The Use of Snow Level Options for Ski Resort Establishments in Norway

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Preface

This master's thesis is an end to a two-year master in Financial Economics. In the process of deciding what to write about, we both agreed that the course Financial Derivatives is the most interesting course we have completed during this two years master program. We were both eager to write our thesis within this area. Climate change is something that interest us, and the idea of weather derivatives came after watching the documentary "Before the flood". We saw how unseasonal rain can destroy an indian farmer's crops and started to discuss how other sectors are exposed to weather. We wanted to check out if some kind of derivative on weather existed and the possible use of this in different sectors. We would like to thank Denis Becker, for saying yes to be our supervisor and making it possible for us to write about something we find interesting. Any errors or omissions are the authors own.

The master's thesis is written as a collaboration between Maren Brenni Gulbrandsen and Hilde Aas.

Abstract

Nearly all businesses are affected by the weather in one way or another. For many, the weather is considered one of the major uncontrollable risk factors (Sharma & Vashishtha, 2007). Especially, winter tourism is one of the sectors most sensitive to weather conditions (Damm & Greuell, 2016). A ski resort's revenues are closely related to the cumulative snow level. A year with low snow levels can lead to a significant reduction in the firms revenues. We will look at the possibility to hedge this downside risk and thereby reduce cash flow volatility, by using weather derivatives as a short term risk management tool.

The weather derivative market is an incomplete market and traditional pricing methods are not applicable. Most of the existing literature on pricing methodology are with respect to temperature derivatives. Few have been conducted on snow level options.

The main objective of this study is to implement some of the few pricing methods that exists on snow level derivatives. These are the indifference pricing method, the method proposed by Alaton, Djehiche, and Stillberger (2002) (ADS) based on Black-Scholes framework, a pricing method using historical densities and pricing by using generalized edgeworth series expansion. All of the above pricing methods are applied on a snow level put option constructed for Vassfjellet Skiheiser AS. The indifference prices for both seller and the buyer Vassfjellet are calculated, and the main assumption in the model saying that the buyer's indifference price must exceed the seller's indifference price, holds for all strike levels. For the most relevant strike levels, the prices generated by the different methods are surprisingly similar. This indicates that these pricing methods may be a good starting point for further application and modification.

By looking at historical data we observe a strong correlation between cumulative snow level data and Vassfjellet's financial data. Further, we check if the volatility in Vassfjellet's operating income the past years could have been reduced by investing in this snow level put option. The findings show that by buying a snow level put option each year from 2009-2015, the volatility in average operating income would decrease. Hence, if wishing to smooth income and hedge downside risk, the purchase of a snow level put option may be a good risk management tool. Our approach can be used on any individual ski resort who is exposed to snow level risk.

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1 Introduction

According to Alexandridis and Zapranis (2013) almost 70 % of all the companies in the US are in some way affected by the weather. Weather is a major uncontrollable risk factor (Sharma & Vashishtha, 2007) and hence an important question is how to hedge this risk (Cao & Wei, 2004). A weather derivative is a financial instrument that can be used to hedge against risk related to the weather. As standard derivatives, they are usually structured as swaps, futures and put and call options (Alaton et al., 2002). Different industries face different weather risks and may therefore want to hedge against different weather variables. It is possible to construct weather derivatives to cover about any weather variable (Alexandridis & Zapranis, 2013) and the most common weather variables are temperature, precipitation, snowfall and wind speed (Alaton et al., 2002). Since these variables cannot be stored or traded, different weather indices are used as underlying variables. The index can be based on for example cumulative rainfall or days with temperature above a certain value during a predetermined period (Alexandridis & Zapranis, 2013).

Issuance of weather derivatives originated first among US energy companies in 1997 as a result of the deregulation in US energy and power industry. This led to increased competition among providers, as well as increased uncertainty about demand. It was quickly understood that weather conditions were the most important factor for uncertainty in revenues (Cao, Wei, & Li, 2003). Weather derivatives were developed and used as an effective tool for hedging against volume risk (Alexandridis & Zapranis, 2013). The weather derivative market grew rapidly the first years with Chicago Mercantile Exchange (CME) offering weather options and futures. At the end of 2009, CME traded weather products written on weather indices in 46 cities around the world (Alexandridis & Zapranis, 2013). Even though 95% of the contracts were written on temperature, CME also offered option and future contracts based on snowfall for different locations in the US (*CME Group*, 2009). The development of weather derivatives did not happen as quickly as many hoped and in mid 2015 the snowfall derivatives were taken off the exchange due to little or no trading (*Fortune*, 2017). Hamisultane (2008) points out that the main explanation for the weather derivative market not to expand is the difficulty in pricing the weather derivatives. For many it is therefore looked at as a risky product and sellers tend to fix a high premium because of the difficulty in evaluating a contract. With regard to the information brought by the market, over the counter (OTC) activity does not allow the standardization of the pricing models because it does not aid price transparency. However, in theory, weather uncertainty and the risk that follows with it should provide an active market for financial derivatives that can insure businesses and others against unexpected weather (*Fortune*, 2017). Despite that snowfall derivatives were taken of official exchanges, they can still be traded OTC.

The aim of this master thesis is to apply some of the existing pricing methods on a snow level put option constructed for a norwegian ski resort. If reasonable results are obtained, we will further examine whether this put option can be used as an efficient risk management tool. In the next section we will go through earlier literature and first look at the advantages of using weather derivatives. Thereafter, studies on different sectors exposed to weather risk will be discussed. Especially studies on the use of snow level derivatives for ski resorts are presented as this is our main objective. Section 3 presents the cumulative snow level data and financial data for Vassfjellet Skiheiser AS which is needed in further application. To obtain the necessary inputs for pricing the put option, the relationship between cumulative snow level data and financial data is examined by a regression analysis. In section 4 the characteristics and obstacles with pricing weather derivatives will be discussed. Because of these obstacles, few studies have been made on pricing precipitation derivatives and especially snow level derivatives. We will briefly go through some of the traditional pricing methods, before examining the four pricing methods on snow level derivatives which we are going to use in our application. In section 5 a snow level put option for Vassfjellet Skiheiser AS is constructed. Then, an attempt to price this by using the indifference pricing method, ADS, Historical densities and Edgeworth densities is done. Since all the methods gave reasonable prices, we further examine whether Vassfjellet Skiheiser AS could have reduced its variance in operating income for the historical years 2009-2015 by investing in this option.

2 Earlier literature

2.1 Why weather derivatives?

For businesses exposed to weather risk, weather derivatives can be a useful tool for several reasons (Brockett, Goldens, & Wen, 2009). Weather derivatives possess advantageous differences compared to traditional insurances used for weather risk management (Sharma & Vashishtha, 2007). Payout from weather derivatives depends on actual weather during a specified period, regardless of the actual damages/loss in revenues. Thus, administration costs are lower than with traditional insurances since there is no need to submit a claim, nor having inspections to demonstrate actual loss (Bossley, 1999).

Moral hazard and adverse selection are problems associated with traditional insurance products. Moral hazard is existent when the insured can affect the payout after the insurance contract is signed. Adverse selection is a situation where the buyer of the insurance has more information regarding the risk involved than the insurer, and therefore is able to better determine a correct risk premium (Sharma & Vashishtha, 2007). These problems are not existent with weather derivatives because the insured outcome is based on an easily observable weather parameter (Turvey, 2001).

(Leggio, 2007, p. 247) also lists several reasons for companies to use weather derivatives. This includes smoothing revenues, cover excess cost, reimburse lost opportunity costs, stimulate sales and diversification. For small businesses with cash flow volatility related to weather, the possibility to smooth revenues can help them obtain financing and also to keep the firm out of financial distress. In the case of unfavourable weather conditions, a weather derivative allows this by transferring the weather risk to a third party (Leggio, 2007). Sharma and Vashishtha (2007) report different studies where weather derivatives have been proven useful in reducing risk by stabilizing the profits. The studies described include one on farmers in Mexico, US dairy production and agriculture districts of Romania. In the latter, the researcher found that the variability in returns was substantially reduced for farmers cultivating under the cover of weather derivatives. Tang and Jang (2011) argue that reducing cash flow volatility is important for various reasons. First, the risk of bankruptcy decreases. Second, it reduces financial distress costs, external-financing costs, underinvestment costs and thus increases firm value. Their own study on publicly listed ski resorts in America demonstrates this by showing that financial hedging, such as the use of weather derivatives, reduce short run snowfall risk for a ski resort (Tang & Jang, 2011, p. 309). In the study of Perez-Gonzalez and Yun (2013), the use of weather derivatives leads to a 12% increase in firm value on average, statistically significant at the 1% level. Jewson and Brix (2005) suggest that a reduction in year-to-year profits will increase a company's share price, and also reduce the interest rate of the company's loans. Unfavourable weather conditions may lead to additional costs for a company. Airports experience higher operational costs on frost days. Winters with above-average snowfall leads to municipal governments having higher snow removal costs (Alexandridis & Zapranis, 2013). Furthermore, businesses exposed to weather risk may experience lost opportunity costs as a result of the unfavourable weather. Extreme wind conditions can cause large costs to an airline company because of cancellations of flights. Similarly, construction companies might not complete a project on a scheduled completion date because of adverse weather conditions (Alexandridis & Zapranis, 2013). Weather can affect businesses cash flow directly by reducing volume of sales. For a ski resort, bad winters with low levels of snow usually result in fewer visitors and lower sale of ski passes. By for example entering a short position in a forward contract, where the underlying index is snow level, the owner of the ski resort receives a predetermined sum that will make up for lost sale when the snow level is low. In this way he is able to simulate sale with the use of weather derivatives (Tang & Jang, 2011). There is a low correlation between weather derivatives and other financial markets (Brockett, Wang, & Yang, 2006). Hence, for investors that wish to diversify their portfolio, weather derivatives can be an attractive new class of assets (Brockett, Wang, & Yang, 2005). Also, including weather derivatives based on uncorrelated weather indices in a portfolio can potentially lower the volatility of the portfolio (Alexandridis & Zapranis, 2013).

2.2 Weather exposure in different industries

From the beginning of the weather derivative market in late 90's and until 2005, the marked was clearly dominated by the energy industry. Since then, the market has grown and the application of weather derivatives has been discussed with respect to many different industries (Alexandridis & Zapranis, 2013). Different industries will experience different risks related to the weather. Thus, they will use different hedging strategies. Some firms will hedge against any deviation from the average, while others will only hedge against extreme weather conditions like hurricanes, droughts and floods (Leggio, 2007).

The Energy sector is highly influenced by the weather. The demand for heating and cooling is strongly related to the changes in weather conditions (Perez-Gonzalez & Yun, 2013). The risk in the electricity industry is characterized as a volume risk as both the demand and the supply for electricity depend on weather related factors. On the supply side, the amount of energy produced will depend on the precipitation for hydro based power generation or the wind speed for windmill power generation. On the demand side, the quantity of electricity demanded for heating or cooling depends on the temperature outside (Sharma & Vashishtha, 2007). If there is an unusual warm winter or a very cold summer, both the price on electricity and the amount of electricity consumed per hour will be reduced. This leads to lower revenues for companies selling electricity.

Most of the research on weather derivatives are done with respect to the energy industry. Eydeland and Wolyniec (2003) present different tools used for hedging in the energy industry. They show that power load is highly affected by the weather, especially by the temperature. They discuss weather derivatives as a possible hedging strategy and conclude that some risk can be reduced by using weather derivatives. Perez-Gonzalez and Yun (2013) use data from different energy companies and compare the value of each firm with and without hedging with weather derivatives. The results show that the use of weather derivatives lead to a positive and significant effect on the value of the firm. They also show that using weather derivatives will increase the firm's debt capacity, the earnings will be smoother and they can invest more. Sharma and Vashishtha (2007) compare weather derivatives to traditional electricity derivatives. They argue that for Indian power sector, weather derivatives are a better tool for risk hedging and are the most effective and sustainable risk-hedging instrument.

Agriculture is also highly linked to weather risk as crops require some certain conditions to grow well. Temperature, precipitation and wind can all affect both the quality and the quantity of a crop (Geyser, 2004). Farmers can get significant crop losses due to extreme temperatures or rainfall. To hedge against some of the weather risk, the farmer can buy a weather derivative based on the amount of precipitation. If the rainfall exceeds a predefined limit, the farmer will receive a certain amount of money. Thus, he is protected against production risk, not price risk (Turvey, 2001). Turvey (2001) examines weather derivatives in Ontario. He shows that weather derivatives can have a significant effect on agricultural production risks and concludes that weather derivatives can be used as a form of agricultural insurance. Xu, Odening, and Musshoff (2007) analyze the effectiveness of precipitation derivatives on wheat production risk for the Brandenburg region of Germany.

Many developing countries have an economy that depends strongly on the agriculture. A bad harvest can affect these countries in a significant way. Historical data show that there has been an increase in extreme weather conditions and that these volatilities in the weather more severely influence farmers in undeveloped areas (Kermiche & Vuillermet, 2016). These features also normally affect most households and companies in the same region and at the same time. In addition, for households and companies in these countries, the financial resources are often limited and thus they can have trouble dealing with weather shocks. Hence, the need for good hedging tools is particularly high in these countries (Hess, Richter, & Stoppa, 2002). There exists several studies on the use of weather derivatives in the agriculture sector in developing countries. Hess et al. (2002) show that it is a huge potential for weather derivatives in this sector in developing countries. Varangis, Skees, and Barnett (2003) discuss potential applications of weather insurance in these countries. Kermiche and Vuillermet (2016) examine weather derivatives as a possible independent long term plan to help African countries moderate their risk associated by the weather. Hess et al. (2002) focus on a case study for rainfall derivatives to protect the cereal producers in Morocco from the financial influence of a season with drought. According to Geyser (2004), farmers in South Africa can use rainfall derivatives during the peak season for maize growing and thereby reduce the yield risk substantially.

In addition to the industries above, some users of weather derivatives are department stores who sell winter coats (Cao & Wei, 2004), wine production (Cyr, Eyler, & Visser, 2013), theme parks (Geyser, 2004), entertainment, restaurants and bars (Cao et al., 2003), insurance firms and banks (Tang & Jang, 2012), golf courses (Leggio, 2007), sports events and sports teams (Ito, Ai, & Ozawa, 2016); (Geyser, 2004) and construction workers (Geyser, 2004); (Jewson, 2004); (Leggio, 2007).

2.3 Studies on the use of snow derivatives for ski resorts

As mentioned in the report by Aaheim et al. (2009) prepared by CICERO, ECON and Vestlandsforskning and commissioned by Climate Adaptation Committee, the tourism industry in Norway is especially vulnerable and sensitive to a variety of climate variables such as temperature, number of hours of sunshine, precipitation, storm and snow level. Particularly mentioned is the increasing temperatures which results in less snow at winter tourist destinations and also the need to find short and long run adaptation measures (Aaheim et al., 2009, p. 52).

With respect to ski resorts, the potential of weather derivatives has been discussed in Beyazit and Koc (2009), Tang and Jang (2011), Tang and Jang (2012) and Bank and Wiesner (2011).

Tang and Jang 2011 analyze how the two different hedging strategies, geographical diversification and financial hedging, can be used as a risk management tool for ski resorts. First, they focus on the effect of geographical diversification on the cash flows exposed to snow fall risk. A ski resort can diversify geographical by investing in several resorts at different places where the weather exposure is different. Hence, the risk is spread on other ski resorts as well. Second, they focus on the effect of financial hedging. That is, hedging with weather derivatives which in this case is a snow derivative. The goal is to provide a method that is based on the interactions between the two different weather risk management strategies. The study is based on snowfall forwards. The peak season in their case starts in November and ends in March for both cash flow and snowfall. They find that snowfall and cash flow is at its highest level at the same time. This indicates a strong correlation between cash flows and snowfall.

They use two different scenarios when examining the geographical diversification effect. A single-property vs. a ski conglomerate, and before vs. after adding a new property to a ski conglomerate. They first compare the weather exposure of a ski conglomerate and single property using a t-test. They use Monte Carlo simulation to estimate all the possible cash flows the companies could have achieved. Furthermore, they regress the cash flows of three ski conglomerates on snowfall and a property acquisition dummy. From this regression they can test the effect of the cash flow exposure to snowfall risk by adding one more property to the company cash flow. When finding the optimal hedge ratio for ski conglomerates they must take into account that they are exposed to multiple basis risks. Basis risk is related to the difference in the amount of snowfall measured at the weather station and the actual amount of snowfall the firm is exposed to. For a single-property ski resort the results show that geographical diversification effectively can reduce the risk exposure to the snowfall. However, the hedging effect would depend on the correlation between the cash flow of the newly added ski resort and snowfall of the original ski resort. As long as this correlation is negative, Tang and Jang (2011) argue that the weather risk exposure can be reduced. To reduce the cash flow volatility due to weather risk, a firm could also buy a weather derivative that hedge against this risk factor. A ski resorts cash flow decrease when the amount of snowfall is low. Thus, they should enter a short position in a snowfall forward to hedge against this risk.

The hedging strategy would depend on the goals of the firm and the availability of capital. If the firm has a small amount of capital and the goal only is to reduce short term snowfall risk, financial hedging would be preferable. For ski conglomerates there will be no significant risk reduction through geographical diversification since they are already geographically diversified, and hence financial hedging may be the best risk management tool.

Tang and Jang (2012) examine how weather derivatives can be used as a risk hedging management tool in nature-based tourism businesses. In their calculations they use a ski resort as an example and the weather variable is snowfall. They argue that the value of risk reduction by using a weather derivative is created through several different sources, including the possibility of achieving a higher debt level, greater tax shield, lower probability of bankruptcy and that the premium required by investors to compensate for the risk would be reduced. The weather risk they want to manage is not extreme low probability weather events, such as hurricanes and floods. It is rather high probability scenarios, for example if snowfall during a year is more or less than anticipated. Hence they are only focusing on short term volatility in snowfall. Further, they compare financial risk management with operational hedging and standard insurance. They conclude that financial risk management requires less capital and is therefore superior for ski resorts that have limited capital and are exposed to short term risks.

In the case study they use forward contracts based on local snowfalls. They point out several reasons for using forwards. With forwards you do not need to calculate any initial price, because there will be no cash exchange up front. The value of a forward is zero when you set the strike as the historical mean. This is because it is then an equal probability for the snowfall to be above or below the strike level, and hence either side has an equal chance of receiving cash flows. Another reason pointed out is that they are over-the-counter contracts and thus can be adapted for local snowfalls. Also, the payoffs are linear. A ski resort's cash flow is positively correlated with snowfall and a contract where the holder receives payoff when the snowfall is low is relevant in the case of a ski resort. Thus, they have to enter a short forward position based on the local snowfall index to hedge against the risk related to the snowfall. They use the average snowfall from 1991 to 2003 as the strike level. The case study is used for Winter Sports Inc, which is a single-property ski resort. The operating cash flow are used to measure the exposure to snowfall risk.

They first present the results based on historical data and show what the effect would have been if they used forward contracts as a hedging strategy during the 1991-2003 period. This is demonstrated by comparing the annual cash flows with and without investing in the snowfall derivative. The ski resort will receive cash if the snowfall is low, and must pay out cash if the snowfall is high, hence the cash flow volatility can be reduced. They use Monte Carlo simulation to simulate the cash flows. According to the results with this simulation, snowfall forwards could reduce ski resorts cash flow volatility up to 25.8%. The hedging is most effective in months when the expected snowfall is high. Thus they suggest that it may be better to hedge cash flow only for the months that the accumulated snowfall is highest, not the whole winter season. They expect small individual ski resorts to benefit most from weather risk hedging because of their concentrated exposure to snowfall risk.

In Bank and Wiesner's study from 2011, 61 ski-lift operators in Austria were interviewed concerning their vulnerability towards unfavourable weather conditions and their knowledge, use and attitude towards weather derivatives as an adaptation measure. When asked on what scale they perceive the consequences of climate change to their business, approximately 63% said they consider climate change as an important issue, whereas 22% said that it is of little or no importance to them. Also, during the warm winter of 2006/2007, a large amount of the respondents said they suffered severe losses which demonstrate the ski resorts vulnerability to snowfall.

Despite these results, only one of the respondents used weather derivatives as an adaptation measure. Highlighted factors for application are reported to be transparency and lack of other insurance products. Lack of awareness and expertise are main factors reported on reluctance towards the use of weather derivatives. Also, many of the ski-operators do not have the resources and/or framework to set up a risk management programme. A high fraction of the respondents showed an interest in the potential use of weather derivatives, and the study concludes that the main challenge is the lack of knowledge regarding risk management and available tools.

Beyazit and Koc (2009) examine how put options on cumulative snow level can reduce the weather risk in winter tourism establishments in Turkey. Tourism is the second largest industry in Turkey and thus, hedging is important for the economy. Beyazit and Koc (2009) analyze data from a ski resort in Palandoken, east in Turkey. They argue that the relationship between the ski resorts business and the snow level in the region is strong, but that the level of snow has an asymmetric impact on profits. In other words, if the snow level is below a certain limit the revenues can be highly reduced, but if the snow level is very high it will not necessarily imply a huge increase in income. They also propose a pricing method for the put options. When calculating the price of the snow level put options they apply three different methods which are the method of Alaton et al. (2002), Historical densities and Edgeworth densities. They use daily snow level data from 1975-2006 and analyze the peak period which they assume to be November to March. The data shows no certain distribution. It has a high volatility and positive kurtosis and skewness. The data is therefore non-normal and hence they use "Generalized Edgeworth Expansion" technique to account for this.

As seen in the articles above, weather derivatives may be an effective tool for ski resorts wishing to hedge risk related to the weather. Studies on snow derivatives are limited to those presented above. Like Beyazit and Koc (2009) we will also examine snow level put options. Since Tang and Jang (2012) argue that small individual ski resorts will benefit the most from using such derivatives, we will in our case study a ski resort with these characteristics. Our purpose is to find a price for a snow level put option and check if a ski resort can smooth earning by investing in one.

3 Data

The weather data is downloaded from the Norwegian Meteorological Institute, whereas the financial data is collected from Brønnøysundsregisteret via "Proff.no". Descriptive statistics of the data will be described in this chapter, together with a regression analysis and main characteristics of the relationship between weather and financial data.

3.1 Snow level data

One of the main challenges with snow derivatives is measuring the snow parameter. As one may think, it is not as easy as putting a measuring stick in the snow. The measuring is dependent on how frequently snowfall are reported. If the observer measures the snow level at 11 am and then waits until 6 pm for the next measuring, the snow has time to settle, drift and melt. An increase in snow level of five centimeters can be a result of two centimeters of melting and seven centimeters of new snow. Hence, reporting five centimeters of new snowfall will be incorrect (*Weatherworks*, 2016). Since snowfall is a function of both precipitation and temperature (Beyazit & Koc, 2009), one way to handle the problem is to obtain both precipitation and temperature data and convert this into snowfall. Unfortunately, this method comes with several faults. When the temperature is close to zero, it will be hard to differ between rainfall and snowfall, hence the measuring of snowfall may be highly incorrect. Because of the challenges in measuring snowfall and therefore obtaining exact and correct data, we will use cumulative snow level when pricing snow derivatives.

3.1.1 Data description

The snow level data is collected from the data download service "eKlima", a service made public by the Norwegian Meteorological Institute. Daily snow level observations, total from ground up and normally measured in the morning, are collected from the weather station "Løksmyr" (station number 68270). Løksmyr is located in Melhus, with an altitude of 173 meters above sea level. The weather station is located approximately 4 km from Vassfjellet.

We have followed Alexandridis and Zapranis (2013) suggested method on how to handle missing values in data sets where there are consecutive missing values. In our data set, there are consecutive missing values during summer months. During the summer, snow levels are zero and reporting has not been taking place. Besides that, missing values are not a problem in the collected data set. One rare missing value can be a result of broken weather equipment, loss of data or lack of reporting the relevant day. Missing values are filled in by using the average snow level of 7 days before and 7 days after the missing value as proposed in Alexandridis and Zapranis (2013):

$$S_{t,avg} = \frac{\sum_{j=1}^{7} S_{t-j} + \sum_{j=1}^{7} S_{t+j}}{14}$$

where S_t is the snow level at day t. They also propose taking the average at that particular day across years. This is not relevant for snow level (unlike temperature) because cumulative snow level exhibits low correlation among the years.

3.1.2 Descriptive statistics

Tang and Jang (2012) and Beyazit and Koc (2009) argue for hedging revenues for the peak season only. Historically, the amount of snow in November has been generally low. Also, during Easter holiday the possibility to generate income is high and therefore this week is regarded one of the most important weeks for a ski resort. Thus, we will focus on the peak season December to April. We will, as Beyazit and Koc (2009) did in their study of winter tourism in Turkey, use cumulative snow level when constructing the weather derivative. The data set for daily snow level starts at December 1st 1996 and continue until April 30th 2016. The cumulative snow level each year is the sum of all daily snow levels for the chosen season December to April. This results in 20 years of cumulative snow level data.

Table 1: Descriptive statistics Cumulative Snow Level 1997-2016

Variable	**Mean**	**(Std. Dev.)**	**Min.**	**Max.**	**N**
Cum snow level cm	4 341.20	(2 324.55)	1141	8799	20

Table 1 summarize the descriptive statistics and figure 1 shows the actual plot of the series of cumulative snow level each year. We observe large variation from year to year, with a minimum value of 1141 cm and a maximum value of 8799 cm. The corresponding standard deviation is 2324 cm. From figure 1 no clear pattern is detected, which makes the winter hard to predict.

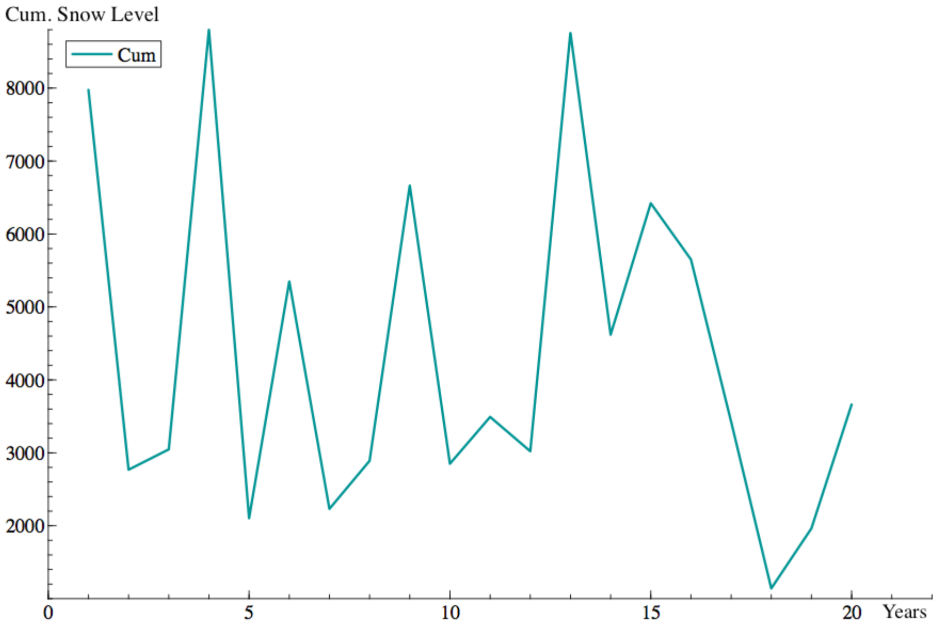

Figure 1: Yearly Cumulative Snow Level

The histogram in figure 2 shows right skewness and we also observe some kurtosis. The values are 0.71 and 2.29 respectively. The purple line is the normal distribution reference and the yellow line is the distribution of the cumulative snow level. From visual inspection cumulative snow level does not appear to be normal distributed. The Shapiro Wilk normality test is rejected at a 5% level and we cannot say that we have a normal distribution. OxMetrics PCGIVE test rejects the test at 7% significance, thus we can with 93% confidence state the data does not fit the normal distribution.

No pattern is observed through the autocorrelation function in figure 3. Hence, cumulative snow level in one year does not dependent on previous years within the considered time horizon. This may not be the case for relatively long time series where a trend can arise because of the long term climate changes. The climate models developed in ToPDAd research project (*ToPDAd*, 2015) predict that snow level in the long run will decrease in most European ski resort destinations due to climate change. We are focusing on short term risk and thus 20 years of data is sufficient to use in this case.

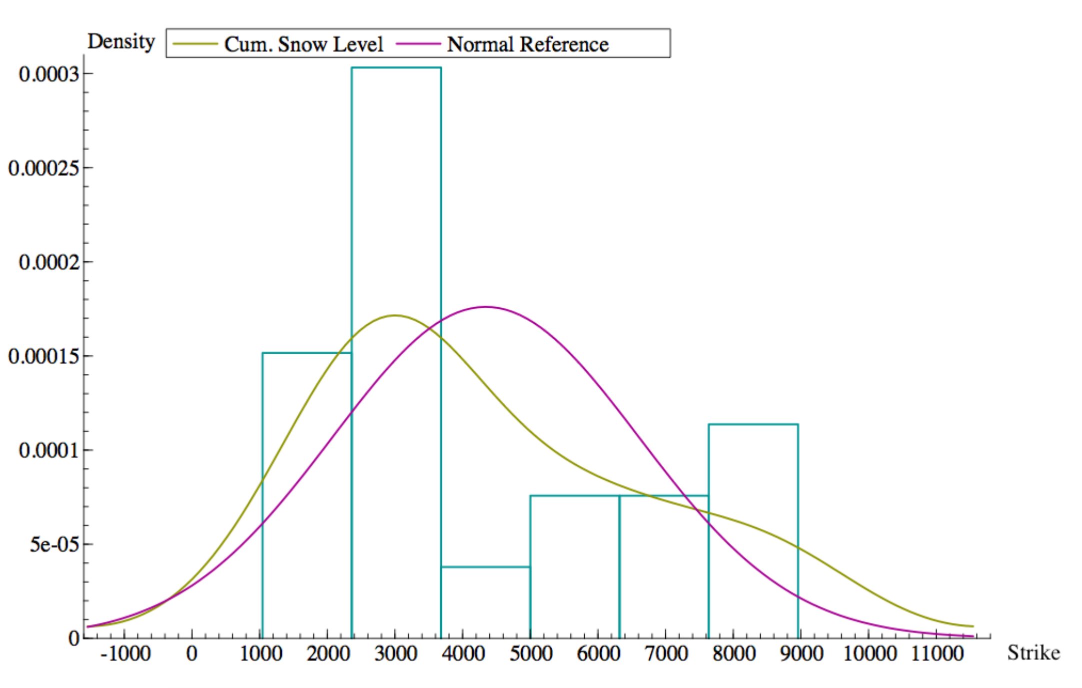

Figure 2: Density: Yearly Cumulative Snow Level

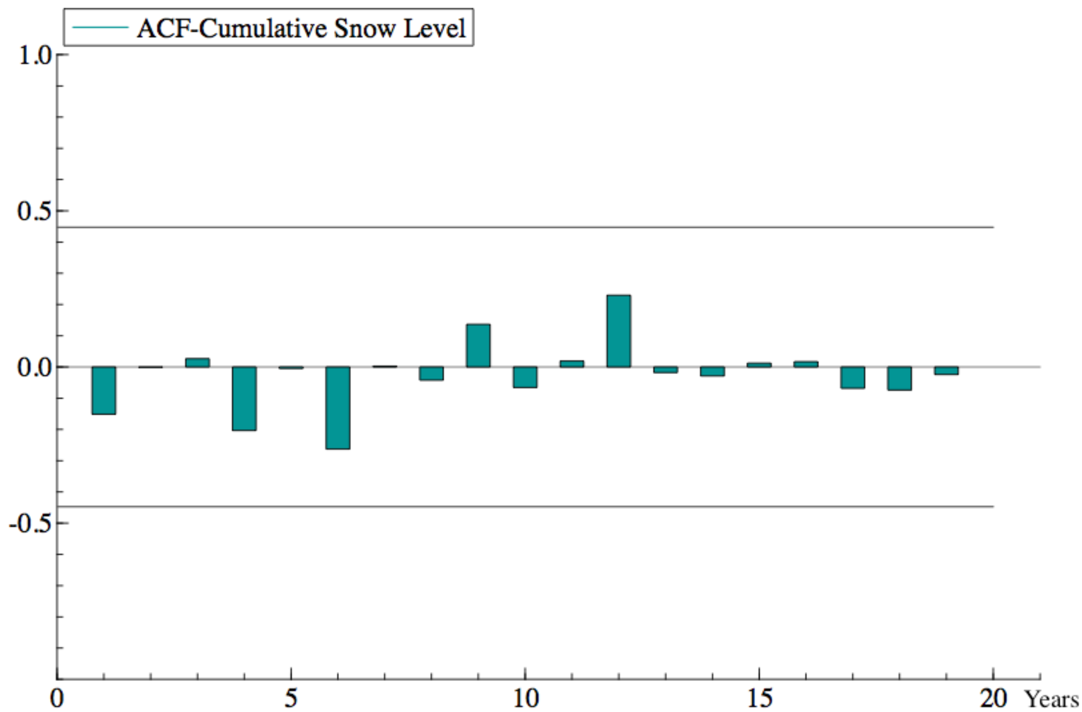

Figure 3: Autocorrelation Function: Yearly Cumulative Snow Level

3.2 Financial data

Annual reports for the years 1997-2015 for Vassfjellet Skiheiser AS was collected from "Brønnøysundsregistereret" (via proff.no). Our focus is to examine if and in what scale the natural snow level impacts profits. Both ski pass revenues and operating income can be possible indicators on this. Low snow level (and thus lower activity) also means lower cost of goods sold, other operating costs/revenues and salary, and we will therefore use operating income in our further discussion and analysis. From here on, Vassfjellet will be used as abbreviation for Vassfjellet Skiheiser AS.

3.2.1 Data description

The annual reports from 1997-2007 are reported on a yearly basis (01.01.xx-31.12.xx), whereas the report in 2008 includes only the months January to August. From there on, the reporting is based on one "accounting year" September-August which includes the whole ski season. In the further discussion we will use the financial data from years 2009-2015 only. It is difficult to compare two different reporting methods. Also, our focus is the winter season and it is therefore more relevant to use the reports where these months are included. By using the earlier reports, the operating income from winter season (December - April) will be split across two different years. Our goal is to detect a relationship between snow level and operating income during each season. Hence, using the reports pre 2009 will not give accurate measurements. Our main concern regarding this is whether the relationship between operating income and snow level for the previous years are similar to our findings for the years 2009 to 2015. One way to get around this problem would be to obtain a seasonal measure that correlates with operating income for the years before 2009. This could be number of ski pass sold, ski pass revenues, amount of cars parked etc. This data could not have been used directly in the analysis, but it would be a good indicator whether or not the relationship found also existed in the previous years. After several attempts to obtain such information with no luck, we must assume similar relationship across the 20 years.

In the annual reports, gain on sale of non-current assets and depreciation on current assets are reported under operating revenues and operating expenses respectively. Our purpose is to examine the operating posts that because of the snow level affects operating income. Hence, gain on sale of non-current assets and depreciation on current assets are deducted from operating income in our reporting.

3.2.2 Descriptive statistics

Descriptive statistics for operating income are reported in the table below. There is a high variation, with a minimum value of 580 042 kr and maximum value of 7 326 398 kr in operating income.

Table 2: Descriptive statistics Operating Income 2009-2015

Variable	**Mean**	**(Std. Dev.)**	**Min.**	**Max.**	**N**
Operating Income kr	5 039 460	(2 531 641)	580 042	7 326 398	7

3.3 Regression analysis of financial and snow data

We will in this section examine the relationship between the weather and financial data. Table 2 summarize the cumulative snow level data and the operating income each year from 2009-2015.

Table 3: Cumulative Snow Level and Operating Income

Variable	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**
CSL	8754	4620	6421	5651	3425	1141	1965
OI	7 326 398	7 019 089	6 898 199	4 9794 409	5 764 932	580 042	2 708 153

The two graphs on right hand side in figure 4 show the deviation from mean for both parameters. The graphs to the left show the data plot for the two parameters for the years 2009-2015. By comparing the graphs in figure 4, there is clearly a strong relationship between operating income and cumulative snow level. We will further try to measure this relationship. As (Beyazit & Koc, 2009), we also observe asymmetric relationship between operating income and the cumulative snow level. At one point there is enough snow for the ski-resort to operate optimal. An increase in snow level may not have much impact on the average skiers decision on whether to visit the ski resort or not. So when this snow level is exceeded, the profits may not increase proportionally. The lower bound of snow level is therefore the crucial one. This characteristic can be observed in the right graphs in figure 4. The blue line represents the mean cumulative snow level of 4341 cm. For the years where cumulative snow level is above this value, the relationship between the two parameters is not that clear. For cumulative snow level values below average the decrease in operating income will be of larger magnitude.

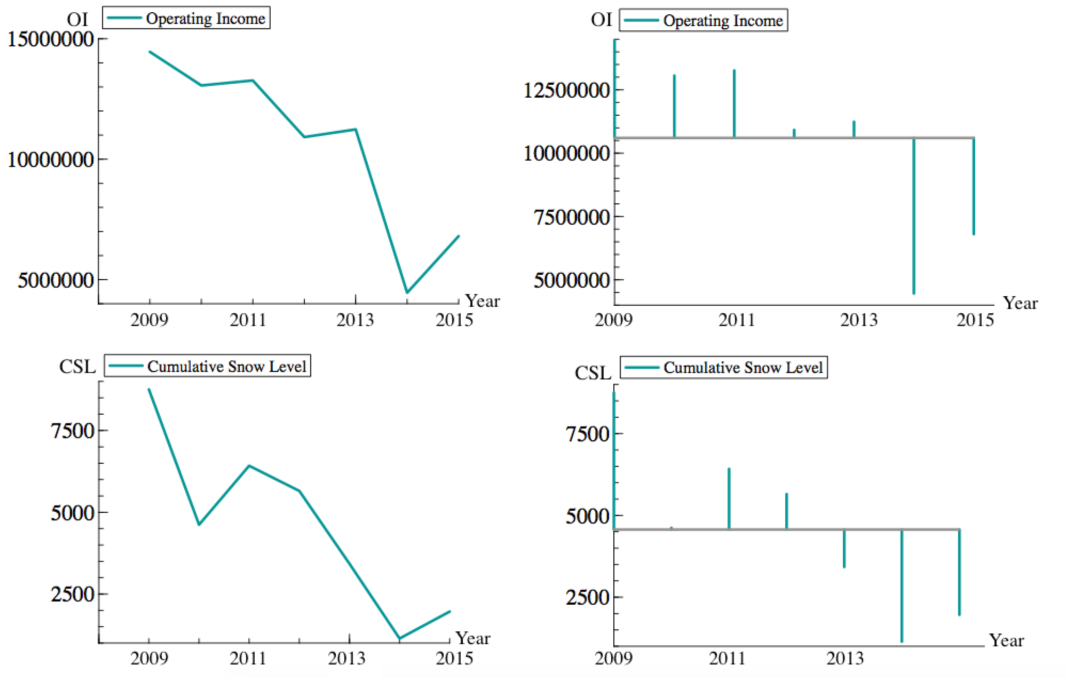

Figure 4: Operating Income and Yearly Cumulative Snow Level: 2009-2015

Figure 5: Scatter Plot: Operating Income and Yearly Cumulative Snow Level

Figure 5 shows the scatter plot of operating income and snow level. The scatter plot may indicate diminishing returns, which is consistent with our above discussion. We will examine three different functions that can describe the relationship between Vassfjellet's operating income and cumulative snow level seen in figure 5.

Figure 6 shows a linear relationship between operating income and cumulative snow level. The correlation coefficient between operating income and cumulative snow level is 0.84. This relative high correlation value proves a close relationship between the two variables and since the correlation reflects a linear relationship, a linear function is a potential candidate to describe this.

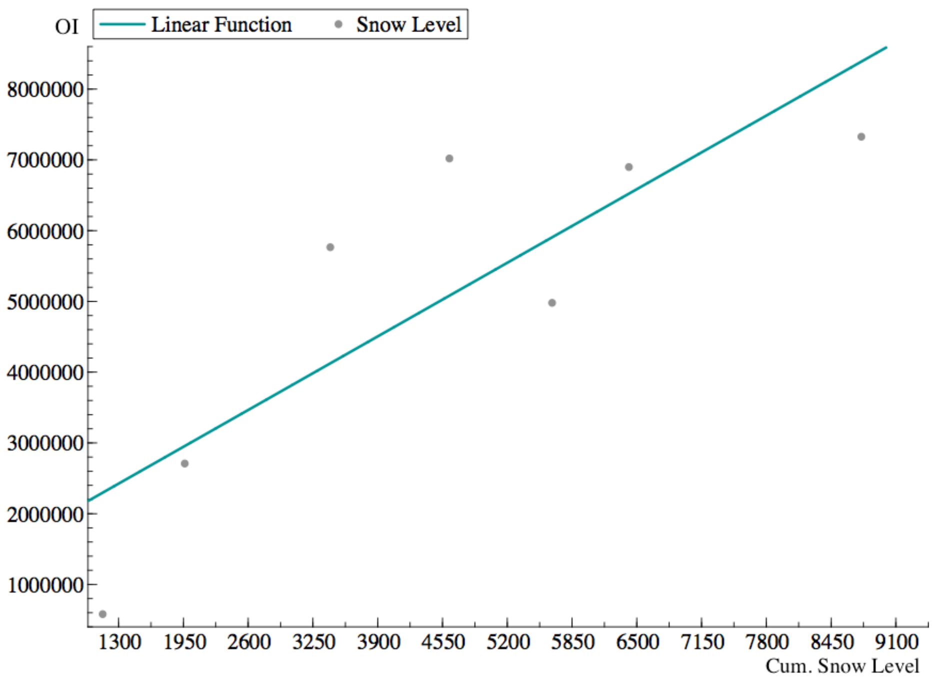

Figure 6: Linear Function

By regression analysis we can write the linear functions as

$$OI = 1384100 + 800 \times CSL \tag{1}$$

where OI corresponds to operating income and CSL to the cumulative snow level for the same year. The regression yields a R^2 of 0.7. In this function, per one cm change in cumulative snow level, operating income changes with 800.2 kr.

A second order- and log function has also been derived. In figure 7 the relationship is computed by a second order function as follows

$$OI = -0.157 \times CSL^2 + 2312.87 \times CSL - 1315000$$

The regression yields a R^2 of 0.85. This function describes an asymmetric relationship between the cumulative snow level and operating income. As discussed above, for snow levels above a certain value, operating income will start to stagnate. The drawback with this function is that it captures a decreasing return after a certain level. We observe from the second order function that for cumulative snow level values above ca. 7300 cm the operating income will start to decrease. Hence, for high values of snow level the second order function will fail to explain the relationship. The probability to reach extreme snow levels (>9000cm) is very low and within the six years it is only one year with a snow level value that is above the maximum point of the second order function. Thus, this function may be suitable within our snow level span. An alternative would be to use the second order function up to a certain point before the maximum point and then use a linear function with slope equal to zero. A log function can also computed in order to try to capture the diminishing return.

In figure 8, a log function is computed as follows

$$OI = -22080000 + 3291300 \times \ln(CSL)$$

The regression has a R^2 of 0.86. As we can see from the figure, this function features diminishing returns to operating income. Thus this model will best explain the relationship in our data. The log function has no maximum point, hence, like the linear function, the operating income will never stop increasing. When choosing a function, it is important to choose one that fits the pricing methods. With a log function, the pricing methods will be more complicated because they must be modified in order to make the tick size a function of the log function. If the functions in figures 7 and 8 are to be used in further application, test of functional form should be conducted. However, in the pricing methods applied later, a linear relationship is assumed. We will therefore proceed using the linear relationship.

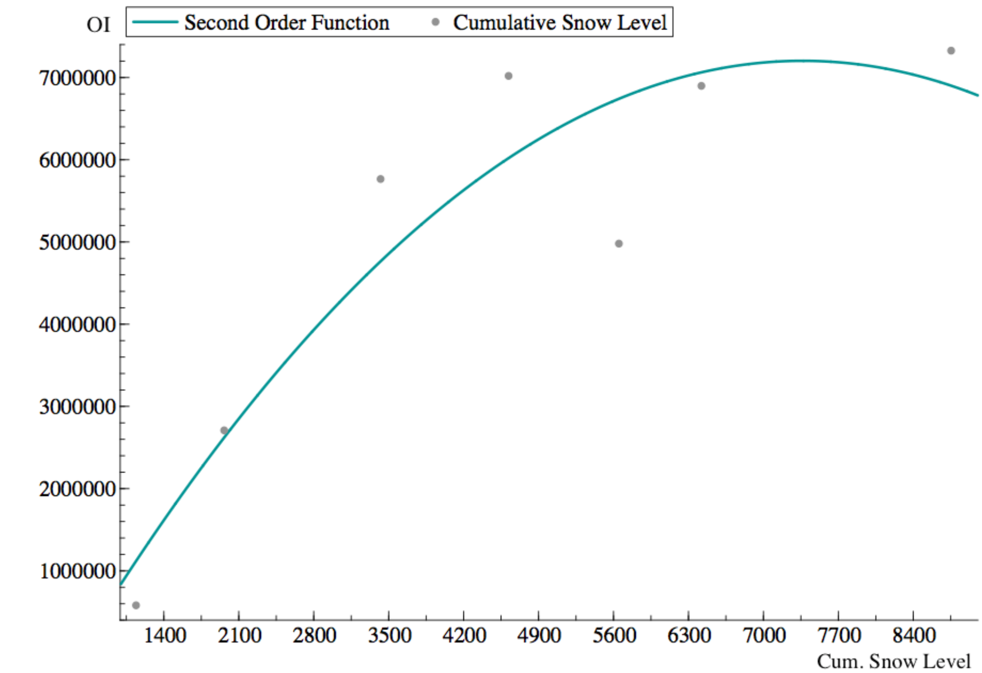

Figure 7: Second Order Function

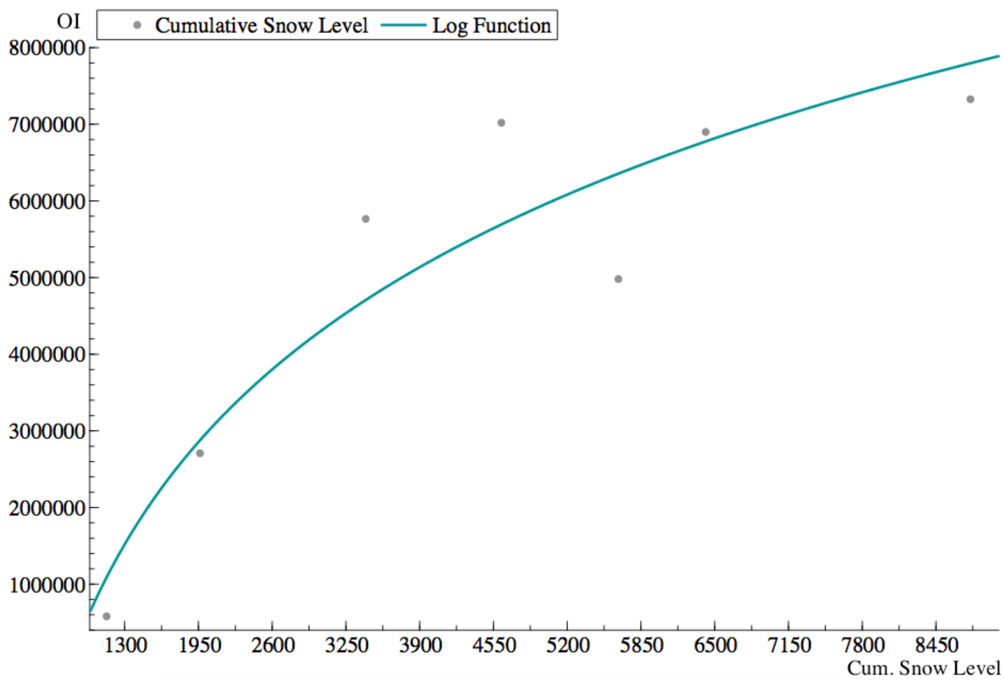

Figure 8: Log Function

4 Methodology

Before moving on to the pricing methodology, we will discuss the characteristics of a weather derivative and obstacles when pricing derivatives based on weather indices. A weather contract is specified by the following parameters (Alaton et al. 2002):

- The contract type (future, swap, put/call option)
- The strike or future price
- The tick size
- The contract period
- The underlying weather index (CAT, HDD, rainfall, snowfall)
- Weather station from which the underlying variable data is obtained
- A premium paid from the buyer to the seller (negotiable)

The payoff of a weather derivative depends on a specified weather index and the magnitude is determined by the strike level and the tick size. The tick size is the amount of money the holder of the contract receives for each unit above or below the strike level, depending on the contract type. The strike level determines whether the option will be exercised or not (Alaton et al., 2002). For example, with a long position in a put option with strike 500, the holder will receive payoff for each unit below the strike 500. The weather derivative contract must define a period in which the underlying index is calculated. This period can vary from a few days and up to many years. Because of the possibility to forecast the weather for a few days ahead, weather derivatives are more usually used for periods longer than a few weeks. The weather derivative contract is based on the observed values of the weather at a specific weather station. Several stations can also be used to measure the weather, but it is most common to use only a single station (Alexandridis & Zapranis, 2013).

From the characteristics listed above, the main difference between a standard financial derivative and a weather derivative is the underlying index. The most common weather indices are based on temperature. Usually they are written on the accumulation of heating degree days (HDD) and cooling degree days (CDD) during a predetermined period (Alaton et al., 2002). HDD measures the number of degrees below a certain reference temperature each day during a specified period, while CDD measures the number of degrees above a certain reference level (Alexandridis & Zapranis, 2013). To measure this deviation, they use the difference between the average temperature for each day and the reference level. The average daily temperature is given by the average of the maximum and minimum temperature that day (Alaton et al., 2002). The reference temperature is often given as 65 °F or 18 °C. The reason for using this as reference value is that for temperatures below or above this value it is common to use either heating or air condition. Hence, HDD is usually used for the winter months and CDD for the summer months (Alexandridis & Zapranis, 2013). A HDD or CDD index future contract is an agreement of buying or selling the value of the HDD or CDD index at a predetermined time in the future. It is possible to construct a call or put option contract on these futures.

It is also possible to construct a call or a put option where the underlying weather index is based on precipitation. This index can for example measure the number of days with precipitation or the total amount of rainfall or snowfall during a period. The buyer must pay a premium when buying the contract and will receive a payoff depending on the index at the expiration day. A contract written on the cumulative amount of precipitation during a period from $t = 0$ to T, has an index:

$$CPR = \sum_{t=0}^{T} PR_t$$

where CPR and PR_t is the cumulative precipitation and the precipitation for day t respectively. If we let K denote the strike level, the payoff W would be:

$$W = \max(K - CPR, 0)$$

Weather swaps are contracts where two parties exchange the weather risk during the contract period. In a standard swap contract, it is common to have several dates where the cash flows are swapped, but for weather swaps it is usually only one swap date (Alaton et al., 2002). Such a weather derivative can be useful when for example one party wants a cold winter with a lot of snow, while the other party wants a warm winter without snow. It is then possible to construct a weather swap which transfers some of the revenues from the "lucky" parties to the "unlucky". In this way both parts will smooth revenues because they get compensation for bad winters and pay an amount when it is a good winter with high profit.

4.1 Obstacles in pricing weather derivatives

The most important difference between weather derivatives and traditional financial derivatives is that their prices reflect weather conditions rather than the price of an underlying variable. Financial derivatives are typically based on the price of a share, bond, currency or other securities and commodities. For these traditional derivatives, the underlying variable can be traded. However, there is no such tradable underlying variable for weather derivatives since the underlying index expresses a certain weather event. Hence, the weather derivative market is an incomplete market (Brockett et al., 2005). Cao and Wei (2004) argue that since the underlying variable is not tradable we cannot form part of a riskless hedge. Thus, it is not appropriate to use the traditional arbitrage free risk-neutral valuations models, such as Black-Scholes, to price weather derivatives. The seller of the derivative will therefore add a risk premium when pricing the derivative (Tang & Jang, 2012).

Local weather indices have a low correlation with prices of other financial assets, such as exchange and interest rate risk. Thus, it is difficult to substitute the underlying with a linked exchanged security to solve the problem with an incomplete market (Hamisultane, 2008). Another problem with weather derivatives is that it is not possible to influence the weather. It is out of human control and we do not know exactly what the weather will be like in the future. Also, it is not possible to store the underlying variable for later use (Tang & Jang, 2011). Weather risk can be managed by estimating the expectations about the future weather states. However, the risk related to the weather is affected by the fact that weather has a high degree of unpredictable fluctuations. Masala (2014) claims that the different weather variables have significantly different properties. There is no direct similarity distributional assumption with respect to weather events in general, so they cannot have any market price. Furthermore, it is very little liquidity in the weather market due to the valuation difficulty (Brockett et al., 2005).

One of the main concerns when using weather derivatives is the basis risk. According to Brockett et al. (2005) the risk effectiveness of a weather derivative is highly dependent on the magnitude of basis risk a firm faces when using this derivative. Alexandridis and Zapranis (2013) divide basis risk into two different components. The first is geographical basis risk. If Vassfjellet buys a snow level put option where the underlying weather index is measured at Løksmyr, the geographical basis risk will reflect the difference in snow level between the weather station and Vassfjellet. The geographical risk increases the further away the business is from the weather station (Tang & Jang, 2012). The second, referred to only as basis risk, is the risk of having estimated the relationship between the snow level and financial data wrong. Weather can vary from one location to another. Because of the possibility of microclimate, a local zone where the climate is different than in the surrounding area, the weather risk is a highly-localized risk (Tang

& Jang, 2011). Geographical basis risk may reduce the effectiveness of the derivative substantially. The demand for the weather derivative is highly dependent on the distance between the weather station where the weather is measured and the place for the hedging company. The geographical risk will always be positive. When the distance increases, the demand will decrease. However, buying weather derivatives from different stations in the area around the hedging company can increase the hedging effectiveness (Alexandridis & Zapranis, 2013). The yield variations a company wish to hedge is not necessarily completely correlated with the weather variable (Xu et al., 2007). However, as this correlation increases the basis risk will decrease (Alexandridis & Zapranis, 2013).

4.2 Obstacles in pricing precipitation vs temperature derivatives

When examining temperature it may be possible to find a trend, seasonality and noise factors in the model. Hence, one may find a stochastic process or a particular distribution for the temperature (Beyazit & Koc, 2009). However, the behavior of precipitation is significantly different from the temperature, so other types of models will be needed when modeling (Benth & Benth, 2012). Rainfall is not a continuous variable, but it is a binary event. It may or may not be raining. Furthermore, the rainfall process is not as smooth as the temperature process, but has a much more irregular distribution. It can go a long time without any rainfall and it can occur in sudden peaks (Cabrera, Odening, & Ritter, 2013). Also, it is not necessarily a high correlation between rainfall amounts in different locations (Xu et al., 2007). Benth and Benth (2012) show that there is no clear pattern in the time series plot of daily rainfall. Both the probability of a rainy day and the amount of daily rainfall varies with the season. It is often more likely to have a rainy day in the winter, but the rainfall has a higher intensity in the summer. The volatility of the amount is also higher in the summer. Furthermore, the probability of a rainy day is higher if it was raining the previous day (Masala, 2014). Precipitation cannot take negative values. Because the measurement period often contains many days without any precipitation, the distribution will contain several zero values. Hence, it will have a highly-skewed distribution and a high kurtosis. This is demonstrated in the histograms of the data in Benth and Benth (2012), where the precipitation has an extreme right-skewness and also an extraordinary high kurtosis. Precipitation is also much more localized than temperature, so the correlation between the weather in different locations is lower than in the case of temperature (Benth & Benth, 2012). This makes geographical basis risk even more important when examining precipitation derivatives that has a relatively distance from the measurement location (Alexandridis & Zapranis, 2013).

Because of all the irregularities mentioned above and the difficulty in measuring the precipitation, it is more complicated to price a weather derivative based on precipitation than temperature. Even though rainfall and snowfall are different weather variables, they have several similar characteristics. Snow level is both a function of precipitation and temperature. The temperature must be lower than about zero to make precipitation into snow. Hence, it is a close and inverse correlation between temperature and snow level (Beyazit & Koc, 2009). Snow level does not have a certain distribution. The snow level one year is weakly correlated with the snow level of other years and Beyazit and Koc (2009) show that the correlation between the snow level in their data set in subsequent years is 0.35.

4.3 Pricing methods

The weather derivative market is an incomplete market and it is still under development. A general accepted method for pricing weather derivatives, as Black Scholes is for financial derivatives, does not yet exist (Alexandridis & Zapranis, 2013). Earlier studies, including (Jewson & Zervos, 2003), (Richards, Manfredo, & Sanders, 2004) have been using the framework of Black Scholes when pricing weather derivatives. Because of the restricting assumptions that follows with it, new and more correct methods have been proposed in later literature. Many studies on the the pricing and modeling of temperature derivatives have been undertaken, but few attempts have been made to price snow level derivatives.

We will in the following focus on pricing methods available for precipitation derivatives. First, a brief overview of earlier pricing methods for snow level contracts will be presented. Thereafter, we will give an in depth description of relevant pricing methods on snow level put options.

4.3.1 Traditional methods

Actuarial pricing methods use the conditional expectation of the snow level derivative's future payoffs to calculate the price (Hamisultane, 2008). The law of large numbers is essential in these models. It states that as the number of samples increase, a more reliable estimate of the true expected value of the observed phenomenon can be obtained (Kordi, 2012).

The expected price can be calculated in different ways. The easiest method to implement is the historical burn analysis (HBO). Historical burn analysis is considered the benchmark approach for pricing temperature derivatives. The main assumption is that history will repeat itself with same likelihood and the model make use of historical distribution of the weather index to calculate historical option payoff (Alexandridis & Zapranis, 2013). For example, to find the price of a put option based on a cumulative snow level index, simply obtain historical snow level records for similar periods in the previous years, $y_1, y_2, \ldots y_n$. Accumulate the values for December – April each year to obtain the series of historical cumulative snow level for these months. Denote this $H_1, H_2 \ldots H_n$. For year i calculate the put option payoff $W_i = \max(H_i - K, 0)$, where K is strike, to obtain the series of historical payoffs. The expectation of the weather derivative's price at time t then corresponds to the average annual payoffs (Benth & Benth, 2012):

$$p(t) = \frac{1}{n} \sum_{i=1}^{n} W_i \tag{2}$$

The method is easy to implement since there is no need to derive the real distribution of the snow level. Thus, it can be implemented on other weather variables as well.

Benth and Benth (2012) address the issue of using HBA on derivatives with aggregated values as the underlying variable. With a long time series the number of data points are reduced drastically when using cumulative values. Furthermore, they state that this again may lead to a very uncertain option value because of the few non-zero payoff values among the data points. Our time series with 20 years of daily snow level observations from Løksmyr will be reduced to only 20 data points when calculating cumulative snow level, where non of the values equal zero. Jewson (2002) points out another disadvantage with the HBO model. Because of the short records of historical data, extreme weather conditions are not well represented in the sample. This can be dealt with by the direct modeling of the weather index. With index modeling, the distribution of the index is modeled and hence, all possible scenarios including extreme values are taken into account (Jewson, 2002). Indices such as HDD, CDD etc. tend to be bounded by zero which results in loss of information when calculating the indices (Alexandridis & Zapranis, 2013).

A better approach, according to Alexandridis and Zapranis (2013), would be daily modeling of the weather parameter. It is considered the most complex of the actuarial methods and involves building a statistical model for the daily snow level itself (Jewson, 2002). The difficulty lies in finding an appropriate stochastic process that captures the observed behaviour of the snow level. Although daily modeling is the most accurate method for modeling the weather process, there is several obstacles attached to it (Alexandridis

& Zapranis, 2013). In the case of temperature, seasonality in the first two moments and the distribution are found. There is also signs of long memory in the autocorrelation. In literature, two different approaches has been suggested for modeling the daily average temperatures; discrete and continuous processes. In the former, the use of an ARMA framework has been suggested from several authors, including Cao and Wei (2004). Others, such as Alaton et al. (2002) and Hamisultane (2008), suggest a continuous process using a stochastic differential equation. Although there exist several studies on the modeling of the daily average temperature, little is found on snow level in the framework of pricing weather derivatives. Alexandridis and Zapranis (2013) emphasize the obstacles in the daily modeling of snowfall versus temperature. Snowfall is not a continuous variable and changes much more irregular and unevenly than temperature. Since snowfall and rainfall share much of the same characteristics, a stochastic process-based model presented in Alexandridis and Zapranis (2013) can be used on both of them. A method for modeling snow level is not presented in the study. The risk of model miss-specification is high, which may lead to sizeable mispricing of weather derivatives when the underlying weather parameter, in this case snow level, is to be modeled (Alexandridis & Zapranis, 2013).

Monte Carlo simulation can be used as an alternative to the indifference method we will discuss later. The method involves simulating several different precipitation scenarios over a prespecified period in order to determine the derivatives possible payoff. In the case of precipitation, Alexandridis and Zapranis (2013) suggest using a two-state, first order markov chain model on historical data. This method is repeated n times and the rainfall index is found by averaging each scenario. From the rainfall index, the payoff and hence, the price of the derivative can be obtained. A similar process can also be done for snow level, but will include a more extensive markov-chain model. The markov-chain model consists of two properties. The first is the different states the weather variable can take. Compared to precipitation which can take on two states (no precipitation, precipitation), a markov-chain model on snow level will have seemingly more states. Second, the order of the chain defines how many previous values the state to state transition probabilities are conditional on. In the case of precipitation a two state, first order markov-chain can be used. For snow level this will include a more complicated markov-chain model.

The weather index is not traded in the market and thus, it is impossible to replicate. Because of this market incompleteness, a risk-neutral probability measure cannot be obtained from the snow level index and must be extracted from elsewhere. Brix, Jewson, and Ziehmann (2002) suggest using already quoted weather contracts whose prices are highly correlated with the underlying of the weather option in valuation. Hamisultane (2008) points out that the obstacle with this strategy is that already quoted contracts are not yet sufficiently liquid. Thus, Monte-Carlo pricing method will give unreliable prices.

Equilibrium pricing approach is a utility-based pricing approach where the problem of market incompleteness is addressed. The pricing method introduced by Cao and Wei (2004), as an extension of Lucas (1978) theory of asset pricing in a one-good, pure exchange economy, incorporates weather as a state variable in the economy. In their model, uncertainties in a two-state economy are explained by the dividend process and a state variable representing the weather condition. Cao and Wei (2004) specialize the model to temperature derivatives by proposing a dynamic system for the daily temperature. The price of the derivative can be found using the specified temperature process, agents preferences and the dividend process. The set-up in Cao and Wei (2000) can ble implemented on other weather variables, including snow level, as long as a dynamic process for the daily weather variable can be suggested.

4.3.2 Indifference pricing method

The indifference pricing method is an utility based approach which has been presented by Brockett et al. (2006) and Xu et al. (2007). Expected utility optimization is an useful framework when deciding whether to take on a project or not. A project is accepted if the expected utility increases. This method is different from other pricing methods because it is based on the basic principle of equivalent utility and makes use of investors risk preferences and a corresponding utility function (Alexandridis & Zapranis, 2013).

An utility function is needed in order to maximize an investor's wealth. In the following, a negative exponential utility function has been used in order to derive the indifference pricing formula:

$$U(X) = -\exp(-\lambda X) \tag{3}$$

where λ is the absolute risk aversion parameter and X is wealth. We follow the framework of Brockett et al. (2006) and Xu et al. (2007) as represented in Alexandridis and Zapranis (2013) where a two-date economy is assumed. The market consists of a buyer and a seller who both wish to construct their portfolio in a way that optimize their terminal wealth at the terminal day T. First, we consider the portfolio choice of the buyer. The buyer has initial wealth X_b in which an amount a_b must be invested in the risky production activity, and the rest $(X_b - a_b)$ in a risk-free asset. The risky production activity depends on weather conditions and has the return r_b. r_f denotes the return of the risk free asset. The value of this portfolio at time T is given by:

$$X_b^{wo} = (X_b - a_b)q_f + a_b q_b \tag{4}$$

where $q_f = 1 + r_f$ and $q_b = 1 + r_b$. Then, we include the possibility of investing an amount in a weather derivative. Here, the buyer can buy k shares of the weather contract for a price F_b. The value of this portfolio at time T is given by:

$$X_b^{w} = (X_b - a_b - kF_b)q_f + a_b q_b + kW_T \tag{5}$$

where W_T is the payoff at time T related to the predetermined weather index. The payoff depends on the tick size θ:

$$W_T = \theta \max\left(K - H_i, 0\right)$$

The setup is similar for the seller, which spends a_s on investing in a market portfolio with risky market return of r_s. Without the possibility to sell k units of a weather contract, the value of this portfolio is given by:

$$X_s^{wo} = (X_s - a_s)q_f + a_s q_s$$

The value of the portfolio when including the possibility to sell k shares of a weather derivative is:

$$X_s^{w} = (X_s - a_s - kF_s)q_f + a_s q_s - kW_T$$

Using this framework we will in the following derive the buyer's indifference price. The optimal portfolio choice a_b is found when the buyer is indifferent between including the weather derivative in the portfolio or not. Hence, we have to equalize the expected utility of the two strategies (4) and (5):

$$sup_{a_b} E[u(X^{wo}_{\ b})] = sup_{a_b} E[u(X_b^{w})] \tag{6}$$

We are interested in a closed form solution of the indifference price. Thus, we need to find the certainty equivalent (CE) of the utility function. The CE can be approximated by using second order Taylor expansion of $U(X)$ evaluated at the point EX. We know that:

$$E[U(\tilde{X})] = U(\hat{X}) \tag{7}$$

where $\hat{X}$ is certainty equivalent wealth and $\tilde{X}$ is the stochastic wealth. Second-order Taylor expansion of $U(\tilde{X})$ evaluated at the point EX gives us:

$$\begin{aligned} E[E(\tilde{X})] &\approx E[U(EX) + U'(EX)(\tilde{X} - EX) + \frac{1}{2}U''(EX)(\tilde{X} - EX)^2] \\ &= U(EX) + \frac{1}{2}U''(EX)E[(\tilde{X} - EX)^2] \\ &= U(EX) + \frac{1}{2}U''(EX)\sigma^2_{\tilde{X}} \end{aligned} \tag{8}$$

By approximation

$$U(\hat{X}) \approx U(EX) + U'(EX)(\hat{X} - EX) \tag{9}$$

Set (8) $\approx$ (9):

$$\begin{aligned} U(EX) + \frac{1}{2}U''(EX)\sigma^2_X &\approx U(EX) + U'(EX)(\hat{X} - EX) \Leftrightarrow \\ \hat{X} &= EX + \frac{1}{2}\frac{U''(EX)}{U'(EX)}\sigma^2_{\tilde{X}} \end{aligned} \tag{10}$$

From the utility function in (3):

$$U'(X) = \lambda \exp(-\lambda X) \quad \text{and} \quad U''(X) = -\lambda^2 \exp(-\lambda X)$$

Inserting this into equation (10):

$$\begin{aligned} \hat{X} &\approx EX + \frac{1}{2}\frac{-\lambda^2 \exp(-\lambda EX)}{\lambda \exp(-\lambda EX)}\sigma^2_{\tilde{X}} \\ &= EX - \frac{1}{2}\lambda\sigma^2_{\tilde{X}} = CE \end{aligned} \tag{11}$$

where EX is the expected wealth and $\sigma^2_{\tilde{X}}$ is the variance at time T. The CE is the guaranteed return an agent require to be indifferent between this safe return and a risky return (Copeland, Weston, & Shastri, 2014, p. 52).

Next, we replace the expected utility in (6) by its certainly equivalent:

$$sup_{a_b}[E(X^{wo}_b) - \frac{\lambda_b}{2}\sigma^2(X^{wo}_b)] = sup_{a_b}[E(X^w_b) - \frac{\lambda_b}{2}\sigma^2(X^w_b)] \tag{12}$$

Setting (4) and (5) into (11), we obtain expression for the certainly equivalent of the wealth, with and without derivative:

$$CE^{wo}_b = x_b q_f + a_b(E(q_b) - q_f) - \frac{\lambda_b}{2}a^2_b\sigma^2_{q_b} \tag{13}$$

$$CE^{w}_b = (x_b - kF_b)q_f + a_b(E(q_b) - q_f) + kE[W] - \frac{\lambda_b}{2}a^2_b\sigma^2_{q_b} - \frac{\lambda_b}{2}k^2\sigma^2_W + \lambda_b a_b k \text{cov}(q_b, W) \tag{14}$$

where $E(q_b)$ is the expected q_b and $E(W)$ is the expected payoff at time T of the weather derivative. $\sigma^2_{q_b}$, σ^2_W and $\text{cov}(q_b, W)$ denotes the variances and covariance between q_b and W, respectively. Using first order conditions on (13) and (14) w.r.t. a_b we obtain the optimal a^*_b:

$$\begin{aligned} CE^{wo\prime}_b(a_b) &= E(q_b) - q_f - \lambda_b a_b \sigma^2_{q_b} = 0 \\ a^{wo*}_b &= \frac{E(q_b) - q_f}{\lambda_b\sigma^2_{q_b}} \end{aligned} \tag{15}$$

$$\begin{aligned} CE^{w\prime}_b(a_b) &= E(q_b) - q_f - \lambda_b a_b \sigma^2_{q_b} + \lambda_b k \text{cov}(q_b, W) = 0 \\ a^{w*}_b &= \frac{E(q_b) - q_f + \lambda_b k \text{cov}(q_b, W)}{\lambda_b\sigma^2_{q_b}} \end{aligned} \tag{16}$$

a^{w*}_b is then the optimal amount of wealth the buyer invests in risky production activity. This amount decreases with higher negative covariance between return on risky production and payoff of the derivative. A higher negative covariance implies receiving payoff from the derivative when it is most needed. Thus, it is more attractive for the buyer to invest in the weather derivative. Setting a_b^{wo*} into (13) and a_b^{w*} into (14) yields:

$$CE_b^{wo*} = x_b q_f + \frac{(E(q_b) - q_f)^2}{\lambda_b \sigma_{q_b}^2} - \frac{\lambda_b (E(q_b) - q_f)^2 \sigma_{q_b}^2}{2\lambda_b^2 \sigma_{q_b}^4}$$

$$CE_b^{wo*} = x_b q_f + \frac{(E(q_b - q_f)^2}{2\lambda_b \sigma_{q_b}^2} \tag{17}$$

and

$$CE_b^{w*} = (x_b - kF_b) q_f + \frac{(E(q_b) - q_f + \lambda_b k \mathrm{cov}(q_b, W))(E(q_b) - q_f)}{\lambda_b \sigma_{q_b}^2} + kE(W)$$
$$- \frac{\lambda_b (E(q_b) - q_f + \lambda_b k \mathrm{cov}(q_b, W))^2 \sigma_{q_b}^2}{2\lambda_b^2 \sigma_{q_b}^4}$$
$$+ \frac{\lambda_b (E(q_b - q_f + \lambda_b k \mathrm{cov}(q_b, W))(k \mathrm{cov}(q_b, W)}{\lambda_b \sigma_{q_b}^2} - \frac{\lambda_b k^2 \sigma_W^2}{2}$$

$$CE_b^{w*} = (x_b - kF_b) q_f + \frac{(E(q_b) - q_f + \lambda_b k \mathrm{corr}(q_b, W) \sigma_{q_b} \sigma_W)(E(q_b) - q_f)}{\lambda_b \sigma_{q_b}^2}$$
$$+ kE(W) - \frac{(E(q_b) - q_f + \lambda_b k \mathrm{corr}(q_b, W) \sigma_{q_b} \sigma_W)^2}{2\lambda_b^2 \sigma_{q_b}^2}$$
$$+ \frac{(E(q_b) - q_f + \lambda_b k \mathrm{corr}(q_b, W) \sigma_{q_b} \sigma_W)(k \mathrm{corr}(q_b, W) \sigma_W)}{\sigma_{q_b}} - \frac{\lambda_b k^2 \sigma_W^2}{2} \tag{18}$$

The two expressions for certainly equivalent, (17) and (18), is now set equal. Solving for F_b yields the indifference price for the buyer [1]:

$$F_b = \frac{1}{q_f}(E(W) + \frac{1}{2}\lambda_b k \sigma_W^2 (\mathrm{corr}^2(q_b, W) - 1) - \frac{\sigma_W}{\sigma_{q_b}}(E(q_b) - q_f)\mathrm{corr}(q_b, W)) \tag{19}$$

$$= \frac{1}{q_f}(E(W) + \pi_b)$$

with

$$\pi_b = \frac{1}{2}\lambda_b k \sigma_W^2 (\mathrm{corr}^2(q_b, W) - 1) - \frac{\sigma_W}{\sigma_{q_b}}(E(q_b) - q_f)\mathrm{corr}(q_b, W))$$

[1] The indifference price here is different from the indifference price found in Alexandridis and Zapranis (2013). This is because of calculation error in Alexandridis and Zapranis (2013). See section A.1 in appendix for full derivation of the indifference price equation.

where $\text{corr}(q_b, W)$ is the correlation between the return on risky production activity and the payoff of the derivative.

The price F_b consists of the discounted value of the expected payoff $E(W)$ and a risk premium π_b. Assuming $\lambda_b > 0$ and $\text{corr}(q_b, W) < 0$, the first term in π_b will always be negative. Also assuming $E(q_b) - q_f > 0$, the second term will be positive. Hence, the sign of the risk premium depends on the size of the parameters. With a high correlation between return on risky investment and derivatives payoff, the first term becomes smaller and the second term becomes larger. With a low risk aversion and a high expected return on production, the risk premium will be positive. The price of the seller can be found in a similar way:

$$F_s = \frac{1}{q_f}(E(W) - \frac{1}{2}\lambda_s k \sigma_W^2(\text{corr}^2(q_s, W) - 1) - \frac{\sigma_W}{\sigma_{q_s}}(E(q_s) - q_f)\text{corr}(q_s, W)) \qquad (20)$$

$$= \frac{1}{q_f}(E(W) + \pi_s)$$

with

$$\pi_s = -\frac{1}{2}\lambda_s k \sigma_W^2(\text{corr}^2(q_s, W) - 1) - \frac{\sigma_W}{\sigma_{q_s}}(E(q_s) - q_f)\text{corr}(q_s, W))$$

The sign of π_s depends on the correlation between payout and market return. If the correlation is negative, the payout is high when the market return is low. Hence, the seller requires a positive premium. If the correlation is positive, the seller's payout is high when the market return is high. The seller will therefore require a lower price on the weather derivative, so the premium is negative. When the seller is risk neutral, $\lambda_s = 0$, and the correlation coefficient is zero, the price of the derivative is equal to the expected value of the payout. Trading of the derivative between seller and buyer can only take place if the price the buyer is willing to pay is higher than the price the seller demand. This can only happen if

$$-\frac{(E(q_b) - q_f)\text{corr}(q_b, W)}{\sigma_{q_b}} > -\frac{(E(q_s) - q_f)\text{corr}(q_s, W)}{\sigma_{q_s}} \qquad (21)$$

There are several drawbacks related to the model. This is a very parameter rich model and many of the parameters that need to be estimated is uncertain. This results in different prices for different estimates. Also, utility functions are dependent on investors risk preferences. Brockett et al. (2009) state that other objective functions such as the power utility function and the mean-variance utility function can be applied. The latter one has been widely cited in literature and also proposed by Brockett et al. (2006). They expect the results would probably differ under different assumptions and suggest that investors adopt other objective functions if more appropriate. We cannot know the real utility function of an investor and extra care must be taken when choosing an utility function.

4.3.3 Pricing by ADS, Historical densities and Edgeworth densities

A pricing methodology for snow level options based on the methodologies of Alaton et al. (2002), Rubinstein (2000) and Stuart and Ord (1987) was proposed by Beyazit and Koc (2009). First, Beyazit and Koc (2009) follow the pricing method proposed in (Alaton et al., 2002) which was derived from the framework of Black and Scholes (1973). The paper focuses on temperature derivatives since temperature is the most used underlying variable. However, the pricing method can be applied on any weather parameter and here we will present it with snow level as the underlying variable. Alaton et al. (2002) start by finding a stochastic process describing the weather parameter, S_t. From the daily snow level values S_t, form H_i as the yearly cumulative snow level for year i. Extract the expected mean and variance from this series. By either defining a process describing daily snow level movements or by using historical data of n years we have that:

$$E(H_i) = \mu_n \quad \text{and} \quad Var(H_i) = \sigma_n^2$$

$H_i = \sum_{t=1}^{T} S_t$, where S_t is daily snow level and H_i is N $\sim (\mu_n, \sigma_n^2)$. As Beyazit and Koc (2009) we will for simplification use historical mean and variance values in our application. Recall from section 4.1 that the discounted payoff of the snow level put option is

$$p(t) = \theta \exp(-r_f(T - t_0))E[\max(K - H_i), 0] \tag{22}$$

where $\theta \max(K - H_i)$ is the payoff the holder of the put option receives if the strike is higher than actual snow level, with tick size θ. The term $\exp(-r_f(T - t_0))$ represents the discount factor. Here, r_f is the risk free rate, T denotes the terminal value and t_0 is present time. Equation (22) represents the discounted payoff of the put option at time t_0. The magnitude of the payoff is dependent on the tick size θ and the strike level K. Each year's cumulative snow level must be standardized by using the following formula

$$x = \frac{H_i - \mu_n}{\sigma_n}$$

The price of the claim (22) is then:

$$\begin{aligned}
&= \theta \exp(-r_f(T-t_0)) \int_0^K (K-x) f_{Hi}(x)dx \\
&= \theta \exp(-r_f(T-t_0))[(K-\mu_n)(\phi(\alpha_n) - \phi(-\frac{\mu_n}{\sigma_n}) + \frac{\sigma_n}{\sqrt{2\pi}}(\exp(-\frac{\alpha_n^2}{2}) - \exp(-\frac{1}{2}(\frac{\mu_n}{\sigma_n})^2)]
\end{aligned} \quad (23)$$

σ_n , μ_n, ϕ denote the standard deviation of cumulative snow level, mean of annual cumulative snow level and cumulative distribution function for standard normal distribution, respectively. The parameter of cumulative distribution function α_n is $\frac{K-\mu_n}{\sigma_n}$. Equation (23) displays the pricing formula of the put option in terms of normal distribution. As mentioned earlier, cumulative snow level is not normal distributed. The mean of cumulative snow level are used in the method of Alaton et al. (2002). Thus, each historical year are therefore given the same weights despite different payoff.

Still assuming normal distribution, Beyazit and Koc (2009) use a version of HBA to modify the pricing method by giving years with different payoffs different weights using historical densities, $a(x)$. First, the probability density function is found using the first two moments from historical data

$$a(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-(x-\mu_x)^2/(2\sigma_x^2)] \quad (24)$$

From the calculated $a(x)$ we observe that years with low snow level values and thus high payoff are given corresponding large weights. The price of the snow level put option is now given by

$$p(t) = \theta \exp(-r_f(T-t_0)) \frac{1}{\sum_{i=1}^{n} a_i(x)} \sum_{i=1}^{n} a_i(x) \max(K - \sum_{t=1}^{T} S_{i,t}, 0) \quad (25)$$

Both of the above pricing formulas (23) and (25) assume a normal distribution in the weather parameter. However, cumulative snow level is not normal distributed and thus, the modified formula (25) is not suitable for direct application (Beyazit & Koc, 2009).

To account for the non-normality in the data, Beyazit and Koc (2009) propose the use of “Generalized Edgeworth Series Expansions”. This method is used by Rubinstein (1994), Rubinstein (2000) and Jarrow and Rudd (1982) in the framework of option pricing. The Black scholes formula assumes that the distribution of the underlying asset is lognormal with skewness $=$ 0 and kurtosis $=$ 3. Following Jarrow and Rudd (1982) the present value of a weather derivative can be approximated by incorporating values of skewness and kurtosis different from the normal distribution by using edgeworth series expansion. The method uses a more manageable alternative distribution, $a(x)$, to approximate the true underlying probability distribution, $f(x)$. Following Stuart and Ord (1987) the adjusted edgeworth density can be written in terms of its nth order cumulants and Hermite polynomials, denoted as k_n and h_n respectively (Lee, 1998, p. 74):

$$\begin{aligned} f(x_i) = [1 &+ \frac{1}{3!}k_3h_3(x_i) + \frac{1}{4!}k_4h_4(x_i) + \frac{10}{6!}k_4^2h_6(x_i) + \frac{1}{5!}k_5h_h(x_i) \\ &+ \frac{35}{7!}k_3k_d4h_7(x_i) + \frac{280}{9!}k^3k_3h_9(k_i) + \frac{56}{8!}k_3k_5h_8(x_i) + \frac{35}{8!}k_4^2h_8(x_i) \\ &+ \frac{2100}{10!}k_3^2k_4h_{10}(x_i) + \frac{15400}{12!}k_3^4h_12(x_i)]a(x_i) \end{aligned} \tag{26}$$

k_n are parameters that can be expressed in terms of the different moments. Expansions higher than the forth moment can lead to fluctuations at the tails of the distribution leading to potential negative values. Hence, in most applications only the first four moments (mean, variance, skewness and kurtosis) are used (Johnson, Kotz, & Balakrishnan, 1994, p. 30). Because the first four moments affect option pricing, Jarrow and Rudd (1982) point out that by including these, the most important influence from the underlying (true) distribution are taken into account. By using skewness and kurtosis extracted from the historical data, $a(x_i)$ can be modified and transformed to general edgeworth density:

$$\begin{aligned} f(x_i) = [1 &+ \frac{1}{6}\xi(x_i^3 - 3x_i) + \frac{1}{24}(\kappa - 3)(x_i^4 - 6x_i^2 + 3) \\ &+ \frac{1}{72}\xi^2(x_i^6 - 15x_i^4 + 45x_i^2 - 15)]a(x_i) \end{aligned} \tag{27}$$

By construction, the first moment of the true distribution is identical to the first moment of the alternative distribution. The difference between the true underlying distribution $f(x_i)$ and the alternative distribution $a(x_i)$ (normal distribution) is then expressed by the general edgeworth series consisting of the higher order moments of both distributions.

A normal distribution is characterized by values of skewness and kurtosis of 0 and 3, respectively. Then, $f(x_i) = a(x_i)$. Our data contains values of skewness and kurtosis that indicate non-normality. The second term in equation (27) adjusts $a(x_i)$ for any differences in skewness between $a(x_i)$ and $f(x_i)$. The third term adjusts $a(x_i)$ for excess kurtosis. The fourth term consists of skewness of second order (Jarrow & Rudd, 1982). The third, fourth and sixth Hermite polynomials are also used in the equation. This is beyond the scope of our thesis, and a description of Hermite polynomials will not be conducted here. Using $f(x_i)$ as the true underlying distribution of the snow level at maturity, the expected value of a snow level put option can be obtained. In (25), replace $a(x_i)$ with the adjusted edgeworth densities $f(x_i)$ from (27). Thus, the snow level put price when incorporating the first four moments can be obtained as follows:

$$p(t) = \theta \exp(-r_f(T - t_0)) \frac{1}{\sum_{i=1}^{n} f_i(x)} \sum_{i=1}^{n} f_i(x) \max\left(K - \sum_{t=1}^{T} S_{i,t}, 0\right) \tag{28}$$

By applying general edgeworth series expansion we have corrected for non-normality and obtained the expected value for the snow level option at maturity in terms of the alternative distribution (normal distribution). Unfortunately, this expansion is only an approximation. Only probabilities for the 20 observed values are taken into account. Therefore, $\sum_{i=1}^{n} f(x_i) \neq 1$ because of possible omitted values. To adjust for this, after the expansion, the probabilities are rescaled by also dividing by $\sum_{i=1}^{n} f(x_i)$ (Rubinstein, 1998).

5 Empirical application to case: The Use of Snow Level Options for Vassfjellet Skiheiser AS

We will in the following apply the same pricing methods as have been applied in earlier studies related to precipitation derivatives. The pricing of weather derivatives related to the case of winter tourism has to our knowledge only been done by Beyazit and Koc (2009). In addition we will compare the approach by Beyazit and Koc (2009) with a pricing method based on the indifference method. Xu et al. (2007) applied the indifference pricing method on rainfall derivatives in the agriculture sector in Germany. To our knowledge, no one has done this to the ski resort sector.

Ski resorts in Norway are highly affected by weather risk (Aaheim et al., 2009). Vassfjellet Skiheiser AS, located 8 km south of the city Trondheim is one of those. During the relevant ski season of December to April, the sum of cumulative snow level in Løksmyr (the weather station near Vassfjellet) varied between 1141 cm and 8799 cm over the last 20 years. The correlation between cumulative snow level and operating income is 0.84. "For Vassfjellet, the amount of people visiting is highly dependent on the snow level", operating manager Per Eyvind Tellungen said to the local news paper Adresseavisa (*Adressa*, 2016). This is also the case for other ski resorts in the area. Arnulf Erdal, managing director at Oppdal Skisenter said they have more guests when there is high levels of natural snow, compared to artificial snow. Looking at the years 1996 to 2015, the standard deviation in cumulative snow level is 2324 cm. Because of high correlation with operating income, a snow level derivative may be used to smooth operating income.

We will in the following calculate both the seller's and Vassfjellet's willingness to sell/buy such a derivative by using the indifference pricing method. Then we will compare these prices with marked prices calculated using the methods proposed in (Beyazit & Koc, 2009). Lastly we will use these prices and check whether Vassfjellet can reduce its volatility in operating income by buying a snow level put option. Following Beyazit and Koc (2009) we will use a linear approach when pricing the snow level put option. Hence, the ski resort receives a fixed amount of money per centimeter of cumulative snow level below the strike level. From the linear relationship between cumulative snow level and operating income represented in equation (1), it is seen that Vassfjellet's operating income decreases by 800 kr per cm of cumulative snow level decreased. We will therefore in the further application set the tick size equal to 800 kr. The pricing methods can be used by all ski resort establishments, but the tick size must be changed to account for the individual ski resort's relationship between snow level and operating income. This relationship will vary for different ski resorts and the tick size is therefore a firm-specific value. Based on this information, a weather derivative on the snow level data from weather station Løksmyr can be designed. In this case, a payoff-profile similar to a put option is relevant. When snow level is below strike, Vassfjellet will recieve compensation. For snow levels above strike, the option will not be exercised. The price of the option must be paid in both cases.

The strike level for Vassfjellet is set equal to 2500 cm. By this, Vassfjellet is hedging against the years with snow level below 2500 cm. This would only cover the worst years. From the historical data it is seen that they would have exercised the option in 2014 and 2015, the two years with the lowest operating income. A cumulative snow level of 2500 cm is far below the average level of 4341 cm. There is several reasons for why we are choosing a relative low strike value. First, choosing a higher strike level will be more expensive for the buyer. The seller of the put option requires a higher premium since the payoff increases. Second, the purpose of the hedge is not to increase average revenues, but to hedge against the worst scenarios. A year with cumulative snow level below 2500 cm may give a significant low operating income, and in the worst case can lead to bankruptcy. Furthermore, we observe from the data plot in figure 5 that snow level values above approximately 3000 cm will not significantly affect the operating income. Hence, hedging against these years may not be relevant. The strike level depends on how risk averse the buyer is. With a strike set to 3500 cm, the buyer is more risk averse, and the option would have been exercised in 11 out of 20 historical years. A strike level of 4500 cm is for extremely risk averse buyers who want to hedge against all bad winters. Later we will compute the prices for strikes between 2500 cm and 4500 cm, but we will use the strike level of 2500 cm in the further application. The payoff received from the put option at the expiration date T depends on the strike level K, the tick size θ and the cumulative snow level index H_i:

$W_T = \max(K - H_i, 0)\theta$

5.1 Indifference pricing method

In the following empirical application, an indifference price for Vassfjellet is found. The utility of Vassfjellet's operating income, X, is maximized using a negative exponential utility function

$$U(X) = -\exp(-\lambda X)$$

The expected payoff $E(W)$ and the payoffs standard deviation σ_w are found using historical data. They are 102 440 kr and 261 893 kr respectively. Furthermore, the risk aversion parameter λ must be defined. The relative risk aversion (RRA) parameter reflects how much risk the market participant is willing to take on (Copeland et al., 2014). In general, it is difficult know the precise estimate of the relative risk aversion parameter (Gandelman & Murillo, 2015). The Federal Reserve Bank of St. Louis estimates it to be within the range -0.1 to 2.5 in Norway (Gandelman & Murillo, 2015). The buyers relative risk aversion is set to average 1.25 and operating income X is set to historical average of 5 039 460 kr. The buyers absolute risk aversion can be found by:

$$ARA(X) = \frac{RRA(X)}{X} = \frac{1.25}{5039460} = 2.381 \times 10^{-7}$$

Following Monoyios (2004, p. 251) the sellers absolute risk aversion is $1 \times 10^{-}6$. Oslo Stock Exchange All-Share Index (OSEAX) approximates as market portfolio. 20 years of closing data was obtained and average total return of 14.9% was found as the risky market return r_s. σ_{r_s} amounts to 0.28. Furthermore, correlation between market return and payoff is calculated to -0.05. The correlation is dependent on the payoff of the derivative and changes when the strike level changes. There is generally a low correlation between the market return and the payoff of the weather derivative (Brockett et al., 2009). Risk free rate is set to 1.5%. This is based on a five year norwegian government bond quoted at 0.89 in year 2016 (*Norges Bank*, 2017). We are experiencing times with particualry low interest rates, and therefore 0.6% is added to this rate. Some market participants argue for increasing the risk free rate further. They claim such low interest rates are not sustainable in the longer term (*PWC*, 2016). Our focus is short term risk, and therefore r_f is set to 1.5%.

Additionally, the buyer's return on production activity r_b and correlation between this return and the payoff of the derivative is needed. Return on net operating assets (RNOA) is chosen as measurement of Vassfjellet's production return. The production activity is the main activity driving Vassfjellet's revenues and RNOA captures the return on the company's assets that are generating revenue.

$$\text{RNOA} = \frac{\text{OI} \times 100}{\text{Avg NOA}} = 42,72\% \tag{29}$$

where net operating assets (NOA) is found by deducting operating liabilities from operating assets. The correlation between r_b and the payoff W is calculated to be -0.40. This parameter changes with the strike.

Table 4: Indifference Pricing Method: Common Put Option Characteristics for Buyer and Seller

Parameters	
Tick size θ	800
Strike level K	2500
Time to maturity T	1
Expected payoff $E(W)$	102 440
Standard deviation σ_W	261 893
Risk-free rate r_f	1.5 %
Contract size k	1

Table 5: Indifference Pricing Method: Individual Put Option Characteristics

Parameters	**Buyer**	**Seller**
Expected return on risky activity, $E(r_b)$ and $E(r_s)$	42%	14%
Standard deviation, σ_{q_b} and σ_{q_s}	38%	28%
Correlation, $\text{corr}(q_b, W)$ and $\text{corr}(q_s, W)$	-0.40	-0.05
Absolute risk aversion, λ_b and λ_s	$2.38 \cdot 10^-7$	10^-6

The balance sheet lists assets, liabilities and stockholders equity. The income statement reports how shareholders' equity change as a result of business activities. We are trying to hedge the snow level, which again affects the operating posts in the annual reports. A reformulation of the income statement and the balance sheet is therefore needed in order to more clearly see the results that come from operations. Following the method of Penman (2012) the reformulations are shown in section A.3 in appendix. This method allows us to reformulate the balance sheet into operating and financial assets, and operating and financial liabilities. From this, net operating assets can be found. In the reformulated income statement we summarize the operating activities and report the operating income. The operating income is combined with income and expenses from financing activities to give the comprehensive income. Thus, this reformulation will give us the operational values we need.

Results

Based on the characteristics in table 4 and 5, equations (19) and (20) determine the indifference price of the buyer and potential seller. With a strike of 2500, the buyer's and the seller's indifference price is 140 775 kr and 140 719 kr, respectively. Table 6 shows the indifference prices for a range of different strike levels. We can see that the price for the buyer always exceeds that of the seller. Hence, equation (21) holds and trading can happen between the seller and the buyer. This can also be seen in figure 9. The area between the two lines reflects a price range of possible prices. These prices are based on a contract size of one.

Table 6: Indifference Pricing Method: Put Price

Strike	Buyer	Seller
2000	63 172	42 144
2250	125 551	78 954
2500	203 219	140 719
2750	283 712	208 762
3000	382 254	292 331
3250	506 654	407 016
3500	643 831	537 349
3750	819 394	687 859
4000	1 024 012	852 638
4250	1 232 075	1 027 373
4500	1 442 451	1 212 062

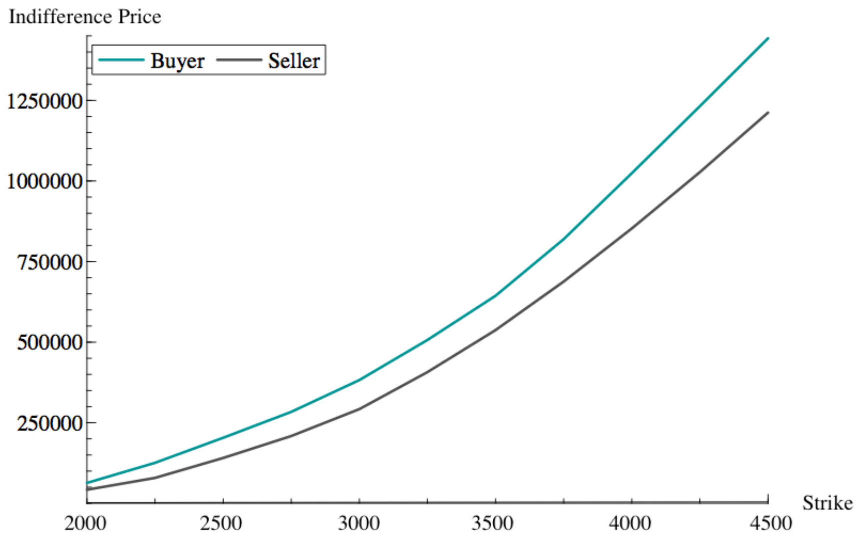

Figure 9: Indifference prices of seller and buyer

Figure 10 graphs the indifference prices for different contract size. By increasing the number of contracts traded between the seller and buyer, the indifference prices will change. Vassfjellet's indifference price decreases with the number of contracts. By buying one contract with strike 2500 cm, they will protect themselves from the worst downside risk. A second contract can be bought if they wish to smooth revenues even more. Then they receive double payoff during bad winters, but also have to pay double price. The second contract is not as crucial as the first one. The variance of the payoff is also very high. Hence, the payoffs are uncertain and the buyer is therefore not willing to pay the same amount per contract when the number of contracts increase. Their indifference price is therefore less per contract. Same reasoning holds for $k > 2$. The seller of the weather derivative requires a higher price for each contract as the number of contracts traded increase. The magnitude of k on the sellers indifference price is

$$\frac{\partial \pi_s}{\partial k} = -\frac{1}{2}\lambda_s \sigma_W^2 (\text{corr}^2(q_s, W) - 1)$$

If $\lambda_s = 0$ and $|\text{corr}(q_s, W)| = 1$ there will be no effect on the sellers indifference price when increasing contract volume. In this case, the seller has a risk aversion of 10^-6 and a correlation coefficient of -0.05. Therefore, the indifference price of the buyer increase with 33 700 kr per additional contract k. Figure 10 shows that for $k < 2.55$, the buyers indifference price is higher than the sellers, and trading can happen. In this area, both parties will benefit from engaging in trading of a contract with price between $F_s(k)$ and $F_b(k)$ (Xu, Odening, & Musshoff, 2008). After this point the seller will require a price higher than what the buyer is willing to give.

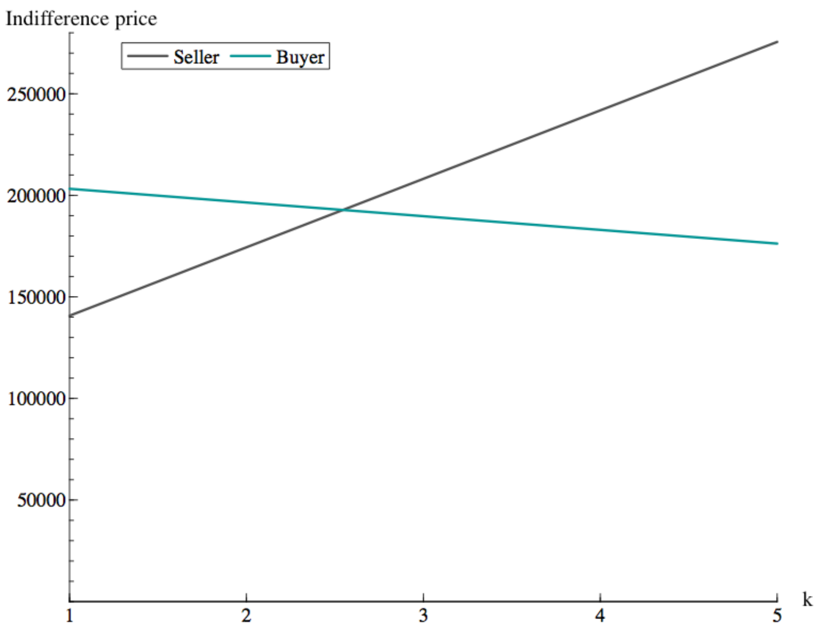

Figure 10: Indifference prices of seller and buyer and contract size

The aim of the case is to check whether Vassfjellet can hedge the downside risk by buying the constructed weather derivative. First, we determine whether Vassfjellet would have exercised the option for all the historical years (2009-2015). The payoff each year will then be $(K - H_i, 0) \times 800$. For the years where the option is exercised, the payoff are added to the operating income and the price of the option is deducted. For the years where $H_i > K$, the operating income will be reduced by the cost of the put option. This gives us the operating income each year when assuming Vassfjellet bought the derivative.

Table 7: Indifference Pricing Method: Operating Income with and without Snow Level Put Option

Year	**Snowlevel**	**Payoff**	**OI with derivative**	**OI without derivative**
2009	8754	0	7 185 623	7 326 398
2010	4620	0	6 878 314	7 019 089
2011	6421	0	6 757 424	6 898 199
2012	5651	0	4 838 634	4 979 409
2013	3425	0	5 624 157	5 764 932
2014	1141	1 087 200	1 526 467	580 042
2015	1965	428 800	2 995 378	2 708 153

As seen in table 7, the winter season in 2014 was extremely bad with 1141 cm of cumulative snow level. If Vassfjellet purchased a put option with a strike level of 2500 cm, they would have exercised the option and realized a payoff of 1 087 200 kr. They would have paid the price of the put option of 140 775 kr and obtained net operating income of 1 526 467 kr. Without a weather contract in place, net operating income was 580 042 kr. During 2013, cumulative snow level was above strike of 2500 cm. The weather option would not have been exercised. Operating income of 5 764 932 kr would have been reduced by the price of the put option, 140 775 kr. This would give an net operating income of 5 624 157 kr. Without a weather contract, operating income would have stayed at 5 764 932 kr. See table 7 for more scenarios. For the years 2009-2015, Vassfjellet's operating income without a contract had a standard deviation of 2 531 641 kr. This amount is reduced to 2 152 529 kr when including a weather contract, a reduction of 379 111 kr.

As mentioned earlier, there is several drawbacks with this method. Many of the parameters are hard to estimate, and different people pricing the exact same option may use different estimates and hence, get different price. The indifference pricing method is also sensitive to changes in the input parameters. Because of this, combined with the difficulty in obtaining correct estimates for the inputs values, we will in the following present a sensitivity analysis on some of the different parameters. The parameters used in the sensitivity analysis are absolute risk aversion for both buyer and seller, return on production, return on market portfolio and the correlation between the two returns and the payoff of the derivative.

Table 8 and 9 show how the indifference prices change with absolute risk aversion and correlation. An agent's risk aversion is hard to measure. We used an estimate of 1.2 based on the average of the range proposed by Gandelman and Murillo (2015). Others may use other sources which will result in a different price, seen in row 5 in both tables. The seller's indifference price range from 104 211 kr to 177 228 kr within the range -0.1 to 2.5. Looking at the correlation between the derivatives payoff and the risky production activity, the buyer's indifference price varies from 120 072 kr to 286 601 kr within the correlation interval of -0.1 to -0.7. The correlation is an important parameter because a small negative increase will make the weather contract more attractive. This parameter must be estimated and depends on the length of the time series of both payoff and the return on risky production. In our case, because of a change in reporting setup, a time series of six years is used when calculating the correlation. If earlier years had been made available for us, the correlation may have been different.

Table 8: Sensitivity Analysis Buyer: Corr(q_b, W), ARA

					ARA			
		-1.98E-08	9.92E-08	1.98E-07	**2.38E-07**	2.98E-07	4.00E-07	4.96E-07
	-0.1	128 701	124 719	121 400	120 072	118 081	114 658	111 444
	-0.2	155 792	151 930	148 712	147 425	145 494	142 174	139 058
	-0.3	182 870	179 209	176 158	174 938	173 108	169 961	167 007
Corr	**-0.402**	210 526	207 154	204 344	**203 220**	201 534	198 635	195 914
	-0.5	236 985	233 968	231 453	230 448	228 939	226 346	223 911
	-0.6	264 022	261 448	259 302	258 444	257 157	254 944	252 866
	-0.7	291 046	288 995	287 285	286 601	285 575	283 812	282 156

Table 9: Sensitivity Analysis Seller: Corr(q_s, W), ARA

					ARA			
		-8.33E-08	4.17E-07	8.33E-07	**1.00E-06**	1.25E-06	1.67E-06	2.08E-06
	0.2	74 250	90 468	103 983	109 389	117 498	131 013	144 528
	0.15	80 194	96 708	110 469	115 973	124 230	137 991	151 753
	0.1	86 152	102 877	116 814	122 389	130 751	144 689	158 626
Corr	**-0.0508**	104 211	121 061	135 103	**140 720**	149 145	163 186	177 228
	-0.1	110125	126850	140 787	146 362	154 724	168 662	182 599
	-0.15	116 154	132 667	146 428	151 933	160 190	173 951	187 712
	-0.2	122 196	138 414	151 929	157 335	165 444	178 959	192 474

Return on risky production was also found from the six years of usable historical data. Therefore, a sensitivity analysis is also conducted on this parameter since it may change when using more years. A change in the risky return of production and risky market return will lead to a significantly change in the buyer's and the seller's indifference price, respectively. In the case of the buyer, an 14% increase from initial value will lead to an increase in her indifferent price by approximately 38 000 kr. The correlation coefficient changes with the strike level. A higher strike will give a higher payoff which results in a higher correlation. This is because the option is more often exercised and less zero values will appear in the payoff. Similar tables for strike level of 3500 is shown in the section A.2 in appendix.

An important reason to form a sensitivity analysis is to check whether for some levels, the price the buyer is willing to give is below the price the seller requires. Table 10 with initial price of 203 220 reflects this. With a decrease in risky production return of approximately 14% and in addition a decrease in correlation from -0.4022 to -0.3, the buyers indifferent price is getting close to the sellers.

Table 10: Sensitivity Analysis Buyer: Corr(q_b, W), q_b

					q_b			
		0.996	1.138	1.280	**1.423**	1.565	1.707	1.850
	-0.1	91 691	101 152	110 612	120 072	129 533	138 993	148 453
	-0.2	90 663	109 583	128 504	147 425	166 346	185 266	204 187
	-0.3	89 795	118 176	146 557	174 938	203 319	231 700	260 082
Corr	**-0.4022**	89 075	127 123	165 171	**203 220**	241 268	279 316	317 365
	-0.5	88 542	135 844	183 146	230 448	277 750	325 051	372 353
	-0.6	88 157	144 920	201 682	258 444	315 206	371 968	428 731
	-0.7	87 933	154 156	220 379	286 601	352 824	419 046	485 269

Table 11: Sensitivity Analysis Seller: Corr(q_s, W), q_s

					q_s			
		-0.114	1.029	1.086	**1.144**	1.200	1.258	1.315
	0.2	342 694	130 767	120 171	109 389	98 978	88 382	77785
	0.15	290 952	132 007	124 060	115 973	108 165	100 218	92271
	0.1	239 041	133 078	127 780	122 389	117 184	111 885	106 587
Corr	**-0.0508**	81 418	135 286	137 979	**140 720**	143 366	146 059	148 753
	-0.1	29 710	135 673	140 971	146 362	151 567	156 866	162 164
	-0.15	-23046	135 899	143 847	151 933	159 741	167 688	175 636
	-0.2	-75 970	135 957	146 553	157 335	167 746	178 342	188 938

Because of the drawbacks with this method, it would be wise to compare the results with other pricing methods. We will in the next section price the same snow level put option following the pricing method in (Beyazit & Koc, 2009).

5.2 Pricing by ADS, Historical densities and Edgeworth densities

The three pricing methods in Beyazit and Koc (2009) is based on different assumptions and will be applied to the case of Vassfjellet.

Pricing method of Alaton, Djehiche and Stillberger (ADS)

The option pricing formula for snow level put option, assuming normal distribution is given in equation (23). $(T - t_0)$ is in our model set to 1, assuming that when winter season end, a contract for next period is entered. The mean of 20 years of cumulative snow level is 4341 cm with corresponding standard deviation of 2324 cm. The parameter of cumulative distribution function α is then -0.79. With a strike level of 2500 and tick size of 800 kr, the price Vassfjellet has to pay for the snow level put option using Alaton et al. (2002)'s pricing formula is calculated to 268 027 kr.

Table 12: ADS: Put Option Characteristics

Parameters	
Time to maturity $T_n - t$	1
Tick size θ	800
Strike level K	2500
Mean μ_n	4341
Std. deviation σ_n	2324
α_n	-0.79

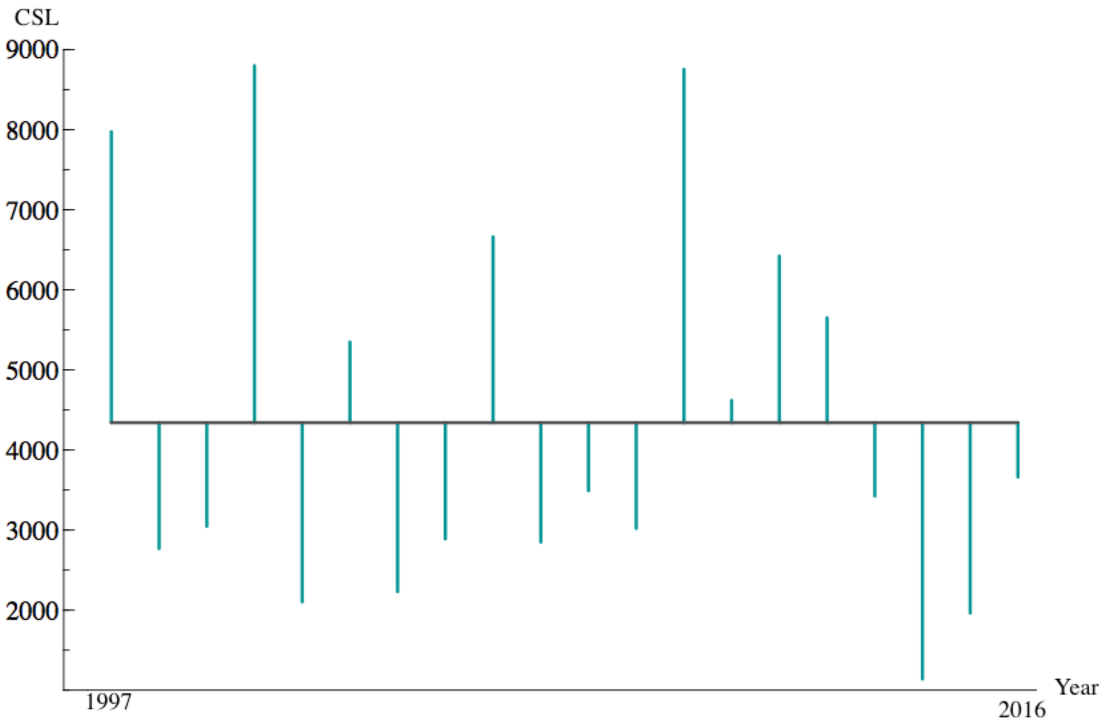

Figure 11: Yearly Cumulative Snow level around Mean

This method assumes a normal distribution on the cumulative snow level. This appears not to be the case. From figure 2 it is seen that cumulative snow level exhibits both right skewness and kurtosis. The yearly cumulative snow level observations appear more frequently below average than above during the 20 years of historical data. Also, more extreme values are found above average of 4341 cm. This can be observed in figure 11. In practice, cumulative snow level will never appear below zero and the possibility of values near zero during a winter season is small. But, extreme winters can occur and therefore the high kurtosis. There are also several observations in our sample with values just below average and hence the right skewness. As discussed in the chapter of data analysis, yearly cumulative snow level fails the normality test and the assumption of normal distribution does not hold in our case.

Historical densities

By still assuming normal distribution in the snow level, we use the historical densities to price the snow level put option. Using the first two moments of the historical snow level data and by standardizing the snow level, the historical densities $a(x)$ were extracted from the probability density function. The yearly values of $a(x)$ can be seen in table 13.

$a(x)$ represents the different weights of the different put option payoffs. To check if the standardization of yearly cumulative snow level is done correctly, the mean and variance of column two in table 13 should be zero and one, respectively.

Table 13: Standarized Snow Level, Historical Densities, Edgeworth Densities

Year	**Standardized snow level**	**Historical densities, a(x)**	**Edgeworth densities, f(x)**
1997	1.563227291	0.000980336	0.001181653
1998	-0.67677615	0.000982102	0.001186226
1999	-0.55675292	0.000982007	0.001103198
2000	1.917704502	0.000980057	0.001273286
2001	-0.962853025	0.000982327	0.001364402
2002	0.433116087	0.000981226	0.00073572
2003	-0.907788604	0.000982284	0.001334323
2004	-0.624292874	0.00098206	0.001149994
2005	0.998386785	0.000980781	0.000915789
2006	-0.641500505	0.000982074	0.001161907
2007	-0.365318019	0.000981856	0.000975477
2008	-0.56793788	0.000982016	0.001110933
2009	1.898345916	0.000980072	0.001270662
2010	0.119937192	0.000981473	0.000756631
2011	0.894710804	0.000980863	0.000868059
2012	0.563463896	0.000981124	0.000755866
2013	-0.394140801	0.000981879	0.000993857
2014	-1.376696565	0.000982654	0.001458157
2015	-1.022649545	0.000982375	0.001393625
2016	-0.292185584	0.000981798	0.000930821

As pointed out in (Beyazit & Koc, 2009), this pricing method using historical densities is a version of the historical burn analysis. In HBA the average payoff is used to determine the option price. By applying $a(x)$ to the pricing formula, different payoff will be given different weights. As observed, the historical densities differ from each other. But, the differences are very small and the put price will be very similar to one calculated with the historical burn method seen in equation (2). The price Vassfjellet have to pay for a snow level put option is calculated to be 101 011 kr using historical densities. In the next section, the non-normality is taken into account by using General Edgeworth Series Expansions.

General Edgeworth Series Expansion

Still assuming x is a standard normal variable, kurtosis and skewness are extracted from historical data. They are 2.29 and 0.71 respectively. Here, $a(x)$ is replaced by the Edgeworth densities $f(x)$. Using this application, Vassfjellet has to pay 130 477 kr for the snow level put option.

Table 14: ADS, Historical Densities, Edgeworth Densities: Put Price

Strike	**ADS**	**Historical densities**	**Edgeworth densities**
2000	207 458	35 305	46 835
2250	234 923	61 573	80 589
2500	268 027	101 011	130 477
2750	307 306	140 449	180 365
3000	353 269	199 285	250 961
3250	406 385	285 322	349 488
3500	467 067	377 312	453 678
3750	535 659	489 206	575 554
4000	612 428	607 486	702 850
4250	697 553	725 766	830 146
4500	791 116	844 046	957 443

It is difficult to know which pricing method that is the best. Beyazit and Koc (2009) emphasize that by using all three described methods, the price range derived reflects the price scale for both buyer and seller. It is seen from figure 12 that for relatively small strike levels, ADS price is higher than the two others. Both Edgeworth Densities and Historical densities will catch up with the ADS price for strike levels of 4000 and 5000, respectively. The Edgeworth densities price is higher than both of the two others for strike levels above 4000. Hence, the ADS price is less sensitive to the strike level. General edgeworth expansions are the only model adjusting for the non-normality in the data. We will therefore use Edgeworh densities when checking if Vassfjellet can smooth revenues by buying this snow level put option.

For the year 2014, assuming Vassfjellet purchased a snow level put option with strike 2500, the option would have been exercised since 1141 cm $<$ 2500 cm. This results in an operating income of 1 536 765 kr. Without the derivative, operating income nets to 580 042 kr. For the year 2013, the snow level option would not have been exercised. The price of the derivative has already been paid and operating income nets to 5 634 455 kr. Without the purchase of the option, Vassfjellet's operating income is 5 764 932 kr. From 2013 - 2014, without a weather derivative operating income varied by 5 184 890 kr. With a weather derivative in place, the variation is 4 097 690 kr. Thus, operating income smooths out with the purchase of the weather contract.

Although this method adjust for the non-normality in the data, it still assumes a linear relationship between snow level and operating income. Improvements of the model could be to incorporate a tick size that changes with the deviation from strike level. So for lower values of H_i, the tick size increase. As discussed in subsection 4.3, the lower bound of cumulative snow level is more crucial for a ski resort, and hence the tick size should be higher for lower values of cumulative snow level.

Table 15: Edgeworth Densities: Operating Income with and without Snow level Put Option

Year	**Snowlevel**	**Payoff**	**OI with derivative**	**OI without derivative**
2009	8754	0	7 195 921	7 326 398
2010	4620	0	6 888 612	7 019 089
2011	6421	0	6 767 722	6 898 199
2012	5651	0	4 848 932	4 979 409
2013	3425	0	5 634 455	5 764 932
2014	1141	1 087 200	1 536 765	580 042
2015	1965	428 800	3 005 676	2 708 153

5.3 Comparing the pricing methods

We have now priced a snow level put option with some of the few pricing models available on snow level that does not require building a dynamic model for the daily snow level. The indifference pricing method reflects the individual buyer's and seller's willingness to pay/sell, whereas the three pricing methods presented in (Beyazit & Koc, 2009) are a measure of the market price of the derivative. The buyer's indifference price derived here reflects Vassfjellet's willingness to buy the constructed snow level option. The price is based on firm-specific characteristics and the tick size of 800 kr is unique for Vassfjellet. Other ski resorts will get a different indifference price on the same option, depending on their firm-specific parameters.

The prices obtained with ADS, Historical densities and General Edgeworth densities are market prices that are the same for buyer and seller. For other buyers than Vassfjellet, the only thing that would differ in the pricing is the tick size. The tick size depends on each firm's sensitivity to the snow level.

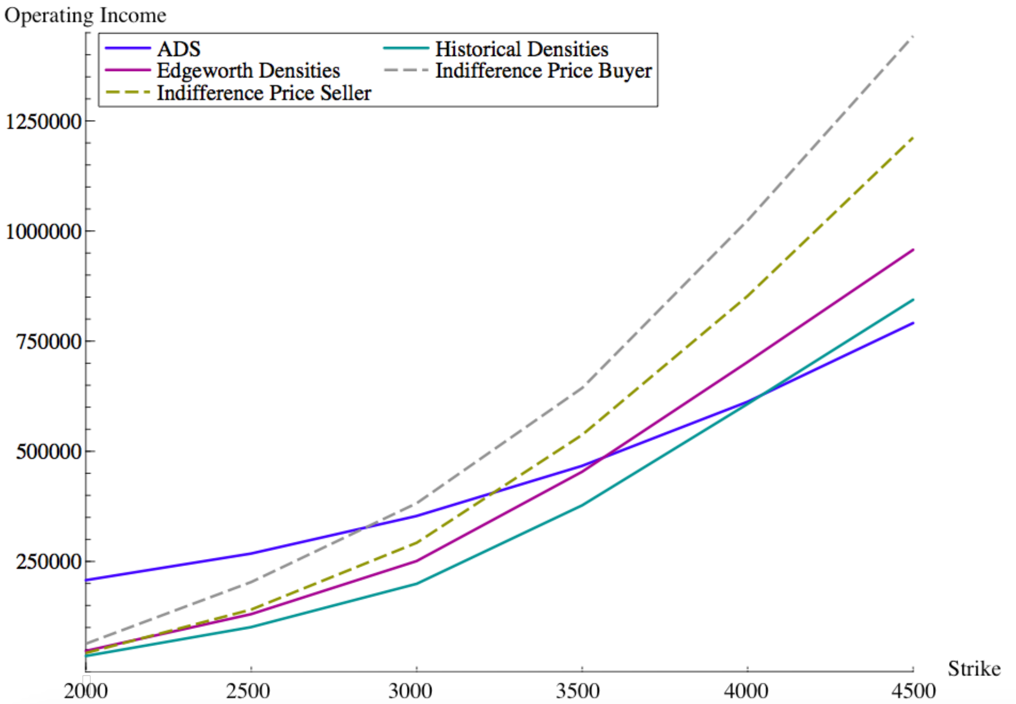

Figure 12: Snow Level Put Option Prices

Despite large differences in the pricing methods, the edgeworth density price and the indifference price for seller are surprisingly similar for the relevant strike level of 2500. The price the seller requires for a snow level put option with strike 2500 is 140 720 kr. The market price for same contract calculated with edgeworth densities is 130 477 kr. If a seller of the contract is calculating the market price using historical densities, he will underestimate Vassfjellet's willingness to pay (distance between dotted grey line and green line). The buyer's price will be lower than the price the seller requires and may result in the trade not happening. The General edgeworth pricing method is the only pricing method adjusting for non-normality in the data, and this price lies in the middle of the price range for all strike levels. The market for snow level derivatives has the last years stagnated, according to Hamisultane (2008) this is partly because of the challenges in correctly pricing snow derivatives. From the graph it is seen that the price of the contract depends on which pricing method the individual agent use.

5.4 Conclusion

Our aim in this study was to price a constructed snow level put option and check if Vassfjellet Skiheiser AS could smooth revenues by using this as a risk management strategy. In order to price the put option, we first needed a tick size for Vassfjellet. This was found by regressing cumulative snow level data on Vassfjellet's operating income. Before pricing the put option with the indifference method, all the relevant parameters were calculated and discussed. Using these, seemingly fair prices for buyer and seller option were found. To check if these prices were reasonable, the price of the put option was also calculated using the methods proposed in Beyazit and Koc (2009). For a strike of 2500, the indifference price of Vassfjellet (buyer) and seller was 293 219 kr and 140 719 kr, respectively. The ADS, Historical Densities and Edgeworth Densities gave prices of 268 077 kr, 101 011 kr, 130 477 kr, respectively.

Further, the prices found using the Indifference method and Edgeworth densities were applied to the case of Vassfjellet. The aim was to check if they can smooth operating income by investing in the constructed snow level put option. As reported, with both the indifference pricing method and the edgeworth density method, the variation in operating income from 2013-2014 decreased with 1 087 200 kr with the purchase of the put option. The standard deviation for operating income during the years 2009-2015 without a weather contract was 2 531 641 kr. Reported in the table, the standard deviation for the same years, but with a weather contract (strike 2500) in place was 2 152 529 kr. Hence, by buying this specific contract, the variance in Vassfjellet's operating income could have been reduced. The purpose of this kind of hedging is to minimize the downside risk, not maximize the cash flows. The result shows that the downside risk was minimized, but Vassfjellet's average operating income would also increase by buying a contract each year from 2009-2015. The operating income without the put option was 5 039 560 kr on average. For comparison, the average operating income with the purchase of the put option was 5 115 142 kr.

Alaton et al. (2002) points out the importance of having a good model for the weather parameter. For further studies or improvements, daily snow level models can be used instead of extracting information from historical data. Also, as discussed in section 4.3, examining the possibility to use a non-linear relationship for the tick size in the models presented in Beyazit and Koc (2009) would also be an interesting approach.

References

Aaheim, H. A., Dannevig, H., Ericsson, T., Oort, B., Innbjør, L., Rauken, T., … Heiberg, E. (2009). *Konsekvenser av klimaendringer, tilpasning og sårbarhet i norge* (Tech. Rep.). CICERO.

Adressa. (2016, Januar). Retrieved 26.04.2017, from `http://www.adressa.no/nyheter/sortrondelag/2016/01/13/Vi-er-helt-avhengige-av-at-det-kommer-sn%C3%B8-i-byen-for-%C3%A5-vekke-opp-folket-12022046.ece`

Alaton, P., Djehiche, B., & Stillberger, D. (2002). On modelling and pricing weather derivatives. *Applied Mathematical Finance*, *9*(1), 1–20.

Alexandridis, A. K., & Zapranis, A. D. (2013). *Weather derivatives - modeling and pricing weather-related risk.* Springer.

Bank, M., & Wiesner, R. (2011). Determinants of weather derivatives usage in the austrian winter tourism industry. *Tourism Management*, *32*, 62–68.

Benth, F. E., & Benth, J. S. (2012). *Modeling and pricing in financial markets for weather derivatives* (O. E. Barndorff-Nielsen, Ed.). World Scientific Publishing Co. Pte. Ltd.

Beyazit, M. F., & Koc, E. (2009). An analysis of snow options for ski resort establishments. *Tourism Management*, *31*, 676–683.

Bossley, L. (1999). *Exposed to weather? corporate finance, issues 170-175.* Euromoney Publication PLC.

Brix, A., Jewson, S., & Ziehmann, C. (2002). Weather derivative modelling and valuation: a statistical perspective. *Climate Risk and the Weather Market. Risk Books, London*, 127–150.

Brockett, P. L., Goldens, L. L., & Wen, C.-C., M-M. Yang. (2009). Pricing weather derivatives using the indifference pricing approach. *North American Actuarial Journal*, *13*(3), 303–315.

Brockett, P. L., Wang, M., & Yang, C. (2005). Weahter derivatives and weather risk management. *Risk Management and Insurance Review*, *8*(1), 127–140.

Brockett, P. L., Wang, M., & Yang, C. (2006). Portfolio effects and valuation of weather derivatives. *The Financial Review*, *41*, 55–76.

Cabrera, B. L., Odening, M., & Ritter, M. (2013). Pricing rainfall futures at the cme. *Journal of Banking Finance*, *37*, 4286–4298.

Cao, M., & Wei, J. (2000). Equilibrium valuation of weather derivatives. *Preprint, Queens University, Kingston, Ontario*.

Cao, M., & Wei, J. (2004). Weather derivatives valuation and market price of weather risk. *The Journal of Futures Markets*, *24*(11), 1065–1089.

Cao, M., Wei, J., & Li, A. (2003). Weather derivatives: A new class of financial instruments.

Cme group. (2009). Retrieved 14.05.2017, from http://investor.cmegroup.com/investor-relations/releasedetail.cfm?ReleaseID=419403

Copeland, T., Weston, J. F., & Shastri, K. (2014). *Financial theory and corporate policy* (4th ed.). Pearson.

Cyr, D., Eyler, R., & Visser, M. (2013). The use of copula functions in pricing weather contracts for the california wine industry.

Damm, A., & Greuell, O. P.-F., W. Landgren. (2016). *Impacts of 2c global warming og winter tourism demand in europa.* Elsevier B.V.

Eydeland, A., & Wolyniec, K. (2003). *Energy and power risk management: New developments in modeling, pricing, and hedging.* Wiley.

Fortune. (2017). Retrieved 14.05.2017, from http://fortune.com/2017/02/09/wall-street-snow-derivatives/

Gandelman, N., & Murillo, R. (2015). Risk aversion at the country level. *Federal Reserve Bank of St. Louis Review*, *97*(1), 53–56.

Geyser, J. M. (2004). Weather derivatives: Concept and application for their use in south africa. *Agrekon*, *43*(4), 444–4644.

Hamisultane, H. (2008). Which method for pricing weather derivatives ?

Hess, U., Richter, K., & Stoppa, A. (2002). Climate risk and the weather market, financial risk management with weather hedges. In (chap. Weather Risk Management for Agriculture and Agri-Business in Developing Coutries). Risk Books.

Ito, H., Ai, J., & Ozawa, A. (2016). Managing weather risks: The case of j. league soccer teams in japan. *The Journal of Risk and Insurance*, *83*(4), 877–912.

Jarrow, R., & Rudd, A. (1982). Approximate option valuation for arbitrary stochastic processes. *Journal of Financial Economics*, *10*, 347–369.

Jewson, S. (2002). Weather derivative pricing and risk management: volatility and value at risk.

Jewson, S. (2004). Weather derivatives and weather derivative pricing. *Risk Management Solutions*.

Jewson, S., & Brix, A. (2005). *Weather derivative valuation: The meteorological, statistical, financial and mathematical foundations.* Cambrigde University Press.

Jewson, S., & Zervos, M. (2003). The black-scholes equation for weather derivatives. *Risk Management Solutions, London*.

Johnson, N. L., Kotz, S., & Balakrishnan, N. (1994). *Continuous univariate distributions* (Second ed., Vol. 1). Wiley.

Kermiche, L., & Vuillermet, N. (2016). Weather derivatives structuring and pricing: A sustainable agricultural approach in africa. *Applied Economics*, *48*(2), 165–177.

Kordi, K. (2012). *Pricing weather derivatives* (Unpublished doctoral dissertation). University of Piraeus.

Lee, T.-W. (1998). *Independent companent analysis: Theory and applications*. Springer.

Leggio, K. B. (2007). Using weather derivatives to hedge precipitation exposure. *Managerial Finance*, *33*(4), 246–252.

Lucas, R. E. (1978). Asset prices in an exchange economy. *Econometrica*, *46*, 1429–1445.

Masala, G. (2014). Rainfall derivatives pricing with an underlying semi-markov model for precipitaion occurrences. *Stoch Environ Res Risk Assess*, *28*, 717–727.

Monoyios, M. (2004). Performance of utility-based strategies for hedging basis risk. *Quantitative Finance*, *4*, 245–255.

Norges bank. (2017). Retrieved 26.04.2017, from `http://www.norges-bank.no/en/Statistics/Interest-rates/Government-bonds-annual`

Penman, S. H. (2012). *Financial statement analysis and security valuation* (5th ed.). McGraw Hill Higher Education.

Perez-Gonzalez, F., & Yun, H. (2013). Risk management and firm value: Evidence from weather derivatives. *The Journal of Finance*, *68*(5), 2143–2176.

Pwc. (2016, December). Retrieved 26.04.2017, from `https://www.pwc.no/no/publikasjoner/verdivurdering/risikopremien-2016.pdf`

Richards, T., Manfredo, M., & Sanders, D. (2004). Pricing weather derivatives. *American Journal of Agricultural Economics*.

Rubinstein, M. (1994). Implied binomial trees. *The Journal of Finance*, *49*(3), 771–818.

Rubinstein, M. (1998). Edgeworth binomial trees. *Journal of Derivatives*.

Rubinstein, M. (2000). On the relation between binomial and trinomial option pricing models. *The Journal of Derivatives*, *8*(2), 47–50.

Sharma, A. K., & Vashishtha, A. (2007). Weather derivatives: Risk-hedging prospects for agriculture and power sectors in india. *The Journal of Risk Finance*, *8*(2), 112–132.

Stuart, A., & Ord, K. (1987). *Kendalls advanced theory of statistic*. Charles Griffin Ltd.

Tang, C.-H., & Jang, S. (2011). Weather risk management in ski resorts: Financial hedging and geographical diversification. *International Journal of Hospitality Management*, *30*, 301-311.

Tang, C.-H., & Jang, S. (2012). Hedging weather risk in nature-based tourism business:

An example of ski resorts. *Journal of Hospitality Tourism Research*, *36*(2), 143–163.

Topdad. (2015). Retrieved 04.05.2017, from `http://www.tmleuven.be/project/topdad/Policybrief2.pdf`

Turvey, C. G. (2001). Weather derivatives for specific even risk in agriculture. *Review of Agricultural Economics*, *23*(2), 333–351.

Varangis, P., Skees, J., & Barnett, B. (2003). Weather indexes for developing coutries. *Forest*, *4*, 3—754.

Weatherworks. (2016, Januar). Retrieved 26.04.2017, from `https://weatherworksinc.com/how-to-measure-snow`

Xu, W., Odening, M., & Musshoff, O. (2007). Indifference pricing of weather insurance.

Xu, W., Odening, M., & Musshoff, O. (2008). Indifference pricing of weather derivatives. *American Journal of Agricultural Economics*, *90*(4), 979–993.

Appendix

A.1 Derivation of Buyers Indifference Price

$$
\begin{aligned}
F_b = {} & \frac{1}{kq_f}(\cancel{-x_b q_f} - \frac{(E(q_b) - q_f)^2}{2\lambda_b \sigma_{q_b}^2} + \cancel{x_b q_f} - \\
& \frac{(E(q_b) - q_f)((E(q_b) - q_f) + (\lambda_b k \text{corr}(q_b, W)\sigma_{q_b}\sigma_W))}{\lambda_b \sigma_{q_b}^2} \\
& - \frac{((E(q_b) - q_f) + (\lambda_b k \text{corr}(q_b, W)\sigma_{q_b}\sigma_W))^2 \cancel{\sigma_{q_b}^2}}{2\lambda_b \sigma_{q_b}^{\cancel{4}2}} \\
& + \frac{((E(q_b) - q_f) + (\lambda_b k \text{corr}(q_b, W)\sigma_{q_b}\sigma_W))k \text{corr}(q_b, W)\sigma_W}{\sigma_{q_b}} - \frac{\lambda_b k^2 \sigma_W^2}{2})
\end{aligned}
$$

$$
\begin{aligned}
F_b = {} & \frac{1}{kq_f}(\cancel{-\frac{1}{2}\frac{(E(q_b) - q_f)^2}{\lambda_b \sigma_{q_b}^2}} + \cancel{\frac{(E(q_b) - q_f)^2}{\lambda_b \sigma_{q_b}^2}} - \frac{(E(q_b) - q_f)(\lambda_b k \text{corr}(q_b, W)\sigma_{q_b}\sigma_W)}{\lambda_b \sigma_{q_b}^2} \\
& + kE(W) \cancel{- \frac{1}{2}\frac{(E(q_b) - q_f)^2}{\lambda_b \sigma_{q_b}^2}} - \frac{2(E(q_b) - q_f)(\lambda_b k \text{corr}(q_b, W)\sigma_W \sigma_{q_b})}{2\lambda_b \sigma_{q_b}^2} \\
& - \frac{1}{2}\frac{(\lambda_b^2 k^2 \text{corr}^2(q_b, W)\sigma_{q_b}^2 \sigma_W^2)}{\lambda_b \sigma_{q_b}^2} + \frac{(E(q_b) - q_f)k \text{corr}(q_b, W)\sigma_W}{\sigma_{q_b}} \\
& + \frac{\lambda_b k \text{corr}(q_b, W)\sigma_{q_b}\sigma_W k \text{corr}(q_b, W)\sigma_W}{\sigma_{q_b}} - \frac{\lambda_b k^2 \sigma_W^2}{2}) \\
F_b = {} & \frac{1}{kq_f}(-\frac{(E(q_b) - q_f)(k \text{corr}(q_b, W)\sigma_W)}{\sigma_{q_b}} + kE(W) - \frac{(E(q_b) - q_f)(k \text{corr}(q_b, W)\sigma_W)}{\sigma_{q_b}} \\
& - \frac{1}{2}(\lambda_b k^2 \text{corr}^2(q_b, W)\sigma_W^2) + \frac{(E(q_b) - q_f)(k \text{corr}(q_b, W)\sigma_W)}{\sigma_{q_b}} \\
& + \lambda_b k \text{corr}(q_b, W)\sigma_W k \text{corr}(q_b, W)\sigma_W - \frac{1}{2}\lambda_b k^2 \sigma_W^2)
\end{aligned}
$$

Which gives us the pricing formula:

$$
F_b = \frac{1}{q_f}(E(W) + \frac{1}{2}\lambda_b k \sigma_W^2(\text{corr}^2(q_b, W) - 1) - \frac{\sigma_W}{\sigma_{qb}}(E(q_b) - q_f)\text{corr}(q_b, W)) \tag{30}
$$

A.2 Tables for Sensitivity Analysis

Table 16: Sensitivity Analysis Buyer: Corr(q_b, W), ARA, Strike 3500

		ARA						
		-1.98E-08	9.92E-08	1.98E-07	**2.38E-07**	2.97E-07	4E-07	4.96E-07
	-0.2	491 756	475 294	461 576	456 089	447 858	433 707	420 423
	-0.3	547 587	531 983	518 979	513 778	505 976	492 562	479 969
	-0.4	603 360	588 957	576 954	572 152	564 951	552 568	540 944
Corr	**-0.52**	670 891	658 402	647 994	**643 831**	637 587	626 851	616 772
	-0.6	714 737	703 763	694 617	690 959	685 472	676 038	667 182
	-0.7	770 339	761 594	754 307	751 391	747 019	739 501	732 444
	-0.8	825 885	819 712	814 567	812 510	809 423	804 117	799 135

Table 17: Sensitivity Analysis Seller: Corr(q_s, W), ARA, Strike 3500

		ARA						
		-8.33E-08	4.17E-07	8.33E-07	1.00E-06	1.25E-06	1.67E-06	2.08E-06
	0.2	316 047	385 178	442 787	465 830	500 396	558 004	615 613
	0.15	328 211	398 602	457 261	480 724	515 920	574 579	633 238
	0.1	340 435	411 726	471 135	494 898	530 544	589 953	649 362
Corr	**-0.068**	382 056	453 730	513 458	**537 349**	573 186	632 914	692 642
	-0.1	389 930	461 221	520 630	544 393	580 039	639 448	698 857
	-0,15	402 453	472 844	531 503	554 967	590 162	648 821	707 480
	-0.2	415 037	484 168	541 777	564 820	599 385	656 994	714 603

Table 18: Sensitivity Analysis Buyer: Corr(q_b, W), q_b, Strike 3500

		q_b						
		0.996	1.138	1.280	**1.423**	1.565	1.707	1.850
	-0.1	340 490	360 022	379 554	399 086	418 618	438 150	457 682
	-0.2	338897	377961	417 025	456 089	495 153	534 217	573 281
	-0.3	337 990	396 586	455 182	513 778	572 374	630 970	689 565
Corr	**-0.5212**	338 420	440 224	542 028	**643 831**	745 635	847 439	949 242
	-0.5	338 234	435 893	533 553	631 213	728 873	826 532	924 192
	-0.6	339 384	456 576	573 768	690 959	808 151	925 343	1 042 535
	-0.7	341220	477944	614 668	751 392	888 115	1 024 839	1 161 563

Table 19: Sensitivity Analysis Seller: Corr(q_s, W), q_s, Strike 3500

		q_s						
		-0.114	1.029	1.086	**1.144**	1.200	1.258	1.315
	0.2	947 514	509 968	488 090	465 830	444 336	422 459	400 581
	0.15	841987	513 828	497 420	480 725	464 604	448 196	431 788
	0.1	735 740	516 967	506 028	494 899	484 151	473 213	462 274
Corr	**-0.0684**	372 510	522 245	529 732	**537 349**	544 705	552 192	559 679
	-0.1	303 552	522 325	533 263	544 393	555 141	566 079	577 018
	-0.15	193 704	521 864	538 272	554 967	571 088	587 496	603 904
	-0.2	83 137	520 683	542 560	564 820	586 315	608 192	630 069

A.3 Reformulation of Income Statement and Balance Sheet

Table 20: Tax distribution. Tax rate 25%

	2015	**2014**	**2013**	**2012**	**2011**	**2010**	**2009**
Taxable Financial Items							
Interest Income	329	289	317	11 249	51 195	137 979	35 326
Other Financial Income	308	21	267	0	17 749	9 799	47 227
Interest Expenses	279 545	334 891	175 618	138 643	26 166	43 728	28 287
Other Financial Expenses	10 131	6 279	13 531	12 421	11 552	25 472	14 946
Net Financial Items	-289 039	-340 860	-188 565	-139 815	31 226	78 578	39 320
Distribution							
Tax Official Income Statement	185 569	-490 938	951 363	874 893	1 474 343	1 574 627	1 525 208
Tax Benefit	80 931	95 441	52 798	39 148	-8 743	-22 002	-11 010
Total Tax	266 500	-395 497	1 004 161	914 041	1 465 600	1 552 625	1 514 198
Tax Other Operating Items	2 240	18 200	0	64 400	28 954	113 094	0
Tax Core Operating Items	264 260	-413 697	1 004 161	849 641	1 436 646	1 439 531	1 514 198

Table 21: Reformulated Income Statement

	2015	**2014**	**2013**	**2012**	**2011**	**2010**	**2009**
Core Operating Income							
Ticket sale	576 0471	3 710 897	9 914 312	9 698 305	11 539 338	11 185 061	12 314 191
Core Sales Revenues	810 792	510 225	1 183 555	1 205 225	1 712 067	1 846 031	2 116 377
Other Operating Revenues	233 496	237 457	141 140	13 817	16 682	30 425	26 904
Cost of materials	414 535	292 172	703 702	625 275	738 682	836 274	915 425
Personnel Expenses	1 807 578	1 799 635	2 561 470	2 785 259	2 961 698	3 124 180	2 945 916
Depreciation	1 739 822	2 183 008	2 178 641	1 951 737	1 783 814	1 893 974	1 929 178
Other Operating Expanses	1 874 493	1 786 730	2 208 903	2 527 404	2 669 508	2 081 974	3 269 733
Core Operating Income before Tax	968 331	-1 602 966	3 586 291	3 027 672	5 114 385	5 125 115	5 397 220
Tax on Core Operating Income	264 260	-413 697	1 004 161	849 641	1 436 646	1 439 531	1 514 198
Core Operating Income	704 071	-1 189 269	2 582 130	2 178 031	3 677 739	3 685 584	3 883 022
Other Operating Items							
Gain on Sale of Non-Current Assets	8 000	65 000	0	230 000	103 407	403 908	0
Total before Tax	8 000	65 000	0	230 000	103 407	403 908	0
Tax (0,28%)	2 240	182 00	0	64 400	28 954	113 094	0
Other Operating Items after Tax	5 760	46 800	0	165 600	74 453	290 814	0
Comprehensive Operating Income	709 831	-1 142 469	2 582 130	2 343 631	3 752 192	3 976 398	3 883 022
Financial Items							
Interest Income	329	289	317	11 249	51 195	137 979	35 326
Other Financial Income	308	21	267	0	17 749	9 799	47 227
Interest Expenses	279 545	334 891	175 618	138 643	26 166	43 728	28 287
Other Financial Expenses	10 131	6 276	13 531	12 421	11 552	25 472	14 946
Net Financial Items before Tax	-289 039	-340 857	-188 565	-139 815	31 226	78 578	39 320
Tax Benefit	80 931	95 440	52 798	39 148	-8 743	-22 002	-11 010
Net Financial Items after Tax	-208 108	-245 417	-135 767	-100 667	22 483	56 576	28 310
Comprehensive Income	501 723	-1 387 886	2 446 363	2 242 964	3 774 675	4 032 974	3 911 332

Table 22: Reformulated Balance Sheet

	2015	**2014**	**2013**	**2012**	**2011**	**2010**	**2009**
Operating Assets							
Fixed Assets	8 666 417	10 319 417	11 384 417	13 365 417	8 404 417	9 317 417	9 307 417
Inventories	53 734	129 929	80 479	103 739	82 136	55 488	17 200
Receivables	132 683	1 451 295	114 930	789 433	314 454	246 659	104 320
Deffered Tax Assets	703 498	595 119	457 644	339 469	430 615	370 513	460 783
Cash	32 820	12 157	22 875	23 644	1 165 980	2 914 366	1 583 408
Financial non-current Asset	0	0	0	0	370 000	0	0
Total Operating Assets	9 589 152	12 507 917	12 060 345	14 621 702	10 767 602	12 904 443	11 473 128
Operating Liabilities							
Tax Payable	0	0	0	0	0	0	0
Value Added Taxes	9 663	37 340	17 785	46 841	52 175	73 787	34 766
Short-term corporate debt	1 088 697	0	3 819 779	2 799 098	5 487 429	6 809 145	5 752 426
Other Short- Term Debt	173 630	262 026	192 161	221 668	158 497	582 020	634 035
Pension Liabilities	0	0	0	0	0	0	0
Trade Creditors	99 124	146 214	395 770	548 087	325 286	524 237	352 702
Total Operating Liabilities	1 371 114	445 580	4 425 495	3 615 694	6 023 387	7 989 189	6 773 929
Net Operating Assets	**8 218 038**	**12 062 337**	**7 634 850**	**11 006 008**	**4 744 215**	**4 915 254**	**4 699 199**
Financial Assets							
Short-term Financial Assets	0	0	0	0	0	0	0
Total Financial Assets	0	0	0	0	0	0	0
Financial Liabilities							
Long-term Debt	4 275 338	7 826 611	2 966 900	6 034 180	0	0	0
Total Financial Liablities	4 275 338	7 826 611	2 966 900	6 034 180	0	0	0
Net Financial Liablities (NFL)	4 275 338	7 826 611	2 966 900	6 034 180	0	0	0
Equity	3 942 700	4 235 726	4 667 950	4 971 828	4 744 215	4 915 254	4 699 199
Equity + NFL	**8 218 038**	**12 062 337**	**7 634 850**	**11 006 008**	**4 744 215**	**4 915 254**	**4 699 199**