



Norwegian University of
Science and Technology

Learning by DIY

Exploring the Pedagogical Potential of the
Serial Two-Tank System in a Control Theory
Context

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Project Description

A challenge in the engineering education is the implementation and execution of laboratory assignments. Typical examples are logistical problems, outdated lab equipment and assignments that are too streamlined to facilitate and motivate learning.

Building on this, the students shall propose ways to elevate the laboratory assignments in university courses influenced by automation, cybernetics or control theory. Summarized, the student shall, in an engineering context:

- Suggest ways to elevate the execution of practical work.
- Propose a dynamical system suited for educational purposes.
- Demonstrate how the proposed system is suited for cybernetics laboratory assignments by presenting a complete walkthrough of the automated system.
- Produce a proof of concept of such a system and describe how it can be utilized in an educational context.

Acknowledgements

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Abstract

With the aim of elevating the educational outcome of practical laboratory work in an engineering context, this thesis proposes a complete review of how to build, analyze, simulate and control a serial two-tank system (STTS). This is then demonstrated by building, analyzing and controlling a physical STTS prototype.

Further, the thesis presents and demonstrates how modern technology and pedagogical models can be combined to better convey knowledge and link theory and practice. A solution is proposed through an educational concept of four learning modules and several instructional videos, all directly related to the STTS. A proof of concept for such a learning module was then produced, and evaluated through a small pilot test.

Finally, the thesis presents and demonstrates a methodology called *design thinking*, being used to solve the human-centered challenges of creating a pedagogical tool.

Sammendrag

Oppgaven tar for seg konstruksjon, analyse, simulering og regulering av et fysisk serielt to-tanks-system (STTS), og forslår bruksområder for dette systemet som et pedagogisk hjelpemiddel for ingeniørstudenter innen reguleringsteknikk.

Videre presenterer og demonstrerer oppgaven hvordan læringsutbyttet av praktisk arbeid kan løftes ved å kombinere ny teknologi og moderne pedagogiske modeller. Målet er å bedre vise studenter koblingen mellom teori og praksis. Oppgaven presenterer videre fire læringsmoduler direkte knyttet opp mot STTS, som er ment å gi en god og stegvis innføring i relevante teknikker rettet mot reguleringsteknikk. Et «proof of concept» på en slik læringsmodul ble testet ut gjennom en liten pilottest.

I tillegg til de tekniske utfordringene ved gjennomføringen av denne oppgaven, var det også flere menneskesentrerte utfordringer som måtte tas hånd om, spesielt med tanke på utformingen av det pedagogiske opplegget. Metodikken *design thinking* ble derfor innlemmet som en viktig del av oppgaven, og vil bli presentert og demonstrert som en del av oppgaven.

Notations, Abbreviations and Terminology

Notations	Explanation
x	Example of scalar notation
\vec{x}	Example of vector notation
X	Example of matrix notation
\dot{x}	Derivative of x with respect to time, dx/dt
Δx	A finite difference, expressed as $x_b - x_a$
$(\cdot)^{m \times n}$	Matrix of dimensions $m \times n$. Note that if either $m = 1$ or $n = 1$ the notation defines a vector, and if $m = n = 1$ the notation defines a scalar
\mathbb{R}^+	Non-negative real numbers, or $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x \geq 0\}$
$\mathbf{0}$	The zero symbol normally represents a normal zero, but might represent a zero-vector or -matrix of the appropriate dimension

Abbreviations and terminology	Explanation
Autonomous system	From [1], an autonomous system is defined as a system that does not depend on time.
ET	External Theory. Used to reference theorems, lemmas and corollaries from external literature.
DIY	Do it yourself
MDS	Motor Driver Shield
PCB	Printed Circuit Board
PWM	Pulse Width Modulation
STTS	Serial Two-Tank System

Software and Hardware used in the Thesis

Software used Throughout the Thesis

Name of software	Version	Developer	License	Usage
Matlab (w/ Simulink)	R2016b	Mathworks	Student	Scripting, GUI, simulation, control design, real-time control
SolidWorks	2016-2017	SolidWorks Corporation	Student	3D modeling and virtual prototyping
DipTrace	3.0	Novarm Limited	Free-ware	Schematic and circuit board design
InkScape	0.92.1	Launchpad	Free-ware	Vector graphics illustrations
Multisim	14.0	National Instrument	Student	Circuit simulation

Hardware used to Observe, Actuate, and Control the System

Model name	Type	Developer
Optiplex 990	Desktop Computer	Dell
Uno	Microcontroller	Arduino
eTape	Water level sensor	Milone Technologies
JT-800	Water pump	Javtop

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1 Introduction

The thesis presents the building, analysis, and automation of a serial two-tank system (STTS). This system is meant to be a proof of concept for a pedagogical tool for college and university courses influenced by control theory, automation or cybernetics.

Further, the thesis proposes a pedagogical approach for implementing this system in an educational setting through several laboratory assignments, showing students the process from hardware to a fully automated system. The thesis presents and demonstrates how the use of modern technology and new pedagogical models can be combined to help modernize and elevate the learning outcomes of practical work and help students to better understand the links between theory and practice.

Finally, since a substantial part of the work behind this thesis is done according to the design thinking methodology, this process is both introduced and documented as part of the report. The methodology was utilized to solve problems that had a more human-centered nature rather than a technical one.

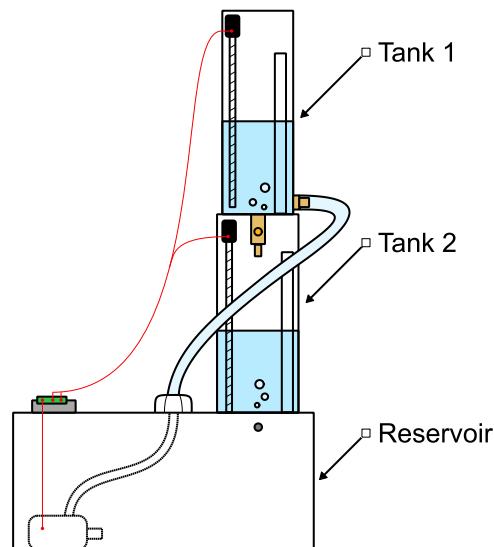


Figure 1-1: Illustration of the final design of the STTS. The figure is meant to clarify the naming convention used refer to the tanks and reservoir, while giving a simplistic view of the system.

1.1 Motivation and Background

The motivation behind this thesis is based on the authors experience as an engineering student, and his personal assessment of the practical laboratory work that has been part of his education. This experience is gained by having studied at four different higher educational institutions, having worked part time jobs as a lab instructor and a substitute teacher, and from two years as a class representative for the two-year master program at the Technical Cybernetics program at the Norwegian University of Science and Technology (NTNU). Based on his experience as a student, an educator, and feedback received both directly from classmates and students he has taught, and indirectly through a yearly survey he managed as a class representative, he has seen a potential for improvement in the way the practical work in classical engineering courses are taught, which is both the motivation, and the aim for this thesis to investigate.

1.2 Structure of the Thesis

The work related to this thesis demanded a fusion of three distinct fields – control theory, pedagogics and design (thinking). With the intent of easily present and discuss the impact from each field, the work related to each field is separated into distinct chapters. This weakens the chronological narrative of the report to some extent, motivating the following description of the thesis structure.

Since readers with an engineering background might be unacquainted with concepts and methods from design thinking and pedagogics, the thesis starts by introducing relevant concepts in Chapter 2 and 3 respectively.

During the initial work with this project, potential systems suited for an educational purpose were investigating, resulting in the further working with the STTS. The process of this investigation is considered a result of the report, and the process of elimination is presented in Chapter 8.1. All relevant theory related for the construction, analysis and control of the STTS is presented in Chapter 0.

Chapter 5 introduces the physical system that was built as part of this thesis, presenting the sensors, actuator and product design of the STTS. A complete summary of the system is presented Chapter 5.2.1, listing the measurements and estimated coefficients used when simulating the system. The process of estimating these coefficients is described later in Chapter 6.2.

All work related to the mathematical modeling, analysis and control of the STTS is presented in Chapter 0. This chapter is the foundation for the system simulator, where the chapter further describes and reviews the attempt to make the simulation imitate the physical behavior of the system. This chapter concludes the work directly related to the STTS.

Based on the theory from Chapter 3 and two interviews presented in Appendix A and Appendix B, a pedagogical concept is proposed in Chapter 0, suggesting four learning modules that can be demonstrated using the STTS. One of these modules was subjected to a small pilot test, which is elaborated in Chapter 8.3.

The material presented in Chapter 8 demonstrates, with several examples, how the design thinking methodology was implemented and executed to elevate the system design and the pedagogical concept.

Finally, Chapter 0 discusses the end results with respect to the proposed system and its pedagogical potential, while Chapter 0 summarizes the findings from a technical and pedagogical standpoint, and gives suggestions for further work.

2 The Design Thinking Methodology

Students in the engineering field are often trained to solve problems in complex systems by making assumptions and generalizations, and are trained to make sure things work in theory before implementing something in real life. This approach has proved itself useful when working solving problems of a technical nature. However, this approach is problematic when designing human centered services and solutions with an end user in mind, as the challenge of finding the right problem can be just as challenging as finding the right answer.

Design thinking shifts the focus from the pure technical problem to a human centered problem., putting the understanding of context and continuous engagement with the end user at the heart of its practice [2]. This human-centric methodology aims to integrate expertise from design, social sciences, engineering, and business [3], with the goal of giving the practitioner(s) a better capacity to innovate [4]. This way of merging fields, and the placement of design innovation, is illustrated in Figure 2-1.

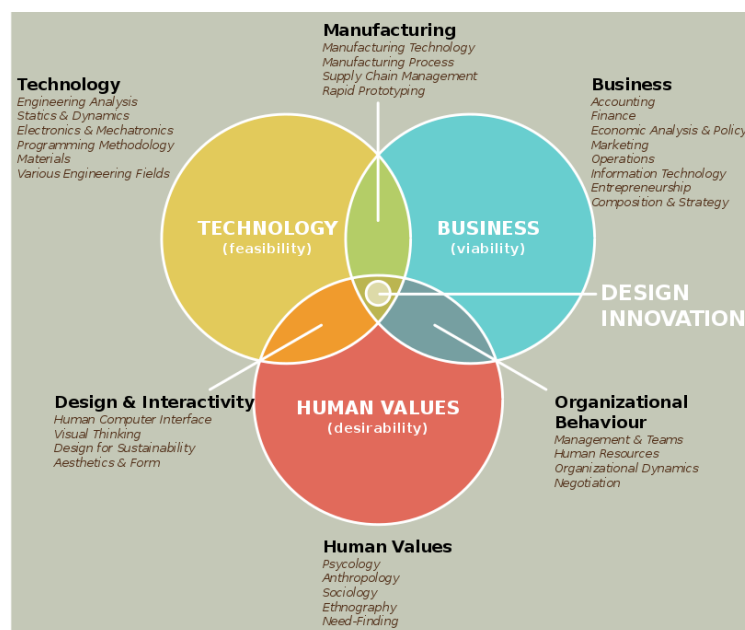


Figure 2-1: Illustration of how design thinking tries helps innovation by combining the mindset of technology, business, and human values.

For this thesis, the methodology was applied as a guideline for how to design a service for students and teachers. While the technical solutions of how to model and control the system is a classical cybernetics problem, deciding on the

best system, designing laboratory assignments and designing a *user experience* requires tools not found in classical engineering. By applying the mindset of design thinking, the priority became “how to design *for* the users” rather than potentially making the ultimate device that no one wants.

2.1 History

The concept of design thinking was partly motivated by Simon A. Herbert and his book “The Sciences of the Artificial”, where he criticizes how scientific fields, like engineering, focuses too much on finding the optimal solution for systems imitating the real world, rather than a satisfactory solution for real life problems. As Herbert stated, “*we must trade off satisficing in a nearly-realistic model against optimizing in a greatly simplified model*” [5].

The term “design thinking” was later coined by Tim Brown and Professor David M. Kelley through their work in the design firm IDEO, and gained increasing popularity, among other after Professor Kelly co-founded the Hasso Plattner Institute of Design at Stanford, better known as the d.school. In recent time, the concept of design thinking has been adopted by several fields such as business and engineering. Though the design thinking literature related to engineering mostly focuses on IT engineering, the methodology is finding its way into other engineering fields, and is currently being offered as a stand-alone course at the Department of Mechanical and Industrial Engineering at NTNU [6].

Defining design thinking is somewhat of a challenge, as the term has been adapted by various disciplines. Different disciplines do, to some extent, interpret the methodology differently, sometimes even arguing whether it in fact is a methodology or a bundle of mindsets and philosophies [7]. For the sake of limiting the scope of this thesis, the concept of design thinking presented here will be based mainly on the model used at the d.school.

Finally, before diving into the methodology of design thinking, it is important to establish a definition of the word *design*. Design in the engineering context is described by [8] as follows: “*Engineering design is a systematic, intelligent process in which designers generate, evaluate, and specify concepts for devices, systems, or processes whose form and function achieve clients’ objectives or users’ needs while satisfying a specified set of constraints*” [8].

2.2 The Five Steps of Design Thinking

Following the model from the d.school, five steps are considered the essence of the design thinking methodology [9]. These are illustrated in Figure 2-2, and are

further explained below. Note that the steps are not necessarily completed sequentially, and each step may be visited several times or skipped entirely, working iteratively to better understand the problem and the end user's need. This is done while proposing and testing out prototypes of solutions, with the aim of continually collecting new feedback from the end user.

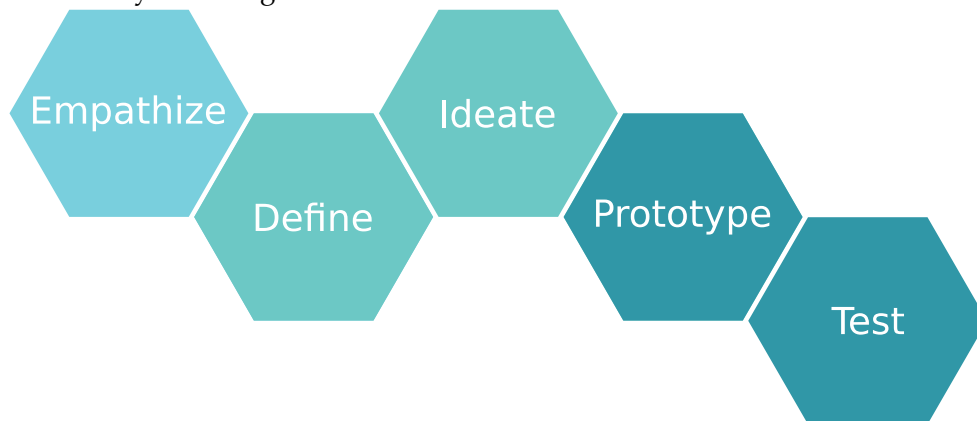


Figure 2-2: The d.school model of the five design thinking steps.

2.2.1 Empathize

The centerpiece of design thinking is to constantly focus on the end user rather than the technical challenges related to the product or service. Through observation, interaction, and immersing oneself in the experiences of the end user, one gets a better understanding of the problem from their point of view (PoV), making it easier to define the problem and later find a solution [9].

2.2.2 Define

Having engaged with the end user through the empathize stage, time should be spent processing and synthesizing the information. The goal is to frame the problem based on the end user's PoV, looking for patterns and making sense of the data collected in the previous stage. The actual problem may be something different than both you and your end user initially thought. As stated by Hasso Plattner, *"Framing the right problem is the only way to create the right solution"* [9].

2.2.3 Ideate

Based on the problem statement from the previous stage, the "ideate" stage is characterized by exploring a wide variety of potential solutions. Instead of focusing on finding the "right" solution, this stage focuses on generating a large

quantity of diverse, viable solutions. By exploring a range of ideas, one can discover unexpected areas of exploration while stepping beyond obvious solutions, and thus increase the potential for innovation. As said by Simon Herbert, *“it is often efficient to divide one’s eggs among a number of baskets. That is, not to follow out one line until it succeeds completely or fails, but to begin to explore several tentative paths, continuing to pursue a few that look most promising at a given moment. If one of the active paths begins to look less promising, it may be replaced by another that had previously been assigned a lower priority.”* [5].

2.2.4 Prototype

Based on the ideas from the previous stage, a collection of ideas of potential solutions are generated. To test out the feasibility of these ideas, one or more prototypes are designed. By giving the ideas a physical form so that people can experience and interact with them, it is possible to both learn and develop more empathy with regards to the end user, the problem, and the solution. Such a prototype can be anything from a physical product to a role-playing activity or a storyboard [9]. Early stage prototypes should be low-resolution that are quick and cheap to make. This process is often called *rapid prototyping*. More effort is usually put into later stage prototypes, when the *need* of the end user (the problem statement) is better defined.

2.2.5 Test

The final stage focuses on testing out the prototype(s) from the previous stage on real users, with the aim of collecting real-life feedback. Observations and feedback from this stage is then used to refine prototypes, gain new insight about the user, and refine your original PoV. As described by Bryan Zmijewski, *“Wrong answers aren’t something to be feared and avoided because they help us get faster at adjusting our thinking through doing. They expose new parts of the problem and help us create better answers. So, we need to push through as many ideas as possible, right or wrong”* [10].

2.3 The Double-Diamond Model of Design

An important part of the design thinking methodology is the process of divergent and convergent thinking. *Divergent thinking* takes place in the concept domain, where concepts or answers themselves do not have “truth values”, that is, they are not necessarily verifiable [8]. The process of gathering data and looking for the problem rather the solution is a typical practice of divergent thinking.

Convergent thinking takes place in the knowledge domain, where the practitioner(s) attempts to converge on and reveal “facts” [8]. As described by Don Norman, “Designers often start by questioning the problem given to them: they expand the scope of the problem, diverging to examine all the fundamental issues that underlie it. Then they converge upon a single problem statement. During the solution phase of their studies, they first expand the space of possible solutions, the divergence phase. Finally, they converge upon a proposed solution” [11].

This workflow can be summarized by the Double-Diamond Model of Design, as shown in Figure 2-3. By actively trying out different approaches and angles before converging to a problem statement or solution, the goal is to expand the so-called problem-space and solution-space as much as possible. This approach helps converging to a more well-informed and feasible solution.

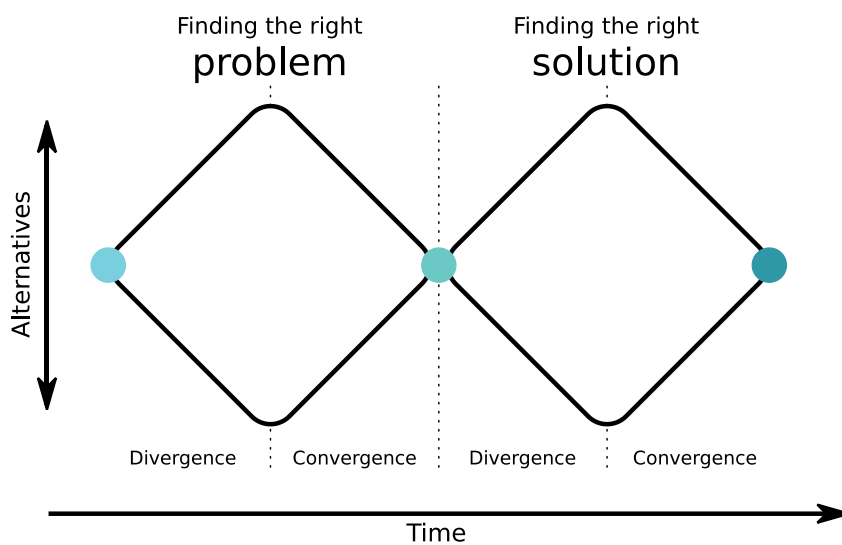


Figure 2-3: The Double-Diamond model of design, as presented in [11].

As mentioned earlier, the design thinking methodology is an iterative approach. The process shown in the Double-Diamond model is often repeated, iteratively expanding the problem- and solution-space while converging towards a better problem statement and solution.

3 Educational and Pedagogical Theory

Since part of this thesis is to present a proposal for how to elevate the implementation and execution of practical work in the engineering curriculum, some pedagogical aspects need to be considered. This chapter therefore presents the highlights from the pedagogical literature research that was found relevant for this thesis.

3.1 The Necessity of Practical Educational Work in an Engineering Context

The importance of including practical and hands-on work as part of the engineering education is something that is recognized by most institutions. As stated in the Norwegian National Guidelines for Engineering Education, part of the skills expected of the engineering graduates are that *“the candidates can apply knowledge and relevant results from research and development to solve theoretical, technical and practical issues within the field of engineering and justify their choices”* [12]. This attitude towards practical and hands-on work is backed up by several other institutions, among others the British Quality Assurance Agency for Higher Education, who concludes that for students to achieve a satisfactory understanding of engineering, a foundation of significant exposure to hands-on laboratory work and substantial individual project work is needed [13].

3.2 Pedagogics for Adults

Implementing practical work is easier said than done. The assignments must be designed in a way that both maximizes the learning outcomes and illustrates practical usage of the theory being lectured. According to Malcolm Knowles [14], who is considered the father of andragogy (pedagogics for adults), there are distinct parts that distinguishes adult education from other types of education. Among others, adults have a rich learning resource through their past experience, making them learn more effectively through experiential learning such as discussions and problem solving, where [15] suggests hands-on learning as one form of experiential learning.

Further, experiences from adult pedagogical initiatives shows, according to [15], that adults have an immediate need to see the benefits of what they learn. Adults also tend to have a traditional view on the structural form of lessons, and often initially expects teacher-led communicative teaching.

3.3 Benefits and Challenges with Educational Practical Work

There are some immediate benefits and challenges that comes with executing practical work. To better summarize these, [13] has highlighted some of the main benefits and challenges with practical assignments, as presented in Table 3-1 and Table 3-2 respectively.

Table 3-1: The main benefits of practical work, according to [13].

<p>Benefits with practical work</p> <ul style="list-style-type: none">• It motivates students and stimulates their interest in the subject• It helps them deepen their understanding through relating theory to practice• It provides opportunities for students to work together on analyzing and solving engineering problems• It helps develop skills and attitudes that will enable graduates to operate efficiently and professionally in an engineering workplace

Table 3-2: The main challenges of practical work, according to [13].

<p>Challenges with practical work</p> <ul style="list-style-type: none">• Laboratory and practical work are expensive to run, sometimes requiring specialized equipment to be purchased that can rapidly become obsolete• Practical sessions are time-consuming to organize, manage and assess• Space and equipment can be limiting• It is often a problem synchronizing the lectures of the associated theoretical concepts with the practical work
--

3.4 Key Learning Goals of Practical Work

Before starting the design of a new laboratory lecture, the facilitator team should first agree and clarify the key learning goals of the practical work and how to achieve these. One goal should, according to [13], be to facilitate students' development as autonomous, lifelong learners. Further, [16] suggests focusing on achieving so-called higher-level aims; lessons that go beyond learning the specific use of equipment. Table 3-3 presents examples of such goals.

Table 3-3: Suggestions for higher level learning goals when designing exercises for practical work, according to [13].

Suggestions for higher level learning goals for practical work
<ul style="list-style-type: none">• Develop experimental, design, problem-solving and analysis skills• Develop data-recording and analysis skills• Familiarize students with equipment, techniques and materials• Develop practical skills• Develop communication and interpersonal skills• Develop technical judgment and professional practice• Integrate theory and practice• Motivate students

When designing practical work, [13] argues that the best way to ensure that creativity and innovation is fostered is to make the students involved in the design of experiments and to develop understanding of the uncertainties and inaccuracies of outcomes from the outset. Further, [13] discusses how classes are often organized around module topics, and concludes that, though this approach may be effective if they mirror the theoretical sessions, they may also prevent students from making links more broadly across the curriculum. Typically, students are required to re-produce routine experiments that yield well-known results rather than developing skills in data recording, analysis and problem solving [13]. To counteract this, [13] suggests several design considerations to consider when designing laboratory activities, adapted from the work of [17]. Table 3-4 presents the suggested overall goals for five types of student laboratory exercises.

Table 3-4: The possible uses and design considerations for five types of student laboratory exercises

Possible uses and design considerations for different approaches of laboratory activities

Demonstration

- Show features of a piece of equipment and how it works
- Demonstrate a particular concept or theory
- Effectively a “show and tell” approach but could be made more interactive by students being asked questions to prompt thinking

Exercise

- Use a skill or technique accurately
- Students follow a procedure to obtain a known outcome - a traditional approach that cannot achieve all goals for practical work and tends to focus on the procedure rather than enquiry
- Students need to know the aims and why it is important to them

Structured enquiry

- Foster deep approach to laboratory learning by encouraging students to take a personal initiative (e.g. planning, experimental design, choice of variables, selection of materials and methods)
- Students are presented with a problem or series of research questions that can be based on real life, together with suggestions for resource materials and a range of equipment/materials to choose from.
- The range of possible outcomes produces individual student solutions and opportunities for collusion and plagiarism are reduced.

Open-ended enquiry

- As above, with more decisions and experimental design considerations resting with the students

Project

- As above, but with major pieces of work that simulate real-life research and development

3.5 Online Education

As mentioned earlier, adults tend to initially expect teacher-led communicative teaching, which is still a commonly used lecturing style. Modern technology has, however, given rise to new ways of conveying knowledge. Online education, also called *eLearning*, is an example of this, where there exist dedicated services focusing purely on this model. Big contributors in this field are Coursera, Udacity, edX, and Khan Academy. These platforms offer services that vary from free courses meant to supplement traditional classroom learning, to specialized, stand-alone courses that results in a course certificate. Within the field of control theory, the private contributor Brian Douglas, with a background in systems engineering, uses YouTube and crowd funding to create free video lectures [18]. These types of videos vary in quality from home office productions to full studio production, and in length from a couple of minutes to full, unedited lectures.

Creating multimedia productions of standard, classroom lectures is also common, and are typically produced by educational institutions. These are often created for internal usage, requiring log-in information to be accessed. In Norway, numerous institutions are trying to exploit online education as part of their lectures, among others NTNU [19, 20], the University of Oslo [21], the University of Agder [22] and Oslo and Akershus College University [23]. These videos are often full-length lectures that are recorded with a minimal amount of post-production, though this is slowly changing. the Department of Electronic Systems at NTNU have exploited the possibilities of online education even further through a pedagogical method called the “flipped classroom”, which will be described in the next chapter.

The existence of these online education services indicates a demand from people who want to be able to access new knowledge at the time and place that suits them best. Focusing on students, [24] found that the primary use of these multimedia productions was in revision and preparation for assessments. This is backed by [25], who found that the students used these videos for remembering better, reviewing old material, helping to solve and understand problems better, and visualization. The usage of step-by-step tutorials was shown to particularly have a beneficial effect [25].

3.6 Flipped Classroom

The “flipped classroom” is a pedagogical model that attempts to repurpose the class time into a workshop where students can inquire about lecture content, test their skills by applying knowledge, and interact with one another in hands-on activities [26]. This model is currently being tried out in practice through the so-

called *Electronic Engineering Ladder*, which is a concept developed at the Department of Electronic Systems at NTNU. The aim of this program is to better show engineering students the relationship between their theoretical work and practical applications, motivating them to become autonomous problem solvers. The concept utilizes the “flipped classroom”, where the use of interactive video lectures is a central part of conveying the material to the students. The co-founder, Professor Lars M. Lundheim, explains in [27] and the interview presented in Appendix A how for each lecture, the students are expected to watch videos and answer quizzes and questions that are design to encourage deeper reflection and understanding of the material [27]. Through their online portal, students can submit questions while watching the videos, and provide feedback to the lecturer if anything is unclear or needs more explanation. The in-class time is typically devoted to exercises, projects, or discussions [26].

A similar concept is implemented at the University College of Southeast Norway, where Dr. Finn A. Haugen merges traditional and video-based lectures in one of his basic control courses [28]. The students are required to finish eight laboratory exercises, working with a desktop-sized air heater system for temperature monitoring and control. Dr. Haugen explains how 26 identical rigs, built in-house, allow all groups to work with the same exercise simultaneously [28].

The *Electronic Engineering Ladder* takes this one step further by requesting all student to buy a specialized kit at the beginning of their first semester. This kit includes a portable oscilloscope and a microcontroller. According to [27], buying and owning the equipment enables them to perform experimental work that traditionally would have required big, expensive and stationary equipment. Now, the student can execute their own experiments when and where they want, in addition to the scheduled class sessions.

3.7 Mayer’s Principles of Multimedia Learning

As explained in the previous chapter, an essential part of the flipped classroom model is the video lecturing. Producing instructional videos for the purpose of online education benefits from following certain guidelines for how to best convey the educational message. With the aim of helping students learn in such a way that they can use the knowledge they have gained to solve new, unforeseen problems, the American educational psychologist Richard Mayer defined twelve principles for learning through multimedia. Based on cognitive theory,

his principles are established using theory regarding constructivism¹, dual coding² and cognitive load³. His principles are presented and elaborated in Table 3-5.

¹ Constructivism is the theory that humans integrate new knowledge through experiencing things and reflecting on those experiences. Students become engaged by applying their existing knowledge and real-world experience, learning to hypothesize, testing their theories, and ultimately drawing conclusions from their findings [48].

² Dual coding is the theory that humans have two distinct subsystems in the brain that handles verbal (sound) and nonverbal (visual) information. The components of these subsystems are thought to be activated when one recognizes, manipulates, or just thinks about words or things.

³ Cognitive load is the theory about an existing limit of how much information the working memory is able to process at any given time. Cognitive load theory is concerned with the manner in which cognitive resources are focused and used during learning and problem solving [49].

Table 3-5: Mayer’s twelve principles for multimedia instructional presentations, according to [29, 30, 31].

Mayer’s principles	Elaboration
Coherence Principle	People learn better when extraneous or ambiguous information is excluded rather than included.
Signaling Principle	People learn better when cues that highlight the structure of the presentation are added. This mainly involves meta information, like informing the students about the name and background of the lecturer, information about the material, and information about how the presentation is related to the other presentations.
Redundancy Principle	People learn better from graphics and narration than from graphics, narration, and printed text.
Spatial Contiguity Principle	People learn better when corresponding words and pictures are placed near each other rather than far from each other on the page or screen.
Temporal Contiguity Principle	People learn better when corresponding words and pictures are presented at the same time rather than in succession.
Segmenting Principle	People learn better when a multimedia lesson is presented in user-paced segments rather than as a continuous unit. Split the teaching material into smaller parts that are more manageable for the students to follow. This gives the learned more time to see, listen to and process the material.
Pre-training Principle	People learn more deeply from a multimedia message when they receive pre-training in the names and characteristics of key components.
Modality Principle	People learn better from graphics and narration than from graphics and printed text.
Multimedia Principle	People learn better from words and pictures than from words alone.
Personalization Principle	People learn better from a multimedia presentation when the words are in conversational style rather than in formal style.
Voice Principle	People learn better when the words in a multimedia message are spoken by a friendly human voice rather than a machine voice. Further, the cognitive load is increased if the narrator is hard to understand
Image Principle	The learning material should be more important than the lecturer, meaning that having a person in the video is of little importance if it takes away the focus from the learning material.

4 Theory Related to the Building, Analysis and Control of the Serial Two-Tank System

This chapter introduces the technical theory needed to model, analyze and control the Serial Two-Tank System (STTS).

4.1 Electronics

This subchapter introduces the theory used when interacting with the sensors of the system and the development of the motor driver shield used to control the pump.

4.1.1 ADC Resolution

Given an ADC with a bit resolution of n bit and a measuring range of $V_m \in [V_{ref}, V_{gnd}]$, the voltage resolution VR is given by

$$VR = \frac{V_{ref} - V_{gnd}}{2^n} \quad (4-1)$$

Assume a sensor reading the physical measurement $X \in [X_{min}, X_{max}]$ as a voltage signal $V_s \in [V_{s,min}, V_{s,max}]$. Given these details, it is possible to find the expected physical measurement per bit, P , from the ADC.

$$P = \frac{X_{max} - X_{min}}{V_{s,max} - V_{s,min}} \cdot \frac{V_{ref} - V_{gnd}}{2^n} \quad (4-2)$$

4.1.2 Optimal Range Using a Voltage Divider

Using classical electrical theory, the resistance in a voltage divider that optimizes the output voltage range can be calculated.

Assume a circuit with one variable resistance, R_v , (typically a sensor where the variable resistance range is given by the manufacturer) and one resistor with a fixed resistance, R_f , chosen by the circuit designer.

$$\begin{aligned} R_v &\in [R_{v,min}, R_{v,max}] \in [\mathbb{R}^+ \quad \mathbb{R}^+] \\ R_f &\in \mathbb{R}^+ \end{aligned}$$

The circuit is consisting of these two resistors connected in series with R_v closest to the reference ground, as illustrated in Figure 4-1.

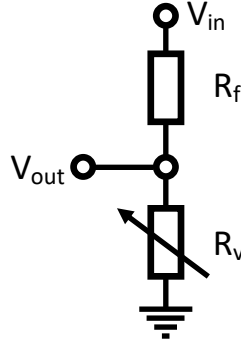


Figure 4-1: Classic example of a voltage divider circuit. The figure shows a resistor in series with a variable resistor, the ladder typically being a sensor

An expression of the voltage level between the two resistors can be found by using Ohm's law, where the two following expressions are true

$$V_{in} = I \cdot (R_f + R_v) \quad (4-3)$$

$$V_{out} = I \cdot R_v \quad (4-4)$$

Solving (4-3) and (4-4) for the current, I , allows the two expressions to be merged into an expression for the output voltage, V_{out} , only dependent on input voltage, V_{in} , and the two resistors, R_f and R_v .

$$V_{out} = \frac{R_v}{R_f + R_v} \cdot V_{in} \quad (4-5)$$

The equation (4-5) is the standard equation for a voltage divider.

The goal is to choose an R_f that maximizes the output voltage *range*, based on the already known resistance range of R_v . Having a big output voltage range allows for a better accuracy when reading the signal. The expression to be optimized is the following

$$\max \left[\left(\frac{R_{v,max}}{R_f + R_{v,max}} - \frac{R_{v,min}}{R_f + R_{v,min}} \right) \cdot V_{in} \right] \quad (4-6)$$

This is easily solved by setting its derivative equal to zero and solve for R_f

$$\frac{d}{dR_f} \left[\left(\frac{R_{v,max}}{R_f + R_{v,max}} - \frac{R_{v,min}}{R_f + R_{v,min}} \right) \cdot V_{in} \right] = 0 \quad (4-7)$$

$$R_f = \pm \sqrt{R_{v,min} \cdot R_{v,max}}$$

From (4-7), the fixed resistance R_f that achieves the largest range can now be calculated. Since no resistance can be negative, the final expression for the optimal, fixed resistance is as follows

$$R_f = \sqrt{R_{v,min} \cdot R_{v,max}} \quad (4-8)$$

4.1.3 Water Level Measurements using an Angular Potentiometer

Given a rod connected to the shaft of a potentiometer, the height of the tip of the rod can be calculated. Assume an angular potentiometer with a linearly varying resistance. This component can be thought of as a voltage divider. Given a potentiometer with a maximum resistance of R_{max} ohm, the variable resistor can be thought of as two variable resistors in series

$$R_{max} = R_1 + R_2 \quad (4-9)$$

Where the resistance in R_1 and R_2 is given by the angular position of the potentiometer

$$\begin{aligned} R_1 &= R_{max} - r(\alpha) \\ R_2 &= r(\alpha) \\ r(\alpha) &\in [R_{min}, R_{max}] \end{aligned} \quad (4-10)$$

The function $r(\alpha)$ is the variable part of the resistance, which is given by the angular position α of the potentiometer shaft. This value is a result of the shaft angle, the angle range, and the resistance range of the potentiometer, as shown in Figure 4-2. An expression for $r(\alpha)$ can be found by realizing that the resistance ratio must be equal to the shaft angle. This can be described as

$$\begin{aligned} \frac{r(\alpha) - R_{min}}{\Delta R} &= \frac{(\alpha - \alpha_{min})}{\Delta \alpha} \\ r(\alpha) &= \frac{(\alpha - \alpha_{min}) \cdot \Delta R}{\Delta \alpha} + R_{min} \\ \alpha &\in [\alpha_{min}, \alpha_{max}] \end{aligned} \quad (4-11)$$

The expression for α_{min} can be described as follows

$$\alpha_{min} = \frac{(R_{min} - R_0) \cdot \Delta \alpha}{\Delta R} \quad (4-12)$$

Where R_0 is the resistance at $\alpha = 0^\circ$. Combining (4-11) and (4-12) results in the following expression for the resistance

$$r(\alpha) = \frac{\Delta R}{\Delta \alpha} \cdot \alpha + R_0 \quad (4-13)$$

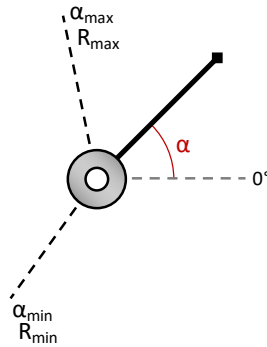


Figure 4-2: Illustration of a potentiometer with a given angular position and resistance range. The black line indicates the angular position shaft, and the grey, dotted line indicates where zero degrees is defined.

Given the input voltage signal V_{in} , the output voltage signal V_{out} is given by the classical voltage divider relation as presented in (4-35), assuming there is no current flowing through the output signal wire. The ladder assumption assumes the resistance between the point of measurement and ground is large compared to the other resistances.

By inserting (4-10) into (4-5), the following expression is achieved for the output voltage of a voltage divider, based on the shaft angle.

$$\begin{aligned}
 V_{out} &= \frac{R_v}{R_f + R_v} \cdot V_{in} \\
 &= \left(\frac{\Delta R \cdot \alpha}{\Delta \alpha \cdot R_{max}} + \frac{R_0}{R_{max}} \right) \cdot V_{in}
 \end{aligned}
 \tag{ 4-14 }$$

Voltage to Height using a Potentiometer

Building on the scenario illustrated in Figure 4-2, assume a shaft with length l attached to an angular potentiometer. Define $\alpha = 0^\circ$ as the angle of the shaft rotated to the lowest point physically allowed (if the shaft can rotate freely, this will equal the shaft hanging downwards). It is desirable to find the height of the tip of the shaft, expressed using the measured voltage. The scenario is illustrated in Figure 4-3.

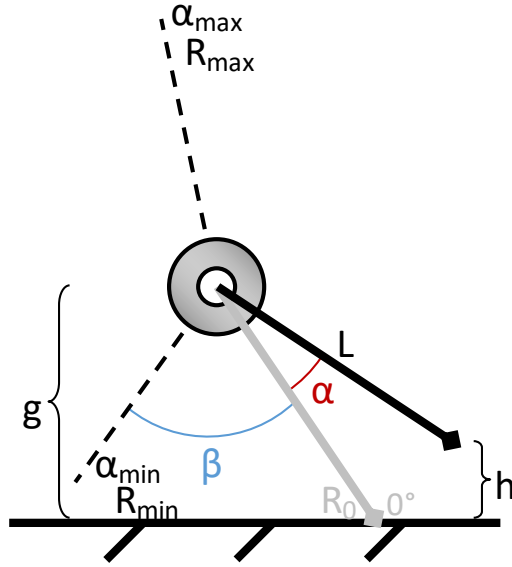


Figure 4-3: Illustration of a potentiometer with a given angular position and resistance range. The black line represents the shaft and its angular position. The grey shaft indicates the lowest angle physically possible. The height h is the height of the tip of the shaft when lifted from the lowest point physically possible.

Using trigonometry, the lowest possible angle can be expressed using the two known lengths and the angle from α_{min} .

$$\frac{g}{l} = \cos(\beta) \quad (4-15)$$

Where g is the length from the potentiometer to the physical constraint. Further, the height of the shaft tip can be found as follows

$$\frac{g-h}{l} = \cos(\alpha(V_{out}) + \beta) \quad (4-16)$$

Where $\alpha(V_{out})$ is a function describing the shaft angle based on the V_{out} illustrated in Figure 4-1. By solving (4-15) for the angle β , it can be combined with (4-16) to give an expression for the height of the shaft tip.

$$h(V_{out}) = g - g \cdot \cos(\alpha(V_{out})) + \sqrt{l^2 - g^2} \cdot \sin(\alpha(V_{out})) \quad (4-17)$$

Where $\alpha(V_{in})$ is found by solving (4-14) for the shaft angle

$$\alpha(V_{out}) = \left(\frac{V_{out}(\alpha_{max} - \alpha_{min})R_{max}}{V_{in}} + \frac{\alpha_{min}R_{max} - \alpha_{max}R_{min}}{(R_{max} - R_{min})} \right) \quad (4-18)$$

The expression for $h(V_{out})$ can be used to calculate the height of the shaft tip based on the measured voltage output from a potentiometer.

Special case

Given the special case where the $R_{min} = 0$ and, the shaft angle (4-18) is reduced to the following expression

$$\alpha(V_{out}) = \left(\frac{V_{out}(\alpha_{max} - \alpha_{min})R_{max}}{V_{in}} + \alpha_{min} \right) \quad (4-19)$$

4.1.4 RC Integrator Cutoff Frequency

Assume a simple RC circuit made up of a resistor, R , and a capacitor, C , in parallel, as described in Figure 4-4.

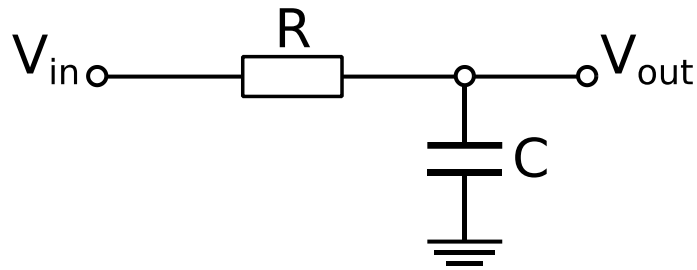


Figure 4-4: Illustration of a classic RC circuit.

This circuit is called an RC integrator, and is a type of low pass filter circuit. An expression of the cutoff frequency for the low pass filter can be found by utilizing the formula for current flowing through an RC circuit

$$i(t) = C \frac{dV_c}{dt} = \frac{V_R}{R} = \frac{V_{in} - V_c}{R} \quad (4-20)$$

Solving (4-20) for the voltage $V_{out} = V_c$ gives the following expression

$$\frac{dV_{out}}{dt} = \frac{1}{RC} (V_{in} - V_{out}) \quad (4-21)$$

By transferring (4-21) to the Laplace domain, the following transfer function is achieved

$$H(s) = \frac{V_{in}(s)}{V_{out}(s)} = \frac{1}{RC \cdot s + 1} \quad (4-22)$$

The cutoff frequency is defined as the frequency where the amplitude of the transfer function response is $1/\sqrt{2}$ (approximately -3dB) of the input amplitude, given an oscillating input signal. This can be described as

$$\left| \frac{V_{in}(s)}{V_{out}(s)} \right| = \frac{1}{\sqrt{1^2 - R^2 C^2 \omega_c^2}} = \frac{1}{\sqrt{2}} \quad (4-23)$$

Where ω_c is the cutoff angular frequency. Solving (4-23) for ω_c gives the following expression

$$\omega_c = \frac{1}{RC} \quad (4-24)$$

The cutoff frequency is defined as $f_c = \omega_c/2\pi$, resulting in the final expression for the cutoff frequency for an RC integrator circuit.

$$f_c = \frac{1}{2\pi RC} \quad (4-25)$$

4.2 Hydraulic Systems

The following chapter derives the general equations used for modelling the STTS.

4.2.1 Mass Balance

When modeling the behavior of the STTS, the mass balance gives a good starting point, assuming that the tank liquid is incompressible and has a conservation of mass and volume (assuming that the mass density is constant). A standard expression for the mass balance is presented bellow

$$\frac{dm(t)}{dt} = W_{in} - W_{out} + W_{generated} - W_{consumed}$$

W represents the mass flow rate and has the unit kg/s . As there is no water generated or consumed inside the tank, this expression simplifies to

$$\dot{m} = W_{in} - W_{out} \quad (4-26)$$

This can be combined with the relationship between mass and volumetric flow rate, where Q represents volume flow rates and has the unit m^3/s .

$$W = \rho Q \quad (4-27)$$

This enables (4-26) to be rewritten for volume flow

$$\dot{m} = \rho Q_{in} - \rho Q_{out} \quad (4-28)$$

By substituting mass with $m = \rho \cdot V = \rho \cdot A[h(t)] \cdot h(t)$, (4-28) can be rewritten to an expression for the height of the liquid level in the tank

$$\begin{aligned} A(h)\dot{h} &= Q_{in} - Q_{out} \\ h(t) &\geq 0 \end{aligned} \quad (4-29)$$

$h(t)$ is the height, or water level, at any given time and $A[h(t)]$ is the surface area of the tank given as a function of the height.

4.2.2 Fluid Resistance

Fluid meets resistance when flowing through a valve or opening. According to [32], the mass flow rate W through a resistance is related to the pressure difference dp across the resistance. The *fluid resistance* R is defined as

$$R = \frac{dp}{dW} \quad (4-30)$$

Put in other words, the resistance is defined as the slope of the pressure p versus the mass flow rate W .

Using Torricelli's principle, an expression for the output flow rate can be achieved. When liquid is flowing out of a tank through a small hole or valve, energy is converted from potential energy in the tank to kinetic energy as water is flowing out. The irrotational Bernoulli expresses this principle in the following equation [33]

$$\frac{1}{2}(v_2^2 - v_1^2) + \frac{p_2 - p_1}{\rho} + (z_2 - z_1)g = 0 \quad (4-31)$$

Where v_i is the liquid velocity, p_i is the pressure and z_i is the height, all according to a reference point $i \in \{1,2\}$. By assuming that the velocity of the liquid inside the tank is so slow that it is negligible, which is a valid assumption according to [32], an expression for the output velocity of the tank is achieved, where reference point 1 is at the bottom of the tank and reference point 2 is right outside of the valve. The reference points are illustrated in Figure 4-5. For further calculations, the distance between the input and output of the valve is assumed to be approximately the same height, that is $z_1 \approx z_2$. This results in the following expression for the output flow.

$$v = \sqrt{2 \cdot \frac{p_1 - p_2}{\rho}} = \sqrt{2 \cdot \frac{\Delta p}{\rho}} = \sqrt{2gh} \quad (4-32)$$

(4-32) can be used together with the definition of volumetric flow rate $Q = v \cdot \alpha$, where α is the orifice area, to get an expression for the mass flow rate out of the orifice.

$$Q_{valve} = C\alpha\sqrt{2gh} \quad (4-33)$$

$C \in [0,1]$

C is a discharge coefficient factor, $C = 1$ being the theoretical maximum volumetric flow rate with perfect energy conversion. Note that (4-33) depends on the orifice area being small enough so that the pressure variation over the orifice area is negligible compared to the average pressure of the orifice [32].

To better relate (4-33) to (4-30), the orifice relation can be rearranged to the following form

$$Q_{valve} = \sqrt{\frac{\rho g h}{R}} \quad (4-34)$$

$$R = \frac{\rho}{2C^2\alpha^2}$$

Where R is the fluid resistance as described in (4-30). However, (4-33) will be used throughout this thesis when estimating the valve coefficient.

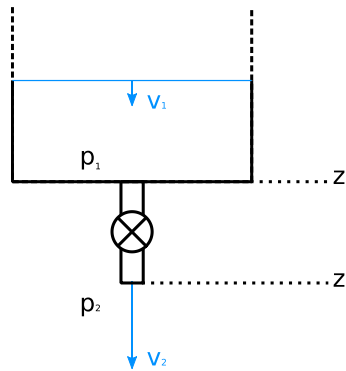


Figure 4-5: Illustration of the parameters used to calculate the flow rate through an orifice opening.

4.3 Function Identification and Parameter Estimation

Function identification, sometimes called *curve fitting*, is the process of finding a function that describes a particular set of data. Parameter estimation is the process of obtaining the coefficients of such a function. Describing data as a function allows us to compute new values at points that were not sampled, with the assumption that the real process behaves smoothly between the data points that were sampled. This is useful when designing e.g. a simulator of a system, as it allows for a continuous way of fetching data rather than using a lookup table. The following methods are based on the theory from [32].

4.3.1 Transforming Binomial, Power, and Exponential Functions to Linearly Shaped Functions

Function identification is rather easy to conduct for linearly shaped data points using the ordinary least square method, which is presented later. There are however many processes that do not behave in a linear fashion. A method for transforming nonlinear signals into linearly shaped signals is therefore desirable, as

it allows us to use methods of linear function identification on nonlinear data. Table 4-1 presents the selected nonlinear functions used either directly in this thesis, or as part of the pilot test assignment presented in Chapter 8.3.

Table 4-1: Three nonlinear functions that can often be used to describe physical phenomena

Name	Function	$y(0)$
Linear function	$y(x) = mx + b$	b
Binomial function	$y(x) = (mx + b)^n$	b^n
Power function	$y(x) = bx^m$	$0, \text{ if } m \geq 0$ $\infty, \text{ if } m < 0$
Exponential function	$y(x) = be^{mx}$	b

Binomial Function

A binomial function can be transformed to have a linear shape in the following way

$$y = (mx + b)^n \quad (4-35)$$

$$Y = \sqrt[n]{y} = mx + b$$

Plotting $\sqrt[n]{y}$ against x gives a straight line with slope m that intersects $\sqrt[n]{y}$ at b .

Power Function

A power function can be transformed to have a linear shape as follows

$$y = bx^m$$

$$\log(y) = m \cdot \log(x) + \log(b) \quad (4-36)$$

$$Y = mX + B$$

Plotting $\log(y)$ versus $\log(x)$, a so-called log-log plot, now gives a straight line with slope m that intercepts $\log(y)$ at b .

Exponential Function

An exponential function can be transformed to have a linear shape as follows

$$y = be^{mx}$$

$$\log(y) = m \log(e) \cdot x + \log(b) \quad (4-37)$$

$$Y = Mx + B$$

Similarly to (4-36), plotting Y versus x gives a straight line with slope M that intercepts $\log(y)$ at B . This corresponds to a semi-log plot, plotting y using a log axis and x using a rectilinear axis.

By doing this pre-processing of the data, all four cases are reduced to the problem of finding the coefficients m and b (or M and B) of a seemingly linear signal.

4.3.2 The Ordinary Least-Squares Method

The ordinary least squares method (OLSM) is a valuable tool in curve fitting. The method estimates the linear function that minimizes the squared, total difference between the observed value and the value that the candidate function predicts. These differences are called the *residuals*.

Assume a signal that has been sampled with n data points. To obtain an estimated function with coefficients m and b for the linear equation $y = mx + b$, the OLSM tries to minimize J .

$$J = \sum_{i=1}^n (mx_i + b - y_i)^2 \quad (4-38)$$

The objective of minimizing J with respect to m and b can be solved by simple derivation along m and b , where the derivatives are set equal to zero to find the global minima of J .

$$\begin{aligned} \min J \\ \frac{\partial J}{\partial m} &= 2 \cdot \sum_{i=1}^n mx_i^2 + 2 \cdot \sum_{i=1}^n bx_i - 2 \cdot \sum_{i=1}^n y_i x_i = 0 \\ \frac{\partial J}{\partial b} &= 2 \cdot \sum_{i=1}^n mx_i + 2 \cdot \sum_{i=1}^n b - 2 \cdot \sum_{i=1}^n y_i = 0 \end{aligned} \quad (4-39)$$

Note that the sum of b can be written as follows

$$\sum_{i=1}^n b = nb$$

The equations (4-39) are then reduced to

$$\begin{aligned} m \sum_{i=1}^n x_i^2 + b \sum_{i=1}^n x_i &= \sum_{i=1}^n x_i y_i \\ m \sum_{i=1}^n x_i + nb &= \sum_{i=1}^n y_i \end{aligned} \quad (4-40)$$

These equations can be used to estimate the coefficients of a linear function based on a set of data points. Further, since the exponential, power and binomial functions can be transformed to a linear shape, as described in Chapter 4.3.1, the OLSM can be applied to these types of functions after transforming the data set. In this case, the linear solution will have to be transformed back to the nonlinear shape after having estimated m and b .

4.3.3 Constraining the Function to Pass Through a Given Point

Many physical applications dictate certain constraints on the model to be estimated. One typical constraint would be that the function must pass through a specific point, e.g. the origin. Because of potential measurement error present in the data, these natural constraints may not always be present in the resulting function estimated with the OLSM. For this reason, it might be necessary to constrain the estimated function.

To obtain a linear function that passes through the point (x_0, y_0) , first subtract this point from all the data points

$$\begin{aligned} (\hat{x}_i, \hat{y}_i) &= (x_i - x_0, y_i - y_0) \\ i &\in [1, n] \end{aligned}$$

Then compute m using as follows

$$m \sum_{i=1}^n \hat{x}_i^2 = \sum_{i=1}^n \hat{x}_i \hat{y}_i \quad (4-41)$$

The estimated function is then given on the following form

$$y = m(x - x_0) + y_0 \quad (4-42)$$

4.4 Stability Analysis of the System

An important property in system control is the stability analysis of the system at hand. By finding the system's equilibrium points and deciding the stability of them, insight can be gained, useful when later designing a controller. There are different approaches for analyzing the stability of dynamical systems. In this thesis, three approaches will be demonstrated, two being characterized in the sense of Lyapunov, a Russian mathematician and engineer who laid the foundation of the theory. The theory presented here is based on the work of Dr. Hassan K. Khalil [1], which assumes the following autonomous system

$$\dot{\vec{x}} = f(\vec{x}) \quad (4-43)$$

Where $f: \mathbb{D} \rightarrow \mathbb{R}^n$ is a locally Lipschitz map from a domain $\mathbb{D} \subset \mathbb{R}^n$ into \mathbb{R}^n . Locally Lipschitz is a criterion that requires both the function and its partial derivative to be continuous. The definition of a locally Lipschitz function is presented in ET 4-1.

If $f(t, \vec{x})$ and $[\partial \vec{f} / \partial \vec{x}](t, \vec{x})$ are continuous on $[a, b] \times \mathbb{D}$, for some domain $\mathbb{D} \subset \mathbb{R}^n$, then \vec{f} is locally Lipschitz in \vec{x} on $[a, b] \times \mathbb{D}$.

ET 4-1: Lemma 3.2 from [1] defining criteria for a function to be locally Lipschitz.

Stability analysis often focuses on equilibrium points or operating points. The different types of equilibrium points are defined in ET 4-2. First, the property of equilibrium points will be presented.

4.4.1 Equilibrium Points

A point $\vec{x} = \vec{x}^*$ in (4-43) is, according to [1], said to be an equilibrium point if it has the property that whenever the state of the system starts at \vec{x}^* , it will remain at \vec{x}^* for all future time. For the system (4-43), the equilibrium points are the real roots of the following equation

$$f(\vec{x}) = 0 \quad (4-44)$$

An equilibrium point may appear alone or as a continuum of equilibrium points.

Common Types of Stability

From [1], a definition of the different types of equilibrium points are given

The equilibrium point $\vec{x} = 0$ of $\dot{\vec{x}} = f(\vec{x})$ is

- Stable if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon) > 0$ such that

$$\|\vec{x}(0)\| < \delta \Rightarrow \|\vec{x}(t)\| < \varepsilon, \quad \forall t \geq 0$$
- Unstable if not stable
- Asymptotically stable if it is stable and δ can be chosen such that

$$\|\vec{x}(0)\| < \delta \Rightarrow \lim_{t \rightarrow \infty} \vec{x}(t) = 0$$

ET 4-2: The most common types of stability for equilibrium points, as defined in Definition 4.1 in [1].

To sum up ET 4-2, an equilibrium point is stable if no initial value smaller than $\pm\delta$ makes the states grow beyond a threshold $\pm\varepsilon$. If the system in addition has the property that initial values smaller than $\pm\delta$ makes the system states return to the origin, the system is said to be asymptotically stable. Asymptotically stability does not say anything about the *rate* of convergence, it only guarantees

that the system will return to the origin as time goes to infinity or faster. These concepts are illustrated in Figure 4-6.

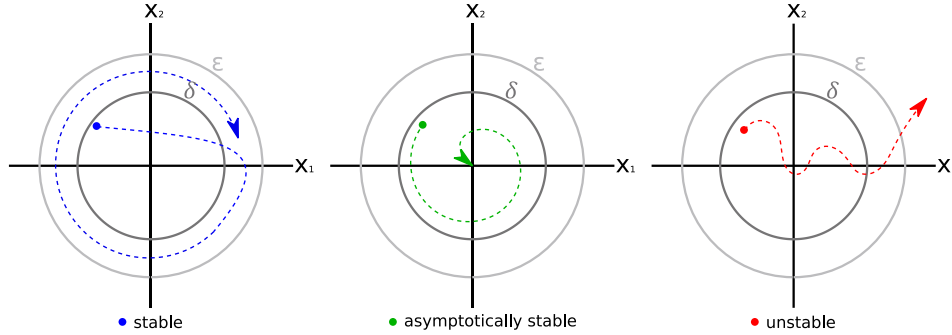


Figure 4-6: Three systems illustrating the stability properties “stable”, “asymptotically stable” and “unstable”, where the origin represent an equilibrium point.

An even more strict definition of stability is called exponential stability. This type of stability ensures that the states will converge to the origin with an exponential rate. The definition of this type of stability is presented below in ET 4-3.

The equilibrium point $\vec{x} = 0$ of $\dot{\vec{x}} = f(\vec{x})$ is exponentially stable if there exists positive constants c , k and λ such that

$$\|\vec{x}(t)\| \leq k\|\vec{x}(t_0)\|e^{-\lambda(t-t_0)}, \quad \forall \|\vec{x}(t_0)\| < c$$

and globally exponentially stable if the expression above satisfies any initial state $\vec{x}(t_0)$.

ET 4-3: Definition of local and global exponential stability, rewritten for autonomous systems, based to Definition 4.5 in [1].

4.4.2 Jacobian Linearization

From [34], a system on the form $\dot{\vec{x}} = f(\vec{x}, \vec{u})$ can be linearized to the standard state space form.

$$\dot{\vec{x}} \approx \mathbf{A}\vec{\delta}_x(t) + \mathbf{B}\vec{\delta}_u(t) \quad (4-45)$$

Where $\vec{x} \in \mathbb{R}^{n \times 1}$ and $\vec{u} \in \mathbb{R}^{p \times 1}$. \mathbf{A} and \mathbf{B} are calculated as follow

$$\mathbf{A} = \left. \frac{\partial f}{\partial \vec{x}} \right|_{\substack{\vec{x}=\vec{x}^* \\ \vec{u}=\vec{u}^*}} \in \mathbb{R}^{n \times n} \quad \mathbf{B} = \left. \frac{\partial f}{\partial \vec{u}} \right|_{\substack{\vec{x}=\vec{x}^* \\ \vec{u}=\vec{u}^*}} \in \mathbb{R}^{n \times p} \quad (4-46)$$

Where $\vec{\delta}_x \in \mathbb{R}^{n \times 1}$ and $\vec{\delta}_u \in \mathbb{R}^{p \times 1}$. (4-45) is called the Jacobian linearization of the original nonlinear system.

4.4.3 Transfer function

A transfer function is a mathematical function relating the output of a linear system to the input. It is briefly mentioned, in addition to showing how the transfer function can be calculated given an ODE on state space form

$$H(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbb{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \quad (4-47)$$

4.4.4 Phase portrait

For systems with three or less states (normally two), the stability properties can be investigated by calculating the trajectories for a large number of initial states spread over the state-line, plane or n -space. This family of trajectories is called the *phase portrait* [1], and is a geometric representation of the trajectories of a dynamical system. This method gives an easy approach to visually investigate the stability properties of a system without proving them. According to Khalil, this is a *qualitative* behavior analysis not a *quantitative*, for each associated solution. The reason for this is that the time is suppressed in the trajectories, making it impossible to recover a solution associated with a given trajectory [1].

4.4.5 Lyapunov's Indirect Method

In the field of engineering, Lyapunov's indirect method is probably the most common approach for checking the local stability of equilibrium points. The following method is based on the theory of [1] and presented in ET 4-4. The idea of ET 4-4 is to linearize the unforced system (4-43) and then check the eigenvalues of the linear system.

Assume $\vec{x} = 0$ is the equilibrium point of the nonlinear system $\dot{\vec{x}} = f(\vec{x})$, where the working set \mathbb{D} is in the neighbourhood of the origin. Let

$$\mathbf{A} = \left. \frac{\partial f}{\partial \mathbf{x}}(\vec{x}) \right|_{\vec{x}=0}$$

Then,

1. The origin is asymptotically stable if $Re\{\lambda_i\} < 0$ for all eigenvalues of \mathbf{A} .
2. The origin is unstable if $Re\{\lambda_i\} > 0$ for one or more of the eigenvalues of \mathbf{A} .

ET 4-4: Lyapunov's indirect method based on Theorem 4.7 in [1]

To better classify the stability, ET 4-5 allows for a potential conclusion of exponential stability.

Let $\vec{x} = 0$ be an equilibrium point of the nonlinear system $\dot{\vec{x}} = f(\vec{x})$, where $f(\vec{x})$ is continuously differentiable in some neighborhood of $\vec{x} = 0$. Let $A = [\partial f / \partial \vec{x}](0)$. Then, $\vec{x} = 0$ is an exponentially stable equilibrium point for the nonlinear system if and only if all eigenvalues of A satisfy $Re\{\lambda_i\} < 0$.

ET 4-5: Corollary for concluding that an asymptotically stable equilibrium point of a *linear* system, also is an exponentially stable equilibrium point for the same linear system, according to Corollary 4.3 in [1]

By combining ET 4-4 with the findings of ET 4-5, one can conclude that an equilibrium point with all negative real eigenvalues can be classified as an exponentially stable equilibrium point in the neighborhood of \mathbb{D} .

Finally, If the same equilibrium point also happens to be the only equilibrium point of the system, the equilibrium point can be said to be globally exponentially stable.

4.4.6 Lyapunov's Direct Method

For highly nonlinear systems, Lyapunov's indirect method may turn out to be insufficient. The structure of the system may cause it to have multiple equilibria, be unlinearizable or contain limit cycles, dead zones or other nonlinear behaviors.

The Lyapunov direct method is based on a realization that the stability of a system can be analyzed by investigating the energy of the system and the rate of change in energy. The idea is that if the energy of the system close to an equilibrium point is decreasing, then the equilibrium point is stable. If it on the other hand is increasing, then the equilibrium point is unstable. A second, less intuitive realization made by Lyapunov was that the stability of the system can be proven without knowing the true physical energy of the system. Instead, a Lyapunov candidate function $V(x)$ can be created. If this candidate satisfies certain conditions, stability can be concluded. Further, not being able to prove stability with a chosen Lyapunov candidate does not prove *instability*, and the user may try a different candidate. The definition and criteria for such a Lyapunov function is described in ET 4-6.

Let $\vec{x} = 0$ be an equilibrium point of the nonlinear system $\dot{\vec{x}} = f(\vec{x})$ and $\mathbb{D} \subset \mathbb{R}^n$ be a domain containing $\vec{x} = 0$. Let $V: \mathbb{D} \rightarrow \mathbb{R}$ be a continuously differentiable function such that

$$\begin{aligned} V(0) &= 0 \\ V(\vec{x}) &> 0 \text{ in } \mathbb{D} - \{0\} \\ \dot{V}(\vec{x}) &\leq 0 \text{ in } \mathbb{D} \end{aligned}$$

Then, $\vec{x} = 0$ is a stable. Moreover, if

$$\dot{V} < 0 \text{ in } \mathbb{D} - \{0\}$$

Then $\vec{x} = 0$ is asymptotically stable.

ET 4-6: Theorem for concluding stability and asymptotical stability using Lyapunov's direct method, according to Theorem 4.1 in [1].

Let $\vec{x} = 0$ be an equilibrium point for $\dot{\vec{x}} = f(\vec{x})$. Let $V: \mathbb{R}^n \rightarrow \mathbb{R}$ be a continuously differentiable function such that

$$\begin{aligned} V(0) &= 0 \quad \text{and} \quad V(\vec{x}) > 0, \quad \forall x \neq 0 \\ \|\vec{x}\| \rightarrow \infty &\Rightarrow V(\vec{x}) \rightarrow \infty \\ \dot{V}(\vec{x}) &< 0 \quad \forall x \neq 0 \end{aligned}$$

Then $\vec{x} = 0$ is globally asymptotically stable.

ET 4-7: Theorem for concluding globally asymptotical stability using Lyapunov's direct method, according to Theorem 4.2 in [1].

Let $\vec{x} = 0$ be an equilibrium point for $\dot{\vec{x}} = f(\vec{x})$ and $\mathbb{D} \subset \mathbb{R}^n$ be a domain containing $\vec{x} = 0$. Let $V: [0, \infty) \times \mathbb{D} \rightarrow \mathbb{R}$ be a continuously differentiable function such that

$$\begin{aligned} k_1 \|\vec{x}\|^a &\leq V(t, \vec{x}) \leq k_2 \|\vec{x}\|^a \\ \dot{V}(\vec{x}) &\leq -k_3 \|\vec{x}\|^a \end{aligned}$$

$\forall t \geq 0$ and $\forall \vec{x} \in \mathbb{D}$, where k_1, k_2, k_3 and a are positive constants. Then $\vec{x} = 0$ is exponentially stable. Further, if $\mathbb{D} = \mathbb{R}^n$, then $\vec{x} = 0$ is globally exponentially stable.

ET 4-8: Theorem for concluding exponential stability and global exponential stability using Lyapunov's direct method, according to Theorem 4.10 in [1].

4.5 Control Theory

This chapter introduces the theory needed to simulate a system using Euler's method, and the theory behind the two control methods implemented in this thesis, namely the PID- and the LQR(I)-controller.

4.5.1 Euler's method for Numerical Integration

Several methods exist for solving ODE systems, where Euler's method is the most basic of the explicit methods. From [33], the solver is given on the form

$$y_{n+1} = y_n + h\dot{y}_n \quad (4-48)$$

Where h is the step size of the solver and \dot{y}_n is the solution to the ODE at step n . The solver requires an initial value for the system, y_0 .

This solver is a first-order method, meaning that its local error is proportional to the square of the step size, and the global error is proportional to the step size. The strength of the solver is that it is easy to implement, and that it is stable for all stable LTI systems, given a small enough step size. From [33], the stability of the solver is guaranteed if the following relation is satisfied between the step size and the eigenvalue of the system

$$-\frac{2}{h} < \lambda < 0 \quad (4-49)$$

4.5.2 Skogestads Internal Model Control for PID Control

The structure of the PID controller in this thesis is mainly centered around the parallel PID controller, given on the following form

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(\tau) d\tau + K_p T_d \frac{de(t)}{dt} \quad (4-50)$$

For convenience, the parallel PID controller (4-50) is stated below in the Laplace domain

$$u(s) = K_p e(s) + \frac{K_p}{T_i} \cdot \frac{1}{s} e(s) + K_p T_d \cdot s \cdot e(s) \quad (4-51)$$

Finally, as the SIMC method was established for the serial PID controller, it too will be presented here, in its Laplace form

$$u(s) = K_p \cdot \frac{T_i s + 1}{T_i s} \cdot (T_d s + 1) \cdot e(s) \quad (4-52)$$

On a side note, in this thesis, mostly PI controllers were implemented, making the parallel controller (4-51) behave identical to serial controller. For the

case of a PID controller (with a derivative part), the tuning parameters can easily be transformed using a serial-to-parallel transformation [35]

$$\begin{aligned}
 K_{p_{parallel}} &= K_{p_{serial}} \left(1 + \frac{T_{d_{serial}}}{T_{i_{serial}}} \right) \\
 T_{i_{parallel}} &= T_{i_{serial}} \left(1 + \frac{T_{d_{serial}}}{T_{i_{serial}}} \right) \\
 T_{d_{parallel}} &= T_{d_{serial}} \cdot \frac{1}{\left(1 + \frac{T_{d_{serial}}}{T_{i_{serial}}} \right)}
 \end{aligned} \tag{4-53}$$

The Skogestad's Internal Model Control (SIMC) method offers a two-step model based approach for finding initial parameters for a PID controller [36, 35] that offers a desired trade-off between performance (“tight” control) and robustness (“smooth” control) [36]. The approach, created by Sigurd Skogestad, is derived using transfer function models, where the method first approximates the plant system as either a first or second order time delay model. Then the parameters for the PID controller can be calculated directly from the approximated transfer function. The approximated model can be obtained either theoretically or experimentally, where this thesis will focus on obtaining them using the system model. A strength with the SIMC is that it works for both simulated and real system, for both a “white box” approach and a “black box” approach.

Model Approximation (Step 1)

The first step of the SIMC method is to approximate the system using either a first-order time delay model

$$H_1(s) = \frac{K}{\tau_1 s + 1} e^{-\theta s} \tag{4-54}$$

Or a second-order time delay model

$$H_2(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\theta s} \tag{4-55}$$

Where K is the process gain, θ is the time delay of the process, τ_1 is the dominant time constant (largest magnitude of the two time constants) and, for the second-order system, τ_2 is the second-order time constants.

When approximating a higher order system as a first- or second-order system, [36] refers to the “half rule”, which compresses a higher order system into a first- or second-order system. The largest neglected time constant is distributed to the effective time delay and the smallest remaining time constant. However, as the system presented in this thesis can be described, after being linearized, as either a second order system or two first-order systems, this method becomes

redundant. Finding this approximated model simply becomes a matter of re-arranging the system equations to the same form as (4-54) or (4-55), and the technical details of the half rule method is therefore left to Appendix G.

The SIMC Tuning Rules for PI and PID Controllers (Step 2)

After approximating a first- or second-order model of the system, the SIMC tuning rules can be used to calculate the initial tuning parameters for a PI or PID controller. The derivation of the SIMC rules can be found in [36].

For a first-order system (4-54), the parameters for the PI controllers (4-51) and (4-52) are given as follows, assuming $T_d = 0$.

$$K_p = \frac{\tau_1}{K(\tau_c + \theta)} \quad (4-56)$$

$$T_i = \min\{\tau_1 \quad 4 \cdot (\tau_c + \theta)\} \quad (4-57)$$

For a second-order system (4-55), the parameters for the PID controller (4-52) are given as

$$K_p = \frac{\tau_1}{K(\tau_c + \theta)} \quad (4-58)$$

$$T_i = \min\{\tau_1 \quad 4 \cdot (\tau_c + \theta)\} \quad (4-59)$$

$$T_d = \tau_2 \quad (4-60)$$

PID controllers (with derivative action) are primarily recommended for processes with dominant second order-dynamics, defined as having $\tau_2 > \theta$ [36]. Further, recall that the SIMC method is intended for serial PID controllers. The parameters of (4-58), (4-59) and (4-60) therefor needs to be transformed using (4-53) when being used with a parallel controller.

The parameter τ_c is the only tuning parameter of the SIMC method. According to [36], the parameter can be chosen more or less freely, but must satisfy $\tau_c \in (-\theta, \infty)$ to ensure that (4-58) gives a positive gain. As a default, [36] suggest $\tau_c = \theta$.

For systems is without a delay, this becomes problematic, as it would result is an infinite proportional gain for (4-56) and (4-58). As a solution for integrating processes like a water tank, [35] suggests setting τ_c equal to a fifth of the time it takes to fill the water tank at a maximum input flow.

$$\tau_c = k \cdot \frac{h_{i,max} \cdot A_i}{Q_{i,max}} \quad (4-61)$$

Where $i \in \{1,2\}$ represents tank 1 or 2, $Q_{i,max}$ is the maximal input flow and k is a scaling variable, suggested by [35] to have an initial value of $k = 0.2$.

4.5.3 Linear Quadratic Regulator

An alternative to the PID controller is the Linear Quadratic Regulator (LQR). This method uses the model of the system to find the optimal state-feedback gain, $\vec{u} = -\mathbf{K}\vec{x}$ that minimizes the following quadratic cost function

$$\begin{aligned} \min J(\vec{u}) &= \int_0^{\infty} \vec{x}^T \mathbf{Q} \vec{x} + \vec{u}^T \mathbf{R} \vec{u} \\ \text{s. t. } \dot{\vec{x}} &= \mathbf{A} \vec{x} + \mathbf{B} \vec{u} \end{aligned} \quad (4-62)$$

Where \mathbf{Q} and \mathbf{R} are symmetric and positive semidefinite⁴ matrices. This can be solved using the *algebraic Riccati equation*

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} + \mathbf{Q} - \mathbf{P} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} = 0$$

Which gives a gain of the form

$$\mathbf{K} = \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P}$$

Since the *algebraic Riccati equation* is quadratic, several solutions for \mathbf{P} are often obtained. However, assuming the system is stabilizable and detectable, the correct solution contains a unique solution for \mathbf{P} which is positive semidefinite. Further, if the system is also observable, \mathbf{P} will be positive definite [37].

For all further work, the Matlab function $\mathbf{K} = \text{lqr}(\mathbf{A}, \mathbf{B}, \mathbf{Q}, \mathbf{R})$ was utilized to find the optimal gain vector.

Checking the controllability and observability of a linear system

As stated above, the system needs to be stabilizable and observable for the *algebraic Riccati equation* to have a unique, positive definite solution. Further, a system that is controllable is by definition stabilizable. The definitions of a controllable and an observable system is briefly mentioned below, where the dimensions n, p are given in (4-45), and the output matrix \mathbf{C} is of the dimensions $\mathbf{C} \in \mathbb{R}^{q \times n}$.

A system is said to be controllable if and only if $\text{rank}(\zeta) = n$, where ζ is given as follows

⁴ Often \mathbf{Q} is required to be positive definite, which is true if all the states are to be considered during the optimization problem. However, this is not the case for the implementation described in this thesis, which will be covered in the Chapter 6.4.6, and a semi definite \mathbf{Q} will be sufficient.

$$\begin{aligned} \zeta &= [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B} \quad \dots \quad \mathbf{A}^{n-1}\mathbf{B}] \\ \zeta &\in \mathbb{R}^{n \times np} \end{aligned} \quad (4-63)$$

A system is said to be observable if and only if $\text{rank}(\mathbf{Q}) = n$, where \mathbf{Q} is given as follows

$$\begin{aligned} \mathbf{Q} &= [\mathbf{C} \quad \mathbf{CA} \quad \mathbf{CA}^2 \quad \dots \quad \mathbf{CA}^{n-1}]^T \\ \mathbf{Q} &\in \mathbb{R}^{nq \times n} \end{aligned} \quad (4-64)$$

State Feedback with Reference Trajectory

The objective of a classic LQR is to make all the states go to zero. This is often not the desired objective for the control scheme, where one either want the system to follow a reference trajectory or to settle at a non-zero state value. This is known as the trajectory tracking problem, where the goal is to design a controller on the form $\vec{u} = \alpha(\vec{x}, \vec{x}_d, \vec{u}_d)$ such that $\lim_{t \rightarrow \infty} (\vec{x} - \vec{x}_d) = 0$ [38].

To design this controller, an *error system* is constructed. The original system is assumed to be on the form $f(\vec{x}, \vec{u}) = f(\vec{x}) + g(\vec{x})\vec{u}$, a nonlinear system with an assumed linear input. Define the error $\vec{e} = \vec{x} - \vec{x}_d$ and the new input $\vec{v} = \vec{u} - \vec{u}_d$. The dynamics of the error is then given by

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{x}_d \\ &= f(\vec{e} + \vec{x}_d) - f(\vec{x}_d) + g(\vec{e} + \vec{x}_d)(\vec{v} + \vec{u}_d) - g(\vec{x}_d)\vec{u}_d \\ &= F(\vec{e}, \vec{v}, \vec{x}_d(t), \vec{u}_d(t)) \end{aligned} \quad (4-65)$$

Where the system in general is a time varying system.

Given a good controller, the error dynamics can be linearized around $\vec{e} = 0$, resulting in the approximated, linear error system

$$\begin{aligned} \dot{\vec{e}} &\approx \mathbf{A}(t)\vec{e} + \mathbf{B}(t)\vec{v} \\ \mathbf{A}(t) &= \left. \frac{\partial \vec{F}}{\partial \vec{e}} \right|_{\vec{x}_d(t), \vec{u}_d(t), \vec{e}=0} \\ \mathbf{B}(t) &= \left. \frac{\partial \vec{F}}{\partial \vec{v}} \right|_{\vec{x}_d(t), \vec{u}_d(t), \vec{e}=0} \end{aligned} \quad (4-66)$$

$\mathbf{A}(t)$ and $\mathbf{B}(t)$ are often only dependent on \vec{x}_d , and [38] suggests the notation $\mathbf{A}(t) = \mathbf{A}(\vec{x}_d)$ and $\mathbf{B}(t) = \mathbf{B}(\vec{x}_d)$ for convenience. This system is valid, according to [38], based on the assumption that \vec{x}_d and \vec{u}_d are constant or slowly varying with respect to the performance criterion.

Using the LQR method for calculating the optimal gain for the system (4-66) results in the following feedback law

$$\vec{v} = \mathbf{K}(\vec{x}_d)\vec{e}$$

By substituting the previously described definitions of e and v , the controller for the original system is then given by

$$\vec{u} = \mathbf{K}(\vec{x}_d)(\vec{x} - \vec{x}_d) + \vec{u}_d \quad (4-67)$$

Where $\mathbf{K}(\vec{x}_d)$ can be used for gain scheduling, where different gains are calculated for different operating regions. \vec{u}_d is a feedforward input signal.

Integral Action

While the controller on the form (4-67) theoretically should be able to stabilize the system at any desired x_d , this only holds based on the assumption that the physical system behaves exactly like the real system. This could be a valid assumption for a simulated system, but falls short when working with real, physical systems where a good control algorithm must be able to compensate for both imperfections in the model and unforeseen disturbances. To account for this, integral action is presented, enabling the system to compensate for the presence of uncertainty.

The basic approach in integral feedback is, according to [38], to create a new state variable, $\dot{z} = e$, that computes the integral of the error signal. This extends the original system

$$\dot{\vec{x}} = f(\vec{x}, \vec{u})$$

To the form

$$\frac{d}{dt} \begin{bmatrix} \vec{x} \\ \vec{z} \end{bmatrix} = \begin{bmatrix} f(\vec{x}, \vec{u}) \\ \vec{e} \end{bmatrix} \quad (4-68)$$

Finally, the control law for the extended system (4-68) can be found using the same approach as presented in the previous subchapter. By extending (4-67) to incorporate the integral action, the final form of the controller is as follows

$$\vec{u} = \mathbf{K}(\vec{x}_d)(\vec{x} - \vec{x}_d) - \mathbf{K}_i \vec{z} + \vec{u}_d \quad (4-69)$$

Where \mathbf{K} is the same state feedback gain as described in the previous chapters of LQR, \mathbf{K}_i is the integral feedback gain and \vec{u}_d is the feedforward gain, either found by calculations on the model or from measurements on the physical system.

4.5.4 Integrated Absolute Error

When comparing control algorithms or tuning parameters, it is important to use an impartial method. As the aim of this thesis is to present several control implementations rather than finding the *best* control, only one method is briefly mentioned, namely the integrated absolute error (IAE). This measure of quality accumulates the magnitude of the error, and can be described as follows

$$IAE = \int |sp - y| dt \quad (4-70)$$

Where sp is the setpoint and y is the measured output of the system.

The same method can also be used to calculate the difference between two signals, enabling an impartial comparison of for example a simulator and a physical system.

$$IAE = \int |y - \hat{y}| dt \quad (4-71)$$

Where y is the measured output of the system, and \hat{y} is the simulated output of the system.

5 The Physical Design of the Serial Two-Tank System

This chapter presents the sensors, actuator and the physical construction of the STTS. Further, as the sensors and motor driver shield were designed with specific pedagogical aspects in mind, these are presented at the end of each subchapter.

5.1 Electrical components

The STTS system was controlled using Matlab and an Arduino Uno as an I/O. This chapter introduces the sensor circuits and the motor driver that was connected to the arduino, enabling feedback and later control over the system.

5.1.1 Float Sensor

A simple float sensor was implemented in the reservoir, used to read out the water level. The float sensor consisted of a small floating device connected to a rod, where the other end of the rod was connected to a potentiometer. Different water levels would lift the floating device to different heights, causing the angle of the potentiometer to change, and thereby its resistance. The circuit consisted of a $0 - 10k\Omega$ potentiometer in series with a 100Ω resistance, where the ladder resistance was added to prevent a potential short circuiting.

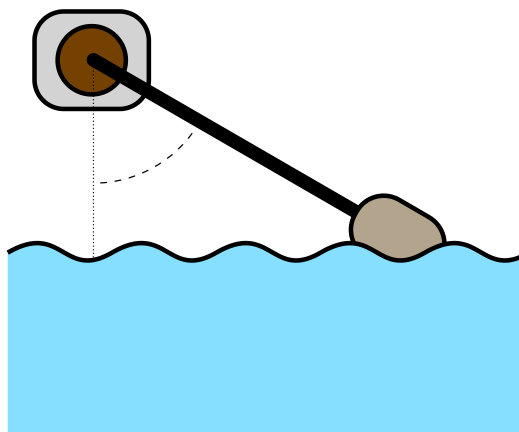


Figure 5-1: Illustration of the float sensor, used to sense the water level in the reservoir.

Pedagogical Aspect

The reason for implementing a floating device was to give students an early introduction to an uncomplicated way of building their own sensor and measure the water level. At an early stage the same sensor was considered being used for both the tanks and the reservoir. However, that would require the tanks to have a considerable wider shape, as the floating device requires physical space to move. Moreover, it was considered a benefit if the students were introduced to more than just one, home-made sensor, which was the motivation for implementing the eTape sensor introduced in the next chapter.

5.1.2 eTape Sensor

The sensor used for the two tanks was the eTape PN-12110215TC-8. The sensor is a solid state, continuous fluid level sensor that works as a variable resistor, changing its resistance when compressed by hydrostatic pressure. The top part of the sensor exposes a small membrane that is used to compare the pressure to the atmospheric pressure. This membrane must not come in contact with water, according to the manufacturer. The key specifications are presented in Table 5-1, based on the datasheet [39], and a picture of the sensor is shown in Figure 5-2.

Table 5-1: Specifications for the eTape, used for sensing the water levels

Part number	PN-12110215TC-8
Nominal Length	8-inch
Sensor Length	10.2" (259 mm)
Active Length	8.4" (213 mm)
Sensor Output	400-1500 Ohm \pm 20%
Ref. resistance	1500 Ohm \pm 20%

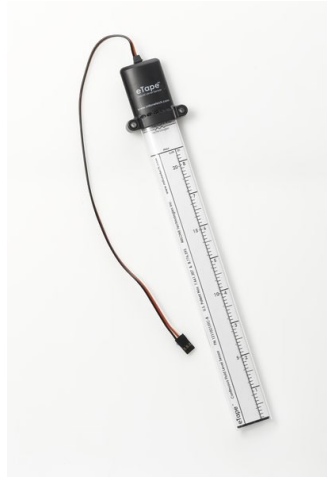


Figure 5-2: The eTape sensor with its corresponding housing cap on top.

From Table 5-1, we see that the sensor acts as a variable resistor, with a resistance varying between 400Ω and 1500Ω . To read the sensor value, a simple voltage divider was designed, based on the description in Chapter 4.1.2. Equation (4-8) allows for calculating the optimal resistance for the fixed resistance in the voltage divider.

$$\begin{aligned}
 R_f &= \sqrt{R_{v,min} \cdot R_{v,max}} \\
 &= \sqrt{400 \cdot 1500} \\
 &\approx 775 [\Omega]
 \end{aligned}
 \tag{ 5-1 }$$

The final voltage divider is summarized in Table 5-2, based on the circuit from Figure 4-1. The result from (5-1) is further illustrated in Figure 5-3, showing a plot of how the voltage range varies when choosing different, fixed resistors for the voltage divider.

Table 5-2: Summarizes the optimal fixed resistance for the variable eTape resistance in a voltage divider circuit.

Voltage divider for the eTape sensor	Resistance
R_f	775Ω
R_v	$400 - 1500\Omega$

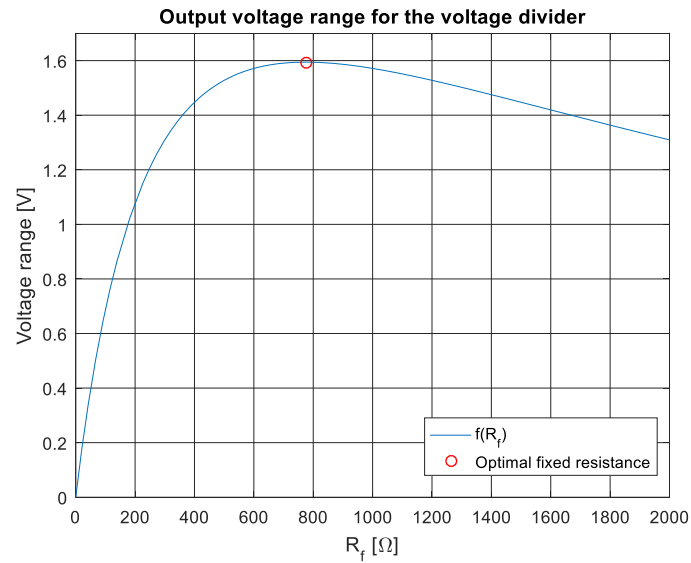


Figure 5-3: Illustrates how the output voltage range in a voltage divider circuit changes when changing the fixed resistance, R_f . The red dot marks the fixed resistance value that maximized output voltage range using the eTape sensor and an input voltage of 5V, which is the supplied voltage input from the Arduino Uno [40].

With the results from Table 5-2, the minimum and maximum output voltage is calculated using (4-35). The results are summarized in Table 5-3 (a). Further, the Arduino Uno operates an ADC with a 10-bit resolution and a voltage reference of $V_{ref} = 5V$, according to [40]. Based on (4-1), the voltage resolution of the eTape water level sensor was calculated and is presented in Table 5-3 (b). Finally, using (4-2), the expected water level per bit is given and presented in Table 5-3 (c), allowing insight into the expected accuracy of each measured sample.

Table 5-3: Summary of the voltage divider. The table presents the expected voltage output from, the voltage resolution, and the water level (height) per bit (accuracy) from a voltage divider using the resistors in Table 5-2.

Sub-section	Parameter name	Value	Unit
(a)	Input voltage	5.0	<i>V</i>
(a)	Voltage at min. variable resistance: $R_v = 400\Omega$	1.7	<i>V</i>
(a)	Voltage at max. variable resistance: $R_v = 1500\Omega$	3.3	<i>V</i>
(a)	Voltage output range compared to voltage input	32	%
(b)	Voltage resolution	4.9	<i>mV/bit</i>
(c)	Water level per bit	0.61	<i>mm/bit</i>

This circuit was implemented for the sensor in tank 1. In tank 2, the same sensor was implemented with a 0-5 VDC resistance-to-voltage module instead. This module is made by Milonetech, the same company that manufactures the eTape sensor.

Pedagogical aspect

Engineering students withing the field of automation often start their studies with introductory courses in electronics. The skills required to make the demonstrated circuit is considered elementary, but illustrates the value of mastering beginners circuit theory to be able to design a circuit for sensor measurement. Further, being able to choose the resistance in a voltage divider that optimizes the voltage range is a useful skill, illustrating the benefits of a theoretical approach to a practical problem.

5.1.3 Motor Driver Shield

A motor driver shield (MDS) was designed to control the pump actuator. A shield is a board that can be plugged directly on top of the Arduino PCB, extending its capability. The MDS is composed of three parts; a low pass filter for smoothing out the PWM signal from the Arduino Uno, a simple transistor circuit to power the pump, and wire routing, connecting the Arduino Uno to the water pump actuator, the 12VDC source and the three sensors. These three parts will be presented next, where the complete circuit for the MDS schematic is shown in Figure 5-4.

Further, as several of the components used on the motor driver shield was not available through the custom library, these parts had to be custom designed for the project. All the project files for the schematic and PCB is available in the associated ZIP-folder. See Appendix I.2 for more information.

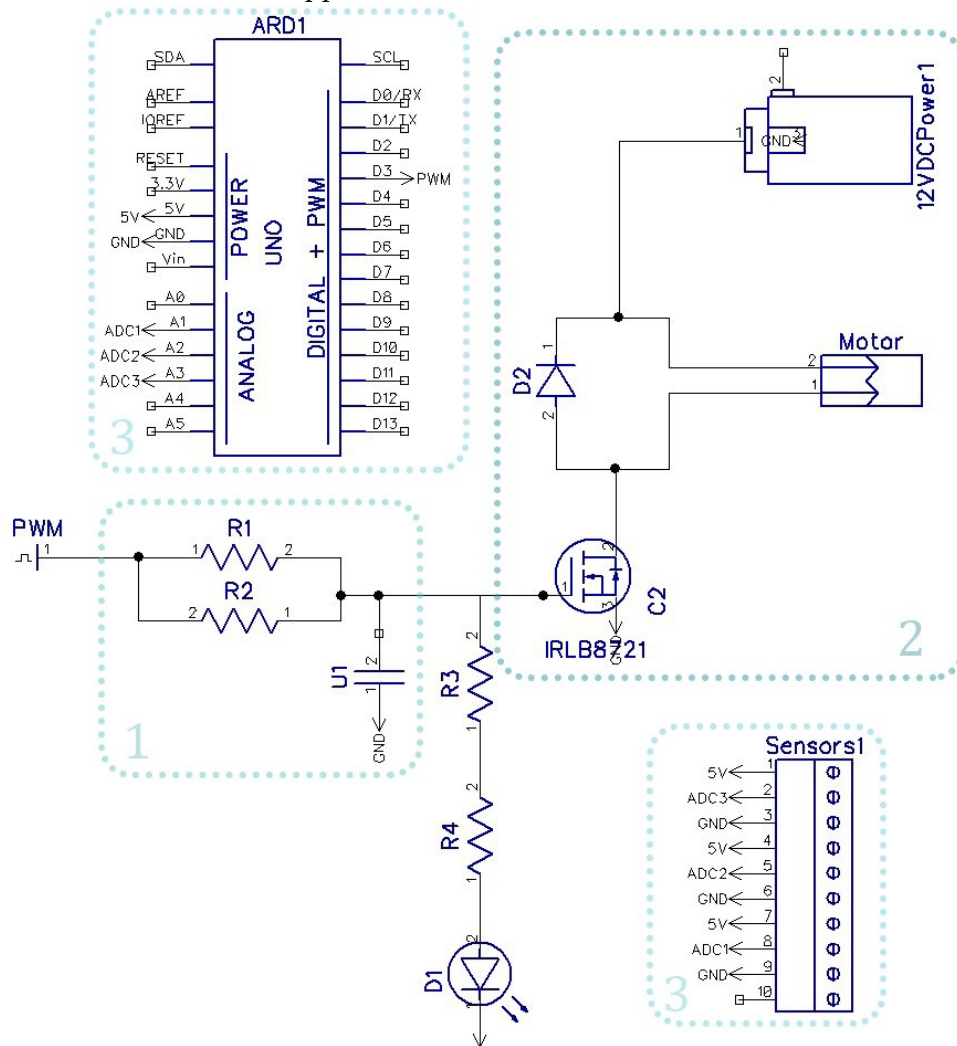


Figure 5-4: The schematics of the motor driver. (1) highlights the simple RC circuit used to smooth out the PWM signal. For educational purposes, the resistance is changed with two resistors in parallel. (2) is the transistor circuit for powering the DC water pump motor. (3) highlights the signal routing that is needed to read from the three sensors. There is also an LED in parallel with the capacitor in (1), which lets the user observe visually the response of the PWM signal, indicated by the brightness of the LED.

PWM Smoothing

The low-pass filter in Figure 5-4 (1) was first simulated using NI Multisim. The purpose of the filter was to provide a smooth control signal to the transistor driving the water pump. The frequency of the PWM signal of the Arduino Uno is approximately 490 Hz [40]. By choosing a cutoff frequency that was around 50 times slower than this, it should ensure a much smoother output signal while still having a reasonable response time. By applying (4-24) and choosing the convenient resistance value of $1k\Omega$, the specifications for the lowpass filter were found, as presented in Table 5-4. Further, the logged response of both the simulated circuit and the real circuit is presented in Figure 5-5 and Figure 5-6 respectively. Though there is a small time delay when working with capacitors, this was later considered negligible with regards to controlling the pump.

Table 5-4: Specifications for the low-pass filter. The filter will smooth out the PWM signal before reaching the transistor that is driving the pump.

Variable name	Variable symbol	Value	Unit
Input PWM signal (5V)	V_{in}	490	Hz
Desired cutoff frequency	f_c	10	Hz
RC resistor	R	1000	Ω
RC capacitor	C	15	μF

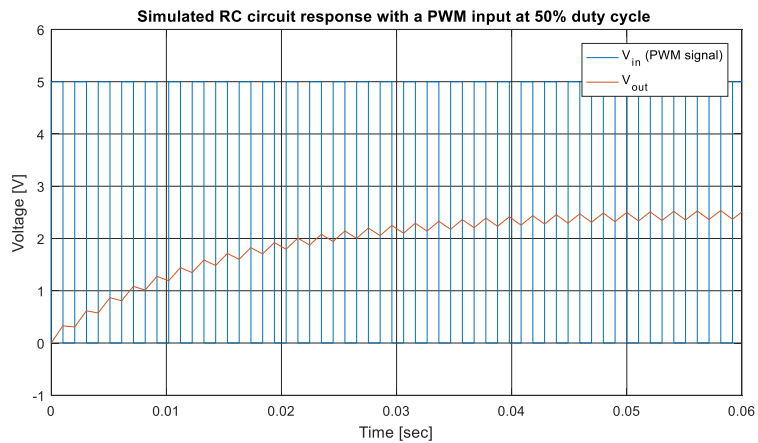


Figure 5-5: The plot shows the simulated response of the low pass filter, with a PWM input signal with a 50% duty cycle.

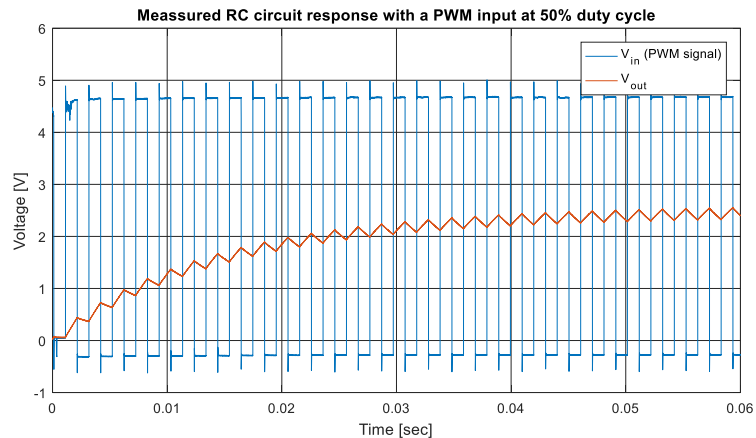


Figure 5-6: The plot shows the measured response of the physical low pass filter, with the Arduino Uno providing a 50% duty cycle PWM input signal.

Motor Driver

The second part of the motor driver, Figure 5-4 (2), is simply a transistor connected in series with the pump and the power source, where the transistor is driven by the smoothed PWM signal. A diode was placed in parallel with the motor to provide a safe path for the potential inductive kickback from the motor. The water pump is a centrifugal pump driven by a DC motor with the working range of 6 – 12V. A power supply of 12 V was therefor applied.

Wire Routing

The final part of the shield was simply a matter of routing the signals to the correct place, Figure 5-4 (3). An illustration of this routing is shown in Figure 5-7.

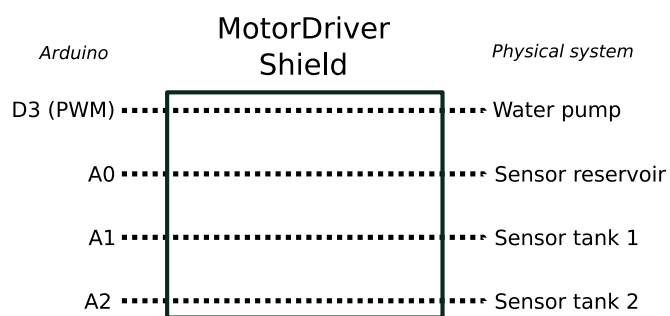


Figure 5-7: Illustration of the wire routing in the motor shield, connecting the sensors and motor controller to the correct pins on the Arduino Uno.

The final design of the motor driver shield is shown in Figure 5-8. This shield is based on the circuit schematic shown in Figure 5-4, containing the PWM low-

pass filter, the motor driver and the wire routing. The key components used in this shield is listed in Table 5-5. A photo of the final shield, and the process of getting there using design thinking is described in Chapter 8.2.

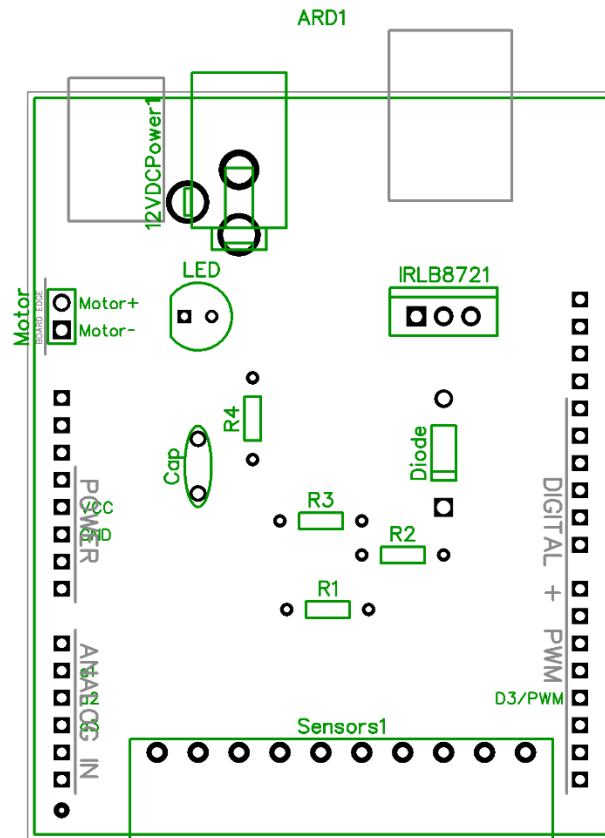


Figure 5-8: The final design of the motor driver, made to fit directly into an Arduino Uno.

Table 5-5: Components used in the motor driver shield circuit.

Component type	Component name
MOSFET	IRLB8721
Transistor	
Power socket	SPC21364
Diode	BYW36-TR

Pedagogical aspect

The motor driver gives an introduction to PWM signals, and illustrates an easy way of controlling an analog actuator using a digital output. The idea is to give the students an early hands-on experience with the electrical components that they later will use on a theoretical level. Depending on the facilities and equipment of the school, the students could be asked to design, produce and/or solder their own motor driver shield. This type of learning goal originates from the higher-level learning goals for practical work, presented in Chapter 3.4, as it will help familiarize the students with equipment, techniques and components.

Regarding the design presented in Figure 5-8, two educational challenges are built in to the schematic. In Figure 5-4, note that two resistors are divided into four – two in parallel and two in series. The reason for this is to challenge the students to choose the correct values for each of them, forcing them to use what they have learned about basic circuit theory.

5.2 The Design of the Physical System

The work of this thesis envisions a learning package where each student receives a flat-packed, physical system when starting the first semester. Through several learning modules, which will be described in Chapter 7.2, the students will learn to build, analyze and control this STTS. This chapters presents the constructed STTS prototype used throughout the thesis.

5.2.1 The Final Product Design

The final design of the STTS was created using SolidWorks and is illustrated in Figure 5-9. The design files are available in the associated ZIP-folder. See Appendix I.1 for more information.

The design of the STTS was created in attempt to embody all the benefits of the STTS as presented in Table 8-2, while additionally dealing with three of the presumed challenges. Firstly, each tank was designed to be separable from the complete system. By designing the tanks to have different sizes, they could then be turned upside down and slipped into the tank below it. The system was still pretty big for a desktop application when fully assembled, but made it possible to compress the size of the system during transportation. Secondly, to prevent the tank from overflowing, an anti-overflow pipe was designed for each tank, making potential overflow water return to the reservoir. Thirdly, to prevent potential leakage from a coupling to a pump placed on the outside of the reservoir, a waterproof pump was purchased that could be placed on the bottom of the reservoir.

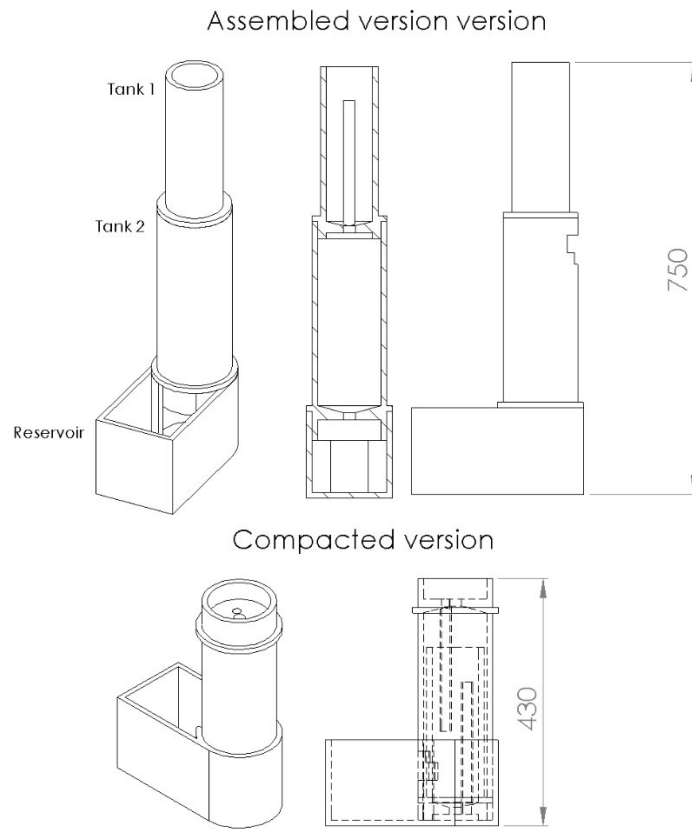


Figure 5-9: Summary of the technical drawings of the STTS, based on the figures in Appendix D. The final design allowed each tank to fit into the next, making it smaller for transportation.

A photo of the final, physical system is presented in Figure 5-10. The two tanks were built using a transparent plastic material, enabling the students to see all the activity happening inside the system. Each eTape sensor was mounted with the membrane directly above the anti-overflow pipes, ensuring that the water level could not cause damage to sensor membrane. Further, to reduce the potential of water splashing from the pump onto this membrane, this input was attached at the bottom of tank 1.



Figure 5-10: Photo of the physical STTS. Water is pumped in at the bottom of upper tank. The input flow to this tank is delivered by a submersible water pump at the bottom of the reservoir. Both tanks have a return pipe preventing overflow.

Summary of the System Parameters

The following table is generated based on physical measurements and parameter estimations, the ladder presented in Chapter 6.2. From Table 5-6, the complete, physical system is summarized.

Table 5-6: Parameters of the physical STTS.

Parameters	Symbol	Value	Unit
General			
Gravitational acceleration	g	9.81	m/s^2
Tank 1			
Height of tank	$h_{1,max}$	0.200	m
Diameter of tank		$95.0 \cdot 10^{-3}$	m
Cross section of tank	A_1	$7.09 \cdot 10^{-3}$	m^2
Diameter of orifice		$13.0 \cdot 10^{-3}$	m
Cross section of orifice	α_1	$1.34 \cdot 10^{-4}$	m^2
Valve discharge coefficient	C_1	0.590	—
Sensor output, ($h_1 = h_{1,max}$)	$V_{1,max}$	3.75	V
Sensor output, ($h_1 = 0m$)	$V_{1,min}$	2.53	V
Tank 2			
Height of tank	$h_{2,max}$	0.200	m
Diameter of tank		$115 \cdot 10^{-3}$	m
Cross section of tank	A_2	$10.4 \cdot 10^{-3}$	m^2
Diameter of orifice		$13.0 \cdot 10^{-3}$	m
Cross section of orifice	α_2	$1.34 \cdot 10^{-4}$	m^2
Valve discharge coefficient	C_2	0.51	—
Sensor output, ($h_2 = h_{2,max}$)	$V_{2,max}$	0.22	V
Sensor output, ($h_2 = 0m$)	$V_{2,min}$	3.81	V
Pump			
Operating voltage		6 – 12	V
Max. specified pump flow	\hat{Q}_{max}	$2.22 \cdot 10^{-4}$	m^3/s
Max. estimated pump flow ($k \cdot \hat{Q}_{max}$)	Q_{max}	$1.73 \cdot 10^{-4}$	m^3/s
Estimated pump flow rate coefficient	C_p	$3.46 \cdot 10^{-5}$	$m^3/s/v$

6 Modeling, Analysis, and Control of the Simulated and Physical STTS

6.1 Modelling the Physical System

This chapter focuses on the mathematical modeling of the STTS. For simplicity, a linear pump input is assumed, with the dynamics presented below.

$$Q_{pump} = C_p u \quad (6-1)$$

u is the input voltage to the motor driver shield controlling the pump.

By combining (4-29) and (4-33), and recalling that the output of tank 1 equals the input of tank 2, the following differential equations are achieved for describing the dynamics of the STTS. The cross-section areas of the tanks are assumed to be constant.

$$\begin{aligned} \dot{\vec{x}} &= f(\vec{x}, u) \\ &= \begin{bmatrix} \frac{1}{A_1} (C_p u - C_1 \alpha_1 \sqrt{2g x_1}) \\ \frac{1}{A_2} (C_1 \alpha_1 \sqrt{2g x_1} - C_2 \alpha_2 \sqrt{2g x_2}) \end{bmatrix} \\ &= \begin{bmatrix} G_p u - G_1 x_1^{1/2} \\ G_2 x_1^{1/2} - G_3 x_2^{1/2} \end{bmatrix} \end{aligned} \quad (6-2)$$

Where $G_p = C_p/A_1$, $G_1 = C_1 \alpha_1 \sqrt{2g}/A_1$, $G_2 = C_1 \alpha_1 \sqrt{2g}/A_2$ and $G_3 = C_2 \alpha_2 \sqrt{2g}/A_2$, all assumed to be positive definite.

Since both the height in tank 1 and tank 2 can be measured directly, the output is given as

$$\vec{y} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (6-3)$$

6.1.1 Equilibrium Points

By following the procedure described in Chapter 4.4.1, the equilibrium point of the unforced system (6-2) is found.

$$\begin{aligned} \dot{\vec{x}} &= f(\vec{x}^*, 0) = 0 \\ \begin{bmatrix} G_p u - G_1 x_1^{1/2} \\ G_2 x_1^{1/2} - G_3 x_2^{1/2} \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned} \quad (6-4)$$

The equilibrium point of (6-4) is $[x_{1,eq} \quad x_{2,eq}]^T = [0 \quad 0]^T$. In other words, the system has only one equilibrium point, which is when there is no water in either of the tanks.

6.1.2 Linearizing the System to State Space Form

Based on the theory described in 4.4.2, the Jacobian state matrices can be calculated, as shown in (4-46), to form the state space model on the form (4-45), linearized around x^* .

$$\begin{aligned} \dot{\vec{x}}(t) &= \mathbf{A}\vec{\delta}_x(t) + \mathbf{B}\vec{\delta}_u(t) \\ &= \begin{bmatrix} -\frac{G_1}{2x_1^{*2}} & 0 \\ \frac{G_2}{2x_1^{*2}} & -\frac{G_3}{2x_2^{*2}} \end{bmatrix} \vec{\delta}_x(t) + \begin{bmatrix} G_p \\ 0 \end{bmatrix} \vec{\delta}_u(t) \\ &= \begin{bmatrix} -a_1 & 0 \\ a_2 & -a_3 \end{bmatrix} \vec{\delta}_x(t) + \begin{bmatrix} b \\ 0 \end{bmatrix} \vec{\delta}_u(t) \end{aligned} \quad (6-5)$$

Where $a_1 = G_1/2x_1^{*2}$, $a_2 = G_2/2x_1^{*2}$, $a_3 = G_3/2x_2^{*2}$, $b = G_p$, $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times p}$ for the linearized system, based on (6-2).

In the case of measuring the water level of *both* tanks, the output of the system results in the following \mathbf{C} matrix.

$$\begin{aligned} \vec{y}(t) &= \mathbf{C}\vec{\delta}_x(t) \\ \mathbf{C} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \quad (6-6)$$

Where $\mathbf{C} \in \mathbb{R}^{q \times n}$.

The feedthrough matrix $\mathbf{D} \in \mathbb{R}^{q \times p}$ is a zero matrix, and therefore not included in the state space representation.

6.1.3 System Transfer Function

To find the transfer function of the two systems, the state space from (6-5) and (6-6) are used according to (4-47), to calculate the transfer functions of the system. This results in two transfer functions for the two separate systems and one for the complete system, where the parameters a_1, a_2, a_3 and b are the same as in (6-5). The systems are presented in the classic form of first and second order systems, as described in (4-54) and (4-55).

Tank 1

$$H_{u \rightarrow x_1}(s) = \frac{Y_1(s)}{U(s)} = \frac{\frac{b}{a_1}}{\frac{1}{a_1} \cdot s + 1} \quad (6-7)$$

Tank 2

$$H_{x_1 \rightarrow x_2}(s) = \frac{Y_2(s)}{Y_1(s)} = \frac{\frac{a_2}{a_3}}{\frac{1}{a_3} \cdot s + 1} \quad (6-8)$$

Complete Open-Loop System⁵

$$H_{u \rightarrow x_2}(s) = \frac{Y_2(s)}{U(s)} = \frac{\frac{ba_2}{a_1a_3}}{(\frac{1}{a_3}s + 1)(\frac{1}{a_1}s + 1)} \quad (6-9)$$

6.1.4 Steady State Input Required to Reach a Desired Steady State

The steady state input needed to stabilize the system at any desired height is useful to know, both for simulation purposes and for feedforward purposes. To calculate the steady state input needed to stabilize either tank 1 or tank 2 at a desired steady state value, the ODE from (6-2) can be solved to find the required inputs.

Tank 1

$$\begin{aligned} \dot{x}_1 &= 0 \\ u &= \frac{C_1 \alpha_1}{C_p} \sqrt{2gx_1} \end{aligned} \quad (6-10)$$

Tank 2

$$\begin{aligned} \dot{x}_1 = \dot{x}_2 &= 0 \\ u &= \frac{C_2 \alpha_2}{C_p} \sqrt{2gx_2} \end{aligned} \quad (6-11)$$

Using the ODE from (6-2), the water level required in tank 1 to keep a desired steady state in tank 2 can also be found

$$x_1 = \left(\frac{C_2 \alpha_2}{C_1 \alpha_1} \right)^2 x_2 \quad (6-12)$$

⁵ The time constants are placed in the order described in Chapter 4.5.1, with the first corresponding to the dominant time constants.

Steady State Output Flow from Tank 2

The output flow of tank 1 for any given height, assuming there is no input flow, is briefly presented here, as it is used later to calculate the tuning parameter of the single-loop PID control scheme in Chapter 6.4.4.

$$Q_1(x_1) = C_1 \alpha_1 \sqrt{2gx_1} \quad (6-13)$$

6.2 Function Identification and Parameter Estimation

Several parameters and functions were estimated based on recorded data, with the intention to make the simulated system resemble the dynamics of the physical system.

6.2.1 Estimate the Valve Coefficient

To estimate the discharge coefficients of the valves, each tank was filled separately with 0.2 meters of water. The tanks were then drained and logged separately, with no input to the tank. This can be formulated as $x(t_0) = x_0 = 0.2$, $t_0 = 0$ and $Q_{in} = 0 \Rightarrow u = 0$.

From (6-2), the general ODE for any of the two tanks with no input is described as follows

$$\dot{x} = -\frac{C \cdot \alpha \cdot \sqrt{2g}}{A} \cdot \sqrt{x} \quad (6-14)$$

Solving (6-14) analytically gives the following expression for the water level of the tank

$$x(t) = \left(\sqrt{x_0} - \frac{1}{2} \cdot \frac{C \cdot \alpha \cdot \sqrt{2g}}{A} t \right)^2 \quad (6-15)$$

Following the theory from Chapter 4.3.1, a new function is created, $g(t) = \sqrt{x(t)}$, which is a linear transformation of the solved ODE.

$$g(t) = mt + b \quad (6-16)$$

Where $m = -C\alpha\sqrt{g}/(\sqrt{2}A)$ and $b = \sqrt{x_0}$. The function to be minimized can now be written as

$$J = \sum_{i=1}^n (g_i - \tilde{g}_i)^2 \quad (6-17)$$

Where \tilde{x}_i is the measured signal at sample i , and $\tilde{g}_i = \sqrt{\tilde{x}_i}$.

By constraining the estimated function to pass through the point (t_0, x_0) , as described in Chapter 4.3.3, the parameter m is then estimated. Finally, to find

the valve coefficients, the definition of m can be used to solve for the valve coefficient C

$$C = -\frac{\sqrt{2}mA}{\alpha\sqrt{g}}$$

The valve coefficients were estimated by draining tank 1 and tank 2 separately, and are presented in Table 5-6 in Chapter 5.2.1. The logged response and the estimated function for both tanks is shown in Figure 6-1, and the calculated IAE for the both observations are presented in Figure 6-2.

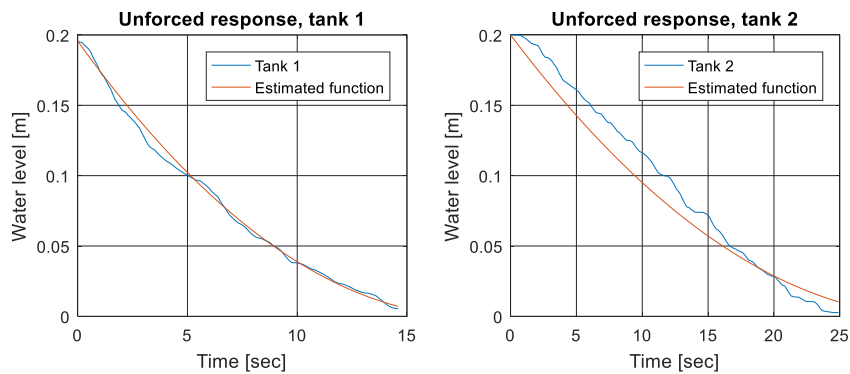


Figure 6-1: The unforced system response of tank 1 and tank 2. Each experiment was done separately, with no input into the tank.

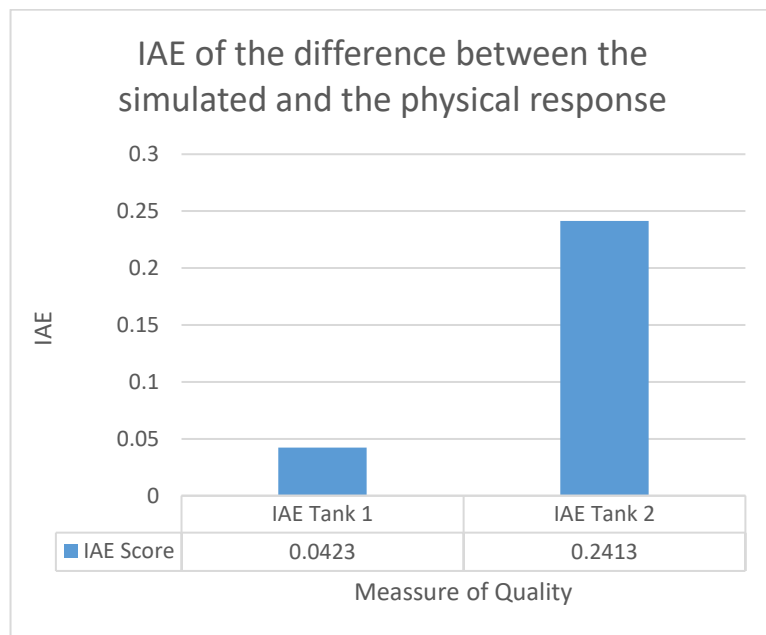


Figure 6-2: IAE calculated from the separate, unforced response presented in Figure 6-1

With the coefficients estimated, the system was simulated with both tanks filled completely and with no input in tank 1. With the newly estimated discharge coefficients of the valves, the draining of the complete system was compared to the behavior of the physical system. Both responses are presented in Figure 6-3.

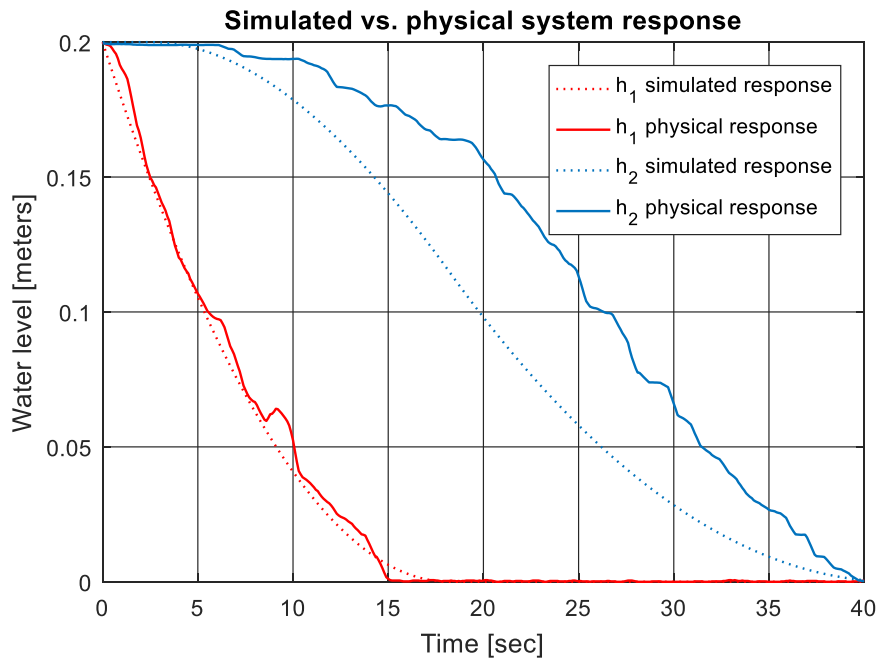


Figure 6-3: The simulated response versus the real response of the STTS, with the same initial water level heights and with no system input. In the experiment, both tanks were drained simultaneously so that the draining of tank 1 became an input for tank 2.'

The IAE is calculated for the system response of tank 1 and 2 to rate the simulator performance. The results are presented in Figure 6-4, where the scores are calculated using (4-71).

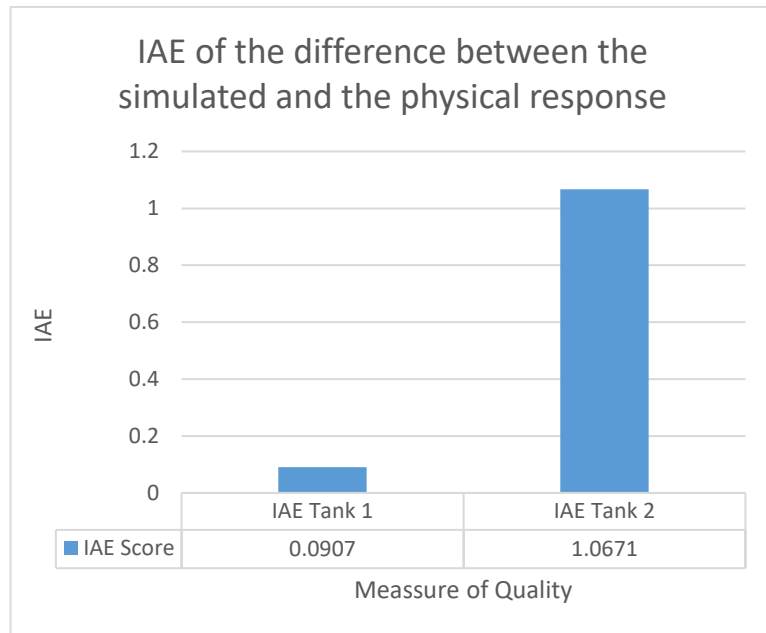


Figure 6-4: The calculated IAE score when comparing the unforced system response for the simulated and the physical system shown in Figure 6-3.

A significant deviation between the physical and simulated response of tank 2 is observed in Figure 6-3 and Figure 6-4. This deviation is mostly due to an accumulated error caused by the poor fit when estimating the valve coefficient shown in Figure 6-1 and Figure 6-2. Since the valve dynamics are assumed to act as orifice openings, this is however the best results achievable when estimating the valve coefficients using the OLSM on the valve dynamics from (4-33).

6.2.2 Estimate a Linear Pump Flow Rate Coefficient

As there was no flow meter available to measure the volumetric flow rate at different voltage inputs, an experiment was designed to estimate the volumetric flow using the available equipment. The valve of tank 1 was closed, and the water level of tank 1 was logged while filling the tank at different input voltage signals. This was done with a voltage output range from the Arduino Uno at 2.5-5 voltage⁶, with an increment of 0.1 voltage for each new experiment.

⁶ The operating range of the water pump was 6-12 V. Assuming a linear gain from the motor driver, this should equal an output range from the Arduino Uno of 2-2.5 V simulated using PWM.

For simplicity, the thesis assumes a linear input, $Q_{in} = C_p u$. For further simplicity, the coefficient C_p is defined as the scalar that transforms the voltage signal from the Arduino Uno (PWM signal simulating a 0 – 5V analog signal) to a volumetric flow rate (potential delays and nonlinearities in the motor driver are neglected).

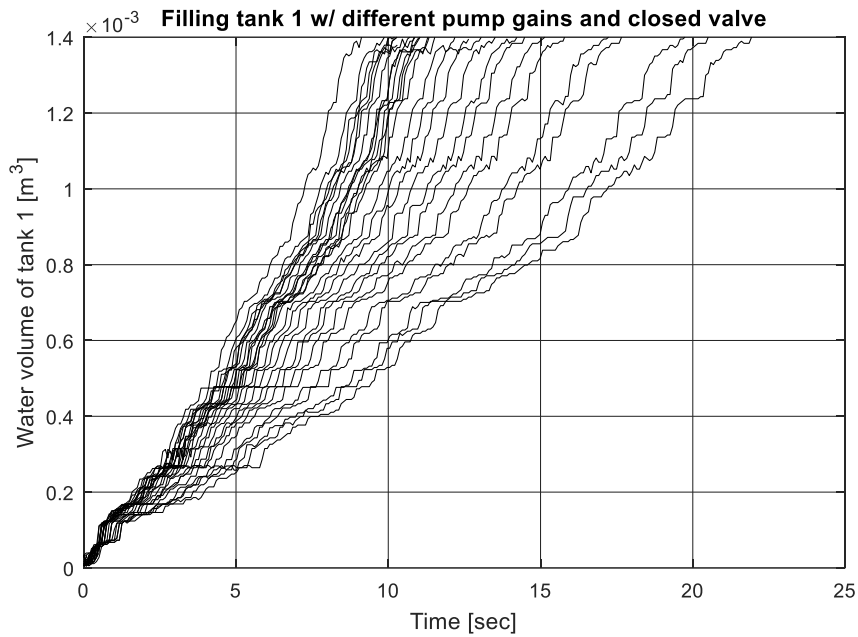


Figure 6-5: The water volume in Tank 1 as the system was filled with water, using different voltage inputs to the pump. The plot does not show the voltage signal used for the different loggings, as this became unreadable between all the lines. However, intuition should let the reader realize that the steeper (rightmost) plots were achieved when applying a higher voltage signal to the pump, causing a higher volumetric input flow that filled the tank quicker.

Next, the volumetric flow for each experiment was calculated as follow

$$Q(v_i) = \frac{A \cdot h_{max}}{t}, \quad v_i \in \{2.5V \quad 2.6V \quad \dots \quad 5.0V\}$$

These experiments gave an approximation of the flow rate. A linear pump flow rate function was then estimated using the OLSM described in Chapter 4.3.2. Further, by constraining it to pass through the origin so that $Q(v = 0) = 0$, as described in Chapter 4.3.3, the coefficient was obtained, scaling the output voltage from the Arduino Uno to an approximated volumetric input flow rate to the system.

The final pump flow rate coefficient was presented earlier in Table 5-6. A corresponding plot is available in Figure 6-6, showing the estimated function with and without the constraint of intersecting the origin, where the ladder was the one implemented in the simulator.

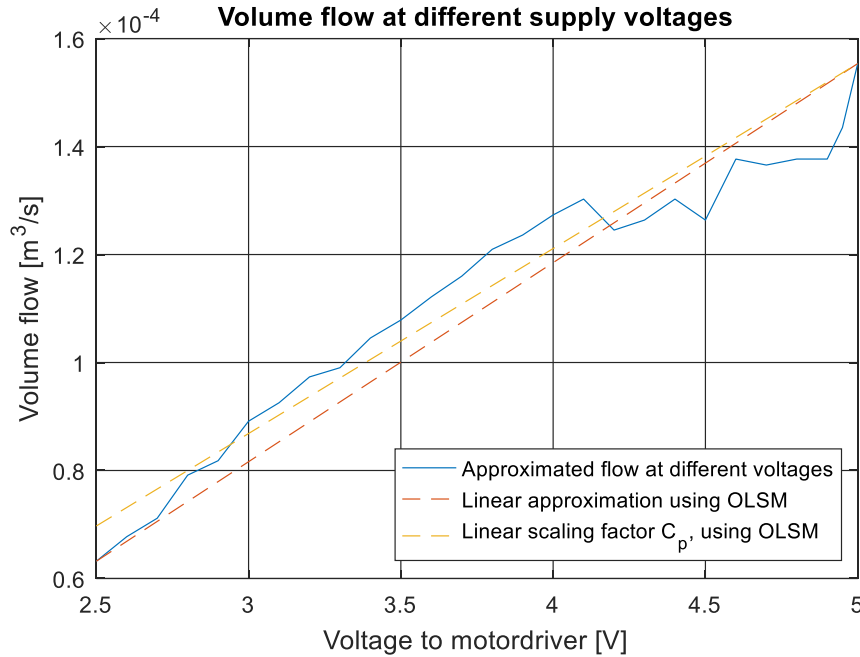


Figure 6-6: The figure shows the measured volume flow based on the data shown in Figure 6-5. The red dotted line shows the best fit linear function based on the OLSM. The yellow dotted line shows the best fit linear function that additionally is forced to pass through the origin. The ladder is the one implemented in the simulation.

6.2.3 Estimate a Linear Voltage-to-Meter Sensor Function

For the sensor signal to represent the actual measurement unit, meters, it first had to be scaled properly using (4-42). Assuming that the sensor is linear, as stated in the documentation [39], the slope was assumed to be $m = (y_2 - y_1)/(x_2 - x_1)$. For simplicity, the following two points were used

$$\begin{aligned}(x_1, y_1) &= (V_{i,min}, 0) \\ (x_2, y_2) &= (V_{i,max}, h_{i,max})\end{aligned}$$

Where $i \in \{1,2\}$ represents tank 1 or 2. This gives the the following function to convert from voltage to meters

$$h(v) = \frac{h_{i,max}}{V_{i,max} - V_{i,min}} \cdot (v - V_{i,min}) \quad (6-18)$$

Where v is the input voltage signal, h is the water level of the tank and the other parameters can be found in Table 5-6.

6.3 Stability Analysis of the Unforced System

The unforced version of the system (6-2) can be written on the form of (4-43), by setting the input to zero

$$\dot{\vec{x}} = f(\vec{x}) = \begin{bmatrix} -G_1 x_1^{1/2} \\ G_2 x_1^{1/2} - G_3 x_2^{1/2} \end{bmatrix} \quad (6-19)$$

This system is locally Lipschitz for the domain $\mathbb{D} = [0, +\infty)$, according to ET 4-1. Based on this unforced system, the stability of the system can be analyzed.

The stability analysis starts of by calculating phase portrait for the the system. Then, in attempt to prove stability, the Lyapunov's indirect method is applied, unsuccessfully, where the method is included to motivate for a more advanced approach. Finally, Lyapunov's direct method is applied, proving and classifying the stability of the system.

6.3.1 Phase Portrait

Based on (6-19), the phase portrait of the modeler STTS was sampled for the working domain $\vec{x} \in \mathbb{E}^{2 \times 1}$, where $\mathbb{E} \in [0 \ 0.2]$. This sampling range was convenient because this is the smallest and biggest possible water level of the physical system, and because the ODE would yield complex answer for negative states. The phase portrait is presented in Figure 6-7, which suggests the system is stable in origin without proving it.

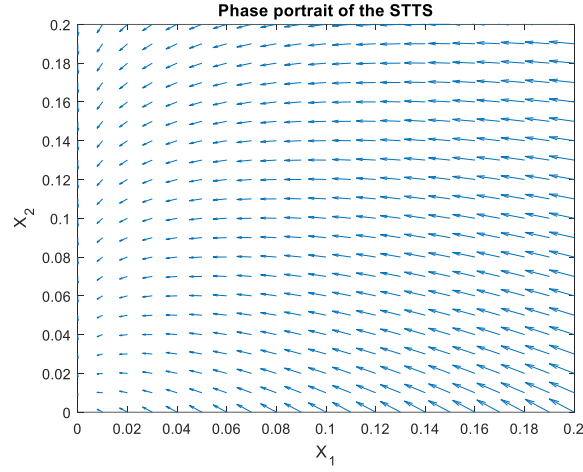


Figure 6-7: Phase portrait of the unforced system, sampled for the whole working domain.

The phase portrait in Figure 6-7 suggests that energy will move from x_1 to x_2 , and then toward the origin. Since the origin also is the equilibrium of the system, according to (6-4), the phase portrait suggests a stable system in the working domain of the STTS.

6.3.2 Lyapunov's Indirect Method

As described in Chapter 4.4.3, the stability of an equilibrium point can be analyzed by investigating the eigenvalues of the linearized system in the equilibrium point. However, this is not possible for this system, and is only presented in this thesis to motivate for the use of more complex, nonlinear stability analysis.

By inspecting the linearized system (6-5) at the equilibrium point found in (6-4), following the method described in ET 4-4, the following problem occurs

$$\begin{aligned} \mathbf{A} &= \left. \frac{\partial f}{\partial \vec{x}}(\vec{x}) \right|_{\vec{x}=0} \\ &= \left. \begin{bmatrix} -\frac{G_1}{2x_1^{*2}} & 0 \\ \frac{G_2}{2x_1^{*2}} & -\frac{G_3}{2x_2^{*2}} \end{bmatrix} \right|_{x=0} \\ \lim_{\vec{x}^* \rightarrow 0} \mathbf{A} &= \begin{bmatrix} -\infty & 0 \\ \infty & -\infty \end{bmatrix} \end{aligned}$$

Applying Lyapunov's indirect method on this system is therefore not possible in the equilibrium point, as the system becomes ill-defined. This proves the necessity for more advanced stability theorems like the direct Lyapunov method.

6.3.3 Lyapunov's Direct Method

Based on the theory presented in Chapter 4.4.6, the following Lyapunov candidate is chosen for the nonlinear water tank system presented in (6-19).

$$\begin{aligned} V(\vec{x}) &= \frac{1}{2} \vec{x}^T \mathbf{P} \vec{x} \\ &> 0, \forall \vec{x} \in \mathbb{D} - \{0\} \\ V(\vec{0}) &= 0 \end{aligned} \quad (6-20)$$

Where $\mathbb{D} = [0, +\infty)$ and $\mathbf{P} = \begin{bmatrix} p_{1.1} & 0 \\ 0 & p_{2.2} \end{bmatrix}$. \mathbf{P} must be positive definite, satisfying $p_{1.1} > 0$ and $p_{1.1}p_{2.2} > 0$. The derivative of $V(\vec{x})$ is given by

$$\begin{aligned} \dot{V}(\vec{x}) &= \vec{x}^T \mathbf{P} \dot{\vec{x}} \\ &= -p_{1.1}G_1x_1^{3/2} - p_{2.2}G_3x_2^{3/2} - p_{2.2}G_2x_1^{1/2}x_2 \\ &< 0, \forall \vec{x} \in \mathbb{D} - \{0\} \\ \dot{V}(\vec{0}) &= 0 \end{aligned} \quad (6-21)$$

Recall from (6-2) that G_1 , G_2 and G_3 are all positive definite.

From (6-20) and (6-21) the Lyapunov properties of ET 4-6 are satisfied, proving asymptotical stability in the domain $\mathbb{D} = [0, +\infty)$. This means that the system will converge to the equilibrium for any initial value $x_0 \in [0, +\infty)$ in the domain \mathbb{D} .

The stronger stability criterion, global asymptotical stability, from ET 4-7 can however not be concluded, as this requires the Lyapunov properties to hold for any real numbers. However, as the states for the real system can only take positive values (as they represent the water level height, which is non-negative), being asymptotically stable within the domain $\mathbb{D} = [0, +\infty)$ will ensure that it holds for the whole physical working domain.

Finally, the potential for exponential stability, as described in ET 4-8, is not satisfied with this Lyapunov candidate. This should however not come as a surprise, as the analytical solution of the system (6-15) shows that the decay of the unforced system follows a second-order polynomial function.

6.4 Implemented Control Schemes

To demonstrate how the STTS has a pedagogical potential within the field of control theory and automation, several control schemes were implemented. Though there exist more advanced control methods that could be well suited for this system (i.e. MPC), the scope of the thesis was limited to implement four control schemes: a single-loop PID for tank 1 and tank 2, cascade PID for tank 2 and LQR for tank 2. The thesis does not put any work into fine-tuning the controller parameters. The aim is to illustrate the procedure of implementation for the different schemes, and compare the simulated response to the physical system.

Before introducing the results of the four control schemes, the implementation of the controller and simulator is briefly described in the next two chapters.

All Matlab-scripts, -functions, -simulations and Simulink files are available in the attached ZIP-folder. See Appendix I.3 for more information.

6.4.1 Real-Time Control using Matlab and Arduino

All logging and control schemes that were implemented on the physical system was done so using Matlab and an Arduino Uno. The Arduino was used to keep the costs low, while Matlab was used both because it is a software that most engineering students are expected to learn and because it offers a language with a low threshold from ideas to implementation, as opposed to lower level programming languages.

This approach required a support package for Arduino hardware to be installed through Matlab. A similar package is also offered to Simulink, but futile attempts showed that that this approach had issues with the serial communication, where measurements often were lost.

A Matlab script was generated that allowed for full measurements and control of the system, while showing a visual real-time plot, as shown in Figure 6-8. For all implementations on the physical system, a sample rate of 10 Hz was used. This is both significantly faster than the dynamics of the STTS, while allowing a uniform sampling from the Arduino Uno.

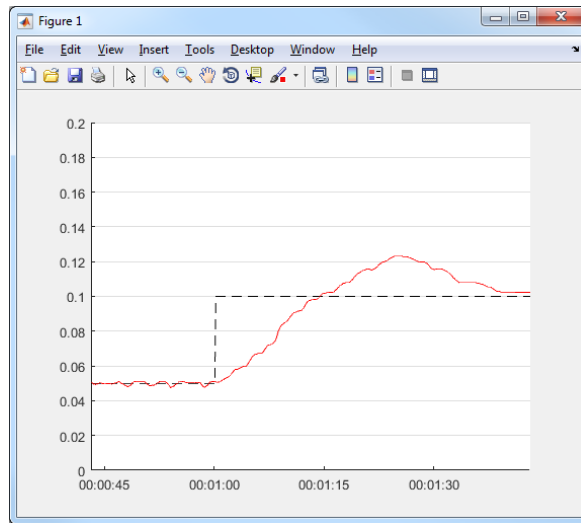


Figure 6-8: The figure shows the real-time when running the Matlab script for controlling the Arduino and the STTS. The plot shows a scenario when the water level of tank 1 was controller while being subjected to a step in the setpoint at $t = 60$ sec. The red line is the water level of tank 1, while the dotted black line is the setpoint.

6.4.2 Simulating the System

The system was simulated both by script and through Simulink. The Simulink simulations where normally used when extensive simulating was required, while the scripted simulation was used to calculate the unforced response and compare this to the measured response. The implementations for the scripted solver is based on Euler's method presented in (4-49). In an attempt to investigate which step size would result in a stable simulation, the eigenvalues of the linearized system were investigated based on (6-9).

$$\begin{aligned}\lambda_1 = a_1 &= -\frac{G_1}{2x_1^{*2}} \\ \lambda_2 = a_3 &= -\frac{G_3}{2x_2^{*2}}\end{aligned}\quad (6-22)$$

From this, we can see that the eigenvalues go towards negative infinity when being linearized around zero, and toward zero when being linearized around an infinite tall tank, as follows

$$\begin{aligned}\lim_{x_1^* \rightarrow 0} \lambda_1 &= \lim_{x_2^* \rightarrow 0} \lambda_2 = -\infty \\ \lim_{x_1^* \rightarrow \infty} \lambda_1 &= \lim_{x_2^* \rightarrow \infty} \lambda_2 = 0\end{aligned}\quad (6-23)$$

In other words, the stability of the simulation cannot be concluded by investigating the linearized eigenvalues. Instead, an empirical approach was tried, simulating the system several times with a decreasing step size until a satisfactory result was achieved, as illustrated in Figure 6-9. Though this approach offers no proof of the stability of the simulation, the empirical results seemed to support a stable simulation for step sizes smaller than $h = 0.5$. To be sure, the scripted simulation used a step size of $h = 0.01$. Further, all simulations done in Simulink used a variable step size with the solver set at “auto”. The Simulink implementation is documented in Appendix H, and all files are available as described in Appendix I.3.

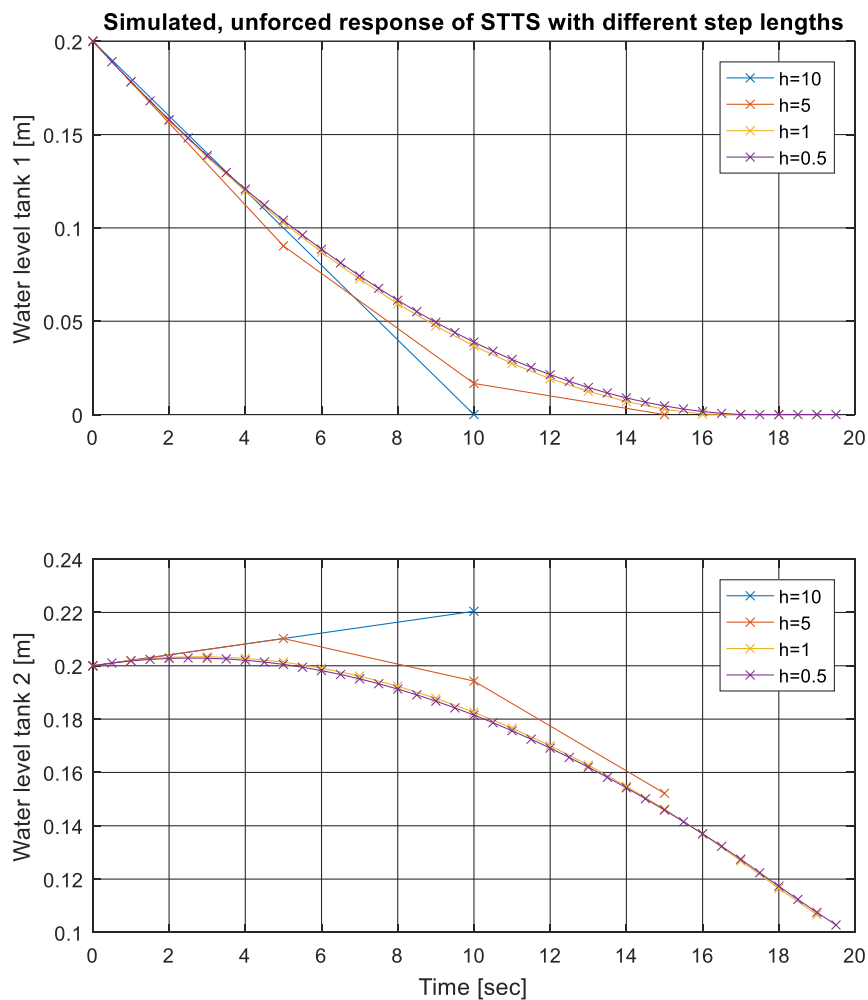


Figure 6-9: Simulated response of the STTS when varying the step size in the numerical integrator, which is based on “Euler’s method”.

6.4.3 Scheme 1: Single- or Inner-Loop Control of Tank 1

To control tank 1, either as a system by itself or as an inner-loop in a cascade scheme (which will be presented later), the approach illustrated in Figure 6-10 was implemented. The PI-controller was implemented on the form (4-51).

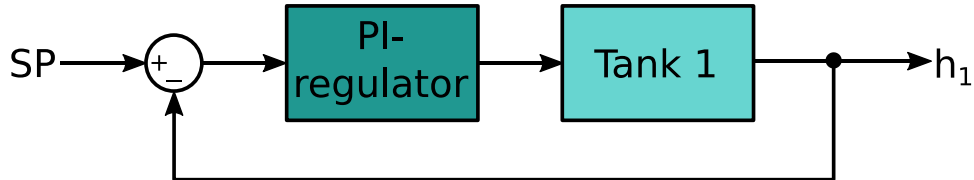


Figure 6-10: Illustration of the control setup when controlling tank 1, alternatively the inner loop in a cascade control scheme.

Tuning Parameters

With the aim of finding good initial PI parameters, the SIMC method from Chapter 4.5.1 was applied. Since the transfer function for Tank 1 from Figure 6-10, described in (6-7), already is described by a first-order model, there is no need for approximating a model, and (6-7) is applied directly. Further, the tuning parameter is chosen according to (4-61), as there is no time delay for this process. The final tuning parameters are presented in Table 6-1, using the constants from (6-5) and the values from the system measurements in Table 5-6. The controller is on the form of (4-51). Note that the controller in this scheme has no derivative part, meaning that it is not necessary to transform the system from serial to a parallel form.

Table 6-1: Tuning parameter for the PI controller for Tank 1

Parameter name	Serial/parallel PID parameters	Parallel values
Tuning parameter	$\tau_c = 0.2 \cdot h_{1,max} \cdot A_1 / Q_{max}$	1.64
Delay	θ	0
Proportional gain	$K_p = (1/a_1) / ((b/a_1) \tau_c)$	125
Integral gain	$T_i = \min\{1/a_1 \quad 4\tau_c\}$	6.57

System Response

Using the tuning parameters from Table 6-1, a PI controller was implemented in both the simulated and the physical system. Both systems were then subject to a step input, from 9cm to 11 cm. The step response of both systems is presented in Figure 6-11. An IAE measure of quality control is presented in Figure 6-19.

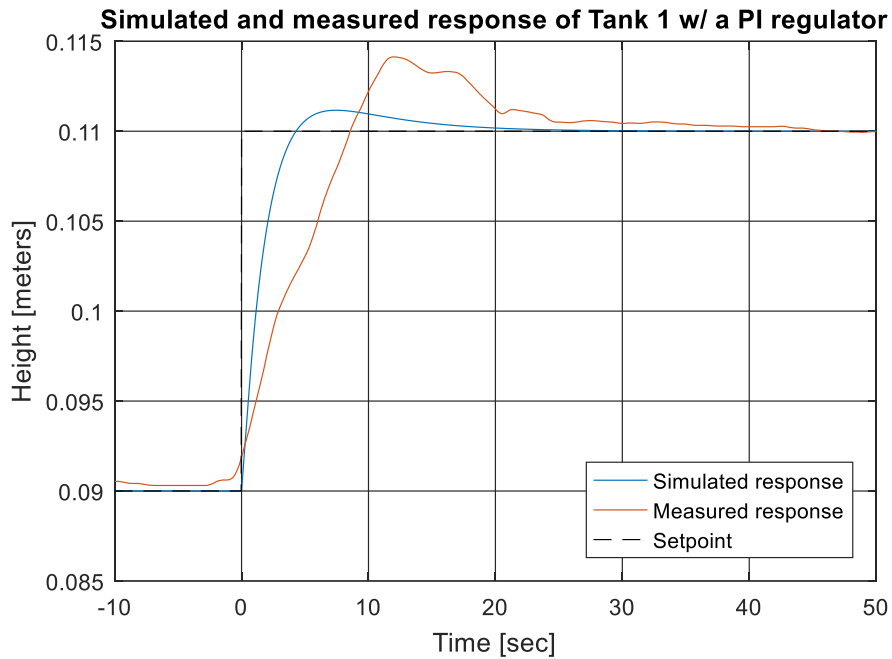


Figure 6-11: The plot shows a step response of the simulated system with its estimated parameters, and the same response for the real, physical system. For this scheme, tank 1 was measured and controlled using a PI controller.

6.4.4 Scheme 2: Single-Loop Control of Tank 2

With a single loop controller, tank 2 was controlled as a second order system (6-9) using a PID controller. This approach is illustrated in Figure 6-12.

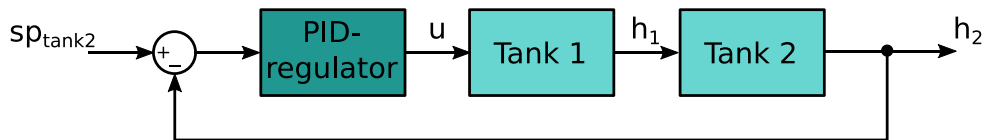


Figure 6-12: Illustration of the control setup when controlling tank 2 with a single loop setup

Tuning Parameters

A PID controller was implemented on the form of (4-51), where the SIMC method was utilized for calculating the PID tuning parameters. Note that as the controller for this scheme includes a derivative part, the transformation (4-53) must be applied.

The tuning parameter τ_c is found in a similar fashion as in the previous chapter, but this time by using the maximum out-flow from tank 1, Q_1 , as the maximum in-flow to tank 2, as described in (6-13).

Table 6-2: Tuning parameters for the single-loop PID controller for Tank 2

Parameter name	Serial PID parameters	Parallel values
Tuning parameter	$\tau_c = 0.2 \cdot \frac{h_{1,max} \cdot A_1}{Q_1(h_{1,max})}$	1.82
Delay	θ	0
Proportional gain	$K_p = \left(\frac{1}{a_3}\right) / \left(\left(\frac{ba_2}{a_1 a_3}\right) \tau_c\right)$	457
Integral gain	$T_i = \min \left\{ \frac{1}{a_3} \quad 4\tau_c \right\}$	20.2
Derivative gain	$T_d = \frac{1}{a_1}$	4.66

System Response

Using the tuning parameters from Table 6-2, a PID controller was implemented in both the simulated and the physical system. Both systems were then subject to a step input, from 9cm to 11 cm. The step response of both systems is presented in Figure 6-13. An IAE measure of quality control is presented in Figure 6-19.

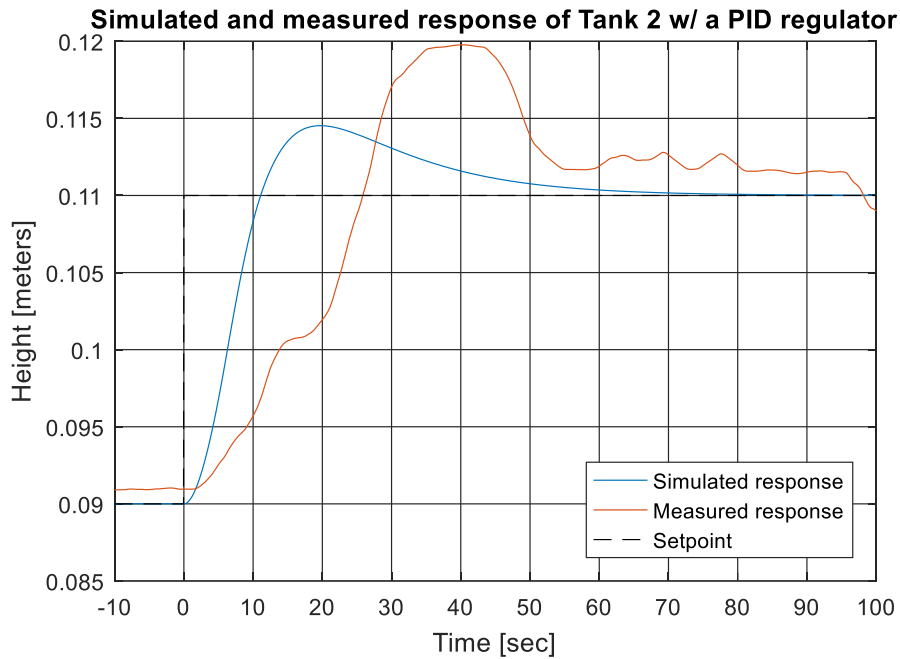


Figure 6-13: The plot shows a step response of the simulated system with its estimated parameters, and the same response for the real, physical system. For this scheme, tank 2 was controlled using a single-loop PID controller.

6.4.5 Scheme 3: Cascade Control of Tank 2

With the objective to control the water level height of tank 2, a second approach was implemented; a cascade control scheme with an inner system identical to the one presented in Chapter 6.4.2. The setup is illustrated in Figure 6-14.

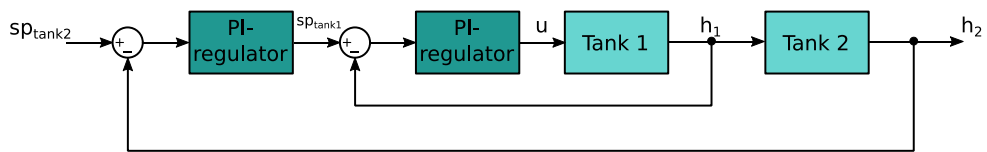


Figure 6-14: Illustration of the control setup when controlling tank 2 with a cascade setup. The outer controller now controls the setpoint of the inner controller.

Tuning Parameters

When tuning a cascade system, one normally starts by tuning the inner loop. This is equivalent to what was done in Chapter 6.4.2. Next, the outer loop can be tuned as if the inner loop represents a gain of one, shown in Figure 6-15. This assumption requires that the inner loop is sufficiently fast compared to the outer

loop. A rule of thumb is having an inner loop that is at least five to ten times faster than the outer loop.

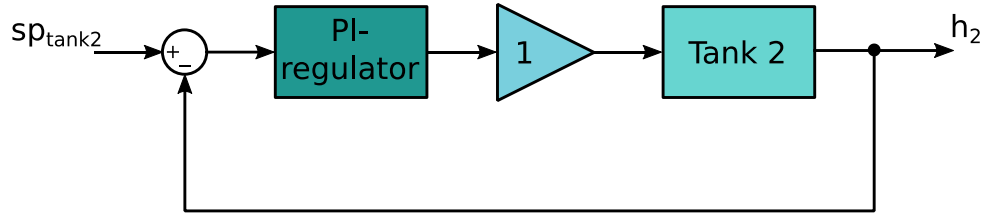


Figure 6-15: Illustration of the assumed control setup when controlling tank 2 with a cascade setup, after the inner loop it tuned sufficiently.

Since the tuning parameters for the inner PI controller was presented in Chapter 6.4.2, only the parameters for the outer PI controller is presented in Figure 6-6.

Table 6-3: Tuning parameters for the outer loop PI controller for Tank 2 in a cascade setup

Parameter name	Serial/parallel parameters	PID Parallel values
Tuning parameter	$\tau_{c,outer} = 5 \cdot \tau_{c,inner}$	21.9
Delay	θ	0
Proportional gain	$K_p = (1/a_3)/((a_2/a_3) \tau_c)$	2.30
Integral gain	$\min\{1/a_3 \quad 4\tau_c\}$	21.9

System Response

Using the tuning parameters from Table 6-1 and Table 6-3, two PI controllers in cascade were implemented in both the simulated and the physical system. Both systems were then subjected to a step input, from 9cm to 11 cm. The step response of both systems is presented in Figure 6-16. An IAE measure of quality control is presented in Figure 6-19.

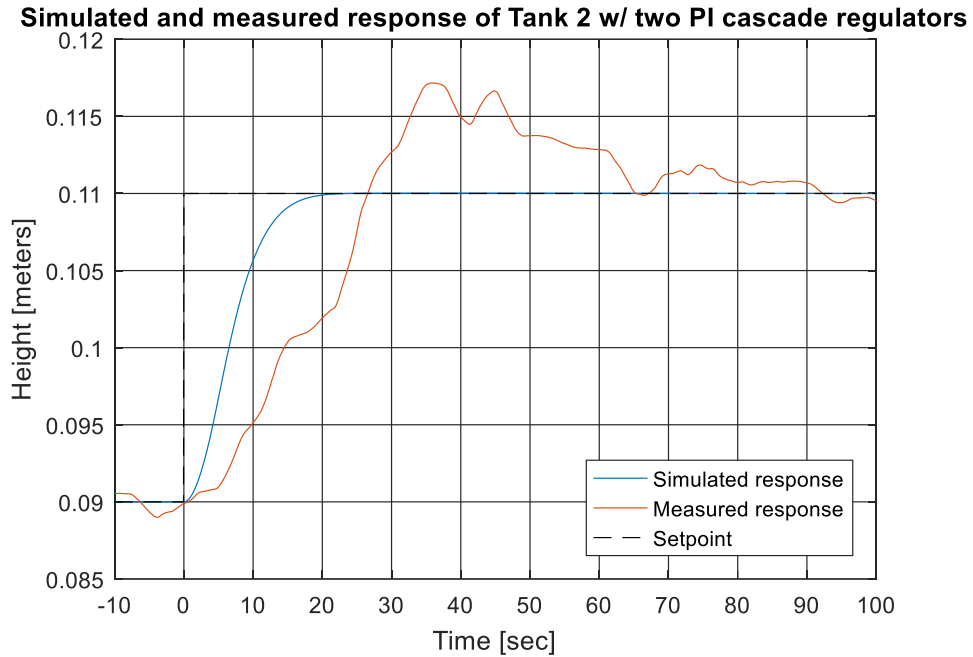


Figure 6-16: The plot shows a step response of the simulated system and the real, physical system. For this scheme, tank 2 was controlled using an inner and an outer PI controller in a cascade scheme.

6.4.6 Scheme 4: LQR with Reference Tracking and Integral Action

With the objective of controlling the water level of tank 2, an LQR with a reference trajectory and integral action was implemented. A modified system was created, containing the error dynamics and integral action of tank 2. The new state matrix ψ has the form

$$\begin{aligned} \vec{\psi} &= [x_1 \quad e \quad z]^T \\ e &= x_2 - x_d \\ \dot{z} &= e \end{aligned} \tag{6-24}$$

And the new input is on the form

$$v = u - u_d \tag{6-25}$$

Where x_1 and x_2 are the water level heights of tank 1 and 2 respectively. This gives a new, modified system that contains the error dynamics and integral action of tank 2.

$$\begin{aligned} \dot{\psi} &= \begin{bmatrix} -G_1 x_1^{1/2} + G_p(v + u_d) \\ G_2 x_1^{1/2} - G_3(e + x_d)^{1/2} \\ e \end{bmatrix} \\ &= F(x_1, e, z, v, x_d(t), u_d(t)) \end{aligned} \quad (6-26)$$

Following the procedure of linearizing the new system around $e = 0$, as described in (4-66), a linearized version of (6-26) is given on the following form

$$\begin{aligned} \dot{\psi} &\approx \mathbf{A}\psi + \mathbf{B}v \\ &\approx \begin{bmatrix} -\frac{G_1}{2x_1^{*1/2}} & 0 & 0 \\ \frac{G_2}{2x_1^{*1/2}} & -\frac{G_3}{2x_d^{*1/2}} & 0 \\ 0 & 1 & 0 \end{bmatrix} \psi + \begin{bmatrix} G_p \\ 0 \\ 0 \end{bmatrix} v \end{aligned} \quad (6-27)$$

With the control law, as described in (4-67)

$$u = -[K_1 \quad K_2 \quad K_i] \begin{bmatrix} x_1 \\ x_2 - x_d \\ z \end{bmatrix} + u_d \quad (6-28)$$

An illustration of the realization of the controller is presented in Figure 6-17.

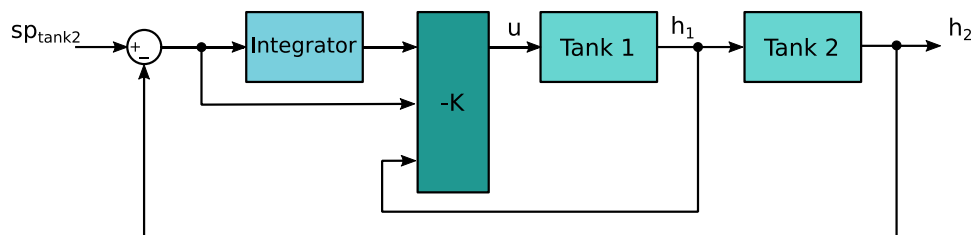


Figure 6-17: Illustration of the implemented LQR controller, with reference tracking and integral action.

To ensure that the *algebraic Riccati equation* has a unique, positive definite solution, the controllability and observability of this system is calculated based on (4-63) and (4-64).

The controllability of the system is shown as follows

$$\begin{aligned}
\text{rank}(\zeta) &= \text{rank}([\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B}]) \\
&= \text{rank} \left(\begin{array}{ccc} K_p & -\frac{G_1 G_p}{2x_1^{*\frac{1}{2}}} & \frac{G_1^2 G_p}{4x_1^*} \\ 0 & \frac{G_2 G_p}{2x_1^{*\frac{1}{2}}} & -G_p \left(\frac{G_1 G_2}{4x_1^*} + \frac{G_2 G_3}{4x_1^{*\frac{1}{2}} x_d^{*\frac{1}{2}}} \right) \\ 0 & 0 & \frac{G_2 G_p}{2x_1^{*\frac{1}{2}}} \end{array} \right) \quad (6-29) \\
&= 3
\end{aligned}$$

Since the rank is equal to the number of states, controllability can be concluded.

The observability of the system is shown as follows.

$$\begin{aligned}
\text{rank}(\mathbf{Q}) &= \text{rank}([\mathbf{C} \quad \mathbf{CA} \quad \mathbf{CA}^2]) \\
&= \text{rank} \left(\begin{array}{ccc} -K_1 & -K_2 & -K_i \\ \frac{G_1 K_1 - G_2 K_2}{2x_1^{*\frac{1}{2}}} & \frac{G_3 K_2}{2x_d^{*\frac{1}{2}}} - K_i & 0 \\ -G_1 \left(\frac{G_1 K_1 - G_2 K_2}{4x_1^*} \right) - \frac{G_2 M}{2x_1^{*\frac{1}{2}}} & \frac{G_3 M}{2x_d^{*\frac{1}{2}}} & 0 \end{array} \right) \quad (6-30) \\
&= 3
\end{aligned}$$

$$\text{Where } M = \left(K_i - \frac{G_3 K_2}{2x_d^{*\frac{1}{2}}} \right).$$

Since the rank is equal to the number of states, observability can be concluded.

The system is both observable and controllable (and thus stabilizable), meaning that the *algebraic Riccati equation* has a unique, positive definite solution.

Tuning Parameters

The tuning parameters for an LQR consists of changing the values of the matrices \mathbf{Q} and \mathbf{R} . Normally only the diagonal values of the matrices are non-zero, as is the case for the implementation presented in this thesis. Further, the LQR minimizes the state values, the first element of \mathbf{Q} , $Q_{1,1}$, must be zero. This element is the tuning parameter related to the first state, x_1 , which would be undesirable if forced to zero, as this represents the water level in tank 1.

Several combinations of \mathbf{Q} and \mathbf{R} were tested out on the simulated system before being implemented on the physical system. The tuning parameters and the corresponding gain matrix \mathbf{K} (which is a vector in this case of a single input system) is presented in Table 6-4. A feed forward gain was implemented for the simulated system using (6-11).

Table 6-4: The tuning parameters and the corresponding gain matrix **K**, calculated for the implemented LQR controller, with reference tracking and integral action

Parameter name	Parameter values
State weights	$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 150 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$
Input weights	$R = 0.001$
Controller gain	$K = [73.44 \quad 355.9 \quad 10.00]$

System Response

An LQR controller with reference tracking and integral action was implemented for both the simulated and the physical system. Both systems were then subject to a step input, from 9cm to 11 cm. The feedforward was turned off for both the simulated and physical experiment, to enable a comparison of only the LQR controller. The step response of both systems is presented in Figure 6-16.

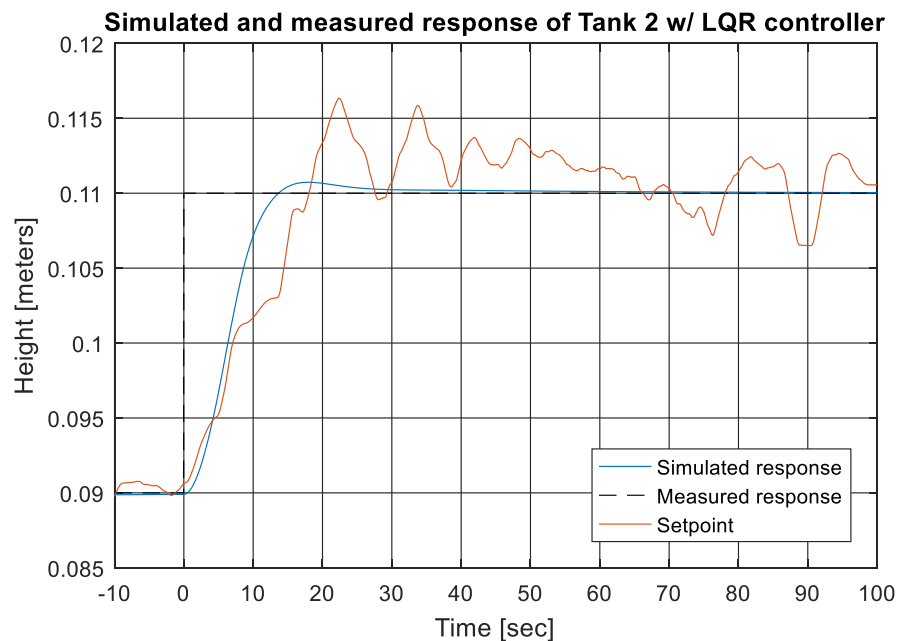


Figure 6-18: The plot shows a step response of the simulated system with its estimated parameters, and the same response for the real, physical system. For this scheme, tank 2 was measured and controlled using an LQR with integral action.

6.4.7 Measurement of Quality

To better compare the algorithms and the correlation of the physical and simulated response, the integrated absolute error (IAE) was calculated. This was calculated both as a measure of the controlled system performance using (4-70), and as a measure of how well the simulator described the behavior of the physical system using (4-71). The results are presented in Figure 6-19, where a lower score indicates a smaller accumulate error.

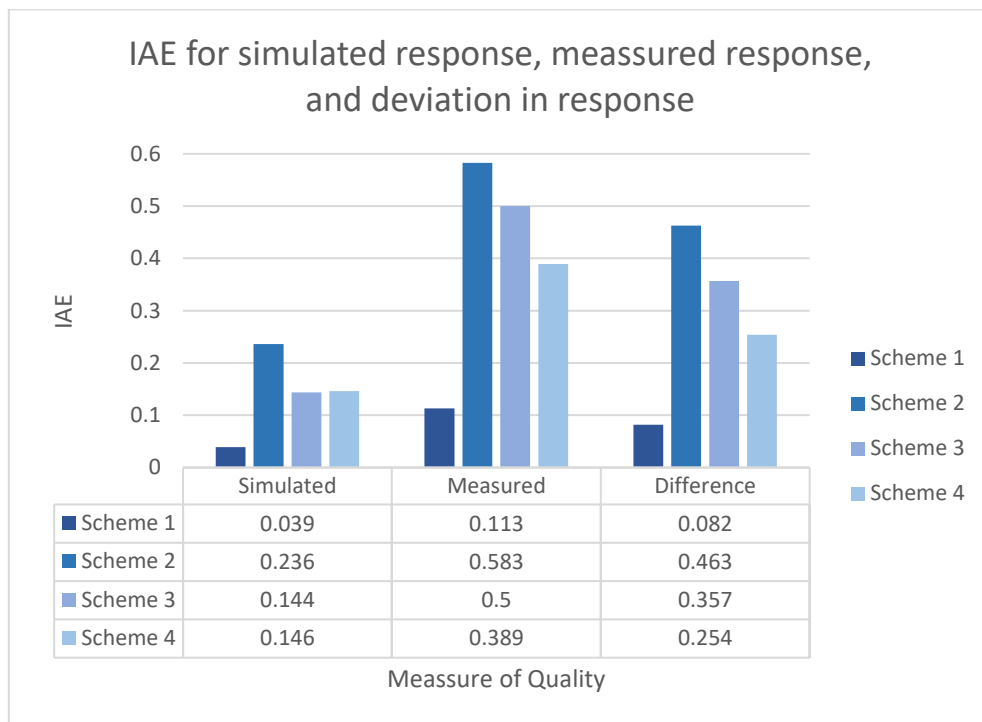


Figure 6-19: The plot presents the integrated absolute error of the four control schemes presented in this thesis. Leftmost is the IAE score of the simulated system response. The middle is the IAE score of the physical system response. The rightmost is the IAE score of the difference between the simulated and the measured response.

7 The Pedagogical Potential of the Serial Two-Tank System

As introduced in Chapter 3, a blend of theoretical and practical work in the engineering education is something that is expected by the National Guidelines for Engineering Education, wanted by the students, and encouraged by pedagogical research. Further, the desire for a modernization of the course implementations is documented both externally, presented in Chapter 3.5, and internally at NTNU by [27] and through the author's own surveys, see Appendix I.5.

So far, the thesis has described the work of building, analyzing and controlling a STTS. However, for the system to have any pedagogical potential, it should be able to demonstrate the theory typically taught in engineering education. Next, the thesis will therefore demonstrate how this work can be separated in to four separate work sessions. These work session, or *learning modules*, are propositions meant to mirror typical courses taught the first four semesters at Norwegian engineering universities and colleges.

To gain insight into what courses are typically taught the first four semesters, the curriculums at four educational engineering institutions are compared and presented. The results of this is presented in the next chapter. Following this, the learning modules is presented and the content of each module is explained briefly.

7.1 Comparing the Curriculum at Various Educational Institutions

A challenge when designing an educational tool for more than just one institution is the fact that all institutions have their own, individual curriculum. Though they often share some common curriculum (typically natural science courses) some courses may vary a great deal. Still, there are some general topics normally are covered by most institutions educating engineers within the field of automation.

To better compare the engineering curriculums at different schools and universities, looking for patterns of courses that have relatively similar curriculum,

a study was done, categorizing courses held by four Norwegian educational institutions⁷ during the first two years or their automation-related program. These categories separate the courses into groups based on the information found in each course description. These categories, especially the first three, were chosen as they are particularly relevant when constructing and controlling a physical system. The result of this study is presented in Figure 7-1.

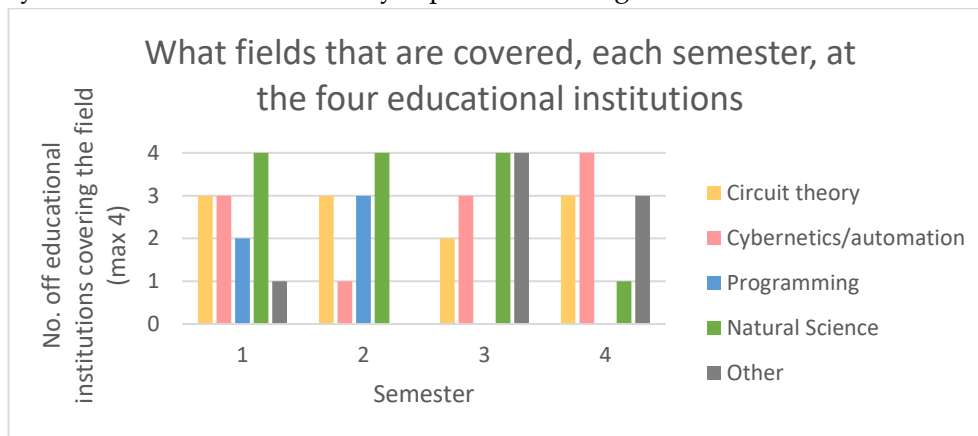


Figure 7-1: The plot shows the fields being covered the first two years at four different Norwegian engineering institutions. The plot is based on the data presented in Appendix C. The plot presents how many of the four institutions that cover each course category, shown for each semester.

Figure 7-1, indicates a subtle trend where natural science and programming typically are taught during the first two or three semesters. Further, automation-related courses seem to be held at the end of this two-year period, while electronics and circuit theory seems to be present during most of the semesters at most of the institutions. Note that some of the courses categorized as “cybernetics/automation” are introductory courses. The full comparison is given in Appendix C.

⁷ The study looked at automation-related engineering degrees at four Norwegian higher educational institution – the Norwegian University of Science and Technology, Oslo and Akershus University College, the University College of Southeast Norway, and what used to be called the University College of Ålesund, now part of NTNU.

7.2 KybTeach – The Proposed Module Based Learning Package

As mentioned in the previous chapter, a challenge when proposing a lab assignment or pedagogical tool for more than one institution is that it should be both general and applicable at the same time. In attempt to solve this, the workload needed to build and control the STTS is segmented into smaller *learning modules*. Each module is envisioned⁸ to include all material needed to complete this work session, while cultivating a deep understanding of theory, practice, and how they are linked.

Further, as the aim of the thesis is to demonstrate how modern technology and pedagogical models can be combined to facilitate this deep learning, the structure of these modules is envisioned as being accompanied with the flipped classroom model, as presented in Chapter 3.6.

The next subchapters present these learning modules, and ends by demonstrating how instructional videos for the flipped classroom model can be generated on a low-cost budget.

This package, containing the physical system, the learning modules, and the online material required for the flipped classroom model was given the project name “KybTeach”. The logo of this concept is presented in Figure 7-2 and its content is illustrated in Figure 7-3. In parallel to the work related to this thesis, the feasibility of turning KybTeach into a viable business model was investigated, though it is not considered within the scope of this thesis, it is briefly mentioned to explain the use of the word “KybTeach” and its logo in some of the documents and videos in the appendices.



Figure 7-2: The KybTeach logo

⁸ The word *envisioned* is emphasized. As the thesis aims to present a proof of concept, only parts of such a module was fabricated as part of the pilot test. However, the achievements still demonstrates an approach for implementing the proposed module based learning package.

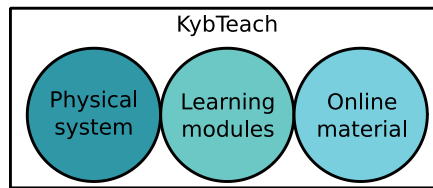


Figure 7-3: Illustration of the KybTeach concept

This concept of having “separated modules with an overall goal” can be thought of as puzzle pieces forming a bigger picture. Like the puzzle pieces, each module has certain constraints that must be satisfied for it to fit well with the next module. These constraints are the tabs for a puzzle piece, and represents either prerequisites or physical hardware needed to start the module. Similarly, each module has a set of specified learning goals and products that works as the input for its natural successor. This is illustrated in Figure 7-4, where the three colors represent the prerequisites, the content, and the product of the learning module.

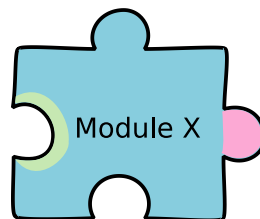


Figure 7-4: General illustration the module summaries presented later in the text.

The green area of the left represents the required prerequisites for starting this module. This includes both theoretical skills and various material. The middle blue part represents the actual module – the implementation and its layout. The right-most pink part represents the learning goals and the tools and/or physical products generated by completing the module.

By executing all modules sequentially, the students are walked through the complete process from hardware to an automated system. However, when segmenting the workload into modules with specified “inputs” and “outputs”, it enables each educational institution to customize, rearrange and implement one or more of the selected learning modules that they find relevant to their curriculum. This concept is illustrated using the puzzle metaphor in Figure 7-5.

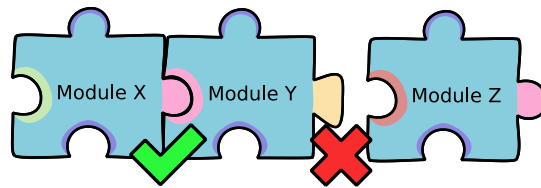


Figure 7-5: Illustration of how the tabs of the puzzle pieces represents suitable and unsuitable prerequisites and learning goals that decides which modules are compatible to be run sequentially.

As mentioned above, the approach of segmenting the workload into smaller learning modules deals with the problem of proposing a general learning package suited for several educational institutions. Additionally, this approach makes it easier to focus on framing and reviewing the content of each individual module, with regards to pre-requisites, content and learning goals. Finally, this approach makes it easier to add new modules by either branching off to dive deeper into a module, or by adding complete new features to the system. This concept is illustrated in Figure 7-6.

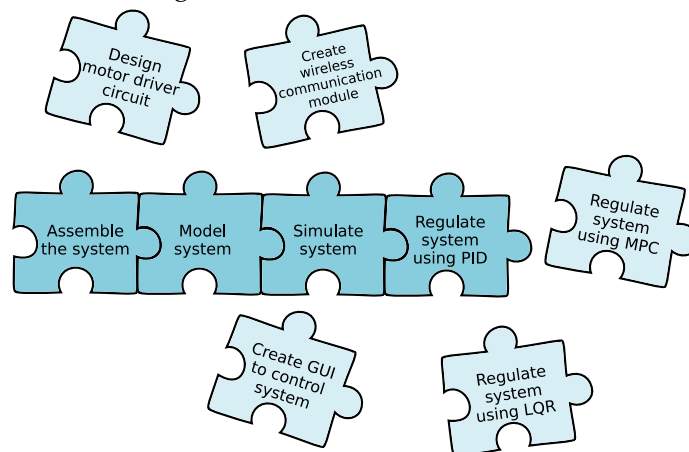


Figure 7-6: Illustrates the concept of the learning modules and how they can be customized to fit individual curriculums and be extended with new modules.

Based on the work needed to reproduce the assembling, analysis and control of the STTS in this thesis, four learning modules are proposed. Each module represents a segment of the total workload, designed to help the students one step closer to a final, automated system. These modules and their suggested order is illustrated in Figure 7-7, where each module is meant to supplement a semester. The modules described and presented separately in the next subchapters.

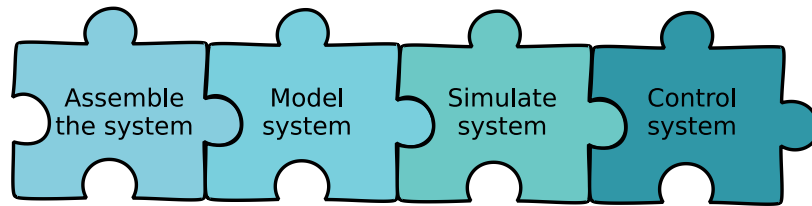


Figure 7-7: An illustration of the four learning modules and their interaction between each other. The leftmost module is proposed for first semester students, while the rightmost is proposed for fourth semester students.

7.2.1 Module 1: Assemble the System

As described in Chapter 3.2, adults have a rich learning resource through their past experience. However, the students entering the university engineering classes the first year often have vastly diverse backgrounds, making it hard to find a structure that fits all. This is especially true in Norway, where people can enter the engineering studies through high school with general, through a dedicated pre-course to engineering or through the so-called Y-way⁹ [41]. The focus of the practical works the first semester should therefore focus on giving a fundamental introduction to hardware, equipment and other material that will be covered in the lectures, ensuring that all the students have the necessary background knowledge.

Further, since most students enter the first semester with few or no friends, the first module has an additional higher level aim of developing communication and interpersonal skills at an early stage. This is an important skill, both in a professional context and with regards to creating a healthy class environment. This is suggested as a higher-level learning goal according to [13], presented in Table 3-3.

This module will primarily be an introduction to mechanical and electrical hardware, while producing the system the students later will learn to analyze and control. This approach is similar to what [13] calls a *demonstration* in Table 3-4, effectively being a “show and tell” approach. Further, [13] suggests that this approach is mostly beneficial when students are being asked questions that prompt thinking and give a better intuition for the dynamical systems they are

⁹ Requiring a vocational background

working with¹⁰, which will be important to keep in mind when defining the concrete assignments.

A visualization that summarizes this module is illustrated in the Figure 7-8.

¹⁰ An anecdotal story related to technical intuition:

I (the author) composed a thought experiment related to the STTS, where students were asked to imagine both tanks being filled completely, with both valves closed, so that no water could exit either of the tanks. When I asked students what would happen if both valves were opened simultaneously, most of them predicted that both tanks would drain simultaneously. I then proceeded to demonstrate this scenario on the physical system. Since tank 1 worked as the input of tank 2, tank 1 kept feeding tank 2 with water while it drained itself, resulting in tank 1 draining much faster than tank 2. After having observed the system response, all the students immediately felt that this response was a much more logical response than what they had proposed, some even feeling embarrassed for not having thought of it themselves. I found this to be an interesting example of how easy and critical it is to both prompt thinking and demonstrate examples when it comes to dynamical systems, as many people do not have a trained intuition for this.

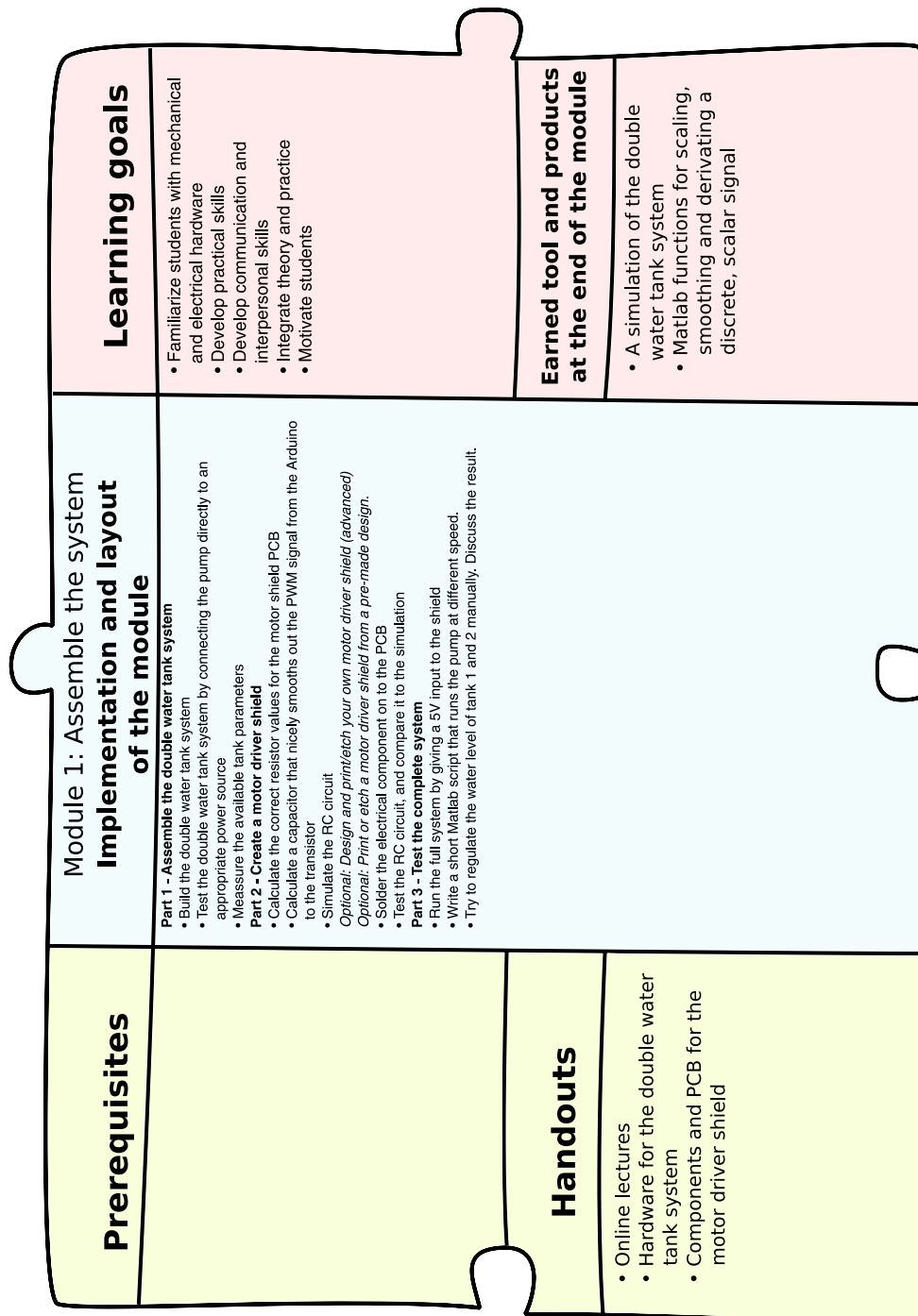


Figure 7-8: Visualization of a first proposed learning module, focusing on the assembling of the STTS. This module is a conceptualization based on the work presented in this thesis.

7.2.2 Module 2: Model the System

Having introduced the students to the physical system through the first module, they should now be better prepared to work with dynamical systems on a theoretical level. This module focuses on deriving the differential equations and analyze the system. The intention is to make it easier for the students to relate the theoretical theory and results to the behavior and properties of the physical system. This module contains the least amount of practical work, but is required as it lays the foundation for creating simulating the system in the next module. Additionally, the module might be motivating as it can demonstrate how to reveal new information about a system that they thought they knew, using a theoretical approach.

A visual summary of the module is illustrated in Figure 7-9.

Prerequisites	Module 2: Model the system Implementation and layout of the module	Learning goals
<ul style="list-style-type: none"> • Pre-calculus • Introduction to differential equations 	<p>Part 1 - Model the system</p> <ul style="list-style-type: none"> • Use the mass balance to create an ordinary differential equation describing the water level in water tank. Assume that the input volume flow rate is linear on the form $Q_{in}=K \cdot u$, and the output volume flow rate described as Q_{out}. • Using Bernoulli's equation, find an expression for the volumetric output velocity • Use the fact that $Q_{out} = v \cdot a$ <p>Where 'v' is the volumetric output velocity and 'a' is orifice area. Based on this, an expression for the volumetric output flow rate.</p> <ul style="list-style-type: none"> • Describe the dynamics of a single tank system by combining the ODE from the mass balance with the new expression for the output flow rate. • Model the second tank. Keep in mind that the input to tank 2 is the same as the output of tank 1 ($Q_{2in} = Q_{1out}$) <p>Part 2 - Analyse the system model</p> <ul style="list-style-type: none"> • Find the equilibrium point of the system • Linearize the system about a point of operation • Find the transfer function of tank 1 and the full system • Plot the Bode-plot of based on the transfer function of tank 1 and the full system. <p>Part 3 - Manual control and logging of physical system</p> <ul style="list-style-type: none"> • Create a script that logs the water level of both tanks, logs the input (PWM voltage from the Arduino) and the time. Further, the user should be able to specify a step input, both the magnitude and the time occurrence. <p>Part 4 - Black box estimation</p> <ul style="list-style-type: none"> • Assume that tank 1 can be approximated using a first order linear ODE. Based on a step response, estimate the model • Discuss the pros and cons of modeling a system using physics versus doing a black box estimation. 	<ul style="list-style-type: none"> • Develop analytical and theoretical skills • Bridge the gap between theory and practice, by modelling a system they know. • Be able use physics to mathematically describe a physical process • Be able to reflect upon the strengths and weaknesses of modeling vs. black box estimation • Motivate students
<p>Handouts</p> <ul style="list-style-type: none"> • Online lectures 		<p>Earned tool and products at the end of the module</p> <ul style="list-style-type: none"> • A mathematical model of the system, which can be used for analysis, and later can be used simulation purposes.

Figure 7-9: Visualization of a second proposed learning module, focusing on modeling the STTS. The module includes the modeling part presented in this thesis.

7.2.3 Module 3: Simulate the System

The third module shows the students how to take the model from the previous module and turn it into a simulation of the system. Further, the students are required to estimate the valve coefficients and the pump coefficient, making the simulated dynamics resemble those of the physical system. This module was also pilot tested, where the work related to the testing is presented in Chapter 8.3.

For the sake of the pilot test, Module 3 is explained a bit more thorough than the other three. This module starts by demonstrating the implementation of the differential equations of tank 1 in Simulink through a video lecture. The students are then asked to replicate this for the tank 2, and connect them together to form the complete system.

Next, the students are asked to investigate ways to estimate the valve parameters. They are asked to plan their experimental design and execute these. This approach is based on the suggested practical work design considerations for *structured inquiry* in Table 3-4. Here, students are presented with a real problem, with a range of material to choose from.

By watching online lectures created specifically for this module, the students are next introduced to the ordinary least squares method. They are then asked to estimate the valve parameters again using this method.

The next objective is to estimate the pump coefficient. Since the students do not have access to a flow meter, they are encouraged to plan, design and execute an experiment. One solution for this is presented in Chapter 6.2.2.

Finally, when the coefficients are estimated, the dynamics of the simulated system should be compared with the one of the physical system. Again, the students are asked to plan, design and execute the experiment. One solution is presented at the end of Chapter 6.2.1.

This module encourages students to take a personal initiative, while incorporating most of the higher-level learning goals presented in Table 3-3. The students also get a valuable lesson about the strengths and weaknesses of a simulated system compared to a physical one. Further, by incorporating several of the suggested design considerations for *structured inquiry* in Table 3-4, the module aims to facilitate students' development as autonomous, lifelong learners by having them plan their own experiments.

A visual summary of the module is illustrated in Figure 7-10.

Prerequisites	Module 3: Simulate the system Implementation and layout of the module	Learning goals
<p>Handouts</p> <ul style="list-style-type: none"> • Online lectures • Tank parameters • Differential equations describing the system 	<p>Part 1 - Getting started</p> <ul style="list-style-type: none"> • Download and install the required Arduino plugins for Matlab and Simulink • Be able to send commands and collect data to the Arduino Board, using Matlab and Simulink <p>Part 2 - Processing the logged data</p> <ul style="list-style-type: none"> • Create a general Matlab function that scales an input signal. Example: $y = \text{signalScale}(x, xMin, xMax, yMin, yMax)$ • Create a moving average filter. Example: $y = \text{signalSmooth}(x, filterSize)$ • Use these two functions to convert the noisy input signal representing voltage to a smooth signal representing water level height. <p>Part 3 - From differential equations to a simulator</p> <ul style="list-style-type: none"> • Get familiar with the common blocks of Simulink • Start by following instructions for how to create a simulation for tank 1, and compress this into a subsystem • Based on the previous work, finish the simulation for tank 2. • Test out the simulation with given parameters (tankParameters.m) <p>Part 4 - Parameter estimation: Make the simulator resemble the real system</p> <ul style="list-style-type: none"> • Create a general function that derives an input signal. • Use the derivative of the logged signal from the real process to estimate the valve coefficients. • By doing a transformation on the ODE, use the ordinary least squares method to make a new estimation of the valve coefficients. • Discuss the pros and cons with the approaches • Discuss ways to estimate the pumping coefficient • Make a function that takes in the voltage signal and outputs the estimated volumetric flow from the pump (A linear approximation will suffice). • With the three unknown parameters now estimated, compare the simulation to the real system • Discuss the similarities and differences between the behavior of real and the simulated system. Further, propose one or more methods to unbiased compare the real and the simulated response and implement one of them. • Use the unbiased implemented method for comparing the simulated system response with the real system response to conclude which parameter estimation approaches worked the best. • Discuss how the simulated system would behave differently if some of the parameters were changed. Then, test this out on the simulation and discuss whether or not the expected result were achieved. • Discuss the pros and cons with working on a simulation compared to a real system. 	<p>Earned tool and products at the end of the module</p> <ul style="list-style-type: none"> • A simulation of the double water tank system • Matlab functions for scaling, smoothing and deriving a discrete, scalar signal

Figure 7-10: Visualization of a third proposed learning module, focusing on simulating the STTS and estimate valve and pump coefficients. This module is a conceptualization based on the work presented in this thesis.

7.2.4 Module 4: Control the System

As the learning modules are aimed at students taking automation-related engineering degrees, this module can be considered the final stage for showing the students the path to an automated system. Using the simulator from the previous module, the students are asked to implement several controller schemes. Further, the students are asked to design a measure for comparing the system response. This, in addition to using the integral absolute error (IAE), is then used to compare the different control schemes, where the students are asked to reflect upon the strengths and weaknesses of their own measure compared to the IAE.

Finally, the students are asked to demonstrate the best controller they can produce. This is a small project, where students can either focus on tuning the best PID scheme even further, or find research of other available controllers (i.e. LQR or MPC). This approach is meant to motivate the autonomous learner, creating an internal competition in the classroom of finding the best controller. This way the students are also able to learn from and be inspired by their peers' work.

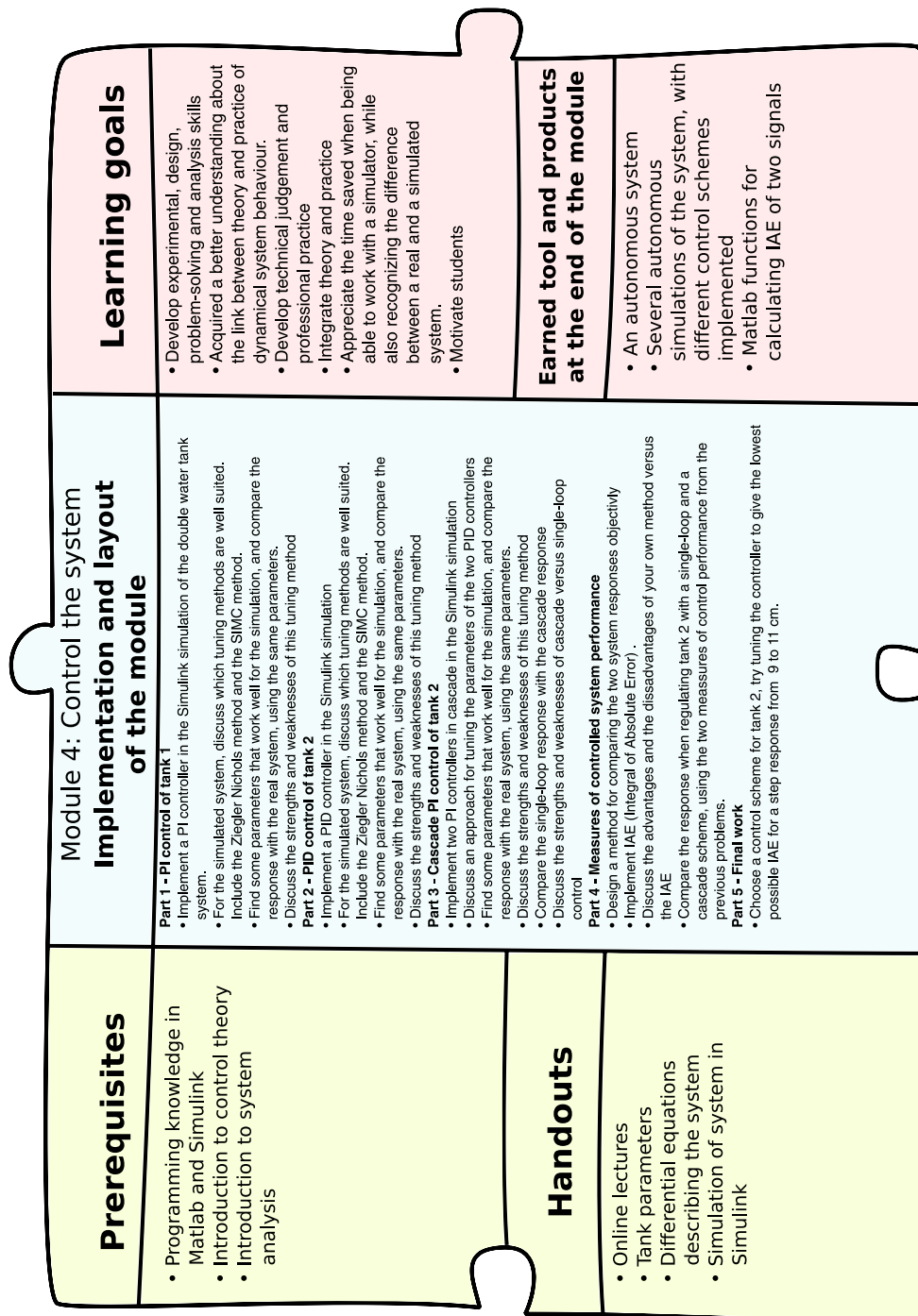


Figure 7-11: Visualization of a fourth proposed learning module, focusing on controlling the STTS. This module is a conceptualization based on the work presented in this thesis.

7.2.5 Online Videos

To accompany the learning modules, it was envisioned an online portal where students could access learning material, answer quizzes and communicate with lecturers and students in the same module. This has successfully been implemented in the *Electronic Engineering Ladder*, where students according to Lars Lundheim “typically watch short video lectures at home before the class sessions, while the in-class time is dedicated to working with exercises, projects and discussion”. See abstract from interview in Appendix A.

A similar approach was adopted by KybTeach, where a total of 18 videos were produced [42] and published on YouTube under the channel name KybTeach. Each video is a segment of a broader lecture, as Mayer’s segmenting principle suggests. The videos are screen recordings, either illustrating concepts by hand using a tablet and pen, or implementations in Matlab or Simulink. This approach is very low-budget, but still manages to follow several of Mayer’s principle. Recording only the screen, the only the essential information is kept, in accordance to the image principle. Further, based on Mayer’s voice and personalization principles, most videos were narrated in post-production (except for the Matlab tutorials) focusing on giving a friendly presentation in a formal form. Post-production also had other advantages, as being able to speed up or cut out unnecessary information. Additionally, prior to every recording, a comprehensive script was created for each video, making it easier to structure the presentations. The scripts for these videos are available in the associated ZIP-folder, see Appendix I.4.

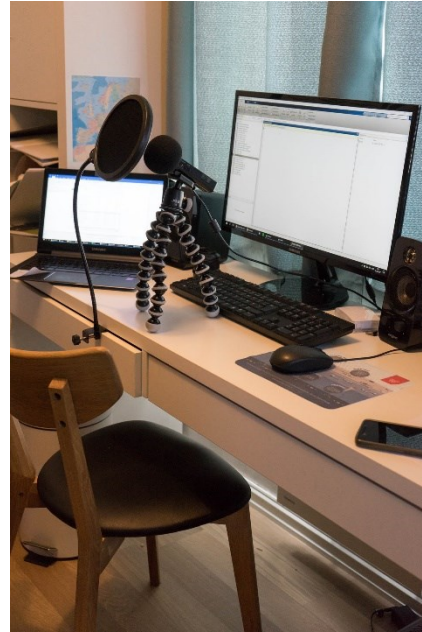


Figure 7-12: Photo of the home-made, low-budget studio, shot during the recordings of the instruc-

8 How the Project Execution and Final Design Was Elevated Using Design Thinking

Designing a system for educational purposes requires more than just finding the right hardware. The process consists of designing a user experience, for the students and the teachers, which best facilitates learning. Thus, the problem becomes more of a human-centric problem rather than a pure technical one. This requires skills and methods that are not part of the cybernetics arsenal, which is the reason for incorporating the design thinking methodology in this thesis.

The next chapters illustrate how design thinking was actively implemented in the development and testing the physical system and the learning modules. Further, the chapters are presented in a slightly narrative way, to illustrate the nature of the design thinking methodology as an iterative process, as described in Chapter 2.

8.1 Decide on a System

As mentioned in Chapter 2, an essential part of design thinking is the obligation to continuously empathize with the end user. The end users in this case was both the students and the lecturers. As the author himself is a student, who have received feedback from students through conversations and the authors own surveys (available in Appendix I.5), it was concluded that the focus should be on getting a better understanding of the teacher role. Thus, as part of the *empathize*-stage from the d.school model for design thinking, two interviews were conducted (see Appendix A and Appendix B). The subjects of the interviews were the co-founder of the *Electronic Engineering Ladder*, Lars Lundheim, and one of the lab engineers at NTNU Kalvskinnnet, Daniel G. Elstad. Additionally, off-the-record mail conversations were initiated with the control theory lecturers Finn A. Haugen at the University College of Southeast Norway and Tiina Komulainen at Oslo and Akershus University College. This information, and the pedagogics theory presented in Chapter 2, a specification list was generated, establishing the criterions that would be used to evaluate potential systems, as presented in Table 8-1. This concluded the first *define*-stage from the d.school model.

Table 8-1: Criteria used to evaluate potential pedagogical systems for the thesis

Criteria for the educational system

- The system should be relatively small and portable, allowing the students to bring it home and enable self-paced learning
- The system should be relatively general, making it easier for students to make links more broadly across the curriculum
- The states of the system should be visible with the naked eye, making them intuitive to observe.
- The dynamic of the system should neither too fast to observe, nor so slow that working with it becomes tedious.
- The system should be robust, withstanding students playing “rough” and making mistakes. Too many moving, mechanical parts are therefore unfavorable
- The system should preferably have two states. This enables most calculations to be done by hand, while simultaneously introducing the students to work with matrix representations (i.e. when calculating the transfer function)
- The systems should be relatively low-budget, allowing the students to purchase their own module as part of one or more courses.
- The system should preferably be relevant to processes used in the industry.

To ideate the solutions-space, several ideas were generated, as shown in Figure 8-1. Every suggestion was tested towards the criterions specified in Table 8-1. The serial two-tank system, a double inverted pendulum, and a balancing table were chosen as the best suited candidates.

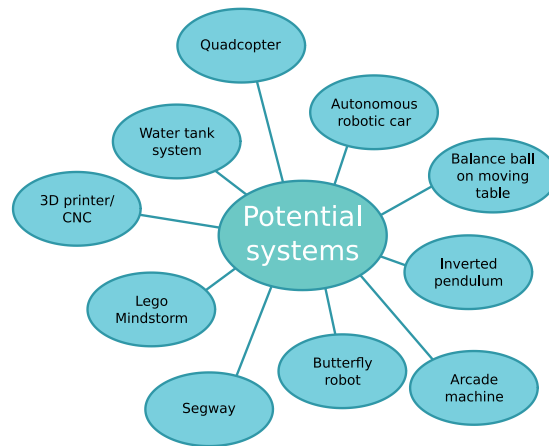


Figure 8-1: The results of brainstorming potential systems that showed potential for illustrating concepts within system analysis and control.

Aiming to converge on a final system, the three systems were considered more in depth by sketching possible implementations, estimating the cost of hardware, and comparing each of them more thoroughly against the criterions of Table 8-1. Examples of such sketches of the candidates are presented in Figure 8-2.

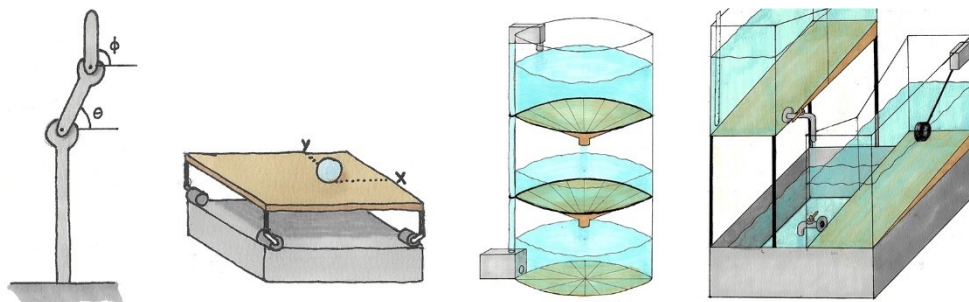


Figure 8-2: Sketches of the three potential system candidates. From the left – a double inverted pendulum, a moving table balancing a ball, and two candidates of a STTS.

The potential benefits and challenges with each system was explored, where the STTS was thought of as showing the most potential. The presumed benefits and challenges related to this system are presented in Table 8-2.

Table 8-2: Presumed benefits and challenges with using the STTS as an educational system. The list is a result of the first *ideate*-stage according to the d.school model for design thinking.

Presumed benefits

- A double water tank has two states: the water level in each tank
- Both states can be observed easily with the naked eye.
- The linearized model of the double water tank behaves like a general second-order system, resembling the dynamics of a mass-spring-damper system or an RLC circuit.
- The system can be viewed as either a first- or second-order system, depending on whether one or two tanks are studied.
- The system behaves nonlinearly, showing the benefits and challenges that comes with linearization.
- The system requires few moving, mechanical parts than easily can break. The pump is the only moving part
- The hardware required for building and controlling the system is relatively cheap
- The system is relevant for the industry, as control of liquid- and gas tanks, and hydraulic systems are widely used.

Presumed challenges

- A double water tank result in a physically big system, making it less portable.
- There might be some HMS challenges:
 - If the system falls, spilling might occur
 - If either of the tanks overflows, spilling might occur
 - If the pump needs to be placed outside or under the reservoir, there might occur leakage in the coupling.
- The states may behave too slow, making the system tiresome to work with.
- Cannot prove stability using Lyapunov's indirect method.

To test out the feasibility of the STTS, a “quick and dirty” prototype was constructed, using a water pump from Biltema and a soda bottle as a tank, as shown in Figure 8-3.

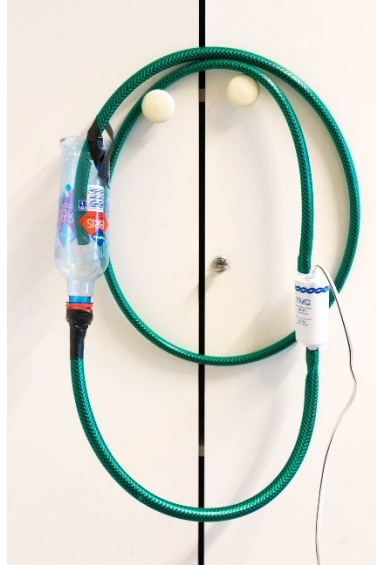


Figure 8-3: The prototype constructed to test out the feasibility of constructing a low-cost STTS. During testing, the pump would fill the bottle of water with water from an external reservoir.

While working with the prototype shown in Figure 8-3, the tests revealed that the pump was very sensitive to how much water it contained, where small air bobbles in the pump would cause it to fail. Except for this, the system seemed realizable

After having converged on a solution for the *type* of system, the *design* of the STTS had to be decided. This triggered a new divergence process, where several design solutions were explored. This was done first as sketches, and thereafter using SolidWorks (see Figure 8-4) to produce 3D-models and technical drawings for the production.

The final model and drawings are attached in Appendix D, and were delivered for production at the mechanical workshop at the Department of Technical Cybernetics, NTNU. A summary of the production drawings was presented earlier in Figure 5-9.

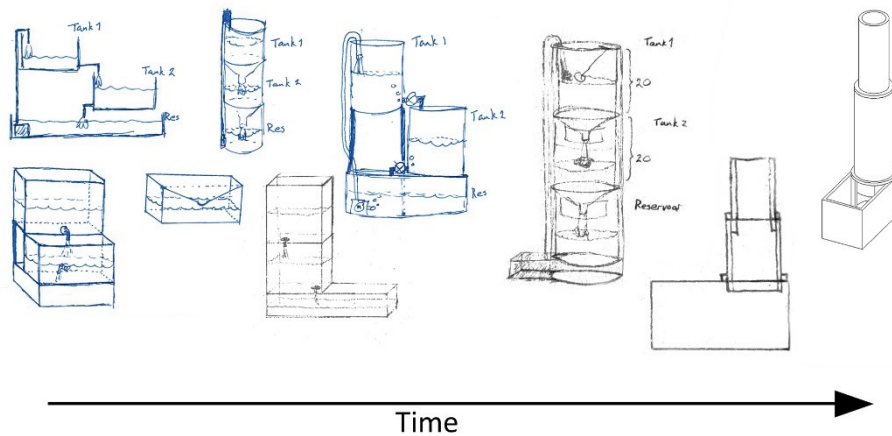


Figure 8-4: A summary of the design sketches for the STTS. In retrospect, one can see how the design gradually looks more like the final design, as more and more challenges were either solved or attempted to be avoided. The latest sketch was designed to tackle the problem of portability, where all three tanks are designed to fit into each other, making it more compact during transportation.

8.2 Designing the Motor Driver Shield

Though the motor driver circuit presented in Chapter 5.1.3 is far from advanced, the iterative process of converging on the final design was a result of several rounds of ideation, prototyping and testing, where some of the prototypes are presented in Figure 8-5.

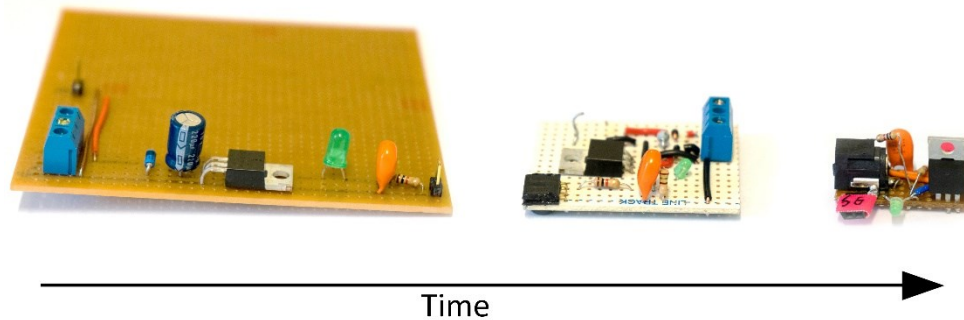


Figure 8-5: Highlights of the prototypes made as part of the iterative design process for creating a motor driver.

Initially, the motor driver was designed as a separate device, only meant to control the pump speed, and first prototype was mostly a proof of concept to that the design worked. Next, through several iterations, new prototypes were created, aiming to make the motor driver as small as possible. However, when

a satisfactory size was achieved, a new problem arose through testing, where the small size of the motor driver made interaction and wiring very cumbersome. The motor driver was therefore redesigned one final time, with the final design shown in Figure 8-6. This design, though being considerably larger in size, had the advantage of fitting directly onto the Arduino Uno as a shield. This was beneficial as it reduced the number of loose devices from two to one. Further, all wire routings between the Arduino Uno and the sensors was now incorporated into the circuit design, removing the wire mess. The wires from the sensors were instead gathered in a plug that could easily be connected and disconnected from the motor driver shield.

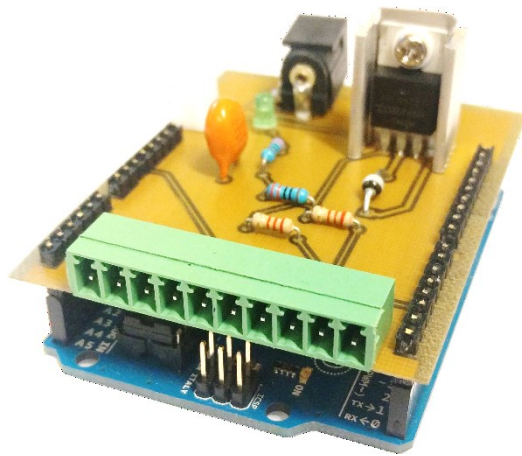


Figure 8-6: Photography of the final design of the motor driver shield, mounted on the Arduino Uno. The sockets facing the front are used to connect the arduino to the three sensors, the white socket in the back connects the Arduino to the pump and the black socket in the back is the power supply of 12 VDC.

8.3 Pilot Testing

Inspired by the design thinking approach of continuously pursuing end user feedback, the work of the thesis did not feel complete without a pilot test of the final learning package. However, finding test subjects during late spring, at a time when most students were occupied with preparing for their exams, proved to be challenging. After sending out an invitation to all students studying technical cybernetics at their first or second year at NTNU, only four students replied. Of these, only two agreed felt they had time to take part in the pilot test. Even though such a limited number of participants does not generate enough data to conclude much with regards to the quality of the learning modules, it was decided that little feedback was better than no feedback, and the design and execution of the pilot test was held.

8.3.1 The Execution

Since there only existed one physical prototype of the system, Module 3 from Chapter 7.2.3 was determined to be the most suited for the pilot test, as it could be completed as a *hands-off* assignment. To respect the limited spare time of the two test subjects, only a small part of Module 3 was tried out as part of the pilot test. The assignment, as presented to the two students, is attached in Appendix E. To summarize it, the students were asked to watch a sequence of short videos. Based on the instructions from these videos and a supplied assignment text, they were then asked to create a simulation of the STTS, and to estimate the two valve coefficients based on measured data. This measured data was the same as the data shown earlier in Figure 6-1.

8.3.2 Test Subject Feedback

After the pilot test, the students were asked to fill out a brief survey, providing some feedback on the pilot test. This feedback is summarized in the Table 8-3, and access to the full survey is gained by following the URL provided in Appendix F. It should be noted that only one of the two students were able to finish the full assignment. Further, only the person who partially answered the problems of the test pilot responded with the correct answer. Including the limited number of test subjects, this fact further limits the value of the results generated from the pilot test.

Table 8-3: Questions and answers from the survey held on after completing the pilot test of the concept KybTeach. The questions and answers have been translated to English for the sake of this thesis.

Questions	Answers
Were you able to complete the whole assignment?	One did, one did not
How many hours did you spend on the pilot test?	Both used three hours
To what degree do you feel you had good enough prerequisites to follow the pilot test?	One felt the level of prerequisites was enough, the other felt it was to level was too low.
How challenging were the problems?	Both answered that the assignment was suitably challenging
How educational did you find the pilot test?	Both scored it 3 on a scale from 1 to 5.
How did you like using video lectures?	One replied that the concept arrangement worked well, the other replied that it arrangement worked <i>very</i> well.
How do you like the idea of being able to work whenever it fits you?	Both liked it
What do you feel worked the way the concept was presented in the pilot test?	#1: <i>"The videos were good"</i> #2: <i>"The focus was very practical. This is the first time I am working with data in this way"</i>
What do you feel did not work in the way the concept was presented in the pilot test?	#1: <i>"The problems were not formulated well enough"</i> #2: <i>"The workload was too heavy to complete during the exam season"</i>
Other comments	"1: <i>"It was a good introduction, but should be more adapted for undergraduates"</i> #2: <i>"Mixing theory and practice in an enjoyable way seems like a really good suggestion. It gives good opportunities for exploring on your own. It might have worked better having some easy exercises at the end of the videos"</i>

9 Discussion

9.1 The Hardware and Physical Design

As stated in the Project Description, the thesis seeks to propose and demonstrate a physical system that is suited for educational purposes in the fields of automation, cybernetics and control theory. The following chapter discusses the construction, analysis and control of the STTS.

Some of the hardware and components proved to be less suited for the STTS than their specifications suggested. The eTape sensors did not measure accurately over their full range, where lower part of the sensor was unable to distinguish between an empty tank and two centimeters filled tank. During the construction, switching to a pressure sensor was considered. However, after the interview with Daniel Elstad in Appendix B, this method seemed to have its own problems, being sensitive to water dripping or flowing irregularly into the tank.

Further, the water pump proved to be a bit undersized for the system. This made the system response slow and made the system unable to create oscillations in tank 2, which could have enabled tuning by Ziegler-Nichols closed-loop method. A solution could be to get a more powerful pump, or by narrowing the radius of each tank, shrinking the volume of each tank. This latter suggestions could also benefit the portability of the system.

Nevertheless, the STTS proved to be a very robust system to work with. The anti-overflow mechanism worked as intended, and both the motor driver shield and the pump worked well, even when subjected to aggressive gains and during extended testing. The same goes for the circuit design, which could drive the pump for several hours of operation with no rest.

9.2 Stability analysis

Since the system is not linearizable in the origin, nonlinear stability analysis must be utilized to prove stability of the STTS. This approach is usually taught in advanced control theory courses, and students at their first or second year can therefore not be expected to prove stability of this system.

An indication of the stability property can be found using the much simpler method of generating a phase portrait. The inability to prove stability with linear system theory is therefore not thought of as a hinder the pedagogical potential, and the phase portrait demonstrates an uncomplicated way of examining the behavior of complex, autonomous systems, even though is students are unable to prove stability.

9.3 Working with Simulators and Physical Systems

The first simulated system that was compared against its equivalent physical system was the RC-circuit. The results from Figure 5-5 and Figure 5-6 show an almost identical response, with the expectation of the

The dynamics of the STTS simulator was comparable, but not identical to those of the physical system. This deviation became most more apparent when tank 2 had no or little water in it, and was caused by the positioning of the two valves. The water beam exiting tank 1 would hit directly into the output valve of tank 2 if tank 2 was empty, causing most water to “shoot” out of tank 2. This dynamic was not considered in the model of the system. Instead of incorporating more advanced hydraulic theory into the system model, this problem could likely be solved by moving the two valves away from their vertical alignment, as shown in Figure 9-1. In other words, it is a decision of adapting the model to reality or adapting reality to the model. Further, with regards to the pedagogical potential of the system, it is not necessarily critical to have a perfect simulator, as it can be a valuable lesson for the students to observe the benefits and limits of a simulated system. Also note that by fixing the alignment-problem of the valves, there are still several factors that where not considered in the simulated model, like preciseness and noise in the sensors, the presence of turbulent flow in the physical system and the fact that the modeled valve is assumed to act as an orifice opening.

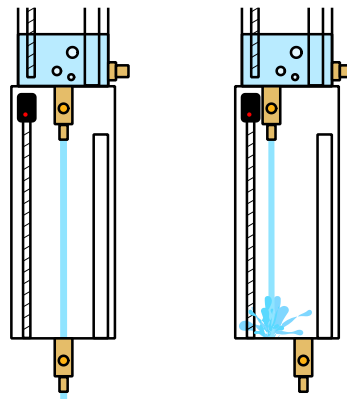


Figure 9-1: Illustration of the problem of the vertically aligned valves causing turbulent flow when tank 2 is empty. To the right is a suggestion for how to fix this problem.

9.4 The Control Schemes

All tuning in this thesis is done purely based on theoretical approaches, focusing on illustrating the implementation of the tuning rather than the control quality.

Regarding the LQR scheme, the STTS system was unable to exploit controller's full potential. While the LQR controller can control multiple-input and multiple-output (MIMO) systems, the STTS demonstrated in this thesis is only a single actuated system. Adding a second actuator could fix this, like adding a new input to tank 2, or by replacing the manual valves with control valves. Working with a multi-input system could also help elevating the educational potential of the system.

9.5 The Flipped Classroom Approach

As a suggestion for how to elevate the execution of practical work, the flipped classroom approach was tried out as part of the pilot test in this thesis. This required a lot of preparational work, where the procedure of creating the assignments, write scripts, and recording and editing video the lectures was a surprisingly time-consuming procedure. The upside is that most videos can be used several times, and by segmenting the lectures into smaller portions, outdated segments can easier be removed or updated.

From the students' perspective, the use of videos as a supplement to ordinary class activity is desired, as documented in Chapter 3.5. This approach offers more freedom and the possibility to structure their learning independently. However, this freedom comes at a cost of more responsibility. Being less constrained to follow conventional, teacher-led teaching, the flipped classroom requires more discipline and maturity from the students, and students with less self-control might easier fall through. This was experienced through the pilot test, as only two of the original four participants were able to initiate and finish the test. However, both the students that did complete gave positive feedback with regards to using videos as an active part of the education.

Finally, in a broader view, there is a technical challenge for institutions striving to implement the flipped classroom approach. For the students to be able to utilize videos as part of a course, they must be able to access them. The educational institution is still responsible for offering access to the ones that do not. This demands a certain accessibility to and capacity at the school library (or wherever computer access is offered). However, as most students own their own computer, this might not be a problem after all.

9.6 The Pedagogical Potential of the Single Two-Tank System

Based on the comparison of curriculums at selected engineering courses (see Chapter 7.1), the technical challenges of building, analyzing and controlling the

STTS seems to resonate well with the courses offered at the different institutions. Further, the system fulfills all the criteria used to evaluate potential pedagogical systems in Table 8-1, though there is room for improvement with regards to its portability and its relatively slow dynamics.

The pedagogical potential of the STTS is further backed up by the fact that a similar single-tank system currently in use at NTNU Kalvskinnet, where it is used to demonstrate several control theory concepts. Further, as the STTS is a slightly more advanced system, it has the potential of resembling both a first-order system equal to the one at NTNU Kalvskinnet, and a second-order LTI system, enabling students to make links more broadly across the control theory curriculum.

The learning modules proposed in this report are proof of concepts, and need further refinement before they can be utilized in a classroom setting. A higher focus on making the students involved in the design of experiments could help elevate the pedagogical potential even further, based on the suggestions from [13] presented in Chapter 3.4.

9.7 The Pilot Test

The number of test subjects in the pilot test is admittedly not large enough to give any reliable results. From the little feedback received, the students seemed to appreciate working with real data. As one of the test subjects wrote, *“The focus was very practical. This is the first time I am working with data in this way”* (see Table 8-3). Further, both students seemed to enjoy using video lectures as a way of conveying new material, though there was a request of *“having some easy exercises at the end of the videos”* (see Table 8-3).

The pilot test was criticized by the students for not having the assignment text formulated well enough, and for not being adapted well adapted enough for their skill level. Further, the pilot test received a mediocre score regarding the experienced learning outcome. This feedback illustrates the importance of planning, and could probably have been improved by coordinating and cooperating better with the course coordinators when designing the assignments.

9.8 Innovation using Design Thinking

Though not a part of the problem statement directly, a human-centric method was considered necessary for produce a proof of concept that illustrated how the STTS could be utilized in an educational context. Initially, these methods feel

like common sense. However, by forcing oneself to actively pursue new information through end user feedback and iterative prototyping, it became easier to focus on the overall goal rather than getting stuck in technical problems.

The best example that illustrates this is through the design of the motor driver shield. Though the first prototype worked well and met all the initial requirements, the design thinking method pushed the design further, resulting in a product that both solved the technical problem and the initially unforeseen problem of cleaning up the loose wires by combining the wire routing into the motor driver shield.

Finally, though the methodology helped push the author to pursue unknown solutions that bore fruit in the end, it is an admittedly time-consuming process where a lot of the work goes into producing things that does not bear any fruit and is not used in the final design. The mindset therefore needs to be that these “failed” experiments help expose new parts of the problem, thus helping to create better answers.

10 Conclusion

10.1 The Applicability of the Serial Two-Tank System as a Pedagogical Tool

The STTS shows promising results as a pedagogical tool within the fields of automation, cybernetics and control theory. Its dynamic is both intuitive and easy to observe, and the mathematical model used to describe it is relatively simple. As the control goal can be either to control the water level in tank 1 or tank 2, the system can be approximated as either a first- or second-order system, making it suitable for illustrating most theory in introductory control theory courses.

10.2 The Walkthrough of the Serial Two-Tank System

The thesis demonstrates how the STTS is suited for cybernetics laboratory assignments by presenting the implementation and execution of the following concepts

- Introduction to essential electrical components and hardware
- Circuit design
- Design of the STTS
- Modeling the STTS
- Valve and pump coefficient estimation
- Simulating the STTS using Matlab scripts or Simulink blocks.
- Several types of stability analysis
- Several control schemes using PID or LQR, implemented both on a simulated and a physical system
- Measuring of control quality

10.3 Combining Learning Modules with the Flipped Classroom approach

The thesis suggests the flipped classroom approach to be used to elevate the execution of practical work. Related to the STTS, this approach is proposed in combination with four separate learning modules that builds on the workload needed to fully automate the STTS. The order of these learning modules are based on the order that relevant courses are taught at four Norwegian Engineering Universities and Colleges.

A proof of concept was created and tried out through a small pilot test, where the combination of the learning modules and the flipped classroom

showed potential as a supplement to standard teacher-led lectures. This approach does however require a certain degree of commitment from the course coordinators, as the implementation of the flipped classroom model is time-consuming, requiring heavy preparational and logistical planning.

10.4 Future Work

This thesis was carried out over a five-month period in the spring of 2017. During this time, parts of the project were found to have potential for improvement or a desire for further work.

10.4.1 The Design of the Serial Two-Tank System

The value of the system simulator would have benefitted from a physical system behaving more like the model prediction. As mentioned in Chapter 9.3, the physical system could probably have been modified to resemble the modelled dynamics by ensuring that valves of the two tanks were not vertically aligned.

Further, to give the students a more diverse introduction to sensors, it could be beneficial to implement different sensor schemes in the two tanks. A suggestion could be to implement a pressure sensor in one of the tanks, which would give a good opportunity to demonstrate how to use physics to transform pressure data to water level data.

10.4.2 The Learning Package

The proposed learning modules needs to be examined and redesigned, preferably in collaboration with a course coordinator. Further, a new pilot test is appropriate, favorably as part of a course.

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Appendix

Appendix A. Interview with Lars Lundheim

A.1. Background

Lars Lundheim, in cooperation with Thomas Tybell, Bjørn B. Larsen and Torbjørn Ekman, had a presentation during the MNT conference in Bergen, 2015 regarding a “stimulus package” for students studying electronic engineering at NTNU. The concept is called the Engineering Ladder (Norwegian: Ingeniørstigen), and it aims to ensure a tighter bond between theoretical and practical work, while at the same time increase the student’s own association within the field they are studying.

A.2. Abstract from Conversation

Lars described that the lecturing style during the Engineering Ladder took use of the pedagogical model called Flipped Classroom. The students typically watch short video lectures at home before the class sessions, while the in-class time is dedicated to working with exercises, projects and discussions [43].

As an example, Lars explained how students in the past would get problem sets with circuits, being asked to calculate different properties of each specific circuit and then move on to the next one. Now, students would be presented with a specific problem, and they are asked to design the circuit that best solves this problem. This, he added, is also more closely related to the way they will need to work with real life problems.

One example of this types of problem they now use at NTNU was a problem where the students were given an individual sound file with added noise in the form of a pure sinusoidal signal with a constant frequency. The object was then for the students to analyze the frequency and design a circuit that filtered out the noise from the original sound clip.

There is one challenge with this type of lecturing style, which is the administrative and logistical work required by the instructors. Lars described how they at NTNU each year create a new project in cooperation with an external institution or organization. Finding the right project and then shape it to fit the level of experience that one can expect from the students takes a lot of work. Lars also mentioned that recording the video lectures was a time-consuming process, though it helped that the same videos could be used for several years. Finally,

there is an ongoing process balancing how much you should supervise and guide the student during their work.

Regarding the choice of hardware and technical equipment, Lars gave a quick demonstration of the package that each student is expected to buy during the first couple of semesters. It consists of a Digilent Analog Discovery 2 and an Arduino Uno Starter Kit. The first is a low-cost multi-function instrument that allows users to measure, visualize, generate, record, and control mixed signal circuits of all kinds. The latter is a low-end microcontroller that has grown popular among hobbyists and tinkerers. This package was sold to the students for 2000 NOK, though Lars felt this was justified by the fact that the courses had no other official lecture material and that the students would use this equipment for at least two years.

Appendix B. Interview with Daniel Gimse Elstad

B.1. Background

Daniel Elstad works as a Department Engineer at the institute of Technical Cybernetics at NTNU Kalvskinnet. There he has the main responsibility for development and construction of lab assignments within the field of automation. One lab he works on developing is a tank system with many similarities to the one in this master project.

B.2. Abstract from Conversation

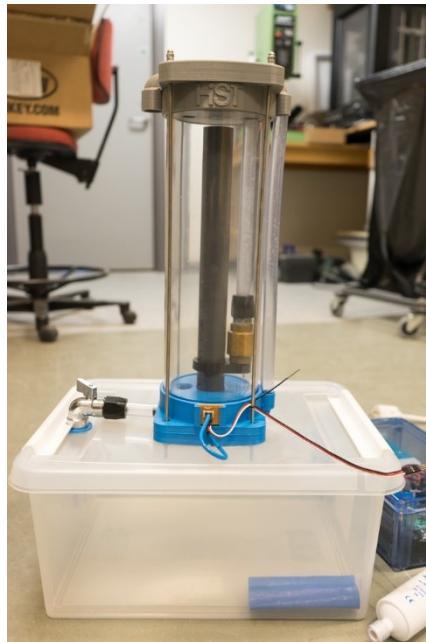
The visit to NTNU Kalvskinnet consisted mainly of Daniel giving a tour of the different tank systems they use in the lab lectures. Most of these tank systems were built with expensive, industrial hardware and had a size that made them hard to transport and move around. There was however one system that stood out, being built mainly by the staff at NTNU Kalvskinnet, and with parts that were much more affordable than the more industrial-looking ones. The focus of this abstract is therefore on this tank system, which was developed at NTNU Kalvskinnet, where the water tank still is being used in lab lectures.

The system, as shown in Appendix Figure B-1, uses a water pump bought at Biltema that pumps water up through a tube on the outside of the tank, and down again on the inside of the tank through a non-return valve. In the middle of the tank there is a tube that returns water directly back to the reservoir, made to prevent overflow. Under normal operation the water exits through a non-linear valve. The tank lid has a screw lid in the middle that enables two or more tanks to be connected in serial.

For sensing the water level, the tank is equipped with a pressure sensor in the bottom of the tank. Daniel mentioned that some of the bigger tanks had used weights to calculate the height of the water. This method had however been prone to measurement errors when water was either flowing or dripping into the tank in an irregular way. Daniel further explained how they had tested out a sensor device called eTape that looks like a long, thin strip of paper, which did not have the problem with measurement errors during dripping. This sensor did however tend to break either if the water level touched a membrane part at the top of the sensor or if the sensor got bent somehow.

The complete system is controlled using an Arduino as the I/O-device with an attached motor driver. The logic is created using National Instrument's LabVIEW, with an add-on for Arduino support.

The structure of the lab lecture was only briefly mentioned by Daniel. The students paired up in groups of two to three students, where each group received one tank system. They then go through a set of challenges presented in a document. The students do not make anything physical, and the lab focuses on applying control theory to a physical system. The tank systems are operated in normal classrooms.



Appendix Figure B-1: The tank system made by staff at former HiST, now called NTNU Kalvskinnet

Appendix C. Comparing the Curriculum of Four Norwegian Educational Institutions

Comparing the curriculums of selected automation-related study programs in Norway during their first two years				
Courses	Kybernetikk og robotikk, 5 years (NTNU Trondheim)	Elektronikk-ingeniør, automatisering, 3 years (HiOA Oslo)	Electronic engineering, cybernetics, 3 years (HSN Oslo)	Automatiseringsteknikk, 3 years (NTNU Ålesund)
Chemistry		S2 (ELPE1100)		S2 (IR102412)
Computer communication and networks		S3 (ELTS2200)		S3 (ID203012)
Computers and digital Control Design	S3 (TDT4160) S4 (TTK4105)		S3, S4	S4 (IE203612)
Cybernetics		S4 (ELFT2400)		
Digital technology		S1 (ELPE1400)	S1,S2 (EFD11101)	
Dynamical systems		S3 (ELTS2300)		
Electronics lvl 1 *	S1 (TFE4101)	S2 (ELPE1300)	S2 (EFEK1200)	S2 (IE100112)
Electronics lvl 2 *	S3 (TTK4240)	S4 (ELTS2200)	S3, S4	
Embedded systems	S4 (TTK4235)			
Ex.phil.	S4 (EXPH0004)			
Industrial control systems (Industielle styresystemer)				S3 (IE203512)
Information technologies Instrumentation and/or measurement techniques (Instrumentering, Måleteknikk)	S1 (TDT4110)			
Introductory course to the field of study	S2 (TTK4101)	S4 (ELFT2500)		S4 (IE203312)
Mathematics lvl 1 *	S1 (TTK4100)	S1 (ELFE1600)		S1 (IE100416)
Mathematics lvl 2 *	S1 (TMA4100)	S1 (ELFE1000)	S1,S2	S1 (IR102512)
Mathematics lvl 3 *	S2 (TMA4105)	S3 (ELTS2000)	S3 (RFMS2101)	S3 (IR201612)
Mathematics lvl 4 *	S2 (TMA4115) S3 (TMA4120)			
Microcontrollers			S1 (EFPM1010)	S1 (IE100212)
Non-object oriented Object-oriented programming	S2 (TDT4102)	S2 (ELPE1500)		S2 (ID101912)
Physics	S3 (TFY4115)	S2 (ELPE1100)	S1,S2 (RFFY1201) S3, S4	S2 (IR102412)
Signal Processing			S3, S4	S4 (IE203412)
Statistics	S4 (TMA4245)	S3 (ELTS2000)	S3 (RFMS2101) S1 (SFIP1100),S4 (MPSE2201)	
Project management				

Circuit theory

Cybernetics/automation

Programming

Natural Science

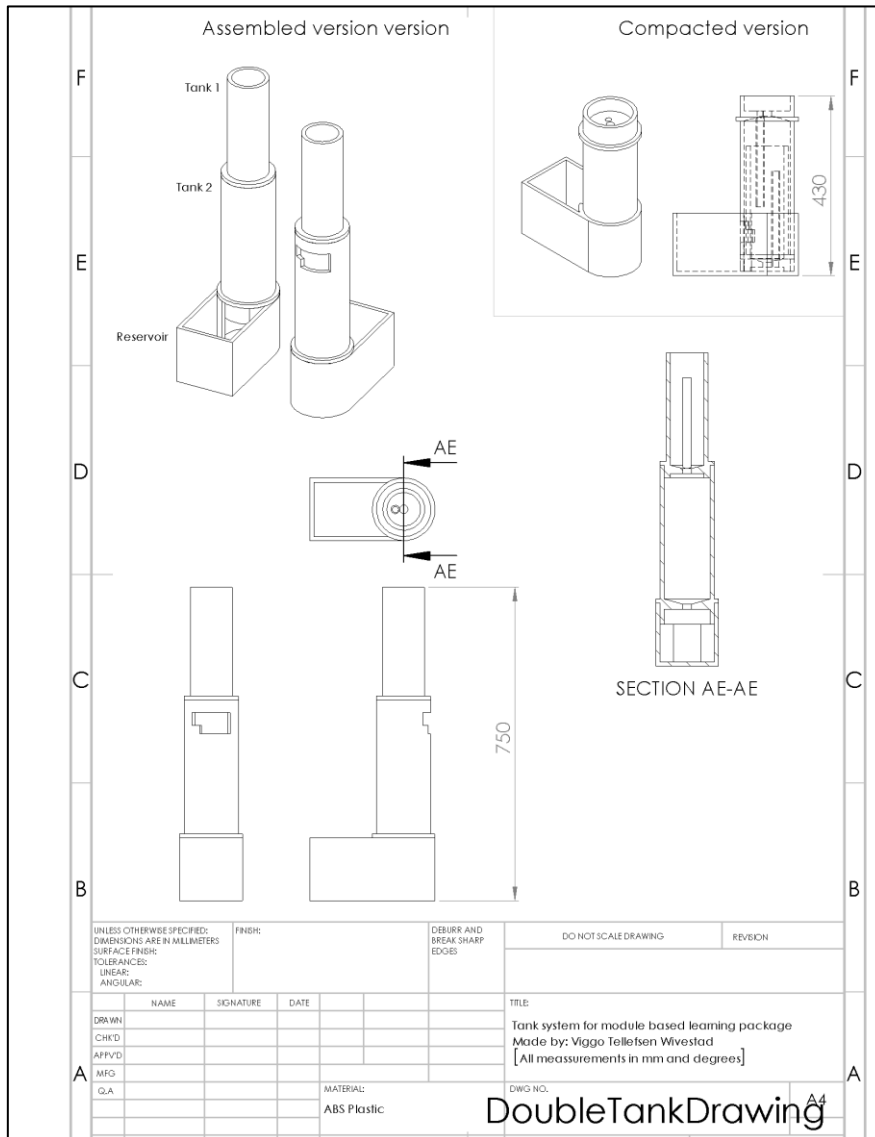
Other

* Courses marked with an asterisk are ranked with a level system. The reason for this is because they in their respectiv institution follow a specific order. This does however not guarantee that the same course will have the same curriculum as a different institution.

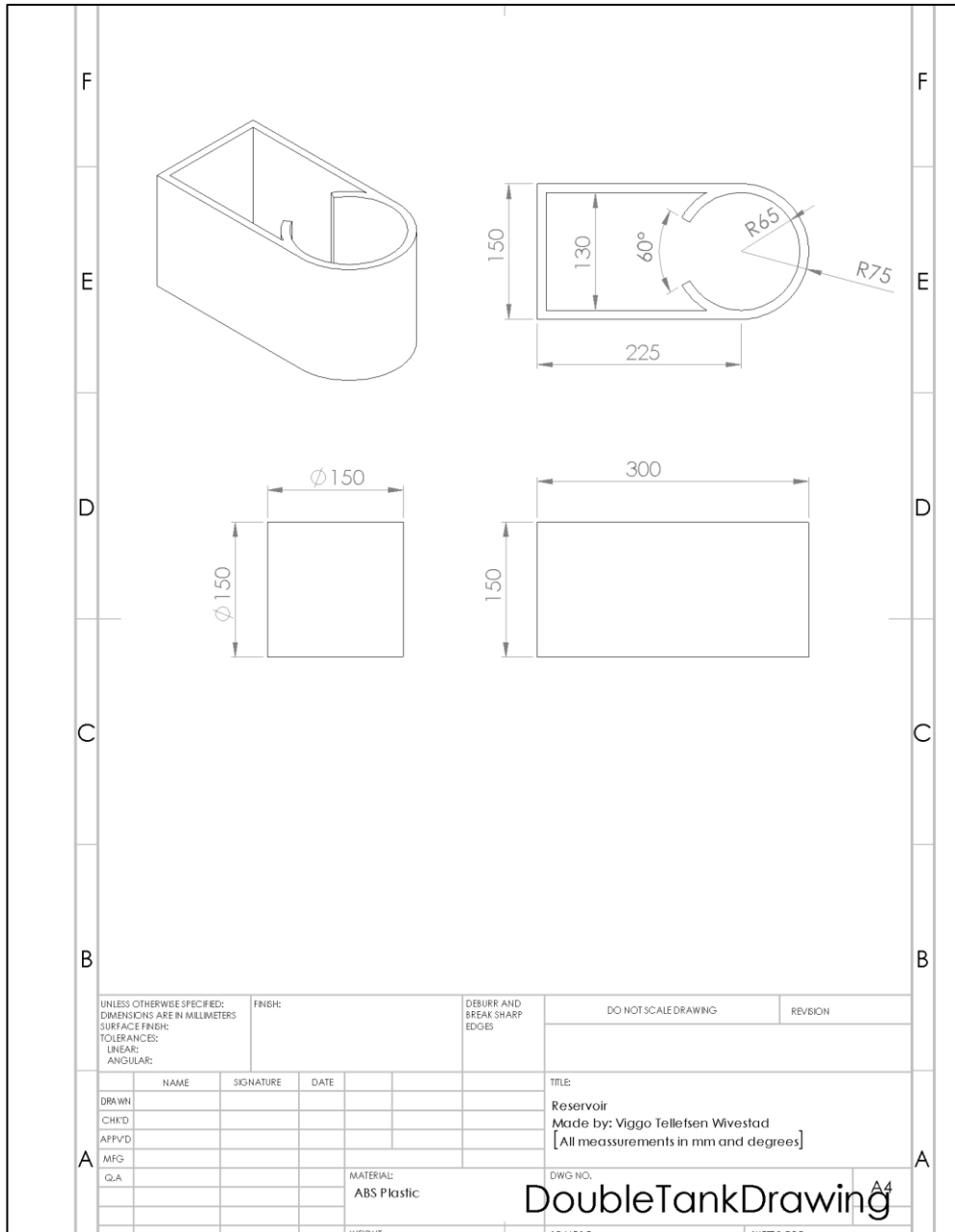
Appendix Table C-1: The table shows the categorization and comparison of different courses that engineering students are required to complete. The categories are created with focus on which field that would fit well with the implementation of the educational package presented in this thesis. For this thesis, four different Norwegian educational institutes were investigated.

Appendix D. Final Version of the 3D Modelled Water Tank System

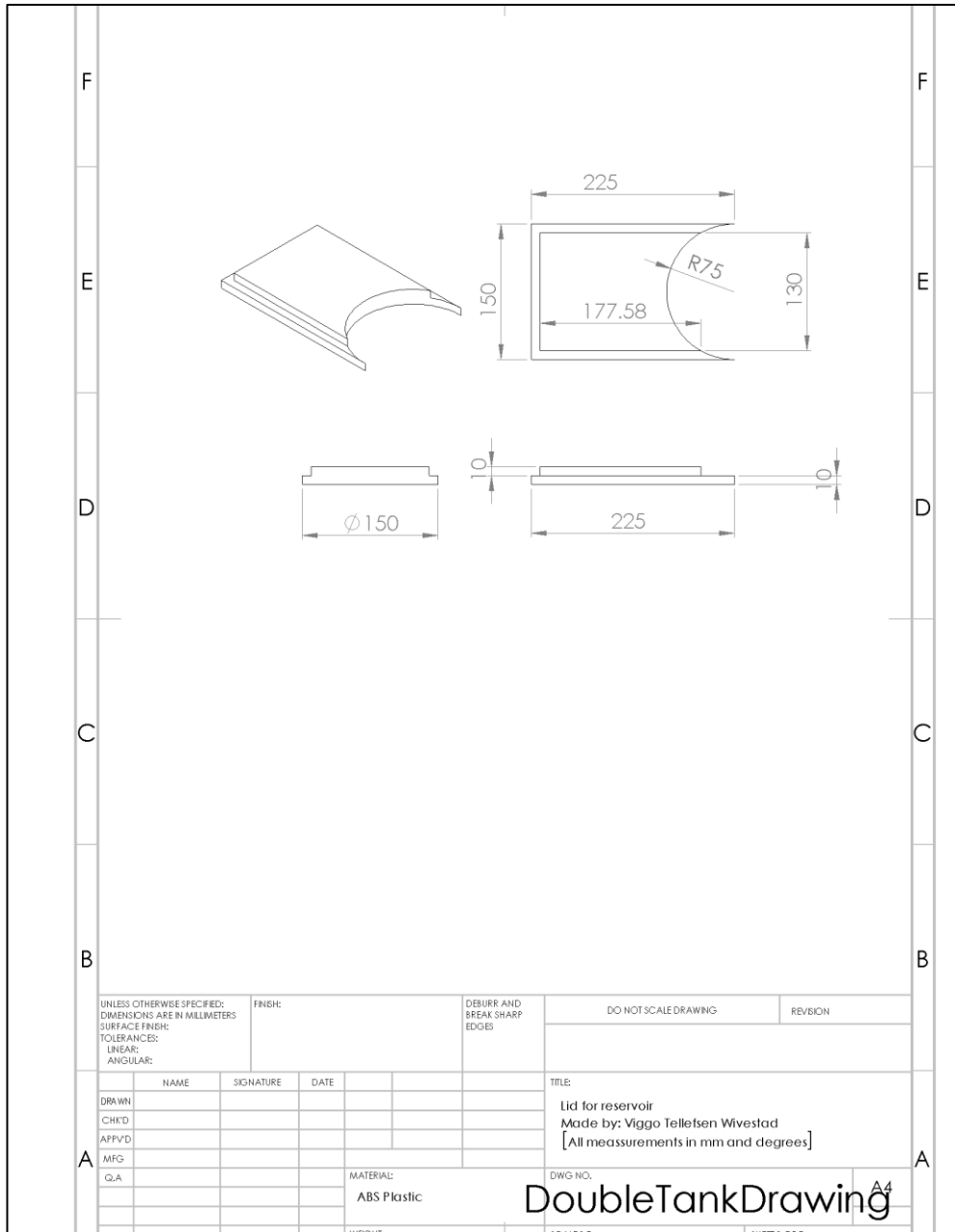
The following images show the technical drawings for the STTS, as they were delivered to the mechanical workshop at the Department of Technical Cybernetics, NTNU.



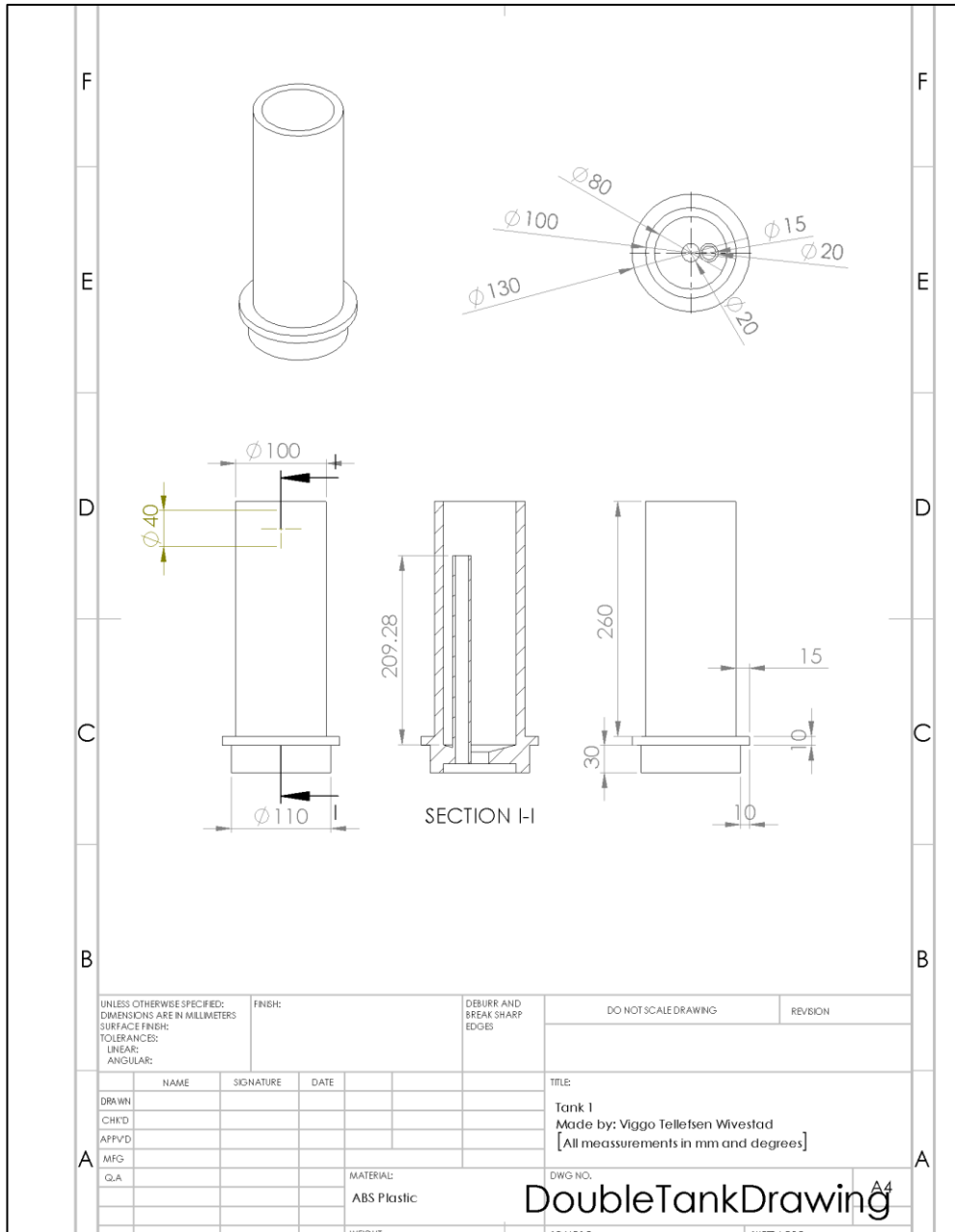
Appendix Figure D-1: Production drawings for the STTS, page 1/5



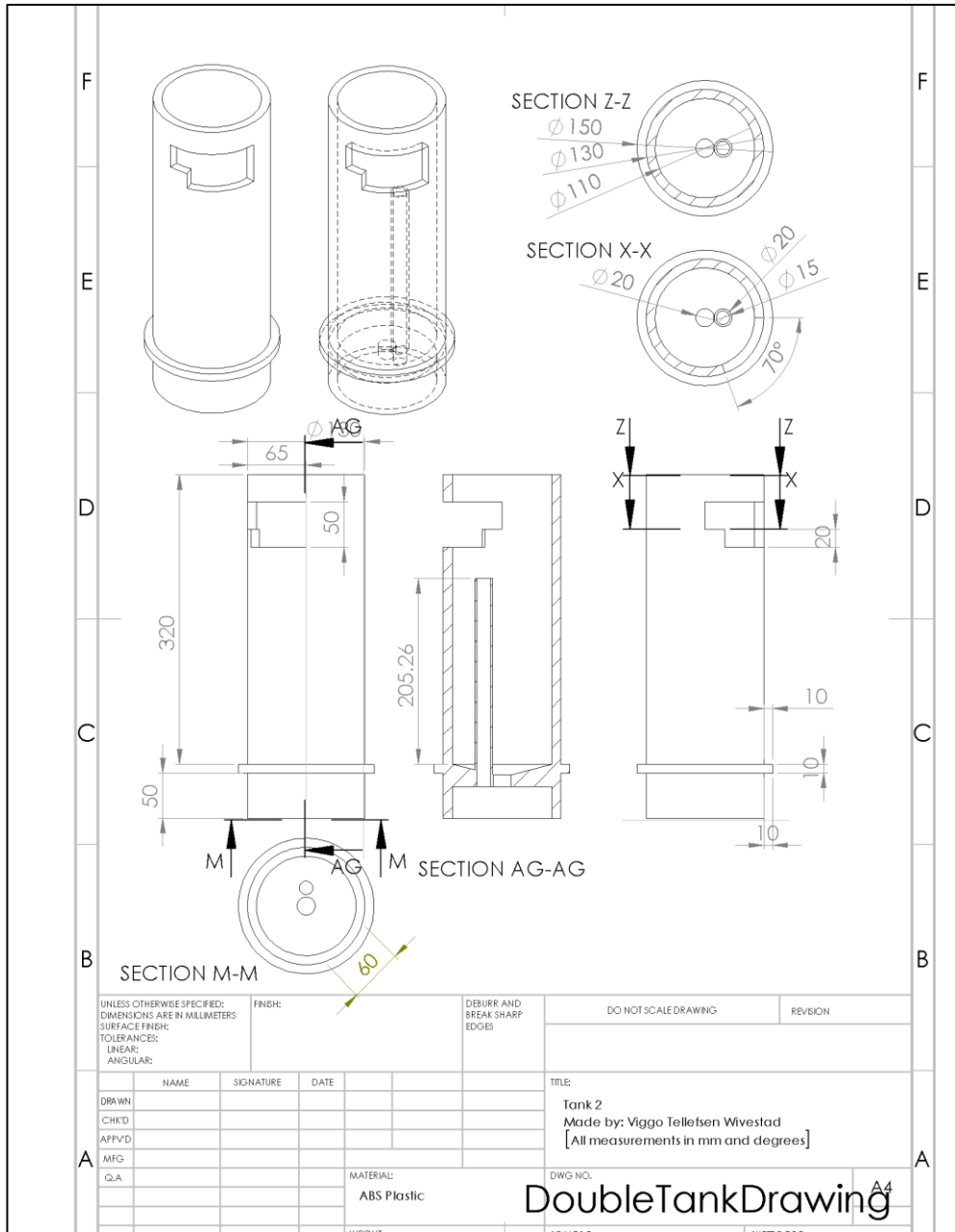
Appendix Figure D-2: Production drawings for the STTS, page 2/5



Appendix Figure D-3: Production drawings for the STTS, page 3/5



Appendix Figure D-4: Production drawings for the STTS, page 4/5



Appendix Figure D-5: Production drawings for the STTS, page 5/5

Appendix E. Assignment for the Pilot Testers of KybTeach.

The following text was handed out to the two test subjects during the pilot test. Since both students were Norwegian, the text for the assignment is also written in Norwegian.

Pilottesting av KybTeach, læringsmodul 3

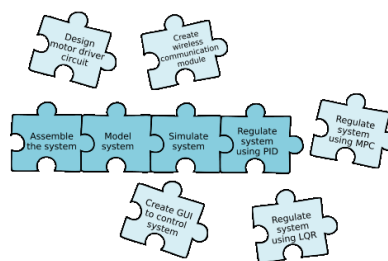
Simulering av «dobbel vanntank»-systemet og parameterestimering

Estimert tid: 1-3 timer. *Trenger ikke være sammenhengende.*

Konseptet bak KybTeach

KybTeach er et pedagogisk verktøy som er ment å gi studenter ved kybernetikk en tidlig hands-on erfaring i studiet sitt, med mål om å gjøre det lettere å koble teorien man lærer opp mot praktiske eksempler. Gjennom flere moduler, eller læringsetapper, skal studentene de fire første semestrene utføre flere praktiske oppgaver knyttet opp mot fagene de har.

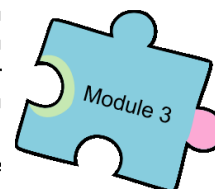
Læringspakken er fremdeles i konseptstadiet, og det er derfor ønskelig å få testet ut en av modulene på en gruppe studenter for å få en tidlig tilbakemelding på hva som fungerer og hva som bør endres.



Modulene

Hver modul kan ses på som en bit av et større pusslespill. Fra illustrasjonen til høyre vil den grønne delen representerer forventet forkunnskap, den blå representerer oppgavene og implementeringen og den rose delen beskriver læringsmålene og hvilke verktøy studenten sitter igjen med etter å ha gjennomført modulen.

En konsekvens av at konseptets tidlige stadium er at det for øyeblikket bare er laget én prototype av den doble vanntanken. Det er derfor bestemt at Modul 3: «Simulering av system og parameter estimering» er den mest hensiktsmessig modulen å gjennomføre som en pilottest på. Modul 3 sin «pusslespillbrikke» er utdypet på neste side.



Pilottesting

Første pilottesting vil bare utføre deler av modul 3, ettersom testdeltakerne nærmer seg eksamenstid. Oppgaven vil bestå i å simulere den doble vanntanken og få den til å ligne på oppførselen til det virkelige systemet. Informasjonen om hvordan dette skal gjøres vil gis gjennom en videoserie som studentene vil få tilgang til.

Oppgavene kan utføres hvor som helst og når som helst (må avsluttes i løpet av pilottesting), og krever kun en datamaskin med internettilkobling og Matlab/Simulink.

Dersom noen av studentene ønsker å lage en regulator til systemet, kan dette gjøres på det simulerte systemet. Regulatoren kan også testes ut på det virkelige systemet dersom de ønsker det, men dette blir ikke en del av pilottesting og må avtales med testansvarlig på forhånd.

Appendix F. Feedback Survey from the Pilot Test

Parts of the third learning module was tried out on two students in the beginning of May 2017. After the pilot test, the students anonymously¹¹ answered a brief survey, reflecting over the experience, the implementation and the potential the concept. To access the full survey, follow this link:

<https://no.surveymonkey.com/results/SM-6K5XXRTH/>

¹¹ The survey was anonymous in the sense that there is no direct way of identifying which person gave what answer to any of the questions. However, the students were asked to deliver a PDF document with the answer to the assignment. Since there was only two pilot testers, the first question therefor becomes an exception to the anonymity.

Appendix G. Approximation of Detailed Model Using the Half Rule

From [36], assume a system that is described by the detailed transfer function in the form

$$g_0(s) = \frac{\prod_j (-T_{j,0}^{-1}s + 1)}{\prod_i (\tau_{i,0}s + 1)} e^{-\theta_0 s} \quad (\text{G-1})$$

Where T^{-1} is the *inverse response time constant*(s) (negative numerator time constant), approximated as a time delay.

$$(-T_o^{-1}s + 1) \approx e^{-T_o^{-1}s}$$

Further, it is assumed that all the given parameters are positive¹² and the time constants are ordered according to their magnitude.

From the model (G-1), the parameters from the first-order model (4-54) can be obtained as follow [36] :

$$\begin{aligned} \tau_1 &= \tau_{1,0} + \frac{\tau_{2,0}}{2} \\ \theta &= \theta_0 + \frac{\tau_{2,0}}{2} + \sum_{i \geq 3} \tau_{i,0} + \sum_j T_{j,0}^{-1} + \frac{h}{2} \end{aligned} \quad (\text{G-2})$$

And the parameters for the second-order model (4-55) can be obtained as follow

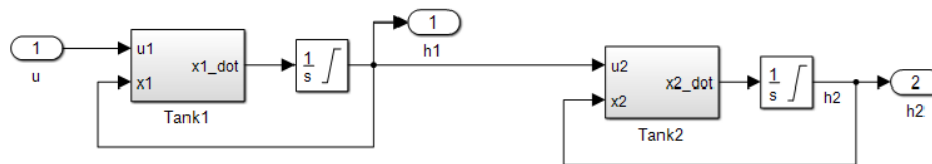
$$\begin{aligned} \tau_1 &= \tau_{1,0} \\ \tau_2 &= \tau_{2,0} + \frac{\tau_{3,0}}{2} \\ \theta &= \theta_0 + \frac{\tau_{3,0}}{2} + \sum_{i \geq 4} \tau_{i,0} + \sum_j T_{j,0}^{-1} + \frac{h}{2} \end{aligned} \quad (\text{G-3})$$

Where h is the sampling period when working with digital implementation. This way, the largest neglected time constant is distributed to the effective time delay and the smallest remained time constant (τ_1 or τ_2 , depending on whether the approximated is a first- or second-order system).

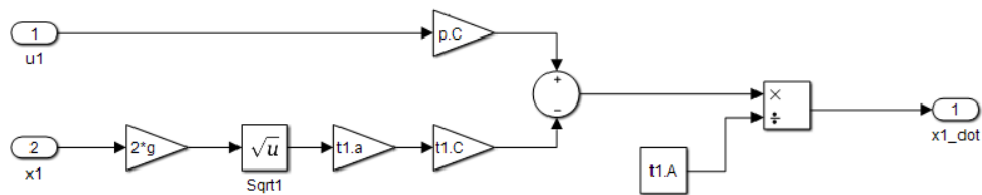
¹² A process model can also contain positive numerator time constants T_0 , where [36] gives a suggestion for how to cancel out the numerator time constant T_0 against a “neighboring” lag time constant τ_0 . This approach was however not necessary for the implementations of this work, and is therefore considered to be outside the scope of this thesis.

Appendix H. Simulation of the Tank System using Simulink

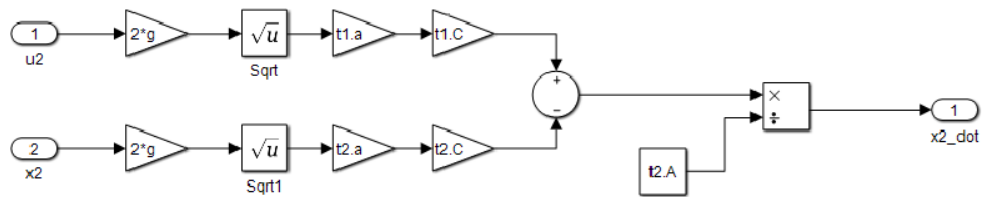
The following figures shows the implementation of the separate tanks, when simulating the STTS in Simulink. The outputs of the tanks were saturated to correspond to physical limitations, being the minimum and maximum water level of each tank. All simulations are available in the associated ZIP-folder, see Appendix I.1.



Appendix Figure H-1: An overview of the Simulink block diagram implementation of the STTS dynamics



Appendix Figure H-2: The Simulink block diagram implementation of the subsystem "Tank 1"



Appendix Figure H-3: The Simulink block diagram implementation of the subsystem "Tank 2"

Appendix I. Files in the Associated ZIP-folder

This thesis is uploaded at DAIM (digital archiving and submission of the master thesis) with an associated ZIP-folder. The material is divided into several sub-folders, and they all include a readme.txt-file that briefly explains the content. The next subchapters briefly describe the attached files.

I.1. Technical Drawings of the STTS

The technical drawings of the system are available in the associated ZIP-folder, in the subfolder named "01 - SolidWorks Model of System". These files are project files for SolidWorks.

I.2. Schematic and PCB Drawing for the Arduino Uno Motor Driver Shield

The schematic and PCB project files for the Arduino Uno Motor Driver Shield can be found in the subfolder named "02 - MotorDriverShield Arduino Uno". The files require the software DipTrace.

I.3. Matlab Scripts, Functions, Simulations and Simulink files

Several Matlab scripts and functions were generated to smooth, scale and in other ways manipulate the measured data. Further, the STTS was simulated using both Matlab scripts and Simulink. Finally, the physical system control schemes were implemented as Matlab code a run directly from the computer using an Arduino Uno as the computer I/O. The files are categorized into four groups

- Matlab Functions
- Simulations in Matlab Script
- Simulations in Simulink
- Controlling the Physical System Using Matlab Script

All associated files can be found in the subfolder named "03 - Matlab Scripts and Simulink Simulations". Further, to demonstrate the functions and scripts, most folder include on or more main-file that illustrates a specific function or task.

I.4. Script for Online Video Lectures, KybTeach

The script for the videos from "Module 3: Simulate system and estimate parameters" can be found in the subfolder named "04 - KybTeach Online Videos Script". This illustrates the amount of preparational work needed to produce a video series.

I.5. Surveys, Two-year Master Program, Technical Cybernetics at NTNU

During the author's work as a class representative, he conducted two surveys, collecting feedback from the students at the two-year program. The findings were presented to the faculty from the Institute of Technical Cybernetics during the semester meeting. Copies of this surveys is found in the subfolder "05 - Survey from the Two-Year Master Program in Technical Cybernetics at NTNU".