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Stochastic Optimisation of Battery System Operation Strategy under different Utility Tariff Structures

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Abstract

This master thesis builds on the project work [1], and develops a stochastic optimisation software for household grid-connected batteries combined with PV-systems. The objective of the optimisation is to operate the battery system in order to minimise the costs of the consumer, and it was implemented in MATLAB using a self-written stochastic dynamic programming algorithm. Load was considered as a stochastic variable and modelled as a Markov Chain. Transition probabilities between time steps were calculated using historic load patterns from up to three previous years, exploiting the repetitive patterns of weekdays and weekends. PV-production was considered deterministic when included. The SDP-model was tested on data from Norwegian households for 2016, and the global optimum solution was used as a benchmark, as found using the dynamic programming model from [1].

As Norwegian households were used as test cases, the Nordic power market Nord Pool Spot sat the scene for market transaction calculations. Day-ahead spot prices were used as market prices, meaning that the prices for the coming day was considered deterministic from noon the present day. The fixed 24-hour horizon optimisation was performed at midnight for each day, yielding an optimal wait-and-see operational policy for the battery system.

The optimisation was investigated under three different utility tariff (UT) structures: Energy based, time based and power based. The energy based UT is a fixed price per kWh, which is what is being used in today's market. The time based UT is a time-of-use tariff, which penalised use during peak demand hours 9-11 and 17-19 in weekdays. The power based UT increases linearly with the demand, designed to limit the power usage at any given time and day. While the energy based UT is what is being used as of today, the widespread roll-out of advanced metering systems (AMS) by 2019 in the Norwegian market will enable UT structures such as the time- and power based ones studied in this thesis.

The global optimal solution achieved 1.2 % (energy based UT), 14.2 % (time based UT) and 8.6 % (power based UT) of operational savings without a PV-system, illustrating the negligible potential for saving under the energy based UT. The developed SDP-model achieved 75-92 and 87-94 % of the global optimal savings without a PV-system under the energy- and time based utility tariff, respectively. This is increased to 92-99 % and 91-96 % with the PV-system installed. Under the power based utility tariff the model shows less promising results, scoring a maximum of 25-44 % of the global optimal solution without a PV-system, and 75-90 % with.

Sammendrag

Denne masteroppgaven bygger på prosjektoppgaven [1], og utvikler et dataprogram for stokastisk optimering av ladestrategien av et batteri i en husstand med og uten et PV-system installert. Målfunksjonen i optimeringen er kostnadene for forbruker, som søkes minimert gjennom riktig opp- og utladning av batteriet. Programmet ble utviklet i MATLAB, og selve optimeringen gjennomføres av en egenutviklet stokastisk dynamisk programmerings-algoritme. Lasten til husstanden ble antatt som stokastisk, og denne ble modellert som en Markov-kjede. Transisjonssannsynlighetene mellom tidssteg ble beregnet med bakgrunn i historiske lastdata fra opp til tre tidligere år ved å utnytte likheten blant ukedager og helgedager. PV-produksjon ble antatt deterministisk når inkludert. Optimeringen ble gjennomført på data for norske husholdninger fra 2016. Ytelsen til den stokastiske løsningen ble målt i oppnådde besparelser sammenlignet med global optimal løsning, funnet ved hjelp av en egenutviklet dynamisk programmerings-algoritme.

Ettersom norske husholdninger ble brukt som kilde for alle lastdata, ble den nordiske kraftbørsen Nord Pool Spot valgt som bakgrunn for alle markedstransaksjoner. Day-ahead spot priser, som blir publisert kl. 12 hver dag, ble brukt i alle kostnadsberegninger. Dette ble utnyttet i den stokastiske optimeringen ved å gjøre spot prisen til en deterministisk variabel. Den stokastiske optimeringen hadde en fast 24-timers optimeringshorisont, og ble gjennomført ved midnatt hver dag.

Optimeringen ble foretatt under tre ulike nettleiestrukturer: Energibasert, tidsbasert og effektbasert. Den energibaserte strukturen gir en fast pris per kWh levert fra nettet, som er hva som benyttes i det norske markedet per dags dato. Den tidsbaserte strukturen varierer med hvilken tid og dag det er, for å gi incentiver til forbrukeren om å flytte last vekk fra timene 9-11 og 17-19 i hverdager. Den effektbaserte nettleien øker lineært med effektuttaket fra nettet, og har som mål å incentivere forbrukeren til å begrense til sitt effektuttak til alle døgnets tider. Selv om det er den energibaserte strukturen som benyttes i dagens marked, så vil utrullingen av AMS-målere i norske husholdninger innen 2019 muliggjøre en overgang til tids- eller effektbaserte nettleiestrukturer som benyttet i denne oppgaven.

Optimale relative besparelser var 1.2 % (energibasert), 14.2 % (tidsbasert) og 8.6 % (effektbasert) uten et PV-system, noe som viser at potensialet for besparelser er neglisjerbart under den energibaserte nettleiestrukturen. SDP-modellen oppnådde 75-92 % og 87-94 % av de optimale besparelsene under henholdsvis den energi- og tidsbaserte nettleiestrukturen uten et PV-system. Disse økte til 92-99 % og 91-96 % med et PV-system inkludert. Under den effektbaserte nettleiestrukturen viser derimot SDP-modellen vesentlig dårligere resultater, og oppnår kun 25-44 % av de optimale besparelsene uten et PV-system, og 75-90 % med.

Preface

This master's thesis has been carried out at the Department of Electric Power Engineering at the Norwegian University of Science and Technology (NTNU) during the spring of 2017, and it was supervised by Professor Magnus Korpås. The preceding project work [1] was carried out during the autumn of 2016, and was supervised by Professor Ole-Morten Midtgård.

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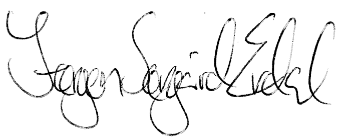
Trondheim, June 13, 2016.



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Abbreviations

AMS Advanced metering systems

BESS Battery energy storage system

DP Dynamic programming

EVPI Expected value of stochastic solution

GHI Global horizontal irradiance

LMT Landbruksmeteorologisk tjeneste

MDP Markov decision process

NPV Net present value

NVE Norges vassdrags- og energidirektorat

PV Photovoltaic

SDDP Stochastic dual dynamic programming

SDP Stochastic dynamic programming

SOC State of charge

STC Standard test conditions

UT Utility tariff

VSS Value of stochastic solution

Chapter 1

Introduction

This master thesis is a continuation of the project work ”En studie av økonomisk potensiale for PV-systemer og batterier i husstander for ulike nettleiestrukturer” [1], which developed a dynamic programming (DP) model to evaluate the economical potential of a household battery, with and without a PV-system. The model was based on deterministic values for load, solar irradiance and spot prices, which made the resulting battery operation impossible to achieve in real life, but rather a demonstration of the global optimal solution.

In this thesis, the DP-model will be developed into a stochastic dynamic programming (SDP) model which optimises the charging strategy in real time based on predicted values for load. The model will be tested on households in central Norway, and the Nord Pool Spot thus set the scene for market transactions. The day-ahead spot price from Nord Pool Spot, which is announced at noon for the coming day, will be used in all consumer cost calculations. The day-ahead announcement will be exploited in the stochastic optimisation, making spot price a deterministic value in the 12-36 hour horizon. The households will be modelled both with and without a PV-system, and when included it will be considered as deterministic, as the focus of this thesis will be the prediction of load rather than PV-production. The model will be tested for three different utility tariff structures: Energy based, time based and power based, in order to investigate how these effect the performance of the SDP-model as well as the global optimal solution.

The motivation for developing the DP-model into a SDP-model is to achieve a battery operation strategy which can be implemented in real life, and to investigate how close to the global optimal solution the SDP-model can perform. The global optimal solution will be calculated ex ante by the use of the DP-model developed in [1].

All programming will take place in MATLAB, and everything will be self-developed. No third party software, for instance readily available optimisation packages, will be used.

Chapter 2

Theory

2.1 Literature Review

The interest in grid-connected batteries has increased rapidly over the recent years, which is evident from the amount of research papers that are published on the topic. In the research for this thesis, numerous papers has been reviewed in order to get a good understanding of the academic scene as of today. In this section, the most relevant papers from this review are highlighted and explained in brief. Curious readers and researchers looking into the sphere of stochastic optimisation of grid-connected batteries are encouraged to look further into these.

In [3], the authors develops a method to optimise the charging strategy of batteries in distribution networks with PV-generation. The PV-generation is considered stochastic, and by modelling it as a Markov Chain with 14 discrete states they are able to use a SDP algorithm. Although the PV-generation is considered as stochastic, the price and load of the system is assumed deterministic, which enables the authors to use a deterministic DP algorithm during night time (when there is no sun) and a SDP algorithm during day time (8 am to 4 pm). Their findings are that the revenue from the battery energy storage system (BESS) increases with its rated capacity and power.

Similar to [3], the authors of [4] models the problem of optimising the battery operation as a Markov decision process, but without the presence of any renewable generation. Using stochastic price and load, both assumed independent from hour to hour, the authors derive a simple threshold-based operation policy. They also use state-dependent charging constraints, allowing for dynamic power constraints dependent on the SOC of the battery.

Both [5] and [6] investigates the economical potential in battery systems without the need of supplying a given load or taking renewable generation into account. In [5] a SDP algorithm is used with a stochastic price which is forecasted using the trailing median along with a first order autoregressive model. [6], which as this thesis takes place in the Nord Pool Spot, utilizes the fact the spot prices are available the day ahead, and thus optimises the charging strategy with deterministic prices using sequential quadratic programming assuming an empty battery at midnight.

The authors of [7] uses the method of stochastic dual dynamic programming (SDDP) to optimise the revenues from a system consisting of a PV-system, a flexible load and a BESS. By using SDDP, one avoids the "curse of dimensionality" due to the discretisation of the stochastic variables. Nevertheless, the authors finds that their proposed SDDP algorithm performs poorer than a standard SDP algorithm below certain level of computational power.

2.2 Stochastic Optimisation

Stochastic programming are a generalisation of deterministic programs, yet where one or several of the variables are not known for certain. The generic mathematical formulation of a stochastic optimisation is presented in equation (2.1). Some variables $x \in X$ are to be decided under the influence of some random variables $\omega \in \Omega$ with their associated probability distribution $F(\omega)$. Each decision has an associated reward or loss dependant on the outcome of ω given by $r(x, \omega)$ [8]. Constraints of the mathematical problem is omitted in equation (2.1) due to simplicity.

$$\max \mathbb{E}_\omega[r(x,\omega)|F] = \max \int_\omega r(x,\omega)dF(\omega) \tag{2.1}$$

The difference to deterministic optimisation is that stochastic programs optimise the expected value of the reward or loss function, due to the uncertainty in the input data. This enables the use of the same solving techniques as deterministic programs with only minor changes, which will be demonstrated in this thesis by the use of *dynamic programming* and *stochastic dynamic programming* as discussed in sections 2.3 and 2.4.

2.2.1 Markov Decision Processes and Markov Chains

As described in [8] and [9], a *Markov Decision Process* (MDP) is a definition of a multi-period sequential stochastic decision problem were the operator is to take actions in discrete time steps. The MDP consists of decision epochs, states, actions, rewards and transition probabilities. The action taken in decision epoch t under a state generates a reward, and determines the state in the following decision epoch $t + 1$ trough the given transition probability.

In each decision epoch, it is assumed that the decision maker knows the present state, the set of possible actions and their associated rewards, and the transition probabilities towards the next decision epoch. The set of actions and associated rewards as well as transition probabilities in every decision epoch are only dependant on the present state, and not on previous sates and actions. MDPs are solved by recursive calculation using dynamic programming, or stochastic dynamic programming when stochastic variables are present.

Load will in this thesis be considered as a stochastic variable, and would thus need to be predicted with a certain probaility in order to solve the optimisation problem. To do this, load will be modelled as a *Markov Chain*. This assumes that can load be considered as *memoryless*, meaning that the probabilities of future load states are only dependent on the current state and not on the path that lead to it [9]. This is an important assumption which enables recursive calculation, as used in dynamic programming and stochastic dynamic programming.

The problem in question in this thesis is indeed a MDP, where the decisions that are to be made are on how to operate the battery in each time step. The decision epochs are the discrete time steps, the states are both the states of the battery (in terms of SOC) and load, the transition probabilities describe the transitions between load states and the ”rewards” are the associated costs for buying power from the grid.

2.2.2 The Value of Information and the Stochastic Solution

When dealing with stochastic optimisation, there are two key metrics which are of special importance. Namely the *expected value of stochastic solution* (EVPI) and the *value of stochastic solution* (VSS). This section is based on [8], and will briefly introduce these concepts and relate them to the optimisation problem in question in this thesis.

The Expected Value of Perfect Information

EVPI quantifies the maximum amount a planner would be willing to pay in order to receive perfect information about the future, and is defined by the following equation:

$$EVPI = RP - WS. \tag{2.2}$$

WS is what is known in theory as the *wait-and-see* solution, given by equation (2.3). The parameter $\boldsymbol{\epsilon}$ is here the set of all possible scenarios (which in this thesis would be different load patterns), z is the objective function and x the set of decisions that are to be made. $\bar{x}(\epsilon)$ is the set of optimal decisions corresponding to the set of scenarios $\boldsymbol{\epsilon}$, which could be calculated using DP. Thus, WS is the sum of the objective functions given the optimal decisions for each scenario multiplied with the probability of each scenario.

$$WS = \mathbb{E}_{\boldsymbol{\epsilon}} z(\bar{x}(\boldsymbol{\epsilon}), \boldsymbol{\epsilon}) \tag{2.3}$$

RP is the *here-and-now* solution corresponding to the recourse problem (RP). This is what will be calculated in this thesis using equation (2.14), and its general definition is given in equation (2.4).

$$RP = \min_{x} \mathbb{E}_{\epsilon} z(x, \epsilon) \tag{2.4}$$

The challenge with calculating the EVPI is the estimation of the probabilities for each load pattern scenario, used to calculate WS. If one were to divide load into 20 states in each hour of the day, this would give 20^{24} load scenarios, where most of these would have a negligible probability. Because of this, it was decided to not further pursue EVPI in this thesis, as the results most probably would have been subject to too many assumptions.

The Value of Stochastic Solution

VSS quantify how good, or bad, a simplified solution is compared to the stochastic solution obtained by (2.4). In order to specify the simplified solution, one first has to define the *expected value problem*, which is given by equation (2.5).

$$EV = \min_{x} z(x, \bar{\epsilon}). \tag{2.5}$$

This is a simplified problem, as the stochastic variables has been replaced by their expected values, denoted by $\bar{\epsilon}$. Let $\bar{x}(\bar{\epsilon})$ be the optimal solution to (2.5), then the *expected result of using the EV solution* would be given by

$$EEV = \mathbb{E}_{\boldsymbol{\epsilon}}(z(\bar{x}(\bar{\epsilon}), \boldsymbol{\epsilon})), \tag{2.6}$$

which is the expected value of using the solution of the expected value problem on all scenarios. Finally, VSS is defined as

$$VSS = EEV - RP. \tag{2.7}$$

In this thesis, the VSS could be calculated by replacing the stochastic load by its expected value and then running this through the DP-model to find $\bar{x}(\bar{\epsilon})$. Calculating EEV, the same probabilities for different load patterns as in WS are needed, which as discussed above is not in the interest of this thesis to look into. Hence will VSS not be further investigated in this thesis.

2.3 Dynamic Programming

Dynamic programming (DP) is an optimisation method which is useful when the problem in question can be divided into discrete states and time steps, breaking one problem into smaller subproblems [10]. The optimisation (maximisation or minimisation) is performed on the *Bellman equation* of the problem, which states the value of the problem as a payoff from some initial choices and the sum of future payoff given a set of optimal decisions, one for each subproblem. This builds on the basis that the problem can be considered memoryless, as described in section 2.2.1, enabling the problem to be optimised by recursive calculation, fulfilling *Bellman's Principle of Optimality* [11].

This is very much the case for the operation of a battery in a household, as the battery can be divided into a number of discrete states of SOC over the optimisation horizon. The optimisation horizon is divided into discrete time steps, which is natural as the input data (e.g. load and price) already are sampled in discrete time steps. In order to find the optimum solution to the desired objective function, DP calculates the value of the objective function for all legal paths recursively and chooses the one that minimises or maximises the objective function. In this section, DP will be explained in the light of this thesis.

The time series will be divided into N_t discrete time steps, i.e. $t = 1, 2, ..., N_t$. Further, the battery will be divided into N_{SOC} discrete states of SOC, i.e. $SOC(t) = 1, 2, ..., N_{SOC}$, where $SOC(t) = 1$ represents an empty battery and $SOC(t) = N_{SOC}$ a full one. This creates a network of $N_t \times N_{SOC}$ *nodes*, which is illustrated in figure 2.1.

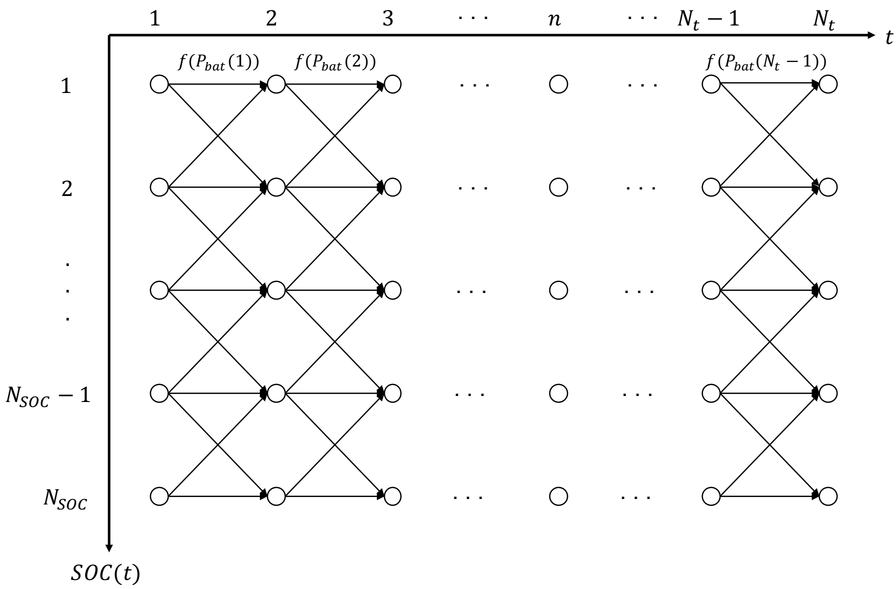

Figure 2.1: Illustration of a DP problem with N_t time steps and N_{SOC} states of SOC.

The nodes are connected by arrows, representing the possible operations of the battery in each time step for all states of SOC. Which decisions that are allowed is dependent on the battery efficiency η_{bat}, maximum power $P_{bat,max}$ and capacity Q_{nom}. The maximum allowable change of states of SOC (ΔSOC_{max}) is given by equation (2.8), and is illustrated in figure 2.2.

$$\Delta SOC_{max} = \frac{\eta_{bat} P_{bat,max}}{Q_{nom}} N_{SOC} \tag{2.8}$$

where

$$\eta_{bat} = \begin{cases} \eta_{ch}, & \text{if } P_{bat} > 0 \\ 1/\eta_{di}, & \text{otherwise.} \end{cases} \tag{2.9}$$

Each decision has its corresponding *transfer cost* given by $f(P_{bat}(t))$, where t indicates in which time step the decision is made. P_{bat} is a function of which state one starts in and to which one transfers to, where a decrease in state corresponds to discharging the battery, zero change to keeping the current SOC and an increase to charging the battery.

In order to achieve a unique optimal solution, one would need to specify both a desired final and initial state of SOC. This would render some nodes and decisions ”illegal”, as they do not lead to the desired final state and/or are not reachable from the initial state.

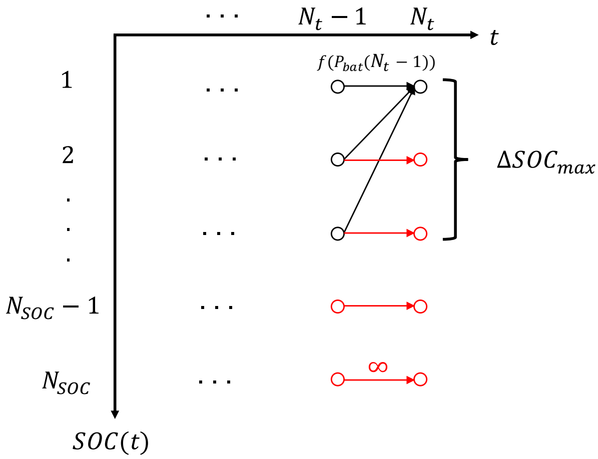

Figure 2.2: Some nodes and decisions will not be allowed because of the desired final state and $P_{bat,max}$.

These illegal nodes and decisions are illustrated in figure 2.2 by a red colour. Illegal decisions would receive an infinite transfer cost[1], and are thus avoided when the optimum path is to be chosen.

By recursive calculation of all transfer costs from $t = N_t - 1$ to $t = 1$, each legal node in each time step will be assigned its minimum or maximum (depending on the objective) *cost-to-go* value C_{tG} and which decision that leads to this solution. When implementing this in e.g. MATLAB, one would need two $N_t \times N_{SOC}$ matrices, where one is the C_{tG}-matrix and the other is the *path matrix*. The path matrix stores to which $SOC \in [1, N_{SOC}]$ to transfer to in the following time step. This ultimately results in one unique optimum solution for each final and initial state of SOC.

The mathematical formulation of the optimisation problem that would need to be solved for all time steps and legal states of SOC in the DP-model is as follows:

$$
\begin{aligned}
\underset{P_{bat}(t)}{\text{Minimise}} \quad & C_{tG}(SOC(t), t) \\
\text{such that} \quad & SOC(t+1) \leq SOC_{max} \\
& SOC(t+1) \geq SOC_{min} \\
& P_{bat}(t) \leq P_{bat,max} \\
& P_{bat}(t) \geq -P_{bat,max} \\
& E_{bat}(t+1) = E_{bat}(t) + \eta_{bat} P_{bat}(t) \Delta t \\
& SOC(t+1) = \frac{E_{bat}(t+1)}{Q_{nom}}
\end{aligned} \tag{2.10}
$$

[1] If one were to maximise the objective function, these would receive a negative infinite transfer cost

where

t	= 1, 2, ... , $N_t - 1$ is the time step,
SOC_{max}	is the maximum allowable SOC,
SOC_{min}	is the minimum allowable SOC,
$P_{bat,max}$	is the maximum power at the battery terminals (in kW),
E_{bat}	is the energy in the battery (in kWh),
η_{bat}	is the battery efficiency given by (2.9), and
Q_{nom}	is the capacity of the battery (in kWh).

The objective function, which is the cost-to-go C_{tG}, is here to be minimised, and will in this thesis be the total cost for the consumer. The cost-to-go is the sum of the cost of the given time step and the cost of all future time steps given optimal battery operation:

$$C_{tG}[SOC(t), t] = f(P_{bat}(t)) + C_{tG}[SOC(t+1), t+1] \tag{2.11}$$

The cost of any given time step and decision is given by the following equation:

$$\begin{aligned} f(P_{bat}) &= C_{el} P_{grid} \\ &= C_{el}[P_{load} + P_{bat} - P_{PV}] \end{aligned} \tag{2.12}$$

where C_{el} is the marginal cost of electricity when buying from the grid ($P_{grid} > 0$) or receives for selling to the grid ($P_{grid} < 0$). C_{el} includes both the spot price as well as the utility tariff, and is further discussed in section 3.2.1.

Using DP, one has to assume that the problem is deterministic, meaning that the planner has perfect information about all input data. This renders the DP-solution in this thesis only implementable in real life if one has perfect information about future prices, load and PV-production. This might be a viable assumption in some cases, like those where load is highly predictable (for instance in industry), spot prices are published the day ahead and no PV-system is present. This is not the case for private households, which are subject to significant seasonal and daily fluctuations in load. Nevertheless, as this thesis will show, there does exist some predictable patterns in load that might be exploited in order to optimise the operation of a household battery by the use of stochastic optimisation.

2.4 Stochastic Dynamic Programming

In order to take the stochastic load into account in the optimisation problem, one would need to use stochastic dynamic programming (SDP) rather than DP [12]. By using information of the probability of future scenarios and assuming that the stochastic variable can be modelled as memoryless, one is enabled to use SDP to optimise the operation of a household battery in much the same way as DP. As presented in section 2.2, the method of SDP is similar to that of DP, but the mathematical formulation of the optimisation problem is slightly altered:

$$\begin{aligned}
\underset{P_{bat}(t)}{\text{Minimise}} \quad & \mathbb{E}[C_{tG}(SOC(t), P_{load}(t), t)] \\
\text{such that} \quad & SOC(t+1) \leq SOC_{max} \\
& SOC(t+1) \geq SOC_{min} \\
& P_{bat}(t) \leq P_{bat,max} \\
& P_{bat}(t) \geq -P_{bat,max} \\
& E_{bat}(t+1) = E_{bat}(t) + \eta_{bat} P_{bat}(t) \Delta t \\
& SOC(t+1) = \frac{E_{bat}(t+1)}{Q_{nom}}
\end{aligned} \tag{2.13}$$

There are two differences between the mathematical formulation of the DP, equation (2.10), and that of the SDP, equation (2.13). These are (1) the usage of the *expected value* of the cost-to-go instead of the cost-to-go itself in the objective function, and (2) the objective function's dependence of the load in time t. The expected cost-to-go is given by equation (2.14), and will be further discussed below. From basic probability theory, the expected value is equal to the probability-weighted sum of all possible values, in order words the sum of products of the probability of a certain scenario and the associated value of the given scenario. In the mathematical problem in question, the uncertainty lies in the load. More specifically the uncertainty of load being in a certain state in the following time step, given its state in the current time step. How these probabilities will be calculated is explained in section 3.4.2.

Similar to what is done with the SOC of the battery, the stochastic variables have to be divided into discrete states in order to be implemented into the SDP. There is no constraint on how many stochastic variables one could include, but the curse of dimensionality (i.e. run-time of the code) will limit the number of stochastic variables and their level of discretisation. In this thesis, only the load of the household will be modelled as a stochastic variable, which limits the run-time and complexity of the model.

Figure 2.3 illustrates the calculations in the chosen SDP-implementation. The load can be in any of N_{load} discrete steps in any time step, in the same manner as the battery SOC can be in any of its N_{SOC} discrete states[2]. This is illustrated in figure 2.3 by the three respective axes. The $load(\tau)$ and $load(\tau+1)$ axis represents the load in time step τ and $\tau+1$, respectively, in the same way as $SOC(\tau)$ and $SOC(\tau+1)$ represents the SOC in either time step. Time is represented in the last axis, denoted by t. In the snapshot of figure 2.3, the iteration has reached load state k and SOC state m in time step τ. The arrows pointing to different load states in the following time step illustrates three of the possible load transitions, here for SOC state n in time step $\tau+1$. Indicated is $p(k,1,\tau)$, $p(k,l,\tau)$ and $p(k,N_{load},\tau)$, which are the *transition probabilities* of the transitions from load state k in time step τ to load state 1, l and N_{load} in time step $\tau+1$, respectively. Notice that the transition probabilities are only dependant on the current load state, the following load state and the current time step, and not on the SOC state.

These transition probabilities, stored in one or several *transition matrices*, are used in the calculation of the expected value of the cost-to-go for time step τ, i.e. the objective

[2]Given that this is a legal state. This is dependant on the chosen final and initial SOC.

function of the SDP. For $SOC(\tau) = m$, $SOC(\tau+1) = n$ and $P_{load}(\tau) = k$, the expected value in time step τ would be given by:

$$\mathbb{E}[C_{tG}(m,k,\tau)] = \underset{n}{min}\left\{ f(m \to n, k) + \sum_{l=1}^{N_{load}} p(k,l,\tau) \times \mathbb{E}[C_{tG}(n, l.\tau+1)] \right\} \tag{2.14}$$

where $m \to n$ indicates $P_{bat}(\tau)$ given the change in SOC, and f is still given by (2.12).

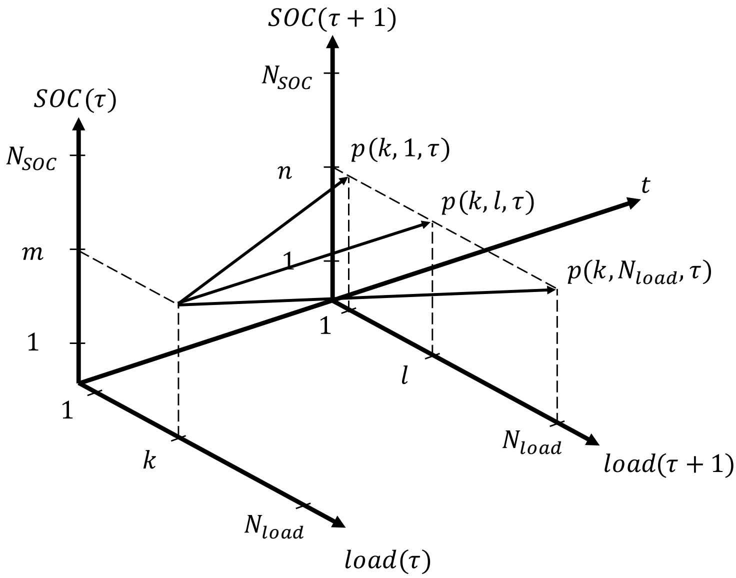

Figure 2.3: Visualisation of the action space of the SDP.

As with the DP-method, the SDP proceeds recursively from $t = N_t - 1$ until $t = 1$ calculating the expected cost-to-go for all loads and legal SOCs. This ultimately results in $N_{SOC} \times N_{load} \times N_t$ optimal decisions, one for each of the combinations of SOC, load and time step. By doing so, one achieves sort of a *wait-and-see* strategy which allows the operator to postpone the decision on how to operate the battery until he or she knows what the load and SOC turns out to be in the respective time steps.

Unlike the DP-model, the SDP-model will benefit from being run for shorter time horizons. The DP-method assumes perfect information, and are thus allowed to optimise over as long time horizons as this assumption holds. This is not the case for the SDP-model, as it is dependent on the spot price information, which is available a maximum of 36 hours in advance, as well as the load state transition probabilities between time steps. These probabilities might vary over the day, week, month or year, depending on the design of the stochastic model which generates the transition matrices and the discretisation of the load.

2.5 The Power Market

This thesis aim at developing a stochastic optimisation model which minimises the cost of supplied energy to a household by the correct operation of a household battery system. Hence, it is important to understand how the household interacts with the power market. Because this thesis will use data from households in central Norway, the Norwegian power market will be used as foundation for this thesis, namely the *Nord Pool Spot*.

2.5.1 Spot Price

Norway is part of the Northern European power exchange Nord Pool Spot, which is owned by the nordic and baltic transmission system operators Statnett SF, Svenska Kraftnät, Fingrid Oyj, Energinet.dk, Elering, Litgrid og Augstsprieguma tikls (AST) [13]. Nord Pool Spot takes care of the bidding process within the day-ahead and intraday markets, and has over 380 participants from 20 countries. As a market operator, one of Nord Pool Spot's mandates is to ensure that all market participants has full access to relevant market information. Because of this, all historic prices, capacities and transactions are available on their websites.

The day-ahead spot prices are set at 12:00 CET the day ahead of production, when all sellers and buyers have submitted their bids to Nord Pool Spot. The bids reflect the participants willingness to sell or pay a given amount of power, hour by hour. Based on these bids, the market operator calculate the spot price in each area, which dependent on the available transmission capacity between the areas [14].

2.5.2 Utility Tariff

In addition to the spot price, the consumer has to pay a fee to his or her local utility company. This fee covers the operation, maintenance and development of the grid, and is called the *utility tariff* (UT) or simply the tariff. In Norway, the UT is decided individually by each utility company which in turn is governed by *The Norwegian Water Resources and Energy Directorate* (abreviated by NVE in Norwegian). NVE controls that no utility company are to receive higher profits than what is allowed, which in turn sets an upper bound on the UT.

Today, Norwegian consumers are subject to an *energy based* UT. This means that the consumer pays the same price per kWh, no matter when it was delivered or at how high a power. Further, the UT is split into an *energy term* and a *fixed term*, where the fixed term is independent of energy delivered [15].

By 2019, all Norwegian households will have a smart meter, known as an Advanced Metering System (AMS), installed. These devices measure electricity consumption in real time, and transmits this information hourly to the utility company. AMS are also capable of two-way communication, which can be used to communicate spot prices to the consumer, giving incentives to adjust consumption in order to avoid high prices. The consumer is allowed to track his or her consumption, for instance with a smartphone, and thus become more aware of energy and power usage [16].

As [17] points out, a widespread installation of AMS allows for new and innovative UT structures which in turn might lead to a more effective usage of the grid. For instance by *time based* and *power based* UT structures. Further, [17] highlights how power based UT structures might accelerate the adoption of batteries in households due to their peak shaving[3] and valley lifting[4] capabilities. This will be investigated in this thesis, as the SDP-model will be tested under three different UT structures, as described in sectin 3.2.2.

2.5.3 The Prosumer Agreement

A consumer which has local electricity generation that sometimes exceeds his or her demand is called a *prosumer*. In March 2010, NVE accepted the *prosumer agreement* which allows prosumers to sell their surplus energy to the market without being subject to the same tariffs as larger producers [18]. It is not mandatory for the utility companies to offer this arrangement to their customers, but it is usual to do so. Prosumers are usually paid the spot price for energy injected into the grid and does not have pay UT for delivered energy. Rather, the prosumer is sometimes paid a premium for limiting losses in the grid, caused by energy being injected closer to consumption.

2.6 Battery Degradation

A battery is degraded as it is being used, and will thus loose performance in terms of capacity and power over time. This is a major concern when profitability calculations are to be made, as there is much uncertainty tied to battery degradation [19]. [20] proposes an energy management system for a battery coupled with a PV-system, and the authors included degradation into the optimisation as a battery degradation cost in order to avoid unnecessary operation of the battery. This cost is estimated by dividing the total installation and maintenance cost over the battery's lifetime by the lifetime energy throughput. Due to the uncertainties in these parameters, and the fact that this thesis does not aim to contribute to quantifying the economical potential in batteries, this cost will not be modelled further in this thesis. Nevertheless, important battery degradation theory is here summarised due to its importance in discussions of deployment of batteries in households.

A battery is usually considered to have reached its end of life when the capacity has reached 80 % of the initial capacity or when the internal resistance has doubled. This is caused by internal chemical reactions in the battery, which are dependent on both the operation of the battery as well as in which environment the battery is being kept. Because of the associated complexity and randomness, it is difficult to analyse and simulate such effects. The degradation of batteries is usually divided into that caused by time (calendar) and use (cycle). The calendar ageing happens spontaneously, and is accelerated by high temperatures and low cell potentials. The cycling ageing is caused by the usage of the battery, and is thus dependant on operational variables like depth-of-discharge, SOC and charge/discharge power. The higher the power, the higher the degradation.

[3] Peak shaving is the process of lowering the consumption during hours of relatively high consumption.
[4] Valley lifting is the process of increasing the consumption during hours of relatively low consumption.

Chapter 3

Methodology

The decision problem in this thesis is a stochastic multi-period sequential optimisation problem, where the periods are connected by the energy storage of the battery. The reward in each period is only influenced by the decision made in that period, but the action space is heavily dependent on decisions in previous periods. Thus, the problem fulfils the criteria of a MDP as described in section 2.2, which is also argued in [3].

The household load will be modelled as a Markov Chain, as it is considered memoryless. By memoryless one assumes that the future load states are only dependent on the present load state, and not on the path that led to it, which is an important assumption in using DP and SDP. Each time step will have a set of transition probabilities which define the probability of discrete load state transitions. Given the transition probabilities and assuming the spot prices and UTs as deterministic over the optimisation horizon, the assumed optimal decision in each time step is calculated using a SDP-method as described in section 2.4.

This chapter will in a step-wise manner describe how this is modelled and implemented in MATLAB.

3.1 Modelling of Household, Battery and PV-system

The household will be modelled as a loss-less bus bar with power directions as indicated in figure 3.1, yielding the power balance given by equation (3.1). All data will be hourly values, and load and PV-production are considered inflexible, meaning that load has to be met at all cost. Load data will be given in kWh, which will be assumed as a met by flat kW-power over that given hour. Thus, 1 kWh for hour τ is equal to $P_{load}(\tau) = 1$ kW. Load is thus analogues to power in this thesis.

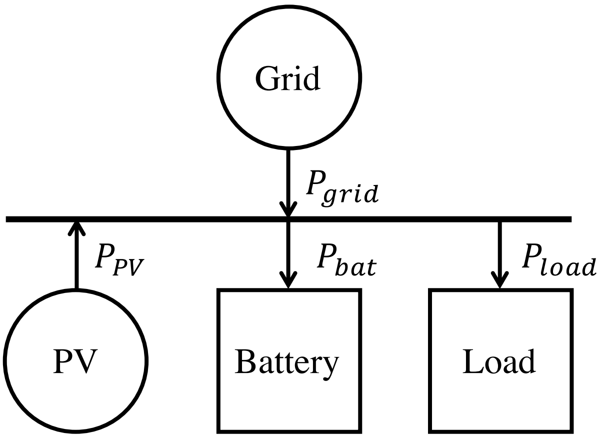

Figure 3.1: Schematic of the household with indicated power directions.

$$P_{grid} + P_{PV} = P_{load} + P_{bat} \tag{3.1}$$

Modelling of battery

The battery in the household will be modelled by the following parameters:

$P_{bat,max}$	the maximal terminal power (in kW),
η_{ch}	the charging efficiency,
η_{di}	the discharging efficiency,
SOC_{max}	the maximum allowable SOC,
SOC_{min}	the minimum allowable SOC, and
Q_{nom}	the capacity of the battery (in kWh).

The battery *State of Charge* (SOC) is a parameter $\in [0, 1]$ which defines the amount of energy that is stored in the battery at any given time, and is defined by the energy in the battery E_{bat} and its capacity Q_{nom}:

$$SOC = \frac{E_{bat}}{Q_{nom}} \tag{3.2}$$

Including a battery into the modelling links the time periods together, as energy can be stored from one time step to another. This linking is defined by equation (3.3), where the battery efficiency η_{bat} is defined by equation (3.4). Δt is the duration of the time steps, which in this thesis will be one hour. Notice that energy depletion over time is not included, which means that energy can be stored for an infinite time.

$$E_{bat}(t+1) = E_{bat}(t) + \eta_{bat} P_{bat} \Delta t \tag{3.3}$$

$$\eta_{bat} = \begin{cases} \eta_{ch}, & \text{if } P_{bat} > 0 \\ 1/\eta_{di}, & \text{otherwise.} \end{cases} \tag{3.4}$$

As discussed in section 2.6, degradation of the battery that occurs during usage will not be taken into account in this thesis, which means that the capacity and maximum terminal power will not degrade as time goes.

Calculation of PV-production

The production from a PV-system (in kW) is given by the following equation [21]:

$$P_{PV} = P_{nom} \eta_{sys} \frac{G_T}{G_{T,STC}} [1 + \alpha_T (T_{cell} - T_{cell,STC})] \tag{3.5}$$

where

P_{nom}	is the nominal installed power (in kW),
η_{sys}	is the system efficiency (in %),
$G_{T,STC}$	is the solar irradiance under standard test conditions (1 kW/m^2),
α_T	is the temperature coefficient (in %/°C),
T_{cell}	is the cell temperature (in °C), and
$T_{cell,STC}$	is the cell temperature under standard test conditions (25°C).

The solar irradiance used in this thesis is the *global horizontal irradiance* (GHI), as this is readily available in public databases. GHI is the sum of direct and diffuse radiation that strikes a horizontal plane, and is a usual input to PV-calculations [22]. Standard Test Conditions (STC) for solar panels is defined in order to enable comparison between different solar panels, and the associated values are as follows [23]:

- Solar irradiance $G_{T,STC} = 1000$ W/m^2
- Cell temperature $T_{cell,STC} = 25$°C
- Air mass ratio AM1.5

The cell temperature is given in °C and will be calculated by the following equation [22]:

$$T_{cell} = T_{amb} + \left(\frac{NOCT - 20°\text{C}}{0.8\text{kW m}^{-2}}\right) G_T \tag{3.6}$$

where

T_{amb}	is the ambient temperature (in °C), and
$NOCT$	is the *normal operating cell temperature*.

$NOCT$ is the expected cell temperature when the ambient temperature is 20°C, the solar irradiance is $0.8\ \text{kW m}^{-2}$ and the wind speed is 1 m/s. $NOCT$, P_{nom} and α_T is given in the data sheet of the PV-panel.

3.2 Power Market Transactions

3.2.1 Consumer Cost

The consumer will have to pay both spot price and UT for energy delivered to the household, but will only be reimbursed with the spot price when selling energy into the grid. This is in accordance with the Norwegian prosumer agreement as presented in section 2.5.3. All transactions will be time dependent, meaning that the transaction for any given time step is calculated using the P_{grid}, spot price and UT for that given hour according to equation (3.7).

$$\begin{aligned} C_{tot}(t) &= C_{el}(t) P_{grid}(t) \\ &= C_{el}(t)[P_{load}(t) + P_{bat}(t) - P_{PV}(t)] \end{aligned} \tag{3.7}$$

where

C_{tot}	is the total cost for the consumer (in NOK) and
C_{el}	is the marginal cost of electricity (in NOK/kWh).

As defined in the prosumer agreement, C_{el} is in this thesis dependent on the sign of P_{grid} as defined in equation (3.8).

$$C_{el} = \begin{cases} C_{spot} + C_{utility}, & \text{if } P_{grid} > 0 \\ C_{spot}, & \text{otherwise,} \end{cases} \tag{3.8}$$

where

C_{spot}	is the spot price (in NOK/kWh) and
$C_{utility}$	is the utility tariff (in NOK/kWh).

3.2.2 Utility Tariff Structures

Three different UT structures will be used in this thesis in order to evaluate their effect on the results: (1) Energy based, (2) time based and (3) power based. These will be described and discussed separately in the following.

Energy Based

An energy based utility structure is what is being used in the Norwegian power market as of today, and is as described in section 2.5.2 only dependent on the amount of energy delivered. This UT structure does not take into account when or at which power the energy was delivered. In today's market, the energy based UT is split in an energy term and a fixed term, but in this thesis will only the energy term be included. This is due to the fact that the fixed term can not be effected by the battery or PV-system, and is thus not of interes in this thesis.

Time Based

A time based UT structure is dependent on the time and day the energy is delivered. Such a UT structure incentivises the consumer to avoid high consumption during certain hours, which might be desired by utility companies. In this thesis, the time based UT will be implemented with three price levels: High, middle and low. The high price level will be used during hours of typically high loading, which in weekdays are in the morning and evening, as shown in figure D.1. The low price level will be used during night, when loading is low. During weekends, the loading is less characterised by such daily patterns, and it thus makes sense to apply a flat tariff for Saturdays and Sundays.

Power Based

A power based UT structure is designed to limit the power at any given time and day. This can be achieved in several ways, and will in this thesis be implemented by having the tariff increase linearly with power. The power based UT will thus be given by equation (3.9), where C_{power} is the power rate (in NOK/kWh^2).

$$C_{utility} = C_{power} P_{grid} \tag{3.9}$$

Notice that this causes C_{tot} to increase exponentially with P_{grid} under the power based UT, in accordance with equations (3.7)-(3.9):

$$C_{tot} = \begin{cases} C_{spot} P_{grid} + C_{power} P_{grid}^2, & \text{if } P_{grid} > 0 \\ C_{spot} P_{grid}, & \text{otherwise.} \end{cases} \tag{3.10}$$

3.3 DP Implementation

The DP implementation which will be used in this thesis is the same as used in [1]. Algorithm 1 describes the DP-algorithm as implemented in MATLAB, and the full code is included in appendix A. The output of the algorithm is the *path matrix*, which for the DP-model is a two dimensional matrix with the dimensions $N_{SOC} \times N_{t,tot}$, where $N_{t,tot}$ is the total number of time steps that are to be optimised. If M days are to be optimised, $N_{t,tot} = M \times N_{t,day}$.

The path matrix stores the optimal path to follow for all legal SOC's and time steps. If the battery is in SOC state n in time step t, path(n, t) stores the optimal SOC state to transition to during the current time step, yielding an optimal P_{bat}. The user can thus decide an initial SOC state after the optimisation, and then find the optimal path to follow, given that the desired initial SOC state is legal. This makes the implementation more flexible and enables studies of the value of initially stored energy. In all calculations in this thesis, the battery will be sat to be empty in the both the first and last time step of the optimisation horizon.

Note that the DP-optimisation is carried out once for all days in question, and not on a daily basis, which will be the case for the SDP-optimisation. More on this in section 3.6.

Algorithm 1: DP algorithm as implemented in MATLAB

cost-to-go = inf($N_{SOC}, N_{t,tot}$);
transitioncost = inf($N_{SOC}, N_{t,tot}, N_{SOC}$);
path = zeros($N_{SOC}, N_{t,tot}$);
Calculate legal SOCs for all time steps;
for $t = (N_{t,tot} - 1 : -1 : 1)$ **do**
  for *all legal soc(t)* **do**
    for *all legal soc($t+1$) reachable from soc(t)* **do**
      Calculate P_{grid} and the associated transition cost;
    end
    Evaluate which transition what minimises the cost-to-go;
    path($soc(t), t$) $\leftarrow soc(t+1)$;
    cost-to-go($soc(t), t$) $\leftarrow \min$(cost-to-go);
  end
end

3.4 Predicting Load

This thesis will model load as a stochastic variable in the optimisation of the operation of the household battery. This section will investigate two different hypotheses which could be used to predict future load based on historic data, which in turn will be the foundation for the stochastic model implementation discussed in section 3.5. TrønderEnergi provided hourly load data from 2013, 2014 and 2015 for three different households in central Norway. This data will be used as input to the stochastic model, which will be used to optimise the battery operation for 2016.

Figure 3.2 shows daily average load values for 2013, 2014 and 2015 for the three loads. Day 1 corresponds to the 1st of January in each year. There are three things worht noting from these plots. First, notice the large day to day variations, as all loads halves or doubles their daily average over the course of just a few days in all three years. Second, notice the difference between the three loads. Load 1 has the lowest daily average for almost all days, while load 2 has the highest values, and load 3 is somewhere in between. This will probably influence the final results, as a bigger load will most likely need a bigger battery in order to achieve the same benefits as a smaller load. Third, there is an evident seasonal variation for all three loads, as all three show lower load during summer compared to winter. All these three observations need to be taken into account by the stochastic model.

In order to use historic data to predict how the load will behave in the future, two hypotheses will be further investigated:

1. **Load follows a similar pattern for similar days.** If this is true, then one could build a stochastic model which recognises the pattern of historic load data and uses this to predict future behaviour given the day and time.

2. **Load is correlated with outdoor temperature.** If there exists a clear correlation between the two, it is possible to predict future load by using weather forecasts.

These two hypotheses will now be discussed in their separate sections.

3.4.1 Hypothesis 1: Load Pattern

To investigate the this hypothesis, it would be beneficial to remove the seasonal variation in load, which is clearly visible in figure 3.2. This will be done by normalising the load for all days against the daily maximum load value. Consider P_{load} as a vector containing all load values of any given day and $max(P_{load})$ as the daily maximum, then the normalised values will be given by equation (3.11).

$$P_{load,norm} = \frac{P_{load}}{max(P_{load})} \tag{3.11}$$

The normalised values will range from 0 to 1, where 1 belongs to the time step(s) being the maximum load of the given day, independent of the season. By plotting these values in a suitable fashion, one might be able to observe daily patterns irrespective of the season.

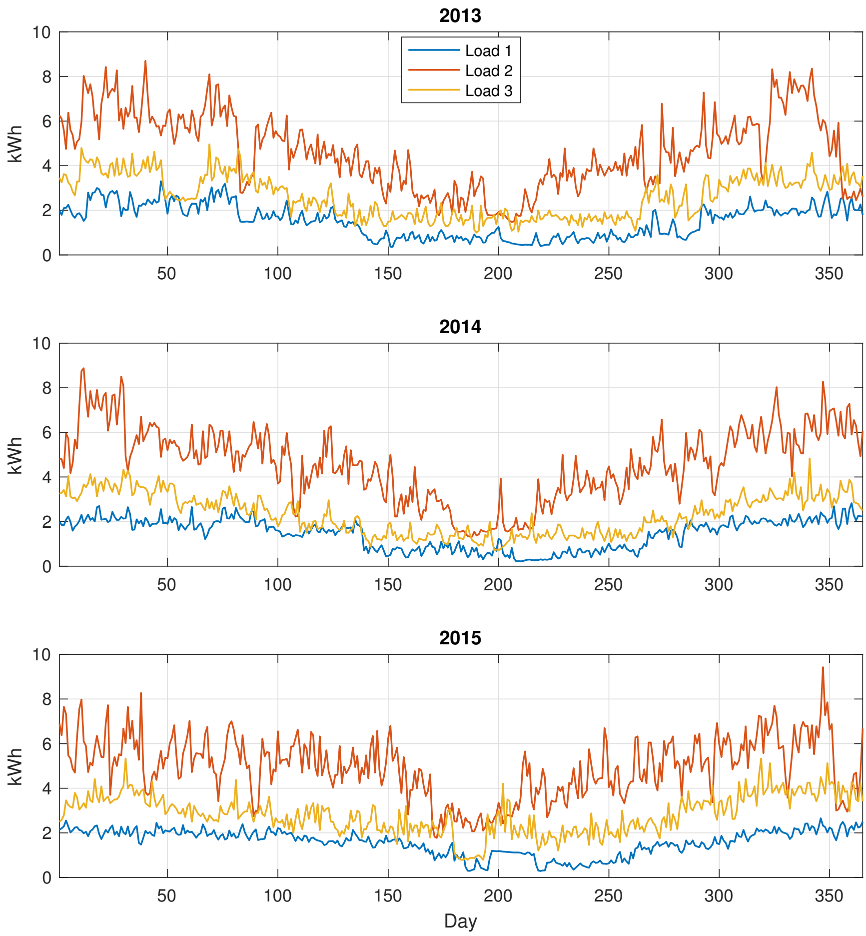

Figure 3.2: Plot of daily average for all loads in 2013, 2014 and 2015.

This is what is visible in figure 3.3, which shows normalised load values for load 1 in 2013 sorted by day. The same plots for all loads and years are included in appendix D.

Figure 3.3 shows clear signs of daily load patterns, and three tendencies are identified during weekdays (Mondays through Fridays): (1) A peak around 8 in the morning, (2) low loading hours 1-8 and 10-16, and (3) higher loads during hours 16-24.

Weekends (Saturdays and Sundays) show less of a characterised pattern, but a flatter load profile during daytime (9-16) followed by higher values after hour 16 seems to be a tendency. The most evident difference between weekdays and weekends is the lack of the peak around 8 in the morning during weekends, which most probably indicate a working week running 8-16 in weekdays. These are differences which should be exploited in the stochastic model, separating weekdays from weekends in order to increase its accuracy.

Load 2 (shown in figures D.5-D.7) shows the same tendencies for weekdays as load 1, but has weekends that look more like weekdays. Load 3 (shown in figures D.9-D.11) on the other hand, does not show as clear patterns as load 1 and 2. There are still tendencies of a peak around 8 in the morning in weekdays, but this is far less visible compared to load 1 and 2. This might be due to a differing working pattern, lifestyle or building quality. It should also be mentioned that it appears that the load patterns of load 3 vary more over the year, showing flatter profiles during spring and winter as compared to summer.

Further, observe how the patterns for the three loads vary over the years. The visible pattern in load 1 for 2013 is far less visible in 2014 and 2015, which might indicate a change of job, lifestyle or a building upgrade. Load 2 look rather similar over the three years, while load 3 gradually turns more predictive from 2013 up until 2015.

Nevertheless, the findings from this section show proof of the presented hypothesis of load following similar pattern for similar days, and that it could yield promising results on a hour-to-hour basis. Implementing a stochastic model in MATLAB which build on this hypothesis would also not be too challenging. Using this hypothesis would also enable the modelling of load as a Markov Chain, as the transition probabilities in each time step could be easily calculated independent of previous states.

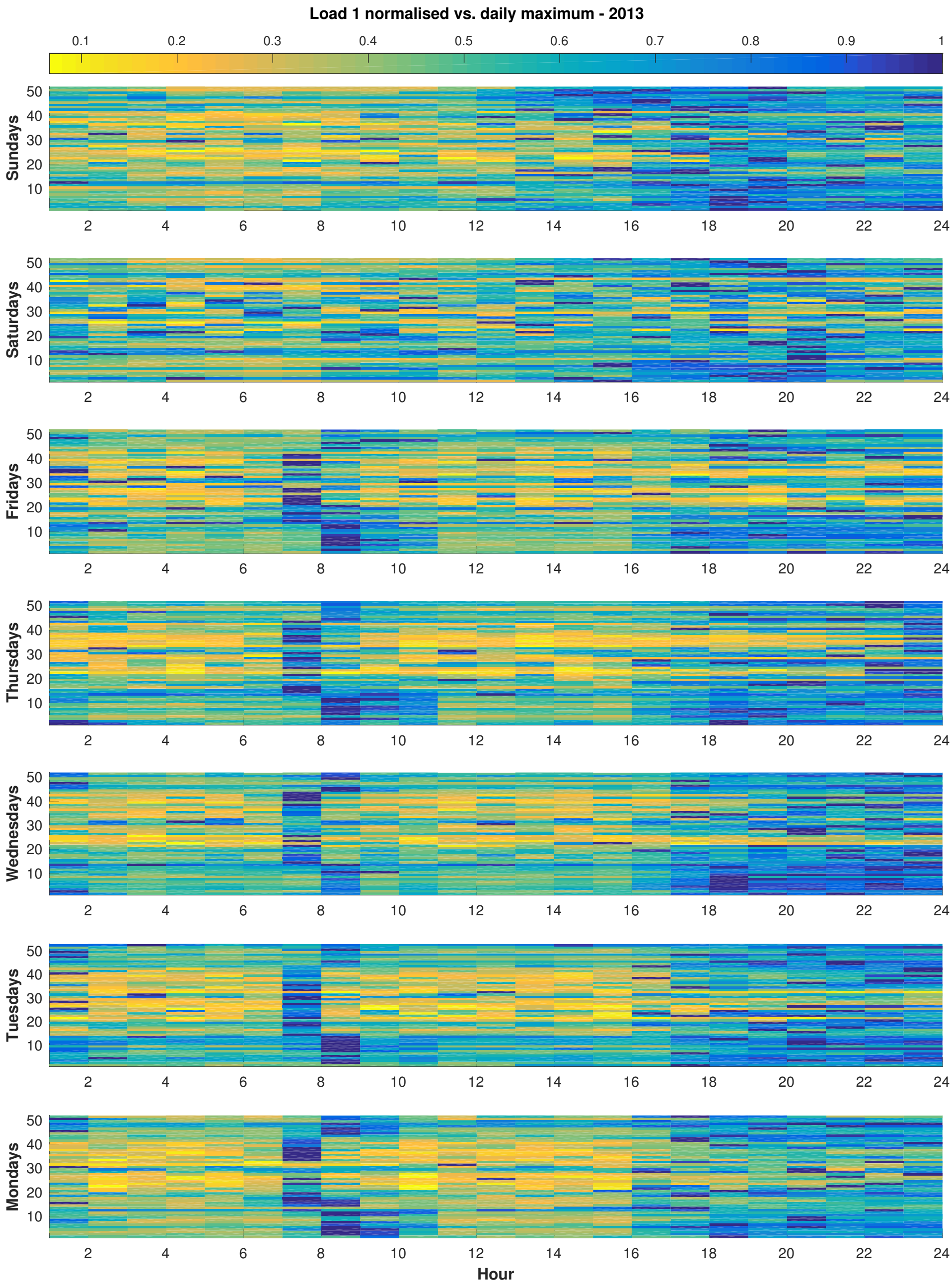

Figure 3.3: Normalised load for load 1 in 2013, sorted by days.

3.4.2 Hypothesis 2: Load-temperature Correlation

As found in [1], loads of households in central Norway show a negative correlation towards outdoor temperature. This will now be further investigated with the three load sets provided by TrønderEnergi, in order to evaluate if the correlation is strong enough to be exploited in the stochastic model of this thesis.

Figure 3.4 shows daily average values of load and outdoor temperature in 2013. It seems like all loads are somewhat negatively correlated with temperature, as the load clearly decreases during summer. This is also somewhat visible in figure 3.5, which shows a scatter plot of load versus temperature for all loads for 2013-2015. The plots show a tendency towards decreased load for higher temperatures, but there is a substantial variance. Observe how the load vary for 0°C for all loads, ranging from close to the yearly minimum to the yearly maximum.

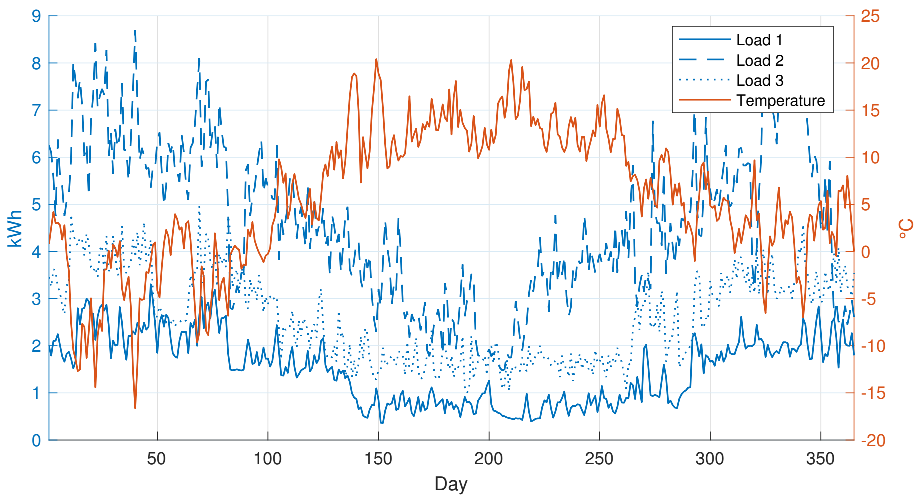

Figure 3.4: Daily average values of load and outdoor temperature in 2013.

Figure 3.6, which is the same plot as 3.5 but with daily average values of load and temperature, shows a more distinct tendency of decreasing load for increasing temperatures. The slope (a) and constant term (b) of the linear fitting[1] to the scatter plots of figure 3.6 are listed in table 3.1. Equation (3.12) relates load (P_{load}), outdoor temperature ($T_{outdoor}$) and the slope and constant term.

$$P_{load} = aT_{outdoor} + b \tag{3.12}$$

As all slope values are negative, all households show a negative correlation towards temperature. This certainly is proof of the hypothesis of load showing a negative correlation towards outdoor temperature, and could thus be interesting to use in a stochastic model for predicting load.

[1] Found using the linear fitting regression in MATLAB.

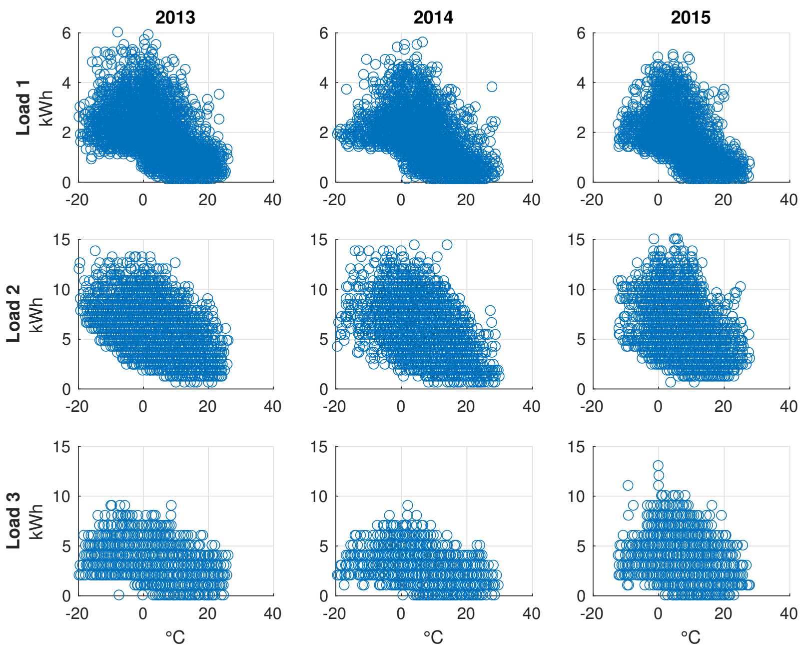

Figure 3.5: Scatter plots of load against temperature.

Figure 3.6: Scatter plots of daily average load against daily average temperature.

Table 3.1: Slope and constant term from linear regression of daily average load and temperature.

	2013		**2014**		**2015**		**Average**	
	a	b	a	b	a	b	a	b
Load 1	-0.080	1.97	-0.082	2.06	-0.086	2.18	-0.083	2.07
Load 2	-0.181	5.67	-0.207	6.13	-0.149	5.98	-0.179	5.93
Load 3	-0.110	3.25	-0.109	3.11	-0.113	3.61	-0.111	3.32

Nevertheless, due to the large variations in hourly values, as shown in figure 3.5, this correlation does not prove to be precise enough to be used for an hour-to-hour load prediction. Rather, it could be used to calculate some kind of load average set-point for the coming day, which could enhance the performance of the SDP when combined with another more precise hour-to-hour model. This is an interesting though, but will not be pursued further in this thesis, and is thus left for future studies to look into.

3.5 Stochastic Model Implementation

Based on the findings and discussion in the previous section, this thesis will proceed with building a stochastic model based on the hypothesis of load following similar patterns for similar days. More specifically that weekdays, Monday till Friday, and weekends, Saturday and Sunday, behave similar and thus can be grouped together.

The chosen implementation in MATLAB was divided into three steps: (1) Sorting of data, (2) discretisation of data and (3) calculation of transition matrices, which now will be described in more detail successively.

Step 1: Sorting data

The first step is to sort the historic load data into two matrices, separating the data from weekdays from weekends. This is done by first splitting the data into the seven separate days, and then joining Mondays, Tuesdays, Wednesdays, Thursdays and Fridays into a weekday matrix and Saturdays and Sundays into a weekend matrix. In both matrices (weekdays and weekends), each row corresponds to one day ($\in [1, N_{days}]$) and each column to one time step ($\in [1, N_{t,day}]$)[2]. One could potentially skip the process of first splitting the data into seven matrices, but by doing so one enables a more flexible solution which can be further investigated, for instance if one were to combine Fridays with weekends rather than weekdays.

Step 2: Discretisation of data

In order to make the stochastic model compatible with SDP, the load data has to be discretised. This is done in the second step of the stochastic model by dividing the load into N_{disc} discrete states. The following process is repeated for both load matrices.

[2] $N_{t,day}$ is the number of time steps in one day. When hourly data is used, $N_{t,day} = 24$

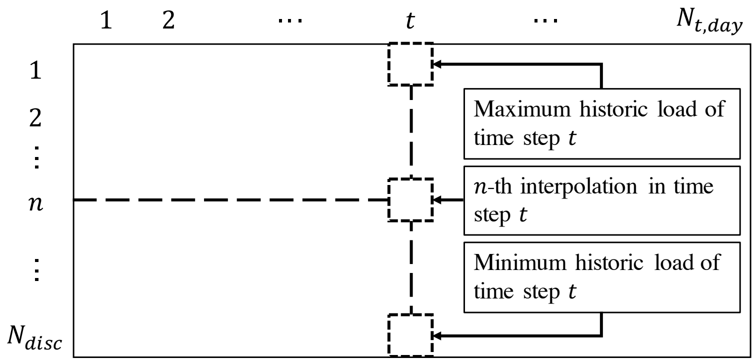

Figure 3.7: Illustration of the discretisation matrix.

First, the maximum and minimum load value of each time step is found. These are stored in two vectors of dimension $1 \times N_{t,day}$, where element i corresponds to namely the maximum and minimum load value for time step i. Next, a matrix of dimensions $N_{disc} \times N_{t,day}$ is created, and the maximum- and minimum-vectors are inserted as the first and last row, respectively. Lastly, the indices in between are assigned the rounded values[3] of a linear interpolation between the maximum and minimum values. This result in the *discretisation matrix*, which contains the discrete load values for the different time steps. An illustration of the discretisation matrix is shown in figure 3.7.

Step 3: Calculating transition matrices

In the third and last step of the stochastic model, the discretisation matrix is used to calculate the $N_{t,day}$ transition matrices, one for each time step. The transition matrices are two-dimensional matrices with the dimensions $N_{disc} \times N_{disc}$, where the first dimension corresponds to the load state in the current time step t and the second dimension to the load in time step $t+1$. The values inside the matrices are the probabilities ($\in [0,1]$) of transitioning between the two associated load states.

The calculation of the transition matrices is performed in two steps. First, the occurrences of each transition in the historic load data is counted and stored in $N_{t,day}$ $N_{disc} \times N_{disc}$ matrices. This process is performed by rounding the actual load in the two time steps to the closest discrete states, as found in the discretisation matrix. Second and last, the probabilities are calculated using equation (3.13).

$$p(k,l,t) = \frac{\text{occ}(k,l,t)}{\sum_{l=1}^{N_{load}} \text{occ}(k,l,t)} \qquad \forall \quad t, k, \tag{3.13}$$

where $p(k,l,t)$ is the transition probability for load state k in time step t to load state l in time step $t+1$, and $\text{occ}(k,l,t)$ is the number of occurrences of the given transition.

[3]Rounded to the same level of detail as the input data.

Because these are probabilities, each row of the transition matrices has to sum up to one, in accordance with equation (3.14).

$$\sum_{l=1}^{N_{load}} p(k,l,t) = 1 \qquad \forall \quad t,k \tag{3.14}$$

The calculated transition matrices for weekdays for load 1 is shown in figure 3.8. Notice that no matrix is made for $t = N_{t,day} = 24$. This is due to the simplification that the optimisation will take place on a daily basis, and this matrix is thus not needed. The rationale behind this decision is explained in section 3.6. Before proceeding to the next section, there are a couple of things to highlight in figure 3.8.

First, notice how the transitions tend to have a higher probability along the diagonal of the matrices. This means that the load tends to go to the same or close to the same discrete load state in following time step, which makes sense. It is important to emphasise that these does not necessarily correspond to the same load values, as the different time steps has their separate load discretisations. There are exceptions to this tendency, like for $t = 6$ and $t = 7$, which are probably due to more varying load patterns during these hours.

Second, observe how there in almost all time steps occur very high probabilities for one specific transition for low indexes of load state in time step t (the vertical axis). This is due to the chosen discretisation process from step 2, as described above. A low index corresponds to a high discrete value of load, and index 1 actually corresponds to the maximum historic load of that time step. The discretisation does not take into account how many times such high loads actually has occurred. This is what is made visible here, as these high probabilities indicate that such high loads have occurred only once or just a few times, yielding very high probabilities for these transitions.

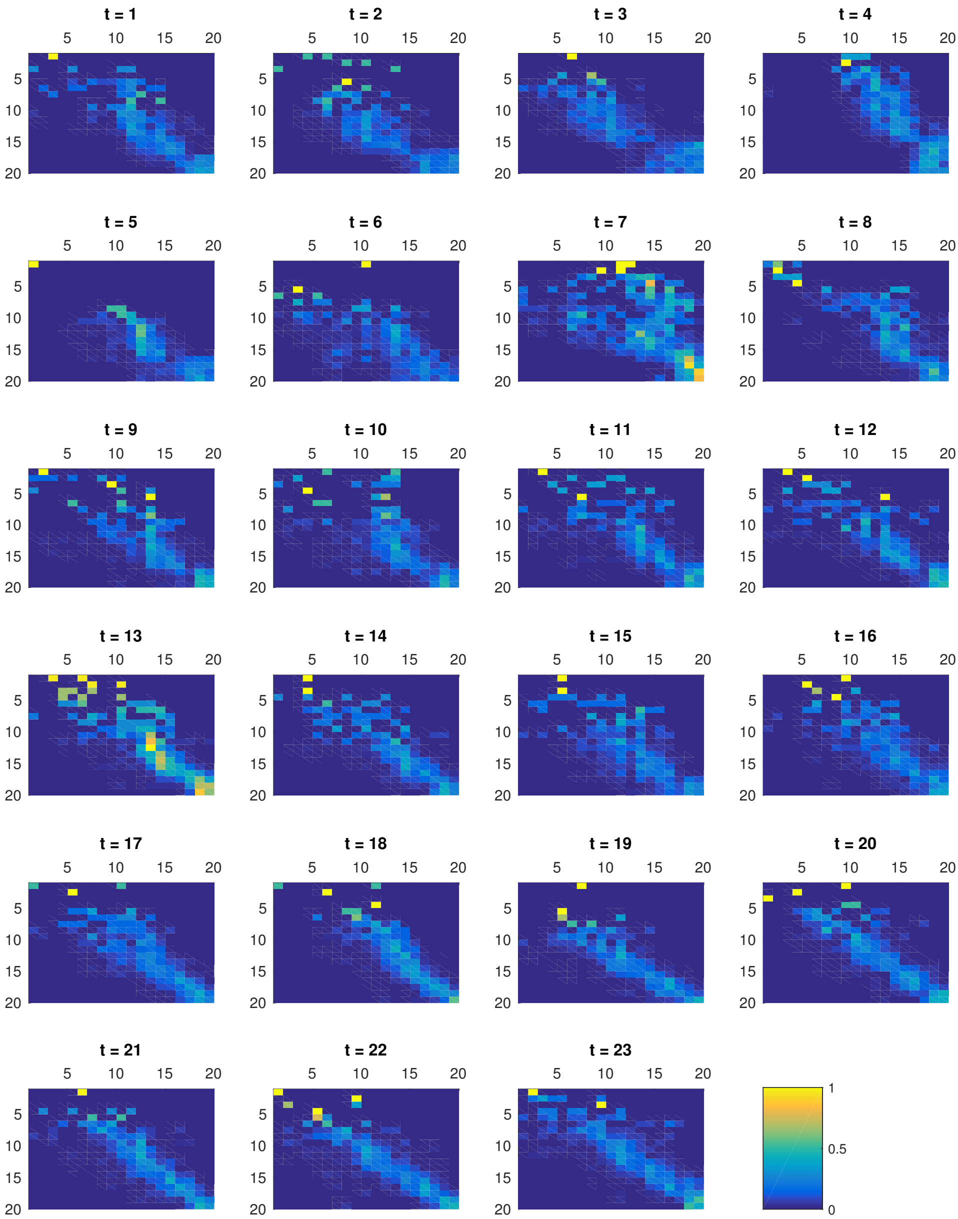

Figure 3.8: Transition matrices for weekdays for load 1 for $N_{disc} = 20$.

3.6 SDP Implementation

This section describes how the SDP has been implemented in MATLAB. First, the decision of the optimisation horizon is discussed, followed by the decision on stored energy at the end of the optimisation horizon. At last, the chosen algorithm is laid out and briefly explained.

3.6.1 Optimisation Horizon

One essential decision to make in the SDP implementation is the optimisation horizon, and if this is to be *rolling* or *fixed*. The optimisation horizon of a stochastic problem, which is to be solved here, must be chosen in accordance with the uncertainty of the input data. If the horizon is chosen too long, the uncertainty in the solution may be too large for it to be useful. On the other hand, if the horizon is chosen too short, the optimisation will carried out too often, which dependent on the run-time of the algorithm might create run-time issues.

A rolling horizon means that an optimisation is carried out more than once for the same horizon, taking into account new information that may have emerged since the last optimisation. Such information might be more accurate weather or demand forecasts, or new market information like spot prices and transaction volumes. Based on this new information, the optimal operation strategy might differ from the previously calculated one. This is taken into account by using a rolling horizon optimisation.

A fixed horizon optimisation is, in contrast to the rolling one, only carried out once for each horizon. For instance once every other hour, once a day or once a month. The optimisation is thus not carried out again if new information is made available, but postponed until the end of the horizon. If the optimisation does not produce a flexible solution, this might lead to sub-optimal results. By a flexible solution, what is meant is the solutions ability to take several scenarios into account, which enables the operator to postpone decisions in the case of new information.

For this thesis, a fixed horizon of one day will be used with a *wait-and-see policy* as the output of the optimisation. With a wait-and-see policy, the operator is left with a three-dimensional look-up table which contains optimal decisions for all state of SOC and load over the optimisation horizon. By using such an optimisation output, one achieves the advantages of both the fixed and rolling horizon methods. Only one optimisation is carried out per day, and new information[4] is taken into account as the day progresses. The length of the horizon is based on the fact that day-ahead spot prices are known 12-36 hours in advance, meaning that the prices for the coming day is always known at midnight. This setup is also an advantage if the production from a PV-system is to be included as a stochastic parameter, due to the repetitive daily pattern of the suns orbit.

[4] In the form of which level of SOC and load state that is realised in each time step.

3.6.2 Stored Energy at the End of the Optimisation Horizon

There are several ways of deciding on the amount of stored energy at the end of the optimisation horizon. These are mainly divided into three categories:

- Fixed level of energy
- Minimal level of energy
- Using a value function

A fixed level of energy is the easiest to implement, and is often used in literature [24]. Usually this is set to the same level as the initial energy at the start of the day. This way, one avoids giving the system the benefit of ”free” initial energy. Sometimes, the final energy is set to be at least the initial one, which belongs to the second category above. This would in the case of this thesis not make sense to implement, as the optimal thing to do would be to have the battery ending on the minimum level, as this minimises the consumer cost for the given day[5].

The third category uses a value function in order to let the optimisation decide the final energy level. These value functions are usually functions of stored energy, but other parameters might also be taken into account. [24] propose a quadratic value function which is dependent on the wind speed and stored energy at the end of the horizon, but concludes that a linear approximation yields a fair results with respect to the battery operational strategy. This makes sense for a system including wind power generation, but this is not the case for this thesis. PV-production is more cyclic than wind, and is rarely present at night, which makes it less useful to include in a value function.

In this thesis, a fixed level of energy will be used in the SDP-model due to the ease of implementation. One could also set a minimum level of energy, but due to the mathematical formulation of the problem which minimises costs in each time step, this would result in the battery ending at the minimum level of energy anyway. It would be interesting to use a value function, but this would demand substantial work in defining the parameters of such a function, and is thus disregarded due to the scope of this thesis.

[5]Assuming that negative spot prices are not present.

3.6.3 SDP Algorithm

Algorithm 2 lists the pseudo-code of the SDP algorithm like it was implemented in MATLAB, and the full code is included in appendix B. The input to the script is the load discretisation and transition matrices, PV-production, spot price and UT structure for the given day, as well as all battery parameters including initial and final SOC. The algorithm is also given which day it is (Monday, Tuesday, etc.), which is needed in the case of the time based UT.

With the chosen implementation, the final SDP-algorithm is made up of five nested for-loops, iterating recursively in time for all state of SOC and load in time steps t and $t + 1$. If PV-production were to be included as a stochastic parameter in the same fashion as load, this would demand two new for-loops to be included. Dependent on the level of discretisation of load and PV-production states, this would increase the run-time of the algorithm. Independent of the discretisation, this would also increase the complexity of the code, and thus increase the risk of making mistakes in the implementation.

The output of the optimisation is a full operational policy of the battery for the given day, stored as a three-dimensional path-matrix with the dimensions $N_{SOC} \times N_{t,day} \times N_{disc})$. Each element is the assumed optimal SOC-value to transfer to in the next time step given the present SOC(t) (first dimension), t (seconds dimension) and load(t) (third dimension), yielding an optimal P_{bat}.

Algorithm 2: SDP algorithm as implemented in MATLAB

```
cost-to-go = inf(N_SOC, N_t,day, N_disc);
transitioncost = inf(N_SOC, N_disc);
path = zeros(N_SOC, N_t,day, N_disc);
Calculate legal SOCs for all time steps;
for t = (N_t,day - 1 : -1 : 1) do
    for all legal soc(t) do
        for load(t)= 1 : N_disc do
            for all legal soc(t + 1) reachable from soc(t) do
                Calculate P_grid and the associated transition cost;
                for load(t + 1)= 1 : N_disc do
                    Calculate expected cost-to-go from t + 1 using equation (2.14);
                end
            end
            Evaluate which transition what minimises the expected cost-to-go;
            path(soc(t), t, load(t)) ← soc(t + 1);
            cost-to-go(soc(t), t, load(t)) ← min(cost-to-go);
        end
        Reset transitioncost matrix to all infinite values;
    end
end
```

3.7 Full Implementation

This section explains how the different parts described above are combined in order to work as a full program. The program is divided into three sections: (1) Initialisation, (2) SDP-loop and (3) finalisation, as shown in figure 3.9. These sections are now explained separately below.

3.7.1 Initialisation

The initialisation starts with setting all parameters: Which load and UT structure to use, how many days to optimise, with or without a PV-system and the desired level of load discretisation (N_{disc}). This is followed by the loading of historic load data, battery- and PV parameters and lastly load and spot price data for 2016. PV-production, if included, is calculated based on PV-system parameters, irradiance and temperature data of 2016. Before entering the SDP-loop, the last step of the initialisation is running the stochastic model for the given historic load data, calculating the load discretisation and transition matrices for weekdays and weekends.

3.7.2 SDP-loop

After the initialisation, the program enters the SDP-loop, optimising day by day of 2016. First, the prices and PV-production of the coming day is loaded into two separate vectors. The SDP-algorithm is run with the given spot prices, UT, PV-production, battery parameters, load discretisation and transition matrices, yielding a path-matrix for the given day. The day is then "executed" by iterating successively over all time steps, finding the assumed optimal decision on how to operate the battery in the path-matrix at the beginning of each time step, as described in section 3.6.3. This process is repeated for all days that one wishes to optimise for, storing the decided SOC, P_{bat} and consequently the P_{grid} for all time steps.

3.7.3 Finalisation

Before calculating the final consumer cost results, the global optimum is calculated by using the DP-algorithm as described in section 3.3. After the DP-algorithm terminates, returning the global optimal values of SOC and P_{bat}, the final consumer cost results are calculated for the basecase, SDP-solution and DP-solution, as described in section 3.2.1. If a PV-system is included, the costs in the case with just a PV-system and no battery are also calculated.

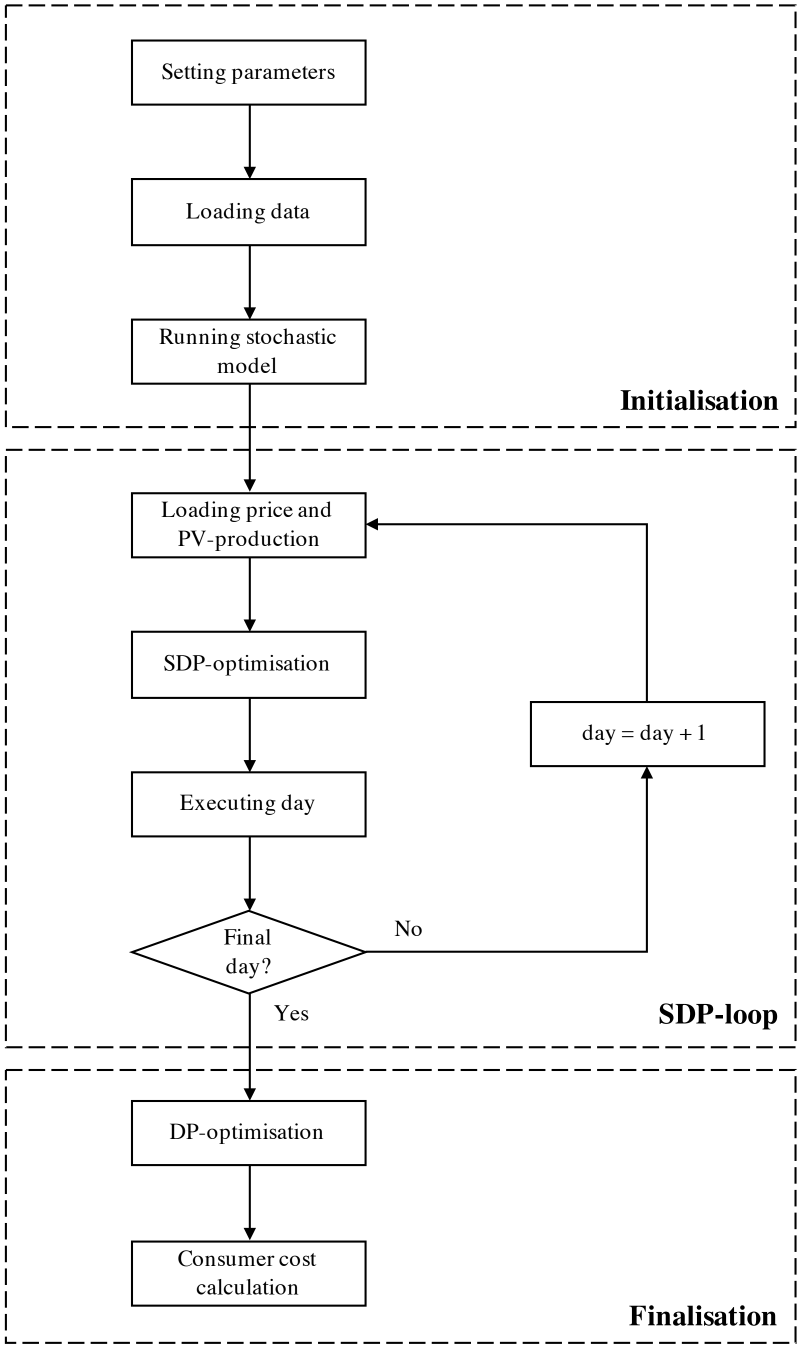

Figure 3.9: Flowchart of full implementation.

Chapter 4

Input Data and Cases

4.1 Input Data

4.1.1 Load

The load data in this thesis will be hourly values for three different households in central Norway, provided by TrønderEnergi. The daily average values for 2013, 2014 and 2015 is shown in figure 3.2. All data from 2013, 2014 and 2015 were made available for all three loads, but only load 3 had available data for the whole of 2016. Load 1 and 2 had available data until day 275 of 2016, i.e. the 1st of October. Figure 4.1 shows the daily average load for all three loads for 2016.

The reader is reminded that load will be considered inflexible in all calculations in this thesis, meaning that load has to be met at all costs.

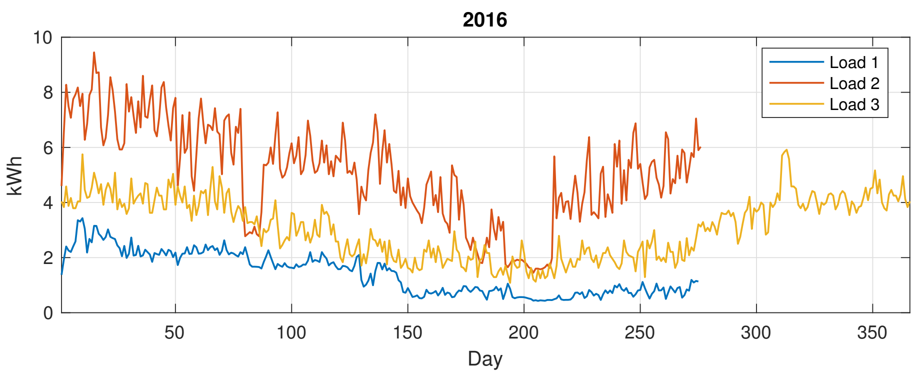

Figure 4.1: Plot of daily average load for 2016. Dashed values indicate extrapolated values.

4.1.2 Spot Price

In this thesis, hourly day-ahead spot prices for Trondheim in central Norway will be used. These can be downloaded freely from Nord Pool Spot's websites [25], and figure 4.2 shows daily average spot prices for the years of 2013-2016. Only the 2016-values will be used in this thesis, but the figure illustrates the high values during the early days of 2016 as compared to the previous years. This will probably contribute to increasing the value of a battery in 2016, as it enables avoiding sudden spot price peaks.

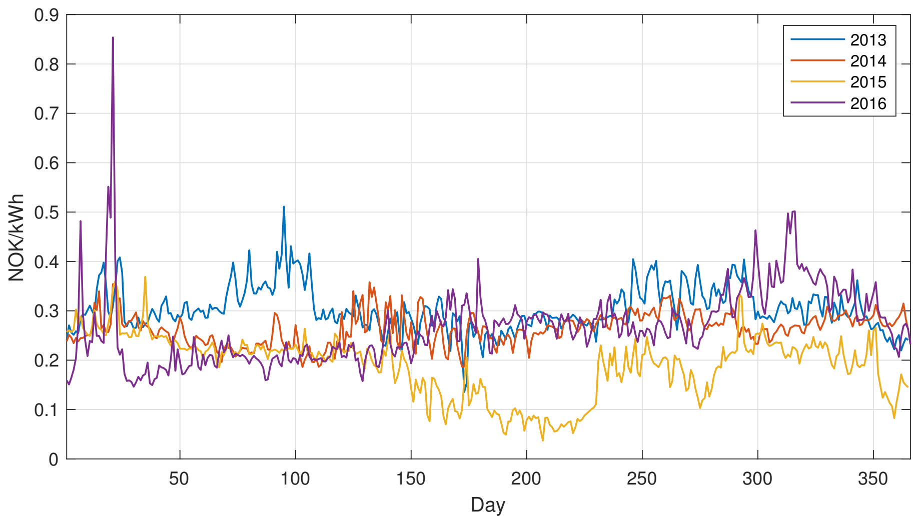

Figure 4.2: Plot of daily average spot prices.

4.1.3 Utility Tariff

Energy based

Average values for the energy based UT can be found on NVE's websites [2], and values for 2013-2016 are listed in table 4.1. The 2016-values will be used for the energy based UT in this thesis, as discussed in section 2.5.2. Notice that the fixed term is included in table 4.1, but it will not be included in the calculations in this thesis as the battery is not able to avoid this and it is thus not of interest to include. Prices are listed including tax and *consumer fee*[1].

Table 4.1: Average values for UT in Trondheim [2].

	2013	**2014**	**2015**	**2016**
Energy term (NOK/kWh)	0.368	0.418	0.437	0.426
Fixed term (NOK)	1879	1902	1908	1907

[1]Norwegian: *forbruksavgift*

Time based

In this thesis there will be used a time based UT with values given in table 4.2, visualised in figure 4.3. The price levels of the time based UT are sat relative to those of the historic energy based UT values, which is approximately 0.4 NOK/kWh [15]. The high price level of the time based UT is 200 % of this, the low price hours 50 %, and the middle and weekend price level to 100 %, i.e. the same value. Weekdays here refer to Mondays through Fridays, while weekends refer to Saturdays and Sundays. The time based UT is designed with inspiration from [26].

Table 4.2: Price levels of the time based UT.

Hour	1-5	6-8	9-11	12-16	17-19	20-22	23-24
Tariff weekdays in NOK/kWh	0.2	0.4	0.8	0.4	0.8	0.4	0.2
Tariff weekends in NOK/kWh	0.4	0.4	0.4	0.4	0.4	0.4	0.4

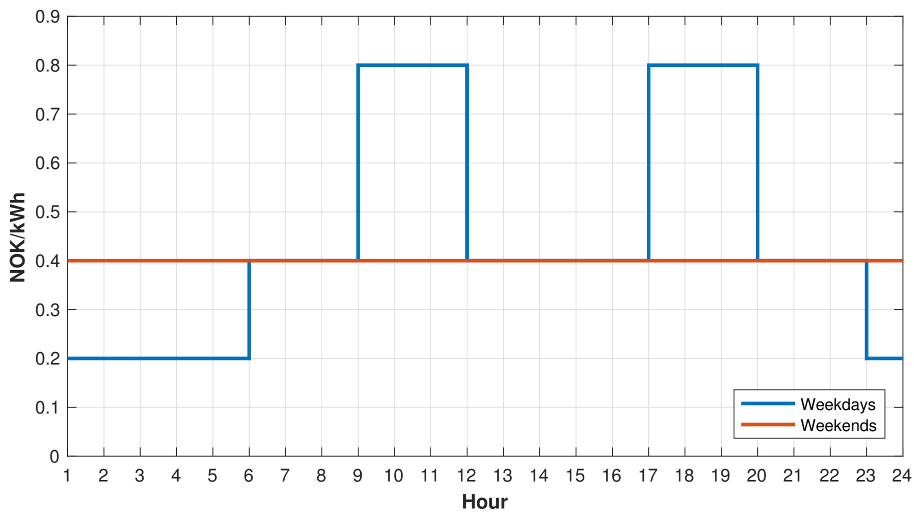

Figure 4.3: Illustration of the time based UT.

Power based

In this thesis, C_{power} will be sat to 0.1 NOK/kWh^2, based on the on the power based UT used in [1]. Remember that this causes C_{tot} to increase exponentially with P_{grid} in accordance with equation (3.10):

$$C_{tot} = \begin{cases} C_{spot}P_{grid} + C_{power}P_{grid}^2, & \text{if } P_{grid} > 0 \\ C_{spot}P_{grid}, & \text{otherwise.} \end{cases} \tag{4.1}$$

4.1.4 PV-panel and Battery parameters

PV-panel

Parameters from SOLARTEK PV-modules will be used in this thesis. This was decided due to the fact that TrønderEnergi has installed these modules in their test systems in Trondheim. The relevant parameters are listed in table 4.3. α_T and $NOCT$ are from the data sheet of the PV-panels[2], P_{nom} is based on the test systems of TrønderEnergi, and the system efficiency of 77 % is from page 323 in [22].

Battery

The battery parameters of this thesis are based on those of the Tesla Powerwall [27], and all parameters are listed in table 4.4. The charge- and discharge efficiency is calculated as the square root of the round-trip efficiency.

Table 4.3: PV-system parameters.

Paramater	Value	Unit
α_T	-0.43	%/K
$NOCT$	47	°C
P_{nom}	5.2	kWp
η_{sys}	77	%

Table 4.4: Battery parameters.

Paramater	Value	Unit
$P_{bat,max}$	7	kW
C_{bat}	13.5	kWh
SOC_{max}	1	-
SOC_{min}	0	-
$\eta_{ch} = \eta_{di}$	95	%

4.1.5 Solar Irradiance and PV-generation

Solar irradiance and temperature for several locations in Norway can be downloaded from the websites of *Landbruksmeteorologisk Tjeneste* (LMT) [28]. Irradiance is measured as *global horizontal irradiance* (GHI), which is the sum of direct and diffuse irradiance that hits a horizontal plane given in W/m^2. As the households in this thesis are located in Trondheim, LMT's measuring station at Skjetlein, just south of Trondheim, will be used as the souce of temperature and irradiance data. Daily average values of temperature and GHI are shown in figure 4.4 in the upper and middle plot, respectively.

Knowing the irradiance, ambient temperature and the PV panel parameters, PV-production can be calculated using equations (3.5) and (3.6). The lower plot in figure 4.4 shows the resulting accumulated daily PV-production in 2016.

[2]Made available by email from TrønderEnergi.

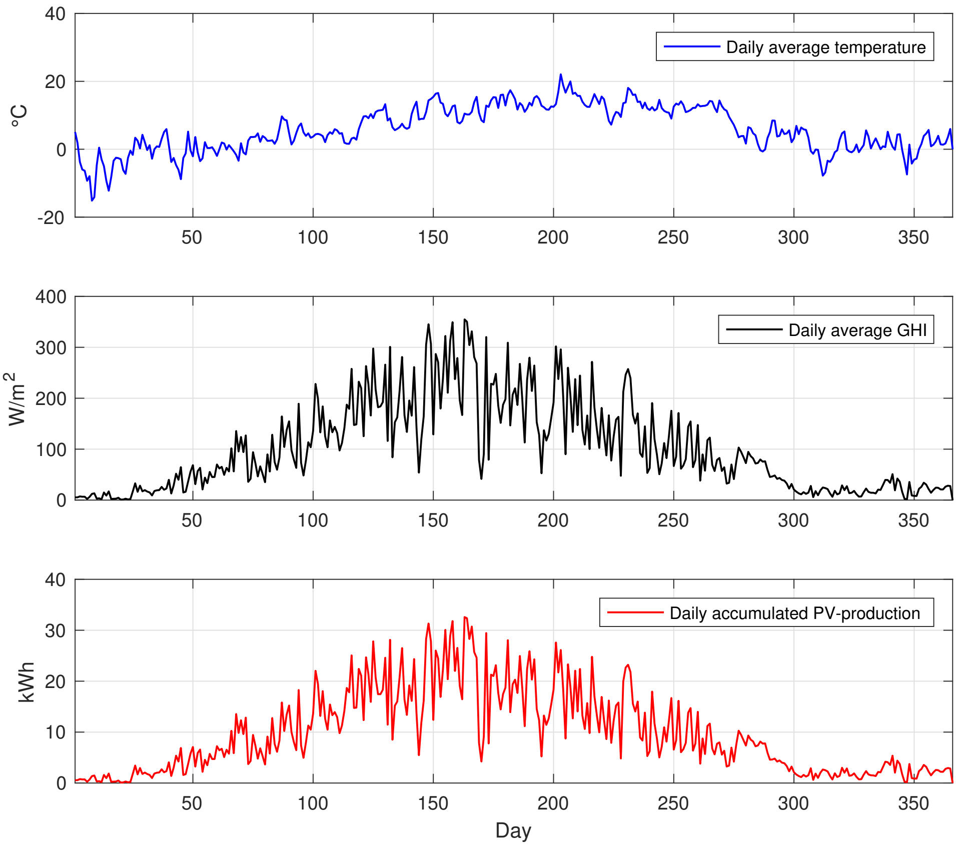

Figure 4.4: Plot of daily average temperature (upper plot) and irradiance (middle plot) in 2016, and the resulting accumulated PV-production.

4.2 Case Studies

The objective of the SDP- and DP-optimisations is to minimise the costs for the consumer in question, and the *relative savings* provided by the two models will be the key performance indicator in this thesis. By relative savings, what is meant is the savings divided by the total costs in the basecase, which is the case without any PV-system or battery. Savings here refer to operational savings, as the investment costs of the PV-system or battery are not taken into account.

Due to the flexibility of the developed program, numerous cases can be studied in order to investigate the effect of changing different parameters. In this section, a set of standard settings are defined, as well as a set of cases which will be run through the program. The results are presented and discussed in chapter 5.

4.2.1 Standard Settings

In order to simplify the results and discussion, a set of standard settings are defined. The standard settings are listed in table 4.5. Whether or not a PV-system is included will thus be specified in each case.

Table 4.5: Standard settings.

Parameter	Setting
Load	3
N_{disc}	20
N_{SOC}	100
Historic load	2013-2015
Optimisation days	366
Battery parameters	See table 4.4

4.2.2 Cases

Different loads and utility tariff structures

With the given data sets, there are a number of interesting cases to look into. How will the different households benefit of installing a battery using both the SDP- and DP-models, and how will this benefit be influenced by different UT structures?

The effect of a PV-system

The households will be investigated both with and without a PV-system installed. When included, the PV-production will be considered deterministic in both the SDP- and DP-model. How will the introduction of a PV-system influence the value of a battery?

Varying historic load data

The real-time SDP-optimisation will be based on the load discretisation and transition matrices, which again is based on historic load data from the given household. It is of interest to see how the variation of the available historic data effects the results. What would the result be if the stochastic optimisation was based on data from just 2013, 2014 or 2015?

Varying load discretisation

In the implementation of the stochastic model, as described in section 3.5, there is a set level of discretisation, namely N_{disc}. The higher this is set, the higher resolution of the model. Thus, it is of interest to investigate how this parameter influences the results. Run-time will also be considered, as a higher N_{disc} will increase the run-time. Is the results, if better at all, justified by the added run-time?

Learning stochastic model

In the standard settings, the stochastic model is only run once, and that is prior to the SDP-loop. This means that the load discretisation and transition matrices does not improve as time goes, which might make the SDP-model outdated over time. Hence, there will be run a case where the past day is added into the stochastic model, yielding updated load discretiastion and transition matrices for each day that passes. Will this yield a more precise stochastic model, and thus a better performing SDP-model?

Different battery parameters

There are mainly two parameters that influence the value of the battery[3], and those are the battery capacity and the maximum allowable terminal power. Four different combinations of these, as described in table 4.6, will thus be investigated. Notice that these values are those of a Tesla Powerwall or twice that value.

Table 4.6: The four different battery combinations that will be tested.

Combination	$\mathbf{C_{bat}}$	$\mathbf{P_{bat,max}}$
1	13.5 kWh	7 kW
2	27.0 kWh	7 kW
3	13.5 kWh	14 kW
4	27.0 kWh	14 kW

[3] If one considers the efficiency as given and something that cannot be changed.

Chapter 5

Results and Discussion

5.1 Run-time

The developed MATLAB-program was run on an Apple MacBook Pro (late 2013) with a 2.6 GHz Intel Core i5 processor. Average run-time was calculated as the average of 10 individual runs of the code in question. The stochastic model had an average run-time of 0.0773 seconds, and the average run-times of the SDP- and DP-model are listed in table 5.1 for the different UT structures. Notice that the values for the SDP-model are for daily optimisation, while the DP is given in both daily and yearly optimisation.

The results in table 5.1 clearly show how much more cumbersome the calculations of the SDP-model are as compared to the DP-model. This added run-time is due to the two extra nested for-loops which is introduced in the SDP in order to consider load as stochastic. The UT does not seem to influence the run-time of neither the SDP- nor the DP-model substantially. This makes sense considering that the difference in implementation in code between them are negligible.

In this thesis, the SDP-solution represent the solution which could have been realised in real life, while the DP-solution represent the global optimum solution, which is basically impossible to achieve in real life. Thus, if the SDP-model were to be implemented in a household it would need to compute in reasonable time for it to be useful. With the hardware used in this thesis, the results of approximately 1.5 seconds would need to be evaluated in real-life experiments in order to evaluate if this is reasonable enough for an appropriate implementation. Thus, this will not be further discussed in this thesis.

Table 5.1: Average run-time for different UT structures without PV. The SDP was run with standard settings without PV.

UT structure	Daily SDP	Daily DP	Yearly DP
Energy	1.5410 s	0.0275 s	9.8677 s
Time	1.4789 s	0.0261 s	9.1061 s
Power	1.6784 s	0.0254 s	8.8227 s

5.2 Accuracy of Stochastic Model

As described in section 3.4.2, the chosen stochastic model uses historic load patterns of similar days and hours in order to predict future load values. By dividing the historic load data into N_{disc} discrete values in each time step and counting the occurrence of each transition between them[1], the transition probabilities was found using equation (3.13). In this section, the accuracy of this stochastic model will be evaluated by comparing the actual load patterns to the calculated transition probabilities.

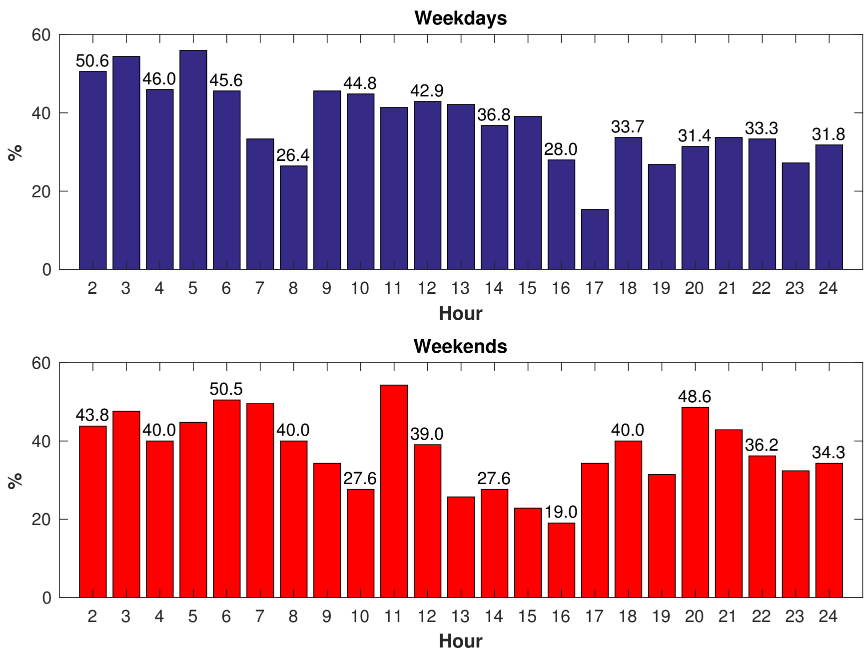

Figure 5.1: Percentage of coincides between load state and the highest probable load state for load 3.

In order to evaluate the accuracy of the stochastic model, figure 5.1 shows the percentage of coincides between the actual load state and the highest probable load state as found in

[1] If the load did not match the exact discrete value, the closest discrete state was used.

the transition matrices for load 3. The upper plot shows the percentage for weekdays, while the lower shows for weekends. Notice that hour 1 is not included due to the chosen daily optimisation horizon. Both weekdays and weekends show a tendency towards a higher accuracy during night time, with approximately 50 % hitting the most probable load state, and a poorer accuracy throughout the day. For weekdays, the model has an especially low accuracy for hours 8 and 17. This might be explained by the load peaks around these hours (see figure D.12), which obviously are not well predicted by the chosen stochastic model.

Figure 5.2 shows the mean and median deviation between actual load state probability and the probability of the most probable load state for weekdays and weekends for load 3. This further emphasises the higher deviations around hour 16, but also uncovers a peak around hour 4. This peak is caused by some very high deviations due to the way the stochastic model is implemented. If the load often has followed the same pattern, but then deviates from this, this results in large probability deviations (sometimes 100 %), which is what is seen in hour 4. Besides these two peaks, the median deviation is below 10 % in most hours, which indicate a high accuracy.

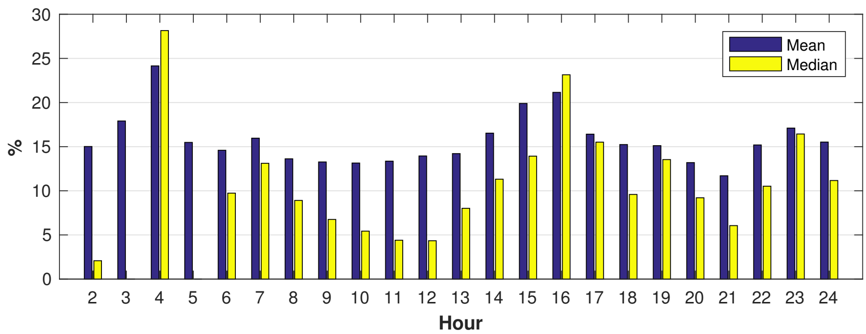

Figure 5.2: Mean and median deviation between actual load state probability and the highest probability load state for load 3.

Figure 5.3 shows the load pattern of load 3 for Wednesday the 27th of January 2016. The x-axis represents the $N_{t,day} = 24$ time steps and the y-axis the $N_{disc} = 20$ discrete load states[2]. The load, here represented by its discrete value, in each time step is indicated by the black line, while the probabilities for each load state is indicated by the coloured contour plot. Remark that load state 1 is equal to the maximum historic value of the given time step and state 20 to the minimum. As the plot shows, the load tend to follow the higher probabilities. Actually in 10 out of 24 hours, the load transitions to the state with the highest probability. Nine out of these ten comes before hour 17, further confirming a higher accuracy in earlier hours.

Figure 5.4 shows the load pattern of load 3 for Friday the 12th of February 2016. Notice how similar the transition probabilities are to the ones in figure 5.3, which is logical as both these days are weekdays and thus use the same discretisation and transition matrices. The latter plot is included as it shows the dependency between the load state in time t and

[2]The load value in kWh is found in the discretisation matrix.

the transition probability. Notice how the probabilities in (for instance) hour 2 and 8 differ from figure 5.3. This is due to the load state being different in hours 1 and 7 in the two days, which yields different probabilities in the following time steps.

The stochastic model also here shows a high accuracy for earlier hours, as the load hits the highest probable transition in 11 out of the first 15 hours. This is not the case for later hours, as the load ends up following a much higher path than what the transition probabilities indicate. This ultimately causes the SDP-model to perform poorly compared to the DP-solution under the power based UT, which will be further discussed in section 5.3.3.

Despite the high accuracy in earlier hours, as shown in figures 5.3 and 5.4, the stochastic model has room for improvement also during these hours. This is obvious from figure 5.5, showing load 3 for Wednesday the 20th of January. Observe how the load misses both 100 %-probabilities in hour 4 and 6. As discussed in section 3.5, these high probabilities occur as an artifact of the way the transition matrices are calculated. Maximum (or close to maximum) load occur very seldom, and this results in very high probability transitions being calculated from these few occurrences. Here, the actual load is the closest to the maximum historic one in hours 3, 5, 8 and 9, and in the case of hour 3 and 5 such high probabilities (100 %, actually) were present in the transition matrices.

In hour 5, observe how there is a zero percent probability for all load states. This is due to the load being in state 4 in hour 4, which it never was in any weekday in the historic data set. Because of the way the transition matrices are calculated (equation (3.13)) this results in zero percent probability for all transitions. This is an obvious weakness in the stochastic model, as this would result in a zero expected cost-to-go value, according to equation (2.14). This would make the best decision look like emptying the battery, as there is zero expected value in keeping any energy stored. This is actually just what happened, which is shown in figure 5.6, showing the resulting SOC from both the DP- and SDP-solution. Observe how the SDP-solution empties the battery from hour 4 to 5, while the DP-solution on the contrary charges the battery during this hour. Also, notice how the SDP acts completely similar to the DP in all hours prior to 4 and again when they meet again at hour 7, demonstrating a high accuracy of the SDP-model for the remaining hours of the day.

The above highlighted problem was caused by failing to incorporate equation (3.14) appropriately. The problem could have been fixed by setting an equal probability of $1/N_{disc}$ for all transitions from load states that was never recorded, which would satisfy equation (3.14). This would need to be fixed by in later revisions of the stochastic model.

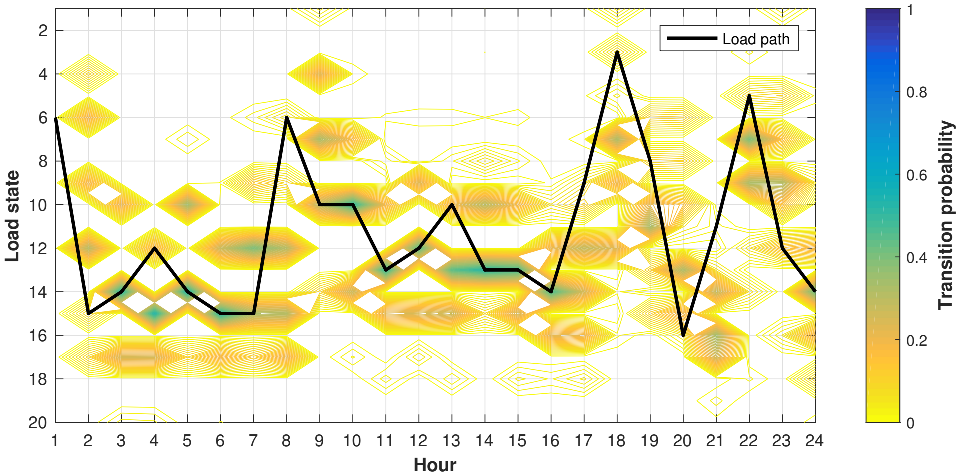

Figure 5.3: Actual discrete load path (black line) and calculated transition probabilities (coloured contour) of load 3 for Wednesday the 27th of January of 2016.

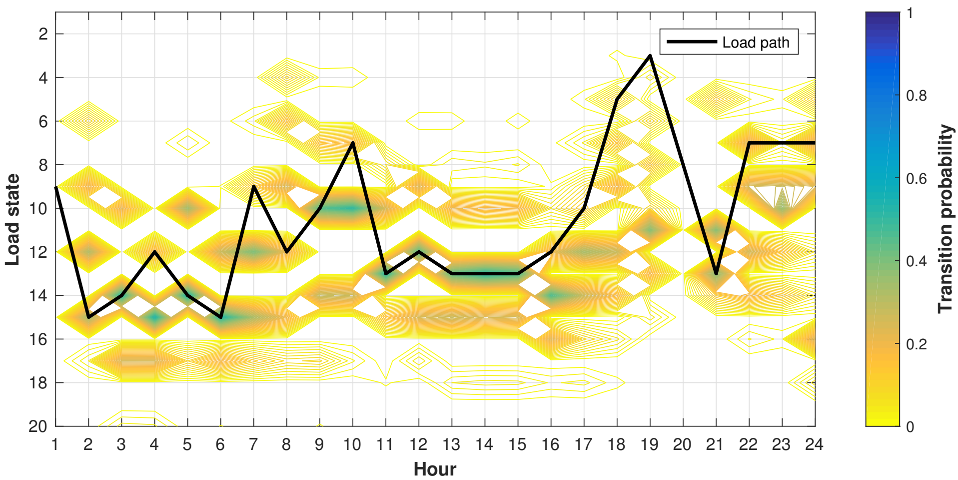

Figure 5.4: Actual discrete load path (black line) and calculated transition probabilities (coloured contour) of load 3 for Friday the 12th of February of 2016.

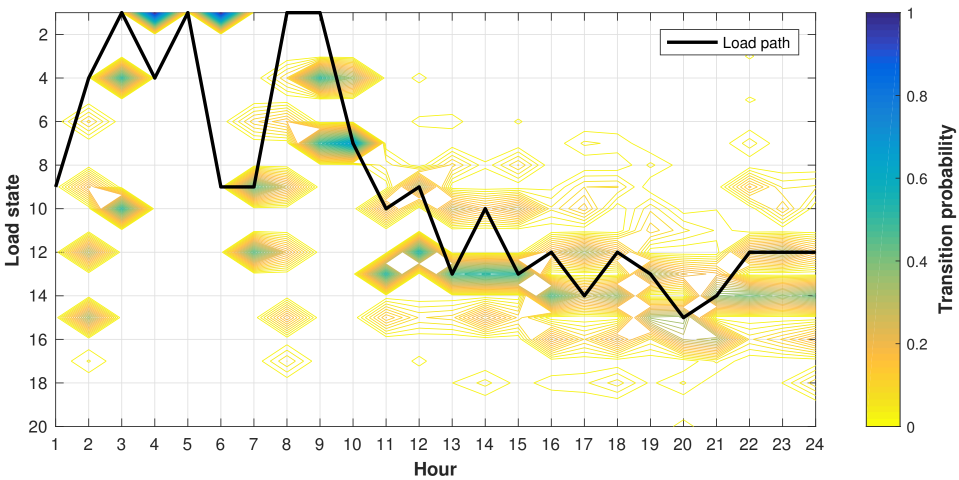

Figure 5.5: Actual discrete load path (black line) and calculated transition probabilities (coloured contour) of load 3 for Wednesday the 20st of January of 2016.

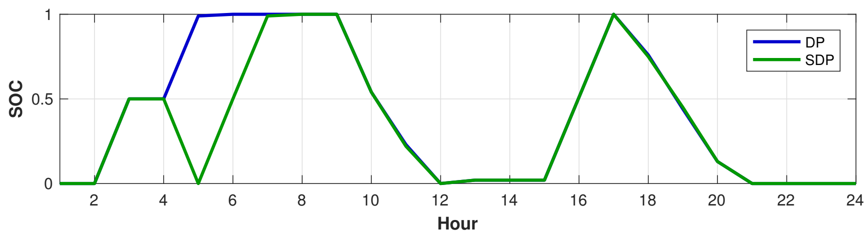

Figure 5.6: Resulting SOC from the DP- and SDP-optimisation of load 3 for Wednesday the 20st of January of 2016. Standard settings under the time based UT.

The results discussed in this section show signs of the stochastic model performing well, and especially in earlier hours of the day. This further supports hypothesis 1 as described in section 3.4.2 - load follows a similar pattern for similar days - which the stochastic model was built upon.

5.3 Consumer Cost

The results for total consumer costs are listed in appendix C in table C.1. The values for load 1 and 2 are for the first 275 days of 2016 (due to available data) while for load 3 the results are for the whole of 2016. Relative savings are listed in table C.2, and values for load 3 without any PV-system is shown in figure 5.7. Savings are the operational savings achieved by different configurations, and relative savings are savings divided by the total costs of the given basecase (no PV-system or battery). The results for the different UT structures in figure 5.7 will now be further discussed in their separate sections.

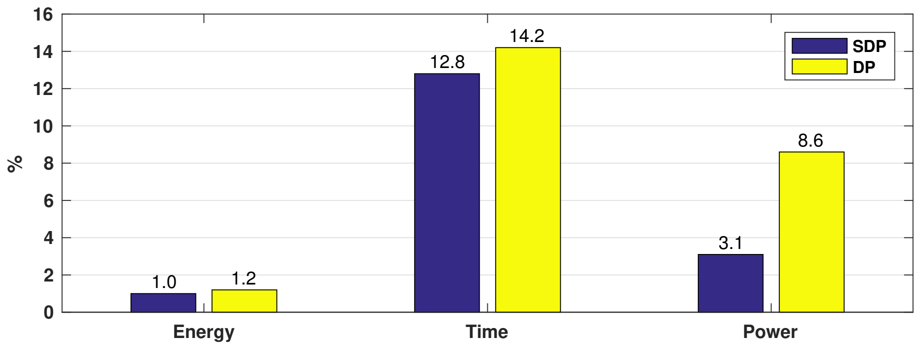

Figure 5.7: Relative savings compared to basecase for standard settings without a PV-system for the three different UT structures.

5.3.1 Energy Based Utility Tariff

The relative savings are almost negigible under the energy based UT, reaching only 1.0 and 1.2 % for the SDP- and DP-solution, respectively. This is over twice the results found in [1], which is partly explained by the variance in the spot price for different years. While 2013 (as used in [1]) had a variance of 0.0034 in spot price, 2016 had 0.0093, nearly three times higher. As the UT is the same for all hours, the battery provides cost savings by ”moving” load. Moving load here refers to charging the battery during low price hours, and then discharging during high price hours, exploiting present spot price arbitrages. This is illustrated in figure 5.8, which shows P_{grid}, SOC and spot price for the 21st of January, the day of the highest spot price of 2016 (2.076 NOK/kWh in hour 9).

In both the SDP- and DP-solution, the battery was empty both at the beginning and end of the day, which makes this a good day to compare the two solutions. In both cases, the battery is fully charged and discharged twice during the day. The first charge takes place at night time during hours 3 through 5. This is followed by a full discharge during hours 8 through 10, in order to avoid the peaking spot price. In both the SDP- and DP-solution, the household sells energy to the grid during these hours in order to benefit of the extraordinary high spot prices. This pattern is repeated during hours 14-20, only this time P_{grid} stays positive in all time steps. The costs for the given day was 147.57 NOK for the basecase, 111.62 NOK for the SDP-solution and 111.39 NOK for the DP-solution. In other words

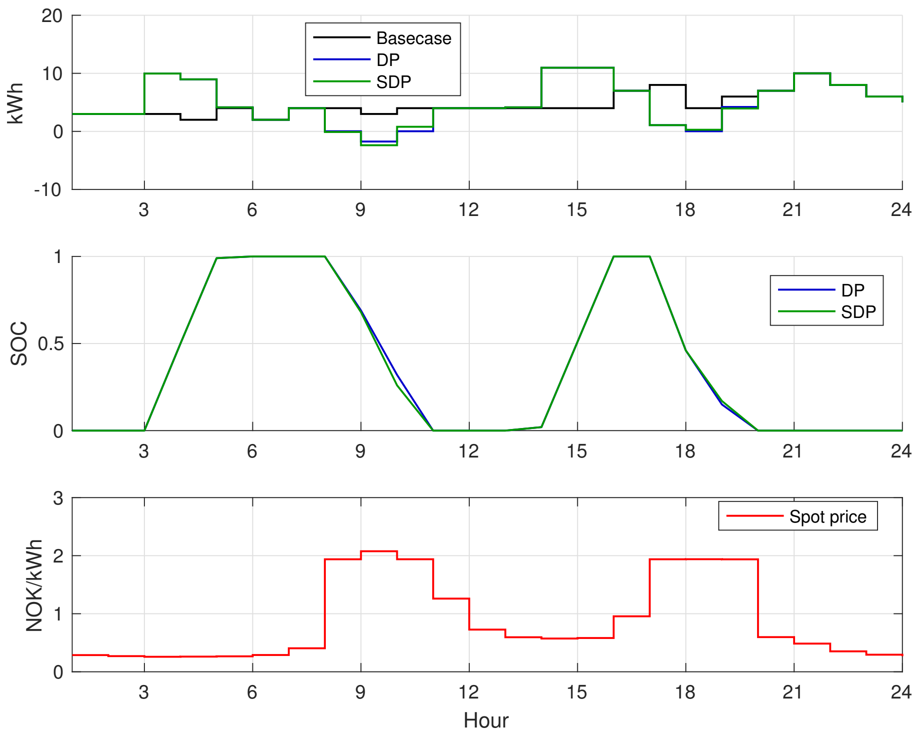

Figure 5.8: Thursday the 21st of January 2016, standard settings without PV under the energy based UT. Upper plot shows P_{grid} for the basecase, DP and SDP. Middle plot shows the SOC of the battery. Lower plot shows spot price.

a cost reduction of more than 24 % in both battery cases. Notice that the SDP-solution achieves 99.4 % of the savings of the DP-solution, which is basically a perfect result.

As shown above, spot price arbitrage is an important lever for achieving cost savings with a battery. Because the consumer is not remunerated the UT when selling to the grid, the spot price difference has to be at least the magnitude of the UT in order to enable price arbitrage. In addition one would need to take into account the losses in the battery, which would further limit its ability to perform arbitrage. To illustrate this, figure 5.9 shows the same plots as above for the 16th of April 2016, the day of the lowest daily spot price variance in the whole of 2016. Observe how the battery is not operated at all, neither in the DP- nor the SDP-solution, and thus stays discharged for the whole day.

In the case of the energy based UT, the battery would need to be used to shift load away between hours with a sufficiently high spot price spread. As the relative savings calculated in this thesis show, this does not occur often enough in order to generate noteworthy cost savings. It should be noted that in a future power system, where renewable energy sources accounts for a bigger share of the generation, the spot prices would probably vary more, thus making a battery more profitable even under a energy based UT.

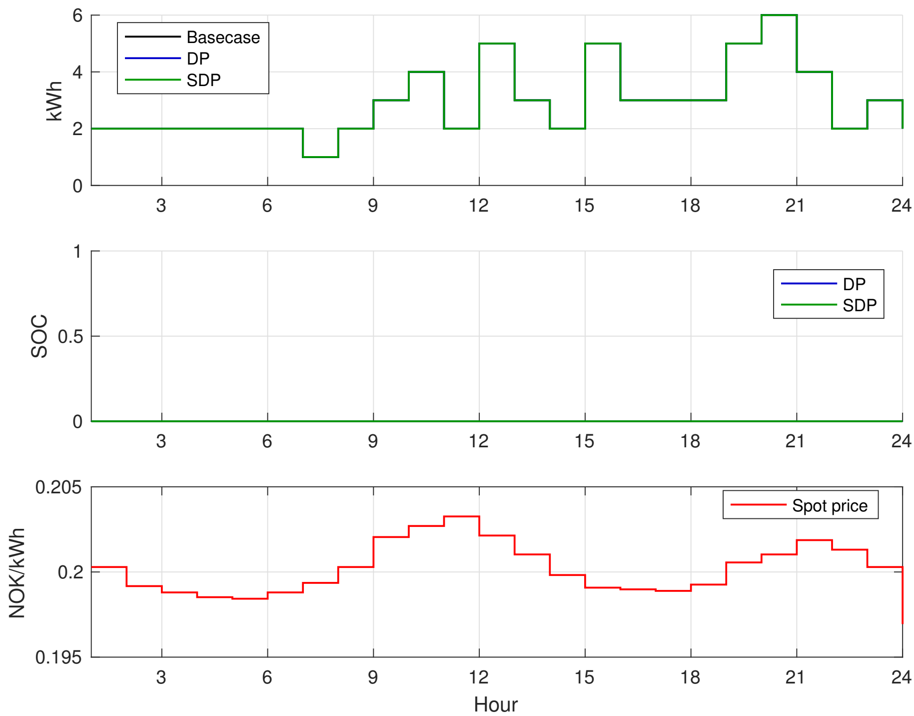

Figure 5.9: Saturday the 16th of April 2016, standard settings without PV under the energy based UT. Upper plot shows P_{grid} for the basecase, DP and SDP. Middle plot shows the SOC of the battery. Lower plot shows spot price.

5.3.2 Time Based Utility Tariff

The savings under the time based UT are vastly higher than the ones of the energy- and power based UTs. Reaching 12.8 and 14.2 % in the case of the SDP- and DP-solution, respectively, accounting for 2 487 and 2 747 NOK of yearly savings. These relatively high savings are caused by the battery shifting load from high to low UT price hours, which occur two times every weekday.

Figure 5.10 shows P_{grid}, SOC, spot price and UT for Tuesday the 13th of April 2016. The behaviour of the battery is much the same as the one observed in figure 5.8, but this time the charging and discharging is triggered by the variations in UT rather than spot price. The first charging takes place prior to hour 6, when the UT doubles from 0.2 NOK/kWh to 0.4 NOK/kWh. The discharge takes place during hours 9 through 11, and fully meets the load of the household, rendering P_{grid} equal to zero[3]. The next charging is performed just before the new doubling of the UT at hour 17, but at slightly different time steps for the DP- and SDP-solution. While the DP-solution charges only in hour 16, the SDP-solution charges both in hour 15 and 16, which leaves the battery with more energy in hour 17. As

[3]The actual values are not completely zero, but this is due to the discretisation of the SOC (DP and SDP) and the load states (SDP).

in the previous high price hours, both the SDP- and DP-solution discharges during hours 17-19 in order to cover the household load, but not selling any energy back to the grid.

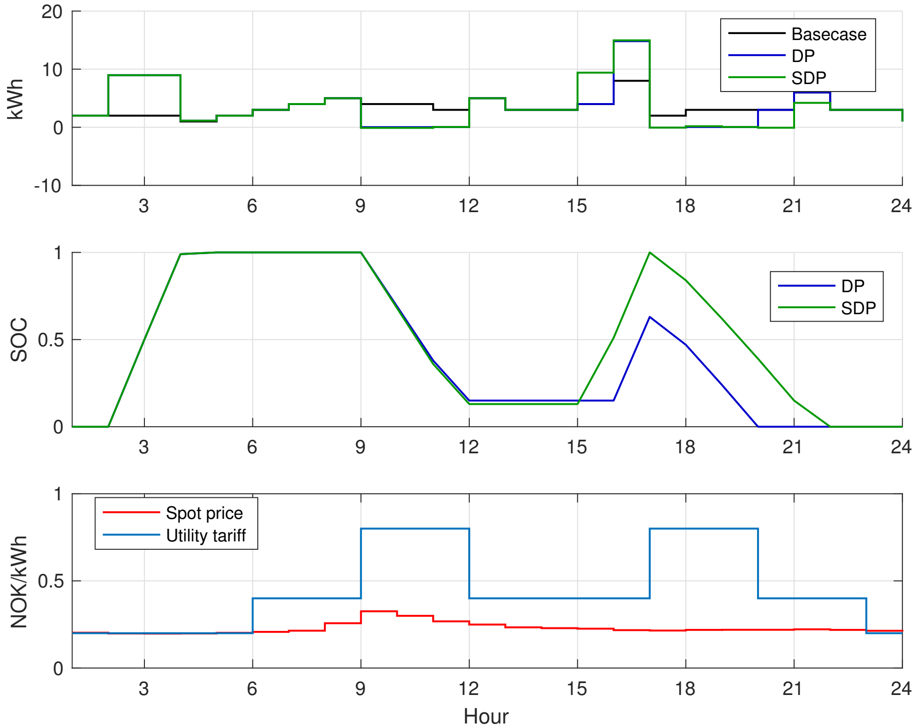

Figure 5.10: Wednesday the 13th of April 2016, standard settings without PV under the time based UT. Upper plot shows P_{grid} for the basecase, DP and SDP. Middle plot shows the SOC of the battery. Lower plot shows spot price and UT.

Under the time based UT, the SDP-solution achieved over 90 % of the savings of the DP-solution, which is the highest of the three different UT structures. This high performance is most likely due to the fact that the SDP-model is able to foresee price variations throughout the day, rendering it able to exploit these. In the case of the time based UT, sufficiently high price variations are present in all weekdays. This causes the battery to be more valuable under the time based UT compared to the energy based UT, which did not offer sufficiently high price variations. As figure 5.10 shows, this often involves fully charging during night time, discharging during hours 9-11, charging in hours 15-16 and then fully discharging in hours 17-19. Thus, it would have been interesting to investigate how a rule-based operational policy following this pattern would perform compared to the SDP- and DP-solutions.

Before ending this discussion, it should be noted that the high-price levels of the time based UT could have been set artificially high. These were set to approximately twice the level of the energy based UT, which might be much higher compared to what is being considered implemented in the Norwegian market. Nevertheless, these results show that such high levels might be needed in order to make batteries profitable enough. Future studies might look into the effect of altering the price levels of the time based UT with respect to the response of the battery.

5.3.3 Power Based Utility Tariff

While the SDP performs close to the DP under both the energy- and time based UT structures, scoring above 81 % and 90 % of the available savings, it is strongly outperformed in the case of the power based UT structure. Achieving only 36 % of the savings of the DP-solution, the SDP-model shows great room for improvement under the power based UT structure.

Figure 5.11 shows P_{grid}, SOC, spot price and UT for the 21st of January of 2016 under the power based UT, the same day as is shown in 5.8. Observe how the battery now charges steadily prior to the two peaks in spot price, as compared to the the fast charging in the case of the energy based UT. This is done in order to avoid high values of P_{grid}, which again would dramatically increase the UT in accordance with equation (3.9). As a result, P_{grid} is flattened out, which might be an interesting result for utility companies, which objective might be to achieve flat power profiles in their grids.

The DP-solution is generally flatter than that of the SDP-model. This might explain some of the difference between the two in terms of cost savings, as the utility costs increase exponentially with P_{grid}. Because of this, any increase in P_{grid} will be a costly affair, and quickly worsen the final result. On January 21st, the SDP-solution actually sold energy back to the grid in hour 9, the hour of the maximum spot price of 2016. This is not the case in the DP-solution, which saves energy in order to cover the load in the evening. The SDP-algorithm did not foresee such high load values, and thus did not fully recharge in order to cover these. It did however plan for the spot price peaks, which is evident from the battery being empty at hour 20, the hour when the prices more than halved itself, returning to more normal values.

The costs for the given day with the power based UT would have been 162.13 NOK in the basecase, 138.85 NOK in the SDP-case and 134.42 NOK in the DP-case. This is 14 and 17 % savings for the SDP- and DP-solution, respectively. Notice that the SDP-solution achieved 84 % of the savings of the DP-solution, which is far better than the 36 % for the full 2016. In the case of figure 5.11, the SDP-model was fed useful information in terms of the coming spot price peaks, and could thus plan for avoiding these, which explains the good performance of the SDP-solution compared to the DP-solution.

The price signals are not always as clear as in the case for Januart 21st. Figure 5.12 shows February 12th, which had negligible spot price variations, but a major peak in load in the evening. The actual load path and transition probabilities of February 12th are shown in figure 5.4, illustrating the low probability of the load achieving such high states several hours in a row for the given day. The DP-solution, which knew this, charged accordingly in order to be fully charged by hour 18. The SDP-solution did not foresee this major load increase (as illustrated in figure 5.4), and did not charge accordingly. This yielded almost no reduction in P_{grid} during the high load hours, and the increased costs were thus not avoided. The total daily costs was 72.40 NOK, 71.27 NOK and 64.43 NOK for the

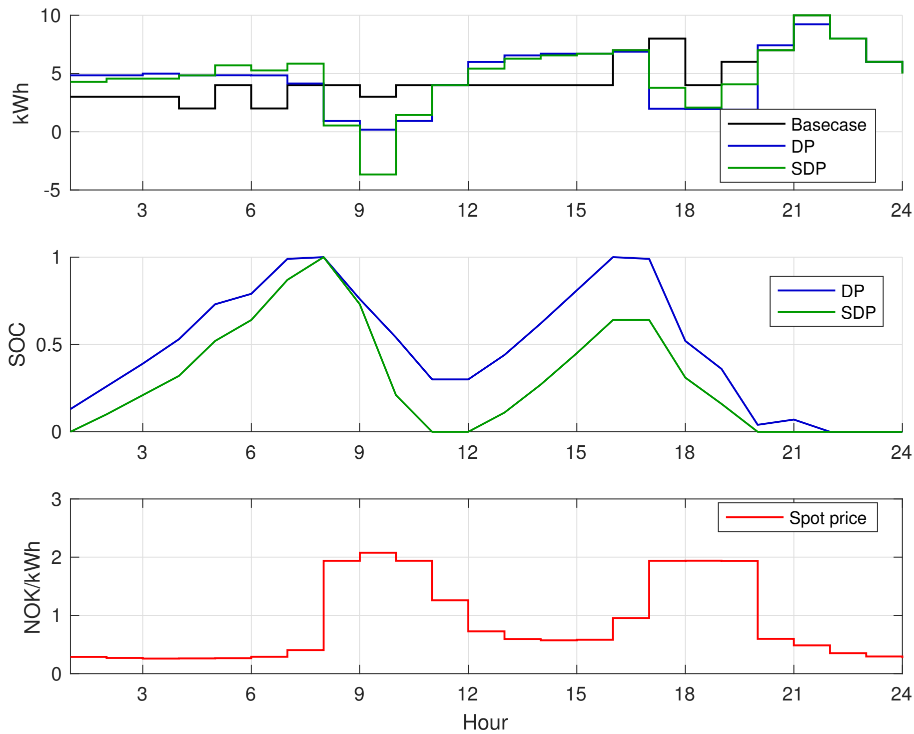

Figure 5.11: Thursday the 21st of January 2016, standard settings without PV under the power based UT. Upper plot shows P_{grid} for the basecase, DP and SDP. Middle plot shows the SOC of the battery. Lower plot shows spot price.

basecase, SDP- and DP-solution, which means that the SDP-solution only achieved 14 % of the cost savings of the DP.

This example, in addition to the generally low spot price variance, explains why the SDP-model is outperformed in the case of the power based UT. The SDP-model foresees price variations, but not load deviations from one of historic patterns. Thus, due to the high costs of increaseing P_{grid}, the SDP-model is heavily penalised for load prediction inaccuracies. In fact, a rule based charging strategy could, if implemented correctly, have yielded better results than the SDP-model. For example by setting a threshold-based policy which charges as long as the load is below a given level, and discharges above a certain load, much like what was found in [4]. This is left for future studies to investigate.

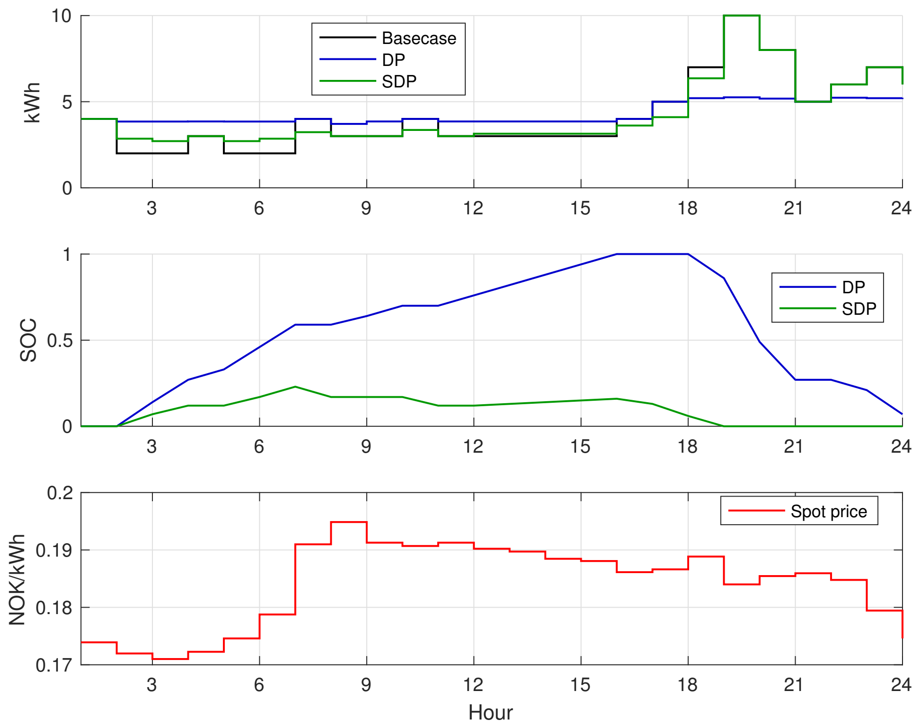

Figure 5.12: Friday the 12th of February 2016, standard settings without PV under the power based UT. Upper plot shows P_{grid} for the basecase, DP and SDP. Middle plot shows the SOC of the battery. Lower plot shows spot price.

5.4 The Effect of a PV-system

Before this reading this section, the reader should be reminded that both the SDP- and DP-algorithm considers PV-production as deterministic.

Figure 5.13 shows relative savings compared to the basecase for different set-ups with a PV-system for standard settings, organised by the three different UT structures. As can be seen, the relative savings provided by just a PV-system are more or less unaffected by the UT structure, scoring in the range 12.2-13.2 % (absolute values range 2 262-2 551 NOK). This means that the deployment of PV-systems in private household in central Norway would most likely not be noteworthy effected by the introduction of time- or power based UT structures.

When combined with a battery, one achieves additional savings in the same range as the ones of just the battery (see figure 5.7), but not exactly. For the energy based UT, the SDP- and DP-optimised battery adds 1.5 and 1.9 % of savings, respectively. This is 50 and 58 % higher than just the battery alone, which means that the battery is more valuable if a PV-system is present in the household. The increased value is caused by the battery being able to charge on free energy from the PV-system, which enables price arbitrage at lower spot price variations, as the consumer do not have to pay UT for this energy.

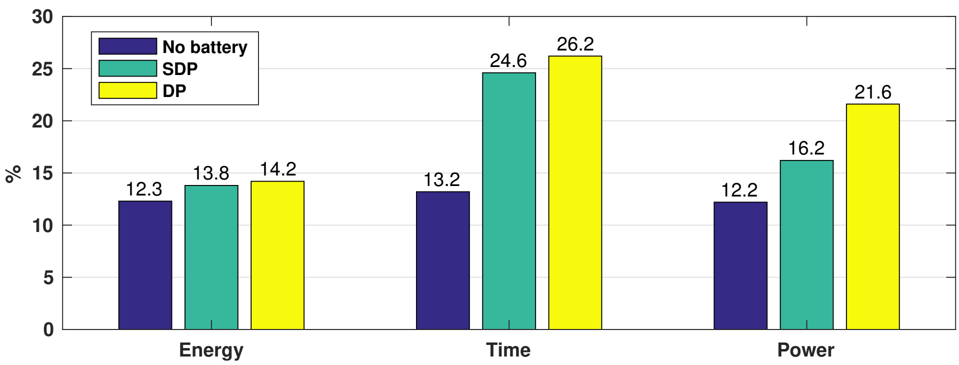

Figure 5.13: Relative savings for different configurations: Just PV, PV with SDP-optimised battery and PV with DP-optimised battery, all under standard settings.

Figure 5.14 shows P_{grid} for the basecase as well as the cases with a SDP- and DP-optimised battery and PV-production P_{PV} (upper plot), SOC (middle plot), spot price and UT (lower plot) for Wednesday the 22nd of June. Notice how in the SDP-solution, the battery is fully charged by hour 14 and can thus not receive more of the free energy from the PV-system, which is thus sold to the grid. The DP-solution foresees this, and charges fully on the excess PV-production. Notice how the SDP- and DP-solutions follow each other closely from hour 16, which indicate that the load followed the most likely pattern until midnight, and that it was not worth saving any energy to the next day.

For the time based UT, the introduction of a battery adds 11.4 and 13 % additional savings in the SDP- and DP-solution, respectively. This is approximately 10 % lower than the

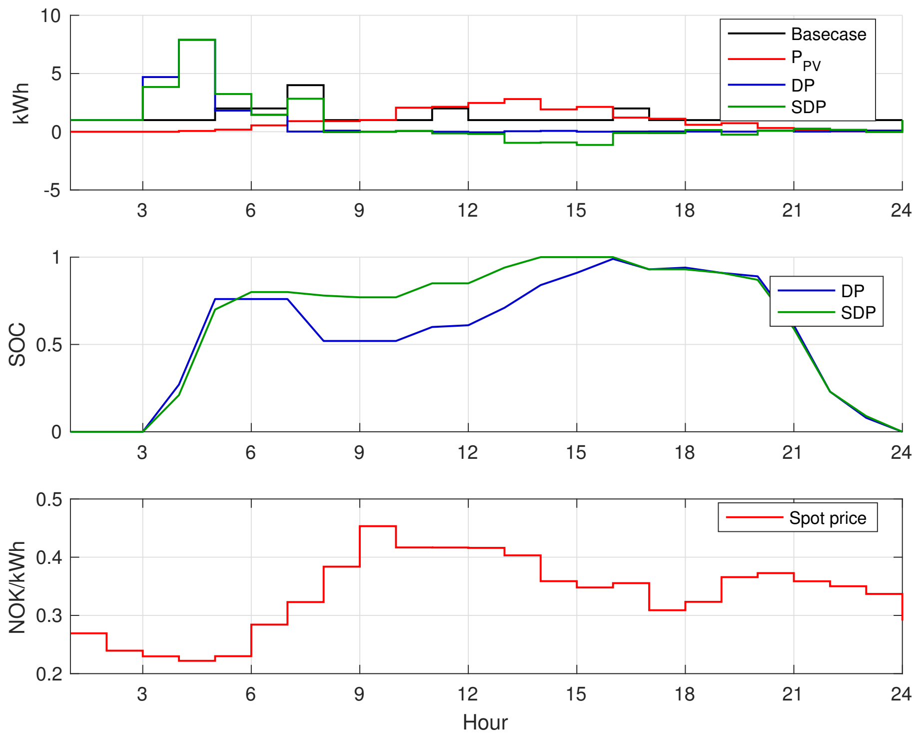

Figure 5.14: Wednesday the 22nd of June 2016, standard settings with PV and the power based UT. Upper plot shows P_{grid} for the basecase, DP and SDP as well as P_{PV}. Middle plot shows the SOC of the battery. Lower plot shows spot price.

savings without a PV-system, which means that in the case of the time based UT, a battery is *less* valuable when combined with a PV-system. This might be explained by the fact that the PV-production will cover some of the household load during high UT hours 9-11 and 17-19. During 2016, the PV-production during these two periods were 908.9 and 422.7 kWh, covering 30 and 9 % of the yearly total load during these hours[4]. This will drive down costs associated with the high UT during these hours, and the battery is thus not as useful as without the PV-system.

Under the power based UT, the battery adds 4 and 9.4 % of additional savings, which is more than without the PV-system. Notice that the SDP thus achieves 43 % of the additional savings of the DP, which is higher than the 36 % without a PV-system. In other words, under the power based UT, the battery is more valuable with a PV-system than without, and the performance of the SDP-model increases relatively to the DP-solution. The reason for the battery being more valuable with a PV-system might be due to the fact that the battery can be charged from the PV-system rather than the grid. Charging from the grid obviously increases P_{grid}, which further increases the UT in accordance with equation (3.9). Thus, if

[4]It should be noted that the PV-production and load could have occurred at different days, but this still illustrates the point.

the battery can be charged without increasing P_{grid}, this would further increase the value of the battery as it can charge at lower costs.

As to why the SDP-model performs better compared to the DP-model when a PV-system is present, this might be explained by figure 5.15, showing the exploited SOC of both the SDP- and DP-solution with and without a PV-system. Using the SDP-model, the battery achieved a higher SOC in 199 out of 366 days when combined with a PV-system[5]. This is visible as the blue part in the upper plot in figure 5.15. For the DP-model, this was only the case in 141 days, which means that it did not get as much more value from the PV-system as the SDP-model. The SOC-differences sum up to 23.37 and 22.92 for the SDP and DP, respectively, which shows that the SDP was able to store relatively more of the energy converted by the PV-system.

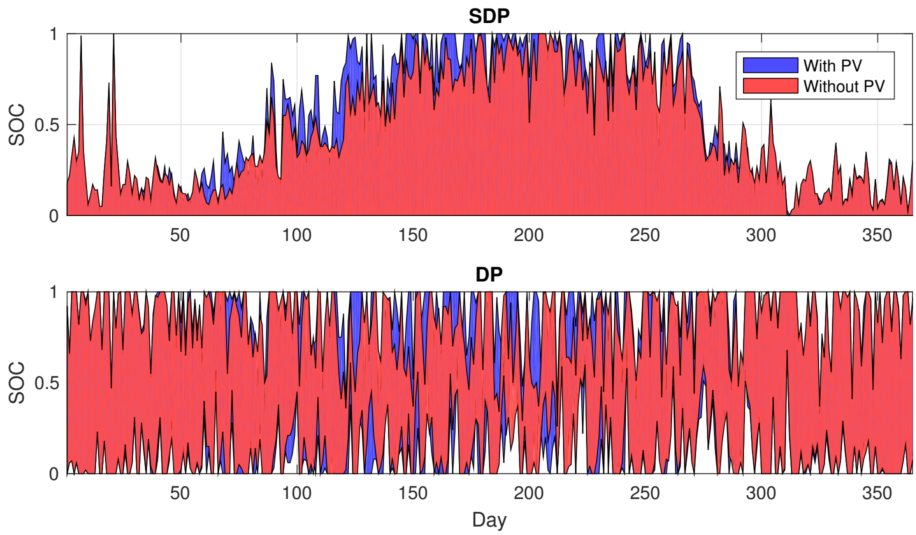

Figure 5.15: Exploited SOC for the SDP (upper) and DP (lower) for standard settings under the power based UT.

5.5 Varying Historic Load Data

The stochastic model in this thesis uses historic load data in order to predict future load patterns, and all the above discussed results were based on historic load data from the years 2013, 2014 and 2015. Table C.3 in appendix C shows the relative savings introduced by a SDP-optimised battery for load 3 without a PV-system for different variations of historic data sets. Figure 5.16 shows the relative savings provided by a SDP-optimised battery under the three different UT structures, namely energy based, time based and power based. These will now be discussed in their separate sections.

[5] As compared to the case without a PV-system

Energy Based Utility Tariff

As can be seen in figure 5.16, under the energy based UT, the highest savings are achieved if just the 2013 data is used in the stochastic model. It should be noted that the span of the results is narrow 0.07 %, which equals just 13 NOK. Thus, the absolute added value for the consumer is almost unaffected by the historic input to the stochastic model. Looking at how the SDP-solution performs compared to the DP-solution using only the 2013-data, this rises more than 5 %, from just short of 82 % up to above 87 %.

Time Based Utility Tariff

As with the energy based UT, the 2013 historic data set yields the best results also for the time based UT, scoring 12.96 % relative savings. Notice that also here, the difference to the lowest result (12.81 % for the 2015 data set) is close to negligible, at just 0.15 %.

Power Based Utility Tariff

The power based UT is the only UT where the SDP-optimised battery does not achieve the best results when based on the 2013 data set, but rather when based the one from 2015. As with the other two UT structures, the spread in the results is almost negligible, reaching just 0.4 % from the 2014- up to the 2015 data set, but looking on the relative difference between the results, the solution based on the 2015 data set performs 15 % better than the one based on the data from 2014, which is not negligible.

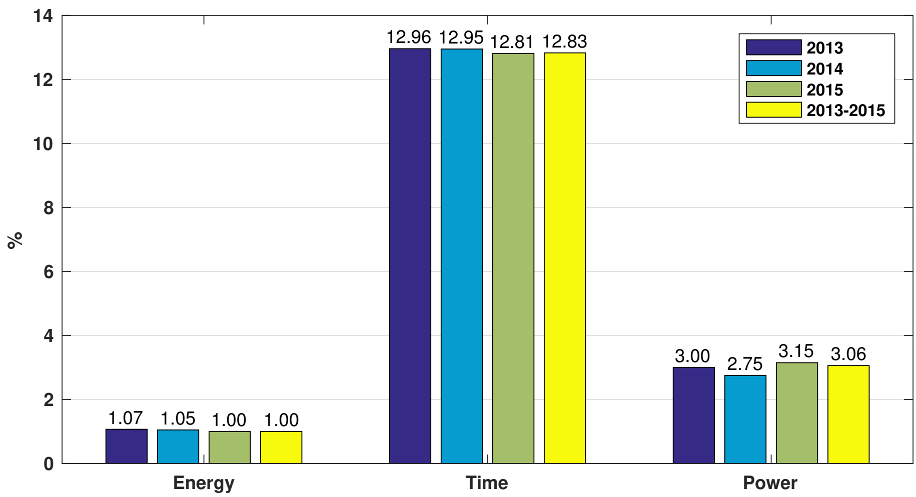

Figure 5.16: Relative savings for the SDP based on different historic load data sets.

As discussed in previous sections, the savings provided by the battery under the energy- and time based UT structures are mostly driven by shifting load from hours of high to hours of low prices. In the case of the power based UT, the savings are more dependant on evening out the load profile. The inaccuracies in the stochastic model is thus heavily penalised, causing the SDP-solution to be outperformed by the DP-solution. This might be what is causing the 2015 data set to perform best as a basis for the stochastic model, as this is probably more similar to the 2016 data.

5.6 Varying Load Discretisation

The stochastic model discretises load into N_{disc} discrete load state. This sections presentes results for different levels of discretisation. Table C.4 in appendix C shows the relative savings for standard settings without a PV-system for different levels of load discretisation (N_{disc}), ranging from 10 to 60, all under the time based UT. Included is also the run-time of the stochastic model, the average run-time of the daily SDP-optimisation[6] and the relationship between the savings achieved by the SDP- and DP-solution. Figure 5.17 shows the relative savings achieved with the SDP-solution (upper plot), the run-time of the stochastic model (middle plot) and the average run-time of the daily SDP-optimisation (lower plot), all as functions of N_{disc}, with linear interpolations between the discrete data points.

The relative savings increases from $N_{disc} = 10$ to 30, before flattening out and actually achieving no further savings above 13.07 % from $N_{disc} = 50$ to 60. It should be noted that the savings achieved with $N_{disc} = 60$ is only 3.6 % higher relative to those with $N_{disc} = 10$, which indicate that the difference is not substantial. The run-time of the stochastic model, which was run once per yearly optimisation, shows a linear relationship towards N_{disc}, while the average run-time of the daily SDP-optimisation shows more of a quadratic relationship. This is critical for the run-time of the whole program, as a doubling of the daily optimisation run-time leads to a more than 700-doubling of the yearly optimisation.

Based on these results and the available computational hardware, an actual implementation of the proposed SDP-model would need to use a suitable N_{disc} which achieves sufficiently high savings while satisfying run-time limitations. For the given model and computer used in this thesis, $N_{disc} = 20$ was set as the standard setting. Increasing this to 30 would have increased the average daily SDP-optimisation run-time by 113 %, but the relative savings by scarce 1.5 %. It would thus have made more sense to adjust N_{disc} down to 10, reducing the average daily SDP-optimisation run-time by two thirds but still achieving 98.3 % of the same savings.

[6] Average of ten individual runs, here rounded to one decimal.

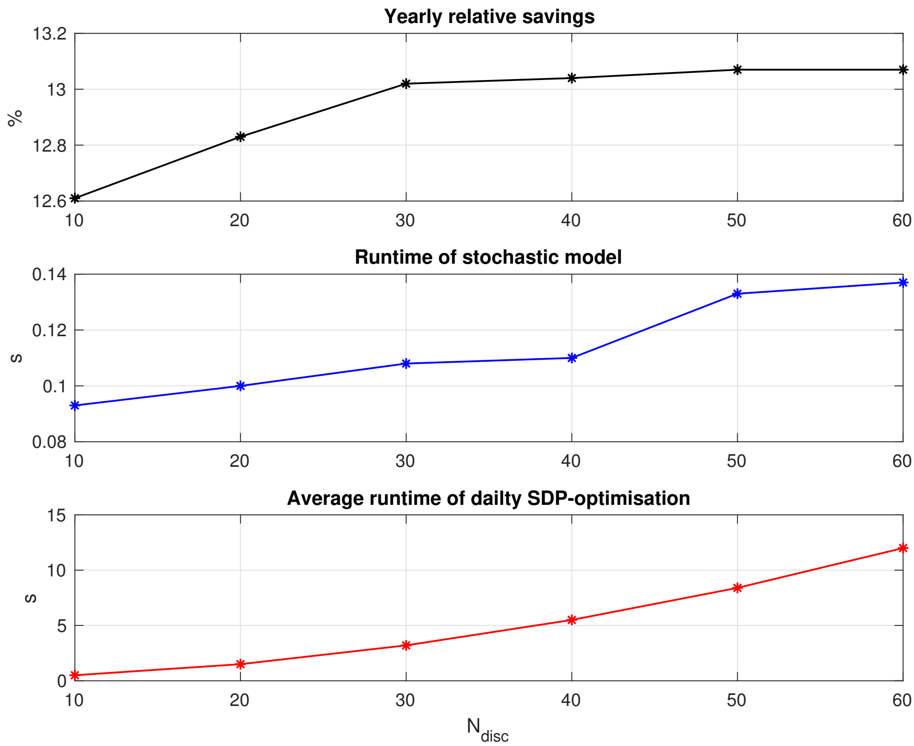

Figure 5.17: Results for the SDP with standard settings without PV under the time based UT. Upper: Relative savings. Middle: Run-time of the stochastic model. Lower: Average run-time of the daily SDP-optimisation.

5.7 Learning Stochastic Model

In all previously discussed results, the load of 2016 was never taken into account in the stochastic model. This section discusses the effect of adding learning to the stochastic model, by updating the load discretisation and transition matrices for every day that passes. Figure 5.18 shows the relative savings achieved by the SDP-model for standard settings without PV under the three UT structures, both with and without learning activated. The relative savings increase under all UT structures when learning is activated, but the improvement differs: 0.03 % under the energy based UT, 0.12 % under the time based UT and 0.13 % under the power based UT. This contributes to closing the gap to the global optimal solution found by DP, with the SDP achieving 84.4 %, 91.4 % and 37.2 % of the savings of the DP-solution.

These results show that a real life implementation of the developed SDP-model should include learning in order to get as close as possible to the global optimal solution. If a learning model were to be implemented, one would need to consider how many days that should be kept in memory at any given time. As section 5.5 shows, the load pattern might change over time, altering the accuracy of the stochastic model. Hence, a learning model could use the last N days to calculate the load discretisation and transition matrices, and - if N is chosen appropriately - also follow the seasonal patterns in the load. This would without doubt be interesting to look closer into, but is left for future studies to investigate.

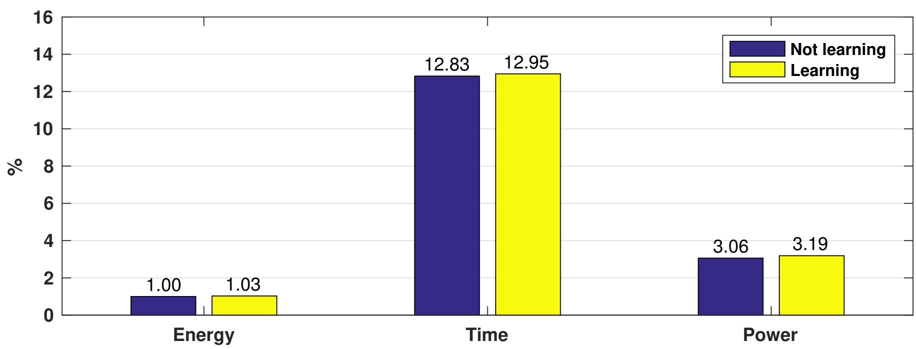

Figure 5.18: Comparison of the relative savings with and without a learning stochastic model, all for standard settings and grouped for the three UT structures.

5.8 Different Battery Parameters

As the parameters of the battery is included into the optimisation, the savings provided by a battery would be dependant on these parameters. Table C.5 in appendix C shows the relative savings for the four different battery configurations as described in section 4.2.2, for both the SDP- and DP-solution. Values are illustrated in figures 5.19-5.21, where the configurations are listed in rising order from left to right.

For load 3, the second configuration achieves 36.4 and 37 % higher savings relative to the first configuration for the SDP- and DP-solution, respectively. The third configuration, which has the same capacity but twice the maximum allowable power, shows less of an improvement with just 0.5 and 0.6 %. The fourth and last configuration has both a doubled capacity and maximum allowable power, but does not perform substantially better than the second configuration, improving this by only 0.7 and 5.5 %.

This shows that for the given household (load 3), a doubling of the battery capacity would be far more important than a doubling of the maximum allowable power. This is obviously dependant on the characteristics of the load in question, which is evident from the same results from load 1 and 2. Load 1, which is the lowest load of the three, does not gain as much as load 3 from the doubling of the capacity. Notice that load 1 with a SDP-optimised battery actually performs worse when the maximum allowable power is doubled, while the DP-solution slightly improves[7]. Load 2, which is the highest of the three, shows an even better effect of doubling the capacity of the battery than load 3, increasing the relative savings by almost 50 % in both the SDP- and DP-solution.

In general, it seems like the savings provided by a battery is dependant on the relative size of the battery compared to the average values of the load in question. Load 1, which has the lowest average values, achieve the highest savings but gain the least from doubling the battery capacity, while load 2, which has the highest average values, achieve the lowest savings, but gain the most from doubling the battery capacity. Load 3, which places in between the other two in terms of average values, also achieve relative savings which are in between those of load 1 and load 2. This shows, at least under the time based UT, that the size of the battery is far more important than the maximum allowable power. These results highlight the importance of a battery being designed appropriately for the household that it is to be installed in.

[7] This might have been caused by the discretisation of SOC (N_{SOC}), which was not adjusted when the power or capacity was altered

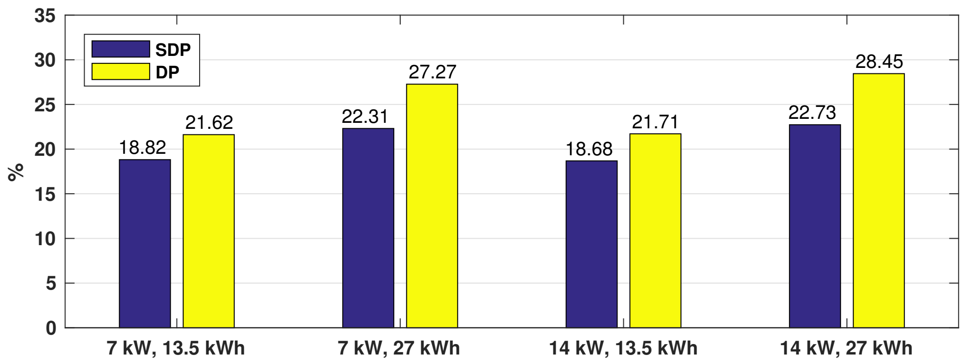

Figure 5.19: Relative savings for different battery configurations for load 1, standard settings, no PV and the time based UT structure.

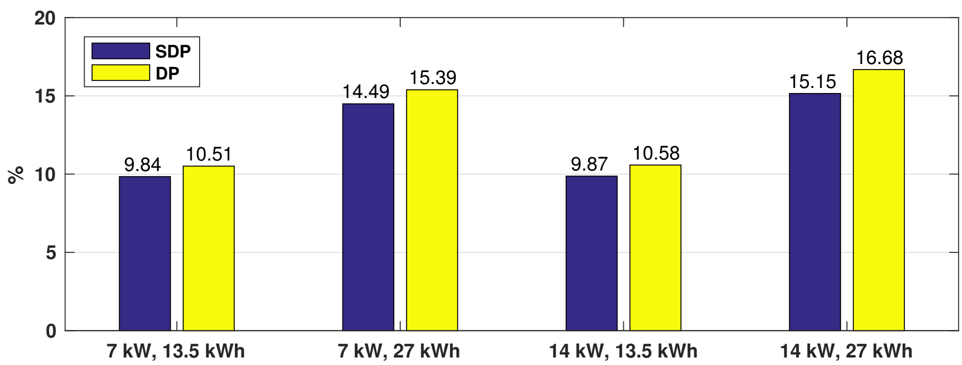

Figure 5.20: Relative savings for different battery configurations for load 2, standard settings, no PV and the time based UT structure.

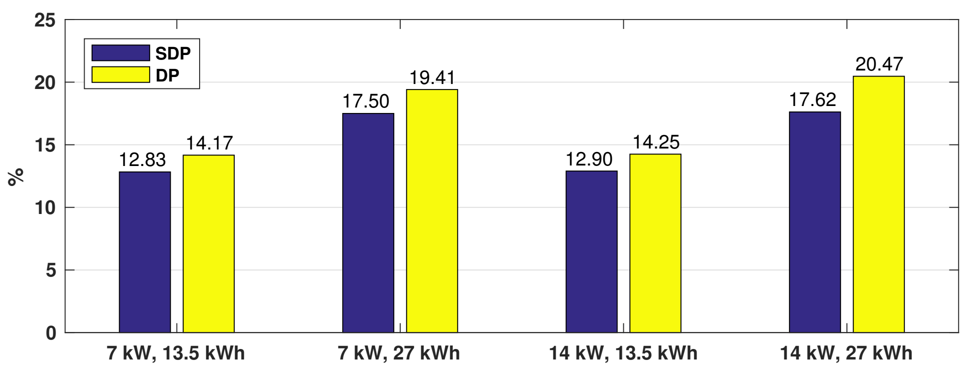

Figure 5.21: Relative savings for different battery configurations for load 3, standard settings, no PV and the time based UT structure.

5.9 Battery Usage

Figure 5.22 shows the total battery usage given in total full charges under standard settings without PV. This was calculated as the sum of all $P_{bat} > 0$ multiplied with the efficiency and normalised towards the capacity as specified in table 4.4:

$$\text{Battery usage} = \text{sum}(P_{bat} > 0)\frac{\eta_{ch}}{C_{bat}} \tag{5.1}$$

The usage of the battery is clearly dependant on the UT structure, and the time based UT, which is the one where the battery is the most valuable, also shows the most use of the battery. This can be explained by looking at figure 5.10, which shows load flow and SOC for Wednesday the 13th of April under the time based UT. As figure 5.10 shows, the battery is usually fully charged and discharged twice each weekday under the time based UT, which explains the high usage of the battery shown figure 5.22. As discussed in section 2.6, this means that the battery would degrade the fastest under the time based UT, which in turn would effect the long term profitability.

Notice that the SDP-solution uses the battery more than the DP-solution only in the case of the energy based UT. There is noe obvious reason for this, but it might be due to the SDP-model optimising day by day. This can lead it to charge the battery more in order to avoid daily spot prive variations, while the DP-solution would charge on one day and discharge on the other. This is somehow visible in figure 5.23, which shows the exploited SOC for standard settings under the energy based UT.

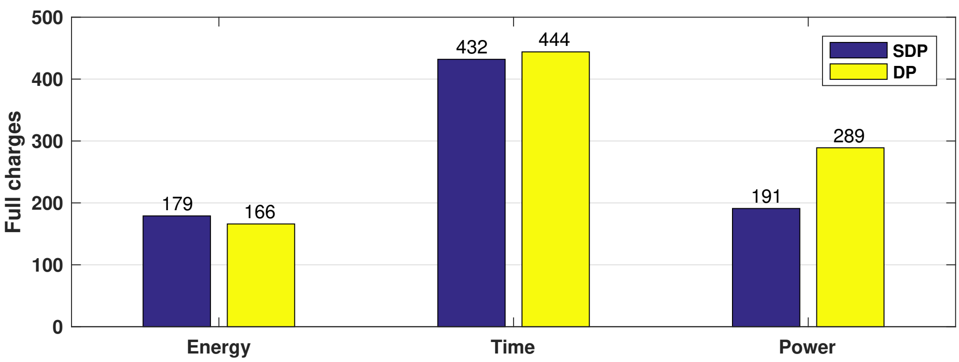

Figure 5.22: Total normalised battery usage counted in number of total recharges under standard settings without PV.

For the power based UT, figure 5.12 might explain why the DP-solution uses the battery more than the SDP-solution. The DP-model, which foresees the sudden rise in P_{load}, charges the battery fully while the SDP-model, who does not foresee this, does not charge accordingly. This results in the DP-model using the battery more than the SDP-model. This higher usage is also clearly visible in figure 5.24, showing the exploited SOC under the power based UT without a PV-system installed. As illustrated, the SDP-solution very rarely manages to fully charge, while the DP-solution manages to do this throughout the whole year. This further emphasises the room for improvement in the SDP-model under the power based UT.

5.9.1 Energy Losses

Due to internal imperfections in the battery, there are losses both during charging and discharging of the battery. In this thesis, these are taken into account by η_{bat} as defined in equation (2.9). The energy losses associated with charging and discharging the battery is linearly dependent on the battery usage, and are thus proportional to the results shown in figure 5.22. These will not be further discussed, as the losses are already taken into account in the consumer costs, which was discussed in detail in section 5.3.

5.9.2 Initial SOC

In the implementation of the SDP-model in this thesis, the battery was set to be empty both at the start of each day. It was also considered using a value function, which would value energy stored at the end of each day, but this was discarded due to the difficulty of defining such a value function. Figure 5.25 shows the distribution of the initial SOC for each day from the DP-solution under standard settings without PV. As can be seen, the distribution vary noteworthy between the three UT structures. For the energy- and time based UT, the battery is empty (SOC = 0) at 58 and 72 % of the days, respectively, representing a majority of the year.

For the power based UT, this is different, as only 11 % of the days started with an empty battery and a vast majority is somewhere in between empty and full. Figure 5.26 shows the initial SOC for all days under the power based UT from the DP-solution, sorted from high to low. It does noe seem to be any clear tendency towards any specific SOC, which could easily have been implemented in the SDP-model. This would thus need more investigation in order to find how the SDP-model could be improved under the power based UT.

A fully charged battery at the start of the day is rare under all three UT structures, with namely 17, 1 and 0 %. This is probably due to the load pattern, as the load is often low after midnight, which is visible in the normalised load plots in appendix D.

Nevertheless, these results show that the decision of setting SOC = 0 at the start of each day is not necessarily that limiting in the case of the energy- or time based UT structure, as the SDP-model achieves 81 and 92 % of the DP-model for these two UT structures. For the power based UT on the other hand, the constraint of an empty at the start of each day seem to be more of a limitation. It should thus be further investigated how to set the initial SOC under the power based UT.

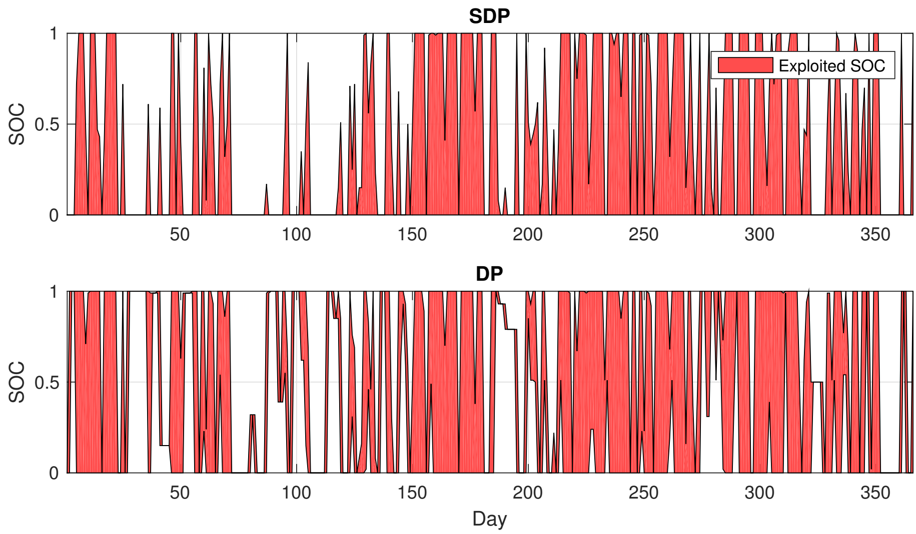

Figure 5.23: Exploited SOC for standard settings without PV under the energy based UT.

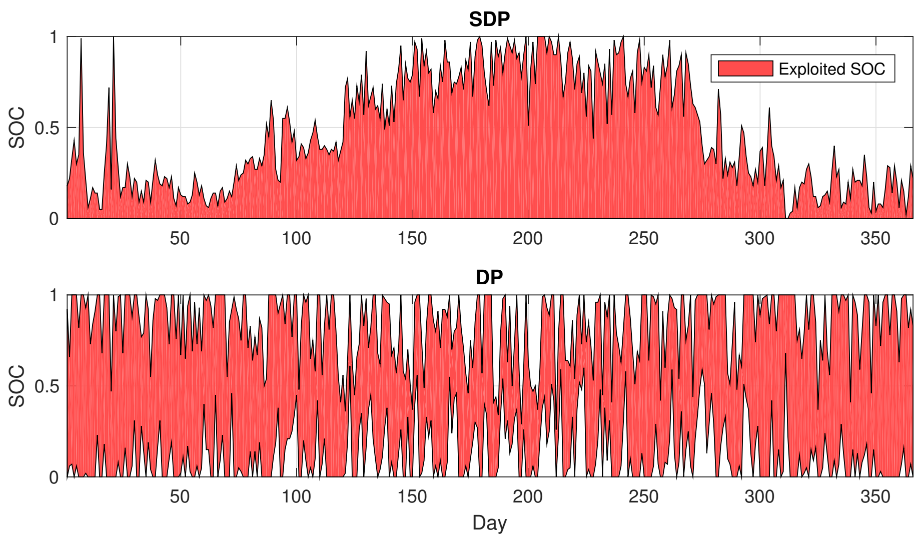

Figure 5.24: Exploited SOC for standard settings without PV under the power based UT.

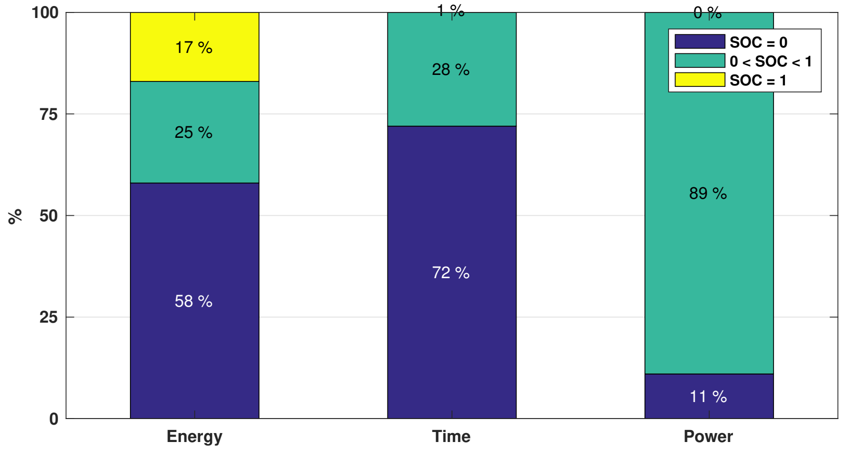

Figure 5.25: SOC at first hour of day from the DP-solution under standard settings without PV.

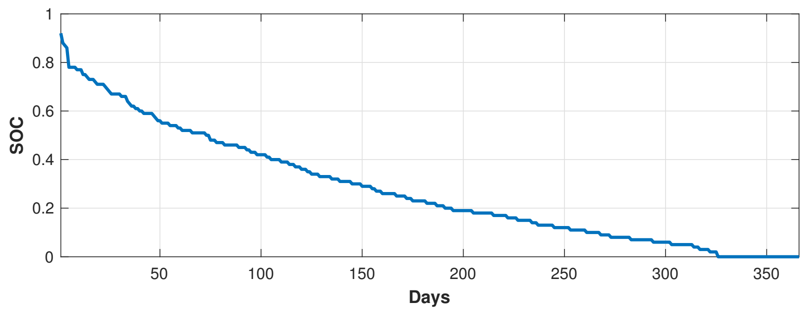

Figure 5.26: SOC at first hour of day from the DP-solution under standard settings without PV under the power based UT, sorted from high to low.

In order to avoid the hard constraint of an empty battery at the end of the day, one possibility could be to perform two SDP-optimisations with overlapping 24-hour horizons. The first optimisation is performed at midnight, when the spot prices for the whole coming day is known. The second one is performed at noon, when the spot prices for the following day are released. As both optimisations are performed before the end of the previous optimisation horizon is reached, there is no single point in time where the battery needs to have a specific SOC (other than the absolute first time step). In the implementation of the stochastic model chosen in this thesis, the transition between the last time step of a day and the first time step of the next day is not taken into account due to the daily optimisation setup. Due to this, such an dual-optimisation method as proposed above were not easily implemented. Thus, the implementation of such a method is left for future studies to investigate.

5.10 Power Flow

In this section, power flow results will be presented and discussed. The values are given in kWh, but due to the hourly values, these can be interpreted as kW-values. Note that no power flow calculations has been performed with respect to voltage and current values, and that no constraints on P_{grid} have been included. Because of this, some of the solutions might violate physical limitations in the local grid of the household. Nevertheless, the results should be of interest for utility companies wanting to investigate how batteries might influence the demand profiles in their grids.

5.10.1 Duration Curves

Figures 5.27-5.29 show the resulting duration curves of P_{grid} for the basecase, SDP- and DP-solution without PV under the energy-, time- and power based UT structures, respectively. Figures 5.30-5.32 show zoomed in versions of figures 5.27-5.29 for the maximum 1 000 hours and minimum 500 hours. Duration curves with PV installed are included in appendix C.2.

The solutions from the energy- and time based UT structures show the same tendencies, that being higher maximum values, lower minimum values and a rather similar middle part, compared to the basecase. The differences in maximum and minimum values are bigger under the time based UT, which is caused by the economic gain by increasing P_{grid} in hours of low UT and decreasing in hours of high UT. This difference, between the energy- and time based UT, is also visible in the usage of the battery, shown in figure 5.22.

Notice how the duration curves from the SDP- and DP-solution coincide for the energy- and time based UT. This further illustrates that the SDP-model performs close to the global optimum under these UT structures. As discussed previously, this is not the case under the power based UT, which is also visible in the duration curves in figure 5.29. Observe how the DP-solution produces an extraordinary flat duration curve, which is substantially lower than the basecase and SDP-solution for maximum values and somewhat higher for middle and lower values. The SDP-solution performs close to the DP-solution between hours 4000-8000, but loses track for higher and lower values. This further emphasises the room for improvement in the SDP-model under the power based UT structure.

The zoomed in figures 5.30-5.32 show that the battery introduces negative minimum values and increased maximum values in the case of the energy- and time based UT structures. For the power based UT, notice how the DP-solution manages to lower the maximum power flow of the year, while the SDP-solution does not. These observations are now further discussed in the following section.

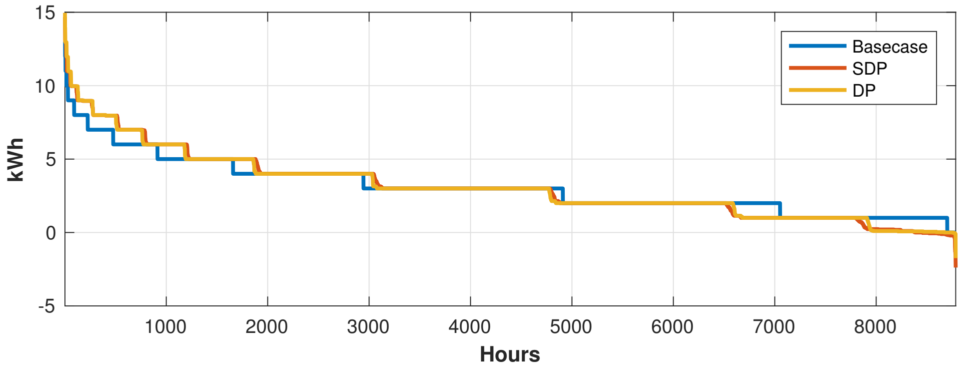

Figure 5.27: Duration curves of P_{grid} for 2016 for load 3 without PV under the energy based UT.

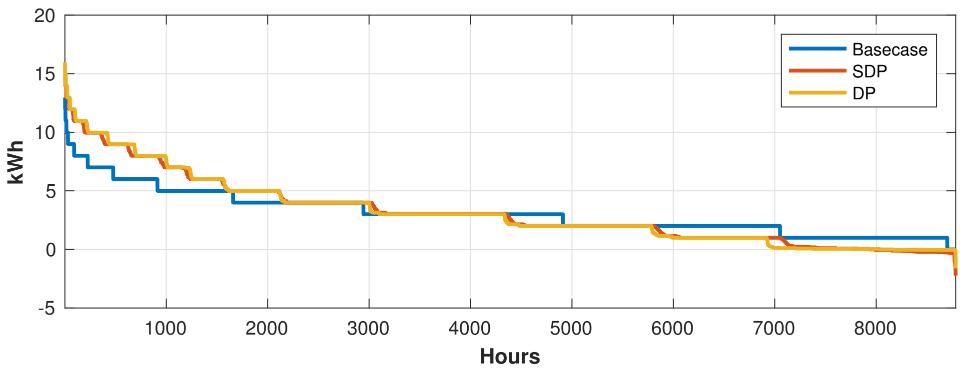

Figure 5.28: Duration curves of P_{grid} for 2016 for load 3 without PV under the time based UT.

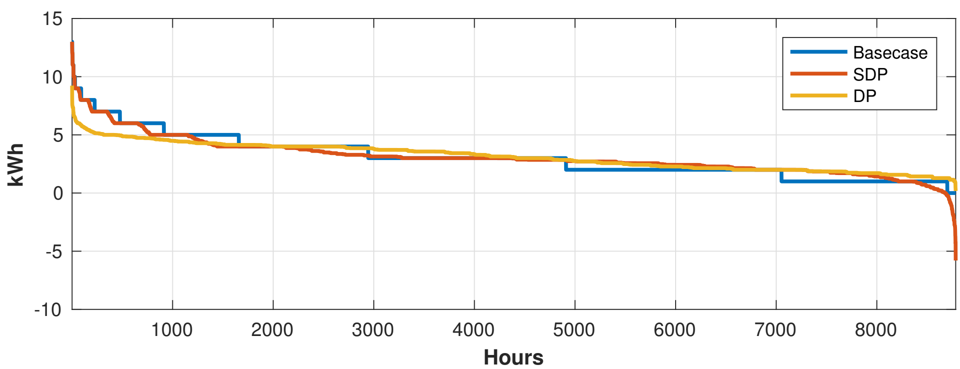

Figure 5.29: Duration curves of P_{grid} for 2016 for load 3 without PV under the power based UT.

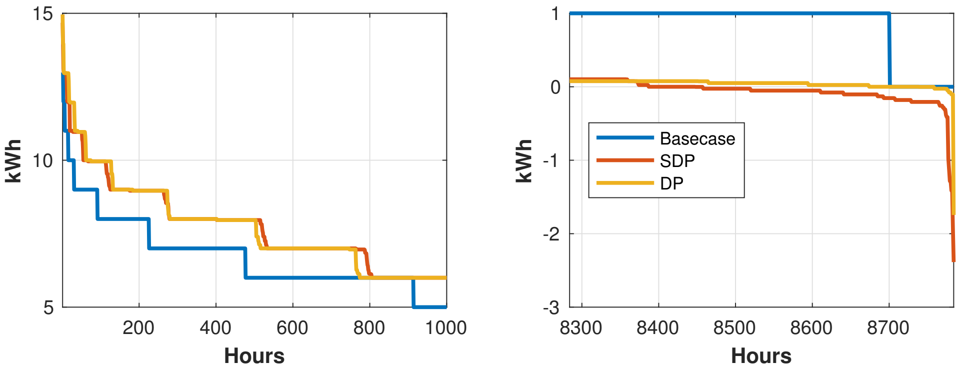

Figure 5.30: Duration curves of P_{grid} for 2016 for load 3 without PV under the energy based UT. Maximum 1000 values and minimum 500 values.

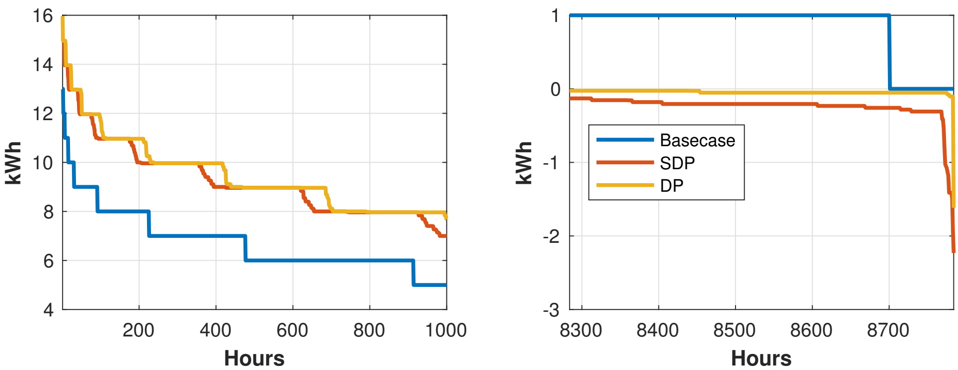

Figure 5.31: Duration curves of P_{grid} for 2016 for load 3 without PV under the time based UT. Maximum 1000 values and minimum 500 values.

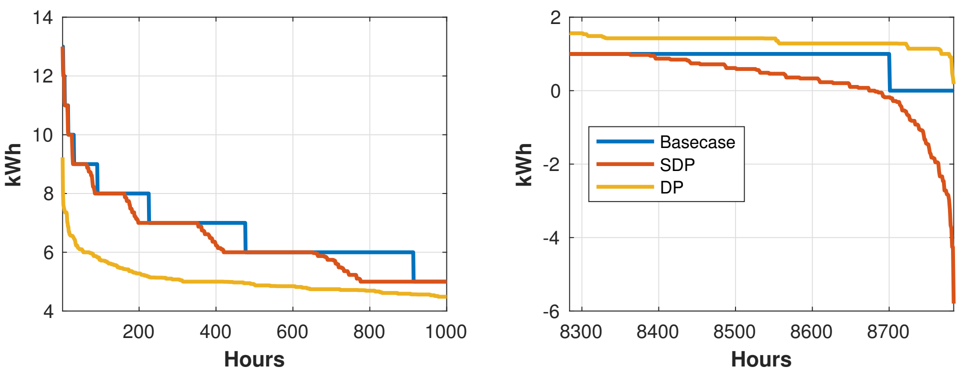

Figure 5.32: Duration curves of P_{grid} for 2016 for load 3 without PV under the power based UT. Maximum 1000 values and minimum 500 values.

5.10.2 Maximum and minimum values

As discussed in the previous section, the configurations with a battery will influence the maximum and minimum power flow values. Figures 5.33 and 5.34 show the maximum and minimum power flow for 2016 for load 3 without PV under different UT structures. Values for load 1 and 2 can be found in table C.6 in appendix C. Both the DP- and SDP-solution increase the maximum load in case of the energy- or time based UT. These increased values are caused by the battery charging in hours of low prices, in order to perform arbitrage. However, this is not the case of the power based UT, where the DP-solution manages to lower the maximum load by almost 30 %, but the SDP-solution does not lower the value of the basecase at all.

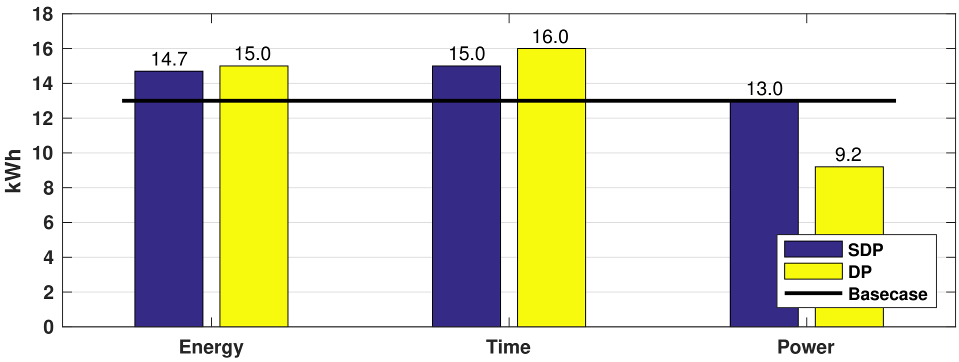

Figure 5.33: Maximum P_{grid} for 2016 for load 3 without PV.

The basecase had a minimum load of 0 kWh, but the introduction of a battery optimised for minimising consumer costs resulted in negative minimum values, which means that power flows from the household and into the grid. The SDP-solution yields higher negative values than the DP-solution in all cases, which is caused by different reasons. In the case of the time- and power based UT, the minimum values of the SDP-solution occurs in hour 23, which indicates that these were caused by the hard constraint that the battery needs to be empty by the end of the day. For the energy based UT, the minimum value occurs at the same time for both the SDP- and DP-solution, which was hour 9 of day 21, the hour of the maximum spot price of 2016 (which can be seen in figure 5.8).

Notice the big difference between the DP- and SDP-solution in the case of the power based UT. While the SDP-solution yields a minimum value of -5.8 kWh (in hour 23 of day 213), the DP-solution yields a *positive* value of 0.2 kWh (in hour 9 of day 21). This further emphasises the room for improvement in the SDP-model when combined with the power based UT. Figure 5.35 shows a full year plot of P_{grid} for both the SDP- and DP-solution. One can clearly see the bigger variance in the SDP-solution, as compared to the much more compact DP-solution. During summer (hours 3000-6500), the SDP-solution frequently yields negative P_{grid}-values, which is often due to the battery discharging during the last hours of the day. This again illustrates that a further development of the SDP-model would need to look into how to avoid the hard constraint of an empty battery at the end of the day.

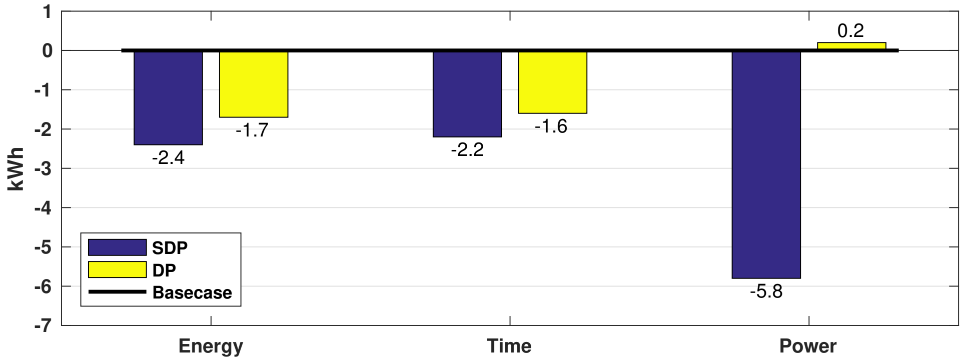

Figure 5.34: Minimum P_{grid} for 2016 for load 3 without PV.

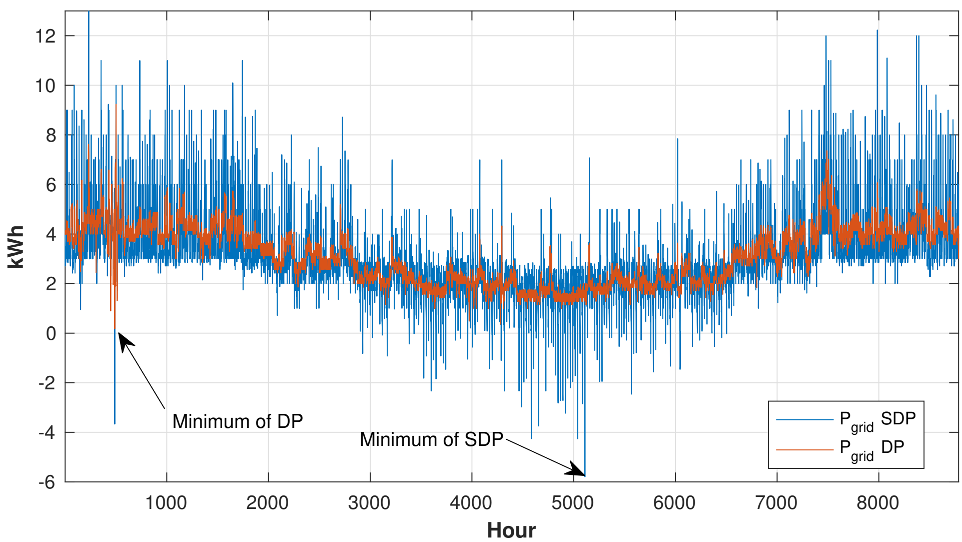

Figure 5.35: P_{grid} with SDP and DP for 2016 for load 3 without PV.

5.11 Net Present Value

All results until now have not considered the investment costs tied to the PV-system or battery. In this section, this will be included in the calculation of the net present value (NPV) for different configurations. Calculations has been performed as described in appendix E.

Table 5.2 shows the results for a PV-system with an investment cost of 30 000 NOK (based on values from test systems of TrønderEnergi), battery system of 62 200 NOK (same as a Tesla Powerwall in Norway [27]), 1 % yearly degradation[8] and 25 years of lifetime. The cash flow in the first year was set to be equal to the savings for the given configuration for 2016 for load 3, as can be derived from table C.1.

Table 5.2: Net present value given in NOK for load 3 under standard settings. All battery configurations are with the SDP-solution. Relative values compared to investment costs can be found in table C.7.

			Discount rate		
PV	**Battery**	**UT structure**	**3 %**	**4 %**	**5 %**
✗	✓	Energy	- 59 255	- 59 543	- 59 791
		Time	- 23 433	- 27 232	- 30 489
		Power	- 53 771	- 54 597	- 55 305
✓	✗	Energy	6 060	2 526	- 503
		Time	9 770	5 873	2 532
		Power	5 256	1 800	- 1 161
✓	✓	Energy	- 51 618	- 55 596	- 59 004
		Time	- 17 839	- 25 126	- 31 372
		Power	- 45 295	- 49 892	- 53 832

As can be seen, there is not a single positive NPV for the configurations with a battery installed. This shows that the initial investment cost for the battery is too high compared to the savings provided by the SDP-model under the proposed UT structures. The closest to breaking even is under the time based UT, achieving -19 and -38 % with and without a PV-system, respectively.

Notice that almost all the NPV values for the configurations with just the PV-system are positive, only turning slightly negative for a 5 % discount rate combined with the energy- and power based UT structures. It should also be mentioned that these calculations have not taken into account subsidies for private household PV-systems, which are offered in Norway. With these, the investment cost could decrease to 13 500 NOK [29], yielding positive NPV values as high as[9] 167, 195 and 155 % for the energy-, time- and power based UT structures, respectively.

[8] For each year that passes, the initial cash flow is reduced with another percentage. After 10 years, the cash flow is reduced with 10 %, and so forth.

[9] Calculated using a 3 % discount rate.

The NPV calculations presented in this section are based on one important assumption, and that is that the savings calculated for 2016 are representative for the coming 25 years. This is probably not a good assumption, which thus renders the NPV results not very representative for what could have been achieved in real life. A more thorough evaluation would need to consider future developments of load patterns and spot price, and especially with regards to the variance. This is not in the scope of this thesis, and is thus left for future studies to look further into.

5.12 Potential Sources of Error

This thesis has proposed a MATLAB program for stochastic optimisation of a household battery system, tested on data from central Norway under different cases. There are several potential sources of error in the presented methodology and results, and here are four that were identified as most important:

1. **Oversimplifications**. The modelling in this thesis has made several simplifications which may render the results unrealistic. For instance that that the battery efficiency is constant and thus independent of SOC and P_{bat}. This is not the case, as is shown in [30], which probably would have effected the results.

2. **Calculation of PV-production.** PV-production has in this thesis been calculated using equation (3.5) with GHI values of solar irradiance from a measurement station that is not located at the exact same location as the households in questions. This means that local shadowing at the location of the households has not been taken into account, and neither has the potential angling of the PV-panels. These simplifications renders the exact PV-production used in this thesis somewhat fictional, but the results still illustrate the potential for savings under different UT structures with and without a battery, which was the objective of this thesis.

3. **Neglecting power flow constraints.** Actual power flow has not been computed in this thesis, which means that some of the calculated solutions from the SDP- or DP-solution might violate voltage or current constraints in the local grid of the households.

4. **Assuming a wait-and-see strategy is feasible.** The developed SDP-model assumes that it is possible for the operator to wait and see what the load turns out to be in each time step before making a decision on how to operate the battery. This could be realistic for loads with slow characteristics, but maybe not for a private household.

5. **Programming errors.** All calculations in this thesis has been performed using self-developed code. The code for SDP- and DP-algorithms are complex pieces of code, even tough they are made of simple functions. This means that there is a high risk for mistakes being made in the implementation. The DP-model which was used in this thesis is, as previously mentioned, a somewhat improved version of what was developed in [1], and it has been tested many times. The SDP-model builds upon this DP-model, so the fact that the DP-model has been through several iterations strengthens the belief that it indeed do work as intended. However, there is always a possibility for human error in such complex implementations.

Chapter 6

Conclusions and Further Work

In this thesis, a stochastic dynamic programming (SDP) optimisation model for a private household battery has been developed and implemented in MATLAB. This is a stochastic multi-period sequential optimisation problem which fulfils the criteria of a Markov Decision Process, where the household load is the stochastic variable and the spot price and utility tariff are assumed known 24 hours in advance, which is the case in the Norwegian power market. The stochastic load was modelled as a Markov Chain, and transition probabilities were calculated using historic load patterns from up to three previous years. Load was considered as inflexible and had to be met in all time steps. A fixed horizon SDP-model was used to optimise 24 hours at a time, running from midnight to midnight, with a hard constraint of an empty battery at the end of each day. Three different utility tariff (UT) structures were developed in order to evaluate their effect on the results: Energy based, time based and power based, as described in section 3.2.2. The results from the stochastic optimisation were bench-marked against the global optimal solution, found by a self-developed deterministic dynamic programming (DP) algorithm.

Conclusions

The model used historic load data for 2013-2015 in order to predict load patterns in 2016, and was run for three different households in central Norway, using actual load, spot price and solar irradiance data. The conclusions are as follows:

- The SDP-model performed close to the optimal global optimal solution in the case of a energy- or time based UT, achieving over 75-94 % of the same savings[1]. These high results are due to the SDP-model foreseeing all price arbitrage possibilities in the optimisation horizon, and not being heavilty penalised for inaccuracies in the load prediction. However, the results are poor under the power based UT, where the SDP-model achieves only 25-44 % of the savings of the DP. This was explained by mainly two factors: First, the stochastic model is not precise enough in order to

[1] No PV-system installed.

foresee sudden load increases and does not charge the battery accordingly in order to cover such ”unexpected” high loads. This causes it to be heavily penalised, which again leads to a poor result compared to the global optimum. Second, the SDP-model was set to have an empty battery at the end of each day. This causes it to discharge in the evening even if it would have been beneficial to store the energy to the following day.

- The savings of installing a PV-system is almost unaffected by the UT, achieving 12.2-13.2 % relative savings. Further, the introduction of a PV-system increases the value of the battery in the case of the energy- or power based UT, but decreases it under the time based UT. The increased value is caused by the ability for the battery to charge for free, without the need to pay both the spot price and UT for this energy. The decreased value under the time based UT is explained by the fact that the PV-system overtaking the cost savings that the battery provided when the PV-system was not present, thus reducing the added value of the battery.

- Varying the available historic load data set gave minor variations in the performance of the SDP, and so did the variation in the level of load discretisation. Increasing the latter from the standard setting of 20 to 60 gave less than 1 % higher savings, but eight times higher run-time of the daily SDP-optimisation. The introduction of a learning stochastic model also gave higher savings, and should thus be included in further developments of the SDP-model.

- The model was investigated for different battery parameters, showing that an increased capacity was far more effective than an increased maximum allowable power.

- The battery is being used substantially more in the case of the time based UT, by almost three times that of the energy based utility tariff. The results from the power based UT scores in between the two. These results shown that a battery under the time based UT would degrade the fastest, which would effect the lifetime of the battery.

- The introduction of a battery in a household would increase the maximum load over the year in the case of the energy- or time based UT, but potentially lower it under the power based UT. A battery could also lead to negative power flow, which should be taken into account by utility companies and others concerned in the development of new UT structures.

- The investment cost of the Tesla Powerwall (as used in this thesis) is still too high in order to produce positive NPV values under any UT structure for load 3, using 3-5 % discount rate, 1 % yearly degradation, 25 years lifetime and an investment cost of 62 200 NOK. On the other hand, installing just a PV-system with a 30 000 NOK investment cost yields positive NPV values for all UT structures when discount rates are below 5 %.

Further Work

As the model used in this thesis is self-developed, there are endless possibilities to extend and further improve it. The following interesting possibilities for further work was identified during the work with this thesis:

- Remove the hard constraint of the battery having to be empty at the start and end of each day in the SDP-model, which was especially a disadvantage under the power based UT. This could be improved in one of the following ways:

  1. Implement a value function for energy stored in the final time step of the day.
  2. Develop the model to perform two 24-hour optimisations a day, one at midnight and another one at noon.

  Number two would demand the stochastic model to be improved as well, as the transition probabilities between days are not calculated with in the present version.

- Develop a threshold- and time-based operation policy for the battery for the power- and time based UT and investigate how close to the DP these could perform.

- Further improve the stochastic model by utilising the temperature-load-correlation found in section 3.4.2. This could help the stochastic model to foresee higher loads due to lower temperatures, and thus work as a ”set-point” for load, even if it was deemed too imprecise for the hour-to-hour optimisation.

- Test the stochastic model with a learning ”rolling horizon” of the N most recent days, which could take the seasonal variations of load into account.

- By using SDP, the state space of the decision variables (the SOC of the battery) had to be discretised. Accordingly, this limits the solution space to the same discrete states. This is avoided if a stochastic dual dynamic programming (SDDP) approach is chosen. Hence, one idea for further work could be to develop the SDP-algorithm to a SDDP-algorithm, as described in [7].

- This thesis has considered PV-production as a deterministic variable, which obviously is not a good assumption. Further work could thus develop a stochastic model for PV-production, and incorporate this into the SDP-algorithm.

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Appendix

A DP-implementation in MATLAB

```
function [totalcost, transitioncost, path, p_bat, SOC] = ...
    optimumCharge(p_load,p_pv,spotprice,utilityprice,structure,p_bat_max...
    ,eff_ch,eff_di,C_bat,SOC_max,SOC_min,delta_soc,firstday)
% Assuming p_load, p_pv and prices has the same dimensions

%% Key parameters
N_soc = 1/delta_soc;
N_days = size(p_load,1);
N_periods = size(p_load,2);
N_tot_periods = N_days*N_periods;

soc_max_change_ch = floor(p_bat_max*eff_ch*N_soc/(C_bat));
soc_max_change_di = floor(p_bat_max*N_soc/(C_bat*eff_di));

%% Initialising arrays
totalcost = inf(N_soc,N_tot_periods);
transitioncost = inf(N_soc,N_tot_periods,N_soc);
path = zeros(N_soc,N_tot_periods);
p_bat = zeros(N_soc,N_tot_periods);
SOC = inf(N_soc,N_tot_periods);

totalcost(1,N_tot_periods) = 0;              % Final state being SOC = 0.
transitioncost(1,N_tot_periods,:) = 0;       % Final cost = 0

soc_max_tp1 = 1;     % First row of SOC-array corresponding to soc_min

day = N_days;
daynum = firstday;
weeks = N_days/7;

if mod(weeks,1) == 0
    % Do nothing, as lastday = firstday
elseif weeks < 1
    daynum = firstday + N_days - 1;
else
    weeks = floor(weeks);
    daynum = firstday + N_days - weeks*7 - 1;
end

if daynum > 7
    daynum = daynum - 7;
end

%% Calculations
for t = (N_tot_periods-1):-1:1

    %% Decide possible SOC
```

```
    soc_max_t = min(N_soc,soc_max_tp1+soc_max_change_di);

    %% Set day and hour with respect to t
    if mod(t,24) == 0
        hour = 24;
        day = day - 1;
        daynum = daynum - 1;
        if daynum == 0
            daynum = 7;
        end
    else
        hour = mod(t,24);
    end

    %% Calculate costs
    for soc_t = 1:soc_max_t

        soc_max_t_tp1 = min(soc_max_tp1,soc_t+soc_max_change_ch);
        soc_min_t_tp1 = max(1,soc_t-soc_max_change_di);

        for soc_tp1 = soc_min_t_tp1:soc_max_t_tp1

            SOC_change = (soc_tp1-soc_t)/N_soc;
            p_bat_temp = SOC_change*C_bat;

            if p_bat_temp > 0
                p_bat_temp = p_bat_temp/eff_ch;  % eff_ch < 1
            else
                p_bat_temp = p_bat_temp*eff_di;  % eff_di < 1
            end

            p_grid = p_load(day,hour) + p_bat_temp - p_pv(day,hour);

            if p_grid > 0
                switch structure
                    case 1
                        % Only spotprice
                        transitioncost(soc_t,t,soc_tp1) = ...
                            spotprice(day,hour)*p_grid;

                    case 2
                        % Flat rate
                        transitioncost(soc_t,t,soc_tp1) = ...
                            (spotprice(day,hour) + utilityprice)*p_grid;

                    case 3
                        % Timely

                        transitioncost(soc_t,t,soc_tp1) = ...
                            (spotprice(day,hour) + ...
                            utilityprice(daynum,hour))*p_grid;

                    case 4
                        % P-based
                        utilityprice_temp = utilityprice*p_grid;
                        transitioncost(soc_t,t,soc_tp1) = ...
                            (spotprice(day,hour) + ...
```

```
                              utilityprice_temp)*p_grid;

                end % switch
            else
                transitioncost(soc_t,t,soc_tp1) = ...
                    spotprice(day,hour)*p_grid;
            end % if p_grid > 0

        end % soc_tp1

        %% Calculate SOC-value and find shortest path
        change_cost = transitioncost(soc_t,t,:);
        change_cost = squeeze(change_cost);

        totalcost_temp = totalcost(:,t+1) + change_cost;

        [min_cost, min_place] = min(totalcost_temp);

        totalcost(soc_t, t) = min_cost;

        path(soc_t,t) = min_place;

    end % soc_t

    %% Preperation for new iteration
    soc_max_tp1 = soc_max_t;
end % t

%% Determining p_bat

for i = 1:N_soc
    soc_from = i;
    for j = 1:N_tot_periods

        soc_to = path(soc_from,j);

        p_bat_temp = (soc_to-soc_from)*C_bat/N_soc;

        if p_bat_temp > 0
            p_bat(i,j) = p_bat_temp/eff_ch;  % eff_ch < 0
        else
            p_bat(i,j) = p_bat_temp*eff_di;  % eff_di < 0
        end

        if soc_from == 1
            SOC(i,j) = 0;
        else
            SOC(i,j) = soc_from*(SOC_max-SOC_min)/N_soc;
        end

        soc_from = soc_to;
    end
end

p_bat(:,N_tot_periods) = 0;

end
```

B SDP-implementation in MATLAB

```matlab
function path = fixedHorizonSDP(load_disc,trans_matrix,p_pv,spotprice,...
    utilityprice,structure,p_bat_max,eff_ch,eff_di,C_bat,SOC_max,...
    SOC_min,N_soc,SOC_initial,SOC_final,daynum)

% Assuming p_pv and prices has the same dimensions, here 1xN_horizon

% load_disc contains the discrete load states, and has the dimensions
% (N_states, N_periods).

% trans_matrix has the dimensions (N_states, N_periods, N_states), and
% gives the probability of the transition from the 1st dim state to the
% 3rd dim state given that you are in the period in the 2nd dim.

%% Setting key parameters
N_periods = size(load_disc,2);  % Number of time steps in the horizon.
N_disc = size(load_disc,1);     % Number of discrete loads.

% Maximum changes in SOC state
soc_max_change_ch = floor(p_bat_max*eff_ch*N_soc/(C_bat));
soc_max_change_di = floor(p_bat_max*N_soc/(C_bat*eff_di));

if SOC_initial == SOC_min
    N_soc_initial = 1;
else
    % Yields which discrete step the initial SOC belongs to
    N_soc_initial = floor(N_soc*(SOC_initial-SOC_min)/(SOC_max-SOC_min));
end

if SOC_final == SOC_min
    N_soc_final = 1;
else
    % Yields which discrete step the final SOC belongs to
    N_soc_final = floor(N_soc*(SOC_final-SOC_min)/(SOC_max-SOC_min));
end

%% Initialising arrays
cost_to_go = inf(N_soc,N_periods,N_disc);
transitioncost = inf(N_soc,N_disc);
path = zeros(N_soc,N_periods,N_disc);

% Calculating all legal SOC states. SOC_legal is a binary matrix, 1 if the
% given SOC is legal for the given period. Else 0.
SOC_legal = socLegalCalc(N_soc,N_periods,N_soc_initial,N_soc_final,...
    soc_max_change_ch,soc_max_change_di);

% Fixed final SOC = 0
cost_to_go(:,N_periods,:) = 0;
transitioncost(N_soc_final,:) = 0;

%% Calculations
for t = (N_periods-1):-1:1 % Period iterated recursively

    %% Calculate costs
```

```
    for soc_t = 1:N_soc % Iterates over all legal soc's in period t

        if SOC_legal(soc_t,t) == 1

            soc_max_tp1 = min(N_soc,soc_t+soc_max_change_ch);
            soc_min_tp1 = max(1,soc_t-soc_max_change_di);

            for load_t = 1:N_disc
                % Iterates over all possible load scenarios in period t.

                for soc_tp1 = soc_min_tp1:soc_max_tp1
                    % Iterates over all legal SOC's in period t+1 which can
                    % be reached from soc_t.

                    if SOC_legal(soc_tp1,t+1) == 1

                        SOC_change = (soc_tp1-soc_t)/N_soc;
                        p_bat = SOC_change*C_bat;

                        if p_bat > 0
                            p_bat = p_bat/eff_ch;  % eff_ch < 1
                        else
                            p_bat = p_bat*eff_di;  % eff_di < 1
                        end

                        p_grid = load_disc(load_t,t) + p_bat - p_pv(t);

                        if p_grid > 0
                            switch structure
                                case 1
                                    % Only spotprice
                                    transitioncost(soc_tp1,load_t) = ...
                                        spotprice(t)*p_grid;

                                case 2
                                    % Flat rate
                                    transitioncost(soc_tp1,load_t) = ...
                                        (spotprice(t) + ...
                                        utilityprice)*p_grid;

                                case 3
                                    % Timely

                                    transitioncost(soc_tp1,load_t) = ...
                                        (spotprice(t) + ...
                                        utilityprice(daynum,t))*p_grid;

                                case 4
                                    % P-based

                                    utilityprice_temp = utilityprice*...
                                        p_grid;
                                    transitioncost(soc_tp1,load_t) = ...
                                        (spotprice(t) + ...
                                        utilityprice_temp)*p_grid;

                            end % switch structure
```

```
                        else
                            transitioncost(soc_tp1,load_t) = ...
                                spotprice(t)*p_grid;
                        end % if p_grid > 0

                        for load_tp1 = 1:N_disc
                            % Add expected cost_to_go using the transition
                            % probabilities in trans_matrix
                            if transitioncost(soc_tp1,load_t) ~= inf
                                transitioncost(soc_tp1,load_t) = ...
                                    transitioncost(soc_tp1,load_t) + ...
                                    cost_to_go(soc_tp1,t+1,load_tp1)*...
                                    trans_matrix(load_t,t,load_tp1);
                            end
                        end

                    end % if SOC_legal(t+1,soc_tp1) == 1
                end % for soc_tp1

                [min_cost, min_soc] = min(transitioncost(:,load_t));

                cost_to_go(soc_t,t,load_t) = min_cost;

                path(soc_t,t,load_t) = min_soc;

            end % for load_t = 1:N_states

        end % SOC_legal(t,soc_t) == 1

        transitioncost = inf(N_soc,N_disc);    % Reset of transitioncosts

    end % for soc_t

end % for t

end % function
```

C Results

C.1 Consumer Costs

Table C.1: Total billing costs in NOK. Load 3 is for all of 2016. Load 1 and 2 uses the first 275 days of 2016.

Settings			Total consumer cost			
Load	**PV**	**Utility Structure**	**Basecase**	**PV**	**SDP**	**DP**
1	✗	Energy	6 251	-	6 144	6 108
		Time	6 341	-	5 147	4 970
		Power	4 101	-	4 053	3 910
	✓	Energy	6 251	4 454	4 237	4 067
		Time	6 341	4 374	3 526	3 258
		Power	4 101	2 802	2 692	2 541
2	✗	Energy	22 381	-	22 197	22 181
		Time	22 741	-	20 503	20 351
		Power	30 148	-	29 249	28 089
	✓	Energy	22 381	20 145	19 954	19 923
		Time	22 741	20 249	18 236	18 062
		Power	30 148	27 009	25 796	24 674
3	✗	Energy	18 871	-	18 683	18 640
		Time	19 384	-	16 897	16 636
		Power	18 576	-	18 008	16 983
	✓	Energy	18 871	16 558	16 268	16 199
		Time	19 384	16 832	14 613	14 309
		Power	18 576	16 315	15 567	14 560

Table C.2: Relative savings compared to basecase. Load 3 is for all of 2016. Load 1 and 2 uses the first 275 days of 2016. SDP vs. DP is equal to the relative savings of the two models divided by each other, indicating the performance of the SDP-model.

Settings			**Relative savings**			
Load	**PV**	**Utility Structure**	**PV**	**SDP**	**DP**	**SDP vs. DP**
1	✗	Energy		1.71%	2.29%	74.65%
		Time		18.82%	21.62%	87.07%
		Power		1.16%	4.65%	24.89%
	✓	Energy	28.75%	32.22%	34.94%	92.22%
		Time	31.02%	44.39%	48.61%	91.33%
		Power	31.67%	34.35%	38.03%	90.31%
2	✗	Energy		0.82%	0.89%	92.02%
		Time		9.84%	10.51%	93.61%
		Power		2.98%	6.83%	43.66%
	✓	Energy	9.99%	10.84%	10.98%	98.74%
		Time	10.96%	19.81%	20.58%	96.28%
		Power	10.41%	14.44%	18.16%	79.52%
3	✗	Energy		1.00%	1.22%	81.75%
		Time		12.83%	14.17%	90.52%
		Power		3.06%	8.58%	35.69%
	✓	Energy	12.26%	13.79%	14.16%	97.40%
		Time	13.16%	24.61%	26.18%	94.00%
		Power	12.17%	16.20%	21.62%	74.92%

Table C.3: Relative savings using SDP for different historic datasets. All values from load 3 and no PV for all of 2016.

		Historic load			
	Utility Structure	**2013**	**2014**	**2015**	**2013-2015**
SDP	Energy	1.07 %	1.05 %	1.00 %	1.00 %
	Time	12.96 %	12.95 %	12.81 %	12.83 %
	Power	3.00 %	2.75 %	3.15 %	3.06 %
SDP vs. DP	Energy	87.48 %	85.97 %	82.03 %	81.75 %
	Time	91.43 %	91.35 %	90.37 %	90.52 %
	Power	34.98 %	32.05 %	36.70 %	35.69 %

Table C.4: Relative savings using SDP for different levels of load discretisation. All values from load 3 and no PV for all of 2016 unter the time based UT. T_{disc} is the run-time of the stochastic model and $T_{SDP,opt}$ is the average run-time of the daily SDP-optimisation, both in seconds.

N_{disc}	T_{disc}	$T_{SDP,opt}$	**SDP**	**SDP vs. DP**
10	0.093	0.5	12.61 %	88.95 %
20	0.100	1.5	12.83 %	90.52 %
30	0.108	3.2	13.02 %	91.88 %
40	0.110	5.5	13.04 %	92.02 %
50	0.133	8.4	13.07 %	92.22 %
60	0.137	12.0	13.07 %	92.23 %

Table C.5: Relative savings using SDP and DP for different battery parameters. All values for standard settings without PV under the time based utility tariff.

	C_{bat}	$P_{bat,max}$	**SDP**	**DP**	**SDP vs. DP**
Load 1	13.5 kWh	7 kW	18.82%	21.62%	87.07%
	27.0 kWh	7 kW	22.31%	27.27%	81.81%
	13.5 kWh	14 kW	18.68%	21.71%	86.05%
	27.0 kWh	14 kW	22.73%	28.45%	79.89%
Load 2	13.5 kWh	7 kW	9.84%	10.51%	93.61%
	27.0 kWh	7 kW	14.49%	15.39%	94.14%
	13.5 kWh	14 kW	9.87%	10.58%	93.26%
	27.0 kWh	14 kW	15.15%	16.68%	90.87%
Load 3	13.5 kWh	7 kW	12.83%	14.17%	90.52%
	27.0 kWh	7 kW	17.50%	19.41%	90.12%
	13.5 kWh	14 kW	12.90%	14.25%	90.53%
	27.0 kWh	14 kW	17.62%	20.47%	86.11%

C.2 Power flow

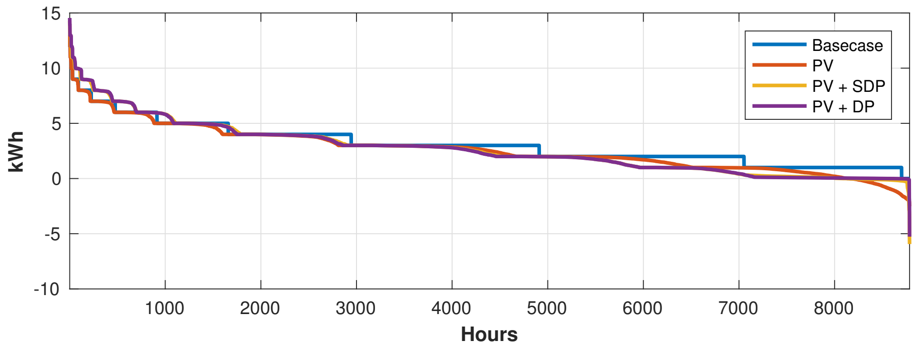

Figure C.1: Duration curves of P_{grid} for 2016 for load 3 with PV under the energy based UT.

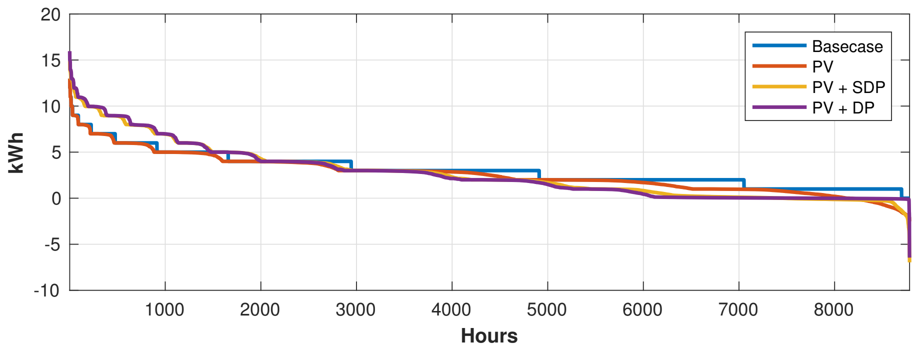

Figure C.2: Duration curves of P_{grid} for 2016 for load 3 with PV under the time based UT.

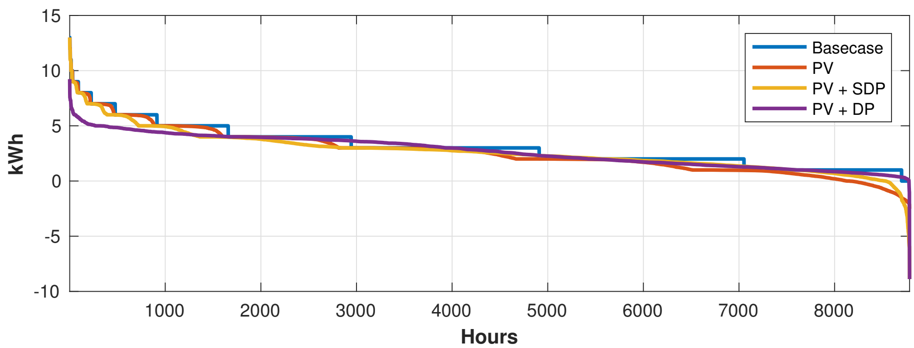

Figure C.3: Duration curves of P_{grid} for 2016 for load 3 with PV under the power based UT.

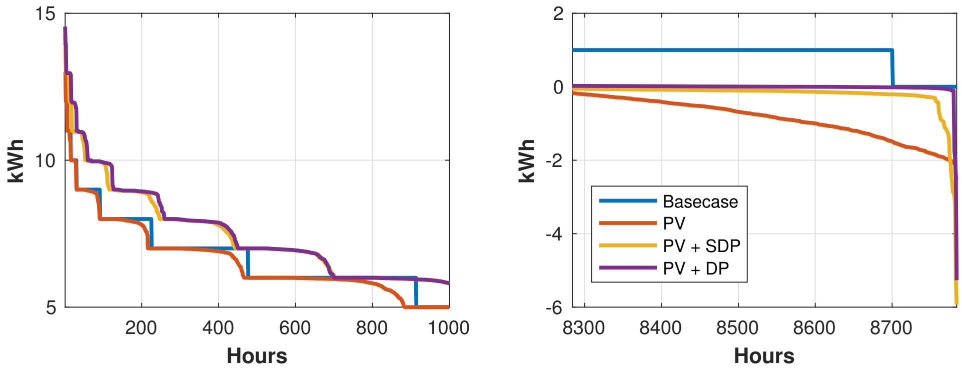

Figure C.4: Duration curves of P_{grid} for 2016 for load 3 with PV under the energy based UT, zoomed in for maximum 1 000 hours and minimum 500 hours.

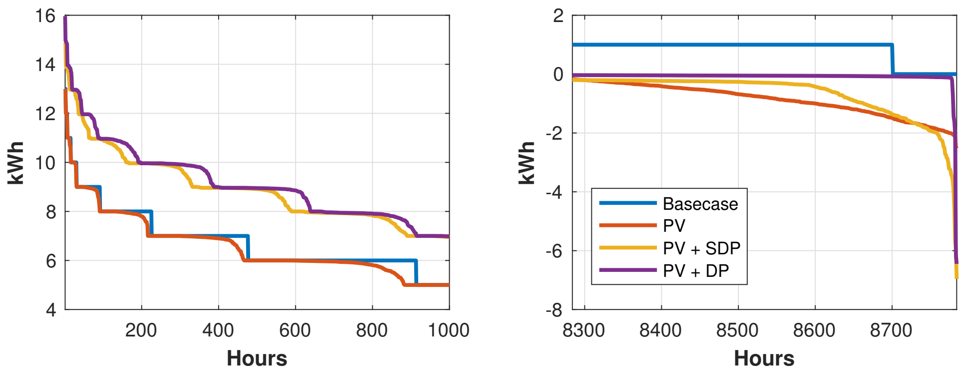

Figure C.5: Duration curves of P_{grid} for 2016 for load 3 with PV under the time based UT, zoomed in for maximum 1 000 hours and minimum 500 hours.

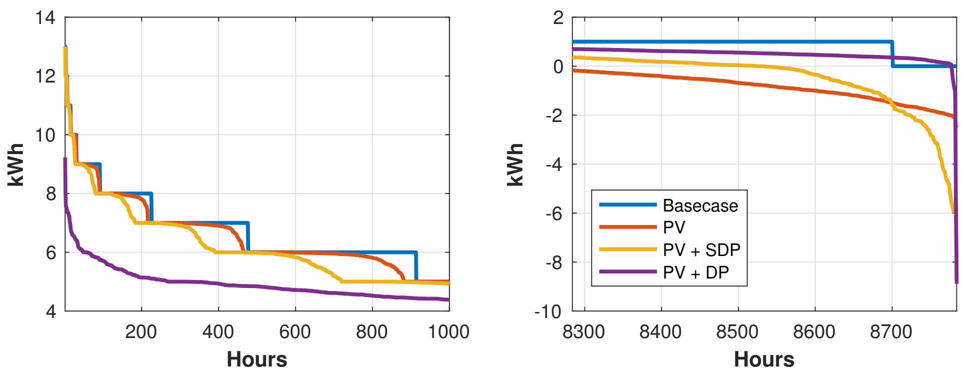

Figure C.6: Duration curves of P_{grid} for 2016 for load 3 with PV under the power based UT, zoomed in for maximum 1 000 hours and minimum 500 hours.

Table C.6: Maximum and minimum hourly load for 2016 for different configurations without PV. All numbers in kWh.

		Max			Min		
Load	**UT**	**Basecase**	**SDP**	**DP**	**Basecase**	**SDP**	**DP**
1	**Energy**	6.3	11.6	11.9	0.1	-6.5	-6.5
	Time	6.3	11.6	11.9	0.1	-6.5	-5.2
	Power	6.3	6.3	6.1	0.1	-6.5	-6.5
2	**Energy**	15.6	17.2	19.6	0.6	-2.1	-1.3
	Time	15.6	19.0	19.6	0.6	-1.1	-0.1
	Power	15.6	15.6	11.7	0.6	-4.8	-0.1
3	**Energy**	13.0	14.7	15.0	0.0	-2.4	-1.7
	Time	13.0	15.0	16.0	0.0	-2.2	-1.6
	Power	13.0	13.0	9.2	0.0	-5.8	0.2

C.3 Net present value

Table C.7: Net present value divided by the investment cost of the different configurations for load 3 under standard settings. All battery configurations are with the SDP-solution.

			Discount rate		
PV	**Battery**	**UT structure**	**3 %**	**4 %**	**5 %**
✗	✓	Energy	-95%	-96%	-96%
		Time	-38%	-44%	-49%
		Power	-86%	-88%	-89%
✓	✗	Energy	20%	8%	-2%
		Time	33%	20%	8%
		Power	18%	6%	-4%
✓	✓	Energy	-56%	-60%	-64%
		Time	-19%	-27%	-34%
		Power	-49%	-54%	-58%

D Normalised load plots sorted by day

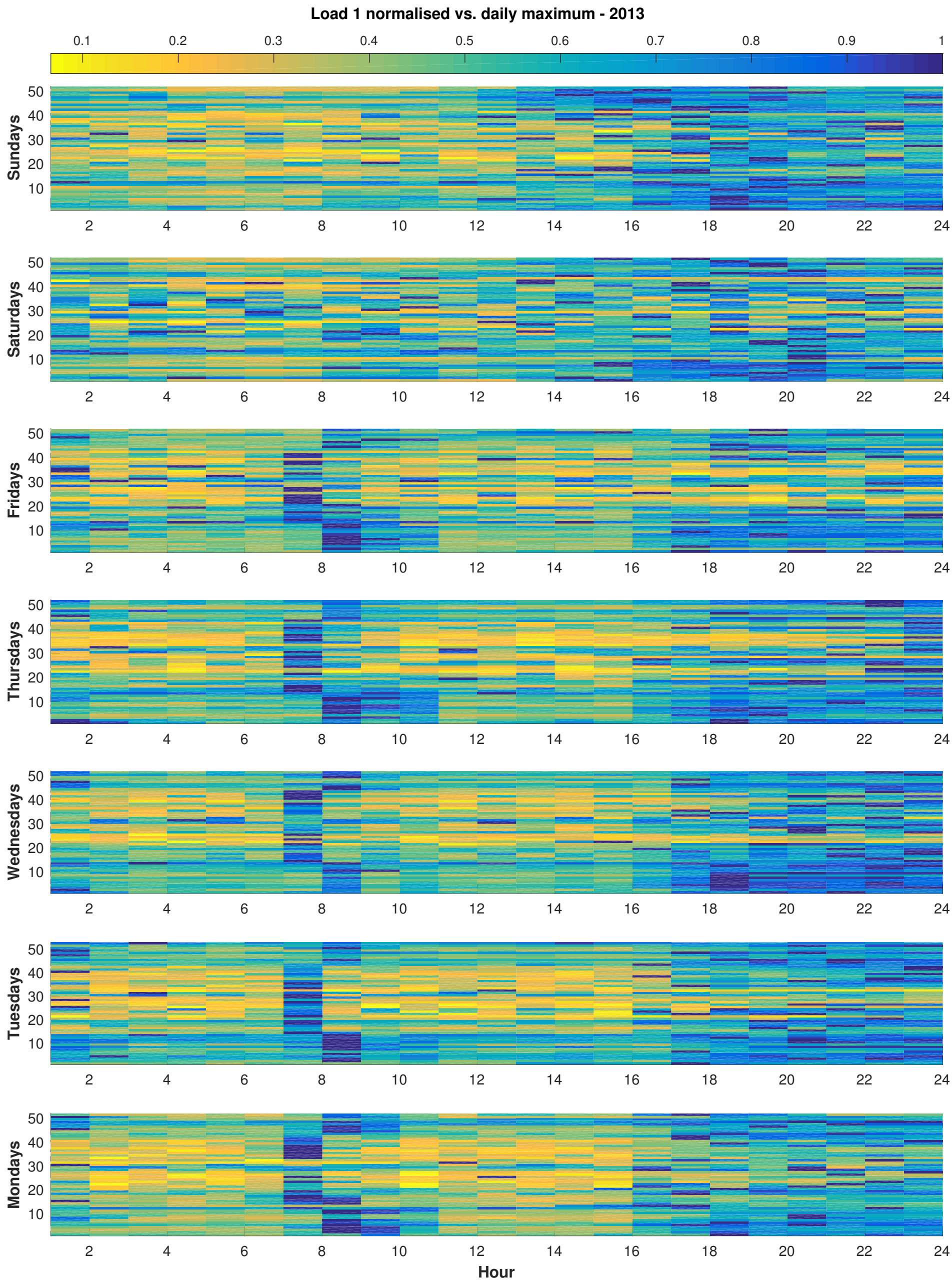

Figure D.1: Normalised load for load 1 in 2013, sorted by days.

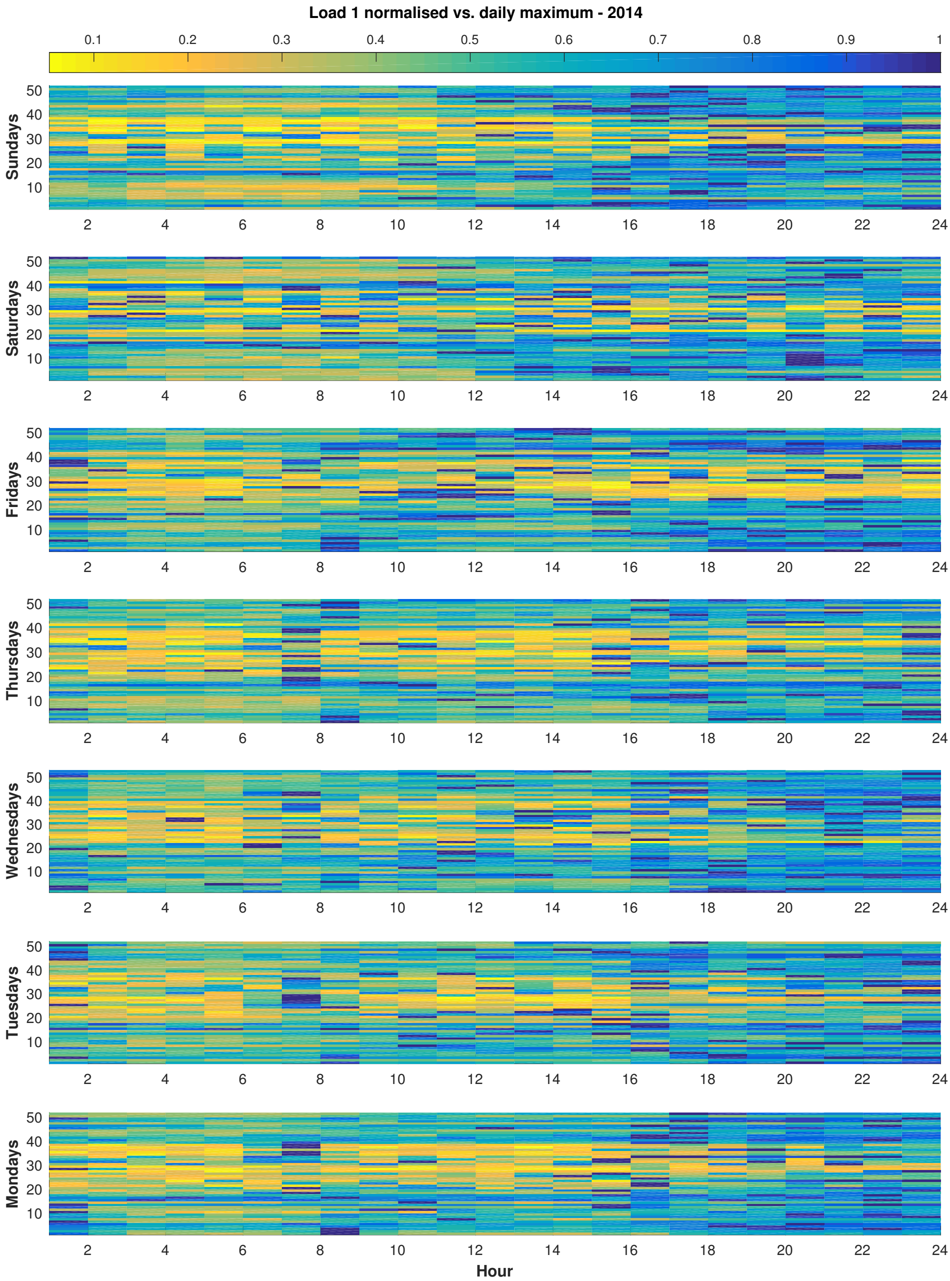

Figure D.2: Normalised load for load 1 in 2014, sorted by days.

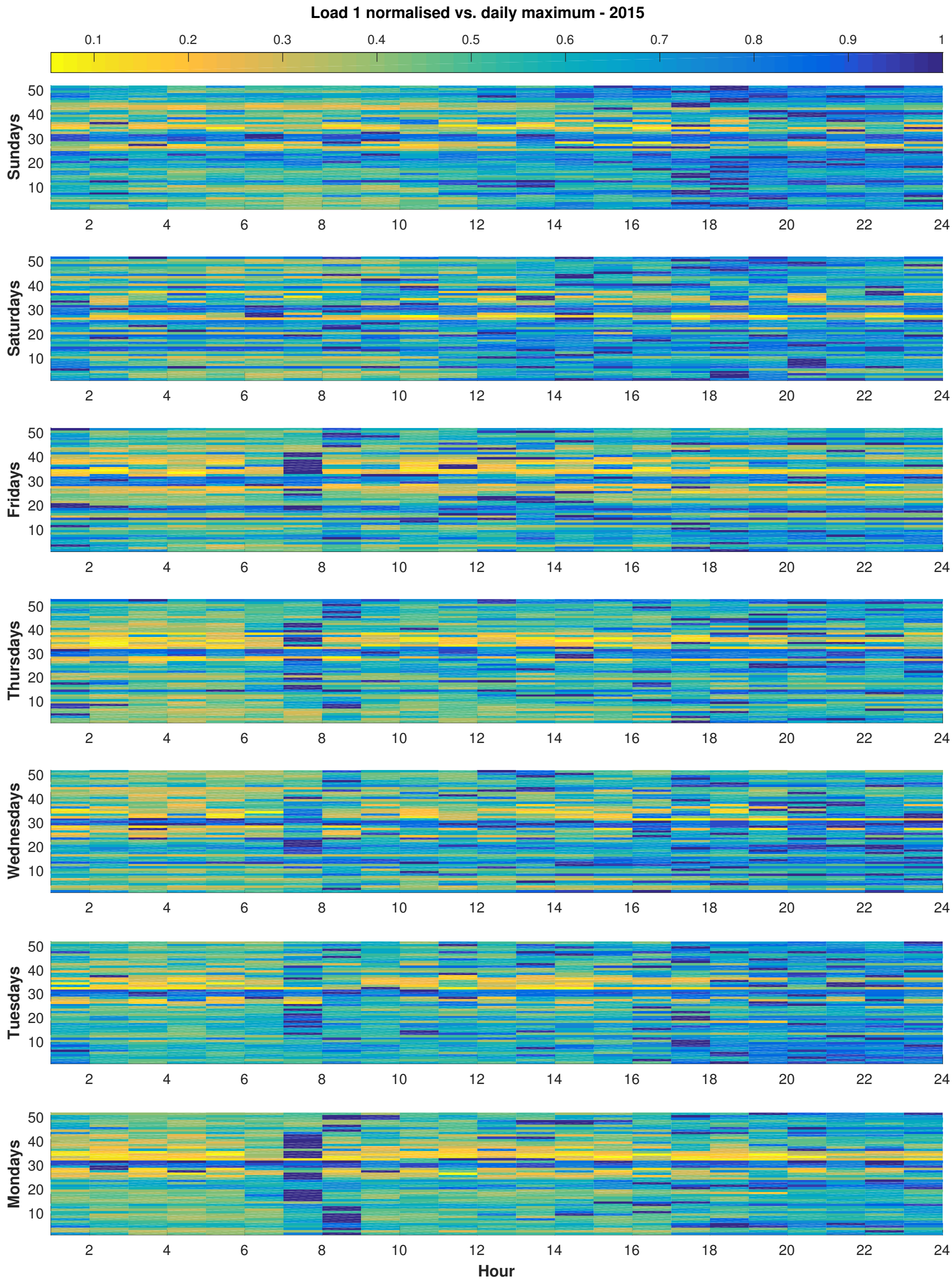

Figure D.3: Normalised load for load 1 in 2015, sorted by days.

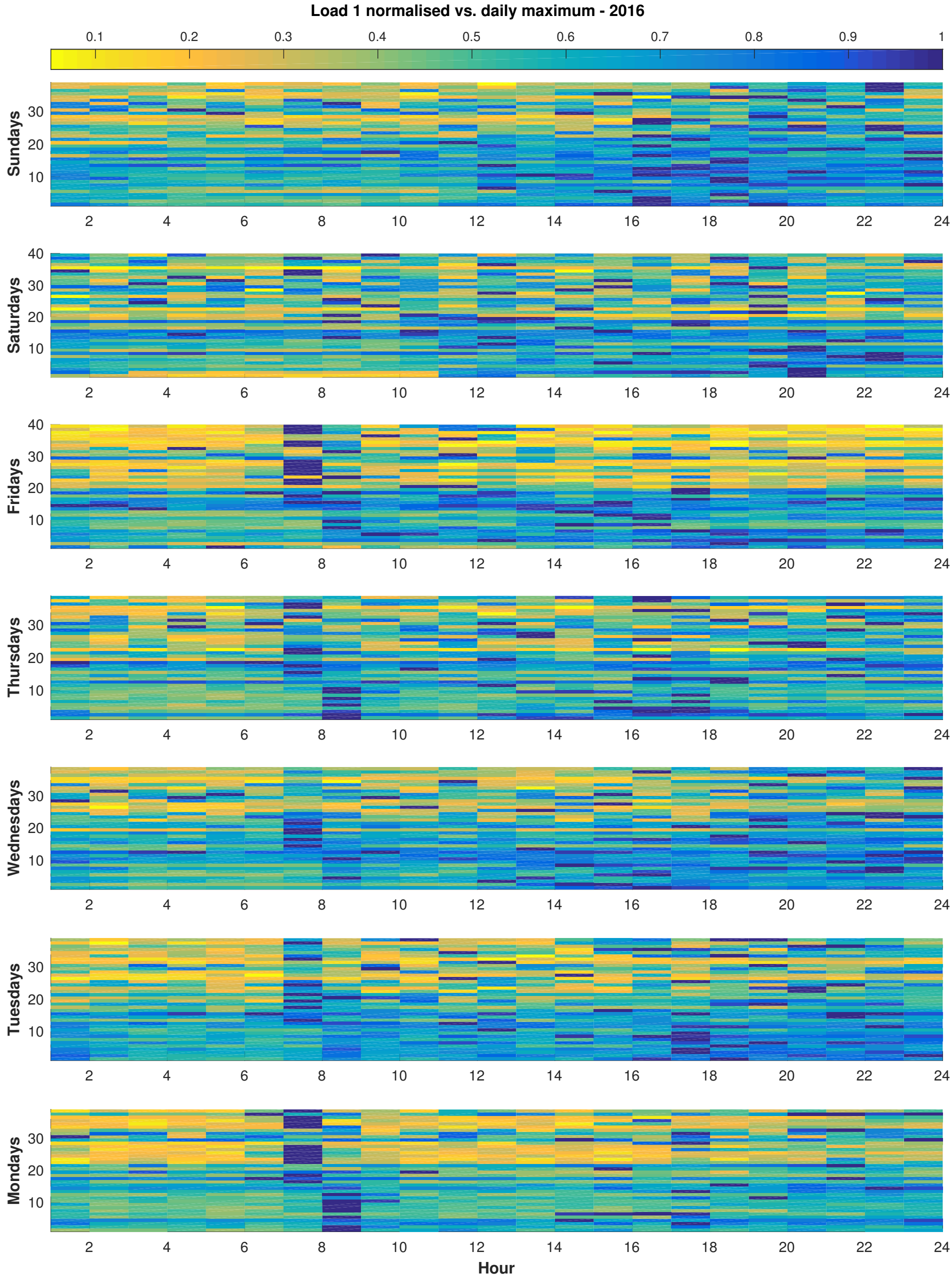

Figure D.4: Normalised load for load 1 in 2016, sorted by days.

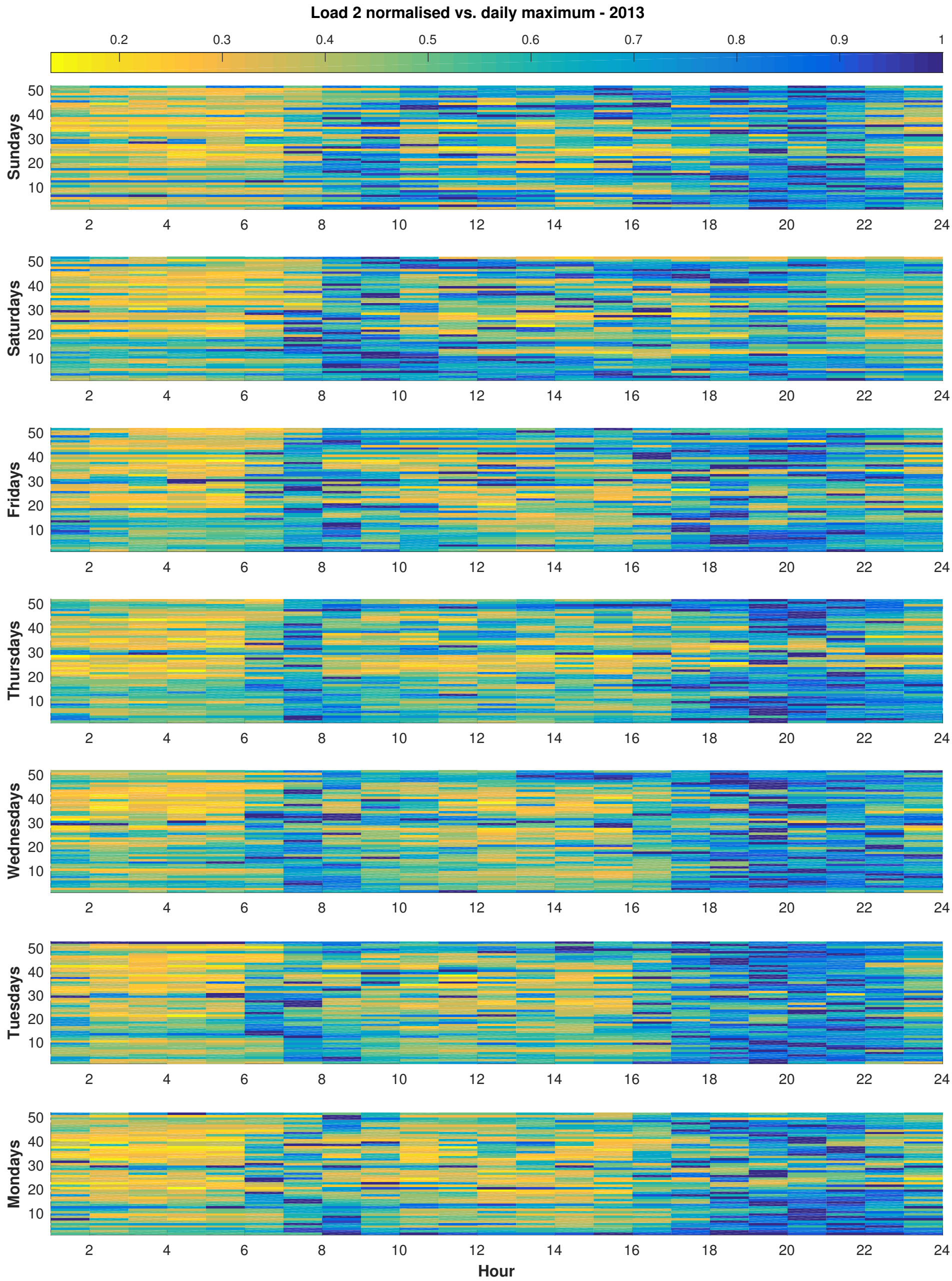

Figure D.5: Normalised load for load 2 in 2013, sorted by days.

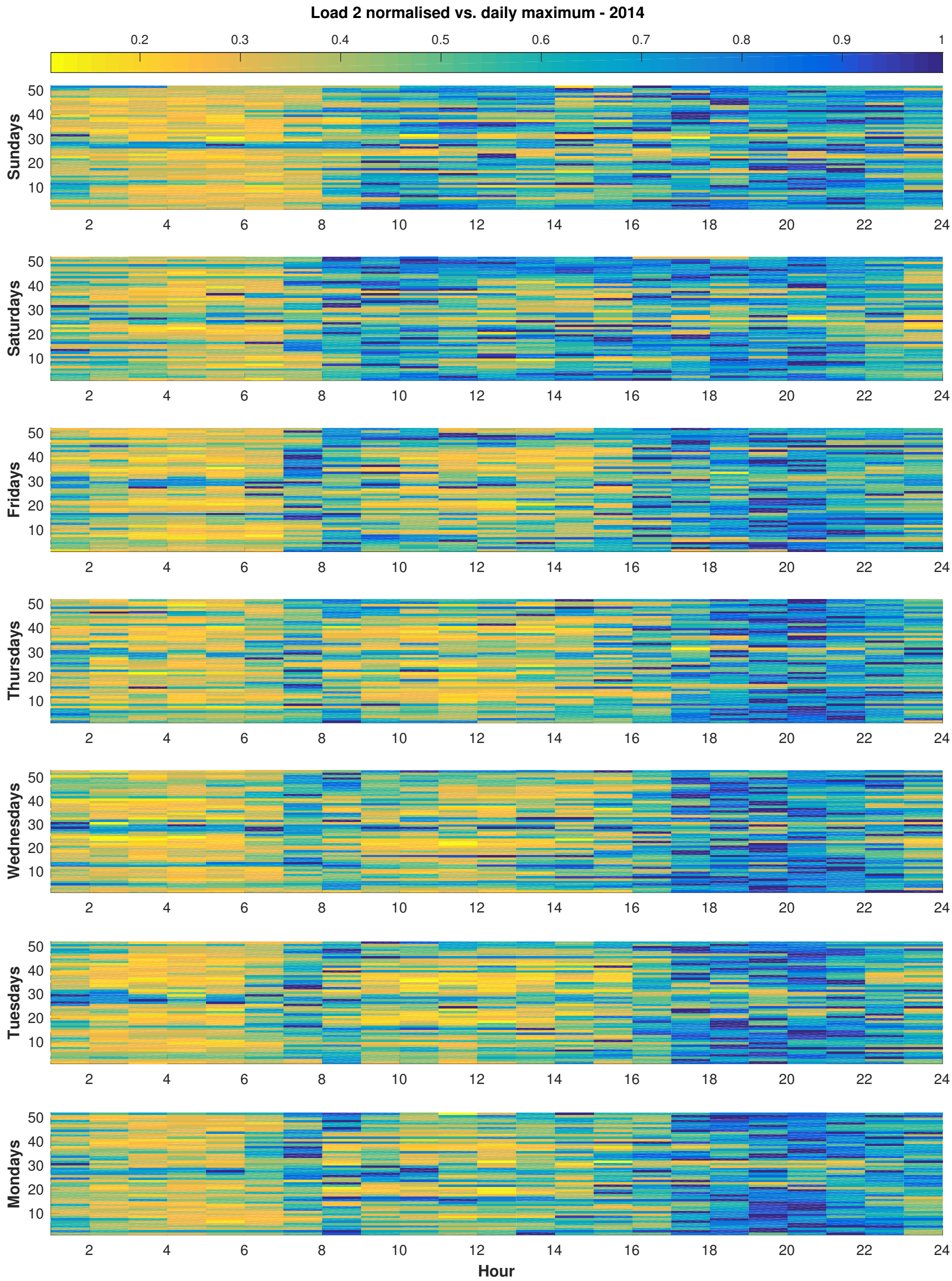

Figure D.6: Normalised load for load 2 in 2014, sorted by days.

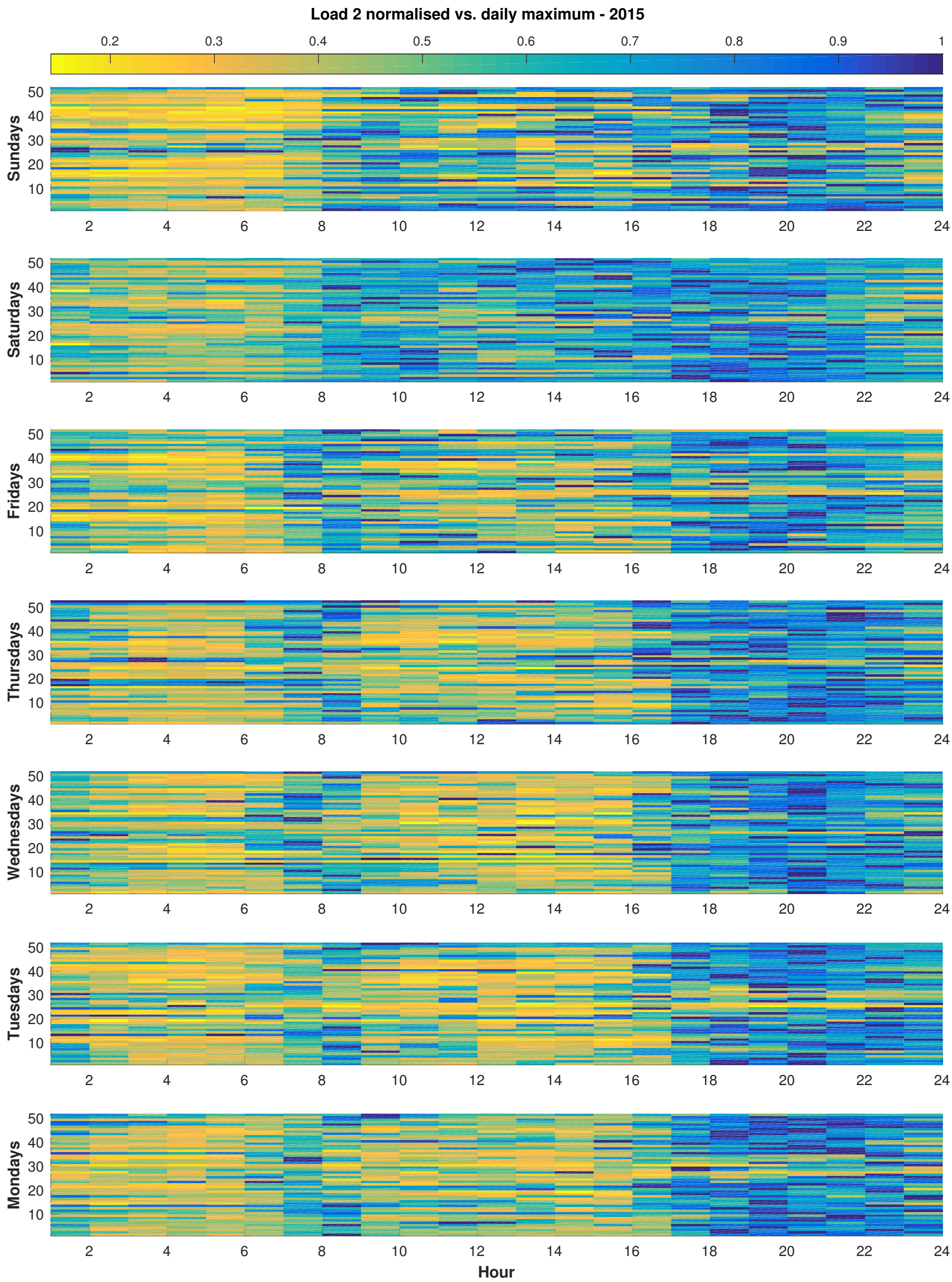

Figure D.7: Normalised load for load 2 in 2015, sorted by days.

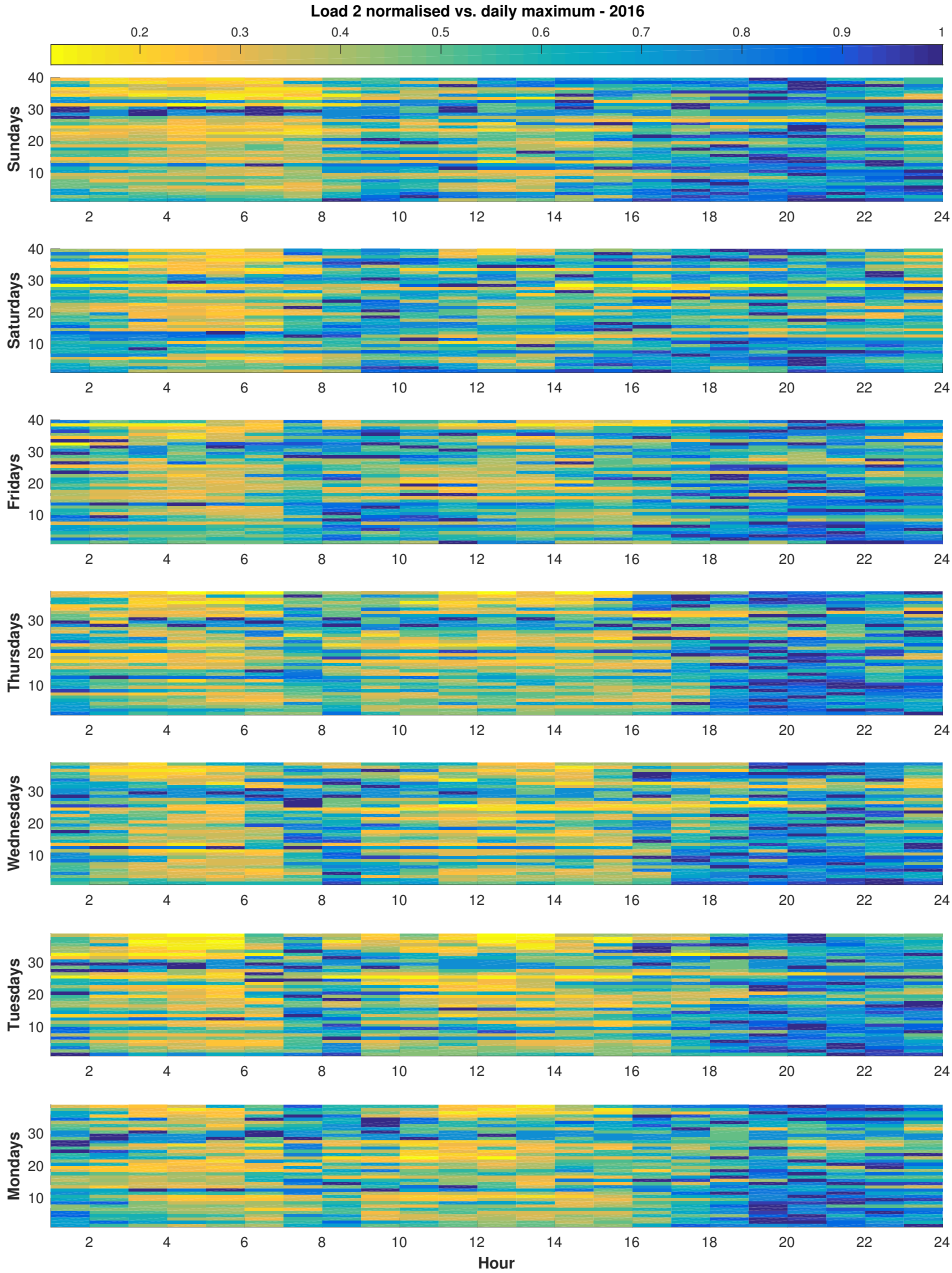

Figure D.8: Normalised load for load 2 in 2016, sorted by days.

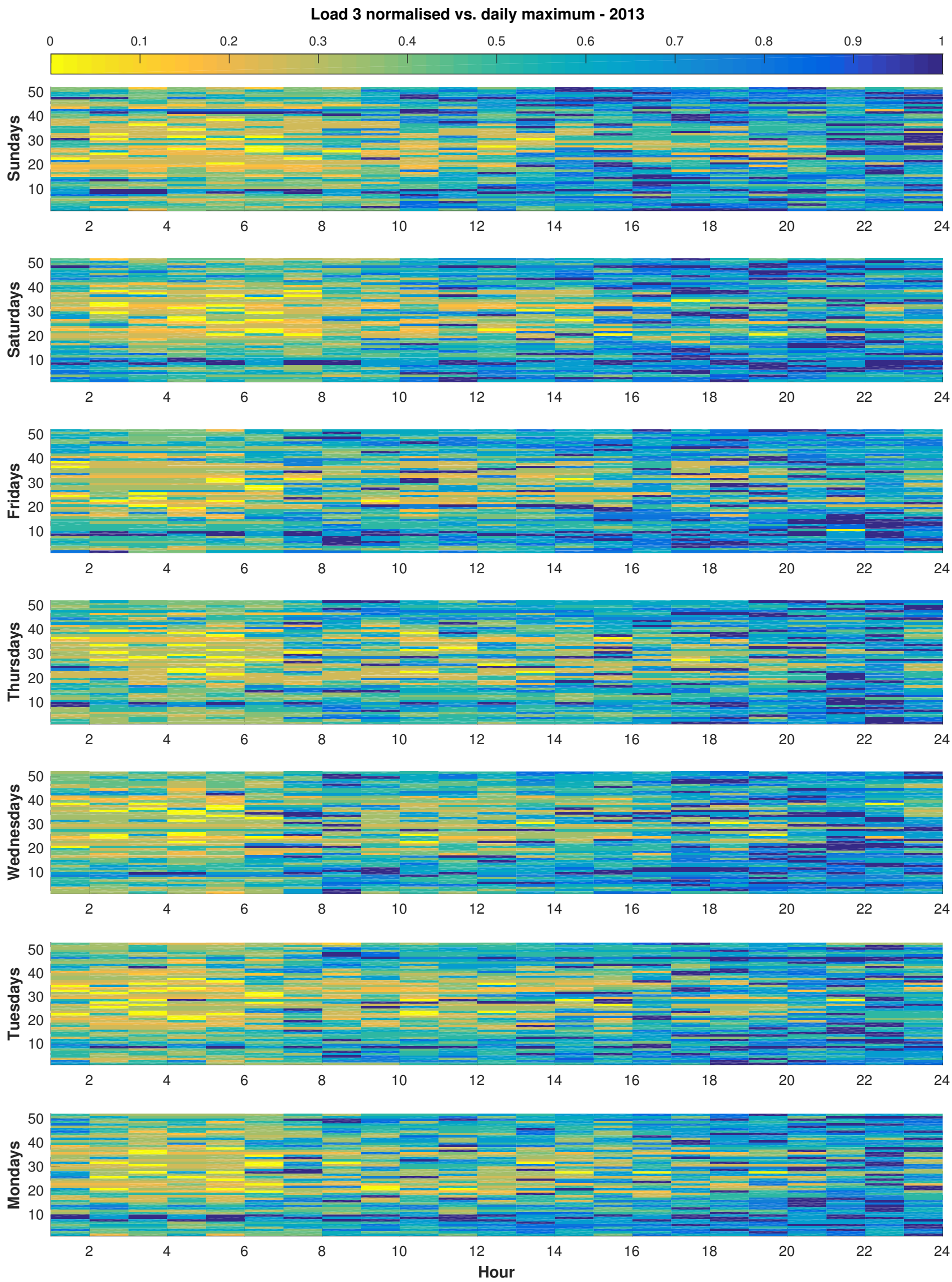

Figure D.9: Normalised load for load 3 in 2013, sorted by days.

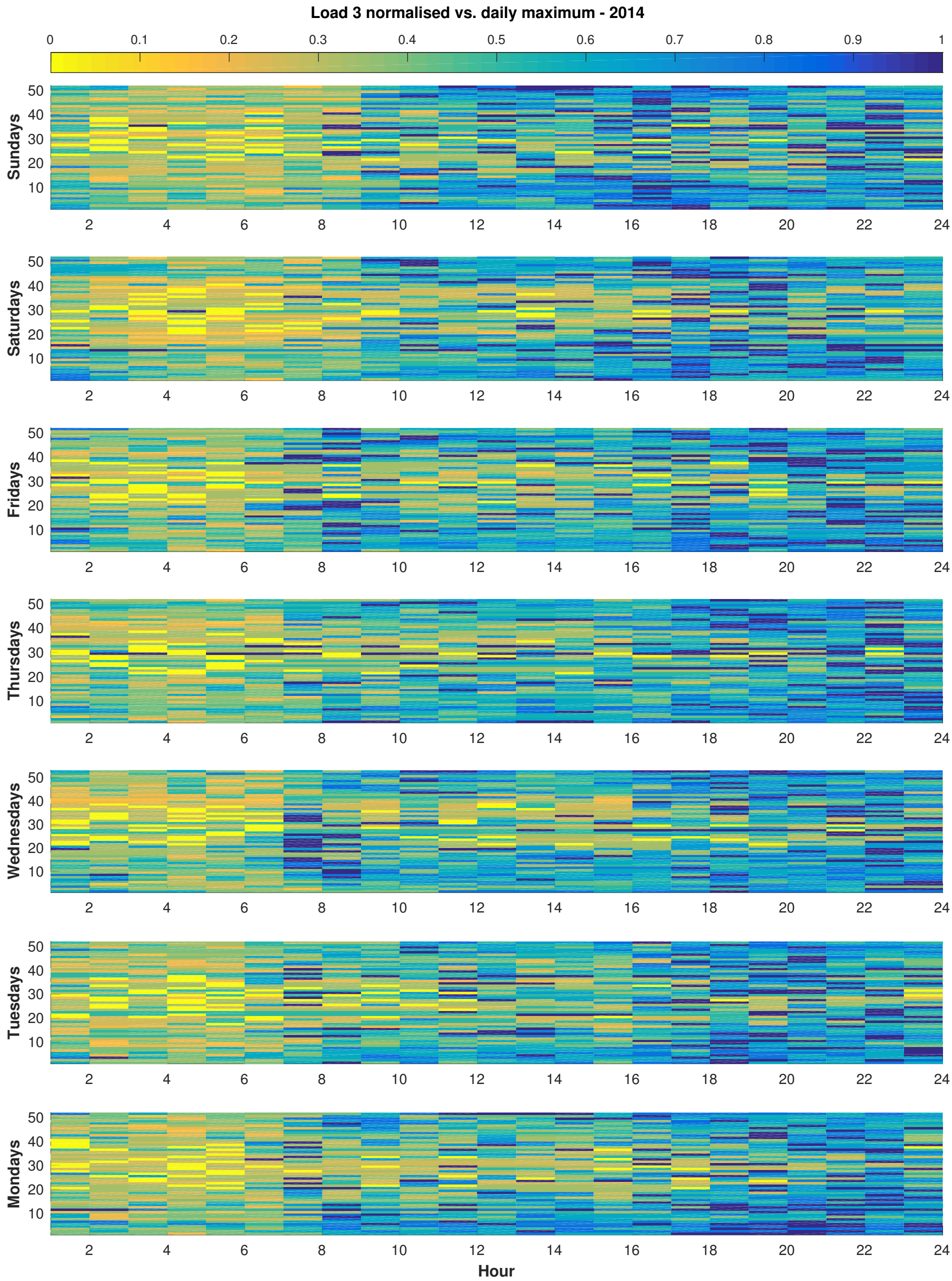

Figure D.10: Normalised load for load 3 in 2014, sorted by days.

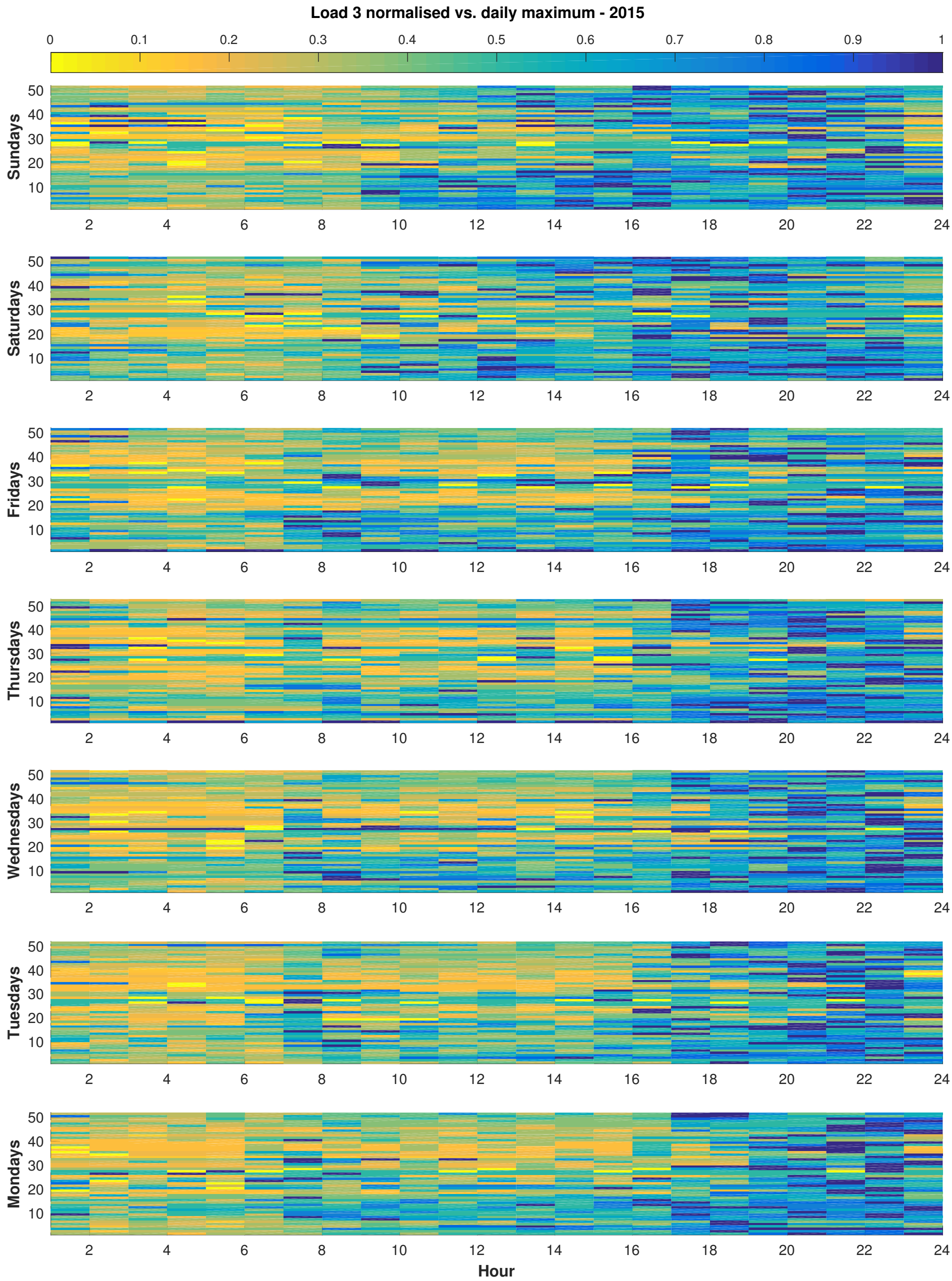

Figure D.11: Normalised load for load 3 in 2015, sorted by days.

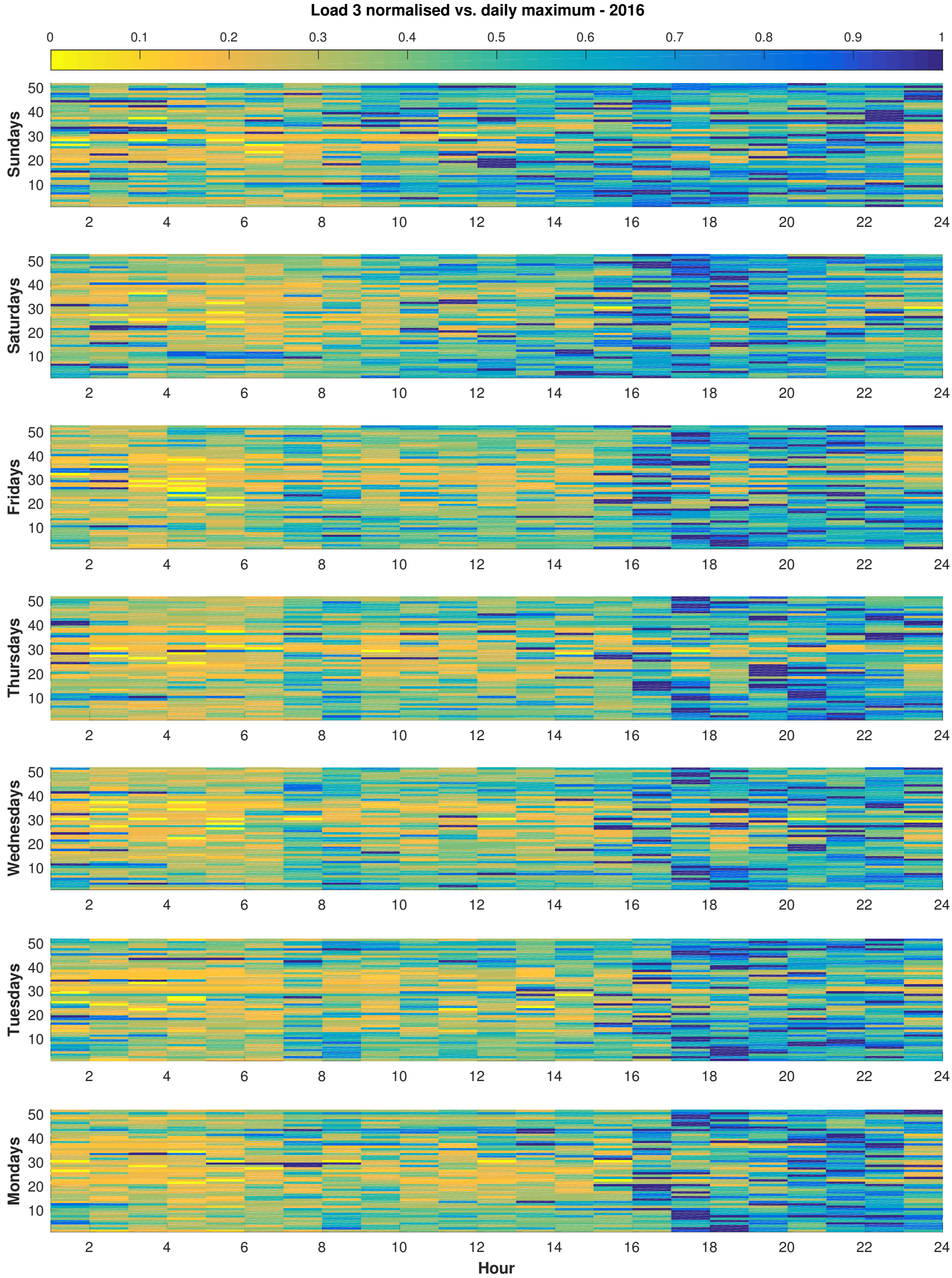

Figure D.12: Normalised load for load 3 in 2016, sorted by days.

E Net present value calculation

Net present value (NPV) is calculated using the following equation:

$$NPV = -CF_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+i)^t}. \tag{6.1}$$

where

CF_t	is the cash flow at the end of year t,
CF_0	is the investment cost at the end of year 0,
i	is the discount rate, and
N	is the lifetime of the investment.

The cash flow in this thesis will be the savings provided by the different configurations of a PV-system and battery, assuming that this comes at the end of the year.