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Power Flow Tracing: Methods and Algorithms

Implementation Aspects

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Abstract

With the advent of deregulation, power systems across the world have undergone major restructuring. The unbundling of generation, transmission and distribution services has led to the emergence of electricity markets. One of the crucial issues encountered in such a scenario concerns the appropriate allocation of transmission costs based on actual usage. Power Flow Tracing (PFT), a method which makes it possible to attribute the power flowing on transmission lines to specific generators and loads, was originally conceived as a means of realising equitable transmission service pricing. Over the last decade and a half, significant attention has been devoted in the power system research community on improving PFT models, techniques and algorithms.

Though PFT finds practical application in the electricity markets elsewhere in the world, it has not yet found widespread use in the European electricity markets. However, of late, it has been identified that the application potential of PFT can be extended to diverse areas of modern power system design and operation, especially in systems with high penetration of renewable energy sources. Taking cue from this, this thesis sets out to build the foundation for an eventual comprehensive framework for applying PFT to practical European power system models, for research at the Department of Electric Power Engineering, NTNU.

The main objective of this Master's thesis is to look into select-few prominent mathematical methods and algorithms in vogue for PFT from the point of view of their comparative efficiency. In-house programming codes in MATLAB for select-few methods – the linear equation-based, the graph-based, and the node test-based methods, have been built, and their implementation aspects studied. Results from the implementation of PFT are presented on the 6 bus Roy Billinton Test System (RBTS) and the 24 bus IEEE Reliability Test System (RTS); additional illustrative systems are considered to demonstrate PFT in meshed systems with circular flows.

Further, the following applications of PFT are illustrated: loss allocation (transmission pricing-related), load shedding (power system operation and reliability-related) and CO_2-emission apportioning (sustainability-related). The latter demonstrative application deals with flow-based market coupling in the Northern European network, and is a joint venture with a fellow Master's student at NTNU, Cecilia Bringedal.

Sammendrag

Etter deregulering av kraftsystemer verden over har også en større restrukturering blitt gjennomført. Ettersom generasjon, transmisjon og distribusjon har blitt separert har dette ført til fremveksten av kraftmarkeder. Et viktig tema som da har oppstått omhandler hensiktsmessig fordeling av transmisjonskostnader basert på faktisk bruk. *Power Flow Tracing* (PFT), en metode som gjør det mulig å tildele kraft som flyter på transmisjonslinjene til spesifikke generatorer og laster, ble originalt opprettet for å kunne realisere en rettferdig transmisjonsprissetting. I løpet av de siste 15 årene har det blitt gitt betydelig oppmerksomhet i forskningsmiljøet innen kraftsystemer til å forbedre PFT-modeller, -teknikker og -algoritmer.

Til tross for at PFT har vist seg nyttig til praktiske bruksområder i kraftmarkeder ellers i verden, er det en metode som enda ikke har blitt særlig brukt i det europeiske kraftmarkedet. Men i det siste har det blitt identifisert at potensialet i bruksområdene til PFT kan utvides til diverse områder av moderne kraftsystemdesign og -operasjon, særlig i systemer med høy forekomst av fornybare energikilder. Med utgangspunkt i dette, ønsker denne Masteroppgaven å bygge et fundament for et fremtidig omfattende rammeverk for å benytte PFT til praktiske europeiske kraftsystemmodeller, til forskning på Institutt for elkraftteknikk, NTNU.

Hovedmålet med denne Masteroppgaven er å utforske enkelte fremtredende matematiske metoder og algoritmer innen PFT, for å kunne sammenligne effektiviteten deres. *In-house* programmeringskoder i MATLAB for enkelte utvalgte metoder - lineær ligning-basert, graf-basert og nodetest-basert metode, har blitt skrevet og implementasjonsaspektene deres har blitt undersøkt. Resultater fra implementasjonen av PFT er presentert på 6 bus *Roy Billinton Test System* (RBTS) og 24 bus *Reliability Test System* (RTS). Andre systemer er også undersøkt for å demonstrere PFT i systemer med maskenettstruktur og sirkulære strømmer.

Videre er følgende bruksområder av PFT illustrert: tapsallokasjon (transmisjonsprisrelatert),

lastutkobling (kraftsystemoperasjons- og pålitelighetsrelatert) og CO_2-utslippsfordeling (bærekraftsrelatert). Sistnevnte er et demonstrativt bruksområde som omhandler *flow-based market coupling* i det nordeuropeiske nettverket, og er et samarbeid med Cecilia Bringedal, en annen Masterstudent ved NTNU.

Preface

This Master's thesis concludes my Master's degree within Energy and Environment at NTNU in Trondheim. The thesis gives an introduction to power flow tracing, and it is my hope that this work will be of interest and help for those wanting to explore the questions that remain unanswered in this field.

I would like to express my sincere gratitude to my supervisor Associate Professor Vijay Venu Vadlamudi for his guidance. His enthusiasm and love for the field of power systems, and the manner in which he strives to be the pedagogical professor his students need, all make him an inspiration to be reckoned with. He has made me understand the importance of fundamental and incremental research; more importantly that a thesis should be written with Quentin Tarantino's filmmaking in mind.

Thanks are also due to Cecilia Bringedal and Associate Professor Hossein Farahmand for their generous support in one of the case studies used in the thesis.

Last but not least, thanks to my family and friends for their continued support.

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Abbreviations

AC Alternating Current

CNTC Coordinated Net Transmission Capacity

CWE Central Western Europe

FBMC Flow Based Market Coupling

ICI Intentional Controlled Islanding

KCL Kirchhoff's Current Law

LSP Load Service Probability

NGDF Nodal Generation Distribution Factor

PFT Power Flow Tracing

PSP Proportional Sharing Principle

RBTS Roy Billinton Test System

RTS Reliability Test System

TGDF Topological Generation Distribution Factor

TLDF Topological Load Distribution Factor

TSO Transmission System Operator

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Chapter 1

Introduction

1.1 Motivation

Since the 1990s, several countries have restructured and deregulated their electricity market. Originally in the US, the utilities were privately owned with public regulation, while in most of Europe the utilities were publicly owned (by centralised State owned utilities or a decentralised utility pattern such as in Scandinavia). After the restructuring, the generation and transmission parts of the market have been privatised. Generally it is accepted that electricity distribution is a natural monopoly, since it does not make sense to build parallel distribution grids to supply the same area [1].

Restructuring and privatisation, leading to competition among the participants responsible for generation and transmission, was done to increase efficiency and secure fair pricing for electricity buyers. However, to ensure a liquid, efficient and competitive market there is a need for transparency [2]. But how can an electricity market have full transparency when no one knows exactly where the power from a generator flows before it ends up at a load? It is commonly known that electricity is untraceable, and thus the conventional way of deciding the relation between power generated by generators and power consumed by loads has been by performing a sensitivity analysis. However, this only gives the impact a generator has on a load, not the actual power flowing in the grid. If it is assumed that all electricity is divided proportionally throughout a network, this might tell us where the power flows. This is the concept of power flow tracing (PFT). PFT first arose as a means to decide the costs of transmission losses in a network, however in the last few years as more research on this topic has been done, several other applications have emerged. There are many proposed ways of performing PFT, but this thesis will focus on PFT using the proportional sharing principle.

1.2 Contributions

- The thesis sets out to develop an initial framework for applying PFT to power system test networks. The work is expected to build a foundation for the subsequent development of a comprehensive PFT framework for research at the Department of Electric Power Engineering, NTNU.

- As it stands now, the thesis aims to highlight the nuances of well-established PFT methods and algorithms, perform comparative case studies, and provide an essential pedagogical treatment of the underlying concepts. The application potential of PFT is demonstrated through illustrative case studies – loss allocation (transmission pricing-related), load shedding (power system operation and reliability-related) and CO_2-emission apportioning (sustainability-related). The latter demonstrative application deals with flow-based market coupling in the Northern European network, and is a joint venture with a fellow Master's student at NTNU, Cecilia Bringedal.

- Select-few prominent mathematical methods in vogue for PFT – the linear equation-based, the graph-based, and the node test-based methods, and their algorithmic execution have been investigated; both from the point of view of their comparative ease and their efficiency of execution.

- In-house programming codes in MATLAB for the aforementioned methods have been built, and their implementation aspects studied. The MATLAB scripts have been released for further internal use and research at the Department of Electric Power Engineering, NTNU.

- Assumptions and Limitations: Only the PFT methods with basis in Proportional Sharing Principle have been studied. The scope of PFT is restricted to active power tracing only; reactive power tracing is not dealt with in this thesis.

1.3 Organisation of Thesis

The organisation of this thesis is as follows:

Chapter 2: An introduction to power flow tracing using proportional sharing is given. Further, the three algorithms focused on in this thesis are presented: the linear equation-based, the graph-based and the node test-based. Modifications to these methods, as found in literature are also highlighted; reactive power tracing is briefly mentioned, and the handling of networks with losses is explained. Finally, an overview of applications of PFT found in literature is given.

Chapter 3: A description on how the three different algorithms have been implemented in MATLAB is given.

Chapter 4: Tracing results are shown for the three algorithms when tracing on the 6 bus RBTS and the 24 bus IEEE RTS. Further, the algorithms are implemented on two systems containing circulating flows. The following applications for tracing are demonstrated: loss allocation, load shedding and CO_2-emission apportioning. Loss allocation is performed for all algorithms, while the two latter applications are demonstrated using only the linear-equation based approach. Discussion on the tracing results and differences between the algorithms are presented.

Chapter 5: Concluding remarks, including a summary of the work performed as well as recommendations for future work are presented.

Chapter 2

Background: Power Flow Tracing

Remark: This work builds on the literature review carried out as part of specialisation project TET4520, and as such there is extensive reproduction/usage of the content therefrom.

This chapter gives a background knowledge of Power Flow Tracing (PFT) with emphasis on proportional sharing methods. The concept of proportional sharing will be introduced, as well as mathematical explanations to three different PFT methods: a linear equation-based method, a graph-based method and a node test-based method. Further, concepts such as circulating flows, reactive power tracing and power flow tracing in literature are discussed.

Power flow tracing (PFT) can be performed on a network where a power flow or a state estimation has been run. After tracing has been performed on the network, it is known how generators contribute to line flow and loads. The idea is to be able to use the tracing result for applications. The idea is illustrated in Figure 2.1.

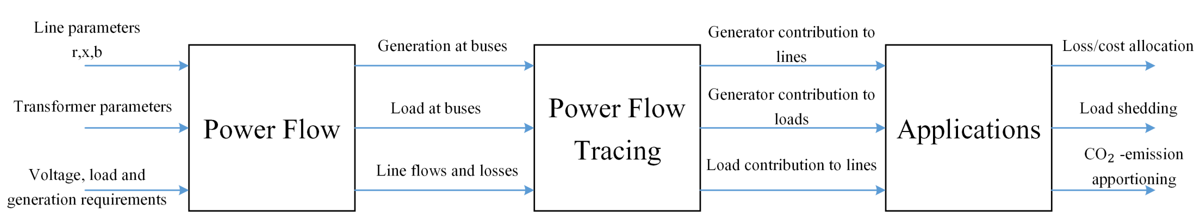

Figure 2.1: PFT: input, output and applications

2.1 Proportional Sharing

Power flow tracing (PFT) methods using proportional sharing are based on the proportional sharing principle (PSP). The benefits of proportional sharing is that it is not dependent on slack bus and it gives only positive costs when used in transmission cost allocation. It also differentiates on the basis of network location [3]. However, since it is based on PSP it can neither be proved nor disproved.

The Beginning

The concept of PSP was introduced in 1996 by J. Bialek [4]. As restructuring of the electricity system began, competition increased and the following question arose: Who is using the different lines and which generator is supplying which load? Since power may flow in any line of the transmission system, it would be interesting to see which lines are being used for which transactions. If transmission system usage can be determined, then losses and costs may be allocated among the participants using the lines. Here enters the PSP as a way of determining the transmission system usage. PSP is a principle which is easy to grasp, but difficult to prove [5]. However, it has been rationalised using game theory and information theory [6]. For a further discussion on the proof of PSP the reader is encouraged to see Ref. [5].

Illustration of Proportional Sharing Principle

Assume a lossless network, with one bus, two incoming lines (1, 2) and two outgoing lines (3, 4) as shown in Figure 2.2.

The proportional sharing principle states that the nodal inflow is proportionally distributed among the nodal outflows [4]. This means that Kirchhoff's current law is obeyed, ensuring that power injected in a bus equals the power extracted from a bus. It is then assumed that line 1 has a share of 40/100 and line 2 has a share of 60/100 in the other lines. This gives the following contributions:

Line 1 injects $\frac{40}{100} \cdot 70 = 28$ in line 3 and $\frac{40}{100} \cdot 30 = 12$ in line 4.

Line 2 injects $\frac{60}{100} \cdot 70 = 42$ in line 3 and $\frac{60}{100} \cdot 30 = 18$ in line 4.

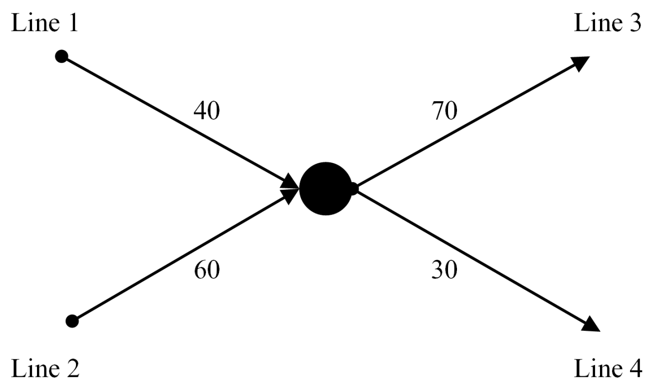

Figure 2.2: Illustration of PSP [4]

2.2 Other PFT Methods

As mentioned, proportional sharing is only one of the many principles used to trace power flow. A brief overview of the other methods mentioned in literature *not* based on the PSP is given in Table 2.1. Other methods not based on the PSP which should be mentioned are the Relative Electrical Distance-technique and game theory.

PFT Technique	**Characteristic**
Circuit Theory	Proximity effect [7]
	Looks at current rather than power [8]
	Includes shunt admittances [9]
	Slack bus independent [9]
	Does not require additional assumptions (such as PSP) [9]
Optimization	Improves fairness model in tracing [10]
	May not converge to a fair solution [11]
	Lacks scalability [11]
	Computationally demanding
Equilateral Bilateral Exchange (EBE)	Independent of slack-bus [12]
	Satisfies KVL and KCL [9]
	Handles counterflows [12]
	Does not have the proximity effect [9]

Table 2.1: Characteristics of different PFT techniques

2.3 Circulating Flows

Circulating flows are phenomena in power systems that occur when power flows in a continuous circle rather than from a source to a sink [13] such as illustrated in Figure 2.3. This means that some of the power flowing through the lines is taking up transmission capacity, but is not being transferred to a load. Circulating flows are also known as loop flows, however the term "loop flows" are used to describe a different concept as well. This other meaning of the term is covered in Section 4.1.3. Therefore, to avoid confusion, circulating flows is the terminology used in this thesis. Circulating flows result in a higher current without actually transferring any power, and is not desirable since transfer capacity is reduced while causing losses in the lines. It may also cause equipment to be loaded higher, maybe even exceeding its limit [13]. Circular flows are often caused by phase-shifting transformers or FACTS devices [14]. It becomes clear that circulating flows may cause a headache when attempting to trace the flow of power, since it is hard to allocate a flow going in a cycle. As will be shown later on, some tracing algorithms are able to handle the circulating flows, and some are not. Numerical examples regarding circulating flows are illustrated in Section 4.1.3.

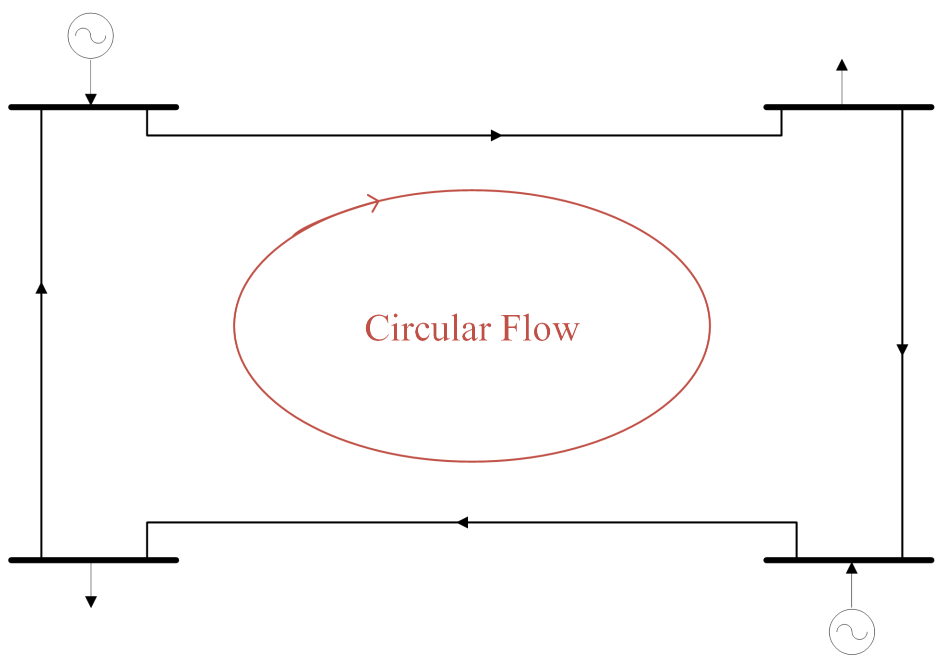

Figure 2.3: Illustration of circulating flow

2.4 Fundamental PFT Methods

There are two basic methods using PSP, well known in literature: The linear equation-based approach presented by Bialek [4] in 1996 and the graph-based approach presented by Kirschen *et al.* in 1997 [15]. In this chapter the algorithms of these two methods will be presented, along with the algorithm of a third power flow tracing method using PSP. This third method is a node test-based method claimed to be able to handle circulating flows, proposed by Abdelkader in 2007 [16]. From here on these methods may also be correspondingly referred to as the Bialek-Method, the Kirschen-method and the Abdelkader-method. Numerical examples, illustrating the step-by-step application of these algorithms are shown on a lossless 4-bus system in Appendix A.

2.4.1 Linear Equation-Based Method

The algorithm presented by Bialek only works on lossless flows when the flows are the same at the beginning and the end of a line. In Ref. [4], Bialek suggests three different ways of achieving this: Gross flows, net flows or average flows. In all three cases, new equivalent networks are constructed, changing generation, load and line flows in the following manner:

Gross flows When using gross flows, an equivalent network is created where the generation is the same as for the actual network. There are no losses in the lines, and the line flow equals the sending end power of the actual line. For KCL to be fulfilled, the loads are then increased. At some buses, it will be necessary to modify the line flow so that KCL is fulfilled. This is taken care of in the algorithm.

Net flows When using net flows, an equivalent network is created where the load is the same as for the actual network. There are no losses in the lines, and the line flow equals the receiving end power of the actual line. For KCL to be fulfilled, the generation at buses is increased. At some buses, it will be necessary to modify the line flow so that KCL is fulfilled. This is taken care of in the algorithm.

Average flows When using average flows, an equivalent network is created where the line flow is the average value of the actual line flow. For example: if a line has a sending end power of 60 MW and a receiving end power of 59 MW (meaning a loss of 1 MW), the average line flow equals 59.5 MW. For KCL to be fulfilled, the generation and load at buses are changed.

When using gross flows, an upstream-looking algorithm is used. When using net flows, a downstream-looking algorithm is used. If using average line flows, both upstream- and downstream-looking algorithms can be used. An upstream-looking algorithm is an algorithm that looks at the *inflows* of a node (hence incoming lines and generation), while a downstream-looking algorithm is an algorithm that looks at the *outflows* of a node (hence outgoing lines and load). In the next section, the general algorithms for upstream- and downstream tracing are presented.

The Bialek-method makes use of topological distribution factors. For the upstream-looking algorithm, a topological generation distribution factor (TGDF) is defined as the portion of generation owing to the kth generator that flows in a line i-l. It represents the share of a particular generator in the total line flow and is presented in (2.5). For the downstream-looking algorithm, a topological load distribution factor (TLDF) is defined as the portion of the kth load demand that flows in a line i-j. It represents the share of the load in a line flow and is presented in (2.11). Both distribution factors are always positive, ensuring no negative costs if this method is used for cost allocation [4].

Upstream-Looking Algorithm using Gross Flows

The upstream-looking algorithm looks at how each generator is contributing to the loads and lines in the system. As mentioned above, using gross flows means that the generation is the same, while the line flows and loads change. It is therefore necessary to define new line flows, loads and nodal throughflows: Gross line flows, gross loads and gross nodal throughflows. A *nodal throughflow* is defined as the total power flowing through a node. The nodal throughflow can either be found from the inflows to a node, or from the outflow from a node. The gross line flow will be either equal to or greater than the sending end power of the line since the network now is lossless and KCL still needs to be satisfied. For a small network, it is possible to decide these quantities by inspection, however for a network of a greater size a more methodical way is necessary. The total gross flow through node i can be expressed as

$$P_i^{\text{gross}} = \sum_{j \in \alpha_i^u} |P_{i-j}^{\text{gross}}| + P_{Gi} \quad , \forall i \tag{2.1}$$

where α_i^u is the set of nodes supplying bus i, i.e. all nodes connected to bus i sending power to bus i,
P_i^{gross} is the gross nodal throughflow of i,
P_{i-j}^{gross} is the gross flow on line i-j,
P_{Gi} is the generation at bus i.

By defining $c_{ji}^{\text{gross}} = \dfrac{|P_{j-i}^{\text{gross}}|}{P_j^{\text{gross}}}$, (2.1) can be written as

$$P_i^{\text{gross}} - \sum_{j \in \alpha_i^u} c_{ji}^{\text{gross}} P_j^{\text{gross}} = P_{Gi} \quad , \forall i \tag{2.2}$$

It is assumed that the transmission losses are small so that $\dfrac{|P_{j-i}^{\text{gross}}|}{P_j^{\text{gross}}} \simeq \dfrac{|P_{j-i}|}{P_j}$. (2.1) can then be written in vector form as

$$\mathbf{A}_u \mathbf{P}^{\text{gross}} = \mathbf{P}_G \tag{2.3}$$

where $\mathbf{P}^{\text{gross}}$ is the vector of gross nodal throughflows,
$\mathbf{P}_G$ is the vector of nodal generations,
$\mathbf{A}_u$ is an upstream distribution matrix, where its elements are determined from (2.4)

$$A_{u_{ij}} = \begin{cases} 1, & \text{for } i = j \\ -c_{ji} = -\dfrac{|P_{j-i}|}{P_j}, & \text{for } j \in \alpha_i^u \\ 0, & \text{otherwise} \end{cases} \tag{2.4}$$

When the gross nodal flows have been decided from (2.3), the gross line flows can be found from (2.5) by using PSP:

$$|P_{i-l}^{\text{gross}}| = \frac{|P_{i-l}^{\text{gross}}|}{P_i^{\text{gross}}} P_i^{\text{gross}} \simeq \frac{|P_{i-l}|}{P_i} \sum_{k=1}^{n} [A_u^{-1}]_{ik} P_{Gk} = \sum_{k=1}^{n} D_{il,k}^{G} P_{Gk} \quad , \quad l \in \alpha_i^d \tag{2.5}$$

where $D^{G}_{il,k}$ is the topological generation distribution factor,
α^{d}_{i} is the set of nodes supplied by bus i,
n is number of buses.

The contribution of generators to a load at node i can be derived in the same manner, giving:

$$P^{\text{gross}}_{L_i} = \frac{P^{\text{gross}}_{Li}}{P^{\text{gross}}_{i}} P^{\text{gross}}_{i} \simeq \frac{P_{Li}}{P_i} \sum_{k=1}^{n} [A_u^{-1}]_{ik} P_{Gk} \tag{2.6}$$

where $P^{\text{gross}}_{L_i}$ is the gross load at node i.

Stepwise Approach Upstream-Looking

1. Determine $\mathbf{A}_u$ from (2.4)
2. Obtain the inverse upstream distribution matrix $\mathbf{A}_u^{-1}$
3. Apply (2.5) to calculate how the generators are contributing to lines
4. Apply (2.6) to calculate how the generators are contributing to loads

A numerical example on a lossless 4-bus system is shown in Appendix A.

Downstream-Looking Algorithm using Net Flows

The downstream-looking algorithm looks at how each load is responsible for the generation and line flow in the system. As mentioned above, using net flows means that the load is the same, while the line flows and generation change. It is therefore necessary to define new line flows, generation and nodal through-flows: Net line flows, net generation and net nodal throughflows. The net line flow will be either equal to or less than the receiving end power of the line since the network now is lossless and KCL still needs to be satisfied. For a small network, it is possible to decide these quantities by inspection, however for a network of a greater size a more methodical way is necessary. The total net flow through node i can be expressed as:

$$P^{\text{net}}_{i} = \sum_{l\in\alpha^{d}_{i}} |P^{\text{net}}_{i-l}| + P_{Li} = \sum_{l\in\alpha^{d}_{i}} c^{\text{net}}_{li} P^{\text{net}}_{l} + P_{Li} \quad ,\forall i \tag{2.7}$$

where $c_{li}^{\text{net}} = \dfrac{|P_{l-i}^{\text{net}}|}{P_l^{\text{net}}}$,
α_i^d = nodes supplied by i.

Rearranging the terms of (2.7) gives

$$P_i^{\text{net}} - \sum_{l \in \alpha_i^d} c_{li}^{\text{net}} P_l^{\text{net}} = P_{Li} \quad , \forall i \tag{2.8}$$

As in the case of the upstream-looking algorithm, it is assumed that the transmission losses are small, so that $\dfrac{|P_{l-i}^{\text{net}}|}{P_l^{\text{net}}} \simeq \dfrac{|P_{l-i}|}{P_l}$. (2.7) can then be written in vector form as

$$\mathbf{A}_d \mathbf{P}^{\text{net}} = \mathbf{P}_L \tag{2.9}$$

where $\mathbf{P}^{\text{net}}$ is the vector of net nodal flows,

$\mathbf{P}_L$ is the vector of nodal demands,

$\mathbf{A}_d$ is the downstream distribution matrix. The elements in the matrix are decided from (2.10)

$$A_{d_{il}} = \begin{cases} 1, & \text{for } i = l \\ -c_{li} = -\dfrac{|P_{l-i}|}{P_l}, & \text{for } l \in \alpha_i^d \\ 0, & \text{otherwise} \end{cases} \tag{2.10}$$

When the net nodal flows have been determined from (2.7), the net line flows can be determined by applying PSP and rearranging the terms of (2.9):

$$|P_{i-j}^{\text{net}}| = \frac{|P_{i-j}^{\text{net}}|}{P_i^{\text{net}}} P_i^{\text{net}} \simeq \frac{|P_{i-j}|}{P_i} \sum_{k=1}^{n} [A_d^{-1}]_{ik} P_{Lk} = \sum_{k=1}^{n} D_{i-j,k}^L P_{Lk} \quad , j \in \alpha_i^u \tag{2.11}$$

where $D_{i-j,k}^L$ is the topological load distribution factor.

The contribution of loads to generation at bus i can be derived in the same manner, giving:

$$P_{G_i}^{\text{net}} = \frac{P_{Gi}^{\text{net}}}{P_i^{\text{net}}} P_i^{\text{net}} \simeq \frac{P_{Gi}}{P_i} \sum_{k=1}^{n} [A_d^{-1}]_{ik} P_{Lk} \tag{2.12}$$

where $P_{G_i}^{\text{net}}$ is the net generation at node i.

Stepwise Approach

1. Determine $\mathbf{A}_d$ from (2.10)
2. Obtain the inverse downstream distribution matrix $\mathbf{A}_d^{-1}$
3. Apply (2.11) to calculate how the loads impact the line flows
4. Apply (2.12) to calculate how the loads impact the generation

A numerical example on a lossless 4-bus system is shown in Appendix A.

Loss Allocation

For the upstream-looking algorithm, the difference between the actual load and the gross load at node i will be the amount of load at node i that contributes to the losses in the system, as shown in equation (2.13).

$$\Delta P_{L_i} = P_{L_i}^{\text{gross}} - P_{L_i} \tag{2.13}$$

For the downstream-looking algorithm, the difference between the actual generation and the net generation at node i will be the amount of generation at node i that contributes to losses in the system, as shown in (2.14).

$$\Delta P_{G_i} = P_{G_i} - P_{G_i}^{\text{net}} \tag{2.14}$$

Intuitively, one might think that the difference between the gross line flow and the actual line flow would yield losses in that line. This is, however, not the case, and important to take notice of when using this approach. Some of the losses incurred in other lines will propagate through the network and add to the adjacent lines. This is expressed in (2.15):

$$|P_{ij}^{\text{gross}}| = |P_{ij}| + \Delta P_{ij} + \Delta P_{ij}^{u} \tag{2.15}$$

where $\Delta P_{ij}$is the transmission loss in line i-j,
$\Delta P_{ij}^{u}$is the unknown accumulated upstream line loss.

In Ref. [4], Bialek also proposes a nonproportional way of allocating losses. The idea is that since transmission loss is proportional to the current squared ($P = RI^2$), it should be allocated in the same manner. This will not be further investigated in this thesis.

2.4.2 Graph-Based Method

The Kirschen-method builds a new topology of the network by defining a state graph consisting of commons and links. New concepts are introduced as below.

Commons Buses reached by the same generator(s), where the buses are connected. A bus may only be in one common. The rank of a common is decided by number of generators supplying the common (fewer generators → higher rank).

Domains The buses reached by a generator (there is only one domain per generator).

Branch A line connecting two buses.
External branch is a line between two buses in different commons.
Internal branch is a line between two buses in the same common.

Link One or more external branches connecting the same commons in the same direction.

State graph Graph illustrating the network, consisting of commons and links.

Domains, commons and links are easy to determine for a small system, however for systems of larger scale an algorithm is beneficial. Ref. [15] gives an algorithm for deciding the domains, and an algorithm using the domains to decide the commons. After this, the links may be determined. A drawback of this method is that since generators and loads are clustered together in commons, even a small change in network topology may change the composition of commons and links.

A recursive method is used to calculate the contributions of generators to lines and loads. The recursive method switches between defining the flow on a line due to a generator, and the relative contribution of a generator to the outflow of a common. The contribution of a generator i to a link j-k is determined by (2.16):

$$F^i_{jk} = C^i_j \cdot F_{jk} \tag{2.16}$$

where F^i_{jk} is the flow on link j-k due to generator at bus i,

C^i_j is the relative contribution of generator at bus i to the outflow of common j,

F_{jk} is the flow on link j-k.

Further, the relative contribution of generator at bus i to the outflow of common k is determined by (2.17):

$$C^i_k = \begin{cases} \dfrac{F_{Gi}}{P_i}, & \text{if } i = k \\ \dfrac{\sum\limits_{j\in D} F^i_{jk}}{P_k}, & \text{otherwise} \end{cases} \tag{2.17}$$

where C^i_k is the relative contribution of generator at bus i to the outflow of common k,

F_{Gi} is the generation at node i,

P_k is the nodal throughflow of common k,

D is set of commons supplying common k.

Stepwise Approach

1. Create a state graph of the system by deciding commons and links
2. Determine P_k for all commons and F_{jk} for all links
3. Use (2.17) to determine relative contribution to one link

   Use (2.16) to determine absolute contribution to the link
4. Repeat previous step for all links, for all generators
5. Find total contribution to load at common k by generator i from (2.18)

$$T^i_k = \begin{cases} C^i_k \cdot P_{L_i}, & \text{if } k = i \\ \sum\limits_{j\in D} F^i_{jk} - \sum\limits_{j\in D} F^i_{kj} = \sum\limits_{j\in D} (F^i_{jk} - F^i_{kj}), & \text{otherwise} \end{cases} \tag{2.18}$$

A numerical example on a lossless 4-bus system is shown in Appendix A.

Loss Allocation

It is assumed that losses may be allocated proportionally to the line flow. Hence, a generator contributes to the losses in a line in the same proportion as the generator contributes to the line flow. The graph-based method does not give information on how to allocate line losses to loads.

2.4.3 Node Test-Based Method

In 2007, Abdelkader published a PFT approach using PSP which is claimed to be applicable even in the presence of circulating flows [16]. The procedure is described for downstream tracing, where a line flow matrix is formed used for identifying *node types*. The advantages are the following: No exhaustive search is required, generator shares are calculated by using only one matrix, no inversion is needed and no additional nodes are required for handling losses. This approach uses nodal generation distribution factors (NGDF) as first introduced in Ref. [17], while the Bialek-method uses topological generation distribution factors (TGDF). In this method, the NGDF is called the *participation factor*. When using NGDF, the transmission losses of the line are taken into account, while when using TGDF the line is assumed to be lossless and therefore the losses are not taken into account.

General Algorithm

All nodes are defined as either a sink, a source, a generation or a load node. A node i in a network containing N_L number of lines is decided from the conditions shown in Table 2.2

Table 2.2: Defining node types in Abdelkader-method

Node type	**Condition**		
Sink node	if	$f_{ij} \leq 0,$	$j = 1, ..., N_L$
Load node	if	$\sum_{j=1}^{N_L} f_{ij} \leq 0,$	$\exists j : f_{ij} > 0$
Generation node	if	$\sum_{j=1}^{N_L} f_{ij} > 0,$	$\exists j : f_{ij} < 0$
Source node	if	$f_{ij} \geq 0,$	$j = 1, ..., N_L$

f_{ij} is the power extracted from bus i by line j. All elements f_{ij} form a line flow matrix $\mathbf{F}$. Outflows are defined as positive and inflows are defined as negative. To calculate how much a generator is contributing to the power flow in a line, a participation factor A_{ij} is defined as:

$$A_{ij} = \frac{\text{power flow in line } j \text{ caused by generator at node } i}{\text{total power flow in line} j} \tag{2.19}$$

This participation factor is the same as the NGDF mentioned earlier.

Creating the A-matrix

For calculations, a participation factor matrix is created. The rows of the participation factor matrix **A** are dependent on which type of node the row represents. It is therefore of great importance that the node types are correctly determined.

- Source node: In **F**, the row of a source node will not contain any negative elements. For each nonzero element in this row, replace the corresponding element in **A** by 1.
- Sink node: No power is injected. All elements in the row corresponding to the sink node are zero.
- Generation node: Elements corresponding to a generation node in **A** are decided from (2.20).

$$A_{ij} = \begin{cases} \dfrac{P_{Gi}}{\sum\limits_{m \in \alpha_P} f_{im}}, & \text{if } f_{ij} > 0 \\ 0, & \text{if } f_{ij} = 0 \\ \dfrac{f_{ij}}{\sum\limits_{m \in \alpha_P} f_{im}}, & \text{if } f_{ij} < 0 \end{cases} \tag{2.20}$$

- Load node: Elements corresponding to a load node in **A** are decided from (2.21).

$$A_{ij} = \begin{cases} \alpha, & \text{if } f_{ij} > 0 \\ 0, & \text{if } f_{ij} = 0 \\ \dfrac{f_{ij}}{\sum\limits_{m \in \alpha_N} |f_{im}|}, & \text{if } f_{ij} < 0 \end{cases} \tag{2.21}$$

where P_{Gi} is the generation at node i,
α_P is the set of positive elements in row i,
α_N is the set of negative elements in row i,
α is a very small, positive number, set to 10^{-8}.

Tracing procedure to eliminate negative elements in A

1. Begin at a row in **A** where all elements are positive (in the starting point, this will be the row of a source node). This is row *i*.

2. Is there a non-zero element in row *i*? If yes, the column of this element is column *j*. If no, go back to step 1 and process a new row containing only positive elements.

3. Does column *j* contain a negative element? If yes, the row of this element is row *m*. If no, go back to step 2 and look at the next non-zero element in row *i*.

4. Does row *m* contain a positive element? If no, move on to the next step. If yes, the column of this element is column *n*. Update **A** by applying (2.22) to elements in column *n*, for all rows not equal to *m*:

$$A_{kn}^{\text{new}} = A_{kn}^{\text{old}} + A_{kj} \cdot |A_{mj}|, \quad \forall k, k \neq m \tag{2.22}$$

   Repeat previous step if there are any more positive elements in row *m*. If there are no more positive elements, element A_{mj} is set to zero.

5. Go to step 3 and finish processing negative elements of column *j*.

Follow these steps until all negative elements in **A** are eliminated. Then all elements in rows corresponding to load nodes are set to zero.

Calculation of contributions

The flow contributed by each generation in each branch can be calculated as

$$\mathbf{T} = \mathbf{A}\,\text{diag}(\mathbf{F}_j) \tag{2.23}$$

where diag($\mathbf{F}_j$) is a diagonal matrix with diagonal elements equal to the power at the sending end of line *j*. The power contributed by each generator to each bus is then calculated from the line contribution as

$$\mathbf{P} = \mathbf{A}\mathbf{F}^{\mathbf{t}} \tag{2.24}$$

Stepwise Approach

1. Determine line flow matrix **F**
2. Create **A**-matrix from (2.20) and (2.21)
3. Eliminate negative elements in **A** from the tracing procedure
4. Find generators' contributions to line flows from (2.23)
5. Find generators' contributions to loads from (2.24)

A numerical example on a lossless 4-bus system is shown in Appendix A.

Loss Allocation

The Abdelkader-method is applicable to a system with losses, and therefore it is not necessary to modify the original network before applying the algorithm. It is assumed that losses in lines may be allocated from the share of flow in a line. Hence, a generator contributes to the losses in a line in the same proportion as the generator contributes to the line flow.

Special Case: Circulating Flows

If the system has circulating flows, they can be detected by the proposed algorithm in the tracing procedure and a different approach is subsequently employed to handle these circulating flows. It should be noted that in Ref. [16] this is only shown for a simple 4-bus system with only one incoming and outgoing line for each node.

At some point, no positive row will be found in Step 1 of the tracing procedure, even though there are still negative elements in the **A**-matrix. To deal with this, the following steps are followed to create a new **A**-matrix:

1. Let x_i be the flow on the outgoing line caused by generation at node i
2. Determine the contribution of generator at node g to the flow of the lines in the system by using (2.25):

$$f_{ij}^{g} = \begin{cases} x_i, & \text{if } g = i \\ \dfrac{f_{ij}}{P_i} \sum_k f_{ki}^{g}, & \text{otherwise} \end{cases} \tag{2.25}$$

where f_{ij} is the flow at the sending end of the line $i-j$,

k is in the set of buses supplying i.

3. Apply KCL at node i to determine the value of x_i
4. Repeat steps 1-3 for all generation nodes
5. Create a new **A**-matrix from the definition of participation factor in (2.19)

2.5 Tracing in a Network with Losses

When tracing in lossless networks, the Kirschen-method and the Abdelkader-method give the same results as the Bialek-method when using upstream tracing. This becomes clear in the numerical example in Appendix A. The difference in the approaches become clear when tracing on networks with losses. In Ref. [4], Bialek is very clear that the algorithm only works on lossless systems, and gives different ways to obtain an equivalent lossless network and how the losses should be handled. The Kirschen-method gives a different tracing result because of the division into commons and links, and in Ref. [15] there is not given any explanation as to how the line losses should be handled in the algorithm. In the subsequent discussion of Ref. [18], Bialek responds to a comment that the Kirshen-method only works on lossless systems. However, in Ref. [15] the algorithm is used on a 30 bus system with losses, and when cross-checking the results it becomes clear that they are obtained without changing the network to an equivalent lossless system. A further explanation as to how the losses are handled in the Kirschen-method and how this affects the results in comparison with the Bialek-method would be beneficial. The Abdelkader-method works on networks with losses and handles them implicitly since it takes into account powers at the sending end and receiving end of a line. This is a clear advantage of this method, and is probably the reason why it provides slightly different results from the Bialek-method.

2.6 Tracing Reactive Power

Both Bialek and Kirschen *et al.* have claimed that it is possible to apply their methods to reactive power tracing, however there are some disagreements around this issue. In Ref. [4], Bialek stated that the linear equation-based method was equally applicable to reactive power, but that there was a significant issue with reactive flows: The reactive power loss of a line may be quite considerable when compared with the reactive flow itself. It is proposed to add fictitious nodes in the middle of each line to act as reactive sinks and sources to solve this problem.

In Ref. [15], Kirschen *et al.* stated that the graph-based approach was applicable to both active and reactive power. In the subsequent discussion of Ref. [15], the authors respond to comments on some of the difficulties with reactive power not yet accounted for in their method: Reactive power may flow into lines from both ends and out of lines to both ends, and reactive power depends heavily on active power flow.

In Ref. [16], Abdelkader stated that his method was applicable also for reactive power. It is also stated that more consideration had to be paid for the loss calculation so as to take into account the effects active and reactive powers have on each other and their impact on line losses. Also, as mentioned above, Abdelkader has published several papers on complex PFT - implicating that tracing active and reactive power separately is not a good enough solution.

So all three methods are said to be applicable to reactive power, but it also becomes clear that not all issues regarding tracing reactive power have been fully resolved. Tracing of reactive power is out of scope for this thesis, and is a recommended future work.

2.7 Modifications to Approaches

After Bialek and Kirschen *et al.* published their papers in 1997 and 1998 respectively, several papers have been published proposing modifications in the PFT procedures. Some are modifications to the algorithms proposed by Bialek and Kirschen *et al.*, some are different approaches based on PSP, and some papers compare the two algorithms. The Abdelkader-method is much more recent and has therefore not been shown as much interest in literature. This section provides a brief overview of some of the papers published on this matter.

2.7.1 Modifications to Linear-Equation Based Approach

Ref. [19] is based on the Bialek-method where additional fictitious nodes are added to the system to account for losses. To calculate reactive power, pi-equivalents for the lines are introduced. Next the line flow is decoupled, leading to a matrix decoupling. This requires less mathematical effort and decreases computational time. In summary, this method suggests an improvement of the Bialek-method, which is illustrated by a comparison between the two, where it is shown that the new method is computationally faster.

Ref. [20] represents the inverted upstream and downstream distribution matrices in the form of matrix power series, and it gives a mathematical proof of the invertibility of the tracing distribution matrix. By representing the matrices in a matrix power series, the paper also provides understanding as to why circulating flows can be detected in the inverted distribution matrix.

In Ref. [21] the Bialek-method is used to trace reactive power. The procedure is as follows: an optimization problem with the objective of minimising total reactive power support for generators is solved, before the reactive power support for generators is re-allocated according to their real power outputs. Tracing is performed to find the generator contributions to lines. It is assumed that the reactive power losses are all incurred by the transmission of real power. Further, two methods for allocating reactive power losses are proposed: a proportional allocation method and a quadratic allocation method.

2.7.2 Modifications to Graph-Based Approach

In 1999, Kirschen and Strbac published a new paper based on the Kirschen-method [22]. This paper picks up the thread from the discussion in Ref. [15] on reactive power. They state that it is reasonable to allocate line capacity solely based on active power, however it is not reasonable to neglect active power when dealing with reactive power flows, since the reactive power flow depend so heavily on active power flow. If a system is heavily loaded, the generators need to produce reactive power to supply the reactive losses caused by the active power flow in the transmission system. The idea in this paper is to calculate the active and reactive parts of the current independently, before translating it back to active and reactive power.

2.7.3 Modifications to Node Test-Based Approach

Abdelkader has published several papers using the node test-based method. In Ref. [23] it is shown how to calculate the load contributions to line flows and losses (as opposed to the generator contributions). Further, Refs. [24], [25] and [26] introduce how to use the node test-based method in complex PFT, allocating losses and also discussing circulating flows. In Ref. [27], published in 2011, Abdelkader presents a new way of handling losses: Dividing the losses into 1) load loss caused by current flow from generators to loads, 2) circulating current loss and 3) network loss.

2.7.4 Approaches based on PSP

In Ref. [17] it is remarked that the two traditional methods proposed by Bialek and Kirschen *et al.* work best for tracing active power, and not so well when tracing both active and reactive power. Because of this, the authors instead introduce the concept of NGDF, determining the share of a particular generator in every line flow. This factor is the same as the factor used in the Abdelkader-method. The calculation of this factor is done based on the proportional sharing principle, but it does not use system matrices. Transmission losses are taken into account. Further, reactive power and active power distribution factors are calculated independently. The method is claimed to work better than the basic methods when tracing both active and reactive power.

In 2000, Wu *et al.* published an article using graph theory to trace the power flow, based on PSP and two lemmas [28]. The method is theoretically efficient [29], and claims to be usable for both reactive and active power transfer allocations. However, it does not work when circulating flows exist in the system.

2.8 Applications of PFT in Literature

In Ref. [4], Bialek suggests several applications for the algorithm. Besides giving insight into how power flows in the network, the applications are mostly related to cost or loss allocation: Apportioning transmission losses, setting tariffs for transmission services (instead of using marginal costs) and using the output of PFT as a tool for reactive power pricing. In Ref. [15], Kirschen *et al.* suggest that the method can be used for geographically-differentiated spot pricing, pricing of transmission services and apportionment of losses. In Ref. [16], Abdelkader says a good application for his method would be an online application to give real time price signals for both power producers and consumers. The subsections of this section present some selected applications where PFT has been used for different purposes, to give the reader a feeling of the variety of applications in which PFT may be useful.

2.8.1 Loss Allocation

Loss allocation has already been mentioned briefly in Section 2.4, when introducing the different PFT approaches. In general, there are several ways proposed to allocate losses in a network (not necessarily using PFT). Ref. [30] gives an overview of different transmission loss allocation algorithms, focusing on *pro rata* procedures, marginal procedures and proportional sharing procedures. *Pro rata* procedures are simple to understand, however they do not take the network into account, in the way that a load located far from a generator is treated in the same way as a load located close to the generator. The standard marginal procedures depend on the choice of slack bus and may end up allocating negative losses. As given in the name, the proportional sharing loss allocation procedures are based on PSP. It is concluded that proportional sharing is recommended as a transmission loss allocation procedure if volatility, negative losses and allocation imbalance are not desired. An illustration on how loss allocation may be performed will be illustrated in Chapter 4.2.

2.8.2 PFT in a European Electricity Network

Ref. [31] applies PFT to a simplified model of a renewable European electricity network. The objective is to assess the grid usage when there is a high share of renewable energy sources in the system. PFT is done based on a method introduced in Ref. [32], which is mainly based on Refs. [15] and [28]. The main idea is to assign the power production to different *entities*. These entities can be decided by power producer, power generation type, node of origin or other criteria [33]. Next, the line usage is determined for each entity, making it possible to decide each entity's share.

Ref. [33] is based on the same approach, but for the German electricity network.

2.8.3 Locational Load Service Reliability

Ref. [34] proposes a novel set of locational load service probability (LSP) indices using the graph-based method. First the "allocate algorithm" defines the domains and commons of a system. Further, the "isolate algorithm" finds the contributions of generators to loads to represent each bus as an isolated bus. Then, the "convolve algorithm" is used to calculate the cumulative outage probability of the generators supplying a load. In this way the customers may be fairly priced based on the LSP they receive from the system operator.

2.8.4 Intentional Controlled Islanding

Ref. [35] presents a novel method to reduce the impact of wide area blackouts in transmission networks. The effects of blackouts can be significantly reduced by splitting the system into smaller islands. Intentional Controlled Islanding (ICI) contains the faults to smaller regions and stop them from cascading further. The method uses the linear equation-based PFT technique to find a boundary around a disturbance, which forms an island that will be disconnected to keep the rest of the system intact.

2.8.5 Under Frequency Load Shedding

Ref. [36] uses dynamic PFT in under frequency load shedding (UFLS). UFLS is an approach to maintain frequency stability to prevent the system from a frequency collapse. By using dynamic PFT, the dynamic changes of load power during faults are taken into account. The approach combines the "kinetic energy theorem of power system" with PFT to obtain the frequency influencing factors of every generator. Further, the contribution to frequency deviation is divided between 1) the mechanical power of the generators, 2) the load power, and 3) the transmission losses. Based on this analysis, a distributed load shedding strategy is proposed.

2.8.6 Cross Border Tracing

Refs. [37] and [38] present similar ways of allocating cross-border flows to the agents causing them, so that the transmission operators may use this information to decide the costs of transmission losses correctly. The idea is to represent each country as one area, with a net import and export, called a supernode. If the goal is to only allocate the flows between certain areas, the remaining areas connected are modelled as a net import/export to the areas considered. The resulting loss charges allocated to exporting and importing areas using the linear equation-based method are different than the ones obtained by using bilateral flows, and considered more fair. It is also concluded that the supernode approach gives a better consideration of the impact a country has on the whole interconnected network.

2.8.7 Probabilistic Pricing Methodology

Ref. [39] proposes a new pricing methodology using PFT. It uses three simultaneous studies: deterministic power flow studies, probabilistic load flow studies and PFT. A reasonable quantification of the intrinsic reliability offered by an existing transmission network structure for a given set of power transactions is provided.

2.8.8 System Splitting Boundary

Ref. [40] proposes a way to split a power system into boundaries when a fault occurs. System islanding may happen automatically, but the islands may end up unbalanced electrically and controlled islanding is therefore preferred to keep stability and avoid large-scale blackouts. The question is how to decide the boundaries of the islands. A method of real-time search for a splitting boundary is proposed, divided into three phases: 1) define the domain of a generator and perform PFT using the graph based-method, 2) determine an initial splitting boundary based on the generator grouping and 3) execute a refinement of the initial splitting boundary.

Chapter 3

Implementation of Algorithms in MATLAB

This chapter gives an overview of how three different PFT algorithms have been implemented in MATLAB, shedding light on additional assumptions which are not necessarily clear from the mathematical introduction to the algorithms.

The three different algorithms - linear equation-based, graph-based and node test-based, have been implemented in MATLAB. The code implemented is a generic code able to trace the active power flow in any network as long as generation, load and line flow of the network are provided as inputs. If a power flow solution is known, tracing can be performed directly. Otherwise, a MATPOWER case is constructed as described below, a power flow is run and then tracing can be performed. The chapter is built in the following way: Section 3.1 describes the construction of cases in MATPOWER. Sections 3.2, 3.3 and 3.4 describe how the different algorithms have been implemented in MATLAB. Section 3.5 mentions the main assumptions made for the implementations in MATLAB.

3.1 MATPOWER 6.0

As mentioned, PFT is an approach performed on a solved power flow or state estimation computation. There exist many types of software able to provide a power flow for a network, such as PSS/E, PowerWorld or PSAT. The implementation of the approaches described in this report has been performed in MATLAB, and power flows have been run in MATPOWER 6.0 as this is a free toolbox of MATLAB and no conversion between programs is needed. For more information on how the loadflow is run in MATPOWER, the reader is encouraged to see the MATPOWER User's Manual [41].

The default solver in MATPOWER for an AC power flow problem is based on a standard Newton Raphson's method, and this is also the solver used for these results. The modelling of the network is done by specifying the bus, generator, line and branch data as described in the MATPOWER User's Manual [41]. The network is modelled in the following way:

- Lines, transformers and phase shifters are all modelled with the same branch model: a pi-equivalent with a series impedance and a total charging susceptance in series with an ideal phase shifting transformer.
- Generators are modelled as complex power injection at buses.
- Loads are modelled as constant power loads of real and reactive power consumed at buses.
- Shunt connected elements (such as an inductor or capacitor) are modelled as fixed impedances to ground at buses.

3.2 Linear Equation-Based Method in MATLAB

The Bialek-method using upstream tracing with gross flows is implemented as a function where the input is a solved power flow: Generation at nodes, load at nodes, line flow between nodes and losses in lines. The function prints the tracing results to an Excel-file, showing generators' contributions to loads and lines. If a node in a network has both generation and load, it is assumed that the load is handled locally so that a node only has either generation or load. For example, if bus i generates 130 MW and has a load of 20 MW, the generation at node 2 becomes 110 MW, and the load 0 MW. This is a consequence of the definition of gross nodal throughflow in (2.1). Figure 3.1 shows a flowchart of the upstream tracing procedure while Figure 3.2 shows a flowchart of the downstream tracing procedure. For a mathematical explanation to how each step is executed, the reader is encouraged to see the mathematics in Section 2.4.

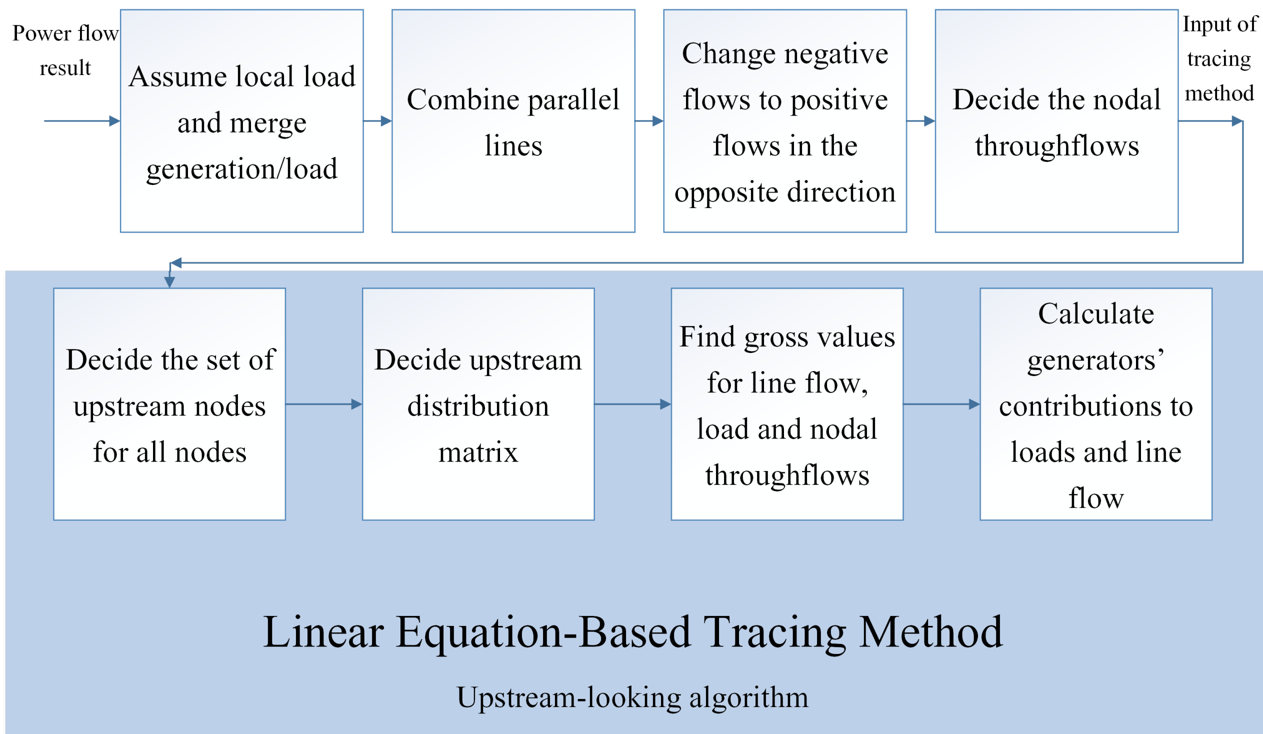

Figure 3.1: Flowchart for the Bialek-method, upstream tracing

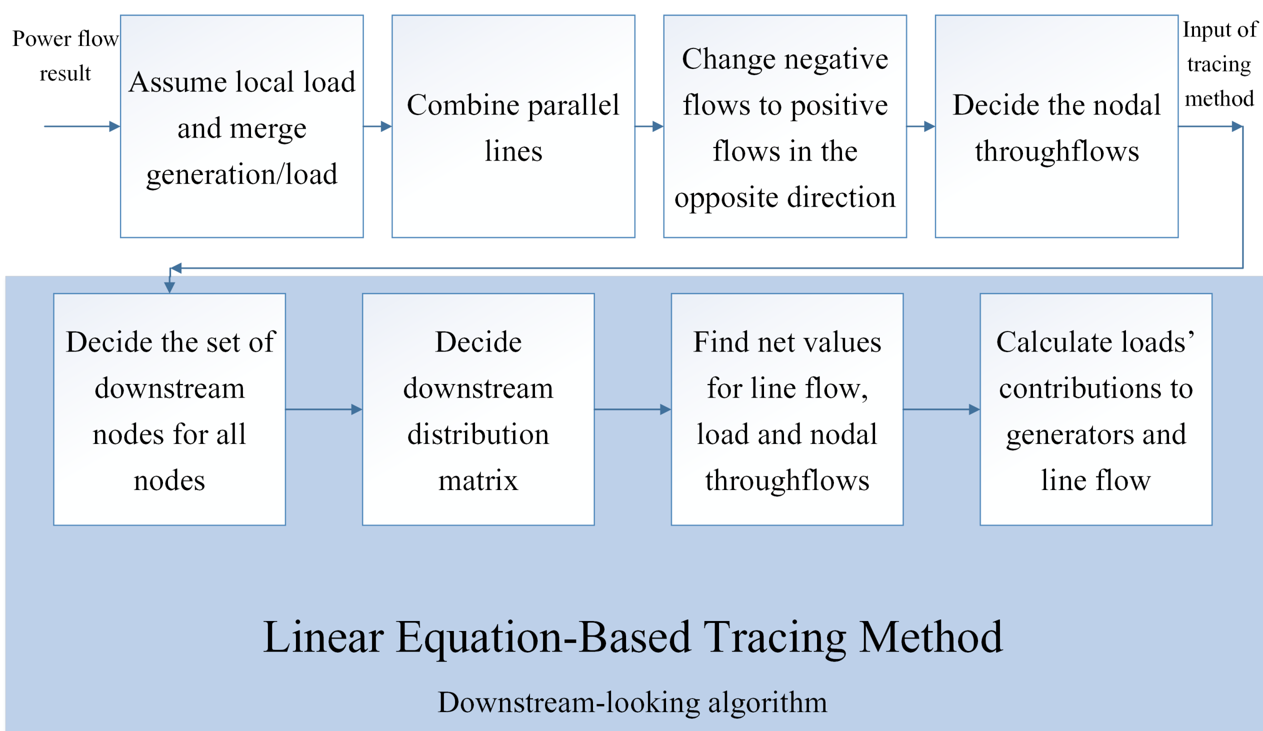

Figure 3.2: Flowchart for the Bialek-method, downstream tracing

Note: When upstream tracing is used, the sum of traced generation should be equal to the actual generation in the system. When downstream tracing is used, the sum of traced load should be equal to the actual load in the system.

3.3 Graph-Based Method in MATLAB

The Kirschen-method is implemented as a function where the input is a solved power flow: Generation at nodes, load at nodes, line flow between nodes and losses in lines. The function converts the network to a state graph consisting of commons and links. Tracing is performed and the results are printed to an Excel-file, showing the commons' relative contributions to loads in commons and commons' absolute contribution to links, along with an overview of the nodes and lines included in commons and links.

Figure 3.3 shows a flowchart of the tracing procedure. For a mathematical explanation to how each step is performed, the reader is encouraged to see Section 2.4. The recursive method of finding relative contributions is shown in Figure 3.3.

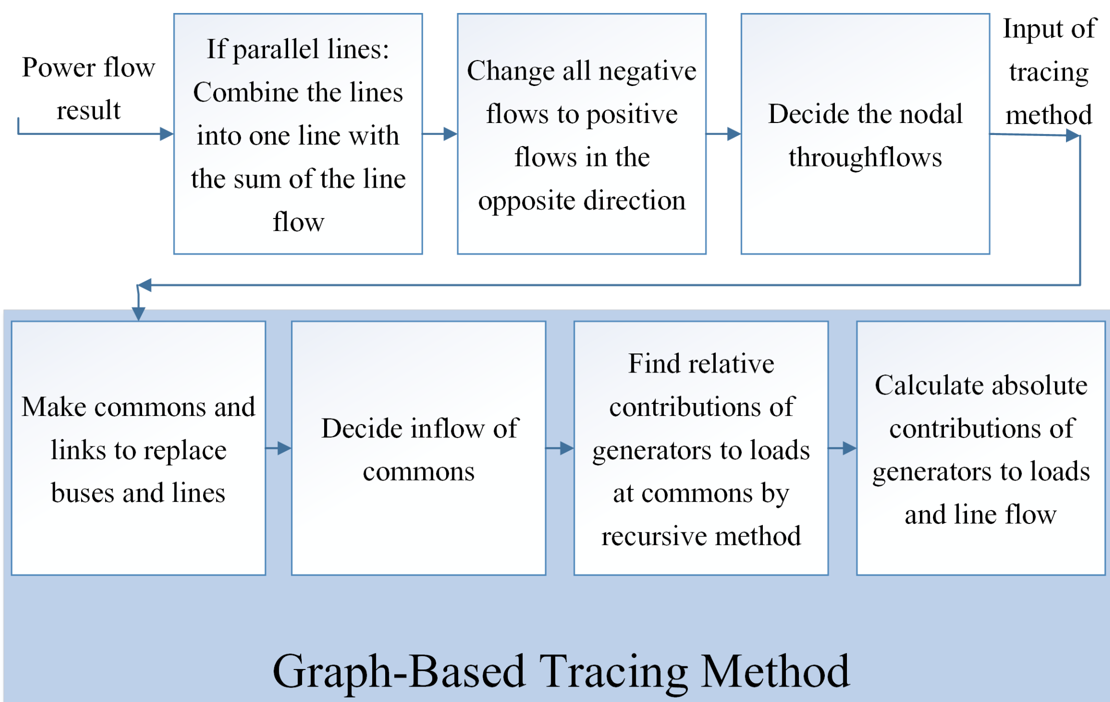

Figure 3.3: Flowchart for the Kirschen-method

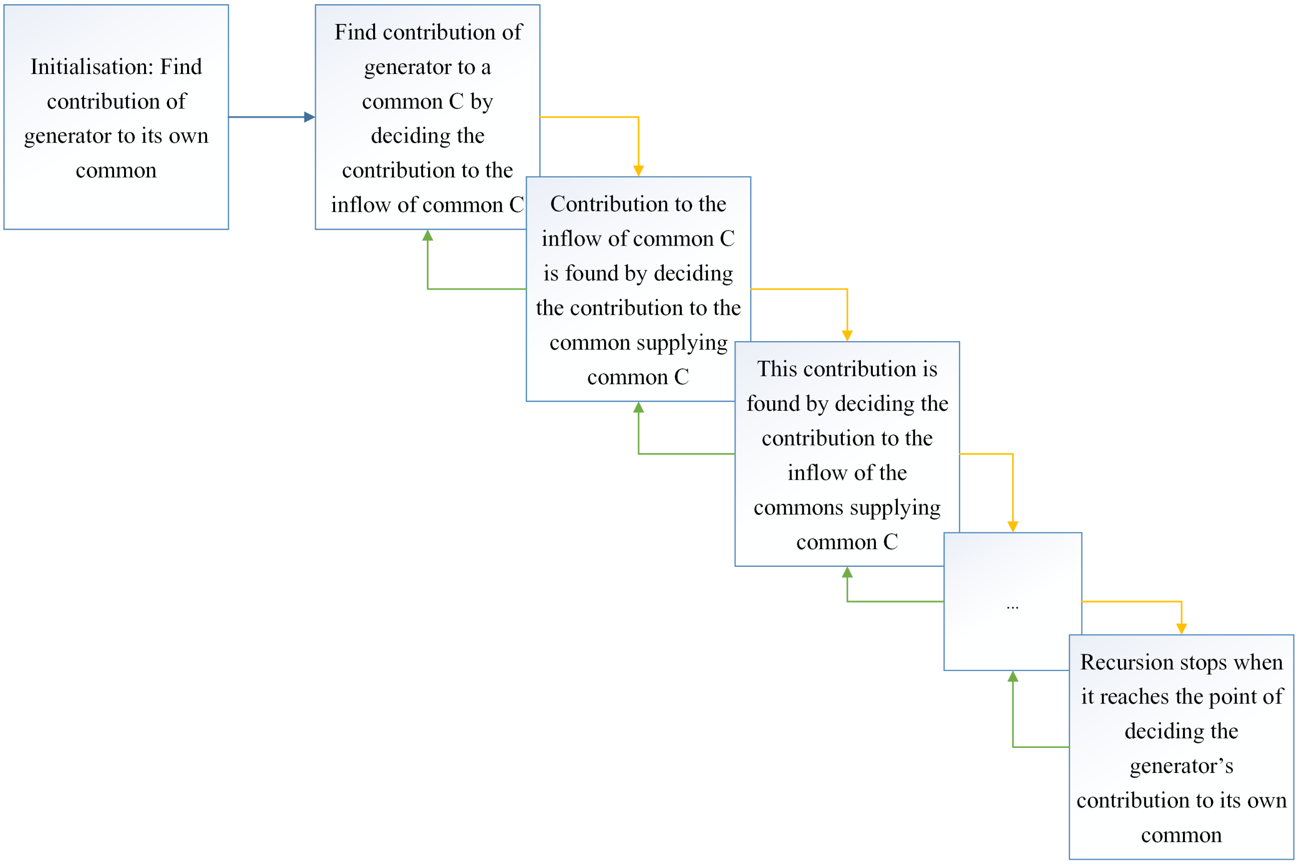

Figure 3.4: Recursion in the Kirschen-method

Note: The output is for commons and links, and not for single generators and loads. To find results for individual nodes and lines, PSP needs to be applied. The sum of traced generation should be equal to the actual generation in the system.

3.4 Node Test-Based Method in MATLAB

The Abdelkader-method is implemented as a function where the input is a solved power flow: Generation at nodes, load at nodes, line flow between nodes and line losses. The function prints the tracing results to an Excel-file, showing generators' contributions to loads and lines. If a node in a network has both generation and load, it is assumed that the load is handled locally so that a node only has either generation or load. Figure 3.5 shows a flowchart of the tracing procedure. For a mathematical explanation to how each step is performed, the reader is encouraged to see

Section 2.4.

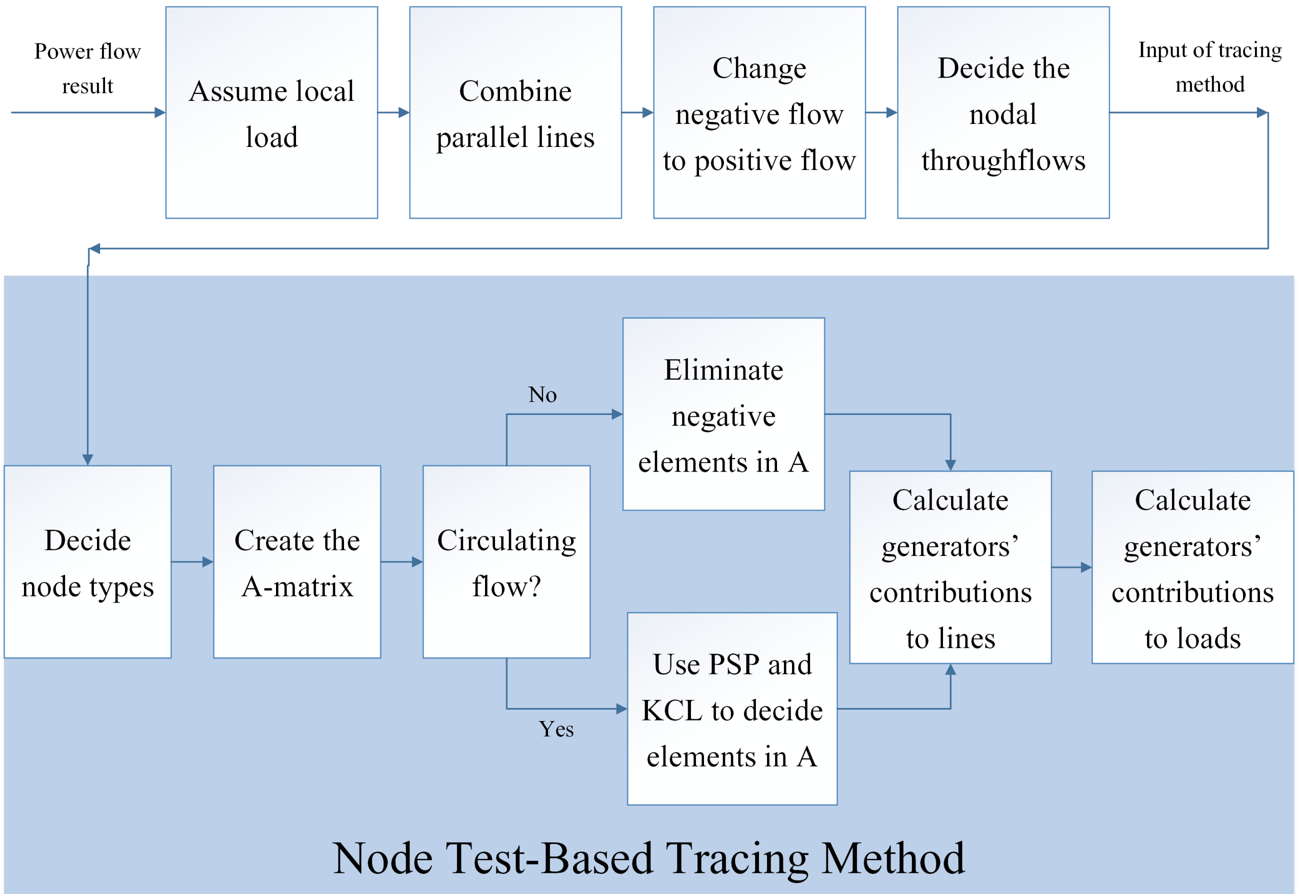

Figure 3.5: Flowchart for the Abdelkader-method

Note: The sum of traced load should be equal to the actual load in the system. The approach of finding the **A**-matrix is dependent on whether a circulating flow is present or not.

3.5 Overview of Assumptions

- Merging generation and load: If a bus has several generators or loads, these are merged to one total generation and one total load. To find the contribution of an individual generator, it is assumed that the generator contributes in proportion to its share in the total generation.
- Parallel lines: If parallel lines exist in the network, they are merged to one line. It is then assumed that the contribution of a generator to one of the parallel lines is proportional to

its share in the merged line. This is assumed in all the three algorithms employed in this thesis.

- Circulating flows: Circulating flows may occur in systems with FACTS-devices or tap-changing transformers, as mentioned in Section 2.3. The Kirschen-method is not able to handle networks containing circulating flows, since the method ends up in an infinite recursion.
- Local load: Local load means that if there is both generation and load at a bus, the generator is responsible for delivering power to the load. This results in all buses in a network having either generation or load, not both at the same time. This is assumed in the Bialek-method and the Abelkader-method.

Table 3.1: Comparison of the tracing methods

	Tracing Method		
	Linear equation-based	Graph-based	Node test-based
Merges generation and load?	Yes	Yes	Yes
Merges parallel lines?	Yes	Yes	Yes
Handles circulating flow?	Yes	No	In some cases
Assumes local load?	Yes	No	Yes

Chapter 4

Case Studies

Tracing results are shown for the three algorithms when tracing on the 6 bus RBTS and the 24 bus IEEE RTS. Further, the algorithms are implemented on two systems containing circulating flows. The following applications for tracing are demonstrated: loss allocation, load shedding and CO_2-emission apportioning. Loss allocation is performed for all algorithms, while the two latter applications are demonstrated using only the linear-equation based approach. Discussion on the tracing results and differences between the algorithms are presented.

4.1 Cases

4.1.1 6 bus RBTS

A Matpower case is constructed from the data of the 6 bus RBTS [42], and a power flow simulation is run, giving the generation, load and line flow as shown in Figure 4.1.1. As mentioned in Chapter 3 it is assumed for all approaches that parallel lines may be merged, resulting in a merger of the two lines between bus 1 and 3 as well as the two lines between bus 2 and 4. This gives the line flow as shown in Table 4.1.

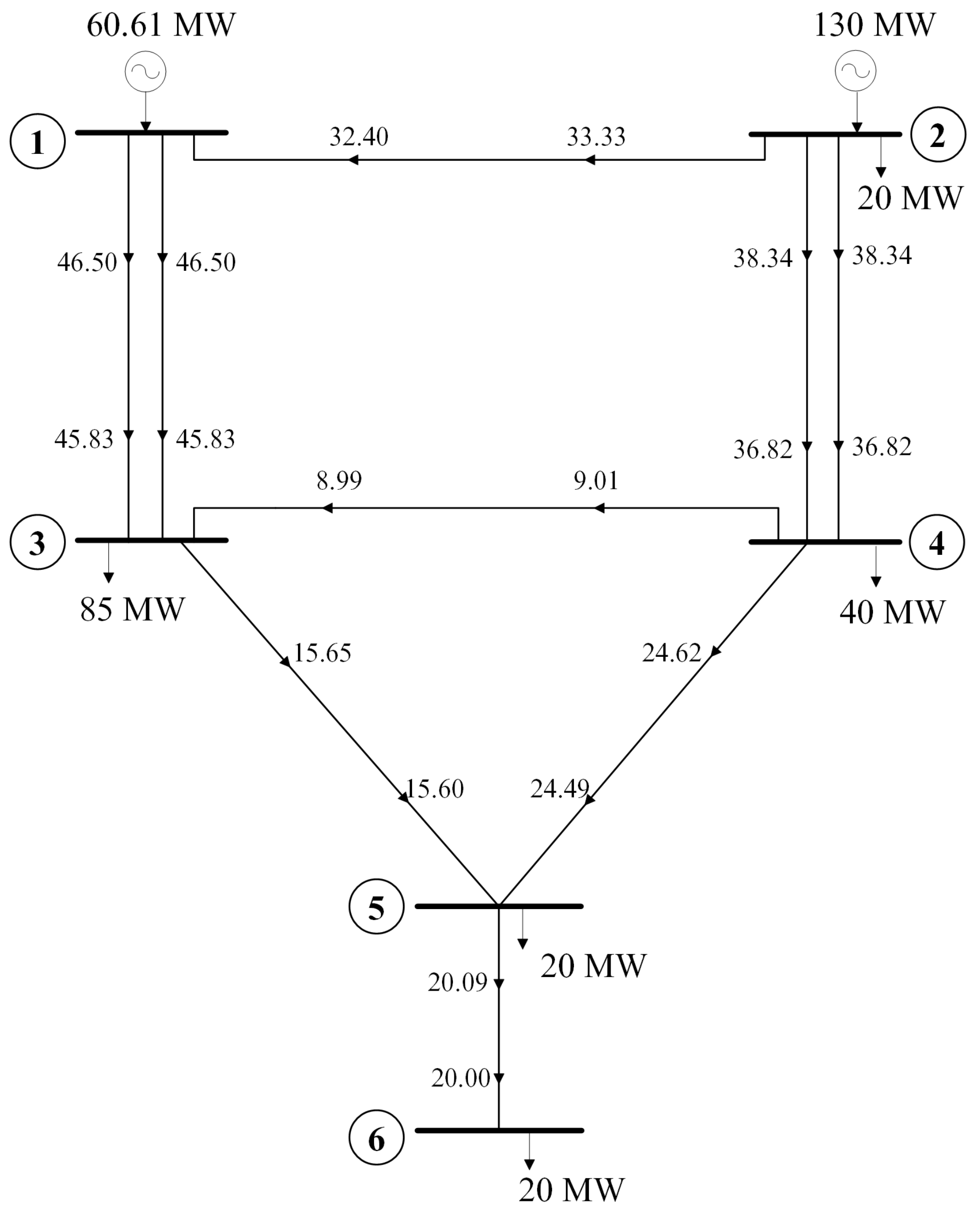

Figure 4.1: 6 bus RBTS [42] power flow result

Table 4.1: 6 bus RBTS, power flow result

Bus	1	2	3	4	5	6
Generation [MW]	60.61	130	0	0	0	0
Load [MW]	0	20	85	40	20	20

Line		**Line Flow [MW]**		
From bus	To bus	Sending end	Receiving end	Loss
2	1	33.33	32.40	0.93
1	3	93.01	91.66	1.35
2	4	76.67	73.63	3.04
4	3	9.01	8.99	0.02
3	5	15.65	15.60	0.05
4	5	24.62	24.49	0.13
5	6	20.09	20.00	0.09

Linear Equation-Based Method

Tracing the generators' contributions to loads using the Bialek-method with upstream tracing gives the result in Table 4.2. Note that bus 2 generates 130 MW and has a load of 20 MW. Since local load is assumed, the generation at node 2 becomes 110 MW, and the load 0 MW.

Table 4.2: 6 bus RBTS, Bialek-method, tracing result

	Load [MW]			
Generator	3	4	5	6
1	51.19	-	4.70	4.72
2	36.07	41.65	16.10	16.17

Line		**Generator** [MW]			**Accumulated line loss** [MW]
From bus	To bus	1	2	Sum	
2	1	-	33.33	33.33	0
1	3	60.61	33.33	93.94	0.93
2	4	-	76.67	76.67	0
4	3	-	9.38	9.38	0.37
3	5	9.43	6.64	16.07	0.42
4	5	-	25.64	25.64	1.02
5	6	4.72	16.17	20.89	0.81

Since upstream tracing is applied, the sum of traced generation is equal to actual generation. It is also clear that the difference between the sum of traced load and he actual load yields total loss in the system.

The sum of traced line flows will be either equal to or greater than the value of the sending end power of the line. This is because the loss accumulates when using gross flows as mentioned in Section 2.4.1. Column "Accumulated line loss" shows this unknown accumulated line loss as calculated from 2.15, which is the difference between the sum of traced line flow and the actual line flow.

Graph-Based Method

Figure 4.2 shows how the 6 bus RBTS is divided into commons and links when the Kirschen-method is applied. Figure 4.3 shows the resulting state graph.

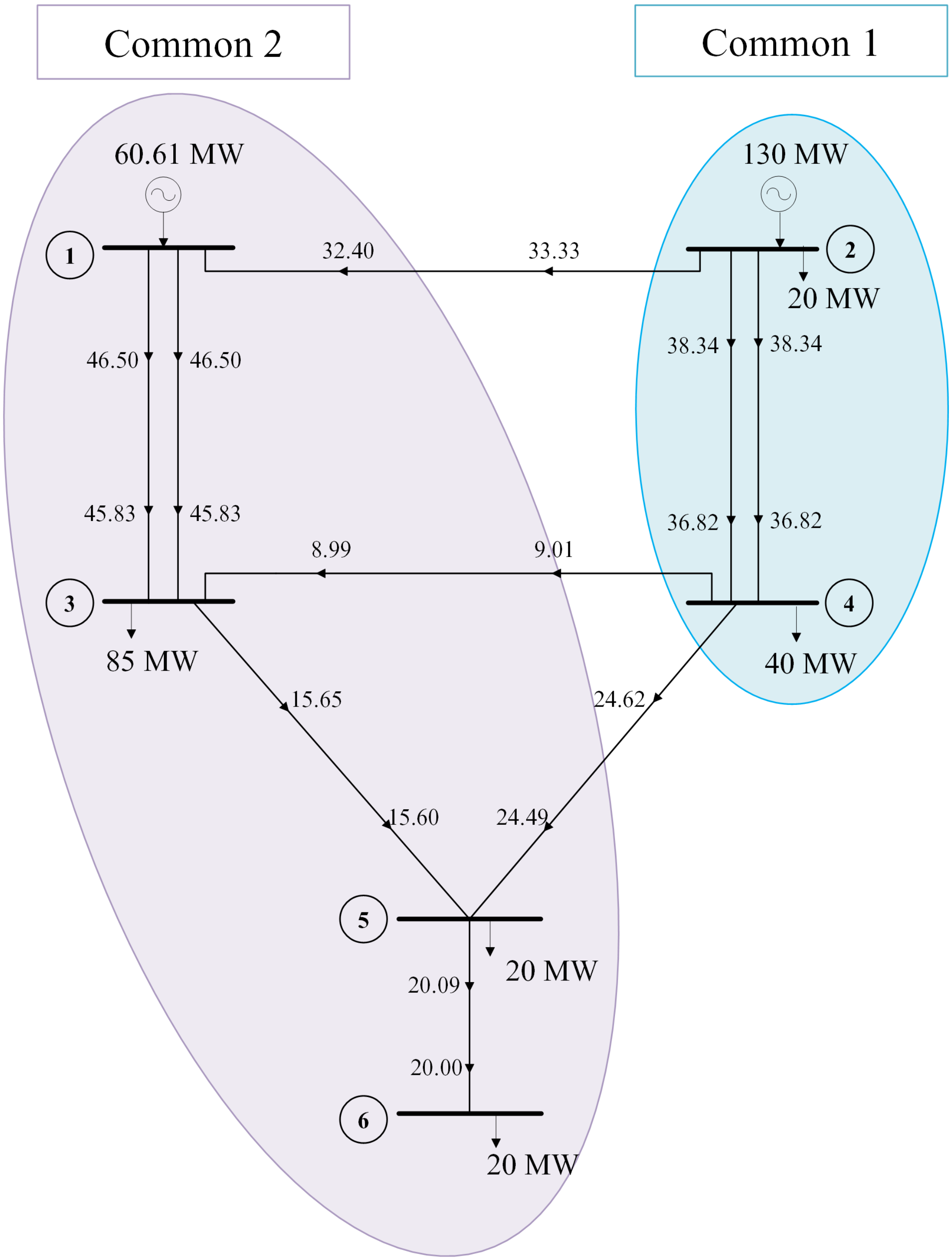

Figure 4.2: 6 bus RBTS with commons

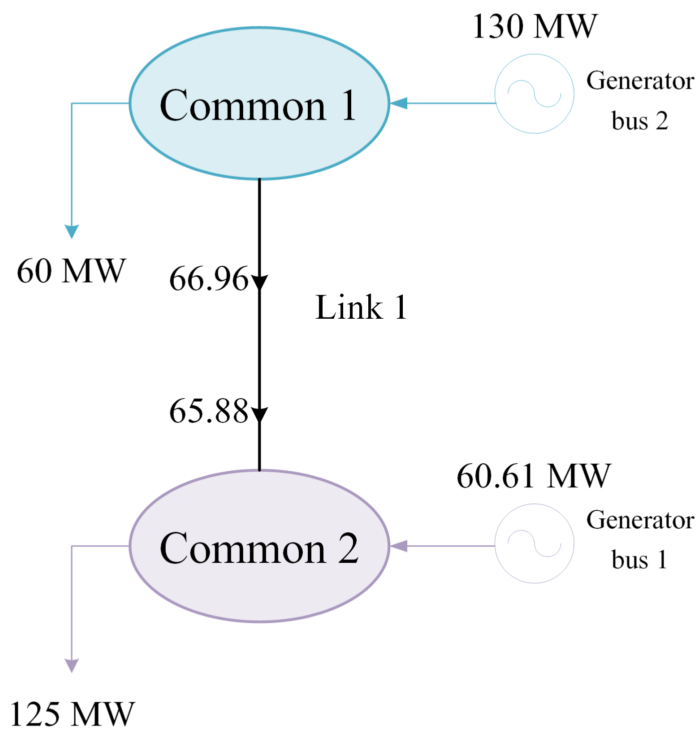

Figure 4.3: Commons and links for 6 bus RBTS

As mentioned in Section 2.4, the rank of a common is decided from the amount of generators contributing to that common. Because of this, generator 2 is in common 1 and generator 1 in common 2. The buses included in each common are shown in Table 4.3, along with total generation and load.

Table 4.3: Commons for 6 bus RBTS

Common #	Generator	Buses in common	Generation [MW]	Load [MW]
1	2	2, 4	130	60
2	1, 2	1, 3, 5, 6	60.61	125

There is only one link since there are two commons. As shown in Figure 4.2 this link consists of the lines 2-1, 4-3 and 4-5. Generator 2 in common 1 is the only generator contributing to this link, and its relative contribution is therefore 1. This gives an absolute contribution to the link of 66.96 MW. The tracing results for loads (relative contributions of generators in commons to loads in commons) are shown in Table 4.4.

Table 4.4: 6 bus RBTS, Kirschen-method, tracing result

	Load common	
Gen. common	1	2
1	1	0.521
2	-	0.479

The contribution to each load in a common is obtained by assuming that each contribution is divided among the loads in equal proportion to the magnitude of the loads. For instance: The load at bus 3 is 85 MW, and bus 3 is in common 2. This means that common 1 (generator 2) contributes to the load at bus 3 with $0.521 \cdot 85 = 44.27$ MW. The same logic is applied for lines: Line 3-5 is carrying 15.65 MW, and is a part of common 2. Hence, generator 2 at common 1 contributes to this line with $0.521 \cdot 15.65 = 8.16$ MW, and generator 1 in common 2 contributes with $0.479 \cdot 15.65 = 7.50$ MW. The results for all loads and lines are shown in Tables 4.5 and 4.6.

Table 4.5: 6 bus RBTS, Kirschen-method, tracing result (lines)

Line		**Generator bus [MW]**		
From bus	To bus	1	2	Sum
2	1	-	33.33	33.33
1	3	44.57	48.44	93.01
2	4	-	76.67	76.67
4	3	-	9.01	9.01
3	5	7.50	8.15	15.65
4	5	-	24.62	24.62
5	6	9.63	10.46	20.09

Table 4.6: 6 bus RBTS, Kirschen-method, tracing result (buses)

	Load bus [MW]				
Generator bus	2	3	4	5	6
1	-	40.73	-	9.58	9.58
2	20	44.27	40	10.42	10.42

Node Test-Based Method

The tracing result from the Abdelkader-method is shown in Table 4.7. Note that bus 2 originally has both generation and load. Since local load is assumed, the generation at node 2 becomes 110 MW, and the load is 0 MW.

Table 4.7: 6 bus RBTS, Abdelkader-method, tracing results

	Load [MW]			
Generator	3	4	5	6
1	50.44	-	4.62	4.62
2	34.56	40	15.38	15.38

Line		**Generator [MW]**		
From	To	1	2	Sum
2	1	-	33.33	33.33
1	3	60.61	32.40	93.01
2	4	-	76.67	76.67
4	3	-	9.01	9.01
3	5	9.29	6.36	15.65
4	5	-	24.62	24.62
5	6	4.64	15.45	20.09

The sum of traced load and actual load are the same since downstream tracing is applied. The difference between sum of traced generation and actual generation is 5.61 MW, equal to the total loss in the system. The sum of traced line flows is equal to the actual line flows, since losses are handled directly in the tracing algorithm.

4.1.2 24 bus IEEE RTS

Matpower 6.0 has several already constructed cases available for free downloading, among them a case constructed for the data in the 24 bus IEEE RTS: "case24_ieee_rts". The data for the system can be found in Ref. [43]. A power flow has been run on this case as a starting point for tracing. The result of the power flow is given in Table 4.8. Note that parallel lines have been merged, and if one bus has several generators, they are merged to one. Figure 4.4 shows the topology of the original 24 bus IEEE RTS.

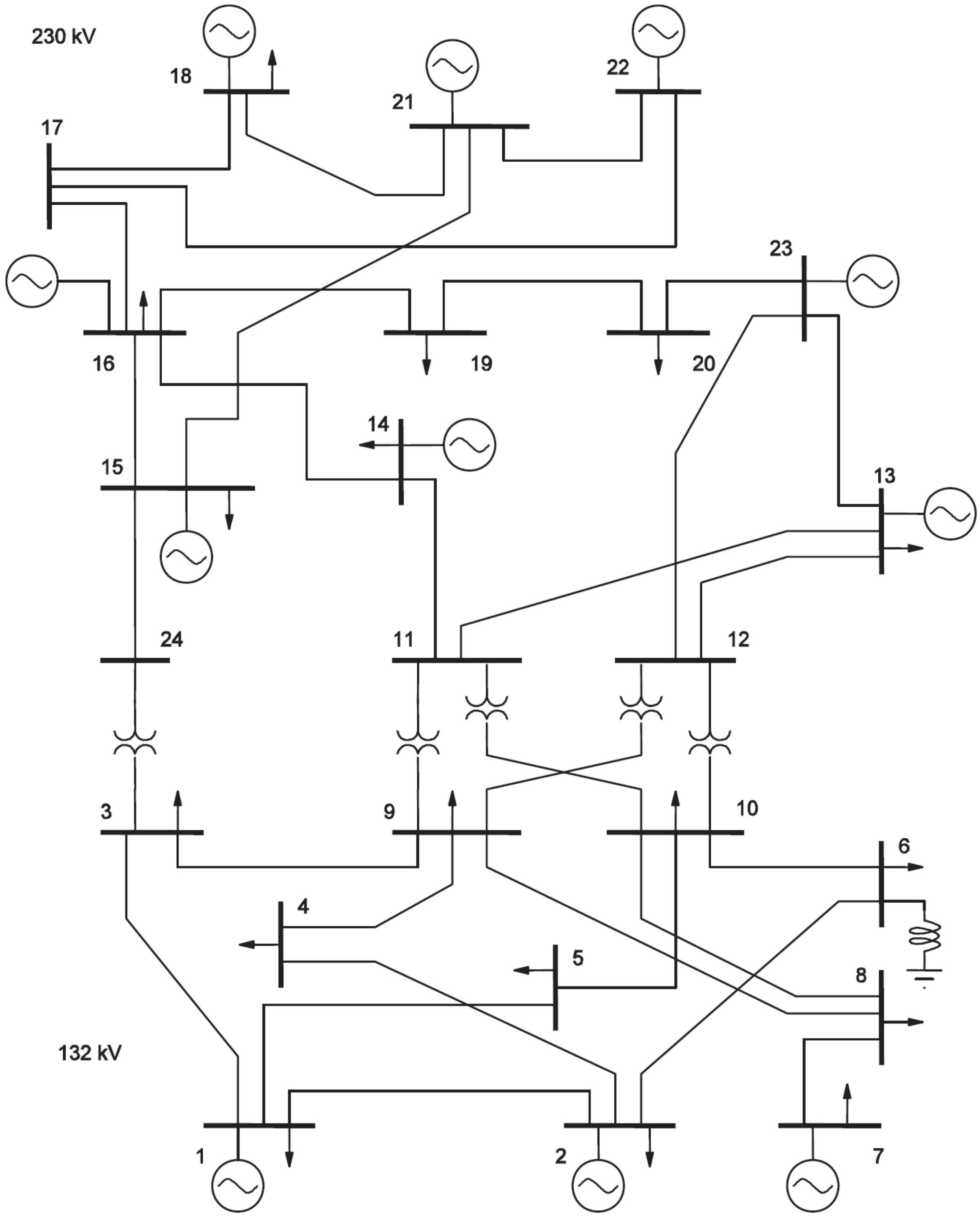

Figure 4.4: 24 bus IEEE RTS [44]

Table 4.8: 24 bus IEEE RTS, power flow result

	Line		**Line Flow [MW]**					
Line #	From	To	Sending	Receiving	Loss	**Bus**	**Generation [MW]**	**Load [MW]**
1	1	2	11.94	11.94	0.00	1	172	108
2	3	1	8.31	7.97	0.34	2	172	97
3	1	5	60.03	59.29	0.74	3	0	180
4	2	4	38.44	37.85	0.59	4	0	74
5	2	6	48.50	47.41	1.09	5	0	71
6	3	9	22.90	22.66	0.24	6	0	136
7	24	3	212.32	211.21	1.11	7	240	125
8	9	4	36.52	36.15	0.36	8	0	171
9	10	5	11.76	11.71	0.05	9	0	175
10	10	6	89.66	88.59	1.07	10	0	195
11	7	8	115.00	112.88	2.12	11	0	0
12	9	8	37.53	36.92	0.60	12	0	0
13	10	8	21.50	21.19	0.30	13	187.25	265
14	11	9	106.20	105.92	0.28	14	0	194
15	12	9	120.84	120.47	0.37	15	215	317
16	11	10	151.72	151.18	0.55	16	155	100
17	12	10	167.38	166.74	0.64	17	0	0
18	13	11	86.76	86.15	0.62	18	400	333
19	14	11	173.55	171.77	1.78	19	0	181
20	13	12	60.79	60.51	0.27	20	0	128
21	23	12	234.10	227.70	6.40	21	400	0
22	23	13	230.74	225.30	5.44	22	300	0
23	16	14	374.60	367.55	7.05	23	660	0
24	15	16	112.30	112.01	0.29	24	0	0
25	21	15	435.66	429.84	5.83			
26	15	24	215.54	212.32	3.22			
27	17	16	326.03	322.68	3.35			
28	16	19	115.08	114.65	0.43			
29	18	17	187.58	186.94	0.64			
30	22	17	141.54	139.09	2.45			
31	21	18	120.80	120.58	0.22			
32	20	19	66.58	66.35	0.23			
33	23	20	195.16	194.58	0.58			
34	22	21	158.46	156.46	1.99			

Linear Equation-Based Method

Local load is assumed, meaning that for buses 1, 2, 7, 13, 15, 16 and 18 it is assumed that the generators at the buses handle the load at the respective buses. Generators' contributions to loads using upstream tracing is shown in Table 4.9. For the tracing results for lines, see Appendix B.

Table 4.9: 24 bus IEEE RTS, generator contribution to loads, Bialek-method

	Load [MW]											
Gen.	3	4	5	6	8	9	10	13	14	15	19	20
1	-	4.69	53.38	5.92	-	-	-	-	-	-	-	-
2	-	33.16	-	41.84	-	-	-	-	-	-	-	-
7	-	-	-	-	115.00	-	-	-	-	-	-	-
16	-	1.20	0.43	3.30	2.02	5.75	7.17	-	22.21	-	12.93	-
18	-	1.46	0.53	4.02	2.46	7.00	8.73	-	27.05	-	15.75	-
21	133.83	6.63	6.48	10.68	8.77	29.59	21.98	-	68.10	74.31	39.64	-
22	53.02	5.71	3.68	12.71	8.68	26.51	27.16	-	84.13	29.44	48.97	-
23	-	23.58	8.30	63.28	39.41	113.02	137.63	79.63	-	-	66.78	128.38
Sum	186.85	76.43	72.80	141.75	176.34	181.87	202.66	79.63	201.49	103.75	184.05	128.38

As for the 6 bus RBTS, the differences between the sum of traced load and actual load sum up to the total losses in the system.

Graph-Based Method

First the system is made into a state graph containing commons and links. An overview of buses in commons, which generators are supplying the commons, and total generation and load is shown in Table 4.10. Figure 4.5 shows the commons of the system. The links are shown in Table 4.11 along with information on the commons they interconnect; the total sending and receiving flows on the link and the lines are included in the link.

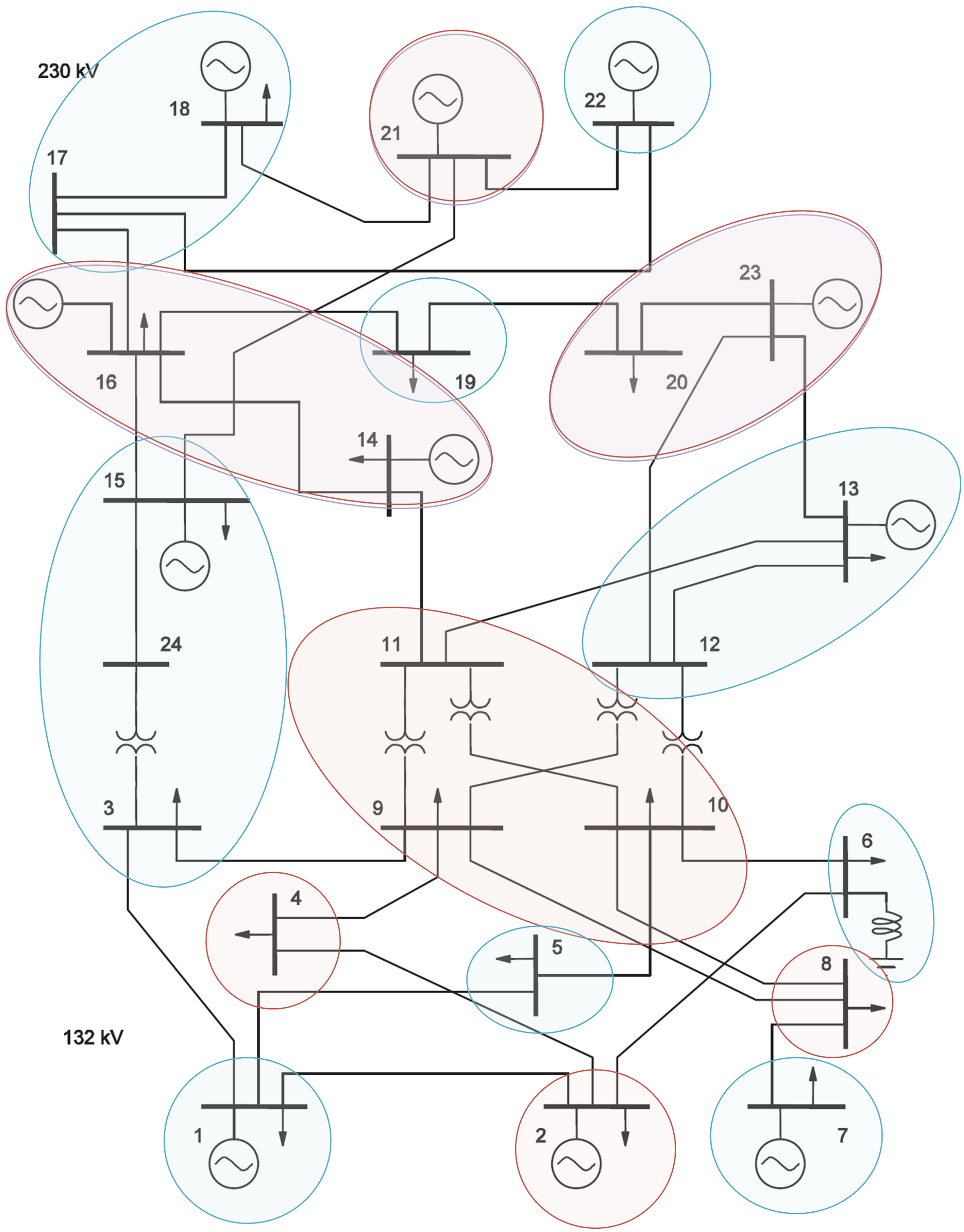

Figure 4.5: 24 bus IEEE RTS with commons [44]

Table 4.10: Commons for the 24 bus IEEE RTS

Common #	Generators supplying common	Buses in common	Generation [MW]	Load [MW]
1	23	20 , 23	660	128
2	22	22	300	0
3	7	7	240	125
4	21 , 22	21	400	0
5	13 , 23	12, 13	187.25	265
6	18 , 21 , 22	17, 18	400	333
7	15 , 21 , 22	3, 15, 24	215	497
8	1 , 15 , 21 , 22	1	172	108
9	15 , 16 , 18 , 21 , 22	14 ,16	155	294
10	1 , 2 , 15 , 21 , 22	2	172	97
11	15 , 16 , 18 , 21 , 22 , 23	19	0	181
12	13 , 15 , 16 , 18 , 21 , 22 , 23	9 , 10 , 11	0	370
13	7 , 13 , 15 , 16 , 18 , 21 , 22 , 23	8	0	171
14	1 , 13 , 15 , 16 , 18 , 21 , 22 , 23	5	0	71
15	1 , 2 , 13 , 15 , 16 , 18 , 21 , 22 , 23	6	0	136
16	1 , 2 , 13 , 15 , 16 , 18 , 21 , 22 , 23	4	0	74

Table 4.11: Links for the 24 bus IEEE RTS

Link #	From	To	Sending [MW]	Receiving [MW]	Lines in link
1	8	10	11.94	11.94	1
2	8	14	60.03	59.29	2
3	10	16	38.44	37.85	3
4	10	15	48.50	47.41	4
5	7	8	8.31	7.97	5
6	7	12	22.90	22.66	6
7	3	13	115.00	112.88	7
8	12	16	36.52	36.15	8
9	12	13	59.02	58.12	9 , 12
10	12	14	11.76	11.71	10
11	12	15	89.66	88.59	11
12	5	12	374.98	373.35	15 , 16 , 17
13	9	12	173.55	171.77	19
14	7	9	112.30	112.01	20
15	9	11	115.08	114.65	23
16	6	9	326.03	322.68	24
17	1	11	66.58	66.35	26
18	4	7	435.66	429.84	27
19	4	6	120.80	120.58	28
20	2	6	141.54	139.09	29
21	2	4	158.46	156.46	30
22	1	5	464.84	453.00	31 , 32

Table 4.12: 24 bus IEEE RTS, commons' relative contribution to loads at commons, Kirschen-method

	Load common															
Gen. common	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	-	-	-	0.708	-	-	-	-	-	0.367	0.465	0.158	0.077	0.303	0.227
2	-	1	-	0.281	-	0.262	0.187	0.008	0.179	0.001	0.113	0.062	0.021	0.017	0.040	0.030
3	-	-	1	-	-	-	-	-	-	-	-	-	0.660	-	-	-
4	-	-	-	0.719	-	0.131	0.479	0.021	0.163	0.001	0.103	0.068	0.023	0.029	0.045	0.034
5	-	-	-	-	0.292	-	-	-	-	-	-	0.192	0.065	0.032	0.125	0.094
6	-	-	-	-	-	0.606	-	-	0.332	-	0.210	0.100	0.034	0.017	0.065	0.049
7	-	-	-	-	-	-	0.333	0.015	0.063	0.001	0.040	0.032	0.011	0.018	0.021	0.016
8	-	-	-	-	-	-	-	0.956	-	0.062	-	-	-	0.798	0.022	0.032
9	-	-	-	-	-	-	-	-	0.263	-	0.166	0.080	0.027	0.013	0.052	0.039
10	-	-	-	-	-	-	-	-	-	0.935	-	-	-	-	0.326	0.478

Commons' contributions to links are shown in Table B.2 in Appendix B. Commons' contributions to loads in commons are given in Table 4.12. To find the contributions of individual generators to individual loads, the procedure is as follows: Take for instance the load at common 6. From Table 4.10 it is found that common 6 consists of buses 17 and 18, and is supplied by generators at buses 18, 21 and 22. From Table 4.8 it is found that the load at bus 17 is 0 MW and at bus 18 is 333 MW. From Table 4.12 it is then found that:
Generator 22 (common 2) contributes with $0.262 \cdot 333$ MW = 87.2 MW,
Generator 21 (common 4) contributes with $0.131 \cdot 333$ MW = 43.6 MW,
Generator 18 (common 6) contributes with $0.606 \cdot 333$ MW = 201.8 MW.

From this example a main difference between the Bialek-method and the Kirschen-method becomes clear. The Bialek-method assumes local load and would therefore say that generator 18 contributes with 333 MW to the load at bus 18, while the Kirschen-method allocates only 201.8 MW from generator at bus 18. Hence this will give different tracing results for all buses.

Node Test-Based Method

When tracing on a large and more complex system, difficulties which are not a problem for a small system often rise to the surface. When using the Abdelkader-method to trace on the 24 bus IEEE RTS, it becomes clear that the choice of defining nodes have a big influence on the tracing process. It might seem like this becomes an issue for nodes which do not have any load or generation, such as nodes 11, 12, 17 and 24. From the definition of nodes given in Table 2.2, these nodes are all defined as load nodes. Following the stepwise approach given in Section 2.4, the eliminated A-matrix found in step 3 should have columns whose entries add up to 1. When tracing using the Abdelkader-method, several columns of the eliminated A-matrix add up to more than 1, indicating that something has gone wrong. The final tracing result of generator contributions to loads is given in Table 4.13, and since Abdelkader uses downstream tracing the sum of traced load should equal the actual load. It becomes clear that this is not the case for loads at buses 4, 5, 6, 8, 9 and 10. Defining the nodes without generation or load as other node types does not give a reasonable answer either. It is therefore not possible to conclude what makes the tracing method fail, and how the error could be rectified. Since the tracing results for loads do not make sense, the tracing result for lines is not included.

Table 4.13: 24 bus IEEE RTS, generator contributions to loads, Abdelkader-method

	Load [MW]											
Generator	3	4	5	6	8	9	10	13	14	15	19	20
1	-	4.62	52.72	5.79	-	-	-	-	-	-	-	-
2	-	32.65	-	40.90	-	-	-	-	-	-	-	-
7	-	-	-	-	112.88	-	-	-	-	-	-	-
16	-	1.15	0.42	3.15	1.93	5.57	6.94	-	21.79	-	12.88	-
18	-	4.88	1.58	11.96	7.84	4.30	3.81	-	26.18	-	15.47	-
21	129.39	10.77	7.37	20.60	15.38	25.16	15.09	-	65.77	73.32	38.87	-
22	50.61	14.38	6.18	32.96	22.36	18.80	13.85	-	80.26	28.68	47.43	-
23	-	29.56	10.06	76.05	48.39	95.30	111.71	77.75	-	-	66.35	128.00
Sum	180.00	98.01	78.33	191.41	208.78	149.13	151.40	77.75	194.00	102.00	181.00	128.00
Actual load	180	74	71	136	171	175	195	77.75	194	102	181	128
Diff.	-	24.01	7.33	55.41	37.78	-25.87	-43.60	-	-	-	-	-

4.1.3 Circulating Flows

As mentioned in Chapter 3, the Kirschen-method cannot handle circulating flows (since the recursive method will end up in an infinite loop) and is therefore not considered in this section. The Bialek-method and Abdelkader-method both claim to be applicable to systems with circulating flows. In this section, the two methods are applied to two different test systems containing circulating flows, one with losses and another without loss.

Circulating Flows in a Lossless 4 Bus System

Figure 4.6 shows a lossless 4 bus system with circulating flow, which is the same as the one used in Ref. [16].

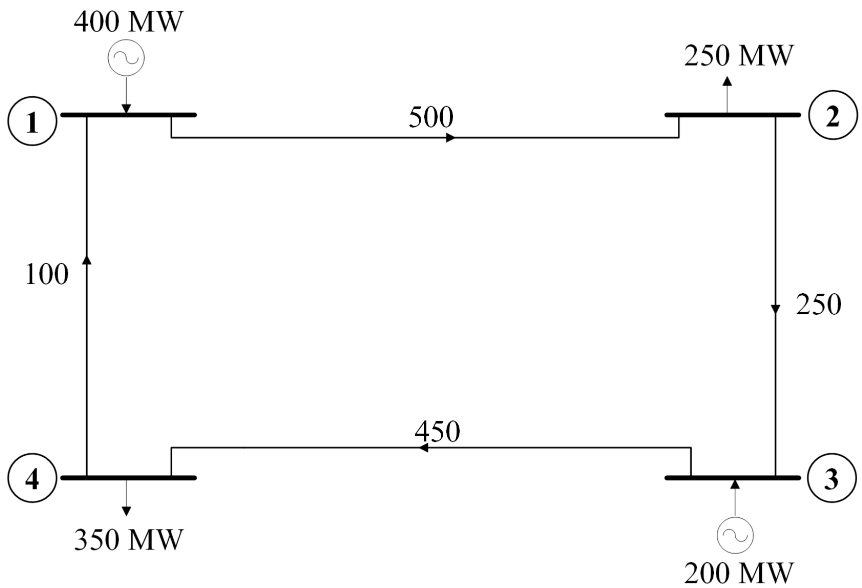

Figure 4.6: 4 bus test system [16]

For this simple lossless system, the Bialek-method and the Abdelkader-method give the exact same results. They are shown in Table 4.14.

Table 4.14: 4 bus system with circulating flow, tracing result

	Load [MW]				**Line**		**Generator [MW]**			
Generator	1	2	3	4	From	To	1	2	3	4
1	-	225	-	175	1	2	450	-	50	-
2	-	-	-	-	2	3	225	-	25	-
3	-	25	-	175	3	4	225	-	225	-
4	-	-	-	-	4	1	50	-	50	-

Circulating Flows in an 8 Bus System with Losses

The tracing algorithms are now applied to an 8 bus system from Ref. [14], shown in Figure 4.7.

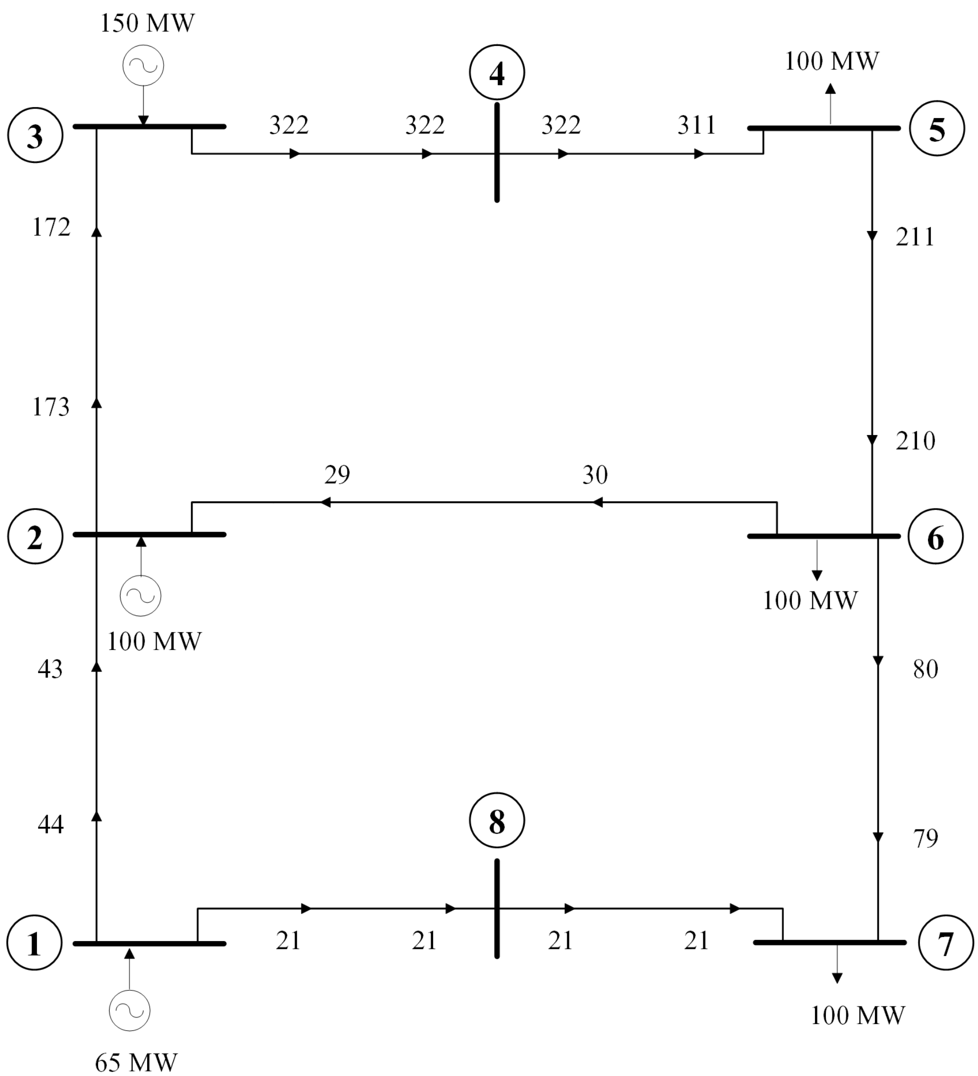

Figure 4.7: 8 bus test system [14]

The tracing results from the Bialek-method are shown in Table 4.15.

Table 4.15: 8 bus system with circulating flow, tracing result, Bialek-method

	Load [MW]		
Generator	5	6	7
1	15.67	15.74	33.59
2	35.61	35.77	28.62
3	53.41	53.66	42.93
Sum	104.68	105.18	105.14

Line		**Generator [MW]**			**Sum [MW]**	**Accumulated line loss [MW]**
From	To	1	2	3		
1	2	44	-	-	44	0
1	8	21	-	-	21	0
2	3	48.72	110.73	16.10	175.55	2.55
3	4	48.72	110.73	166.10	325.55	3.55
4	5	48.72	110.73	166.10	325.55	3.55
5	6	33.06	75.13	112.69	220.87	9.87
6	2	4.72	10.73	16.10	31.55	1.55
6	7	12.59	28.62	42.93	84.14	4.14
8	7	21	-	-	21	0

Note the contribution of generators 2 and 3: For the lines included in the circulating flow, the generators contribute with more than what they are generating. To find the "actual" contributed flow, it would be necessary to define the size of the circular flow. When tracing using the upstream linear equation-based method, an upstream distribution matrix, $\mathbf{A}_u$ is formed as shown in Section 2.4. There should be no elements greater than 1 in the inverted matrix, $\mathbf{A}_u^{-1}$, since this would indicate that a generator is contributing to a load with more than 100% of its generation. When a circulating flow exists in the system, the diagonal elements corresponding to the nodes involved in the circulating flow will be greater than 1. The value of the circulating flow is then equal to the smallest flow in the cycle [14]. In the 8 bus case this would be the flow in line 6-2: 30 MW in the sending end. If line 6-2 was to be removed, the network would no longer have circulating flows and the generator contributions would not exceed 100%.

This network is an example of a network that it seems the Abdelkader-method is not able to handle. In Ref. [16], it is stated that circulating flows will be detected when there is no source node to be found for the tracing procedure. For this network, however, since node 1 is a source node, the elimination of the A-matrix is possible, but the final tracing results does not make sense, since the elements of each column of the eliminated A-matrix do not add up to 1. It is also not described how to handle networks where the nodes have several ingoing or outgoing lines. Abdelkader concludes in his paper that the Abdelkader-method needs more work to be able to handle more complicated circulating flows, and it seems this 8 bus system is an example of such a system that the algorithm is not yet able to handle. The tracing results are not shown.

4.1.4 Discussion

All three algorithms have assumptions made which should be emphasised. Firstly, for all the methods, it is assumed that parallel lines may be merged into one line and that several generators may be merged to one generator. To find the contribution to an individual line it is assumed that it is proportional to how the parallel lines are contributing to the merged line. The same logic applies for the generators. Since this thesis looks at PFT using proportional sharing, it seems reasonable to assume this proportional division, although it should be pointed out that this is yet another assumption made.

Two of the algorithms are assuming that load is handled locally: the linear equation-based and the node test-based. As stated in Ref. [7], the graph-based method does not assume this because the authors feel that "*... preserving the individuality of all generators and loads is necessary to properly allocate the usage of the transmission system*". It might seem rational that a generator at a bus also handles the load at a bus, to minimise unnecessary usage of transmission lines, however if this is done in real life or not is a different question. From the tracing results in Tables 4.2, 4.6 and 4.7, it becomes clear that this assumption has great impact on the tracing result, and is probably the main reason for the differences in results between Kirschen-method and the other methods.

Further the three algorithms have different ways of handling tracing in networks containing losses. The Bialek-method needs to convert the system to an equivalent lossless system either by using gross flows, net flows or average line flows. The Kirschen-method does not convert the system to a lossless system (although it has been claimed that it is necessary), but traces the inflows to nodes (and thus the receiving ends of the lines). The Abdelkader-method takes the losses into account when tracing (taking both the sending and the receiving ends of the lines into account).

The Kirschen-method is the only method which changes the topology - creating commons, links and a state graph. This makes the method more intuitive and hence more easy to understand conceptually [45]. When tracing for the 24 bus IEEE RTS it becomes obvious that more conversion between tracing results and results for individual generators and loads are required, as opposed to the other methods which give results directly for the individual components. Further, the graph-method has received criticism because the state graph may change drastically even when there is only a slight change in the power flow of the system, or a bus is removed/added [19, 29]. Again, the contribution to an individual load may be found by using proportional sharing in a common, but this also raises the question whether this is a valid assumption or not.

A drawback of the Bialek-method is the inversion of the distribution matrix, since it can become quite large, hence the inversion may be time-consuming [29]. An advantage of the Bialek-method is the use of linear equations, making it easy to program [45].

The Abdelkader-method does not require an exhaustive search, no inversion of matrices are needed and the algorithm handles losses implicitly in the algorithm, leaving no need for creating a lossless system before tracing.

The 6 bus RBTS has generation and load at bus 2, revealing the differences when assuming locally supplies load. For comparison, the tracing results from generators to loads for all the three algorithms can be viewed in Table 4.16 (the load at bus 2 is left out of these results):

Table 4.16: 6 bus RBTS, all load tracing results

	Bialek-method [MW]		**Kirschen-method** [MW]		**Abdelkader-method** [MW]	
Load	Gen. 1	Gen. 2	Gen. 1	Gen. 2	Gen. 1	Gen. 2
3	51.19	36.07	40.73	44.27	50.44	34.56
4	-	41.65	-	40	-	40
5	4.70	16.10	9.58	10.42	4.62	15.38
6	4.72	16.17	9.58	10.42	4.62	15.38

As the tracing results show, the Abdelkader-method and the Bialek-method are very similar. The small difference observed is explained by the difference in handling losses and the fact that Abdelkader uses NGDF while Bialek uses TGDF. It is also noted that the Kirschen-method allocates the same contribution to loads at buses 5 and 6, just as in the Abdelkader-method. Bialek gives different answers for these two, most likely because of the handling of losses in the algorithm. Further, the assumption of local load gives quite a different answer for the Kirschen-method when compared to the two other methods.

The 24 bus IEEE RTS has several buses with both generation and load, as well as buses with no generation or load. This is a case which gives good insight into how the different tracing algorithms work. From the tracing results of Abdelkader-method, it becomes clear that the algorithm has difficulties handling networks where there are nodes purely used for interconnecting transmission lines, where there is no generation or load. Further, it might seem as the differences between the tracing results of the Bialek-method and the Kirschen-method are mostly due to the different ways of treating local load, just as in the case of the 6 bus RBTS.

As for the two cases containing circulating flows, there are two questions to be answered: Firstly, can the algorithm detect the circulating flow and successfully carry out tracing in networks where circulating flows are present? And secondly, how does the algorithm allocate the flows when circulating flows are present? Should the circulating flow be detected, "removed" from the system to create an acyclic graph, before tracing is performed again? If that is the case, the circulating flow will still exist in the system creating losses, and someone still needs to be held responsible for this. Should circulating flows be the responsibility of TSOs or should the losses incurred in the system because of the circulating flow be allocated to the generators or loads using the lines where it flows?

The 8 bus test system also gives a good illustration on how the methods handle circulating flow. While both the Abdelkader-method and the Bialek-method can handle the circulating flow of the 4 bus system, the 8 bus system is shown to be more difficult for the Abdelkader-method. Since the system has buses which are not in the path of the circular flow, the circular flow is not detected by the algorithm. Further, since this network has buses with more than one line going in or out, it does not have a way of calculating the contributions. It therefore becomes clear that more work needs to be done before the Abdelkader-method can be applicable to all systems with circulating flows. The Bialek-method is able to trace the generators' contributions to loads. The traced generation adds up to actual generation, and the traced load sums up to actual load + losses in the system.

The question on how to allocate generator contributions to line flows remains. From Table 4.15, generator 2 is said to contribute with 110.73 MW to line 2-3, however the generation at 2 is only 100 MW. This is because of the circulating flow, and there has not been given a good explanation on how to allocate the generators' contributions to lines when circular flows are present. A suggestion is to remove the line with the least flow (hence line 6-2) to create an acyclic graph, and then trace again. That would be an easy fix for this system, however it might create problems for tracing in a bigger network where there are several circulating flows adjacent to each other. This topic requires more attention.

4.2 Applications

In this section, selected applications of PFT are illustrated to give examples on the potential use of and applicability of PFT. In Section 4.2.1 loss allocation is performed using all the three methods. In Section 4.2.2 PFT is applied on the 6 bus RBTS with an overloaded line to illustrate how PFT may be used for load shedding. In Section 4.2.3 PFT is applied on a Northern European System where flow based market coupling is used to find the power flow between zones. After PFT is applied, it can be decided which generators are supplying which loads, and from this the total CO_2-emission from the generating resources of a country can be found.

4.2.1 Loss Allocation

Because of different ways of handling losses in the tracing algorithms, the allocation of losses will also differ. The tracing results from the 6 bus RBTS used in Chapter 4 is used in this section to allocate the losses to loads.

Linear Equation-Based Method

From the tracing results in Table 4.2 in Section 4.1.1, the sum of traced load is found. The difference between this and the actual load, gives the loss allocated to a load as shown in Table 4.17.

Table 4.17: 6 bus RBTS load loss allocation, Bialek-method

	Load bus [MW]			
	3	4	5	6
Sum Traced Load	87.26	41.65	20.81	20.90
Actual Load	85	40	20	20
Loss Allocated	2.26	1.65	0.81	0.90

If it assumed that the losses in lines may be allocated to generators in the same proportion as a generator contributes to the line, the results are given in Table 4.18. As expected, the sum of losses adds up to the total loss, 5.61 MW.

Table 4.18: 6 bus RBTS line loss allocation, Bialek-method

Line		**Generator [MW]**	
From bus	To bus	1	2
2	1	0	0.93
1	3	0.87	0.48
2	4	0	3.04
4	3	0	0.02
3	5	0.03	0.02
4	5	0	0.13
5	6	0.02	0.07
Sum		0.92	4.69

Graph-Based Method

Losses are calculated by using the relative contributions shown in Table 4.4 from Section 4.1.1. Noting that generator 1 is in common 2 and generator 2 in common 1, the loss allocation for each generator is shown in Table 4.19.

Table 4.19: 6 bus RBTS line loss allocation, Kirschen-method

Line		**Generator [MW]**	
From bus	To bus	1	2
2	1	0	0.93
1	3	0.65	0.70
2	4	0	3.04
4	3	0	0.02
3	5	0.03	0.03
4	5	0	0.13
5	6	0.04	0.05
Sum		0.71	4.90

Kirschen *et al.* do not give any instructions on how to allocate the losses to loads in the system.

Node Test-Based Method

The contribution of generators to lines was shown in Table 4.7. Assuming that the line losses are allocated in the same proportion as the contribution to lines, the losses allocated to generators are shown in Table 4.20.

Table 4.20: RBTS 6 bus line loss allocation, Abdelkader-method

Line		**Generator [MW]**	
From	To	1	2
2	1	0	0.93
1	3	0.88	0.47
2	4	0	3.04
4	3	0	0.02
3	5	0.03	0.02
4	5	0	0.13
5	6	0.02	0.07
Sum		0.93	4.68

Table 4.7 also gives the sum of traced contribution of generators to loads, again shown in Table 4.21. This result may also be used to find the generator loss allocation. Since this is a downstream-looking approach, it is only possible to find the generators' contributions to line losses. If an upstream-looking algorithm is used instead, it is possible to allocate the losses to the loads in the system by an algorithm similar to the Abdelkader-method. For further reading on the upstream-looking algorithm, the reader is referred to Ref. [23].

Table 4.21: Generator loss allocation, Abdelkader-method

	Generator [MW]	
	1	2
Sum traced generation	59.58	105.32
Actual generation	60.61	110
Loss allocated	0.93	4.68

Comparing Algorithms

A summary of the loss allocated for each generator for each tracing method is given in Table 4.22.

Table 4.22: Loss Allocated to Generators

	Loss Allocated [MW]		
	Bialek-method	Kirschen-method	Abdelkader-method
Gen. 1	0.92	0.71	0.93
Gen. 2	4.69	4.90	4.68

The loss allocation results follow the tracing results already discussed. Since the Bialek-method and the Abdelkader-method have very similar tracing results, they also allocate losses similarly. The Kirschen-method on the other hand, not assuming local load, allocates less to generator 1 than the others since generator 1 has less responsibility for the line flow from the tracing results.

The Bialek-method is the only method which can allocate losses to loads in the same time as allocating to generators. As mentioned, Abdelkader has published a paper showing how to perform upstream-tracing as well, while Kirschen has not. Loss allocated to loads are therefore not covered here.

4.2.2 Load Shedding

Concept

In power system reliability studies for composite generation and transmission systems, adequacy indices such as Loss of Load Expectation and Expected Energy Not Served are quantified through analytical simulations of contingencies in the network [46]. The consequence analysis for each contingency simulation carried out greatly impacts the numerical values of these indices. Corrective actions are employed to alleviate the voltage and the line-overload violations (steady state security criteria); the actions include generation rescheduling, phase shifters, transformer tap setting adjustments or reactive power injection. Controlled load curtailment is resorted to as a last measure. The amount of load to be shed to ensure steady state security is usually obtained through optimisation techniques. PFT has been shown to afford the convenience of applying non-optimisation based techniques for load shedding, which are computationally quick and efficient.

In Ref. [47], a method is shown for the application of PFT for load shedding. This paper is built on the same idea as in Ref. [48]: If one knows which buses are receiving power from a line, then one also knows which buses should shed load to decrease the power flowing in the line. The linear equation-based tracing method is used in this application; both upstream and downstream tracing are considered. The load distribution factor is found from downstream tracing and is defined as

$$\alpha^{L}_{l_i,k} = \frac{\text{contribution of line } l_i \text{ to load at } k}{\text{total load at } k} \tag{4.1}$$

while the generation distribution factor is found from upstream tracing and is defined as

$$\alpha^{G}_{l_i,j} = \frac{\text{contribution of generator } j \text{ to line } l_i}{\text{total generation at } j} \tag{4.2}$$

The load necessary to shed at k because of an overload ΔP_{l_i} in line l_i is then given by:

$$P_{\text{shed},k} = \frac{\text{overload at line } l_i}{\text{load distribution factor for load } k\text{, line } l_i} = \frac{\Delta P_{l_i}}{\alpha^{L}_{l_i,k}} \tag{4.3}$$

It is beneficial to shed as little load as possible. This load is given by the highest load distribution factor, and the generator needing to decrease its generation is given by the highest generation distribution factor. If the load with the highest load distribution factor is denoted k_{max}, then

$$P_{\text{shed},k_{\text{max}}} = \frac{\text{overload at line } l_i}{\text{highest load distribution factor , line } l_i} = \frac{\Delta P_{l_i}}{\alpha^L_{l_i,k_{\text{max}}}} \tag{4.4}$$

Further, λ denotes the maximum share of load possible to shed at load k_{max}, if this is a restriction. The necessary load to shed would then be decided from:

$$P_{\text{shed},k_{\text{max}}} = \min\left\{\frac{\Delta P_{l_i}}{\alpha^L_{l_i,k_{\text{max}}}}, \quad \lambda P_{k_{\text{max}}}\right\} \tag{4.5}$$

If the generator with the highest generation distribution factor is denoted j_{max}, then the necessary decrease in generation at bus j_{max} is

$$P_{\text{decrease},j_{\text{max}}} = P_{\text{shed},k_{\text{max}}} \tag{4.6}$$

Load shedding on the 6 bus RBTS with line outages

Consider the 6 bus RBTS as introduced in Chapter 4 but with one of the parallel lines between bus 1-3 and one of the parallel lines between 2-4 removed, for the purpose of simulating a contingency. After a power flow has been run on this system, the flow is as shown in Figure 4.8.

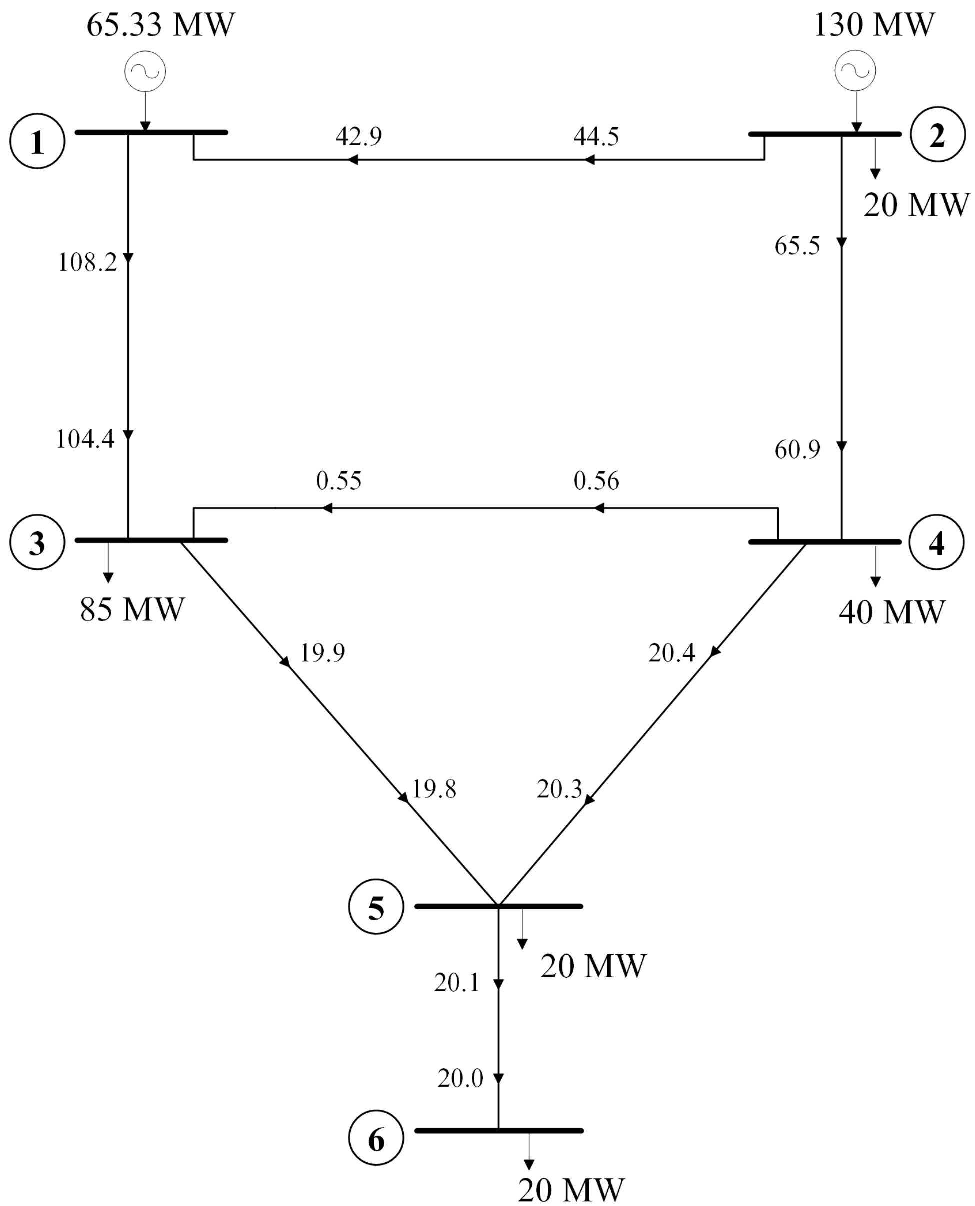

Figure 4.8: 6 bus RBTS with line outages

The capacity of the remaining line between 1-3 (line 2) is 85 MW (as given in Ref. [42]), but as Figure 4.8 shows the line flow on this line after the outages is 108.2 MW. In other words the line is overloaded by 23.2 MW. To find the amount of load shedding necessary to avoid overloading, downstream-tracing is performed to decide the load distribution factor. The tracing results using the downstream Bialek-method are given in Table 4.23.

Table 4.23: 6 bus RBTS overload, downstream tracing result, Bialek-method

	Generator [MW]	
Load	1	2
3	51.06	33.94
4	0	40
5	5.94	14.06
6	5.94	14.06

Line		**Load** [MW]			
From bus	To bus	3	4	5	6
2	1	13.73	16.19	5.69	5.69
1	3	84.55	0	9.84	9.84
2	4	20.21	23.81	8.37	8.37
4	3	0	0.37	0.09	0.09
3	5	16.14	0	1.88	1.88
4	5	0.15	13.37	3.40	3.40
5	6	0	0	10.02	10.02

(4.1) gives the following load distribution factors:

$$\alpha^L_{l_2,3} = \frac{84.55}{85} = 0.99$$

$$\alpha^L_{l_2,4} = \frac{0}{40} = 0$$

$$\alpha^L_{l_2,5} = \frac{9.84}{20} = 0.49$$

$$\alpha^L_{l_2,6} = \frac{9.84}{20} = 0.49$$

The maximum load distribution factor is 0.99 for load 3. The load necessary to shed at load 3 is then, from (4.4):

$$P_{\text{shed},3} = \frac{23.2}{0.99} = 23.43 \text{ MW.}$$

For this case there is no restriction on the amount of load that can be shed, hence λ is set to

1. From (4.5), the amount of load to be shed becomes:

$P_{\text{shed},3} = \min\{23.43, 85\} = 23.43$ MW.

To decide the generator which has to decrease its generation, upstream tracing is performed giving the results in Table 4.24.

Table 4.24: 6 bus RBTS overload, upstream tracing result, Bialek-method

	Load [MW]			
Generator	3	4	5	6
1	52.92	-	6.19	6.22
2	36.54	43.00	15.19	15.26

Line		**Generator [MW]**	
From bus	To bus	1	2
2	1	-	44.51
1	3	65.33	44.51
2	4	-	65.49
4	3	-	0.60
3	5	12.41	8.57
4	5	-	21.89
5	6	6.22	15.26

(4.2) gives the following generation distribution factors (Note: the Bialek-method assumes local load, and the generation at bus 2 is therefore 110 MW):

$$\alpha^G_{l_2,1} = \frac{65.33}{65.33} = 1.0$$

$$\alpha^G_{l_2,2} = \frac{44.51}{110} = 0.405$$

The maximum generation distribution factor is 1.0 for generator 1 (meaning that all power generated at bus 1 goes through line l_2). From equation (4.6) the necessary decrease in generation becomes:

$P_{\text{decrease},1} = P_{\text{shed},3} = 23.43$ MW.

So the load shedding necessary to avoid the overloading of 23.2 MW on line l_2 is 23.43 MW at load 3, obtained by a decrease of 23.43 MW at generator 1.

Thus, a PFT-based load shedding strategy can be realised, and subsequently employed in the analytical contingency simulations to be run in the course of power system adequacy studies. It must be noted that such PFT-based reliability studies have not been conducted in this thesis; this illustration was used to depict the potential employability of PFT in adequacy studies.

4.2.3 CO_2-Emission Apportioning in a Northern European Market Model

Note: The results in this section are obtained in conjunction with the work done by Cecilia Bringedal during her Master's project at the Department of Electric Power Engineering, NTNU [49].

Concept

When transmission system operators (TSOs) are calculating the amount of power that may be transferred in the electrical grid, a model is used. Irrespective of the way the coupling is performed, three phases need to be identified [50]:

1. Pre-market coupling: Capacity calculation done by the TSOs.
2. Market coupling: Actual solving of the market done by the power exchange.
3. Post-market coupling: Verification and analysis of operational security performed by the TSOs.

In phase 1 of the coupling process, the current market coupling algorithm used in Norway and in most of Europe is the coordinated net transmission capacity (CNTC) model. This model does not account for the actual power flow in the grid, but the capacity is decided by the TSO, and commercial exchanges are considered. Loop flows are not accounted for by the market model, and therefore need to be handled by the TSOs separately. Note that the "loop flows" mentioned here denote the unscheduled flows created by a transaction (*not* circulating flows) as illustrated in Figure 4.9 to the right. If a generator in zone B sends power to a load in zone B, some of the power may flow through lines in zones A and C. These flows are called loop flows. If a generator in zone B sends power to a load in zone C, some of the power may flow through the lines in zone A. These flows are called transit flows, as illustrated in Figure 4.9 to the left.

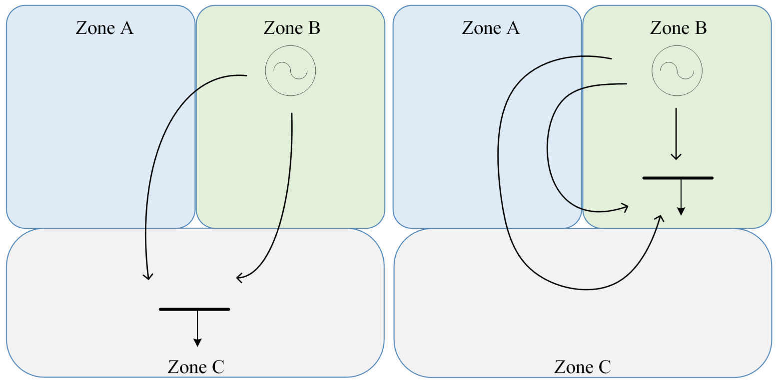

Figure 4.9: Transit flows (left) and loop flows (right) [51]

A different market model called flow based market coupling (FBMC) was launched for the first time by five countries in Central Western Europe (CWE) in May 2015 [52]. FBMC is based on the physical flow in the system, taking the reactances of the lines into account. When the TSOs need to decide the amount of capacity in a network using FBMC, a factor called PTDF (power transfer distribution factor) is used. The $\mathrm{PTDF}_{ij,n}$ gives the impact of injection at node n on the flow of power on line i-j [53], and is defined in (4.7).

$$\mathrm{PTDF}_{ij,n} = \frac{1}{x_{ij}}(Z_{\mathrm{bus}_{in}} - Z_{\mathrm{bus}_{jn}}) \tag{4.7}$$

where x_{ij} is the reactance in line $i-j$,
$Z_{\mathrm{bus}_{in}}$ is the ith row, nth column element of the bus impedance matrix, Z_{bus},
$Z_{\mathrm{bus}_{jn}}$ is the jth row, nth column element of the bus impedance matrix, Z_{bus}.

When the PTDFs have been calculated, the maximum amount of power allowed to transfer may be decided. How this is calculated is shown in detail in Ref. [50]. It is also shown that the FBMC domain is always equal to or greater than the CNTC domain - meaning that by using FBMC it is possible to transfer more power but with the same level of system security. For a greater understanding on how the PTDFs are derived and the ideas behind the calculation of the FBMC domain, the reader is encouraged to see Ref. [50].

The idea for this application is to use FBMC to decide the power flow in a network. Then, tracing is performed to find generator contributions to loads. Finally, using conversion factors for CO_2-emission of various generators, the CO_2-emission apportioning of each load may be found.

Stepwise Approach

1. To find the optimal flow and generation in a network where FBMC is applied, the following information is given as input to a computer program in GAMS [54]: zonal PTDFs, hydro reservoir information, load per hour per zone, topology of network (zone division), generator information (max/min generation, start-up costs, fixed costs), transmission capacity of lines between zones. A DC power flow based on FBMC is run in GAMS, giving prices, generation, AC and DC flow as output per hour per zone.

2. When flow and generation are decided, they are (along with the load) given as input to trace the power flow for one specific hour. Generators and loads for each zone are added together so that there is only one generation and one load per zone. If using the linear equation-based method with upstream tracing, the output of the tracing algorithm is the generator contribution to lines and loads.

3. The contribution to loads along with CO_2-emission information for each generator type is then used to calculate the CO_2-emission responsibility of each load for that specific hour.

A flowchart showing how these steps are performed is shown in Figure 4.10. Steps 1 and 3 have been performed by Cecilia Bringedal [49]; Step 2 is the contribution of the present thesis work.

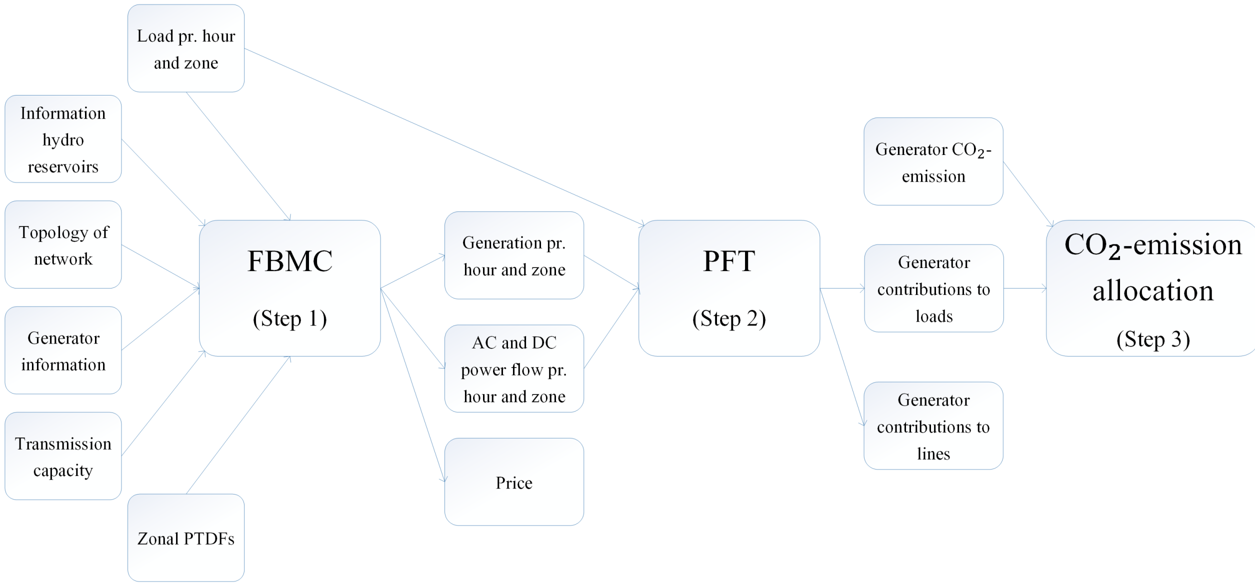

Figure 4.10: Flowchart CO_2-emission apportioning

Power Flow in a Northern European Network Model using FBMC

A representation of how the northern European network is expected to look in 2020, divided into 29 zones, is shown in Figure 4.11, taken from Ref. [55]. Note that this figure was originally made for a 2010 scenario, and therefore some HVDC-lines are not marked on the figure which are included for the 2020 scenario. This yields the following HVDC-lines:

G4 - G5

G2 - N5

D1 - NL.

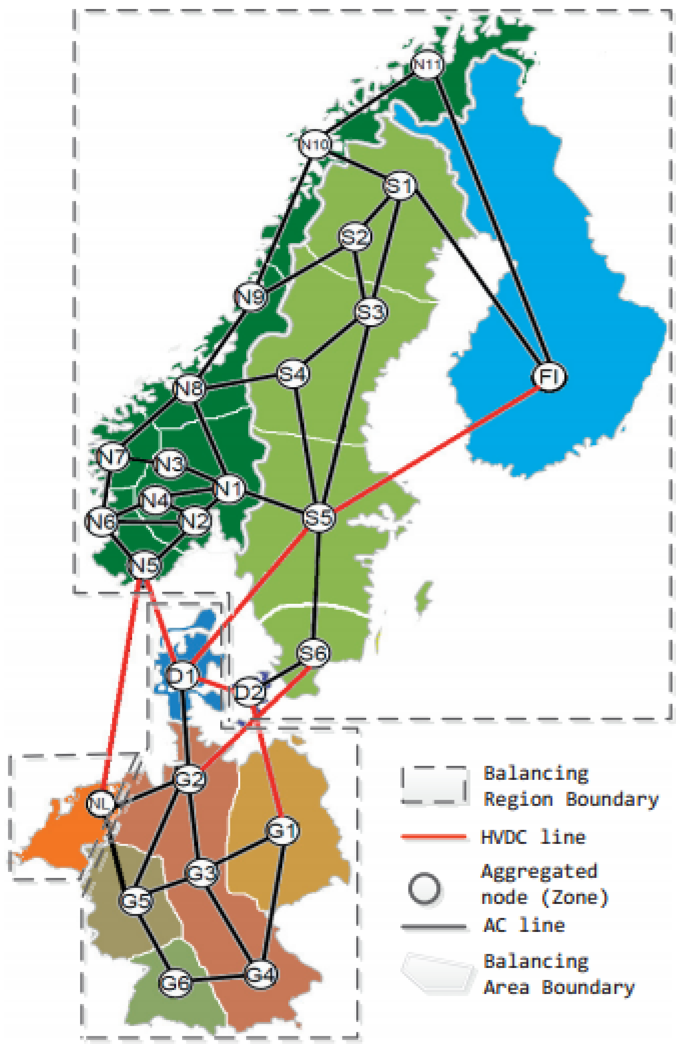

Figure 4.11: Northern European Network Zones [55]

All the zones are given bus numbers from 1 to 29 as in Table 4.25.

Table 4.25: Overview of zones in the Northern European Network Model

Zone	Bus #	Country	Zone	Bus #	Country
N1	1	Norway	S5	16	Sweden
N2	2	Norway	S6	17	Sweden
N3	3	Norway	FI	18	Finland
N4	4	Norway	D2	19	Denmark
N5	5	Norway	Ds	20	Denmark
N6	6	Norway	D1	21	Denmark
N7	7	Norway	G1	22	Germany
N8	8	Norway	G2	23	Germany
N9	9	Norway	G3	24	Germany
N10	10	Norway	G4	25	Germany
N11	11	Norway	G5	26	Germany
S1	12	Sweden	G6	27	Germany
S2	13	Sweden	NL	28	Netherlands
S3	14	Sweden	NLs	29	Netherlands
S4	15	Sweden			

The different generators in the different zones are divided into the following types:

- Gas
- Oil
- Misc. non-renewable (other non-renewable energy sources, mostly combined oil and gas)
- Hard coal
- Nuclear
- Lignite coal
- Misc. renewable (other renewable energy sources, mostly bio)
- Hydro
- Wind
- PV

A DC power flow and FBMC are performed on the system to obtain the optimal line flows between zones as well as generation for each zone. The data is shown in Appendix C. For further information on how the algorithmic implementation has been performed, the reader is referred to Cecilia Bringedal's Master's thesis [49].

Tracing Results

Loads in zones, generation, and line flow are then used to trace the flow using the linear equation-based method. This method is preferred since there are nodes in the network without generation and load, and from the tracing results obtained on the 24 bus IEEE RTS in Chapter 4 it was discovered that the Abdelkader-method had some issues with these nodes. When testing the Abdelkader-method satisfying results were not obtained and because of time constraints, this was not investigated further. Further, the Kirschen-method takes longer time since there is a conversion needed from the commons and links to buses and lines. Because of this, the Bialek-method was the preferred tracing approach. The different generation in each zone is added up to one generation per zone, and the same is done for loads. Tracing results for loads and line flow are shown in Appendix C.

CO_2-Emission Apportioning

The hour with most load in Norway is January 23rd, hour 9. As shown in Table 4.25, Norway consists of buses 1-11. To find the total apportionment of CO_2-emissions for Norway, the contributions of generations to loads are needed. These contributions are found in the tracing results shown in Appendix C. From these contributions and CO_2-conversion factors for generators, the results are shown in Table 4.26. The conversion factors are retrieved from Ref. [56]. It is then found that the total emission for Norway day 23, hour 9 adds up to 1119 g CO_2.

Table 4.26: CO_2-emission Norway, winter hour (d23, h9)

Zone	**Emission CO_2 [g]**										
	Gas	Oil	Misc. Non-Renewable	Hard Coal	Nuclear	Lignite Coal	Misc. Renewable	Hydro	Wind	PV	Sum [g CO_2]
1	-	-	-	-	-	-	-	-	-	-	-
2	3.341	-	-	148.127	-	444.767	-	-	-	-	596.236
3	3.341	-	-	-	-	-	-	-	-	-	3.341
4	1.236	-	-	71.022	-	229.554	-	-	-	-	301.813
5	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-	-	-
7	1.529	-	-	54.797	-	151.479	-	-	-	-	207.804
8	0.132	-	-	2.771	-	5.212	-	-	-	-	8.115
9	-	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	1.329	-	-	-	-	-	-	1.329
Total sum											1 118.639

Discussion

FBMC is beneficial to use when cross-zonal capacity between bidding zones is highly interdependent. However, in regions where the cross-zonal capacity is less interdependent, it has been shown that FBMC does not bring any added value and CNTC may be kept as the market model [57]. The advantage of FBMC is that it maximises the use of inter-regional transmission networks while providing the market with more detailed information [52]. Some disadvantages of FBMC are occasional non-intuitive market results [52] as well as the possibility of flow-factor competition leading to competition bias [58]. If FBMC becomes the preferred market coupling algorithm, then *combined with PFT* it may increase fairness and transparency in the European network. One of the possible applications for this combination is CO_2-emission apportioning; however the combination of FBMC and PFT may also be used for other applications, such as deciding zone delineation for a fair congestion management [51]. If transmission capacity in countries are scarce, the loop flows mentioned may create a problem. If one country uses the transmission lines in other countries for transactions made in its own country, who should pay for the congestion made? Is it fair that other countries may take up scarce transmission capacity? By using PFT it is possible to show which countries are utilising the capacity in lines, leading to perhaps a more fair division of costs.

The concept of apportioning CO_2-emissions may also be an incentive for consumers and governments in Europe. For example, Norway is a country where 98 % of electricity production comes from renewable energy sources [59]. But during periods of high demand, and depending on the price of electricity in Europe, electricity is imported from other non-renewable energy sources. The Paris climate agreement is a hot topic these days, especially after the US decided to withdraw from it. In the agreement, all the signing countries agreed to work to limit global temperature rise to well below 2 degrees Celsius, and to strive for 1.5 degrees Celsius [60]. Tracing may be a way to make consumers and governments realise how they are contributing to the CO_2-emissions. The tracing result shows that even Norway, where almost all production is renewable, gets electricity from non-renewable sources and therefore needs to be held accountable appropriately. In other words we cannot just look the other way as long as we are a part of an interconnected electricity network where non-renewable energy sources are available.

Chapter 5

Concluding Remarks

This thesis is a documentation of *preliminary* investigation conducted, to the extent it was feasible in the limited time frame, on the power flow tracing methods that use the proportional sharing principle.

Power flow tracing algorithms are mathematically intensive, apparently cryptic in presentation in literature, with the nuances not extensively highlighted for all the varied cases. The contribution of this thesis lies in the endeavoured pedagogical clarity and precision for conducting PFT studies. The work done in the thesis is meant to lay a foundation for those wishing to embark on research in the domain of PFT. Though the thesis does not create a comprehensive framework for power flow tracing, a foundation has been built, which can be expanded on and used as a springboard for further dedicated research in this field at the Department of Electric Power Engineering, NTNU; PFT has not received much attention at NTNU thus far.

The Master's thesis highlights the nuances of well-established PFT methods and algorithms, performs comparative case studies, and provides an essential pedagogical treatment of the underlying concepts. In-house programming codes in MATLAB for the aforementioned methods have been built, and their implementation aspects studied. The MATLAB scripts have been released for further internal use and research at the Department of Electric Power Engineering, NTNU.

Originally, power flow tracing was conceived as a decision making tool for equitable transmission cost allocation in deregulated power systems. Though PFT finds practical application in the electricity markets elsewhere in the world, it has not yet found widespread use in the European electricity markets. However, of late, it has been identified that the application potential of PFT can be extended to diverse areas of modern power system design and operation. By illustrating some of these applications in this thesis, though on a limited scale because of the time constraints encountered, it is hoped that the need for and potential in power flow tracing has been satisfactorily demonstrated.

5.1 Summary of Results

A detailed discussion of the results obtained from the conducted PFT studies, and the consequent implications have been presented in Chapter 4. A brief summary is presented here.

- The linear equation-based tracing method is preferred because of its ability to handle complex networks as well as circulating flows.
- The graph-based tracing method separates itself from the other two tracing methods by not assuming local load modelling. It also uses commons and links in the tracing approach, and is not applicable to systems with circulating flows.
- The node test-based method has the advantage of handling losses directly in the algorithm, but encounters issues when the system has nodes that are neither generator buses or load buses, i.e., pure interconnecting buses. In its present form, the method is also not well-suited to handle circulating flows.
- When combined with PFT, Flow Based Markets can be made more transparent, contributing to fair apportioning of transmission costs when cross-border loop flows are present.
- PFT makes possible the attribution of the power flowing on transmission lines to specific generators and loads. This will prove invaluable in quantifying CO_2-emission apportioning in power systems, making consumers and governments monitor their responsibility to the Paris Climate Agreement (with respect to power systems). Using PFT for CO_2-emission apportionment enables the quantification of carbon footprint from electricity imports, which may incentivise consumers in supporting renewable energy technologies.

- PFT has potential employability in power system reliability studies. PFT affords the convenience of applying non-optimisation based techniques for determining the amount of load to be shed during analytical contingency simulations; this is a computationally quick and efficient application.

5.2 Future Work

From the PFT studies conducted in this thesis, an obvious issue that emerges to the fore is that of circulating flows. There is a pressing need for future research on circulating flows to answer the following questions encountered in PFT:

- How can the node test-based method be improved for handling circulating flows in complex interconnected networks? What modifications must be made to this method when some nodes in the network have neither generators nor loads connected?

- How should the generator contributions to line flows be allocated when using linear equation-based method in networks containing circulating flows?

- How can equitable transmission pricing be realised in the presence of circulating flows?

Since most electricity is transferred using AC, reactive power is an intrinsic feature of power transmission. This thesis has looked solely at active power transfers without taking reactive power into account. As mentioned in Sections 2.6 and 2.7, reactive power tracing has been discussed in literature, but without definitive methodologies in place. Some argue that active and reactive power are so closely intertwined that it does not make sense to trace them separately. Also, since active and reactive power may flow in different directions in a line, tracing them simultaneously is not straightforward. This is a topic that still requires more research.

There is potential for utilising PFT in the domain of power system reliability studies; more research is warranted.

Flow based market coupling is a novel market coupling algorithm with many advantages and may very well be the future for the rest of Europe since CWE already has it implemented. One possible application of using FBMC and PFT in tandem has been shown in this thesis. There is immense future potential for deploying PFT for addressing the issues of zone delineation for fair congestion management in FBMC.

The in-house programming codes built in MATLAB for the purpose of PFT studies on test networks are *simple* in that they faithfully execute the underlying algorithms. However, there is a subsequent need for optimising the code and making it computationally efficient; this is expected to be realised in the future when further developing the comprehensive framework for applying PFT to practical European power system models, at the Department of Electric Power Engineering, NTNU.

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Appendix A

Illustration of Methods

The three different methods are illustrated on the lossless 4-bus system shown in Figure A.1.

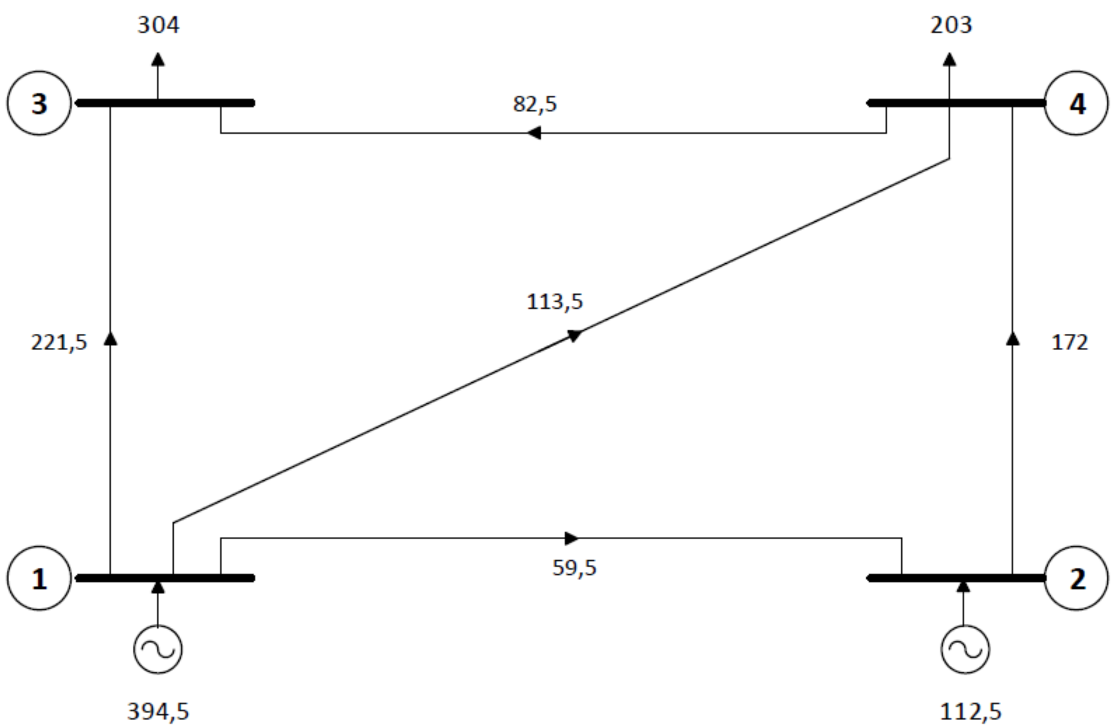

Figure A.1: 4-bus system without loss [4]

A.1 Illustration of Linear Equation-Based Algorithm

A.1.1 Upstream-Looking Algorithm

Step 1: $\mathbf{A}_u$

$$\mathbf{A}_u = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.1508 & 1 & 0 & 0 \\ -0.5615 & 0 & 1 & -0.2890 \\ -0.2877 & -1 & 0 & 1 \end{bmatrix}$$

Step 2: $\mathbf{A}_u^{-1}$

$$\mathbf{A}_u^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.1508 & 1 & 0 & 0 \\ 0.6882 & 0.2890 & 1 & 0.2890 \\ 0.4385 & 1 & 0 & 1 \end{bmatrix}$$

Step 3: Contribution to line

Example: Flow in line 2-4 caused by generation (from (2.5)):

$$|P_{2-4}| = \frac{|P_{2-4}|}{P_2} P_2 = \frac{|P_{2-4}|}{P_2} \sum_{k=1}^{4} [A_u^{-1}]_{2k} P_{Gk} \tag{A.1}$$

where $\mathbf{P}_G = [394.5 \quad 112.5 \quad 0 \quad 0]$.

Line 2-4 is transmitting

$\frac{172}{172} \cdot 0.1508 \cdot 394.5 = 59.5$ from generator 1 and

$\frac{172}{172} \cdot 1 \cdot 112.5 = 112.5$ from generator 2.

Step 4: Contribution to load

Example: Total load at bus 3 (from (2.6)):

$$P_{L_3} = \frac{P_{L3}}{P_3} P_3 = \frac{P_{L3}}{P_3} \sum_{k=1}^{4} [A_u^{-1}]_{3k} P_{Gk} \tag{A.2}$$

Load 3 receives

$\frac{304}{304} \cdot 0.6882 \cdot 394.5 = 271.5$ from generator 1 and

$\frac{304}{304} \cdot 0.289 \cdot 112.5 = 32.5$ from generator 2.

Results for all lines and loads are shown in Table A.1:

Table A.1: Results from upstream-looking algorithm

	Load [MW]		**Line [MW]**				
Generator	3	4	1-2	1-3	1-4	2-4	4-3
1	271.5	123	59.5	221.5	113.5	59.5	50.0
2	32.5	80.0	0	0	0	112.5	32.5

A.1.2 Downstream-Looking Algorithm

The 4-bus system in Figure A.1 is used to illustrate the downstream-looking algorithm.

Step 1: $\mathbf{A}_d$

$$\mathbf{A}_d = \begin{bmatrix} 1 & -0.3459 & -0.7286 & -0.3975 \\ 0 & 1 & 0 & -0.6025 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -0.2714 & 1 \end{bmatrix}$$

Step 2: $\mathbf{A}_d^{-1}$

$$\mathbf{A}_d^{-1} = \begin{bmatrix} 1 & 0.3459 & 0.8931 & 0.6060 \\ 0 & 1 & 0.1635 & 0.6025 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0.2714 & 1 \end{bmatrix}$$

Step 3: Impact on line flow

Example: Flow in line 2-4 because of loads (from (2.11)):

$$|P_{2-4}| = \frac{|P_{2-4}|}{P_2} P_2 = \frac{|P_{2-4}|}{P_2} \sum_{k=1}^{4} [A_d^{-1}]_{2k} P_{Lk} \tag{A.3}$$

where $\mathbf{P}_L = [0 \quad 0 \quad 304 \quad 203]$.

Line 2-4 is sending

$\frac{172}{172} \cdot 0.1635 \cdot 304 = 49.7$ because of load 3 and

$\frac{172}{172} \cdot 0.6025 \cdot 203 = 122.3$ because of load 4.

Step 4: Impact on generation

Example: Total generation at bus 1 because of loads (from (2.12)):

$$P_{G_1} = \frac{P_{G1}}{P_1} P_1 = \frac{P_{G1}}{P_1} \sum_{k=1}^{4} [A_d^{-1}]_{1k} P_{Lk} \tag{A.4}$$

Generator 1 is producing

$\frac{394.5}{394.5} \cdot 0.8931 \cdot 304 = 271.5$ because of load 3 and

$\frac{394.5}{394.5} \cdot 0.606 \cdot 203 = 123$ because of load 4.

Results for all lines and generators are shown in Table A.2.

Table A.2: Results from downstream-looking algorithm

	Generator [MW]		**Line [MW]**				
Load	1	2	1-2	1-3	1-4	2-4	4-3
3	271.5	32.5	40.9	152.4	78.1	49.7	23.8
4	123	80	18.6	69.1	35.4	122.3	58.7

A.2 Illustration of Graph-Based Algorithm

The 4-bus system in Figure A.1 is used to illustrate the graph-based algorithm.

Step 1: Make a state graph

A 4-bus system is not the best system to demonstrate this method, since its advantages become clearer when viewing a large system with many nodes and lines. If this was to be done "properly" as it is defined in Ref. [15], then:

- Common 1 = Buses reached by generator 1 = Bus 1.
- Common 2 = Buses reached by generators 1 and 2 = Bus 2, 3 and 4.

- Link 1 = Branches 1-2, 1-3, 1-4 connecting the two commons.
- State graph would have two commons and one link between them as in Figure A.2.

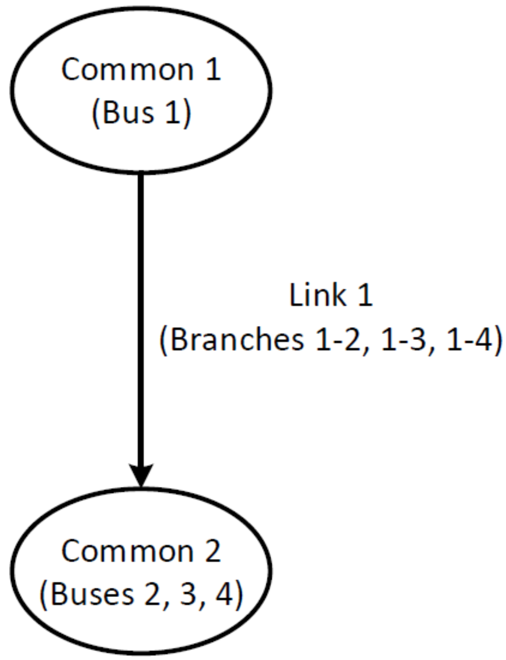

Figure A.2: State graph original case [15]

From the state graph, one of the disadvantages of this method becomes clear. By defining commons as clusters of buses, computations will also be made for these clusters, not the individual buses. To compare this method to the Bialek-method, a state graph looking like the original system is created. Then the system becomes:

- Common i = Bus i, for $i = 1,2,3,4$
- Link 1 = Branch 1-2
- Link 2 = Branch 1-3
- Link 3 = Branch 1-4
- Link 4 = Branch 2-4
- Link 5 = Branch 4-3
- State graph now looks like the original 4-bys system, shown in Figure A.3.

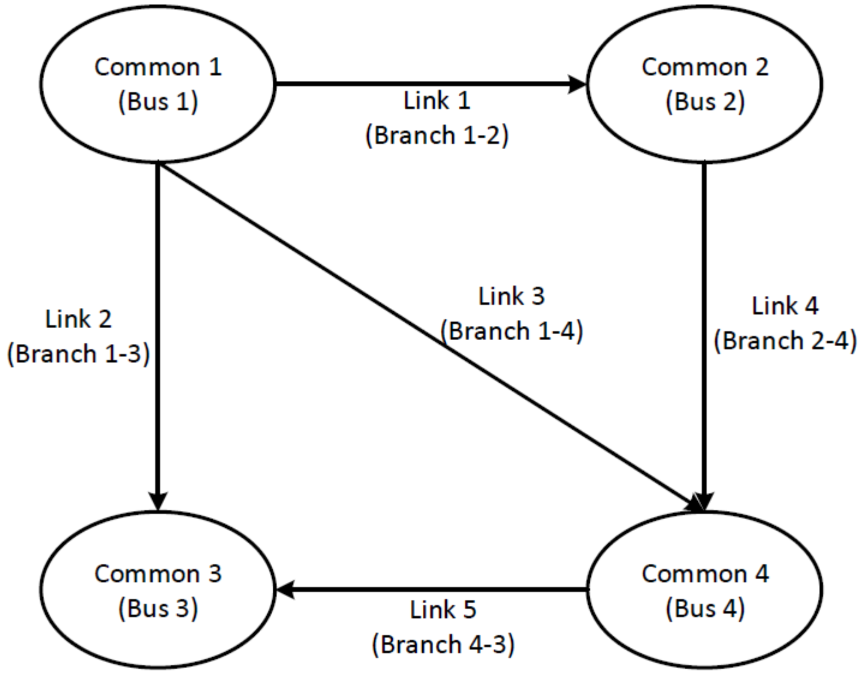

Figure A.3: State graph 4-bus case [15]

Step 2: Decide inflows of commons and flow on links

Inflows	**Flow on links**
$P_1 = 394.5$	$F_{12} = 59.5$
$P_2 = 172$	$F_{13} = 221.5$
$P_3 = 304$	$F_{14} = 113.5$
$P_4 = 285.5$	$F_{24} = 172$
	$F_{43} = 82.5$

Step 3 and 4: Decide absolute and relative contributions to lines by recursive method

By recursion, the contributions of generators to the lines of the system are calculated. The procedure for generator 1 and 2 is shown in Table A.3 and A.4 respectively.

Line	**Relative contribution**	**Absolute contribution [MW]**
Line 1-2	$C_1^1 = \frac{F_{G1}}{P_1} = \frac{394.5}{394.5}$	$F_{12}^1 = C_1^1 \cdot F_{12} = 1 \cdot 59.5 = 59.5$
Line 1-3	$C_1^1 = 1.0$	$F_{13}^1 = C_1^1 \cdot F_{13} = 1 \cdot 221.5 = 221.5$
Line 1-4	$C_1^1 = 1.0$	$F_{14}^1 = C_1^1 \cdot F_{14} = 1 \cdot 113.5 = 113.5$
Line 2-4	$C_2^1 = \frac{F_{12}^1}{P_2} = \frac{59.5}{172} = 0.346$	$F_{24}^1 = C_2^1 \cdot F_{24} = 0.346 \cdot 172 = 59.5$
Line 4-3	$C_4^1 = \frac{F_{14}^1 + F_{24}^1}{P_4} = \frac{113.5 + 59.5}{285.5} = 0.606$	$F_{43}^1 = C_4^1 \cdot F_{43} = 0.606 \cdot 82.5 = 50.0$

Table A.3: Relative and absolute contributions for generator 1

Line	**Relative contribution**	**Absolute contribution [MW]**
Line 2-4	$C_2^2 = \frac{F_{G2}}{P_2} = \frac{112.5}{172} = 0.654$	$F_{24}^2 = C_2^2 \cdot F_{24} = 0.654 \cdot 172 = 112.5$
Line 4-3	$C_4^2 = \frac{F_{24}^2}{P_4} = \frac{112.5}{285.5} = 0.394$	$F_{43}^2 = C_4^2 \cdot F_{43} = 0.394 \cdot 82.5 = 32.5$

Table A.4: Relative and absolute contributions for generator 2

Step 5: Find total contribution to loads

Using (2.18) to find total contributions. Contribution by generator 1 to load 3:

$$T_3^1 = \sum_k (F_{k3}^1 - F_{3k}^1) = 221.5 + 50 - 0 = 271.5 \tag{A.5}$$

Contribution by generator 1 to load 4:

$$T_4^1 = \sum_k (F_{k4}^1 - F_{4k}^1) = 113.5 + 59.5 - 50 = 123 \tag{A.6}$$

Contribution by generator 2 to load 3:

$$T_3^2 = \sum_k (F_{k3}^2 - F_{3k}^2) = 32.5 - 0 = 32.5 \tag{A.7}$$

Contribution by generator 2 to load 4:

$$T_4^2 = \sum_k (F_{k4}^2 - F_{4k}^2) = 112.5 - 32.5 = 80 \tag{A.8}$$

A.3 Illustration of Node Test-Based Algorithm

The 4-bus system from Figure A.1 is used to illustrate the node test-based algorithm.

Step 1: Determine F

$$\mathbf{F} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 1-2 & 1-3 & 1-4 & 2-4 & 4-3 \end{array} \\ \begin{pmatrix} 59.5 & 221.5 & 113.5 & 0 & 0 \\ -59.5 & 0 & 0 & 172 & 0 \\ 0 & -221.5 & 0 & 0 & -82.5 \\ 0 & 0 & -113.5 & -172 & 82.5 \end{pmatrix} \end{array}$$

Step 2: Determine A

Row 1 corresponds to a source node, row 2 to a generation node, row 3 to a sink node and row 4 to a load node.

$$\mathbf{A}=\begin{array}{c} \\ 1\\2\\3\\4\end{array}\!\!\begin{array}{c}\begin{array}{ccccc}1-2 & 1-3 & 1-4 & 2-4 & 4-3\end{array}\\ \begin{pmatrix} 1 & 1 & 1 & 0 & 0\\ \frac{-59.5}{172} & 0 & 0 & \frac{112.5}{172} & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \frac{-113.5}{113.5+172} & \frac{-172}{113.5+172} & \alpha \end{pmatrix}\end{array} = \begin{array}{c} \\ 1\\2\\3\\4\end{array}\!\!\begin{array}{c}\begin{array}{ccccc}1-2 & 1-3 & 1-4 & 2-4 & 4-3\end{array}\\ \begin{pmatrix} 1 & 1 & 1 & 0 & 0\\ -0.346 & 0 & 0 & 0.654 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -0.398 & -0.602 & 10^{-8} \end{pmatrix}\end{array}$$

Step 3: Eliminate negative elements in A

$$\mathbf{A}=\begin{array}{c} \\ 1\\2\\3\\4\end{array}\!\!\begin{array}{c}\begin{array}{ccccc}1-2 & 1-3 & 1-4 & 2-4 & 4-3\end{array}\\ \begin{pmatrix} 1 & 1 & 1 & 0.346 & 0.606\\ 0 & 0 & 0 & 0.654 & 0.394\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 10^{-8} \end{pmatrix}\end{array}$$

$$\mathbf{A}=\begin{array}{c} \\ 1\\2\\3\\4\end{array}\!\!\begin{array}{c}\begin{array}{ccccc}1-2 & 1-3 & 1-4 & 2-4 & 4-3\end{array}\\ \begin{pmatrix} 1 & 1 & 1 & 0.346 & 0.606\\ 0 & 0 & 0 & 0.654 & 0.394\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}\end{array}$$

Step 4: Calculate generators' contributions to line flows

$$\mathbf{T}=\mathbf{A}\,\mathrm{diag}(\mathbf{F}_j)=\begin{bmatrix} 1 & 1 & 1 & 0.346 & 0.606\\ 0 & 0 & 0 & 0.654 & 0.394\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 59.5 & 0 & 0 & 0 & 0\\ 0 & 221.5 & 0 & 0 & 0\\ 0 & 0 & 113.5 & 0 & 0\\ 0 & 0 & 0 & 172 & 0\\ 0 & 0 & 0 & 0 & 82.5 \end{bmatrix}$$

$$= \begin{bmatrix} 59.5 & 221.5 & 113.5 & 59.5 & 50.0 \\ 0 & 0 & 0 & 112.5 & 32.5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 5: Calculate generators' contributions to loads

$$\mathbf{P} = \mathbf{AF^t} = \begin{bmatrix} 1 & 1 & 1 & 0.346 & 0.606 \\ 0 & 0 & 0 & 0.654 & 0.394 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 59.5 & -59.5 & 0 & 0 \\ 221.5 & 0 & -221.5 & 0 \\ 113.5 & 0 & 0 & -113.5 \\ 0 & 172 & 0 & -172 \\ 0 & 0 & -82.5 & 82.5 \end{bmatrix} = \begin{bmatrix} 394.5 & 0 & -271.5 & -123 \\ 0 & 112.5 & -32.5 & -80 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Results

	Load [MW]		**Line** [MW]				
Generator	3	4	1-2	1-3	1-4	2-4	4-3
1	271.5	123	59.5	221.5	113.5	59.5	50
2	32.5	80	0	0	0	112.5	32.5

Table A.5: Generators' contribution to lines and loads

Appendix B

Tracing Results 24 bus IEEE RTS

B.1 Linear Equation-Based Method

Table B.1: Generator contribution to lines, Bialek-method

Line		**Generator [MW]**												
From	To	1	2	7	16	17	18	19	20	21	22	23	Sum	Acc.
1	2	10.62	-	-	-	-	-	-	-	1.02	0.41	-	12.05	0.11
3	1	-	-	-	-	-	-	-	-	6.18	2.45	-	8.62	0.32
1	5	53.38	-	-	-	-	-	-	-	5.15	2.04	-	60.58	0.55
2	4	4.69	33.16	-	-	-	-	-	-	0.45	0.18	-	38.49	0.05
2	6	5.92	41.84	-	-	-	-	-	-	0.57	0.23	-	48.56	0.06
3	9	-	-	-	-	-	-	-	-	17.02	6.74	-	23.77	0.87
24	3	-	-	-	-	-	-	-	-	157.03	62.21	-	219.24	6.92
9	4	-	-	-	1.20	-	1.46	-	-	6.17	5.53	23.58	37.95	1.43
10	5	-	-	-	0.43	-	0.53	-	-	1.33	1.64	8.30	12.22	0.46
10	6	-	-	-	3.30	-	4.02	-	-	10.11	12.49	63.28	93.18	3.52
7	8	-	-	115	-	-	-	-	-	-	-	-	115	0
9	8	-	-	-	1.23	-	1.50	-	-	6.35	5.69	24.24	39.00	1.47
10	8	-	-	-	0.79	-	0.96	-	-	2.42	2.99	15.17	22.34	0.84
11	9	-	-	-	8.18	-	9.96	-	-	25.08	30.99	36.59	110.80	4.61
12	9	-	-	-	-	-	-	-	-	-	-	124.25	124.25	3.41
11	10	-	-	-	11.69	-	14.24	-	-	35.84	44.27	52.27	158.30	6.58
12	10	-	-	-	-	-	-	-	-	-	-	172.11	172.11	4.73
13	11	-	-	-	-	-	-	-	-	-	-	88.86	88.86	2.09
14	11	-	-	-	19.87	-	24.20	-	-	60.92	75.26	-	180.25	6.70
13	12	-	-	-	-	-	-	-	-	-	-	62.25	62.25	1.47
23	12	-	-	-	-	-	-	-	-	-	-	234.10	234.10	0
23	13	-	-	-	-	-	-	-	-	-	-	230.74	230.74	0
16	14	-	-	-	42.07	-	51.25	-	-	129.02	159.39	-	381.73	7.13
15	16	-	-	-	-	-	-	-	-	81.82	32.41	-	114.23	1.93
21	15	-	-	-	-	-	-	-	-	313.17	124.06	-	437.22	1.56
15	24	-	-	-	-	-	-	-	-	157.03	62.21	-	219.24	3.70
17	16	-	-	-	-	-	67.00	-	-	86.83	175.94	-	329.78	3.75
16	19	-	-	-	12.93	-	15.75	-	-	39.64	48.97	-	117.27	2.19
18	17	-	-	-	-	-	67.00	-	-	86.83	34.40	-	188.23	0.65
22	17	-	-	-	-	-	-	-	-	-	141.54	-	141.54	0
21	18	-	-	-	-	-	-	-	-	86.83	34.40	-	121.23	0.43
20	19	-	-	-	-	-	-	-	-	-	-	66.78	66.78	0.20
23	20	-	-	-	-	-	-	-	-	-	-	195.16	195.16	0
22	21	-	-	-	-	-	-	-	-	-	158.46	-	158.46	0

B.2 Graph-Based Method

Table B.2: 24 bus IEEE RTS commons' contribution to lines, Kirschen-method

Link #	**Line**		**Generator common [MW]**										
	From	To	1	2	3	4	5	6	7	8	9	10	Sum
1	8	10	-	0.10	-	0.25	-	-	0.18	11.41	-	-	11.94
2	8	14	-	0.50	-	1.27	-	-	0.89	57.37	-	-	60.03
3	10	16	-	0.02	-	0.05	-	-	0.04	2.38	-	35.94	38.44
4	10	15	-	0.03	-	0.07	-	-	0.05	3.01	-	45.35	48.50
5	7	8	-	1.56	-	3.98	-	-	2.77	-	-	-	8.31
6	7	12	-	4.29	-	10.97	-	-	7.63	-	-	-	22.90
7	3	13	-	-	115	-	-	-	-	-	-	-	115
8	12	16	16.99	2.25	-	2.50	7.02	3.67	1.19	-	2.90	-	36.52
9	12	13	27.46	3.64	-	4.04	11.35	5.92	1.92	-	4.69	-	59.02
10	12	14	5.47	0.73	-	0.80	2.26	1.18	0.38	-	0.94	-	11.76
11	12	15	41.71	5.53	-	6.13	17.24	9.00	2.91	-	7.13	-	89.66
12	5	12	265.31	-	-	-	109.67	-	-	-	-	-	374.98
13	9	12	-	31.08	-	28.27	-	57.58	10.99	-	45.62	-	173.55
14	7	9	-	21.05	-	53.81	-	-	37.44	-	-	-	112.30
15	9	11	-	20.61	-	18.75	-	38.18	7.29	-	30.25	-	115.08
16	6	9	-	85.50	-	42.84	-	197.69	-	-	-	-	326.03
17	1	11	66.58	-	-	-	-	-	-	-	-	-	66.58
18	4	7	-	122.50	-	313.17	-	-	-	-	-	-	435.66
19	4	6	-	33.97	-	86.83	-	-	-	-	-	-	120.80
20	2	6	-	141.54	-	-	-	-	-	-	-	-	141.54
21	2	4	-	158.46	-	-	-	-	-	-	-	-	158.46
22	1	5	464.84	-	-	-	-	-	-	-	-	-	464.84

Appendix C

Northern European Network

Table C.1: Line flow day 23, hour 9

From	To	Line flow [MW]	From	To	Line flow [MW]
1	2	345.39	18	11	21.01
1	3	246.10	18	12	653.71
1	8	44.68	19	20	890.73
2	4	40.18	20	17	593.99
3	7	504.05	22	24	2685.43
4	1	49.30	22	25	1238.47
5	2	1004.80	23	21	1720.96
5	6	1555.36	24	23	2152.93
6	2	425.28	24	25	329.94
6	4	900.00	24	26	2125.53
6	7	447.20	25	26	149.01
8	7	54.88	25	27	3707.96
8	15	145.83	26	23	249.17
9	8	963.14	26	27	16.70
9	13	122.53	26	28	89.33
10	9	46.45	28	23	9.08
10	11	25.16	5	28	69.51
10	12	559.29	18	16	1100.00
12	13	2063.90	21	5	391.63
12	14	2217.57	21	16	740.00
13	14	2169.58	21	19	341.27
14	15	1200.00	22	19	38.61
14	16	86.50	23	5	1400.00
16	1	586.87	23	17	615.00
16	15	107.70	28	21	700.00
16	17	688.11			

Table C.2: Generation and load, day 23, hour 9

Bus	**Generation [MW]**	**Load [MW]**
1	3639.62	3639.62
2	564.82	2300.11
3	3423.20	3165.25
4	1453.88	2344.76
5	2886.80	2048.76
6	1693.34	1476.22
7	1480.15	2486.28
8	2562.56	3369.23
9	2082.14	1042.92
10	1691.28	1060.38
11	301.93	348.11
12	4655.62	1587.15
13	2365.00	2381.86
14	74.79	3175.43
15	475.03	1928.55
16	12691.52	13235.35
17	1694.67	3591.77
18	15374.73	13600.00
19	510.86	0
20	2064.12	2360.86
21	2759.81	3707.86
22	17357.38	13394.87
23	7471.94	6147.16
24	9014.13	7091.16
25	10008.89	7720.33
26	27495.19	29414.53
27	8971.61	12696.28
28	17108.26	16558.02
29	0	0

Table C.3: Tracing results load, Bialek-method

	Load [MW]													
Generator	2	4	7	8	11	13	14	15	16	17	20	21	26	27
3	-	-	257.95	-	-	-	-	-	-	-	-	-	-	-
5	439.50	249.18	131.80	0.89	-	-	-	0.54	1.91	4.54	1.04	8.65	-	-
6	54.02	105.65	57.06	0.33	-	-	-	0.06	-	-	-	-	-	-
9	0.37	0.01	50.48	737.96	-	0.90	82.25	165.37	0.65	0.82	-	-	-	-
10	1.79	0.04	3.56	33.17	25.17	2.12	397.49	160.45	3.13	3.96	-	-	-	-
12	9.75	0.21	7.18	1.03	-	11.41	2160.60	839.74	17.02	21.53	-	-	-	-
18	180.00	3.95	132.53	19.07	21.01	2.43	460.30	243.80	314.14	397.49	-	-	-	-
19	-	-	-	-	-	-	-	-	-	340.67	170.19	-	-	-
22	367.68	189.77	125.23	4.31	-	-	-	12.92	61.30	400.09	46.23	278.21	1091.82	1384.96
23	342.30	177.62	115.82	3.83	-	-	-	11.39	54.01	345.29	29.40	245.12	-	-
24	262.14	135.32	89.26	3.07	-	-	-	9.19	43.63	266.80	23.74	198.00	752.91	138.91
25	2.97	1.47	1.06	0.05	-	-	-	0.15	0.69	3.24	0.38	3.15	74.61	2200.80
28	74.76	27.66	34.21	2.96	-	-	-	9.91	47.36	112.68	25.77	214.93	-	-

Table C.4: Tracing results line flow, Bialek-method

Line		**Generator [MW]**												
From	To	3	5	6	9	10	12	18	19	22	23	24	25	28
1	2	-	8.60	3.17	0.38	1.84	9.98	184.17	-	41.62	36.98	29.63	0.45	28.58
1	3	-	6.13	2.26	0.27	1.31	7.11	131.23	-	29.65	26.35	21.11	0.32	20.36
1	8	-	1.11	0.41	0.05	0.24	1.29	23.82	-	5.38	4.78	3.83	0.06	3.70
2	4	-	10.18	1.25	0.01	0.04	0.23	4.17	-	8.51	7.93	6.07	0.07	1.73
3	7	257.95	6.13	2.26	0.27	1.31	7.11	131.23	-	29.65	26.35	21.11	0.32	20.36
4	1	-	13.79	5.85	0.00	0.00	0.01	0.22	-	10.50	9.83	7.49	0.08	1.53
5	2	-	321.62	-	-	-	-	-	-	243.97	228.41	173.97	1.89	34.94
5	6	-	497.85	-	-	-	-	-	-	377.65	353.56	269.29	2.93	54.08
6	2	-	119.45	52.09	-	-	-	-	-	90.61	84.83	64.61	0.70	12.98
6	4	-	252.79	110.25	-	-	-	-	-	191.76	179.53	136.74	1.49	27.46
6	7	-	125.61	54.78	-	-	-	-	-	95.28	89.20	67.94	0.74	13.64
8	7	-	0.06	0.02	50.21	2.26	0.07	1.30	-	0.29	0.26	0.21	0.00	0.20
8	15	-	0.16	0.06	133.41	6.00	0.19	3.45	-	0.78	0.69	0.55	0.01	0.53
9	8	-	-	-	921.93	41.21	-	-	-	-	-	-	-	-
9	13	-	-	-	117.29	5.24	-	-	-	-	-	-	-	-
10	9	-	-	-	-	46.45	-	-	-	-	-	-	-	-
10	11	-	-	-	-	25.16	-	-	-	-	-	-	-	-
10	12	-	-	-	-	559.29	-	-	-	-	-	-	-	-
12	13	-	-	-	-	269.61	1 479.17	315.13	-	-	-	-	-	-
12	14	-	-	-	-	289.68	1 589.30	338.59	-	-	-	-	-	-
13	14	-	-	-	116.38	272.73	1 467.77	312.70	-	-	-	-	-	-
14	15	-	-	-	31.83	153.83	836.19	178.14	-	-	-	-	-	-
14	16	-	-	-	2.29	11.09	60.28	12.84	-	-	-	-	-	-
16	1	-	2.06	-	0.70	3.38	18.36	339.01	-	66.15	58.28	47.08	0.75	51.10
16	15	-	0.38	-	0.13	0.62	3.37	62.21	-	12.14	10.70	8.64	0.14	9.38
16	17	-	2.41	-	0.82	3.96	21.53	397.49	-	77.56	68.34	55.20	0.88	59.92
18	11	-	-	-	-	-	-	21.01	-	-	-	-	-	-
18	12	-	-	-	-	-	-	653.71	-	-	-	-	-	-
19	20	-	3.11	-	-	-	-	-	510.86	138.76	88.24	71.27	1.13	77.37
20	17	-	2.08	-	-	-	-	-	340.67	92.53	58.84	47.53	0.76	51.59
22	24	-	-	-	-	-	-	-	-	2 685.43	-	-	-	-
22	25	-	-	-	-	-	-	-	-	1 238.47	-	-	-	-
23	21	-	0.13	-	-	-	-	-	-	643.60	610.35	459.13	4.48	3.26
24	23	-	-	-	-	-	-	-	-	1 254.57	-	898.36	-	-
24	25	-	-	-	-	-	-	-	-	192.26	-	137.68	-	-
24	26	-	-	-	-	-	-	-	-	1 238.60	-	886.93	-	-
25	26	-	-	-	-	-	-	-	-	55.27	-	5.32	88.42	-
25	27	-	-	-	-	-	-	-	-	1 375.46	-	132.36	2 200.14	-
26	23	-	-	-	-	-	-	-	-	141.74	-	97.74	9.69	-
26	27	-	-	-	-	-	-	-	-	9.50	-	6.55	0.65	-
26	28	-	-	-	-	-	-	-	-	50.82	-	35.04	3.47	-
28	23	-	0.28	-	-	-	-	-	-	0.87	0.20	0.60	0.05	7.08
5	28	-	22.25	-	-	-	-	-	-	16.88	15.80	12.03	0.13	2.42
18	16	-	-	-	-	-	-	1 100.01	-	-	-	-	-	-
21	5	-	3.57	-	-	-	-	-	-	114.92	101.26	81.79	1.30	88.78
21	16	-	6.75	-	-	-	-	-	-	217.15	191.33	154.55	2.46	167.76
21	19	-	3.11	-	-	-	-	-	-	100.15	88.24	71.27	1.13	77.37
22	19	-	-	-	-	-	-	-	-	38.61	-	-	-	-
23	5	-	0.11	-	-	-	-	-	-	523.57	496.52	373.50	3.65	2.65
23	17	-	0.05	-	-	-	-	-	-	230.00	218.11	164.07	1.60	1.16
28	21	-	21.96	-	-	-	-	-	-	66.83	15.60	46.47	3.56	545.58

Appendix D

MATLAB Code

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