1

On Capacity and Delay of Multi-channel Wireless Networks with Infrastructure Support

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Abstract-In this paper, we propose a novel multi-channel network with infrastructure support, called an MC-IS network, which has not been studied in the literature. To the best of our knowledge, we are the first to study such an MC-IS network. Our proposed MC-IS network has a number of advantages over three existing conventional networks, namely a single-channel wireless ad hoc network (called an SC-AH network), a multichannel wireless ad hoc network (called an MC-AH network) and a single-channel network with infrastructure support (called an SC-IS network). In particular, the network capacity of our proposed MC-IS network is $\sqrt{n \log n}$ times higher than that of an SC-AH network and an MC-AH network and the same as that of an SC-IS network, where n is the number of nodes in the network. The average delay of our MC-IS network is $\sqrt{\log n/n}$ times lower than that of an SC-AH network and an MC-AH network, and $\min\{C_I, m\}$ times lower than the average delay of an SC-IS network, where C_I and m denote the number of channels dedicated for infrastructure communications and the number of interfaces mounted at each infrastructure node, respectively. Our analysis on an MC-IS network equipped with omni-directional antennas only has been extended to an MC-IS network equipped with directional antennas only, which are named as an MC-IS-DA network. We show that an MC-IS-DA network has an even lower delay of $\frac{c}{|\frac{2\pi}{L}| \cdot C_I}$ compared with an SC-IS network and our MC-IS network. For example, when $C_I = 12$ and $\theta = \frac{\pi}{12}$, an MC-IS-DA can further reduce the delay by 24 times lower that of an MC-IS network and reduce the delay by 288 times lower than that of an SC-IS network.

I. INTRODUCTION

How to improve the network performance, in terms of the network capacity and the average delay, has been a key issue in recent studies [1]. Conventional wireless networks typically consist of nodes that share one single channel for communications. It is found in [2], [3] that in a random ad hoc network with n nodes, each node has a throughput capacity of $\Theta(W/\sqrt{n\log n})$ (where W is the total network bandwidth) and the average delay of this network is $\Theta(\sqrt{n/\log n})$. When the number of nodes increases, the per-node throughput decreases and the average delay increases. One major reason is that all the nodes within the network share the *same* medium. When a node transmits, its neighboring nodes in the same

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channel are prohibited from transmitting to avoid interference. Besides, multi-hop and short-ranged communications are preferred in this network in order to minimize the interference and achieve the high network capacity [2]. However, the multi-hop communications inevitably lead to the high end-to-end delay. Furthermore, every node equipped with a single interface cannot transmit and receive at the same time (i.e., the half-duplex constraint). We name this single-channel ad hoc network as an *SC-AH* network.

One approach to improve the network performance is to use multiple channels instead of a single channel in a wireless network. The experimental results of [4]–[9] show that using multiple channels can significantly improve the network throughput. One possible reason for the improvement is that using multiple channels can separate multiple concurrent transmissions in frequency domains so that the interference can be mitigated. Another reason is that multiple simultaneous transmissions/receptions are supported by multiple network interfaces mounted at a wireless node, consequently leading to the improved frequency reuse and the increased throughput. However, it is shown in [2] [8] that each channel (or up to $O(\log n)$ channels) must be utilized by a dedicated interface at a node in order to fully utilize all the channels simultaneously so that the network capacity can be maximized. When the condition is not fulfilled, the capacity degrades significantly. Besides, the average delay of an MC-AH network is also $\Theta(\sqrt{n/\log n})$, which increases significantly with the increased number of nodes. We call this multi-channel wireless ad hoc network as an MC-AH network.

Recent studies [10]–[15] investigated the performance improvement by adding a number of infrastructure nodes to a wireless network. Specifically, as shown in [10], [14], deploying infrastructure nodes in the wireless network can significantly improve the network capacity and reduce the average delay. But, every node in such a network equipped with a single interface cannot transmit and receive at the same time. Besides, only one single channel is used in such a network. We call this single-channel networks with infrastructure support as an *SC-IS* network.

In this paper, we propose a novel multi-channel network with infrastructure support that overcomes the above drawbacks of existing networks. This network consists of *common nodes*, each of which has a single interface, and *infrastructure nodes* (or base stations), each of which has multiple interfaces. Both common nodes and base stations can operate on different channels. This multi-channel wireless network with infrastructure support is called an *MC-IS* network that has the following characteristics.

TABLE I
COMPARISON WITH OTHER EXISTING WIRELESS NETWORKS

	Pure Ad Hoc	Ad Hoc with Infrastructure
Single Channel	SC-AH networks	SC-IS networks
	[2], [3]	[10]–[17]
Multiple Channels	MC-AH networks	MC-IS networks
	[4]–[9]	(this paper)

- Each common node is equipped with a single network interface card (NIC). Each base station is equipped with multiple NICs.
- There are multiple non-overlapping channels available.
 Each NIC at either a common node or a base station can switch to different channels quickly.
- Base stations are connected via a wired network that has much higher bandwidth than a wireless network.
- Each common node with a single NIC can communicate with either another common node or a base station, where a communication with another common node is called an ad-hoc communication and a communication with a base station is called an infrastructure communication. But, a common node supports only one transmission or one reception at a time. Besides, it cannot simultaneously transmit and receive (i.e., it is in a *half-duplex* mode).
- Each base station with multiple NICs can communicate
 with more than one common node. In addition, a base
 station can also work in a *full-duplex* mode, i.e., transmissions and receptions can occur in parallel.

Our proposed *MC-IS* networks have provided a solution to the new applications, such as *Device-to-Device* (D2D) networks [18], wireless sensor networks (WSNs), smart grid and smart home [19], [20]. For example, the theoretical analysis on the throughput and the delay of *MC-IS* networks can be used to analyze the performance of the *overlaid* D2D networks (refer to Section VII-C for more details).

Table I compares our proposed MC-IS networks with other existing networks, where one can observe that MC-IS networks can fully exploit the benefits of both MC-AH networks and SC-IS networks and can potentially have a better network performance (in terms of the network capacity and the delay) than other existing networks. However, to the best of our knowledge, there is no theoretical analysis on the capacity and the average delay of an MC-IS network. The goal of this paper is to investigate the performance of an MC-IS network and to explore the advantages of this network. The primary research contributions of our paper are summarized as follows.

- (1) We formally identify an MC-IS network that characterizes the features of multi-channel wireless networks with infrastructure support. To the best of our knowledge, the capacity and the average delay of an MC-IS network have not been studied before.
- (2) We propose a *general* theoretical framework to analyze the capacity and the average delay. We show that other existing networks can be regarded as special cases of our *MC-IS* network in our theoretical framework. Besides, we find that our *MC-IS* networks are limited by *four requirements* (to be defined in Section IV) *simultaneously* but the existing networks are only limited by subsets

- of them (not all of them). This means that studying the performance of our *MC-IS* networks is more challenging but it is more useful and realistic to consider four requirements simultaneously since they exist naturally in real life applications.
- (3) Our proposed MC-IS network has a lot of advantages over existing related networks. In particular, an MC-IS network can achieve the *optimal* per-node throughput W, which is $\sqrt{n \log n}$ times higher than that of an SC-AH network and an MC-AH network and the same as that of an SC-IS network, while maintaining the smallest delay, which is $\sqrt{\log n/n}$ times lower than that of an SC-AH network and an MC-AH network, and $\min\{C_I, m\}$ times lower than that of an SC-IS network. The performance improvement mainly owes to the multiple NICs at a base station, compared with a single NIC at a base station in SC-IS networks. As a result, our MC-IS networks have a better performance than SC-IS networks though the theoretical analysis is also more complicated than that of SC-IS networks.
- (4) We also extend our MC-IS networks with the consideration of using directional antennas instead of omnidirectional antennas. Specifically, all aforementioned networks (i.e., SC-AH networks, MC-AH networks, SC-IS networks and our MC-IS networks) are equipped with omni-directional antennas but the extended MC-IS networks have both the base stations and all common nodes equipped with directional antennas. We name the extended MC-IS networks as MC-IS-DA networks. We show that an MC-IS-DA network can have an even lower delay of $\frac{c}{\left|\frac{2\pi}{L}\right| \cdot C_I}$ compared with both an MC-IS network and an SC-IS network, where θ is the beamwidth of a directional antenna mounted at the base station (usually $\theta < 2\pi$). Consider the case of $C_I = 12$ and $\theta = \frac{\pi}{12}$ that is feasible in Millimeter-Wave systems [21]. An MC-IS-DA can further reduce the delay by 24 times lower than that of an MC-IS network and reduce the delay by 288 times lower than that of an SC-IS network.

The remainder of the paper is organized as follows. Section II presents a survey on the related studies to our *MC-IS* network. We present the models used in this paper in Section III. Section IV then summarizes our main results. We next derive the capacity and the delay contributed by *ad hoc communications* in an *MC-IS* network in Section V. Section VI presents the capacity and the delay contributed by *infrastructure communications* in an *MC-IS* network. We extend our analysis with the consideration of directional antennas as well as the mobility and provide the implications of our results in Section VII. Finally, we conclude the paper in Section VIII.

II. RELATED WORKS

We summarize the related works to our study in this section. The first network related to our proposed *MC-IS* network is an *SC-AH* network. An *SC-AH* network has a poor performance due to the following reasons: (i) the interference among multiple concurrent transmissions, (ii) the number of simultaneous transmissions on a single interface and (iii) the multi-hop communications [2], [3].

The second network related to our MC-IS network is an MC-AH network, in which multiple channels instead of a single channel are used. Besides, each node in such a network is equipped with multiple NICs instead of single NIC. This network has a higher throughput than an SC-AH network because each node can support multiple concurrent transmissions over multiple channels. However, this network suffers from the high delay and the increased deployment complexity. The average delay of an MC-AH network is the same as that of an SC-AH network, which increases significantly with the number of nodes. The deployment complexity is mainly due to the condition [8] that each channel (up to $O(\log n)$ channels) must be utilized by a dedicated interface at a node so that all the channels are fully utilized simultaneously. When the condition is not fulfilled, the capacity degrades significantly.

The third network related to our *MC-IS* network is an *SC-IS* network [10]–[17], [22]. It is shown in [10], [14] that an *SC-IS* network can significantly improve the network capacity and reduce the average delay. However, an infrastructure node in such a network equipped with a single interface cannot transmit and receive at the same time (i.e., the half-duplex constraint is still enforced). Thus, the communication delay in such an *SC-IS* network is still not minimized. Besides, such *SC-IS* networks also suffer from the poor spectrum reuse.

The fourth network related to our *MC-IS* network is a multi-channel wireless mesh network with infrastructure support (called an *MC-Mesh-IS* network) [23]–[28], which is the evolution of multi-channel multi-interface wireless mesh networks (called an *MC-Mesh* network) [29], [30]. An *MC-Mesh-IS* network is different from our *MC-IS* network due to the following characteristics of an *MC-Mesh-IS* network:

- (i) a typical *MC-Mesh-IS* network consists of *mesh clients*, *mesh routers* and *mesh gateways* while an *MC-IS* network consists of common nodes and infrastructure nodes.
- (ii) different types of communications exist in the multi-tier hierarchical MC-Mesh-IS network, which are far more complicated than an MC-IS network. For example, there are communications between mesh clients, communications between mesh gateways, and communications between a mesh gateway and a mesh router.
- (iii) an MC-Mesh-IS network uses wireless links to connect the backbone networks (corresponding to the infrastructure network in an MC-IS network). As a result, the assumption of the unlimited capacity and the interferencefree infrastructure communications in an MC-IS network does not hold for an MC-Mesh-IS network.
- (iv) the traffic source of an MC-Mesh-IS network is either from a mesh client or from the Internet while the traffic always originates from an MC-IS network.

Therefore, the analytic framework on the capacity and the delay of such *MC-Mesh-IS* networks is significantly different from that of an *MC-IS* network.

In this paper, we analyze the capacity and the delay of an *MC-IS* network. Although parts of the results on the analysis on the capacity and the delay contributed by ad hoc communications have appeared in [31], our analysis in this paper significantly differs from the previous work in the following aspects.

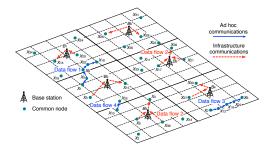


Fig. 1. Network topology of an MC-IS network

- We derive the capacity and the delay of an *MC-IS* network contributed by infrastructure communications in this paper while [31] only addresses the capacity and the delay contributed by ad hoc communications.
- We fully investigate the capacity and the delay of an *MC-IS* network with consideration of both infrastructure communications and ad hoc communications. Specifically, we also analyze the average delay and the optimality of our results, all of which have not been addressed in [31].
- We also compare our results with other existing networks, such as an SC-AH network, an MC-AH network and an SC-IS network and analyze the generality of our MC-IS network in this paper.
- We extend our analysis with consideration of using directional antennas in an MC-IS network. Discussions on the mobility are also presented in this paper.

III. MODELS

We adopt the asymptotic notations [32] in this paper. We then describe the *MC-IS* network model in Section III-A. Section III-B next gives the definitions of the throughput capacity and the delay.

A. MC-IS Network Model

Take Fig. 1 as an example of MC-IS networks. In this network, n common nodes are randomly, uniformly and independently distributed on a unit square plane A. Each node is mounted with a single interface that can switch to one of C available channels. Each node can be a data source or a destination. All the nodes are homogeneous, which means that they have the same transmission range. In addition, there are b infrastructure nodes, which are also called base stations interchangeably throughout the whole paper. We assume that b can be expressed as a square of a constant b_0 (i.e., b_0^2) where b_0 is an integer in order to simplify our discussion. Each base station is equipped with m interfaces and each interface is associated with a single omni-directional antenna, which can operate on one of C channels. The plane Ais evenly partitioned into b equal-sized squares, which are called BS-cells. Similar to [10], [14], [15], we also assume that a base station is placed at the center of each BS-cell. Unlike a node, a base station is neither a data source nor a destination and it only helps forwarding data for nodes. All the base stations are connected through a wired network without capacity constraint and delay constraint.

There are two kinds of communications in an MC-IS network: (i) Ad hoc communications between two nodes, which often proceed in a multi-hop manner; (ii) Infrastructure communications between a node and a base station, which span a single hop. An infrastructure communication consists of an uplink infrastructure communication from a node to a base station, and a downlink infrastructure communication from a base station to a node.

In the following, we describe two major components for network communications. The first component is the routing strategy. The second component is the interference model.

1) Routing Strategy: In this paper, we consider the H-maxhop routing strategy, in which, if the destination is located within $H(H \ge 1)$ hops from the source node, data packets are transmitted through ad hoc communications. Otherwise, data packets are forwarded to the base station through infrastructure communications (i.e., the uplink infrastructure communication). The base station then relays the packets through the wired network. After the packets arrive at the base station that is closest to the destination node, the base station then forwards the packets to the destination node (i.e., the downlink infrastructure communication). Take Fig. 1 as the example again. Data flow 1 starts from node X_1 to node X_{16} in the multi-hop ad hoc manner since node X_{16} is within H hops from node X_1 . With regard to Data flow 2, since destination node X_{28} is far from source node X_{36} , data packets are transmitted from source node X_{36} to its nearest base station B_3 first and then are forwarded through the wired network till reaching base station B_5 that finally sends the data packets to destination node X_{28} .

The H-max-hop routing strategy can avoid the problem that arises by using the k-nearest-cell routing strategy in the case of two nodes near the boundary of two adjacent BS-cells. For example, Data flow 4 as shown in Fig. 1 starting from node X_{10} to destination node X_{25} will be transmitted in one hop by ad hoc communications according our H-max-hop routing strategy. However, in the k-nearest-cell routing strategy [10], node X_{10} has to transmit to its nearest BS (i.e., B_3) first and then B_3 forwards the data packets through the wired network till they reach B_2 , which is the nearest BS to node X_{25} . This problem may result in inefficient use of bandwidth resources.

It is obvious that when there is an uplink communication, there is always a downlink communication. We then divide the total bandwidth of W bits/sec into three parts: (1) W_A for ad hoc communications, (2) $W_{I,U}$ for uplink infrastructure communications and (3) $W_{I,D}$ for downlink infrastructure communications. Since $W_{I,U}$ is equal to $W_{I,D}$, it is obvious that $W = W_A + W_{I,U} + W_{I,D} = W_A + 2W_{I,U}$. To simplify our analysis, we use W_I to denote either $W_{I,U}$ or $W_{I,D}$. Corresponding to the partition of the bandwidth, we also split the C channels into two disjoint groups C_A and C_I , in which C_A channels are dedicated for ad hoc communications and C_I channels are dedicated for infrastructure communications. Thus, $C = C_A + C_I$. Besides, each base station is mounted with m NICs, which serve for both the uplink traffic and the downlink traffic. It is obvious that the number of NICs serving for the uplink traffic is equal to the number of NICs serving for the downlink traffic. So, m must be an even number.

2) Interference model: In this paper, we consider the interference model [2], [8], [10]–[12], [14]. When node X_i transmits to node X_j over a particular channel, the transmission is successfully completed by node X_j if no node within the transmission range of X_j transmits over the same channel. Therefore, for any other node X_k simultaneously transmitting over the same channel, and any guard zone $\Delta>0$, the following condition holds.

$$dist(X_k, X_j) \ge (1 + \Delta)dist(X_i, X_j)$$

where $\operatorname{dist}(X_i, X_j)$ denotes the distance between two nodes X_i and X_j . Note that the *physical interference* model [2] is ignored in this paper since the physical model is equivalent to the interference model when the *path loss exponent* is greater than two (it is common in a real world [2], [33]).

The interference model applies for both ad hoc communications and infrastructure communications. Since ad hoc communications and infrastructure communications are separated by different channels (i.e., C_A and C_I do not overlap each other), the interference only occurs either between two ad hoc communications or between two infrastructure communications.

B. Definitions of Throughput Capacity and Delay

The notation of throughput of a transmission from a node X_i to its destination node X_j is usually defined as the number of bits that can be delivered from X_i to X_j per second. The aggregate throughput capacity of a network is defined to be the total throughput of all transmissions in the network. The per-node throughput capacity of a network is defined to be its aggregate throughput capacity divided by the total number of transmissions (or all nodes involved in transmissions). In this paper, we mainly concentrate on the per-node throughput capacity and the average delay, which are defined as follows.

Definition 1: Feasible per-node throughput. For an MC-IS network, a throughput of λ (in bits/sec) is feasible if by ad hoc communications or infrastructure communications, there exists a spatial and temporal scheme, within which each node can send or receive λ bits/sec on average.

Definition 2: Per-node throughput capacity of an MC-IS network with the throughput of λ is of order $\Theta(g(n))$ bits/sec if there are deterministic constants h>0 and $h'<+\infty$ such that

$$\lim_{n\to\infty} P(\lambda=hg(n) \text{ is feasible})=1 \text{ and } \lim_{n\to\infty}\inf P(\lambda=h'g(n) \text{ is feasible})<1.$$

In this paper, the per-node throughput capacity of an MC-IS network is expressed by $\lambda = \lambda_a + \lambda_i$, where λ_a and λ_i denote the throughput capacity contributed by the ad hoc communications and the infrastructure communications, respectively. Besides, we use T, T_A , T_I to denote the feasible aggregate throughput, the feasible aggregate throughput contributed by ad hoc communications, and the feasible aggregate throughput contributed by infrastructure communications, respectively.

Definition 3: Average Delay of an MC-IS network. The delay of a packet is defined as the time that it takes for the packet to reach its destination after it leaves the source [3]. After averaging the delay of all the packets transmitted in

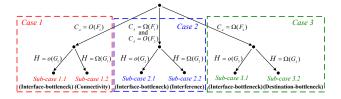


Fig. 2. All possible sub-cases considered

the whole network, we obtain the average delay of an MC-IS network, denoted by D.

The average delay of an MC-IS network is expressed by $D=D_a+D_i$, where D_a and D_i denote the delay contributed by ad hoc communications and the delay contributed by infrastructure communications, respectively. To derive the average delay in this paper, we consider the *fluid model* proposed by A. El. Gamal et al. in [3]. In this model, the packet size is allowed to be arbitrarily small so that the time taken for transmitting a packet may only occupy a small fraction of one time slot, implying that multiple packets can be transmitted within one time slot. The fluid model can be easily extended to the case of the packet with constant size as shown in [34]. Note that we do not count the delay caused by the infrastructure communications within the wired network. Besides, we also ignore the queuing delay in this model.

In order to compare the optimality of our results with the existing ones, we introduce the *optimal* per-node throughput capacity λ_{opt} , which is the maximum achievable per-node throughput capacity, and the *optimal* average delay D_{opt} , which is the average delay when the optimal per-node throughput capacity λ_{opt} is achieved.

IV. MAIN RESULTS

We first present the four requirements that limit the capacity of an *MC-IS* network in Section IV-A. Section IV-B then gives the main results.

A. Four Requirements

We have found that the capacity of an MC-IS network is mainly limited by four requirements simultaneously: (i) Connectivity requirement - the need to ensure that the network is connected so that each source node can successfully communicate with its destination node; (ii) Interference requirement - two receivers simultaneously receiving packets from two different transmitters must be separated with a minimum distance to avoid the interference between the two transmissions for the two receivers; (iii) Destination-bottleneck requirement - the maximum amount of data that can be simultaneously received by a destination node; (iv) Interface-bottleneck requirement - the maximum amount of data that an interface can simultaneously transmit or receive. Besides, each of the four requirements dominates the other three requirements in terms of the throughput of the network under different conditions on C_A and H.

Our findings are significantly different from the previous studies in SC-AH networks, MC-AH networks and SC-IS networks, which are limited by only subsets of the four

requirements. For example, the capacity of SC-AH networks and SC-IS networks is limited by Connectivity requirement and Interference requirement as shown in [2] and [10] while the capacity of MC-AH networks is limited by Connectivity requirement, Interference requirement and Interface-bottleneck requirement [8]. As a result, our analysis on an MC-IS network is far more challenging than those in the previous studies.

More specifically, as shown in Fig. 2, C_A can be partitioned into 3 cases: Case 1 corresponding to the case when $C_A = O(F_1)$, Case 2 corresponding to the case when $C_A = \Omega(F_1)$ and $C_A = O(F_2)$, and Case 3 corresponding to the case when $C_A = \Omega(F_2)$, where $F_1 = \log n$ and $F_2 = n(\frac{\log\log{(H^2\log{n})}}{\log{(H^2\log{n})}})^2$.

Under each of the above cases, H can be partitioned into two sub-cases. Under $Case\ 1$, H is partitioned into 2 sub-cases, namely $Sub\text{-}case\ 1.1$ and $Sub\text{-}case\ 1.2$. $Sub\text{-}case\ 1.1$ is when $H=o(G_1)$ and $Sub\text{-}case\ 1.2$ is when $H=\Omega(G_1)$, where $G_1=n^{\frac{1}{3}}/\log^{\frac{2}{3}}n$. Under $Case\ 2$, H is partitioned into 2 sub-cases, namely $Sub\text{-}case\ 2.1$ and $Sub\text{-}case\ 2.2$. $Sub\text{-}case\ 2.1$ is when $H=o(G_2)$ and $Sub\text{-}case\ 2.2$ is when $H=\Omega(G_2)$, where $G_2=n^{\frac{1}{3}}C_A^{\frac{1}{6}}/\log^{\frac{1}{2}}n$. Under $Case\ 3$, H is partitioned into 2 sub-cases, namely $Sub\text{-}case\ 3.1$ and $Sub\text{-}case\ 3.2$. $Sub-case\ 3.1$ is when $H=o(G_3)$ and $Sub\text{-}case\ 3.2$ is when $H=\Omega(G_3)$, where $G_3=n^{\frac{1}{2}}/\log^{\frac{1}{2}}n$. Fig. 2 shows all possible sub-cases we consider.

Each requirement dominates the other at least one sub-case under different conditions as follows.

- Connectivity Condition: corresponding to Sub-case 1.2 in which Connectivity requirement dominates.
- *Interference Condition*: corresponding to *Sub-case 2.2* in which *Interference requirement* dominates.
- Destination-bottleneck Condition: corresponding to Subcase 3.2 in which Destination-bottleneck requirement dominates.
- Interface-bottleneck Condition: corresponding to Subcase 1.1, Sub-case 2.1, or Sub-case 3.1, in which Interface-bottleneck requirement dominates.

B. Summary of Results

We summarize the main results as follows.

1. Throughput and Delay for an MC-IS network

Theorem 1: The per-node throughput λ for an MC-IS network has four regions as follows.

- i) When Connectivity Condition is satisfied, $\lambda = \Theta\left(\frac{W_A}{H\log n}\right) + \Theta\left(\min\{\frac{b}{n}, \frac{bm}{nC_I}\}W_I\right)$, where $\lambda_a = \Theta\left(\frac{W_A}{H\log n}\right)$ and $\lambda_i = \Theta\left(\min\{\frac{b}{n}, \frac{bm}{nC_I}\}W_I\right)$; ii) When Interference Condition is satisfied, $\lambda = \frac{1}{N}$
- ii) When Interference Condition is satisfied, $\lambda = \Theta\left(\frac{W_A}{C_A^{\frac{1}{2}}H\log^{\frac{1}{2}}n}\right) + \Theta(\min\{\frac{b}{n}, \frac{bm}{nC_I}\}W_I)$, where $\lambda_a = \Theta\left(\frac{W_A}{C_A^{\frac{1}{2}}H\log^{\frac{1}{2}}n}\right)$ and $\lambda_i = \Theta\left(\min\{\frac{b}{n}, \frac{bm}{nC_I}\}W_I\right)$;
- iii) When Destination-bottleneck Condition is satisfied, $\lambda = \Theta\left(\frac{n^{\frac{1}{2}}\log\log(H^2\log n)W_A}{C_AH\log^{\frac{1}{2}}n\cdot\log(H^2\log n)}\right) + \Theta(\min\{\frac{b}{n},\frac{bm}{nC_I}\}W_I),$ where $\lambda_a = \Theta\left(\frac{n^{\frac{1}{2}}\log\log(H^2\log n)W_A}{C_AH\log^{\frac{1}{2}}n\cdot\log(H^2\log n)}\right)$ and $\lambda_i = \Theta\left(\min\{\frac{b}{n},\frac{bm}{nC_I}\}W_I\right);$

iv) When Interface-bottleneck Condition is satisfied,
$$\lambda = \Theta\left(H^2\frac{\log n}{n}\cdot\frac{W_A}{C_A}\right) + \Theta(\min\{\frac{b}{n},\frac{bm}{nC_I}\}W_I)$$
, where $\lambda_a = \Theta\left(H^2\frac{\log n}{n}\cdot\frac{W_A}{C_A}\right)$ and $\lambda_i = \Theta\left(\min\{\frac{b}{n},\frac{bm}{nC_I}\}W_I\right)$.

Theorem 2: The average delay of all packets in an MC-IS network is $D = \Theta\left(\frac{H^3 \log n}{n}\right) + \Theta\left(\frac{c}{\min\{C_I, m\}}\right)$, where $D_a = \Theta\left(\frac{H^3 \log n}{n}\right)$ and $D_i = \Theta\left(\frac{c}{\min\{C_I, m\}}\right)$.

2. Overview of Our Proof

Since ad hoc communications and infrastructure communications are carried in two disjoint channel groups C_A and C_I , we will derive the bounds on the capacity and the delay contributed by the two communications separately. In particular, we first obtain the bounds on the the capacity contributed by ad hoc communications in Section V. More specifically, we will derive the upper bounds on the capacity by consideration of the aforementioned four requirements and then prove the lower bounds by constructing the cells, designing routing scheme and TDMA scheme properly. Although our approach is the integration of the previous studies on SC-IS networks [14] and MC-AH networks [8], our solution is non-trivial due to the following reasons: (i) the capacity of MC-IS networks is limited by the aforementioned four conditions simultaneously while those of SC-IS networks and MC-AH networks are only limited by subsets of the four conditions; (ii) as a result, we need to redesign the cell construction, the routing scheme and the scheduling scheme based on various factors (such as H, C_A and n), which are not straight-forward. Details about our proof on ad hoc communications will be given in Section V. We will next derive the capacity contributed by infrastructure communications in Section VI. Similarly, we need to construct BS-cells, design routing scheme and TDMA scheme in this phrase while these constructions are different from those of ad hoc communications. The complete proof of Theorem 1 and Theorem 2 will be given in Section VI.

3. Generality of MC-IS Networks

Our proposed *MC-IS* network offers a more general theoretical framework than other existing networks. In particular, other networks such as an *SC-AH* network [2], an *MC-AH* network [8], and an *SC-IS* network [14] can be regarded as special cases of our *MC-IS* network under the following scenarios.

- (A) An SC-AH network is a special case of our MC-IS network: The theoretical bounds in the SC-AH network [2] are consistent with our bounds when our configuration is set to the one for the SC-AH network. Specifically, the configuration is that H is set to $\Theta(\sqrt{n/\log n})$, $C_A=1$, $W_A=W$ and $W_I=0$. In that configuration, the total bandwidth is assigned for ad hoc communications ($W_A=W$ and $W_I=0$), there is a single channel available ($C_A=1$) corresponding to that of an SC-AH network [2].
- (B) An MC-AH network is a special case of our MC-IS network: The theoretical bounds in the MC-AH network [8] are consistent with our bounds shown in Theorem 1, when our configuration is set to the one for the MC-AH network, in which H is set to $\Theta(\sqrt{n/\log n})$, corresponding to that of an MC-AH network [8].

In particular, we have the following cases:

- Case I: when $C_A = O(\log n)$ and $H = \Theta(\sqrt{n/\log n})$ (Connectivity Condition is satisfied), the per-node throughput $\lambda = \Theta(W/\sqrt{n\log n})$ and the average delay $D = \Theta(\sqrt{n/\log n})$, which matches the result of an MC-AH network [8];
- Case II: when $C_A = \Omega(\log n)$ and $C_A = O\left(n\left(\frac{\log\log\left(H^2\log n\right)}{\log\left(H^2\log n\right)}\right)^2\right)$, and $H = \Theta(\sqrt{n/\log n})$ (Interference Condition is satisfied), the per-node throughput $\lambda = \Theta(W/\sqrt{C_A n})$ and the average delay $D = \Theta(\sqrt{n/\log n})$, which matches the result of an MC-AH network [8]:
- Case III: when $C_A = \Omega\left(n\left(\frac{\log\log\left(H^2\log n\right)}{\log\left(H^2\log n\right)}\right)^2\right)$ and $H = \Theta(\sqrt{n/\log n})$ (Destination-bottleneck Condition is satisfied), the per-node throughput $\lambda = \Theta(\frac{n\log\log nW}{C_A\log n})$ and the average delay $D = \Theta(\sqrt{n/\log n})$, which matches the result of an MC-AH network [8].

Note that we do not consider the capacity contributed by infrastructure communications in the above four cases.

(C) An SC-IS network is a special case of our MC-IS network: Similarly, the theoretical bounds in the SC-IS network [14] are consistent with our bounds when our configuration is set to the one for the SC-IS network.

In particular, we have the following cases:

- Case I: when $C_A = 1$ and $H = \Omega(n^{\frac{1}{3}}/\log^{\frac{2}{3}}n)$ (Connectivity Condition is satisfied), $\lambda = \Theta(\frac{W_a}{H\log n} + \frac{b}{n}W_i)$ and $D = \Theta(\frac{H^3\log n}{n} + c)$, which matches the result of an *SC-IS* network [14];
- Case II: when $C_A=1$ and $H=o(n^{\frac{1}{3}}/\log^{\frac{2}{3}}n)$ (Interface-bottleneck Condition is satisfied), $\lambda=\Theta(H^2\frac{\log n}{n}\cdot\frac{W_a}{C_a}+\min\{\frac{b}{n},\frac{bm}{nC_i}\}W_I)$ and $D=\Theta(\frac{H^3\log n}{n}+c)$, which matches the result of an SC-IS network [14].

4. Optimality of Results

We analyze the optimality of the per-node throughput capacity λ and the average delay D of an MC-IS network. Specifically, the analysis is categorized into two cases: (1) when λ_a dominates λ_i ; (2) when λ_i dominates λ_a .

Case 1: when λ_a dominates λ_i (i.e. $W_A \to W$ and $W_I/W \to 0$). We obtain the maximum per-node throughput capacity as the following sub-cases: (i) $\lambda = \Theta\left(\frac{W}{H \log n}\right)$ with Connectivity condition; (ii) $\lambda = \Theta\left(\frac{W}{C^{\frac{1}{2}} H \log^{\frac{1}{2}} n}\right)$ with Interference condition; (iii) $\lambda = \Theta\left(\frac{n^{\frac{1}{2}} \log \log(H^2 \log n)W}{CH \log^{\frac{1}{2}} n \cdot \log(H^2 \log n)}\right)$ with Destination-bottleneck condition; (iv) $\lambda = \Theta\left(\frac{H^2 W \log n}{Cn}\right)$ with Interface-bottleneck condition. In all the above sub-cases, we always have the average delay $D = \Theta\left(\frac{H^3 \log n}{n}\right)$. The results imply that we should assign most of channel bandwidth to ad hoc communications in order to obtain the maximum capacity and the minimum delay. However, we show next that the above results are not optimal compared with Case 2.

Case 2: when λ_i dominates λ_a (i.e. $W_I \to W/2$ and $W_A/W \to 0$). In this case, the maximum per-node throughput capacity $\lambda = \Theta(\frac{b}{n}W)$ and the average delay $D = \Theta(\frac{c}{\min\{C_I,m\}})$. It implies that when when λ_i dominates λ_a ,

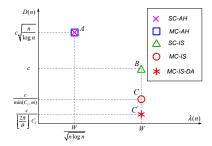


Fig. 3. Capacity and delay regions under different networks. The scales of the axes are in terms of the orders in n

to maximize the capacity, most of the channel bandwidth should be assigned for infrastructure communications. At this time, increasing the number of base stations can significantly improve the network capacity. More specifically, if $b = \Omega(n)$, then $\lambda = \Theta(W)$, which is significantly higher than those in Case 1. This is because the multi-hop ad hoc communications may lead to the capacity loss due to the higher interference of multiple ad hoc communications. Meanwhile, the minimum average delay D in this case is bounded by $\Theta(\frac{c}{\min\{C_I,m\}})$, where c is a constant and $\frac{c}{\min\{C_I,m\}}$ is independent of n. It is obvious that $\frac{c}{\min\{C_I,m\}} = o(\Theta(\frac{H^3 \log n}{n}))$ since H is determined by the number of nodes n. Intuitively, we have much lower delay than that of Case 1. The reason behind this lies in the higher delay brought by the multi-hop communications in Case 1. In summary, MC-IS networks have the optimal per-node throughput capacity $\lambda_{opt} = \Theta(W)$ and the optimal average delay $D_{opt} = \Theta(\frac{c}{\min\{C_I,m\}})$.

We compare our results with other networks (an MC-AH network, an SC-IS network, and an SC-AH network) in terms of the optimal per-node throughput λ and the optimal average delay D. As shown in Fig. 3, an MC-IS network can achieve the optimal per-node throughput $\lambda_{opt} = \Theta(W)$ (point C in Fig. 3), which is $\sqrt{n \log n}$ times higher than that of an MC-AH network and an SC-AH network (point A in Fig. 3), and the same as that of an SC-IS network (point B in Fig. 3), implying that there is no degradation in the optimal per-node throughput of an MC-IS network.

Besides, an MC-IS network can achieve the smallest delay $\Theta\left(\frac{c}{\min\{C_I,m\}}\right)$ (point C in Fig. 3) when the optimal pernode throughput capacity $\lambda=\Theta(W)$ is achieved. It is shown in [3] that in an SC-AH network and an MC-AH network, the increased capacity pays for the higher delay due to the multi-hop transmissions. However, an MC-IS network and an SC-IS network can overcome the delay penalty by transmitting packets through infrastructure, inside which there is no delay constraint. Furthermore, an MC-IS network can achieve an even shorter delay than an SC-IS network by using multiple NICs at each base station, which can support multiple simultaneous transmissions. Specifically, as shown in Fig. 3, an MC-IS network (point C) has a delay reduction gain of $\frac{1}{\min\{C_I,m\}}$ over an SC-IS network (point B). For example, an MC-IS network with $C_I=m=12,$ in which we assign a dedicated interface for each channel, has a delay 12 times lower than an SC-IS network. Besides, when we extend our analysis on an MC-IS network equipped with omni-directional antennas only to an MC-IS network equipped with directional antennas only, which are named as an MC-IS-DA network, we can obtain an even lower delay of $\frac{c}{\lfloor \frac{2\pi}{\theta} \rfloor \cdot C_I}$ as shown in point C', where θ is the beamwidth of a directional antenna mounted at the base station (usually $\theta < 2\pi$). Consider the same case of $C_I = 12$ and $\theta = \frac{\pi}{12}$ that is feasible in most of mmWave systems [21]. An MC-IS-DA can further reduce the delay by 24 times lower that of an MC-IS network and reduce the delay by 288 times lower than that of an SC-IS network. Details on this extended work will be addressed in Section VII.

V. CAPACITY CONTRIBUTED BY AD HOC COMMUNICATIONS

We first derive the upper bounds on the network capacity contributed by ad hoc communications in Section V-A and then present the constructive lower bounds on the network capacity contributed by ad hoc communications in Section V-B, which have the same order of the upper bounds, implying that our results are tight. Wenext give the aggregate throughput capacity in Section V-C.

A. Upper Bounds on Network Capacity Contributed by Ad Hoc Communications

The network capacity contributed by ad hoc transmissions in an MC-IS network, denoted by λ_a , is mainly affected by (1) Connectivity requirement, (2) Interference requirement, (3) Destination-bottleneck requirement and (4) Interface-bottleneck requirement. We first derive the upper bounds on the per-node throughput capacity under Connectivity Condition. Before presenting Proposition 1, we have Lemma 1 to bound the expectation of the number of hops denoted by \overline{h} .

Lemma 1: The expectation of the number of hops \overline{h} is bounded by $\Theta(H)$.

Proof. We first denote P(h=i) by the probability of the event that a packet traverses h=i hops. According to the H-max-hop routing scheme, P(h=i) is essentially equal to the probability that a packet traverses at most h=i hops with the exclusion of the event that a packet traverses no more than h=i-1 hops, where i>0. Thus, P(h=i) is equal to the ratio of the area of a disk with radius $(i-1)\cdot r(n)$ to the area of a disk with radius $i\cdot r(n)$, where r(n) is the distance of a hop. As a result, $P(h=i)=\frac{(i^2-(i-1)^2)\cdot \pi r^2(n)}{\pi i^2r^2(n)}$.

We then have

$$\overline{h} = E(h) = \sum_{i=1}^{H} \frac{i \cdot (i^2 - (i-1)^2) \cdot \pi r^2(n)}{\pi H^2 r^2(n)}$$
(1)

Since $i[i^2 - (i-1)^2]$ in Eq. (1) are the series of hexagonal numbers, then Eq. (1) can be simplified as follows

$$\overline{h} = \frac{\frac{1}{6}H(H+1)(4H-1)}{H^2} = \frac{4H^3 + 3H^2 - H}{6H^2}.$$
 (2)

It is obvious that \overline{h} is a function of H as shown in Eq. (2). The limit of $\overline{h}(H)$ as H approaches ∞ is $\lim_{H\to\infty}\overline{h}(H)=\Theta(H)$, which can be directly derived from the definition of the asymptotic notation $\Theta(\cdot)$ and Eq. (2).

We then have Proposition 1 that bounds the per-node throughput capacity contributed by ad hoc communications under Connectivity condition,

Proposition 1: When Connectivity requirement dominates, the per-node throughput capacity contributed by ad hoc communications is $\lambda_a = O(\frac{nW_A}{H^3 \log^2 n})$.

Proof. We first calculate the probability that a node uses the ad hoc mode to transmit, denoted by P(AH), which is the probability that the destination node is located within H hops away from the source node. Thus, we have $P(AH) = \pi H^2 r^2(n)$.

Since each source generates λ_a bits per second and there are totally n sources, the total number of bits per second served by the whole network is required to be at least $n \cdot P(AH) \cdot \overline{h} \cdot \lambda_a$. We next prove that $n \cdot P(AH) \cdot \overline{h} \cdot \lambda_a$ is bounded by $\frac{\tilde{k}_1}{\Delta^2(r(n))^2}W_A$. Denote the maximum number of simultaneous transmissions on a particular channel by $N_{\rm max}$. As proved in Lemma 5.4 in [2], N_{max} is upper bounded by $\frac{k_1}{\Delta^2(r(n))^2}$, where $k_1 > 0$ is a constant, independent of n. Note that each transmission over the ϖ channel is of W_A/C_A bits/sec. Adding all the transmissions taking place at the same time over all the C_A channels, we have that the total number of transmissions in the whole network is no more than $\frac{k_1}{\Delta^2(r(n))^2}\sum_{\varpi=1}^{C_A}\frac{W_A}{C_A}=\frac{k_1}{\Delta^2(r(n))^2}W_A$ bits/sec. Therefore, we have $n\cdot P(AH)\cdot \overline{h}\cdot \lambda_a\leq \frac{k_1}{\Delta^2(r(n))^2}W_A$.

Combining the above results with Lemma 1 yields $\lambda_a \leq \frac{k_1}{\Delta^2 r^2(n)} \cdot \frac{W_A}{n\pi H^3 r^2(n)} \leq \frac{k_2 W_A}{nH^3 r^2(n)}$, where k_2 is a constant. Besides, to guarantee that the network is connected with high probability (*w.h.p.*), we require $r(n) > \sqrt{\frac{\log n}{\pi n}}$ [2]. Thus, we have $\lambda_a \leq \frac{k_3 n W_A}{H^3 \log^2 n}$, where k_3 is a constant.

We then derive the upper bounds on the per-node throughput capacity under Interference Condition.

Proposition 2: When Interference requirement dominates, the per-node throughput capacity contributed by ad hoc communications is $\lambda_a = O\left(\frac{nW_A}{C_A^{\frac{1}{2}}H^3\log^{\frac{3}{2}}n}\right)$

Proof. We present a proof of the bound in Appendix A. Before proving the upper bounds on the throughput capacity under Destination-Bottleneck condition, we have Lemma 2 to bound the number of flows towards a node under the H-maxhop routing scheme.

Lemma 2: The maximum number of flows towards a node under the H-max-hop routing scheme is $D_H(n) =$ $\Theta\left(\frac{\log(H^2\log n)}{\log\log(H^2\log n)}\right)$ w.h.p.

Proof. Let $N_i (1 \le i \le n)$ be a random variable defined as

 $N_i = \left\{ \begin{array}{ll} 1 & \text{if source node } i \text{ transmits to its destination node;} \\ 0 & \text{otherwise.} \end{array} \right.$

Let N_t be a random variable representing the total number of source nodes transmitting in ad hoc mode. We have $N_t = \sum_{i=1}^n N_i$. Thus, the expected number of source nodes transmitting in ad hoc mode is $E(N_t) = E\left(\sum_{i=1}^n N_i\right) =$ $\sum_{i=1}^{n} E(N_i)$. Since $f(N_i = 1) = P(AH) = \pi H^2 r^2(n)$ and r(n) needs to be $\Theta(\sqrt{\log n/n})$ to ensure that the network is connected, we have $E(N_i) = 1 \cdot \pi H^2 r^2(n) + 0 \cdot (1 - 1)$

 $\pi H^2 r^2(n) = \pi H^2 r^2(n)$, i.e., $E(N_i) = \Theta(\pi H^2 \frac{\log n}{n})$. Therefore, $E(N_t) = n \cdot \pi H^2 \frac{\log n}{n} = \pi H^2 \log n$.

Recall the Chernoff bounds [35], we have

- $\begin{array}{ll} \bullet \ \ \mbox{for any} \ \ \delta \ \ > \ 0, \ \ P(N_t \ \ > \ (1 + \delta)\pi H^2 \log n) \ \ < \\ \ \ \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\pi H^2 \log n}; \\ \bullet \ \ \mbox{for any} \ \ 0 \ < \ \delta \ \ < \ 1, \ P(N_t \ < \ (1 \delta)\pi H^2 \log n) \ \ < \\ \ \ \ e^{-\pi H^2 \log n \cdot \delta^2/2}. \end{array}$

In summary, for any $0 < \delta < 1$, we can obtain $P(|N_t \pi H^2 \log n > \delta \pi H^2 \log n < e^{-\varepsilon \pi H^2 \log n}$, where $\varepsilon > 0$. Thus, when $n \to \infty$, the total number of source nodes transmitting in ad hoc mode is $\Theta(H^2 \log n)$ w.h.p. Besides, it is proved in [36] that the maximum number of flows towards any given node in a random network with N nodes, denoted by D(N), is upper bounded by $\Theta(\frac{\log N}{\log \log N})$, w.h.p. Combining the two results leads to the above result.

We then prove the upper bounds on the per-node throughput capacity under Destination-bottleneck Condition.

Proposition 3: When Destination-bottleneck requirement dominates, the per-node throughput capacity contributed by ad hoc communications is $\lambda_a = O\left(\frac{n^{\frac{3}{2}}\log\log(H^2\log n)W_A}{C_AH^3\log^{\frac{3}{2}}n\cdot\log(H^2\log n)}\right)$. **Proof.** Since each node has one interface that can support at

most $\frac{W_A}{C_A}$ and Since each node has at most $D_H(n)$ flows under the H-max-hop routing scheme, the data rate of the minimum rate flow is at most $\frac{W_A}{C_A D_H(n)}$, where $D_H(n)$ is bounded by $\Theta\left(\frac{\log(H^2\log n)}{\log\log(H^2\log n)}\right)$ by Lemma 2. After calculating all the data rates at each node times with the traversing distance, we have

we then have $\lambda_a \leq \frac{W_{A}n}{C_A D_H(n)} \cdot 1$.

We then have $\lambda_a \leq \frac{W_A}{C_A D_H(n) P(AH)\overline{h}r(n)} \leq \frac{W_A}{C_A T_H(n) P(AH)\overline{h}r(n)} \leq \frac{W_A}{C_A \pi H^3 r^3(n) \cdot \frac{\log(H^2 \log n)}{\log\log(H^2 \log n)}}$. This is because $\overline{h} = \Theta(H)$ and $P(AH) = \pi H^2 r^2(n)$ are derived in Lemma 1 and in the proof of Proposition 1, respectively. Since $r(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ as proved in [2], we then prove the result.

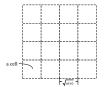
Finally, we prove the upper bounds on the per-node throughput capacity under Interface-bottleneck Condition.

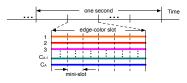
Proposition 4: When Interface-bottleneck requirement dominates, the per-node throughput capacity contributed by ad hoc communications is $\lambda_a = O(\frac{W_A}{C_A})$.

Proof. In an *MC-IS* network, each node is equipped with only one interface, which can support at most $\frac{W_A}{C_A}$ data rate. Thus, λ_a is also upper bounded by $\frac{W_A}{C_A}$. Note that this result holds for any naturals at V_A . for any network settings.

B. Constructive Lower Bounds on Network Capacity Contributed by Ad Hoc Communications

We then derive the lower bound on the network capacity by constructing a network with the corresponding routing scheme and scheduling scheme when each requirement is considered. The derived orders of the lower bounds are the same as the orders of the upper bounds, meaning that the upper bounds are tight. In particular, we first divide the plane into a number of equal-sized cells. The size of each cell is properly chosen so that each cell has $\Theta(na(n))$ nodes, where a(n) is the area of a cell (Sec. V-B1). We then design a routing scheme to assign





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Fig. 4. Plane divided into Fig. 5. TDMA transmission schedule a number of cells.

the number of flows at each node evenly (Sec. V-B2). Finally, we design a *Time Division Multiple Access (TDMA)* scheme to schedule the traffic at each node (Sec. V-B3).

1) Cell Construction: We divide the plane into 1/a(n) equal-sized cells and each cell is a square with area of a(n), as shown in Fig. 4. The cell size of a(n) must be carefully chosen to fulfill the three requirements, i.e., the connectivity requirement, the interference requirement and the destination-bottleneck requirement. In particular, similar to [8], we set $a(n) = \min\left\{\max\left\{\frac{100\log n}{n}, \frac{\log^{\frac{3}{2}}n}{C_A^{\frac{1}{2}}n}\right\}, \frac{\log^{\frac{3}{2}}n\cdot\log(H^2\log n)}{n^{\frac{3}{2}\cdot\log\log(H^2\log n)}}\right\}$. Note that the interface-bottleneck requirement is independent of the size of a cell.

The maximum number of nodes in a cell can be upper bounded by the following lemma.

Lemma 3: If $a(n) > \frac{50 \log n}{n}$, then each cell has $\Theta(n(a(n)))$ nodes w.h.p..

Proof. Please refer to [8].

We next check whether all the above values of a(n) are properly chosen such that each cell has $\Theta(n(a(n)))$ nodes w.h.p. when n is large enough (i.e., Lemma 3 is satisfied). It is obvious that $\frac{100\log n}{n} > \frac{50\log n}{n}$ and $\frac{\log^{\frac{3}{2}}n}{C_A^{\frac{3}{2}}n} > \frac{50\log n}{n}$ since we only consider C_A in Connectivity Condition and Interference Condition. Besides, $\frac{\log^{\frac{3}{2}}n \cdot \log(H^2\log n)}{n^{\frac{3}{2} \cdot \log\log(H^2\log n)}}$ is also greater than $\frac{50\log n}{n}$ with large n since $\frac{\log(H^2\log n)}{\log\log(H^2\log n)} > 1$ and $\frac{\log^{\frac{3}{2}}n}{n^{\frac{3}{2}}} > \frac{50\log n}{n}$ when n is large enough.

Besides, the number of interfering cells around a cell is bounded by a constant, given by Lemma 4 as follows.

Lemma 4: Under the interference model, the number of interfering cells of any given cell is bounded by a constant k_5 , which is independent of n.

Proof. The detailed proof is stated in Appendix B.

2) Routing Scheme: To assign the flows at each node evenly, we design a routing scheme consists of two steps: (1) Assigning sources and destinations and (2) Assigning the remaining flows in a balanced way.

In Step (1), each node is the originator of a flow and each node is the destination of at most $D_H(n)$ flows, where $D_H(n)$ is defined in Lemma 2. Thus, after Step (1), there are at most $1 + D_H(n)$ flows. We denote the straight line connecting a source S to its destination D as an S-D lines. In Step (2), we need to calculate the number of S-D lines (flows) passing through a cell so that we can assign them to each node evenly. Specifically, we have the following result.

Lemma 5: The number of S-D lines passing through a cell is bounded by $O(nH^3(a(n))^2)$.

Proof. The detailed proof is stated in Appendix C.

As shown in Lemma 3, there are $\Theta(n \cdot a(n))$ nodes in each cell. Therefore, Step (2) will assign to any node at most $O\left(\frac{nH^3(a(n))^2}{n\cdot a(n)}\right) = O(H^3a(n))$ flows. Summarizing Step (1) and Step (2), there are at most $f(n) = O(1 + H^3a(n) + D_H(n))$ flows at each node. On the other hand, $H^3a(n)$ dominates f(n) since H > 1 and a(n) is asymptotically larger than $D_H(n)$ when n is large enough. Thus, we have $f(n) = O(H^3a(n))$.

3) Scheduling Transmissions: We next design a scheduling scheme to transmit the traffic flows assigned in a *routing scheme*. Any transmissions in this network must satisfy the two additional constraints simultaneously: 1) each interface only allows one transmission/reception at the same time, and 2) any two transmissions on any channel should not interfere with each other.

We propose a TDMA scheme to schedule transmissions that satisfy the above two constraints. Fig. 5 depicts a schedule of transmissions on the network. In this scheme, one second is divided into a number of *edge-color* slots and at most one transmission/reception is scheduled at every node during each edge-color slot. Hence, the first constraint is satisfied. Each edge-color slot can be further split into smaller *mini-slots*. In each mini-slot, each transmission satisfies the above two constraints. Details are described as follows.

- (i) Edge-color slot: First, we construct a routing graph in which vertices are the nodes in the network and an edge denotes transmission/reception of a node. In this construction, one hop along a flow is associated with one edge in the routing graph. In the routing graph, each vertex is assigned with $f(n) = O(H^3a(n))$ edges. It is shown in [8], [37] that this routing graph can be edge-colored with at most $O(H^3a(n))$ colors. We then divide one second into $O(H^3a(n))$ edge-color slots, each of which has a length of $\Omega(\frac{1}{H^3a(n)})$ seconds and is stained with a unique edge-color. Since all edges connecting to a vertex use different colors, each node has at most one transmission/reception scheduled in any edge-color time slot.
- (ii) *Mini-slot*: We further divide each edge-color slot into mini-slots. Then, we build a schedule that assigns a transmission to a node in a mini-slot within an edge-color slot over a channel. We construct an *interference graph* in which each vertex is a node in the network and each edge denotes the interference between two nodes. We then show as follows that the interference graph can be vertex-colored with $k_7(na(n))$ colors, where k_7 is a constant defined in [8].

Lemma 6: The interference graph can be vertex-colored with at most O(na(n)) colors.

Proof. By Lemma 4, every cell has at most a constant number of interfering cells. Besides, each cell has $\Theta(na(n))$ nodes by Lemma 3. Thus, each node has at most O(na(n)) edges in the interference graph. It is shown that a graph of degree at most k_0 can be vertex-colored with at most $k_0 + 1$ colors [8] [37]. Hence, the interference graph can be vertex-colored with at most O(na(n)) colors.

We need to schedule the interfering nodes either on different channels, or at different mini-slots on the same channel since two nodes assigned the same vertex-color do not interfere with each other, while two nodes stained with different colors may interfere with each other. We divide each edge-color slot into mini-slots on every channel, and assign the minislots on each channel from 1 to $\left\lceil \frac{k_7 na(n)}{C_A} \right\rceil$. A node assigned with a color s, $1 \le s \le k_7 na(n)$, is allowed to transmit in mini-slot $\left| \frac{s}{C_A} \right|$ on channel $(s \mod C_A) + 1$.

We next prove the constructive lower bounds of the capacity.

Proposition 5: The achievable per-node throughput capacity λ_a contributed by ad hoc communications is as follows.

- 1) When Connectivity requirement dominates, λ_a is $\Omega\left(\frac{nW_A}{H^3\log^2 n}\right)$ bits/sec;
- When Interference requirement dominates, λ_a is $\Omega\left(\frac{nW_A}{H^3C_A^{\frac{1}{2}}\log^{\frac{3}{2}}n}\right)$ bits/sec;
- 3) When Destination-bottleneck requirement dominates, λ_a is $\Omega\left(\frac{n^{\frac{3}{2}}\log\log(H^2\log n)W_A}{C_AH^3\log^{\frac{3}{2}}n\cdot\log(H^2\log n)}\right)$ bits/sec;

 4) When Interface-bottleneck requirement dominates, λ_a is

Proof. Since each edge-color slot with a length of $\Omega(\frac{1}{H^3a(n)})$ seconds is divided into $\left\lceil \frac{k_T n a(n)}{C_A} \right\rceil$ mini-slots over every channel, each mini-slot has a length of $\Omega\left(\left(\frac{1}{H^3a(n)}\right)/\left\lceil\frac{k_7na(n)}{C_A}\right\rceil\right)$ seconds. Besides, each channel can transmit at the rate of $\frac{W_A}{C_A}$ bits/sec, in each mini-slot, $\lambda_a = \Omega\left(\frac{W_A}{C_AH^3a(n)\cdot\left\lceil\frac{k_7na(n)}{C_A}\right\rceil}\right)$ bits can be transported. Since $\left\lceil\frac{k_7na(n)}{C_A}\right\rceil \leq \frac{k_7na(n)}{C_A} + 1$, we have $\lambda_a = \Omega\left(\frac{W_A}{k_7H^3a^2(n)n+H^3a(n)C_A}\right)$ bits/sec. Thus, $\lambda_a = \Omega\left(\frac{W_A}{H^3a^2(n)n}, \frac{W_A}{H^3a(n)C_A}\right)$ bits/sec. Recall that a(n)is min $\left\{\max\left\{\frac{100\log n}{n}, \frac{\log^{\frac{3}{2}}n}{C_A^{\frac{1}{2}}n}\right\}, \frac{\log^{\frac{3}{2}}n\cdot\log(H^2\log n)}{n^{\frac{3}{2}\cdot\log\log(H^2\log n)}}\right\}$. Substituting the three values to λ_a , we have the results 1), 2) and 3). Besides, each interface can support the rate of $\frac{W_A}{C_A}$ bits/sec. Thus, $\lambda_a = \Omega(\frac{W_A}{C_A})$, which is the result 4).

C. Aggregate Throughput Capacity

It is shown in [14] that the total traffic of ad hoc communications is $n\pi H^2 r^2(n) \lambda_a$. Combining Propositions 1, 2, 3, and 5 with the total traffic leads to the following theorem.

Theorem 3: The aggregate throughput capacity of the network contributed by ad hoc communications is

- 1) When Connectivity requirement dominates, T_A is $\Theta(\frac{nW_A}{H\log n})$ bits/sec.
- 2) When Interference requirement dominates, T_A is $\Theta(\frac{nW_A}{\frac{1}{2}H\log^{\frac{1}{2}}n})$ bits/sec.
- 3) When Destination-bottleneck requirement dominates,
- T_A is $\Theta(\frac{n^{\frac{3}{2}}\log\log(H^2\log n)W_A}{C_AH\log^{\frac{1}{2}}n\cdot\log(H^2\log n)})$ bits/sec. 4) When Interface-bottleneck requirement dominates, T_A is $\Theta(H^2\log n\cdot \frac{W_A}{C_A})$ bits/sec.

VI. CAPACITY CONTRIBUTED BY INFRASTRUCTURE **COMMUNICATIONS**

We first derive the upper bounds of the capacity in Section VI-A and give the constructive lower bounds of the capacity in Section VI-B. We give the aggregate capacity contributed by infrastructure communications in Section VI-C. Finally, Section VI-D gives the proof of Theorem 1 and Theorem 2.

A. Upper Bounds of Network Capacity Contributed by Infrastructure Communications

We derive the upper bounds of the throughput capacity contributed by infrastructure communications as follows.

Proposition 6: Under the H-max-hop routing scheme, the throughput capacity contributed by infrastructure communications, denoted by T_I , is:

- $\begin{array}{ll} \text{(1)} \ \ \text{When} \ C_I \leq m, \ T_I = O(bW_I). \\ \text{(2)} \ \ \text{When} \ C_I > m, \ T_I = O(b\frac{m}{C_I}W_I). \end{array}$

Proof. Since each packet transmitted in the infrastructure mode will use both the uplink and the downlink communications, we only count once for the throughput capacity.

Case (1) when $C_I \leq m$. It is obvious that the m interfaces at each base station can support at most W_I bandwidth. In other words, the C_I channels are fully utilized by the m interfaces. Counting all the b base stations, we have $T_I = O(bW_I)$.

Case (2) when $C_I > m$. The number of interfaces is smaller than the number of channels, implying that not all the C_I channels are fully used. In fact, at most m channels can be used at a time. Besides, each channel can support at most $\frac{W_I}{C_I}$ bits/sec. Thus, each base station can support at most $\frac{m}{C_I}W_I$ bits/sec. Counting all the b base stations, we have $T_I = O(b\frac{m}{C_I}W_I)$.

B. Constructive Lower Bounds of Network Capacity Contributed by Infrastructure Transmissions

The lower bounds are proved by constructing a routing scheme and a transmission scheduling scheme on a regulartessellated BS network. The derived orders of the lower bounds are the same as the orders of the upper bounds, implying that the upper bounds are tight.

- 1) BS-Cell Construction by Regular Tessellation: There are b base stations regularly placed in the plane dividing the plane into a number of equal-sized BS-cells. Note that the size of each BS-cell may not be necessarily equal to the size of a cell. Besides, Lemma 4 still holds even if the base stations are regularly placed in the plane. So, the number of interfering BS-cells is also bounded by a constant, denoted by k_8 , which is also independent of b.
- 2) Routing and Scheduling Schemes: The routing scheme for the infrastructure traffic is simple, i.e., to forward the traffic to a base station (uplink) and to forward the traffic from a base station (downlink). We propose the following TDMA scheduling scheme Σ_1 to schedule the BS-cells to be active in a round-robin fashion.
- (1) Divide the plane into b equal-sized BS-cells.
- (2) We group the b BS-cells into a number of clusters. Each cluster has $(k_8 + 1)$ BS-cells. We then split the transmission time into a number of time frames. Each frame consists of $(k_8 + 1)$ time slots that correspond to the number of BS-cells in each cluster. In each time slot, one BS-cell within each cluster becomes active to transmit and the BS-cells in each cluster take turns to be active.

Proposition 7: Under the TDMA scheme Σ_1 , the throughput capacity T_I , is:

- (1) When $C_I \leq m$, $T_I = \Omega(bW_I)$.
- (2) When $C_I > m$, $T_I = \Omega(b \frac{m}{C_I} W_I)$.

Proof. Since each packet transmitted in the infrastructure mode will use both the uplink and the downlink, we only count once for throughput capacity.

Case (1) when $C_I \leq m$: Under Σ_1 , each BS-cell is active every (k_8+1) time slots. When a BS-cell is active, there are at most C_I channels available. Thus, the total bandwidth of W_I of those C_I channels are fully used, implying that the per-cell throughput λ_i is lower bounded by $\frac{W_I}{k_8+1}$. Counting all the b base stations, we have $T_I = \Omega(\frac{bW_I}{k_8+1})$.

Case (2) when $C_I > m$: Similarly, each BS-cell is active to transmit every (k_8+1) time slots in case (2). But, when a BS-cell is active, only m channels available at a time and each channel can support at most $\frac{W_I}{C_I}$ data rate. Thus, the per-cell throughput λ_i is lower bounded $\frac{mW_I}{C_I(k_8+1)}$. Counting all the b base stations, we have $T_I = \Omega(\frac{bmW_I}{C_I(k_8+1)})$.

C. Aggregate Throughput Capacity

Combining Proposition 6 and Proposition 7, we have *Theorem 4:* The aggregate throughput capacity of the network contributed by infrastructure communications is

- (1) When $C_I \leq m$, $T_I = \Theta(bW_I)$.
- (2) When $C_I > m$, $T_I = \Theta(b \frac{m}{C_I} W_I)$.

It is shown in Theorem 4 that the optimal throughput capacity contributed by infrastructure communications $T_I = \Theta(bW_I)$ is achieved when $C_I \leq m$. Generally, we have $C_I = m$. If $C_I \neq m$, some interfaces are idle and wasted. It implies that to maximize T_I , we shall assign a dedicated interface per channel at each base station so that all the C_I channels can be fully utilized.

D. Proof of Theorem 1 and Theorem 2

We finally give the proof of Theorem 1 as follows.

Proof of Theorem 1

We first have the aggregate throughput capacity $T = T_A + T_I$, where T_A is the aggregate capacity contributed by ad hoc communications and T_I is the aggregate capacity contributed by infrastructure communications given by given by Theorem 3 and Theorem 4, respectively. Since there are at most n nodes in the network, we then divide T by n and finally have the results in Theorem 1. This completes the proof.

We then derive the average delay of an MC-IS network contributed by ad hoc communications and infrastructure communications as follows.

Proof of Theorem 2

We first derive the bound on the delay when the packets are transmitted in the infrastructure mode. As shown in [14], the average delay for the packets transmitted in the infrastructure mode in an SC-IS network is bounded by $\Theta(c)$, where c is a constant depending on the transmitting capability of the base station. Different from an SC-IS network, where each base station is equipped with a single interface supporting at most one transmission at a time, each base station in an MC-IS

network can support $\min\{C_I, m\}$ simultaneous transmissions at a time. This is because when $C_I \leq m$, a base station with m interfaces can support at most C_I simultaneous transmissions; when $C_I > m$, a base station with m interfaces can support at most m simultaneous transmissions. Thus, the average delay for the packets transmitted in the infrastructure mode in an MC-IS network is bounded by $\Theta(\frac{c}{\min\{C_I, m\}})$.

We then derive the bound on the delay when the packets are transmitted in ad hoc mode. The expectation of \overline{h} under H-max-hop routing strategy is bounded by $\Theta(H)$ as proved by Lemma 1. Since the time spent by a packet at each relay is bounded by c_1 , the average delay is of the same order as the average number of hops, i.e., $D=c_1\cdot\overline{h}=\Theta(H)$. It is shown in the proof of Lemma 2 that the number of transmitters in the ad hoc mode is $\pi H^2\log n$ w.h.p. Then the number of transmitters in the infrastructure mode is $(n-\pi H^2\log n)$ w.h.p. After applying the above analysis, we have the average delay of all packets $D=\Theta\left(\frac{\pi H^2\log n\cdot H + (n-\pi H^2\log n)\cdot \frac{c}{\min\{C_I,m\}}}{n}\right)$. Note that $\frac{n-\pi H^2\log n}{n}$ is bounded by $\Theta(1)$. Thus, we have Theorem 2.

VII. DISCUSSIONS AND IMPLICATIONS

In this section, we first extend our analysis to the scenarios of using directional antennas in *MC-IS* networks in Section VII-A. We then discuss the impacts of mobility models in Section VII-B. Finally, we present the implications of our *MC-IS* networks in Section VII-C.

A. Using Directional Antennas in MC-IS networks

Conventional wireless networks assume that each node is equipped with an omni-directional antenna, which radiates signals in all directions including some undesired directions. Recent studies such as [38], [39] show that applying directional antennas instead of omni-directional antennas to wireless networks can greatly improve the network capacity. The performance improvement mainly owes to the reduction in the interference from undesired directions since directional antennas concentrate radio signals on the desired directions. Although directional antennas have numerous advantages, the bulky size and the impacts of directionality also restrict the application of directional antennas to wireless networks. However, with the evolution of wireless communication technologies, these challenging issues will finally be solved. In fact, a directional antenna has become a necessity in order to compensate for the tremendous signal attenuation in millimeter-wave (mmWave) communication systems [40]. It is feasible to deploy directional antennas at both base stations and mobile devices in mmWave communication systems since their size will be quite compact due to the fact that the antenna size is inversely proportional to the radio frequency (the frequency band is ranging from 30GHz to 300GHz in mmWave communication systems [41]).

We extend our analysis on an MC-IS network with omnidirectional antennas (in the previous part of this paper) to that with directional antennas. In particular, we name an MC-IS network equipped with directional antennas as an MC-IS-DA

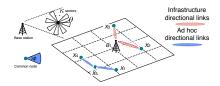


Fig. 6. Network topology of an MC-IS-DA network in a BS-cell

network. Fig. 6 shows an example of MC-IS-DA networks, in which each base station is equipped with multiple directional antennas and each common node is equipped with a single directional antenna. Similar to an MC-IS network, there are two types of communications in an MC-IS-DA network: ad hoc communications between common nodes and infrastructure communications between a common node and a base station. Differently, both ad hoc communications and infrastructure communications in an MC-IS-DA network consist of directional communication links only.

In this paper, we consider a *flat-top* antenna model [9], [38], [42], in which sidelobes and backlobes are ignored. Our antenna model assumes that a directional antenna gain is within a specific angle, i.e., the beamwidth of the antenna, which is ranging from 0 to π . The gain outside the beamwidth is assumed to be zero. In our MC-IS network, each common node is mounted with a single interface, which is equipped with a directional antenna with beamwidth ϕ . Each base station is mounted with m interfaces, each of which is equipped with a directional antenna with beamwidth θ , where each directional antenna at each base station is identical. Note that the beamwidth ϕ of an antenna at a common node is not necessarily equal to the beamwidth θ of that at a base station.

1. Capacity of an MC-IS-DA network

The capacity of an MC-IS-DA network contributed by infrastructure communications is the same as that of an MC-IS network. However, an MC-IS-DA network has different capacity regions on the per-node throughput capacity λ_a from an *MC-IS* network.

Corollary 1: The per-node throughput λ for an MC-IS-DA network has four regions as follows.

- i) When Connectivity Condition is satisfied, $\lambda = \Theta\left(\frac{4\pi^2}{\phi^2} \cdot \frac{W_A}{H \log n}\right) + \Theta\left(\min\{\frac{b}{n}, \frac{bm}{nC_I}\}W_I\right)$, where $\lambda_a = \Theta\left(\frac{4\pi^2}{\phi^2} \cdot \frac{W_A}{H \log n}\right)$ and $\lambda_i = \Theta\left(\min\{\frac{b}{n}, \frac{bm}{nC_I}\}W_I\right)$;
- ii) When *Interference Condition* is satisfied, $\lambda = \Theta\left(\frac{2\pi}{\delta}\right)$
- iii) When $hac y c reference Condition is statisfied, <math>\lambda = O\left(\frac{b}{\phi} \frac{W_A}{C_A^{\frac{1}{2}} H \log^{\frac{1}{2}} n}\right) + \Theta\left(\min\left\{\frac{b}{n}, \frac{bm}{nC_I}\right\}W_I\right)$, where $\lambda_a = \Theta\left(\frac{2\pi}{\phi} \cdot \frac{W_A}{C_A^{\frac{1}{2}} H \log^{\frac{1}{2}} n}\right)$ and $\lambda_i = \Theta\left(\min\left\{\frac{b}{n}, \frac{bm}{nC_I}\right\}W_I\right)$;

 iii) When Destination-bottleneck Condition is satisfied, $\lambda = \Theta\left(\frac{n^{\frac{1}{2}} \log \log(H^2 \log n)W_A}{C_A H \log^{\frac{1}{2}} n \cdot \log(H^2 \log n)}\right) + \Theta\left(\min\left\{\frac{b}{n}, \frac{bm}{nC_I}\right\}W_I\right)$,

 where $\lambda_a = \Theta\left(\frac{n^{\frac{1}{2}} \log \log(H^2 \log n)W_A}{C_A H \log^{\frac{1}{2}} n \cdot \log(H^2 \log n)}\right)$ and $\lambda_i = \Theta\left(\min\left\{\frac{b}{n}, \frac{bm}{nC_I}\right\}W_I\right)$;

 iv) When Interface-bottleneck Condition is satisfied, $\lambda = O\left(\frac{n^{\frac{1}{2}} \log \log(H^2 \log n)W_A}{n^{\frac{1}{2}} \log \log(H^2 \log n)}\right)$
- iv) When Interface-bottleneck Condition is satisfied, $\lambda =$ $\Theta\left(H^2 \frac{\log n}{n} \cdot \frac{W_A}{C_A}\right) + \Theta\left(\min\left\{\frac{b}{n}, \frac{bm}{nC_I}\right\} W_I\right), \text{ where } \lambda_a = \Theta\left(H^2 \frac{\log n}{n} \cdot \frac{W_A}{C_A}\right) \text{ and } \lambda_i = \Theta\left(\min\left\{\frac{b}{n}, \frac{bm}{nC_I}\right\} W_I\right).$

Proof. The detailed proof is presented in [43].

As shown in Corollary 1, an MC-IS-DA network has four capacity regions similar to an MC-IS network. However, compared with an MC-IS network, an MC-IS-DA network has the higher throughput capacity than an MC-IS network when Connectivity requirement and Interference requirement dominate. In particular, when Connectivity Condition is satisfied, an MC-IS-DA network has a capacity gain $\frac{4\pi^2}{\phi^2}$ over an MC-IS network. When Interference Condition is satisfied, an MC-IS-DA network has a capacity gain $\frac{2\pi}{\phi}$ over an MC-IS network. This result implies that using directional antennas in an MC-IS network can significantly improve the capacity contributed by ad hoc communications. The capacity improvement may owe to the improved network connectivity and the reduced interference. One thing to note that the capacity of MC-IS-DA network contributed by infrastructure communications λ_i is the same as that of an MC-IS network, implying that using directional antennas at base stations will not improve the capacity. However, our following analysis will prove that using directional antennas at base stations can significantly reduce the delay contributed by infrastructure communications.

2. Delay of an MC-IS-DA network

Recall in Section VI-C that $C_I \leq m$ so that the maximum throughput capacity contributed by infrastructure communications can be achieved. We usually have $C_I = m$ so that there is no waste of interfaces, implying that we shall assign a dedicated interface per channel at each base station so that all the C_I channels can be fully utilized. However, as the radio spectrum is becoming more congested and scarce [44], it is extravagant and impractical to let $C_I = m$. Thus, we extend our analysis to the case with $C_I < m$.

We first equally divide m antennas into κ groups, each of which has $\frac{m}{\kappa}$ antennas (m is assumed to be divisible by κ though this analysis can be easily extended to the case that mis not divisible by κ). Within each group, the $\frac{m}{\kappa}$ antennas are pointed to the same direction so that their beams cover each other, as shown in Fig. 6. We name each group of antennas as a sector. It is obvious that each sector will cover θ . There is no overlapping between any two adjacent sectors. Therefore, there is no conflict between any transmissions from two adjacent sectors. The conflict only happens between the antennas within the same sector. To avoid conflicts, we can assign C_I channels to the conflicting transmissions within the same sector. In an MC-IS-DA network, each base station with multiple directional antennas can support more simultaneous transmissions than that of a typical MC-IS network. Intuitively, an MC-IS-DA network can have a better performance than a typical MC-IS network. In particular, we have the following result.

Corollary 2: The average delay of all packets in an extended MC-IS network is $D = \Theta\left(\frac{H^3 \log n}{n}\right) + \Theta\left(\frac{c}{\lfloor \frac{2\pi}{\theta} \rfloor \cdot C_I}\right)$, where $D_a = \Theta\left(\frac{H^3 \log n}{n}\right)$ and $D_i = \Theta\left(\frac{c}{\lfloor \frac{2\pi}{\theta} \rfloor \cdot C_I}\right)$. **Proof.** The detailed proof is presented in [43].

It is shown in Corollary 2 that using directional antennas at base stations in an MC-IS network can further reduce the average delay contributed by infrastructure communications D_i in the case $C_I < m$ since obviously $\lfloor \frac{2\pi}{\theta} \rfloor C_I > C_I$. Besides, Corollary 2 also shows that the narrower antenna beamwidth θ is, the lower average delay D_i is. This result also implies that using directional antennas in an MC-IS network can significantly improve the spectrum reuse. For example, suppose that we only have only one channel available, i.e. $C_I=1$, which can only be used by one omni-directional antenna in an MC-IS network. However, in an MC-IS-DA network where each base station is equipped with 12 directional antennas each with beamwidth $\frac{\pi}{6}$ (i.e., 30 °), this single channel can be simultaneously used by 12 antennas.

B. Impacts of Mobility

Multi-hop and short-ranged ad hoc communications inevitably result in the low throughput and the high delay due to the interference among multiple concurrent transmissions and the time spent on multi-hop relays. As shown in [45], to allow a mobile node to serve as the relay between the source and the destination can greatly reduce the interference and consequently lead to the higher throughput than the network without mobile relays. In MC-IS networks, we can also employ mobile nodes to serve as the relays similar to [45]. Note that the mobility can only be applied to common nodes instead of base stations since all the base stations are connected through a wired network and they are usually fixed. When there is the similar assumption on the mobile model (i.e. random walk) to [45], we shall be able to derive the higher throughput capacity contributed by ad hoc communications, which shall be bounded by $\Theta(W_A)$ as suggested in [45].

In addition to random walk model, more realistic mobility models, such as random way-point model [46] and Brownian motion model [47]. can also be used in our *MC-IS* networks. It is not the focus of our paper to consider mobility in our *MC-IS* networks due to the following reasons: (1) most of existing mobility models can be directly used in ad hoc communications in our *MC-IS* networks, which basically have the similar features to conventional ad hoc networks; (2) introducing mobile relay nodes to the network also brings the higher delay no matter which mobility model is used, as indicated in [3], [47]. This is because it always takes a long time for relay nodes to move from the source to the destination.

C. Implications of our results

The penetration of wireless communications with mobile intelligent technologies is significantly changing our daily lives. It arises a diversity of scalable smart communication systems, e.g., wireless sensor networks (WSNs), smart grid and smart home [19], [20]. The smart communication systems require smart devices (smart-phones, smart appliances, sensors, robots, surveillance devices) connected together. Due to the heterogeneity of devices and applications, heterogeneous traffics are generated. Take the smart grid as an example. It may require the narrower bandwidth to transmit power consumption information from smart meters to the operation center than that to transmit surveillance videos. The heterogeneity of the network performance requirements of various applications leads to the new research challenges in this area [48], e.g., how to improve the throughput capacity by offloading the traffic at base stations. Our MC-IS networks provide a

solution to the above raised challenges. When there are a large number of low-volume traffics, e.g., transmitting monitored temperature information from sensors to sinks in a WSN, we need to let ad hoc communications dominate, i.e. λ_a dominates λ_i , as implied from our results. On the other hand, when there are high-volume traffics, such as transmitting images or surveillance videos obtained from autonomous cameras to the controlling center of a smart grid, we need to let infrastructure communications dominate, i.e. λ_i dominates λ_a . When there are some hybrid traffics of high-volume data and low-volume data, we need to assign ad hoc communications and infrastructure communications proportionally. There is an interesting question: how to assign the traffics to either infrastructure communications or ad hoc communications according to different bandwidth requirements of various applications.

Device-to-Device (D2D) communications have recently attracted great attentions since this technology can offload the network traffic, improve the spectrum reuse and increase the throughput capacity [18], [49]. However, there are a number of challenges in D2D networks, such as the interference management, relay management and the spectrum allocation. D2D networks have the common features of our MC-IS networks - there are two kinds of communications in a D2D network: (i) D2D communications between devices (similar to ad hoc communications in our MC-IS networks) and (ii) cellular communications between devices and base stations (similar to infrastructure communications in our MC-IS networks). Thus, our theoretical analysis on MC-IS networks can be used to analyze the performance of D2D networks. For example, we can allocate C_A channels for multi-hop D2D communications and allocate C_I channels for cellular communications in D2D networks. The throughput and the delay of D2D networks shall have the same bounds as our MC-IS networks. Meanwhile, our proposed H-max-hop routing scheme can be applied to D2D networks to solve the relay (routing) issues with multi-hop D2D communications [50], [51] since it is more practical than conventional ad hoc routing schemes, which often traverse the whole network while our H-max-hop routing scheme can localize the communications within H hops.

VIII. CONCLUSION

In this paper, we propose a novel MC-IS network. We derive the upper bounds and lower bounds on the capacity of an MC-IS network. Besides, we find that an MC-IS network has a higher optimal capacity and the lower average delay than an MC-AH network and an SC-AH network. In addition, we show that an MC-IS network has the same optimal capacity as an SC-IS network while maintaining a lower average delay than an SC-IS network. Moreover, since each common node in an MC-IS network is equipped with a single interface only, we do not need to make too many changes to conventional ad hoc networks while obtaining high performance. We extend our analysis on an MC-IS network equipped with omni-directional antennas only to an MC-IS network equipped with directional antennas only, which are named as an MC-IS-DA network. We show that an MC-IS-DA network has an even lower delay of $\frac{c}{\lfloor \frac{2\pi}{a} \rfloor \cdot C_I}$ compared with an SC-IS network and our MC-IS network.

APPENDIX A

Proof of Proposition 2

Let the average distance between a source and a destination be \overline{l} , which is roughly bounded by $\overline{h} \cdot r(n)$. In the network with n nodes and under the H-max-hop routing scheme, there are at most $n \cdot P(AH)$, where P(AH) is the probability that a node transmits in ad hoc mode. Within any time period, we consider a bit $b, 1 \leq b \leq \lambda n P(AH)$. We assume that bit b traverses h(b) hops on the path from the source to the destination, where the b-th hop traverses a distance of r(b,h). It is obvious that the distance traversed by a bit from the source to the destination is no less than the length of the line jointing the source and the destination. Thus, after summarizing the traversing distance of all bits, we have $\lambda_a \cdot n\overline{l} \cdot P(AH) \leq \sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} r(b,h)$.

Let T_h be the total number of hops traversed by all bits in a second and we have $T_h = \sum_{b=1}^{n\lambda_a P(AH)} h(b)$. Since each node has one interface which can transmit at most $\frac{W_A}{C_A}$, the total number of bits that can be transmitted by all nodes over all interfaces are at most $\frac{W_A n}{2C_A}$, i.e., $T_h \leq \frac{W_A n}{2C_A}$.

On the other hand, under the interference model, we have

On the other hand, under the interference model, we have $\operatorname{dist}(X_1-X_2)\geq \frac{\Delta}{2}(\operatorname{dist}(X_3-X_4)+\operatorname{dist}(X_1-X_2)),$ where X_1 and X_3 denote the transmitters and X_2 and X_4 denote the receivers. This in-equality implies that each hop consumes a disk of radiums $\frac{\Delta}{2}$ times the length of the hop. Therefore, we have $\sum_{b=1}^{n\lambda_a P(AH)}\sum_{h=1}^{h(b)}\frac{\pi\Delta^2}{4}(r(b,h))^2\leq W_A$, which can be rewritten as

$$\sum_{b=1}^{n\lambda_a P(AH)} \sum_{b=1}^{h(b)} \frac{1}{T_h} (r(b,h))^2 \le \frac{4W_A}{\pi \Delta^2 T_h}.$$
 (3)

Since RHS of this in-equality is convex, we have

$$\left(\sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} \frac{1}{T_h} r(b,h)\right)^2 \le \sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} \frac{1}{T_h} (r(b,h))^2.$$
(4)

Joining Eq. (3) and Eq. (4), we have $\sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} r(b,h) \leq \sqrt{\frac{4W_A T_h}{\pi \Delta^2}}.$ Since $T_h \leq \frac{W_A n}{2C_A}$, we have $\sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} r(b,h) \leq \sqrt{\frac{4W_A T_h}{\pi \Delta^2}}.$

Since
$$T_h \leq \frac{NAh}{2CA}$$
, we have $\sum_{b=1}^{MA} r(b,h) \leq W_A \sqrt{\frac{2n}{\pi\Delta^2 C_A}}$. Besides, since $\lambda_a \cdot n\overline{l} \cdot P(AH) \leq \sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} r(b,h)$, we have $\lambda_a \leq \frac{W_A \sqrt{\frac{2n}{\pi\Delta^2 C_A}}}{n\overline{l} \cdot P(AH)} = \frac{W_A \sqrt{\frac{2n}{\pi\Delta^2 C_A}}}{n\overline{l} \cdot r(n)\pi H^2(r(n))^2} \leq \frac{W_A \sqrt{\frac{2n}{\pi\Delta^2 nC_A}}}{\pi H^3(r(n))^3}$. Since $r(n) > \sqrt{\frac{\log n}{\pi n}}$, we final prove the result.

APPENDIX B

Proof of Lemma 4

Consider any cell in Fig. 4. The distance between any transmitter and receiver within the cell can not be more than $r_{\rm max} = \sqrt{2a(n)}$. Under the interference model, a transmission can be successful if no node within distance $d_s = (1+\Delta)r_{\rm max}$ of the receiver transmits at the same time. Therefore, all the interfering cells must be contained within a disk D. The number of cells contained in disk D is thus bounded by $k_5 = \frac{(\sqrt{2}d_s)^2}{a(n)} = \frac{(\sqrt{2}(1+\Delta)r_{\rm max})^2}{a(n)} = 4(1+\Delta)^2$, which is a constant, independent of n.

APPENDIX C

Proof of Lemma 5

Consider a cell S contained in a disk of radius $R_0 = \frac{\sqrt{a(n)}}{2}$. Suppose S_i lies at distance x from the center of the disk. The angle α subtended at S_i by the disk is no more than $\frac{k_7}{x} \cdot \sqrt{\frac{a(n)}{2}}$. It the destination node D_i is not located within the sector of angle α , the line l_i cannot intersect the disk containing the cell S. Thus, the probability that L_i intersects the disk is no more than $\frac{k_8H^2(r(n))^2}{x} \cdot \sqrt{\frac{a(n)}{2}}$.

Since each source node S_i is uniformly distributed in the plane of unit area, the probability density that S_i is at a distance x from the center of the disk is bounded by $2\pi x$. Besides, $R_0 \leq x \leq H \cdot r(n)$. In addition, to ensure the successful transmission, the transmission range $r(n) \leq 4R_0 = \sqrt{8(a(n))}$. As a result, we have

$$P\left(L_i \text{ intersects } S \text{ and the transmission along } L_i \text{ is using bandwidth } \frac{W_A}{C_A}\right) \leq \int_{R_o}^{H \cdot r(n)} \frac{H^2}{x} \cdot ((a(n))^{\frac{3}{2}} \cdot 2\pi x dx \leq k_6 H^3(a(n))^2.$$

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