too complex and impossible to solve. Even problems with 9 nodes and 2 products can quickly become hard to solve. Computational time is therefore important as the problem size increases. The model is not applicable if it presents an answer long after it is needed, this makes it necessary to simplify the model to make it efficient enough. Therefore, the model is based on several major assumptions and this may lead to significant factors not being properly represented.

7.1 Validation of the Model

The final model was created through an iterative optimization approach. As it evolved and more features were added, it was constantly tested and the results validated. This was done by keeping the data sets small (3-4 nodes, 1-2 products). With small data sets it is possible to find the optimal solution and compare it with the solution from the model. This way the errors could easily be identified.

The objective function is what decides the logic of the model. Based on the objective function, the model will know what to prioritize and what gives the most utility. In the current objective function penalties are dominating the model. This leads to the model being motivated towards first delivering the minimum amount to all installations, then increasing the amount if there is room. With revenue being constant for all installations, the model has no reason to fill one platform before another after the minimum delivery volumes has been reached. This can be adjusted, but in the cases presented in this thesis, the revenue for delivery is constant for all installations. The model has only been tested on a limited set of installations. With further testing on a larger data case it may produce different results.

With respect to the objective function, it is currently a profit-maximizing function, but could also be made into a cost-minimizing function. These two are similar, but a cost-minimizing function may represent the real-life system better.

In its current design, the model will first try to avoid as many penalties as possible. This is done by delivering the minimum amount to as many installations as possible. This can cause problems if the vessel only delivers the minimum on many round trips, causing a shortage to build up. However, it can be adjusted by reducing penalties and varying/increasing revenue for delivery. It should also be noted that when the model has found the optimal route (where few penalties will dominate slightly), it will focus on delivering as much product as possible. For this to become a problem there must be several cases in a row that makes the model deliver the minimum amount. One may also assume that the logistical operators will request more if a small load is received from the last delivery.

7.2 The Value of the Model

This model is far from a perfect planning tool. It is a decision support tool that may be used to get an initial proposal for a round-trip. Base personnel will then have to look at the context and real-life factors not included in the model. The most important of these factors are that it does not account for backload, time windows or jobs where the vessel must stay at an installation for an extended period, these will be further discussed in section 7.3. It is also dependent on logistical personnel at installations giving their bulk demands with flexibility.

However, this model will provide a suggested solution as early as possible. This may help the logistical coordinators on land and in turn the offshore personnel. In the current system, the logistical coordinators on land would first start planning when they have all demand requests, the planning may take some time, and then plans are sent to personnel on installations. If everything was done on a digital platform and the model found the suggested solution early, the planning process would become more efficient.

Logistical coordinators may use this model to test cases and get insight into logistical problems. By studying trends of different cases and seeing the effects of adjusting different variables, the logistical coordinators can see what the limiting factors are. It may also possibly be used by offshore personnel to test cases to see what adding their demand would do to a specific route. The model may also help logistic personnel consider all possibilities as the number of variables increase. As it quickly gives a solution, it may also help personnel to respond to uncertainty which was identified as one of the largest challenges.

It is a generic model that may be adapted to various problems. This was a specific object in the thesis that was achieved. However, it will need object functions and parameters that are adapted to the specific problems it is to be used on.

In drilling for oil and gas wells, the operation often deviates from the plan. This makes offshore logistics hard to plan for as there are so many changes from the original plan. This model does not solve that problem. However, it may give a slight improvement by making the planning more efficient and as changes happen, the model can quickly generate new solutions.

This model is at an operational planning level, meaning it plans for the next few days. Models planning for 1 year or more are on a strategic level. The model presented in this thesis may be used in combination with a strategic planning model such as fleet size and mix. This model may test the utility of different designs on an operational level. A fleet size and mix model may then test the best way to deploy a fleet of these vessels. This model may also be used in procurement and design decisions by testing the different designs.

7.3 Problems Not Addressed in the Model

There are several problems and factors not addressed in the model. This may be because they are too complex to model, or assumed to be of low significance. The most significant ones are addressed in this section.

After use, the bulk tanks must be washed if there is a change in products or differing properties between the same product. This is not included in the model. It could potentially be included in a future version of the model. However, most newer vessels have the capability of washing their own tanks(Aas et al., 2009). This reduces the cost significantly and it is expected that all newer vessels will have this capability. It is therefore assumed to not be a significant problem going forward.

Some of the bulk products supplied to installations can produce gasses which are easily ignitable or toxic which is a safety hazard. The products should therefore not be in tanks on the vessel for a long time. This may become a problem if this model is used and the installations wrongly estimate how much they need or have capacity to receive and the excess is just left on the vessel. This should be avoided if this model is to be used. This can also be avoided by using inert gasses to mitigate, an example of this is a layer of Hydrogen on top of the product working as a “gas-blanket”.

The model does not declare in which direction the vessel should sail the route (1 then 2 then 3, or 3 then 2 then 1). It is often easy for base personnel to decide which direction is best. This also gives the operators the ability to choose based on necessity. However, since an optimal model should at least give a suggestion, this is a limitation with the current model. It is to be noted that this would be adjusted if there were time windows, which can be added in a future model.

It is only a model for bulk supply trips. It does not account for any deck cargo transport, any activities as storage vessel or other jobs. This makes the model relatively limited in its use.

However, this is by design, the objective was to make a model for bulk supply cases. In future versions of the model, other jobs could be included.

Additionally, the model does not account for backload. This makes the backload a manual job for base personnel, which is inefficient. It also may cause storage problems during the trip, especially on the early deliveries on the routes, as there may not be enough space to receive backload on the vessel before it has delivered certain loads. This should be included in future models.

Lastly, a factor that is not addressed appropriately in this thesis is that in offshore drilling there are often parallel activities. Parallel operations may cause problems and delays as installations cannot serve several vessels at the same time. If the model could take such factors into account, the planning could be even more efficient. However, such factors come with high uncertainty and thus, they will always be hard to plan for.

7.4 Value of Flexibility

Increased flexibility gives better solutions and more efficient transport through reduced costs, increased revenue and increased vessel utilization. It may improve the system, but it will require several changes in how the system is operated today. These changes may come at a high cost.

Introducing flexibility to the demands of the installations leads to uncertainty. In the current system, the logistical personnel on installations request a specific quantity of a product and that is what they expect to receive unless they are told otherwise. With flexibility, they would not know what quantity they would get on the next delivery before the plans are set on base. One also should consider to what degree the installations has the ability and opportunity to give flexible requests.

8 Conclusion

In this thesis, the offshore bulk system and problem is analyzed thoroughly, though one can always expand the analysis. A lot of the thesis is spent grasping and understanding the system and problem, this is an important part and makes the model more accurate. The authors understanding is based on experience by the author as a logistical coordinator in the industry, interviews with representatives from the industry, research and literature review.

The model is based both on earlier models in relevant topics, and original solutions by the author. It solves the selective and flexible delivery problem, but it has several limitations. The model is generic with the intention that it is easily adaptable to other similar problems.

The model improves planning by quickly giving a solution based on the data provided, if the number of nodes are not higher than 9. The model may also give logistical personnel the ability to better respond to uncertainty in demand, as it quickly finds new solutions.

Based on the results from the computational study one can reduce costs and improve logistical efficiency by introducing flexibility. However, introducing flexibility also increases uncertainty and may be hard to do in the real system. The results should therefore be considered with caution.

The proposed model may also be used to test different designs.

9 Further Work

The work performed in this thesis may serve as the basis of several future works. This section describes some of the work that may be done.

As mentioned in section 7.3, the model does not address the load it is to pick-up, time windows or the load on the vessel during sailing. In future models this should be a feature. A feature that also could be added to the model is delays or storage operations. Sometimes a vessel is needed as storage on a platform for a few hours, or it is to deliver a product, then wait and receive a different product after a few hours. This could be added to the model to make it usable in cases where this is relevant.

In further work, this model could be modified to be more generic and intuitive. One could also develop a digital platform. This platform could be used for the logistical personnel to report their demands and flexibilities. The base personnel could then use the results from the model and adjust as necessary. If this is done, one should focus on making it easy to use and adapt to other systems and problems.

An important factor to keep in mind if such a vessel with increased bulk capacity is introduced, is that it could lead to the demands changing drastically. A specialized bulk vessel has larger bulk capacity, which means that larger bulk volumes can be transported. As this model is tested on historical demand based on standard PSV's the bulk volumes may be smaller than optimal. This could lead to different demands and results. It could therefore be interesting to investigate whether the offshore installations would request larger bulk demands, if it was a possibility. If this is the case, a specialized bulk vessel may meet the new demand better than expected.

9.1 Single Vehicle Selective and Flexible Delivery Dynamic Routing Problem

The following is a proposed model with a dynamic problem. It does not work as of now and is only a suggestion. As of now, sequential demand variable does not update. Further, there were difficulties with several constraints multiplying variables with other variables which causes the program to stop. The model is intended to find the optimal route, delivery and cargo mix in a sequential setting.

Hypothesis: The route and cargo mix chosen in stage one will depend on the following stages.

If a demand is not chosen in stage one, it must be added to the next stages. The model will take this into account.

Sets
$N - Set\ of\ nodes, indexed\ by\ \text{"i"}\ and\ \text{"}j\text{"}$
$P - Set\ of\ products, indexed\ by\ \text{"}p\text{"}$
$S - Set\ of\ stages, indexed\ by\ \text{"}s\text{"}\ \ (Each\ stage\ is\ seen\ as\ one\ journey)$

Parameters
$F_s \quad - Weighting\ between\ stages\ \text{"s"}$

C_{ij} – *cost of travelling from* "i" *to* "j" [NOK]
R_{pj} – *Revenue of delivering product* "p" *at node* "j" [NOK/m^3]
D_{spj} – *Stage specific demand in stage* "s" *at node* "j" for product "p" [m^3]
T_j^L – *Loading time at* "j" [h]
T_{ij}^S – *Sailing time from* "i" *to* "j" [h]
Q_p^{MAX} – *Capacity of vessel for product* "p", [m^3]
K_j – *Cost penalty for not visiting plattform* "j" [NOK]
F_p – *Flexibility on product* "p" [%]
T^{MAX} – *Max time for round trip*, [h]
N_p^{tanks} – *Number of different tanks that can carry product* "p"

Variables
x_{sij} – 1 *if vessel travels from* "i" *to* "j" *in stage* "s", 0 *if not*
q_{spj} – *Amount of product* "p" *delivered to* "j" *in stage* "s", [m^3]
y_{sj} – 1 *if* "j" is visited *in stage* "s", 0 if not
t_{sj} – *Time at which service on* "j" is started *in stage* "s", [h]
z_{spj} – 1 *if* "p" is delivered to "j" *in stage* "s", 0 if not
d_{spj} – *Total demand in stage* "s" *at node* "j" for product "p", [m^3]

Model

$$MAX \sum_{s=1}^{S} F_s \left(\sum_{p \in P} \sum_{j \in N} R_{pj} q_{spj} - \sum_{i \in N} \sum_{j \in N} C_{ij} x_{sij} - \sum_{j \in N} \left(1 - y_{sj}\right) K_j \right) \tag{1}$$

Subject to:

$$\sum_{i \in N} x_{sij} = \sum_{i \in I} x_{sji}, \qquad j \in N \backslash \{1\}, s \in S \tag{2}$$

$$\sum_{i \in N} x_{sij} = y_{sj}, \qquad j \in N, s \in S \tag{3}$$

$$\sum_{j \in N} x_{s1j} = \sum_{j \in N} x_{sj1} = 1, \qquad i \in N, s \in S \tag{4}$$

$$d_{spj} = D_{spj} + d_{(s-1)pj} - q_{(s-1)pj}, \qquad p \in P, j \in N, s \in S \backslash \{1\} \tag{5}$$

$$d_{1pj} = D_{1pj}, \qquad j \in N, p \in P \tag{6}$$

$$q_{spj} \geq d_{spj} F_p z_{spj}, \qquad p \in P, j \in N, s \in S \tag{7}$$

$$q_{spj} \leq d_{spj} \left(1 + F_p\right) z_{spj}, \qquad p \in P, j \in N, s \in S \tag{8}$$

$$\sum_{j \in N} q_{spj} \leq Q_p^{MAX}, \qquad p \in P, s \in S \tag{9}$$

$$t_{sj} \geq t_{si} + T_{ij} + T_j^L - \left(1 - x_{sij}\right) T^{MAX}, \qquad i \in N, j \in N \backslash \{1\}, s \in S \tag{10}$$

$$t_{s1} = 0, \qquad s \in S \tag{11}$$

$$\sum_{j\in N} Z_{spj} \leq \; N_p^{tanks}, \qquad p \in P, s \in S \tag{12}$$

$$q_{spj} \leq \; Z_{spj} Q_p^{MAX}, \qquad p \in P, j \in N, s \in S \tag{13}$$

$$\sum_{i\in N} x_{sij} \leq \sum_{p\in P} z_{spj}\,, \qquad j \in N, s \in S \tag{14}$$

$$\sum_{p\in P} z_{spj} \geq y_{sj}, \qquad j \in N, s \in S \tag{15}$$

$$t_{sj} \leq T^{MAX}, \qquad j \in N, s \in S \tag{16}$$

$$x_{sij} \in \{0,1\}, \qquad (i,j) \in N\backslash i \neq j, s \in S \tag{17}$$

$$q_{spj} \geq 0, \qquad p \in P, j \in N, s \in S \tag{18}$$

$$y_{sj} \in \{0,1\}, \qquad j \in N, s \in S \tag{19}$$

$$z_{spj} \in \{0,1\}, \qquad p \in P, j \in N, s \in S \tag{20}$$

$$t_{sj} \geq 0, \qquad j \in N, s \in S \tag{21}$$

Status: Not working, sequential demand variable does not update. Difficulties with several constraints multiplying variables with variables.

10 List of References

AAS, B., HALSKAU SR, Ø. & WALLACE, S. 2009. The role of supply vessels in offshore logistics. *Maritime Economics & Logistics,* 11**,** 302-325.

ARCHETTI, C., HERTZ, A. & SPERANZA, M. 2007. Metaheuristics for the team orienteering problem. *Journal of Heuristics,* 13**,** 49-76.

BJØRNAR, A., IRINA, G., ØYVIND, H., SR. & ALEXANDER, S. 2007. Routing of supply vessels to petroleum installations. *International Journal of Physical Distribution & Logistics Management,* 37**,** 164-179.

BOULY, H., DANG, D.-C. & MOUKRIM, A. 2010. A memetic algorithm for the team orienteering problem. *A Quarterly Journal of Operations Research,* 8**,** 49-70.

CHRISTIANSEN, M., FAGERHOLT, K., NYGREEN, B. & RONEN, D. 2013. Ship routing and scheduling in the new millennium. *European Journal of Operational Research,* 228**,** 467-483.

DEFRYN, C., SÖRENSEN, K. & CORNELISSENS, T. 2016. The selective vehicle routing problem in a collaborative environment. *European Journal of Operational Research,* 250**,** 400-411.

DUSAVIK 2016. Interview with Dusavik base personnell, logistical coordinators from Statoil.

ENGH, H. D. & ERIKSTAD, S. O. 2015. Optimization of an Upstream Supply Chain - Developing the Optimal Supply Chain for Exploration Drilling Operations on the NCS. NTNU.

FAGERHOLT, K. & LINDSTAD, H. 2000. Optimal policies for maintaining a supply service in the Norwegian Sea. *Omega,* 28**,** 269-275.

GIBSON, V. 2009. *Supply ship operations,* Aberdeen, La Madrila Press.

GRIBKOVSKAIA, I., LAPORTE, G. & SHYSHOU, A. 2008. The single vehicle routing problem with deliveries and selective pickups. *Computers and Operations Research,* 35**,** 2908-2924.

GULLBERG, E. V. A. 2016. Skype meeting with Statoil representatives. *In:* MARTIN OTTERAAEN, Y. W. A. R. R. (ed.).

HALVORSEN-WEARE, E. E., FAGERHOLT, K., NONÅS, L. M. & ASBJØRNSLETT, B. E. 2012. Optimal fleet composition and periodic routing of offshore supply vessels. *European Journal of Operational Research,* 223**,** 508-517.

HVATTUM, L. M., FAGERHOLT, K. & ARMENTANO, V. A. 2009. Tank allocation problems in maritime bulk shipping. *Computers and Operations Research,* 36**,** 3051-3060.

KORSVIK, J. & FAGERHOLT, K. 2010. A tabu search heuristic for ship routing and scheduling with flexible cargo quantities. *Journal of Heuristics,* 16**,** 117-137.

LUNDGREN, J., RÖNNQVIST, M. & VÄRBRAND, P. 2010. *Optimization,* Lund, Studentlitteratur.

MAGNUSSEN, E., AMDAHL, J. & FUGLERUD, G. 2014. *Marin teknikk grunnlag : TMR4105 : kompendium,* Trondheim, Akademika forlag Kompendieforlaget.

NOG, N. O. O. G. 2015. Environmental report.

NPC, O. O. S. 2011. Subsea drilling, well operations and completions. Operations & Environement Task Group of National Petroleum Council.

NPD, N. P. 2016. *Gjenværende reserver / Remaining reserves* [Online]. http://www.norskpetroleum.no/fakta/gjenvaerende-reserver/: Norsk Petroleumsdirektorat. Available: http://www.norskpetroleum.no/fakta/gjenvaerende-reserver/ [Accessed 21.03 2017].

ORSZULIK, S. 2016. *Environmental Technology in the Oil Industry,* Cham, Springer International Publishing: Cham.

OSMUNDSEN, P., ROLL, K. H. & TVETERÅS, R. 2010. Exploration drilling productivity at the Norwegian shelf. *Journal of Petroleum Science and Engineering,* 73**,** 122-128.

OTTERAAEN, M. 2016 2016. *RE: From experience working as a logistical coordinator for Statoil ASA.*

PSAC, P. S. A. O. C. 2017. *Industry Overview* [Online]. Petroleum Services Association of Canadas' website: PSAC. Available: http://www.psac.ca/business/industry-overview/ [Accessed 27.03 2017].

SKRAM, T. 2017. Skype interview with Statoil representative. *In:* WINDSLAND, M. O. A. Y. (ed.).

TING, C.-K. & LIAO, X.-L. 2013. The selective pickup and delivery problem: Formulation and a memetic algorithm. *International Journal of Production Economics,* 141**,** 199-211.

WESTSHORE. 2016. *Spot rates of PSV in North Sea* [Online]. Westshore.no. Available: http://www.westshore.no/ [Accessed 17.11 2016].

ZOVEIDAVIANPOOR, M. S., A. & SHADIZADEH, S. 2012. Overview of Environmental Management by Drill Cuting Re-Injection Through Hydraulic Fracturing in Upstream Oil and Gas industry, Sustainable Development. Available: http://www.intechopen.com/source/html/38137/media/image1_w.jpg [Accessed 05.05.2017].

Appendix

The following section contains the appendix. This includes;

A.1 Problem Description

MASTER THESIS IN MARINE TECHNOLOGY

SPRING 2017

For stud.techn.

Martin Vikøren Otteraaen

Single Vehicle Flexible and Selective Delivery Problem in Offshore Bulk Shipping

Background

Drilling for oil and gas is a capital-intensive activity. Improving planning and routing can decrease logistical cost.

Planning in offshore bulk shipping today is done by a logistical/drilling responsible at each platform sends a request for delivery/pick-up of specific products at specific volumes to the logistical coordinators at the depot onshore. The logistical coordinators will then try to serve as many of the requests as possible with the vessels and routes available within the current time frame.

Objective

The objective of this thesis is to investigate ways to improve planning in offshore bulk shipping. This will be done by creating a model to solve Single Vehicle Selective and Flexible Delivery Routing problem in offshore bulk shipping. This thesis will focus on how inventory considerations and single vehicle routing and pick-up/delivery models can be used in offshore bulk shipping to improve planning and cut costs.

Further, the objective is also to acquire a thorough understanding of what drives the demand, supply and which factors that will influence the model. This will require the author to learn about the offshore supply chain and drilling operations.

Tasks

The candidate shall/is recommended to cover the following tasks in the master thesis:

1. Research the current offshore bulk logistics system.
    a. How is it done today?
    b. Understand the offshore supply chain
    c. Understand relevant offshore operations
2. Discuss the problem
    a. What kind of operations/platforms is it relevant for?
    b. What influences the demand and supply?
    c. Which factors are the system affected by?
    d. What can be improved?
3. Review state of art within the topic of Single Vehicle Flexible and Selective Delivery Problems. Document what others have done and published previously on the topic.
    a. Decide which model to use and what data is needed from Statoil.
4. Make a model for Single Vehicle Delivery Problem and implement it into relevant software.

    a. The first model should include routing and delivery decision model. This will decide in which order to solve the given tasks and how much bulk is to be delivered at each node.
5. Expand the model, include different factors.
    a. Several different products; The platforms demand different products.
    b. Flexibility; adjusting it to see how it affects the model.
    c. Soft/hard limits; often one can delay a delivery, but one should aim at avoiding this. This can be done by introducing soft limits with a penalty if the given limit is broken.
6. Discuss model, and how this could be implemented in real-life planning and what impact this could have.
    a. What could be implemented today?
    b. What would be the benefits and drawbacks of this system?
7. The master thesis shall in addition define alternative scope of further work.

Plan, Structure and Allocation of Time

01.03.17 – A model should be ready. Background is analyzed.

01.04.17 – Main model should be ready. A case to investigate should also be ready. System and problem description should be complete

16.05.17 – All modelling work is to be done, focus on finishing the thesis.

Outputs envisaged

A Single Vehicle Selective and Flexible Delivery model for offshore bulk logistical system. A discussion on whether this model may improve planning or reduce costs, and if so, under which circumstances.

General

In the thesis the candidate shall present his personal contribution to the resolution of a problem within the scope of the thesis work.

Theories and conclusions should be based on a relevant methodological foundation that through mathematical derivations and/or logical reasoning identify the various steps in the deduction.

The candidate should utilize the existing possibilities for obtaining relevant literature.

The thesis should be organized in a rational manner to give a clear statement of assumptions, data, results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Telegraphic language should be avoided.

The thesis shall contain the following elements: A text defining the scope, preface, list of contents, summary, main body of thesis, conclusions with recommendations for further work, list of symbols and acronyms, reference and (optional) appendices. All figures, tables and equations shall be numerated.

The supervisor may require that the candidate, in an early stage of the work, present a written plan for the completion of the work.
The original contribution of the candidate and material taken from other sources shall be clearly defined. Work from other sources shall be properly referenced using an acknowledged referencing system.

Deliverables

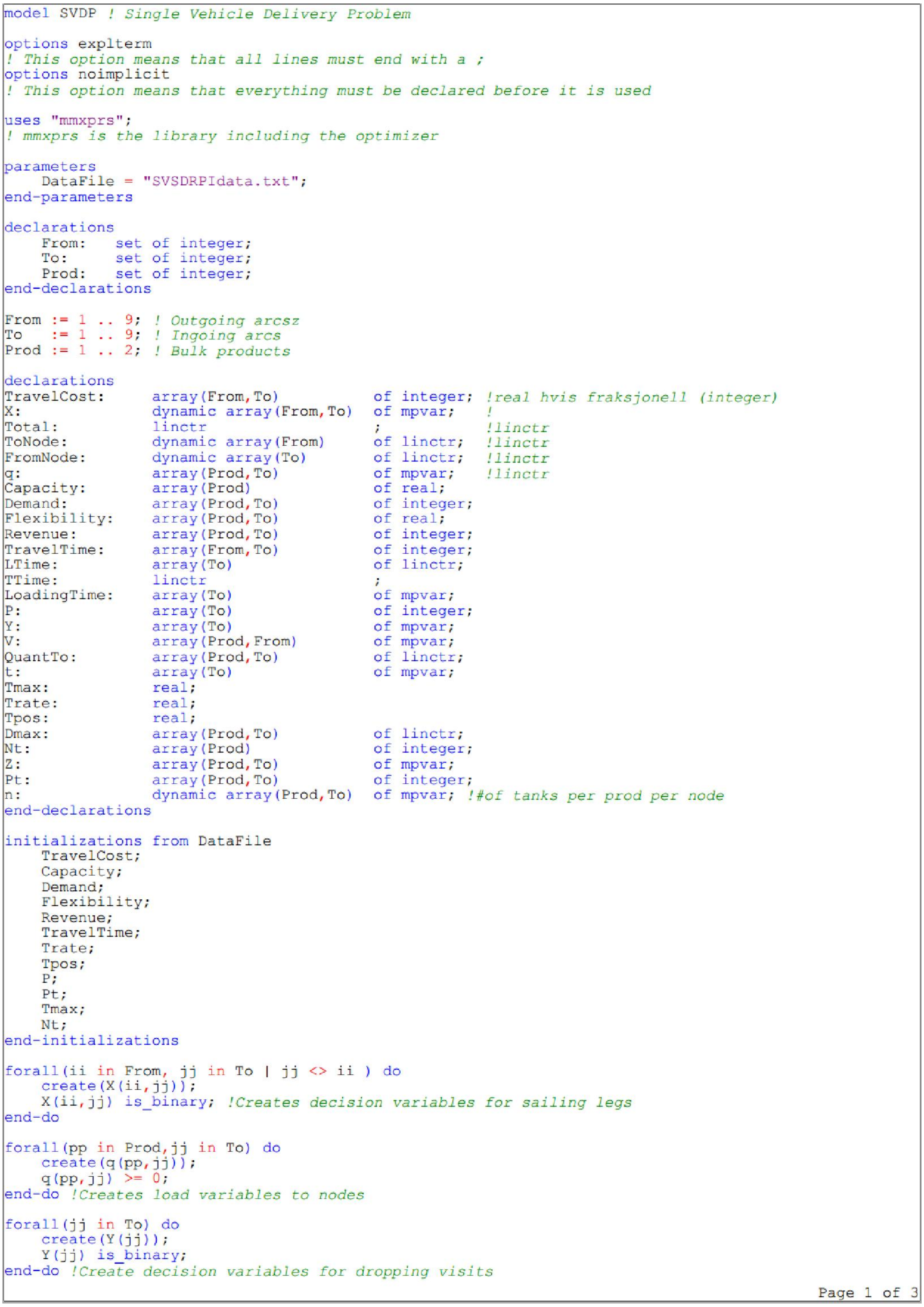
- The thesis shall be submitted in two (2) copies:
- Signed by the candidate
- The text defining the scope included
- In bound volume(s)
- Drawings and/or computer prints that cannot be bound should be organized in a separate folder.
- The bound volume shall be accompanied by a CD or DVD containing the written thesis in Word or PDF format. In case computer programs have been made as part of the thesis work, the source code shall be included. In case of experimental work, the experimental results shall be included in a suitable electronic format.
- A poster shall be submitted

Supervision:

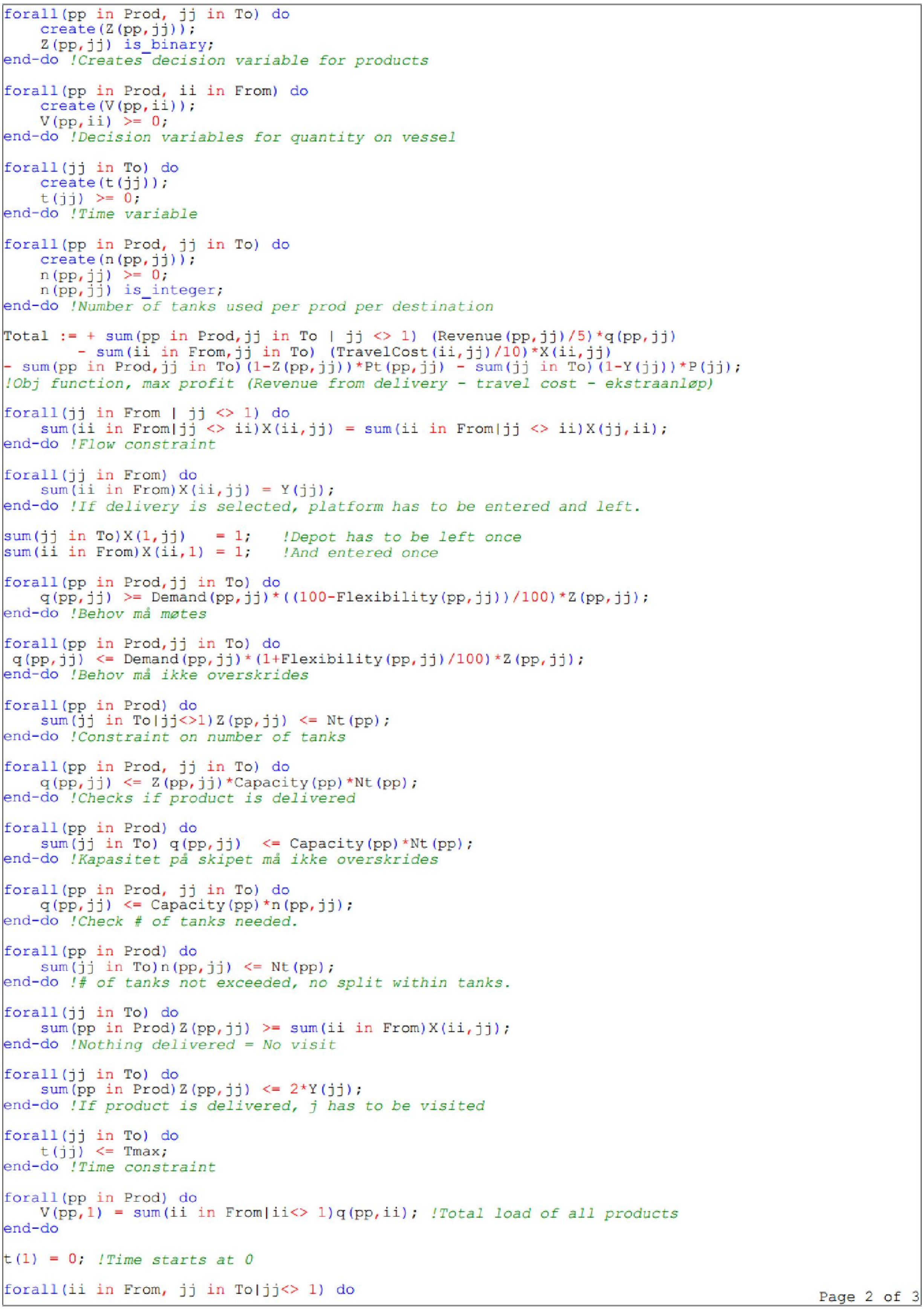
Main supervisor: Bjørn Egil Asbjørnslett

Deadline: 11.06.2017

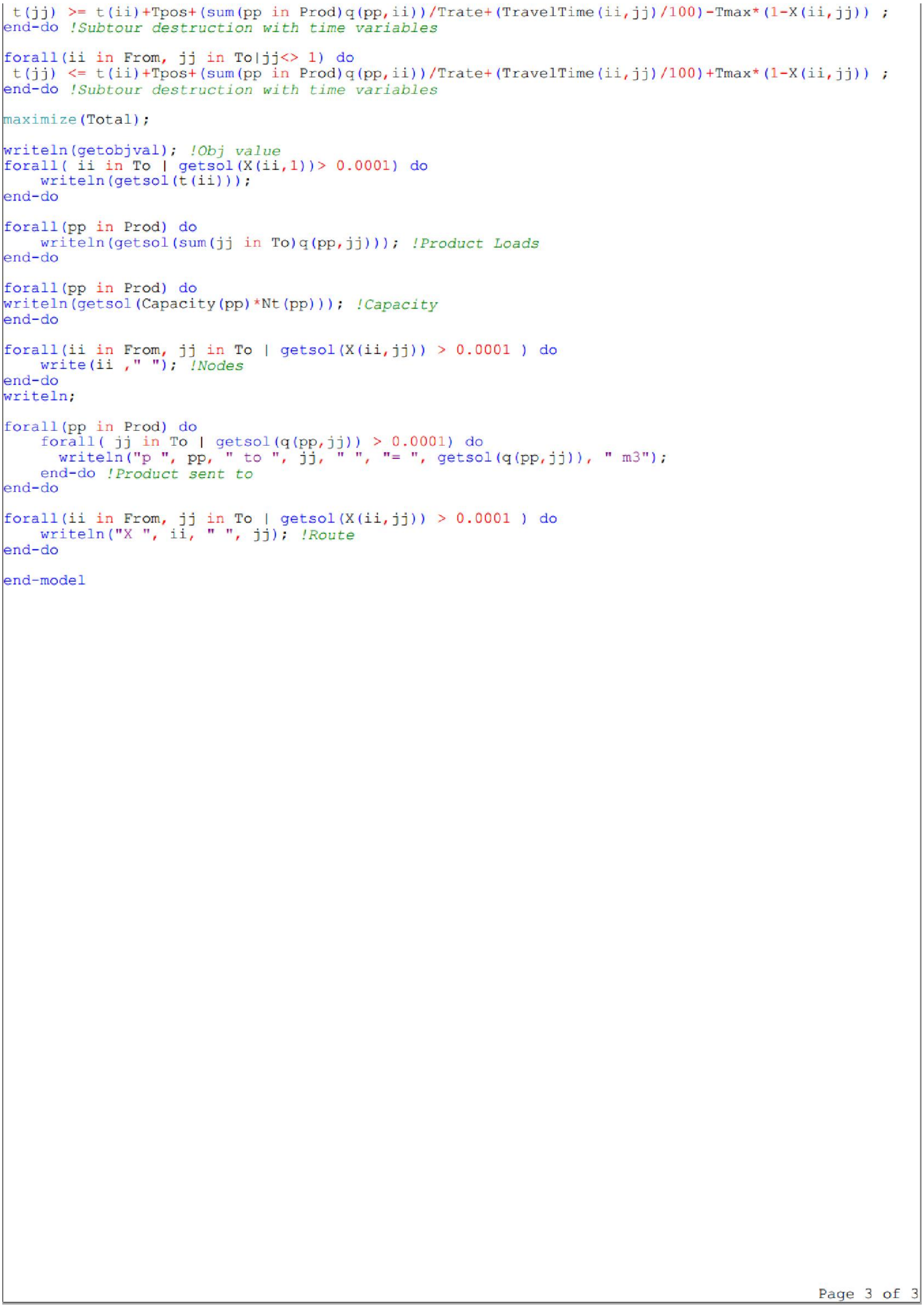
A.2 Code for the Model



```
model SVDP ! Single Vehicle Delivery Problem

options explterm
! This option means that all lines must end with a ;
options noimplicit
! This option means that everything must be declared before it is used

uses "mmxprs";
! mmxprs is the library including the optimizer

parameters
    DataFile = "SVSDRPIdata.txt";
end-parameters

declarations
    From:   set of integer;
    To:     set of integer;
    Prod:   set of integer;
end-declarations

From := 1 .. 9; ! Outgoing arcsz
To   := 1 .. 9; ! Ingoing arcs
Prod := 1 .. 2; ! Bulk products

declarations
TravelCost:     array(From,To)          of integer; !real hvis fraksjonell (integer)
X:              dynamic array(From,To)  of mpvar;   !
Total:          linctr                  ;           !linctr
ToNode:         dynamic array(From)     of linctr;  !linctr
FromNode:       dynamic array(To)       of linctr;  !linctr
q:              array(Prod,To)          of mpvar;   !linctr
Capacity:       array(Prod)             of real;
Demand:         array(Prod,To)          of integer;
Flexibility:    array(Prod,To)          of real;
Revenue:        array(Prod,To)          of integer;
TravelTime:     array(From,To)          of integer;
LTime:          array(To)               of linctr;
TTime:          linctr                  ;
LoadingTime:    array(To)               of mpvar;
P:              array(To)               of integer;
Y:              array(To)               of mpvar;
V:              array(Prod,From)        of mpvar;
QuantTo:        array(Prod,To)          of linctr;
t:              array(To)               of mpvar;
Tmax:           real;
Trate:          real;
Tpos:           real;
Dmax:           array(Prod,To)          of linctr;
Nt:             array(Prod)             of integer;
Z:              array(Prod,To)          of mpvar;
Pt:             array(Prod,To)          of integer;
n:              dynamic array(Prod,To)  of mpvar; !#of tanks per prod per node
end-declarations

initializations from DataFile
    TravelCost;
    Capacity;
    Demand;
    Flexibility;
    Revenue;
    TravelTime;
    Trate;
    Tpos;
    P;
    Pt;
    Tmax;
    Nt;
end-initializations

forall(ii in From, jj in To | jj <> ii ) do
    create(X(ii,jj));
    X(ii,jj) is_binary; !Creates decision variables for sailing legs
end-do

forall(pp in Prod,jj in To) do
    create(q(pp,jj));
    q(pp,jj) >= 0;
end-do !Creates load variables to nodes

forall(jj in To) do
    create(Y(jj));
    Y(jj) is_binary;
end-do !Create decision variables for dropping visits
                                                                    Page 1 of 3
```



```
forall(pp in Prod, jj in To) do
    create(Z(pp,jj));
    Z(pp,jj) is_binary;
end-do !Creates decision variable for products

forall(pp in Prod, ii in From) do
    create(V(pp,ii));
    V(pp,ii) >= 0;
end-do !Decision variables for quantity on vessel

forall(jj in To) do
    create(t(jj));
    t(jj) >= 0;
end-do !Time variable

forall(pp in Prod, jj in To) do
    create(n(pp,jj));
    n(pp,jj) >= 0;
    n(pp,jj) is_integer;
end-do !Number of tanks used per prod per destination

Total := + sum(pp in Prod,jj in To | jj <> 1) (Revenue(pp,jj)/5)*q(pp,jj)
        - sum(ii in From,jj in To) (TravelCost(ii,jj)/10)*X(ii,jj)
- sum(pp in Prod,jj in To)(1-Z(pp,jj))*Pt(pp,jj) - sum(jj in To)(1-Y(jj))*P(jj);
!Obj function, max profit (Revenue from delivery - travel cost - ekstraanløp)

forall(jj in From | jj <> 1) do
    sum(ii in From|jj <> ii)X(ii,jj) = sum(ii in From|jj <> ii)X(jj,ii);
end-do !Flow constraint

forall(jj in From) do
    sum(ii in From)X(ii,jj) = Y(jj);
end-do !If delivery is selected, platform has to be entered and left.

sum(jj in To)X(1,jj)    = 1;    !Depot has to be left once
sum(ii in From)X(ii,1)  = 1;    !And entered once

forall(pp in Prod,jj in To) do
    q(pp,jj) >= Demand(pp,jj)*((100-Flexibility(pp,jj))/100)*Z(pp,jj);
end-do !Behov må møtes

forall(pp in Prod,jj in To) do
 q(pp,jj) <= Demand(pp,jj)*(1+Flexibility(pp,jj)/100)*Z(pp,jj);
end-do !Behov må ikke overskrides

forall(pp in Prod) do
    sum(jj in To|jj<>1)Z(pp,jj) <= Nt(pp);
end-do !Constraint on number of tanks

forall(pp in Prod, jj in To) do
    q(pp,jj) <= Z(pp,jj)*Capacity(pp)*Nt(pp);
end-do !Checks if product is delivered

forall(pp in Prod) do
    sum(jj in To) q(pp,jj)  <= Capacity(pp)*Nt(pp);
end-do !Kapasitet på skipet må ikke overskrides

forall(pp in Prod, jj in To) do
    q(pp,jj) <= Capacity(pp)*n(pp,jj);
end-do !Check # of tanks needed.

forall(pp in Prod) do
    sum(jj in To)n(pp,jj) <= Nt(pp);
end-do !# of tanks not exceeded, no split within tanks.

forall(jj in To) do
    sum(pp in Prod)Z(pp,jj) >= sum(ii in From)X(ii,jj);
end-do !Nothing delivered = No visit

forall(jj in To) do
    sum(pp in Prod)Z(pp,jj) <= 2*Y(jj);
end-do !If product is delivered, j has to be visited

forall(jj in To) do
    t(jj) <= Tmax;
end-do !Time constraint

forall(pp in Prod) do
    V(pp,1) = sum(ii in From|ii<> 1)q(pp,ii); !Total load of all products
end-do

t(1) = 0; !Time starts at 0

forall(ii in From, jj in To|jj<> 1) do
                                                                        Page 2 of 3
```



```
 t(jj) >= t(ii)+Tpos+(sum(pp in Prod)q(pp,ii))/Trate+(TravelTime(ii,jj)/100)-Tmax*(1-X(ii,jj)) ;
end-do !Subtour destruction with time variables

forall(ii in From, jj in To|jj<> 1) do
 t(jj) <= t(ii)+Tpos+(sum(pp in Prod)q(pp,ii))/Trate+(TravelTime(ii,jj)/100)+Tmax*(1-X(ii,jj)) ;
end-do !Subtour destruction with time variables

maximize(Total);

writeln(getobjval); !Obj value
forall( ii in To | getsol(X(ii,1))> 0.0001) do
    writeln(getsol(t(ii)));
end-do

forall(pp in Prod) do
    writeln(getsol(sum(jj in To)q(pp,jj))); !Product Loads
end-do

forall(pp in Prod) do
writeln(getsol(Capacity(pp)*Nt(pp))); !Capacity
end-do

forall(ii in From, jj in To | getsol(X(ii,jj)) > 0.0001 ) do
    write(ii ," "); !Nodes
end-do
writeln;

forall(pp in Prod) do
    forall( jj in To | getsol(q(pp,jj)) > 0.0001) do
      writeln("p ", pp, " to ", jj, " ", "= ", getsol(q(pp,jj)), " m3");
    end-do !Product sent to
end-do

forall(ii in From, jj in To | getsol(X(ii,jj)) > 0.0001 ) do
    writeln("X ", ii, " ", jj); !Route
end-do

end-model
```

Page 3 of 3

A.3 Data Used in the Computational Study

Travel time:

	1	2	3	4	5	6	7	8	9
Node	FMO	GFA	GFC	OSB	OSC	OSS	STB	STC	SDO
1	0	869	849	687	684	715	975	966	417
2	869	0	32	455	383	507	106	108	459
3	849	32	0	461	389	515	130	120	442
4	687	455	461	0	74	61	522	556	347
5	684	383	389	74	0	132	455	486	311
6	715	507	515	61	132	0	568	605	397
7	975	106	130	522	455	568	0	59	564
8	966	108	120	556	486	605	59	0	562
9	417	459	442	347	311	397	564	562	0

Travel cost:

	1	2	3	4	5	6	7	8	9
Node	FMO	GFA	GFC	OSB	OSC	OSS	STB	STC	SDO
1	0	217	212	172	171	179	244	242	104
2	217	0	8	114	96	127	26	27	115
3	212	8	0	115	97	129	33	30	111
4	172	114	115	0	18	15	130	139	87
5	171	96	97	18	0	33	114	121	78
6	179	127	129	15	33	0	142	151	99
7	244	26	33	130	114	142	0	15	141
8	242	27	30	139	121	151	15	0	140
9	104	115	111	87	78	99	141	140	0

Demand and penalties for cases:

Case 1	1	2	3	4	5	6	7	8	9
Installation	FMO	GFA	GFC	OSB	OSC	OSS	STB	STC	SDO
Demand obm	0	0	219	230	0	126	195	98	0
Demand bento	0	0	0	0	0	0	81	89	0
Penalty Visit	0	0	40	40	0	40	40	40	0
Penalty OBM	0	0	400	400	0	40	40	40	0
Penalty Bento	0	0	0	0	0	0	50	50	0

Case 2	1	2	3	4	5	6	7	8	9
Installation	**FMO**	**GFA**	**GFC**	**OSB**	**OSC**	**OSS**	**STB**	**STC**	**SDO**
Demand obm	0	0	384	0	0	207	0	42	0
Demand bento	0	0	138	0	0	0	120	0	115
Penalty Visit	0	0	40	0	0	40	40	40	40
Penalty OBM	0	0	400	0	0	400	0	40	0
Penalty Bento	0	0	50	0	0	0	50	0	50

Case 3	1	2	3	4	5	6	7	8	9
Installation	**FMO**	**GFA**	**GFC**	**OSB**	**OSC**	**OSS**	**STB**	**STC**	**SDO**
Demand obm	0	0	0	0	0	238	25	0	170
Demand bento	0	0	0	0	85	0	0	0	146
Penalty Visit	0	0	0	0	40	40	40	0	40
Penalty OBM	0	0	0	0	0	400	40	0	40
Penalty Bento	0	0	0	0	50	0	0	0	50

Case 4	1	2	3	4	5	6	7	8	9
Installation	**FMO**	**GFA**	**GFC**	**OSB**	**OSC**	**OSS**	**STB**	**STC**	**SDO**
Demand obm	0	164	0	402	307	108	0	285	0
Demand bento	0	92	0	59	64	0	116	70	0
Penalty Visit	0	40	0	40	40	40	40	40	0
Penalty OBM	0	40	0	400	400	40	0	400	0
Penalty Bento	0	50	0	50	50	0	50	50	0

Case 5	1	2	3	4	5	6	7	8	9
Installation	**FMO**	**GFA**	**GFC**	**OSB**	**OSC**	**OSS**	**STB**	**STC**	**SDO**
Demand obm	0	0	258	0	74	0	0	385	372
Demand bento	0	0	0	0	71	0	0	0	130
Penalty Visit	0	0	40	0	40	0	0	40	40
Penalty OBM	0	0	400	0	40	0	0	400	400
Penalty Bento	0	0	0	0	50	0	0	0	50

High demand case									
Case 6	1	2	3	4	5	6	7	8	9
Installation	FMO	GFB	GFC	OSB	OSC	OSS	STB	STC	SDO
Demand obm	0	250	258	150	74	180	120	385	372
Demand bento	0	115	200	50	71	160	80	150	130
Penalty Visit	0	40	40	40	40	40	40	40	40
Penalty OBM	0	400	400	40	40	40	40	400	400
Penalty Bento	0	50	50	50	50	50	50	50	50

A.4 Results from Computational Study

Case 1 | Test	1	2	3	4
Flex +/-	0 %	5 %	10 %	15 %
Comp time (s)	0,3	0,4	0,3	0,3
Profit (kNOK)	71	79	87	96
Prof %	0,0 %	12,0 %	23,9 %	35,9 %
Rev (kNOK)	843	885	927	969
Rev %	0,0 %	5,0 %	10,0 %	15,0 %
Cost (kNOK)	-773	-806	-840	-874
Costs %	0,0 %	-4,4 %	-8,7 %	-13,1 %
OBM (m3)	673	707	740	774
OBM %	0,0 %	5,0 %	10,0 %	15,0 %
Bentonite (m3)	170	179	187	196
Bentonite %	0,0 %	5,0 %	10,0 %	15,0 %
Time	36	34	37	35
Utilization	60,2 %	63,2 %	66,2 %	69,2 %
Visits	1 3 4 6 7 8	1 3 4 6 7 8	1 3 4 6 7 8	1 3 4 6 7 8
Obj Val	70,5	78,93	87,36	95,79
Total time	36,3529	34,13	37,3771	35,0214
Total load	673 170	706,65 178,5	740,3 187	773,95 195,5
Cap	900 500	900 500	900 500	900 500
Visits	1 3 4 6 7 8	1 3 4 6 7 8	1 3 4 6 7 8	1 3 4 6 7 8
Node deliveries	p 1 to 3 = 219 m3 p 1 to 4 = 230 m3 p 1 to 6 = 126 m3 p 1 to 8 = 98 m3 p 2 to 7 = 81 m3 p 2 to 8 = 89 m3	p 1 to 3 = 229,95 m3 p 1 to 4 = 241,5 m3 p 1 to 6 = 132,3 m3 p 1 to 8 = 102,9 m3 p 2 to 7 = 85,05 m3 p 2 to 8 = 93,45 m3	p 1 to 3 = 240,9 m3 p 1 to 4 = 253 m3 p 1 to 6 = 138,6 m3 p 1 to 8 = 107,8 m3 p 2 to 7 = 89,1 m3 p 2 to 8 = 97,9 m3	p 1 to 3 = 251,85 m3 p 1 to 4 = 264,5 m3 p 1 to 6 = 144,9 m3 p 1 to 8 = 112,7 m3 p 2 to 7 = 93,15 m3 p 2 to 8 = 102,35 m3
	X 1 3 X 3 8 X 4 6 X 6 1 X 7 4 X 8 7	X 1 6 X 3 1 X 4 7 X 6 4 X 7 8 X 8 3	X 1 3 X 3 8 X 4 6 X 6 1 X 7 4 X 8 7	X 1 6 X 3 1 X 4 7 X 6 4 X 7 8 X 8 3

Case 2 | Test	1	2	3	4
Flex +/-	0 %	5 %	10 %	15 %
Comp time (s)	0,3	0,4	0,3	0,4
Profit (kNOK)	70	79	88	167
Prof %	0,0 %	12,7 %	25,4 %	138,0 %
Rev (kNOK)	891	936	980	1125
Rev %	0,0 %	5,0 %	10,0 %	26,2 %
Cost (kNOK)	-821	-857	-892	-958
Costs %	0,0 %	-4,3 %	-8,7 %	-16,7 %
OBM (m3)	633	665	696	728
OBM %	0,0 %	5,0 %	10,0 %	15,0 %
Bentonite (m3)	258	271	284	397
Bentonite %	0,0 %	5,0 %	10,0 %	53,8 %
Time	42	36	43	39
Utilization	63,6 %	66,8 %	70,0 %	80,3 %
Visits	1 3 6 7 8 9	1 3 6 7 8 9	1 3 6 7 8 9	1 3 6 7 8 9
Obj Val	70,1	79,01	87,92	166,83
Total time	41,7686	36,32	43,0414	38,7257
Total load	633	664,65	696,3	727,95
	258	270,9	283,8	396,7
Cap	900	900	900	900
	500	500	500	500
Visits	1 3 6 7 8 9	1 3 6 7 8 9	1 3 6 7 8 9	1 3 6 7 8 9
Node deliveries	p 1 to 3 = 384 m3	p 1 to 3 = 403,2 m3	p 1 to 3 = 422,4 m3	p 1 to 3 = 441,6 m3
	p 1 to 6 = 207 m3	p 1 to 6 = 217,35 m3	p 1 to 6 = 227,7 m3	p 1 to 6 = 238,05 m3
	p 1 to 8 = 42 m3	p 1 to 8 = 44,1 m3	p 1 to 8 = 46,2 m3	p 1 to 8 = 48,3 m3
	p 2 to 3 = 138 m3	p 2 to 3 = 144,9 m3	p 2 to 3 = 151,8 m3	p 2 to 3 = 158,7 m3
	p 2 to 7 = 120 m3	p 2 to 7 = 126 m3	p 2 to 7 = 132 m3	p 2 to 7 = 138 m3
	X 1 6	X 1 9	X 1 6	p 2 to 9 = 100 m3
	X 3 9	X 3 8	X 3 9	X 1 9
	X 6 7	X 6 1	X 6 7	X 3 8
	X 7 8	X 7 6	X 7 8	X 6 1
	X 8 3	X 8 7	X 8 3	X 7 6
	X 9 1	X 9 3	X 9 1	X 8 7
	X 9 1	X 9 3	X 9 1	X 9 3

Case 3 | Test	1	2	3	4
Flex +/-	0 %	5 %	10 %	15 %
Comp time (s)	0,3	0,3	0,3	0,3
Profit (kNOK)	76	82	89	96
Prof %	0,0 %	8,8 %	17,5 %	26,3 %
Rev (kNOK)	664	697	730	764
Rev %	0,0 %	5,0 %	10,0 %	15,0 %
Cost (kNOK)	-588	-615	-641	-668
Costs %	0,0 %	-4,5 %	-9,0 %	-13,5 %
OBM (m3)	433	455	476	498
OBM %	0,0 %	5,0 %	10,0 %	15,0 %
Bentonite (m3)	231	243	254	266
Bentonite %	0,0 %	5,0 %	10,0 %	15,0 %
Time	36	34	34	32
Utilization	47,4 %	49,8 %	52,2 %	54,5 %
Visits	1 2 4 5 6 7 9	1 2 5 6 7 9	1 4 5 6 7 9	1 5 6 7 9
Obj Val	75,7	82,34	88,98	95,62
Total time	35,6714	33,89	34,1586	32,3771
Total load	433	454,65	476,3	497,95
	231	242,55	254,1	265,65
Cap	900	900	900	900
	500	500	500	500
Visits	1 2 4 5 6 7 9	1 2 5 6 7 9	1 4 5 6 7 9	1 5 6 7 9
Node deliveries	p 1 to 6 = 238 m3	p 1 to 6 = 249,9 m3	p 1 to 6 = 261,8 m3	p 1 to 6 = 273,7 m3
	p 1 to 7 = 25 m3	p 1 to 7 = 26,25 m3	p 1 to 7 = 27,5 m3	p 1 to 7 = 28,75 m3
	p 1 to 9 = 170 m3	p 1 to 9 = 178,5 m3	p 1 to 9 = 187 m3	p 1 to 9 = 195,5 m3
	p 2 to 5 = 85 m3	p 2 to 5 = 89,25 m3	p 2 to 5 = 93,5 m3	p 2 to 5 = 97,75 m3
	p 2 to 9 = 146 m3	p 2 to 9 = 153,3 m3	p 2 to 9 = 160,6 m3	p 2 to 9 = 167,9 m3
	X 1 6	X 1 6	X 1 6	X 1 6
	X 2 9	X 2 9	X 4 5	X 5 7
	X 4 5	X 5 7	X 5 7	X 6 5
	X 5 7	X 6 5	X 6 4	X 7 9
	X 6 4	X 7 2	X 7 9	X 9 1
	X 7 2	X 9 1	X 9 1	X 7 2
	X 9 1			X 9 1

Case 4 | Test	1	2	3	4
Flex +/-	0 %	5 %	10 %	15 %
Comp time (s)	0,1	0,3	0,3	0,2
Profit (kNOK)	-321	-310	-261	-199
Prof %	0,0 %	3,6 %	18,8 %	37,9 %
Rev (kNOK)	1137	1195	1238	1296
Rev %	0,0 %	5,1 %	8,9 %	14,0 %
Cost (kNOK)	-1458	-1504	-1499	-1495
Costs %	0,0 %	-3,2 %	-2,8 %	-2,6 %
OBM (m3)	795	836	869	874
OBM %	0,0 %	5,1 %	9,3 %	10,0 %
Bentonite (m3)	342	359	370	422
Bentonite %	0,0 %	5,0 %	8,0 %	23,4 %
Time	43	44	44	45
Utilization	81,2 %	85,3 %	88,5 %	92,6 %
Visits	1 2 4 5 6 7 8	1 2 4 5 6 7 8	1 2 4 5 6 7 8	1 2 4 5 6 7 8
Obj Val	-321,1	-309,58	-260,84	-199,27
Total time	42,96	43,7057	44,2529	45,0021
Total load	795	835,5	868,8	874,2
	342	359,1	369,5	421,95
Cap	900	900	900	900
	500	500	500	500
Visits	1 2 4 5 6 7 8	1 2 4 5 6 7 8	1 2 4 5 6 7 8	1 2 4 5 6 7 8
Node deliveries	p 1 to 4 = 402 m3	p 1 to 4 = 422,1 m3	p 1 to 2 = 150 m3	p 1 to 2 = 150 m3
	p 1 to 6 = 108 m3	p 1 to 5 = 300 m3	p 1 to 5 = 300 m3	p 1 to 5 = 300 m3
	p 1 to 8 = 285 m3	p 1 to 6 = 113,4 m3	p 1 to 6 = 118,8 m3	p 1 to 6 = 124,2 m3
	p 2 to 2 = 92 m3	p 2 to 2 = 96,6 m3	p 1 to 8 = 300 m3	p 1 to 8 = 300 m3
	p 2 to 5 = 64 m3	p 2 to 5 = 67,2 m3	p 2 to 2 = 100 m3	p 2 to 2 = 100 m3
	p 2 to 7 = 116 m3	p 2 to 7 = 121,8 m3	p 2 to 4 = 64,9 m3	p 2 to 4 = 67,85 m3
	p 2 to 8 = 70 m3	p 2 to 8 = 73,5 m3	p 2 to 7 = 127,6 m3	p 2 to 5 = 73,6 m3
	X 1 2	X 1 2	p 2 to 8 = 77 m3	p 2 to 7 = 100 m3
	X 2 8	X 2 8	X 1 2	p 2 to 8 = 80,5 m3
	X 4 6	X 4 6	X 2 8	X 1 2
	X 5 4	X 5 4	X 4 6	X 2 8
	X 6 1	X 6 1	X 5 4	X 4 6
	X 7 5	X 7 5	X 6 1	X 5 4
	X 8 7	X 8 7	X 7 5	X 6 1
	X 7 5	X 6 1	X 8 7	X 7 5
	X 8 7	X 7 5	X 7 5	X 8 7
		X 8 7	X 8 7	X 8 7

Case 5 | Test	1	2	3	4
Flex +/-	0 %	5 %	10 %	15 %
Comp time (s)	0,3	0,2	0,3	0,3
Profit (kNOK)	-270	-261	-252	-243
Prof %	0,0 %	3,4 %	6,8 %	10,2 %
Rev (kNOK)	918	964	1010	1056
Rev %	0,0 %	5,0 %	10,0 %	15,0 %
Cost (kNOK)	-1188	-1225	-1262	-1298
Costs %	0,0 %	-3,1 %	-6,2 %	-9,3 %
OBM (m3)	717	753	789	825
OBM %	0,0 %	5,0 %	10,0 %	15,0 %
Bentonite (m3)	201	211	221	231
Bentonite %	0,0 %	5,0 %	10,0 %	15,0 %
Time	37	34	35	35
Utilization	65,6 %	68,9 %	72,1 %	75,4 %
Visits	1 3 5 8 9	1 3 5 8 9	1 3 5 8 9	1 3 5 8 9
Obj Val	-270,1	-260,92	-251,74	-242,56
Total time	36,5771	34,245	34,7971	35,3493
Total load	717	752,85	788,7	824,55
	201	211,05	221,1	231,15
Cap	900	900	900	900
	500	500	500	500
Visits	1 3 5 8 9	1 3 5 8 9	1 3 5 8 9	1 3 5 8 9
Node deliveries	p 1 to 3 = 258 m3	p 1 to 3 = 270,9 m3	p 1 to 3 = 283,8 m3	p 1 to 3 = 296,7 m3
	p 1 to 5 = 74 m3	p 1 to 5 = 77,7 m3	p 1 to 5 = 81,4 m3	p 1 to 5 = 85,1 m3
	p 1 to 8 = 385 m3	p 1 to 8 = 404,25 m3	p 1 to 8 = 423,5 m3	p 1 to 8 = 442,75 m3
	p 2 to 5 = 71 m3	p 2 to 5 = 74,55 m3	p 2 to 5 = 78,1 m3	p 2 to 5 = 81,65 m3
	p 2 to 9 = 130 m3	p 2 to 9 = 136,5 m3	p 2 to 9 = 143 m3	p 2 to 9 = 149,5 m3
	X 1 5	X 1 9	X 1 9	X 1 9
	X 3 9	X 3 8	X 3 8	X 3 8
	X 5 8	X 5 1	X 5 1	X 5 1
	X 8 3	X 8 5	X 8 5	X 8 5
	X 9 1	X 9 3	X 9 3	X 9 3

Case 6 | Test	1	2	3	4
Flex +/-	0 %	5 %	10 %	15 %
Comp time (s)	1	22	1007	428
Profit (kNOK)	-363	-359	-345	-330
Prof %	0,0 %	-1,0 %	-5,0 %	-9,0 %
Rev (kNOK)	1580	1797	1869	1942
Rev %	0,0 %	13,7 %	18,3 %	22,9 %
Cost (kNOK)	-1943	-2156	-2214	-2272
Costs %	0,0 %	-11,0 %	-14,0 %	-17,0 %
OBM (m3)	954	1088	1132	1177
OBM %	0,0 %	14,0 %	18,7 %	23,4 %
Bentonite (m3)	626	709	737	765
Bentonite %	0,0 %	13,3 %	17,7 %	22,2 %
Time	53	54	55	49
Utilization	73,1 %	83,2 %	86,5 %	89,9 %
Visits	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8
Obj Val	-363,10	-359,46	-344,92	-330,38
Total time	52,7657	53,6471	54,5714	49,3
Total load	954	1087,65	1132,3	1176,95
	626	709,05	737,1	765,15
Cap	1200	1200	1200	1200
	960	960	960	960
Visits	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8
Node deliveries	p 1 to 2 = 250 m3	p 1 to 2 = 262,5 m3	p 1 to 2 = 275 m3	p 1 to 2 = 287,5 m3
	p 1 to 3 = 258 m3	p 1 to 3 = 270,9 m3	p 1 to 3 = 283,8 m3	p 1 to 3 = 296,7 m3
	p 1 to 5 = 74 m3	p 1 to 4 = 150 m3	p 1 to 4 = 150 m3	p 1 to 4 = 150 m3
	p 1 to 9 = 372 m3	p 1 to 8 = 404,25 m3	p 1 to 8 = 423,5 m3	p 1 to 8 = 442,75 m3
	p 2 to 2 = 115 m3	p 2 to 2 = 120 m3	p 2 to 2 = 120 m3	p 2 to 2 = 120 m3
	p 2 to 4 = 50 m3	p 2 to 3 = 210 m3	p 2 to 3 = 220 m3	p 2 to 3 = 230 m3
	p 2 to 5 = 71 m3	p 2 to 4 = 52,5 m3	p 2 to 4 = 55 m3	p 2 to 4 = 57,5 m3
	p 2 to 6 = 160 m3	p 2 to 5 = 74,55 m3	p 2 to 5 = 78,1 m3	p 2 to 5 = 81,65 m3
	p 2 to 7 = 80 m3	p 2 to 6 = 168 m3	p 2 to 6 = 176 m3	p 2 to 6 = 184 m3
	p 2 to 8 = 150 m3	p 2 to 7 = 84 m3	p 2 to 7 = 88 m3	p 2 to 7 = 92 m3
	X 1 9	X 1 3	X 1 3	X 1 6
	X 2 8	X 2 8	X 2 8	X 2 3
	X 3 2	X 3 2	X 3 2	X 3 1
	X 4 6	X 4 6	X 4 6	X 4 5
	X 5 4	X 5 4	X 5 4	X 5 7
	X 6 1	X 6 1	X 6 1	X 6 4
	X 7 5	X 7 5	X 7 5	X 7 8
	X 8 7	X 8 7	X 8 7	X 8 2
	X 9 3			

Tmax (h)	20	25	30	35	40	45	50	55	60	65

Case 7 | Test	0	1	2	3	4	5	6	7	7	8
Flex +/-	15 %	15 %	15 %	15 %	15 %	15 %	15 %	15 %	15 %	15 %
Comptime (s)	19,7	262	811	2838	4213	3867	3487	428	1	0,2
Profit (kNOK)	-879	-707	-707	-597	No sol.	No sol.	No sol.	-330	-291	-278
Prof %	0,0 %	-19,6 %	-19,6 %	-32,1 %	0	0	0	-62,4 %	-66,9 %	-68,4 %
Rev (kNOK)	1148	1318	1318	1668	0	0	0	1942	1940	2005
Rev %	0,0 %	14,8 %	14,8 %	45,3 %	0	0	0	69,2 %	69,0 %	74,6 %
Cost (kNOK)	-2027	-2025	-2025	-2265	0	0	0	-2272	-2231	-2283
Costs %	0,0 %	-0,1 %	-0,1 %	11,7 %	0	0	0	12,1 %	10,1 %	12,6 %
OBM (m3)	878	855	855	1129	0	0	0	1177	1161	1177
OBM %	0,0 %	-2,6 %	-2,6 %	28,6 %	0	0	0	34,1 %	32,2 %	34,1 %
Bentonite (m3)	270	463	463	539	0	0	0	765	780	828
Bentonite %	0,0 %	-2,6 %	-2,6 %	28,6 %	0	0	0	34,1 %	32,2 %	34,1 %
Time	24	29	29	34	42% gap	26% gap	13% gap	49	58	58
Utilization	53,1 %	61,0 %	61,0 %	77,2 %	0	0	0	89,9 %	89,8 %	92,8 %
Visits	1 2 3 8	1 2 3 7 8	1 2 3 7 8	1 2 3 7 8	0	0	0	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
Obj Val	-1354,41	-879,3	-706,68	-596,68	No sol.	No sol.	No sol.	-330,38	-291,02	-278,18
Total time	19,0064	23,5007	29,4	34,1	42% gap	26% gap	13% gap	49,3	58,2571	58,4886
Total load	584,2	877,75	854,85	1128,95	0			1176,95	1160,75	1176,95
	362,25	270,25	463,25	539,15	0			765,15	779,65	827,65
Cap	1200	1200	1200	1200	0			1200	1200	1200
	960	960	960	960	0			960	960	960
Visits	1 2 3	1 2 3 8	1 2 3 7 8	1 2 3 7 8	0			1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
Node deliveries	p 1 to 2 = 287,5 m3	p 1 to 2 = 215,7 m3	p 1 to 2 = 212,5 m3	p 1 to 2 = 287,5 m3				p 1 to 2 = 287,5 m3	p 1 to 2 = 287,5 m3	p 1 to 2 = 287,5 m3
	p 1 to 3 = 296,7 m3	p 1 to 3 = 219,3 m3	p 1 to 3 = 219,3 m3	p 1 to 3 = 296,7 m3				p 1 to 3 = 296,7 m3	p 1 to 3 = 296,7 m3	p 1 to 3 = 296,7 m3
	p 2 to 2 = 132,25 m3	p 1 to 8 = 442,75 m3	p 1 to 8 = 423,05 m3	p 1 to 7 = 102 m3				p 1 to 4 = 150 m3	p 1 to 4 = 133,8 m3	p 1 to 4 = 150 m3
	p 2 to 3 = 230 m3	p 2 to 2 = 97,75 m3	p 2 to 2 = 97,75 m3	p 1 to 8 = 442,75 m3				p 1 to 8 = 442,75 m3	p 1 to 8 = 442,75 m3	p 1 to 8 = 442,75 m3
	X 1 2	p 2 to 8 = 172,5 m3	p 2 to 3 = 170 m3	p 2 to 2 = 120 m3				p 2 to 2 = 120 m3	p 2 to 2 = 120 m3	p 2 to 2 = 120 m3
	X 2 3	X 1 3	p 2 to 7 = 68 m3	p 2 to 3 = 199,65 m3				p 2 to 3 = 230 m3	p 2 to 3 = 230 m3	p 2 to 3 = 230 m3
	X 3 1	X 2 8	p 2 to 8 = 127,5 m3	p 2 to 7 = 92 m3				p 2 to 4 = 57,5 m3	p 2 to 5 = 81,65 m3	p 2 to 5 = 81,65 m3

	X 3 2	X 1 3	p 2 to 8 = 127,5 m3		p 2 to 5 = 81,65 m3	p 2 to 6 = 136 m3	p 2 to 6 = 184 m3
	X 8 1	X 2 7	X 1 3		p 2 to 6 = 184 m3	p 2 to 7 = 92 m3	p 2 to 7 = 92 m3
		X 3 2	X 2 7		p 2 to 7 = 92 m3	p 2 to 9 = 120 m3	p 2 to 9 = 120 m3
		X 7 8	X 3 2		X 1 6	X 1 9	X 1 9
		X 8 1	X 7 8		X 2 3	X 2 8	X 2 8
			X 8 1		X 3 1	X 3 2	X 3 2
					X 4 5	X 4 6	X 4 6
					X 5 7	X 5 4	X 5 4
					X 6 4	X 6 1	X 6 1
					X 7 8	X 7 5	X 7 5
					X 8 2	X 8 7	X 8 7
						X 9 3	X 9 3

Case 7 uses the same input data as case 6, but it varies time constraint and keeps flexibility constant at +/- 30%.