Sensor Fault Tolerance in Output Feedback Nonlinear Model Predictive Control

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***Abstract*— This paper presents an active output feedback fault-tolerant model predictive control (MPC) scheme for systems with sensor faults. The proposed control scheme actively steers the system in order to prevent loss of observability caused by a sensor fault. To this end, the standard tracking objective of the MPC controller is augmented with an observability cost term which strongly penalizes unobservable state and input trajectories. A numerical example illustrates the use of the proposed approach on a target estimation and tracking control problem with faulty sensors.**

***Index Terms*— Sensor faults, Model predictive control, Nonlinear observability, Output feedback**

I. Introduction

Model predictive control (MPC) is a model-based optimal-control scheme, capable of handling complex multiple-inputs multiple-outputs (MIMO) systems with hard constraints on control inputs [1]. At every time step, an MPC controller uses a model of the system to compute the optimal constrained finite horizon state prediction that minimizes a given performance index, and then applies only the first optimal input to the system. Since the model of the system, the constraints, and the performance index can be updated at every time step, the MPC control scheme is particularly suited for embedding fault tolerance [2].

Fault tolerant model predictive control (FTMPC) schemes have been constructed with a variety of fault-diagnosis and reconfiguration approaches, and for many different applications. The majority of FTMPC schemes are active fault-tolerant control (FTC) schemes, based on reconfiguration of the internal MPC model, the constraints or the objective function subject to fault in an actuator (e.g. [3], [4], [5]) or in a sensor [6], [7]. FTMPC schemes may also be designed using a passive approach, thus relying on robustness of the MPC controller, see e.g. [2], [8]. Some advances have also been made towards embedding fault detection and isolation within an MPC controller, see e.g. [9].

In this paper, we consider *sensor* fault tolerance in MPC schemes, which conceptually is more complex to address than actuator and component faults [2]. In particular, we consider the problem of handling sensor faults in output feedback nonlinear MPC (NMPC), where a state feedback NMPC controller is employed in combination with a state observer [10]. Sensor faults may cause loss of observability, and thereby result in poor or divergent state estimates. These estimates are used as initial conditions for the MPC prediction, and may therefore deteriorate the performance of an output feedback MPC scheme, or in worst case jeopardize stability and safety of the system. Loss of observability may in fact be a challenge for output feedback NMPC without consideration of sensor faults, see e.g. [11], [12], [13].

In this paper, we propose an active FTMPC scheme for handling sensor faults which, upon external detection of a sensor fault, actively steers the system so as to preserve observability with the remaining healthy sensors. This is achieved by exploiting the fact that for nonlinear systems, contrary to linear systems, observability depends on the state and inputs of the system, and can therefore be influenced by the control action. In particular, we augment the standard performance index of the MPC controller with an observability index, which we parametrize on the specific fault scenario to force the controller to take into account the set of healthy outputs when computing the input signals for the system.

We organize the reminder of the paper as follows: In section II we present the system and problem structure. Section III contains a brief introduction to nonlinear observability and presents the set-up of the proposed FTMPC scheme, while section IV presents a simulation example to demonstrate merits of our proposed approach. Finally, section V ends the paper with concluding remarks.

II. Problem Statement

A. System Description

Consider the nonlinear system

$$\dot{x}(t) = f(x(t), u(t)), \qquad x(0) = x_0 \tag{1a}$$
$$y(t) = h(x(t)), \tag{1b}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ the input, and $y(t) \in \mathbb{R}^p$ the measured output. The inputs and states are subject to polytopic constraints,

$$x(t) \in \mathcal{X} \subseteq \mathbb{R}^n, \tag{2a}$$
$$u(t) \in \mathcal{U} \subseteq \mathbb{R}^m. \tag{2b}$$

We assume that the system has multiple output measurements, i.e. $p > 1$, but not available measurements for all of the states n. Hence, we consider output feedback NMPC as illustrated in Fig. 1, where an observer provides state estimates to a state feedback NMPC controller. In particular, we

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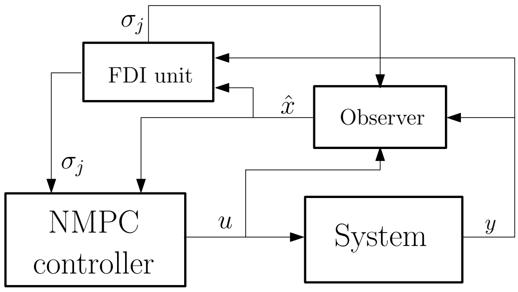


Fig. 1: Schematic diagram of the sensor fault-tolerant NMPC scheme.

consider a sampled-data receding horizon strategy, solving at each sampling instant $t_k \in \mathcal{T} := \{t_0, t_1, \ldots\}$ the following finite-horizon optimal-control problem:

$$\min_{\bar{u}(\cdot)} \int_0^{T_\mathrm{p}} l(\bar{x}(\tau), \bar{u}(\tau)) d\tau + l_T(\bar{x}(T_\mathrm{p})) \tag{3a}$$

$$\text{s.t.} \quad \dot{\bar{x}}(\tau) = f(\bar{x}(\tau), \bar{u}(\tau)), \tag{3b}$$

$$\bar{x}(0) = \hat{x}(t_k) \tag{3c}$$

$$(x(\tau), u(\tau)) \in \mathcal{X} \times \mathcal{U} \tag{3d}$$

$$\bar{x}(T_\mathrm{p}) \in \mathcal{E}_x \tag{3e}$$

In (3), the cost functional (3a) is defined by a performance index $l(\bar{x}, \bar{u})$ over the prediction horizon T_p, and by a terminal state cost $l_T(\bar{x})$. The bar over the input and states denotes internal MPC controller variables, where $\bar{x}(\cdot)$ constitutes the solution to (1a) with initial condition $\hat{x}(t_k)$ driven by the input $\bar{u}(\cdot) \in \mathcal{U}$ in the time interval $\tau \in [0, T_\mathrm{p}]$. The set $\mathcal{E}_x \subseteq \mathbb{R}^n$ in (3e) defines a terminal set. Observe that in (3c), the initial condition $\hat{x}(t_k)$ is the state estimate provided by the observer.

Corresponding to the conventional receding horizon control policy, the solution $\bar{u}^*(\cdot; \hat{x}(t_k))$ to the optimal control problem (3) is applied to the system from time t_k up until the next sampling instant $t_k + \delta$, defining the implicit NMPC feedback control law

$$u(t; \hat{x}(t_k)) := \bar{u}^*(\tau = 0; \hat{x}(t_k)), \tag{4}$$

where δ is the sampling time which we assume constant.

B. Problem Description

The NMPC controller's tracking performance and stability properties depend on the accuracy of the state estimate $\hat{x}(t)$ provided by the observer [10], and hence by properties of the observer design. A fault in one or several sensors may cause loss of observability or detectability, and hence impede reconstruction of the state from the output measurements [14]. This may cause a poor state estimate to propagate in time through the internal NMPC prediction model (3b), eventually diverging the observer state estimates and thereby destabilizing the system. In particular, for nonlinear output feedback control system, this may cause finite-time escape of an unstable state [15]. Yet, in nonlinear systems, observability is a local concept in the sense that with a given output configuration, certain regions of the state space may be observable whilst others remain unobservable. This motivates the following fault-reconfiguration problem for sensor faults in output feedback NMPC:

Problem 1. Given a fault in $l < p$ number of outputs, control the system (1) using the NMPC controller (3)–(4) such that the full state $x(t)$ can be reconstructed using the remaining $p - l$ healthy outputs.

We focus in this paper on the problem of control-system reconfiguration after faults in one or several sensors. Consequently, we make the following assumption:

Assumption 1. If there is a fault in a sensor $j \in \{1, \ldots, p\}$, a fault-detection and isolation (FDI) unit is able to detect and isolate the fault, and sends a switching signal $\sigma_j \in \{0, 1\}$ to the NMPC controller.

Fig. 1 illustrates the couplings between the FDI unit, the observer, the NMPC controller and the system. Note that the observer may as well be an integrated part of the FDI unit. For simplicity, we limit the study to full sensor faults. For details on fault-diagnosis methods for sensor faults, we refer the reader to e.g. [16].

III. SENSOR FAULT-TOLERANT NMPC APPROACH

To alleviate the danger of state-estimate divergence and destabilization of an output feedback NMPC controller during a sensor fault, the controller (3)–(4) must be modified or reconfigured to compensate for the possible loss of state observability. The salient feature of our proposed approach is to impose an explicit measure of the observability of (1) in the NMPC optimization problem.

A. Nonlinear Observability

We use the following definition of nonlinear observability for designing an observability index for the NMPC controller (e.g. [17], [18], [19]).

Definition 1. Let

$$Q(x,u) := \begin{bmatrix} y(t) \\ \dot{y}(t) \\ \vdots \\ y^{(r-1)}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{L}_f^0 h(x) \\ \vdots \\ \mathcal{L}_f^{r-1} h(x) \end{bmatrix} \tag{5}$$

denote the observability map of (1), where $\mathcal{L}_f h(x) = \dot{y}(t)$ denotes the Lie derivative of h in the direction of f, and where

$$\mathcal{L}_f^j h(x) = \frac{\partial \mathcal{L}_f^{j-1} h(x)}{\partial x} f(x,u) \tag{6}$$

is the jth order Lie derivative, with $\mathcal{L}_f^0 h(x) = h(x)$. The observability rank condition is said to hold at a point $x = x_0$ if the $rp \times n$ observability matrix

$$O(x,u) = \frac{\partial Q(x,u)}{\partial x} = \left[\frac{\mathcal{L}_f^0 h(x)}{\partial x}, \ldots, \frac{\mathcal{L}_f^{r-1} h(x)}{\partial x} \right]' \tag{7}$$

evaluated at (x_0, u_0) has full column rank.

While the observability of a system is a global concept [18], nonlinear observability is normally considered locally, as establishing global observability for nonlinear systems is generally difficult. If a system satisfies the observability rank condition at a point x_0, then it is said to be locally weakly observable at this specific state [18], which essentially means that we can instantaneously distinguish x_0 from all other points x in a neighborhood of x_0. Note that the observability rank condition is a sufficient, but not necessary requirement for local weak observability [18]. Furthermore, for nonlinear systems there is no universal law for choosing the number of derivatives $r-1$ in the definition of $O(x,u)$ in (7).

There are several possible measures of unobservability based on (7), both for detecting rank deficiency and for evaluating the difficulty of estimating the state from the observations. To detect rank deficiency and hence singularity of (7), we may use the determinant or the smallest singular value, which are both zero at an unobservable point. The condition number of $O(x,u)$, which gives a measure of the numerical sensitivity of the inverse of the map from (x,u) to $Q(x,u)$ in (5), provides a measure of (local) ill-conditioning of the state estimation problem. Other possible observability measures include the trace of $O(x,u)$, and approaches based on the Fisher information matrix or the Kalman Filter covariance matrix [11]. The most suitable observability measure depends on the application, in particular the system size and the degree of nonlinearity of f and h, as well as the necessary number of derivatives $r-1$ in (5) to get a well defined observability matrix.

B. Output Parametrization for Sensor Faults

Consider a fault in sensor j, transmitted from an FDI unit to the NMPC through the binary switching signal σ_j set to 0, cf. Fig. 1. To integrate the fault signal σ_j in an observability measure, we formulate the output parametrization

$$\tilde{y}(\sigma,t) := \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_p \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}. \tag{8}$$

Using the parametrization $\tilde{y}(\sigma,t)$ of active (healthy) outputs, we define a σ-parametrized observability map,

$$Q(x,u,\sigma) := \left[\tilde{y}(\sigma,t), \dot{\tilde{y}}(\sigma,t), \dots\right]', \tag{9}$$

and correspondingly a σ-parametrized observability matrix,

$$O(x,u,\sigma) := \frac{\partial Q(x,u,\sigma)}{\partial x}, \tag{10}$$

according to Definition 1. The parametrization (10) of the nonlinear observability matrix allows us to impose an observability measure in the NMPC problem that depends on the set of active outputs. To this end, we decompose the performance index $l(x,u)$ in (3a) into a tracking cost $l_{tt}(x,u)$ and an observability index $l_O(x,u,\sigma)$,

$$l(x,u,\sigma) := l_{tt}(x,u) + l_O(x,u,\sigma). \tag{11}$$

To design the observability index (or cost) $l_O(x,u,\sigma)$, we use a measure based on the determinant of $O(x,u,\sigma)$, which is generally easier to compute than its minimum singular value or condition number. In particular, we impose the observability index as

$$l_O(x,u,\sigma) = \frac{k}{\sqrt{\det\left(O(x,u,\sigma)'O(x,u,\sigma)\right) + \varepsilon^2}}, \tag{12}$$

where $k > 0$ is a scalar tuning parameter and $\varepsilon > 0$ is small smoothing parameter. The $n \times n$ matrix $O(\cdot)'O(\cdot)$ is positive definite with non-zero determinant if and only if $O(x,u,\sigma)$ satisfies the observability rank condition for a given output parametrization σ. If the system is rendered unobservable in its current state by a fault in sensor j, the smoothing parameter ε will prevent singularity of $l_O(\cdot)$. Observe that the proposed determinant-based observability index circumvents the need to choose which n rows of $O(\cdot)$ to use for defining its determinant, considering that $O(\cdot)$ typically is non-square.

The tracking cost $l_{tt}(x,u)$ in MPC is normally a quadratic performance index penalizing the distance to a desired state or trajectory. Combining the tracking and observability cost in the performance index $l(x,u,\sigma)$ naturally lends itself to a dual or multi-objective NMPC formulation. Through a dual-objective formulation, we seek to retain, if achievable, the tracking performance also in fault-tolerant mode, in particular if the system actually remains observable when a sensor fails.

The output-parametrized performance index designed in (11) and (12) can be used to specify four operating modes of the proposed FTMPC scheme:

Mode 1. (Nominal mode)
In nominal, fault-free mode, the user may set $\sigma \equiv 0$, leaving $l_O(x,u) = \frac{k}{\varepsilon}$, which is a constant and thereby results in the conventional MPC tracking objective.

Mode 2. (Nominal mode with observability index)
Also in nominal mode, one may choose to set $\sigma_j = 1, \forall j \in \{1,\dots p\}$, and hence encourage the controller to steer the system through good observable trajectories. Even if the system is (globally) observable, favoring good observable trajectories may possibly increase the quality of the observer state estimates.

Mode 3. (Fault mode)
The fault signal $\sigma_j = 0$, transmitted from the FDI to the NMPC controller and the observer, triggers a switching of the controller and a re-parametrization of $l_O(x,u,\sigma)$, leaving $\sigma_i = 1, \forall i \in \{1,\dots p\} \setminus j$. This fault mode defines the controller reconfiguration upon a sensor fault.

Mode 4. (Planned faults)
In this mode, the FDI unit may set $\sigma_j = 1$ only to the indices associated with sensors that it "trusts" or knows are healthy. As an effect, using this mode the scheme can also be applied to *planned* (or predicted) sensor faults. Planned sensor faults may for instance be loss of a sensor for some time during maintenance or replacement, or due to physical barriers hampering the reliability of a measurement.

The resulting closed-loop behavior in these different modes depends on the system characteristics and the form and relative scaling of the two objective terms in (11). The augmented observability index $l_O(\cdot)$ may cause the system to be steered to a steady state or along a trajectory different from the reference trajectory, thereby causing a tracking offset. A different possible outcome is that the controller prevent the system from operating in steady state, causing periodic or oscillatory behavior with excitation from the reference in order to retain observability. We demonstrate this by an example in the next section.

Remark 1. We impose the observability index $l_O(x,u,\sigma)$ by offline computing the algebraic expression for (12) through symbolic computations (e.g. Matlab Symbolic Toolbox). Symbolically computing the nonlinear observability matrix (7) is naturally limited to relatively small systems, and the $r-1$ number of derivatives in (5) must be low. Adding an algebraic observability term may significantly increase the computational complexity and the number of local optima. Yet, smoothness of the closed-loop trajectories and reduced solution times can often be achieved by properly warm-starting the MPC solver. For larger systems, it would be possible to use automatic differentiation to include $l_O(x,u,\sigma)$ in the MPC performance index.

Remark 2. It is important to notice that, while the proposed scheme encourages the system to undertake good observable trajectories, the existence of such trajectories is a structural problem. Clearly, there is no guarantee that a certain subset or even a singleton of the feasible region of the NMPC controller is observable with the remaining $p-l$ healthy measurements. This issue should be considered during the controller design, by for instance analyzing offline the observability measure (12) for a given sensor configuration.

IV. Numerical Example

In this section, we demonstrate the proposed sensor FTMPC scheme on a unicycle-like follower-target problem adopted from [13] and [20], where the objective is to track a target and estimate its position. The dynamics of the follower vehicle is given by

$$\dot{p} = \begin{pmatrix} v \ \cos\theta \\ v \ \sin\theta \end{pmatrix}, \qquad \dot{\theta} = \omega, \tag{13}$$

where $p = [p_x, p_y]'$ is position, θ heading, v linear velocity and ω angular velocity of the follower vehicle. The input velocities $u = [v, \omega]'$ are bounded by the box constraints

$$|v| \leq 2, \qquad |\omega| \leq \pi. \tag{14}$$

We assume that the dynamics of the target can be described by the same model as the follower vehicle, i.e. with unicycle dynamics. Since the MPC controller will require a prediction of the position of the target, we must parametrize the future input of the target vehicle and estimate these in the observer. Note that the future input of the target is of course unknown to the FTMPC controller. Still, in the controller design, we *assume* that target-vehicle velocities are slowly varying, and hence approximate these parametrized inputs as constants. See [13] for more details. These assumptions results in the target vehicle dynamics

$$\dot{p}_t = \begin{pmatrix} v_t \ \cos\theta_t \\ v_t \ \sin\theta_t \end{pmatrix}, \qquad \dot{\theta}_t = \omega_t, \tag{15a}$$

$$\dot{v}_t = 0, \qquad \dot{\omega}_t = 0, \tag{15b}$$

where $p_t = [p_{tx}, p_{ty}]'$ is position, θ_t heading, v_t linear velocity and ω_t angular velocity of the target vehicle.

We assume that the follower observes the target position through a bearing measurement using an omnidirectional camera, and through a range measurement, both located at the center of the follower vehicle. The bearing measurement $y_b(t) \in \mathbb{R}^2$ provides the relative direction of the target vehicle but no information about the distance, and can be modeled as the perspective observation model (e.g. [13])

$$y_b(\sigma_1, t) = \sigma_1 \frac{p_t - p}{||p_t - p||}. \tag{16}$$

The range measurement $y_r(\sigma_2, t) \in \mathbb{R}$, on the other hand, provides the distance to the target but not the direction. We model this measurement as

$$y_r(\sigma_2, t) = \sigma_2 \, ||p_t - p||. \tag{17}$$

We assume that a Global Positioning System (GPS) or an Inertia Measurement Unit (IMU) provides a continues measurement

$$y_f = h_f(p_x, p_y, \theta) \in \mathbb{R}^3, \tag{18}$$

of the position and heading of the follower vehicle. Combined, the outputs from the bearing, range and follower-position measurements provides observability for the estimation of the target position. However, if either the bearing or range measurement is lost due to a sensor fault, there are motions of the follower-target system that are known to render the target position unobservable [21], in particular, any parallel motion of the follower and target vehicle.

We apply an Extended Kalman filter (EKF) to estimate the position, heading and parametrized inputs of the target vehicle modeled in (15), using the measurements (16)–(18). We add small noise terms $r_s = 10^{-2}$ and $q_s = 10^{-3}$ as standard deviation of the measurement and process noise, respectively. For the observability index $l_O(x,u,\sigma)$ in (12) we set $k = 4 \times 10^4$ and $\varepsilon = 10^{-2}$. The target-tracking applies the controller design proposed in [22] for $l_{tt}(\cdot)$, $l_T(\cdot)$ and ε_x, with prediction horizon $T_p = 0.3$s and sampling time 0.1s. See [22] for details. The FTMPC scheme is implemented in Matlab, using ACADO [23] to solve the optimal control problems.

To illustrate the fault-tolerant properties of the proposed FTMPC controller, we simulate the following fault scenario. After $t = 10$s, the range measurements fails, i.e. $\sigma_2 = 0$, and is fixed again at $t = 68$s. Then at $t = 85$s, the bearing measurement fails, i.e. $\sigma_1 = 0$, and is fixed again at $t = 120$s. The initial velocities of the target vehicle are set to $v_t(0) = 1$ and $\omega_t(0) = 0$. To study the controller and observer performance subject target motions that differs from the assumed future target inputs (15b), we impose the following target motion unknown to the controller: After $t = 17$s, the target suddenly stops, and resumes its motion at $t = 50$s, but with a velocity $v_t = 0.5$m/s. Then at $t = 105$s, the target doubles its linear velocity. These particular target motions act as type of disturbance in the Kalman filter. Fig. 2 displays the

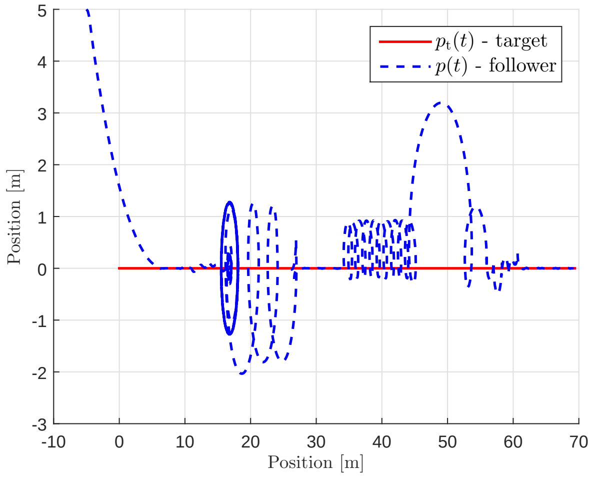
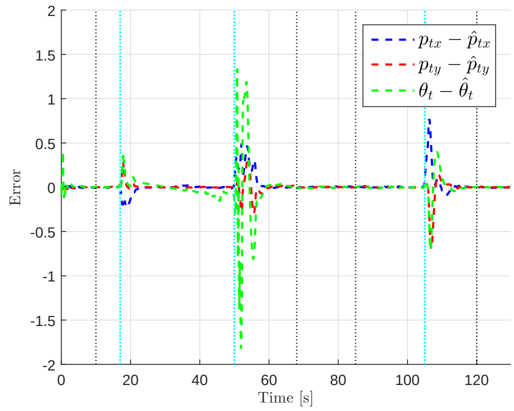


Fig. 2: Trajectories of the target and follower vehicle with the proposed FTMPC approach.

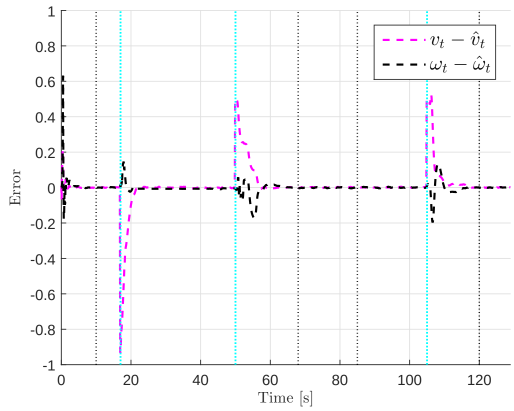
motion of the follower and target vehicles. For the case where all measurements are healthy, the NMPC controller operates in Mode 1, in which the follower is seen to tightly track the target. When the range measurement fails, the controller switches to Mode 3, and the follower starts making small oscillations around the target trajectory to retain observability and estimate the target position. When the target stops, the follower starts orbiting around the target (seen by the thick blue line in Fig. 2) to continuously estimate its position, and continues this motion when the target starts moving again. The follower then tightly tracks the target once the range measurement becomes healthy (around $p_x = 30$m, Mode 1). When the bearing measurement fails, the follower switches to Mode 3 and again starts orbiting around the target, however, with a different pattern than during loss of the range measurement. When the target accelerates, the follower vehicle makes a large turn to regain observability, thereby clearly sacrificing the tracking performance for some time.

The target estimation errors with the proposed FTMPC approach are shown in Fig. 3, where the vertical black-doted lines indicate the time of a fault or repair of an output measurement as described above, while the vertical cyan-doted lines indicates a change in the target motion. The figure demonstrates that the proposed FTMPC controller efficiently retains good state estimates during the simulated time intervals of sensor faults. The follower clearly undertakes motions that deteriorate the tracking performance in order to preserve observability and thereby compensate for the faulty sensor. When the target motions differs from those assumed in the model (15), both the target state and input estimates oscillates for some time before quickly converging to zero.

For comparison, we show in Fig. 4 the trajectory of the follower without any observability measure in the NMPC formulation, corresponding to Mode 1 of the proposed FTMPC scheme, and as such with decoupled observer and NMPC



(a) Target state-estimation errors



(b) Target input-estimation errors

Fig. 3: Estimation errors for the states and parametrized inputs of the target vehicle with the proposed FTMPC approach. The black-doted vertical lines indicates the time of a fault or repair of a measurement, while the vertical cyan-doted lines indicates a change in the target motion, unknown to controller. The loss of a sensor is seen not to affect the estimation errors, while a change in target motion (cyan vertical lines) gives a transient estimation error which quickly converges to zero.

design. The corresponding target state and input estimates are displayed in Fig. 5. When the range measurement fails at time $t = 10$s, the controller retains good tracking performance and state estimates for a short time simply by keeping the control input of the follower as before the fault. However, once the target stops or accelerates, the fault in the range or bearing measurement (respectively) can be seen to cause both the target position estimation and hence the target tracking to fail. This clearly demonstrates that, without compensating for the loss of observability due to a sensor fault, the output feedback NMPC controller fails.

V. Concluding Remarks

In this paper, we have presented a sensor fault-tolerant NMPC scheme. The scheme is based on a parametrization of healthy outputs together with an auxiliary observability index in the NMPC controller. Through simulations of a unicycle follower and target system, we have demonstrated that the

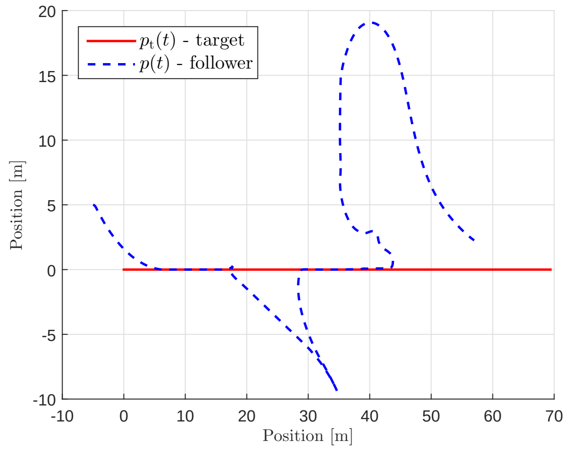
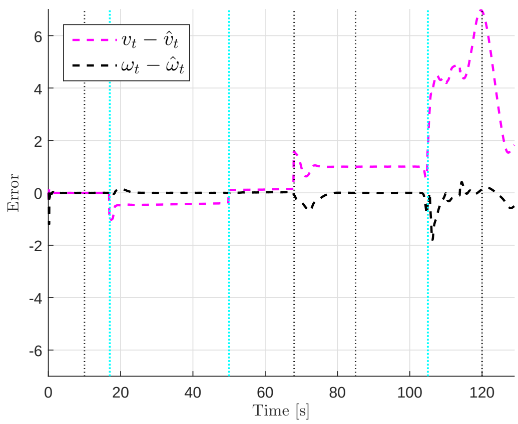
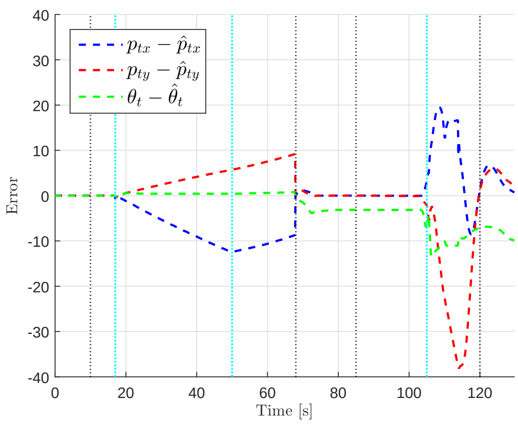


Fig. 4: Trajectory of target and follower vehicle *without* fault-tolerant scheme, i.e. without any observability measure in the NMPC formulation.



(a) Target state-estimation errors



(b) Target input-estimation errors

Fig. 5: Estimation errors for the states and parametrized inputs of the target vehicle *without* including an observability index in the NMPC controller. Observe that the y-axis resolution is different from Fig. 3.

proposed approach efficiently mitigates loss of observability due to a sensor fault, by computing control inputs that forces the system to follow observable trajectories.

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