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Implied Volatility Indices and Volatility Forecasting

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PROBLEM STATEMENT

The purpose of this thesis is to expand upon current techniques in volatility forecasting. We look into two key issues:

1. Volatility forecasting in the U.S. market has been extensively studied for equity indices, but not for individual stocks. We apply modern techniques to forecast the volatility of individual stocks, and then study how they can be improved upon with information from individual implied volatility indices and earnings announcement dates.

2. We construct and evaluate the potential value of an implied volatility index for the Norwegian market. Implied volatility indices have become immensely popular internationally, but none exists for the Norwegian market.

PREFACE

This thesis is submitted in partial fulfillment of the requirements for the degree Master of Science at the Norwegian University of Science and Technology.

Our objective is to expand the current literature on volatility modelling of financial securities. This active area of research first came to our attention as part of our project thesis, and since then our understanding of the subject has been expanded through multiple conversations with our supervisor Peter Molnar, DNB Markets and Oslo Børs. Understanding volatility is a vital part of risk management, portfolio optimization, pricing of derivatives, hedging of portfolios and more. Modelling volatility has become a rich subject in academic research, with great progress made over the last decade.

Our research has culminated in two articles. One where we introduce a new implied volatility index for the Norwegian market, and another where we seek to improve volatility forecasts of single stocks in the U.S. through self-constructed volatility indices. Both articles explore new terrain in the field of volatility modelling, and we hope our contribution can be of interest to the academic community.

We would like to thank our supervisor, Post. Doc. Peter Molnar at the Department of Industrial Economics and Technology Management for making this thesis possible. Your ideas and support have been invaluable. We also wish to thank Øyvind Skar and Oslo Børs for providing us with essential data for this project. Finally, we would like to thank our friends and families for putting up with us while we have been working on this thesis.

ABSTRACT

This thesis consists of two articles that study volatility forecasts and the value of implied volatility indices.

In the first paper, we construct implied volatility indices for all stocks in the Dow Jones Industrial Average Index, and study how they can improve volatility forecasts for the individual stocks. In addition, we utilize information about earnings announcement dates to account for these stock specific events. We find that both measures improve the volatility forecasts significantly. The implied volatility indices for individual stocks improve the forecasts more than the general VIX index. However, after we adjust for earnings announcements, we find that the VIX is equally useful as the individual implied volatility indices. On average, we are able to reduce the forecast errors by 9% compared to our benchmark model.

In the second paper, we construct and evaluate the NOVIX - an implied volatility index for the Norwegian market created according to the VIX methodology. Implied volatility indices have been an active area of research since the introduction of the VIX by CBOE in 1993. Since then, more and more exchanges have introduced their own implied volatility indices with great success, yet there exists no official index for the Norwegian market. We evaluate the relationship between the NOVIX and returns and realized volatility of the underlying OBX index, and find that the ability of the NOVIX to capture information has improved consistently over the last decade. For the most recent years, we find that the NOVIX has similar properties to the popular VIX and VDAX-NEW volatility indices from the U.S. and German markets.

SAMMENDRAG

Vi har skrevet to artikler om volatilitetsprognoser og verdien av indekser for implisitt volatilitet som til sammen utgjør vår masteroppgave.

I den første artikkelen lager vi indekser for implisitt volatilitet for alle aksjene i den amerikanske DIJA indeksen, og studerer hvordan disse kan brukes til å forbedre volatilitetsprognoser for enkeltaksjene. Sammen med disse bruker vi informasjon om når kvartalsrapportene til selskapene publiseres for å forklare aksjespesifikke hendelser. Vi finner at begge informasjonskildene kan brukes til å signifikant forbedre volatilitetsprognosene, og at de aksjespesifikke volatilitetsindeksene forbedrer volatilitetsprognosene mer enn den generelle VIX-indeksen. Etter at vi har justert for datoer for kvartalstall, finner vi at VIX-en er like nyttig som de aksjespesifikke volatilitetsindeksene. I snitt er vi i stand til å redusere feilen i volatilitetsprognosene med 9%.

I den andre artikkelen lager vi NOVIX - en volatilitetsindeks for det norske markedet som baserer seg på VIX-metoden. Volatilitetsindekser for implisitt volatilitet har vært et aktivt forskningsområde siden VIX-en ble introdusert av CBOE i 1993. Siden da har stadig flere børser lansert egne, suksessfulle volatilitetsindekser. Likevel finnes det ingen offisiell volatilitetsindeks for det norske markedet. Vi evaluerer forholdet mellom NOVIX og avkastning samt realisert volatilitet på den underliggende OBX-indeksen. Vi finner at NOVIX har blitt konsistent bedre til å absorbere informasjon over det siste tiåret. For de siste årene har NOVIX vist tilsvarende egenskaper som de populære VIX og VDAX-NEW indeksene for det amerikanske og tyske markedet.

VOLATILITY FORECASTING FOR SINGLE STOCKS

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Abstract

We construct implied volatility indices for all stocks in the Dow Jones Industrial Average Index, and study how they can improve volatility forecasts for the individual stocks. In addition, we utilize information about earnings announcement dates to account for these stock specific events. The forecast models are based on the HAR-RV model of Corsi (2009). We find that both measures improve the volatility forecasts significantly. The implied volatility indices for individual stocks improve the forecasts more than the general VIX index. However, after we adjust for earnings announcements, we find that the VIX is equally useful as the individual implied volatility indices. Our results are significant not only statistically, but also from a practical perspective, with forecast errors that are on average 9% smaller than forecast errors from the benchmark HAR-RV model.

1 INTRODUCTION

The Heterogeneous Autoregressive Realized Volatility model (HAR-RV) of Corsi (2009) has emerged as a preferred specification for realized volatility forecasting. While the HAR-RV model is based only on realized volatility from high frequency data, it can be extended with other sources of information. In this paper, we create implied volatility indices for each of the individual stocks in the Dow Jones Industrial Average (DJIA) index, and examine their value together with the VIX and information about earnings announcements.

The use of implied volatility in volatility forecasting has been widely studied across different markets, and implied volatility is often referred to as the market's own volatility forecast. Implied volatility is generally accepted as a relevant source of information about future volatility for equity markets, with a notable exception in Canina and Figlewski (1993).

Blair, Poon, and Taylor (2001) compare the information in daily index returns, daily VIX observations and intraday returns for the S&P 100 index with ARCH models. They find that the VIX index provides the most accurate forecasts for all forecast horizons, and that intraday returns provide little additional information. Later, Koopman, Jungbacker, and Hol (2005) perform another study on the S&P100, but with long memory models, GARCH, and stochastic volatility models instead of ARCH. In a comparison of historical volatility, the VIX and realized volatility, they find that realized volatility is the most relevant when forecasting volatility.

Busch, Christensen, and Nielsen (2011) study the role of implied volatility as a complementary source of information beyond that of realized volatility, by forecasting foreign exchange, stock and bond markets volatility. They use the HAR-RV model with Black-Scholes implied volatility as an additional variable, and conclude that implied volatility is important for forecasting all three markets. Moreover, they suggest that implied volatility should be used alone when forecasting realized volatility on a monthly horizon for all three markets.

In general, several measures can be used as implied volatility in the volatility forecasting models. Busch, Christensen, and Nielsen (2011) use

Black-Scholes implied volatility, Blair, Poon, and Taylor (2001) and Koopman, Jungbacker, and Hol (2005) use the model-based old VIX methodology, and Becker, Clements, and White (2006) use the model-free new VIX methodology.

The Chicago Board Option Exchange (CBOE) introduced the CBOE volatility (VIX) index in 1993 (Whaley, 1993). This first official implied volatility index was based on the Black-Scholes pricing model (Black and Scholes, 1973), and calculated as the average Black-Scholes implied volatility from S&P100 options. In total, the method uses eight near-the-money puts and calls for the nearby and second most nearby maturity. Although it captures more info than the implied volatility of a single strike, it does not capture all the information in the wide range of strikes available. Also, the method still depends on the assumptions of the Black-Scholes formula.

A decade later, the VIX was revised in a collaboration with Goldman Sachs with the purpose of providing exchange-traded volatility derivatives. The underlying index changed from S&P100 to the S&P500. More importantly, the method for calculating the index was replaced by a model-free approach. The concept of model-free implied variance is based on work by Derman and Kani (1994), Dupire (1994, 1997) and Rubinstein (1994), and was first coined by Britten-Jones and Neuberger (2000). They use no-arbitrage conditions to extract common features of all stochastic processes that are consistent with observed option prices. This has the advantage of not depending on any particular option-pricing model, and extracts information from all relevant option prices (Jiang and Tian, 2005).

Jiang and Tian (2005) compare the traditional concept of implied volatility with model-free implied volatility. Their results for the S&P500 with a monthly forecast horizon suggest that the model-free implied volatility subsumes all information contained in the Black–Scholes implied volatility. Further, their results show that past realized volatility is a more efficient forecast for future realized volatility than implied volatility. Becker, Clements, and White (2006) examine whether the VIX contains any information relevant to future volatility beyond that available from a wide range of model based volatility forecasts. Their findings indicate that the VIX index does not contain any such information, and they conclude that the S&P500 options market cannot anticipate movements in volatility unanticipated by model based forecasts.

Volatility forecasting has mostly been studied for equity indices. Our study is the first on the value of individual stocks' implied volatility indices in volatility forecasting. Motivated by the previous research on equity indices, we choose to use a model-free implied volatility index together with realized volatility. Only three of the stocks in the DJIA index have an official implied volatility index, and we therefore construct new implied volatility indices for each of the stocks in the DJIA index.

Since we study individual stocks, there are certain special dates that can be incorporated into the forecasts. These dates are not relevant for equity indices, and therefore not much studied yet. Most important is earnings announcements. The Securities & Exchange Commission requires all companies with securities traded on a U.S. based stock market to release quarterly earnings. The seminal work of Beaver (1968) established that both trading volume and return volatility increase at the time of earnings announcements. Later, Patell and Wolfson (1979) captured the ex ante information content of annual earnings announcements, and found that the future release of annual earnings numbers does affect security prices. More recently, Landsman and Maydew (2002) examine the evolution of the information content of quarterly earnings announcements. Their results suggest that the informativeness of earnings announcement dates has increased over time.

Information about earnings announcement dates is easily available in advance, and thus we argue it should be included when forecasting the volatility of individual stocks.

With this in mind, we augment the HAR-RV model of Corsi (2009) with the identified sources of information — the individual stocks' implied volatility indices, the VIX and the earnings announcement dates. The HAR-RV model has shown remarkably good forecasting performance. It exploits the information inherent in high-frequency data described by Andersen et al. (2003) through the realized volatility measure, and has arguably emerged as the most popular model for realized volatility based forecasting.

We forecast daily, weekly and monthly volatility for the stocks on DJIA from 2006 to 2014. To evaluate the forecasts, we use the Model Confidence Set (MCS) procedure of Hansen, Lunde, and Nason (2011) to identify the set of best models at a given confidence level. We get the best results from including earnings announcement dates together with any of the two sources of implied volatility. When combined with earnings announcement dates, both sources are equally useful. Without earnings announcement dates, we find that the individual stock's implied volatility indices give better results than the VIX. This indicates that the stock's implied volatility indices include information about earnings announcements that the VIX does not. Our results are significant not only statistically, but also from a practical perspective. Forecast errors for the models that include both earnings announcement dates and an implied volatility index, either the VIX or the implied volatility index for a particular stock, are on average 9% smaller than forecast errors from the benchmark HAR-RV model.

The rest of the paper is organized as follows. Section 2 describes the data and data processing we use in this paper together with characteristics of the data. Section 3 proceeds to describe the volatility models we use in our study. We then present the forecasting results in section 4, before we provide a final conclusion in section 5.

2 DATA

We calculate implied volatility indices and realized volatility for the individual stocks in the DJIA index[1] in the period January 3rd 2010 to December 31st 2014[2]. The index is considered as one of three major indicators for the US stock market, along with Nasdaq Composite and Standard & Poor's 500 (S&P500). Together, these indices are referred to as the Security Market Indicator Series. The DJIA index is a price-weighted average of 30 stocks traded on the New York Stock Exchange and Nasdaq. The included stocks are leaders in their industries and widely held by both individual and institutional investors. Together, the 30 stocks in the DJIA represent about 20% of the overall market value of all US stocks, and thus it reflects a large part of the US economy. All 30 stocks in the DJIA are also included in the S&P500.

We use three main sources of data for our study. First, we use high-frequency prices on all the individual stocks to compute the realized volatility (RV). The RV measure is the basis of the HAR-RV model of Corsi (2009). Further, we use the complete set of all put and call options issued on each stock on a daily frequency to compute implied volatility indices for each stock. Finally, we use information about earnings announcement dates for the stocks.

2.1 Realized Volatility

If we let a trading day be split into m equidistant intervals, the intraday return r_i over the time period $[i - 1/m, i]$ is given by

$$r_i = \ln\left(\frac{P_i}{P_{i-1/m}}\right), \quad \text{for} \quad i = 1/m, 2/m, \ldots, 1, \tag{1}$$

1 We also include AAPL and AMZN since they both have an implied volatility index provided by CBOE. This gives us a total of 32 stocks in our study.

2 We calculate realized volatility from 2006 and use it to incorporate earnings announcements, but for the actual forecasts we use data from 2010.

where P_i is the price at time i. By taking the square root of the sum of the m squared intraday returns over a given trading day, we obtain daily realized volatility

$$RV^D = \sqrt{\sum_{i=1}^{m} r_i^2} \times \sqrt{250}. \tag{2}$$

We quote daily RV using the conventional annual conversion by the square-root-of-time rule, using 250 trading days a year. In the literature, the acronym RV is by convention used interchangeably for realized volatility and realized variance. We follow Corsi (2009), and reserve it for realized volatility.

RV constructed from high-frequency intraday returns permits the use of traditional time series procedures for modeling and forecasting of volatility (Andersen et al., 2001). Throughout the paper, we use RV as a proxy for the true volatility as proposed by Andersen and Bollerslev (1998).

It is an established fact that RV yields a perfect estimate of volatility in ideal situations where prices are observed continuously and without measurement errors (Hansen and Lunde, 2006; Merton, 1980). While the theory suggests that RV should be based on intraday returns sampled at the highest possible frequency, it runs into the challenge of market microstructure in real world applications (Zhang, Mykland, and Aït-Sahalia, 2005). This stems from the fact that efficient prices cannot be observed directly. Empirical work suggests that the estimate seems to diverge if RV is calculated using too frequent observations (Andreou and Ghysels, 2002; Bai, Russell, and Tiao, 2001; Bandi and Russell, 2008).

We do not include overnight returns in the calculation of RV. There is no consensus in the literature for whether to include overnight returns when calculating RV or not. Ahoniemi and Lanne (2013) find that measures that do not incorporate overnight returns performs best for individual stocks, although the overnight returns may improve in-sample performance.

We calculate RV for the 30 stocks in the DJIA index and for AMZN and GOOGL using high frequency trade data obtained from the NYSE

TAQ database. The trade data is time stamped with precision of one second, and there are typically multiple trades recorded per second. In total, the data set consists of approximately 150 million observations of raw data per stock, in the period from 2010 to 2014.

We have no guarantee for accuracy of the NYSE TAQ raw data. Therefore, a cleaning of the raw high-frequency data is a prerequisite for conducting a meaningful data series analysis, as discussed by Brownlees and Gallo (2006). Our procedure for cleaning the data is based on the steps proposed by Barndorff-Nielsen et al. (2009). The cleaning procedure is outlined below.

1) Delete entries with a time stamp outside the window 9:30am to 4:00pm when the exchange is open.

2) Delete entries with a transaction price equal to zero.

3) Retain entries from a single exchange (NASDAQ), delete all other.

4) Delete entries with corrected trades.

5) Delete entries with abnormal sales condition.

6) If multiple transactions have the same time stamp, use the median price.

7) Delete entries for which the price deviated by more than 10 mean absolute deviations from a rolling centered median of 50 observations (25 before, 25 after).

We first identify the relevant open-to-close data in step 1. Then we remove serious errors in the database in steps 2 and 5, such as misrecorded prices and timestamps that may be way off. With step 3 we reduce the impact of time-delays in reporting of trade updates. A large fraction of the trades was executed on NASDAQ, and thus it does not significantly reduce our data sets. Some version of step 6 is inevitable due to the lack of precision in the timestamps, and this step leads to the largest reduction of data. Finally, we use step 7 to remove serious outliers in the data. The choice of 50 observations for the window is *ad hoc*, but in line with Barndorff-Nielsen et al. (2009).

2.2 Implied Volatility Indices

The interest in implied volatility indices has been growing ever since the Chicago Board Option Exchange (CBOE) introduced the CBOE Volatility Index (VIX) in 1993. The VIX now estimates the 30 calendar-day expected volatility of the S&P500 index. Whaley (1993) proposed that these indices can help the investment community in at least two different ways. First, they provide reliable estimates of expected short-term stock market volatility. Second, they offer a market volatility "standard" upon which derivative contracts may be written. After the VIX was updated to a model-free approach, this became a reality when futures and options on the VIX became tradable products in 2004 and 2006. Trading and hedging in volatility have become very popular, where the combined trading activity in VIX options and futures is over 800,000 contracts per day (Chicago Board Options Exchange, 2015).

CBOE alone publishes 28 volatility indices for stock indices, ETFs, interest rates, commodities, currencies and individual stocks. Gradually, other derivatives exchanges have begun offering volatility indices for their respective markets. Some notable examples are Deutsche Börse with the VDAX (1994), the French Marche des Options Negociables de Paris (MONEP) with VX1 and VX6 (1997) and NYSE Euronext with the FTSE 100 Volatility Index (2008).

CBOE publishes single stock volatility indices for 5 stocks — AMZN, GOOG and three of the stocks in the DJIA index. To the best of our knowledge, there have not been created any individual stock volatility indices for the remaining 27 stocks in the index. We have therefore created and implemented one for each of the stocks in the DJIA index, as well as for AMZN and GOOGL. Our implied volatility indices are all based on the model-free VIX methodology, and calculated from daily close option data gathered from the OptionMetrics database.

2.3 Creating Implied Volatility Indices

When the VIX was updated in 2003, it changed from measuring the expected 30-day volatility of S&P100 to the 30-day expected S&P500 volatility. More importantly, it changed to a model-free methodology. Demeterfi et al. (1999) showed theoretically how a portfolio of standard options could replicate a variance swap and that the cost of this replicating portfolio is the fair price of a variance swap. The new VIX methodology is based on a discretization of this method for pricing a variance swap, and is now computed directly from observed option prices. Our volatility indices are computed accordingly, with the formula (Chicago Board Options Exchange, 2015)

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2, \qquad (3)$$

where

σ	VIX/100
T	Time to expiration in years
F	Forward level of underlying
K_0	First strike below F
K_i	Strike price of the i-th out-of-money option
ΔK_i	$1/2 \times (K_{i+1} - K_{i-1})$
R	Risk-free rate
$Q(K_i)$	Midpoint of bid-ask spread for option with strike K_i

Following Chicago Board Options Exchange (2015), we compute the implied volatility estimate σ for two selected maturities, a *near-term* and *next-term* maturity, representing options expiring before and after the desired 30-day horizon. For each maturity, we screen all options by using specified selection criteria to select the options to include in the calculation.

First, we find the forward level from the option prices by identifying the strike with the smallest absolute put-call price difference and applying the formula

$$F = \text{Strike} + e^{RT} \times |\text{Call Price} - \text{Put Price}|$$

We define K_0 as the first strike below F, and consider the option pair with strike K_0 as at-the-money. Then, we discard all in-the-money options. That is, we only consider the at-the-money options, the call options with strikes $K_i > K_0$ and the put options with strikes $K_i < K_0$. Intuitively, the demand for out-of-the-money options can be interpreted as a need for insurance by investors, which in turn reflects the market volatility. Further, we exclude all out-of-the-money options with a zero bid price, and all options following two zero bid prices in a row, when the options are ordered by type as increasingly out-of-the-money.

We have now obtained a complete set of options for each maturity. We apply Equation 3 to each of these sets, when the options are ordered by increasing strikes. We obtain the desired 30-day volatility estimate from an interpolation between the two results. We refer to the CBOE's White Paper (Chicago Board Options Exchange, 2015) for a more detailed options selection scheme, variable descriptions and calculations.

To verify that our implementation of the VIX-methodology is correct, we have tested the code on the example data in the CBOE White Paper (Chicago Board Options Exchange, 2015) as well as a more comprehensive verification by comparing the five equity volatility indices provided by CBOE to those we have calculated. The results are satisfying, although CBOE does not publish their raw data and we have some minor deviations that may be the result of different input data.

2.4 Earnings Announcement

The dates of the earnings announcements are known in advance. We can therefore adjust the volatility forecasts around these dates to this information prior to the actual announcements. We gather the quarterly earnings announcement dates for the DJIA stocks from the Compustat

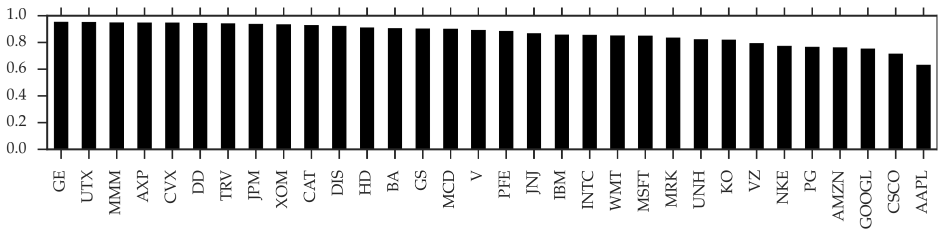

Figure 1: Correlation of returns between the VIX and the individual stock implied volatility indices for the period January 4th 2010 to December 31st 2014.

database. Earnings announcements can be released either before market opens or after it closes. In order to adjust for this, we shift the release date for stocks that announce their quarterly earnings after market close by one day. This makes the resulting data set correspond to the first trading day when the quarterly results will be known in the market.

2.5 Data Characteristics

The correlations between the VIX and the individual stock implied volatility indices are displayed in Figure 1. The highest correlations are those of GE and UTX, which both have correlation with the VIX of more than 95%. With the exception of AAPL with 63% correlation, all other volatility indices have a correlation with the VIX above 70%. The high correlations imply that our implied volatility indices capture much of the same systematic risk as the VIX.

Figure 2 displays RV and implied volatility index time series for five of the stocks. The implied volatility index is consistently above the RV. This upward bias for the implied volatility indices is a feature for equity assets, and can be explained by the required risk premium that is demanded by investors for bearing unwanted risk (Doran and Ronn, 2005). Further, we observe a cyclical fluctuation in Figure 2 for the five stocks, with reoccurring sudden spikes for RV and a gradual increase for the implied volatility indices around the same dates. These patterns match the firms' release dates of quarterly earnings. It is evident that both

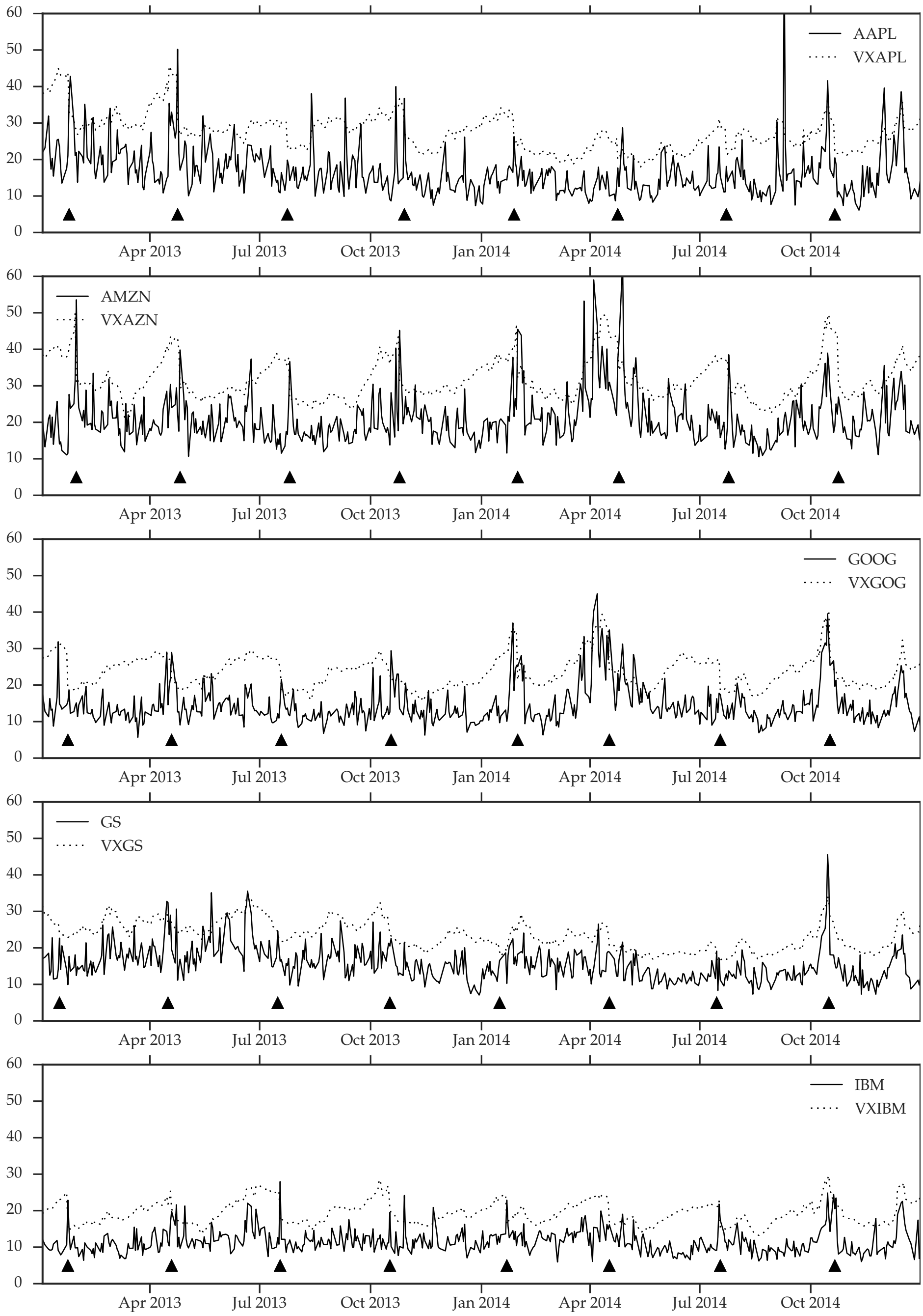

Figure 2: Realized volatility and CBOE's implied volatility indices for five stocks, quoted in percent. Each of the implied volatility indices uses CBOE's naming scheme, all starting with VX. The triangles in the bottom of each graph indicate quarterly earnings announcements.

the RV and the implied volatility measures are affected by the earnings announcements.

Figure 3 displays the average level of RV, the implied volatility indices and the VIX over a period from 15 days prior to the release dates to 15 days after. The averages are calculated using events between January 4th 2010 and December 31st 2014 for all the 32 stocks. From Figure 3, we observe that the RV increases significantly on days close to earnings announcement events and is at its highest at the release date. We also see that the implied volatility rises gradually as it heads into an announcement, before we see a drop after the earnings are announced. There is a simple explanation for why RV is at its highest and implied volatility at its lowest at the earnings announcement date: RV is the ex post volatility over the first trading day when the earnings are known, whereas IV is the observed value at the end of this day after all options prices have adjusted to the news. The implied volatility pattern demonstrates a significant risk premium embedded in the prices of traded options before

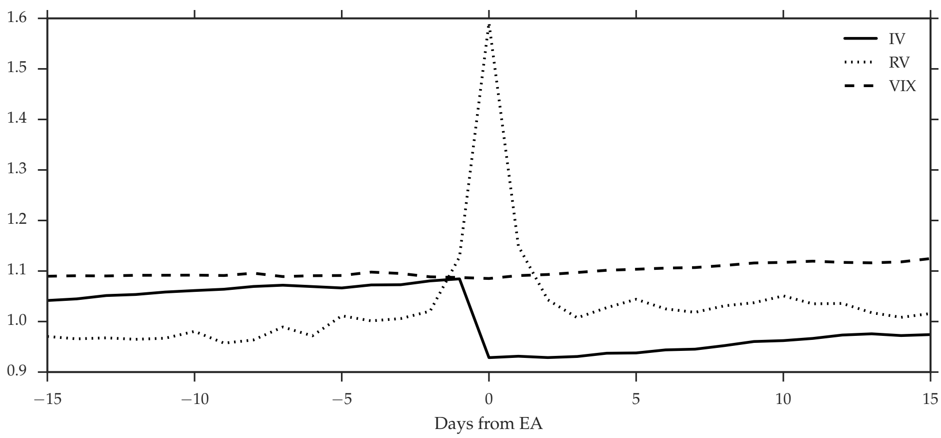

Figure 3: Average level of implied volatility, RV and VIX over a period of +/- 15 days around the release dates, calculated using quarterly earnings announcements between January 4th 2010 and December 31st 2014 for all 32 stocks. The values have been normalized such that the implied volatility, RV and VIX are relative to their average values over the entire sample.

these events, indicating that traders are averse to the increase in market uncertainty related to the announcement. These characteristics are in line with previous studies like Donders, Kouwenberg, and Vorst (2000). Further, we see that the VIX is stable, but consistently above its mean for the 30 day period. Since all the stocks in the DJIA release their quarterly earnings during a given time interval, this is likely why the VIX is elevated in this period.

3 MODEL

We use the Heterogeneous Autoregressive realized volatility model (HAR-RV) of Corsi (2009) as the base model in our study. The HAR-RV model is inspired by the Heterogeneous Market Hypothesis introduced by Müller et al. (1997), which claims that there is heterogeneity across traders with different investment horizons in the way they perceive, react and cause different types of volatility components. The model captures this with three heterogeneous volatility components that span different time horizons.

Traditionally, volatility forecasting has been done using the GARCH models, introduced by Engle (1982) and Bollerslev (1986). However, in recent years there have emerged several new methods for modeling volatility, and the introduction of realized volatility by Andersen and Bollerslev (1998) was a significant step forward for volatility modeling. The realized volatility measure exploits information in high-frequency data, and one can in effect treat volatility as observable. Today, the HAR-RV model is arguably one of the most popular models for realized volatility based forecasting. The model predicts variance better than traditional GARCH models that do not take advantage of high frequency data (Andersen et al., 2003). The HAR-RV model has also shown noteworthy good forecasting results compared to the more complicated Autoregressive Fractionally Integrated Moving Average model (Andersen, Bollerslev, and Diebold, 2007).

Since the introduction of the HAR-RV model, several extensions and variants have been developed that all seek to improve the base model. Andersen, Bollerslev, and Diebold (2007) incorporate jumps in their HAR-RV-J model. Patton and Sheppard (2015) argue that a Semivariance HAR model that separates between variance caused by positive and negative high-frequency returns, performs better than the HAR-RV-J model. Recently, Bollerslev, Patton, and Quaedvlieg (2016) introduced the HARQ model, that further expands the HAR-RV model by incorporating a time-varying variance of the measurement errors, called realized quarticity.

We choose to apply the standard HAR-RV model and extend the model in two ways. First, we add an implied volatility component to the HAR-RV model. Second, we add an earnings announcement component. The extended models follow a simple naming scheme. We add IV or VIX as suffixes to indicate that either the implied volatility index of a single stock or the VIX is included, and EA to indicate that the earnings announcement component is included.

Following Corsi (2009), we include three different volatility components based on different horizons: a short-term (daily), a medium-term (weekly) and a long-term (monthly) component, and specify the one-day ahead HAR-RV model as

Model 1 (HAR-RV)**.**

$$RV_{t+1}^{D} = \alpha + \beta_1 RV_t^D + \beta_2 RV_t^W + \beta_3 RV_t^M + \epsilon_{t+1}, \tag{4}$$

where RV_t^D, RV_t^W and RV_t^M are daily, weekly and monthly multi-period volatilities, respectively. These multi-period volatilities are defined as simple backward averages of the daily RV. That is, weekly and monthly RV using 5 trading days a week and 22 per month are

$$RV_t^W = \frac{1}{5}\sum_{i=0}^{4} RV_{t-i}^D \qquad \text{and} \qquad RV_t^M = \frac{1}{22}\sum_{i=0}^{21} RV_{t-i}^D.$$

Our first extension to the model is to add an implied volatility component similar to Haugom et al. (2014). We add ν_t to Model 1 and let it represent either the implied volatility index for the stock (IV) or the VIX at time t:

Model 2 (HAR-RV-IV and HAR-RV-VIX)**.**

$$RV_{t+1}^{D} = \alpha + \beta_1 RV_t^D + \beta_2 RV_t^W + \beta_3 RV_t^M + \beta_4 \nu_t + \epsilon_{t+1}. \tag{5}$$

Our second extension to the model is to incorporate information from earnings announcement dates. We propose a procedure to do this that is adapted to the features of the HAR-RV model. The forecast and the three heterogeneous volatility components that span different time horizons are all affected by earnings adjustment dates. In our proposed solution, we correct for these separately. First, we define an indicator function to indicate intervals in which the earnings announcements occur

$$\mathbb{1}_{A(t_1,t_2)} = \begin{cases} 1, & \text{if EA occurs in the interval } [t_1, t_2] \\ 0, & \text{otherwise.} \end{cases}$$

Next we introduce four independent variables to adjust forecasts values that in some way are affected by the earnings announcements. If an earnings announcement is due to occur within the forecast horizon, we expect an elevation in the future volatility beyond what lagged RV measures can predict. To account for this known future event, we add an independent variable $\widehat{\beta}_{EA}^{+h}$ to adjust the forecast for earnings announcements that will occur within the forecast horizon h. Conversely, if an announcement has occurred within any of the past multi-period volatility variables, these variables will be inflated. To counteract this past event, we add three independent variables, for the possibilities that EA occurred the past day, the past week or the past month, denoted $\widehat{\beta}_{EA}^{-h}$ with $h = 1, 5$ and 22, respectively.

We include earnings announcement variables that apply for each day. The daily model is specified as

Model 3 (HAR-RV-EA)**.**

$$RV_{t+1}^{D} = \alpha + \beta_1 RV_t^D + \beta_2 RV_t^W + \beta_3 RV_t^M + \beta_4 \widehat{EA}_{t+1}^{D} + \epsilon_{t+1}, \quad (6)$$

where

$$\widehat{EA}_{t+1}^{D} = \mathbb{1}_{A(t+1,t+1)} \widehat{\beta}_{EA}^{+1} + \sum_{h \in \{1,5,22\}} \mathbb{1}_{A(t+1-h,t)} \widehat{\beta}_{EA}^{-h}.$$

Finally, we extend this model with implied volatility measures similar to how Model 2 was defined,

Model 4 (HAR-RV-EA-IV and HAR-RV-EA-VIX)**.**

$$RV_{t+1}^{D} = \alpha + \beta_1 RV_t^{D} + \beta_2 RV_t^{W} + \beta_3 RV_t^{M} + \beta_4 \widehat{EA}_{t+1}^{D} + \beta_5 \nu_t + \epsilon_{t+1}, \tag{7}$$

where $\widehat{EA}_{t+1}^{D}$ is defined as in Model 3 and ν_t is defined as in Model 2.

The $\widehat{\beta}_{EA}^{+/-h}$ variables are estimates from a separate regression that we perform prior to the forecasting. If we had a long history of data available, these variables could be estimated directly for each stock. However, earnings announcements only occur four times per year for each stock, and we only have a few years of data. Therefore, we pool data together and estimate the variables for all stocks simultaneously in a separate panel OLS regression. In the EA estimation below, we add subscript s to distinguish between the stocks, and estimate the following for all stocks s simultaneously

Model 5 (EA-estimation)**.**

$$RV_{s,t+1}^{D} = \alpha + \beta_1 RV_{s,t}^{D} + \beta_2 RV_{s,t}^{W} + \beta_3 RV_{s,t}^{M} + \beta_{EA}^{+1} \mathbb{1}_{A(s,t+1,t+1)} + \sum_{h \in \{1,5,22\}} \beta_{EA}^{-h} \mathbb{1}_{A(s,t+1-h,t)} + \epsilon_{t+1}, \tag{8}$$

where the estimated values for the $\beta_{EA}^{+/-h}$ coefficients become the independent $\widehat{\beta}_{EA}^{+/-h}$ variables in the HAR-RV-EA model. This estimation is performed with a window of 1000 days for each day of the actual forecast regressions.

In addition to daily forecasts, we modify Model 1, Model 2, Model 3 and Model 4 to produce weekly and monthly forecasts. To do this, we change the left-hand-side variables in all the models to forward weekly and monthly averages, given by

$$RV^W_{t+1} = \frac{1}{5}\sum_{i=1}^{5} RV^D_{t+i} \qquad \text{and} \qquad RV^M_{t+1} = \frac{1}{22}\sum_{i=1}^{22} RV^D_{t+i}.$$

We update Model 3 and Model 4 by changing $\widehat{EA}^D_{t+1}$ to $\widehat{EA}^W_{t+1}$ or $\widehat{EA}^M_{t+1}$ for the weekly and monthly forecasts, with the following definitions

$$\widehat{EA}^W_{t+1} = \mathbb{1}_{A(t+1,t+5)}\widehat{\beta}^{+5}_{EA} + \sum_{h\in\{1,5,22\}} \mathbb{1}_{A(t+1-h,t)}\widehat{\beta}^{-h}_{EA}$$

$$\widehat{EA}^M_{t+1} = \mathbb{1}_{A(t+1,t+22)}\widehat{\beta}^{+22}_{EA} + \sum_{h\in\{1,5,22\}} \mathbb{1}_{A(t+1-h,t)}\widehat{\beta}^{-h}_{EA},$$

where the only change is the expansion of the forward interval for the indicator function prior to the $\widehat{\beta}^{+h}_{EA}$ variable, now matching the forecast horizon. The estimation of the $\widehat{\beta}^{+/-h}_{EA}$ variables is done similarly as for daily.

3.1 Model Robustness

There are many ways the information in earnings announcement dates can be included in the HAR-RV model. There are no standard way of doing this in the literature, and if its done inefficiently it will not capture the full value of knowing the earnings announcement dates. Our goal is capture the effect in an intuitive and computationally efficient manner. We have explored several methods for doing this, where we have fully tested and implemented two concepts in addition to the one we present. The first was to introduce estimated independent EA variables, similar to what we do now, but where each EA variable represents a particular day away from the announcement. The goal was to capture the increase seen in volatility on the days prior to and after the announcements as well. The method worked well for the daily forecast, but the results results were not satisfactory for longer forecast horizons.

The second concept we explored in-depth was to remove the average increase in RV over a given interval around the earnings announcement from the time series. Then we simply forecasted with these adjusted time series. Afterwards, we added back the removed increase to our forecast values for the relevant days. This method worked well, but not as well as our current method. Also, the method became less intuitive for the longer horizons, where we adjusted the daily observation and added back an increase for the weekly or monthly forecast values.

We acknowledge the opportunity that there are better and more sophisticated methods for capturing the information about earnings announcements. The topic has sparse coverage in current literature, and could be an interesting subject for further research.

One potential for improvement of our implementation that deserves some attention is the estimation of the $\widehat{\beta}_{EA}^{+/-h}$ variables. They are estimated with panel OLS for all stocks simultaneously. The argument for doing this is the lack of EA observation, which is limited to four observations a year per stock. However, the lack of data is not an issue for the estimation of α and β's in Model 5. Thus, we have tested the effect of improving the estimation of the $\widehat{\beta}_{EA}^{+/-h}$ variables as an robustness check by allowing for stock-specific α and β's similar to how they are estimated in the actual forecasts

Model 6 (EA-estimation with stock-specific coefficients)**.**

$$RV_{s,t+1}^{D} = \alpha_s + \beta_{s,1} RV_{s,t}^{D} + \beta_{s,2} RV_{s,t}^{W} + \beta_{s,3} RV_{s,t}^{M} + \\ \beta_{EA}^{+1} \mathbb{1}_{A(s,t+1,t+1)} + \sum_{h \in \{1,5,22\}} \beta_{EA}^{-h} \mathbb{1}_{A(s,t+1-h,t)} + \epsilon_{s,t+1}, \tag{9}$$

This can not be solved with regular panel OLS. Instead, we estimate the $\widehat{\beta}_{EA}^{+/-h}$ coefficients with a quadratic optimization program. The optimization program allows for stock-specific α and βs, but the underlying concept and loss function are similar to a panel OLS. Table 7 in Appendix A contains the specified optimization program for the day-ahead forecasts. We set up similar optimization programs for the weekly and monthly forecasts as well. The parameters are estimated for each day using a rolling window of the previous 1 000 observations, similarly as with the panel OLS.

The method slightly improves the utilization of information in the earnings announcement dates, based on comparing mean square errors when using panel OLS. However, given the small improvement we feel that the computationally efficient panel OLS that is available in statistical software is a more practical choice. As such, we present only results that are calculated with the panel OLS.

4 RESULTS

4.1 In-Sample Evaluation

We begin our evaluation of the forecasting models by considering the in-sample results. Although the out-of-sample results are the natural focus of the paper, we use the in-sample results to confirm that the variables for implied volatility indices and earnings announcements are significant. Furthermore, the estimated coefficients help us build an intuition for the behaviour of the model.

Table 1 summarizes the daily horizon in-sample results by averaging the mean squared error (MSE) and adjusted R^2 across all 32 stocks. The MSE and adjusted R^2 measures improve when either IV, VIX or EA is included in the base model. If we include only one component, the VIX is slightly less useful than the other two. We get the best results from including both IV or VIX and EA, with an average improvement in MSE and the adjusted R^2 of approximately 9% from the base model.

Table 1: Summary of in-sample evaluations for daily forecasts using MSE and adjusted R^2

Model	$\overline{\text{MSE}}$	$\overline{R^2_{Adj}}$
HAR-RV	1.000	0.473
HAR-RV-IV	0.939	0.504
HAR-RV-VIX	0.957	0.495
HAR-RV-EA	0.949	0.502
HAR-RV-IV-EA	0.901	0.526
HAR-RV-VIX-EA	0.906	0.525

The average MSE have been normalized to the MSE of the HAR-RV model.

In Table 2, we present the one-day horizon in-sample results for two of the stocks. Appendix A contains the in-sample results for the remaining 30 stocks, found in Table 8 to Table 13. The behaviour of the two stocks

Table 2: In-sample evaluation for one-day ahead forecasts.

		Regression coefficients							
Ticker	Model	$\hat{\alpha}$	Daily	Weekly	Monthly	EA	IV/VIX	R^2_{adj}	AIC
AAPL	HAR-RV	0.035** (0.007)	0.52 ** (0.075)	0.065 (0.078)	0.216** (0.06)			0.390	-2.580
	HAR-RV-IV	−0.01 (0.008)	0.462** (0.082)	0.006 (0.074)	0.075 (0.064)		0.305** (0.046)	0.418	-2.625
	HAR-RV-VIX	0.03 ** (0.008)	0.511** (0.075)	0.06 (0.08)	0.17* (0.07)		0.089 (0.046)	0.393	-2.583
	HAR-RV-EA	0.035** (0.007)	0.521** (0.075)	0.062 (0.076)	0.223** (0.059)	0.883** (0.183)		0.405	-2.604
	HAR-RV-IV-EA	−0.005 (0.008)	0.469** (0.081)	0.01 (0.072)	0.096 (0.064)	0.672** (0.176)	0.27 ** (0.046)	0.426	-2.639
	HAR-RV-VIX-EA	0.029** (0.008)	0.511** (0.075)	0.056 (0.077)	0.172* (0.069)	0.901** (0.184)	0.097* (0.046)	0.409	-2.609
AMZN	HAR-RV	0.036** (0.009)	0.398** (0.044)	0.277** (0.07)	0.171** (0.062)			0.442	-2.616
	HAR-RV-IV	−0.007 (0.01)	0.342** (0.047)	0.181** (0.063)	0.034 (0.065)		0.314** (0.037)	0.480	-2.684
	HAR-RV-VIX	0.044** (0.009)	0.354** (0.043)	0.219** (0.066)	−0.04 (0.088)		0.358** (0.077)	0.465	-2.657
	HAR-RV-EA	0.033** (0.009)	0.381** (0.045)	0.305** (0.068)	0.176** (0.06)	1.495** (0.214)		0.485	-2.693
	HAR-RV-IV-EA	0.0 (0.01)	0.342** (0.047)	0.226** (0.063)	0.07 (0.063)	1.199** (0.215)	0.24 ** (0.031)	0.504	-2.732
	HAR-RV-VIX-EA	0.041** (0.008)	0.334** (0.044)	0.244** (0.064)	−0.044 (0.085)	1.532** (0.209)	0.374** (0.074)	0.510	-2.743

The parentheses below the regression coefficients display Newey-West standard errors.
(**) Significant at the 1 percent level. (*) Significant at the 5 percent level.

is illustrative for the general behaviour of the model, but all remarks are made considering the entire sample of stocks. From the lagged values of daily, weekly and monthly RV, we see that all models put the most weighting on the daily variable.

The daily, weekly and monthly RV coefficients decrease in value and significance when we add either IV, VIX or EA. This holds true for all stocks, but the magnitude of decrease in RV coefficients varies across the stocks. The effect is also considerably larger for weekly and monthly variables compared to the daily variables. After VIX or IV are added, at most two of the weekly and monthly coefficients are significant at 5%.

More importantly, when including either the IV, VIX or EA alone, their coefficients are significant at 1% for all stocks, except when VIX is added for AAPL. The IV, VIX and EA variables are also significant at 1% across all stocks when IV or VIX and EA are added together, except when VIX and EA is added for AAPL. This suggests that the variables contain different information, and that including both will be useful in the out-of-sample forecasting.

4.2 Out-of-Sample Evaluation

We now turn our attention to the out-of-sample forecasts produced by the models. The out-of-sample forecasts let us directly compare the performance of the models to the actual observed values. Our results support the initial findings from the in-sample evaluation, and show that the models that include both EA and IV/VIX outperform the other models for the majority of the stocks. However, which combinations of IV/VIX and/or EA that are the best performing across the stocks varies between the three horizons.

We perform the forecasts using a moving window regression from June 6th 2010 to December 31th 2014 with a window of 125 trading days[3]. For each new forecast, we roll the window one day forward while the window size is kept constant. All the models have been tested with window sizes ranging from 85 to 500 days. The results are consistent across different window sizes, and for that reason we only present results for the equivalent of half a year in trading days in the paper (125 days).

We evaluate all forecasts using the Model Confidence Set (MCS) procedure by Hansen, Lunde, and Nason (2011). This procedure examines a given set of competing models simultaneously. From this set of competing models, the procedure identifies a subset that contains the unknown best model with some level of confidence, known as the MCS. The other models contained in the MCS are then not significantly different from the true best model.

There are several properties of the MCS procedure that makes it attractive for forecast evaluation. The remaining set of models contains the best model with a given confidence level, hence the method provides us with a traditional statistical conclusion. Many evaluation methods that

3 The first window start on January 4th 2010.

report p-values for multiple pairwise model comparisons lack this feature, such as pairwise tests for equal predictive accuracy of Diebold and Mariano (1995) and West (1996). Further, the procedure allows for the possibility that more than one model is the best. The method recognizes limitations of the data, where informative data will result in a MCS that contains only the best model, while less informative data may result in a MCS that contains several or even all models evaluated. The MCS procedure allows for an arbitrary loss function specified by the user, which makes it a flexible procedure. Alternatives to the MCS are available in the form of the reality check of White (2000) and the test for superior predictive ability (SPA) of Hansen (2005). However, both these procedures require the specification of a benchmark forecast, which is not the case for the MCS procedure. Since the MCS procedure is relatively new and not so frequently employed in the literature, we present a brief description. The algorithm of the procedure is

MCS algorithm:

Step 0 Let the initial set $\mathcal{M}$ contain all competing models with m elements..

Step 1 We perform an equivalence test with the null hypothesis that all models in the set $\mathcal{M}$ are equally good. This means that the set of superior models equals $\mathcal{M}$. A significance of α is used.

Step 2 If the null hypothesis is accepted, we define $\widehat{\mathcal{M}}^*_{1-\alpha} = \mathcal{M}$ as the MCS with a confidence level of $1-\alpha$. Otherwise, we eliminate the most inferior model in $\mathcal{M}$ using an elimination rule and return to *Step* 1.

The equivalence test determines if the forecast errors between all models in $\mathcal{M}$ are significantly different from each other. First, a suitable loss function for the forecast errors must be chosen. For our purposes, we choose the commonly used loss functions MSE and QLIKE. Patton (2011) argues that MSE and QLIKE are the only robust loss functions when it comes to volatility forecast error. Thus, we define the loss functions, $l_{i,t}$ for model i at time t, in two ways

$$l_{i,t} = (RV_t - \widehat{RV}_{i,t})^2 \qquad \text{(MSE)}$$

$$l_{i,t} = \log(\widehat{RV}_{i,t}) + \frac{RV_t}{\widehat{RV}_{i,t}} \qquad \text{(QLIKE)}$$

where $\widehat{RV}_{i,t}$ is the forecast value for time t for model i and RV_t is the actual observed RV at time t. The i-th models loss relative to all other models j in the set $\mathcal{M}$ at time t is

$$d_{i,t} = \frac{1}{m-1} \sum_{j \in \mathcal{M}} d_{ij,t} \qquad \forall \quad i \in \mathcal{M}, \quad t \in T,$$

where $d_{ij,t}$ is the difference between loss functions for model i and j, i.e. $(l_{i,t} - l_{j,t})$. The procedure assumes the expected value of $d_{i,t}$, denoted $\mathbb{E}(d_i)$, to be finite and independent of time, and the null hypothesis for the equivalence test is given by:

$$H_{0,\mathcal{M}} : \mathbb{E}(d_i) = 0 \qquad \forall \quad i \in \mathcal{M}.$$

The alternative hypothesis is that one or more of the models in the set $\mathcal{M}$ have an expected value different from zero, which means that all models in the set are not equally good. Hansen, Lunde, and Nason (2011) then formulate the following statistic for the hypothesis testing

$$t_i = \frac{\bar{d}_i}{\sqrt{\widehat{\text{var}}(\bar{d}_i)}} \qquad i \in M,$$

where $\bar{d}_i = (m-1)^{-1} \sum_{j \in \mathcal{M}} \bar{d}_{ij}$ with $\bar{d}_{ij} = m^{-1} \sum_{t \in T} d_{ij,t}$, while $\widehat{\text{var}}(\bar{d}_i)$ is a bootstrapped estimate of var($\bar{d}_i$). Following Hansen, Lunde, and Nason (2011), we perform a block-bootstrap procedure of 5,000 resamples, with a block length determined by the maximum number of significant parameters from fitting all loss differences, d_{ij}, to an AR(p) process. The test statistic for the null hypothesis is then given by

$$T_{\max,\mathcal{M}} = \max_{i \in \mathcal{M}}(t_i)$$

The asymptotic distribution of test statistic is estimated using a bootstrap procedure similar to that of $\text{var}(\bar{d}_i)$, since it is non standard. Finally, as long as the null hypothesis is rejected, the set $\mathcal{M}$ is reduced by eliminating $e_{\max,\mathcal{M}} = \arg\max_{i\in\mathcal{M}}(t_i)$. For a more detailed outline of the steps and theoretical background of the MCS procedure reference is made to Hansen, Lunde, and Nason (2011).

4.2.1 *Results*

In Table 3 we present a summary of the MCS results for daily, weekly and monthly forecast horizons using the MSE loss function. We report the number of times each of the models is included in the MCS in total across all 32 stocks, using both $\alpha = 0.15$ and $\alpha = 0.25$ as significance levels.

The HAR-RV-EA model appears in the MCS significantly more frequently than the benchmark HAR-RV. The improvement holds for both daily, weekly and monthly forecasts. We also see that the models with

Table 3: Out-of-sample evaluation for daily, weekly and monthly horizons using the Model Confidence Set (MCS) procedure of Hansen, Lunde, and Nason (2011). The table show how many times each model is included in the MCS. We use a square error loss function with significance levels $\alpha = 0.15$ and 0.25.

		Daily		Weekly		Monthly	
Model	$\alpha =$	0.15	0.25	0.15	0.25	0.15	0.25
HAR-RV		7	3	3	3	19	16
HAR-RV-IV		25	21	26	26	29	27
HAR-RV-VIX		21	17	14	13	21	19
HAR-RV-EA		26	24	20	19	26	26
HAR-RV-IV-EA		31	29	28	28	32	30
HAR-RV-VIX-EA		30	29	29	28	31	31

A window size of 125 observations is used when producing the forecasts.

Table 4: Out-of-sample evaluation for daily forecasts using mean square errors (MSE) evaluation. The MSE losses have been normalized relative to the HAR-RV model. We use a window size of 125 observations for the forecasts.

Ticker	HAR-RV	HAR-RV-IV	HAR-RV-VIX	HAR-RV-EA	HAR-RV-EA-IV	HAR-RV-EA-VIX
AAPL	1.00	0.96*	0.99	0.98	0.95**	0.97
AMZN	1.00	0.95**	0.97	0.94**	0.92**	0.91**
AXP	1.00	0.96**	0.93**	0.96**	0.93**	0.88**
BA	1.00	0.94*	0.96*	0.96*	0.92**	0.92**
CAT	1.00	0.95	0.99	0.90**	0.88**	0.90**
CSCO	1.00	0.92**	0.95	0.90**	0.86**	0.85**
CVX	1.00	0.96*	0.95**	1.00	0.96*	0.95**
DD	1.00	0.98**	0.96**	0.97**	0.96**	0.93**
DIS	1.00	0.93**	0.91**	0.94**	0.89**	0.85**
GE	1.00*	0.98**	0.96**	0.98**	0.96**	0.94**
GOOGL	1.00**	0.98**	1.00**	0.99**	0.98**	0.99**
GS	1.00	0.94**	0.90**	0.96**	0.92**	0.87**
HD	1.00	0.95**	0.95**	0.91**	0.88**	0.86**
IBM	1.00	0.96**	0.94**	0.94**	0.92**	0.88**
INTC	1.00*	0.97**	1.00*	0.95**	0.93**	0.95**
JNJ	1.00	0.97	0.96	0.93**	0.92**	0.90**
JPM	1.00	0.93	0.90**	1.00	0.93	0.90**
KO	1.00	0.96	0.97	0.90**	0.89**	0.87**
MCD	1.00	0.97	0.97	0.93	0.91**	0.90**
MMM	1.00	0.93**	0.91**	0.95**	0.92**	0.87**
MRK	1.00*	1.00*	0.98*	0.96**	0.96**	0.94**
MSFT	1.00	0.96**	0.97	0.96**	0.94**	0.93**
NKE	1.00	0.95**	0.94**	0.90**	0.89**	0.85**
PFE	1.00	0.92**	0.95	0.97	0.91**	0.93
PG	1.00	0.94**	0.93**	0.94**	0.92**	0.88**
TRV	1.00**	1.00**	0.98**	0.83**	0.85**	0.81**
UNH	1.00	0.95	0.98	0.92**	0.89**	0.91**
UTX	1.00	0.94**	0.92**	0.93**	0.90**	0.85**
V	1.00**	0.97**	0.99**	1.00**	0.98**	0.99**
VZ	1.00	0.95	0.97	0.93	0.90**	0.90**
WMT	1.00	0.95**	0.96**	0.94**	0.91**	0.90**
XOM	1.00*	0.95**	0.97*	1.00*	0.96*	0.98*
Average	1.00	0.95	0.96	0.95	0.92	0.91

The MSE values are marked with */** if the model is included in the superior set according to the Model Confidence Set (MCS) procedure of Hansen, Lunde, and Nason (2011), using squared error loss function.

* Included in superior set with significance $\alpha = 0.15$.

** Included in superior set with significance $\alpha = 0.25$.

implied volatility included, HAR-RV-IV and HAR-RV-VIX, appear in the MCS more frequently than HAR-RV. HAR-RV-IV perform better than HAR-RV-VIX, which may indicate that the calculated IV contain stock specific information the VIX does not possess.

The models that include both IV/VIX and EA, HAR-RV-IV-EA and HAR-RV-VIX-EA, are the best performing models for both the daily, weekly and monthly forecasts. This indicates that both implied volatility and earnings announcements each contain unique information that is useful for the forecasts. Further, the results show that HAR-RV-IV-EA and HAR-RV-VIX-EA appears in the MCS approximately the same number of times. For this reason, the relative improvement from adding EA to HAR-RV-VIX is greater than the improvement from adding EA to HAR-RV-IV. Our interpretation is that although HAR-RV-IV is improved when we add EA, IV contain some information about stock specific earnings announcements, while VIX does not. We conclude that the difference between HAR-RV-IV-EA and HAR-RV-VIX-EA is small, and when we add EA to the forecast it is sufficient to use VIX as the information source for implied volatility.

In Table 4, Table 5 and Table 6, we present relative MSE values for the daily, weekly and monthly forecasts for all the 32 stocks. Almost all our extensions to HAR-RV improve the forecasts. The average reductions in MSE for HAR-RV-VIX-EA compared to the benchmark HAR-RV are 9% for the daily and weekly forecasts, and 10% for the monthly forecasts. The average reductions in MSE for HAR-RV-IV-EA compared to HAR-RV are 8% for the daily and weekly forecasts, and 6% for the monthly forecasts. At the most, we get an 26% improvement in MSE for the monthly forecasts of the TRV stock with the HAR-RV-IV-EA model.

We find no strong relationship between the stocks' correlation with VIX, shown in Figure 1, and the performance of the models that include VIX for any of the forecast horizons. The results from the MCS procedure are similar with a QLIKE loss function and supported by pairwise comparisons from the Diebold-Mariano test and Mincer-Zarnowitz regression that have been omitted from the paper.

Table 5: Out-of-sample evaluation for weekly forecasts using mean square errors (MSE) evaluation. The MSE losses have been normalized relative to the HAR-RV model. We use a window size of 125 observations for the forecasts.

Ticker	HAR-RV	HAR-RV-IV	HAR-RV-VIX	HAR-RV-EA	HAR-RV-EA-IV	HAR-RV-EA-VIX
AAPL	1.00	0.95**	0.99	0.98**	0.95**	0.97**
AMZN	1.00	0.92**	0.94**	0.92**	0.91**	0.86**
AXP	1.00	0.94**	0.93**	0.94**	0.92**	0.88**
BA	1.00	0.92**	0.97	0.99	0.92**	0.95
CAT	1.00	0.96**	1.03	0.95**	0.94**	0.99
CSCO	1.00	0.89**	0.95	0.90**	0.87**	0.85**
CVX	1.00	0.93**	0.91**	1.00	0.94	0.92**
DD	1.00	0.95**	0.97	0.98	0.94**	0.95**
DIS	1.00	0.89**	0.87**	0.94	0.88**	0.81**
GE	1.00**	0.99**	0.96**	0.98**	0.97**	0.94**
GOOGL	1.00	0.94**	1.00	0.97**	0.94**	0.97**
GS	1.00**	0.98**	0.84**	0.95**	0.95**	0.81**
HD	1.00	0.90	0.94	0.92	0.86**	0.86**
IBM	1.00	0.96**	0.91**	0.97**	0.96**	0.89**
INTC	1.00	0.93**	1.03	0.94**	0.90**	0.98
JNJ	1.00	0.91	0.93	0.87**	0.86**	0.85**
JPM	1.00	0.93	0.83	0.98	0.92	0.82**
KO	1.00	0.92**	0.95	0.95	0.91**	0.92**
MCD	1.00	0.95**	0.93**	0.93**	0.91**	0.87**
MMM	1.00	0.95**	0.95**	1.00	0.97	0.94**
MRK	1.00	1.02	0.99	0.94**	0.97	0.94**
MSFT	1.00	0.93**	1.01	0.95**	0.91**	0.96**
NKE	1.00	0.92**	0.96	0.91**	0.90**	0.88**
PFE	1.00	0.90**	0.95	0.97	0.90**	0.92*
PG	1.00	0.91**	0.90**	0.92**	0.88**	0.84**
TRV	1.00	0.94	0.93	0.86**	0.83**	0.80**
UNH	1.00	0.90**	0.97	0.93**	0.86**	0.91**
UTX	1.00	0.94**	0.95**	0.97	0.94**	0.93**
V	1.00**	0.90**	1.01**	0.99**	0.89**	0.99**
VZ	1.00	0.93	0.96	0.94	0.90**	0.90**
WMT	1.00	0.89**	0.95*	0.95*	0.88**	0.90**
XOM	1.00	0.94**	0.96**	0.99	0.95**	0.96**
Average	1.00	0.93	0.95	0.95	0.92	0.91

The MSE values are marked with */** if the model is included in the superior set according to the Model Confidence Set (MCS) procedure of Hansen, Lunde, and Nason (2011), using squared error loss function.

* Included in superior set with significance $\alpha = 0.15$.

** Included in superior set with significance $\alpha = 0.25$.

Table 6: Out-of-sample evaluation for monthly forecasts using mean square errors (MSE) evaluation. The MSE losses have been normalized relative to the HAR-RV model. We use a window size of 125 observations for the forecasts.

Ticker	HAR-RV	HAR-RV-IV	HAR-RV-VIX	HAR-RV-EA	HAR-RV-EA-IV	HAR-RV-EA-VIX
AAPL	1.00**	0.98**	0.96**	0.97**	1.00**	0.93**
AMZN	1.00**	0.99**	0.94**	0.97**	1.00**	0.88**
AXP	1.00**	0.99**	1.00**	0.96**	0.98**	0.95**
BA	1.00**	0.99**	0.98**	0.97**	0.99**	0.96**
CAT	1.00	0.96**	0.99	0.98	0.95**	0.96**
CSCO	1.00	0.94	1.01	0.82**	0.84*	0.80**
CVX	1.00**	0.97**	0.92**	0.98**	0.95**	0.89**
DD	1.00	0.94**	0.94**	0.97	0.93**	0.91**
DIS	1.00**	0.98**	0.94**	0.88**	0.86**	0.80**
GE	1.00**	0.99**	0.96**	1.01**	1.02**	0.96**
GOOGL	1.00	0.92**	1.02	0.93**	0.92**	0.94**
GS	1.00**	0.98**	0.78**	0.98**	0.98**	0.78**
HD	1.00*	0.98*	0.98*	0.88**	0.91*	0.86**
IBM	1.00*	0.97**	0.95**	0.97**	0.98**	0.93**
INTC	1.00	0.89**	1.04	0.94	0.89**	0.98
JNJ	1.00	0.93**	1.00	0.92**	0.89**	0.94**
JPM	1.00**	0.93**	0.79**	1.03**	0.99**	0.81**
KO	1.00**	0.96**	0.98**	1.00**	0.98**	0.99**
MCD	1.00**	0.98**	0.96**	0.98**	0.97**	0.93**
MMM	1.00**	1.01**	0.97**	0.99**	1.02**	0.95**
MRK	1.00**	0.99**	0.95**	0.89**	0.90**	0.85**
MSFT	1.00	0.94**	1.02	0.95**	0.94**	0.95**
NKE	1.00**	0.97**	1.00**	0.99**	1.00**	0.98**
PFE	1.00	0.92	0.96	0.88	0.83**	0.84**
PG	1.00	0.93**	0.93**	0.89**	0.86**	0.82**
TRV	1.00	0.87	0.94	0.80	0.74**	0.76**
UNH	1.00	0.90**	0.95	0.96	0.90**	0.91**
UTX	1.00**	0.97**	1.00**	0.97**	0.94**	0.97**
V	1.00	0.92**	1.00	0.93**	0.88**	0.91**
VZ	1.00	0.93**	0.99	0.91**	0.88**	0.91**
WMT	1.00*	1.01*	1.01*	0.85**	0.84**	0.81**
XOM	1.00**	1.01**	0.95**	1.00**	1.01**	0.95**
Average	1.00	0.96	0.95	0.95	0.94	0.90

The MSE values are marked with */** if the model is included in the superior set according to the Model Confidence Set (MCS) procedure of Hansen, Lunde, and Nason (2011), using squared error loss function.

[*] Included in superior set with significance $\alpha = 0.15$.

[**] Included in superior set with significance $\alpha = 0.25$.

5 CONCLUDING REMARKS

We propose that HAR models for individual stocks should be augmented by including information about earnings announcement dates and implied volatility, which yields a significant improvement for both in- and out-of-sample forecasts on a wide number of stocks in the DIJA index.

The connection between IV and RV has been studied for equity indices, currencies and commodities, but we are the first to study it for individual stocks. Since there only are a few stocks that have an official implied volatility index, we create individual implied volatility indices for all the stocks in the DJIA index for to get a more complete sample.

The inclusion of EA is a natural consequence of working with single stocks, and earlier work on this effect is limited. Earnings announcements are typically known in advance, and have a large impact on volatility for a few days. We show how this information can be incorporated in the HAR-RV model. This significantly improves the forecasts, and complements the improvements we get from IV.

After we adjust for earnings announcement dates, we find that including the VIX is at least as good for volatility forecasts as the stocks' own implied volatility index. Therefore, we recommend including the VIX in volatility models for stocks in the DJIA index. This main advantage is that one does not need to first construct an individual volatility index for the stock. The VIX is easily available, and can be used for all stocks.

Finally, many other types of improvements to the HAR-RV model have been proposed. We believe the benefits of including implied volatility and earnings announcements should translate well into these HAR-based models, but leave the study of any joint performance improvements to future studies.

A APPENDIX

This appendix contains the optimization problem described in section 3, found in Table 7 and the in-sample results described in section 4, found in Table 8 to Table 13.

Table 7: Quadratic optimization program for Model 6 to determine earnings announcement variables for daily forecasts.

Sets and indices	
$s \in S$	Stock s
$t \in T$	Observation t
Parameters	
$RV^{H}_{s,t}$	Daily, weekly and monthly multi-period RV for stock s at time t, $H = D, W, M$
$\mathbb{1}_{A(s,t_1,t_2)}$	Indicator function with value 1 if stock s has an earnings announcement in the interval $[t_1, t_2]$
Variables	
$\widehat{RV}^{D}_{s,t}$	Forecast value of daily RV for stock s at time t
α_s	Intercept for stock s
$\beta_{s,1}$	Daily RV coefficient for stock s
$\beta_{s,2}$	Weekly RV coefficient for stock s
$\beta_{s,3}$	Monthly RV coefficient for stock s
β^{h}_{s}	Coefficient for EA that occurred within h future $(h > 0)$ or past $(h < 0)$ days for stock s
β^{h}_{EA}	Equal β^{h}_{s} for all stocks

Objective

$$\text{minimize} \quad \sum_{s \in S} \sum_{t \in T} (\widehat{RV}^{D}_{s,t} - RV^{D}_{s,t})^2$$

Constraints

$$\widehat{RV}^{D}_{s,t+1} = \alpha_s + \beta_{s,1} RV^{D}_{s,t} + \beta_{s,2} RV^{W}_{s,t} + \beta_{s,3} RV^{M}_{s,t} + \beta^{+1}_{s} \mathbb{1}_{A(s,t+1,t+1)} + \sum_{h \in \{1,5,22\}} \beta^{h}_{s} \mathbb{1}_{A(s,t+1-h,t)} \quad \forall \quad s \in S, t \in T$$

$$\beta^{h}_{s} = \beta^{h}_{EA} \quad \forall \quad s \in S, h \in \{+1, -1, -5, -22\}$$

Table 8: In-sample evaluation for one-day ahead forecasts, part I

		Regression coefficients							
Ticker	Model	$\hat{\alpha}$	Daily	Weekly	Monthly	EA	IV/VIX	R^2_{adj}	AIC
AXP	HAR-RV	0.015**	0.447**	0.22**	0.244**			0.582	-3.089
		(0.005)	(0.08)	(0.08)	(0.069)				
	HAR-RV-IV	−0.003	0.351**	0.087	−0.154		0.474**	0.617	-3.177
		(0.005)	(0.083)	(0.072)	(0.079)		(0.047)		
	HAR-RV-VIX	0.004	0.358**	0.129	−0.04		0.501**	0.612	-3.163
		(0.006)	(0.068)	(0.075)	(0.094)		(0.087)		
	HAR-RV-EA	0.013**	0.439**	0.225**	0.259**	1.01 **		0.605	-3.145
		(0.005)	(0.082)	(0.078)	(0.069)	(0.212)			
	HAR-RV-IV-EA	−0.003	0.351**	0.102	−0.111	0.887**	0.438**	0.635	-3.223
		(0.005)	(0.084)	(0.072)	(0.079)	(0.204)	(0.047)		
	HAR-RV-VIX-EA	0.002	0.351**	0.135	−0.024	1.001**	0.498**	0.635	-3.222
		(0.006)	(0.07)	(0.073)	(0.094)	(0.204)	(0.087)		
BA	HAR-RV	0.022**	0.299**	0.313**	0.26 **			0.429	-2.882
		(0.006)	(0.067)	(0.097)	(0.077)				
	HAR-RV-IV	−0.009	0.21 **	0.145	−0.136		0.536**	0.473	-2.962
		(0.007)	(0.068)	(0.093)	(0.08)		(0.058)		
	HAR-RV-VIX	0.02 **	0.234**	0.204*	−0.036		0.449**	0.464	-2.944
		(0.006)	(0.056)	(0.086)	(0.083)		(0.065)		
	HAR-RV-EA	0.02 **	0.305**	0.316**	0.263**	0.889**		0.449	-2.917
		(0.006)	(0.066)	(0.095)	(0.077)	(0.168)			
	HAR-RV-IV-EA	−0.008	0.221**	0.16	−0.105	0.73 **	0.497**	0.486	-2.986
		(0.007)	(0.067)	(0.092)	(0.081)	(0.163)	(0.059)		
	HAR-RV-VIX-EA	0.019**	0.239**	0.206*	−0.033	0.892**	0.449**	0.484	-2.981
		(0.006)	(0.056)	(0.084)	(0.083)	(0.158)	(0.064)		
CAT	HAR-RV	0.015**	0.488**	0.207*	0.227**			0.629	-2.963
		(0.005)	(0.071)	(0.103)	(0.076)				
	HAR-RV-IV	−0.011*	0.395**	0.094	−0.043		0.406**	0.657	-3.041
		(0.006)	(0.069)	(0.096)	(0.084)		(0.047)		
	HAR-RV-VIX	0.002	0.423**	0.133	0.036		0.415**	0.645	-3.006
		(0.006)	(0.07)	(0.098)	(0.086)		(0.069)		
	HAR-RV-EA	0.012*	0.496**	0.213*	0.227**	1.326**		0.660	-3.050
		(0.005)	(0.069)	(0.101)	(0.076)	(0.194)			
	HAR-RV-IV-EA	−0.01	0.415**	0.115	−0.007	1.168**	0.351**	0.681	-3.112
		(0.006)	(0.066)	(0.096)	(0.084)	(0.197)	(0.046)		
	HAR-RV-VIX-EA	0.0	0.433**	0.141	0.042	1.305**	0.402**	0.675	-3.094
		(0.006)	(0.068)	(0.097)	(0.085)	(0.189)	(0.066)		
CSCO	HAR-RV	0.022**	0.422**	0.155*	0.298**			0.442	-3.062
		(0.006)	(0.045)	(0.072)	(0.07)				
	HAR-RV-IV	−0.005	0.382**	0.09	0.167*		0.241**	0.469	-3.110
		(0.008)	(0.039)	(0.06)	(0.073)		(0.045)		
	HAR-RV-VIX	0.033**	0.366**	0.079	−0.023		0.379**	0.470	-3.112
		(0.006)	(0.038)	(0.064)	(0.106)		(0.077)		
	HAR-RV-EA	0.019**	0.42 **	0.139	0.334**	1.172**		0.483	-3.137
		(0.006)	(0.045)	(0.084)	(0.073)	(0.162)			
	HAR-RV-IV-EA	−0.001	0.391**	0.092	0.23 **	1.01 **	0.183**	0.497	-3.165
		(0.008)	(0.04)	(0.073)	(0.075)	(0.164)	(0.044)		
	HAR-RV-VIX-EA	0.03 **	0.363**	0.061	0.005	1.197**	0.391**	0.512	-3.195
		(0.006)	(0.038)	(0.074)	(0.102)	(0.146)	(0.074)		
CVX	HAR-RV	0.014**	0.556**	0.204*	0.144*			0.622	-3.539
		(0.004)	(0.074)	(0.092)	(0.071)				
	HAR-RV-IV	−0.008	0.463**	0.113	−0.057		0.353**	0.643	-3.596
		(0.005)	(0.069)	(0.082)	(0.082)		(0.053)		
	HAR-RV-VIX	0.009*	0.499**	0.166	−0.018		0.236**	0.634	-3.572
		(0.004)	(0.07)	(0.091)	(0.086)		(0.049)		
	HAR-RV-EA	0.014**	0.555**	0.212*	0.143*	0.309**		0.625	-3.546
		(0.004)	(0.075)	(0.093)	(0.072)	(0.1)			
	HAR-RV-IV-EA	−0.009	0.463**	0.12	−0.057	0.287**	0.35 **	0.645	-3.602
		(0.005)	(0.069)	(0.084)	(0.082)	(0.096)	(0.052)		
	HAR-RV-VIX-EA	0.008*	0.498**	0.174	−0.021	0.318**	0.238**	0.637	-3.579
		(0.004)	(0.07)	(0.093)	(0.086)	(0.096)	(0.049)		

Newey-West standard errors are displayed in parentheses below the regression coefficients.
(**) Significant at the 1 percent level. (*) Significant at the 5 percent level.

Table 9: In-sample evaluation for one-day ahead forecasts, part II

		Regression coefficients							
Ticker	Model	$\hat{\alpha}$	Daily	Weekly	Monthly	EA	IV/VIX	R^2_{adj}	AIC
DD	HAR-RV	0.017** (0.005)	0.403** (0.068)	0.271** (0.082)	0.218** (0.078)			0.546	-3.103
	HAR-RV-IV	−0.002 (0.005)	0.358** (0.068)	0.203** (0.076)	0.049 (0.073)		0.274** (0.044)	0.562	-3.138
	HAR-RV-VIX	0.008 (0.005)	0.338** (0.061)	0.175* (0.076)	0.019 (0.083)		0.377** (0.056)	0.568	-3.152
	HAR-RV-EA	0.016** (0.005)	0.408** (0.068)	0.275** (0.081)	0.219** (0.078)	0.768** (0.158)		0.560	-3.134
	HAR-RV-IV-EA	−0.002 (0.005)	0.366** (0.067)	0.211** (0.075)	0.063 (0.073)	0.704** (0.159)	0.253** (0.044)	0.574	-3.165
	HAR-RV-VIX-EA	0.007 (0.005)	0.343** (0.06)	0.179* (0.075)	0.022 (0.083)	0.762** (0.163)	0.376** (0.055)	0.582	-3.186
DIS	HAR-RV	0.022** (0.005)	0.38 ** (0.051)	0.273** (0.066)	0.209** (0.072)			0.445	-3.071
	HAR-RV-IV	−0.011 (0.006)	0.31 ** (0.042)	0.161** (0.058)	−0.18 (0.105)		0.49 ** (0.078)	0.490	-3.155
	HAR-RV-VIX	0.027** (0.005)	0.29 ** (0.039)	0.15 ** (0.058)	−0.215* (0.098)		0.528** (0.067)	0.490	-3.155
	HAR-RV-EA	0.02 ** (0.005)	0.35 ** (0.054)	0.29 ** (0.066)	0.237** (0.072)	0.969** (0.156)		0.474	-3.123
	HAR-RV-IV-EA	−0.009 (0.006)	0.29 ** (0.042)	0.185** (0.057)	−0.121 (0.104)	0.825** (0.145)	0.447** (0.081)	0.511	-3.195
	HAR-RV-VIX-EA	0.025** (0.005)	0.255** (0.039)	0.163** (0.051)	−0.199* (0.097)	1.017** (0.142)	0.545** (0.07)	0.522	-3.218
GE	HAR-RV	0.014** (0.004)	0.521** (0.059)	0.115 (0.097)	0.283** (0.077)			0.608	-3.037
	HAR-RV-IV	−0.004 (0.005)	0.44 ** (0.056)	0.015 (0.093)	−0.029 (0.082)		0.387** (0.052)	0.630	-3.095
	HAR-RV-VIX	0.0 (0.005)	0.452** (0.059)	0.059 (0.084)	0.057 (0.078)		0.404** (0.064)	0.625	-3.082
	HAR-RV-EA	0.012** (0.004)	0.515** (0.06)	0.128 (0.099)	0.285** (0.077)	0.835** (0.154)		0.622	-3.073
	HAR-RV-IV-EA	−0.004 (0.005)	0.439** (0.056)	0.033 (0.095)	−0.008 (0.083)	0.745** (0.154)	0.362** (0.052)	0.641	-3.124
	HAR-RV-VIX-EA	−0.001 (0.005)	0.447** (0.06)	0.073 (0.087)	0.065 (0.078)	0.807** (0.162)	0.393** (0.063)	0.638	-3.117
GOOGL	HAR-RV	0.028** (0.006)	0.448** (0.046)	0.265** (0.057)	0.119* (0.052)			0.455	-3.135
	HAR-RV-IV	0.002 (0.007)	0.409** (0.046)	0.209** (0.055)	0.065 (0.053)		0.189** (0.03)	0.472	-3.166
	HAR-RV-VIX	0.027** (0.006)	0.413** (0.044)	0.236** (0.062)	−0.026 (0.067)		0.193** (0.052)	0.469	-3.160
	HAR-RV-EA	0.027** (0.006)	0.448** (0.046)	0.265** (0.055)	0.125* (0.051)	0.563** (0.155)		0.465	-3.152
	HAR-RV-IV-EA	0.005 (0.008)	0.414** (0.046)	0.215** (0.054)	0.077 (0.052)	0.428** (0.165)	0.166** (0.031)	0.477	-3.175
	HAR-RV-VIX-EA	0.026** (0.006)	0.413** (0.044)	0.236** (0.06)	−0.02 (0.067)	0.567** (0.158)	0.194** (0.051)	0.478	-3.177
GS	HAR-RV	0.021* (0.008)	0.304** (0.117)	0.248** (0.082)	0.339** (0.094)			0.451	-2.397
	HAR-RV-IV	0.012 (0.007)	0.225* (0.1)	0.095 (0.069)	−0.049 (0.106)		0.434** (0.075)	0.490	-2.469
	HAR-RV-VIX	0.004 (0.008)	0.254** (0.095)	0.194** (0.071)	0.103 (0.098)		0.458** (0.1)	0.479	-2.448
	HAR-RV-EA	0.019* (0.008)	0.296** (0.114)	0.255** (0.076)	0.349** (0.096)	1.007** (0.129)		0.466	-2.423
	HAR-RV-IV-EA	0.011 (0.007)	0.223* (0.099)	0.11 (0.066)	−0.017 (0.107)	0.827** (0.118)	0.408** (0.077)	0.500	-2.488
	HAR-RV-VIX-EA	0.003 (0.008)	0.246** (0.092)	0.201** (0.065)	0.116 (0.099)	0.984** (0.121)	0.452** (0.101)	0.493	-2.475

*N*ewey-West standard errors are displayed in parentheses below the regression coefficients.
(**) Significant at the 1 percent level. (*) Significant at the 5 percent level.

Table 10: In-sample evaluation for one-day ahead forecasts, part III

Ticker	Model	Regression coefficients							
		$\hat{\alpha}$	Daily	Weekly	Monthly	EA	IV/VIX	R^2_{adj}	AIC
HD	HAR-RV	0.018** (0.005)	0.402** (0.059)	0.272** (0.077)	0.216** (0.064)			0.512	-3.449
	HAR-RV-IV	−0.008 (0.007)	0.319** (0.048)	0.125 (0.079)	−0.085 (0.083)		0.461** (0.074)	0.555	-3.541
	HAR-RV-VIX	0.026** (0.005)	0.321** (0.051)	0.191** (0.073)	−0.118 (0.097)		0.384** (0.07)	0.542	-3.512
	HAR-RV-EA	0.015** (0.005)	0.415** (0.058)	0.264** (0.075)	0.229** (0.06)	1.074** (0.123)		0.558	-3.548
	HAR-RV-IV-EA	−0.006 (0.007)	0.345** (0.047)	0.144 (0.079)	−0.021 (0.083)	0.902** (0.12)	0.381** (0.076)	0.586	-3.613
	HAR-RV-VIX-EA	0.023** (0.005)	0.335** (0.05)	0.184** (0.07)	−0.103 (0.092)	1.07 ** (0.123)	0.382** (0.067)	0.588	-3.618
IBM	HAR-RV	0.019** (0.005)	0.471** (0.063)	0.232** (0.079)	0.154* (0.074)			0.490	-3.695
	HAR-RV-IV	−0.006 (0.006)	0.406** (0.063)	0.154* (0.071)	−0.003 (0.074)		0.307** (0.038)	0.517	-3.748
	HAR-RV-VIX	0.021** (0.004)	0.399** (0.056)	0.163* (0.074)	−0.111 (0.089)		0.271** (0.043)	0.518	-3.750
	HAR-RV-EA	0.018** (0.005)	0.461** (0.065)	0.254** (0.081)	0.15 * (0.074)	0.7 ** (0.117)		0.515	-3.744
	HAR-RV-IV-EA	−0.002 (0.006)	0.41 ** (0.063)	0.186* (0.074)	0.024 (0.075)	0.556** (0.117)	0.249** (0.037)	0.531	-3.778
	HAR-RV-VIX-EA	0.02 ** (0.004)	0.389** (0.057)	0.185* (0.076)	−0.114 (0.089)	0.7 ** (0.118)	0.271** (0.042)	0.542	-3.802
INTC	HAR-RV	0.025** (0.006)	0.468** (0.049)	0.169* (0.072)	0.223** (0.063)			0.466	-3.132
	HAR-RV-IV	−0.003 (0.007)	0.419** (0.052)	0.117* (0.06)	0.029 (0.06)		0.305** (0.041)	0.489	-3.175
	HAR-RV-VIX	0.03 ** (0.006)	0.426** (0.047)	0.123 (0.068)	0.031 (0.065)		0.25** (0.04)	0.483	-3.163
	HAR-RV-EA	0.024** (0.006)	0.462** (0.05)	0.18 * (0.072)	0.227** (0.061)	0.741** (0.167)		0.483	-3.163
	HAR-RV-IV-EA	−0.0 (0.007)	0.421** (0.052)	0.133* (0.062)	0.057 (0.059)	0.597** (0.167)	0.265** (0.038)	0.500	-3.195
	HAR-RV-VIX-EA	0.029** (0.006)	0.419** (0.048)	0.134* (0.068)	0.032 (0.064)	0.748** (0.168)	0.252** (0.04)	0.500	-3.196
JNJ	HAR-RV	0.019** (0.004)	0.435** (0.073)	0.221* (0.09)	0.178* (0.079)			0.425	-3.666
	HAR-RV-IV	−0.0 (0.005)	0.392** (0.069)	0.165* (0.084)	−0.028 (0.089)		0.317** (0.068)	0.443	-3.696
	HAR-RV-VIX	0.015** (0.004)	0.403** (0.07)	0.193* (0.084)	0.041 (0.092)		0.139** (0.037)	0.439	-3.690
	HAR-RV-EA	0.018** (0.004)	0.429** (0.072)	0.244** (0.09)	0.169* (0.079)	0.62 ** (0.146)		0.447	-3.703
	HAR-RV-IV-EA	0.001 (0.005)	0.39** (0.07)	0.192* (0.085)	−0.016 (0.086)	0.572** (0.137)	0.287** (0.062)	0.461	-3.728
	HAR-RV-VIX-EA	0.014** (0.004)	0.396** (0.07)	0.217* (0.086)	0.032 (0.092)	0.62 ** (0.145)	0.14 ** (0.037)	0.461	-3.728
JPM	HAR-RV	0.017** (0.006)	0.489** (0.082)	0.157* (0.071)	0.266** (0.07)			0.583	-2.808
	HAR-RV-IV	0.007 (0.006)	0.384** (0.069)	0.044 (0.074)	−0.096 (0.091)		0.419** (0.05)	0.618	-2.895
	HAR-RV-VIX	0.003 (0.006)	0.4 ** (0.07)	0.084 (0.072)	0.009 (0.087)		0.526** (0.082)	0.611	-2.877
	HAR-RV-EA	0.016** (0.006)	0.492** (0.081)	0.163* (0.071)	0.262** (0.07)	0.602** (0.127)		0.589	-2.822
	HAR-RV-IV-EA	0.007 (0.006)	0.39 ** (0.069)	0.053 (0.074)	−0.085 (0.091)	0.432** (0.132)	0.404** (0.052)	0.621	-2.902
	HAR-RV-VIX-EA	0.003 (0.006)	0.405** (0.07)	0.091 (0.072)	0.009 (0.086)	0.55 ** (0.128)	0.517** (0.082)	0.616	-2.889

Newey-West standard errors are displayed in parentheses below the regression coefficients.
(**) Significant at the 1 percent level. (*) Significant at the 5 percent level.

Table 11: In-sample evaluation for one-day ahead forecasts, part IV

		Regression coefficients							
Ticker	Model	$\hat{\alpha}$	Daily	Weekly	Monthly	EA	IV/VIX	R^2_{adj}	AIC
KO	HAR-RV	0.018** (0.004)	0.482** (0.081)	0.201** (0.075)	0.166* (0.066)			0.474	-3.961
	HAR-RV-IV	−0.007 (0.005)	0.422** (0.083)	0.12 (0.068)	−0.003 (0.065)		0.344** (0.046)	0.499	-4.010
	HAR-RV-VIX	0.016** (0.004)	0.455** (0.075)	0.185* (0.079)	0.085 (0.075)		0.094** (0.025)	0.483	-3.978
	HAR-RV-EA	0.017** (0.004)	0.493** (0.079)	0.212** (0.074)	0.155* (0.065)	0.872** (0.091)		0.529	-4.072
	HAR-RV-IV-EA	−0.003 (0.005)	0.443** (0.08)	0.146* (0.069)	0.02 (0.063)	0.801** (0.092)	0.275** (0.044)	0.545	-4.105
	HAR-RV-VIX-EA	0.015** (0.004)	0.465** (0.073)	0.196* (0.078)	0.071 (0.074)	0.875** (0.093)	0.096** (0.024)	0.539	-4.092
MCD	HAR-RV	0.017** (0.004)	0.445** (0.058)	0.148* (0.066)	0.261** (0.071)			0.425	-4.014
	HAR-RV-IV	−0.005 (0.005)	0.386** (0.054)	0.033 (0.069)	0.047 (0.082)		0.375** (0.057)	0.457	-4.069
	HAR-RV-VIX	0.025** (0.004)	0.384** (0.051)	0.094 (0.055)	−0.046 (0.094)		0.218** (0.038)	0.455	-4.066
	HAR-RV-EA	0.015** (0.004)	0.44 ** (0.059)	0.167* (0.066)	0.266** (0.07)	0.759** (0.083)		0.471	-4.096
	HAR-RV-IV-EA	−0.005 (0.005)	0.388** (0.054)	0.063 (0.068)	0.076 (0.082)	0.703** (0.083)	0.333** (0.056)	0.495	-4.142
	HAR-RV-VIX-EA	0.023** (0.004)	0.379** (0.052)	0.113* (0.055)	−0.04 (0.093)	0.759** (0.083)	0.218** (0.037)	0.501	-4.153
MMM	HAR-RV	0.013** (0.004)	0.393** (0.046)	0.241* (0.096)	0.268** (0.085)			0.526	-3.385
	HAR-RV-IV	−0.016** (0.005)	0.3 ** (0.043)	0.111 (0.089)	−0.099 (0.081)		0.507** (0.074)	0.563	-3.466
	HAR-RV-VIX	0.003 (0.004)	0.28 ** (0.037)	0.082 (0.081)	−0.108 (0.091)		0.536** (0.075)	0.567	-3.475
	HAR-RV-EA	0.011** (0.004)	0.398** (0.046)	0.252** (0.095)	0.265** (0.085)	0.842** (0.108)		0.551	-3.437
	HAR-RV-IV-EA	−0.014** (0.005)	0.315** (0.042)	0.134 (0.09)	−0.059 (0.081)	0.683** (0.117)	0.45 ** (0.079)	0.579	-3.501
	HAR-RV-VIX-EA	0.001 (0.004)	0.287** (0.035)	0.094 (0.081)	−0.104 (0.091)	0.819** (0.108)	0.527** (0.075)	0.591	-3.529
MRK	HAR-RV	0.022** (0.005)	0.376** (0.051)	0.27** (0.1)	0.205* (0.086)			0.437	-3.299
	HAR-RV-IV	0.001 (0.007)	0.338** (0.042)	0.204* (0.084)	−0.04 (0.094)		0.323** (0.065)	0.459	-3.338
	HAR-RV-VIX	0.019** (0.005)	0.353** (0.044)	0.251** (0.093)	0.088 (0.095)		0.141** (0.04)	0.446	-3.314
	HAR-RV-EA	0.02 ** (0.005)	0.373** (0.05)	0.292** (0.101)	0.2 * (0.085)	0.708** (0.135)		0.457	-3.334
	HAR-RV-IV-EA	0.001 (0.007)	0.338** (0.042)	0.23 ** (0.086)	−0.026 (0.095)	0.648** (0.144)	0.299** (0.065)	0.475	-3.368
	HAR-RV-VIX-EA	0.017** (0.005)	0.35 ** (0.043)	0.273** (0.094)	0.081 (0.095)	0.714** (0.137)	0.144** (0.039)	0.466	-3.350
MSFT	HAR-RV	0.03 ** (0.007)	0.418** (0.064)	0.229** (0.076)	0.173** (0.057)			0.397	-3.249
	HAR-RV-IV	0.001 (0.008)	0.368** (0.065)	0.135 (0.069)	−0.012 (0.061)		0.34 ** (0.042)	0.426	-3.298
	HAR-RV-VIX	0.041** (0.007)	0.366** (0.057)	0.17 * (0.075)	−0.088 (0.073)		0.275** (0.046)	0.424	-3.294
	HAR-RV-EA	0.028** (0.007)	0.398** (0.066)	0.26 ** (0.078)	0.175** (0.057)	0.887** (0.144)		0.427	-3.300
	HAR-RV-IV-EA	0.004 (0.008)	0.358** (0.066)	0.175* (0.072)	0.019 (0.061)	0.753** (0.145)	0.287** (0.041)	0.447	-3.335
	HAR-RV-VIX-EA	0.04 ** (0.007)	0.343** (0.058)	0.2 ** (0.077)	−0.095 (0.073)	0.918** (0.146)	0.286** (0.046)	0.456	-3.351

Newey-West standard errors are displayed in parentheses below the regression coefficients.
(**) Significant at the 1 percent level. (*) Significant at the 5 percent level.

Table 12: In-sample evaluation for one-day ahead forecasts, part V

Ticker	Model	Regression coefficients							
		$\hat{\alpha}$	Daily	Weekly	Monthly	EA	IV/VIX	R^2_{adj}	AIC
NKE	HAR-RV	0.021** (0.006)	0.459** (0.057)	0.233** (0.077)	0.184* (0.084)			0.503	-3.220
	HAR-RV-IV	−0.006 (0.007)	0.413** (0.056)	0.176* (0.074)	0.015 (0.086)		0.286** (0.041)	0.529	-3.273
	HAR-RV-VIX	0.025** (0.005)	0.38 ** (0.047)	0.155* (0.07)	−0.128 (0.111)		0.398** (0.063)	0.534	-3.284
	HAR-RV-EA	0.019** (0.006)	0.441** (0.057)	0.259** (0.082)	0.186* (0.084)	1.18 ** (0.144)		0.549	-3.315
	HAR-RV-IV-EA	0.001 (0.007)	0.412** (0.055)	0.216** (0.078)	0.069 (0.084)	1.011** (0.139)	0.197** (0.035)	0.560	-3.340
	HAR-RV-VIX-EA	0.023** (0.005)	0.366** (0.048)	0.182* (0.074)	−0.118 (0.111)	1.158** (0.145)	0.387** (0.063)	0.578	-3.382
PFE	HAR-RV	0.024** (0.006)	0.247** (0.074)	0.435** (0.109)	0.169 (0.089)			0.415	-3.202
	HAR-RV-IV	0.01 (0.005)	0.185** (0.066)	0.299** (0.094)	−0.145 (0.108)		0.419** (0.069)	0.455	-3.272
	HAR-RV-VIX	0.027** (0.005)	0.202** (0.066)	0.367** (0.1)	−0.07 (0.104)		0.284** (0.053)	0.439	-3.244
	HAR-RV-EA	0.021** (0.006)	0.255** (0.073)	0.439** (0.109)	0.178* (0.089)	0.863** (0.139)		0.442	-3.248
	HAR-RV-IV-EA	0.008 (0.005)	0.197** (0.066)	0.313** (0.093)	−0.114 (0.11)	0.762** (0.144)	0.387** (0.07)	0.475	-3.309
	HAR-RV-VIX-EA	0.024** (0.005)	0.211** (0.065)	0.372** (0.098)	−0.059 (0.105)	0.857** (0.136)	0.281** (0.053)	0.465	-3.291
PG	HAR-RV	0.022** (0.004)	0.392** (0.058)	0.278** (0.085)	0.136 (0.072)			0.384	-3.822
	HAR-RV-IV	−0.005 (0.006)	0.34 ** (0.052)	0.238** (0.073)	−0.077 (0.083)		0.346** (0.052)	0.412	-3.868
	HAR-RV-VIX	0.021** (0.004)	0.363** (0.051)	0.252** (0.083)	−0.003 (0.089)		0.123** (0.033)	0.398	-3.846
	HAR-RV-EA	0.02 ** (0.004)	0.38 ** (0.059)	0.311** (0.088)	0.135 (0.073)	0.736** (0.157)		0.421	-3.884
	HAR-RV-IV-EA	−0.003 (0.006)	0.338** (0.055)	0.274** (0.077)	−0.044 (0.081)	0.652** (0.16)	0.291** (0.051)	0.440	-3.917
	HAR-RV-VIX-EA	0.019** (0.004)	0.35 ** (0.052)	0.284** (0.085)	−0.01 (0.088)	0.749** (0.152)	0.129** (0.031)	0.437	-3.912
TRV	HAR-RV	0.01 * (0.005)	0.536** (0.098)	0.174 (0.106)	0.214** (0.08)			0.627	-3.791
	HAR-RV-IV	0.009* (0.005)	0.483** (0.091)	0.138 (0.109)	−0.042 (0.103)		0.222** (0.045)	0.644	-3.838
	HAR-RV-VIX	0.009* (0.005)	0.459** (0.086)	0.135 (0.108)	−0.024 (0.104)		0.266** (0.053)	0.646	-3.842
	HAR-RV-EA	0.007 (0.004)	0.563** (0.095)	0.149 (0.093)	0.233** (0.071)	1.094** (0.129)		0.692	-3.983
	HAR-RV-IV-EA	0.006 (0.004)	0.522** (0.088)	0.123 (0.096)	0.04 (0.092)	1.035** (0.129)	0.167** (0.04)	0.702	-4.014
	HAR-RV-VIX-EA	0.006 (0.004)	0.489** (0.082)	0.112 (0.096)	0.005 (0.094)	1.081** (0.128)	0.254** (0.051)	0.710	-4.039
UNH	HAR-RV	0.022** (0.006)	0.335** (0.062)	0.179* (0.083)	0.365** (0.085)			0.426	-2.616
	HAR-RV-IV	−0.02 (0.012)	0.295** (0.056)	0.135 (0.074)	0.038 (0.12)		0.434** (0.108)	0.462	-2.680
	HAR-RV-VIX	0.016* (0.007)	0.303** (0.055)	0.16 * (0.076)	0.135 (0.116)		0.322** (0.081)	0.444	-2.647
	HAR-RV-EA	0.017** (0.006)	0.333** (0.061)	0.223** (0.084)	0.352** (0.081)	1.845** (0.251)		0.492	-2.738
	HAR-RV-IV-EA	−0.019 (0.012)	0.298** (0.058)	0.182* (0.076)	0.072 (0.117)	1.713** (0.255)	0.373** (0.107)	0.519	-2.791
	HAR-RV-VIX-EA	0.012 (0.006)	0.304** (0.056)	0.205** (0.077)	0.145 (0.113)	1.798** (0.252)	0.29 ** (0.078)	0.507	-2.766

Newey-West standard errors are displayed in parentheses below the regression coefficients.
(**) Significant at the 1 percent level. (*) Significant at the 5 percent level.

Table 13: In-sample evaluation for one-day ahead forecasts, part VI

Ticker	Model	Regression coefficients $\hat{\alpha}$	Daily	Weekly	Monthly	EA	IV/VIX	R^2_{adj}	AIC
UTX	HAR-RV	0.017**	0.421**	0.282**	0.18 **			0.520	-3.381
		(0.005)	(0.058)	(0.079)	(0.069)				
	HAR-RV-IV	−0.007	0.35 **	0.164*	−0.069		0.385**	0.549	-3.443
		(0.006)	(0.057)	(0.075)	(0.078)		(0.052)		
	HAR-RV-VIX	0.017**	0.331**	0.19 **	−0.152		0.419**	0.554	-3.453
		(0.005)	(0.053)	(0.071)	(0.088)		(0.055)		
	HAR-RV-EA	0.015**	0.431**	0.287**	0.18 **	1.016**		0.556	-3.458
		(0.005)	(0.056)	(0.077)	(0.068)	(0.14)			
	HAR-RV-IV-EA	−0.006	0.367**	0.181*	−0.041	0.925**	0.342**	0.579	-3.510
		(0.006)	(0.054)	(0.073)	(0.077)	(0.143)	(0.052)		
	HAR-RV-VIX-EA	0.015**	0.341**	0.196**	−0.147	1.002**	0.413**	0.589	-3.535
		(0.005)	(0.051)	(0.069)	(0.086)	(0.14)	(0.054)		
V	HAR-RV	0.032**	0.283**	0.233**	0.306**			0.299	-2.147
		(0.008)	(0.063)	(0.077)	(0.08)				
	HAR-RV-IV	−0.008	0.243**	0.076	−0.198		0.615**	0.354	-2.228
		(0.011)	(0.049)	(0.062)	(0.126)		(0.134)		
	HAR-RV-VIX	0.024**	0.267**	0.191**	0.038		0.359**	0.316	-2.171
		(0.008)	(0.056)	(0.058)	(0.109)		(0.091)		
	HAR-RV-EA	0.03 **	0.276**	0.25 **	0.307**	0.889**		0.310	-2.162
		(0.008)	(0.063)	(0.078)	(0.081)	(0.239)			
	HAR-RV-IV-EA	−0.009	0.239**	0.095	−0.182	0.729**	0.595**	0.361	-2.239
		(0.011)	(0.05)	(0.063)	(0.128)	(0.233)	(0.137)		
	HAR-RV-VIX-EA	0.022**	0.26 **	0.208**	0.034	0.908**	0.364**	0.328	-2.188
		(0.008)	(0.055)	(0.059)	(0.109)	(0.235)	(0.091)		
VZ	HAR-RV	0.03 **	0.313**	0.314**	0.151*			0.309	-3.543
		(0.005)	(0.064)	(0.089)	(0.07)				
	HAR-RV-IV	0.006	0.261**	0.211**	−0.098		0.397**	0.345	-3.596
		(0.007)	(0.057)	(0.074)	(0.086)		(0.059)		
	HAR-RV-VIX	0.033**	0.289**	0.286**	−0.002		0.135**	0.324	-3.565
		(0.005)	(0.059)	(0.088)	(0.088)		(0.034)		
	HAR-RV-EA	0.027**	0.31 **	0.334**	0.156*	1.005**		0.369	-3.633
		(0.005)	(0.061)	(0.087)	(0.07)	(0.176)			
	HAR-RV-IV-EA	0.007	0.265**	0.244**	−0.057	0.924**	0.34 **	0.396	-3.675
		(0.007)	(0.054)	(0.075)	(0.085)	(0.171)	(0.057)		
	HAR-RV-VIX-EA	0.03 **	0.284**	0.305**	−0.002	1.015**	0.14 **	0.386	-3.659
		(0.005)	(0.056)	(0.086)	(0.088)	(0.176)	(0.033)		
WMT	HAR-RV	0.025**	0.341**	0.35 **	0.1			0.365	-3.993
		(0.004)	(0.068)	(0.07)	(0.069)				
	HAR-RV-IV	0.0	0.289**	0.244**	−0.072		0.356**	0.394	-4.040
		(0.007)	(0.06)	(0.066)	(0.085)		(0.065)		
	HAR-RV-VIX	0.031**	0.305**	0.316**	−0.105		0.141**	0.385	-4.025
		(0.004)	(0.058)	(0.066)	(0.096)		(0.031)		
	HAR-RV-EA	0.023**	0.353**	0.338**	0.117	0.696**		0.409	-4.064
		(0.004)	(0.067)	(0.069)	(0.067)	(0.1)			
	HAR-RV-IV-EA	0.002	0.307**	0.249**	−0.032	0.632**	0.305**	0.430	-4.100
		(0.006)	(0.06)	(0.065)	(0.084)	(0.102)	(0.065)		
	HAR-RV-VIX-EA	0.03 **	0.315**	0.302**	−0.098	0.714**	0.149**	0.431	-4.102
		(0.004)	(0.058)	(0.065)	(0.093)	(0.104)	(0.03)		
XOM	HAR-RV	0.019**	0.482**	0.239*	0.146*			0.516	-3.340
		(0.005)	(0.109)	(0.104)	(0.074)				
	HAR-RV-IV	−0.004	0.399**	0.166*	−0.139		0.408**	0.542	-3.396
		(0.006)	(0.093)	(0.083)	(0.098)		(0.079)		
	HAR-RV-VIX	0.015**	0.43 **	0.198*	−0.049		0.243**	0.531	-3.373
		(0.004)	(0.102)	(0.096)	(0.092)		(0.059)		
	HAR-RV-EA	0.018**	0.48 **	0.245*	0.146*	0.196		0.516	-3.341
		(0.005)	(0.109)	(0.104)	(0.074)	(0.11)			
	HAR-RV-IV-EA	−0.004	0.398**	0.171*	−0.138	0.175	0.406**	0.543	-3.397
		(0.006)	(0.092)	(0.084)	(0.099)	(0.115)	(0.08)		
	HAR-RV-VIX-EA	0.014**	0.428**	0.203*	−0.049	0.197	0.243**	0.532	-3.374
		(0.004)	(0.101)	(0.096)	(0.092)	(0.113)	(0.059)		

Newey-West standard errors are displayed in parentheses below the regression coefficients.
(**) Significant at the 1 percent level. (*) Significant at the 5 percent level.

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INTRODUCING THE NORWEGIAN VOLATILITY INDEX

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Abstract

In this paper we construct and evaluate the NOVIX - an implied volatility index for the Norwegian market created according to the VIX methodology. Implied volatility indices have been an active area of research since the introduction of the VIX by CBOE in 1993. Since then, more and more exchanges have introduced their own implied volatility indices with great success, yet there exists no official index for the Norwegian market. We evaluate the relationship between the NOVIX and returns and realized volatility of the underlying OBX index, and find that the ability of the NOVIX to capture information has improved consistently over the last decade. For the most recent years, we find that the NOVIX has similar properties to the popular VIX and VDAX-NEW volatility indices from the U.S. and German markets.

1 INTRODUCTION

Implied volatility indices based on equity index options have become immensely popular during the two decades they have existed. Investors use them as an expectation of future volatility, a gauge of market sentiment, and as a way to buy and sell volatility itself. In this paper, we introduce the NOVIX - a volatility index for the Norwegian market based on the VIX methodology. The NOVIX is part of a larger trend, in which more and more exchanges have introduced their own implied volatility indices. In addition to the many official volatility indices, there are several academic studies that construct and evaluate volatility indices for markets without official volatility indices, see e.g. Skiadopoulos (2004) and González and Novales (2009).

Internationally, the interest in implied volatility indices has been growing since the Chicago Board Option Exchange (CBOE) introduced the CBOE Volatility Index (VIX) in 1993. Whaley (1993) proposed that these indices can help the investment community in at least two different ways. First, they provide reliable estimates of expected short-term stock market volatility. Second, they offer a market volatility "standard" upon which derivative contracts may be written. The potential to hedge against volatility risk and for profit trading in volatility has led to successful introductions of markets for volatility derivatives and exchange traded products that replicate implied volatility indices.

Today, the combined trading activity in VIX options and futures is over 800,000 contracts per day (Chicago Board Options Exchange, 2015). CBOE alone publishes 28 volatility indices for stock indices, ETFs, interest rates, commodities, currencies and individual stocks. Gradually, other derivatives exchanges have begun offering volatility indices for their respective markets. Some notable examples are Deutsche Börse with the VDAX (1994), later updated to VDAX-NEW (2005), the Marche des Options Negociables de Paris (MONEP) with VX1 and VX6 (1997) and NYSE Euronext with the FTSE 100 Volatility Index (2008).

There is no official implied volatility index for Oslo Børs or the Norwegian market. Oslo Børs is an independent exchange, and the only regulated market for securities trading in Norway (Oslo Børs, 2015). It is internationally recognized as a global leader in the segments energy, shipping and seafood. The main objective of Oslo Børs is to be the central marketplace for listing and trading of financial instruments in the Norwegian market, and nearly all Norwegian companies regard Oslo Børs as the natural place to list.

We construct the NOVIX from options on the OBX Total Return Index (OBX). The OBX is a stock market index composed of the 25 most traded securities on Oslo Børs, and a natural choice as the underlying for an implied volatility index. Oslo Børs has offered options on OBX since 1990, and futures since 1992. On May 22nd 2016, the 25 stocks in OBX had a total market capitalization of NOK 1,437bn compared to the total market capitalization of NOK 1,810bn for all stocks listed on Oslo Børs, representing over 75% of the total value.

Considering the central position of Oslo Børs, we believe the NOVIX can be used as a reference for both practitioners and academic studies about volatility in the Norwegian market. As a way to facilitate for further research, we calculate and provide continuously updated 5-minute intraday values for the NOVIX available at https://novix.xyz.

We use the VDAX-NEW and the VIX from the German and US markets as reference indices to evaluate the properties and behaviour of the NOVIX. In general, we find that the NOVIX exhibits many of the same characteristics as the reference indices, and that its relevance has increased in the most recent years. The degree of negative correlation between OBX returns and NOVIX returns has increased consistently over the last decade, and approaches the level seen in the other markets. This increases the potential of NOVIX derivatives as a tool for risk management. Further, we find that the NOVIX exhibits an asymmetric leverage effect, which is in line with the findings for VDAX-NEW and VIX. However, the asymmetric effect is more pronounced in our reference indices.

Finally, we study how useful the NOVIX is for predicting future volatility in the Norwegian market. For this, we use realized volatility from high-frequency OBX data as a proxy for the true volatility, and include the NOVIX in the Heterogeneous Autoregressive model (HAR-RV) of Corsi (2009). Our out-of-sample results show that the NOVIX adds information beyond the information that is captured by past realized volatility.

The rest of the paper is organized as follows. In section 2, we describe the NOVIX and how the index is created. Section 3 presents characteristics of the NOVIX. Section 4 is an analysis of the relationship between the NOVIX and OBX returns and the leverage effect. Section 5 then proceeds to describe the potential the NOVIX has to forecast realized volatility, before we give a conclusion in section 6.

2 IMPLIED VOLATILITY INDICES

The Chicago Board Option Exchange (CBOE) introduced the VIX volatility index in 1993 as a measure of the expected 30-day future market volatility (Whaley, 1993). This original VIX index was based on the Black-Scholes (BS) pricing model (Black and Scholes, 1973), and calculated as the average BS implied volatility from S&P 100 put and call options. In total, this method uses eight near-the-money puts and calls for the nearby and second most nearby maturity. The original VIX depends on the assumptions of the BS model, and is therefore a model-based implied volatility index. Although it captures more info than the implied volatility of a single strike, it does not capture all the information in the wide range of strikes available.

A decade after its introduction, the VIX was revised in a collaboration with Goldman Sachs. The purpose was to provide exchange-traded volatility derivatives. Still a measure of the expected 30-day future market volatility, the underlying index changed from the S&P 100 to the S&P 500. More importantly, the method for calculating the index was replaced by a model-free approach. The concept of model-free implied variance was first coined by Britten-Jones and Neuberger (2000), and is based on work by Derman and Kani (1994), Dupire (1994, 1997) and Rubinstein (1994). They use no-arbitrage conditions to extract common features of all stochastic processes that are consistent with observed option prices. This has the advantage of not depending on any particular option-pricing model, and extracts information from all relevant option prices (Jiang and Tian, 2005). Demeterfi et al. (1999) show theoretically how a portfolio of standard options can replicate a variance swap and that the cost of this replicating portfolio is the fair price of a variance swap. The VIX methodology is essentially a discretization of the formula for the fair value of a variance swap. Other exchanges have followed CBOE, and like the VIX, the VDAX was updated with a similar model-free approach in 2005 and renamed VDAX-NEW.

2.1 Creating the NOVIX

The NOVIX is constructed with the model-free VIX methodology, according to the formula (Chicago Board Options Exchange, 2015)

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2, \tag{1}$$

where

σ	NOVIX/100
T	Time to expiration in years
F	Forward level of underlying
K_0	First strike below F
K_i	Strike price of the i-th out-of-money option
ΔK_i	$1/2 \times (K_{i+1} - K_{i-1})$
R	Risk-free rate
$Q(K_i)$	Midpoint of bid-ask spread for option with strike K_i

Following Chicago Board Options Exchange (2015), we compute the implied volatility estimate σ for two selected maturities, a *near-term* and *next-term* maturity, that represents the options expiring before and after the desired 30-day horizon. For each maturity, we select a subset of options to include in the calculation by the procedure below.

We determine the forward level from the option prices by first identifying the strike with the smallest absolute difference in put-call price and then applying the formula

$$F = \text{Strike} + e^{RT} \times \left|\text{Call Price} - \text{Put Price}\right|.$$

We define K_0 as the first strike below F, and consider the option pair with strike K_0 as at-the-money. Then, we discard all in-the-money options. That is, we only consider the at-the-money options, the call options with strikes $K_i > K_0$ and the put options with strikes $K_i < K_0$. Intuitively, the demand for out-of-the-money options can be interpreted as a need for insurance by investors, which in turn reflects the market volatility. Further, we exclude all out-of-the-money options with a zero bid price, and all options following two zero bid prices in a row, when the options are ordered by type as increasingly out-of-the-money.

We obtain the final desired 30-day volatility estimate from a linear interpolation between the near-term and next-term results,

$$\text{NOVIX} = 100 \times \sqrt{\left(T_1 \sigma_1^2 \left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} \right] + T_2 \sigma_2^2 \left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}} \right] \right) \times \frac{N_{365}}{N_{30}}},$$

where the subscripts 1 and 2 represent the *near-term* and *next-term* options, respectively. The NOVIX is calculated using a time precision of a minute, where T is the time to expiration in years quoted as minutes to expiration over minutes in a year. The remaining time terms are

$$\begin{aligned} N_{T_1} &= \text{Number of minutes to settlement of } \textit{near-term} \text{ options} \\ N_{T_2} &= \text{Number of minutes to settlement of } \textit{next-term} \text{ options} \\ N_{30} &= \text{Number of minutes in 30 days} \\ N_{365} &= \text{Number of minutes in a year (365 days).} \end{aligned}$$

For more details on the VIX method, we refer to the CBOE White Paper (Chicago Board Options Exchange, 2015).

Our implementation of the VIX-methodology has been tested on the examples in the CBOE White Paper (Chicago Board Options Exchange, 2015). In addition, it has been more comprehensively tested by calculating equity implied volatility indices on the five stocks provided by CBOE. We use daily close option data for this, gathered from the OptionMetrics database, and compare the results to the actual indices published by CBOE. The comparison shows that our implementation is correct. Code and documentation can be provided upon request.

2.2 Data

The NOVIX is calculated from OBX options traded on Oslo Børs. All the data was provided by Oslo Børs, and consist of daily close data on all available call and put options for the period January 3rd 2000 to February 22nd 2016. We use the Norwegian Interbank Offered Rate (NIBOR) as the risk-free interest rate, and interpolate yield to maturity from the two closest values around each term expiration.

There are some potential problems with the VIX methodology that are addressed in Jiang and Tian (2007). The VIX methodology introduces truncation and discretization errors due to the limited number of strike prices available. They suggest that interpolation and extrapolation over strikes can improve the accuracy of the model-free approach. This was implemented by Ting et al. (2007) for the Korean stock market, where there were especially large steps between strikes (8%). In the period used, the relevant OBX options have a strike interval of 3%. Based on this, we consider these problems as less critical for the calculation of NOVIX. Further, by staying with the standard VIX methodology, we have comparable numbers to other markets. However, we shortened the data period from initially starting from January 3rd 2000 to starting from January 3rd 2006 instead. This was done due to low volumes in option trading during the first years, where the issues above are especially relevant.

To understand NOVIX better, we use the VDAX-NEW and VIX together with their respective underlying equity indices DAX and S&P500 as reference indices. The VIX index is the most popular volatility index, and financial products based on the VIX are by far the most traded among those based on volatility indices. VDAX-NEW is a recognized volatility index in a major market with strong relations to the Norwegian market. Daily close prices for the VDAX-NEW were downloaded from the Frankfurt Stock Exchange, and daily VIX close prices from CBOE.

3 CHARACTERISTICS OF THE NOVIX

Figure 1 displays the OBX and the NOVIX for the period from January 2000 to February 2016. There is a clear negative relationship between the OBX index and the NOVIX: when the OBX trends downward the NOVIX rises, and vice versa. The largest spikes in the NOVIX correspond to negative geopolitical events, such as the terrorist attack in 2001, the financial crisis in 2008 and the EU debt crisis in August 2011. The largest movements are all responses to global events, and not specific for the Norwegian market.

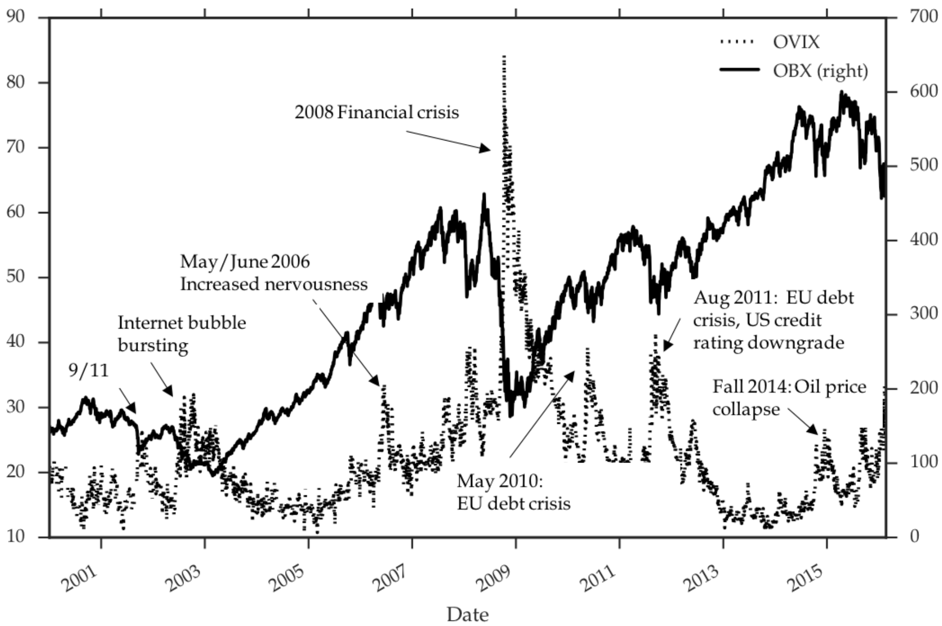

Figure 1: NOVIX levels (left axis) and OBX levels (right axis) from January 3rd 2000 to February 22nd 2016. In addition, we highlight a selection of financial and geopolitical events in the figure.

Over the full-sample, the correlation between NOVIX and OBX returns is -0.30. In comparison, the correlations between the VDAX-NEW and VIX and their underlying stock indices are 0.70 and 0.65. The difference between the NOVIX and the other indices may appear large, but it has changed considerably over time. As we see in Figure 2, the VDAX-NEW and VIX have a stable, large negative correlation with their underlying index returns, while the negative correlation between NOVIX and OBX returns has increased in magnitude over the sample. The higher degree of correlation between NOVIX and OBX returns indicates that the NOVIX absorbs information better, and increases the relevance of the NOVIX in risk management.

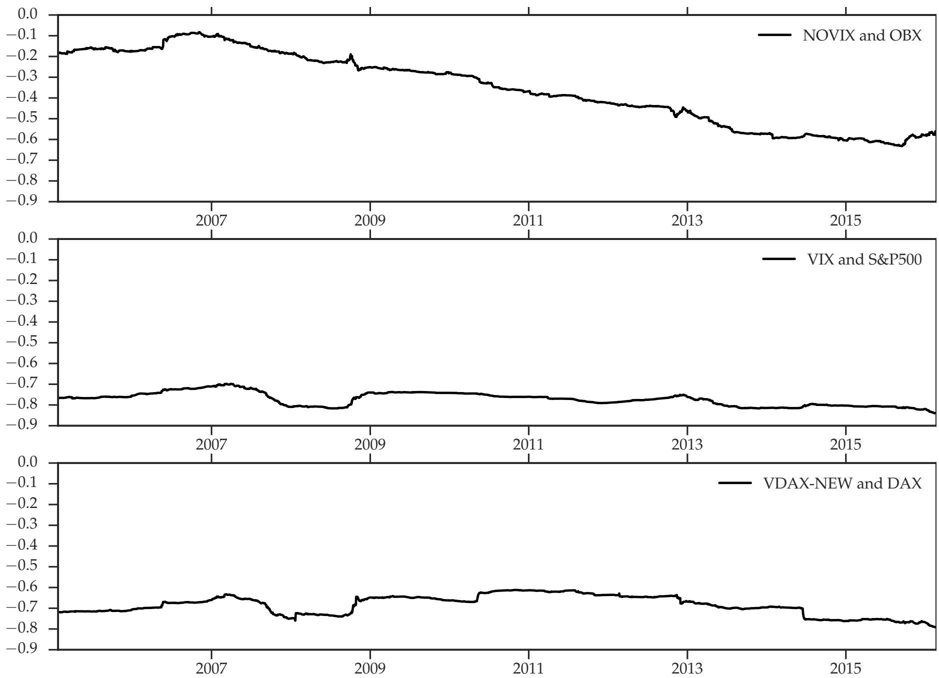

Figure 2: Evolution of correlation between log returns of implied volatility indices and underlying stock indices, using a 1000 day moving window.

Table 1 displays descriptive statistics of the implied volatility indices (IV) for both levels and log returns. The NOVIX has similar distributional characteristics as VDAX-NEW and VIX. Both levels and returns are right skewed and leptokurtic for all indices, but the two measures are more pronounced for levels. The average level of NOVIX is 24.45.

Table 1: Descriptive statistics for the implied volatility indices NOVIX, VDAX-NEW and VIX, for levels and returns. The period from January 3rd 2006 to February 22nd 2016 is used, with 2439 observation for each series.

Index	Mean (%)	Std. (%)	Min (%)	Max (%)	Skew	Ex. Kurt.
NOVIX	24.45	10.01	11.19	84.25	1.86	5.22
VDAX-NEW	23.43	9.06	12.13	83.23	2.31	7.60
VIX	20.55	9.85	9.89	80.86	2.31	7.04
R^{NOVIX}	0.00	5.34	−27.34	29.22	0.37	3.66
$R^{VDAX\text{-}NEW}$	0.01	7.24	−35.06	49.60	0.65	3.23
R^{VIX}	0.01	5.68	−26.65	30.57	0.49	2.02

R^{IV} denotes the IV returns, computed as log differences.

This is close to VDAX-NEW (23.43), but higher than VIX (20.55). The highest value for NOVIX of 84.25 was recorded on September 20th 2008 during the financial crisis, and is of similar magnitude to the highest values of the other indices. The log returns for all IV indices have an average close to zero. NOVIX has a similar magnitude of largest and lowest observed returns as VIX, while the maximum observed return of VDAX-NEW is higher.

We use the Augmented Dickey-Fuller (ADF) unit root test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) stationarity test to test for stationarity in the implied volatility indices. Table 2 shows the results for the entire time period, as well as for two non-overlapping subsamples. As common for volatility, we assume no time trend in the long run, and present the test results without any deterministic trend[1].

With levels we get different results for the indices, while log returns are stationary for all indices. Both tests conclude that NOVIX levels are non-stationary for all samples. The results for VDAX-NEW and VIX are inconclusive. For the entire-sample, the ADF test rejects non-stationarity

1 Results are not sensitive to the number of lags used, and do not change significantly with an added time trend.

Table 2: Stationarity tests using Augmented Dickey Fuller (ADF) test and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for the implied volatility (IV) indices at both levels and log returns. The ADF and KPSS test statistics are indicated with star(s) if the null-hypothesis is rejected. The null-hypothesis for the ADF test is non-stationarity, whereas the null-hypothesis for the KPSS test is stationarity.

		Levels		Returns	
IV index	Sample	ADF	KPSS	ADF	KPSS
NOVIX	2006 - Feb. 2016	−2.79	2.79**	−23.42**	0.06
	2011 - Feb. 2016	−1.73	1.33**	−17.32**	0.12
	2006 - 2010	−2.15	1.14**	−23.87**	0.11
VDAX-NEW	2006 - Feb. 2016	−3.45**	0.68*	−13.96**	0.03
	2011 - Feb. 2016	−2.47	0.79**	−10.37**	0.05
	2006 - 2010	−2.70	1.21**	−10.88**	0.07
VIX	2006 - Feb. 2016	−3.33*	1.27**	−23.97**	0.04
	2011 - Feb. 2016	−2.80	1.29**	−10.66**	0.03
	2006 - 2010	−2.19	1.55**	−11.76**	0.11

The number of lags is determined according to the formula $\sqrt[4]{12 \times (n/100)}$ of Schwert (2002).
* Significant at the 5% level.
** Significant at the 1% level.

at the 1% significance level for VDAX-NEW, whereas the KPSS test rejects the null hypothesis of stationarity at the 5% level. For entire-sample of the VIX, the ADF test rejects non-stationarity at the 5% level, and the KPSS test rejects stationarity at the 1% level.

Intuitively, we would expect the implied volatility levels to be stationary. The volatility is naturally bounded from above and below, and volatility indices are usually assumed to be mean-reverting. However, the sample may be too short to capture long cycles and their variance is not necessarily constant. From the stationarity tests, it is not obvious whether we should use levels or returns in the subsequent analysis of the NOVIX. Therefore, we proceed with both for completeness.

4 LEVERAGE EFFECT

The leverage effect refers to the well-established relationship between volatility and equity returns — volatility increases as stock prices fall. We observe this negative relationship in the plot of NOVIX and OBX in Figure 1, and in Figure 2 where we display the correlation between NOVIX and OBX returns. In this section, we study this leverage effect explicitly, and compare the NOVIX with our reference indices.

The leverage effect was first discussed in Black (1976) and Christie (1982). The term *leverage* refers to the economic interpretation that when asset prices decline, companies become more leveraged as their debt-to-equity ratio increases. As a result, one expects their stock to become more risky, and hence more volatile. However, the magnitude of the effect seems too large to be attributable solely to an increase in financial leverage. Figlewski and Wang (2000) noted among other findings that there is no apparent effect on volatility when leverage changes because of a change in debt or number of shares, only when stock prices change. This questions whether the effect is linked to financial leverage at all.

In previous literature it has been documented that the effect is generally asymmetric, meaning that the increase in volatility is higher for negative returns than the reduction in volatility for positive returns of the same magnitude. The degree of asymmetry depends on the volatility proxy employed in the estimation, with options' implied volatility generally exhibiting much more pronounced asymmetry (see e.g. Bates (2000), Wu and Xiao (2002), Eraker (2004)).

Several different parametric models and volatility-return regressions can be employed for empirically assessing the leverage effect. We use a regression model that is able to capture the asymmetric effect, by separating positive and negative returns. The regression is specified for IV as both levels and log returns:

$$\begin{aligned} IV_t &= \alpha + \beta IV_{t-1} + \gamma_1 R_t^+ + \gamma_2 R_t^- + \epsilon_t && \text{(Levels)} \\ R_t^{IV} &= \alpha + \gamma_1 R_t^+ + \gamma_2 R_t^- + \epsilon_t, && \text{(Returns)} \end{aligned} \tag{2}$$

where R^{IV} denotes the log difference for the IV index and R_t the log return of the underlying index. We separate the log returns into positive and negative returns as

$$R_t^+ = \max(R_t, 0) \qquad \text{and} \qquad R_t^- = \min(R_t, 0).$$

IV_t is the close value of IV at the end of day t, and we include its lagged value in order to account for the strong temporary dependencies in volatility in the levels regression. R_t is the return from the end of day $t-1$ to the end of day t. Negative γ values indicate an opposite movement to returns in the IV index. We expect both γ_1 and γ_2 to be negative, and a larger magnitude of γ_2 will indicate asymmetry.

Table 3 displays the results of the regressions. From the levels regression we see that the NOVIX has negative γ coefficients for the entire sample, with the largest magnitude for negative returns. More interestingly, we see a change in the leverage effect over the two sub-samples. Both γ coefficients increase in magnitude and γ_1 become significant in the second sample, which reflect that NOVIX has become more sensitive to changes in OBX. The γ coefficients of the reference indices have a larger magnitude and the asymmetric relationships are more pronounced.

We see similar effects when modeling IV as log returns. The magnitude of the γ coefficients have increased from the first to the second sub-sample, and a clear asymmetry is evident in the second sub-sample. Again, the asymmetry is more pronounced for the reference indices and the explanation power from the returns regression is lower for NOVIX.

Our results show that NOVIX reacts less to market changes than VDAX-NEW and VIX. This smaller response may imply that the Norwegian option market is less efficient than the German and U.S. markets. At the same time, we see it as a positive sign that the response has increased from the first to the second sub-sample.

Table 3: Leverage effect regression for Equation 2 with levels and returns. Results for the entire time period, as well as two sub-samples.

LHS	Sample	Regression coefficients $\hat{\alpha}$	IV_{t-1}	R_t^+	R_t^-	$R^2_{Adj.}$
NOVIX	2006 - Feb. 2016	0.00** (0.00)	0.98** (0.01)	−0.13* (0.06)	−0.37** (0.08)	0.98
	2011 - Feb. 2016	0.00** (0.00)	0.98** (0.01)	−0.26** (0.08)	−0.43** (0.09)	0.96
	2006 - 2010	0.01* (0.00)	0.98** (0.01)	−0.08 (0.07)	−0.35** (0.10)	0.97
VDAX-NEW	2006 - Feb. 2016	0.00** (0.00)	0.96** (0.01)	−0.31** (0.11)	−1.21** (0.08)	0.98
	2011 - Feb. 2016	0.00* (0.00)	0.98** (0.01)	−0.58** (0.07)	−1.07** (0.08)	0.98
	2006 - 2010	0.01* (0.00)	0.96** (0.01)	−0.13 (0.15)	−1.31** (0.12)	0.98
VIX	2006 - Feb. 2016	0.00** (0.00)	0.97** (0.01)	−0.93** (0.08)	−1.50** (0.08)	0.99
	2011 - Feb. 2016	0.00** (0.00)	0.96** (0.01)	−0.89** (0.14)	−1.71** (0.09)	0.98
	2006 - 2010	0.00** (0.00)	0.97** (0.01)	−0.94** (0.10)	−1.42** (0.09)	0.99
R^{NOVIX}	2006 - Feb. 2016	−0.00 (0.00)		−0.70** (0.14)	−0.88** (0.15)	0.06
	2010 - Feb. 2016	−0.00 (0.00)		−1.39** (0.30)	−1.73** (0.32)	0.12
	2006 - 2010	−0.00 (0.00)		−0.42** (0.15)	−0.64** (0.16)	0.04
$R^{VDAX\text{-}NEW}$	2006 - Feb. 2016	−0.01** (0.00)		−1.88** (0.28)	−3.51** (0.18)	0.45
	2010 - Feb. 2016	−0.01** (0.00)		−2.61** (0.26)	−4.03** (0.25)	0.55
	2006 - 2010	−0.01** (0.00)		−1.34** (0.35)	−3.11** (0.21)	0.38
R^{VIX}	2006 - Feb. 2016	−0.00* (0.00)		−3.67** (0.31)	−4.65** (0.41)	0.54
	2010 - Feb. 2016	−0.00 (0.00)		−5.12** (0.65)	−7.37** (0.49)	0.64
	2006 - 2010	−0.00* (0.00)		−2.97** (0.28)	−3.78** (0.37)	0.54

R^{IV} denotes the IV returns, computed as log differences. Newey-West standard errors in parenthesis below the regression coefficients, using five lags.
* Significant at the 5% level.
** Significant at the 1% level.

5 NOVIX IN VOLATILITY FORECASTING

We use realized volatility as a proxy for the true volatility as suggested by Andersen and Bollerslev (1998), and assess the incremental value of the information in the NOVIX for realized volatility forecasts. The realized volatility measure exploits information in high-frequency data, and one can in effect treat volatility as observable.

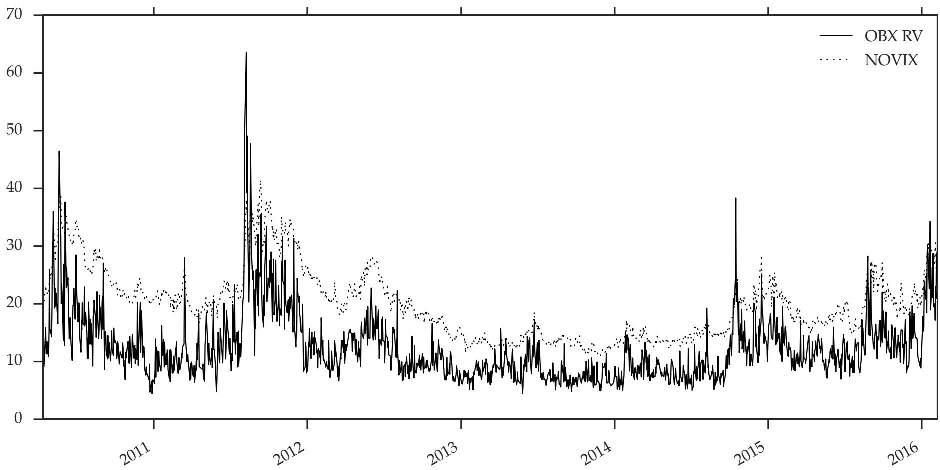

Figure 3: The NOVIX and OBX realized volatility in the period from April 13th 2010 to February 8th 2016, quoted in percentage values.

In the literature, the value of implied volatility indices for realized volatility forecasts has been studied extensively. Jiang and Tian (2005) find that the model-free implied volatility of S&P500 options subsumes all information contained in past realized volatility, and is a more efficient forecast for future realized volatility. We see clear signs of a relationship between the NOVIX and realized volatility in Figure 3, and they clearly capture much of the same information.

5.1 Calculation of Realized Volatility

If we let one trading day be split into m equidistant intervals, the intra-day return r_i over the time period $[i - 1/m, i]$ is given by

$$r_i = \ln\left(\frac{P_i}{P_{i-1/m}}\right), \quad \text{for} \quad i = 1/m, 2/m, \dots, 1, \tag{3}$$

where P_i is the price at time i. By taking the square root of the sum of the m squared intraday returns over a given trading day, we obtain daily realized volatility

$$RV^D = \sqrt{\sum_{i=1}^{m} r_i^2}. \tag{4}$$

We annualize RV^D by the conventional square-root-of-time rule with 250 trading days a year, i.e. $RV^D \times \sqrt{250}$.

The literature suggests that RV should be based on intra-day returns sampled at the highest possible frequency. However this runs into the challenge of market micro-structure in real world applications (Zhang, Mykland, and Aït-Sahalia, 2005). This stems from the fact that efficient prices cannot be observed directly. Empirical work suggests that the estimate seems to diverge if RV is calculated using too frequent observations (Andreou and Ghysels, 2002; Bai, Russell, and Tiao, 2001; Bandi and Russell, 2008). We follow the common practice of using a sampling frequency of 5 minutes that allows us to ignore much of the microstructure noise.

The RV calculations were implemented by resampling a clean dataset to 5-minute intervals using a calendar time sampling scheme with equidistant samples. Our procedure for cleaning the high-frequency OBX data is based on the steps proposed by Barndorff-Nielsen et al. (2009). Oslo Børs is open between 09:00 and 16:20, giving us 88 five-minute intra-day returns over a 440-minute typical trading day.

Our data set spans the period from April 13th 2010 to February 8th 2016. On average, there were 15,000 trades per day. For the DAX and S&P500, we obtain RV from the Oxford-Man Institute of Quantitative

Finance (2016). Their values have been calculated with a 5-minute sampling interval, an equivalent procedure to ours, and cleaned as described in Barndorff-Nielsen et al. (2009).

5.2 Model

We use the Heterogeneous Autoregressive model of Realized Volatility (HAR-RV) by Corsi (2009) to forecast volatility on OBX, and augment the model with an additional independent variable for the NOVIX. The HAR-RV model is a long-memory model utilizing RV calculated from high-frequency data. Empirically, the volatility forecasts calculated from the HAR-RV model have performed much better than traditional GARCH models (Andersen et al., 2003), and perform well compared to other more complicated long memory models (Andersen, Bollerslev, and Diebold, 2007). Following Corsi (2009), we include three different volatility components based on different horizons: daily, weekly and monthly. The one-day ahead HAR-RV model is specified as

$$RV_{t+1}^{D} = \alpha + \beta_1 RV_t^{D} + \beta_2 RV_t^{W} + \beta_3 RV_t^{M} + \epsilon_{t+1}, \tag{5}$$

where the superscripts D, W and N denote daily, weekly and monthly RV. These multi-period volatilities are defined as simple backward averages of the daily RV. Thus, weekly and monthly RV using 5 trading days a week and 22 per month is

$$RV_t^{W} = \frac{1}{5}\sum_{i=0}^{4} RV_{t-i}^{D} \qquad \text{and} \qquad RV_t^{M} = \frac{1}{22}\sum_{i=0}^{21} RV_{t-i}^{d}.$$

We augment the model similarly as Haugom et al. (2014) when we include the IV component, and denote this model HAR-RV-IV. The model is then specified as

$$RV_{t+1}^{D} = \alpha + \beta_1 RV_t^{D} + \beta_2 RV_t^{W} + \beta_3 RV_t^{M} + \beta_4 IV_t + \epsilon_{t+1}. \tag{6}$$

In addition to daily forecasts, we use the models in Equation 5 and Equation 6 to produce weekly and monthly forecasts. For this, the left-hand-side variables in both models are changed to the forward averages of the monthly and weekly RVs, i.e.

$$RV_t^W = \frac{1}{5}\sum_{i=0}^{4} RV_{t+i}^D \quad \text{and} \quad RV_t^M = \frac{1}{22}\sum_{i=0}^{21} RV_{t+i}^D.$$

The models are estimated with OLS regression.

5.3 In-sample evaluation

In-sample results are displayed in Table 4. We first consider the Norwegian market. For the HAR-RV model, we see that all lagged values of daily, weekly and monthly RV coefficients are significant. The model puts the most weighting on the lagged variable matching the model's horizon. Moving to the HAR-RV-IV model, we see that the NOVIX coefficient is significant for all horizons and of relatively large magnitude. Consequently, the daily, weekly and monthly RV coefficients are reduced in both statistical significance and coefficient value. The largest decrease is observed for the monthly RV, which is no longer significant for the daily and weekly horizons. Also, the adjusted R squared is increased across all three horizons when the NOVIX is included. Thus, our initial finding is that the NOVIX may be a useful predictor of future RV.

Turning to our reference indices, we observe similar dynamics when modeling future RV for DAX and S&P500. However, we note that the magnitude of the IV coefficients relative to the lagged RV coefficients in the HAR-RV-IV model is always greater for the reference indices compared to the NOVIX. This may indicate that the VDAX-NEW and VIX are more useful than the NOVIX when predicting future RV in their respective markets.

Table 4: In-sample regression results for HAR-RV and HAR-RV-IV models where the left-hand-side (LHS) variable in Equation 5 and Equation 6 are solved using OLS regression, using daily (Panel A), weekly (Panel B) and monthly (Panel C) horizons.

Panel A: Daily horizon

		Regression coefficients					
LHS	Model	Intercept	RV^d_{t-1}	RV^w_{t-1}	RV^m_{t-1}	IV_{t-1}	R^2_{Adj}
OBX RV	HAR-RV	0.01** (0.00)	0.43** (0.07)	0.36** (0.08)	0.15* (0.07)		0.672
	HAR-RV-IV	−0.00 (0.00)	0.36** (0.07)	0.33** (0.07)	−0.03 (0.08)	0.23** (0.05)	0.683
DAX RV	HAR-RV	0.01** (0.00)	0.37** (0.08)	0.39** (0.10)	0.17 (0.09)		0.613
	HAR-RV-IV	−0.02** (0.00)	0.25** (0.08)	0.24** (0.09)	−0.14 (0.09)	0.57** (0.06)	0.641
SP500 RV	HAR-RV	0.01** (0.00)	0.34** (0.07)	0.34** (0.07)	0.21** (0.07)		0.517
	HAR-RV-IV	−0.03** (0.01)	0.19** (0.07)	0.19** (0.06)	−0.15 (0.08)	0.66** (0.07)	0.600

Panel B: Weekly horizon

		Regression coefficients					
LHS	Model	Intercept	RV^d_{t-1}	RV^w_{t-1}	RV^m_{t-1}	IV_{t-1}	R^2_{Adj}
OBX RV	HAR-RV	0.01** (0.00)	0.31** (0.08)	0.33** (0.07)	0.25** (0.07)		0.704
	HAR-RV-IV	0.00 (0.00)	0.23** (0.06)	0.31** (0.07)	0.07 (0.08)	0.23** (0.05)	0.715
DAX RV	HAR-RV	0.02** (0.00)	0.26** (0.07)	0.35** (0.06)	0.26** (0.08)		0.673
	HAR-RV-IV	−0.00 (0.01)	0.15** (0.06)	0.25** (0.05)	0.05 (0.08)	0.41** (0.06)	0.693
SP500 RV	HAR-RV	0.02** (0.00)	0.23** (0.05)	0.31** (0.06)	0.29** (0.07)		0.580
	HAR-RV-IV	−0.01 (0.01)	0.11** (0.04)	0.19** (0.06)	0.01 (0.08)	0.51** (0.07)	0.625

Panel C: Monthly horizon

		Regression coefficients					
LHS	Model	Intercept	RV^d_{t-1}	RV^w_{t-1}	RV^m_{t-1}	IV_{t-1}	R^2_{Adj}
OBX RV	HAR-RV	0.03** (0.00)	0.16** (0.05)	0.18** (0.05)	0.42** (0.05)		0.618
	HAR-RV-IV	0.02** (0.00)	0.11* (0.04)	0.16** (0.05)	0.28** (0.07)	0.17** (0.04)	0.626
DAX RV	HAR-RV	0.04** (0.01)	0.15** (0.05)	0.23** (0.05)	0.36** (0.07)		0.586
	HAR-RV-IV	0.03** (0.01)	0.10* (0.05)	0.18** (0.06)	0.26** (0.08)	0.20* (0.08)	0.591
SP500 RV	HAR-RV	0.04** (0.00)	0.12** (0.03)	0.20** (0.06)	0.35** (0.07)		0.503
	HAR-RV-IV	0.02** (0.01)	0.05 (0.03)	0.12* (0.06)	0.17* (0.09)	0.32** (0.08)	0.529

Newey-West standard errors in parenthesis below the regression coefficients, using five lags.
* Significant at the 5% level.
** Significant at the 1% level.

5.4 Out-of-sample evaluation

Out-of-sample forecasts let us directly compare the performance of the models relative to the actual observed values. The forecasts use a rolling window of 500 observations for the time interval January 4th 2010 to February 22nd 2016.

We use mean squared errors (MSE) to evaluate the two models. MSE is a robust loss function when it comes to volatility forecast errors, as described in Patton (2011). The loss function is defined as

$$\text{MSE} = \frac{1}{T}\sum_{t=1}^{T}(\widehat{RV}_t - RV_t)^2, \tag{7}$$

where $\widehat{RV}_t$ is the forecast value for time t and RV_t is the actual observed value at time t. Besides MSE, we employ the statistical test of Diebold and Mariano (1995) (DM) to determine if the model that includes the NOVIX provides significantly more accurate forecasts than the model

Table 6: Out-of-sample regression results for HAR-RV-IV models specified in Equation 6. We use a window 500 observations, and evaluate the forecasts with mean squared errors (MSE) for daily, weekly and monthly horizons. The MSE values are normalized relative to HAR-RV.

	MSE		
LHS	Daily	Weekly	Monthly
OBX RV	0.955*	0.921**	0.944*
DAX RV	0.919**	0.914**	0.973*
SP500 RV	0.820**	0.832**	0.921**

The MSE values are marked with */** if the HAR-RV-IV model is significantly more accurate than the corresponding HAR-RV model according to the test of Diebold and Mariano (1995), using squared error loss function.

* Significant at the 5% level.

** Significant at the 1% level.

without. For consistency with the MSE measure, we specify the loss function in the DM test as squared errors.

Table 6 show the out-of-sample results for all forecast horizons. The MSE values improve by 4.5%, 7.9% and 5.6% for the daily, weekly and monthly horizons when we add the NOVIX as an additional variable. The DM test rejects the null hypothesis of equal predictive ability at the 5% level for daily and monthly forecasts, and at the 1% level for weekly forecasts. The alternative hypothesis is that the HAR-RV-IV model produces more accurate forecasts.

Turning to our reference indices, we see that the VDAX-NEW and VIX both improve their respective volatility forecasts with regards to MSE more than the NOVIX across all horizons, except when VDAX-NEW is included in monthly forecasts for the DAX RV. Overall, the VIX has the largest positive impact on the volatility forecasts.

6 CONCLUDING REMARKS

The NOVIX is an academic, model-free implied volatility index for the Norwegian market. We construct the NOVIX from options on the OBX index and analyze its properties in the period between January 3rd 2006 and February 22nd 2015. Throughout the paper we study the NOVIX in light of the popular VIX and VDAX-NEW implied volatility indices for the U.S. and German markets. We show that the index contains the same characteristics as VIX and VDAX-NEW when we study features such as stationarity and leverage effect. In order to facilitate for further research, we also calculate and provide continuously updated 5-minute intraday values for the NOVIX at https://novix.xyz.

We find that the potential value of the created Norwegian implied volatility index has increased steadily over the last 15 years, as the NOVIX is more efficient at absorbing market information today than it was a decade ago. The correlation between NOVIX and OBX returns shows an increased negative relationship, and today the correlation between NOVIX and OBX returns is similar to that of VIX and S&P500 returns.

We also find evidence of improved OBX volatility forecasts based on high-frequency data when we include NOVIX. The HAR-RV-IV forecasts that include implied volatility yield a statistically improved performance over HAR-RV for daily, weekly and monthly forecast horizons. However, the magnitude of improvement is larger for the German and U.S. markets, and this may indicate that these options markets are more efficient.

Based on our findings we argue that the created index can be of great interest to market participants as a tool for risk management purposes. First, the index provide an estimate of expected short-term Norwegian stock market volatility. Second, we argue that a future official Norwegian volatility index created in a similar fashion as the NOVIX, could create a basis for volatility derivative products and Exchange Traded Products (ETPs) for the Norwegian stock market. However, this will require further studies in many areas such as the liquidity and efficiency of the Norwegian options market.

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