Carl Haakon Waadelphia

Rhythmic Movements and Moveable Rhythms

Syntheses of Expressive Timing by Means of

Rhythmic Frequency Modulation

Dissertation for the Degree Doctor Artium

Department of Musicology

Norwegian University of Science and Technology

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Abstract

In an attempt at gaining new insight into basic aspects of rhythmic performance of music, based on the point of view that performance (and experience) of rhythm is intimately related to movements of the body, a main goal of the present project is to formulate a new description of musical rhythmic activity, through which gestural aspects of performed rhythm are taken crucially into account. We try to do this by establishing a terminology and model for rhythm analysis and rhythm synthesis where movement activities of the rhythm performer are fundamentally implemented. A basic idea in this respect is to view performed rhythm as a result of mutual interactions of different movements (oscillations), and to construct a theoretical model describing rhythmic activity by means of frequency modulated rhythms, where mathematical, trigonometric functions are representatives of "atomic" movements. This model represents a new description of rhythm performance and is useful for constructing musical syntheses approximating human rhythmic behavior.

Examples of rhythm performances that can be simulated in our model include various "deviations from the exact" characterizing live performance of music; e.g. performances of Vienna waltz accompaniment, different musical dialects in Norwegian folk music, and ways of performing "grooves" in jazz rhythm sections. Moreover, we present simulations of rhythmic accelerandi and ritardandi, and give an example showing how "phasing", applied as a compositional technique, by Steve Reich, among others, may be created using rhythmic frequency modulation (RFM). A MIDI-based computer program has been constructed (in collaboration with Sigurd Saue) converting the theoretical model into audible syntheses of rhythm, and a CD-recording providing several sounding examples of rhythm syntheses created by means of the computer realization of RFM has been made.

A theoretical interpretation offered by our model construction is to describe representations of expressive timing as continuous transformations of rhythmic structure; or, to put it another way, to view expressive timing as a result of rhythmic structure being "stretched" and "compressed" by actions of movements. Thus, in our presentation, new musical syntheses of live performances of rhythm are achieved by means of model constructed "moveable rhythms".

This research project, "Rhythmic Movements and Moveable Rhythms", is based on interactions between a scientific problem formulated within musicology and constructions of models and syntheses using mathematics and computer technology as theoretical and technological tools. Moreover, in approaching our objectives, we try to incorporate experiences of the performing musician.

Key words: Rhythm, performance, expressive timing, movement, frequency modulation
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I am employed at the Trondheim Conservatory of Music, and my office at the conservatory has been my daily place of work during my doctoral studies. I would like to thank my colleagues and the staff at the conservatory for their understanding of my devotion to this scientific research.

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Carl Haakon Waadeland
Prologue:

"It Don’t Mean a Thing If It Ain’t Got That Swing"

In The New Grove Dictionary of Jazz, Vol. Two, 1988, we read:

Swing (i). (1) A quality attributed to jazz performance. Although basic to the perception and performance of jazz, swing has resisted concise definition or description. Most attempts at such refer to it as primarily a rhythmic phenomenon, resulting from the conflict between a fixed pulse and the wide variety of actual durations and accents that a jazz performer plays against that pulse (......).

This brief description points at some fundamental characteristics of the concept 'swing' as understood within the study of jazz performance. Subject to this understanding, swing is conceived as a quality of music performance, related to a process through which the musicians, both individually and in an interactive context of playing together, make a musical phrase, a rhythm, or a melody "come alive", by creating a performance that in varying degrees involves playing "against" a "fixed pulse". From this point of view, swing does not exist on the level of musical syntax, e.g. in a written score of music consisting of quarter notes, eighth notes and different subdivisions and ties thereof. A "swing rhythm", as played on a cymbal by a jazz drummer, is most often written in one of the following three ways:

(1)

```
[Diagram of rhythmic pattern 1]
```

(2)

```
[Diagram of rhythmic pattern 2]
```

---

1 Another meaning of this concept is 'swing' used to denote a jazz style that developed in the United States during the 1930's. Among the most influential musicians from this period were Roy Eldridge, Coleman Hawkins, and "King of Swing" Benny Goodman. Dance music played by Big Bands became very popular during the swing period, and famous Big Band leaders were Duke Ellington, Count Basie, and Benny Goodman.
To be played with a "swing feeling":

All of these written alternatives represent notated approximations, or; quantizations\(^2\), of various (possible) performances of a swing rhythm, where (3) in many respects may seem to be the most adequate, since this alternative indicates that the written notes are not to be understood too "literally", but should be given a "swing feeling" by the performing musician. To the experienced jazz drummer, however, the choice of notation is rather unimportant. Based on a musical acquaintance with the jazz tradition, combined with a performing musician's understanding of musical style, the jazz drummer will make the written cymbal rhythm swing in his own specific way, related to the tradition of which the specific musical performance is a part.

How then, do we, as an audience, or as performing musicians, experience that music is swinging? When does music swing, and, when is it not swinging? What constitutes the qualitative basis for the perception of swing? These are questions that, stated in somewhat various ways, have dominated different branches of modern rhythm research. On a purely intuitive basis the observations made by the composer and jazz historian, Günter Schuller, seem relevant, when he asserts that a rhythm is perceived as swinging when:

... a listener inadvertently starts tapping his foot, snapping his fingers, moving his body or head to the beat of the music. (Schuller, 1989, p.223)

In this characterization of swing Schuller refers to common phenomena that many of us experience and recognize in our own behavior: When exposed to music that we perceive as swinging, we often want to tap our foot, clap our hands, move our body, or, perhaps, dance to the music. In this way we experience how swinging and "groovy" music initializes "energy" and generates movements in our body, thus, various body movements may be seen as a consequence of an experience of swing.

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\(^2\) Quantization in this context meaning quantitative approximation, i.e. an adjustment of durations to the nearest (naturally readable) note value.
On the other hand, body movements also represent a condition, somewhat, prior to the performance of musical rhythm as a whole, in the sense that a rhythmic performance is created through a process where an "internal rhythmic intention" (whatever that may be) is physically articulated by movements of the body, interacting with a musical instrument. An understanding and description of swing in particular, and sounding musical rhythm on a more general basis, may thus seem to be fundamentally dependent on discussing both performance and experience of rhythm closely related to aspects of the phenomenon 'movement'.

Even though 'swing' is most commonly associated with the performance of jazz music, the qualities characterizing a swinging performance, briefly suggested on an intuitive basis in the presentation above; giving "life" to the music, or, making you want to move along with the rhythm of the music, may indeed be typical of musical performances belonging to other traditions than the jazz tradition, as well. Therefore, it also makes sense to assert that various performances of "The Brandenburg Concertos" by J.S. Bach, "Le Sacre du printemps" by Stravinsky, a Vienna waltz, a samba from Brazil, or a "springal" in Norwegian folk music may swing, each and every performance in its own specific way. Or; to quote Irving Mills' lyrics to the famous composition by Duke Ellington, "It Don't Mean a Thing If It Ain't Got That Swing"\(^3\):

\[
\text{...It makes no difference if it's sweet or hot,-} \\
\text{Just give that rhythm ev'ry thing you got.} \\
\text{It don't mean a thing if it ain't got that swing,-...}
\]

The work here presented is in a quite basic way inspired by the lyrics quoted above, not the least by the statement: "Just give that rhythm ev'ry thing you got." – When making a musical performance swing, the performing musician is giving "life" to the rhythm through a process by which (more or less) conceptualized structural properties of rhythm are transformed into live performances of rhythm. Such a process of rhythmic "transformation", often denoted 'timing', or 'expressive timing' (cf., Clarke, 1999, pp.489-490), may be seen as a result of the musician performing "against the pulse", or, as we prefer to put it: being caused by the musician moving in various "non-metronomical" ways. Sounding musical consequences of expressive timing are "artistic deviations", or, "deviations from the exact", as denoted by C.E. Seashore (cf., H.G. Seashore, 1937, p.155). Various such deviations have

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\(^3\) Cf., Chuck Sher (PUBL.ED.): The New Real Book, Volume Two, 1991
been studied in empirical rhythm research (see, e.g. Bengtsson, Gabrielsson & Thorsén, 1969; Bengtsson, 1974; Bengtsson & Gabrielsson, 1977, 1983; Keil, 1987, 1995; Pröger, 1995; Alén, 1995), and it is shown that different kinds of rhythmic deviations are characteristic of different musical styles of performance.

In an attempt at gaining new insight into basic aspects of rhythmic performance of music, based on the point of view that performance (and experience) of rhythm is intimately related to movements of the body, a main goal of the present project is to formulate a new description of musical rhythmic activity, through which gestural aspects of performed rhythm are taken crucially into account. We try to do this by establishing a terminology and model for rhythm analysis and rhythm synthesis where movement activities of the rhythm performer are fundamentally implemented. A basic idea in this respect is to view performed rhythm as a result of mutual interactions of different movements, and to construct a theoretical model describing rhythmic activity by means of frequency modulated rhythms, where mathematical, trigonometric functions are representative of "atomic" movements. The construction of this model is done in a stepwise manner, finding solutions to the following problems:

(A) Present a model of rhythmic structure, where information of note values is represented as continuous movements through attack points.

(B) Construct a model of expressive timing, where performed rhythm is viewed as a result of continuous interactions of movements.

A theoretical interpretation offered by the constructions carried out in (A) and (B) is to describe representations of expressive timing as non-linear continuous transformations of rhythmic structure; or, to put it another way, to view expressive timing as a result of rhythmic structure being "stretched" and "compressed" by actions of movements. Thus, in our presentation, new musical syntheses of live performances of rhythm are achieved by means of model constructed "moveable rhythms".

This, somewhat complementary relationship between rhythmic movements characterizing rhythmic performance of music, and moveable rhythms used as a new technique of making rhythm synthesis, constitute an important axis in the development of our concepts and models. An illustration of the basic idea we are trying to pursue is given in the figure on the next page.
The left hand side of this figure illustrates expressive timing, understood as a process where structural properties of rhythm are transformed into live performances of rhythm, whereas the right hand side indicates the construction of our models (A) and (B). The elements of model (B) are various non-metronomical movement curves, naturally interpretable as movements associated with syntheses of rhythm performances. Thus, our idea is that rhythmic movements typical of live, rhythmic performances are approximated by moveable rhythms.

Examples of rhythm performances that can be simulated in our model (B) include various "deviations from the exact" characterizing live performance of music; e.g. performances of Vienna waltz accompaniment, different musical dialects in Norwegian folk music, and ways of performing "grooves" in jazz rhythm sections. Moreover, we present simulations of rhythmic accelerandi and ritardandi, and give an example showing how
Prologue

"phasing", applied as a compositional technique, by Steve Reich, among others, may be created using rhythmic frequency modulation, RFM. In the construction of our various syntheses, it is interesting to investigate at least two different approaches in searching for answers to the following questions:

(i) How "close to reality" can we get, when applying our theory to live performances of rhythm?

(ii) How "far from reality" are we able to come; or, to put it another way, "how bad can things get" within the limits of our theory?

Whereas the aim of the first approach is to make syntheses approximating live rhythmic performances as studied by empirical investigations, the second approach investigates the potential of the theory in making syntheses of "pathological" rhythmic performances, and might, on the basis of representing various "non-real" unfoldings of rhythm, also generate new ideas of applying our synthesis technique in descriptions of "real" performances of rhythm.

Within our theoretical model the various syntheses of live rhythmic performances that we are able to create exist on a purely formal and rather axiomatic level, where mathematical functions and graphic illustrations are interpreted as different representations of rhythmic performances of music. The musical performance itself, however, "exists" in an interaction with a temporal unfolding of sound. It would, therefore, be very interesting if we were able to "convert" our theoretical model into sounding syntheses of rhythm. This we do by presenting a computer realization of rhythmic frequency modulation, where audible musical information (represented in the MIDI language) may be manipulated by applying RFM to movement curves. The construction of this computer program is a joint work of Sigurd Saue, who has done the programming part of the construction, and the author of this book. An enclosed CD attached to the back cover of the book provides sounding examples of rhythm syntheses made by means of the computer realization of RFM.

The work we are presenting is organized in the following way:

Chapter 1: Divergencies in Use of the Concept 'Rhythm': We start by discussing the etymological origin of the rhythm concept, rhythmos. The ancient meaning of 'rhythmos' constitutes an important motivation for the further development of our
theories and discussions. Moreover, we demonstrate the multidimensional aspects of 'rhythm', used as a concept of everyday speech as well as in scientific language.

Chapter 2: Some Main Results in Modern Rhythm Research: In this chapter we try to give an overview of the major directions in the rather diverse field of rhythm research, hoping to be able to establish a sufficient basis for our work as developed in the subsequent chapters.

Chapter 3: A Possible Convergence of 'Rhythm': Based on the many different applications of the rhythm concept demonstrated in the first two chapters, we now pose the following question: Are there any existing features common to all the different ways of using the concept 'rhythm'? We propose an affirmative answer to this question, thereby pointing at a possible concept "summarizing" various common features of rhythmic phenomena. Some interesting consequences of the identification of such a unifying concept is suggested in Chapter 8.

Chapter 4: Rhythmic Movements: In developing notions basic to a discussion of body movements as related to rhythmic performances of music, we present the eurhythmics of Emile Jaques-Dalcroze. Different states of rhythmic movements are discussed, and the notion of timbral aspects is introduced as relevant to a description of rhythmic performance; the idea being that different kinesthetic movements of the body are expressions of different "rhythmic timbres". An important distinction is made between attack-point rhythm and gestural rhythm, the former being related to a discrete representation of rhythmic performance, whereas the latter is applied in descriptions of rhythm as a continuous unfolding.

Chapter 5: Moveable Rhythms: In this chapter we present our two models; (A): MPR: a model of metronomic performances of rhythm, and (B): LPR: a model of live performances of rhythm. LPR is constructed as a continuous transformation of MPR, applying a new technique of rhythm synthesis; rhythmic frequency modulation, RFM. It should be noted that although the motivation for the models MPR and LPR to a large extent is based on the ideas of Emile Jaques-Dalcroze, as presented in Chapter 4, the models themselves are constructed on a purely theoretical basis.
Chapter 6: From Theoretical Models to Sounding Rhythms: A computer realization of RFM is created (in collaboration with Sigurd Saue), providing an answer to the question: What do our rhythm synthesers sound like?

Chapter 7: Syntheses of Expressive Timing by Means of Rhythmic Frequency Modulation: Applying the computer implementation of RFM, we present numerous examples of audible rhythm synthesers approximating live performances of musical rhythm, as documented and investigated by empirical rhythm research. Every example presented in this chapter is recorded on the attached CD.

Epilogue: If This Means Something....: Here we try to draw some conclusions. Moreover, we point at several open questions and problems, and suggest some ideas for future developments and research.

The different chapters are related as illustrated by the following figure:
At this point of the introduction it should be mentioned that in developing our synthesis technique and constructing our models, as described above, the approach chosen is strongly influenced by the author's background and experiences related to music performance and scientific research. I, myself, am a performing drummer playing mainly jazz, rock and folk music. For many years I have been touring in Scandinavia (mostly in Norway) playing concerts, as well as performing at most of the major Norwegian music festivals. I have also participated in many recordings, playing the drum set (e.g. with the American jazz saxophone player Warne Marsh, with the Greek composer Mikis Theodorakis, and with the Norwegian rock artist Åge Aleksandersen, as well as with two jazz groups I am playing with for the moment; "Siri’s Svale Band" and "Coloured Moods"). In addition to performing music, I am also employed at Trondheim Conservatory of Music, Norwegian University of Science and Technology, teaching drum set playing, rhythm (as a discipline of ear training), and music technology. Also, some years ago, I studied mathematics at The University of Trondheim (today called Norwegian University of Science and Technology), and wrote a thesis in mathematics for the degree cand. real. (Waeland, 1977).

The experience from these, in some sense, rather complementary disciplines; music performance, music teaching, and mathematics, has substantially influenced this present work. This research project, "Rhythmic Movements and Moveable Rhythms", is fundamentally based on interactions between a scientific problem formulated within musicology, and constructions of models and syntheses using mathematics and computer technology as theoretical and technological tools. Moreover, in approaching our objectives, we try to incorporate experiences of the performing musician.
1

Divergencies in Use of the Concept 'Rhythm'

1.1. Order in Movement

To place our focus on rhythm and the rhythmic in a more general context, it might be both interesting and relevant to achieve some acquaintance with the origin of the concept 'rhythm'. Etymologically 'rhythm' originates from *rhythmos*, which is a concept born of the ancient Greeks' understanding of music, of which a general overall feature is that music (*mousikē/mousikē techne*) and the "musical" have a dominating and integrated role in social life, human co-existence and the understanding of reality as a whole. This is demonstrated in a beautiful way in the myth of the birth of the Muses, as told by Pindar in the 5th century B.C., in his hymn to Zeus (see Otto, 1955, p.28):


The Muses, brought to birth to complete and perfect the existing, performed through singing, dancing, poetry and playing instruments, and provided existence with a voice capable of reporting the glory of the world. According to Greek antiquity, "musical" activities denoted the totality of performing activities executed by the Muses, and were as such characterized by the following features:

1. "Musical" activities unfold through time.
2. "Musical" activities are intimately related to the phenomenon 'movement'.

---

1 Other subject related concepts also originating from antiquity, are 'music', 'harmony', 'tone', 'melody' and 'symphony'.
3. "Musical" activities unfold through a process or progression and are related to a development of action.

4. "Musical" activities represent a form of social fellowship.

Of major importance for the theoretical formulation of antiquity's understanding of music are the philosophers Pythagoras, Plato and Aristotle. In this respect it seems adequate to assert Pythagoras as being the most extreme. For him and his disciples, the Pythagoreans, the understanding of music and the understanding of reality itself were two views of the same matter. The Pythagoreans regarded music as a sonorous realisation of harmonic relations between numbers, and these numerical relations represented a key to an understanding and comprehension of "the being".²

As pointed out above, movement is a crucial component of the art forms performed by the Muses. On the other hand movement was also seen as a characteristic of cosmos (e.g. in speaking of the "beautiful ringdance" of the sun, the moon and the stars) as well as an important quality of the human soul (microcosmos), where different states of mental moods were seen to be related to different "movements" of the soul. Essential in this respect is that the patterns of movements occurring in the "musical" were seen as similar to or, more precisely, as an imitation (mimesis) of corresponding patterns of movements in as well cosmos as the human soul. The "musical" is thus, according to this antique thinking, governed by the same "laws of movements" which determine processes of cosmos and the human soul. This view reflects the dominating analogy (correspondance) thinking typical of much of antiquity's understanding of reality and its philosophy in general, and the Pythagorean thinking in particular.

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² Founded on Pythagoras' interpretation of the universal and "cosmic" meaning of numerical relations, the Pythagoreans supposedly stated: "All things resemble number" (see Barker, 1989(b), p.36). Pythagoras is sometimes also quoted by the statement: "All is number!". These are statements which in many respects have gained renewed validity today, viewed in light of the dominating role digital representations have achieved in technological transformations of information and communication.
Chapter 1

Through the phenomenon movement, music and the art forms of the Muses thus become an important link, or connection, between cosmos and the human soul (see Fig. 1.1). A beautiful expression of this is found in antiquity’s thinking of the well-ordered, harmonic cosmos pictured as a tuned lyre: Through man’s performance of the ”musical” art forms the inner ”strings” of the soul will be so tuned that when the cosmic lyre is played, the strings of man’s soul will be brought in sonoric resonance. (See O.K. Sundberg, 1980, pp.129-130.)

In light of the importance of the concept ‘movement’ in ancient philosophy and analogy thinking, the meaning of rhythmos becomes extremely interesting. According to Paul Fraisse (1982, pp.149-150):

Rhythmos appears as one of the key words in Ionian philosophy, generally meaning “form,” but an improvised, momentary, and modifiable form. Rhythmos literally signifies a ”particular way of flowing.” Plato essentially applied this term to bodily movements, which, like musical sounds, may be described in terms of numbers. He wrote in The Banquet ”The system is the result of rapidity and of slowness, at first opposed, then harmonized.” In The Laws he arrived at the fundamental definition that rhythm is ”the order in the movement.”

Or, to quote Plato himself, translated from ”The Laws” 664b-671a, by Andrew Barker:

The name for [665a] order in movement is rhythm, and order of the voice, where high and low are mixed together at once, is given the name harmonia, while the combination of the two is called choreia. (See Barker, 1984(I), p.149.)

In The New Grove Dictionary of Music and Musicians (1980), under the heading ”Rhythm”, we likewise read:

Etymologically the word probably implies ‘not flow, but the arresting and the firm limitation of movement’ (Jaeger, 1959). The widely accepted view of rhythmos as deriving from rheó (’flow’) has now lost ground in favour of an older derivation from the root ry (ery) or w’ry (’to pull’). The history of the word rhythmos shows that it was close in meaning to schéma (’shape’, ’form’, ’figure’ - Leemans, 1948). Peterson (1917) has characterized rhythmos as ”immobile form which arises through motion”, thus suggesting an artistic origin of the word as well.

Hence we understand that rhythmos is a fundamental concept in antiquity’s understanding of reality and in ancient philosophy in general.

---

1 As a consequence of this view, music, not to say performance of music, also becomes important in education and ethics. This is explicitly formulated by the theorist and pedagogue Damon in the 5th century B.C., and later in works by Plato ("The Republic" and "The Laws") and Aristotle ("Politics", book VIII).
1.2. A Natural Attraction to Rhythmic Behavior and Experience of Rhythm

The view of rhythmos as a phenomenon representing a structuring and ordering of patterns of movements has much in common with the meaning many of us today, on a purely intuitive basis, will assign to the concept 'rhythm'. In Cappelens Musikkleksikon, volume 5, 1980, under the term "Rytme", the late Swedish musicologist and founder of the empirical rhythm research project in Uppsala, Ingmar Bengtsson, writes that empirical-psychological research on rhythm as behavior and experience show:

For rhythm experiences and rhythm behavior to occur, some kind of temporal structuring involving grouping and differences in accentuation seems to be necessary. The "well-ordering" thus achieved seems moreover to be dependent on some kind of experienced or perceived regularity ("return of the similar"), which, within some tolerable "variance", should occur on a temporal level of the musical unfolding. (Author's translation.)

Also exposed to a course of equal events temporally divided by constant time intervals, as when listening to a series of simple, regular beats where all the beats have the same volume and the same sound, it has been shown that the listener, more or less spontaneously, "assigns" extra accent, duration, or "weight", to some of the beats, thereby experiencing the events as groups of events of 2, 3, or 4 beats rather than as single events in succession. This grouping of perceived events is often referred to as subjective rhythmization in the literature (see for instance Fraisse, 1982, pp.155-156, or Elliot, 1986, p.5).

The need of structuring and grouping events in time, space, and movement seems to be deeply founded in man, and in human communication in general. Our whole life is fundamentally formed and organized within well-ordered patterns of movements and repeating cycles of events:

- The heartbeat gives a basic pulse, establishing a reference for elapsing time.
- Breathing represents a continuous cycling between tension and relaxation (breathe in/breathe out).
- The rotation of the earth gives a repeating alternation of night and day.
- The gravitational attraction between the earth and the moon causes tidal ebb and flow.
- The earth evolving around the sun causes the cycle of the year.
These are examples of basic biological and astronomical cycles that we all are subject to. In addition we make our own everyday routines and "working rhythms", and develop our own "life patterns", which at different times and in varying degree we may break or ignore, but which we are still governed by and relate our actions to. Thus it seems reasonable to say that human life, and in general all living things are subject to a chain of structured temporal events and well-ordered patterns of movements.

Is it equally valid to maintain that all living things are subject to a chain of rhythmic cycles? The answer to this question is heavily dependent on how general we want the concept 'rhythm' to be, and this will be discussed more thoroughly below. Nevertheless, inspired by the ancients' analogy thinking, it is here tempting to call attention to the fact that principles of structuring and grouping of events in time and space that constitute our natural environments and to which our daily life is related seem to be similar to characteristic features of the phenomenon 'rhythm'. Viewed against this background, we understand that an attraction to experiences of rhythm and rhythmic behavior is deeply rooted in man's nature.

1.3. A Wilderness of "Definitions" - a Labyrinth of Connections

In our everyday speech the concept 'rhythm' is assigned many different meanings, and related to various situations. As mentioned above, the terms "working rhythms" and "the rhythm of life" may be used in a meaningful way. Some may speak of "biological clocks" or "bio-rhythms", while the trainer of a football team may say that the reason for the team losing the match was its lack of co-action and "rhythm". Furthermore, most people will agree that rhythm is a basic component in music, and many may find it natural to say: "This music has got a good dance rhythm. It's groovy, I feel like dancing to it!" These various ways of using 'rhythm' clearly show how in our everyday speech assign "multidimensional" meanings to this concept. In addition, the miscellaneous applications of 'rhythm' mirror how different aspects of a, more or less precise, rhythm concept is seen relevant in characterizing events belonging to a generous selection of everyday relations.

A multidimensionality in 'rhythm' is moreover present in scientific language and scientific applications of the concept, a matter which is demonstrated in Evans & Clynes: "Rhythm in Psychological, Linguistic and Musical Processes" (Evans & Clynes, 1986). This book contains articles written by authors related to different fields of science, where the focus on rhythmic aspects in the various scientific fields is the common theme. Here, Payne, Jr. and
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Holzman present results showing rhythm as a factor in memory\(^4\), Evans points at a connection between dysrhythmia and disorders of learning and behavior\(^5\), Condon emphasizes that human communication is fundamentally synchronous and rhythmic\(^6\), while Elliott in his contribution comments:

Though rhythm had been defined as a psychophysical phenomenon as far back as 1886 (\(...\)), recent research is showing that rhythmicity is likely basic to human perception and that an understanding of the nature of rhythm may be the key to an understanding of the human perceptual process. (Elliott, 1986, p.5.)

It is extremely important to be aware of this multidimensionality, this "pregnancy of meanings", of the concept 'rhythm'. As a direct consequences, it becomes necessary in any discussion of rhythm and rhythm features claiming a scientific level of precision\(^7\) to *come to a decision* as to *which aspects* of rhythm one wishes to illuminate. To the same extent that such a choice is made and certain features are focused upon, other aspects of rhythm will be excluded. Particularly since the concept is assigned so many different meanings, it is crucial that this decision is made explicit.\(^8\)

Let us now look at some examples of such decisions, by presenting different "definitions" of rhythm.\(^9\)

- **Definitions involving units of time:**

  ...regular units of time or pulses. (Spohn, 1977, p.62.)

  ...a patterned cycle of events. (Bond, 1959, p.260.)

  ...some sort of recurring and within limits, predictable event...

  Rhythm in music is the pattern of organized sounds and silences. (Radocy, 1980, p.98.)

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\(^4\) In Payne, Jr. and Holzman (1986, p.49) we read: "The studies of temporal spacing in conjunction with those of duration indicate that rhythmic organization is an important factor in acquiring the important human language skill of reading."

\(^5\) Evans, 1986.

\(^6\) Condon, 1986, p.75: "Synchrony and rhythm are primary aspects of human individual and interactional behavior. They are not separate forms added to the structure of behavior, but are forms of organization discovered in behavior. They are elements in the structure of behavior."

\(^7\) What is a "scientific level of precision"? This is a question which in a fundamental way depends on the attitude taken towards the question: "What is a scientific method?", and it is as such deeply founded in the philosophy of science (e.g. Popper, 1957). For the time being it is sufficient to say that on a scientific level of precision (as we choose to use the term) the degree of ambiguity is less than in everyday speech.

\(^8\) The literature shows many examples of the lack of such decisions or decisions that are made implicit rather than explicit, and hence cause misunderstandings and obscure a discussion that otherwise might have been fruitful.

\(^9\) More examples are found in, for instance, Elliott's article in Evans & Clynes, 1986, pp.8-11.
• **Definitions involving time and space:**

Motor rhythm. A periodic succession of events in time and space. (Schwanda, 1969, p.568.)

Movement is a space-time organization of events. (Smoll and Schultz, 1978, p.838.)

• **Definitions involving movement in time:**

Rhythm and motion may be analytically distinguished, the former meaning movement in time and the latter movement in space (pitch). (Apel, 1969, p.729.)

...movements of parts of the body or the whole body repeated in the same form at least three times at regular short intervals of about a second or less. (Thelen, 1981, p.238.)

• **Definitions involving the organization of music:**

...the organizational and dynamic force in music. (Radocy and Boyle, 1979, p.69.)

...a generic term which includes a variety of concepts that bear directly or indirectly upon the organization of musical sounds in the dimension of time. (Petzold, 1966, p.184.)

• **Definitions involving a subjective organization:**

...the subjective grouping of objectively separate events. (Demany, McKenzie, and Vurpillot, 1977, p.718.)

...the perception of a series of stimuli as a series of groups of stimuli. (Smoll, 1973, p.232.)

• **Definition involving a match or a motor response to an external source:**

...to be at a specific point in space (spatial accuracy) at a specific point in time (temporal accuracy). (Smoll and Schultz, 1978, p.838.)

• **Definition involving movement quality:**

...movement that is aesthetically pleasing to the observer as well as to the one being observed. (Schwanda, 1969, p.573.)

It may seem unjust, or at the worst misleading, to present these definitions separated from the context within which they are formulated. The intention at this point however, is just to use the examples above to show some of the complexities and the vast degree of *divergency* related to the study of rhythm and rhythmic phenomena.- Later, in Chapter 3 we will try to demonstrate some possible *convergencies* of the concept 'rhythm'.

16
1.4. Different Aspects of Rhythm in Music

In the previous section the multidimensionality of the rhythm concept was demonstrated on two levels: (i) In use of 'rhythm' in everyday speech related to various everyday situations. (ii) In scientific applications of the concept, associated with different fields of science. If we try to make the concept more precise by looking at *musical rhythm* instead of discussing rhythm as a phenomenon of interscientific interest, a multidimensionality on a third level becomes apparent. The simple descriptive model below may illustrate this case.\(^{10}\)

![Diagram](image)

*Figure 1.2. A general framework illustrating processes of musical communication. (See further explanation in Bengtsson, 1973.)*

To the left in the diagram is a representation of one or more musicians performing some music, in the middle we have the sound sequence generated by their instruments (as a purely acoustical phenomenon), and to the right is the listener's response to this music. Since the performer himself (in most common cases) also is a listener, and hence is responding to the music he is playing, there is "feedback" from RESPONSE to PERFORMANCE. Moreover, the performer will be musically influenced by interactions with his fellow musicians and a possible audience, causing other kinds of feedback to occur. In addition, a possible "transformation" or change of status, from *responding to performing* subject may take place if the "audience" starts to sing or play, and thus becomes a participating element in the performing process. Depending then on which aspect of the musical communication process is being discussed, different features of musical rhythm will be illuminated.\(^{11}\)

\(^{10}\) This model is found in Gabrielson (1986), and is a special case of models of the musical communication process, presented in Bengtsson (1973). The model, as given above, may be expanded in different ways to include other aspects of the musical communication process; e.g. the composer, a possible transformation process from written music to the performer, the influence of the conductor (if there is any), "feedback" from commercial interests etc. However, for the time being, the model as presented above, seems adequate.

\(^{11}\) Bengtsson (1973) is in this respect quite precise, using different notations denoting the various aspects of musical rhythm; e.g. \(rhythm_s\) denoting experience of rhythm; \(rhythm_{pa}\) denoting physical aspects of rhythm related to physical properties of the instruments being played, as well as acoustical properties of the sound sequence; \(rhythm_n\) denoting the notated representations of rhythm and rhythmic structures in written music.
Gabrielsson (1986, pp.139-140) points at a natural and clarifying sub-division of rhythm response according to the following categories:

1. **Rhythm as experience**, referring to perceived grouping, accents, pulse and tempo, various motion characters (e.g. "swinging", "dancing", "walking"), or feelings of excitement, tension, calmness, release, etc.

2. **Rhythm as overt behavior** includes, for instance, tapping one’s feet, shaking one’s head, clapping one’s hands, and dancing.

3. **Rhythm as psychophysiological response** refers to change in breathing, heart rate, muscle activity, activity in the brain, etc.- that is, various physiological responses which may appear in connection with listening to music.

In the classification above Gabrielsson emphasizes, however, that there are no strict limits between these categories; they are, on the contrary, "different aspects of the "same" phenomenon, the rhythm response, and are interrelated in extremely complex ways" (Gabrielsson, 1986, p.140). As an example it might be added that an experience of rhythm may cause the listener to start dancing, and through this *performed action of dancing* a new *experience* of rhythm is achieved.

A multidimensionality within rhythm as experience is also suggested in (1) above. More explicitly, experience of rhythm may be regarded as comprising the following qualities:12

- Qualities related to the *experienced structure* of the rhythms; e.g. meter, position and strength of accents, uniformity vs. variation, simplicity vs. complexity.

- Qualities related to the *experienced motion character* of the rhythms, such as overall rate (related to the density of all sound events), tempo (rate of the underlying beat), forward motion (e.g. in "galloping" rhythm), and various other motion characters as "swinging", "dancing", "walking", "rocking", etc.

- Qualities related to the *emotional character* of the rhythms, as for instance "vital" vs. "dull", "excited" vs. "calm", "rigid" vs. "flexible", "solemn" vs. "playful".

Strongly emphasizing the large degree of interactions between the different aspects of rhythm response, Figure 1.3. might summarize the classifications so far presented.

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12 See Gabrielsson (1986, pp.140-141).
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Rhythm response

Experience

Overt behavior

Psychophysical

Structure

Motion character

Emotions

Figure 1.3. Schematic outline of sub-divisions and classifications of aspects of rhythm response.

Having discussed various dimensions of rhythm response, what can be said about the *expressive performance* of a piano sonata by Mozart, or the jazz drummer’s swinging cymbal rhythm *performed* on his ride cymbal? More generally: What about the component PERFORMANCE of the musical communication process presented in the beginning of this section?- *Physiological* processes as changes in muscle activity, activity in the brain, and changes in breathing are necessary biological/neurological conditions for performance of music. The *behavior* of the performing musician is, on a physical level, his musical performance, and *into his performance* the musician projects subjective *experiences* of structural, *motional* and *emotional* character. Hence, viewed in this light, a sub-division of aspects of rhythm *performance* similar to the above classification of rhythm response may seem relevant.

On the other hand, it can be maintained that particularly in the case of the jazz drummer and related to situations where the musicians are improvising, the music is created "spontaneously" and the musicians are "at the same time" delivering and receiving musical information. Thus the separation of the components PERFORMANCE and RESPONSE as well as the further sub-divisions of the two, might in these situations obscure rather than illuminate important features of musical rhythm. However, in this respect it might be adequate, once again, to underline the importance of making the subject of your presentation and investigation precise and explicit. Thereby aspects and features *not* being discussed will also be made visible.

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13 Peter Reinholdson’s doctoral dissertation: "Making Music Together" (Reinholdson, 1998) investigates and discusses music-making among players in jazz groups. Very interesting analyses of sociomusical interactions are here presented.
In "The Performance of Music" (Gabrielsson, 1999) empirical research of music performance and related matters is outlined. In his presentation of the different topics, Gabrielsson makes a certain expansion and reconstruction of the model of the musical communication process. The topics addressed follow a kind of chronological order, as Gabrielsson himself points out;

...beginning with the planning of performance, proceeding to various aspects of the performance itself (sight-reading, improvisation, feedback, motor processes, measurements, and models), then to physical, psychological, and social factors that may influence the performance, and finally to performance evaluation. (Gabrielsson, 1999, p.501.)

Through this classification and discussion of various aspects related to music performance, the focus of discussion is put on various features of the performing musician's interactions with musical, cultural, physical, neurological, motory, psychological and social relations, all being factors that in varying degree and at different times may influence the music performance in a fundamental way. Thus, to a large extent the dichotomy between PERFORMANCE and RESPONSE is seen as subordinate to a discussion of music performance as governed by different kinds of interactions. This point of view reflects much of empirical research of rhythm, as will be commented on more thoroughly in Chapter 2. A similar presentation is made by Caroline Palmer in the article "Music Performance" (Palmer, 1997). As an introduction to her discussions, Palmer points out that:

The majority of studies focus on the performance of musical compositions for which notation is available, thus providing unambiguous performance goals. (Palmer, 1997, p.116.)

Moreover, she emphasizes that the performing musician possesses two basic skills that in a crucial way characterize the music performance: (i) **Cognitive skills** related to musical interpretation, knowledge of musical style and understanding of the syntax of musical structure, and the musician's ability of "translating" musical structure in written music into musical expression of the performance; (ii) **Motor skills** associated with the performer's ability of transforming conceptual musical information into adequate movements of the body.

If we inherit Gabrielsson's and Palmer's way of classifying music performance in general, and apply this classification to a study of rhythm performance in particular, an outline of basic features of rhythm performance as studied in empirical rhythm research will take the following schematic form (see Figure 1.4.):
Figure 1.4. Outline of various aspects of rhythm performance as studied in empirical rhythm research: Empirical research of rhythm apply measurements, descriptions, analyses, models and syntheses in the study of (i) Factors that have an influence on the performance, (ii) The rhythm performance itself, and (iii) Evaluation of the rhythm performance. Each of these topics have various sub-topics that are made the subject for discussions and investigations, as illustrated above. Mutual interactions between the different aspects is crucial, and should be taken into account.

The presentation of the different aspects of rhythm in music, schematically outlined in Figures 1.2., 1.3. and 1.4, will constitute a framework for further discussion and presentations of empirical rhythm research in the next chapter.
1.5. Summary

‘Rhythm’ is a concept used in a wide variety of contexts. It is given various meanings and refers to quite different situations in everyday life, in different branches of science, and in music. For some it is meaningful to speak of “the rhythm of life”, “working rhythms” or “bio-rhythms”; in other contexts one may learn about rhythm as basic to memory, or rhythm as an important factor in the process of learning; and listening to music, a groovy rhythm performed by a group playing salsa may make one want to dance. The origin of the rhythm concept is the Greek rhythm, which was one of the key words in Ionia philosophy, and by Plato given the meaning “order in movement”. This view of rhythm as a phenomenon representing an ordering of patterns of movements has much in common with the meaning many today will assign to a general concept ‘rhythm’. Based on biological, astronomical, and, not the least, habitual cycles and well-ordered patterns of movements and events that govern our everyday environments, one might even say that an attraction to rhythm, viewed in this general sense, is fundamentally rooted in human nature.

These many applications of ‘rhythm’ point to the fact that the different meanings assigned to this concept, apart from having something general in common, may be quite divergent. Moreover, these divergencies reflect what may be called the multidimensionalities, or rather the many different aspects of rhythm. To avoid misunderstandings and obscurities, it is of major importance that the choice is made explicit and precise as to which aspect of rhythm is the subject of discussions and investigations.

In musicology, an often used model describing musical processes is the model of the musical communication process, where a kind of dialectic relation between rhythm performance and rhythm response is established. Further sub-divisions of response, as well as performance of rhythm, are also made. This classification may be useful for many purposes, where a study of the various aspects more or less isolated from one another is motivated as meaningful and relevant. However, the different aspects of rhythm interact and are inter-related in quite complex ways. Another approach of rhythm research is thus to investigate the musician’s interactions with various musical, cultural, physical, neurological, motory, psychological and social relations; all being factors that influence the music performance, as well as the music response, in crucial ways.
2

Some Main Results in
Modern Rhythm Research

As discussed in the previous chapter, the rhythm concept is quite complex with rather divergent meanings assigned to different situations of use. This state of affairs is also reflected in the many attitudes taken in the research of the various aspects of rhythm. In the present chapter we will outline some main results in modern rhythm research (the earliest investigations and results presented dating back to the second half of the 19th century). In reading this presentation it is important to be aware of the fact that our aim in this respect is not to be complete and inclusive as to cover all aspects of modern rhythm research. We hope, however, to be able to give an overview of this field which, on the one hand, gives the reader a relevant picture of some achieved results and basic definitions, and on the other establishes a sufficient basis for further discussion in the following chapters. Complementary presentations may be found in Fraisse (1982), Gabrielsson (1986), Palmer (1997), Clarke (1999), and Gabrielsson (1999).

2.1. Research Related to Structural Aspects of Rhythm

2.1.1. Rhythmic Structures in Sequences of Sounds

Experimental research on rhythm started in the last decades of the 19th century as part of early developments in experimental psychology. Wilhelm Wundt, one of the fathers of experimental psychology, discussed rhythm quite extensively in his book: "Grundzüge der Physiologischen Psychologie" (Wundt, 1911). Wundt started by describing the perception of an isochronous sequence of identical sounds. Although there were no physical dissimilarities between the sounds, the listener tended to perceive a grouping of them. This is a phenomenon that has been empirically documented through many different experiments, and is most
commonly referred to as *subjective rhythmization* (see also 1.2.). The next step was to introduce *systematic variations of physical variables*; i.e. to change the duration of certain sounds, increase or decrease the length of the interval between two sounds, and to change intensity, pitch or timbre of some of the sounds. The purpose was to investigate how such changes would affect the perceived grouping, accentuation, and other phenomena. The results from these different investigations were not always consistent, but indicated that any of the variables could affect the perceived grouping in different ways. For example: Increased intensity on a certain sound tended to make this sound begin the group, while an increased duration tended to make the corresponding sound terminate the perceived group.

Determining conditions and "laws" of perceptual grouping of sound sequences has been a major task of research related to structural aspects of rhythm *perception* and rhythm *experience*. A considerable contribution to our understanding of these matters has been given by Paul Fraisse (to a large extent summarized in Fraisse, 1982). In the article "Rhythm and Timing in Music", Eric F. Clarke gives an overview of research relating to the temporal dimension in music. In his presentation Clarke emphasizes the importance of Fraisse’s work by saying:

> Amongst this literature, the work of Paul Fraisse stands out above the research of any other single individual in both its scope and the manner in which it foreshadows the preoccupations of a great deal of the more contemporary work in the area. (Clarke, 1999, p.473.)

Fraisse shows his classical heritage by adopting the definition of rhythm given by Plato: "The order in the movement", and he repeatedly stresses the dependence of rhythm on our body movements:

> La psychologie du rythme commence avec celle des mouvements humains ordonnés. (Fraisse, 1974, p.10.)

According to Fraisse, both animals and people move about with rhythmic movements characteristic of their species, and the very principles of temporal structuring in rhythm originally derives from these movements. In studying spontaneous tapping, Fraisse observed

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14 Fraisse, however, does not approve of this expression, pointing out that: "This expression, which appeared at the end of the nineteenth century (....), must today be considered inadequate, because all perceived rhythm is the result of an activity by the subject since, physically, there are only successions." (Fraisse, 1982, p.156.)

15 In this case, when physical differences are introduced into a sequence of elements, Fraisse speaks of *objective rhythmization* (Fraisse, 1982, p.157.)

16 Clarke (1999).

17 Fraisse (1982, p.150): "We will adopt this definition, which, even in its generality, conveys different aspects of rhythm."
that by far the most ubiquitous relationship between successive tapped intervals was a ratio close to 1:1 (i.e. near-isochronous, corresponding to pendular motion). Fraisse regarded this as intimately connected with anatomical and motor properties, most notably the bilateral symmetry of the body, the pendular movements of the limbs in walking and running, and the regular alternation of exhalation and inhalation in breathing. The natural speed of tapping, the spontaneous tempo, is characteristic of the individual, with 600 msec as the length of the interval between two successive taps being the "most representative". Fraisse distinguishes between spontaneous tempo and preferred tempo, the latter corresponding to "the speed of a succession of sounds or of lights that appears to be the most natural — that is to say, to a regular succession judged as being neither too slow nor too fast" (Fraisse, 1982, p.153). As Fraisse points out:

It is striking that the rhythm of the heart, of walking, of spontaneous and of preferred tempo are of the same order of magnitude (intervals of from 500 to 700 msec). (Fraisse, 1982, p.154.)

In discussing possible durations of rhythmic groups, Fraisse arrives at a "duration limit" for perceiving successive elements as a unity. This limit, observed by Fraisse as being about 5 seconds, corresponds to what has been called the psychological present. Clarke (1999, p.474) points out that the psychological present, or rather "the perceptual present" as he denotes this entity, makes it possible for Fraisse to draw a distinction between the perception of time and the estimation of time, the former being temporally limited in duration by the perceptual present, the latter being dependent on reconstruction of temporal estimates from information stored in memory. In Fraisse (1982, p.165) the following, rather rhetoric, question is asked:

If rhythm is order, arrhythmia is disorder (i.e., it is a priori, a sequence of continuous sounds where no temporal organization is perceptible). A computer can create this type of sequence. Can man?

Fraisse found some very interesting answers to this. He constructed experiments where subjects were asked to produce an uninterrupted series of taps as irregularly as possible. Then, in contrast, they were asked to produce patterns of sounds having an internal structure of their choice. Having gathered the results of these tests, Fraisse studied the temporal structures by calculating the successive ratios between durations by computing the shorter of the two intervals to the longer. In Fraisse’s own words, the conclusions made on this basis were the following (Fraisse, 1982, p.165):

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18 Fraisse (1982, p.153.)
Chapter 2

The first characteristic fact, in rhythm as well as in arrhythmia, is that a ratio of near-equality between two successive intervals predominates (40% of the ratios are less than 1.2). It is as though every sequence were based on a tendency to produce an interval equal to the preceding one, which is evidently the easiest and the most economical activity.

Rhythmic and arrhythmic sequences are constructed on the basis of this regularity. However, the way of breaking regularity is different in the two cases. In arrhythmia, the higher the ratio the less frequent it is. The rupture with equality then happens by a lengthening (or by a decrease) of the preceding interval: small differences become numerous, large ones become rare. In rhythm, on the contrary, small differences are rare. When the subject has broken the regularity, he or she produces a new interval of a noticeable duration. The difference forms about a ratio of one to two.

Thus, rhythmic tapping, in contrast to arrhythmic tapping, exploits a principle of identity or clear differentiation between time intervals, and demonstrates the existence of two separate regions of durations in the subjects'appings; by Fraisse given the names temps longs (long times/long durations) and temps courts (short times/short durations). Moreover, the approximate relation between temps longs and temps courts is 2:1. To quote Fraisse himself (Fraisse, 1982, p.167):

Briefly, patterns are characterized by a composition of basically two sorts and only two sorts of time: short times of 200 to 300 msec and long times of 450 to 900 msec.

Perceptually, temps courts correspond to a perception of "collection" (we have no real sense of the passage of time during each event, but perceive collections of the events grouped together as unities), while temps longs correspond to perception of "durations" (the passage of time during such an interval is perceivable). Hence these categories of time intervals are not just quantitatively but also qualitatively different.

Through his heavy underlining of the mutual interactions between perceptual capacities, motor actions and body movements, Fraisse represents a holistic view of rhythm, where aspects of rhythm perception, rhythm performance and rhythmic movements are seen as fundamentally dependent on one another. He develops his results and theories of rhythm in music through empirical experiments and investigations, all of which are carefully analyzed and discussed. The experiments which he studied are basically directed towards "simple" perceptions triggered by stimuli comprising of identical sounds, sounds differing by one parameter (either duration or intensity or pitch), or taps. This is a deliberate choice made by Fraisse, and indeed, it should be added, a very clever one: Precisely by making the experimental situation more simple, the analysis of the results can be made more exact. However, in such simplifications there are always possibilities of arriving at results that may seem plausible under the given conditions, but when applied to the more complex situation; in
our case: musical rhythm as part of a musical communicative interaction, become more or less irrelevant. Fraisse himself is extremely well aware of this point, and with reference to his experiments and analyses described in this section, his comments are the following:

The above analyses have permitted us to extract the laws characteristic of rhythm perception. However, the stimuli used were far from musical, since these researches used only taps, identical sounds, or at best, two types of sound of different duration, intensity, or pitch. Musical rules, however, do not escape the fundamental laws that we have demonstrated. Without doubt, these laws do not explain music any more than gravity explains the art of architecture. But there is no architect who ignores gravity any more than there is a musical rhythm that does not respect perceptual laws. (Fraisse, 1982, p.170.)

2.1.2. Rhythmic Structures in Music

An influential musicological text on rhythm is Cooper and Meyer: "The Rhythmic Structure of Music" (Cooper & Meyer, 1960). This book was written as a theoretical textbook for students of music, where the structural aspects of rhythm were heavily emphasized. In the first chapter we read:

To experience rhythm is to group separate sounds into structured patterns. Such grouping is the result of the interaction among the various aspects of the materials of music: pitch, intensity, timbre, texture, and harmony — as well as duration. (Cooper & Meyer, 1960, p.1)

The music analyzed by Cooper and Meyer was classical music of the Western tradition, for which a written score exists. They proposed the following definition of rhythm:

Rhythm may be defined as the way in which one or more unaccented beats are grouped in relation to an accented one. (ibid., p.6)

and postulated that there are five basic rhythmic groupings, denoted by terms traditionally associated with prosody: Jamb (\(^-\)), anapest (\(^^-\)), trochee (\(^^\)), dactyl (\(^^\)), and amphibrach (\(^^-\)). Cooper and Meyer asserted that rhythmic organization is architectonic, and they consistently applied the five basic rhythmic groupings above in an analysis of the musical score on different architectonic levels. The examples range from simple tunes up to movements of a symphony, but the basic rhythmic units remain the same.

Different criticisms have been raised against Cooper and Meyer’s approach, one objection concerning their attempt at extending the use of the rhythmic groupings to much larger formats than usual: When their basic rhythmic groups are applied to higher architectonic levels, the time span of the segmented parts of the music exceeds the limit of the
psychological present, in which case one would rather talk about "conceived rhythm", i.e. "The rhythm is ... inferred from a mental construction" (Fraisse, 1982, p.150). As Gabrielsson remarks:

It seems much more reasonable to use concepts like "structure" or "form" for such purposes rather than add more to the already large confusion concerning the rhythm concept. (Gabrielsson, 1986, p.145)

Another theoretical starting point is taken by Yeston (1976). He applies ideas presented and developed by the music theorist Henrich Schenker, and describes how the rhythmic "foregrounds" are products of deeper rhythmic strata (levels) of interaction.19 Interesting is Yeston's distinction between "rhythmic consonance", in which the rate of any level can be expressed as a multiplication or division by an integer greater than 1 of the rate of any other level, and "rhythmic dissonance", in which this is not the case (e.g. in polyrhythms).

A major contribution to the theoretical and conceptual development of music theory, which has also influenced later empirical investigations, is given by Lerdahl and Jackendoff's "A Generative Theory of Tonal Music" (Lerdahl & Jackendoff, 1983). Lerdahl and Jackendoff present their basic attitude towards music theory quite explicitly by saying:

The present study will justify the view that a piece of music is a mentally constructed entity, of which scores and performances are partial representations by which the piece is transmitted........ Seen in this way, music theory takes a place among traditional areas of cognitive psychology such as theories of vision and language. (Lerdahl & Jackendoff, 1983, p.2)

Related to our discussion in this section, a significant contribution of Lerdahl and Jackendoff is their distinction between grouping and meter. They pointed out that rhythm in the tonal/metric music of the Western tradition is structured according to two independent principles: Grouping structure "expresses a hierarchical segmentation of the piece into motives, phrases, and sections" (Lerdahl & Jackendoff, 1983, p.8), and metrical structure "expresses the intuition that the events of the piece are related to a regular alternation of strong and weak beats at a number of hierarchical levels" (ibid.). Although theoretically independent of one another, there is a connection between grouping and meter, made apparent by the fact that strong points in the meter often coincide with group boundaries. Another important point to notice is that grouping structure is concerned with phenomena that extend over specified

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19 See also Gabrielsson (1986, pp.145-146).
durations, whereas meter is concerned with theoretically durationless moments in time, as pointed out by Clarke (1999, p.478).

Basic to Lerdahl and Jackendoff’s theory of tonal music is a proposed set of principles, or rules, to account for the segmental structure in music. The rules of their theory are divided into two distinct types: Well-formedness rules, "which specify the possible structural descriptions" (Lerdahl & Jackendoff, 1983, p.9), and preference rules, "which designate out of the possible structural descriptions those that correspond to experienced listeners’ hearings of any particular piece" (ibid.). Lerdahl and Jackendoff offer no empirical evidence for the operation of these preference rules. Deliège (1987), however, investigated their empirical validity in the context of both highly reduced experimental materials and extracts of real music from Bach to Stravinsky. Her investigations demonstrated the validity of the predictions made by the preference rules, and provided some evidence for the relative strength of the different rules.20

Todd (1994a) developed a model of rhythmic grouping based on the idea that the functioning of the auditory system can be seen as the operation of a number of energy-integrating low-pass filters with differing time constants.21 Through applying this low-pass filtering to a standard audio recording of a professional performer, Todd extracts so-called rhythmograms that bear a striking resemblance to tree diagrams that depict grouping analyses like those theoretically developed by Lerdahl and Jackendoff. Todd’s approach and model indicate some very interesting perspectives for future investigations, as commented upon by Clarke (1999, p.482):

What is interesting and provocative about the model,..., is the amount of grouping and sectional structure it can recover from real performances despite its "knowledge-free" approach. It suggests powerfully that rather more structural information is available within the acoustical signal itself than has hitherto been recognized and that processes in the more peripheral parts of the auditory system may be more important for rhythm perception than was at one time believed.

2.1.3. Perception of Meter

The area of rhythm research that has attracted the most consideration since about 1980 is meter, a fact Clarke (1999) ascribes to the dominant influence of metrical structure in the music of the Western tradition and in most popular music. Research on meter has taken two

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20 See Clarke (1999, p.479) for complementary comments on these matters.
21 See also Clarke (1999, p.480).
forms: (i) Empirical investigations, and (ii) computational models. As mentioned above, Lerdahl and Jackendoff viewed metrical structure as the regular, hierarchical pattern of strong and weak beats to which the listener relates musical events. For Lerdahl and Jackendoff three kinds of accents become important in the perception of meter:

- **Phenomenal accent:** "any event at the musical surface that gives emphasis or stress to a moment in the musical flow. Included in this category are attack points of pitch-events, local stresses such as sforzandi, sudden changes in dynamics or timbre, long notes, leaps to relatively high or low notes, harmonic changes, and so forth." (Lerdahl & Jackendoff, 1983, p.17)

- **Structural accent:** "an accent caused by the melodic/harmonic points of gravity in a phrase or section – especially by the cadence, the goal of tonal motion." (ibid.)

- **Metrical accent:** "any beat that is relatively strong in its metrical context." (ibid.)

Lerdahl and Jackendoff point out that phenomenal, structural, and metrical accents relate and interact in various ways. Phenomenal accents function as perceptual inputs to metrical accents, which is to be understood in the sense that the moments of musical stress in the raw signal (caused by physical properties of the stimulus such as changes in intensity, simultaneous note density, register, timbre, or duration) serve as "cues" from which the listener attempts to extrapolate a regular pattern of metrical accents. Once a clear metrical pattern has been established, a sense of meter is perceived. Clarke (1999, p.482) summarizes this characterization of meter perception by saying:

In general terms, perceiving meter is characterized by Lerdahl and Jackendoff as a process of detecting and filtering phenomenal and structural accents so as to discover underlying periodicities. These constitute the rates of repetition (...) that define the meter and confer metrical status on regularly recurring phenomenal (and structural) accents.

It is important to note that in Lerdahl and Jackendoff's terminology metrical accent "is a mental construct, inferred from but not identical to the patterns of accentuation at the musical surface." (Lerdahl & Jackendoff, 1983, p.18) – This view of meter as a construct that has no reality in the stimulus itself has caused some authors to prefer to talk of meter induction rather than meter perception.

Of the empirical investigations into meter, the vast majority have been concerned with the influence of durational factors. Important in this respect is the work of Fraisse as discussed in section 2.1.1. Povel (1981), starting from a position based on Fraisse’s work,
developed a model that demonstrates important elements of many of the more recent accounts of meter perception. His "beat-based" model proposed that perception of rhythmic sequences depends on two steps: (i) The segmentation of the sequence into parts of equal length (beats), based on the detection of regularly occurring accents; (ii) the identification of individual events as specific subdivisions of these beats into a small number (usually only two or three) of equal parts, or parts relating to one another in a ratio of approximately 1:2. The model is developed further in Povel and Essens (1985), the essential idea of which is that a rhythmic sequence induces an "internal clock" with a period that captures the primary metrical level of the sequence. Complementary comments on this model may be found in Clarke (1999, pp.483-484). Clarke (ibid., pp.483-489) presents, discusses, and comments several other interesting models of meter perception. We will only briefly present some of these models here, and refer to Clarke for complementary information.

Pamcutt (1994) proposes a theory of meter perception based on the salience of different possible "pulse trains," and the perceived accentual strength of individual pulses, in music. The model is based on principles that are close to classical psychophysics, and can be summarized as consisting of the following series of processes: (a) individual event durations are converted into phenomenal accents; (b) an absolute tempo factor and pattern-matching process select the most salient pulse trains (i.e. sequences of isochronous phenomenal accents) — the single most salient level of pulsation being identified as the tactus (the level at which a listener is most likely to tap his/her foot); (c) the three or four most salient and mutually consonant pulse trains are superimposed to create a metrical hierarchy for the sequence, with an associated overall salience. Clarke (1999) points out that the great strengths of this model are its systematic simplicity and the fact that to a large extent it is a quantitative model, thus permitting systematic and rigorous testing.

Desain (1992) provides another approach that owes its origin to connectionism. The model is an extension of Desain and Honing's (1989) connectionist approach to what they describe as the "quantization problem" — the extraction of discrete durational values in reasonable relationships with one another (essentially equivalent to standard Western rhythmic notation) from a string of continuously variable durations (equivalent to the raw data of human performance). The method by which this is done is to use a constructed connectionist "quantizer" to "clean up" messy timing data so that the meter may be inferred. The quantizer works to adjust durations so that every pair of durations is adjusted toward an integer ratio, if it is already close to one.
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A very interesting model for meter perception and rhythm response is presented by Large and Kolen (1994). Their basic idea is to view the perception of metrical structure as a dynamic process where the temporal organization of external musical events synchronizes, or entrains, a listener's internal processing mechanisms. The starting point for their mathematical model of entrainment is resonance theory, in which the behavior of oscillatory units that continuously adjust their phase and period to the rhythmic characteristics of a stimulus sequence is used to model the human response to rhythm. The fundamental idea is that neural units with differing natural resonances, and "tuned" to be more or less sensitive/restrictive in terms of what they will adjust to, adapt to the periodicities of external stimulus events. The authors show that a system using six oscillators covering a resonance range from 600 msec to 2560 msec will successfully track the meter of a fairly complex piece of real musical performance.

Using a strongly biologically motivated approach, Todd (1994a, 1994b; Todd & Brown, 1996) has proposed a model for meter perception arising out of the filter-based model for grouping described in 2.1.2. In essence, Todd proposes that a frequency-domain multiscale filter system capable of detecting metrical structure exists in parallel with the time-domain multiscale filtering that is responsible for detecting the grouping properties of rhythmic structures.

Dixon (1999) has described an interesting computer implemented beat tracking system which analyzes acoustic data, detects the salient note onsets, and then discovers patterns in the intervals between the onsets, from which the most likely inter-beat interval is induced. The system employs a bottom-up approach to beat tracking, assuming no prior knowledge of the music such as the time signature or approximate tempo.

2.1.4. Transformations from Musical Structure to Expressive Performance

Many findings have established a causal relationship between musical structure and patterns of performance expression (Clarke, 1988; Palmer, 1989; Sloboda, 1983). Palmer (1997) points out that one of the most well-documented relationships is the marking of group boundaries, especially phrases, with decreases in tempo and dynamics (Henderson, 1936).

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It is tempting, as well as interesting, to view the approach of Large and Kolen as a contemporary analogue to the correspondance thinking discussed in section 1.1.; the mathematical resonance model of Large and Kolen being a modern parallel to the ancient idea of "tuning" the inner strings of man's soul such that when the cosmic lyre is played, the strings of the soul will be brought in sonoric resonance.
Patterns of rubato often indicate a hierarchy of phrases, with amount of slowing at a boundary reflecting the depth of embedding (Shaffer & Todd, 1987; Todd, 1985, 1989). The greatest correspondence between expressive timing and intensity in performance is found at an intermediate phrase level (Palmer, 1996a; Todd 1992a), and performers' notated and sounded interpretations tend to differ most at levels lower than the phrase (Palmer, 1989; Repp, 1992a). Metrical structure also influences performance expression. Palmer (1997) comments upon some of the achieved results on these matters (e.g., Henderson, 1936; Palmer & Kelly, 1992; Sloboda, 1983, 1985), summarizing by saying that these findings suggest that there is no one set of necessary and sufficient expressive cues to denote meter.

Palmer (1997) raises the question: Do performers use a syntax or formal set of rules to generate expression? According to the view that musical structure is related to performance expression in terms of explicit generative principles, systematic patterns of expression result from transformations of the performer's internal representation of musical structure (Clarke, 1993, 1995). Palmer (1997) points out that three types of evidence support the view that structure systematically generates expression: (i) The ability to replicate the same expressive timing profile with very small variability across performances (cf. Henderson, 1936; Seashore, 1938); (ii) the ability to change an interpretation of a piece and produce different expression with little practice (Palmer, 1989, 1996b); and (iii) the ability to perform unfamiliar music from notation (sight-read) with appropriate expression (Palmer, 1988; Shaffer, 1981; Sloboda, 1983).

A more thorough presentation and discussion of research on the rhythm performance in itself will be given in section 2.3. of this below. Some further comments on the transformations, or mappings, of rhythmic structure into expressive performance will also be given there. With these remarks we end our presentation of research related to structural aspects of rhythm, and change our focus to motional and emotional qualities of the rhythm experience.

2.2. Motional and Emotional Aspects of Rhythm Experience

The majority of investigations and studies of rhythm experience and rhythm response has been directed towards experience of the structural aspects of rhythm. This is not surprising, since the motional and emotional qualities by their very nature may seem less
accessible to explicit analyses and experiments than the more quantitative structural aspects. However, interesting results have been achieved, some of which will be briefly outlined here.

Among the motional-emotional aspects of the rhythm experience, tempo is in many respects the most familiar and significant. Tempo is very important for the listener's impression of the music. As Fraisse points out\(^{23}\), the possibility of rhythmic perception depends on tempo in a fundamental way, demonstrated by the fact that when the tempo slows down too much, the rhythm and also the melody disappears. It is, moreover, difference in tempo that most frequently is noted first when different performances of the "same" piece of music are compared. However, 'tempo' is a concept that may have different meanings, and how rapid the music is perceived is not just a function of a given metronomic value, but also depends on many other factors such as the density of sound events, the specific structure of the rhythm pattern, melodic and harmonic progressions, etc. On these matters Gabrielsson (1986, p.148) comments:

> It seems preferable to make a distinction between (perceived) tempo and rate (rapidity, speed), so that tempo designates the perceived rate of the beat or pulse, while the latter terms refer to the "total" impression of speed (.....); note also that metronomic tempo and perceived tempo may be different.

It might be relevant to add as an example that a listener at the same time may perceive a slow tempo caused by the "basic pulse" in the ballad "Parker's Mood" played by a jazz quartet featuring Charlie Parker\(^{24}\) and a fast tempo, or rapid "motion", as a result of Charlie Parker's "double tempo feeling" and the high density of sound events in his playing.\(^{25}\)

Although tempo is important, there are many other motion characters of rhythm experience. The importance of giving the music the proper motion character is demonstrated in Irving Mill's title to a melody of the famous big band leader, Duke Ellington: "It Don't Mean a Thing If It Ain't Got That Swing". As Gabrielsson puts it:

> "Swing" in jazz and dance music is one of the best known examples of motion character. (Gabrielsson, 1986, p.150)

Different dances are associated with different specific motion characters, sometimes hinted at by their names, e.g., "rock n'roll", "twist", "cha-cha-cha", and in the musical performance of the corresponding rhythmic patterns these various motion qualities must be articulated

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\(^{23}\) See Fraisse, 1982, p.151.

\(^{24}\) Charlie Parker (1920-1955), an outstanding saxophone player, of major importance for the development of be-bop in jazz.

properly in order to make the music swing in the "right" way. A considerable amount of
research has been directed towards documenting and classifying how various parameters of
musical performance influence characteristic features of different musical styles. This will be
discussed further in section 2.3.

As to emotional aspects of rhythm experience, older investigations of Hevner (1937)
and Rigg (1940) indicated, as one might have guessed, that faster tempi were associated with
judgements like "pleasant", "happy", "exciting", "restless", and slower tempi mostly with the
opposite judgements. Partly similar results were also found by Motte-Haber (1968), Behne
(1972), and Gabrielsson (1973); but, as pointed out by Gabrielsson (1986, p.149), "the
question is complex for several reasons, among them the possible dependence on other factors
in the music". In this respect recent research by Juslin (1997a, 1997b), applying a method of
analysis and synthesis of musical performances, has shown that both performers and listeners
can use a number of cues in the performance (e.g., tempo, sound level, and timbre) to encode
or decode particular emotions (e.g., "sadness", "anger", "happiness"). Gabrielsson & Juslin
(1996) reported that the performer's expressive intention has considerable effects on the so
called timing patterns of the performances, while Juslin & Madison (1998) show that listeners
may also use such timing patterns to decode emotional expressions.

A very interesting approach in classifying motional aspects of rhythm experience is
given by empirical studies which demonstrate that listeners can recover motion information in
the musical acoustic signal and convert this information into overt movements of the body
(e.g., Sievers, 1924; Becking, 1928; Truslit, 1938; Clynes, 1977, 1983, 1986a, 1986b, 1992;
Clynes & Walker, 1982; Todd, 1992a, 1992b, 1992c, 1993). In these studies the principal
methodology for demonstrating that in fact music does convey movement information is the
reconstitution of an analogous spatial movement by a human listener. The listener's body thus
acts as a transducer for the coding of musical movement. Through strongly emphasizing the
connections and interactions between the motional-emotional qualities of rhythm and the
body movements of the performer as well as the listener, these investigations represent a
holistic view of performance and experience of rhythm that in many respects resembles the
classical correspondence thinking and the idea of movement as fundamental in rhythm (cf.
Section 1.1.). Patrick Shove and Bruno H. Repp: "Musical motion and performance:
thoretical and empirical perspectives", (Shove & Repp, 1995) give a very instructive
presentation of research fundamentally related to this approach. They point to Eduard Sievers,
Gustav Becking, and Alexander Truslit as three German "pioneers" who developed a
vocabulary of different movement curves as geometric, or rather "kinematic", representations
of: (i) literary works (Sievers: "Schallanalyse"), (ii) compositions by different composers (Becking's table of conducting curves for selected German composers), and (iii) musical dynamics and agogics (timing variations) (Truslit's three basic types of movement curves portraying the melodic dynamics in space). Whereas Sievers and Becking mainly studied movements of the arms and fingers, Truslit was concerned primarily with movements of the whole body. (See Shove & Repp, 1995, pp.65-72, for complementary presentation of the work of Sievers, Becking, and Truslit.)

Following Shove & Repp (1995), Manfred Clynes and Neil Todd can in many respects be seen as two modern successors of these German pioneers. Over a number of years, Clynes (1977) developed the notion of essentia forms, dynamic time forms which characterize basic emotions. To measure them, he devised an apparatus called the sentograph, consisting of a button sensitive to finger pressure in vertical and horizontal directions, and a computer registering the pressure over time and averaging successive pressure cycles. It was found that subjects who imagine certain basic emotions (love, anger, grief, etc.) while pressing on the sentograph, produce quite different pressure curves for different emotions. The same technique has also been used to study the "motor output" of auditory and musical rhythm. For instance, the subject listens to repeated two-pulse patterns with certain characteristics of durations and amplitudes, and "follows" them by pressing on the sentograph. The resulting "motor pulses" for various cases differ in general form. (See Clynes & Walker, 1982, and also Gabrielsson, 1986, p.151.)

Another application of the sentograph is demonstrated in Clynes study of composers' "inner pulse" (Clynes, 1977, 1983; Clynes & Walker, 1982). To determine graphic representations of these inner pulses the subjects, in this case several professional musicians, were asked to press rhythmically on the sentograph while imagining various works of Beethoven, Mozart, Schubert and others. It was then shown (cf. Clynes, 1977) that the average vertical pressure curves show striking differences between different composers, and considerable agreement within composers across different subjects and different pieces. As an explanation of this observation, Clynes pursued the idea that composers' personal pulses must somehow be manifested in the expressive microstructure of an expert performance (cf. Clynes 1983, 1986a, 1986b).

The basic idea applied by Clynes is very interesting. It consists of:

1. To "translate" motional-emotional qualities of rhythm into visual, graphic representations (in Clynes' case: the registrations on the sentograph) caused by the listener's body movements.
2. To classify motional and emotional qualities by means of a classification of the corresponding visual representations.

Through the application of this idea Clynès shows that he works in the tradition of the earlier German pioneers, especially Becking, presented above. A fundamental necessary condition for the significance of this approach is the existence of some kind of similarity, or analogy, between the classification of visual representations and the classification of motional-emotional qualities of rhythm. However, to what extent this method is valuable in providing relevant information is heavily dependent upon the nature of the "correspondance" established. Although Clynès' approach and the results he obtains are both interesting and convincing, it might in this respect be questioned whether the pressure of a finger is the most adequate expression of motional-emotional qualities of rhythm.

As Shove & Repp (1995, p.75) point out, Neil Todd, like Truslit, is concerned primarily with motion at the level of the whole body, rather than of the limbs or fingers. Todd appeals, as did Truslit before him, to physiological evidence concerning two distinct motor systems, the ventromedial and lateral systems (Todd, 1992b; see also Todd, 1992a, 1992c, 1993). The former controls body posture and motion, and is closely linked with the vestibular system. Since larger masses are to be moved, the movements are slower than those possible with feet, hands and fingers, which are controlled by the lateral system. Typically their cycles extend over several seconds, whereas the pulse microstructure studied by Clynès (and executed by finger pressure on the sestograph) is contained within cycles roughly one second in duration, which may be nested within the larger cycles described by Truslit and Todd.

Truslit and Todd's view of the whole body as a transducer for coding motional and emotional qualities of rhythm seems intuitively more appropriate than Clynès' registration of finger pressure. On the other hand, registrations of movements of the whole body, not least discussions and model constructions of causal relations between body movements and motional-emotional qualities of rhythm involving a larger number of parameters to be considered, appear more difficult and ambiguous than establishing correlations between finger pressure and motional-emotional aspects. These are matters that will be further commented below (especially in 2.5. and Chapter 4).

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26 Hence, again, the link to classical correspondance thinking is apparent.
27 This comparison of Clynès and Todd is found in Shove & Repp, 1995, pp.75-76.
2.3. From Seashore to SYVAR

2.3.1. "Deviation from the Exact"

Around 1930, a large group of researchers headed by Carl E. Seashore studied music performance at the University of Iowa. Most of their reports appeared in two volumes edited by C.E. Seashore (1932, 1937), both classics in music performance literature.\(^{28}\) A characteristic feature of music performance, soon discovered by C.E. Seashore and his coworkers, is different kinds of variabilities in performance and various deviations from presumed properties according to musical notation, e.g. deviations in relative note duration compared to simple ratios like 2:1, 3:1, etc. defined in the notation of music, fluctuations in tempo, and various dynamic deviations.\(^{29}\) As an "explanation" of these facts C.E. Seashore stated a principle concerning "deviation from the exact" as a characteristic feature of artistic expression:

The unlimited resources for vocal and instrumental art lie in artistic deviation from the pure, the true, the exact, the perfect, the rigid, the even, and the precise. This deviation from the exact is, on the whole, the medium for the creation of the beautiful - for the conveying of emotion. (Cited from H.G. Seashore, 1937, p.155.)

Similar ideas were expressed in Germany by von Kries (1926) and Truslit (1938). They both stated that it is the performed deviations from accuracy which provide "life to music":

Diese feinen Abstufungen in den Verhältnissen der Zeitdauer und der Stärke der Töne, die der Künstler durch die Tätigkeit seines Gestaltens hineinbringt, sind es vor allem, durch die er aus dem Ton lebendige Musik erstehen lässt. (Truslit, 1938, p.29)

Truslit generally meant that these phenomena in music performance are reflections of "inner motion", and that "Bewegung ist das Urelement der Musik" (ibid., p.53). As we know from the foregoing section, Truslit, moreover, proposed three basic types of movement curves portraying the melodic dynamics in space.

The above statements, underlining the importance of various "deviations" in order to give "life" to music, mainly refer to motional and emotional aspects of rhythm. As discussed

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\(^{28}\) As pointed out by Gabrielson (1999).

\(^{29}\) Even earlier, Sears (1902) measured the tone durations in organists' performance of five hymns, using an electromechanical device which permitted the registration of the key depressions on a "kymograph"; while Hartman (1932) used "player rolls", produced for reproduction by mechanical pianos, to measure performances by a pianist. Both Sears and Hartman found considerable variations in the durations of tones designated by the same note value in the score. (See also Gabrielson, 1986, p.138.)
in section 2.2., Truslit was concerned with transforming motional qualities of music into overt movements of the body; and in the above quotation from Seashore, deviation from the exact is correlated to "conveying of emotion". However, as pointed out in 2.1.4., various transformations from musical structure to "expressive performance" have also been studied and documented, thus demonstrating basic dependencies between different kinds of "deviations" and structural aspects of rhythm. Clarke (1999) emphasizes this correlation by asserting that a distinction must be made between the structural properties of rhythm, which according to Clarke are based on integer ratios of durations as defined in the notation of music, and their so-called expressive properties – "continuously variable temporal transformations of the underlying rhythmic structure" (ibid., p.489). Clarke goes on to say:

These temporal transformations, referred to by some authors (…) as expressive microstructure, are what the term "timing" identifies, and there has been considerable attention paid to the nature and origins of these timing properties in performed music, as well as a rather smaller literature on their perceptual consequences for listeners (ibid., p.489).

Clarke’s view of timing as continuous transformations of rhythmic structure is extremely interesting. It seems to pinpoint the very essence of the processes by which conceptualized musical information, through an interaction of cognitive skills and motor skills, is transformed into live performances of music. 39 A schematic illustration of such processes of temporal transformation is given in Figure 2.1 below.

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39 Cf. also Palmer (1997) and section 1.4.
RHYTHMIC STRUCTURE:

Patterns of:

* Durations
* Dynamics
* Pitch
* Timbre
* Harmonics

Inducing conceptualized structures of:

(a) Meter
(b) Grouping

(Live Performances of Rhythm)

\[ \theta_1 \]
\[ \theta_2 \]
\[ \theta_3 \]
\[ \theta_k \]

Characterized by different kinds of "deviation from the exact"

(cf. H.G. Seashore, 1937)

Figure 2.1. Illustration of temporal transformations of conceptualized rhythmic structure into live performances of rhythm.

\( \theta_1, \theta_2, \theta_3, \ldots, \theta_k \) are different temporal transformations, or mappings, of conceptualized rhythmic structure into various live performances of rhythm. These different transformations, referred to by some as expressive microstructure (e.g., Clynes, 1987, 1983; Repp, 1992a), and by Clarke as timing, are characterized by stylistic features as well as individual preferences and physical constraints (e.g. physical corporal constraints of the performer, physical constraints of the instrument being played, or of the physical environment). A result of, and also a motivation for, the various temporal transformations is communication of motional and emotional musical qualities from the performer to the listener.

Clarke goes on discussing "timing" as temporal transformations by saying that the distinction he makes between "rhythm" (in a very narrow sense as structure derived from ratios according to duration values in musical notation) and "timing" is perceived through a perceptual mechanism of "categorical perception". According to Clarke (1999, p.490) both Clarke (1987) and Schulze (1989) have demonstrated empirically the existence of categorical perception. Moreover, Clarke explains:
In general terms, the idea is that listeners assign the continuously variable durations of expressive performance to a relatively small number of rhythmic categories. The pattern of these categories constitutes the rhythmic structure of the sequence, and the departure of each duration in the original performance from its appropriate categorical target value is understood as expressive timing (Clarke, 1999, p.490).

Thus, the notion ‘expressive timing’ seems to be used in order to make the former ‘timing’ more precise. Based on Clarke’s explanation above, we understand ‘expressive timing’ as denoting a process by which conceptualized structural properties of rhythm are transformed into live performances of rhythm. Hence, viewed in this manner, each transformation, 0j above, is an example of expressive timing. We will adopt this notion of expressive timing, which appears as an interesting and useful concept in describing characteristic features of rhythm performance. However, we will not restrict ‘rhythm’ to include only formalized structure, as Clarke seems to do in his distinction between rhythm and expressive timing.31

Sounding musical consequences of expressive timing are “artistic deviations”, as Seashore denotes these phenomena (see previous quotation). Other terms used in contemporary research are, for instance, “expressive deviations” and “systematic variations”.32 During recent years a considerable amount of research has been directed towards specifying the principles governing expressive performances (different overviews are given in, e.g., Shove & Repp, 1995; Palmer, 1997; Clarke, 1999; Gabrielson, 1999). This research has used a mixture of empirical measurement and simulations, today to a great extent heavily dependent on computer technology for accurate registrations of performances and different constructions of models. In spite of the modern equipment and techniques used, however, much of contemporary research on music performance has been foreshadowed by the studies of C.E. Seashore and his coworkers. Referring to the investigations undertaken at the University in Iowa in the 1930s, Gabrielson (1999, p.532) makes the following point:

Readers familiar with contemporary performance studies will realize that much of the data and results in today’s investigations was in fact presented already in these early studies, which seem unknown to many.

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2.3.2. The SYVARD Hypothesis

Some 40 years ago, around 1960, the Swedish musicologist Ingmar Bengtsson initiated a research project on musical rhythm in Uppsala, Sweden. He was soon joined by the psychologist Alf Gabrielsson. Presentations of this project are given in Bengtsson, Gabrielsson & Thorsén (1969) and Bengtsson (1974a). A basic starting point for Bengtsson and Gabrielsson was to investigate relations between performed and experienced musical rhythm, by means of analyzing the acoustic sound sequence generated by the instruments of the performers.\textsuperscript{33} Hence, the three basic components of the musical communication process presented in section 1.4.; PERFORMANCE, SOUND SEQUENCE, and RESPONSE are all the focus of this approach. The main hypothesis is that live music performance is usually characterized by some kind of \textit{systematic variations} (SYVAR) of \textit{durations} (D) compared to chronometric regularity; the so-called \textit{SYVARD hypothesis}.\textsuperscript{34} Systematic variations were described in terms of deviations from a norm. As a reference norm for variations of durations Bengtsson and Gabrielsson used the temporal relations of standard musical notation, where e.g. a quarter note \textit{by definition} has a duration equal to twice the duration of an eighth note. Hence, the Uppsala approach, headed by Bengtsson and Gabrielsson, used systematic variations of durations compared to simple integer relations in musical notation in attempts at describing and classifying characteristic features of performed and experienced musical rhythm.

Examples showing how such systematic rhythmic deviations are characteristic for different \textit{styles} of performance are found in the descriptions of Viennese waltzes and some Swedish folk music, see Bengtsson (1974a,b), Bengtsson & Gabrielsson (1977, 1983), and Bengtsson, Gabrielsson & Thorsén (1969). \textit{Analytical methods} for describing musical performances based on the SYVARD hypothesis were developed by Bengtsson & Gabrielsson (1983), and in the same article perceptual effects of changing timing and articulation parameters by means of \textit{synthesized versions} of the waltz in "Die Fledermaus" by Johann Strauss Jr. are demonstrated.

In all the above mentioned examples systematic deviations were found at \textit{different rhythmic levels}; at the sound event level (i.e. for single tones or groups of tones within a beat), at the beat level, at the measure level, at the phrase level, etc. (see also Bengtsson &

\textsuperscript{33} See also Bengtsson (1973, p.165).

\textsuperscript{34} Systematic variations of \textit{intensity} were also encountered, but not to the same extent as duration (see also Gabrielsson, 1999, pp.332-338).
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Gabrielsson, 1983). It is interesting to note that these different levels of deviation represent performed correspondances to the different hierarchical levels of rhythmic structure, as described by Lerdahl & Jackendoff (1983). Furthermore, it was shown that the deviations may be characteristic for the individual, being different for different performers, or style specific, related to the type of music performed. Referring to demonstrated deviations from mechanical performance in a pianist’s performance of the theme in Mozart’s Piano Sonata in A major, Gabrielsson makes the following remark:

The deviations may relate to various structural aspects of the rhythm experience, e.g., they may help in clarifying the intended meter, accents, sub-divisions, etc. However, the main impression is that they essentially affect motional and emotional qualities (Gabrielsson, 1986, p.156).

Different kinds of systematic deviations in duration and dynamics characterizing various live performances of music are also documented by, among others, Clarke (1982, 1985) related to different piano performances of compositions by Erik Satie; Palmer (1989) and Behne & Wetekam (1993) studying performances of the theme in Mozart’s Piano Sonata in A major; Repp (1990) comparing different pianists’ performances of a Beethoven minuet; Repp (1992a) analyzing different performances of Schumann’s ”Träumerei”; Povel (1977), Cook (1987), Shaffer & Todd (1987), and Shaffer (1995) studying various performances of the first prelude in C major in J.S.Bach’s ”Das wohltemperierte Clavier”. All of these results, as well as some other related investigations, are further commented upon by Gabrielsson (1999, pp.532-538).

2.3.3. SYVARD in Non-Notation-Bound Music

The majority of the empirical studies carried out by Bengtsson and Gabrielsson have focused on performances for which notation is available, thus providing the musician with explicit guidelines that to a great extent govern the performance. The same is true for the other investigations referred to in section 2.3.2. above. However, the existence of a score guiding the musician’s performance imposes some obvious limitations as to the possible rhythmic displacements and deviations in the performance. Extremely interesting it thus becomes to try to classify possible relations between rhythmic deviations and individual or

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35 Compare also Figure 2.1. illustrating the transformations $\theta_1$, $\theta_2$, ..., $\theta_6$ of rhythmic structure into performance of rhythm.

36 Observe that all the studies mentioned in section 2.3.2. focus on piano (or keyboard) performances of written music.
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style specific characteristics of non-notation-bound music, where an even stronger correlation of systematic rhythmic deviations and typical performance features seems likely to appear.\(^{37}\)

An attempt at studying non-notated, improvised music applying the SYVARD hypothesis as a basic premise is found in Reinholdsson (1985). Reinholdsson defines his task as twofold: on the one hand, to analyze an eight-bar drum solo by the jazz drummer Roy Haynes by means of SYVARD detection and on the other, to participate in developing a scientific method suitable for this kind of investigation. Related to our presentation and discussion of SYVARD, what is most interesting in Reinholdsson’s research seems to be his very problem identification, and the choice he makes as to what style and what musician is to be in focus. The focus of Reinholdsson’s empirical studies is a style of music fundamentally based on rhythm and rhythmic expression; jazz music, or viewed in a broader context, Afro-American music. Moreover, as an expressive performer of jazz music, Reinholdsson chooses to study the drummer, the musician that within the various genres of Afro-American music contributes most to, and even generates, the rhythmic unfolding. On these matters Reinholdsson himself comments:

\[\ldots\text{this pilot study may have served its purpose in calling attention to an apparently neglected category of performers and performances in the line of jazz analysis, as well as rhythm research (Reinholdsson, 1987, p.120).}\]

However, precisely viewed against Reinholdsson’s focus on the jazz drummer, it appears a little surprising that the object of his study is a drum solo and not the drummer playing the “groove”: the expressively performed rhythmic basis establishing a musical, temporal reference for the other musicians in the band. It is primarily groove-playing that characterizes the drummer’s role in major parts of Afro-American music, and it is, moreover, related to the performance of the groove one would expect to find relations between SYVARD and individual or style specific features of the performance most explicitly articulated.

The reasons for Reinholdsson’s study of a drum solo instead of a groove are basically of methodological nature, a fact that is also pointed out by Reinholdsson himself.\(^{38}\) A fundamental necessary condition for testing the SYVARD hypothesis is the access to accurate registrations and measurements of the acoustic sound sequences generated by the musicians’ instruments. Reinholdsson’s registrations were in this case done by means of the apparatus MONA (from: monophonic analysis), which basically was constructed for measuring musical

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\(^{37}\) A similar point is made by Gabrielsson (1988, p.29).

\(^{38}\) See Reinholdsson (1985).
monophonic sound sequences, and had limitations in precision of registration compared to the task formulated by Reinholdsson. (Also Bengtsson and Gabrielsson to a great extent have applied MONA and POLLY (from: polyphonic analysis) in their acoustical registrations.)

An example of empirical research on non-notation-bound music which in fact does focus on the drummer’s groove-playing is provided by Alén (1995).\textsuperscript{39} In these studies Alén investigates and classifies rhythmic deviations in the performance of different ostinato patterns played by Cuban percussion ensembles. By detecting SVYARD on different rhythmic levels of the performance, Alén shows his debt to Bengtsson, Gabrielsson and the Uppsala approach in empirical rhythm research. However, Alén’s methods of registration are different from the methods previously described. The apparatus used by Alén was a so-called “Winckel-repeater”, which converts a sound’s temporal duration to a measurable physical length on a tape recorder.

2.3.4. Use of Computer Technology in Search of SYVAR

The former tools used for registration of the acoustic sound sequences used by Bengtsson, Gabrielsson, Reinholdsson and Alén described above may seem old-fashioned today. The very approach of applying various kinds of SYVAR detection (i.e. registrations of systematic variations of different parameters of musical expression such as duration, dynamics, intensity, timbre, pitch) in classification of characteristic rhythmic features of live performances of music is, on the other hand, still valid and useful in contemporary rhythm research. However, through the application of computer technology, where samplers\textsuperscript{40} are used for recording and manipulation of audio signals, and MIDI events\textsuperscript{41} are recorded and edited by sequencers\textsuperscript{42}, empirical rhythm research has gained methodological possibilities that were both qualitatively and quantitatively out of reach just a few decades ago:

\textsuperscript{39} This article is a translation of Chapter 5 in Olavo Alén: “La Música de las Sociedades de Tumba Francesa en Cuba” (Alén, 1986).
\textsuperscript{40} A sampler is a device for digital recording, editing, and playback of audio signals.
\textsuperscript{41} MIDI: Musical Instrument Digital Interface, is a digital “language” communicating musical information between electronic instruments.
\textsuperscript{42} A sequencer is a device for recording, editing, and playback of MIDI signals.
1. The *level of precision* in the registrations and measurements of acoustic sound sequences using samplers and digital technology by far exceeds the possibilities offered by MONA and other previous registration tools.

2. Being able to handle a large number of parameters simultaneously, computer technology offers new possibilities in measuring and demonstrating *interrelations* between different musicians. Hence, the individual performance can be studied and analyzed both isolated from other musicians and in a context of several musicians playing together.

3. Applications of samplers, sequencers and synthesizers represent unique possibilities of constructing *syntheses* of different musical unfoldings, where the various rhythmic parameters may be systematically controlled and varied.

Some examples of empirical research that applies computer technology in SYVAR detection will be outlined in the next section, where focus is on various kinds of *asynchronization* between different musicians playing together.

### 2.4. Participatory Discrepancies

Alén’s results, as presented in 2.3.3., are by Charles Keil (1995) referred to as a concrete example of a phenomenon Keil denotes as ”participatory discrepancies”\(^{43}\). A focus of ethnomusicological studies and investigations carried out by Keil is folk music, learned by ear, and, more specifically, jazz music. A starting point for Keil’s approach in studying and describing characteristic features of performance and experience of jazz music is a criticism of Leonard Meyer’s ”Emotion and Meaning in Music”\(^{44}\), where Meyer, based on a structuralistic, syntactic point of view, with focus mainly directed towards Western classical music, tries to answer the question: What is a musical experience? Keil argues that the answers proposed by Meyer are *not* relevant for African and African-derived genres of music, and a major task for Keil becomes to discuss aspects of music that he feels have been neglected in Meyer’s study. To cite Keil (1995, p.1)\(^{45}\):

\[\text{\textsuperscript{43}} \text{See also Keil (1987): "Participatory Discrepancies and the Power of Music".}\]

\[\text{\textsuperscript{44}} \text{Meyer (1956)}\]

\[\text{\textsuperscript{45}} \text{See, moreover, Keil (1966), where the criticism of Meyer is formulated. This article as well as Keil (1987) are both included in Charles Keil & Steven Feld "Music Grooves" (1994).}\]
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So I set out to overturn Meyer's paradigm completely, put syntax at the bottom and "vital drive" or groove on top. ...; music is not so much about abstract emotions and meanings, reason, cause and effect, logic, but rather about motions, dance, global and contradictory feelings; it's not about composers bringing forms from on high for mere mortals to realize or approximate, it's about getting down and into the groove, everyone creating socially from the bottom up.

A fundamental concept for Keil in his description of expressive live performances is participatory discrepancies (PD). This concept points to the empirical fact that in a situation of several musicians playing together, the different musicians are in varying degree asynchrony with one another; for example, in playing a swing rhythm in jazz, the drummer might be a bit "ahead" of the beat, while the bass player could be "laid back" ("behind" the beat), both being compared to a metronomic pulse. And, according to Keil, "the power of music" lies precisely in this asynchronicity. As Keil puts it (in Keil & Feld, 1994, p.96):

The power of music lies in its participatory discrepancies, and these are basically of two kinds: processual and textural. Music, to be personally involving and socially valuable, must be "out of time" and "out of tune."

For participatory discrepancy one could substitute "inflection," "articulation," "creative tension," "relaxed dynamism," or "semiconscious or unconscious slightly out of syncness." For process one could say "groove," "beat," "vital drive," "swing," "pulse," or "push," and for texture, "timbre," "sound," "tone qualities," "as arranged by," and so forth.

In Keil's heavy underlining of "out of time" and "out of tune" as basic qualities of music that is "personally involving" and "socially valuable", we see obvious parallels to Seashore's "deviation from the exact" as a characteristic feature of artistic expression. However, Seashore is primarily concerned with the individual artist's deviations from a defined reference norm, whereas Keil to a greater extent focuses on the performance and experience of music as a social process, where the different interactions among the musicians and the audience are characterized by various manifestations of participatory discrepancies. Moreover, through Keil's emphasis of the processual and social aspects of live music, a link between Keil's approach and the classical characterization of the "musical" art forms becomes apparent (cf. section 1.1.).

According to Keil, an understanding of the processual features of music depends on concrete detection and classification of PD, and must be based on empirical investigations. Keil shows his analytical debt to the SYVARD hypothesis by saying:

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46 Keil uses the notion 'engendered feeling' in talking about the processual/motor/improvised qualities of music, in contrast to 'embodied meaning' referring to the syntactic/mental/composed aspects, as discussed by Meyer (see Keil, 1966).
..., something approaching complete comprehension of the processual aspect will only be possible when we are able to determine accurately the placement of notes along the horizontal dimensions (Keil, 1966, p.345).

As mentioned in section 2.3.3., Alén’s investigations of the performance of Cuban percussion ensembles provide such registrations and measurements of temporal placements of notes. Another example of empirical research where PD is measured is given by J.A. Prögler. In his article "Searching for Swing: Participatory Discrepancies in the Jazz Rhythm Section" [47], Prögler takes Keil’s statement above as a challenge in his own investigations. Using computer technology involving digital registration of audio signals, Prögler describes PD between the bass player and the drummer in a jazz rhythm section. Prögler comments upon the results he obtained by saying:

Briefly, what I found is that participatory discrepancies are observable at the subsyntax level and they can be precisely measured. This allows us to say something concrete about swing or groove as crucial elements of musical style (Prögler, 1995, p.21).

It is interesting to note that results similar to Prögler’s PD demonstrations are also found by Reinholdsson (1987) in his detection of SYVARD related to performances of improvised basslines in jazz (more specifically: so-called “walking bass”). In these investigations Reinholdsson also uses sampling and computer technology in the measurements and registrations. Research on different kinds of asynchrony among jazz musicians playing together is also given by Rose (1989) and Ellis (1991). Rose used a sampler to measure durations in jazz rhythm section (piano, bass, drums) performances of swing, jazz ballad, and Latin jazz; while Ellis used an electronic saxophone and MIDI technique to study three saxophonists’ performances in swing style. Both Rose and Ellis demonstrated various kinds of systematic deviations, some style-specific, others characteristic of the individual musician.

Classifying typical features of performances of Norwegian folk music, the musicologist Tellef Kvifte uses computer technology to detect systematic variations along a line closely related to Bengtsson and the Uppsala approach. [48] Kvifte’s presentation of the different characteristics of the Norwegian folk dance “springar” is very interesting, and will be discussed in more detail in Chapter 7. Some further examples of registrations and classifications of asynchronization in live musical performances are given by Gabrielsson (1999, pp.543-544). Complementary information on the matters described in this section may be found there.

— Prögler (1995)

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Concluding this presentation of research relating PD to typical features of performance of music, it should be mentioned that questions have been raised as to the relevance of PD detection in "approaching complete comprehension of the processual aspect" of performance and experience of music, as is one of Keil's main concerns (cf. Keil, 1966, p.345). Ingrid Monson discusses Keil's approach, and concludes:

Keil is absolutely right that the interactive and processual are crucial areas to explore in our ethnomusicological thinking; I just think that in this case he has mistaken a product (measurements of discrepancies) for the culturally, bodily, musically and socially interactive processes by which human beings create them. (Monson, 1995, pp.88-89)

Suggesting an alternative starting point for investigating the processual, social aspects of musical communication, Monson, moreover, asserts (Monson, 1995, p.88):

The social metaphor of "conversation" is something that both Berliner and I encountered in our work with musicians, and it seems to me a better ethnographic point of departure for theorizing the relationality of musical processes, grooves, and social life than measuring small gaps between players and reductively positing a theory of participation on the basis of them (see Berliner 1994; Monson 1991).

A similar "point of departure" in studying social interrelations among musicians playing together is chosen by Peter Reinholdsson in his doctoral dissertation "Making Music Together. An Interactionist Perspective on Small-Group Performance in Jazz". Reinholdsson applies quite an interdisciplinary approach, where he integrates theoretical ideas of the interactionist perspective with jazz analytic, social psychological, ethno- and sociomusicological ideas.

In spite of the criticism raised by Monson, PD, as SYVAR detection nevertheless does provide interesting information for an understanding of essential features of performance and experience of music. This is a fact that has been documented through empirical investigations along a line from Seashore, via the Uppsala approach, to the contemporary rhythm research of today. However, the fundamental question seems to be: To what questions, or what problems, does detection of PD and SYVAR provide answers? Certainly, it is empirically valid, based on several investigations, to say that typical for Viennese waltzes are rhythmic deviations of this and this kind; in certain folk music the deviations are such and such; characteristic for this specific musician are deviations of this and this type; or, in this jazz rhythm section the bass player's phrasing is so and so many milliseconds "behind" the drummer, etc. All of this does, indeed, provide important as well as interesting information in the classification of style-

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specific and individual features of performance, and experience, of music. What this information does not tell us too much about, according to Monson, is basic features of the processual and social aspects of playing and experiencing music together. Whether one agree with Monson or not, it does not, however, seem right to question the very correlations between systematic deviations and different styles of performance, since they are strongly documented empirically.

2.5. Motor Skill and Kinematics

Although many investigations demonstrate characteristic SYVAR and PD in live performances of music, this research, as presented in 2.3 and 2.4 above, does not seem to provide a complete understanding of the different conditions governing and affecting the live expressions of these deviations. The investigations in the tradition of Seashore and the Uppsala approach have mainly documented the existence of correlations between deviations and style-specific or individual features, whereas social/processual and motor/kinematic "reasons" for the deviations have not been taken sufficiently into account. Monson's very criticism of Keil cited in section 2.4 seems precisely to pinpoint a lack of sufficient explanation of social and processual aspects of live music performance. In this section we will present some examples of research investigating the motor and kinematic qualities of performance and experience of rhythm.50

Through focusing on research demonstrating various relations between music and movement, a link connecting ancient Greek philosophy and modern rhythm research again becomes apparent. Several important examples of investigations that in varying degree are related to the Greek comprehension of rhythm, or rhythmos, as "order in movement", and the ancient correspondence thinking have already been presented above, e.g. Fraisse (section 1.1. and 2.1.1.), Large & Kolen (section 2.1.3.), Sievers, Becking, Truslit, Clynes, and Todd

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50 While the meaning of 'motor skill' and 'motor qualities' hopefully, and probably, is familiar to the reader, the concept 'kinematics', or 'kinematic qualities' might perhaps need some explanation. 'Kinematics', as a concept of physics, belongs to the field of mechanics, which is sometimes divided into two subfields: Kinematics and dynamics. Whereas dynamics is the study of moving bodies under the influence of various forces (e.g. gravitation), kinematics represents a more geometric description of the moving body's trajectory, without taking the actions of the different forces into account. Kinematics is thus concerned with studying a body's movements by representing the body's spatial coordinates as mathematical functions of time. Thereby, the moving body's velocity and acceleration may be calculated by differentiations of the position with respect to time.
(section 2.2.). When discussing the causal relationships between rhythm and human movement, a division into two conceptually separated relations may be appropriate:

1. Movement can generate rhythm and timing; i.e. rhythm and timing can be seen as a consequence of movement.
2. Rhythm and timing can generate movement; i.e. rhythm and timing can be seen as the source of, or motivation for, movement.

In this respect it seems right to assert that relation (1) mainly concerns rhythm performance. Obviously the musician's movements of fingers, hands, and feet, etc., and the different physical ways by which the musician interacts with the instrument crucially determine and generate the rhythm and timing of the performance. The relation (2), on the other hand, appears more relevant for rhythm response, or the experience of rhythm, the most striking examples of movements generated by rhythm being the audience tapping their feet, clapping their hands, or dancing to the music. Another example which shows that rhythm can generate movements is given by the use of work songs, where a primary function of rhythmic singing is to coordinate the activities and movements of people working together, and through this collective rhythmic action to optimize the efficiency of physical work. The movements of the performing musician and the movements of the audience are, indeed, usually quite different. Nevertheless, these different movements may be seen as basically related, all being in some way associated to the physical sound structure transmitted from the performer to the listener. Shove & Repp (1995) comment upon this interesting point by saying:

Music is made by moving hands, fingers or extensions thereof over an instrument, and the dynamic time course of these movements is reflected to some extent in the resulting stream of sounds. Conversely, people listening to music frequently perform coordinated movements ranging from foot tapping to elaborate dance. Although these movements on the listener's side are not the same as those of the performer, they are certainly not unrelated. At the very least, they share a rhythmic framework which is transmitted from player to listener via the sound structure. (Shove & Repp, 1995, p.64)

The relationship (1) movement generating timing is to a great extent an issue of motor control and motor skill. The basic idea underlining theories of motor skill is that conceptualized structural information is the input to a motor system, which then produces some kind of temporally structured behavior, resulting in expressive timing. According to Gabrielsson (1999), four theories of motor skill may be discerned:51 (i) The closed-loop

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51 This brief overview of the different theories are closely related to Gabrielsson's more thorough presentation in Gabrielsson, 1999, pp. 518-519.
theory, postulating that sensory information produced from the movement is feed back to the central nervous system and compared with an internal referent to check for discrepancies between the intended and the actually produced movement. (ii) Open-loop or motor program theory: A motor program contains representations of an intended action and processes that translate these into a movement sequence. The main idea is that a sequence of movements can be coordinated in advance of its execution, thus not relying on sensory feedback in the real-time control of movement. (iii) Schema theory assumes that there exist some kind of abstract representations of classes of motor actions, generalized motor programs so to speak, out of which can be generated a wide variety of movements in new situations. (iv) The Bernstein approach, named after the Russian physiologist Nicolai Bernstein, emphasizes that muscles are not individually controlled but function in muscle linkages, or coordinative structures. An important point of this approach is that groups of muscles are constrained to act as functional units, thus reducing the degree of freedom and thereby also the number of movement parameters. As Gabrielsson (1999) points out, however, the Bernstein approach is still little discussed in relation to music.52

According to Gabrielsson (ibid.), the most thorough investigations of music performance in relation to motor skill theories have been conducted by Shaffer and his coworkers.53 Shaffer’s starting point is a theory of motor programming (cf. (ii) above). Timing is handled by the motor system and an “internal clock”, acting as a reference. The motor system itself acts as a “timekeeper” by translating a given time interval into a movement trajectory with the corresponding duration. Complementary information on the work of Shaffer as well as presentations of other empirical investigations of motor skill in relation to musical performance may be found in e.g. Gabrielsson (1999), Clarke (1999), and Palmer (1997).

Turning now to relationship (2); rhythm and timing generating movement, this view is reflected in the proposition that music perception (as well as music performance) has its origin in the kinematic and dynamic characteristics of typical motor actions (see, e.g. Shove & Repp, 1995). If one accept this proposition, it makes sense that listeners exposed to expressive performances of music may want to respond by physical, bodily movements, the reason being that the perception of music is fundamentally related to basic features of motor behavior, and

52 Some interesting ideas and examples related to music that, indeed, may be seen as following the Bernstein approach are presented in Chapter 5.

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thus different kinds of movements are naturally "triggered" when music performed with good timing is perceived. This is a point which is strongly emphasized by Shove & Repp (ibid.). They even end their discussions of musical motion and performance by asserting that the lack of audience to much of contemporary composed music is due to a lack of motion qualities and kinematic considerations in branches of modern music:

Much music composed in this century encourages only primitive forms of motion or inhibits natural motion altogether. Many twentieth-century composers focus on sound qualities or on abstract tonal patterns, and performers of their compositions often neglect whatever kinematic potential the music may have. The absence of natural motion information may be a significant factor limiting the appreciation of such music by audiences. While compositional techniques and sound materials are subject to continuous change and exploration, though not without perceptual and cognitive constraints of their own (…), the laws of biological motion can only be accepted, negated or violated. If more new music and its performers took these laws into account, the size of audiences might increase correspondingly. (Shove & Repp, 1995, p. 79)

Obviously, this criticism does not apply to rock music, Latin-American music, African and African-related music, for example, where a characteristic feature is precisely the intimate relation between music and dance.

A common feature of kinematic models used as an attempt at describing performance and experience of musical expression is to construct trajectories or "movement curves" within a purely physical/mathematical or rather mechanical framework, and view these trajectories as analogues to corresponding evolutions of "movement" in musical expressions. Musical motion and movement (in some cases defined as the ways by which a musical performance changes in tempo, intensity or dynamics, for example, in other cases as the concrete physical, human movements of the performer or the listener) are thus "translated" from a context of musical unfolding to a formalized situation where mathematics and the laws of mechanics may be applied. In a strictly kinematic model the idea is to study time-dependent parameters of position, velocity and acceleration related to a formalized trajectory, and show how these resemble corresponding parameters of musical motion. In a dynamic model, on the other hand, physical forces (e.g. gravitation) are taken into account as analogues to "gravitational" aspects of musical motion.

Kinematic models were first applied to the large decelerations in performance tempo that commonly occur at the ends of pieces, called the final ritard. As Palmer (1997) points out, pianists' final ritards were modeled in two parts; a variable timing curve followed by a systematic, constant decrease in tempo (called linear tempo) (Sundberg & Verrillo, 1980).
Feldman et al. (1992) modeled both ritards and positive accelerations that occurred throughout performances, while Repp (1992a,b) modeled the expressive timing of a short melodic gesture in piano performances of a Schumann piece. Although most models of motion in performance address timing, some apply to changes in dynamics, or intensity, as well. Todd: "The dynamics of dynamics: A model of musical expression\(^{54}\) is in this respect very interesting. Based on a proposition stating that musical dynamics is coupled to tempo, Todd shows that a model of performance timing and dynamics based on the speed and force of movements of objects moving under the influence of gravity can give a good account of the expression found in spontaneous musical performances. It is, moreover, interesting to note that Todd, like Sundberg & Verrillo (1980), proposes that musical expression induces a percept of self-motion in listeners. Windsor & Clarke (1997) use Todd’s model as an analytical tool in discussing expressive timing and dynamics in real and artificial musical performances. In their concluding remarks they note that Todd’s model fails to account for every aspect of human performance, but as they put it: "... these failures are seen as positive because they highlight different aspects of musical expression" (ibid., p.149).

Another very interesting article by Todd is "The kinematics of musical expression"\(^{55}\) Referring to antiquity, Todd begins by discussing the relationship between musical motion and physical movement. He then proceeds to construct a kinematic model of musical expression. The trajectories in his model, however, "... are not those of the performers limbs in physical space, but those of an abstract movement relative to a metrical grid associated with a musical score" (Todd, 1995, p.1940). Applying this model, Todd is able to extend the studies of Sundberg et al. to include the accelerandi as well as the ritardandi in performances of complete pieces. One of Todd’s conclusions is that "... the variation of tempo in music can be reasonably compared with velocity in the equations of elementary mechanics" (ibid., p.1940). Moreover, extremely interesting is Todd’s attempt at relating the abstract trajectories and "movements" in his model to physical motions of the human body. In doing so, he constructs an auditory-motor model of rhythm perception able to detect meter and rhythmic grouping on the basis of an acoustic sound sequence.\(^{56}\) Fundamental to this model construction is a proposed separation of the sensory-motor process into two interacting components relating to two different kinds of human movements:

\(^{54}\) Todd (1992a).
\(^{55}\) Todd (1995).
\(^{56}\) This model construction has been briefly described above, see section 2.1.2 and 2.1.3.
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- One movement representing "the natural speed of tapping", or the spontaneous tempo as observed by Fraisse (1982)\textsuperscript{57} with 600 msec as the length of the interval between two successive taps being the "most representative"\textsuperscript{58}.

- The other movement is associated with whole body motion, e.g. body sway, having a natural period of about 5 sec (Todd, 1993, 1994a).

The modeled "interaction" between these two movements is formalized and constructed as two weakly coupled mass-spring-damper systems. Todd himself comments upon these matters by saying:

> In the first part of this paper the purely musical quantity of tempo was seen as corresponding in some way to a more abstract quantity, i.e., the speed of a moving body. The above separation of the global dynamics of the motor system into two components enables us to see how this abstract motion may be related to real physical or concrete motions. These real motions are also of two kinds.

> The first kind is just the pendular sort of motion inferred from a constant tempo rhythm with a strong beat. In this case the abstract speed corresponds to the frequency of the concrete pendular or sinusoidal motion. The second kind of motion is a more gestural kind associated with a tempo rabato. A gesture is a single motion which has a beginning, a point of maximum speed and an end. In this case the abstract speed corresponds more directly to the concrete speed of motion. (Todd, 1995, p.1948)

Most arguments against kinematic models suggest that physical notions of energy cannot be equated with psychological concepts of musical energy (see Desain & Honing, 1991, 1992). This, indeed, is a very important and relevant remark, and it is also a point that Todd is aware of and comments upon. Windsor & Clarke, however, view the different attempts at describing and explaining musical expression as reflecting the multidimensionailities of the very matter and, as such, the various approaches may all in varying degrees be valuable:

> Hence we see no conflict between approaches that start from a reductionist standpoint (such as Todd, 1992) and those that build in some notion of flexibility or diversity from the outset (such as Desain & Honing, 1991). It is surely heartening that the empirical study of musical expression provides a domain that constantly resists simple and systematic explanations and hence extends our understanding of human behavior beyond limits imposed by less challenging and complex tasks. (Windsor & Clarke, 1997, p.150)

Another approach to modeling rhythmic movements using mathematical and physical tools is demonstrated by Beek, Peper & van Wieringen (1992). The focus of this study is

\textsuperscript{57} See also section 2.1.1.  
\textsuperscript{58} Fraisse (1982, p.153).
coordinated rhythmic movements such as breathing and walking (i.e. the adjustment of respiratory to locomotory rhythms), cascade juggling, and polyrhythmic tapping. In modeling these phenomena, Beek et al. borrow concepts and tools from classical analytical dynamics and modern qualitative dynamics. From this point of departure Beek et al. are able to model "frequency locking" between breathing and running, and "frequency modulation" in rhythmic hand movements. Interesting additional work along this line is presented by Peper, Beek & van Wieringen (1995), Peper & Beek (1999), Haken, Peper, Beek & Daffertshofer (1996), and Daffertshofer, van den Berg & Beek (1999). Among these model constructions and findings, the ideas leading to frequency modulations will be further commented upon in Chapter 5.\textsuperscript{59}

A considerable amount of research has been done on biological motion and human motor control. These investigations cover a large area of different human activities, and the obtained results may in varying degree be relevant for describing performance and experience of music. A branch of this research that does seem relevant is that of Paolo Viviani and his collaborators (see Viviani, 1990, and Viviani & Laissard, 1991). Of particular interest in this respect is the findings of Viviani et al. that state correlations between kinematic and geometric variables in natural human hand movements (Viviani, 1990). This observation seems to impose conditions on models attempting to simulate physical, bodily aspects of music performance, and will be commented further in Chapter 5.

\textbf{2.6. Summary}

During the twentieth century, research on the various aspects and characteristics of rhythm in music has developed into a vast scientific field demonstrating a great diversity of focused issues, thereby also presenting quite different approaches in describing the various features of rhythm. The huge field of rhythm research has thus been more or less naturally divided into many subfields, and subfields thereof again. Some branches of this field of investigations have been presented and discussed in this chapter.

\textsuperscript{59} Frequency locking, also known as "resonance", is also studied and modeled in investigations of various other rhythmic movements, see Treffner & Turvey (1993), and is, moreover, applied in models of rhythm perception, cf. for instance Large & Kolen (1994).
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Structural aspects of rhythm have attracted a major interest in research of experience as well as performance of rhythm. Influential for both model constructions and empirical investigations on these matters has been Lerdahl & Jackendoff (1983), pointing out that rhythm in the tonal/metric music of the Western tradition is structured according to two somewhat independent principles: grouping structure and metrical structure. Various models have been constructed to account for meter perception, ranging from rule-based systems to auditory models, and some models attempting to describe perception of grouping have also been developed. Moreover, many empirical findings have established a relationship between musical structure and patterns of performance expression. Clarke (1999) emphasizes this correlation by introducing the notion 'timing', or 'expressive timing', to denote the different transformations of rhythmic structure (in the sense of Lerdahl & Jackendoff, it seems) into live performances of rhythm. We will adopt this definition of expressive timing, which seems relevant for discussions of typical features of rhythm performance.

Whereas rhythmic structure on the syntactic level, or rather the level of notated music, is confined to a great extent to exact values related to a defined norm (this is particularly true of note durations, where, for instance, a quarter note by definition has a duration equal to twice the duration of an eighth note), live performances of rhythm are heavily characterized by different kinds of deviations from presumed properties according to musical notation. This is an empirical fact that has been documented and studied in a large number of experiments, and which caused C.E. Seashore to postulate deviation from the exact as a characteristic feature of artistic expression. Many of the ideas of C.E. Seashore and his coworkers have been inherited and further developed in modern rhythm research, especially by Bengtsson, Gabrielson and the Uppsala approach – an approach which formulates the SYVAR hypothesis and correlates various, empirically registrated, SYVAR ("systematic variations") with style-specific and individual features of performance. To a large extent, Seashore's ideas are also demonstrated by different detections of PD ("participatory discrepancies", a notion introduced by Keil, 1987) in interactive contexts of several musicians playing together.

Although rhythmic structure is mirrored in expressive timing and different occurrences of SYVAR, the main impression seems to be that the various deviations in performance are first of all important in order to give "life" to music, and thus serve as fundamental to communicating motional and emotional aspects of rhythm. Research on the motional and emotional qualities of perception and performance of rhythm has been more limited in quantity than the, perhaps, more easily accessible and describable structural properties. However, results showing various ways of communicating emotions from the performer to the
listener (e.g. using tempo, sound level or timbre) have been obtained. Especially interesting in this regard is the attempt at classifying motional/emotional aspects of rhythm experience by investigating empirical studies which demonstrate that listeners can recover motion/emotion information in the musical acoustic signal and convert this information into overt movements of the body. Through strongly emphasizing the connections and interactions between the motional-emotional qualities of rhythm and the body movements of the listener (as well as the performer), these investigations represent a holistic view of performance and experience of rhythm that in many respects resemble the classical correspondence thinking and the idea of 'movement' as fundamental in rhythmos. Viewed in the light of the presentation given in this chapter, it seems appropriate to regard the contributions of Sievers, Becking, Truslit, Fraisse, Shove & Repp, Clynes, and Todd as belonging to a methodological tradition fundamentally related to antiquity's ideas of the nature of rhythm.

Connections between ancient Greek philosophy and modern rhythm research are also apparent in investigations of the motor and kinematic qualities of performance and experience of rhythm. Different theories of motor skill have been developed, the basic underlining idea seeming to be that conceptualized structural information is the input to a motor system, which then produces some kind of temporally structured behavior, resulting in expressive performance. A common feature of kinematic models used as an attempt at describing performance and experience of musical expression is to construct trajectories, or "movement curves", to a large extent within a purely physical/mathematical framework, and view these trajectories as analogues to corresponding evolutions of "movement" in musical expressions. In most kinematic models the trajectories are seen as representing some kind of abstract "movement" corresponding to the ways by which a musical performance changes in, for instance, tempo, intensity or dynamics, but are also by some, Todd (1995) for example, regarded as possible representations of movements of the human body.
3

A Possible Convergence
of 'Rhythm'

The many different applications of the concept 'rhythm' demonstrated in the preceding chapters clearly point at the fact that the various meanings assigned to this concept may be quite divergent. Moreover, these divergences in use of 'rhythm' reflect what is often called the multidimensionalities, or rather, the many different aspects of rhythm. Having presented and discussed some of these rhythmic "dimensions", it now seems natural to ask: Are there any existing features of rhythm common to all the different applications of the concept 'rhythm'? Is it possible, in a meaningful way, to identify a "core" of the rhythm concept, being basic to the various divergent applications, making it plausible to denote seemingly quite different events, from everyday life, music or scientific investigations, by the common term, rhythmic?

In the following discussion we will try to give an affirmative answer to this question by pointing at a possible process of convergence of concepts. Thereby we arrive at a concept 'X' "summarizing" various common features of rhythmic phenomena. An identification of such a concept seems interesting for several reasons: On one hand, an articulation of a common rhythm concept establishes an important basis for model constructions and syntheses of rhythmic events, the possibilities of general interpretations of the model being dependent on to what extent the model meets the demands of the concept 'X'. On the other hand, this concept might represent a key to a discovery of a possible "code" in the understanding of rhythm as phenomenon, and thereby contribute to a greater comprehension of human perception as such. As formulated by Elliott (1986, p.5):

..... an understanding of the nature of rhythm may be the key to an understanding of the human perceptual process.
3.1. Relevant Invariance

Searching for common features of the various applications of ‘rhythm’, one may run the risk of discovering that the characteristics of the rhythm concept shared by the diverging definitions of ‘rhythm’ are to such extent general, or even trivial, that the possible “core” of the rhythm concept one is left with is without relevant content of meaning. Therefore an important task in these investigations is to alternate between an identification of rhythm characteristics common to the various manifestations of rhythm, and, of equal significance, to argue that the identified characteristics are relevant to the description of rhythm. A characteristic of rhythm that is common to the different manifestations, and which is consequently contained in both everyday use and scientific applications of the concept will be called an invariant rhythm characteristic (IRC). The following question immediately arises: Do invariant rhythm characteristics exist? and, if so: What does their relevance mean?

Criteria for distinguishing relevant information from irrelevant information is by no means given in any unique and unambiguous way. Making distinctions between relevance and irrelevance touches upon problems of philosophical analysis and the philosophy of science, and is, moreover, dependent on scientific, or for that matter, cultural or everyday points of view. A very interesting discussion of ‘relevance’ from an ethnomusicological point of view is given by Simha Arom in his book “African Polyphony & Polyrhythm. Musical Structure and Methodology” in his book Arom studies and describes music of the Central African Republic. Having developed a method and technique for transcribing and analyzing polyphony, it becomes important to achieve a transcription which contains primarily “relevant elements”. To quote Arom:

For an element to be relevant, it must be an essential and inalienable part of a musical structure. Our problem is, first of all, to determine what makes a given element in a musical structure relevant, and then to identify the structural levels at which different sets of elements are assigned this relevance. (Arom, 1994, p.137)

A major point in this respect is “... what degree of significance should be assigned to each item of information the ethnomusicologist collects?” (ibid., p.138). Crucial for making decisions on this matter are the cultural references and social framework within which the

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60 Experiencing this situation is similar to the findings of Peer Gynt, a famous character in a play by the Norwegian poet Henrik Ibsen: Peer is peeling an onion, looking for the onion’s core, only to find eventually that no such core exists.

61 Arom (1994).
studied events naturally take place, and the kind of meaning designed to the event under consideration by the internal users and participants of the culture. Arom concludes his discussions by saying:

We therefore define as relevant everything in this system of references which is meaningful to its users. (ibid., p.146)

Several of Arom's thoughts on 'relevance' made on an ethnomusicological basis seem valuable and useful also to our general discussion and study of the rhythm concept. The focus of Arom's research are different characteristics of African music. The strategy chosen by Arom in his investigations is to observe musical performances as unfoldings of cultural activity, where "relevant musical elements" are identified. Furthermore, these relevant elements are used as pieces in the puzzle Arom tries to solve in order to discover the hidden code underlying the music of the Central African Republic. Arom makes the following point:

The ethnomusicologist has no prior knowledge of the system he intends to study, or of its underlying code. Unlike the musicologist analyzing a message in a code he knows, he must work out a method which can take him from a description of the message to the discovery of the code. (ibid., p.160)

The focus of our present discussion are various characteristics of rhythm. Our choice of strategy is to observe and present different applications of the rhythm concept belonging to various traditions of music, science, and everyday language. In doing so we will try to identify "relevant rhythmic elements" common to the many manifestations of rhythm. Moreover, these relevant elements are used as pieces in the puzzle we hope to solve, attempting to discover an underlying "core" of the concept 'rhythm'. Eventually we wish to investigate to what extent a discovery of a such conceptual "core" might increase our understanding of a code related to rhythm as a phenomenon.

Viewed in this manner, our situation seems to resemble that of Arom. However, an important difference is that, initially at least, we are concerned with investigating applications of concepts, not unfoldings of cultural activities, and our focus is therefore transformed to a metalevel compared to the investigations of Arom. Another distinction is that I myself am a user of some of the concepts being discussed, and, hence, I am familiar with some of the "cultural references" which give the concepts meaning. On the other hand, there is no explicitly expressed common code underlying the various rhythm concepts, and therefore our situation seems closer to that of the ethnomusicologist than that of the musicologist, as Arom distinguishes between these. Inspired by Arom's notion of 'relevance', we now propose the
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following transformation to our situation: A rhythm characteristic is said to be relevant if the users of a concept 'rhythm' regard this characteristic as important in making distinctions between rhythm and arrhythmia. Compared to Arom's definition, we observe that being relevant in Arom's sense is dependent on being meaningful, whereas relevance in our sense depends on being important in making distinctions between rhythm and arrhythmia. We therefore use a more "narrow" or "strengthened" notion of relevance than Arom. This is deliberately done to account for our specific focus of discussion; obviously, a characteristic of arrhythmia does not belong to the "core" of the rhythm concept.

On the basis of the above presentation, a relevant, invariant rhythm characteristic (RIRC) is a characteristic of rhythm that is common to the different manifestations of rhythm, and which users of a concept 'rhythm' regard as important in making distinctions between rhythm and arrhythmia. We now pose the following question: Do relevant, invariant rhythm characteristics exist? Affirmative answers to this question might lead us closer to a core of the concept 'rhythm'. If, moreover, a process of convergence of rhythm concepts is detected, the "limit" of this process will be a concept 'X' "summarizing" the various common features of rhythmic phenomena. The designatum of this concept, i.e. the events referred to by 'X', will then represent a possible "code" in the comprehension of rhythm as a phenomenon. Hence, the situation illustrated in Figure 3.1. below would be achieved:

![Diagram](image)

Figure 3.1. Illustration of a rhythm "code" underlying various manifestations of rhythm.

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The structure of investigations and discussions as outlined so far is the following:

A "limit" of a possible convergence of 'rhythm':

Relevant, invariant rhythm characteristics:

Formulation of concepts:
- 'Rhythm' in everyday speech
- 'Rhythm' in psychology
- 'Rhythm' in physiology
- 'Rhythm' in music
  - Ways of walking, running, etc.
  - Ways of musical performance
  - Processes of language development
  - Processes of learning
  - Alternations of night and day
  - Structuring of work, "working rhythms"
  - Interactions and coordinate movements of e.g. football players
  - Ways of musical response

Activities/phenomena

Figure 3.2. Illustration of a possible process of convergence of rhythm concepts. The various activities and phenomena listed above are to be understood as examples of manifestations of rhythm taken from different situations of everyday life and science. These examples should by no means be considered as representing a complete list of rhythmic phenomena. It could, moreover, be said that the rhythm concept might certainly appear in relation to several other concept formulations than those mentioned above. However, these remarks have no influence on the very structure of the approach outlined here.
3.2. Rhythm Inside or Outside Time

When trying to find a concept containing common characteristics of the different rhythm concepts, our first task is to substitute the lower question marks in Figure 3.2. with relevant, invariant rhythm characteristics. Thus, a main question is: Which characteristics of activities and phenomena commonly denoted as "rhythmic" are globally present; i.e. present in all manifestations of rhythm? Referring to musical rhythm, Ingmar Bengtsson comments upon this.

For rhythm experiences and rhythm behavior to occur, some kind of temporal structuring involving grouping and differences in accentuation seems to be necessary. The "well-ordering" thus achieved seems moreover to be dependent on some kind of experienced or perceived regularity ("return of the similar"), which, within some tolerable "variance", should occur on a temporal level of the musical unfolding. (Author's translation.)

In this brief outline of characteristic features of rhythm in music we find several candidates that might be labeled "relevant, invariant rhythm characteristic". A basic presumption underlying the rhythm concept, as presented by Bengtsson above, is that experience as well as performance of rhythm unfolds in time. Bengtsson points out (cf., above): "... temporal structuring, .... regularity .... should occur on a temporal level ..." (our underlining). That rhythm, so to speak, "takes time", or, maybe better put, "consumes time", is something most of us take for granted in our everyday use of the concept 'rhythm'. Based on everyday speech, it might even be valid to say that "time consumes rhythm", in the sense that our very experience of time is fundamentally subject to various "rhythmizations" and "rhythmic" structuring of perceived events. This is, for example, demonstrated in the use of the clock, dividing evolution of time into hours, minutes and seconds, and in various physical, biological, or conventional regularities naturally constructing "rhythmic" cycles of events that time evolution, as such, is related to; e.g. alternations of day and night, the cycle of the year, the beating of the heart, cycling between tension and relaxation in breathing, various working rhythms, and daily routines structuring everyday activities. Viewed in this manner, the contents of the concepts 'rhythm' and 'time' are to a large extent interdependent in everyday speech.

That rhythm "consumes time" seems to be a basic presupposition in musical and scientific language, as well. This is already illustrated as related to musical rhythm by

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62 In Cappelens Musikkleksikon, volume 5, 1980, under the term "Rytme".
Bengtsson's characterization given above. Furthermore, in the article "Rhythm" in *Science de la Musique*\(^6\) we read:

Rhythm is the temporal aspect of music, the source of its temporal unity. It is what gives oneness to the flow of music. It provides the underlying coherence holding together the moments into which music can be broken down by analysis, and allows it to exist .... But rhythm involves the structuring of a sequence of musical acts. It is the temporal form of music, the particular way in which it achieves its existence in time ....

In the presentation of some different traditions and approaches in modern rhythm research given in Chapter 2 above, the basic temporal characteristic of rhythm is underlined in various ways. This is especially true of the approaches indebted to antiquity, where the definition of rhythm as "order in movement" was postulated. (Sievers, Becking, Truslit, Fraisse, Shove & Repp, Clynes, and Todd belong in varying degree to this tradition. See Chapter 2 for complementary comments and presentations.) Since movement as a phenomenon exists as a spatial/temporal unfolding, it is obvious that views reflecting rhythm as fundamentally related to movement also presuppose time (and space) as basic to rhythm.

Also in scientific applications of the rhythm concept not relating to music, it seems to be more or less taken for granted that 'time' and 'rhythm' are fundamentally related. This is exemplified by Evans in the article "Dysrhythmia and Disorders of Learning and Behavior", where we read:

Somewhere among these rhythms seems to be a basic reference rhythm, which may function as the "pacemaker"- an internal clock, the timing of which could be fundamental to one's perception of time. (Evans, 1986, p.253)

In Elliott's article "Rhythmic Phenomena – Why the Fascination?"\(^6\) a variety of examples demonstrating views asserting temporality as a characteristic feature of rhythm are given in several "definitions" of rhythm.\(^6\)

Based on the discussion and the examples given above, we hereby identify our first relevant, invariant rhythm characteristic:

**RIRC 1: Time is a relevant, invariant rhythm characteristic.**

It might be necessary to make some further comments on this identification of RIRC 1:


\(^6\) Elliott (1986)

\(^6\) Some of these "definitions" are presented in 1.3.
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First comment: In this context 'time' is regarded as a basic concept, and not a subject for
discussion or definition. Different "definitions" of time exist, some related to the natural
sciences, other to the social sciences and humaniora. We will not touch upon these here, but
instead take an intuitive experience and comprehension of time as granted. The psychologist
Masanao Toda has written:

It is a fool's errand to try to "define" time. Defining a notion is to find for it an equivalent
ideational construct made of some other, usually more primitive, notions. The
prerequisite for a successful definition, however, is that every aspect of the target notion
is represented by some of the component notions used for the definition. Any attempt at
defining time, therefore, is bound to be ridiculous, as there is nothing in this world that
even remotely resembles time. (Toda, 1978, pp.371-372) 66

Second comment: Even if it seems to be accepted on a general basis that rhythm as a
phenomenon unfolds in time, or maybe it is appropriate to say: inside time, Iannis Xenakis
also discusses a rhythm concept he places "outside time" ("hors-temps"). As Xenakis puts it
(Xenakis, 1992, p.264):

Music participates both in space outside time and in the temporal flux.

Xenakis asserts that in the formulation of concepts and constructions of representations of
music, for instance in formal constructions of scales and note durations, in the notation of
music, or in physical-mathematical representations of music, we are taking "snapshots" of
musical unfoldings, and these "snapshots" exist outside time, uninterrupted by time evolution.
To quote Xenakis himself:

Thus, the scales of pitch; the scales of the church modes; the morphologies of higher
levels; structures, fugal architectures, mathematical formulae engendering sounds or
pieces of music, these are outside time, whether on paper or in our memory. The
necessity to cling against the current of the river of time is so strong that certain aspects
of time are even hauled out of it, such as the durations which become commutable. One
could say that every temporal schema, pre-conceived or post-conceived, is a
representation outside time of the temporal flux in which the phenomena, the entities, are
inscribed. (Ibid., p.264)

As we understand Xenakis, his point seems to be that our conceptual abstractions of music
exist outside time—these abstractions being constructed representations of concrete, musical

66 Many might also assert perhaps that it is a fool's errand to try to "define" rhythm. Apel (Harvard Dictionary
of Music, 1945, p.639) comments on this: "It would be a hopeless task to search for a definition of rhythm which
would prove acceptable even to a small minority of musicians and writers on music." However, in this respect it
is important to be aware of that our aim in this chapter is not to postulate any "definition" of rhythm, but rather
try to identify global characteristics of rhythm having general relevance for the various rhythm concepts.
realizations that exist inside time. To us, this appears a very appropriate way of enlightening the distinctions between theories of music on the one hand, and performance and experience of music on the other. The former existing outside, the latter inside time. Moreover, it is interesting to note that the process of realization that converts abstract musical structure existing outside time into concrete musical performances existing inside time is analogous to the transformations of rhythmic structure into live performances of rhythm, denoted by expressive timing in section 2.3.1. above. On the basis of the notions of Xenakis presented in this section, the discussions of the present chapter may be seen as an attempt at making the following achievements:

(a) To identify a process of convergence of rhythm concepts, hopefully arriving at a "limit concept" 'X' existing outside time.

(b) To use the concept 'X' as a possible "tool" for obtaining increased understanding of rhythm as phenomenon inside time.

3.3. From Rhythm to Difference

So far, we have identified time as a global characteristic of rhythm. Another feature that seems to be shared by the various applications of 'rhythm' is that 'rhythm experience', as well as 'rhythm performance', as these concepts are being used, point at phenomena that presuppose an unfolding of differences or contrasts. In everyday speech "the rhythm of breathing" refers to a cycling between the contrasting states tension - relaxation (caused by alternations of breathing in - breathing out). "Working rhythm" may not only point to regular changes of different activities of work but also to successions of specific, distinct patterns of human actions related to one particular task, e.g. to dig a ditch: (1) Put the spade in the ground, (2) Lift up some soil, (3) Throw the soil away. Repeated "rhythmic" executions of the three activities of movement, (1), (2), (3), will then eventually result in the work being done, the digging of the ditch is finished! At a soccer match, the ways the players move in different patterns related to each other may give a good "rhythm" to the team, while differences in duration, pitch, dynamics and timbre are basic characteristics of rhythm in music. The latter is not just a fact related to everyday speech, but is also explicitly articulated in musicological, as well as ethnomusicological research and writings. This is, for instance, demonstrated in Bengtsson's presentation of the rhythm concept, quoted in the previous section, where he
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points out the importance of "differences in articulation" for rhythm experiences and rhythm behavior to occur.\(^{67}\) Arom also emphasizes the existence of contrasting features as fundamental to rhythmic unfoldings by saying:

For there to be rhythm, sequences of auditive events must be characterised by contrasting features. This contrast may be created in three different ways: by accents, tone colours, or durations. (Arom, 1994, p.202)

This statement becomes important in Arom's attempt at making the rhythm concept precise, and is crucial to his study of characteristic aspects of African music.\(^{68}\) In scientific terminology related to fields of research other than music, it also seems to be taken more or less for granted that the presence of a sequence of differences is a necessary feature in phenomena denoted as 'rhythmic'. This is illustrated, for example, in Evans' chapter "Dysrhythmia and Disorders of Learning and Behavior"\(^{69}\), as well as in the other chapters of Evans & Clynes' book "Rhythm In Psychological, Linguistic And Musical Processes".\(^{70}\)

Motivated by these examples taken from various applications of the rhythm concept, we now postulate our second relevant, invariant rhythm characteristic:

RIRC 2: Difference is a relevant, invariant rhythm characteristic.

As the many applications of the concepts show, it may seem appropriate to assert that unfolding of differences is a necessary condition for unfolding of rhythm. Or, to put it in another way: Where no differences are unfolded, no rhythm exists (i.e. according to the way these concepts are used, in everyday speech, in music and in scientific language). This is a point that is explicitly expressed by Arom through the clear distinction he makes between the concepts 'pulsation' and 'rhythm'. To quote Arom (1994, p.202):

By pulsation we mean the isochronous, neutral, constant, intrinsic reference unit which determines tempo.

Moreover, Arom asserts that: "The pulsation as defined above is not rhythm" (ibid., p.202).

What is missing, according to Arom, is the presence of contrasting events (cf., Arom above). Hence, an unfolding of contrasts (or differences as we choose to put it) is a characteristic of

\(^{67}\) Cooper & Meyer make a similar point in their "definition" of rhythm (see section 2.1.2), while the importance of an unfolding of differences to the perception of rhythm is heavily underlined by Lerdahl & Jackendoff in their discussions of different types of accents as important in the perception of meter (cf., 2.1.3. above).

\(^{68}\) As presented in Arom (1994).

\(^{69}\) Evans (1986).

\(^{70}\) Evans & Clynes (1986).
rhythm that, as Arom views the matter, is important in making distinctions between rhythm and arrhythmia. Thus, difference as a characteristic of rhythm fulfills the criterion for being labeled "relevant rhythm characteristic" according to our use of terminology, as discussed in 3.1.71

Based on the previous discussions, where time and difference have been identified as relevant, invariant rhythm characteristics, we now make the following definition:

**Definition:**

*A sequence of differences (SD) is an unfolding of perceivable differences over a closed interval of time.*

It might, indeed, be appropriate to make some comments on this definition:

*First comment:* By the term "perceivable differences" we understand events, states of affairs, or objects which, when exposed to some kind of perception, are perceived as distinguishable or different. Thus, as the unfolding evolves, the events, states of affairs, or objects are identified and experienced as different. Consequently, a sequence of differences *exists in an interaction between an "external, physical" and an "internal, mental" reality.* To name an example: An unfolding of variations of air pressure as a purely acoustic phenomenon is, by definition, *not a sequence of differences unless* these physical variations are identified and experienced as differences in, for instance, dynamics, pitch or timbre.

These considerations of an entity, a "sequence of differences", obtaining existence through a reciprocal interaction of perception and the "external" world, are closely related to the philosophical issue of *phenomenology* and the *phenomenological method* of thought (cf. for instance, Husserl, 1913 (Engl. transl., 1982); Merleau-Ponty, 1945; Clifton, 1983). According to Clifton (1983, p.50):

Phenomenology is neither the study of pure logic, which would make it a philosophy of mind, nor the study of discrete particulars, where it would reduce to a materialistic empiricism. It affirms the essential reciprocity of contingent experience and rational certainty. Without this reciprocity, human life tends to be interpreted as determined by external causes, or conversely, the natural world becomes translated into statements of formal logic. Phenomenology is, in brief, the logic of experience.

In this context it is important, once more, to stress that our main interest in this chapter is to search for common characteristics of different concepts of rhythm, which, hopefully, will

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71 At least this is true if we adopt Arom’s rhythm concept.
result in the discovery of a "convergence of concepts". In doing so, we touch upon aspects of philosophy and philosophical analysis, but philosophical analysis, as such, is not the subject of our discussions. Hence, we choose not to question the meaning of concepts like 'perception', 'perceivable', 'phenomenon', or 'existence', which indeed are basic to an understanding and discussion of phenomenology. So far, the only concept whose meaning is questioned and discussed is the very concept 'rhythm' itself.

*Second comment:* From the definition it follows that an SD is dependent on both perceptual ability and social/cultural references. This is demonstrated by the fact that when exposed to the same physical unfolding of events, two different subjects may react quite differently as to an identification of the events as an SD or not. The following examples illustrate this:

Example (i): A blind person is not able to identify visual differences, whereas deaf people have difficulties distinguishing contrasts in sound (unless the sounds are loud enough to cause vibrations in the floor, for instance, which a deaf person can feel). Moreover, several animals are able to experience contrasting events that are not identified as sequences of differences by human beings. For instance, dogs are able to identify sounds belonging to a frequency region exceeding the limits of human capacity.

Example (ii): Music originating from foreign cultures may to our "untrained" ears sound monotonous, for lack of the "usual" musical differences in sound, pitch, timbre, harmonics, etc. common to us. However, this only demonstrates that we do not know this music; we are not familiar with the cultural and social references within which this music is made and performed. To the users and performers of this music, on the other hand, the music is rich with qualities and nuances that we, captured by our cultural and social background, are not able to identify, not to say value.\(^7\)

*Third comment:* An SD is an unfolding of perceivable differences over a closed interval of time. By this formulation we emphasize the following two points:

(i) An SD occupies an interval of time, that is to say: an SD has a temporal length. In other words, an SD does not exist in singular, discrete points of time.\(^7\)

\(^7\) Thus, it becomes an important task for the ethnomusicologist to achieve an understanding of the cultural and social references of this music, thereby being able to identify relevant contrasts and differences characteristic of the musical performance. Eventually, this might lead to a discovery of the underlying "code" of the music (cf. Arom, as quoted in section 2.1).

\(^7\) According to Xenakis' terminology, it seems correct to say that in discrete points of time only differences outside time exist.
(ii) Being defined over a closed temporal interval, an SD has a beginning and an end. Thus, an SD is not an infinite unfolding of perceivable differences. Viewed against "First comment" where we pointed out that an identification of a sequence of differences is fundamentally dependent on perception by some living creature, this limitation seems appropriate.

On a purely descriptive level, a graphic representation of an SD may take a form as illustrated in Figure 3.3 below:

![Diagram of perceived events over time](image)

Figure 3.3. A descriptive illustration of an SD related to a specific act of perception.

Note that the SD illustrated in Figure 3.3 is dependent on a specific, individual act of perception. The perception starts at time $t_1$, and ends at $t_2$. The total temporal unfolding covers the closed interval $[t_1, t_2]$. The perception, as such, may be a perception of sound (in which case Figure 3.3 illustrates perception of differences in, for instance, pitch, timbre, durations, or dynamics); color (different colors being perceived; red, blue, "blueish", yellow, etc.); taste (sweet, sour, bitter, etc., being recognized); movements of the body (experiencing differences in, for example, walking, jumping, running, dancing), as well as perception of several other kinds of events.

At this point, it might be interesting to suggest some possible unfoldings of events that, when exposed to various kinds of perception, may be perceived and identified as a sequence of differences. We will do so by presenting a list of "perceivable differences". It should be noted that the examples given below by no means are meant to be complete, covering all possible SDs. Furthermore, the various classes of perceivable differences exemplified below are not necessarily disjoint, i.e. one kind of perceived differences may very well be
perceivable as differences of another kind; for instance, differences of movements may also be perceived as differences in energy.

**Some examples of perceivable differences:**

(I) Audible differences:
   Loud – soft (dynamics), long duration – short duration, high – low (pitch), differences in timbre (e.g. the sound of a saxophone compared to the sound of a trumpet).

(II) Visual differences:
   (a) Differences in color
   (b) Differences in geometric form:
       E.g. circle – square – triangle, ball – cube – pyramid, straight – curved, large – small.
   (c) Differences in position in space:
       E.g. left – right, up – down, in front – behind, inside – outside.

(III) Differences in smelling and tasting:
   Sweet – sour – bitter – salt, etc.

(IV) Differences experienced through touch:
   Hot – cold (temperature), soft – solid, wet – dry, smooth – rough, etc.

(V) Differences in movement:
   (a) Different qualities of movement:
       E.g. fast – slow, speeding up – slowing down, curved movement – straight movement.
   (b) Different movements of the body:
       For instance, walk – run, march – dance, jump – slide; or: bend the knee – shake the hand – nod the head, etc.

(VI) Biological differences:
   E.g. old – young, alive – dead.

(VII) Differences given by astronomical changes:
   Day – night, ebb – tide, different seasons, etc.

(VIII) Differences in energy:
   Differences in potential or kinetic energy; tension – relaxation (high level of energy – low level of energy).

(IX) Social/economic/cultural/geographic differences:
   Rich – poor, conservative – radical, industrial countries – developing countries.
3.4. From Difference to Rhythm?

In the previous sections of this chapter we have indicated various examples taken from different branches of everyday life, music and science, which demonstrate rhythmic performance and experience to be fundamentally dependent on temporal unfoldings of differences. As a preliminary achievement in our search for a possible "convergent" rhythm concept, this dependency has motivated our definition of a sequence of differences. Moreover, according to the use of the concept 'rhythm', it seems correct to assert that the presence of a sequence of differences is a necessary condition for unfoldings of rhythm, i.e.

*If U is an unfolding of rhythm, then U is a sequence of differences.*

Given this relation, it is tempting to ask if this implication can be reversed. In other words; is it valid to say the following: *If U is an SD, then U is an unfolding of rhythm* (i.e. according to the common use of the concept 'rhythm')? If this can be answered in the affirmative, an SD will be a necessary and sufficient condition for an unfolding of rhythm, and, hence, an SD will represent a possible "convergent" rhythm concept.\(^74\) Let us approach an answer to this question by examining various examples of SDs. The first examples will be taken from musical situations, later examples illustrate some non-musical unfoldings.

*Example 1:*

A drummer is playing on one drum. His playing may be notated according to traditional notation in the following way:

\[(*)\]

\[\begin{array}{cccccccccccc}
\wedge & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
\end{array}\]

As the notation indicates, the drummer is playing a sequence of drum beats where the temporal distances between the beats are experienced as constant, some beats are perceived as accentuated, others not. Thus, the drummer creates an unfolding consisting of perceivable differences between loud and soft (strong and weak) sounds, and the unfolding is consequently, by definition, a sequence of differences.

At this point, it should be noted that the notation of the drummer's playing as given in (\(\ast\)) above is not an SD. This notation is just a constructed representation of an experienced

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\(^{74}\) Or, maybe better put; an SD would represent a possible "limit" of convergency of rhythm concepts.
unfolding of events, and is therefore, to use Xenakis' terminology, only a "snapshot" outside time of an unfolding of events inside time.

Now, given that the drummer creates an SD, is it reasonable in this case to say that the drummer creates a rhythm? As the concepts are used, it indeed seems right to answer Yes. Through the drummer's regular accentuation of every third beat, a perception of meter, or induction of meter as some would put it, is even likely to take place. According to the way Western music is notated, the following notation of the drummer's playing would be appropriate:

![Notation of drummer's playing]

If, on the other hand, the drummer were playing isochronous identical beats with no accentuations, the created auditive events would contain no contrasting features, and the drummer would, according to Arom, not create a rhythm but rather a pulse (or "pulsation"). However, in making such distinctions between unfoldings containing differences and unfoldings where no "contrasting features" are present, it is of major importance to underline whether the unfolding is considered over an interval of time or just as a sequence of discrete points of time. In the latter case, it certainly seems appropriate to assert that the isolated beats played by the drummer are without contrasts and thus do not create a rhythm. (i.e. unless we take into account the effects of "subjective rhythmization", where an experience of a sequence of isochronous, identical beats is very often perceived as groups of beats rather than as isolated beats in succession, cf., 2.1.1.) However, if we, as is our concern, regard the drummer's playing as a continuous unfolding of events, it becomes apparent that also in the performance of the pulse (at least) two different phenomena are unfolded; beat (here: the instant when the drumstick (or hand) hits the drum) and non-beat (the event temporally separating one beat from the next). Experienced in this way, the unfolding of the pulsation as

---

75 Arom, for instance, asserts that the drummer plays a rhythm since the sequence of drum beats are characterized by contrasting features (cf. Arom, 1994, p.202; or section 3.3 above). According to Cooper & Meyer, the performance is rhythmic since a grouping of unaccented beats in relation to accented ones is implemented (see Cooper & Meyer, 1960, p.6; or section 2.1.2 above), while Lerdahl & Jackendoff would probably say that the playing is rhythmic due to the strong metrical structure induced by the performance (cf. Lerdahl & Jackendoff, 1983, p.8; or 2.1.2 above).

76 See 2.1.3 above.

77 Arom (1994, p.202); or section 3.3 in this chapter.

78 A similar point is made by Fraisse (1982, p.151) in stating: "The problem remains: Is rhythm the arrangement of durable elements, or is it the succession of more or less intense elements, the upbeat and the fall, the axis and the thesis of the Greeks being the most simple example?"
given above is indeed a sequence of differences. Whether it is reasonable to regard pulsation, viewed as a continuous unfolding, as **rhythm** will be discussed later.

**Example 2:**

The drummer is still playing on one drum, but this time his playing may be notated in the following way:

(***):

```
> x x x x x x x x x x x x x x x x x
```

As in example 1, the intervals between the beats are perceived as identical, some beats are experienced as accentuated, others not. The notation (***), thus represents an unfolding that commonly (by most people) is identified as an SD. Is it equally common to identify this as a realization of **rhythm**? The answer may in this case seem less obvious than in example 1. According to Arom, this performance creates a rhythm since the playing is characterized by a presence of audible contrasts. Others might say: "Yes, the performance is perhaps rhythmic, but I don't recognize the rhythm!" If in this example we should try to induce a **meter** where the accented beats represent the beginning of each measure, the notation would be as follows:

```
\[ \frac{2}{4} \frac{2}{4} \frac{2}{4} \frac{3}{4} \frac{2}{4} \frac{3}{4} \frac{3}{4} \frac{4}{4} \]
```

Similar unfoldings of frequent changes of meter due to irregular accentuations are found in modern Western compositions, such as in the music of Stravinsky, while successive chains of different groups of 2 and 3 beats are typical for branches of East European folk music. In Bulgarian folk music we might, for instance, hear the following pattern of accents:

```
\[ \text{2} \quad \text{2} \quad \text{3} \]
```

However, in this last case the accents are **ordered**, constructing repeating **cycles** of beat groups: 2-2-3, 2-2-3, 2-2-3, etc.
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Example 3:

This time the drummer is using two drums, the drums being tuned to sound differently. Let us assume that the following is an adequate notation of the drummer’s playing:

\[ (***) \]

In this situation perceivable differences in pitch (or maybe it is more appropriate to say \textit{timbre}) are created, and consequently the unfolding is (to those being able to experience these differences) an SD. If we adopt Arom’s definition, (***) definitely represents a rhythm since, as in the previous examples, contrasting features are unfolded. Because of the drummer’s regular alternations of tone color (the lowest sounding drum on every third beat), a notation of the performance given as 3 beats to the measure would also be appropriate according to Western notation (compare with example 1).\footnote{In Lerdahl & Jackendoff’s sense it seems right to say that the lowest sounding drum represents a \textit{phenomenal accent} serving as a cue from which the listeners extrapolate a regular pattern of \textit{metrical accents}, and unless a clear metrical pattern has been established, a sense of \textit{meter} is perceived. (See Lerdahl & Jackendoff, 1983, pp.17-18, and section 2.1.3 above.)}

Example 4:

Once more the drummer is playing on two drums, but now the notation is given by the following picture:

\[ (***) \]

This notation represents an unfolding of contrasts in tone color (pitch/timbre) and duration, with no contrasts in dynamics. Hence, (***) is a notated representation of an SD, and since the unfolding consists of audible events, this is a rhythm according to Arom. However, in this case the differences in tone color and duration are to such extent “irregularly” performed, that when exposed to this performance, it is probable that many would say: “I don’t understand anything of this rhythm!” Others might say: “The drummer is playing un rhythmically!” Or perhaps: “This is not a rhythm at all!” In this example it does not seem reasonable to induce
any obvious numbers of "beats to the measure" as was the case in the previous examples. The only "metrical reference" naturally present in this unfolding, appears to be that all note durations are integer multiples of a smallest note value, in this case being a sixteenth note. Thus, the underlying temporal "stream" of sixteenth notes constitutes an "isochronous, neutral, constant, intrinsic reference unit which determines tempo" (as Arom puts it, 1994, p.202), and is, in Arom's meaning of the word (ibid., p.202), a pulsation that the performance is ordered in relation to.\footnote{Music characterized by the presence of an underlying pulsation, where each note value is an integer multiple of the duration of the pulse beat, is denoted "measured music" by Arom:}

\[
\ldots \text{measured music, i.e., music comprised of durations with proportional values. (ibid., p.179)}
\]

As Arom uses the concepts, measured music is performed related to a constant tempo, the tempo being given by the pulsation. In this context it is important to emphasize that in Arom's investigations of African music "measured music" is his object of study, and the rhythm concept as used by Arom is defined for measured music only.

\textit{Example 5:}

As mentioned in section 3.3, Arom asserts that rhythm (in measured music) is unfolded as sequences of auditive events characterized by contrasts created by dynamic accents (A), tone colors (T), or durations (D).\footnote{As we have discussed in section 2.3, live performances are characterized by different kinds of deviations from various norms given in the notation of music. Nevertheless, the present isochronous, temporal norm is important as a "grid" related to which the performed deviations are constructed.} Arom emphasizes that these three parameters usually operate together in a wide variety of ways, but maintains that for theoretical and methodological purposes it might be fruitful to study their impact on rhythmic phenomena separately.

In a sequence of auditive events differences caused by A, T, and D can occur according to the following possibilities.\footnote{It is important to note that "accents" in Arom's sense seem to denote dynamic, phenomenological accents in Lerdahl & Jackendoff's meaning of the words. Differences in dynamics, as well as differences in tone color or duration represent various kinds of phenomenological accents according to Lerdahl & Jackendoff. (Lerdahl & Jackendoff, 1983, p.17; and section 2.1.3 above.)}

\footnote{This is our way of making the classification. Arom does this slightly differently (Arom, 1994, pp.202-204).}
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Dynamic accents can occur in one of the following ways:

- $A_0 = \text{No dynamic accents (i.e. constant level of dynamics)}$
- $A_r = \text{Regular repetitions of accents (cf. Ex.1)}$
- $A_{ir} = \text{Irregular repetitions of accents (cf. Ex.2)}$

Differences in tone color can occur in one of the following ways:

- $T_0 = \text{Constant tone color (cf. Ex.1 and Ex.2)}$
- $T_r = \text{Regular changes in tone color (cf. Ex.3)}$
- $T_{ir} = \text{Irregular changes in tone color (cf. Ex.4)}$

Differences in duration can occur in one of the following ways:

- $D_0 = \text{No differences in duration (Ex. 1,2 and 3)}$
- $D_r = \text{Regular changes in duration. This is not exemplified above, but the following is a simple example of a } D_r:$

```
\_____\_____\_____\_____\_____\_____\_____\_____\_____\_____\_____\_____\_____\_____\_____\_____
```

- $D_{ir} = \text{Irregular changes in duration (Ex.4)}$

Whether an unfolding is identified as having regular or irregular features might indeed be a question of both context and perceptual ability. At this point we will adopt Aром’s distinction between regular and irregular, given on a rather intuitive basis.

By combining the parameters ($A$, $T$, $D$), where each parameter can take one out of three different values (0, r, ir), we get a totality of $3^3 = 27$ possible sequences of audible contrasts. According to Aром, all of these except $A_0T_0D_0$ is a rhythm. $A_0T_0D_0$, on the other hand, is a pulsation as Aром views the matter. Figure 3.4 below presents Aром’s classification of possible rhythms in measured music (Aром, 1994, p.203, Fig.1). As we can see, Aром distinguishes nine different cases. However, by inspection we find that Aром’s classification includes all of our 27 possible SDs, except $A_0T_0D_0$. Aром’s "Type a" is identical to (*) in our Example 1, "Type b" is our (**) in Example 2, "Type c" is (***) of Example 3, and "Type h" is (****) in our Example 4.
Figure 3.4. Arom's presentation of the nine possible ways of producing rhythm in measured music (Arom, 1994, p.203, Fig.1).

Arom makes some very interesting comments upon this classification by saying that types (a) and (b) are rarely used systematically outside children's game songs (type (a)) or some American "repetitive music" (types (a) and (b)). Types (c) and (d), on the other hand, are often found in traditional African music, particularly in polyrhythmic ensembles. The most frequent types of rhythm are, according to Arom, types (e) and (f). They can be used both simultaneously and in succession. Type (e) is typical in Western music of the classical period. Type (f) is current both in *ars nova* polyphony and in modern composed works (e.g. in compositions by Stravinsky). Types (g) and (h) are frequently used by African percussion
groups, while type (I) is characteristic of ancient Greek music and the rhythm of some types of African music. Arom emphasizes that any of these types can appear in succession in a piece of monody, and simultaneously in the case of polyphony. See Arom (1994, p.204) for supplementary comments.

Example 6:

Let us again listen to the drummer playing on two drums. This time his playing may be notated in the following way:

\[ X \times X \times X \times X \times X \times X \times X \times X \times X \times X \times X \times X \times X \]

Note that the distribution of sounds from the low and the high drum follows the same succession as (***) in Example 4 (i.e. low – high – low – high – low – low – low -, etc.). However, as indicated by the lack of note stems and note flags, no underlying pulsation is present in this example, no constant tempo is perceived. Therefore, this performance is not measured music in Arom’s sense, and consequently not the subject of Arom’s discussions.83

Since the drummer also in this case creates an unfolding of perceivable differences (differences in tone color and duration), the notation above clearly represents an SD. Does it also make any reasonable sense to say that the drummer’s performance is rhythmic? We are tempted to answer: No, and yes! Because of the irregular unfolding of changes in both tone color and duration, and due to the lack of any tempo constituting a temporal reference for the performance, many listeners will undoubtedly experience the drummer’s playing as a sequence of arbitrarily executed drum strokes, where the different drum beats are unrelated to each other as to musical or communicative content. Listeners having this experience are likely to say that the drummer is not creating any rhythm in this case. If we adopt this point of view, we hereby obtain an example showing that being SD is not a sufficient condition for being rhythmic.

On the other hand, it might well be the case that the drummer himself views this performance as rhythmic. In spite of no constant tempo and a rather irregular unfolding of changes in tone color and duration, the drummer might experience that he is creating and controlling a musical unfolding that to him has perceivable qualities that make it plausible for him to say that his playing is rhythmic. For instance, the drummer might be tacitly thinking of

83 As presented in Arom (1994, pp.179-216).
a melody to which his pulsationless playing is "rhythmically” related. Given this situation, this exemplifies a case where the performer and the listener might react quite differently as to identifying an SD as a realization of rhythm or not.

As a further comment to Example 6, it is important to stress that we do not assert (nor even mean to suggest) that only music comprised of durations with proportional values, i.e., measured music, is perceivable as rhythmic (to others than the performing musician himself). What is special in this example is that the lack of constant tempo in combination with an irregular unfolding of changes in tone color and duration creates a sequence of audible events where no expressed, explicitly given, familiar temporal reference is present. Therefore, many will experience this performance as unrhythmic. Let us, on the other hand, assume that the drummer is playing according to the following notation:

\[
\begin{array}{cccccccccccc}
& x & x & x & x & x & x & x & x & x & x & x & x \\
\end{array}
\]

Also in this performance no underlying pulsation is present, and the music is consequently not measured music. In this case, however, the changes in tone color are unfolded in quite a regular way: A low drumbeat is followed by two high beats in a repeating cycle. The drumbeats are thus not arbitrarily distributed in time but are ordered, creating a pattern of different events. For many listeners this ordering of auditive events might be sufficient for a perception of the performance as rhythmic.

In spoken language, as in music, differences in dynamic accents, tone color and durations are unfolded. Our speech is usually not "measured" in the sense of being comprised of temporal parameters with proportional values. Nevertheless, the differences unfolded through speech constitute various ordered patterns and regular changes in audible events with which we are able to familiarize, and that makes it possible to understand different languages and distinguish between different dialects. Perhaps the process of pattern recognition, fundamental to human communication as such, is the very common feature of the perception of various phenomena denoted as "rhythmic". And if so, maybe this motivates the use of the concept ‘rhythm’ in investigations of linguistic and psychological processes.\(^\text{84}\)

\(^{84}\) As is demonstrated in e.g. Evans & Clynes (1986).
Example 7:

The examples 1 to 6 have all demonstrated SDs related to unfoldings of music. Viewed against the many examples of "perceivable differences" given in section 3.3, numerous examples of SDs taken from other scientific fields or everyday contexts can also be presented. We shall here confine ourselves to two examples:

(a) A visual SD:

A movie shows a transformation/deformation of geometrical objects according to the following illustrated sequence of events:

![Diagram of geometric shapes transformation](image)

As indicated by this illustration, a transformation of visual differences in size (large – small) and geometric shape (square – circle) is here unfolded. This sequence of events is thus a sequence of differences. Due to the cycling pattern created by these changes (large square – small square – small circle – large circle, back to large square, and over again), this unfolding represents an ordered sequence of events where the changes after some time become predictable to the observer. To some viewers this might even be sufficient to denote this sequence of visual events as "rhythmic".

(b) An SD in sports:

A soccer team has trained in different ways of performing corner kicks. One example is demonstrated in the diagram below: Player A makes a short pass to player B. Player B lifts the ball to player C, who comes running and heads the ball into the goal!

![Diagram of soccer corner kick](image)
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This sequence of events consists of different activities of movements carried out by three different people, and is therefore an SD. A characteristic feature of this SD is a coordinated interaction between the three players A, B and C. A necessary condition for a successful performance of the corner kick is that the players’ movements are adjusted and timed in relation to each other. To the same extent that the soccer players succeed in their interactions, the chances of scoring a goal will increase. If such coordinated movements are described as "rhythmic" (which indeed is usual among many sport trainers and journalists), it seems appropriate to assert that well performed rhythmic patterns of movements are fundamentally basic to success in the sports!

3.5. What Is Missing?

In the beginning of section 3.4 we asked if a sequence of differences is necessarily an unfolding of rhythm. We have tried to approach an answer to this question by examining various examples of SDs, one by one discussing whether it makes sense to denote the SD as "rhythm" or not. The examples 1 to 5 were all taken from musical situations, where the music was played related to a constant tempo. If we accept Arom’s classification of audible contrasts (consisting of combinations of contrasts in A: Dynamical accents; T: Tone color; D: Duration), we obtain a total number of 26 possible SDs. (A,T,D is, as mentioned earlier, not an SD according to Arom.) All of these SDs are also rhythms if we follow Arom. Thus, restricted to this class of events, taken from unfolded performances of measured music, the concepts ‘rhythm’ and ‘sequence of differences’ coincide according to Arom.

Example 6 was also taken from a musical situation, but in this case the music was played without any constant tempo. In our discussion of this example we argued that the lack of tempo in combination with the irregular unfolding of differences in tone color and duration creates a sequence of audible events (i.e. different drum beats), which when exposed to a listener, is most likely perceived as an arbitrarily distributed sequence of drum strokes, where the different drum beats seem unrelated to each other. Subject to this experience, it does not appear reasonable to denote the performance as "rhythmic". If we accept this argumentation, we also obtain an example of an SD not being a rhythm. Hence, in this case a negative answer must be given to the question posed at the beginning of section 3.4.
If we adopt this view, a new question naturally arises: What is "missing" in a sequence of differences that is not a rhythm? Or, to put it in another way: If an unfolding, U, is a sequence of differences, which additional qualities must be present if U is to be an unfolding of rhythm? Some possible answers to this question are implicitly given in the examples of the preceding section. Comparing Example 6 to the other examples, it becomes apparent that common to examples 1 to 5 is a varying degree of perceivable structure constituting a basic reference for the musical unfolding. In these examples the structure provides a temporal reference, or more precisely, an underlying pulsation, to which both the performance and the experience of music is related. In some of these examples additional structural features are present, e.g. in Example 1: Regular occurrences of dynamic accents, and in Example 3: Regular change of tone color. In Example 6, on the other hand, no obvious, explicitly expressed temporal, dynamic or timbral structure is unfolded. The lack of such structural information in the sequence of audible events makes it difficult, not to say impossible, for the listener to induce any regularity, "well-ordering" or basic reference unit in this sequence of differences.\(^5\) Precisely for this reason many people will argue that the unfolding discussed in Example 6 is not a realization of rhythm.\(^6\) Thus, it seems appropriate to say that structure is a characteristic of rhythm that is common to the different manifestations of rhythm, and which a large number of the users of the rhythm concept regard as important in making distinctions between rhythm and arrhythmia.

Hence, based on the examples given in section 3.4 and the discussion above, we have identified our third relevant, invariant rhythm characteristic:

**RIRC 3: Structure is a relevant, invariant rhythm characteristic.**

In Example 7 of section 3.4 we illustrated two different SDs taken from non-musical situations (one visual SD, and one SD in sports). In both of these brief illustrations we said that if some kind of ordering or pattern of differences is recognized, ordinary use of language

\(^5\) Of course, various kinds of subjective rhythmizations "adding" structural features to the sequence of audible events might be induced also in the experience of this performance, but, to our knowledge, investigations demonstrating principles of inter-individual validity in this case have not been carried out. On the other hand, subjective rhythmization related to experience of unfoldings of constant tempo is well documented empirically, as mentioned earlier in chapters 1 and 2.

\(^6\) This view seems to be supported by Bengtsson in his article in Capella Musikklotet (Volume 5, 1980) (quoted in the beginning of section 3.2), by Cooper & Meyer's "definition" of rhythm (1960, p.6) (quoted in section 2.1.2), by Lerdahl & Jackendoff's heavy underlining of various structural principles fundamental to music of the Western tradition (Lerdahl & Jackendoff, 1983; see also section 2.1.2), not to mention by the classical comprehension of rhythm as order in movement, and the many modern successors of this ancient point of view (cf. Chapter 7).
makes it plausible to denote each of these events as "rhythmic". This suggests that RIRC 3 is valid on a more general basis than just applied to musical situations.  

3.6. Rhythm Object - a Possible Limit of Convergence

Based on RIRC 1 and RIRC 2, we have defined a sequence of differences as an unfolding of perceivable differences over a closed interval of time. Furthermore, through discussions of various examples we have reached the conclusion that a sequence of differences is not necessarily an unfolding of rhythm. The characteristic distinguishing a sequence of differences from rhythm that seems to be "missing" is RIRC 3.

This makes us ready to propose the following definition of a possible limit of convergence of rhythm concepts:

Definition:
A rhythm object (RO) is a sequence of differences where the differences are perceived as ordered according to some familiar structure.

It is important to make some comments on this definition:

First comment: Since a rhythm object is by definition a sequence of differences, every comment made in section 3.3 related to an SD is also valid addressing an RO. In particular it should be emphasized that an identification of a rhythm object is dependent on an act of perception. This dependency has "double weight" in the recognition of a rhythm object: On the one hand, an unfolding of events must be perceived as an SD; and, on the other hand, whether this SD is experienced as a rhythm object is heavily dependent on abilities in identifications of "familiar" structure and perception of some "order". These are abilities that are governed by cultural as well as social conditions, and which to some extent can be developed and expanded. For example, structural characteristics of Chinese music might be difficult to recognize for Western, and in this respect, untrained ears, while people with other

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87 Further support to regarding structure as a basic characteristic of rhythmic phenomena outside the field of musicology or ethnomusicology is given in Evans & Clynes (1986).
88 As touched upon above, the same might be said related to perception of differences.
social and cultural references than us might feel unfamiliar with our ways of organizing information. A fundamental key to an understanding and appreciation of a foreign culture is precisely an ability to familiarize with new ways of organizing cultural and social interactions and communication: learning a new language; becoming familiar with new habits, new customs, new rituals, a new religion, learning new ways of organizing and performing music, etc.

Second comment: Being an SD, a rhythm object is an unfolding over a closed interval of time. Thus, an RO does not exist in discrete, isolated points of time. Let us return to the drummer playing on two drums. This time we suppose his performance may be notated as follows:

```
\[ \begin{array}{ccccccc}
  & x & x & x & x & x & x \\
\end{array} \]
```

As mentioned earlier (cf. Example 3 in section 3.4), this represents a rhythm according to Arom’s classification of possible rhythms in measured music. The very notated representation of the performed drum beats is an example of rhythm “outside time” in Xenakis’ terminology, whereas the audible unfolding of drum beats perceived over a closed interval of time \([t_1, t_2]\) is to a large number of people a rhythm object. The discrete sequence of isolated drum beats is, on the other hand, not a rhythm object, according to the definition.

If the drummer is playing as indicated above, but using only one drum instead of two, he is creating a pulsation in Arom’s terminology (cf. Example 1 in section 3.4). According to Arom, this pulsation is not a rhythm. As mentioned in Example 1, however, also in this performance two different phenomena are unfolded; beat and non-beat (the event temporally separating one beat from the next). A possible perception of this unfolding as a rhythm object may thus be induced through an experience of a regularity in the alternations between these differences; beat – non-beat – beat – non-beat, etc. Consequently, it does indeed make sense to regard pulsation perceived as an unfolding over an interval of time as a rhythm object, or maybe some would say, as a rhythm.¹⁰

Third comment: In his doctoral dissertation: “Formalization and epistemology”, Rolf Inge Godøy defines the concept ‘musical object’ in the following way (Godøy, 1993, p.33):

¹⁰ It should be observed that this process of perception is somewhat different from subjective rhythmization, the latter involving a perception of some kind of “weight” added to some of the drum beats.
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A musical object is any segment of sonorous unfolding, within defined temporal limits, which we have for our consideration.

This concept is fundamentally basic to Godøy in his studies and discussions. To quote Godøy (ibid., p.33):

The notion of the musical object is essential in this study. It represents the convergence of epistemological, hermeneutical and formalist ideas, and is a standpoint in favour of cultivating the global qualities of delimited segments of musical unfolding rather than the cultivation of the more abstract kinds of structures encountered in some music theory.

Godøy’s concept ‘musical object’ and our ‘rhythm object’ share many common features. First of all, both concepts denote phenomena existing in an interaction between an "external, physical" and an "internal, mental" reality. As Godøy puts it: "....sonorous musical unfolding, ...., which we have for our consideration" (cf. above, as well as our "first comment" related to the definition of a sequence of differences in section 3.3). Referring to Schaeffer’s concept ‘sound object’ ⁹⁰, Godøy points at this interaction between physical and mental characteristics of the musical object by emphasizing what the musical object is not:

....these qualifications apply equally well to the musical object:
- "The sound object is not the sounding body."
- "The sound object is not the physical signal.”
- "The sound object is not a fragment of a recording.”
- "The sound object is not a symbol notated in a score."
- "The sound object is not a state of the soul.‖ (Godøy, 1993, p.97)

From the definition of rhythm object and the comments made above, it should be clear that the remarks made related to Schaeffer’s sound object, which Godøy claims valid for the musical object, also apply to our rhythm object.

Another feature shared by ‘musical object’ and ‘rhythm object’ is that both concepts are defined over a delimited interval of time. If we borrow Godøy’s formulation, it may therefore seem appropriate to say that our notion of the rhythm object represents “a standpoint in favour of cultivating the global qualities of delimited segments of musical unfolding” (cf. above).

However, there is one essential point where an important distinction between Godøy’s ‘musical object’ and our ‘rhythm object’ is clear: All musical objects are sonorous musical unfoldings, whereas a rhythm object may be an unfolding of events being of another kind than just auditive (cf. the examples of "perceivable differences" given in section 3.3). On the other hand, it does seem right to say that the class of auditive rhythm objects is a subclass of the class of musical objects.

⁹⁰ As presented in Schaeffer (1966).
Fourth comment: A rhythm object is a sequence of differences. On the other hand, there exist sequences of differences that are not a rhythm object. Rhythmic unfoldings of music are examples of rhythm objects, but there do exist rhythm objects that are not audible (for instance, Example 7 in section 3.4). The following illustration might help to make these relations more explicit:

Figure 3.5. Illustration showing the relations between the concepts 'sequence of differences', 'rhythm object' and 'sequence of auditory events'.
3.7. Summary

In the beginning of this chapter we posed the following questions: Are there any existing features of rhythm common to all the different applications of the concept ‘rhythm’? Is it possible in a meaningful way to identify a “core” of the rhythm concept basic to the various divergent applications, making it plausible to denote seemingly quite different events from everyday life, music or scientific investigations by the common term: rhythmic?

Applying a step-wise process of presenting examples, formulating concepts and making definitions, we have tried to approach an answer to these questions. Discussing how the rhythm concept is used in everyday speech, in musical and in scientific language, we have identified three relevant, invariant rhythm characteristics (RIRC): RIRC 1: Time; RIRC 2: Difference; and RIRC 3: Structure. We have argued that these characteristics are common to the various manifestations of rhythm, and are important in making distinctions between rhythm and arrhythmia. A concept ‘X’ “summarizing” common features of rhythmic phenomena should therefore include all three characteristics, RIRC 1, RIRC 2 and RIRC 3.

Our proposed definition of rhythm object is made as an attempt at meeting this demand. A rhythm object may therefore be seen as a possible “limit of convergence” of rhythm concepts. Accepting this view, we also arrive at an affirmative answer to the questions asked in the introduction to this chapter. Hence, the illustration given in Figure 3.2 of section 3.1 now takes the following form:

A "limit" of a possible convergence of 'rhythm':

![Diagram](image)

Figure 3.6. Illustration of a process identifying a "limit of convergence" of rhythm concepts.

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Chapter 3

As mentioned several times in this chapter, our efforts in the discussions of examples and formulations of concepts have not been aimed at giving any definition of rhythm. Instead we have tried to extract common features of different applications of 'rhythm', and unite these in a convergent rhythm concept. Thus, we are not asserting: "Rhythm is ...", but rather: "According to the use of the concept 'rhythm', rhythm is fundamentally characterized by ...". An identification of a convergent rhythm concept seems interesting for several reasons: On the one hand, an articulation of a common rhythm concept establishes an important basis for model constructions and syntheses of rhythmic events. The possibilities of making various interpretations of the model and hence the very validity of the model itself are fundamentally dependent on to what extent the model takes the different rhythm characteristics into account. It is therefore interesting to have these characteristics of rhythm, as well as a possible convergent rhythm concept, explicitly at hand. On the other hand, the designatum of this concept, i.e. the events referred to by rhythm object, may represent a key to the discovery of a possible "code" in the understanding of rhythm as a phenomenon, and thereby contribute to greater comprehension of human perception as such.

In our following studies and discussions we will apply the concept 'rhythm object' to an investigation of characteristics of rhythmic performance of music. Subsequently this will constitute a basis for model constructions and syntheses of performance of rhythm. Our main focus will be on rhythm in music, but by the nature of the approach in our identification of the rhythm object, some interpretations of our model to situations outside music will also be hinted at.
Rhythmic Movements

Having identified 'rhythm object' as a convergent rhythm concept, we now use this concept as a point of departure in our further studies of rhythmic phenomena. As pointed out in the previous chapter, rhythm objects constitute a class of temporal events belonging to a wide variety of perceivable unfoldings. Some of these are audible, others visual; some are experienced through movement, others through touch or taste; some are given by biological or astronomical changes, others may be dependent on social, economic or cultural differences, to name just a few. Our main focus in this chapter will be directed towards the subclass of rhythm objects containing sequences of events denoted as human rhythmic activities, and in particular, rhythmic performance of music. These are activities that involve a performing, acting human being, \( H_{perform} \), creating an unfolding perceived as an ordered, structured sequence of differences by a human being, \( H_{perceive} \). In those situations where the performer himself experiences his activities as ordered according to some familiar structure, we obtain \( H_{perform} = H_{perceive} \). This, however, need not necessarily be the case, according to our definitions of the concepts involved.

4.1. Rhythmic Activities and Movements of the Body

One type of rhythmic activity is overt rhythmic behavior. This includes the behavior of human beings or animals, expressed as ordered patterns of movement in, for instance, walking, running, jumping, or in the moving wings of a flying bird.\(^9\) Human movements related to various kinds of human activities of physical work are also included; e.g. rhythmic movements involved in digging a ditch, or rhythmic co-action among people helping each other to lift a heavy rock.\(^9\) In sports, rhythmic behavior is expressed in ordered patterns of

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\(^9\) The very structured patterns of an animal's movements are fundamentally related to the animal's ability to survive.

\(^9\) Cf. the use of work songs, where a primary function of rhythmic singing is to coordinate the activities and movements of people working together, thereby optimizing the efficiency of physical work.
movements, crucial to optimizing speed in moving from one place to another (for instance, in swimming, running, skiing, skating, etc.), and in rhythmic interactions between soccer players, important to the success of the team. And, not the least, in the experience of music various kinds of overt rhythmic behavior may include well-known things as tapping the feet, shaking the head, clapping the hands, and dancing. Common to all these types of rhythmic activities is an unfolding of ordered, structured changes in movements of the body.

Various movements of the body are also fundamental to rhythmic performance of music. Precisely in the interaction between the musician and his instrument different movements of the body are performed that to a varying degree create, or are influential to, the musical unfolding - the performance of the music. The correlations between body movements and sounding music may differ to a great extent, and are dependent on both the performing musician and the instrument being played. It seems right to say that this correlation is strongest in the case of the drummer playing with one hand on one drum. The sound of the drum and the rhythm being created is in this situation closely in correspondence with the physical movements of the hand. To the same extent that the performed music is perceived as rhythmic, the movements of the drummer’s hand may be called rhythmic movements. If the drummer is playing with both hands on one drum, a rhythmic performance of music will also be related to rhythmic movements of the drummer’s hands. However, in this latter case the relation is by no means as unambiguous as when the drummer is playing with only one hand. This is exemplified by the fact that:

\[
\begin{array}{cccccc}
& & R & R & L & R \\
\downarrow & & & & & \\
\end{array}
\]

and

\[
\begin{array}{cccccc}
L & L & R & R & L & \\
\downarrow & & & & & \\
\end{array}
\]

played on the same drum, with the same tempo and dynamics, may be experienced by a listener as the same rhythm. To the performing drummer, on the other hand, these two performances are based on different patterns of rhythmic movements in the right (R) and left (L) hand. Hence, in performing these patterns many drummers will have an experience of different rhythms, or, maybe better formulated, as different ways of phrasing the same rhythm. The choice of phrasing and how the musician experiences the corresponding
performance are both cultural dependent and related to musical style and genre. The effect of the different choices in right and left hand becomes more obvious if the drummer is playing on two drums, the right hand playing on one drum, the left on another, or if right and left hand are performed with different dynamics.

For musicians playing other instruments than drums or percussion, there also exist fundamental and significant correlations between patterned movements of the body and rhythmic performance. Such correlations may seem easiest to observe and describe when studying musicians that are primarily using their hands and feet: For instance, when playing a keyboard, the movements of the musician's hands and fingers are intimately related to the rhythmic qualities of the musical performance, as is also the case when playing guitar, bass, or violin. When playing a wind instrument, the situation may seem somewhat different. In this case the sound is produced through an interaction between movements of hands and fingers, and movements of the lip, tongue and breath. Even if body movements are certainly basic to the performance of musical unfoldings also in this situation, the correspondence between rhythmic movements of the body and performance of rhythm is far from simple and unambiguous. This is perhaps even more true for singers, where the very process of sound creation is hidden "inside" the body. In playing a bagpipe, it might even be the case that different body movements may seem more or less independent of one another: One kind of movements in correspondence with the musical unfolding (for instance, movements of fingers), other movements related to filling air in the bag. And this latter need not, as it often seems, be related to the rhythm of the music.

In addition to body movements performed in a conscious way to create the musical unfolding, the musician may also move his body in ways that are more or less unconscious, being in varying degree "in time" with the rhythm of the music. When listening to a jazz band, we might, for instance, observe that some of the musicians "take off", moving their heads and feet in ways that seem to be only vaguely correlated to the rhythmic patterns of the music. In this case the musician also expresses an obvious responding relation to the musical performance of which he himself is a participant. In a similar way to a listener, the musician gets "carried away". Lifted by the music, a listener might even, perhaps more often than the musician, start dancing to the music. While shaking the head and tapping the feet may be expressions of unconscious rhythm response, dancing, at least in our culture, is a way of expressing rhythm response in a basically conscious manner.99 Precisely in dancing the

99 In many African cultures, where music and dance are closely related to religion, dance as part of a religious ceremony will often lead to a state of (unconscious) trance.
correlation between sequences of rhythmic events in the music and patterns of rhythmic movements of the body are just as important as significant.

Hence, fundamental to both performance and experience of rhythm is a connection between musical rhythm and patterned movements of the body. Through this connection body movements are given a rhythmic form, and rhythm in music is related to ordered movements of the body. In this respect Gabrielson (1986, pp.160-161) asserts:

With regard to auditory-musical rhythm we have seen attempts at relating the structural aspects to principles from Gestalt psychology, but also to kinaesthetics and motor functioning. The "origin of rhythm" has been sought in the rate and regularity of the heart-beats (mother's heart-beats are felt by us even before birth), the breathing cycle, various electrophysiological processes in the nervous system, and not the least in the pendular movements of our limbs (e.g., in walking) or other body movements. All of these candidates are more or less plausible. ....

That rhythm is somehow deeply rooted in the construction and functioning of the body seems beyond any doubt, and the apparent and multi-faceted connections between movements and rhythm speak very much for the movement alternative (which does not mean exclusion of other alternatives).

Especially in music education and pedagogies of rhythm it seems very important to underline the basic and "multi-faceted" connections between movements and rhythm. This is strongly emphasized in eurhythmics, which is a method developed by Emile Jaques-Dalcroze (1921 (new ed. 1967)), but also in other well-known systems in music pedagogy, as the Orff school, the Kodaly system and the Carubbo-Cone method, to name some.94 Gabrielson (1986, p.159) makes the following general comment on these approaches to music education:

The very concrete dealing with rhythm in these settings in terms of body movements to sound and music -- walking, jumping, running, bending, clapping hands, playing various games, etc. -- provides a fundamental sensori-motor understanding of many musical concepts (tempo, pulse, accents, grouping, phrasing, dynamics, and, of course, rhythm in general), even long before the verbal learning of such concepts. It is tempting to believe that this also naturally contributes to the understanding of the motional and emotional aspects of rhythm, which tend to be suppressed in rhythm training which is solely based on reading of notated patterns (.....).

For the further discussions and investigations it now becomes important to have a closer look at the method of Emile Jaques-Dalcroze.

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94 See, e.g., Mark (1978) for an overview.
4.2. The Eurhythmics of Emile Jaques-Dalcroze

Emile Jaques-Dalcroze (1865-1950) was a Swiss pedagogue, composer and performing musician. His educational method, **eurhythmics**, is an approach to music education based on the premise that rhythm is the primary element in music, and that the source for all musical rhythm may be found in the natural rhythms of the human body. Although the pedagogy of Jaques-Dalcroze is often referred to as “eurhythmics”, his total method consists of three parts: **eurhythmics, solfège** (some kind of melodic/harmonic ear-training), and **improvisation**. In his book “Rhythm, Music & Education”, published for the first time in 1921 (new edition in 1967), Jaques-Dalcroze presents 13 chapters written during the period from 1897 to 1919. These are presented in chronological order, recording his ideas, experiments and the stepwise development of his pedagogic method. In the introduction to the new edition of 1967 Keith Falkner writes:

Music education has advanced out of all knowledge in the past sixty years. It is no longer regarded as an “extra”, it has become an integral part of general education.

Among the pioneers who have contributed to this change of outlook, Emile Jaques-Dalcroze is outstanding. His unique approach to music education undoubtedly stemmed from the fact that he came to teaching as a composer and creative artist. He quickly realised that the musical element of primary appeal to children is rhythm; that the natural response to rhythm is physical, and that the body should be the child’s first instrument through which to reflect and interpret the movement and nuances in music. (Ibid., p.v)

It is interesting to notice that Falkner makes a point of the relation between Dalcroze’s approach to music education and the fact that Jaques-Dalcroze himself was a performing artist. Indeed, this remark seems both relevant and important and is, furthermore, largely in accordance with our strong emphasis on the connection between rhythmic performance of music and rhythmic patterns of body movements, as presented above.

In the foreword of the 1921 edition of “Rhythm, Music & Education” we learn that Emile Jaques-Dalcroze made his début in pedagogy in the early 1890s as professor of harmony at the Conservatory of Geneva. After a few lessons he discovered that many of his students, though theoretically well trained, were unable to appreciate and hear the chords which they had to write. From this he soon concluded that the flaw in the conventional method of training of that time was that students were not given experience of chords at the

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95 *Eurhythmics*; from Greek "good rhythm".
96 In Jaques-Dalcroze, 1921 (1967), p.65, we read: "The study of SOLFÈGE awakens: the sense of pitch and tone-relations and the faculty of distinguishing tone-qualities."
97 See Jaques-Dalcroze (1921 (1967)).

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beginning of their studies. Therefore, he started preceding his lessons in written harmony by special exercises of a physiological nature, aimed at developing the hearing faculties of his students. Doing this, Jaques-Dalcroze noticed that the exercises he presented were quite spontaneously appreciated by children, whereas older students often reacted with some resistance, hindered by intellectual preconceptions. For this reason he set about training the ears of his pupils as early as possible. Through conducting concrete, practical exercises Dalcroze now observed that the hearing faculties of the children developed in a remarkable way, and once the ear was well trained, there were less problems related to the various processes of reading and writing music and solving tasks of composition.

Nevertheless, even if the students’ abilities to hear harmonies were well developed, Jaques-Dalcroze noticed that some students had problems estimating and articulating variations of time and rhythmic grouping. This led Dalcroze to the conclusion that the motive and dynamic element in music depends not only on hearing, but also on another sense. Having observed that metrical finger exercises contributed to the pupil’s progress in experiencing and articulating rhythm, Dalcroze at first thought that the fundamental sense in need of musical training was the sense of touch. However, as formulated by Jaques-Dalcroze himself, in the foreword of the 1921 edition of his book:98

Presently, however, a study of the reactions produced by piano-playing in parts of the body other than the hands – movements with the feet, oscillations in the trunk and head, a swaying of the whole body, etc. – led me to the discovery that musical sensations of a rhythmic nature call for the muscular and nervous response of the whole organism. I set my pupils exercises in stepping and halting, and trained them to react physically to the perception of musical rhythms. That was the origin of my “Eurhythmics”……

To test his theories, Jaques-Dalcroze conducted a series of experiments where he observed his students in the execution of different exercises of body movements related to musical performance and response. At an early stage of his investigations Dalcroze thought that a solution to many rhythmic problems consisted in teaching his students rhythmic behavior through training the muscles to contract and relax according to the following three parameters:

1. **Time**: A body movement (a muscular activity) should be placed (executed) in time in accordance with the *speed* or *tempo* of a sequence of sounds.

2. **Space**: The physical space occupied by a body movement should correspond to the *duration* of a sound.

3. Force: A body movement should be executed with a particular force corresponding to the dynamic energy of a sound.

Or, to quote Dalcroze from an article written in 1907: “The Initiation into Rhythm” (see Jaques-Dalcroze, 1921 (1967), p.39):

Muscles were made for movement, and rhythm is movement. It is impossible to conceive a rhythm without thinking of a body in motion. To move, a body requires a quantum of space and a quantum of time. The beginning and end of the movement determine the amount of time and space involved. Each depends on the gravity, that is to say (in relation to the limbs set in motion by the muscles), on the elasticity and muscular force of the body.

Moreover, in the same article Dalcroze asserts:

..... a properly executed rhythm requires, as a preliminary condition, complete mastery of movements in relation to energy, space, and time. (ibid., p.39)

And, to conclude at this point:

To sum up: music is composed of sound and movement. Sound is a form of movement of a secondary, rhythm of a primary, order. Musical studies should therefore be preceded by exercises in movement. Every limb – first separately, then simultaneously; finally the whole body – should be set in rhythmic motion; the resulting formations; i.e. the relations between the energy, space, and time involved, being carefully collated and regulated. (ibid., p.44)

However, by further observations of his students Dalcroze soon found out that even if his students were able to physically move their bodies in precise and comfortable ways, the students' ways of performing body movements as response to music were quite divergent.- In “The Approach of Emile Jaques-Dalcroze” (Abramson, 1986, pp.32-33) we read:

He began by playing musical rhythms suggesting the motion of walking and asked his students to respond to what they heard by regulating the positions and movements of their walk to express the speed, the durations, and the accents they heard in the music. ...Some students responded by moving too quickly; some, too slowly. Some students performed well only at certain tempi and not at others. Many students performed well at moderate tempi, but had difficulty when asked to change to a faster or slower speed, in spite of their obvious desire to make the change. Students had difficulty remembering a tempo or a duration. Sometimes the students would begin a walking exercise smoothly and confidently, then suddenly become confused and unable to continue accurately.

On these observations Jaques-Dalcroze, himself, comments:

I soon discovered that, out of ten children, at most two reacted in a normal manner; that the motor-tactile consciousness, the combination of the senses of space and movement, exist in a pure state as rarely as the perfect sense of hearing that musicians call “absolute pitch.” I saw the lack of musical rhythm to be the result of a general “a-rhythm,” whose
cure appeared to depend on a special training designed to regulate nervous reactions and effect a co-ordination of muscles and nerves; in short, to harmonise mind and body. (Jaques-Dalcroze, 1921 (1967), Foreword, p.viii)

Thus, Jaques-Dalcroze recognized that something crucial was still missing from his method. It is not sufficient to understand a rhythm and to have the physical, muscular ability to execute patterned movements of the body. His students had been trained at both, but many were still unable to perform rhythm in agreement with the very aim of Dalcroze’s method: To perform musical rhythm in an accurate, expressive and comfortable way by means of patterned movements of the body where energy, time, and space are interrelated as to obtain the maximum effect by a minimum of effort. In addition to a conceptual understanding and a physical ability to execute certain patterns of body movements, some kind of “co-ordination of muscles and nerves”, as formulated by Dalcroze, is necessary. This co-ordination represents a sort of rapid communication between the brain, which conceives and analyzes, and the muscles which perform. Hence, eurhythmics should also include the training of this system of communication, hopefully with the result of obtaining what has so far been missing; “to harmonise mind and body” (cf. Dalcroze, as quoted above).

4.3. A Kinesthetic Process

Abramson (1986, p.33) asserts:

Jaques-Dalcroze now returned to his earlier speculations on the role of muscular sensations in rhythmic learning. He postulated that whenever the body moves, the sensation of movement is converted into feelings that are sent through the nervous system to the brain which, in turn, converts that sensory information into knowledge. The brain converts feelings into sensory information about direction, weight, force, accent quality, speed, duration, points of arrival and departure, straight and curved flow paths, placements of limbs, angles of joints, and changes in the center of gravity. The brain judges the information and issues orders to the body again through the nervous system. These orders are given to protect the organism from injury and to find the most effective ways to move through the mental phenomena of attention, concentration, memory, willpower, and imagination.

Today this process is called the kinesthetic sense.

Exercises aimed at training this kinesthetic sense had been missing up to this point in the eurhythmics of Jaques-Dalcroze. When a kinesthetic process is incorporated as being fundamental to eurhythmics, the basic relations between performance and experience of

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99 To our knowledge, Dalcroze himself does not use the term ‘kinesthetic’.
music and rhythmic movements of the body are further strengthened and emphasized. Figure 4.1, adopted from Abramson (1986, p.34), summarizes the kinesthetic process:

![Diagram of the kinesthetic process]

Figure 4.1. Schematic description of the kinesthetic process (adopted from Abramson, 1986, p.34)

A major problem facing Dalcroze in developing exercises aimed at training the kinesthetic sense was that kinesthesia generally works on a subconscious level. We are seldom aware of the kinesthetic process during ordinary activities: riding a bicycle, brushing one's teeth, singing, kicking a ball, clapping one's hands are different activities carried out without being consciously aware of the communicative processes of information exchange between the body and the brain, the "exterior" and the "interior" (cf. Figure 4.1 above). In most cases it is even an advantage, not to say a matter of life or death, that the connection between brain and body are made largely automatic in such a way that time does not have to be spent thinking of setting a movement into action. For instance, moving around in a crowded city you make a series of rapid changes in speed and direction of movement; you walk, stop, run - as well as various changes in body balance. All these changes are made in order to fulfill the intention of reaching a specific destination, e.g. a certain restaurant or a concert hall, without being run over by a car or being run into by other people on the way. If it was necessary for you to think and make conscious judgements for every move you made, this would slow down your movements in a considerable way, and probably appear confusing to the people you met simply because they would consider your movements as unsecure and hesitant. Precisely for this reason the very chances of you being run over by a car might also increase! Or, if you run for your life followed by a furious dog and arrive at a fence, you are
not likely to use much time thinking of what kinds of movements of your hands and feet you should activate in order to jump over the fence. You just jump (hoping that the dog won’t do the same)!

To a large extent a similar situation is present with respect to musical activities. A musician playing a piano concert should not spend time during the concert thinking of which movements of his fingers and hands are necessary to produce the various musical results. (On the other hand, this may indeed be a relevant activity during rehearsals.) To the piano player it is extremely important that a large number of musical patterns of body movements are made automatic, in such a way that he can use all his attention and energy solely on the expressive musical performance, giving “life” to the music! Especially in improvised, in various kinds of jazz and folk music for example, it is of fundamental importance that the musicians have internalized a repertoire of musical phrases and patterns of body movements. When improvising, the musician is at the same time performing and responding, and the musical communication and the various interactions between the musicians (and the musicians and the audience) happen instantaneously. An improvising musician must therefore be able to perform spontaneously, without having to think of the various phrases and movements of the body in advance.

Being a musician himself, these facts were given much attention by Jaques-Dalcroze. It became an important task in his eurhythmics to develop an educational method where the unconscious processes governing a musical performance were trained and stimulated. Dalcroze attempted to achieve this by conducting exercises that contributed to an increase of the student’s repertoire of unconscious rhythmic movements. The following provides an example:

- A certain rhythm may appear difficult the first time you try clapping it.- You have to concentrate very hard, thinking about how you should move your hands and consciously adjusting the movements of your hands in relation to some pulse or some other basic rhythm.
- You repeat the rhythm many times, clapping it over and over again. Each time you pay careful attention to performing the rhythm as comfortably and naturally as possible.
- Little by little you experience that the movements of your hands are executed “by themselves”; you don’t have to think as much as before performing the rhythm.
Chapter 4

- After some time you might even be able to clap the rhythm while thinking of something else! Or, perhaps you succeed in clapping the rhythm simultaneously as you are tapping another rhythm (which you have rehearsed earlier) with one of your feet.

Through this process a rhythmic pattern of body movements (the performed clapping of the rhythm) has been made automatic. In the beginning it was necessary to think about the different movements involved in clapping the rhythm, and the various patterns of body movements were to a large extent carried out in a conscious way. At a later stage of the exercise, the movements are made automatic and the rhythm can be performed without thinking of the different hand movements. Hence, by means of this exercise the repertoire of unconscious rhythmic movements has increased.

Moreover, while clapping a rhythm you have learned through a process as above, you are able to listen to yourself "from the outside". I.e.; through an experience of yourself performing the rhythm you become consciously aware of your unconscious movements, and you are able to apply your unconscious movements in a conscious way simultaneously with other unconscious movements. This is, for instance, what happens when a drummer performs different rhythmic movements with his hands and feet playing a rhythm on the drum set. On these matters Dalcroze comments:

The aim of all exercises in eurhythmics is to strengthen the power of concentration, to accustom the body to hold itself, as it were, at high pressure in readiness to execute orders from the brain, to connect the conscious with the sub-conscious, and to augment the sub-conscious faculties with the fruits of a special culture designed for that purpose. In addition, these exercises tend to create more numerous habitual motions and new reflexes, to obtain the maximum effect by a minimum of effort, and so to purify the spirit, strengthen the will-power and install order and clarity in the organism. (Jaques-Dalcroze, 1921 (1967), p.62, first presented in the article "Rhythmic Movement, Solfèège, and Improvisation", written in 1914.)

Moreover, he continues:

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100 A usual way for drummers to practice bebop drumming (bebop, a jazz style developing around 1940, where the drummer most often performs with a large degree of independence between the hands and the feet) is to play a solid and "constant" rhythm on the ride-cymbal with the right hand, to play 2's and 4's on the hi-hat with the left foot, and simultaneously use the left hand and the right foot to play syncopated and accentuated rhythmic phrases on the snare drum and the bass drum. When performing this exercise, it is important that the drummer is able to play these rhythms without the rhythmic figures on snare and bass drum "disturbing" the rhythms on ride-cymbal and hi-hat. The point in this respect is to be able to play the ride-cymbal and hi-hat in an "automatic" way such that the drummer's focus playing with other musicians can be directed towards communicating with the piano player, the saxophone player, etc. through various choices of rhythmic phrases on the snare drum and the bass drum.
The aim of eurhythmics is to enable pupils, at the end of their course, to say, not “I know,” but “I have experienced,” and so to create in them the desire to express themselves; for the deep impression of an emotion inspires a longing to communicate it, to the extent of one's powers, to others. The more we have of life, the more we are able to diffuse life about us. “Receive and give!” is the golden rule of humanity; and if the whole system of rhythmic training is based on music, it is because music is a tremendous psychic force: a product of our creative and expressive functions that, by its power of stimulating and disciplining, is able to regulate all our vital functions. (Ibid., p.63)

On reading these statements, we understand that Jaques-Dalcroze was of the opinion that his eurhythmics had the potential of being a method that could have important applications beyond the field of music education. Abramson (1986, p.35) writes:

It was, in fact, a general education using music as a humanizing force. It was designed to teach students to use all of their faculties in solving problems. This aspect of the use of Eurhythmics as an educational force goes beyond music into the fields of therapy, rehabilitation, and special education. It also becomes valuable for the training of dancers, actors, athletes, poets, and painters. Perhaps this explains the wide differences of understanding, definition, range, interests, and goals of teachers using the method of conscious kinesthesia invented by Jaques-Dalcroze. His methods are complementary to many other methods and fields of study.

4.4. Different States of Rhythmic Movement

After several years of experimentation and observations Emile Jaques-Dalcroze and his associates finally found a process and method that involved a constant spiral of learning, relating hearing to moving; moving to feeling; feeling to sensing; sensing to analyzing; analyzing to reading; reading to writing; writing to improvising; and improvising to performance. By 1905 Jaques-Dalcroze had worked out numerous exercises and games linking music, intense listening, and consciously improvised movements, and his method was introduced into the regular curriculum of the Geneva Conservatory. The very basic idea using rhythmic body movements as fundamental to an educational method of expressions of music was rather revolutionary at the time of his experiments. Today this theory has been absorbed in music curricula throughout the world, and is particularly evident in the methodologies of Carl Orff, Maria Montessori, and Zoltán Kodály.

101 In addition to students and pupils who were extremely important and necessary to the experiments, Dalcroze was also partly working with the Swiss psychologist Édouard Claparède. Claparède was founder of the Institut Jean-Jaques Rousseau for the study of child development, and teacher of Jean Piaget.
102 At least as a part of Western educational systems in music. In many other cultures, e.g. in African music, various patterns of body movements have always been indispensably connected to teaching and performance of music.
However, exercises in eurhythmics may be applied in ways not resulting in the desired effects, according to the ideas of Daleroze. In this respect Abramson (1986, p.38) points out that it is the walking and clapping exercises that Jaques-Daleroze invented that are most frequently used in the general practice of music teaching, but these are quite often used ineffectively. If the teacher thinks that the goal of the exercises is to make the students be able to clap their hands or tap their feet in unison with each other, this will be achieved when the teacher hears unison, simultaneously performed claps or taps from the students. This, however, only requires that the attack points of the movements (the instant the hands touch in a clap, or the instant the foot touches the floor) coincide. Hence, the teacher might in this case indeed succeed in producing a sensation of tactility (touch), but the necessary sensation of kinesthesias (rhythmic movement) is likely to be missing. Referring to this situation Abramson comments:

This error prevents the student from developing a clear kinesthetic feeling for the process of preparation, attack, and prolongation involved in the performance of each musical beat. (Abramson, 1986, p.38)

Abramson continues by emphasizing that a total kinesthetic sensation must be invoked in every Jaques-Daleroze movement experience. With reference to a simple movement as clapping, a full sequence of rhythmic kinesthetic movements should involve the following states (ibid., p.38):

1. **Preparation**
   breathe, along with a lifting swing of the arms and shoulders away from the center of the body measuring the tempo (time-space) of the beat (inhaling)

2. **Attack**
   the instant of striking the hands together (exhaling)

3. **Prolongation**
   pulling the hands apart to feel and measure kinesthetically the full length of the beat

Abramson also includes a fourth state (ibid., p.38):

4. **Return to preparation**
   lifting the arms upward and outward and breathing (inhaling for recycling of energy)

However, in performing repeated claps in succession, the fourth state is identical with the first. A rhythmic performance of body movements involved in clapping is therefore completely described as a cyclic unfolding of the states 1-2-3 above.

Abramson’s classification of the different states of a kinesthetic performance is very interesting in relation to our general focus on the connections between rhythmic performance of music and patterned movements of the body. For our following discussions and
investigations it is, however, necessary to make some of Abramson’s concepts a little more precise, as well as to add some further comments to the various states involved in rhythmic movements.

Let us make a graphic representation of a single movement of clapping where the distance between the hands is displayed along the vertical axis, and elapsed time is shown along the horizontal axis. By doing this we consider the following figure:

![Graph of a single movement of clapping](image)

Figure 4.2. A graphic representation of a single movement of clapping.

The graph in Figure 4.2 shows a possible movement involved in the performance of one clap. The observation of this movement starts at time $t_0$. Without loss of generality we let $t_0 = 0$. We furthermore assume that at time $t_0$ the hands are in a start position, where the distance between the hands is $d_0$. Moreover, once the movement has started we assume that the distance between the hands is increasing to a maximum distance, then decreasing to a minimum ($d = 0$) when the hands touch in the clap, whereafter the distance again increases, as the hands return to the start position. I.e. we do not consider a situation as the following:
Referring to Figure 4.2, we now make the following definitions:

**Preparation time:** $t_{\text{prep}} = \text{The time elapsed from the start of the movement until the hands begin moving towards each other.}$

**Execution point:** $p_{\text{ex}} = \text{The point on the movement curve corresponding to the instant the hands begin moving towards each other to execute the clap.}$

**Maximum point:** $p_{\text{max}} = \text{A point on the movement curve where the distance between the hands, d, is the largest.}$

**Attack time:** $t_{\text{at}} = \text{The time elapsed from the instant the hands start moving towards each other until the hands touch.}$

**Attack point:** $p_{a} = \text{The first (earliest) point on the movement curve where the hands touch (d = 0).}$

**Prolongation time:** $t_{\text{pro}} = \text{The time elapsed from the attack point until the hands again are in start position (i.e., when d = d_0).}$

**Movement time:** $t_{\text{move}} = \text{The total amount of time used in the performance of one hand clap (i.e. the time elapsed performing one movement cycle).}$

From these definitions it is obvious that $t_{\text{move}} = t_{\text{prep}} + t_{\text{at}} + t_{\text{pro}}$. Moreover, subject to the conditions given immediately after Figure 4.2, stating that the execution of a clap starts from a position where the distance between the hands is the largest, it follows that $p_{\text{ex}}$ is a maximum point. It is also interesting to note that the cyclic movement of the hands performing a series of claps in succession, hands apart — hands together — hands apart — hands together — etc., which is represented as alternations of rise and fall (up and down) on the movement curve (cf. Figure 4.2), is closely related to alternations of arsis and thesis of ancient Greek thinking. Every movement cycle involved in the performance of hand claps includes an "up-part" and a "down-part", an "arsis" and a "thesis". A full movement cycle related to the performance of one hand clap, as described above, will in the following be denoted a rhythmic single-swing.

By means of graphic representations as exemplified in Figure 4.2, and the definitions given above, important differences between various performances of clapping movements can
now be demonstrated and discussed. With reference to Abramson’s definitions we first make the following remarks:

(1) Abramson’s state preparation is unfolded during our preparation time. In this state the hands are pulled apart to prepare the execution of the hand clap.

(2) Abramson’s attack, which he defines as “the instant of striking the hands together (exhaling)” seems to correspond most closely to our attack point, \(p_e\). On the other hand, Abramson also suggests that “attack” denotes exhaling, and since it seems most reasonable to assert that the process of exhaling starts at the execution point, \(p_{ex}\). Abramson’s concept ‘attack’ is somewhat ambiguous. This ambiguity is basically due to the fact that Abramson does not make the distinction between \(p_{ex}\) and \(p_e\) explicit. This is rather unfortunate, since \(p_{ex}\) and \(p_e\) correspond to two quite different states of energy, being a maximum point and a minimum point on the movement curve respectively. Another shortcoming of the concepts defined by Abramson is that none of his concepts seem to take our attack time into account. Important information of the force (dynamics, velocity) involved in the clapping movement may be obtained through knowledge of the attack time. And, as pointed out in section 4.2, force, together with time and space, is a parameter of major importance in the eurhythmics exercises conducted by Jaques-Dalcroze.

(3) Abramson’s state prolongation is unfolded during our prolongation time. In this state the hands are pulled apart after the clap and eventually return to the start position.

According to the eurhythmics of Dalcroze; crucial in “developing a clear kinesthetic feeling” as Abramson puts it (see previous quotations), it is of fundamental importance that \(t_{prop}\), \(t_{int}\), and \(t_{pool}\) are adjusted and related to rhythmic phrasing and tempo in such a way that the body movements of the performer in a comfortable, "economic", and "natural" manner are expressing the speed of the sound sequence, as well as the duration and energy of the sound. Let us now consider an example where three persons are performing one clap in such a way that the sound of the three claps is simultaneous, but the body movements are quite different:
Example: Simultaneous claps - different movements:

In the figure below we present graphic representations of movement curves related to the performance of one clap, executed by three different persons; I, II and III.

Figure 4.3. Graphic representation of movement curves related to the performance of one clap, executed simultaneously by three different persons, I, II and III. The three performances have the same attack points, but the body movements involved are quite different for I, II and III.
We observe from Figure 4.3 that the movement curves of I, II and III have a common attack point, \( p_a \). Hence, in performing the clap, the hands of the persons I, II and III touch at the same time. However, the hand movements performed by I, II and III are strikingly different:

1. First of all we observe that the total amount of time used by I, II and III performing one movement cycle is different:

   \[ t_{\text{move}(III)} < t_{\text{move}(I)} < t_{\text{move}(II)} \]

   If this one clap were to be followed by other claps in a rhythmic succession, \( t_{\text{move}} \) is of major importance: Person III is ready to perform a second clap earlier than both person I and person II. Thus, the movement performed by III is better adjusted to clap beats of short duration, i.e. notes with small note values, than the movements executed by I and II. If, on the other hand, the clap is supposed to express a beat of longer duration (relatively speaking), then a movement as performed by I or II is preferrable from a kinesthetic point of view. In this latter case III does not take into account that the movement in the state of preparation should be "...measuring the tempo (time-space) of the beat", and in the prolongation state the performer should "...feel and measure kinesthetically the full length of the beat", as Abramson formulates these matters (Abramson, 1986, p.38; see also above).

2. Secondly, we notice that the time used on the separate states of the clapping movement is different in the three performances. In this respect we especially point to the fact that the attack time, \( t_{\text{att}} \), differs substantially in the performances I, II, III. The shape of the movement curve during \( t_{\text{att}} \) conveys information of the velocity of the movement striking the hands together, executing the clap.\(^{103} \) The larger the velocity is at the instant the hands touch (i.e. at the point \( p_a \)), the louder the clap. Hence, the shape of the movement curve between the points \( p_{\text{ex}} \) and \( p_a \) is of basic importance to the energy of the clap. Since the movement curves related to II and III approach \( p_a \) with a "steeper slope" than the curve associated with I, it seems reasonable to assert that the claps performed by II and III will be louder than the clap performed by I.

3. We also observe that in the movement performed by II the hands are further apart at the point \( p_{\text{ex}} \) than in the performances of I and III. In relation to the performance of clap

\(^{103} \) More precisely; in mathematical language, the velocity is given as the derivative of \( d(t) \) with respect to \( t \).
movements it seems natural to associate the distance between the hands at $p_{ex}$ to two distinct parameters of performance:

(a) *Loudness*: In performing a loud clap, there is often a larger distance between the hands than in a performance of a softer clap.

(b) *Frequency of movement*: If several claps are to be performed in succession at rapid speed (i.e. with small intervals of time between each clap), there is not time to move the hands as far apart as when the temporal distances between the separate claps are larger (i.e. the tempo is slower).

(4) Furthermore, we notice that in the performance II $d = 0$ over an interval of time, not just in one point ($p_3$) as is the case in I and III. Thus, in the performance of the clap the person II keeps his hands together some time after the attack point, before his hands again are brought back to the start position. Keeping the hands together some time after the clap might indeed make the clap sound somewhat different from the sound of a performance where the hands touch only at a point of time (this, however, need not always be very noticeable to a listener). Even more so, if the graphs of Figure 4.3 were representations of the movements involved in pressing a key on a piano, the differences between II on the one hand and I and III on the other would be quite substantial. Whether a piano key is kept down for some time or not makes an important difference in the sound of the performance.

This simple example demonstrates and illuminates a very basic idea of Jaques-Dalcroze's eurhythmics: Body movements are of fundamental importance in the performance and articulation of musical rhythmic unfoldings. Even stronger, Dalcroze asserts that rhythmic behavior and expressions of rhythm are created in a fundamental way in relation to patterns of rhythmic body movements; i.e. an experience of kinesthetic movements of the body is basic to an expressive performance of rhythm. Or, to put it in another, more straightforward way: *Different movements of the body create different performances of rhythm*. Using the approach of Emile Jaques-Dalcroze to describe and discuss rhythmic performance of music, it is therefore of major importance to include a description of rhythmic movements of the body.\textsuperscript{104}

\textsuperscript{104} It should be noted that in the example of simultaneous claps performed with different movements we consider the three movement curves without consideration of some possible pulsation or meter to which the performances may be related. In Figure 4.3 we simply regard a situation where three different people, performing three different patterns of hand movements, execute a clap sounding at the same time. In the subsequent remarks, however, we discussed how the different performances in varying degree are naturally related to different parameters of musical performance, such as tempo, duration, and velocity.
4.5. "Timbral" Aspects of Rhythmic Performance

In elementary acoustics we learn that a musical sound\textsuperscript{105} is characterized in a fundamental way by the following three parameters:

- Pitch
- Loudness
- Timbre

The pitch of a sound is related to the sound's frequency; the loudness is correlated with amplitude; whereas musical timbre is dependent on the shape of the sound's waveform, as given in the graphic representation of the sound.\textsuperscript{106} Differences in timbre are what makes a trumpet and a saxophone sound different, even if they are played producing the same pitch and the same loudness. Being related to both the particular instrument being played and to the specific musician playing the instrument, timbre is an important parameter of musical sound conveying information that make it possible to distinguish between instruments as well as between musicians. In an attempt to make these concepts more precise and clear, we look at the following figure:

![Graphic representation of a square wave (I) and a sawtooth wave (II) with the same frequency and amplitude.](image)

\textsuperscript{105} In this context, a musical sound is a sound for which a definite pitch is recognizable.

\textsuperscript{106} Further explanations may be found in Dodge & Jerse (1985)
Figure 4.4 shows a graphic representation of two sounds; (I) is commonly denoted a square wave, and (II) a sawtooth wave. The sounds (I) and (II) are perceived as having the same pitch since their frequencies are equal, and the same loudness because their amplitudes are the same (= A). Nevertheless, the square wave and the sawtooth wave sound different because their waveforms differ.\textsuperscript{107}

In graphic representations of various rhythmic movements, as illustrated in the previous section, interesting \textit{rhythmic "waveforms"} become apparent, having many mathematical features in common with graphic representations of acoustic sound. Having made this observation, it is very tempting to investigate and discuss if there are any parameters important in the classification of acoustic sound that \textit{in a musical meaningful and relevant} way can be "translated", or "transformed" into a context of musical rhythm, providing important and interesting information about \textit{rhythmic performance of music}. Immediately, a translation of the concept 'frequency' appears relevant in describing several \textit{temporal} aspects of rhythmic performance. Analogous to the way acoustic sound is characterized, a natural meaning of 'frequency' related to rhythmic performance may be: \textit{the number of performed beats during some (prescribed) unit of time.} Understood in this way, frequency as a parameter of rhythmic performance conveys information of the \textit{speed or tempo} of a performance, as well as of the \textit{duration} of the different beats (rather: information of the distance between the attack points of two beats in succession). The concept 'amplitude', on the other hand, is naturally associated with \textit{the maximum distance between the hands} in the performance of a clap, and is thus related to both \textit{dynamics} and \textit{velocity} of the rhythm performance (cf. remark (3) proceeding the example of simultaneous claps with different movements, in the previous section).- What, then, about the concept 'timbre'?

The timbre of a sound is reflected in the \textit{shape} of the waveform in the graphic representation of the sound.- In making graphic representations of \textit{rhythmic performance}, we have pointed out above that the shape of the associated movement curve conveys valuable information of kinesthetic qualities of the performance. In the different "rhythmic waveforms", differences in body movements important in the kinesthetic experience of the performance become apparent. In applying a collection of concepts translated from the field of acoustics into a study of rhythmic performance of music, it thus seems relevant to assert

\textsuperscript{107} As is well known, the shape of a sound's waveform is intimately related to the \textit{spectrum} of the sound; i.e. to the sound's \textit{partials} as given in the \textit{Fourier decomposition} of the sound. At this point of our investigations it is not necessary to make further comments on the theory of spectral decomposition. Instead, we refer to Dodge & Jerse (1985) for more information on these matters.
that kinesthetic movements of the body are intimately related to "timbral" aspects of rhythmic performance. Furthermore, from this point of view, different kinesthetic movements of the body are expressions of different "rhythmic timbres".

Based on these ideas, where concepts fundamental to a classification of acoustic sound are transformed into new concepts applied in a description of rhythmic performance, the following question more or less poses itself: Can any of the methods or techniques used in the analysis and synthesis of acoustic sound be transformed and adopted as valuable and relevant methods to provide new ways of making analyses and syntheses of rhythmic performance of music? This is a question which will be thoroughly investigated and discussed in Chapter 5. First, however, we will make some comments on differences between discrete and continuous aspects of rhythmic performance.

4.6. Attack-Point Rhythm and Gestural Rhythm - Discrete and Continuous Aspects of Rhythmic Performance

If the goal of an exercise in rhythmic clapping is to be able to produce sounding claps at the "right" time in relation to some temporal reference, such as a metronome or some sequence of musical events, this goal is achieved when the attack points of the moving hands are placed "correctly" along the axis of time. In this case, what happens in between the separate attack points is irrelevant, except for the fact that the temporal distances between each touch of the hands should be "correct". An exercise aiming at this goal is not in accordance with the eurhythms of Emile Jaques-Dalcroze. As emphasized several times in section 4.4, the movements performed between the individual attack points are of major importance to the kinesthetic experience of the rhythmic performance, which is fundamental to Dalcroze's eurhythms.

Similar considerations can also be made in a broader context of musical performance, than the example of clapping. Related to musical performance in general, an attack point of a sound is commonly understood as the instant the sound starts (or sometimes; the instant the beginning of a sound is perceived). In performing a clap, the instant the sound starts coincides with the instant the hands touch. Our definition of attack point presented in section 4.4 is thus
in accordance with the definition given in a more general situation.\textsuperscript{108} With reference to this general meaning of 'attack point', \textit{attack-point rhythm} is rhythm conceived as a temporal unfolding of attack points and durations (see, for instance, Graybill, 1990). In scientific investigations of rhythm experience and rhythm performance the large majority of studies have been directed towards investigating characteristics of attack-point rhythm. One obvious explanation of this fact is that attack points and durations are rather easy accessible to measurements and quantitative judgements. Another reason, of a more musicological/psychological nature, is stated in "Analysis and synthesis of musical rhythm", where Bengtsson and Gabrielsson assert:

> There are many reasons to believe that the durational characteristics of the sound sequences are of primary importance for the rhythm response. (Bengtsson & Gabrielsson, 1983, pp.28-29)

Bengtsson and Gabrielsson point out that it is necessary to distinguish between different \textit{durational variables}, thereby making various different aspects of duration clear. We consider the following schematic representation of a sound sequence consisting of two tones: \textit{tone 1} and \textit{tone 2}:

\textbf{Figure 4.5.} Schematic illustration of a two tone sequence. \textit{Tone 1} starts at $t_{1,\text{on}}$ and ends at $t_{1,\text{off}}$, whereas \textit{tone 2} starts at $t_{2,\text{on}}$ and ends at $t_{2,\text{off}}$. \textit{Tone 1} precedes \textit{tone 2}. \textit{Tone 2} lasts longer than \textit{tone 1}.

\textsuperscript{108} Strictly speaking, an attack point as defined in section 4.4, is a point \textit{on the movement curve}, whereas an attack point in the general sense is a point \textit{on the axis of time}. However, if the correspondence between points on the movement curve associated with the creation of musical sounds and points on the axis of time is made clear, it should not create any confusion whether an attack point is considered as a specific point of time or as a particular point of movement. We will adopt both points of view.

\textsuperscript{109} A similar representation is found in Bengtsson & Gabrielsson (1983, p.29).
We now make the following definitions:

**Note on:** The instant the tone starts. (I.e. the attack point, also denoted the onset of the tone\(^{110}\).)

**Note off:** The instant the tone ends.\(^{111}\)

\(D_{io}:\) "Duration in-out"\(^{112}\)

The duration from note on of a tone to note off of the same tone. Expresses how long the tone lasts.

\(D_{oi}:\) "Duration out-in"

The duration from note off of one tone to note on of the subsequent tone. Expresses (in combination with \(D_{io}\)) to what extent the tone sequence is staccato or legato.

\(D_{ii}:\) "Duration in-in"

The duration from note on of a tone to note on of the subsequent tone. Measures the temporal distance between the attack points of two tones in succession.

In the literature, \(D_{ii}\) is often called the interonset interval, abbreviated IOI (see, for instance, Parnut (1994)).

From these definitions we immediately obtain: \(D_{ii} = D_{io} + D_{oi}\)

In research on rhythm, \(D_{ii}\) is often assigned the largest importance. Bengtsson and Gabrielsson (1983) also underline the fundamental significance of \(D_{ii}\) to investigations of musical rhythms, but in addition they strongly emphasize that both \(D_{io}\) and \(D_{oi}\) are crucial parameters in musical performance, and are of major importance in affecting the listener’s experience of motion character of rhythm. This latter point is exemplified in a very interesting way by synthesized versions of Swedish folk music, Vienna waltz accompaniment, and “Die Fledermaus” by Johann Strauss Jr. (see Bengtsson & Gabrielsson, 1983. Listening to the enclosed sound examples is indeed extremely interesting and enlightening.) Interesting examples of research demonstrating systematic variations of \(D_{ii}\) as important to the

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111 "Note on" and "note off" are adopted from the MIDI-terminology.
112 These different classifications of the durational variables are adopted from Bengtsson & Gabrielsson (1983, p.30).
performance of musical rhythms are given in the various cases of SYVAR and PD detection presented and discussed in sections 2.3 and 2.4 above. Through these examples we learn how the different systematic "deviations from the exact" in musical performances "give life to" the music and make it possible to recognize performances as "typical Vienna waltz", "characteristic of Norwegian folk music from Valdres", "typical Cuban percussion rhythm", "characteristic of swing in jazz music", etc.

At this point, however, a basic question of epistemological nature should be posed. In studying attack-point rhythm the subject of study is attack points and durations; i.e. note on and note off information. The total collection of all note ons and note offs of a rhythmic performance constitute a finite set of discrete points along the axis of time. Hence, a description of rhythmic performances which only takes the attack points and durations into account, is fundamentally based on information of discrete points on a one-dimensional axis. The very phenomenon under consideration - the rhythmic performance of music - is, however, created through an interaction between the musician and his instrument, expressed as continuous movements in time and space. The musical performance as such is thus basically a continuous, multi-dimensional phenomenon.

A question of fundamental importance is therefore: What kind of knowledge of rhythmic performances is obtained through a study of attack-point rhythm? Or, perhaps even more interesting; what knowledge is not obtainable in a study of attack points and durations? i.e.; what information is lost by projecting the continuous, multi-dimensional phenomenon of rhythmic performance onto a discrete, one-dimensional registration of points of time? Let us consider an example: A jazz drummer is playing a swing rhythm on a ride cymbal. His rhythmic phrasing on the cymbal is generated through a process where an "internal rhythmic intention"113 is physically articulated by continuous movements of the body, having as a result that the drum stick hits the ride cymbal at distinct, isolated points of time. An attack-point study of this performance involves investigating the temporal placements of the instants the drum stick hits the cymbal, and will thus provide a numeric description of a result of a musical process of performance. The various cymbal strokes and the fact that the drum stick hits the cymbal is indeed very important to the musical performance (if the drum stick did not hit the cymbal, no sound would be created, or, we might hear some other sound resulting from the drum stick hitting something else). However, a knowledge of the temporal placements of

113 Whatever that might be!-
the drum stick hitting the cymbal conveys little or no information of *the processes through which the articulation of the cymbal strokes are created*. In other words; essential qualities of a "timbral" nature are lost in this description of the rhythmic performance.

To the performing musician a knowledge, or rather *feeling*, of kinesthetic movements of the body related to interactions between the musician and his instrument, as well as connected to processes of communication among several musicians playing together, is of vital importance. If I, being a drummer myself, would like to play like the famous jazz drummer Elvin Jones, it is far from sufficient for me to know that the cymbal strokes in a typical performance of Elvin Jones are so and so many milliseconds behind (and/or ahead of) some metronomic reference. It is of far greater importance for me to know how to hold the drum sticks in a comfortable way, how to place and adjust my drums and cymbals, and how to sit relaxed while playing. To be able to play like Elvin Jones, I also have to practice many different rhythmic movements of hands and feet, in separate as well as coordinated patterns of movements. Not least, I have to listen a lot to Elvin Jones’ way of playing, and I must observe Elvin Jones in action. And even after having obtained all this skill and knowledge, I have no guarantee that I will be able to play like Elvin Jones!

If we want to achieve a comprehension of rhythmic performance as a process, and if we wish to acquire an understanding of *timbral aspects* of the musical performance, a description of rhythmic performances based solely on a study of attack-point rhythm is unsufficient. In addition, what is needed, is to investigate rhythmic performance as a continuous phenomenon where different patterns of kinesthetic movements of the body are taken into account. The very approach of Emile Jaques-Dalcroze is a main example of an educational method aimed at obtaining this increased comprehension and feeling of rhythm. Another example, greatly inspired by Jaques-Dalcroze, is introduced in relation to the concept *gestural rhythm* as presented by Roger Graybill in the article "Towards a Pedagogy of Gestural Rhythm". In this article we read:

> While this concept of rhythm as a neutral series of attack points and durations may be appropriate in certain contexts, it does not accord very well with our experience of rhythm in its fullest sense. When we respond to rhythm as listeners or feel a rhythm as performers, we experience something vital and dynamic — a flow of energy through time, one might say. This flow is not uniform and undifferentiated, but is rather characterized

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114 A similar point is made by Ingrid Monson (1995, pp.88-89) when in reference to Charles Keil and his theory of "participatory discrepancies", she asserts: "...I just think that in this case he has mistaken a product (measurements of discrepancies) for the culturally, bodily, musically and socially interactive processes by which human beings create them."
by a dynamic interplay of ebb and flow, of intensification and relaxation. Since this kind of rhythm is often associated with physical movement, I will call it *gestural rhythm*;....
(Graybill, 1990, p.1)

Shortly after, in the same article, Graybill makes this concept somewhat more precise and concrete by saying (ibid., pp.1-2):

*Gestural rhythm*, ..., is broader in scope than attack-point rhythm, and includes the latter within its domain. It is concerned not merely with the measurement of discrete musical elements, but also with the continuous dynamic flow through these elements.

With this assertion Graybill argues that the concept ‘attack-point rhythm’ is contained within the concept ‘gestural rhythm’. ‘Gestural rhythm’ is a more inclusive concept, applied in a description of rhythm as a continuous unfolding “through” attack points.- Graybill, moreover, shows his close pedagogic relationship with Emile Jaques-Dalcroze by presenting an educational method of teaching gestural rhythm, where several exercises involving kinesthetic movements of the body are demonstrated (ibid.).

The importance of focusing on body movements in studies of musical performance and perception is also emphasized by Jane Davidson. Through empirical studies Davidson (1993, 1994a) showed that body movements are highly informative in the perception of expressive differences in musical performance. Moreover, Davidson (1994b) studied which areas of a pianist’s body convey information about expressive intention to an audience.

Another very interesting example of an approach describing musical performance as intimately related to continuous movements of the body is presented by Jan-Petter Blom (1961, 1981, 1993; Blom & Kvifte, 1986). In studying expression of rhythm in Norwegian folk music, Blom strongly emphasizes the role of the dancer’s movements in creating style-specific rhythmic performances. Blom’s point of departure is his strong belief in the motor theory of musical rhythm. Thus, Blom shows his debt to the classical Greek comprehension of *rhythmos* and several later investigations belonging to a methodological tradition fundamentally related to ancient ideas of the nature of rhythm (e.g. Sievers, Becking, Truslit, Fraisse, Shove & Repp, Clynes, and Todd, as presented in Chapter 2). Blom, moreover, makes the following point:

Moving from the psycho-physical to the cultural level this perspective on rhythm logically implies that culture specific movement styles and kinesthetic experiences of a social group are supposed to be embedded in its musical forms of expression, thus constituting the implicit and shared background knowledge from which socially appropriate rhythmic action/reaction is generated. (Blom & Kvifte, 1986, p.502)
Based on this theoretical position, Blom makes some very interesting observations on characteristic features of the dancer’s movements, showing that so-called graphic libration curves of space-force-time relationships in the way the dancers move are useful in depicting and explaining dialect differentiation in Norwegian folk dance meter (Blom, 1961, 1981). Furthermore, Blom points at body movements as fundamental to the interplay between the dancer and the performing musician by predicting:

.....traditional fiddlers who are constantly involved in situations of dance in the role of leader, synchronizer and inspirer, are expressing and communicating the particular rhythm of the swinging bodies and therefore also experience their own and others performances accordingly. Hence the striking isomorphism between music and dance can be inferred as a case of symbolic transformation. (Blom & Kvite, 1986, p.589)

Whereas Blom’s graphic libration curves are constructed on a somewhat descriptive level, Turi Mård has applied computer technology in empirical investigations and measurements of body movements, dance steps and foot pressure in Norwegian folk dance. In her results interesting temporal relations and discrepancies between the folk musician and the dancer are demonstrated (see Mård, 1999).

4.7. Summary

Fundamental to both performance and experience of rhythm in music is a connection between musical rhythm and patterns of body movements. Through this connection body movements are given a rhythmic form, and rhythm in music is related to ordered movements of the body. The correspondence between rhythmic movements and musical rhythm by no means need to be simple and unambiguous, being dependent on the instrument being played as well as on the person performing and listening to the music. Particularly in music education and pedagogics of rhythm it seems very important to underline these basic and multi-faceted connections between movements and rhythm. This is heavily emphasized in aurhythmics, which is an educational method developed by the Swiss pedagogue, composer and performing musician, Emile Jaques-Dalcroze, in the beginning of the twentieth century. Dalcroze conducted exercises training his students to obtain mastery of movements in relation to energy, space and time, and eventually in such a way that the student’s kinaesthetic sense was
stimulated. In order to develop a clear kinesthetic feeling in the performance of a rhythm, it is of fundamental importance that the various states of a rhythmic movement, i.e. preparation, attack and prolongation, are adjusted and related to rhythmic phrasing and tempo in such a way that the body movements of the performer in a comfortable, "economic", and "natural" manner are expressing the speed of the sound sequence, as well as the duration and energy of the sound. A graphic representation of a single movement of clapping is presented and some new parameters are defined, being important to our further descriptions and discussions of rhythmic movements as related to the performance of music. An example discussing three different performances of simultaneous claps shows the significance of our suggested definitions. Moreover, on the basis of the examples and discussions presented in this chapter, the notions of timbre and timbral aspects are introduced as relevant to a description of rhythmic performance, the idea being that different kinesthetic movements of the body are expressions of different "rhythmic timbres".

The majority of research on rhythm has been concerned with investigating attack-point rhythm, i.e. attack points and durations, and is, as such, fundamentally based on information of a finite number of discrete points along the one-dimensional axis of time. However, the very object of study, the musical performance, is basically a continuous multi-dimensional phenomenon. Hence, it seems both natural and important to pose the following epistemological question: What information is lost by projecting the continuous, multi-dimensional phenomenon of rhythmic performance onto a discrete, one-dimensional registration of points of time? Our suggested answer to this question is that essential qualities of a "timbral" nature are lost in the attack-point description of rhythmic performance. In addition to investigating attack points and durations, we argue that it is important to view musical performance as a continuous phenomenon and take gestural aspects of the performance crucially into account. The concept gestural rhythm has been applied in descriptions of rhythm as a continuous unfolding "through" attack points. In our further discussions we adopt this concept.

A major task in our following studies is to describe characteristic features of gestural rhythm. In the next chapter we approach this task by constructing a model providing continuous multi-dimensional representations of musical performances of rhythm.
Moveable Rhythms

In the previous chapter various aspects of rhythmic movements fundamental to rhythmic performance and experience of music were discussed. Basic to our discussions was a presentation of the thorough and highly interesting work of Emile Jaques-Dalcroze in developing the educational method of eurhythms. Strongly motivated by the ideas of Dalcroze and the argumentation carried out in the preceding chapter, we now wish to develop a terminology and a model describing characteristic features of rhythmic performance of music where movement activities of the rhythm performer are fundamentally implemented. Our main focus in this respect will be on the performance of measured music (in Arom’s sense of the word), i.e. music comprised of durations with proportional values (see Arom, 1994, p.179). Moreover, on the basis of our previous discussions it is important that the model fulfills the following requirements:

(M1) The model should represent a description of rhythm where rhythmic performance of music is intimately related to movements of the body.

(M2) The model should represent a description of gestural rhythm, i.e. rhythm as a continuous unfolding through attack points.

(M3) The model should include descriptions of expressive timing, i.e. various "deviations from the exact" being characteristic of live performances of music.

If we succeed in constructing a model fulfilling these requirements, this model will represent new and rather unique possibilities of making syntheses of live rhythmic performances of music, where interesting relations between variations of model constructed control parameters and different approximations of live performances of rhythm may be illuminated. The construction of such a model will be our main concern in this chapter. In the stepwise presentation of the model we define our task as being twofold:
Chapter 5

(A) Present a model of *rhythmic structure*, where information of note values is represented as continuous movements through attack points.

(B) Construct a model of *expressive timing*, where performed rhythm is viewed as a result of continuous interactions of movements.

A theoretical interpretation offered by the constructions carried out in (A) and (B) is to conceive expressive timing as a result of rhythmic structure being "stretched" and "compressed" by actions of movements. Thus, in our presentation, new syntheses of live musical performances of rhythm are achieved by means of model-constructed "moveable rhythms".

5.1. A Continuous Model of "Exact" Rhythmic Performance

As mentioned above, our main focus in developing terminology and models of rhythmic performance will be on the performance of *measured music*, i.e. music characterized by the presence of an underlying *pulsation* determining a *tempo reference* for the performance. According to Arom's use of these concepts, a pulsation is "the isochronous, neutral, constant, intrinsic reference unit which determines tempo" (see Arom, 1994, p.202). Arom also points out (ibid.,p.202):

Pulsations are an uninterrupted sequence of reference points with respect to which rhythmic flow is organised. All the durations in a piece, *whether they appear as sounds or silences*, are defined in relationship to the pulsation. In terms of the temporal organisation of a polyphonic ensemble, the pulsation is also the common denominator for all the parts.

Hence, in Arom's sense, pulsation is a *discrete* sequence of temporal reference points, an example of an audible pulsation being the clicks from a metronome. Fundamental to our approach in making models and descriptions of rhythmic performance of music is, as underlined in (M1) and (M2) above, to view performance of rhythm closely related to movements of the body, where rhythm is seen as a continuous unfolding through attack points. Based on this approach it also seems natural, not to say necessary, to regard the very temporal reference for the performance; the pulsation, or rather the *pulse* as we prefer to say, as a *continuous* unfolding through the isolated, discrete temporal reference points denoted as 'pulsation' in Arom's terminology. A question of major importance to our model constructions is therefore: *How should 'pulse' be defined as a continuous unfolding through*
the isolated points of pulsation (in Aron's sense)? A simple observation of rhythmical movements of the body may help us find an answer to this question:

When tapping the hand against a table in perfect synchronization with a metronome, a possible curve describing the hand's movement could be something like the following:

![Graph of a possible movement curve](image)

**Figure 5.1.** Graphic illustration of a possible movement curve when tapping the hand against a table in perfect synchronization with a metronome. Time is displayed along the horizontal axis, and the hand's distance from the table is measured along the vertical axis.

Observe that the points where the curve touches the horizontal axis correspond to the instants when the hand touches the table, producing the audible tap. This curve is, of course, not the only possible movement curve, but certainly a plausible one. To be quite explicit, the curve above is given by the mathematical function:

\[ p(t) = A[l + \sin(ft)] , \]

where \( t \) is time, \( f \) is the frequency, i.e. a measure of the speed by which the tapping is performed, and \( 2A \) is a measure of the hand's maximum distance from the table. (In the figure above \( f = 1 \) and \( A = 1 \).) Moreover, we observe that the function:

\[ t \rightarrow p(t) = A[l + \sin(ft)] \]

is a continuous function with minimal points coinciding with the clicks from a metronome, i.e. with the discrete points of pulsation. Hence, the function \( p(t) \) as given above fulfills the requirement of representing a continuous unfolding through the isolated points of pulsation, and is, as such, a possible candidate for the definition of a 'pulse'. Motivated by this simple example, we now make the following general definition:

**5.1.1. Definition of a pulse:**

A pulse is any function, \( t \rightarrow p(t) \), where:

\[(*)\] \[ p(t) = A[B + \sin(ft + \phi)], \quad t \in [t_i, t_j] \]

\[ 115 \] The symbol "\( \in \)" is a mathematical symbol meaning "belongs to". I.e. in the above: "\( t \) belongs to the closed interval of time from \( t_i \) to \( t_j \)."

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Related to this definition we immediately observe the following:

(i) \( B \) represents a vertical displacement, and \( \phi \), commonly denoted the phase angle or just the phase, represents a horizontal displacement of the curve associated with \( p(t) \).

(ii) The curve illustrated in Figure 5.1 is a special example of pulse as defined in (i), which is obtained by setting \( B = 1 \), and \( \phi = 0 \).

(iii) A pulse is a continuous function defined over a closed interval of time, \([t_1, t_2]\). The distance between successive minimal points of \( p(t) \) is constant, and each minimal point represents an isolated point of pulsation (as defined by Arom, 1994, p.202).

As mentioned in (ii), the curve in Figure 5.1 represents an example of a pulse according to our definition. Let us now look briefly at some other examples of pulses:

5.1.2. Examples:

(a) \( p(t) = 2\sin(3t), \ t \in [0, 6\pi] \)

Compared to (*) this corresponds to \( A = 2, B = 0, f = 3 \), and \( \phi = 0 \).

The graph of this pulse is the following:

(b) \( p(t) = \cos(2t), \ t \in [-\pi/4, 23\pi/4] \). Corresponds to: \( A = 1, B = 0, f = 2, \phi = \pi/2 \).

\[ ^{116} \text{Since} \ sin(\theta + \pi/2) = \cos(\theta). \]
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(c) \( p(t) = 1 - \cos(t), t \in [-3\pi/2, 21\pi/2] \).

Compared to (*) this corresponds to \( A = B = 1, f = 1 \), and \( \phi = 3\pi/2 \).\(^{117}\)

Here, the graph is:

![Graph of \( p(t) \)]

Observe that the pulse in example (c) above is identical with a displacement of the pulse illustrated in Figure 5.1, where the displacement is \( 3\pi/2 \) in the negative direction along the horizontal axis. In our following model constructions carried out in this section it will be preferable to work with pulses of the form:

(**) \( p(t) = A[1 - \cos(fu)] \)

as "building blocks". The pulse in example (c) is obviously an example of such a pulse. Among the properties of pulses of the form (**), we note that the minimal points of these pulses are identical with the points where the curve touches the horizontal axis, thus, the isolated points of pulsation are the points where \( p(t) = 0 \). Moreover, \( p(0) = 0 \) for all pulses of the form (**). Hence, \( t = 0 \) represents an isolated point of pulsation for all these pulses in the sense of Amor. Most often we regard (**) as being related to a rhythm performance where the first sound - the first pulse beat - is made at time \( t = 0 \). When working with pulses of the form (**), we therefore assume that \( t \) belongs to a closed interval of time, \( I \), containing \( t = 0 \).

5.1.3. Interpretations:

(a) One interpretation of a pulse, as mentioned earlier, is a movement created by tapping the hand against a table in perfect synchronization with a metronome. A quite similar interpretation is a drummer playing equal beats with equal distances using one hand to hit one drum, performing in a robot-like way, synchronized with a metronome.

(b) Another interpretation of a pulse is a "metronomic" performance of hand claps. If the pulse is of the form (**), the vertical axis in this case displays the distance between the hands, and the instants when the hands touch, i.e. the instants when a clap is produced, occurs when \( p(t) = 0 \).

\(^{117}\) Since \( \sin(\theta + 3\pi/2) = -\cos(\theta) \).
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(c) A third, more general, interpretation of a pulse is to view a pulse as a representation of a constant, static, isochronous alternation between two states of energy, e.g., tension-relaxation, breath in-breath out, ebb-tide, fast-slow, hot-cold. Subject to this interpretation a pulse exceeds the domain of music and performance of musical rhythm. We will return briefly to this general interpretation later in our presentation of the model constructions.

5.1.4. Different states of a pulse:

Let us now look more closely at the pulse \( p(t) = 1 - \cos(t) \), where \( t \) is restricted to the interval \( t \in [-\pi/2, \pi/2] \). The graph is given by Figure 5.2 below:

![Graph of the pulse](image)

Figure 5.2. A graph of the pulse \( p(t) = 1 - \cos(t) \), restricted to the interval \([-\pi/2, \pi/2]\), where the notions preparation time, attack time, and prolongation time are illustrated.

The graph in Figure 5.2 shows a possible movement associated with the performance of one drum beat, one hand clap, or one alternation of energy states, all dependent on the choice of interpretation. With one hand clap, we observe that the curve describes an execution where the hands move from a start position, \( d_0 \) (related to this specific graph \( d_0 = 1 \)), through the performance of one clap, whereafter the hands return to the start position, \( d_0 \). Thus, during the elapsed time from \( t = -3\pi/2 \) to \( t = \pi/2 \) a "full movement cycle", or one rhythmic single-swing, is performed (cf. the definition of these concepts in section 4.4). Moreover, we observe that the sounding clap occurs at the time \( t = 0 \), which corresponds to the (first) instant when the hands touch. As we know, the corresponding point on the movement curve is commonly denoted the attack point of the curve (see section 4.4). It is well known that the notion 'attack point' is also used to describe characteristics of a sound, denoting the instant when the sound starts (cf. section 4.6). It should not cause any confusion whether we use 'attack point'
addressing a point on a movement curve or denoting the beginning of a sound, since in our situations the instant the sound starts coincides with the instant the attack point of a movement curve occurs.

In section 4.4 different states of rhythmic movements were defined. With reference to Figure 5.2 we find that:

- **Preparation** is performed during the time interval \([-\pi/2, -\pi]\).
- **Attack** is performed during the time interval \([-\pi, 0]\).
- **Prolongation** is performed during the time interval \([0, \pi/2]\).

Hence, in this particular performance of one hand clap:

\[ t_{\text{prep}} = t_{\text{post}} = \frac{1}{3} t_{\text{att}} \text{, and;} \quad t_{\text{move}} = t_{\text{prep}} + t_{\text{att}} + t_{\text{post}} = 2\pi. \]

5.1.5. Subdivisions of a pulse:

Let us consider the pulse \( p(t) = A[1 - \cos(ft)] \), \( t \in I \). Various interpretations of this pulse have been given, one being a drummer playing with one hand on one drum, in perfect synchronization with a metronome. Without loss of generality we may assume that the drummer is playing a sequence of quarter notes, e.g.:

\[
\begin{align*}
\text{\_} & \quad \text{\_} & \quad \text{\_} & \quad \text{\_} & \quad \text{\_]}
\end{align*}
\]

Subject to one of our overall aims in this chapter; to present a model of rhythmic structure, where information of note values is represented as continuous movements through attack points, it is now necessary to define an operation constructing subdivisions of a pulse, thus making it possible to construct continuous representations of rhythmic sequences consisting of a mixture of quarter notes, eighth notes, triplets, sixteenth notes, etc. According to this musical interpretation, a subdivision of the pulse, \( p(t) \), should be an algorithm which when applied to \( p(t) \), constructs new pulses musically interpreted as:

\[
\begin{align*}
\text{\_\_\_} & \quad \text{\_\_\_]} & \quad \text{\_\_\_} & \quad \text{\_\_\_]}
\end{align*}
\]

118 From the viewpoint of model construction, it is quite irrelevant whether \( t_{\text{move}} = 2\pi \), seconds, or minutes, or hours. This is all a matter of scaling the temporal axis. However, in the various concrete interpretations of the model, it is certainly important whether one hand clap (for instance) is performed within seconds or hours!

119 Here, as in the following of this section, \( t \) denotes a closed interval of time containing \( t = 0 \).
An algorithm fulfilling this requirement will represent a *subdivision* of the pulse, $p(t)$, in accordance with the common understanding of subdivision in musical terminology.

### 5.1.6. Definition:

Let $p_f(t) = A[1 - \cos(ft)]$, $t \in I$.

A subdivision of $p_f$ in $k$, $k = 1, 2, 3, \ldots$, is a pulse, $p_{f_k}$, with frequency $kf$, such that $p_f$ and $p_{f_k}$ have a common minimal point.

Observe that the condition stating that $p_f$ and $p_{f_k}$ should have a common minimal point assures that every isolated point of pulsation belonging to $p_f$ is also a point of pulsation belonging to $p_{f_k}$. This condition is obviously in accordance with the musical interpretation of $p_{f_k}$.

### 5.1.7. Example:

Let $p_3(t) = 1 - \cos(3t)$, $t \in [-\pi/2, 13\pi/6]$.

A subdivision of $p_3$ in 2 is: $p_{3^*2}(t) = 1 - \cos(6t)$, $t \in [-\pi/2, 13\pi/6]$. See Fig. 5.3 below:

Figure 5.3. An illustration showing the pulse $p_{3^*2} = p_6$ as a subdivision of the pulse $p_3$ in 2.
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The situation illustrated in example 5.1.7 is quite representative for subdivisions. This is stated in the following proposition:

5.1.8. Proposition:

Let \( p_f(t) = A[1 - \cos(ft)] \), \( t \in I \), and let \( k = 1, 2, 3, \ldots \).

Then \( p_{rf} = p_{rf}' = A[1 - \cos(kft)] \), \( t \in I \), is a subdivision of \( p_f \) in \( k \).

Proof:

The proof of this proposition is quite simple. First of all it is obvious that \( p_{rf} = A[1 - \cos(kft)] \) has frequency \( kf \). Moreover, since \( p_{rf}(0) = 0 = p_f(0) \), \( p_{rf} \) and \( p_f \) have a common zero and hence a common minimal point (cf. the remarks related to example 5.1.2.(c)). Thus, by definition 5.1.6, \( p_{rf} \) is a subdivision of \( p_f \) in \( k \). Q.E.D.\(^{120}\)

5.1.9. Example:

We will look briefly at one more example of subdivision, further illustrating prop. 5.1.8:

Let \( p_3 = 1 - \cos(5t) \), \( t \in [-3\pi/10, 17\pi/10] \).

A subdivision of \( p_3 \) in 3 is: \( p_{3r}(t) = p_{15}(t) = 1 - \cos(15t) \), \( t \in [-3\pi/10, 17\pi/10] \). See Figure 5.4 below:

![Graph of p_3(t) and p_{15}(t)](image)

Figure 5.4 An illustration showing the pulse \( p_{15} \) as a subdivision of \( p_3 \) in 3. Observe that if the pulse \( p_3 \), \( t \in [-3\pi/10, 17\pi/10] \) is interpreted as a sequence of 5 quarter notes, then \( p_{3r} = p_{15} \) is naturally interpreted as a sequence of 15 eighth note triplets.

\(^{120}\) Q.E.D.: Quod erat demonstrandum (Latin): "Which was to be proved".
5.1.10. Making ties of pulse beats:

Given a definition of pulse as a continuous function and an algorithm constructing subdivisions of the pulse, we are now able to represent complex sequences of mixtures of quarter notes, eighth notes, triplets, etc. as continuous movements through attack points. The sequence of note values:

\[
\begin{array}{cccc}
\cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

might, for instance, be represented as:

\[
\begin{array}{cccc}
1 - \cos(t) & 1 - \cos(3t) & 1 - \cos(2t) & 1 - \cos(4t)
\end{array}
\]

However, a representation of a sequence of note values as, for example,

\[
\begin{array}{cccc}
\cdot & .\cdot & .\cdot & \cdot \cdot \\
\end{array}
\]

consisting of dotted values, ties and syncopations, is not yet included in our model. In order to be able to represent these ties and syncopations, we need to define an operation making ties of the isolated beats of a pulse.

Again, we let \( p(t) = A[1 - \cos(\theta t)], \) \( t \in I. \) Without loss of generality we now assume that this pulse may be interpreted as representing the movements associated with a performance of a sequence of eighth notes:

\[
\begin{array}{cccc}
\cdot & .\cdot & .\cdot & \cdot \cdot \\
\end{array}
\]

According to this musical interpretation, an operation making ties of the pulse beats must be an algorithm which when applied to the pulse \( p(t), \) constructs new pulses naturally interpreted
as the following sequence of notes:

*Making ties of two pulse beats:*

\[ \text{The tie starts on the first beat} \]

\[ \text{The tie starts on the second beat} \]

*Making ties of three pulse beats:*

\[ \text{The tie starts on the first beat} \]

\[ \text{The tie starts on the second beat} \]

\[ \text{The tie starts on the third beat} \]

Etc.

Figure 5.5. An illustration demonstrating the operation of making ties as being dependent on the pulse beat on which the tie is to start.

Observe that whereas a subdivision of a pulse is given in a unique way (see prop.5.1.8), the operation of making ties of pulse beats is *multivalued*, being dependent on the pulse beat on which the tie is to start. If, for instance, we wish to make ties of \( n \) pulse beats, \( n = 1, 2, 3, \ldots \), we have the following \( n \) possibilities: The tie may start on the first, the second, the third, \( \ldots \), or the \( n \)-th beat. Since the pulse \( p_f(t) \) has frequency \( f \), a tie of \( p_f \) making ties of \( n \) pulse beats should be a new pulse with frequency \( f/n \) according to the musical interpretation illustrated in Figure 5.5. Thus, we make the following definition:
5.1.11. Definition:

Let \( pq(t) = A[1 - \cos (ft)], t \in I, \) and let \( n = 1, 2, 3, \ldots \)

A **n-tie of \( p_f \)** is a pulse, \( p_{f/n} \), with frequency \( f/n \), such that \( p_f \) and \( p_{f/n} \) have a common minimal point.

Although the operation of making ties is multivalued, Figure 5.5 indicates that the \( n \)-ties of \( p_f \) are pulses that differ from each other only by a translation in the horizontal direction (e.g. the 3-tie of \( p_f \) starting on the second beat is just a transposition "one-beat-to-the-right" of the 3-tie starting on the first beat). This observation is expressed in the following Lemma:

5.1.12. Lemma:

Let \( pq(t) = A[1 - \cos (ft)], t \in I, n = 1, 2, 3, \ldots \)

Then any \( n \)-tie of \( p_f \) is of the following form:

\[
\tau_l(t) = A \left[ 1 - \cos \left( \frac{ft}{n} + \phi \right) \right], l = 1, 2, 3, \ldots, n
\]

**Proof:**

By definition 5.1.11. any \( n \)-tie of \( p_f \) has frequency \( f/n \). Moreover, the \( n \) different \( n \)-ties of \( p_f \) are all horizontal translations of each other, where the phase \( \phi \) ensures that \( p_f \) and \( p_{f/n} \) have a common minimal point. Q.E.D.

5.1.13. Example:

Let \( p_2(t) = 1 - \cos (2t), t \in [-3\pi/4, 4\pi/4] \).

According to lemma 5.1.12, any 3-tie of \( p_2 \) is of the form:

\( \tau_l(t) = 1 - \cos (2t/3 + \phi) \), \( l = 1, 2, 3 \).

In Figure 5.6 below we see that \( \tau(t) = 1 - \cos (2t/3 - 4\pi/3) \) is a 3-tie of \( p_2 \) starting on the **third** pulse beat of \( p_2 \). In this case the phase is: \( \phi = -4\pi/3 \).

![Figure 5.6](image)

**Figure 5.6.** The upper graph shows \( p_2(t) \), the lower displays \( \tau(t) \). Observe that if \( p_2 \) is interpreted as a performance of eighth notes, \( \tau \) is naturally interpreted as a performance of dotted quarter notes (i.e. a sequence of three eighth notes tied together).
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Example 5.1.13 is a special case of a more general result describing the operation of making ties of pulse beats. Before stating this result we need to number the different beats of a pulse. Again we let \( p_f(t) = A[1 - \cos(\Omega t)] \), \( t \in I \), where \( I \) is a closed interval of time containing \( t = 0 \).

A graphic illustration of the different pulse beats of \( p_f(t) \) is given by the following figure:

![Graph of pulse beats](image)

Figure 5.7. A graphic illustration of the isolated beats of pulsation related to the pulse \( p_f(t) \). In this figure \( f = 5 \) and \( A = 1 \).

The pulse beats occur precisely when \( p_f(t) = 0 \); i.e. when \( \cos(\Omega t) = 1 \), which is equivalent to

\[ t = 2k\pi f, \quad k = \ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots \] (see Figure 5.7 above).

Based on this situation, we choose the following numbering of pulse beats corresponding to the non-negative values of \( t \):

- The first pulse beat, \( b_1 \), of \( p_f \) is the beat corresponding to \( t = 0 \).
- The second pulse beat, \( b_2 \), of \( p_f \) is the beat corresponding to \( t = 2\pi f \).
- The third pulse beat, \( b_3 \), of \( p_f \) is the beat corresponding to \( t = 4\pi f \).

... 

- The \( n \)-th pulse beat, \( b_n \), of \( p_f \) is the beat corresponding to \( t = 2(n-1)\pi f \).

We are now ready to state a proposition which in general terms describes the operation of making ties of pulse beats:

5.1.14. Proposition:

Let \( p_f(t) = A[1 - \cos(\Omega t)] \), \( t \in I \), \( n = 1, 2, 3, \ldots \)

Then:

\[ r(t) = A\left[1 - \cos\left(\frac{\Omega t}{n} - \frac{2(l-1)\pi}{n}\right)\right], \quad l = 1, 2, 3, \ldots, n \]

is a \( n \)-tie of \( p_f \) with minimal points on the pulse beats of \( p_f \) with numbers:

\( l, l + n, l + 2n, l + 3n, \ldots \)

\[ \text{\textsuperscript{121} At this point we do not assign any numbering to pulse beats corresponding to negative values of } t. \]
Proof:
\( r_t(t) \) has frequency \( f/n \) and is therefore subject to some translation by phase angle, \( \phi \), a
n-tie of \( p_f \) (cf. lemma 5.1.12). It is easy to verify by inspection that the phase angle \( \phi = -2(l-1)\pi/n \) gives the prescribed result. Q.E.D.

Observe that proposition 5.1.14 states that \( r_t \) is an n-tie of \( p_f \) starting on the l-th pulse beat, \( b_l \),
of \( p_f \). It might also be interesting to note that the operations of making ties and constructing
subdivisions of a pulse are inverse operations in the sense that:

\[
p_f \xrightarrow{\text{make n-tie}} r_t \xrightarrow{\text{subdivide in n}} p_f
\]

i.e. if you start with a pulse, \( p_f \), make \( n \)-ties and subdivide the \( n \)-ties in \( n \), this chain of
operations will bring you back to \( p_f \). However, due to the multivalued nature of making ties,
if you start with \( p_f \), subdivide in \( n \), and make \( n \)-ties of the subdivision, you need not get \( p_f \)
back. (You will, though, get a phase-translation, i.e. a horizontal displacement, of \( p_f \).) Let us
now look at some further examples of making ties of a pulse, demonstrating the content of
proposition 5.1.14:

5.1.15: Example:

Let \( p_f(t) = 1 - \cos(4t), t \in I = [-3\pi/8, 45\pi/8] \).

The 5-ties of \( p_f \) are, according to proposition 5.1.14, given by the formula:

\[
\tau_i(t) = 1 - \cos \left[ \frac{4t}{5} - \frac{2(l-1)\pi}{5} \right], l = 1,2,3,4,5.
\]

Hence:

\[
\begin{align*}
\tau_1(t) & = 1 - \cos(4t/5) \\
\tau_2(t) & = 1 - \cos(4t/5 - 2\pi/5) \\
\tau_3(t) & = 1 - \cos(4t/5 - 4\pi/5) \\
\tau_4(t) & = 1 - \cos(4t/5 - 6\pi/5) \\
\tau_5(t) & = 1 - \cos(4t/5 - 8\pi/5)
\end{align*}
\]

The graphs of \( p_f \) and all these 5-ties are given in Figure 5.8 below.
Figure 5.8. Illustration showing the five 5-ties of \( p_d(t) = 1 - \cos(4t) \), as given by proposition 5.1.14.
We observe that Figure 5.8 indicates that:

- \( \tau_1 \) starts on the first pulse beat of \( p_d \).
- \( \tau_2 \) starts on the second pulse beat of \( p_d \).
- \( \tau_3 \) starts on the third pulse beat of \( p_d \).
- \( \tau_4 \) starts on the fourth pulse beat of \( p_d \).
- \( \tau_5 \) starts on the fifth pulse beat of \( p_d \).

which is all in accordance with proposition 5.1.14.

5.1.16. Amplitude as a function of speed and dynamics:

When performing a sequence of beats with one hand on a drum, the distance between the musician's hand and the drum is closely related to at least two distinct parameters of performance.\footnote{A similar point was made related to the performance of hand claps in section 4.4.}

(a) Speed (frequency of movement): If several drum beats are to be performed in succession at rapid speed (i.e. with small intervals of time between each drum beat), there is not time to move the hand as far apart from the drum as when the temporal distances between the separate drum beats are larger.

(b) Dynamics (loudness): When performing a loud drum beat, a larger distance between the hand and the drum must often be used than in the performance of a softer beat.

At this point, these considerations have not been implemented into our model. As mentioned in 5.1.3, one interpretation of a pulse is a drummer performing drum beats in a robot-like way in perfect synchronization with a metronome. Having defined the operations of subdivision and making ties of pulse beats, we are now in a position to allow our robot-drummer to play various series of more complex rhythmic patterns in a strictly metronomic way. For instance, a performance of the following sequence of notes:

\[
\begin{array}{cccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]
could be executed by our rhythmic robot, according to the following continuous movement curve:

![Movement Curve Diagram](image)

**Figure 5.9.** An illustration showing the connections between a movement curve, a sequence of notes, and a mathematical representation of pulses as continuous functions, all related to a robot-like rhythmic performance executed in perfect synchronization with a metronome. The horizontal axis displays time, $t$, where the first beat occurs at time $t = 0$.

Note that every pulse in the figure above, and hence every section of the movement curve is given by repeated applications of the operations of subdivision and tie-making as described in propositions 5.1.8 and 5.1.14. If we let quarter notes be mathematically represented by the pulse $p_1(t) = 1 - \cos(t)$ (and phase-translations thereof), the calculations are as follows:\textsuperscript{123}

- The dotted quarter note may be regarded as a 3-tie of eighth notes. Since quarter notes are represented by pulses of frequency 1, the eighth notes are represented by pulses of frequency 2. The pulse representing the dotted quarter note has therefore frequency $2/3$. Since the dotted quarter note starts on the first beat (where we conventionally set $t = 0$), the phase angle of the corresponding pulse is 0. Hence, the pulse representing the dotted quarter note is $1 - \cos(2t/3)$.
- The two eighth notes are both represented by the pulse $p_2(t) = 1 - \cos(2t)$. No phase-translation is necessary since these eighth notes start on beats belonging to $1 - \cos(2t)$. Of similar reasons the first quarter note is represented by $1 - \cos(t)$.

\textsuperscript{123} The explanation of these calculations will hopefully shed some light on the technical/mathematical side of this model construction, but is not essential to an understanding of the overall ideas of our theory.
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- The second and third quarter notes, however, both start on beats occurring one eighth note after beats belonging to \( p_1(t) = 1 - \cos(t) \). Some phase-translation of \( p_1 \) is therefore necessary in this case. If we look at a quarter note as a 2-tie of eighth notes, proposition 5.1.14 informs us that the possible phase angles in this case are: \( \phi = -2(l-1)\pi/2, \ l = 1, 2 \). Since \( l = 1 \) gives phase angle \( 0 \), we must choose \( l = 2 \), which results in phase angle \( -\pi \). Hence the relevant pulse in this case is \( 1 - \cos(t-\pi) \).

- The two quarter notes tied together at the end of the note series must obviously be represented by a pulse of frequency 1/2. This note starts one eighth note off, compared to the regular counting of quarter notes from the beginning of the rhythm: “One-two-three-four-five..” (which is represented by the pulse \( 1 - \cos(t) \)). Some phase-translation is therefore necessary. Since the rhythmic displacement is one eighth note, the phase-translation must be related to an eighth-note-pulse instead of a quarter-note-pulse. Therefore, we represent the two quarter notes tied together as a 4-tie of eighth notes, in this case starting on the fourth pulse beat of \( p_2 \). According to proposition 5.1.14 we now get the following phase angle: \( \phi = -2(l-1)\pi/4, \ l = 4; \ i.e., \ \phi = -3\pi/2 \). Consequently, \( 1 - \cos(t/2-3\pi/2) \) is the relevant pulse in this case.

Even though our rhythmic robot is capable of performing complex rhythms by hitting his hand on a drum, the performance is indeed quite unhuman. First of all our model, so far, describes a completely "exact" performance of rhythm in the sense that each beat, i.e. every attack point, is synchronized with a metronomic pulse and subdivisions and ties thereof. This is deliberately done as a preliminary step in our stepwise construction of a model of expressive timing as outlined in the introduction to this chapter. Secondly, we note that at this point our model does not take into account that the distance between a drummer's hand and the drum is closely related to the speed and dynamics of the performance. In our model as constructed so far, rhythms are represented as combinations of various pulses, all of the following form:

\[
(***) \ p_f(t) = A[1 - \cos(ft)] \ , \ t \in I.
\]

Let us again look at the graph of \( p_f \) (with \( f = 1 \ , \ A = 1 \)):
A is commonly denoted as the amplitude of $p_f$, and we note that if $p_f$ represents a metronomic performance of playing a drum with one hand, the maximum distance between the drummer's hand and the drum is 2A. In an attempt to implement some relations between hand distance and speed and dynamics into our model, it now seems reasonable to assert that the amplitude, $A$, should be some function of speed and dynamics. Since the frequency, $f$, is a measure of the speed by which the drumming is performed, we should have $A = A(f, \text{dyn})$, denoting that the amplitude is dependent on the frequency and the dynamics, $\text{dyn}$. Hence, instead of pulses of the form (***)³, we should consider combinations of pulses of the following form:

$$\text{(***) } p_{f,\text{dyn}}(t) = A(f, \text{dyn})[1 - \cos(ft)] , \ t \in I.$$ 

So far, representations of dynamics have not at all been discussed in our model. Some possible ways of including considerations of dynamic differences in various concrete applications of our model will be presented in the following chapters. As to relations between $A$ and $f$, we have already commented that when several drum beats are performed in succession at rapid speed, there is not time to move the hand as far apart from the drum as when the frequency of drum beats is lower. It thus seems reasonable to impose some relation between $A$ and $f$ such that:

"Large" $f$ gives "small" $A$, and "small" $f$ gives "large" $A$.

One obvious, purely mathematically motivated, relation fulfilling these requirements is:

Relation 1: $A(f) = 1/f$.

By using this relation, we would find that our rhythmic robot would perform eighth notes lifting his hand half as high as when performing quarter notes; eighth note triplets would be performed with maximum distance between hand and drum being one third of the maximum distance in the performance of quarter notes, etc. A modeled performance of

\[
\begin{align*}
\text{\textbullet} & \quad \text{\textbullet} & \quad \text{\textbullet} & \quad \text{\textbullet} & \quad \text{\textbullet} & \quad \text{\textbullet} & \quad \text{\textbullet}
\end{align*}
\]
would now take the following form:

Figure 5.10. Illustration of a representation of note values as continuous movements through attack points, subject to the relation $A(f) = 1/f$ between amplitude and frequency.

Comparing this movement curve to the curve in Figure 5.9, we observe that the only difference between the two is given by the imposed dependency between amplitude and frequency. Another dependency relation than the one exemplified above, making the amplitude differences of performing different note values somewhat smaller, might be the following modification of relation 1:

**Relation 2:** $A(f) = (1/f)^{1/2}$

Applying this latter relation to a construction of a movement curve representing the note values above, we obtain the following figure:

Figure 5.11. Movement curve representing the same note values as in Figure 5.10, but now subject to the relation $A(f) = (1/f)^{1/2}$. 
Of course, there is no unique relation between A and f that is representative for live performances of rhythm on a general basis. Obviously, different drummers may lift their hand at different heights in the performance of the “same” rhythm, and the individual drummer may use different movements in repeated performances of the same rhythmic pattern. As mentioned in Chapter 4, this is all basically a matter of differences in kinesthetics, or, as we like to put it, differences in rhythmic timbres. However, subject to different interpretations of our model, certain relations between A and f will be more relevant than others. Which relation to choose in the various concrete applications should primarily be decided on the basis of empirical investigations of live performances of rhythm.

5.1.17. A continuous model of rhythmic structure:

In our traditional notation system of Western music the duration of every note value and every rest is given as an integer multiple of some smallest value of duration. In a strictly metronomic performance of any sequence of note values and rests, the corresponding sequence of discrete attack points will therefore occur as isolated pulse beats of subdivisions and ties of some basic pulse constituting the metronomic reference for the performance. Fundamental to our model construction is to represent this basic pulse and the various subdivisions and ties as continuous movements through the isolated pulse beats. Through this shift of focus from note values to attack points and further to continuous movements, we have thus obtained a new description of rhythmic structure, where information of note values is represented as continuous movements through attack points. This transformation of focus might be illustrated by the following figure:

![Figure 5.12. Illustration of a transformation of representations of rhythmic structure, from note values to continuous movements through attack points.](image)

Several important and interesting points related to this transformation of representations of rhythmic structure can now be made:
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1. The model represents a shift from a discrete to a continuous representation of note values.
2. Structure of durations is transformed into structure of movements.
3. Structure of attack-point rhythm is transformed into structure of gestural rhythm.
4. Written representations of music are transformed into representations of “exact” (i.e. “metronomic”) performance of music.
5. There is a shift from a one-dimensional to a two-dimensional representation.

On the basis of these remarks and the situation illustrated in Figure 5.12, we now claim to have obtained a solution of problem (A) formulated in the introduction of this chapter: “Present a model of rhythmic structure, where information of note values is represented as continuous movements through attack points.” However, it should be observed that note rests are not properly treated in our model. For instance, two different sequences of note values and rests as:

\[
\begin{align*}
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \\
\end{align*}
\]

and

\[
\begin{align*}
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\\cdot \quad \cdot \\
\end{align*}
\]

are both represented by the same movement curve:

This curve is a movement curve through attack points, and since the attack points corresponding to the two different sequences of notes and rests given above are the same, these different sequences of notes and rests may be represented by the same movement curve. This, indeed, is a very important point. As underlined by Bengtsson and Gabrielsson (1983, pp. 29-30), it is necessary to distinguish between different durational variables, thereby making various different aspects of duration clear. (See section 4.6, where a discussion of these different durational variables is presented.) Our model describes metronomically

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performed gestural rhythm as continuous movements through attack points. Hence, the only durational variable taken into account so far is the temporal distance between two attack points in succession; $D_{ii}$: “Duration in-in”, also called “the interonset interval” (cf. section 4.6). As mentioned in section 4.6, $D_{ii}$ is often assigned the largest importance in rhythm research. However, Bengtsson and Gabrielsson (1983) strongly emphasize that both $D_{io}$ (“Duration in-out”) and $D_{oi}$ (“Duration out-in”) are crucial parameters in musical performance, and are of major importance in affecting the listener’s experience of motion, as well as emotion character of rhythm. In Chapter 6, where a computer implementation of our model is presented, we try to incorporate the parameters $D_{io}$ and $D_{oi}$.

Keeping these latter remarks in mind, we now turn to task (B) presented in the beginning of this chapter: Construct a model of expressive timing, where performed rhythm is viewed as a result of continuous interactions of movements.

5.2. A Continuous Model of Rhythmic Deviation

In the preceding section we managed to construct a model describing strict metronomic performances of rhythm as continuous movements through attack points. A possible interpretation of this model is a robot-drummer beating his hand against a drum, performing various rhythmic patterns in perfect synchronization with a metronome. Such “exact” performances of rhythm are seldom, if ever, found in real life.\(^{124}\) As formulated in a poetic and beautiful way by C.E. Seashore, live artistic performances, rather than being “exact” and metronomic, are characterized by various deviations from the exact:

The unlimited resources for vocal and instrumental art lie in artistic deviation from the pure, the true, the exact, the perfect, the rigid, the even, and the precise. This deviation from the exact is, on the whole, the medium for the creation of the beautiful – for the conveying of emotion. (Cited from H.G. Seashore, 1937, p.155.)

Different kinds of deviations from the exact have been investigated and discussed in empirical rhythm research through detecting systematic variations of durations, SYVARD (cf. the large amount of rhythm research carried out in Uppsala headed by Bengtsson and Gabrielsson, see Bengtsson, Gabrielsson & Thorsén, 1969; Bengtsson, 1974a; Bengtsson & Gabrielsson, 1977,\(^{124}\) Except, of course, in “performances” executed by computers, as for instance by a MIDI sequencer.

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1983), as well as in discussions and detections of participatory discrepancies, PD (cf. Keil, 1987, 1995; Prögler, 1995; Alén, 1995). These deviations may in varying degree be related to, and typical of stylistic and/or social features as well as individual preferences and physical constraints, and contribute in fundamental ways to a communication of motional and emotional musical qualities from the performer to the listener and within the group of performing musicians. A presentation and discussion of rhythm research investigating various kinds of “deviations from the exact” is given in sections 2.3 and 2.4.

Being able to present a model that simulates live performances of rhythm is of major importance to us. The model constructed in section 5.1 is thus far from satisfactory, since the only performances of rhythm that may be simulated here are strictly metronomic performances. Nevertheless, the model constructed so far, representing rhythmic structure as a continuous unfolding, seems to be an interesting, as well as relevant starting point for further constructions. As pointed out in 2.1.4, various transformations from musical structure to “expressive performances” have been studied and documented empirically, thus demonstrating basic dependencies between different kinds of deviations and structural aspects of rhythm (e.g. Clarke, 1988, 1993, 1995; Palmer, 1988, 1989, 1997; Sloboda, 1983, 1985; Shaffer, 1981; Shaffer & Todd, 1987; Todd, 1985, 1989, 1992a, 1995; Repp, 1992a). Clarke (1999) emphasizes this correlation by asserting that a distinction must be made between the structural properties of rhythm and their so-called expressive properties — “continuously variable temporal transformations of the underlying rhythmic structure” (ibid., p.489). Clarke continues by saying:

These temporal transformations referred to by some authors (…) as expressive microstructure, are what the term “timing” identifies, … (ibid., p.489)

As mentioned in section 2.3.1 we find Clarke’s view of timing as continuous transformations of rhythmic structure extremely interesting. It seems to pinpoint the basic essence of the processes by which conceptualized musical information, through an interaction of cognitive skills and motor skills, and subject to a social context of communication, is transformed into live performances of music. In Figure 2.1 an illustration of the processes of “temporal transformations” (to use Clarke’s terminology) is given. Let us now look at this figure once more:
Figure 5.13. Illustration of temporal transformations of conceptualized rhythmic structure into live performances of rhythm (this figure is identical with Figure 2.1).

$\theta_1$, $\theta_2$, $\theta_3$, ..., $\theta_n$ are different temporal transformations, or mappings, of conceptualized rhythmic structure into various live performances of rhythm. These different transformations are referred to as expressive microstructure (e.g. Clynes, 1983, 1987; Repp, 1992a), and by Clarke denoted as timing, or rather, expressive timing (see Clarke, 1999, p.490; and section 2.3.1). Sounding musical consequences of expressive timing are "artistic deviations", as denoted by Seashore (see quotation in the beginning of this section). Other terms used in contemporary research are, for instance, "expressive deviations" and "systematic variations"\textsuperscript{125}, and also "participatory discrepancies"\textsuperscript{126}.

The model we have constructed up to this point is a continuous model of rhythmic structure, primarily, at least in our discussions and interpretations so far, focusing on conceptualized structure induced from patterns of durations (or rather, note values). Let us in the following refer to our model presented in the preceding section as MPR (metronomic

\textsuperscript{125} Cf. Gabrielsson (1999, p.531).
\textsuperscript{126} See Keil (1987).
performances of rhythm). The elements of MPR are movement curves constructed as combinations of subdivisions and ties of various pulses, as defined in section 5.1. As pointed out in 5.1.17, every sequence of note values may be represented by elements of MPR. Motivated by the description of expressive timing as continuous transformations of rhythmic structure, it now seems natural to try to construct a model of live performances of rhythm, LPR, where the elements of LPR are given as transformations of elements of MPR. Our basic idea in the construction of a model of live performances of rhythm is therefore as follows:

Syntheses of live performances of rhythm are constructed as continuous transformations of syntheses of metronomic performances of rhythm.

This idea may be illustrated by the following figure:

![Diagram](image)

Figure 5.14. Illustration of an idea for a construction of a model of live performances of rhythm. (Note the resemblance with Figure 5.13.)

Observe that if \( m \) is an element in MPR; i.e. \( m \) is a movement curve associated with a metronomic performance of rhythm, our idea suggests that \( \delta_i(m) \), \( i = 1, 2, 3, \ldots, k \), is a movement curve simulating a live performance of rhythm, characterized by different kinds of “deviations from the exact”, as Seshore puts it. The very fundamental question in our further model construction thus becomes: Are there any plausible or reasonable ways of defining the transformations, \( \delta_1, \delta_2, \delta_3, \ldots, \delta_k \), such that these transformations applied to movement curves in MPR create relevant approximations to live performances of rhythm?
5.2.1. Interactions of movements:

In the model MPR every movement curve is created by connecting together various pulses, all of the form: \( p(i) = A[1 - \cos(\phi t + \phi)] \). This mathematical trigonometric function is naturally interpreted as the movement performed when tapping the hand against a table in perfect synchronization with a metronome. In MPR these basic, or "atomic" movements are allowed to exist without any disturbance, so to speak, of any other movements, the result being the construction of a model of metronomic performances of rhythm. In a live performance of rhythm, however, whether related to musical rhythm or rhythm in a more general meaning of the word, the various actions of different movements do indeed interact in a wide variety of ways that may crucially affect the rhythm performance. When tapping the hand against a table, for instance, there is an interaction of movements between the different segments of the arm; when playing a drum set, movements of the arms and feet interact in substantial ways; when running, various interactions of breathing and body movements influence the way the running is performed, just to give a few examples. Moreover, in performances of music the various expressive intentions (e.g. related to the performer’s understanding of musical style, individual preferences of the performer, or associated with the performer’s communication of musical emotions) may rhythmically "move" the performance in such a way that deviations from an exact, metronomic performance are created. In an attempt at constructing a model of live performances of rhythm based on some "transformations" of metronomic performances, it thus seems both natural and tempting to try to define the transformations, \( \delta_1, \delta_2, \delta_3, \ldots \), in such a way as to represent some kind of operations where movements interact in some way or another. Reflecting the knowledge obtained from empirical investigations of live performances of rhythm (in particular detections of SYVARD and PD), the term "movements" should in this context be understood both as movements of the body and in some more general manner. We will return to a discussion of this later.
5.2.2. Rhythmic frequency modulation:

The question we now address is the following: How should our model define interaction of movements? In MPR, movements are defined as pulses, which in general form may be expressed as: \[ p(t) = A[B + \sin(\alpha t + \phi)], \quad t \in I. \] Defining interaction of movements should thus mean defining how one pulse, \( p'(t) \), affects or interacts with another pulse, \( p(t) \). Hence, what we are looking for is a binary operation, \( * \), such that \( (p * p')(t) \) is a continuous function, which in a natural way may be interpreted as a movement.\(^{127}\) There may indeed be several possible choices of definition of this binary operation, and some of these may certainly be more relevant to interpretations in terms of rhythm performance than others. Searching for one possible and, hopefully, relevant choice of such a binary operation, we will borrow some ideas from synthesis techniques of other aspects of music performance than the performance of rhythm.

It is well known that mathematical trigonometric functions are quite useful as "atomic" oscillations in analyses and syntheses of musical sound. According to Fourier theory, any musical sound can be decomposed into an (infinite) series of pulses (as these are defined in our model), and various ways of combining these pulses create different syntheses of musical sound.\(^{128}\) In section 4.5 various concepts fundamental to a classification of acoustic sound were transformed into new concepts applied in a description of rhythmic performance. Based on the ideas presented there, and viewed in the light of the very construction of our model MPR, the following question more or less poses itself: Can any of the methods or techniques used in the analysis and synthesis of acoustic sound be transformed and adopted as valuable and relevant methods providing new ways of making analyses and syntheses of rhythmic performance of music? In the following we will try to give this question an affirmative answer.

In constructing MPR, our primary focus is on patterns of durations (note values) as basic to rhythmic structure. Moreover, note values are represented as frequencies of the pulses fundamental in our model. As mentioned several times, one characteristic of live performances of rhythm is that note values are performed with various deviations compared to strict metronomic regularity. In making a model of live performances of rhythm, it thus seems plausible to impose some operation on pulses creating the effect that the frequency of the

\(^{127}\) i.e., may be interpreted as a movement in a "similar way" as the pulse \( p(t) \) is interpreted as a movement.

\(^{128}\) See, e.g., Dodge & Jerse (1985).
pulse is "affected" or "distorted" in such a way as to make the pulse beats occur at times deviating from metronomic regularity. A well known technique of sound synthesis using various alterations or distortions of the frequency of an oscillator in order to achieve parameter control over the spectral richness of the sound is frequency modulation, FM, pioneered by John Chowning. FM synthesis is based on interactions of various oscillators. A commonly used flowchart for an oscillator is given in Figure 5.15 below:

Figure 5.15. Flowchart symbol for an oscillator (adopted from Dodge & Jerse, 1985, p. 65, Figure 3.2).

The symbol inside the oscillator in Figure 5.15 (in this case, WF) designates the waveform of the oscillator, while the controls applied to an oscillator determine the amplitude, frequency and phase. If the oscillator is sinusoidal, the waveform is a sinus, and we write WF = sin. If, in this case, AMP = A, FREQ = \( f \), and PHASE = \( \phi \), the output of the oscillator illustrated above is: \( A\sin(f + \phi) \).

The most basic FM-instrument consists of two sinusoidal oscillators interacting as diagrammed in Figure 5.16:

Figure 5.16. Basic FM-instrument (see also Dodge & Jerse, 1985, p.106, Figure 4.1).

\(^{129}\) Chowning (1973).
With reference to Figure 5.16, we note that the output of the *modulating oscillator* is added to the argument of the *carrier oscillator*. The output of this whole operation is thus:

\[ A \sin[f_c t + d \sin(f_m \phi)] \]

Related to this expression the following notions are commonly used (cf. Dodge & Jerse, 1985, p.106):

- \( f_c \) : carrier frequency
- \( f_m \) : modulating frequency
- \( d \) : peak frequency deviation

Observe that if \( d = 0 \), there is no modulation and the output from the carrier oscillator is simply a sine wave with frequency \( f_c \). This very situation resembles, at least on a purely theoretical level, the situation we are trying to establish for syntheses of rhythm: When there is no interaction of movements, i.e. no "rhythmic modulation", the result is a strict metronomic (i.e. sinusoidal) performance. When, on the other hand, "modulation" occurs, some kind of interaction of movements is present and various deviations of frequency are created, resulting in different kinds of "deviations from the exact" in the modeled performance.

Strongly motivated by this observation, we now set out to investigate to what extent the technique of frequency modulation, previously applied to syntheses of sound, may be transformed and adjusted to a technique of frequency modulation of rhythm, thus providing a new tool for creating new syntheses of live performances of rhythm. Pursuing this idea, we first make the following definition:

### 5.2.3. Definition of frequency modulated rhythms:

Let \( p_f \) and \( q_f \) be the following pulses:

\[ p_f(t) = A[1 - \cos(\phi t)], \quad q_f(t) = \sin(f_f' t + \phi), \quad t \in I. \]

A frequency modulation of \( p_f \) by \( q_f \) is any function:

\[ r(t) = A[1 - \cos(f_f' t + d \sin(f_f' t + \phi))], \quad t \in I. \]

We use the notation:

\[ r = p_f \oplus_d q_f \]

to denote that \( r \) is a frequency modulation of \( p_f \) by \( q_f \) with peak frequency deviation, \( d \).
Observe that pulses of the form \( p_f(t) = A[1 - \cos(f t)] \) are the basic pulses in MPR, used as building blocks in the constructions of subdivisions and ties. These pulses are naturally interpreted as atomic movement curves associated with a metronomic performance of pulse beats. The pulses \( q_f(t) = \sin(f \cdot t + \phi) \), on the other hand, represent oscillations that are naturally interpreted as movement curves related to continuous alternations of two states of energy, oscillating in a symmetric way around a state of equilibrium, \( q_f(t) = 0 \). Thus, the definition above gives a description of how one kind of movement, \( p_f \), is affected by the action of another kind of movement, \( q_f \). Since \( p_f \) is naturally interpreted as a (metronomically performed) rhythm, we denote the function \( r \) above as a frequency modulated rhythm. In the following, moreover, we will denote the technique of frequency modulation applied to synthesis of rhythm as rhythmic frequency modulation, RFM. A flowchart for the most basic RFM, as defined in 5.2.3, is given in Figure 5.17 below (compare this illustration with Figure 5.16):

![Flowchart](image)

**Figure 5.17. Flowchart for basic rhythmic frequency modulation.** The pulse \( p_f(t) = A[1 - \cos(f t)] \) is modulated by the pulse \( q_f(t) = \sin(f \cdot t + \phi) \), with peak frequency deviation \( d \). The result output is: \( r(t) = A[1 - \cos(f t + \phi)] \).

As noted in a comment to Figure 5.16, we again observe that if \( d = 0 \), there is no rhythmic modulation (i.e., as we see it; no interaction of movements) and the output of the operation illustrated in the figure above is simply \( p_f \). To give a brief illustration of what might happen when \( d \neq 0 \), i.e., when some non-trivial rhythmic frequency modulation occurs, we now look at some concrete examples:
5.2.4. Some preliminary examples of rhythmic frequency modulation:

(a) Let \( p_3(t) = 1 - \cos(3t) \).

The graph of \( p_3 \) is the following:

The pulse beats of \( p_3 \), as illustrated above, occur at \( t = \frac{2k\pi}{3} \), \( k = 0, 1, 2, 3, \ldots \), and the distance between two successive pulse beats is \( 2\pi/3 \) for every pair of successive beats. A possible interpretation of this pulse is a drummer playing with one hand on one drum, in perfect synchronization with a metronome. Without loss of generality we may assume that the drummer is playing a sequence of quarter notes.

Now we let \( q_{1,0}(t) = \sin(t + 0) = \sin(t) \). It will be useful to apply double index, \( q_{r,s} \), in this example, to denote both the frequency and phase of \( q \). At first we have phase \( = 0 \).

A frequency modulation of \( p_3 \) by \( q_{1,0} \) with peak frequency deviation \( d = 1 \) is:

\[
    r(t) = (p_3 \circ q_{1,0})(t) = 1 - \cos[3t + \sin(t)]
\]

The graph of \( r \) is given below:

Looking at the graph of this frequency modulated rhythm, we immediately observe that the distance between the modulated pulse beats is no longer the same for every pair of successive beats. The first, fourth, seventh, eleventh,... modulated pulse beats occur at \( t = 0, 2\pi, 4\pi, 6\pi, \ldots \) resp. and coincide with the corresponding beats of the unmodulated pulse. The second, fifth, eighth,... modulated pulse beats are, however, somewhat early, whereas the third, sixth,
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... modulated beats are a little late compared to the corresponding beats of the unmodulated pulse. If, for the following discussion, we let $\Delta_j$ denote the distance between the pulse beats $j$ and $j+1$ (modulated or not), we are also able to prove that $\Delta_1=\Delta_3$, and that $r$ is a periodic function with period $T=2\pi$. Hence: $\Delta_1=\Delta_3=\Delta_4=\Delta_6=\Delta_7=\ldots$ (approximately equal to $0.28T$), and $\Delta_2=\Delta_5=\ldots$ (approximately equal to $0.44T$). Based on the interpretation of the unmodulated pulse as a drummer playing a sequence of quarter notes in perfect synchronization with a metronome, it is now tempting to interpret this modulated pulse as a movement curve associated with a new, non-metronomic performance, theoretically constructed by applying an "interaction" of one movement, $p_1$, with another movement, $q_{1,0}$. Since the modulated pulse is periodic with three beats in each cycle, it seems reasonable to interpret $r$ as a performance of quarter notes in $\frac{3}{4}$ meter, where the first and third beats are performed shortened and the second lengthened, compared to a strict metronomic performance. Observe, however, that the length of the measure in the non-metronomic performance is $T$ ($=2\pi$), which is equal to the length of the measure in the metronomic performance interpreted as a performance in $\frac{3}{4}$ meter. Let us now look at some other examples of rhythmic frequency modulation.

(o) We let $p_2$ be as in example (a) above, but now we investigate the result of modulating $p_2$ by $q_{1,0}(t) = \sin(t + \pi) = -\sin(t)$, with peak frequency deviation $d = 0.5$. Hence, we look at the following rhythmic frequency modulation:

$$r(t) = (p_2 \odot q_{1,0})(t) = 1 - \cos[3t - 0.5\sin(t)]$$

The graph of $r$ is thus as follows:

\[\text{Graph of } r(t)\]

\[\Delta_1=0.35T \quad \Delta_2=0.3T \quad \Delta_3=0.35T \quad T=2\pi\]

---

130 These facts are fairly well illustrated in the graph of $r$. The mathematical proof of these matters is quite straightforward, and is not included here.
As in example (a) we are able to prove that \( \Delta_1 = \Delta_3 \), and that \( r \) is a periodic function with period \( T = 2\pi \). A natural interpretation of \( r \) in this situation is thus a performance of quarter notes in \( \frac{3}{4} \) meter, where the first and the third beats are slightly lengthened and the second shortened, compared to a metronomic performance. Observe that whereas the distances between successive modulated pulse beats in example (a) make the pattern: short-long-short; short-long-short; …etc., the distances between the modulated pulse beats in example (b) make the pattern: long-short-long; long-short-long; …etc.

(c) Again we let \( p_3 \) be as in the examples above, but this time we modulate by applying \( q_{1.2}(t) = \sin(t + \pi/2) = \cos(t) \) with peak frequency deviation \( d = 1 \). The result is:

\[
r(t) = (p_3 \oplus q_{1.2})(t) = 1 - \cos[3t + \cos(t)]
\]

and the graph of \( r \) is now:

This \( r \) is also periodic, with period \( T = 2\pi \). Within each cycle there are three beats, thus an interpretation of \( r \) as a non-metronomic performance of quarter notes in \( \frac{3}{4} \) meter seems plausible also in this case. However, this example demonstrates some interesting features not present in the examples (a) and (b). First of all, we observe that the modulated first pulse beat occurs \textit{before} \( t = 0 \). Hence, a performance in accordance with the movement curve of this example would perform the first beat of every measure a little early compared to a strict metronomic performance where the first beat is at \( t = 0 \). Secondly, we note that within one cycle the distances between successive modulated pulse beats all differ: \( \Delta_1 \) is here approximately \( 0.42T \), \( \Delta_2 \) is close to \( 0.32T \), whereas \( \Delta_3 \) is approximately \( 0.26T \).
Consequently, these distances make the pattern: L(long)-I(intermediate)-S(short); L-I-S; L-I-S; ... etc.

It might also be interesting to note that if we apply modulation by \( q_{1.3\pi/2}(t) = \sin(t+3\pi/2) = -\cos(t) \), with peak frequency deviation \( d = 1 \), we obtain:

\[
r(t) = (p_3 \oplus q_{1.3\pi/2})(t) = 1 - \cos(3t - \cos(t))
\]

with the following graph:

![Graph showing modulation pattern]

\( \Delta_1 = 0.26T \quad \Delta_2 = 0.32T \quad \Delta_3 = 0.42T \)

In this case we find that the modulated first beat is too late compared to a metronomic performance with first beat at \( t = 0 \), and the distances between the modulated pulse beats make the following pattern: S-I-L; S-I-L; ... etc. Note also that \( \Delta_1 \) in the latter case equals \( \Delta_3 \) in the case of modulation by \( \cos(t) \), and \( \Delta_3 \) in the modulation by \( -\cos(t) \) equals \( \Delta_1 \) in the modulation by \( \cos(t) \). \( \Delta_2 \) remains in these examples the same, whether \( p_3 \) is modulated by \( \cos(t) \) or \(-\cos(t)\). Thus, the modulation of \( p_3 \) by \(-\cos(t)\) seems to be somewhat "equivalent to" running through the movement curve of the modulation of \( p_3 \) by \( \cos(t) \) in a *reversed direction*.113

(d) So far, the examples of rhythmic frequency modulation have all been related to various modulations of \( p_1(t) = 1 - \cos(3t) \) by pulses \( q_f \) with frequency \( f' = 1 \).

Let us now look at \( p_8(t) = 1 - \cos(8t) \), modulated by \( q_{2\pi}(t) = \sin(2t) \). This time we apply the peak frequency deviation \( d = 4 \). Hence, we now get the following mathematical function:

\[
r(t) = (p_8 \oplus q_{2\pi})(t) = 1 - \cos(8t + 4\sin(2t))
\]

Below, the graphs of \( p_8 \) and \( r \) are displayed in the same illustration:

---

113 At this point we just make this comment, with no further definitions or proofs.
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The upper graph shows the pulse $p_B$, the lower graph displays $r$. In this situation we find that the first, third, fifth, seventh,... beats of $p_B$ and $r$ coincide (occurring at $t = 0, 4\pi/8, 8\pi/8, 12\pi/8,...$ resp.), whereas the second, sixth, tenth,... beats of $r$ are too early, the fourth, eighth, twelfth,... beats of $r$ are too late, compared to the metronomic performance associated with $p_B$. Moreover, we are able to prove that $r$ is a periodic function with period $T = \pi$, $\Delta_1 = \Delta_4$ (approximately equal to 0.13T), and $\Delta_2 = \Delta_3$ (approximately 0.37T). Thus, the distances between the modulated pulse beats in this case make the following pattern: S-L-L-S; S-L-L-S; ... etc.\(^{122}\) Furthermore, since each cycle of the modulated pulse contains four beats, it seems reasonable to interpret $r$ as a modulated movement curve associated with a non-metronomic performance of quarter notes in 4/4 meter.\(^{133}\)

Looking at the graph of $r$, we also notice that the movement curve "flats out" at the third, seventh, eleventh,... modulated pulse beats. This feature reflects the fact that the modulation in this example is somewhat at a limit. We will return to a discussion of this shortly.

\(^{122}\) Again, the proof of these facts is quite straightforward, and thus omitted.

\(^{133}\) Or, if we wish, we may regard $r$ as the movement curve related to a non-metronomic performance of quarter notes in 8/4 meter, where the manner by which the performance is executed divides the cycle of 8 beats into two equal parts.
5.2.5. Some remarks related to the choice of rhythmic frequency modulation as basic to our model:

We have now defined rhythmic frequency modulation as a possible tool for creating new syntheses of live performances of rhythm. The definition of frequency modulated rhythms, as given in 5.2.3, is made on the basis of two fundamental ideas:

(i) Syntheses of live performances of rhythm are sought constructed by means of movements interacting with metronomic performances of rhythm.

(ii) The operation of movements interacting is defined by adopting a well known technique of sound synthesis, FM, and adjusting this to a new technique of rhythm synthesis.

Before we proceed in our further model constructions, it is at this point very important to emphasize that the choice of the pulses, \( p(t) = A[B + \sin(ft + \phi)] \) as atomic movements, and the proposed definition of interaction of movements given as rhythmic frequency modulation, is here given in an axiomatic manner, on a purely theoretical basis. Certainly, the work of Emile Jaques-Dalcroze in developing an educational method of perception and performance of rhythm, fundamentally based on exercises of kinesthetic movements of the body, is a major motivation for our focus on interactions of movements as being basic to rhythm performance. Moreover, the importance of approaching performance of musical rhythm as a continuous unfolding through attack points, i.e. investigating gestural rhythm rather than attack-point rhythm, is strongly motivated by the performing musician's own experiences, myself being a drummer playing the drum set. And, not the least, the very emphasis on the connections and interactions between performance (and experience) of rhythm and body movements basically represents a view of rhythm that in many respects resembles the Greek correspondence thinking and the idea of 'movement' as being fundamental to rhythmos (cf. Chapter 1). Viewed as such, our approach to synthesis and analysis of rhythm belongs to a tradition of rhythm research fundamentally related to ancient ideas of the nature of rhythm. Through this basic connection to antiquity, our model constructions are also to some extent related to ideas presented in some more recent contributions to rhythm research, e.g. Sievers (1924), Becking (1928), Truslit (1938), Fraisse (1982), Clynes (1977, 1983, 1986a, 1986b, 1992), Repp (1992a,b), Todd (1992a,b,c, 1993, 1994a,b,1995), and Blom (1993). (See chapters 2 and 4 for a presentation of these investigations.)
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However, prior to our choice of the pulses $p_i$ as atomic movements, and to the definition of rhythmic frequency modulation giving meaning to the operation "interaction of movements", we have not carried out any empirical investigations establishing empirical motivation for our proposed definitions. Although it would indeed be highly interesting if we were empirically able to obtain results that could support our definitions, such research is beyond the scope of our present study. However, what we intend to do is to continue the construction of our model of expressive timing, and then eventually test our theoretically constructed model against the findings of empirical rhythm research. That is to say, what we propose to do is to suggest various interpretations of our model and to investigate to what extent these interpretations may be applied as syntheses of rhythm approximating live rhythmic performances of music, as documented by empirical rhythm research. Some such applications of rhythmic frequency modulation will be presented and discussed in Chapter 7, but what is already suggested by example 5.2.4, is that various "deviations from the exact", as documented in investigations of SYVARD and PD, may be simulated by continuous manipulations of continuous movement curves, applying the technique of rhythmic frequency modulation. Hence, a possible interpretation of our model as presented up to this point is to regard this model as a continuous model of rhythmic deviation.

Even though we have not made any empirical research prior to our model constructions, there are, nevertheless, some support for our proposed definitions found in research on human movements. It is, for instance, very interesting to note that Paolo Viviani (1990, pp.356-357) asserts that:

... sinewaves are among the simplest predictable motions and have been used extensively in tracking experiments. Both adults and children above the age 8 or 9 years are able to produce spontaneously very good approximations to this waveform (...). ... . Whether we consider wrist or forearm movements, normal adults can pursue a 1-dimensional sinewave very accurately up to frequencies of about 1 or 2 Hz, depending on the amplitude (...).

As stated by Viviani, sinewaves are easy to approximate by human movements, and are among the simplest predictable motions. A choice of pulses, defined in 5.1.1, as atomic movements in our model constructions, seems, therefore, quite natural.

As to our definition of frequency modulation as a mathematical operation corresponding to interaction of movements, it is interesting to observe that Beek, Peper and van Wieringen
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(1992), modeling coordinated rhythmic movements such as breathing and walking, cascade juggling, and polyrhythmic tapping, conclude that:

Constrained movement involving more than one limb segment often leads to modulation.

(ibid., p.604)

The approach of Beek et al. is to borrow concepts and tools from classical analytical dynamics ("the force balance" and "the Lagrangian") and modern qualitative dynamics ("the circle map" and "the Farey tree"), in order to demonstrate their usefulness for the movement scientist. Applying a system of differential equations modeling constrained hand movements, they arrive at the following interesting conclusion (ibid., p.606):

What this example shows is that the presence of one (or a few) configuration constraint(s) may be sufficient to induce frequency modulation in a Lagrangian system. In human movement systems, of course, such constraints abound — hence, the ubiquity of frequency modulation in such systems.

This conclusion of Beek et al. indeed seems to support our choice of frequency modulation as a mathematical expression related to interaction of movements.

Along this line, it is also interesting to recall that among the different theories of motor skill, briefly presented in section 2.5, the Bernstein approach emphasizes that muscles are not individually controlled but function in muscle linkages or coordinative structures. An important point of this approach is that groups of muscles are constrained to act as functional units, thus reducing the degree of freedom and thereby also the number of movement parameters. Through emphasizing the presence of constrained movements and coordinative structures in rhythmic movements, the Bernstein approach shows a relationship to the models of Beek et al., and is thereby also somewhat related at least on a theoretical level, to our strategy in the construction of syntheses of live performance of rhythm, when we are applying the technique of rhythmic frequency modulation.

Having made these remarks on the definitions of pulse and rhythmic frequency modulation, we now proceed with our construction of a model of live performance of rhythm.

5.2.6. Some mathematical results of value to our model:

In all the examples of 5.2.4 the number of modulated beats over any interval, \( J \), of length \( 2\pi \) is equal to the number of unmodulated beats over the same interval. This need not
generally be the case. In our discussion of this fact we choose a quite concrete musical interpretation: Let \( p_f(t) = A[1 - \cos(f t)] \) be a mathematical representation of a metronomic performance of pulse beats. A graphic illustration is as follows (with \( f = 8 \)):

\[
\begin{array}{c}
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\end{array}
\]

\( \Delta_1 = 2\pi f \)  \( \Delta_2 = 2\pi f \)  \( \Delta_3 = 2\pi f \)

The distance between any pair of successive beats of \( p_f \) is \( 2\pi f \). Thus, \( \Delta_1 = \Delta_2 = \ldots = \Delta_l = \ldots \). Therefore, if \( f \) is a positive, natural number, i.e. \( f \in \mathbb{N} = \{1, 2, 3, \ldots \} \), the number of beats over any interval of length \( 2\pi \) is \( f \). Defining \( q_f(t) = \sin(f' t + \phi) \), we are now able to prove the following:

**5.2.6 (i) Lemma:**

With \( p_f \) and \( q_f \) as above, let \( f, f' \in \mathbb{N} = \{1, 2, 3, \ldots \} \):

If \( 0 \leq d \leq \frac{f'}{f} \), then \( p_f \oplus q_f \) has exactly \( f' \) beats over any interval of length \( 2\pi \).

The mathematical proof of this lemma is given in Appendix I. Instead of looking at the mathematical aspects, we note that some musical interpretations of this lemma are demonstrated in the examples given in 5.2.4. In 5.2.4 (a), (b) and (c) the number of modulated beats over any interval of length \( 2\pi \) is 3, which equals the number of unmodulated beats over any interval of the same length; whereas in 5.2.4 (d) the number of beats (modulated or not) over any interval of length \( 2\pi \) is 8. Observe, however, that in example (d) \( f = 8, f' = 2, d = 4 \), hence, \( d = \frac{ff'}{f} \). Therefore, this example represents a limit case related to the premise of the lemma stating that \( d \leq \frac{ff'}{f} \). To illustrate what might happen when \( d > \frac{ff'}{f} \), let us look at a modulation of \( p_8 \) by \( q_{2.0} \) with peak frequency deviation \( d = 6 \). The result is:

\[
r(t) = (p_8 \oplus q_{2.0})(t) = 1 - \cos[8t + 6\sin(2t)]
\]

and the graph of \( r \) is as follows (compare this to the graphs in 5.2.4 (d)).
We observe that in this case 4 "new" modulated beats are created: nb₁, nb₂, nb₃, nb₄, contained in every interval, J, of length 2π. Thus, in this situation we have 12 modulated beats in any interval of length 2π, whereas the number of unmodulated beats in the same interval is 8.

Another mathematical result which is useful in our later applications of rhythmic frequency modulation is:

**5.2.6 (ii) Lemma:**

Let \( p_f \) and \( q_f \) be as in the previous, \( f, f' \in \mathbb{N} = \{1, 2, 3, \ldots\} \).

Then \( p_f \oplus q_f \) is a periodic function with period \( \frac{2\pi}{\gcd(f, f')} \)

\( (\gcd(f, f') = \text{greatest common divisor of } f \text{ and } f' ) \)

**Proof:** See Appendix I.

Looking at the examples of 5.2.4, we find that this result is reflected in (a), (b), and (c) by reason of the modulation of \( p_f \) by \( q_f \) being a rhythmic cycle of 3 modulated beats with period \( T=2\pi \) (since \( \gcd(3,1)=1 \)); whereas in (d) we see that the modulation of \( p_8 \) by \( q_2 \) yields a rhythmic cycle of period \( 2\pi/\gcd(8,2) = 2\pi/2 = \pi \). Moreover, since there are 8 modulated beats in each interval of length \( 2\pi \) (by lemma 5.2.6 (i)), and the modulation in this case is periodic with period \( \pi \), there are 4 modulated beats to each cycle. (A possible interpretation of example (d) is, as noted above, a non-metronomic performance of quarter notes in 8/4 meter, where the manner by which the performance is executed divides the cycle of 8 beats into two equal parts with 4 beats in each part.)
5.3. Modulations of Complex Rhythms

In the previous section we introduced rhythmic frequency modulation as a possible technique useful for making new syntheses of live performances of rhythm. Some preliminary examples of rhythmic frequency modulation of pulses were presented in 5.2.4, thereby also suggesting how live performances of pulse beats, characterized by various deviations from metronomic regularity, may be simulated in our model by applying different manipulations of continuous movement curves. However, in order to be able to create syntheses of live performances of more complex rhythms, we need to investigate rhythmic modulation of subdivisions and ties of pulses as well.

5.3.1. Proposition:

Let $p_r(t) = \Delta[1 - \cos(ft)], q_r(t) = \sin(f't + \varphi), k = 1, 2, 3, \ldots$, and $0 \leq d \leq \frac{f}{f'}$.

Then $p_r \otimes q_r$ is a modulated pulse such that:

i) Every beat of $p_r \otimes q_r$ is also a beat of $p_r \otimes q_r$.

ii) For every beat of $p_r \otimes q_r$ there are $k$ beats of $p_r \otimes q_r$.

Proof: See Appendix I.

An illustration of the relevance of this proposition to musical interpretations is given in the figure below:

Figure 5.18. The upper graph shows a modulation of $p_2$ by $q_{1,0}$ with peak frequency deviation $d=1$. The lower graph shows a modulation of $p_{2x2}$ by $q_{1,0}$ with peak frequency deviation $d=2x1$. Observe that the lower graph represents a modulated subdivision of the upper graph in 2.
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In Figure 5.18 the upper graph illustrates the modulation:

\[(p_3 \oplus q_{1, \pi/2})(t) = 1 - \cos[3t + \sin(t + \pi/2)] = 1 - \cos[3t + \cos(t)]\]

This is identical with the modulation exemplified in 5.2.4 (c), and, as noted in the discussion of 5.2.4 (c), this modulated pulse is periodic with period \(T = 2\pi\), within each cycle there are three beats, and the distances between successive beats make the following pattern: L-L-S; L-L-S; ... etc. A plausible interpretation of this pulse is a non-metronomic performance of quarter notes in \(\frac{3}{4}\) meter. The lower graph in Figure 5.18 shows an illustration of:

\[p_{3k} \oplus q_{1, \pi/2}, \text{ where } k = 2.\]

Thus, displayed in the lower graph is:

\[(p_{6} \oplus q_{1, \pi/2})(t) = 1 - \cos[6t + 2\cos(t)]\]

From Figure 5.18 we observe the following:

1) Every beat of \(p_3 \oplus q_{1, \pi/2}\) is also a beat of \(p_6 \oplus q_{1, \pi/2}\)
2) For every beat of \(p_3 \oplus q_{1, \pi/2}\) there are \(k = 2\) beats of \(p_6 \oplus q_{1, \pi/2}\)

which is in accordance with proposition 5.3.1.

Hence, if the upper graph of Figure 5.18 is interpreted as a movement curve related to a non-metronomic performance of quarter notes, the lower graph is naturally interpreted as a non-metronomic performance of a subdivision of the quarter notes in 2; i.e. as a non-metronomic performance of eighth notes. Observe, however, that in this subdivision a modulated quarter note is not divided into two notes of equal length, that is to say, the distance between two successive quarter-note-beats is not divided in two equal halves by this subdivision. This is a very interesting observation, which shows that the feature of "deviating from the exact" is somewhat inherited by this operation of subdividing modulations.

On the basis of the discussion above, the following definition seems quite natural:

**5.3.2. Definition:**

Let \(p_r\) and \(q_r\) be as in proposition 5.3.1, \(k \in \{1,2,3,\ldots\}\)

We denote the modulated pulse, \(p_r \oplus q_r\), a subdivision of \(p_r \oplus q_r\) in \(k\).

We use the notation \((p_r \oplus q_r)_k\) for subdivisions of \(p_r \oplus q_r\) in \(k\). Thus, by definition:

\[(p_r \oplus q_r)_k = p_{rk} \oplus q_{rk}\]
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Observe that if \( d = 0 \), proposition 5.3.1 reduces to proposition 5.1.8, and the situation illustrated in Figure 5.18, reduces to example 5.1.7.

To make the definition 5.3.2 explicit, a subdivision, \( r \), of \( p_T \oplus q_T \) in \( k \) is given by the formula:
\[
(S): \quad r(t) = \lambda [1 - \cos(k \varphi + kd \sin(f' t + \varphi))]
\]

5.3.3. Example:

Let us look briefly at one more example of subdivisions of modulations, providing further illustration of prop.5.3.1. The present example should be compared to example 5.1.9, which shows the unmodulated case \( (d = 0) \).

Consider the modulation \((p_T \oplus q_T)(t) = 1 - \cos(5t + 2 \sin(2t))\).

A subdivision of \( p_T \oplus q_T \) in 3 is \((p_T \oplus q_T)_3(t) = 1 - \cos(15t + 6 \sin(2t))\).

The graphs of these modulations are given below:

![Graphs of modulations](image)

A possible interpretation of this situation is as follows:
The upper graph shows a seemingly rather strange movement curve related to a performance of a rhythmic ostinato of 5 beats with cycle length (period) \( T = 2\pi \) (cf., lemmas 5.2.1 (i) and (ii)). The lower graph shows a movement curve associated with a non-metronomic subdivision of the 5 beats in 3. Observe that both graphs of this specific example are symmetric with axis of symmetry; \( t = \pi \), and that the distances between successive beats in the upper movement curve make the pattern: I-L-S-L-I; I-L-S-L-I, .... etc.
Having defined subdivisions of modulations as modulations of subdivisions, it is tempting to do something similar when defining ties of modulations. Again, we let $p_f$ be the pulse: $p_f(t) = A[1 - \cos(ft)]$. We recall from prop. 5.1.14 that with $n = 1, 2, 3, \ldots$

$$\tau_l(t) = A\left[1 - \cos\left(\frac{f}{n} - \frac{2(l-1)}{n}\right)\right], l = 1, 2, 3, \ldots, n$$

is a $n$-tie of $p_f$ with minimal points on the pulse beats of $p_f$ with numbers: $l, l + n, l + 2n, l + 3n, \ldots$ etc. For defining ties of modulations, we need to choose a numbering of the beats of a modulated pulse. In doing so, we inherit the numbering of beats of an unmodulated pulse, in the sense that if $b_l$ is the $l$-th beat of $p_f$, the modulated transformation of $b_l$ is the $l$-th beat of $p_f$ modulated by the pulse $q_f$. We are now able to prove the following:

### 5.3.4. Proposition:

Let $p_f$ and $\tau_l$ be as above, $q_f(t) = \sin(f't + \varphi), n = 1, 2, 3, \ldots$ and $0 \leq d \leq \frac{f}{f'}$.

Then $\tau_l \oplus_{\varphi}^d q_f$ is a modulated pulse such that:

i) Every beat of $\tau_l \oplus_{\varphi}^d q_f$ is also a beat of $p_f \oplus_{\varphi} q_f$.

ii) For every beat of $\tau_l \oplus_{\varphi}^d q_f$ there are $n$ beats of $p_f \oplus_{\varphi} q_f$.

iii) The beats of $\tau_l \oplus_{\varphi}^d q_f$ coincide with the beats of $p_f \oplus_{\varphi} q_f$ with numbers:

$l, l + n, l + 2n, l + 3n, \ldots$ etc.

**Proof:** See Appendix I.

This proposition is a "modulated version" of prop. 5.1.14 (i.e. with $d=0$, prop. 5.3.4 reduces to prop. 5.1.14). Based on prop. 5.3.4, it is natural to define ties of modulations as "modulated ties". Thus:

### 5.3.5. Definition:

Refering to the use of notation above, we denote the modulated pulse:

$\tau_l \oplus_{\varphi}^d q_f$ an $n$-tie of $p_f \oplus_{\varphi} q_f$.

To make this definition explicit, a modulated $n$-tie, $r_l$ is given by the following formula:

$$(T) \quad r_l(t) = (\tau_l \oplus_{\varphi}^d q_f)(t) = A \left[1 - \cos\left(\frac{f}{n} + \frac{d}{n} \sin(f't + \varphi) - \frac{2(l-1)}{n}\right)\right], l = 1, 2, 3, \ldots, n.$$ 

To obtain a better understanding of how this works, let us look at an example of modulated n-ties:
5.3.6. Example:

Let \( p_4(t) = 1 - \cos(4t) \), \( q_{l, \pi/8}(t) = \sin(t + \pi/8) \). A modulation of \( p_4 \) by \( q_{l, \pi/8} \) with peak frequency deviation \( d = 2 \), gives the following result:

\[
(p_4 \circledast q_{1, \pi/8})(t) = 1 - \cos[4t + 2\sin(t + \pi/8)]
\]

According to the formula, (7), above, a 5-tie of this modulated pulse is given by:

\[
r_l(t) = 1 - \cos\left[\frac{4t}{5} + \frac{2}{5}\sin(t + \frac{\pi}{8}) - \frac{2\pi}{5}(l - 1)\right], \quad l = 1, 2, 3, 4, 5.
\]

The graph of \( p_4 \circledast q_{1, \pi/8} \) along with all the 5-ties, \( r_l \) are given in the figure below:

Figure 5.19. Illustration of modulated 5-ties. The uppermost graph is the graph of the modulation of \( p_4 \) by \( q_{l, \pi/8} \) with peak frequency deviation \( d=2 \). The other graphs are, in downward order, the five 5-ties: \( r_1, r_2, r_3, r_4, r_5 \). Compare this figure with Fig.5.8 of example 5.1.15, which shows the corresponding unmodulated case (i.e. \( d=0 \)).
Let $p$ be the modulation of $p_i$ by $g_{i,w}$, with $d=2$. As indicated in Fig.5.19, we find that:

$r_1$ has a beat on the first beat of $p$,  
$r_2$ has a beat on the second beat of $p$,  
$r_3$ has a beat on the third beat of $p$,  
$r_4$ has a beat on the fourth beat of $p$,  
$r_5$ has a beat on the fifth beat of $p$,

which is all in accordance with prop.5.3.4.

It is interesting to note the resemblance between prop.5.3.1, identifying modulated subdivisions, and prop.5.3.4, identifying modulated $n$-ties. In fact, if we in the formula (S), defining subdivisions in $k$, allow $k$ to be a rational number, the formula (T), defining $n$-ties, will be a special case of (S), except for the term $-2(l+1)\pi/n$, which reflects the multivalued nature of the operation of making $n$-ties. We may thus combine the results of prop.5.3.1 and prop.5.3.4 into one theorem, stating an algorithm useful for making syntheses of live performances of complex rhythms:

5.3.7. Theorem:

Algorithm for constructing modulated subdivisions and ties:

Let $\psi(t) = A[1 - \cos(f t)]$, $\phi(t) = \sin(f t + \phi)$, $0 \leq d \leq \frac{f}{f'}$, $m, n \in N = \{1,2,3,\ldots\}$.

Then:

$$(\alpha): r(t) = A \left[ 1 - \cos \left( \frac{m}{n} \right) f t + \left( \frac{m}{n} \right) d \sin(f t + \phi) - \frac{2(l+1)\pi}{n} \right], \ l = 1,2,3,\ldots,n$$

is an algorithm constructing subdivisions and ties of the pulse $\psi \oplus \phi \psi$.

If $n = 1$, $(\alpha)$ creates subdivisions in $m$.
If $m = 1$, $(\alpha)$ creates $n$-ties.
If $n > 1$, $m > 1$, $(\alpha)$ creates a combination of subdivision in $m$ and $n$-ties. In this case, the beats of the $n$-tie, $r_l$ coincide with the beats with numbers $l, l+n, l+2n, l+3n,\ldots$ of the modulated $m$-subdivided pulse.

Proof: By prop.5.3.1 and prop.5.3.4. Q.E.D.
In the following we will say that \( (\alpha) \) creates a subdivision of \( p_r \ominus q_r \) in \( m \). Hence, \( (\alpha) \) is an algorithm constructing rational subdivisions, and an \( n \)-tie may also be called a subdivision in \( 1/n \).

5.3.8. Example:

Let us look at an example illustrating the effect of a modulated subdivision in \( m/n \), where \( n \geq 1, m \geq 1 \). In all the previous examples, the pulse frequencies, \( f \) and \( f' \), have been natural numbers (i.e. among the numbers \( 1, 2, 3, 4, \ldots \)). This is not a premise in theorem 5.3.7. Consider the pulse \( p(i) = 1 - \cos(i) \), modulated by \( q \) i.e., \( s(i) = \sin((0.4)i + \pi/4) \) with peak frequency deviation \( d = 1.5 \). The result is:

\[
p(i) = 1 - \cos[(t + (1.5)\sin((0.4)i + \pi/4))]
\]

We now investigate a subdivision of this modulated pulse, \( p \), in \( 2/3 \). This operation corresponds to a subdivision of \( p \) in 2, followed by a construction of 3-ties.

Note that if the pulse \( p_1 \) is interpreted as a movement curve associated with a metronomic performance of quarter notes, the modulated pulse, \( p \), may be regarded as a movement curve related to a non-metronomic performance of quarter notes. A subdivision of \( p \) in 2 yields a movement curve of a non-metronomic performance of eighth notes, whereas a construction of 3-ties thereof results in tying together 3 eighth notes. The output of this entire operation is therefore a movement curve related to a non-metronomic performance of dotted quarter notes.

According to theorem 5.3.7 a subdivision of \( p \) in \( 2/3 \) is:

\[
r_l(i) = 1 - \cos[(2/3)i + (2/3)(1.5)\sin((0.4)i + \pi/4) - (2\pi/3)(i - 1)], \ l = 1, 2, 3.
\]

The graphs of \( p \) and \( r_l \), \( l = 1, 2, 3 \), are given in Figure 5.20 below. To illustrate the positions of the minimal points (i.e. the beats) of \( r_l \) related to the beats of the subdivision of \( p \) in 2, we also show the graph of the modulated 2-subdivided pulse:

\[
(p_1 \ominus 3 q_{9/4,5/4})s.
\]
Figure 5.20. Illustration of rational subdivisions of a modulated pulse.

According to theorem 5.3.7:

$r_1$ has minimum point coinciding with the first beat of the modulated 2-subdivided pulse, $(p)_2$

$r_2$ —— the second ———

$r_3$ —— the third ———

These facts are all demonstrated in the figure above.
5.3.9. Syntheses of non-metronomic performances of complex rhythms:

In section 5.1 we presented a continuous model of "exact", i.e. metronomic performance of rhythm. This model is denoted MPR, and the elements of MPR are combinations of pulses, subject to the operations of making subdivisions and ties. In section 5.2 and the present section we have made a "modulated version" of this model. Rhythmic frequency modulation (RFM) of pulses, subdivisions and ties has been defined, a result being the construction of a new tool useful for making syntheses of non-metronomic performances of rhythm. Possible applications of RFM in making syntheses of non-metronomic performances of pulse beats have been demonstrated in 5.2.4, whereas the effect of RFM on subdivisions and ties has been illustrated in Figure 5.18, and the examples 5.3.3, 5.3.6, and 5.3.8 above. Based on repeated applications of theorem 5.3.7, combining constructions of modulated subdivisions and ties, we are now in a position to create continuous movement curves representing non-metronomic performances of any rhythm of measured music. To give an example of how this works, let us look at a "modulated version" of the movement curve constructed in 5.1.16 as a representation of the following sequence of notes:

\[
\begin{array}{cccccc}
\text{||} & \text{||} & \text{||} & \text{||} & \text{||} & \text{||}
\end{array}
\]

As presented in Figure 5.9, the connections between this note sequence, a mathematical representation of pulses as continuous functions, and an associated movement curve are given as illustrated by the following:

![Movement Curve Illustration](image)

Figure 5.21. Illustration of a movement curve associated with a metronomic performance of a specified note sequence. The horizontal axis displays time, \(t\), where the first beat occurs at \(t=0\).
The curve given in Figure 5.21 shows a possible movement curve related to a robot-like rhythmic performance executed in perfect synchronization with a metronome. Observe that in this movement curve quarter notes are mathematically represented by the pulse $p_i(t) = 1 \cdot \cos(t)$ (and phase-translations thereof). By manipulating this movement curve by applying the technique of RFM, we are now able to create various new movement curves representing different non-metronomic performances of the note sequence above. For instance, if we use the modulation of example 5.3.8 to every pulse in the metronomic representation, i.e., every pulse illustrated in Figure 5.21 is modulated by $q_{0, \pi, \pi/4}$ with peak frequency deviation $d = 1.5$, we obtain the following situation:

![Diagram of movement curve with modulations](image)

1. $1 - \cos[(2/3)t + \sin((0.4)t + \pi/4)]$
2. $1 - \cos[2t + 3\sin((0.4)t + \pi/4)]$
3. $1 - \cos[(t/2 + (0.75)\sin((0.4)t + \pi/4) - 3\pi/2)]$
4. $1 - \cos[(t + (1.5)\sin((0.4)t + \pi/4) - \pi)]$
5. $1 - \cos[(2t + 3\sin((0.4)t + \pi/4))$
6. $1 - \cos[(t + (1.5)\sin((0.4)t + \pi/4))$

Figure 5.22. An illustration showing the connections between a note sequence, modulated pulses, and a movement curve associated with a non-metronomic rhythmic performance. This figure is a modulated version of Fig. 5.21.

Since metronомically performed quarter notes in this example are represented by the pulse $p_i(t) = 1 - \cos(t)$ (and phase-translations thereof), the distance between any quarter note beat and the next following beat in the metronomic performance is: $\Delta_{\text{quat}} = 2\pi$. Consequently: $\Delta_{\text{dotted~quat}} = 3\pi$, $\Delta_{\text{half}} = 4\pi$, $\Delta_{\text{eighth}} = \pi$, and $\Delta_{\text{sixteenth}} = \pi/2$ in the metronomic performance illustrated in Fig. 5.21. With reference to Fig. 5.22, on the other hand, we find that:
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\[ \Delta_1 > \Delta_{\text{dotted quart}} \quad \Delta_2 < \Delta_{\text{eighth}} \quad \Delta_3 < \Delta_{\text{quart}} \]
\[ \Delta_4 > \Delta_{\text{eighth}} \quad \Delta_5 < \Delta_{\text{quart}} \quad \Delta_6 < \Delta_{\text{quart}} \]
\[ \Delta_7 > \Delta_{\text{half}} \]

Moreover, we note that:

\[ \Delta_2 = \Delta_{\text{modulated first eighth}} < \Delta_4 = \Delta_{\text{modulated second eighth}}, \quad \text{and that} \]
\[ \Delta_3 = \Delta_{\text{mod. first quart}} = \Delta_6 = \Delta_{\text{mod. third quart}} < \Delta_5 = \Delta_{\text{mod. second quart}} \]

Thus, if the movement curve of Figure 5.22 is representative of a performance of drum beats, the quarter note beats, as well as the eighth note beats, are performed with different durations, in varying degree "deviating from the exact". It is also interesting to observe that:

\[ \Delta_{\text{mod. first quart}} < \Delta_{\text{mod. second eighth}}, \]

that is to say, in this (hypothetic) performance of drum beats the first quarter beat is performed with a duration shorter than the second eighth beat. Therefore, a listener will most likely perceive this performance as a performance of some other rhythmic pattern than the pattern represented by the note sequence in Fig.5.22. As noted by Clarke (1999, p.490), a mechanism of categorical perception has been empirically demonstrated, the idea being that the listener assigns the variable durations of expressive performances to a relatively small number of rhythmic categories. In the present performance \( \Delta_{\text{mod. first quart}} \) is approximately equal to \((0.7)\Delta_{\text{quart}}\), whereas \( \Delta_{\text{mod. second eighth}} \) is close to \((0.8)\Delta_{\text{quart}}\). Consequently, this performance may be interpreted as a performance where the "first quarter" and the "second eighth" belong to the same rhythmic category, both being "a bit short" compared to a quarter note performed in perfect synchronization with a metronome.

The modulation applied in the example above is quite "dramatic", creating a shift of rhythmic categories compared to the original pattern of note values. Starting with the same sequence of note values as in the previous example, it might be interesting to look at the different movement curves created by rhythmic frequency modulations applying different strength of modulation, i.e. using different values of peak frequency deviation, \( d \). In Figure 5.23 such movement curves are displayed. In all these curves the modulating pulse is \( q_{0.4, \pm 4} \).

Starting with \( d=0 \), which corresponds to the unmodulated, i.e. metronomic performance of the note sequence, we show the effect of applying different peak frequency deviations, ending up with the "limit case", \( d = 2.5 = f_{\text{upper}} = 1/(0.4) \) (cf. theorem 5.3.7). Note that the movement curves below are horizontally translated such that the first beat of every curve occurs "at the same time" as the first beat of the other curves.

---

\(^{134}\) All of this can be calculated using, for instance, a graphic calculator as found on most Macintosh computers.
Figure 5.23. A demonstration of the effect of modulating, applying different peak frequency deviations. \( d = 0 \) corresponds to a performance of the note sequence at the top in perfect synchronization with a metronome, whereas \( d = 2.5 \) shows the "limit case" of theorem 5.3.7.
With reference to Figure 5.23, it might be interesting to observe that modulating with $d=2.5$, we get:

- $\Delta_1$ approx. equal to (0.99)$\Delta_{\text{half}}$
- $\Delta_2$ approx. eq. to (0.55)$\Delta_{\text{eighth}} = (1.1)\Delta_{\text{sixteenth}}$
- $\Delta_3$ approx. eq. to (1.1)$\Delta_{\text{eighth}}$
- $\Delta_4$ approx. eq. to (1.2)$\Delta_{\text{quart}}$
- $\Delta_5$ approx. eq. to $\Delta_{\text{oned-eighth}}$
- $\Delta_6$ approx. eq. to (1.1)$\Delta_{\text{eighth}}$
- $\Delta_7$ approx. eq. to (1.1)$\Delta_{\text{half}}$

Thus, based on the idea of categorical perception, a possible interpretation of a rhythmic performance in accordance with the case $d=2.5$ above is the pattern:

\[\lower{1.5pt}{\text{\[\hspace{1em}\]} \hspace{1em}\text{\[\hspace{1em}\]}}\]

Hence, Figure 5.23 illustrates how an application of rhythmic frequency modulation, utilizing increasing values of peak frequency deviation, creates a shift between different rhythmic perceptual categories. This transformation of rhythmic structure develops through a process of rhythmic morphing of one rhythmic pattern into another rhythmic pattern. In this specific example the transformation is between the following two patterns:

\[\lower{1.5pt}{\text{\[\hspace{1em}\]} \hspace{1em}\text{\[\hspace{1em}\]}}\]

\[\lower{1.5pt}{\text{\[\hspace{1em}\]} \hspace{1em}\text{\[\hspace{1em}\]}}\]
5.4. The Model LPR

A main concern of the present chapter is to construct a new model of live performances of rhythm. The approach we have chosen in solving this task is first to construct a model, MPR, of metronomic performances of rhythm, and then apply a technique of rhythmic frequency modulation to the elements of MPR, creating movement curves which represent rhythmic performances characterized by various deviations from metronomic regularity. Whether our construction of syntheses of non-metronomic rhythmic performances also yields relevant approximations to live performances of rhythm, is a question which can be answered only on the basis of empirical investigations. Some such empirical rhythm research has indeed been undertaken, especially in the various detections of SYVARD and PD (cf. sections 2.3 and 2.4). To what extent the findings of these empirical studies may be modeled by continuous movement curves applying rhythmic frequency modulation, will be focused on in Chapter 7, where different applications of RFM are demonstrated. However, some interesting indications of the possibilities of applying RFM in making syntheses of live performances of rhythm may be mentioned at this point as well:

(1) Empirical rhythm research has documented that in performances of music in \( \frac{3}{4} \) meter, various patterns of beat durations; e.g. S-L-I, or I-I-S, are characteristic of musical style (for instance, characterizing performances of Vienna waltzes\(^{135}\)), as well as typical of musical dialects (e.g. in Norwegian folk music\(^{136}\)). In example 5.2.4 (a), (b), and (c) we demonstrated how different patterns of beat durations may be constructed by applying RFM, thus suggesting that RFM may be useful in making syntheses of live performances of, e.g. Vienna waltzes and various dialects of Norwegian folk music.

(2) Playing a cymbal ostinato in jazz music, different drummers may utilize various different phrasings of a rhythm which may be written as:

\[\begin{array}{cccc}
\text{\( \frac{3}{4} \)} & \text{\( \text{\( \frac{3}{4} \)} \)} & \text{\( \text{\( \frac{3}{4} \)} \)} & \text{\( \text{\( \frac{3}{4} \)} \)}
\end{array}\]

\(^{135}\) Cf. e.g. Bengtsson & Gabrielsson (1983).
\(^{136}\) See Kvife (1995).

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A specific rhythmic performance of this ostinato may be characteristic of the individual drummer, it might be related to style, and is also dependent on the tempo of the performance. Sometimes this ostinato is played according to a subdivision of the quarter note in triplets, other times a rather "flat" performance is heard (quarter note – two eighth notes; quarter note – two eighth notes; … etc.). Moreover, some cymbal beats may be performed "ahead", others "behind" a metronomic reference. In 5.3.9 we presented an example showing that an application of RFM, through a process of rhythmic morphing, may create a shift between different rhythmic categories. Applying this technique, it seems possible to make approximations of different performances of cymbal ostinatos in jazz by using RFM in manipulations of continuous movement curves.

(3) If two identical rhythmic patterns are modulated by different pulses, or with different peak frequency deviations, the beats of the (originally) identical patterns are "moved" in different ways, thus creating various occurrences of non-synchronization between the two patterns. Applied in this manner, RFM may be useful in creating different out-of-sync phenomena demonstrated in empirical detections of PD\textsuperscript{137}.

These three indications of possible applications of RFM in making syntheses of live performances of rhythm will be discussed in more detail in Chapter 7.

Motivated by our many examples showing that different applications of RFM create various continuous movement curves modeling a large number of non-metronomic performances of rhythm, and combined with a knowledge of empirical investigations demonstrating various "deviations from the exact" as being typical of live performances of rhythm, it seems appropriate to assert that the technique of RFM, indeed, may be quite useful in making syntheses approximating live performances of rhythm. We are thus ready to present a formal definition of our model, LPR:

\textbf{5.4.1. Definition:}

\begin{align*}
\text{LPR} &= \text{MPR} \otimes d \quad q \\
&= \{ m \otimes d \quad q \quad m \in \text{MPR} , \quad q \quad \text{is a pulse} : \quad q(t) = \sin(f' t + \varphi) \quad , \quad \text{and } \quad d \quad \text{is a peak frequency deviation} \}
\end{align*}

As argued above, our assertion is that LPR is an adequate model of live performances of rhythm. Having made this definition of LPR, we have also proved to be consistent with our idea presented in 5.2, of creating a model of live performances of rhythm where: Syntheses of live performances of rhythm are constructed as continuous transformations of syntheses of metronomic performances of rhythm.

In 5.1.17 we argued that MPR is a model of rhythmic structure where information of note values is represented as continuous movements through attack points. Through our stepwise constructions of the models MPR and LPR, we have thus arrived at a situation as illustrated by the following figure (compare this figure with Figures 5.12 and 5.14):

![Diagram showing the relationship between MPR and LPR](image)

Figure 5.24. Illustration of interrelations between rhythmic structure and the models MPR and LPR.

In the figure above, $\delta_1, \delta_2, \ldots, \delta_k$ are transformations given by: $\delta_i(m) = m \oplus q, m \in \text{MPR}$. 

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Observe that if $m \in \text{MPR}$, then $m$ is a movement curve associated with a metronomic performance of rhythm, and $\delta(m)$ is a movement curve associated with a non-metronomic performance of rhythm, which, in some cases, also represents a relevant simulation of live performances of rhythm. Moreover, it should be noted that on the basis of the defined interrelations between structure and movement curves, illustrated in Fig. 5.24, the transformations, $\delta_1, \delta_2, \delta_3, \ldots, \delta_k$, may be understood as representations of expressive timing, as Clarke defines this notion (Clarke, 1999, p.490; see also Fig.5.13 in the previous).

Hence, we now claim to have obtained a solution of problem (B) formulated in the introduction of this chapter: "Construct a model of expressive timing, where performed rhythm is viewed as a result of continuous interactions of movements." A theoretical interpretation offered by our model construction is to describe representations of expressive timing as non-linear, continuous transformations of rhythmic structure; or, to put it another way, to view expressive timing as a result of rhythmic structure being "stretched" and "compressed" by actions of movements. However, it is important to recall that the only durational variable taken into account in the model MPR so far is the interonset interval, $D_0$ (cf. 5.1.17). According to the way LPR is defined, the same applies for LPR. This is indeed an important point to make, since, as emphasized by Bengtsson & Gabrielsson (1983), both $D_0$ and $D_{oi}$ are crucial parameters in live musical performances. In Chapter 6 we try to incorporate $D_0$ and $D_{oi}$ into a computer implementation of our model LPR.

5.4.2. Some kinesthetic considerations:

As strongly emphasized in the eurhythms of Emile Jaques-Dalcroze, it is of fundamental importance in a performance of a rhythm that the various states of a rhythmic movement are adjusted and related to rhythmic phrasing and tempo in such a way that the body movements of the performer in a comfortable, "economic", and "natural" manner are expressing the speed of the sound sequence, as well as the duration and energy of the sound. As yet, these considerations of kinesthetic nature have not been sufficiently implemented in our model LPR. As pointed out in 5.1.16, one consequence of such kinesthetic considerations related to the model MPR is that the amplitude of the pulse, $p(t) = A[1 - \cos(ft)]$, should be a function of frequency and dynamics; $A = M(f, dyn)$. A similar implication may be made.
applied to the model LPR. Whereas some possible ways of including considerations of dynamics will be presented in the next chapter, it might be interesting at this point to give an example of how some plausible relations between amplitude and frequency might be introduced in LPR. If, as used above, $\Delta t$ denotes the distance between the modulated pulse beats, $j$ and $j+1$, it is a reasonable suggestion, based on kinesthetic considerations, that if $\Delta t$ is "large", the amplitude should be "large", and if $\Delta t$ is "small", the amplitude should be "small". (Simply because when, e.g., a series of drum beats are performed at rapid speed, there is not time to move comfortably the hand as far apart from the drum as when the temporal distances between the drum beats are larger.) An illustration of two different ways of making the amplitude a function of beat distance is given in the figure below:

![Diagram of pulse beats with amplitude variation](image)

Figure 5.25. Two simulations of a non-metronomic rhythmic performance of the note sequence at the top. Both curves are constructed applying the modulating pulse $d = 1.5 \Delta$ with peak frequency deviation $\Delta = 1.5$ (as was also applied in Fig. 5.22). The amplitudes of the upper graph are related to the beat distances as $A = \Delta$, whereas in the lower graph the relation is: $A = (\Delta)^{1.5}$. Observe that the vertical axes of the graphs are given different scales.

As pointed out in 5.1.16, which relation between amplitude and beat distance to choose in the various concrete applications should primarily be decided on the basis of empirical investigations of live performances of rhythm.
5.4.3. More complex algorithms:

In all our examples of rhythmic frequency modulation up to this point, we have used an algorithm as diagrammed in the following flowchart:

This diagram represents the most basic application of RFM, involving only one modulating pulse. It seems rather remarkable that by using this quite simple algorithm, we are able to construct a wide variety of movement curves, creating syntheses of rhythm that make interesting approximations to live rhythmic performances. As is well known from applications of FM to syntheses of sound, the construction of more complex algorithms, involving several modulations together with time-dependent amplitudes and peak frequency deviations, have proved to be successful in making syntheses of many known sounds as well as in creating new sounds not associated with any former known instruments. An obvious idea related to rhythmic FM is therefore to investigate to what extent some of these more complex algorithms, formerly used in sound syntheses, may be transformed and adopted as valuable in making new simulations of rhythmic performances of music. This is indeed a very interesting task, which, however, requires a lot of research and discussion to be properly answered. Some such studies of applications of complex RFM will be the subject of our later investigations. In this present project, we will only look at some new examples of modulations involving two modulating pulses. An illustration of an algorithm we will apply in the next two chapters, in addition to the basic applications of RFM already demonstrated, is given in Figure 5.26 below.
Figure 5.26. Illustration of a more complex algorithm creating "modulations of modulations". The construction of this algorithm obviously carries on to more general algorithms of modulations of modulations (e.g., "modulations of modulations of modulations ... of modulations"). Observe also that we here indicate that the peak frequency deviation, $d$, may be made dependent on time; $d = d(t)$. 
5.5. Summary

The main concern of the present chapter is to develop a model describing characteristic features of rhythmic performance of measured music, where movement activities of the rhythm performer are fundamentally implemented. A basic idea in this respect is to view performed rhythm as a result of the mutual interaction of different movements, and to construct a mathematical model describing rhythmic activity by means of frequency modulated rhythms. The construction of this model is done in a stepwise manner, finding solutions to the following problems:

(A) Present a model of rhythmic structure, where information of note values is represented as continuous movements through attack points.

(B) Construct a model of expressive timing, where performed rhythm is viewed as a result of continuous interactions of movements.

In solving (A), we define pulses, \( p(t) = A[1 - \cos(bt)] \) as mathematical representations of elementary; or, atomic movements, an interpretation of which being a movement curve created by tapping the hand against a table in perfect synchronization with a metronome. Furthermore, subdivisions and ties of \( p(t) \) are defined, reflecting the common understanding of 'subdivisions' and 'ties' in musical terminology. Having made these definitions, a model of metronomic performances of rhythm, MPR, is constructed, consisting of combinations of pulses subject to the operations of making subdivisions and ties. We argue that every sequence of note values may be represented by continuous movement curves using elements of MPR, and a solution of problem (A) is thus obtained.

Motivated by a description of expressive timing as continuous transformations of rhythmic structure (see Clarke, 1999, p.490; and section 2.3.1), we approach a solution of problem (B) by pursuing our following basic idea: Syntheses of live performances of rhythm are constructed as continuous transformations of syntheses of metronomic performances of rhythm. On the basis of this idea we address the fundamental question: Are there any plausible or reasonable ways to define transformations, \( \delta_1, \delta_2, \delta_3, \ldots, \delta_n \), such that these transformations applied to movement curves of MPR create relevant approximations to live performances of rhythm? An affirmative answer to this question is suggested by introducing a new technique of rhythm synthesis; rhythmic frequency modulation, RFM, adopting and
adjusting well known ideas of FM-synthesis as formerly applied to the synthesis of sound (cf. Chowning, 1973).

The notation \( p_f \oplus \omega q_r.\phi \) is used, denoting that \( p_f \) is modulated by \( q_r.\phi \), with peak frequency deviation, \( d \); the formal definition of this expression being:

\[
(p_f \oplus \omega q_r.\phi)(t) = A[1 - \cos(f t + d \sin(f' t + \phi))]
\]

It has been observed that whereas \( p_f \) may be interpreted as a movement curve associated with a metronomic performance of pulse beats, a modulated pulse is naturally interpreted as a movement curve associated with a non-metronomic performance, where the distances between the isolated pulse beats no longer are the same for every pair of successive beats. Hence, a result of RFM is that one movement curve is "affected" by another movement curve, creating various deviations from metronomic regularity. The binary operation of one movement interacting with another movement, as announced in problem (B), is therefore understood in our theoretic constructions as rhythmic frequency modulation of pulses; one pulse modulating another pulse. Subdivisions and ties of modulated pulses are defined as modulated versions of the corresponding definitions in the unmodulated case, and our proposed model of live performances of rhythm, LPR, is defined as:

\[
LPR = MPR \oplus \omega q
\]

In accordance with this definition, the transformations, \( \delta \), mapping syntheses of metronomic performances into syntheses of live performances are given by:

\[
\delta \circ (m) = m \oplus \omega q, \quad m \in MPR.
\]

Throughout this chapter numerous examples showing the effect of different applications of RFM have been presented. Compared with empirical findings in detections of SYVARD ("systematic variations of durations", cf. 2.3), these examples indicate that the technique of rhythmical frequency modulation may be quite useful in making syntheses of live performances of, e.g. Vienna waltzes, various dialects in folk music, and cymbal ostinatos as played by different jazz drummers, and might also be used to create different "out-of-sync" phenomena, demonstrated in empirical studies of PD ("participatory discrepancies", cf. 2.4). All of these examples, along with some additional applications of RFM, will be presented and discussed in further detail in Chapter 7.
Based on our definition of LPR and supported by the various examples indicating that RFM may indeed be applied to construct interesting approximations of live rhythmic performances of music, we claim to have obtained a solution of problem (B). Thereby, we have also constructed a model fulfilling the requirements presented in the introduction of this chapter. A theoretical interpretation offered by our model construction is to describe representations of expressive timing as non-linear, continuous transformations of rhythmic structure. Several important and interesting points related to our approach in modeling live performances of rhythm can now be made:

1. The model LPR represents a shift from a discrete to a continuous representation of note values.
2. Structure of durations is transformed into structure of movements.
3. Structure of attack-point rhythm is transformed into structure of gestural rhythm.
4. Written representations of music are transformed into representations of performance of music.
5. There is a shift from a one-dimensional to a two-dimensional representation.

However, it should be observed that the only durational variable taken into account in the model LPR, up to this point, is the interonset interval, \(D_0\). This is an important point to make, since, as emphasized by Bengtsson & Gabrielsson (e.g. 1983), both \(D_0\) and \(D_a\) are crucial parameters in live musical performances. In Chapter 6 we will try to incorporate \(D_0\) and \(D_a\) into a computer implementation of our model LPR.

In the present chapter LPR has been constructed on a purely theoretical basis. Certainly, the work of Jaques-Dalcroze in developing an educational method of perception and performance of rhythm, fundamentally based on exercises of kinesthetic movements of the body, is a major motivation for our focus on interactions of movements as being basic to rhythm performance. Moreover, the importance of approaching performance of musical rhythm by investigating gestural rhythm rather than attack-point rhythm is strongly motivated by the performing musicians' own experiences. And, not least, the very underlining of the connections and interactions between performance and experience of rhythm and body movements represents in a basic way a view of rhythm that in many respects resembles the Greek correspondence thinking and the idea of 'movement' as being fundamental to rhythmos (cf. Chapter 1). However, in our studies and model developments we have not carried out any empirical research or investigations establishing empirical evidence or motivation prior to our
choice of the pulses, \( p_n \), as "atomic movements", or to the definition of rhythmic frequency modulation giving meaning to the operation "interaction of movements". Given this epistemological basis, it is extremely interesting to note that some such support to our proposed definitions is indeed found in research on human movements. As stated by Viviani (1990), for instance, sinewaves are easy to approximate by human movements, and are among the simplest predictable motions. Thus, our definition of pulses as atomic movements seems quite natural. As to our definition of frequency modulation as a mathematical operation corresponding to interaction of movements, it is interesting to observe that Beek, Peper and van Wieringen (1992), in their modeling of coordinated rhythmic movements such as breathing and walking, cascade juggling, and polyrhythmic tapping conclude that: "Constrained movement involving more than one limb segment often leads to modulation." (ibid. p.604). This conclusion of Beek et al. seems to support our choice of frequency modulation as a mathematical expression related to interaction of movements. Moreover, it should be noted that, at least on a theoretical level, our model LPR seems somewhat related to the Bernstein approach as a theory of motor skill (cf. Gabrielsson, 1999, pp.518-519).
6

From Theoretical Models to Sounding Rhythms

By means of rhythmic frequency modulation we are now able to construct continuous movement curves simulating a large number of non-metronomic performances of rhythm. As underlined in the previous chapter, it should be noted that up to this point our various syntheses of rhythmic performance exist on a purely formal and rather axiomatic level, where mathematical functions and graphic illustrations are interpreted as different representations of rhythmic performances of music. However, many will certainly assert that musical rhythm is perceived on an emotional rather than a rational level, where the experience of (and participation in) a "swinging" rhythm makes one want to clap, tap the feet, or perhaps dance to the music. It would therefore be very interesting if we were able to "transform", or "translate", as it were, our model into the acoustical dimensions where musical rhythm "exists" as a temporal unfolding of sound. In other words, we would like to find an answer to the following question: What do our rhythm syntheses sound like?

In the present chapter we will try to find an answer to this question, by presenting a computer realization of rhythmic frequency modulation, where audible musical information may be manipulated by applying RFM to continuous movement curves. The construction of this computer program is a joint work of Sigurd Saue, who has done the programming part of the construction, and the author. At this point of our presentation we outline the basic features and possibilities offered by the computer program in rather general and non-technical terms. In Appendix II a more technical description of the program is given by Sigurd Saue.\textsuperscript{138}

6.1. From Musical Information to Movement Curves to Sound

Our computer implementation of RFM is carried out using the programming language C++, and an ordinary PC with Windows 98 and a sound card. The basic ideas of the computer implementation are:

\textsuperscript{138} A brief overview of the computer implementation is also given in Wandeland & Saue (1999).
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- A multi-track, polyphonic MIDI recording\(^{139}\) is imported into the computer as a MIDI file (e.g. from any MIDI sequencer), or as a text file.
- The computer calculates RFM and gives a graphic representation of modulated pulses according to the definitions given in the previous chapter.
- The computer "runs through" the movement curves created by means of RFM and in every pulse beat (modulated or not) the computer makes a sound on the basis of the following MIDI parameters: MIDI channel, note number, duration (note on/note off), and velocity.

To get a better understanding of how the computer program is constructed, it might indeed be necessary to make some comments on these basic features of the program, as listed above.

1. First of all; in order to be able to manipulate music by applying a computer, musical information has to be translated into a language the computer "understands"; i.e. the various musical parameters have to be given a digital representation. One such digital musical language is MIDI. MIDI is a well-known specification communicating musical information (MIDI messages) between electronic musical instruments (such as synthesizers, samplers, MIDI-implemented acoustical instruments, and computers). Many textbooks explaining MIDI are available, one of these is, e.g. Braut (1994). For us it is sufficient at present to point out that MIDI communicates on 16 MIDI channels, and that in the computer implementation of RFM we will be using the following MIDI parameters:

   **Note number:** Every key on a synthesizer (or any other MIDI controller) is given a note number. The note number corresponding to middle C (with frequency approximately 261.6 Hz) is 60. Middle C is in the MIDI language also commonly denoted C3. C#3 has note number 61, D3 is 62, C4 is 72, and B2 is 59, ... etc. The MIDI range consists of 128 (= \(2^7\)) notes (i.e. ten and a half octaves from C-2 with note number 0, to G8 with note number 127). Observe that the MIDI note numbers need not address only definite pitches. Which sound is being triggered by a given note number is dependent on the sound module being addressed. Collections of non-tempered pitches, various drum

sounds, or different "pitchless" sounds stored in a sampler, might, for instance, also be activated by the MIDI language.

*Note On message:* The Note On message is transmitted at the instant a key on a MIDI keyboard is pressed, and, more generally, whenever any physical action is applied to a MIDI controller in order to trigger the production of a sound. When this message is received, the sound generator plays the note that has the corresponding note number.

*Note Off message:* The Note Off message is transmitted when a key value is released. When this message is received, the sound generator stops playing the corresponding note.

*Velocity:* In the MIDI specification, velocity indicates the speed with which a note is being pressed (or the speed with which a note is being released). Its values range from 0 to 127. Velocity might affect dynamics and timbral parameters of the sound module being addressed (but should not be confused with the MIDI parameters "volume" and "expression"). The velocity of a Note On message is also known as *attack velocity*, whereas the velocity of a Note Off message is known as *release velocity*. It should also be noted that a Note On message with velocity 0 means the same as a Note Off message.

2. MIDI information can be imported into our RPM computer program in two different ways: as a MIDI file or as a text file. To begin with the latter, consider the following short melody:

![Musical notation](image)

Underneath each note we have written the corresponding MIDI note number. The melody above may be given the following representation as a text file:

```
0 0
67 65 1 1
67 65 1 1
64 65 1 2
65 65 1 2
67 65 2 1
64 65 1 1
0 0 0 0
```

0 0 at the top of the text file indicates that the first note starts on the first beat of the first measure (i.e. no temporal translation), whereas 0 0 0 0 at the bottom of the text file
indicates the end of the melody. The other parameters in the text file are quadruples of the form \((x_1, x_2, x_3, x_4)\), where:

\[
\begin{align*}
  x_1 &= \text{MIDI note number} \\
  x_2 &= \text{MIDI velocity (in the example above every note is given velocity 65)} \\
  x_3 &= n \text{ in theorem 5.3.7 in Chapter 5 (tie)} \\
  x_4 &= m \text{ in theorem 5.3.7 (subdivision)}
\end{align*}
\]

Importing a text file into the RFM program, the following options are offered:\footnote{Observe that in the RFM program the instructions are here given in Norwegian. Moreover, in the following it will become apparent that a mixture of Norwegian and English is applied in this first version of the program. In later versions all instructions will be given in English.}

\begin{itemize}
  \item[a)] Here we specify how long each tone should last. The value of "duration in-out", \(D_{io}\) (which equals the temporal distance between the Note On and the Note Off message of the same MIDI note event), is given as a fraction of the interonset interval, \(D_i\). In the example above \(D_{io} = 0.9D_i\). The possible choices of this fraction given by the computer program are decimal numbers between 0.01 and 1.
  \item[b)] Here we enter the meter. In this example a natural choice may be \(\frac{3}{4}\) meter.
  \item[c)] Tempo is entered as number of beats pr. minute. In our example above, the tempo is 120 quarter note beats pr. minute. (The tempo can be changed later on.)
\end{itemize}

\textit{Construction of movement curves:}
On the basis of the parameter values \(x_3\) and \(x_4\) (i.e. the values determining ties and subdivisions), and subject to the choice of meter, the computer program applies
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combinations of pulses of the form \( p(t) = A[1 - \cos(ft)] \) constructing the following movement curve as a graphic representation of a metronomic performance of our short melody:

![Diagram of movement curve]

The vertical lines represent the beginning of each measure. Observe that the calculations carried out by the computer in the construction of this movement curve are the same as the calculations given in the algorithm for constructing subdivisions and ties in theorem 5.3.7. Note also that the movement curve above is given subject to the amplitude adjustment: \( A = (\Delta)^{0.5} \), where \( \Delta \) denotes the distance between two successive pulse beats.

**Generating sound:**

On the basis of the MIDI parameters \( x_1, x_2 \) and \( a \), and subject to the choice of tempo given in c), the computer transmits MIDI messages in every pulse beat, \( b_n \), of the movement curve illustrated above. When these MIDI messages are received by a MIDI sound module (e.g. a synthesizer, a sampler, or a sound card in the computer), the sound module plays the notes with the corresponding values of note number, velocity and duration. For instance, given the parameters in the example of our short melody above, the sounds produced in the pulse beats \( b_1, b_2, \ldots, b_7 \) are:

<table>
<thead>
<tr>
<th>Pulse beat</th>
<th>Note number</th>
<th>Velocity</th>
<th>( D_{ia} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>67</td>
<td>65</td>
<td>(0.9)x(0.5sec)=0.45sec.(^{141})</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>67</td>
<td>65</td>
<td>0.45 sec.</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>64</td>
<td>65</td>
<td>0.225 sec.</td>
</tr>
<tr>
<td>( b_4 )</td>
<td>65</td>
<td>65</td>
<td>0.225 sec.</td>
</tr>
<tr>
<td>( b_5 )</td>
<td>67</td>
<td>65</td>
<td>0.9 sec.</td>
</tr>
<tr>
<td>( b_6 )</td>
<td>64</td>
<td>65</td>
<td>0.45 sec.</td>
</tr>
<tr>
<td>( b_7 )</td>
<td>0</td>
<td>0</td>
<td>(no sound)</td>
</tr>
</tbody>
</table>

Observe that which sound is actually produced in the different pulse beats is dependent on the sound module being used. It might, for instance, be a piano sound.

---

\(^{141}\) The tempo in this example is 120 quarter note beats per minute. Hence, for every quarter note in this metronomic performance, \( D_{ia} \) is 60 sec. divided by 120, i.e. 0.5 sec. According to our choice of the parameter \( a \) we therefore get: \( D_{ia} = (0.9)D_{ia} = 0.45 \) sec. for every quarter note.
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a xylophone sound, a drum sound, or any other sound stored in a synthesizer or a sampler (including, of course, sounds not associated with any traditional musical instruments).

3. As mentioned at the beginning of comment 2 above, MIDI information can also be imported into our RFM program as a MIDI file. Using the short melody of comment 2 as an example, the procedures for doing this are the following:

(i) The melody is recorded on (any) MIDI sequencer.\(^{142}\)

(ii) The recording is saved as a MIDI file; i.e., as a file containing MIDI information of a "global" (that is, "manufacturer independent") nature, that may be exchanged between any MIDI sequencers. The MIDI file of our short melody contains a MIDI event list, which may include the following MIDI parameters:\(^{143}\)

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---

\(^{142}\) A MIDI sequencer is a device for recording, editing, and playing back MIDI messages. The recording can be done: (a) as a real time recording, in which case the melody is played in "real time" on a MIDI instrument and the sequencer makes a MIDI recording of the performance, or, (b) as a step time recording, where the different MIDI events are entered into the sequencer one by one, without actually making any real time performance of the melody. (See, e.g. Braut, 1994, for further details.)

\(^{143}\) Here we show an event list as displayed in the program Cakewalk Pro Audio.
Observe the similarities between this MIDI event list and the text file exemplified in comment 2. Both these lists contain information of two basically different kinds: One type of information addresses MIDI sound generation (MIDI channel number, note number, velocity, duration (Da)), whereas the other type of information concerns the temporal distribution of the different MIDI events along the axis of time (i.e., various ways of displaying temporal placements of events, parameters of subdivisions and ties, and tempo).

(iii) The MIDI file is imported into the RFM program. The parameter values related to sound generation and temporal distribution, as exemplified above, are kept unchanged by this operation.

Construction of movement curves:
On the basis of information of temporal distribution of MIDI events, the computer calculates frequencies associated with subdivisions and ties of some chosen reference pulse (this reference pulse is often taken as the pulse representing quarter notes). When applying combinations of pulses of the form \( p(t) = A[1 - \cos(\theta)] \), the RFM program creates continuous movement curves representing performances of our short melody, in accordance with the definitions and algorithms given in Chapter 5.

Generating sound:
Given a movement curve and a well-ordered collection of MIDI information, sound is created in the same manner as described in comment 2: In every pulse beat of the movement curve the computer transmits MIDI messages, which, when received by a MIDI sound module, causes the sound module to produce a sound.

4. It should be noted that whereas the description of how MIDI information can be imported into our RFM computer program, as presented above, is related to a monophonic musical performance, it is an extremely valuable feature that the RFM

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144 A more detailed description of these calculations is given by Sigurd Saue in Appendix II.
program also offers possibilities of importing multi-track, polyphonic MIDI recordings, imported as MIDI files. Given such possibilities, we may import a multi-voice MIDI performance (e.g. melody line and accompaniment, or, drum and bass patterns as used in jazz rhythm sections for instance) and apply different rhythmic frequency modulations to the different voices, thus simulating various occurrences of non-synchronization that are characteristic of live performances of music. Some such examples of rhythm syntheses will be demonstrated in Chapter 7.

So far we have not explained how rhythmic modulation is implemented in our RFM computer program. This we do in the next section, where a basic "RFM-instrument" is presented.

6.2. A Basic RFM-Instrument

The flowchart for basic RFM is presented in Figure 5.17 of Chapter 5. This algorithm is implemented into the computer program in a somewhat more general form, as illustrated below:

![Flowchart](image)

**Figure 6.1. A basic RFM-instrument as designed in a PC using C++.**
The parameters for the RFM-instrument of Fig. 6.1. are:

P1: Carrier frequency, displays number of (reference) beats to the measure. Permits only natural numbers.
P2: Modulating waveform. The program permits a choice between \textit{sine}, \textit{triangle}, \textit{sawtooth}, and \textit{square}. So far, we have used only \textit{sine}.
P3: Modulating frequency, denoted \( f' \) in Fig. 5.17. Permits decimal numbers.
P4: Phase of modulating waveform divided by \( 2\pi \). Permits decimals. If we enter \( P4 = x \), then \( 2\pi x = \phi \), where \( \phi \) is the phase as given in Fig. 5.17.
P5: Exponent of modulating waveform. Permits only natural numbers. Makes it possible to modulate by, e.g. \( \sin^n \), \( n = 1, 2, 3, \ldots \)
P6: Peak frequency deviation, denoted \( d \) in Fig. 5.17. Permits decimal numbers.

In this RFM-instrument, as in every RFM-instrument of the present computer program, the carrier waveform is a pulse of the form \( p(t) = A[1 - \cos(\theta)] \). This is in accordance with the choice of \( p(t) \) as "building blocks" in the construction of the model MPR. If the parameter values given in Fig. 6.1. are applied in a modulation of our short melody exemplified in comment 2 of the previous section, the computer creates the following modulated movement curve:

![Diagram](image)

\( \Delta_1 = 0.42T \quad \Delta_2 = 0.26T \quad \Delta_1 + \Delta_2 \quad \Delta_3 \quad T \)

Observe that the parameter values given in Fig. 6.1. are identical to the parameter values used in example 5.2.4 (c), where some different effects of modulating the pulse \( p(t) = A[1 - \cos(\theta)] \) are illustrated. As commented in example 5.2.4 (c), one result of this specific modulation is that the modulated first beat of every measure is a little early compared to a strict metronomic (unmodulated) performance. (The vertical lines in the graphic illustration above represent the temporal placement of the first beats in the first three measures of a metronomic performance.) Another consequence of this modulation is that the distances
between the modulated pulse beats of $p_3$ ($\Lambda_1, \Lambda_2, \Lambda_3$; i.e. the $D_i$'s of the modulated quarter notes) make the pattern L(long)-I(intermediate)-S(short), as illustrated above. The measure length (the period), $T$, of the modulated performance is, in this case, identical to the measure length of the unmodulated performance. Observe also that in the movement curve above, the amplitude adjustment $A = (\Delta)^{0.5}$ has been made, where $\Delta$ (as before) denotes the distance between two successive pulse beats.

By means of the computer program's ability of transmitting MIDI messages in every pulse beat of modulated movement curves, we are now in a position to create **sounding, audible syntheses of non-metronomic performances of rhythm** - that is to say, we are offered the opportunity to **hear what our RFM synthesizes sound like**. Several examples of such syntheses will be presented in Chapter 7, and audible illustrations of how these syntheses sound are included on the audio CD attached to this book.

### 6.3. Modulated Polyphonic Performances

As mentioned in section 6.1, it is a valuable feature of the RFM computer implementation that the program offers possibilities of importing multi-track, polyphonic MIDI recordings, imported as MIDI files. The different voices of such polyphonic recordings may be manipulated subject to **different values of RFM parameters**, thereby creating various occurrences of **non-synchronization** between the voices. We exemplify this by making a metronomic sequencer recording of three identical voices of our short melody of section 6.1, and import this recording as a MIDI file into the RFM program. These three voices; voice 1, voice 2, and voice 3, are modulated by applying the following different collections of parameter values:

**Voice 1:**

- P1 = 3 (our short melody is regarded as a melody in ¾ meter)
- P6 (= $d$ (peak deviation)) = 0; that is: **no modulation**; i.e. voice 1 represents a metronomic performance. A graphic illustration of a corresponding movement curve is:
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Voice 2:
P1 = 3  P2 (modulating waveform) = sine  P3 (mod. frequency) = 1
P4 (phase/2\pi) = 0.25  P5 (exponent of mod. wave) = 1  P6 = 1
A corresponding movement curve is:

Voice 3:
P1 = 3  P2 = sine  P3 = 1
P4 = 0.75  P5 = 1  P6 = 1
A corresponding movement curve is:

All three movement curves above are given subject to the amplitude adjustment: A = (Δ)^0.5
(Δ = distance between successive pulse beats). Observe, moreover, that whereas voice 1 is
unmodulated, voice 2 and voice 3 are modulated by applying identical parameter values,
except for the parameter P4, determining the phase of the modulating pulse. In the modulation
of voice 2, P4 = 0.25, which gives the phase $\phi_2 = \pi/2$. In the modulation of voice 3, on the
other hand, P4 = 0.75, resulting in the phase $\phi_3 = 3\pi/2$. These values of modulation
parameters chosen in voice 2 and voice 3 are the same as the values illustrated in example
5.24 (e). As commented in example 5.24 (e), the distances between the quarter note beats of
voice 2 make the pattern: L(long)-I(intermediate)-S(short), whereas the distances between the
quarter note beats of voice 3 make the reversed pattern: S-I-L. When displaying the three
movement curves of voice 1, 2, and 3 in the same graphic illustration, the various occurrences
of non-synchronization between the three corresponding performances become apparent:

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Figure 6.2. Illustration of movement curves of three identical voices of a polyphonic performance where non-synchronization is created by applying different parameter values of RFM to the different voices. Voice 1 (black): unmodulated (i.e. metronomic), voice 2 (blue): modulated, creating the pattern L-I-S of quarter note beats, voice 3 (red): modulated, creating the pattern S-I-L of quarter note beats.

When different MIDI channels are assigned to the different voices of a polyphonic MIDI recording, the different voices may address different sounds of a MIDI sound module. An illustration of how this is done in the computer implementation of RFM is given in the figure below:

![MIDI Channel Assignment Table]

Figure 6.3. An example showing how the different voices of a polyphonic MIDI recording may address different sounds of a MIDI sound device.

In this example, voice 1 is assigned MIDI channel 1, addressing the sound "grand piano" ("flygel"), voice 2 is assigned MIDI channel 2, addressing "marimba", whereas voice 3 is assigned MIDI channel 3, addressing the sound "guitar with nylon strings".

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Since the MIDI language communicates on 16 MIDI channels, 16 different voices of a polyphonic MIDI recording may address 16 different sounds of a MIDI sound device, as suggested in Figure 6.3. The sounds available are any sounds stored in a MIDI sound module (e.g. any digitally sampled sound, or any sound of a synthesizer). Several sounding examples of rhythmic frequency modulation will be presented and discussed in Chapter 7, and are included as examples that may be listened to on the CD.

6.4. Some More Complex RFM-instruments

In section 5.4.3 some possible extensions of RFM synthesis to more complex algorithms were suggested. Some such extensions are included in the computer implementation of RFM. These are:

(i) Serial rhythmic modulation
(ii) Parallel rhythmic modulation

A flowchart for serial rhythmic modulation, as implemented in the RFM program, is the following:

![Flowchart for serial rhythmic modulation](image)

Figure 6.4. An example of a flowchart for serial rhythmic frequency modulation.

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Compare Fig. 6.4 to Fig. 5.26 and observe that the RFM-instrument illustrated in Fig. 6.4, creates "modulations of modulations". Using general notation for frequencies (i.e. \( f, f', f'' \)), phases \( \phi, \phi_2, \phi_3 \), etc. instead of the specific choice of parameter values given in Fig. 6.4, and applying sine as modulating waveform at each level of modulation, the output of serial modulation, as illustrated above, is:

\[
(*) \quad r(t) = A[1 - \cos(fn + d_1\sin^m(f't + \phi_1 + d_2\sin^p(f''t + \phi_2))] \]

As shown in Fig. 6.4, we have 11 parameters to manipulate in this case of modulation. Some examples of applications of serial rhythmic modulation in approximating live performances of rhythm will be given in Chapter 7, and audible examples are included on the CD.

A flowchart for parallel rhythmic modulation, as constructed in the RFM program, is the following:

![Flowchart](image)

**Figure 6.5.** An example of a flowchart for parallel rhythmic frequency modulation.

Applying sine as modulating waveforms, the output of parallel modulation, using general notation, is:

\[
(**) \quad r(t) = A[1 - \cos(fn + d_1\sin^m(f't + \phi_1) + d_2\sin^p(f''t + \phi_2))] \]

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6.5. Some Remarks on Kinesthetics and Durational Variables

As pointed out in 5.1.16 and 5.4.2, we recall that if the movement curves of our model LPR are to reflect kinesthetic performances of rhythm, as discussed by Emile Jaques-Dalcroze (1967), Abramson (1986), and Graybill (1990) (cf. Chapter 4), the amplitudes of the pulses (whether modulated or not) should be functions of the distance between successive beats, $\Delta$, and dynamics, $dym$; $A = A(\Delta, dym)$. Examples of making $A$ a function of $\Delta$ have been given above. In all these examples $A = (\Delta)^a$, where $a$ is a non-negative rational number. Such an option of making $A = (\Delta)^a$ is also offered in the computer implementation of RFM.

Dynamics are not taken into account in the construction of the model LPR in Chapter 5. When musical information is imported into the RFM program either as a text file or as a MIDI file, dynamical features of the performance are reflected in the MIDI message velocity.\footnote{As pointed out in 6.1, velocity indicates the speed with which a note on a MIDI controller is pressed (or the speed with which a note is released). Even though velocity should not be confused with the MIDI parameters "volume" and "expression", velocity may affect both dynamics and timbral parameters of the sound module addressed.} Suggesting a possible way of including considerations of dynamics in our computer implementation of RFM, the program offers possibilities of making $A$ a function of velocity, $vel$, in a similar way as $A$ may be made a function of $\Delta$, namely; $A = (vel)^b$, where $b$ is a non-negative rational number. A such relation between $A$ and $vel$ seems at least plausible on the basis of simple observations of live performances of rhythm, since a performance of, e.g. a loud hand clap (which in our MIDI implementation will be represented by "large" velocity value) most often requires an execution where the hands are brought further apart (i.e. "large" amplitude, $A$), than in a performance of a softer clap (represented by "smaller" velocity value). In the computer program, these possibilities of making $A$ a function of $\Delta$ and $vel$ is expressed in the following combined form:

\begin{equation}
(*) \quad A = (\Delta)^a(vel)^b; \text{ where } a \text{ and } b \text{ are non-negative rational numbers.}
\end{equation}

At this point, it should be strongly emphasized that which values of $a$ and $b$ one should choose in the different applications and interpretations of our model, and even more so: whether the relation $(*)$ is at all a relevant expression of relations between $A$, $\Delta$, and $vel$, should be decided on the basis of empirical investigations of live performances of rhythm. However, at this stage of our research, our intention is just to suggest some possible ways of
implementing kinesthetic considerations into our model LPR. Empirical studies of rhythmic performances will show to what extent these suggested adjustments of our constructed movement curves are able to capture and simulate characteristic features of “real” movements, as found in live performances of rhythm.

As underlined several times in Chapter 5, our model LPR and the technique of rhythmic frequency modulation is fundamentally based on representing rhythmic structure and rhythmic performances as continuous movement curves through attack points. Making such representations of rhythm, our theoretic model takes only $D_{ii}$ (the interonset interval) into account when constructing syntheses of live performances of rhythm. As demonstrated in the present chapter, however, the durational variable $D_{io}$ (and thereby also $D_{oi} = D_{ii} - D_{io}$) is indeed incorporated into our computer implementation of RFM. $D_{io}$ is represented as the temporal distance between Note On and Note Off messages of a MIDI note event, and may be entered into the computer program when importing musical information in any of the two possible ways; as a text file or as a MIDI file. Thus, the RFM computer program enables us to make sounding simulations of live rhythmic performances where the different durational variables, as defined by Bengtsson & Gabrielsson (1983), are all considered. Several such examples will be given in the next chapter, and are included on the CD.

6.6. Summary

The basic question raised in this chapter is: What do our rhythm syntheses created by means of RFM sound like? An answer to this question is given by presenting a computer realization of rhythmic frequency modulation, where audible musical information may be manipulated by applying RFM to continuous movement curves. The construction of this computer program is the joint work of Sigurd Saue, who has done the programming part of the construction, and the author of this book. In this chapter basic features of the RFM computer program are outlined in rather general and non-technical terms. In Appendix II a more technical description of the program is given by Sigurd Saue.

Our computer implementation of RFM is carried out using the programming language C++, and an ordinary PC with Windows 98 and a sound card. The basic ideas of the computer
implementation are: (i) Musical information (represented according to MIDI) is imported into the computer program, (ii) the computer calculates RFM and constructs graphic representations of modulated pulses, and (iii) MIDI information is transmitted in every pulse beat, causing any MIDI sound module (e.g. a synthesizer or a sampler) to make a sound on the basis of the MIDI parameters: MIDI channel, note number, note on/note off, and velocity.

MIDI information may be imported into the computer program as a text file or as a MIDI file, the latter created from a multi-track, polyphonic MIDI recording made on any MIDI sequencer. We give a brief description of how this works in each case. A basic RFM-instrument is presented, being the computer implementation of the RFM algorithm defined in Chapter 5, and the various parameters for the RFM-instrument are listed. Moreover, we show how the different voices of a polyphonic performance may be manipulated subject to different values of RFM parameters, thereby creating various occurrences of non-synchronization between the voices. Furthermore, we present two examples of more complex RFM-instruments that are implemented in the computer program: (a) Serial rhythmic modulation, and (b) parallel rhythmic modulation, and the flowcharts for these two algorithms are briefly illustrated.

In an attempt at making the movement curves of the model LPR reflect kinesthetic features of rhythmic performance, we suggest a way of making the pulse amplitudes, $A$, functions of beat distance and velocity; $A = A(\Delta, \text{vel})$. It should be noted that even though our suggested relation between $A$, $\Delta$, and $\text{vel}$ seems plausible on the basis of simple observations of rhythmic performances, additional empirical investigations of live performances of rhythm are necessary in order to determine the relevance of the proposed relation in different applications and interpretations of RFM.

Whereas $D_{ni}$ is the only durational variable taken into account in the construction of LPR, as presented in Chapter 5, the computer implementation of RFM incorporates $D_{ni}$ as well (and thereby also $D_{ni} = D_{ni} - D_{no}$). Hence, all durational variables defined by Bengtsson & Gabrielsson (1983) may be considered when making sounding syntheses of live rhythmic performances by means of the RFM computer program.

In the next chapter several audible examples of applications of rhythmic frequency modulation are presented.
7

Syntheses of Expressive Timing
by Means of Rhythmic Frequency Modulation

Applying the technique of rhythmic frequency modulation, as developed theoretically in
Chapter 5, we are able to make simulations of various live performances of music by means
of continuous movement curves "interacting" with each other. Moreover, with the computer
implementation of RFM, presented in Chapter 6, we are offered the possibility of
"translating" our theoretic simulations of rhythmic performances into temporal unfoldings of
sound, thereby making new audible syntheses that approximate live performances of musical
rhythm.

In this chapter we present several applications of rhythmic frequency modulation used
as a technique of making syntheses of expressive timing, characteristic of various live
performances of rhythm. In choosing our examples, we try to show how RFM may be useful
in simulating features of live rhythmic performances as documented and discussed in
empirical rhythm research through the detection of SYVARD and PD (cf. the presentation of
this research in Chapter 2). We also present simulations of rhythmic accelerandi and
ritardandi, and give an example showing how "phasing", applied as a compositional technique
by Steve Reich, for instance, may be created using RFM synthesis.

In the construction of our various syntheses it is interesting to investigate (at least) two
different approaches, searching for answers to the following questions:

(i) How "close to reality" can we get, applying our theory to live performances of
    rhythm?

(ii) How "far from reality" are we able to come; or, to put it another way, "how bad can
    things get" within the limits of our theory?

The aim of the first approach is to make syntheses approximating live rhythmic performances
as studied by empirical investigations. The second approach, on the other hand, investigates
the potential of the theory in making syntheses of "pathological" rhythmic performances, and
might, on the basis of representing various "non-real" unfoldings of rhythm, also generate new ideas of applying RFM synthesis in descriptions of "real" performances of rhythm. Whereas most of our examples given in the present chapter address question (i), we will also give some examples that may shed some light on question (ii) above.

By means of the computer realization of RFM, a recording is made of every example presented in this chapter. These recordings may be listened to on the attached CD. An overview of its contents and a description of how the recordings are made are given in Appendix III.

Since our examples and discussions will refer to the parameters of the computer implementation of RFM, we recall the flowchart for a basic RFM-instrument, previously given in Figure 6.1:

![Flowchart and parameters for a basic RFM-instrument.](image)

Figure 7.1. Flowchart and parameters for a basic RFM-instrument. (This figure is identical with Figure 6.1.)

From section 6.2 we recall that the parameters for the RFM-instrument of Fig. 7.1 are:

- **P1**: Carrier frequency, displays number of (reference) beats to the measure.
- **P2**: Modulating waveform. The program permits a choice between sine, triangle, sawtooth, and square.
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P3: Modulating frequency, denoted $f'$ in Fig. 5.17. Permits decimal numbers.
P4: Phase of modulating waveform divided by $2\pi$. Permits decimals. If we enter
   $P4 = x$, then $2\pi x = \phi$, where $\phi$ is the phase as given in Fig. 5.17.
P5: Exponent of modulating waveform. Permits only natural numbers. Makes it
   possible to modulate by, e.g., $\sin^n, n = 1, 2, 3,\ldots$
P6: Peak frequency deviation, denoted $d$ in Fig. 5.17. Permits decimal numbers.

7.1. Syntheses of SYVARD and PD

We start off by illustrating how rhythmic frequency modulation may be used in making
syntheses approximating live performances characterized by SYVARD (systematic variations
of durations), as investigated and discussed in the large amount of rhythm research carried out
in Uppsala headed by Bengtsson and Gabrielsson (see, e.g. Bengtsson, Gabrielsson &
Thorsen, 1969; Bengtsson, 1974; Bengtsson & Gabrielsson, 1977, 1983), as well as studied

7.1.1. "Asymmetries" in rhythmic performances in $\frac{3}{4}$ meter:

Empirical rhythm research has documented that in performances of music in $\frac{3}{4}$ meter,
various patterns of beat durations; e.g. S-L-I, or L-I-S, are characteristic of musical style (for
instance, characterizing performances of Vienna waltzes\textsuperscript{146}, as well as typical of musical
dialects (e.g. in Norwegian folk music\textsuperscript{147}). In example 5.2.4 of Chapter 5 we demonstrated
how different patterns of beat durations may be constructed by applying RFM, thus
suggesting that RFM may be useful in making syntheses of different occurrences of non-equal
beat durations, sometimes called "asymmetries" (cf. Kvifte, 1995), in rhythmic performances
in $\frac{3}{4}$ meter.

(a) Vienna waltz accompaniment:

As noted by Bengtsson & Gabrielsson (1983), a well-known feature of performances of
Vienna waltzes occur at the beat level in the accompaniment; the first beat is shortened and
the second beat is lengthened, whereas the third beat is close to one third of the measure

\textsuperscript{146} Cf. e.g. Bengtsson & Gabrielsson (1983).
\textsuperscript{147} See Kvifte (1995).
length. These deviations from metronomic regularity may, indeed, vary throughout the performance of one singular waltz, and may also change in proportional values in different performances. However, as pointed out by Bengtsson & Gabrielson (ibid.p.42), a typical distribution of beat durations in a Vienna waltz accompaniment may be: first beat: 25-27%, second beat: 40-42%, third beat: close to 33% of the total measure. But, as commented: "Still more extreme deviations may occur, ..." (ibid.p.42). Such patterns of beat durations may now be simulated by continuous movement curves, applying RFM. We first choose the following parameter values:

\[
P1 = 3 \quad P2 = \text{sine} \quad P3 = 1
\]
\[
P4 = 0.25; \; \text{i.e.,} \; \phi = \pi/2 \quad P5 = 1 \quad P6 = 1
\]

These parameter values make an RFM-instrument which creates a modulation identical to the situation discussed in example 5.2.4(c) of Chapter 5. Thus, we obtain a movement curve interpretable as a performance of quarter note beats in \(\frac{3}{4}\) meter, making a cyclic pattern: L-I-S of beat distances, where \(L = 42\%\), \(I = 32\%\), \(S = 26\%\) of the total measure:

![Figure 7.2. Illustration of movement curve with beat distances making the pattern L-I-S.](image)

Hence, if a Vienna waltz accompaniment starts on the third beat of this movement curve, the following cyclic permutation of L-I-S occurs: S-L-I, where, as before, \(S = 26\%\), \(L = 42\%\), \(I = 32\%\). That is to say: a continuous simulation of characteristics of a Vienna waltz accompaniment is created (compare the findings of Bengtsson & Gabrielson cited above). Observe that if \(P6\) (the peak deviation) is allowed to vary between \(P6 = 0\) (no modulation, i.e. metronomic performance) and the maximum value \(P6 = 3\) (given by the condition \(d \leq \frac{1}{10}\) in the algorithm (2) in Theorem 5.3.7), the values of S-L-I are changed between the limits:

\[
S = 33\%, \; L = 33\%, \; I = 33\% \quad \rightarrow \quad S = 17\%, \; L = 61\%, \; I = 22\%
\]
It should be noted that the values of the beat durations above, refer to $D_0$ ("duration in-in"), i.e. the duration from the onset of a tone to the onset of the following tone. As strongly emphasized by Bengtsson & Gabrielsson (ibid.), the values of $D_0$ ("duration in-out"), i.e. the duration from the onset of a tone to the end of the same tone are also of crucial importance in rhythmic performance of music. In the computer implementation of RFM, $D_0$ corresponds to the distance between the MIDI messages: Note on - Note off. To make a simulation of a Vienna waltz accompaniment sound right, $D_0$ should be "long" for the first beat and "short" for the second and third beat of every measure (cf. Bengtsson & Gabrielsson, ibid.).

*Synthesis of Vienna waltz accompaniment*: 148

We first make a MIDI recording of a performance of the following accompaniment:

The performance is made by the author and is carried out in such a way that $D_0$ is "long" for the first beat, and "short" for the second and third beat of every measure. It should be noted that "long" and "short", as applied to characteristics of $D_0$, are not given subject to any *quantitative measurements* in this example. The differences in performances of $D_0$ are, however, made in a *conscious* way by the author's performance, reflecting a (general) knowledge of characteristic features of Vienna waltz accompaniments. The note onsets of this performance are *quantized* in the sequencer to quarter notes. In other words: every onset

148 This synthesis, with corresponding sound examples, was first presented at the conference: DIDEROT FORUM on Mathematics and Music, Computational and Mathematical Methods in Music, Vienna, December 2-4, 1999; see Waechtland (1999). This example should also be compared to the simulations of Vienna waltz accompaniment given by Bengtsson & Gabrielsson (1983).
(attack point) occurs in exact synchronization with metronomic quarter note beats. The D₀’s, however, are not quantized (i.e. the variations in performance of "long" and "short" beats are kept unchanged).

The performance of the accompaniment, quantized as described above, will be denoted V₀ (Vienna₀), and we call this performance a **metronomic performance** (with reference to the metronomic performance of note onsets). To summarize, the characteristics of V₀ are:

- Note onsets are quantized in such a way that every attack point occurs in synchronization with metronomic quarter note beats. Hence, the interonset interval, D₀, in the performance of every quarter note is constant throughout this performance.

- D₀’s vary in accordance with the performer’s (in this case; the author’s) general knowledge of characteristics of Vienna waltz accompaniment (the D₀’s make the cyclic pattern: "long" - "short" - "short").

This metronomic performance, V₀, may be heard as **track 1** of the CD (\( \square = 160 \)).

V₀ is imported as a MIDI file into the RFM computer program, and different rhythmic frequency modulations of V₀ are constructed, creating various simulations of Vienna waltz accompaniments.

**First simulation:**

An RFM-instrument with the following parameters is used:
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An associated movement curve related to a performance of quarter notes modulated according to these choices of parameters is given in Figure 7.2. We now make the waltz start on the third beat of this movement curve. The result is as illustrated below:

Figure 7.3. Illustration of a movement curve associated with a modulated performance of the first two measures of the accompaniment. Observe that an amplitude adjustment has been made \( A = (\Delta)^{85} \).

As documented by Bengtsson & Gabrielson in their investigations and discussions of SYVARD (e.g. 1983), this RFM synthesis approximates typical rhythmic features of Vienna waltz accompaniment. The modulated performance constructed in this example will be denoted \( V_f \).

\( V_f \) is recorded as track 2 on the CD.

Second simulation:

Another modulated performance, \( V_2 \), approximating live performances of Vienna waltz accompaniment is created by applying the same parameters as in the construction of \( V_f \), except for peak frequency deviation, where this time we use \( P6 = 0.8 \). With a smaller value of peak frequency deviation than in the first simulation, the effect of the modulation is somewhat less than in the previous example. As in the example above, we again obtain beat distances making the pattern S-L-L-I, but in this case: \( S = 0.28T, \ L = 0.40T, \ I = 0.32T \), where \( T \) is the length of one period (i.e. here: the length of one measure). Thus, the difference between \( S \) and
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L is a little smaller than in the previous example, whereas I is still "close to" 1/3 of the measure length. According to the findings of Bengtsson & Gabrielsson (ibid.) \( V_2 \) exhibits typical features of Vienna waltz accompaniment.

\( V_2 \) may be heard as *track 3* on the CD.

*Third simulation:*

Whereas the modulated performances \( V_1 \) and \( V_2 \) both represent relevant approximations to Vienna waltz accompaniments, we now present a rhythm synthesis where the effect of modulation yields a modulated performance "stretched" and "compressed" beyond the limits of what is likely to be perceived as a Vienna waltz accompaniment. We do so by applying the same parameters for modulating waveform, modulating frequency, phase and exponent of modulating waveform as in the previous examples, but this time we modulate with *maximum strength*, using peak frequency deviation \( d (=-P6) = 3 \) (cf. the condition \( d \leq \frac{1}{2}f \) in theorem 5.3.7). The result of this modulation, denoted \( V_3 \), is illustrated below:

![Diagram of modulated waveform](image)

*Figure 7.4. Example of a movement curve associated with a modulated performance "stretched" and "compressed" beyond the natural limits of a Vienna waltz accompaniment.*

Observe that on the basis of the beat distances of \( V_3 \), the attack points of this modulated performance are performed "close to" a "slight" modulation of the rhythm pattern:
However, on *listening* to the recording of $V_3$ (*track A*), it does *not* seem reasonable that this performance will be perceived as a performance in $5/8$. A more likely way to perceive it is to regard $V_3$ as a "non-exact" performance of the following rhythm pattern:

These questions of different ways of perceiving $V_3$ (as well as other performances of rhythm), are a matter of judgements based on classification according to different rhythmic perceptual categories (cf. Clarke, 1999, p.490), and are, as such, dependent on as well individual, as cultural and stylistic background and preferences. It should be strongly emphasized that the suggested perceptual classifications above are based on my own subjective judgements of the recorded version of $V_3$, made on the basis of my experience as a drummer performing jazz, rock and folk music.

*Fourth simulation:*

In our various rhythmic modulations of Vienna waltz accompaniment presented so far, we have used a basic RFM-instrument consisting of *one* carrier waveform modulated by *one* modulating waveform. Moreover, the parameters for the RFM-instrument are kept unchanged throughout the entire synthesized performance. This creates, as we have seen, different syntheses of non-metronomic performances where the beat distances make various *periodic* patterns (e.g. S-L-I, as in Vienna waltz simulations). These periodic, cyclic patterns are repeated *unchanged* throughout the entire synthesis. If we were able to apply *different* values of the RFM parameters throughout the simulation of *one* single performance, e.g. by making the peak deviation, $\delta$, a function of time, the relations between beat differences could be changed throughout the performance, and additional features of expressive rhythmic performances, as found in, for instance, live performances of Vienna waltz accompaniments, might be demonstrated. These ideas represent a very natural extension of our model *LPR* of live performances of rhythm, and will certainly be investigated and developed in our future work on this subject. Such extensions will also, eventually, be implemented in the computer.
realization of RFM. What we are able to do, at this point, is to make the relations between beat distances vary throughout the simulation of one single performance by applying a more complex RFM algorithm, namely serial rhythmic modulation, involving modulation of modulation. To illustrate the effect of such algorithms, we consider the following RFM-instrument:

![Diagram of a RFM algorithm](image)

The result of applying this modulation to the Vienna waltz accompaniment is denoted $V_4$. $V_4$ is recorded as track 5. It should be noted that the choice of the parameters in the algorithm above are made by the author in an attempt at providing a sounding example of a piano accompaniment, where the fluctuations in beat distances throughout the performance are easily heard. $V_4$ is not a good simulation of a Vienna waltz accompaniment. Listening to the recording of $V_4$, we notice that the performance starts off with beat distances making the pattern S-L-1 (in some varying ways), whereas from measure 5 the first beat is longer than the third, and something like L-L-S followed by L-S-S is performed. The graphic illustration in the following figure shows a movement curve of measure 5 and 6 of $V_4$.
(b) Different dialects of "springar" as performed in Norwegian folk music:

Another example of "asymmetries" in rhythmic performances in ¾ meter is found in Norwegian folk music. In performing the dance "springar", which, if written, is commonly notated in ¾ meter, different musical dialects show different practices of performance. In Telemark the "springar" is performed according to a pattern L-I-S of beat distances, whereas in Valdres the pattern S-L-I is typical. In both these practices we find that the values L = 7/18, I = 6/18 = 1/3, S = 5/18 of the total measure, give a good approximation to the live performances. The performances of various dialects of "springar", thus show striking similarities to various performances of Vienna waltz accompaniments. We will now demonstrate how these different dialects of "springar" may be simulated by means of rhythmic frequency modulation. As an example of "springar", we choose the Norwegian folk melody: "Kjerringa med staven". This is a well-known song in Norway. A score of this melody is as follows:

\[ \text{Score of "Kjerringa med staven"} \]

---

\[ \text{Cf. Kvifte (1995).} \]
A metronomic (quantized) MIDI recording of this melody has been made and imported as a MIDI file into the RFM program. A graphic representation of an unmodulated movement curve associated with this metronomic performance of the melody is given below:

This metronomic performance of the melody of "Kjerringa med staven" is denoted $K_0$ ($Kjerringa$), and is recorded as track 6 ($\downarrow = 148$).
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First simulation (Telemark):

A simulation of a performance according to the Telemark dialect, with pattern L-I-S of beat distances, is created by the following RFM-instrument:

![RFM-instrument diagram]

Observe that the only difference between these parameters and the parameters used in the first, second and third Vienna waltz simulations is the parameter choice made for peak deviation, which, in the present case, is $d = 0.7$. The modulated performance constructed by means of the RFM-instrument above will be denoted $K_4$, and is recorded as track 7. A corresponding movement curve is:

![Movement curve diagram]

L = 0.39T   I = 0.33T   S = 0.28T   (T: measure length)
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This is close to the relation 7:6:5 of beat distances, being typical of the Telemark performance (cf. Kvifte, 1995).

Second simulation (polyphonic Telemark):

To illustrate how rhythmic modulation might effect a polyphonic, multitrack recording, we make a MIDI recording of the following simple 3 voice arrangement of "Kjerringa med staven":

Kjerringa med staven (3 voice arrangement)

A metronomic version (quantization of attack points (note onsets)) of this arrangement is imported as a MIDI file into the RFM program. When applying the same parameters as in the first simulation, $K_1$, to each of the three voices, we obtain a simulation, $K_3$, of a multi-voice
performance of "springar" in the style of the Telemark tradition. Observe that since the three voices of this example are given the same modulation, the voices are in synchronization with each other, all making the pattern L-I-S of beat distances, as described in first simulation above.

K₂ may be heard as track 8. The three voices are assigned to different MIDI channels, addressing different sounds on a GM (General MIDI) sound device:

Voice 1: "melody": Guitar with nylon strings
Voice 2: "afterbeat": Muffled guitar
Voice 3: "bass": Acoustic bass

Third simulation (polyphonic Valdres):

As mentioned above, the pattern S-L-I of beat distances is typical of the performance of "springar" in Valdres. To simulate the Valdres dialect, we make the metronomic, polyphonic version of "Kjerringa med staven" start on the third beat of an unmodulated movement curve, and then apply the same parameters of modulation as in second simulation, K₂. In this way the pattern S-L-I is constructed as a permutation of L-I-S (cf. the way Vienna waltz accompaniments were simulated).

This modulated performance is denoted K₂, and is recorded as track 9.

Fourth simulation (non-synchronization between the voices, Telemark):

In the second and third simulation above, the three voices were modulated subject to the same values of parameters. This creates synchronization between the voices, even though each voice deviates from metronomic regularity. In live performances involving multiple voices and different musicians, perfect synchronization between the voices seldom, if ever, occurs. When applying different parameters of modulation to the different voices of a polyphonic performance, various such occurrences of non-synchronization, by Keil (1987, 1995) called participatory discrepancies (PDs), may be simulated by means of RFM. To give an example of how this might work, we now use different parameters creating different rhythmic modulations of the three voices of "Kjerringa med staven":

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Let us imagine three musicians playing the prescribed arrangement of "Kjerringa med staven".

1. The musician playing the melody is familiar with the Telemark tradition of playing, and is performing the melody close to a 7:6:5 ratio between the distances of the quarter note beats. *This is simulated by applying the parameters of K₁ to the melody.*

2. The musician playing the afterbeat (voice 2) knows nothing about the Telemark way of playing, and is trying to play "straight" 2's and 3's as metronomically "correct" as possible. *This is simulated by applying no modulation to voice 2.*

3. The bass player has minor experience with playing "springar", but has been told that according to the Telemark tradition the first beat of every measure is supposed to be "long" and the third beat "short". In an attempt at performing according to this knowledge, the bass player exaggerates, making the first beat too long, while the third beat is performed too short. *A simulation of this is created by applying the parameters of K₁, except that we now strengthen the modulation, choosing d = 3.*

The modulated performance simulating the musical meeting of these three musicians is denoted $K_d$, and may be heard as *track 10*.

**Fifth simulation:**

As a final example of "springar" synthesis, illustrating how various PDS in synthesized performances of "Kjerringa med staven" may be created, we now make the non-synchronization more extreme by applying different modulations to the different voices in such a way that the distances between quarter note beats of voice 1 make the pattern L-I-S, the distances between quarter note beats of voice 3 make the pattern S-I-L (the "reverse" of voice 1), whereas voice 2 is unmodulated. Explicitly, voice 1 and voice 3 are modulated according to the following parameters:

*Voice 1:*

\[
P_1 = 3 \quad P_2 = \text{sinc} \quad P_3 = 1 \quad P_4 = 0.25 \quad P_5 = 1 \quad P_6 = 1.5
\]

*Voice 3:*

\[
P_1 = 3 \quad P_2 = \text{sinc} \quad P_3 = 1 \quad P_4 = 0.25 \quad P_5 = 1 \quad P_6 = -1.5
\]

Observe that the only difference in modulation parameters is the parameter choice of $P_6$ (peak deviation). A graphic illustration of movement curves associated with this simulation is given.
below. Voice 1 (melody) is represented by the black curve, voice 2 (afterbeat) is illustrated by the red curve, and voice 3 (bass) is represented by the blue curve.

Figure 7.6. Movement curves associated with a 3 voice performance of "Kjerringa med staven", illustrating various occurrences of non-synchronization between the voices.

This synthesized performance is denoted $K_s$, and is recorded as track 11. Listening to $K_s$ and looking at the movement curves in Figure 7.6, we notice that "the bass player" is performing the first beat of every measure too late compared to both the melody as well as metronomic regularity (metronomic first beat is indicated by the vertical lines in the figure above), whereas the third beat is performed too early compared to metronomic regularity (in this case represented by a performance of afterbeat), as well as related to the melody. Although the melody is performed as an exaggerated version of the Telemark dialect, it does not seem reasonable to consider this performance, $K_s$, as belonging to any specific dialect of "springar", due to the lack of common consistency in the rhythmic performance of the three voices.
7.1.2. Performance of cymbal swing rhythms in jazz:

Playing a cymbal "swing" ostinato in jazz music, different drummers may utilize various different phrasings of a rhythm which is often written as:

A specific rhythmic performance of this ostinato may be characteristic of the individual drummer, it might be related to style, and is also dependent on the tempo of the performance. Sometimes this ostinato is played according to a subdivision of the quarter note in triplets, other times a rather "flat" performance is heard (quarter note – two eighth notes; quarter note – two eighth notes; ...etc.). A simulation of different performances of this cymbal rhythm is constructed in the following way: First, we make a recording of the ostinato on a MIDI sequencer, where every MIDI Note On event is quantized in eight note triplets; i.e. we create a metronomic performance according to the following notation:

Secondly, we import this recording as a MIDI-file into the computer program, applying RFM with the following parameter values:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>4</td>
</tr>
<tr>
<td>P2</td>
<td>sine</td>
</tr>
<tr>
<td>P3</td>
<td>2</td>
</tr>
<tr>
<td>P4</td>
<td>0</td>
</tr>
<tr>
<td>P5</td>
<td>2</td>
</tr>
<tr>
<td>P6</td>
<td>between −1 and 1.</td>
</tr>
</tbody>
</table>

Choosing P1 = 4, P2 = 2, we obtain a movement curve dividing the 4/4 measure into two equal halves; i.e. the performance of the third and fourth beat (with subdivisions) equals the performance of the first and second beat. This appears to be in accordance with the situation of a live performance, reflecting a drummer's consistency in performing the rhythm. It should, moreover, be noted that with the parameters above, the modulation has effect on the subdivisions of the quarter notes, whereas the quarter note beats themselves (the 1, 2, 3, and 4 of every measure) are not affected. Observe that if P6 = 0, no modulation occurs, and the RFM-instrument generates a metronomic performance according to a subdivision in triplets.
If, on the other hand, \( P6 = -1 \), a performance simulating a subdivision in sixteenth notes is created, whereas \( P6 = 1 \) results in a performance simulating subdivisions in eighth notes (i.e. a "flat" performance). The movement curves related to the choices \(-1\) and \(1\) are given in the figure below. Note that (again) an amplitude adjustment has been made, reflecting the fact that in a performance of cymbal beats where the elapsed time between each beat is "short", the drum stick is kept closer to the cymbal than when the temporal distances between the cymbal beats are "larger".

![Figure 7.7](image)

**Figure 7.7. Illustration of movement curves associated with two different modulations of the same rhythmic pattern. The upper corresponds to \( P6 = -1 \), in the lower \( P6 = 1 \). In this figure the amplitude, \( A \), is made a function of beat distance, \( \Delta; A = (\Delta)^{1/2} \).**

Using other values of \( P6 \), or applying modulation of modulations, we are able to simulate various performances of this cymbal rhythm where the subdivisions may fluctuate between sixteenth notes and eighth notes, which, indeed, is the case in live performances of this rhythm in jazz.

Referring to the choice of parameters above, the following simulations of performances of cymbal swing rhythms are created and recorded on the CD (\( \text{\textbullet} = 130 \)):

- \( S_9 \) (swing9): \( P6 = 0 \); i.e., no modulation, and a metronomic performance according to a subdivision in triplets is created: *Track 12*.

- \( S_{17} \): \( P6 = -1 \); simulation of subdivision in sixteenth notes. Observe that the quarter note beats (the 1, 2, 3, and 4 of every measure) are performed in synchronization with a metronomic performance of quarter notes: *Track 13*. 

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$S_2$: $P6 = 1$; simulation of subdivision in eighth notes. Again, the quarter note beats are not affected by this modulation: Track 14.

Applying modulation of modulations, we are able to obtain synthesized performances where the subdivisions may fluctuate between sixteenth notes and eighth notes:

$S_3$: This modulated performance is created by applying the following algorithm of serial modulation:

The effect of this modulation, which may be heard as track 15, is a performance beginning with a subdivision close to eighth notes, and, during four measures of performance, ending with a performance of the swing rhythm where a subdivision close to sixteenth notes is applied. Listening to the recording, or looking at the illustration of an associated movement curve, given in the figure below, we notice that, in contrast to the previous syntheses of swing, the quarter note beats are indeed affected by this modulation.
Figure 7.8. A movement curve associated with a performance of a swing rhythm, beginning with a simulation of subdivision in eighth notes, and ending with subdivision close to sixteenth notes. (A sounding illustration is given on track 15.)

With reference to the example above, it is also interesting to point out that $S_3$ illustrates a process of rhythmic morphing, where one rhythmic pattern:

![Rhythmic Pattern 1]

is gradually transformed into another rhythmic pattern:

![Rhythmic Pattern 2]

In the present example, this transformation is both a transformation of movement curves and a transformation of rhythmic perceptual categories.

We give two more examples of syntheses of swing applying modulation of modulations. In the first example we create a periodic modulated performance with period equal to two measures, in the second we create a performance with period larger than 4 measures (which is the number of measures in the present recording of the swing rhythm). It can be shown that the period of a modulated modulation is dependent on the ratio between the frequencies of the two modulating waveforms. (The proof of this is, however, not a subject of the present presentation.)
$S_4$: We use the following parameters of modulation:

An associated movement curve is:

$S_4$ is recorded as *track 16*. Observe that the *first* beat of every measure is in synchronization with a metronomic (unmodulated) performance, whereas the other quarter note beats (the 2, 3, and 4) are "moved" by this modulation. We also notice, as mentioned above, that $S_4$ is periodic with length of period equal to *two measures*. 
$S_5$: This time the following RFM-instrument is used:

An associated movement curve is:

$S_5$ may be heard as track 17. In this example the period exceeds the duration of the performance and the first beat of every measure is "moved" by the modulation.

Having presented some examples of various simulations of performances of cymbal swing rhythms in jazz, we will now try to make some illustrations of how different characteristic features of a drummer and a bass player, playing together in a jazz rhythm section, may be simulated applying the technique of RFM synthesis.
7.1.3. **Bass and drums in a jazz rhythm section:**

As discussed by Keil (1987, 1995), and empirically investigated by, e.g. Prögler (1995) and Alén (1995), typical features of playing "grooves" and making music "swing" are various *participatory discrepancies* in the performance of musicians playing together.\(^{150}\) For instance, when playing a swing rhythm in jazz, the drummer might be a bit "ahead" of the beat, while the bass player could be "laid back" ("behind" the beat), both being compared to a metronomic pulse. According to Keil, *"the power of music"* lies precisely in this asynchronizity. As Keil puts it (in Keil & Feld, 1994, p.96):

> The power of music lies in its participatory discrepancies, and these are basically of two kinds: processual and textural. Music, to be personally involving and socially valuable, must be "out of time" and "out of tune."

In an attempt at simulating various such PDs in a jazz rhythm section by means of rhythmic frequency modulation, we first make a quantized, multi-track MIDI recording of the following "walking" bass line and cymbal rhythm:

**A Bass and Drums Swing Rhythm**

\(^{150}\) As commented above (cf. section 2.4), such different occurrences of asynchronicity are typical of expressive live performances on a more general basis, but we *exemplify* these phenomena here by focusing on bass and drums in a jazz rhythm section.
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Observe that in the notation above we have chosen to use triplet notation denoting the swing rhythm, instead of the more usual:

\[
\begin{array}{cccc}
\hline
\hline
\hline
\end{array}
\]

The reason for this choice is that the MIDI recording of the performance of this bass and drums swing rhythm (performed by the author), is (to begin with) quantized to eighth note triplets, and the notation applied in the score above is therefore the best representative of the quantized performance used as a basis in our further RFM synthesis. At this point we again emphasize that the quantization is carried out on note onsets (MIDI Note On events), whereas durations and dynamics are not affected by the quantization. It should also be noted that the drum part contains a performance of “closed hihat” (which, when playing the drum set, is performed with a foot) on “2” and “4” of every measure (except in the last measure at the end). This is in accordance with common practice of performing swing rhythms in jazz, at least in the “mainstream” tradition of the jazz styles swing and bebop. This quantized version of the bass and drums recording, denoted \( B&D_b \) (bass\&drums\(_b\)), is imported as a MIDI file into the RFM computer program, and may be heard on the CD as track 18 (\( \text{\textbf{\textdagger}} = 130 \)).

We now create the following rhythmic modulations of \( B&D_b \), which may be seen as approximations to live performances of this groove in a jazz rhythm section:

\( B&D_1 \): With the application of the modulation parameters of \( S_1 \) in the cymbal swing synthesis of the previous section to both the bass track and the drum track, the bass and drums are still in synchronization, but now performing according to a subdivision of the quarter note beat in sixteenth notes. \( B&D_1 \) is recorded as track 19.

\( B&D_2 \): A simulation of a synchronized performance between bass and drums where both voices are performed with an eighth note subdivision is created by applying the modulation parameters of \( S_2 \) to both bass and drums. \( B&D_2 \) is heard as track 20.

\( B&D_1 \) and \( B&D_2 \) demonstrate two different performances synchronized subject to different subdivisions of the quarter note beat. These syntheses were created by using the same parameters in the modulation of both voices. If, instead, we apply one set of
RFM-parameters to the bass line, and another set of parameters to the cymbal rhythm, various occurrences of participatory discrepancies may be simulated:

**B&D**

**B&DD**

**B&D**

**B&D**

**B&D**

The bass is given the same modulation as in B&D, i.e. the bass line is performed with sixteenth note subdivision, the quarter note beats are *not* affected. *The drums*, however, are now modulated according to the following RFM-instrument:

![RFM-instrument diagram](image)

A graphic illustration of movement curves resulting from these modulations is (bass: **black**, drums: **red**):

![Movement curves](image)

**Title:** B&D (drums "ahead")

In this case we have a simulation of a performance where both musicians are performing according to a sixteenth note subdivision. However, the drummer is now "rushing", playing "ahead" of the bass player. As a matter of fact, in this modulated performance the drummer and the bass player are *never* in perfect synchronization with one another. Observe that in this example the discrepancies between the two voices are
uniform (i.e. unchanged) throughout this entire performance. \textit{B&D}; may be heard as track 21.

\textbf{B&D} : An example of a modulated performance where the discrepancies between the voices vary throughout the performance is created by applying the following set of parameters:

\textit{The drums} are modulated subject to the following parameters:

The result of this modulation is a performance of the cymbal rhythm where each quarter note beat is performed in synchronization with the metronomic beat, whereas a subdivision "in between" sixteenth notes and eighth note triplets is applied in a consistent (i.e. uniform) manner throughout the performance.

\textit{The bass} is affected by modulation of modulation as follows:

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We notice that this latter RFM-instrument is constructed by applying the same parameters of modulation as in the swing synthesis S₉. An illustration of movement curves associated with this simulation is given in the figure below. The bass is represented by the black curve, whereas the cymbal rhythm is illustrated by the red curve.

Figure 7.9. Example of a synthesized version of a bass and drum swing rhythm where the drummer is playing "on" the metronomic beat, applying a subdivision "in between" sixteenth notes and triplets, whereas the bass player fluctuates between various subdivisions of the quarter notes, performing in varying degrees out of sync with the drummer.

This synthesized performance is denoted B&D₉, and is recorded as track 22. On listening to B&D₉ and looking at the movement curves in the figure above, we notice that whereas "the drummer" is performing in a consistent way, "the bass player" is not at all consistent in his performance, varying in a seemingly unstable and "unnatural" way between different subdivisions of the quarter note beat, being in different degrees "too late" on the beat compared to the drummer. The performance of the bass line gives the impression of "the bass player" having difficulties in his playing. If this were a real performance, these difficulties might be those of a technical nature, or related to sight reading, for instance.

This evaluation is related to my own personal judgements of the performance, based on my own experience as a performing drummer, familiar with playing and listening to jazz swing rhythms.
In the previous example we argued that the bass player was inconsistent in his playing, being in varying degrees too late on the beat compared to the drummer. If we now, as an experiment, turn this around, and claim instead that the bass player should be used as a reference, the inconsistency is executed by the drummer rather than the bass player. To illustrate how a swing rhythm might sound if the bass performance of B&D₃ is taken as an "ideal", we suggest a new esthetics of swing performance by creating the following swing rhythm synthesizes:

_B&D₃:_ Both bass and drums are given the modulation of the bass in the previous example. This creates a performance where the bass and drums are synchronized with one another, both being, in an identical manner, out of sync with a metronomic performance. B&D₃ is recorded as track 23.

_B&D₄:_ The bass is modulated as in the two previous examples, whereas the cymbal performance is modulated according to the following:

Observe that the parameters of this RPF-instrument are the same as for the modulation of the bass, except that in this cymbal modulation we have chosen a smaller value of peak deviation for the second modulating waveform (0.5 instead of 1.5). The result of this modulation is that the drums are still perceived as "close" to the bass performance (although not synchronized), whereas the subdivision applied by "the drummer" is in this case somewhat more "usual" compared to ordinary standards of swing performance. B&D₄ may be heard as track 24.

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7.2. Syntheses of Accelerando and Ritardando

In the previous section we have demonstrated various modulations that create rhythmic deviations on the beat level and measure level. In all these examples, the ratio between the carrier frequency and the frequency of the first modulating waveform, \( f_c/f_m \), was either 3 or 2 (this ratio was 3 in the Vienna waltz and "springer" simulations, whereas a ratio of 2 was used in the different syntheses of swing performances). Applying a modulating waveform with frequencies making a larger ratio with the carrier frequency, we are able to simulate different performances of accelerandi and ritardandi.

7.2.1. Performing accelerando and ritardando of quarter notes:

A sequence of 64 quarter notes (16 measures of 4/4) is imported into the RFM-program as a text file. When given no modulation, the quarter notes are performed metronomically, with constant distance between the attack points of successive quarter note beats. A simulation of an accelerando followed by a ritardando is created by applying the following RFM-instrument:
We observe that in this case \( f_c/f_m = 4/0.0625 = 64 \), and the result of this particular modulation is: An accelerando is created in the performance of the first \( 64/2 = 32 \) quarter notes, followed by a ritardando in the performance of the last \( 64/2 = 32 \) quarter notes. In other words, the first 8 measures in 4/4 are accelerated, the last 8 measures in 4/4 are performed with ritardando. A graphic illustration of an associated movement curve is given below:

Note that in this illustration the first quarter note beat of every measure is marked by a vertical line. This synthesized performance of accelerando and ritardando is denoted \( A\&R \), and may be heard as track 25.

7.2.2. Practising a drum roll:

One way of practicing the performance of a drum roll is by performing the following exercise (R: right hand (or, rather; right drum stick), L: left hand (left drum stick)):

\[ R \quad R \quad L \quad L \quad R \quad R \quad L \quad L \quad R \quad R \quad L \quad L \quad \]

\( P.a.p. \) accelerando, then \( p.a.p. \) ritardando.

A performance of right-right-left-left-.... without any accelerandi or ritardandi (i.e. constant tempo) is recorded on two tracks of a sequencer (right hand: track 1, left hand: track 2), and
imported as a MIDI file into the RFM-program. A simulation of a performance of the roll exercise with accelerando and ritardando is created by applying the following RFM-instrument:

Observe that in this example \( f_{eff} = 4/0.015625 = 256 \), which creates an accelerando in the performance of the first \( 256/2 = 128 \) drum beats, followed by a ritardando in the performance of the last 128 beats. A graphic illustration of movement curves related to this synthesis of the drum roll exercise is given below (right hand: black, left hand: red):

Tittel: Drum roll exercise
In the figure above we note that, as in many previous examples, an amplitude adjustment has been made \((A = (\Delta)^{0.5})\), reflecting kinesthetic aspects of live rhythmic performances.\(^{152}\) It should also be noted that in the present illustration of the drum roll exercise it seems, on the basis of my own experiences as a performing drummer, plausible to assert that the movement curves given above show larger similarities to the physical movements of the \textit{drum sticks} than to the movements of the drummer’s hands. This simulation of a performance of the roll exercise is denoted \textit{DR (drum roll)}, and is recorded as \textit{track 26}.

7.3. Syntheses of Rhythmic "Phasing"

If the different voices of a multi-track polyphonic MIDI recording are modulated subject to different RFM parameters in such a way that the differences in "stretching" or "compressing" the associated movement curves are "small", various occurrences of rhythmic "phasing" may be simulated.

7.3.1. Simulating the "phasing" technique of Steve Reich:

"Phasing" is a technique used in many compositions by Steve Reich (e.g. Steve Reich: "Drumming", composed in 1971). The basic idea of this technique is that two or three identical instruments play the same repeating rhythmic pattern, and then gradually move out of synchronization with each other, creating new interlocking resultant patterns. As an illustration of an RFM-synthesis of this effect, we consider the pattern:

\[\text{illustration of phasing pattern}\]

\(^{152}\) We again emphasize that the particular choice of this amplitude adjustment is in the present example made as an illustration of how kinesthetic considerations might be implemented into our model. That is to say, we have, at this point, not carried out any empirical investigations that would provide information as to \textit{which} amplitude adjustment should be made.
Two identical metronomic versions, V₁, V₂, of this pattern, played over and over again for 64 measures, are recorded on separate tracks on a sequencer, and imported into the computer program. V₁ is kept unmodulated, whereas V₂ is modulated according to the parameters:

\[ P_1 = 4, \quad P_2 = \text{sine}, \quad P_3 = 0.015625 (= 1/64), \quad P_4 = 0, \quad P_5 = 3, \quad P_6 = 3 \]

The result is: V₁ and V₂ start off in synchronization. V₁ has constant tempo, whereas V₂ accelerates, gradually moving \textit{out of sync} with V₁. The accelerando of V₂ lasts for 16 measures (measured in relation to the metronomic V₁), whereafter V₂ makes a ritardando, gradually moving \textit{back into sync} with V₁. After a cycle of 32 measures V₁ and V₂ are in synchronization again. This synthesis of moving out of sync – back into sync is then repeated in the last 32 measures (again, measured in relation to V₁). This example of rhythmic phasing is denoted \( P_3 \), and is recorded as \textit{track 27}.

7.3.2. A "pathological" two-part Bach invention:

To give an example of how synthesis of rhythmic phasing, when applied to \textit{non-identical} voices of a polyphonic piece of music, might create rather "weird" or "pathological" performances, we import a metronomic MIDI recording of J. S. Bach’s composition "2-Part Invention No. 13 in A minor" as a MIDI file into the RFM computer program.\(^{133}\) The voices played by the right and left hand are given different rhythmic modifications, in the following way:

\textit{Right hand (1\textsuperscript{st} voice)}:
\[ P_1 = 4, \quad P_2 = \text{sine}, \quad P_3 = 0.0625 (=1/16), \quad P_4 = 0, \quad P_5 = 3, \quad P_6 = 16 \]

\textit{Left hand (2\textsuperscript{nd} voice)}:
\[ P_1 = 4, \quad P_2 = \text{sine}, \quad P_3 = 0.0625, \quad P_4 = 0, \quad P_5 = 3, \quad P_6 = -16 \]

Observe that the modulation parameters used in this example are the same as the parameters used in the Steve Reich phasing simulation above, except for the choice of P3 and P6. We also note that the only difference in modulation of the right hand voice and the left hand voice is given by the different choices of P6 (peak frequency deviation).

\(^{133}\) The sequencer recording of this two-part Bach invention is included as a demo-recording in the program \textit{Cakewalk Pro Audio 7}, and unlike the other audio examples of this chapter, is \textit{not} performed by the author of the present book.
The result of this modulation, which is denoted BI (Bach invention) and may be heard as track 28, is that the voices played by the right and left hand are moving out of sync and into sync with each other in a somewhat "symmetric" way (due to the choice of modulation parameters, where the peak frequency deviation of the two voices have identical absolute values, but opposite signs). Whenever the 1st voice is making an accelerando, the 2nd voice is making a ritardando, and vice versa. The two voices "meet" (i.e. are synchronized) at different occasions during the performance, and also at the end.¹⁵⁴

With reference to the two examples of rhythmic phasing presented in this section, it is an interesting observation that whereas the synthesis of the Steve Reich phasing technique represents a simulation of two drummers playing together, creating a performance which can be heard in real, live performances of music, the synthesized performance of the Bach invention, on the other hand, does not, according to common standards of piano playing, appear as very "playable" by a "real" musician. Thus, the example BI may be seen as an example of a "non-real" rhythmic performance of music.

7.4. Summary

In this chapter we have presented several examples which illustrate how the technique of rhythmic frequency modulation may be useful in making syntheses of rhythm, simulating various live rhythmic performances as documented and discussed in empirical rhythm research through the detection of SYVARD and PD. We have, moreover, presented simulations of rhythmic accelerandi and ritardandi, and showed how different occurrences of rhythmic "phasing" may be created by applying RFM synthesis. All of these examples are recorded for listening on the attached CD. An overview of the content on the CD, with track numbers assigned to the different examples, is given in Appendix III.

It is interesting to note that the various deviations created in these different examples of synthesis occur at different levels in the hierarchical organization of the rhythmic structure of the performances:

¹⁵⁴ This is created by the specific choice of the parameters P3 and P5, which is a matter of mathematical investigation, and will not be discussed further at this point.
1. At the beat level: In the syntheses of Vienna waltz accompaniment (first, second, and third simulation), as well as in the simulations of different dialects of "springar" as performed in Norwegian folk music, the rhythmic frequency modulation affected the beats of every measure, creating deviations from metronomic regularity in the performance of the beats (as well as in the performance of subdivisions of the beats). The measure length, however, was not affected by modulation in these examples. In all of these synthesized performances every measure was performed with the same length of duration.

2. At the level of subdivisions of the beat: Simulating performances of cymbal swing rhythms in jazz, the examples $S_1$ and $S_2$ illustrate how subdivisions of quarter notes might be modulated, whereas the quarter note beats themselves (in these cases the 1, 2, 3, and 4 of every measure) are not affected by the modulation.

3. At the measure level: Applying serial modulation (i.e. modulation of modulation), we are able to create simulations of performances where the duration in the performance of one measure is varying throughout the performance. This is illustrated in the examples $S_3$ and $S_5$ of cymbal swing rhythm, as well as in the examples $B&D_1$, $B&D_3$, and $B&D_5$ of bass and drums in a jazz rhythm section.

4. Extending the measure: In the examples where we are making syntheses of accelerandii and ritardandii, and constructing simulations of rhythmic phasing, the effect of rhythmic frequency modulation is of a more "global" nature, affecting the tempo of the performance. This is in contrast to the situations described in 1., 2., and 3. above, where the modulation is of a rather "local" nature, affecting beats, subdivisions and measures, whereas the "overall tempo" is (more or less) unaffected.

As pointed out in the introduction to the present chapter, two different approaches might be applied when creating rhythm syntheses by means of RFM: One approach is to use RFM in an attempt at making approximations to live performances of rhythm, another strategy is to investigate how "far from reality" we are able to come within the limits of our synthesis technique. Examples illustrating both these approaches have been presented in this chapter. Whereas the first approach is interesting from the point of view of making new models and descriptions of live musical performances of rhythm, the second approach might, on the one
hand, generate new ideas of describing various "pathological" performances, and will, on the other hand, point at new practices of "non-real" rhythmic performances, or, to put it another way, suggest new "languages" and new "dialects" of rhythmic expression.

Some possible extensions of the RFM technique have been mentioned in this chapter. These will be discussed further in the following epilogue, where we also try to point at some overall conclusions and consequences of our work here developed.
Epilogue:
If This Means Something....

A main concern of this project has been to give a new description of rhythmic performance of music where fundamental aspects of movements are incorporated. In doing so we have focused on gestural rhythm understood as a continuous unfolding through attack points, rather than attack-point rhythm where a finite number of discrete registrations of rhythmic performance are investigated. Our approach has been motivated by basically the following factors:

(a) The ancient concept *rhythmos*.
(b) The *eurhythms* of Emile Jaques-Dalcroze.
(c) The author's experience from music performance, music education, and mathematics.
(d) An attempt at modeling *timbral* aspects of rhythmic performance by translating well-known techniques of sound synthesis into new synthesizes of rhythm.

An important point of departure in our discussions, concept constructions, and model developments is antiquity's ideas of a strong correspondance between *rhythmos* and movements (Chapter 1). In choosing this classical understanding of rhythm as a basic reference for our study, we make our investigations a part of a tradition of modern rhythm research previously developed in various ways by the contributions of Sievers, Becking, Truslit, Fraisse, Repp, Clynes, Todd, and Blom (cf. chapters 2 and 4). As we see it, valuable insight into rhythm performance may be gained by studying movement curves (trajectories) associated with the performance. However, whereas most kinematic models regard the trajectories as representing some kind of abstract movements corresponding to the ways by which a musical performance changes in, for instance, tempo, intensity or dynamics (cf. Chapter 2), our approach is to use simple, concrete movements of the body as atomic elements in the theoretical development of our model. Moreover, in order to make our rhythm synthesizes "come alive" thereby approximating live performances of rhythm, we introduce a new idea of *rhythmic frequency modulation*, RFM, as a construct correlated to mutual
Epilogue

interactions of atomic movements. In general terms, the major achievements of our investigations may be summarized as follows:

(1) A model of *metronomic performance* of rhythm, MPR, is constructed where information of note values is represented as continuous movements through attack points (cf. Chapter 5). From this construction we also obtain that:

- Structure of *durations* is transformed into structure of *movements*.
- *Written* representations of music are transformed into representations of *metronomic performance* of music.

(2) A model of *live performance* of rhythm, LPR, is presented where expressive timing is simulated by applying rhythmic frequency modulation as a new technique of rhythm synthesis (Chapter 5). The model MPR is naturally embedded in LPR. Interesting consequences of this latter model construction are:

- Structure of *attack-point rhythm* is transformed into structure of *gestural rhythm*. Thereby we suggest a shift from a *discrete* to a *continuous* representation of rhythmic performance.
- *Written* representations of music are transformed into representations of *live performances* of music.
- The technique of RFM is shown to provide a very *simple temporal control* over movement curves associated with quite complex rhythms. In other words: by manipulating a small number of parameters we are able to create a large variety of different simulations of live performances of rhythm.

(3) A theoretical interpretation offered by the models MPR and LPR is to describe representations of expressive timing as non-linear continuous transformations of rhythmic structure. Subject to this interpretation our models reflect processes by which conceptualized musical information, through an interaction of cognitive skills and motor skills, is transformed into live performances of music. It seems appropriate to say that such processes are fundamental to musical performance on a general basis (cf. Palmer as presented in section 1.4, and Clarke in 2.3. above).
(4) A computer implementation of rhythmic frequency modulation is constructed in collaboration with Sigurd Saue. Using this computer program we are able to translate our theoretical models into sounding rhythm syntheses (cf. Chapter 6). Concrete applications of the computer realization include (cf. Chapter 7):

- Various audible examples are presented which illustrate how RFM synthesis may be useful in making simulations that approximate live rhythmic performances as documented in empirical rhythm research.
- RFM synthesis may be used to create rhythmic morphing between different rhythmic perceptual categories.
- Apart from approximating real performances, an interesting application of RFM is also to create various unreal unfoldings of rhythm. This has been demonstrated by creating a "pathological" performance of a two-part Bach invention. The construction of such pathological performances is interesting for several reasons. On the one hand, an understanding of parameters determining a pathological performance may give valuable insight into what kind of adjustments should be made to make the pathological performance non-pathological. Moreover, if we were able to correlate these model-constructed adjustments to physical movements of the performing musician's body, this knowledge would indeed be quite significant to music education. On the other hand, by appreciating the pathological performance as valuable in its own right a new standard, or maybe put more appropriately, a new esthetics of rhythmic performance is suggested.
- Both the construction of rhythmic morphing and the possibilities of creating various unreal rhythmic performances indicate that RFM synthesis may be applied as a new compositional tool of electro-acoustic music. This is also illustrated in our simulation of the phasing technique of Steve Reich.

Even though our concepts and models have been developed and motivated on the basis of the ancient Greek comprehension of rhythmos, the educational ideas of Emile Jaques-Dalcroze, and the performing musician's experiences, the model LPR and the technique of
rhythmic frequency modulation have been constructed on a purely theoretical basis. As commented above, various interpretations and applications of the model have been tested against empirical findings indicating that LPR and RFM synthesis may provide new insight of value to an enlarged understanding of rhythmic performance of music. However, in the stepwise development of our theory as well as in the discussion of the various concrete applications, we are left with several open questions and unsolved problems, most of which we hope to investigate in our future research on rhythm. Among the many open questions suggesting interesting ideas for future research are:

(I) What can empirical investigations tell us about the relevance of our model-constructed movement curves to physical movements of the body? Can empirical studies of movement patterns supply sufficient evidence to make it plausible to assert that frequency modulation gives a significant description of characteristic gestural features of rhythmic performance and unfoldings of rhythm?

Up to this point, the applications of RFM synthesis have demonstrated that the technique of rhythmic frequency modulation represents a new tool for making continuous simulations of rhythmic performance as documented by empirical investigations of attack-point rhythm. By studying patterns of physical movements of the body we may gain additional information telling us to what extent RFM can also create continuous simulations of empirically investigated gestural rhythm. Various computer-based technical equipment for carrying out such empirical investigations of body movements has been developed and is extensively used within the fields of human movement science and sport sciences. Of major interest to our future research on rhythm is to apply such equipment in empirical investigations of rhythmic performance of music.

(II) Based on empirical findings of rhythmic movements of the body, various adjustments and further development of RFM and the model LPR might be suggested. In future studies we will pursue the following ideas:

(i) Kineesthetic implementation: As strongly emphasized in Chapter 4, the eurhythmics of Emile Jaques-Dalcroze is not just concerned with kinematic aspects of movement curves. Even more so, it is of fundamental importance to develop a kineesthetic feeling in the performance of a
rhythm. In an attempt at implementing kinesthetic considerations into our model we have suggested that the amplitude of the pulses (whether modulated or not) should be a function of the distance between successive beats, \( \Delta \), and dynamics, \( \text{dyn} \); \( A = A(\Delta, \text{dyn}) \) (cf. chapters 5 and 6). On a theoretical basis we have also indicated some plausible ways of defining this function in order to reflect kinesthetic aspects of performances. However, which relation between \( A, \Delta \), and \( \text{vel} \) one should choose in the different applications and interpretations of our model, and also, which additional parameters are important to kinesthetic considerations, can be decided on the basis of empirical investigations only.

(ii) More complex algorithms: Each example here presented has been constructed either by means of a basic RFM-instrument consisting of one carrier oscillator affected by one modulating oscillator, or by using serial modulation in its most simple form where two modulating oscillators are involved. In addition, we have also suggested an RFM algorithm of parallel modulation (cf. chapters 5, 6 and 7). Looking at well-known applications of FM in the syntheses of sound, many additional algorithms providing new syntheses of rhythm may give interesting results, for instance, algorithms where the peak frequency deviation, \( d \), is made a function of time, \( d = d(t) \). To what extent these additional algorithms are relevant to RFM syntheses of live performances of rhythm should, again, be decided when tested against empirical findings. However, various advanced RFM algorithms may also be quite valuable independent of empiri. As powerful tools in theoretical and computer-based manipulation of rhythm capable of creating new unreal rhythms, such complex algorithms of RFM may be interesting to both musicians and composers of electro-acoustic music.

(iii) Implementing stochastic variables: The model LPR is basically a deterministic model where the elements are continuous functions representing movement curves associated with rhythmic performances of music. In several examples we have demonstrated that the elements of LPR provide approximations to various style specific features of live
performance of music, such as systematic variations of duration. Moreover, we are able to simulate some overall features of participatory discrepancies (cf. chapters 5 and 7). However, within each musical style and underlying various overall features of musicians playing together, the rhythmic deviations of the individual musicians may be rather diverse. In a performance of a Vienna waltz accompaniment, for instance, the beat durations, S(short) – L(long) – I(intermediate), may vary throughout the performance of one singular waltz, and may also change in proportional values in different performances. To account for such individual variances in live performance of rhythm it would be interesting to implement non-deterministic elements in the model LPR. A possible way of doing this could be to make the peak frequency deviation dependent on a probability distribution, or to introduce a stochastic variable as an argument of the modulating pulse(s), for example. This will be investigated in our future research.

(iv) From live performance to atomic movements: In developing our theory and constructing the different syntheses we have applied atomic movements (pulses) and a binary operation of frequency modulation to create rhythmic unfoldings that approximate live performances of music. It would be interesting to know whether this construction can be reversed. In other words: Given a live performance of rhythm - to what extent are we able to detect the atomic movements that are involved in a (possible) RFM synthesis of this performance? Apart from being interesting from a theoretical point of view, an answer to this question could also be significant to methodological aspects of music education. We comment upon this latter circumstance below.

(III) Does RFM and LPR offer relevant information to music education and pedagogies of rhythm performance?

As mentioned above, the eurhythmics of Emile Jaques-Dalcroze has been an important inspiration to our present work. Since a basic motivation for Dalcroze was to apply eurhythmics as a method of music education (and eventually also as
an educational method of general application beyond the field of music), it seems natural to ask whether our findings of rhythmic frequency modulation as valuable to synthesizes of live performance of music also might be of some value to music pedagogy and performance training. As we see it, there may be different answers to this question. On the one hand, an acceptance of the classical premise asserting a strong connection between rhythm and movement certainly makes our focus on body movements, movement curves, and timbral aspects of rhythm performance interesting to methods of music education. In this respect our contribution represents an approach where ancient ideas are combined with fundamental aspects of Dalcroze's educational method, and are further developed and articulated by applying modern techniques of sound synthesis. If movement is regarded as fundamental to music performance, our construction of concepts and terminology and particularly the distinction we make between discrete and continuous aspects of rhythmic performance seem important to educational methods in music on a general basis. On the other hand, whether our specific idea of RFM has concrete applications to music pedagogy and performance training requires careful empirical research on body movements of performing musicians to be answered properly. However, what can be said is that if some correlation between frequency modulation of model-constructed atomic movements and interactions of physical movements of the body can be empirically established, then the model LPR will certainly be very interesting to educational methods of rhythm performance. In this case, an understanding of atomic movements and how they interact will be of importance to an understanding of live performance of rhythm.

(IV) In chapters 1 and 2 the many different applications of the concept 'rhythm' were demonstrated pointing to the fact that the various meanings assigned to this concept may be quite divergent. Having presented and discussed the multi-dimensionalities of rhythm, in Chapter 3 we posed the following question: Is it possible in a meaningful way to identify a "core" of the rhythm concept basic to the various divergent applications, making it plausible to denote seemingly quite different events from everyday life, music or scientific investigations by the common term: *rhythmic*? We proposed an affirmative answer to this question by defining *rhythm object* as a possible "limit of convergence" of rhythm concepts.
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Since a rhythm object "summarizes" common features of rhythmic phenomena, an identification of this concept is interesting with respect to our model construction and synthesis of rhythmic events. In Chapter 3 various examples of rhythm objects outside the field of music were suggested when we presented a list of perceivable differences (cf. section 3.3). Aside from audible differences and differences in movement, both typical of rhythm in music, such perceivable differences may include visual differences, differences in smelling and tasting, differences experienced through touch, biological differences, differences given by astronomical changes, differences in energy, and social/economic/cultural/geographic differences, to give some examples. Exposed to this variety of rhythmic unfoldings within different fields of science and everyday life the following intriguing question more or less poses itself: Can our synthesis technique and model construction be applied in a significant way to unfoldings of rhythm beyond performance and experience of music? At present we make no attempt at answering this question. However, it is here interesting to make a point of the fact that the atomic elements of the model LPR are oscillators that may naturally be interpreted as continuous alternations between energy states of a far more general kind than just different positions of a drummer's hand when performing a rhythm. It therefore seems appropriate to suggest that it is not improbable that an affirmative answer exists to the question above.

Viewed in the light of the many open questions and unsolved problems briefly presented above, a full understanding and comprehensive application of the RFM technique seems only in its beginning. Various theoretical developments of our model can certainly be made and new interpretations and applications to a larger class of rhythmic unfoldings than here presented may find their support in empirical investigations. By using the different unsolved problems discussed above as points of departure for scientific investigations, further insight into the multidimensional aspects of rhythm might be gained. In our future research we hope to contribute to a such development.

The prologue was entitled "It Don't Mean a Thing If It Ain't Got That Swing". With address to our work here presented it is tempting to close this epilogue with the following logical restatement: If This Means Something, Then This Has Got That Swing.
Appendix I

Some Mathematical Proofs

With reference to Chapter 5, we here present some mathematical proofs.

\textbf{Proof of lemma 5.2.6 (i):}

\[ p_f(t) = A[1 - \cos(f t)] \]\[ q_f(t) = \sin(f \ t + \phi) \] \( f, f' \in \mathbb{N} = \{1, 2, 3, \ldots\} \).

By definition:

\[ (p_f \oplus q_f)(t) = A[1 - \cos(f t + d \sin(f' t + \phi))] \]

The beats of \( p_f \oplus q_f \) are solutions of the equation:

\[ \cos[\tilde{f} + d \sin(f' t + \phi)] = 1 \], which is equivalent to:

\[ (*) \quad \tilde{f} = 2k\pi - d \sin(f' t + \phi), \quad k = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \]

\[ g(t) \quad h_0(t) \]

Now \( \frac{dh_0(t)}{dt} = -df' \cos(f' t + \phi) \), and since by assumption \( 0 \leq d \leq f \), we get:

\[ (**) \quad \left| \frac{dh_0(t)}{dt} \right| \leq df' \leq f = \frac{dg(t)}{dt} \]

The situation is therefore as illustrated in the graphs below:
From (***) it follows that $g$ intersects each $h_k$ once and only once. Since $h_{k+1}$ is a translation by $2\pi$ of $h_k$ along the vertical axis, $g(t) = ft$ intersects $f$ successive functions $h_k$ over any interval of length $2\pi$. In other words, (*) has exactly $f$ solutions over any interval of length $2\pi$.

Q.E.D.
Proof of lemma 5.2.6 (ii):

With \( p', q' \), \( f \) and \( f' \) as above, we let \( m = \gcd(f, f') \).
Then \( m \) is the greatest natural number such that:
\[
f = km \quad \text{and} \quad f' = k'm \quad \text{for some} \quad k, k' \in \mathbb{N}.
\]
Since the functions sine and cosine are periodic with period \( 2\pi \), it easily follows that \( T = 2\pi/m \) is the smallest positive number such that:
\[
(p' \oplus q')(t + T) = (p' \oplus q')(t)
\]
Hence, \( p' \oplus q' \) is periodic with period \( 2\pi/m \).

Q.E.D.

Proof of proposition 5.3.1:

We let \( p' \) and \( q' \) be as above, \( k \in \mathbb{N} \) and \( 0 \leq d \leq f'f \).
By definition:
\[
(p' \oplus q')(t) = A[1 - \cos(f + d\sin(f't + \phi))]
\]
\[
g(t)
\]
\[
(p' \oplus q')(t) = A[1 - \cos(kf + kd\sin(f't + \phi))]
\]
\[
kg(t)
\]

(i) Suppose that \( b_0 \) is a beat of \( p' \oplus q' \) occurring at time \( t \).

Then:
\[
A[1 - \cos(g(t))] = 0, \quad \text{and therefore} \quad \cos(g(t)) = 1.
\]
As a consequence we get:
\[
g(t) = 2\pi m, \quad m \in \mathbb{Z}, \quad \text{and thus} \quad kg(t) = 2\pi km, \quad km \in \mathbb{Z}, \quad \text{and therefore}:
\]
\[
A[1 - \cos(kg(t))] = 0,
\]
which means that \( b_0 \) is a beat of \( p' \oplus q' \) occurring at time \( t \).

This proves proposition 5.3.1 (i).
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(ii) Since $0 \leq d \leq \frac{a}{f'}$, lemma 5.2.6 (i) implies that

$p_f \oplus_n q_f$ has exactly $f$ beats over any interval of length $2\pi$, and that

$p_{f'} \oplus_n q_{f'}$ has exactly $k f'$ beats over any interval of length $2\pi$.

In (i) above we proved that every beat of $p_f \oplus_n q_f$ is also a beat of $p_{f'} \oplus_n q_{f'}$, and therefore it follows that for every beat of $p_f \oplus_n q_f$ there are $k$ beats of $p_{f'} \oplus_n q_{f'}$.

This proves proposition 5.3.1 (ii).

Q.E.D.

**Proof of proposition 5.3.4:**

We let $p_f$ and $q_f$ be as above, $n \in N$ and $0 \leq d \leq \frac{a}{f'}$.

By definition:

$$(r \oplus_n q_f)(t) = \left[A \left[1 - \cos \left(\frac{1}{n} \left[ft + d \sin(f't + \varphi) - 2(l - 1)\pi\right]\right]\right]

= A \left[1 - \cos \left(\frac{1}{n} [g(t) - 2(l - 1)\pi]\right]\right]\n
$$

(i) Suppose that $b_i$ is a beat of $r \oplus_n q_f$ occurring at time $t_i$.

Then:

$$A \left[1 - \cos \left(\frac{1}{n} [g(t_i) - 2(l - 1)\pi]\right]\right] = 0, \text{ and therefore}\n
(1/n)[g(t_i) - 2(l - 1)\pi] = 2\pi m, m \in \mathbb{Z}, \text{ and thus } g(t_i) = 2\pi m', m' \in \mathbb{Z}.

Consequently:

$$A[1 - \cos[g(t_i)]] = 0,$$

which implies that $b_i$ is a beat of $p_f \oplus_n q_f$.

This proves proposition 5.3.4 (i).
(ii) From lemma 5.2.6 (i) it now follows that for every beat of $\tau_i \oplus^{d^i}_n q_r$ there are $n$ beats of $p_f \oplus \nu q_r$, cf. the proof of proposition 5.3.1 (ii) above.

This proves proposition 5.3.4 (ii).

(iii) Suppose that $b_t$ is a beat of $\tau_i \oplus^{d^i}_n q_r$ occurring at time $t$.

As above, we let $g(t) = ft + d\sin(\pi t + \phi)$. Then (cf. the proof of prop. 5.3.4 (i) above):

\[
(*) \ g(t) = 2\pi[mn + (l-1)], \ m \in \mathbb{Z}.
\]

By our choice of numbering of beats, cf. the comments preceding prop. 5.3.4, the beats of $p_f \oplus \nu q_r$ with numbers $l, l+n, l+2n, \ldots$ are given as modulations of the beats of $p_f$ at $2\pi(l-1), 2\pi(l-1+n), 2\pi(l-1+2n), \ldots$

From (*) it therefore follows that the beats of $\tau_i \oplus^{d^i}_n q_r$ coincide with the beats of $p_f \oplus \nu q_r$ with numbers $l, l+n, l+2n, l+3n, \ldots$ etc.

This proves proposition 5.3.4 (iii).

Q.E.D.
Appendix II
Implementing Rhythmic Frequency Modulation

by Sigurd Saue

In order to investigate the implications of the theory of Rhythmic Frequency Modulation (RFM), a computer program was developed. The purpose was to allow interactive experimentation with modulation parameters together with a graphical presentation of movement curves and real-time playback of rhythmically modulated MIDI\textsuperscript{1}-sequences. The application was written in C++ using the Microsoft Visual C++ development system and Microsoft Foundation Class library (MFC)\textsuperscript{2}. It should run on any Windows-based computer (95/98/NT).

The application was written in Norwegian. Menus and dialog boxes will therefore be shown with Norwegian text. The name of the application is FMrhythm, which translates to FMRhythm (Frequency Modulated Rhythm). It is very likely that we will make an English version later.

The present appendix will give a detailed description of how this computer program is designed and implemented. It will start with a general overview of the design, presenting the two main structures and how they are related. The next two sections will describe the two structures (MIDI-songs and FM-songs) separately in greater detail. File import and MIDI playback will be treated together with the MIDI Song structure. Modulation, computation and plotting will be covered in the section on the FM Song structure.

The graphical notation used in the sections below is based on the Unified Modeling Language (UML), an emerging standard for modeling and design of object-

\textsuperscript{1} Musical Instrument Digital Interface. Specification 1.0 (1983) can be ordered from the MIDI Manufacturers Association (MMA)

\textsuperscript{2} Trademarks of Microsoft Corporation. An excellent introduction to Visual C++ and MFC is given in Kruglinski (1997)
oriented software. It should be noted that UML or any related tool was not used in the design of the program. In order to make the implementation easier to understand the names of all classes, attributes and operations have been given an English translation. The elements shown are but a subset of the complete implementation. Code fragments in C++ are simplified for clarity and will not work as listed.

AII.1. Overview

The entire application is build up around two main structures. One closely related to sequences of MIDI-events, the other to sequences of notes or pulses in which the temporal spacing is modulated by Frequency Modulation:

1. MIDI Song – for playing. This structure is basically the input to the program. It contains information about melodic content and is playable as any ordinary sequence of MIDI data.

2. FM Song – for computing modulations and for drawing. This structure is generated from the input MIDI Song, but only as a sequence of temporally distributed pulses. Each sequence of pulses is assigned a modulation operator, allowing modifications of the temporal distribution. The FM Song structure handles all computations and also graphic rendering of movement curves.

The two main structures are interconnected only through time references, one for each note in the FM Song structure. When computing movement curves these references are updated influenced by the modulating operators. During playback the timing of each MIDI event in the MIDI Song is proportional to the corresponding time reference. We will study this mechanism in further detail later on, but for now the simple figure below should suggest the connection between the two structures. One or more MIDI events in the MIDI Song is connected to each time reference in the

---

3 Visit Rational Corporation’s UML resource center at http://www.rational.com/ for further information
FM Song, where the time reference reflects the distance between successive pulse beats in the modulation curve.

![Diagram showing MIDI Song, FM Song, and time references]

**Figure AII. 1.** Connecting the MIDI and FM Song. Typical MIDI events are note on (hatched) and note off.

### AII.1.1. The application

FMytme is a standard MFC Multiple Document Interface (MDI) Windows application, which means that several documents can be shown in separate windows of the same application. The basic classes in the application framework are:

- CFMytmeApp – the application class
- CMainFrame – the single MDI FrameWnd class
- CCChildFrame – the MDI ChildWnd class
- CFMytmeView – the View class
- CFMytmeDoc – the Document class

Figure AII. 2 indicates how these classes work together in the specific application. Only the most important attributes and operations are shown. CMainFrame is the application window handling functionality on a global level. It contains a single CMidiPlay object, which controls playback through a CMidiDevice.
When switching between documents in the application, different MIDI sequences are given to the CMidiPlay object. CMainFrame also contains some overall application settings, such as choice of MIDI device, computation resolution \( n\_n\_Period = \) points per period in the movement curve) and display colors.

![Diagram of application structure](image)

**Figure AII. 2. Fmrtme application structure**

The application document (CFmrtmeDoc) manages all data structures including the MIDI Song (CMidiSong) and the FM Song (CFMSong). It imports data from MIDI or text files into the MIDI Song structure, initializes FM Song structures, prepares modulating operators, and computes movement curves. The actual drawing of movement curves is handled by the document view (CFmrtmeView), which also manages the different dialogs used to configure the modulation data. CChildFrame is a necessary document frame with no specific functionality in this application.

### AII.2. The MIDI Song structure

We will now take a closer look at the MIDI implementation. We will describe the different elements involved, and how they relate to each other. Importing data from MIDI or text files will be covered in this section. In the last subsection the playback of MIDI sequences is explained. The following section will show how the MIDI and FM Song structures are related. One should notice that the two structures interact only through time references and can be explained relatively independent of each other.
All.2.1. The MIDI event

Figure All. 3 gives an outline of the most important elements in our MIDI implementation. The basic element is the singular MIDI event (CMidiEvent). It contains up to 4 bytes of data, typically one status byte and two data bytes (e.g. the Note On message with note number and velocity as data bytes). The MIDI data can be retrieved with a GetEventData() operation and sent directly to the MIDI Device using the latter’s SendMidi() operation.

![Diagram of MIDI elements]

**Figure All. 3. The Midi Song structure**

The CMidiTimeEvent is a subclass of the CMidiEvent class with added time information. When recording MIDI events they are given a time tag relative the previous event recorded. Consequently a MIDI stream consists of a sequence of MIDI events, each related to the previous event with a temporal distance. For simultaneous events all but the first listed will have a time distance of zero. It is important to notice that MIDI events have no duration. Therefore two events with the same note number
are needed to fully describe a note, one for note start (Note On) and one for note end (Note Off). The temporal distance between these two events signifies the note duration.

The time information in CMidiTimeEvent objects is given as a floating point number (m_dTime) and a reference to a CTimeRef object. If this reference points to a valid object then the event time tag is multiplied with the reference time:

```cpp
double CMidiTimeEvent::GetTime()
{
    if (m_pTimeRef != NULL)
        return m_dTime * m_pTimeRef->GetTime();
    else
        return m_dTime;
}
```

Through the CTimeRef mechanism, the timing between events can easily be modified external to the MIDI structure. We will later show how this is utilized.

![Figure AII.4. The MIDI orchestration dialog](image)

### AII.2.2. The MIDI song

One or (usually many) more CMidiTimeEvent objects are collected into a linked list as part of a CMidiSequence object. This is equivalent to an instrument part in a musical arrangement. The sequence is assigned a name (m_strTitle), a MIDI channel between 1 and 16 (m_nChannel), and a sound (represented as a MIDI program, m_nProgram).
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An array of one or more CMidiSequence objects constitutes the CMidiSong object. This is then equivalent to the entire musical arrangement. Each sequence in the song may have a separate MIDI channel and a separate sound. The CMidiSong object also contains a song title and a time signature. The latter has some influence on the default modulation parameters. The MIDI song can be given different orchestrations through a Song Edit Dialog, reassigning MIDI channels and programs to each MIDI sequence (see Figure AII.4 above).

AII.2.3. Importing MIDI data from a MIDI file

The program imports MIDI data into the MIDI Song structure from files formatted according to the Standard MIDI Files format\(^4\). Three different formats are specified: 0, 1 and 2. Our application can read the first two. Format 0 MIDI files contain all data in a single multi-channel track. The entire track is read into a CMidiSequence object. Since each MIDI sequence should address one MIDI channel only, the sequence is split up into several single channel sequences (if necessary). All sequences containing CMidiTimeEvent objects are added to the current CMidiSong object (m_pSong):

```cpp
void CMidiFile::ReadFormat0()
{
    CMidiSequence* pSeq = ReadTrack();
    if (pSeq != NULL && pSeq->GetCount() != NULL)
    {
        int nTrack = 0;
        CMidiSequence* pSubSeq;
        do {
            pSubSeq = pSeq->SplitSequence();
            m_pSong->AddSequence(pSub, nTrack);
            nTrack++;
            pSeq = pSubSeq;
        } while (pSeq->GetCount() > 0);
        m_pSong->SetTracks(nTrack);
    }
}
```

Format 1 MIDI files contain one or more tracks of data, where the data on each track addresses one MIDI channel only. A split is therefore not necessary. Each track is read into a CMidiSequence object. Some tracks may however have non-MIDI information only (such as title, time signature, etc.). These tracks should not be added to the MIDI Song:

```cpp
void CMidiFile::ReadFormat1()
{
    int nTrack = 0;
    for (int i=0; i<m_nTracks; i++)
```  

\(^4\) See the Standard MIDI Files 1.0 specification distributed by the International MIDI Association

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When importing MIDI files the user is asked to specify a time resolution. MIDI Note On messages with a time separation smaller than this resolution are considered to belong to the same simultaneous chord. In performance recordings chord attacks unavoidably will be imprecise, and we do not want them to be treated as a collection of separated notes very close together. The importance of this will be obvious when we convert MIDI events into FM notes. The actual timing of events is not changed, just the way they are handled in the conversion process.

A11.2.4. Importing data from text files

An alternative format for importing data is a very simple text file with 4 numeric parameters describing each note. The underlying model is a continuous stream of notes. The format does not support simultaneous notes. Each note is described with a MIDI note number, a MIDI velocity and a temporal distance from the previous note. The latter is specified through two parameters related to the length of a quarter note: the numerator \( n \) and denominator \( m \) of a fractional factor \( n/m \). In musical terms they constitute a tie and a subdivision respectively. We also recognize these two parameters from the RFM model. In this terminology a quarter note is given as 1/1, a half note as 2/1, an eighth note as 1/2, and so on. The numerator \( n \) may be a decimal number. If the denominator \( m \) is zero, \( n \) is taken to be a factor by itself (avoiding numerical error).

The implementation reads each note in the file as a temporal distance (td, 2 numbers) followed by the two MIDI note parameters:

\[
<\text{td} \quad \text{note number} \quad \text{velocity}> \\
<\text{td} \quad \text{note number} \quad \text{velocity}>
\]

... 

It might be easier to reformulate this as an initial distance followed by notes described with note number, velocity and note duration (equivalent to the distance to the next note):

\[
<\text{td} > \\
<\text{note number} \quad \text{velocity} \quad \text{td}>
\]

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<note number  velocity  td>

...  

This corresponds better with the natural notation, and is transparent to the program. The actual duration of notes is specified as an overall fraction of the temporal distance between notes (d\text{Fraction}, see code fragment below). Each note is then split up in two MIDI events, a Note On and a Note Off. The distance from a Note On event to the corresponding Note Off event is given as: \( d\text{Fraction} \cdot td \). The distance from the Note Off event to the next Note On event is given as: \( (1 - d\text{Fraction}) \cdot td \).

File reading exits when it reads a MIDI velocity of zero or at end of file. Each file is converted into a single CMidiSequence object:

```c
C2mFile::readMidi(pTrack, double d\text{Fraction}, int nChannel)
{
  m_file >> m_dTime >> m_nSubdivision >> m_nPitch >> m_nVelocity;
  if (m_file.eof() || m_nVelocity == 0)
    return;
  if (m_nSubdivision != 0) m_dTime /= m_nSubdivision;
  CMidiSequence* pSeq = new CMidiSequence(nChannel);

  // First note on
  CHdrTimeEvent* pEvent = new CHdrTimeEvent(NOTEON+nChannel, m_nPitch, m_nVelocity);
  pEvent->SetTime(m_dTime);
  pSeq->AddEvent(pEvent);
  // First note off
  pEvent = new CHdrTimeEvent(NOTEOFF+nChannel, m_nPitch, m_nVelocity);
  pSeq->AddEvent(pEvent);
  bool bEnabled = FALSE;
  m_file >> m_dTime >> m_nSubdivision >> m_nPitch >> m_nVelocity;
  while (!m_file.eof()) {
    if (m_nSubdivision != 0) m_dTime /= m_nSubdivision;
    if (m_nVelocity == 0) {
      bEnabled = TRUE;
      break;
    }
    pEvent->SetTime(d\text{Fraction} * m_dTime);  // duration of previous note
    pEvent = new CHdrTimeEvent(NOTEON+nChannel, m_nPitch, m_nVelocity);
    pEvent->SetTime((1.0 - d\text{Fraction}) * m_dTime);
    pSeq->AddEvent(pEvent);
    pEvent = new CHdrTimeEvent(NOTEOFF+nChannel, m_nPitch, m_nVelocity);
    pSeq->AddEvent(pEvent);
    m_file >> m_dTime >> m_nSubdivision >> m_nPitch >> m_nVelocity;
  }
  pEvent->SetTime(bEnabled ? m_dTime : 1.0);
}
```

When importing MIDI files a new MIDI Song is created, and any previous data is discarded. This is not the case with text files. Each file is read into the same CMidiSong structure, and merely adds data to the current song. When importing the first sequence in a song, the user is asked to supply a time signature and a playback
tempo (this information is usually present in MIDI files). It is also possible to remove
sequences from the song after it is imported (no matter what file format it originated
from).

### AII.2.5. Playback of MIDI sequences

Due to the clear separation of the MIDI and FM structures, we can consider the
playback of modulated MIDI sequences without knowing exactly how the modulation
works. As a matter of fact rhythmic modulation parameters can be updated while the
sequences are playing, and the changes should be audible in close to real time (the
plotting of movement curves is much slower).

Going back to Figure AII. 3 we see that both CMidiSong and CMidiSequence
are subclasses of the CMidiPlayObject class and thereby inherits the listed
operations: GetMidiTime(), InitPlay(), Play() and ExitPlay(). This means that
objects of both classes are playable from the CMidiPlay object. The latter is actually
a high-resolution timer controlling a precise playback of MIDI events. As mentioned
in the section on page 254 about the application, this object is owned by CMainFrame
and is called whenever the user starts or stops playback (the two buttons to the left in
Figure AII. 5 below). The CMidiPlay object contains a CMidiDevice object, and
supplies a reference to the device when playing a CMidiPlayObject.

![Play controls in FMrytime (slightly edited)](image)

It is possible to loop playback (pressing the third button in Figure AII. 5), so
that it keeps playing until explicitly stopped. Normally playback involves all
sequences of a song, which implies that the current CMidiSong object is loaded into
CMidiPlay as a CMidiPlayObject. If one wants to listen to (and look at) just a single
sequence, one can press the SOLO button and select the sequence or voice in the spin
control at the very right in Figure AII. 5. The chosen sequence is then loaded into
CMidiPlay.

The playback tempo can be changed while playing. Each CMidiSong object
owns a CMidiTime object which instantiates a subclass of CTimeRef. Every
CMidiSequence contained in the CMidiSong has a reference to the same CMidiTime
object. When the user changes the playback tempo in the left spin control in Figure

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All 5, a new metronome value is entered into the CMidiTime object and a corresponding time factor is calculated. The CMidiPlay object accesses the time factor every time it calculates the send-out time for the next event. Finding initial playback time delay is done like this:

```cpp
CMidiPlay::Play()
{
    double time = m_pObj->InitPlay() * m_pObj->GetMidiTime()->GetTime();
    if (time < 0.0) return;
    while (time < m_nMinTime) {
        time = m_pObj->Play(m_pDevice)
            * m_pObj->GetMidiTime()->GetTime();
        if (time < 0.0) return;
    }
    ...
}
```

The CMidiPlayObject (m_pObj) in the code fragment above returns a time delay that is multiplied with the CMidiTime time factor. If the resulting time value is negative, playback is stopped. If it is less than the minimal time resolution of the counter (m_nMinTime), the event is handled immediately and a new time delay is asked for. If it is larger than m_nMinTime, a timer starts counting down the delay and later asks for a new time value (notice the m_bStopped and m_bLoop variables):

```cpp
CMidiPlay::PlayNext()
{
    if (m_bStopped) return;
    double time;
    do {
        time = m_pObj->Play(m_pDevice)
            * m_pObj->GetMidiTime()->GetTime();
        if (time < 0.0) {
            if (m_bLoop) {
                time = m_pObj->InitPlay() * m_pObj->GetMidiTime()->GetTime();
            } else return;
        } while (time < m_nMinTime);
    ...
}
```

A CMidiSequence object sends a MIDI event to the MIDI device when called with the Play(…) operation, and then returns the time delay until next event (this delay will be affected by rhythmic modulation):

```cpp
double CMidiSequence::Play(CMidiDevice* pDevice)
{
    pDevice->SendMidi(GetNextEvent())->GetEventData();
    if (m_pObj == NULL) // No more events in this sequence
        return -1.0;
    else
        return GetEvent()->GetTime();
}
```

The CMidiSong object must keep and update a sorted list of time delays for the CMidiSequence objects it contains. Whenever the Play(…) operation is called the
sequence with the shortest time delay performs its Play-operation, the sorted list is updated again, and the new shortest time delay is returned to the CMidiPlay object. When the last sequence is empty, the CMidiSong object returns a negative value.

### AII.3. The FM Song structure

In this section we will examine FM Song structure in closer detail. We will describe the different elements involved, and how they interact. We begin with looking at the modulation operators and their implementations. We then show how the FM Song structure is generated from a MIDI Song structure. Next we give a description of how the FM sequences are computed, followed by an explanation of how we synchronize the different sequences in a song. Finally we show how the movement curves are plotted.

![Diagram of FM Song structure](image)

**Figure AII.6. The modulation operators**

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AII.3.1. The modulation operator

The basic element in the FM Song structure is the COperator (see Figure AII. 6 above). It is an abstract base class containing two important operations: GetValue() and UpdatePeriod(). The first operation takes a (possible large) integer argument and computes a floating-point value based on some internal function. This internal function is usually continuous and must be sampled with some kind of resolution. The second operation UpdatePeriod() specify this resolution for a periodic function. Its floating-point argument defines the number of samples per period.

COperator has two subclasses, CFMOscillator and CSumOp. Objects of the latter class implements the sum of two COperator objects (m_pLeftOp and m_pRightOp):

```cpp
double CSumOp::GetValuelong time const
    return m_pLeftOp->GetValuelong time) + m_pRightOp->GetValuelong time);
```

CSumOp objects are used to implement parallel modulation algorithms.

CFMOscillator is a far more interesting class. It represents an oscillator with several parameters and implements the basic modulator in our FM algorithms. The available parameters are shown in the dialog in Figure AII. 7:

- m_nFunction: periodic waveform - sine, triangular, sawtooth and square
- m_nExponent: restricted to natural numbers, e.g. sin^2 x
- m_dAmplitude: equivalent to peak frequency deviation
- m_dFrequency
- m_dPhase: given as a fraction of the period

In addition the CFMOscillator class has a reference to another COperator object allowing further frequency modulations. Using a sine function the output of the modulating oscillator can be formulated as:

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\[ m_\_dAmplitude \cdot \sin^{m_\_nExponent}[2\pi \left(\frac{m_\_dFrequency}{m_\_dPeriod} \cdot \text{time} + m_\_dPhase\right) + m_\_pFreqMod\_mod(time)] \]

Expressed in C++ code this looks like:

```cpp
CFMOscillator::getValue(long time) const
{
    double dE = 2.0 * m_dPhil * (m_dFrequency/m_dPeriod) * time + m_dPhase;
    if (m_pFreqMod != NULL)
        dE += m_pFreqMod->getValue(time);
    return m_dAmplitude * pow(sin(dE), m_nExponent);
}
```

The other waveforms are equivalent, except that there is no \(2\pi\) factor. Removing this factor and exchanging \(\sin()\) with one of the following functions gives the output of the other functions:

```cpp
CFMOscillator::triangle(double expr) const
{
    expr = floor(expr); // now inside the 0.0-1.0 interval
    if (expr < 0.25)
        return 4.0 * expr;
    else if (expr > 0.75)
        return 4.0 * expr - 4.0;
    else
        return 2.0 - 4.0 * expr;
}

CFMOscillator::sawtooth(double expr) const
{
    expr = floor(expr); // now inside the 0.0-1.0 interval
    if (expr < 0.5)
        return 2.0 * expr;
    else
        return 2.0 * expr - 2.0;
}

CFMOscillator::square(double expr) const
{
    expr = floor(expr); // now inside the 0.0-1.0 interval
    if (expr == 0.0 || expr == 0.5)
        return 0.0;
    else if (expr < 0.5)
        return 1.0;
}
```

The triangle function can be particularly interesting when squared, since it then turns into a smooth parable.

CFMNote is a specialized subclass of CFMOscillator. The waveform is fixed to the expression \((1 - \cos x)\) and there is no exponent. Two additional parameters influence the operator expression: m_dLength and m_nElocity. They are directly linked to the MIDI note information. m_nElocity is identical to the MIDI velocity and relates to the CFMNote oscillator amplitude. m_dLength is equivalent with the
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temporal distance between following notes (given as the fraction \( m/n \) in the RFM model and in the text file format). The operator output is given as:

\[
expr = 2\pi \left( \frac{m \_ dFrequency}{m \_ dPeriod} \times m \_ dPhase + m \_ pFreqMod(time) \right)
\]

\[
output = m \_ nVelocity \cdot \text{dVelocityFactor} \cdot \left( 1 - \cos\left( \frac{expr}{m \_ dLength} \right) \right)
\]

The \text{dVelocityFactor} is a global floating-point variable in the range [0.0-1.0] specifying how strongly the MIDI velocity should influence the resulting amplitude. It will be the same for all CFMNote objects. Expressing the output in C++ code looks like:

```cpp
double CFMNote::GetValue(long time) const
{
    double de = 2.0 * m_dPI * (m_dFrequency/m_dPeriod) * time + m_dPhase;
    if (m_pFreqMod != NULL)
        de += m_pFreqMod->GetValue(time);
    return pow(m_nVelocity, dVelocityFactor) * (1 - cos(de/m_dLength));
}
```

When computing the movement curve another amplitude factor is involved, \text{dLengthFactor}. Through this factor the note amplitude is proportional to the actual length of the note (after modulation). Because we need to calculate the modulated note to find this length it is more natural to keep this factor outside the note object and apply it afterwards (see AII.3.4 Calculating movement curves below).

![Diagram of modulators](image)

Figure AII. 8. Complex modulation operators
CFMNote is actually the carrier oscillator in the RFM model. There is one CFMNote object for every temporally unique MIDI Note On event (chords are assigned just one object). All parameters of the CFMNote object are extracted from MIDI event data. We will return to this later. For now it is important to notice that each CFMNote object contains an object of the CFMTime class which is a subclass of CTimeRef (see Figure AII. 6). All CMidiTimeEvent objects corresponding to a certain CFMNote object get a reference to this CFMTime object. This is the only connection between the two main structures (see also Figure AII. 9 below).

Figure AII. 7 shows a simple frequency modulator with one carrier oscillator and one modulating oscillator. Since the COperator class defines a standard interface between all modulation operators, the program in principle allows any combination of CFMOscillator and CSumOp objects as modulators for the CFMNote object. For simplicity we have limited the choices to serial and a parallel frequency modulation with one added modulating oscillator as shown in Figure AII. 8.

AII3.2. The FM Song

Figure AII. 9 shows how the FM Song is structured. It should be obvious that it parallels the MIDI song structure, and there is a constructional dependency on both song and sequence level. However, as soon as the FM Song structure is established, the only connection between the two structures goes through the CFMNote – CFMTime – CTimeRef – CMidiTimeEvent chain (at the bottom of Figure AII. 9).

CFMSequence contains a linked list of at least one, usually several CFMNote objects. For a monophonic voice, this is equivalent to the sequence of notes in standard notation, and each CFMNote corresponds to two MIDI events (Note On and Note Off). In polyphonic voices, notes with simultaneous attacks are represented by just one CFMNote object, and all CMidiTimeEvent objects involved refer to this single CFMNote object. The CFMSequence object also contains a large floating-point array (m_dCurve) that holds the complete movement curve for this voice. Each CFMNote object in the sequence represents and calculates a segment of the curve – the segment between downbeats (illustrated in Figure AII. 1). Another important element of CFMSequence is the modulator, a reference to a single COperator object. This could be a CFMOscillator (as in simple or serial frequency modulation) or a CSumOp (in parallel frequency modulation). When loading a new modulator into the
sequence, each CFMNote object gets a reference to it. Subsequent calculations of movement curves will involve this modulator reference (see \( a_{pF\text{SeqMod}} \) in the equations in the previous section).

![Diagram of CFMSequence and FMPlotInfo classes](image)

Figure AII. 9. The FM Song structure

Arrays of CFMSequence objects are compiled into CFMSong objects. This class organizes the sequences, their appearance and synchronization. A special utility class called FMPlotInfo keeps all necessary information for plotting the song, such as number of lines, measures per line, beats per measure, synchronization of voices and amplitude factors. The CFMSong object manages all computations, and makes sure that only the necessary sequences are updated when the user changes modulation parameters. Modulation operators are assigned on the sequence level. It is possible to specify the same modulation on all sequences of a song, but this is implemented with identical modulators on each sequence, not a common modulator for the entire song.
11.3.3. Creating the FM structure from a MIDI Song

After importing MIDI data from file the entire CMidiSong object is time normalized so that all temporal distances are expressed as (possible fractions of) quarter notes. Then the FM Song is constructed through the call CFMSong::LoadSong(CMidiSong* pSong, double dMinTime). We notice the presence of the time resolution (dMinTime) that the user must supply when importing MIDI files (see Importing MIDI data from a MIDI file on page 258). The construction process converts each MIDI sequence into a CFMSSequence object. This process is rather complex, so we will only give an informal description here.

The algorithm runs through all events in the CMidiSequence object, and creates a CFMNote object for every rhythmically significant MIDI event. The latter is defined as a MIDI Note On event with a temporal distance to the previous significant event that is larger than the time resolution specified in dMinTime. The following parameters are set for each CFMNote:

- m_dFrequency: This is equivalent to the carrier pulse, which initially is equal to the number of beats per measure in the song (retrieved from the MIDI Song as the numerator of the time signature). This parameter is the same for all CFMNote objects in a sequence, but different sequences are allowed to have different carrier pulses (allowing different modulation periodicity between sequences).

- m_dPhase: The position of each note in the sequence is given as the accumulated time distance from the beginning (0.0) and up to this event. The CFMNote phase is equal to the negated position: m_dPhase = -pos.

- m_dDistance: This is equal to the temporal distance to the previous significant event. This number is updated with modulations, and will later influence the temporal spacing of MIDI events through the time reference CFMTime.

- m_dLength: This is equal to the temporal distance to the next significant event.

- m_nVelocity: This is equal to the MIDI velocity of the next significant event (think of the movement curve of each note as a preparation to hit the next note). m_nVelocity is involved in calculations of oscillator amplitude through the dVelocityFactor (see page 266).
Among these parameters only \texttt{m\_dFrequency} can be modified later by the user (in the FM Oscillator dialog, see Figure AII. 7). \texttt{m\_dDistance} will be recalculated for every change of modulation. The rest of the parameters represent information necessary to calculate the movement curve for this particular CFMNote and will not be changed.

In order to terminate the CFMSequence properly the very last MIDI Note Off event is considered to be significant and is assigned a CFMNote object with no length. It merely functions as a time reference for MIDI through the \texttt{m\_dDistance} variable.

![Figure AII. 10. MIDI event - FM note relationship](image)

How are the MIDI events connected to the CFMNote objects? A significant CMidiTimeEvent is assigned a CFMNote object and its internal time factor ($m\_dTime$) is set to 1.0. This means that the MIDI event's \texttt{GetTime()} operation returns 1.0 times the \texttt{m\_dDistance} variable of its time reference. All non-significant MIDI events (Note Off, Note On in chord, MIDI controller, etc.) in between the significant ones, are assigned the time reference belonging to the CFMNote for the following significant event. Their internal time factor ($m\_dTime$) is set to a fraction between 0.0 and 1.0, corresponding to their relative distance between the significant events. If the \texttt{m\_dDistance} variable is stretched, then all the connected MIDI events are stretched in a proportional fashion. Figure AII. 10 above illustrates the connection between events and notes. The number next to the events signifies the internal time factor. The significant events (checkered) have a time factor of 1.0. In the example above we assume that the time resolution ($dm\_lnTime$) is larger than 0.01, so that the second event is considered to be in a chord with the first one.

The RFM model is a representation of rhythmic performance. The movement curve do not start with the beat, but with the preparation before the beat, e.g. the raising of the hand. The first figure on the next page shows how we define the start of
the movement. It starts before time 0.0 in the MIDI event list. When calculating movement curves we have to take the start figure into account. Adding a CFMNote object at the front of the FM sequence solves this. This object has a quarter note length and position -1.0. It is not connected with the CMidiTimeEvent list. Due to the need for synchronization between sequences, this object is necessary even for sequences where the actual movement starts later.

If the sequence doesn't start at time 0.0 we have to add one or two extra CFMNote objects to track the movement curve from the start figure. In the example given in the next figure, the first note in the sequence starts at the last eighth note in the first measure. The arrows below the curve show how we assign CFMNote objects. One object captures the common movement start as before. The next object represents the temporal distance from time 0.0 to the beat before the first note. If the first note starts on the second beat or earlier, this object is not necessary. The third object captures the actual start of the movement for this sequence. It is defined to start at the beat before the note. These three CFMNote objects are only used for calculating movement curves and are not related to the MIDI events.

**AII.3.4. Calculating movement curves**

After the user imports new data or updates the modulation parameters, the movement curves are recalculated. The calculations are carried out on one CFMNote object at the time, tracking the curve from pulse beat (trough) to pulse beat, which is the segment represented by each CFMNote object. The length of the entire curve will be equal to the number of measures in the song multiplied with the number of sample points per measure. The time argument used in the COperator objects (see the section
on The modulation operator on page 264) is the numeric sample point counted from 0 at time 0.0 (tracking the start figure therefore involves negative arguments).

The tracking process is done in this order:

1) Find the pulse beat (trough) on the curve closest to time 0.0 using the first CFMNote object (the start figure). When using a modulating function with a phase offset, the pulse beat will move away from time 0.0. We call this sample offset the modulation offset (m_nModOffset).

```
CFMNote* pNote = GetNote();
m_nModOffset = 0;
double dPrevious = pNote->GetValue(0);
double dNow = pNote->GetValue(-1);
while (dNow < dPrevious) {
  dPrevious = dNow;
  m_nModOffset--;
  dNow = pNote->GetValue(m_nModOffset + 1);
}
dNow = pNote->GetValue(m_nModOffset+1);
while (dNow < dPrevious) {
  dPrevious = dNow;
  m_nModOffset++;
  dNow = pNote->GetValue(m_nModOffset + 1);
}
```

2) Track the start figure backward from the first pulse beat (found above). Looking at the CFMNote object, we see that its time function is proportional to \((1 - \cos x)\) which varies between 0.0 and 2.0. We define the start figure to start at the value 1.0 (see figure on previous page). The sample distance of the start figure is called the synchronization offset (m_nSyncOffset).

```
BOOL bTop = FALSE;
for (UINT j=1;j<=m_nSyncOffset;j++) {
  dNow = pNote->GetValue(m_nModOffset-j);
  if (bTop && dNow < dPrevious) {
    bTop = TRUE;
  } else if (bTop && dNow < 1.0) {
    bTop = FALSE;
    dPrevious = dNow;
  }
  m_nSyncOffset = --j;
}
```

3) Adjust offsets according to how the sequences in a song should be synchronized (see the next subsection).

4) Track the start figure and assign to the movement curve. This time the amplitude of the movement curve is a function of the actual length of the curve segment. This function is a fractional power with the globally chosen dLengthFactor (between 0.0 and 1.0) as exponent: 

\[
    amp = \left( \frac{\text{curve length}}{\text{hor length}} \right)^{\text{dLengthFactor}}.
\]

a) If the sequence start at time 0.0 track the first CFMNote object again.
Appendix II

b) Else if it starts later use the two extra CFMNote objects to track until the beginning of the sequence. Find the actual start of movement as the movement curve of the second object passes through 1.0.

5) For all the rest of the CFMNote objects, track from pulse beat to pulse beat to find the length (i - nPreviousTime in the code below). The length of each curve segment is normalized and expressed in terms of quarter notes. Retrack using this length to calculate the curve amplitude (not shown). The length is assigned to the CFMNote m_dDistance variable, by which it immediately changes the timing of the corresponding MIDI events.

```c
pNote = GetNextNote();
pNote->SetDistance((i - nPreviousTime) / (double) GetBeatLength());
nPreviousTime = i;
OSC.bTop = FALSE;
prevTop = 0.0;
for (i = LastSample; i < LastSample + LastSample; i++)
    dnew = pNote->GetValue(i + m_nModOffset);
    if (bTop && dnew < dPrevious)
        bTop = TRUE;
    else if (bTop && dnew > dPrevious)
        break;
    a_dCurve[mOffset + i] = dnew;
    dPrevious = dnew;
```

### All3.5. Synchronizing sequences

Unmodulated sequences all start the movement at the same point (identical synchronization offset) and they all pass through zero (the pulse beat) at time 0.0 (modulation offset equal to 0). Differently modulated sequences do not have this property. It is therefore necessary to indicate how they should be synchronized with respect to each other. We define a sequence offset, noffset, so that:

- noffset = 0, if the sequence has a pulse beat at time 0.0
- noffset < 0, if the first pulse beat comes before time 0.0
- noffset > 0, if the first pulse beat arrives after time 0.0

The user is given three choices, with corresponding consequences for sequence offsets:

1) **Common zero** – they all have a pulse beat at time 0.0:
   - noffset = 0

2) **Common startpoint** – they all start in the same point. One sequence is selected to be the reference and this sequence will be given a pulse beat at time 0.0:
   - noffset = m_nSyncOffset - pRefSeq->m_nSyncOffset

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3) **No synchronization** – the sequences are moved with respect to each other by modulation. One sequence can still be selected as reference and given a pulse beat at time 0.0 (the other ones are moved correspondingly):

- \( n\text{Offset} = m\_n\text{ModOffset} - p\text{RefSeq->m}_n\text{ModOffset} \) (with reference)
- \( n\text{Offset} = m\_n\text{ModOffset} \) (without reference)

Figure All. 11 shows the consequences of each choice on a pair of movement curves. The synchronization also affects the playback of the sequences.

![Diagram](image)

**Figure All. 11. Examples of song synchronization**

**All.3.6. Plotting**

Finally, after importing, modulating and computing data, the movement curves are ready to be plotted. This is done after all the sequences in a song are calculated, and after the modulations are audible in the MIDI playback. The plotting itself is time-consuming. With a fine-grained sampling of the curve (more than a thousand points per period) and several voices of many measures each, the graphical update may take more than half a minute. The choice of curve sampling should therefore be a sensible trade-off between time resolution and plot time. The sampling period is identical to the measure, so a measure of only two beats could be sampled less frequently than one with 6 beats and still maintain the same resolution.

A few choices have been made when it comes to the plot layout. The entire song is shown at once in the view. The curves are shown on three lines, so that the number of measures on each line varies with the length of the song. Songs of more than 30 measures will definitely look crowded on this display. The user may hide the bars between measures. The carrier pulse of each sequence is by default equal to the number of beats per measure in the song. The former is strongly connected with the periodicity of the modulations, and it can be changed for each sequence individually and thereby also deviate from the beat count. Conversely, the number of beats per measure can be changed with or without updating the carrier pulse for all sequences in the song.
The different sequences can be assigned different colors to help separate them visually (similar to the orchestration of sequences that help separate them audibly). The color selection remains as application default in the Windows registry, together with the selected MIDI device and the sample rate period count. The factors controlling how strong the MIDI velocity and the note length should influence the curve amplitude and the selection of song synchronization are also saved in the registry as document independent defaults.

![Figure 11.12. Display with position markers](image)

Instead of setting up enumerated axes on the curve display, we allow the user to retrieve numerical information through the use of markers and the mouse. When moving the mouse across the curve display, the sequence position (x) and the curve amplitude (y) is shown on the status bar at the bottom of the display (see Figure 11.12). The sequence position is given in terms of measures. The bar starting measure 1 will be 1.00, the bar separating measure 1 and 2 will be 2.0. Any position in measure 1 will be a number between these two. Markers are useful when one wants to find the distance between two pulse beats, especially after modulation. Clicking the left mouse button at the desired position sets a left marker, and the right marker is set similarly clicking the right mouse button. The value of each marker is shown on the status bar together with the difference between them \((x_{\text{diff}} = x_{\text{right}} - x_{\text{left}}, y_{\text{diff}} = y_{\text{right}} - y_{\text{left}})\).
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Figure AII.12 shows an example with two markers in the second measure (circled on the figure) and a corresponding status bar ($x_{\text{left}} = 2.01$, $x_{\text{right}} = 2.66$, $x_{\text{diff}} = 0.65$).

AII.4. References

Appendix III

Contents on the Audio Recording
of RFM Syntheses

Each example on the enclosed CD is created by applying the computer realization of RFM installed on an ordinary PC with Windows 98 and MIDI implementation. (Cf. Chapter 6 and Appendix II.) The sounds are taken from the GM (General MIDI) library of a non-expensive Korg X 5D synthesizer. No manipulations of the sounds have been made, i.e. the sounds are used as preset from the factory. All the examples are first recorded on a DAT cassette by the author, whereafter they are copied and mastered on CD by studio engineer Roger Valstad. With reference to Chapter 7 where the construction of each example is explained, the contents on the CD are as follows:

<table>
<thead>
<tr>
<th>Track 01: $V_0$:</th>
<th>Metronomic performance of Vienna waltz acc.</th>
<th>Sound: Piano.</th>
<th>0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Track 02: $V_1$:</td>
<td>Vienna waltz acc. S-L-I (26-42-32)</td>
<td>Sound: Piano.</td>
<td>0.10</td>
</tr>
<tr>
<td>Track 03: $V_2$:</td>
<td>Vienna waltz acc. S-L-I (28-40-32)</td>
<td>Sound: Piano.</td>
<td>0.10</td>
</tr>
<tr>
<td>Track 04: $V_3$:</td>
<td>Vienna waltz acc. <em>extreme</em> S-L-I (17-61-22)</td>
<td>Sound: Piano.</td>
<td>0.10</td>
</tr>
<tr>
<td>Track 05: $V_4$:</td>
<td>Vienna waltz acc. where beat distances vary throughout the performance (modulation of modulation).</td>
<td>Sound: Piano.</td>
<td>0.10</td>
</tr>
</tbody>
</table>
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Track 06: $K_6$ : Metronomic performance of "springar":
The melody: "Kjerringa med staven"
Sound: Piano.

Track 07: $K_7$ : Telemark dialect of "Kjerringa..." (melody)
L-I-S (39-33-28 (7-6-5))
Sound: Piano.

Track 08: $K_8$ : 3 voice Telemark performance of "Kjerringa...
L-I-S (7-6-5) Synchronization between the voices.
Sounds: 1: Guitar w. nylon str., 2: Muffled guitar,
3: Acoustic bass.

Track 09: $K_9$ : 3 voice Valdres performance of "Kjerringa..."
S-L-I (5-7-6) Synchronization between the voices.
Sounds: Same as $K_2$.

Track 10: $K_{10}$ : 3 voice performance of "Kjerringa..."
Non-synchronization between the voices.
To some extent in the Telemark tradition.
Sounds: Same as $K_2$.

Track 11: $K_{11}$ : 3 voice performance of "Kjerringa..."
Non-synchronization between the voices.
Lack of common consistency.
Sounds: Same as $K_2$.

Track 12: $S_6$ : Metronomic performance of cymbal swing rhythm.
Subdivision in eighth note triplets.
Sound: Ride cymbal.

Track 13: $S_7$ : Modulation creating subdivision in sixteenth notes.
Sound: Ride cymbal.

Track 14: $S_8$ : Modulation creating subdivision in eighth notes.
Sound: Ride cymbal.

Track 15: $S_9$ : Rhythmic morphing from subdivision in eighths
to subdivision in sixteenths by means of
modulation of modulation.
Sound: Ride cymbal.

Track 16: $S_{10}$ : Subdivision varies, the first beat of every measure
is metronomic, the other quarter note beats are
moved. Periodic, period equals two measures.
Sound: Ride cymbal.

Track 17: $S_{11}$ : Subdivision varies, all quarter note beats are moved.
Sound: Ride cymbal.

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Track 18: B&D₆: Metronomic performance of a bass and drums swing rhythm.
Subdivision in eighth note triplets.
Sounds: Acoustic bass, ride cymbal and hi-hat.

Track 19: B&D₇: Modulation creating subdivision in sixteenth notes.
Synchronization between the voices.
Sounds: Same as B&D₆.

Track 20: B&D₈: Modulation creating subdivision in eighth notes.
Synchronization between the voices.
Sounds: Same as B&D₆.

Track 21: B&D₉: The drummer is "rushing", playing "ahead" of the bass player, both are subdividing in sixteenths.
Non-synchronization between the voices, but the discrepancies are uniform (i.e. unchanged).
Sounds: Same as B&D₆.

Track 22: B&D₁₀: The discrepancies vary throughout the performance.
The drummer is consistent in his playing, whereas the bass player is varying between different subdivisions.
Sounds: Same as B&D₆.

Track 23: B&D₁₁: Bass and drums are synchronized, both being, in an identical manner, out of sync with a metronomic performance. Suggests a "new esthetics of swing performance".
Sounds: Same as B&D₆.

Track 24: B&D₁₂: The bass player is performing as in B&D₁₁, whereas the drummer is performing in a somewhat more "usual" way compared to ordinary standards.
Non-synchronization between the voices, varying discrepancies.
Sounds: Same as B&D₆.

Track 25: A&R: Accelerando and ritardando.
Sound: Acoustic bass.

Track 26: DR: A drum roll exercise.
Sound: Snare drum.

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Track 27:  
P: "Phasing" à la Steve Reich.
Two identical voices moving out of sync –
back into sync.
Sounds: Percussion.  

Track 28:  
BI: A "pathological" two-part Bach invention:
"2-Part Invention No.13 in A minor"
Different modulations are given to the right
and left hand.
Whenever the 1st voice is making an accelerando,
the 2nd voice is making a ritardando, and vice versa.
Exemplifies a non-real rhythmic performance.
Sound: Piano.
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