NTNU
Norwegian University of
Science and Technology

Green Investment under Policy Uncertainty and Bayesian Learning

Peder Andreas Olsen Dalby
Gisle Ryan Gillerhaugen
Tord Leth-Olsen

Industrial Economics and Technology Management
Submission date: June 2016
Supervisor: Verena Hagspiel, IØT

Norwegian University of Science and Technology
Department of Industrial Economics and Technology Management

Problem description

The costs of electricity generation from most green sources are significantly above average European market prices for electricity. To ensure competitiveness of renewable electricity production, various support schemes have been introduced. The implementation of support schemes have led to a new source of uncertainty for renewable energy investments, as several unexpected and retroactive revisions of subsidy payments have occurred. These revisions make policy uncertainty a significant challenge for investors in renewable electricity plants. Investors' belief regarding a change in support policy is therefore thought to be of great importance in investment decisions.

This thesis will examine how learning about the timing of revision or termination of a support scheme can affect investment behavior. We incorporate Bayesian learning into a real options model, and analyze the case where an investor updates her probabilistic belief about the timing of a revision.

Preface

We submit this thesis as the concluding part of our Master of Science degrees in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). The thesis is written as a scientific paper and will be submitted to a peer-reviewed academic journal. We aim for one of the following journals: Energy or Energy Policy.

We would like to thank our supervisor, Associate Professor Verena Hagspiel, for her guidance and valuable comments throughout the work of this thesis. The main topic of the thesis was proposed by Verena Hagspiel, and we have appreciated working with a topic at the forefront of real options theory.

We also want to express our gratitude to Professor Jacco Thijssen at the University of York for his valuable insight and helpful suggestions. We are grateful to Professor João Janela at the University of Lisbon for mathematical guidance on the Finite Elements Method. We would also like to thank Assistant Professor Cláudia Nunes at the University of Lisbon for helpful feedback.

Finally, we are pleased that our supervisor has offered to present our paper at the INFORMS Annual Meeting in Nashville, USA, November 13-16, 2016.

Trondheim, June 10, 2016

Peder Andreas Dalby Gisle Ryan Gillerhaugen Tord Leth-Olsen

Abstract

In the last decades, several countries have introduced support schemes to accelerate investments in renewable energy (RE). When support schemes have served their purpose, and production goals are met, retraction or revision of currently employed support schemes becomes more likely, as confirmed by several policy changes in European countries.

Investors in RE are greatly affected by the risk of subsidy changes when considering an investment opportunity. Therefore, they need to evaluate the likelihood of potential subsidy retractions and revisions, and take it into account when making an investment decision.

The main purpose of this paper is to examine how investment behavior is affected by updating a subjective belief on the timing of a subsidy revision. We incorporate Bayesian learning into a real options modeling approach. Subsidies in the form of fixed feed-in tariffs (FIT) are considered, and we analyze a scenario where a retroactive downward adjustment of the FIT is expected through a regime switching model.

We find that investors are less likely to invest when the arrival rate of a policy change increases. Further, investors prefer a lower FIT with a long expected lifespan, while policy makers prefer a higher FIT with shorter life span.

In addition, we consider an extension of our model where a FIT scheme is retroactively retracted and followed by a regime under which electricity is sold on a free market.

We find that if policy uncertainty is high, an increase in the FIT will be less effective at accelerating investments. However, if policy risk is low, FIT schemes can be very effective for accelerating investment, even in highly volatile electricity markets.

Sammendrag

De siste tiårene har det i flere land blitt innført støtteordninger med mål om å øke investeringsaktivitet innen fornybar energi. Tilbaketrekning eller endring av støtteordningene er blitt mer sannsynlig etter hvert som ordningene har tjent sitt formål og produksjonsmål for fornybar energi er nådd. Dette bekreftes av flere eksempler på tilbaketrekninger og revisjoner i Europa.

Lønnsomheten til prosjeker innen fornybar energi er i stor grad avhengig av subsidieordninger. Investorer må derfor anslå sannsynligheten for en endring i støtteordninger og ta høyde for dette i investeringsbeslutninger.

Formålet med denne artikkelen er å undersøke hvordan investeringsbeslutninger påvirkes av at en investor over tid endrer sin oppfatning av risiko, gjennom læring. Vi utvider en tradisjonell realopsjonmodell ved å inkludere bayesiansk læring og muligheten for et regimeskifte. Et scenario der en tilbakevirkende nedjustering av faste innmatingstariffer presenteres.

Vi finner at investeringsnivået reduseres når sannsynligheten for en endring i tilskuddsordninger øker. Videre konkluderer vi med at investorer foretrekker en lavere innmatingstariff over en lang tidshorisont, mens politikere foretrekker en ordning med høyere tariff og kortere levetid.

I tillegg utvider vi modellen, og ser på et scenario der innmatingstariffen blir fjernet med tilbakevirkende kraft slik at produsenter må selge elektrisitet i markedet. Vi viser at hvis den politiske usikkerheten er høy, vil en økning i innmatingstariff ikke påvirke investeringsaktiviteten i særlig grad. Derimot, hvis den politiske usikkerheten er lav, kan innmatingstariffer være svært effektive for å øke investeringsaktiviteten, selv i volatile kraftmarkeder.

Contents

1 Introduction

We consider an investment in a renewable energy (RE) project. Policy uncertainty[1] in the form of adverse revisions of support schemes, has a large impact on RE investments. It is therefore important that investors try to assess this risk through learning. In standard real option models, learning is an implicit consequence of postponing investment. We allow for a more realistic modeling of the investment environment, where information is received, processed and incorporated into the decision making.

The member states of the European Union have agreed to reduce the emission of greenhouse gases substantially by 2050. Specific targets, like EU2020 and EU2030, have been set in order to reach this long-term goal. One of the objectives of The European Strategic Energy Technology Plan (SET-Plan) is to accelerate investments in renewable energy technologies. As a consequence of the deregulation of the electricity markets in Europe, it is private investors with an objective of maximizing profit who choose whether to invest in an RE project or not (Abadie and Chamorro, 2014). At the same time, the costs of electricity generation from most renewable energy sources are significantly above average European market prices of electricity (Klessmann et al., 2013). Therefore, several European countries have changed their policies and introduced various support schemes to ensure competitiveness of renewable energy production and encourage investment.

Support schemes can be characterized as either quantity-driven[2] or price-driven[3]. The price-driven feed-in schemes are the most commonly used support mechanism. In 2015 nearly 80 countries had employed feed-in tariffs (FIT) as support policy (REN21, 2015). FITs are considered to be the most effective scheme for accelerating development of renewable energy sources (Couture and Gagnon (2010); del Rio and Mir-Artigues (2012); Ritzenhofen and Spinler (2016)).

Under a FIT scheme, producers are often guaranteed a market independent and fixed price for every unit of electricity generated, over the lifetime of a project (Couture and Gagnon, 2010). However, the problem for investors is that unexpected and retroactive revisions of vital subsidy payments have occurred in several countries during the last years (REN21, 2015). Underlying factors for revisions include changes of government, budget constraints and the fact that investment targets have already been met. When targets are met, incentives for accelerating investment are not required anymore.

In Bulgaria, Germany, Greece, Italy, and Switzerland, the FIT rate was reduced during 2014, and in Ukraine, the tax exemption for companies that sell renewable electricity has

[1]In line with other research in this field (e.g. Boomsma and Linnerud (2015); Ritzenhofen and Spinler (2016); Yang et al. (2008)), the terms "uncertainty" and "risk" will be used interchangeably in this paper.

[2]Quantity-driven schemes include electricity certificates, where producers of renewable energy are given a number of certificates based on the quantity of electricity supplied to the market.

[3]Price-driven schemes include feed-in schemes, which can be implemented either as a price premium paid on top of the electricity price, or as a fixed tariff paid to producers instead of the electricity price. The fixed tariff is independent of the electricity price.

been removed (REN21, 2015). In Spain, Belgium, the Czech Republic, Bulgaria and Greece, the size of subsidy payments was retroactively adjusted, thereby reducing the profitability of already operating plants (Boomsma and Linnerud, 2015).

According to an estimate, the revision in Spain caused a 40 % cut in expected income for a large amount of RE projects (The Institute of Energy for South East Europe, 2014). These cuts made the investors unable to meet their debt payments. As a consequence, several lawsuits against the Spanish government were filed. In one lawsuit it was concluded that plaintiff investors could not legitimately expect the FIT scheme to remain unchanged throughout the life of their RE plants, and that the investors could have easily foreseen the prospect of a revision.

The possibility of an unexpected subsidy revision has introduced a new source of uncertainty for investors, since the profitability of RE investments is largely or entirely dependent on consistent government policy (Helm et al., 2003). White et al. (2013) states that policy uncertainty is a significant challenge for actors in the renewable energy sector. This is in line with Europe's largest producer of renewable energy, Statkraft, who states in its annual report of 2014 that uncertainty related to framework conditions, such as taxes, fees and political regulations are highly accentuated in investment decisions (Statkraft, 2014). This is consistent with Canada's Rural Partnership, who states the importance of policy support being consistent, long term, and predictable to avoid boom and bust cycles[4] (White et al., 2013). Investors' subjective belief regarding a change in support policy is therefore of great importance for investment decisions in the renewable energy sector.

For investors it is important to acquire experience on how to learn effectively about policy uncertainty in the energy sector (Fuss et al., 2008). Fuss et al. (2009) find that increasing policy uncertainty leads to an increase in the expected value of information. As changes in policies may be harmful to investors, it is reasonable to assume that investors try to learn about the likelihood of policy revision or retraction. This is in accordance with White et al. (2013), who argue that a prognosis for future policies must be made before investments are carried out.

In our model, a risk-neutral profit maximizing investor has the option to invest in an RE project. Motivated by recently observed revisions, we examine a situation where an adverse retroactive transition between two regimes of fixed FIT is expected. The transition can be thought of as a downward adjustment of FIT received by RE producers. Furthermore, we extend the model and analyze a scenario where investors expect a retroactive transition from a regime of fixed FIT to a free-market regime, where the electricity produced must be sold on the spot or futures market at a price that varies over time.

[4] Boom and bust cycles involve periods with high investment rates followed by periods with significantly lower investment rates.

Similar to Boomsma et al. (2012), Adkins and Paxson (2016), Boomsma and Linnerud (2015) and Ritzenhofen and Spinler (2016), we consider a single subsidy revision. Our model distinguishes itself from the formerly mentioned in that the transition rate between the subsidy regimes is unknown. Through Bayesian learning, the investor updates her subjective belief about the value of the transition rate, based on the arrival of exogenous signals. This is a problem of sequential hypothesis testing, where observations are made until a hypothesis is accepted, and must not be confused with a quickest detection problem, where the goal is to detect a change in an observed process as quickly as possible. Specifically, we consider a fixed but unknown transition rate and not a changing and unknown rate.

The specification of active learning varies among researchers. In this work, we define the processing of new information and explicit updating of belief in the decision making as active learning. This terminology corresponds with Martzoukos (2003), who uses the term active learning for actions that improve the information available to investors and/or add value. In our context, observations and research of markets and framework conditions give rise to active learning. Kolstad (1996) and Kelly and Kolstad (1999) use a more narrow definition of active learning, and refer to this terminology when the rate of information arrival can in some degree be influenced. The authors use passive learning for exogenous arrival of information.

A real options approach allows us to acknowledge the characteristics of RE investments. First, investment costs are often considered project specific and therefore sunk. Second, the project value is uncertain, and depends on factors such as fluctuating electricity prices and changing subsidy schemes. Third, the investor can choose to postpone the project if the current framework conditions do not justify immediate investment. The investor has an option to invest in the project, i.e. the right, but not the obligation to invest.

This approach is further motivated by a recent study by Linnerud et al. (2014) who suggest that the investment behavior of professional investors in RE projects reflects real options investment rules, and that uncertainty associated with support schemes delays investment.

A model formulation assuming retroactive policy revisions and endogenous likelihood and timing of a revision, is an area for further analysis according to Ritzenhofen and Spinler (2016). Our model is a step in this direction. Assuming non-retroactive policy revisions may no longer be realistic (Ritzenhofen and Spinler, 2016).

The goal of this paper is to examine how learning affects investor's behavior under policy uncertainty. In standard real options models, the value of the underlying project often varies according to a stochastic variable, e.g. electricity price. In our model, the value of the underlying project is not stochastic. Therefore, the value of the option to invest varies only with a stochastic belief process, describing the investor's expectation about the lifetime of the currently high FIT scheme. The explicit opportunity to learn about the lifetime of the subsidy scheme, motivates the investor to postpone investment.

An analytical expression for the option value is found, based on the method of Frobenius. The application of this method is relatively new in the financial literature (Pinto et al., 2009).

In an extension of our model, the value of the option to invest varies in both the stochastic belief process and the stochastic electricity price. Volatility in electricity prices and the opportunity to learn about the policy uncertainty, motivates the investor to postpone investment. We derive a system of second order partial differential equations (PDE) that the option to invest must satisfy. The complexity of this system does not allow the derivation of an analytical solution, and must be solved by numerical methods. We apply the finite element method (FEM), which has been widely used in other fields of science and engineering (Andalaft-Chacur et al., 2011).

To analyze the results of our model we present a case study based on a wind power project in Europe. Sensitivity in the option value and the investment threshold is then examined for selected parameters. The optimal investment strategy is characterized by a threshold on the probabilistic belief of the high FIT scheme having a long lifespan. In the extended model, the investment decision is dependent on both this probabilistic belief and the electricity price. We find that the investment threshold increases in the arrival rate of a policy change. Hence, investors are more hesitant to invest when a subsidy scheme is expected to be short-lived compared to an RE project's lifetime. Investors who choose to invest will prefer a low FIT with a long expected lifespan, over a higher FIT with a shorter lifespan. When a termination of the FIT scheme is expected at some point in time, high volatility in electricity prices has little effect on the investment decision if the perceived policy uncertainty is low.

Our results also contribute with valuable insight to policy makers. Uncertainty related to support schemes might drive the renewable energy sector in an unintentional direction. If a transition to a free market is expected, the investor's perception of policy uncertainty significantly affects the investment rate. White et al. (2013) state that sudden, unexpected policy changes make it more challenging to attract investors.

We conclude that policy makers can have a large impact on the investment rate by a relatively small change in the FIT, when the policy uncertainty is low. The effect is significantly lower when the policy uncertainty is high, so a more generous subsidy is required to achieve the same investment rate. Active learning can greatly reduce the perceived policy uncertainty, and thereby increase the effectiveness of subsidy schemes.

According to the European Commission (2015), the energy system in Europe is underperforming, and the current market design is not able to facilitate sufficient investment.

2 Literature review

Real options theory has been applied by several authors to problems regarding uncertain market conditions and policy change in the energy sector. With this work we contribute to two strands of literature. First, we extend the traditional real options model by including exogenous arrival of information in the decision making. Second, we allow for a more realistic analysis of RE investments under policy uncertainty, by including an evolving subjective belief about the timing of a policy revision.

Fuss et al. (2008) analyze the effects of market and policy uncertainty on investment in a coal-fired electricity generation facility. The price of electricity and CO_2 are assumed to be stochastic. Policy uncertainty is incorporated by an uncertain drift rate in the price process of CO_2, and the true value is revealed at a given date. The authors find that investors will postpone their decision until the true value of the drift of CO_2 prices is revealed. The results indicate that uncertainty related to government policies affects investment decisions more than uncertainty in market prices.

Boomsma et al. (2012) examine investment timing and capacity choice under three stochastic variables; capital cost, electricity price and subsidy payment. They analyze subsidies in the form of feed-in tariffs and energy certificates. In addition, the authors look at the possibility of a shift from one support scheme to another, where the change is modeled by Markov switching. With policy risk, it is shown that the project value under the current scheme depends on the value under the alternative scheme and the transition probability. Compared to the case without policy risk, the risk exposure of energy investors increases. Numerical results show that the value of waiting is higher when policy uncertainty is present compared to the static case.

Adkins and Paxson (2016) consider a perpetual option to construct a renewable energy facility at a fixed investment cost. Both the amount of electricity sold and the price per unit of electricity follow geometric Brownian motions. The authors compare the effectiveness of different subsidy schemes. A Poisson jump process with a constant intensity factor is used to model a sudden introduction or retraction of subsidies. It is shown that the option value always is greater in the presence of a government subsidy than in its absence. They argue that a subsidy having unexpected withdrawal motivates earlier investment, compared to the case without subsidies.

Closest to our paper is the work of Boomsma and Linnerud (2015) and Ritzenhofen and Spinler (2016).

Boomsma and Linnerud (2015) examine how investors in energy projects respond to possible termination or revision of current support schemes. Policy uncertainty is modeled as a Markov process with a given jump intensity. They show that the risk of subsidy retraction will slow down the investment rate if it is retroactively applied, but otherwise increase the rate. The authors also conclude that policy uncertainty may add substantial risk to investments in the energy sector.

Ritzenhofen and Spinler (2016) consider a regime switching model, in which regulators are considering a shift from a FIT scheme to a free market regime. They apply a lattice approach to model the regime switching situation, where a pentanomial lattice is constructed to reflect the combined development of the underlying project under two different regimes. Their results suggest that policy uncertainty has little impact on investment projects when current FIT regimes are sufficiently attractive. In contrast, when FIT levels are near electricity market prices, regulatory uncertainty reduces the investment rate.

Motivated by the recent revisions of support schemes in Europe, we consider, in contrast to Ritzenhofen and Spinler (2016), a model with retroactive revision.

Neither of the aforementioned papers consider learning. Within a framework using a time homogeneous Markov process or a Poisson jump process to model policy changes, the implicit assumption is made that investors have no information regarding the dynamics governing the changing policy scheme. Our paper aims to contribute to the existing literature on policy uncertainty in the renewable energy sector by incorporating Bayesian learning in the investment decision.

Learning in the real options literature can be characterized by how information arrives and what generates the information. Learning may be continuous in the sense that at every point in time some probabilistic belief changes, or discrete in the sense that changes only occur at discrete points in time. Information can be generated by a process directly related to the underlying, such as a price or profit process. Alternatively, information arrives independent of the underlying from some exogenous source, such as news or market research.

Pawlina and Kort (2005) analyze the case where a policy change arrives when a stochastic state variable reaches a certain trigger level. The decision maker has incomplete information and knows only the probability distribution of the state variable, and not the trigger level itself. Based on the realization of the state variable and whether or not a policy change has occurred, the decision maker updates her belief about the probability of a policy change. They show that there is a non-monotonic relationship between the optimal investment threshold and the trigger level uncertainty. When the uncertainty is high, the threshold increases with the uncertainty, and the opposite is true when the uncertainty is low.

Harrison and Sunar (2015) examine investment planning in a continuous-time Bayesian framework. A firm is considering investment in a project with unknown value. However, the uncertainty about project value can be reduced by several means of learning. Information gathering in any learning mode follows a Brownian motion with exogenously given drift and incurs a given cost. The optimal learning policy is dependent on the drift and corresponding cost of a learning mode, versus the signal quality. As our model, this can also be classified as a sequential testing problem.

Jensen (1982) develops a model on the adoption behavior of a firm facing the option to invest in a new innovation. The probability of the innovation being profitable is unknown. In each time period the decision maker receives a signal indicating the profitability of the project, and the probabilistic belief is updated in a Bayesian manner. A positive signal increases both the belief in a good project and the probability of receiving a positive signal the next period.

Thijssen et al. (2004) examine a firm with the option to invest in a project of unknown profitability. The decision maker has a prior belief of the project being profitable. The belief is updated based on the arrival of signals indicating the profitability of the project. Arrival of signals follows a Poisson process and signals can be both good or bad, and accurate or inaccurate. Contrary to Jensen (1982), the quality of the signals is independent of the past. An explicit expression for the critical value of the belief process is obtained.

Our starting point is discrete arrival of signals, as in Thijssen et al. (2004). However, we use a random walk approximation to derive a stochastic differential equation (SDE) describing the investor's belief process where the arrival of signals is continuous. Shiryaev (1967) and Peskir and Shiryaev (2006) obtain an SDE which is similar to ours, when studying the problem of minimizing the cost of error when sequentially testing a hypothesis on the unknown drift rate of a one-dimensional Brownian motion. Further, the SDE of Shiryaev (1967) is the starting point for Ryan and Lippman (2003) and Kwon and Lippman (2011) in analyzing decision making under Bayesian learning. Both formulate their sequential hypothesis testing problem as an optimal stopping problem.

Kwon and Lippman (2011) study a pilot project, with an option to expand or abandon. The cumulative profit of the project follows an arithmetic Brownian motion. Based on the observed profit stream, a Bayesian framework is used to update the probability that the drift rate of the profit is either in a high or low state. The investor has to decide whether to abandon or expand the pilot project. The optimal decision policy is given by two thresholds for the posterior probability; a lower for abandonment and a higher for expansion.

Ryan and Lippman (2003) analyze optimal abandonment policy for an ongoing project whose profit stream is subject to imperfect information. Cumulative profit is modeled as an arithmetic Brownian motion, where the drift is either a positive or a negative constant. The investor has an initial probabilistic belief of the profit process having a positive drift rate. Based on observations of the realized profit, the decision maker updates the posterior probability regarding the value of the drift.

Learning related to energy has, to the best of our knowledge, only examined learning in relation to global warming over time. Examples include Kolstad (1996) and Kelly and Kolstad (1999).

Kolstad (1996) examines optimal climate-related policy, where the damage due to global warming is uncertain. The focus is to reduce this uncertainty about CO_2-related damages through learning. The world is modeled as having a finite set of states, and there is a finite set of exogenous signals that can arrive for each state. Learning evolves according to the arrival of these signals. Their numerical results show that the effect of irreversible abatement capital is stronger than the effect of irreversible environmental damages.

Kelly and Kolstad (1999) model the relationship between greenhouse gas levels and global mean temperature in a Bayesian framework. Based on observations of the climate records, agents update their belief about the state of the uncertain climate response parameter. The authors show that the rate at which agents can detect evidence of climate changes, depends on the noise in realized temperature and on the emission policy.

To the best of our knowledge, we are the first to consider Bayesian learning and policy uncertainty in green investments.

3 Model

We consider a continuous time model, where time is denoted by $t \geq 0$. A risk neutral and profit maximizing investor has the option to invest in an RE project. At some random point in time, regulators are expected to revise the current subsidy scheme. The current fixed FIT scheme offers a subsidy payment of K_0, while the subsequent scheme will offer a subsidy payment of K_1. A change is adverse to investors so that $K_0 > K_1$.

The state of the world can be in one of two states, a Good state or a Bad state, denoted by G and B respectively. In the Good state, the duration of the current subsidy scheme is expected to be long. Conversely, the lifespan of the current subsidy scheme is expected to be short in the Bad state. Thus, by using λ as notation for the arrival rate of a subsidy change, we have $\lambda_B > \lambda_G > 0$.

The true state of the world is not known to the investor *ex-ante*. At time t, the probabilistic belief of being in state G is given by the belief process $X_t \in [0, 1]$ so that $\mathbb{P}(Good) = X_t$ and $P(Bad) = 1 - X_t$. The investor has a subjective prior belief X_0. For ease of notation, we will usually drop the subscript t on X.

To improve the chances of making the best possible decision, the investor should exploit all available information about the true state of the world. Information can be drawn from numerous sources, such as news and media sources, observation of the political environment, communication with other investors and the actions of other players in the market. Each piece of information can be interpreted as a positive or negative signal that either strengthens or weakens a hypothesis that the investor has about the likelihood of policy change. To simplify, we assume that the investor can observe one aggregate stream of such signals, comprising of the different sources of information. Similarly to Harrison and Sunar (2015), we assume that the frequency of signal arrivals is sufficient to model the situation as a Brownian motion where signals arrive continuously. Following a Bayesian approach, the signals are used to update the investor's belief about the world.

When X is high, the investor expects the current subsidy scheme to be long-lived and that there is a low risk of an adverse revision. Learning is modeled as a change in X, such that the perceived policy uncertainty will vary over time.

In the following, we start by deriving and explaining the learning process in Section 3.1. In Section 3.2, we introduce how policy uncertainty is modeled. In Section 3.3, the model of a private investor updating the subjective belief of a policy change is introduced.

3.1 Derivation of the belief process

To construct the Brownian motion we follow Dixit (1993), starting with a random walk approximation and then look at the limit as the time steps become infinitesimally small.

In each time step, a positive or negative signal is received. The probability of receiving a correct signal about the true state of the world, is $p > \frac{1}{2}$.[5]

$$\begin{bmatrix} P(Pos \mid Good) & P(Neg \mid Good) \\ P(Pos \mid Bad) & P(Neg \mid Bad) \end{bmatrix} = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}.$$

Given the prior belief at time t, $P(Good) = X_t$, the process evolves according to Bayes' rule,

$$\begin{bmatrix} P(Good \mid Pos) & P(Good \mid Neg) \\ P(Bad \mid Pos) & P(Bad \mid Neg) \end{bmatrix} = \begin{bmatrix} \frac{p\,X_t}{p\,X_t+(1-p)\,(1-X_t)} & \frac{(1-p)\,X_t}{(1-p)\,X_t+p\,(1-X_t)} \\ \frac{(1-p)\,(1-X_t)}{p\,X_t+(1-p)\,(1-X_t)} & \frac{p\,(1-X_t)}{(1-p)\cdot X_t+p\,(1-X_t)} \end{bmatrix}.$$

Thus, the belief of being in the Good state after the arrival of one signal can be expressed as

$$X_{t+\Delta t} = \begin{cases} \dfrac{p\,X_t}{p\,X_t + (1-p)\,(1-X_t)} & \text{if Pos signal,} \\ \dfrac{(1-p)\,X_t}{(1-p)\,X_t + p\,(1-X_t)} & \text{if Neg signal.} \end{cases}$$

We want a convenient way to express the current value of the process in terms of the previously realized values. We therefore consider the probability ratio process defined as $Z_t = \frac{X_t}{1-X_t}$, which gives

$$Z_{t+\Delta t} = \begin{cases} \dfrac{p}{1-p} \cdot Z_t & \text{if Pos signal,} \\ \dfrac{1-p}{p} \cdot Z_t & \text{if Neg signal.} \end{cases}$$

However, it is difficult to compute the expectation and variance of the product of a sequence of random variables. So, instead we consider the logarithm of the probability ratio process, defined as $Y_t = \ln Z_t = \ln \frac{X_t}{1-X_t}$ so that

[5]If $p < \frac{1}{2}$, the same analysis can be done by replacing p with $1-p$. If $p = \frac{1}{2}$, the signals have no value, and the investor faces a now-or-never decision, by the net present value rule and using the prior belief X_0.

$$Y_{t+\Delta t} = \begin{cases} Y_t + \ln\left(\dfrac{p}{1-p}\right) & \text{if Pos signal,} \\ Y_t - \ln\left(\dfrac{p}{1-p}\right) & \text{if Neg signal.} \end{cases}$$

So far we have only considered the evolution over one time step. Using a binomial tree, we now consider the evolution over a time interval $[0, T]$ which we divide into n time steps of equal length Δt, so that $n = \frac{T}{\Delta t}$. We define time step i to be $[(i-1)\Delta t, i\Delta t]$.

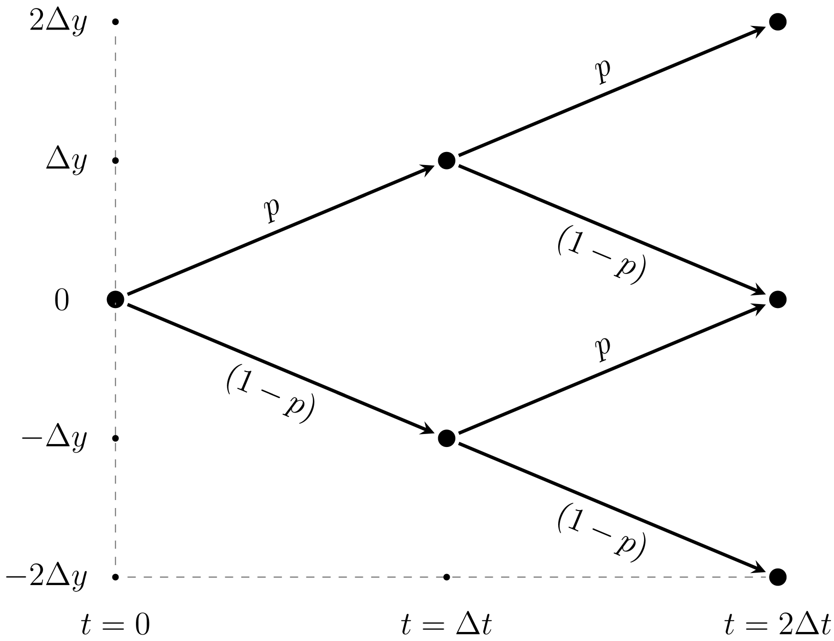


Figure 2: Binomial tree showing the possible paths of Y over two time steps.

Since the dynamics of the process are different depending on the state of the world, we treat the Good state and the Bad state separately.

At time $t = 0$, given that the state is good, the process starts evolving according to the following scheme: $Y_{t+\Delta t} = Y_t + \Delta y$ with probability p, and $Y_{t+\Delta t} = Y_t - \Delta y$ with probability $1 - p$, where $\Delta y = \ln\left(\frac{p}{1-p}\right)$. The increments of the process are independent.

Let $(Z_i)_{i=1}^n$ be a sequence of independent Bernoulli random variables such that $P(Z_i = 1) = p$ and $P(Z_i = -1) = 1 - p$.

Then for some initial value, $Y_0 = y_0$, the process can be expressed as the sum of Bernoulli random variables

$$Y_T = y_0 + \Delta y \sum_{i=1}^{n} Z_i.$$

Since the increments of the process are independent, the expectation and variance over the time horizon are given by (see Appendix A)

$$\begin{aligned}\mathbb{E}\left[Y_T - Y_0\right] &= \frac{T}{\Delta t}\Delta y\left(2p-1\right),\\ Var\left(Y_T - Y_0\right) &= \frac{T}{\Delta t}\left(\Delta y\right)^2 4p(1-p).\end{aligned}$$

While keeping the time horizon fixed at T we let n approach infinity in such a way that the process $(Y_i)_{i=1}^n$ converges to a continuous-time limit $(Y_t)_{t\in[0,T]}$, where $\mathbb{E}[Y_T] = \mu T$ and $Var(Y_T) = \sigma^2 T$. To accomplish this, we choose the parameters Δy and p such that the expectation and variance stay finite while taking the limit, that is

$$\begin{aligned}\lim_{\Delta t\to 0}\frac{T}{\Delta t}\Delta y(2p-1) &= \mu T,\\ \lim_{\Delta t\to 0}\frac{T}{\Delta t}(\Delta y)^2 4p(1-p) &= \sigma^2 T.\end{aligned}$$

For this to hold, we must have $\frac{(\Delta y)^2}{\Delta t} = \sigma^2$, which yields

$$\Delta y = \sigma\sqrt{\Delta t}, \tag{3.1}$$

$$p = \frac{e^{\sigma\sqrt{\Delta t}}}{1+e^{\sigma\sqrt{\Delta t}}}. \tag{3.2}$$

The process converges to an arithmetic Brownian motion with the desired properties, given that the step size and probability are consistent with (3.1) and (3.2) (see Appendix B). Similarly we find, by interchanging the probabilities for up and down increments, the process given the bad state, so that

$$dY = \begin{cases}\mu\, dt + \sigma\, dW & \text{Good state,}\\ -\mu\, dt + \sigma\, dW & \text{Bad state.}\end{cases}$$

where dW is the increment of a Wiener process[6]. Applying Itô's Lemma (see Appendix C), we obtain

$$dX = \begin{cases}\sigma^2 X(1-X)^2\, dt + \sigma X(1-X)\, dW & \text{Good state,}\\ -\sigma^2 X^2(1-X)\, dt + \sigma X(1-X)\, dW & \text{Bad state.}\end{cases}$$

By symmetry, the Bellman equation is the same regardless of the state of the world (see Appendix D), which allows us to only consider

$$dX = \sigma_X X(1-X)\, dW. \tag{3.3}$$

[6] Also called standard Brownian motion.

where the substitution, $\sigma = \sigma_X$, is made to avoid confusion later.

Some observations can be made from Equation (3.3), where we interpret dX as the rate of learning. It is evident that the rate of change in the posterior belief is governed by the value of $\sigma_X X(1-X)$. Firstly, the rate of learning increases as the signal strength, σ_X, increases, because each individual signal carries more information. Secondly, the term $X(1-X)$ reaches its maximum at $X = \frac{1}{2}$, which means that the rate of learning is highest when the investor has an equal belief of being in either state. Lastly, the rate of learning decreases as X moves toward its upper or lower bound. If $X = 0$ or $X = 1$, then $dX = 0$ and the process is in an absorbing state.

3.2 Policy uncertainty

Policy uncertainty involves the possibility of a change or termination of the current support scheme. These events occur at discrete points in time, and policy uncertainty is modeled as a Markov process, $(\delta_t)_{t\geq 0}$, with two regimes $\{0, 1\}$, such that

$$\delta_t = \begin{cases} 1, & \text{if a policy change has occurred in the time interval } [0,\, t), \\ 0, & \text{otherwise,} \end{cases}$$

with $\delta_0 = 0$.

Subsidies are normally intended to accelerate investments, as a step to meet production goals from renewable sources. As technology becomes more mature and cost-effective, and production goals are met, the need for high subsidy payments decreases. Reduction of subsidy payments are therefore permanent, and will not be followed by an increase to previous levels. We only consider one revision, as Boomsma and Linnerud (2015) and Ritzenhofen and Spinler (2016). If the relevant costs continue to decrease in the long-run, several revisions might however be expected.

The transition rates of the Markov process are denoted by λ_{ij}, where

$$\lambda_{ij} = \begin{cases} \lambda & \text{if } i = 0,\ j = 1, \\ 0 & \text{if } i = 1,\ j = 0, \end{cases}$$

and $\lambda \in \{\lambda_G,\, \lambda_B\}$.

3.3 Model formulation

When an investor has obtained a license to develop and operate a power plant, she owns the exclusive right to install the project within a given time frame. For analytical tractability we assume that once granted, this exclusive right will be available forever.[7]

Similar to Boomsma and Linnerud (2015) and Ritzenhofen and Spinler (2016), we assume that the lifetime of the project is finite and denoted by T, construction is instantaneous and the generating capacity is exogenous. The expected production is constant, and there is no operational flexibility or other options embedded in the facility.

Renewable electricity generation is largely dependent on weather conditions, making production highly variable in both short- and medium-term. However, according to Boomsma et al. (2012), production is less variable in the longer time scales, e.g. yearly. Less variation in production in the long-term justifies the assumption of constant expected production.

In contrast to conventional power plants, most of the costs of owning and operating RE plants are known with great certainty prior to investment (European Wind Energy Association, 2009). For wind, solar and hydropower, the operation and maintenance (O&M) costs are relatively low since the energy input is freely available. Capital costs as interest and depreciation can be predicted with a high accuracy at the time of investment, and are known for sure once the plant is built and financed. Therefore, the risk is low with regards to cost assessments in RE plants.

In the current research in this field, O&M costs are often assumed constant and included in the investment cost, as in Boomsma and Linnerud (2015), or neglected, as in Fleten et al. (2007). We assume constant operating costs and can therefore include them in the irreversible and fixed investment cost denoted by I.

Electricity markets may be considered incomplete due to lack of suitable hedging instruments for volume risk and risk of revision/retraction of the current support scheme (Boomsma et al., 2012; Boomsma and Linnerud, 2015). As a consequence, risk-neutral valuation may not be possible. We therefore assume an exogenously given real discount rate, denoted by r.

The investor is a price-taker in the relevant markets. Furthermore, we consider subsidies in the form of fixed FIT payments. However, we believe the model easily can be extended to include static FIT degression.

[7] A project where the investment decision must be made within a finite amount of time usually demand a numerical solution, as seen in e.g. Ritzenhofen and Spinler (2016).

3.3.1 Transition between two regimes of FIT

We consider two regimes, which are characterized by the subsidy scheme in place,

- Regime 0: A change in FIT payment has not yet occurred, the project value is denoted by $V_0(X)$, the option to invest is denoted by $F_0(X)$ and the instantaneous revenue is denoted K_0,

- Regime 1: A change in FIT payment has occurred, the project value is denoted by V_1, the option to invest is denoted by F_1 and the instantaneous revenue is denoted K_1, where $K_1 < K_0$.

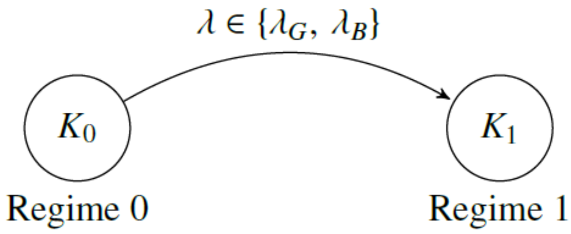


Figure 3: Illustration of the transition between the two regimes. Under regime 0 the FIT payment is K_0 and under regime 1 the FIT payment is K_1. The transition rate, $\lambda \in \{\lambda_G, \lambda_B\}$, is unknown to the investor.

With retroactive revision of the subsidy scheme and starting in regime 0, for a given λ, the project value, calculated as revenue per MWh of electricity produced, is given by

$$\begin{aligned} V_0 &= \mathbb{E}\left[\int_0^T K_0 1_{\{\delta_t=0\}} e^{-rt}\,dt \,\middle|\, \delta_0 = 0\right] + \mathbb{E}\left[\int_0^T K_1 1_{\{\delta_t=1\}} e^{-rt}\,dt \,\middle|\, \delta_0 = 0\right] \\ &= K_0 \int_0^T e^{-rt} P(\delta_t = 0)\,dt + K_1 \int_0^T e^{-rt} P(\delta_t = 1)\,dt \\ &= K_0 \int_0^T e^{-rt}\left(1 - 1 + e^{-\lambda t}\right)\,dt + K_1 \int_0^T e^{-rt}\left(1 - e^{-\lambda t}\right)\,dt \\ &= K_0 \frac{1 - e^{-(r+\lambda)T}}{r+\lambda} + K_1 \left[\frac{1 - e^{-rT}}{r} - \frac{1 - e^{-(r+\lambda)T}}{r+\lambda}\right]. \end{aligned}$$

The third equality holds since the time to termination is exponentially distributed.

Under regime 1, the project value is given by

$$\begin{aligned} V_1 &= \mathbb{E}\left[\int_0^T K_1 e^{-rt}\,dt \,\middle|\, \delta_t = 1\right] \\ &= K_1 \left[\frac{1 - e^{-rT}}{r}\right]. \end{aligned} \tag{3.4}$$

For $\delta_t = 0$, the belief of being in the Good state at time t is given by the process X_t,

$$\begin{aligned} P(Good) &= P(\lambda = \lambda_G) = X_t, \\ P(Bad) &= P(\lambda = \lambda_B) = 1 - X_t. \end{aligned}$$

Starting in regime 0, and considering the two possible transition probabilities, the expected value of the project is

$$\begin{aligned} V_0(X) = &\; X \left[K_0 \frac{1 - e^{-(r+\lambda_G)T}}{r + \lambda_G} + K_1 \left(\frac{1 - e^{-rT}}{r} - \frac{1 - e^{-(r+\lambda_G)T}}{r + \lambda_G} \right) \right] \\ &+ (1 - X) \left[K_0 \frac{1 - e^{-(r+\lambda_B)T}}{r + \lambda_B} + K_1 \left(\frac{1 - e^{-rT}}{r} - \frac{1 - e^{-(r+\lambda_B)T}}{r + \lambda_B} \right) \right]. \end{aligned} \tag{3.5}$$

We assume that $V_0(0) - I < 0$, otherwise there would be no value of waiting and the investor would invest as long as the net present value (NPV) is positive. This assumption is further motivated by the retroactive changes of the subsidy regime in 2014 in Spain, which led to a significant decrease in profitability for RE producers and a drastic slowdown in investments (CSPWorld (2014); REN21 (2015)).

At every point in time the investor has to decide whether to invest, paying the investment cost I and start accumulating profits in accordance with $V_0(X)$, or to delay investment and continue learning.

We want to find the threshold of the subjective belief at which it is optimal to invest, X^*, i.e. the free-boundary of the continuation region. The free-boundary separates the continuation region from the stopping region. In the continuation region, $X < X^*$, postponing investment and learning is more valuable than immediate investment. Therefore, the option value is higher than the expected payoff from immediate investment and the optimal decision is to postpone. In the stopping region, $X \geq X^*$, the expected gain from immediate investment is greater than or equal to the option value, and the optimal decision is to invest.

Starting in regime 0, the value of the option to invest must satisfy the Bellman equation, which represents the two choices available to the investor

$$F_0(X) = max \left\{ V_0(X) - I, \; \frac{\mathbb{E}\left[1 - \lambda\, dt\right]}{1 + r\, dt} \mathbb{E}\left[F_0(X + dX)\right] + \frac{\mathbb{E}\left[\lambda\, dt\right]}{1 + r\, dt} \mathbb{E}\left[F_1\right] \right\}.$$

In the continuation region, the following must hold

$$(1 + r\, dt) F_0 = \mathbb{E}\left[1 - \lambda\, dt\right] \mathbb{E}\left[F_0 + dF_0\right] + \mathbb{E}\left[\lambda\, dt\right] \mathbb{E}\left[F_1\right].$$

where

$$F_1 = max\,\{V_1 - I,\, 0\} = 0,$$

and

$$\mathbb{E}\,[\lambda] = X\lambda_G + (1-X)\lambda_B.$$

The probability of a change in subsidy payment during a short time interval dt is $\mathbb{E}\,[\lambda\, dt]$, and the probability that a change will not occur is $\mathbb{E}\,[1-\lambda\, dt]$. In regime 1, the revenue is a fixed tariff of K_1 for the remaining lifetime of the facility. The fixed tariff makes the option to postpone investment worthless, since there is no uncertainty. In addition, the net present value is assumed to be negative, therefore the value of the option to invest in regime 1 is zero.

Applying Itô's lemma and rearranging terms, we obtain the following second order linear ODE (see Appendix D), which holds when continuation is optimal

$$\frac{1}{2}\sigma_X^2 X^2(1-X)^2\frac{\partial^2 F_0}{\partial X^2} - \Big(X\lambda_G + (1-X)\lambda_B + r\Big)F_0 = 0, \tag{3.6}$$

Equation (3.6) does not have a closed form solution. The differential equation is singular at $X = 0$ and $X = 1$, thus no solution exists for these values of X. We are, however, only interested in a solution on the interval $X \in (0,1)$, since $X = 0$ and $X = 1$ are absorbing and not reachable from any other state.

We find an analytical solution in the form of a power series (see Appendix E)

$$F_0(X) = A_1 X^c \sum_{n=0}^{\infty} a_n(c)\, X^n,$$

where

$$c = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(\lambda_B + r)}{\sigma_X^2}},$$

and

$$a_0(c) = 1,$$

$$a_1(c) = \frac{\sigma^2 c(c-1) - \lambda_B + \lambda_G}{\frac{1}{2}\sigma^2 c(c+1) - \lambda_B - r},$$

$$a_n(c) = \frac{2\big[\sigma_X^2(n+c-1)(n+c-2) - \lambda_B + \lambda_G\big]a_{n-1}(c) - \big[\sigma_X^2(n+c-2)(n+c-3)\big]\,a_{n-2}(c)}{\sigma_X^2(n+c)(n+c-1) - 2(\lambda_B + r)}, \qquad n \geq 2.$$

Following Dixit and Pindyck (1994), the option to invest, $F_0(x)$, must satisfy the value-matching and smooth pasting conditions given by Equation (3.7) and (3.8), respectively. These equations allow us to solve for the investment threshold X^* and the coefficient A_1.

Since the expression for $F_0(X)$ is not on a closed-form, some numerical technique must be used.

$$F_0(X^*) = V_0(X^*) - I \tag{3.7}$$

$$\left.\frac{\partial F_0}{\partial X}\right|_{X=X^*} = \left.\frac{\partial V_0}{\partial X}\right|_{X=X^*} \tag{3.8}$$

Pinto et al. (2009) also encounters a differential equation with no closed form solution, and they use a mixed analytical/numerical solution process based on the method of Frobenius. As us, they solve the value-matching and smooth pasting conditions numerically.

4 Numerical results

In this section we obtain the investment threshold and the option value numerically for a case study based on a wind power project. We also examine sensitivity of the investment threshold and option value to changes in selected parameters.

4.1 Case study

Our case study focuses on an investment in a single onshore wind turbine. Wind is globally the most important source of renewable energy for electricity generation, and onshore wind represents the largest fraction (REN21, 2015). Although we focus on a single wind turbine, the results extend to for instance an investment in a wind park containing several turbines or an investment in solar power. The parameter values used in our calculations below are summarized in Table 1.

The parameters are based on a typical 2 MW wind turbine installed in Europe (European Wind Energy Association, 2009). The investment cost and the project life of the wind power turbine are set to $I = 3\,320\,000$ EUR and $T = 20$ years. The investment cost include upfront costs and operations and maintenance (O&M) costs, and is calculated using a risk-adjusted nominal discount rate of 7.5 %. The O&M costs are set equal to 15 EUR per MWh of generated electricity (McKenna et al., 2014). The capacity factor of an electricity generating facility is the amount of electricity generated during a year divided by the amount of electricity generated with the facility running at maximum power output in all 8 760 hours of a year. For wind turbines the typical capacity factors are in the range 20 – 35 %. We set the capacity factor to $F_{Cap} = 30$ %, which is in line with Boccard (2009). The exact capacity factor of a plant can be estimated to a high degree of accuracy by analytical tools and simulations, and will depend on e.g. wind conditions and the specific technology used.

Our model is solved using dynamic programming, which entails setting an exogenous risk-adjusted discount rate in the analysis. The risk-adjusted discount rate is calculated as a sum of the risk-free interest rate and a risk premium reflecting the risk embedded in the project. As Boomsma and Linnerud (2015), we set the risk adjusted real discount rate equal to 5 %. The real discount rate corresponds to a nominal rate of 7.5 % and an inflation rate of 2.5 %. Since I is constant over time, we implicitly assume that the investment cost will grow with the inflation rate.

The FIT is $K_0 = 65$ EUR/MWh and $K_1 = 30$ EUR/MWh, for regime 0 and regime 1, respectively. The FIT under regime 0 is in line with the rates in Spain and Germany as reported by the European Wind Energy Association (2009). The FIT under regime 1 corresponds to the average day-ahead price of electricity for the period April 2012 to

April 2016, based on weekly data from the Nordic electricity exchange Nord Pool[8].

The transition rates are set to $\lambda_G = 0.05$ and $\lambda_B = 0.2$, implying an expected regime change in 20 years and 5 years, respectively. Hence, in the Good state, the investor expects to receive subsidy payments throughout the project lifetime.

The signal strength of the belief process is set to $\sigma_X = 0.3$. The signal strength affects the rate of learning and the value of the option.

Table 1: Parameter values in base case.

Parameter	Value	Unit	Description
I	3 320 000	EUR	Investment cost
K_0	65	EUR/MWh	FIT in regime 0
K_1	30	EUR/MWh	FIT in regime 1
T	20	Years	Lifetime of RE project
λ_G	0.05	-	Rate of revision, Good state
λ_B	0.2	-	Rate of revision, Bad state
σ_X	0.3	-	Signal strength, belief process
r	0.05	-	Real discount rate
Capacity	2	MW	Capacity of power plant
F_{Cap}	0.3	-	Capacity factor

4.2 Results

Based on the values in the presented case study (see Table 1), the investment threshold and option value are calculated numerically.[9]

We obtain an investment threshold of $X^* = 0.799$. Hence, the investor must have a strong belief in the subsidies of regime 0 being long-lived before she is willing to invest. We show in Figure 4 how the value of the option and the NPV varies with X. The investment threshold, X^*, lies at the tangency point of the option value and the NPV. In a now-or-never scenario, the investor will invest if X is greater than or equal to 0.693. For lower values of X, the project will be rejected even though it might turn out to be profitable at a later point in time.

The difference between the NPV of investing at the optimal threshold and investing suboptimally, is called the value of waiting (Dixit and Pindyck, 1994).

We show that the NPV rule can be very misleading and that the value of waiting can be substantial up to the optimal threshold (see Figure 4) .

[8]http://www.nordpoolspot.com - Nord Pool is Europe's leading market for physical and financial power contracts. The day-ahead market consists of about 360 buyers and sellers of power, and is the main arena for trading. The electricity price is determined by supply and demand.

[9]All numerical results are obtained using MATLAB R2015a. $F_0(X)$ is expanded to n=1000 terms, so that the error is of order $<< 10^{-10}$.

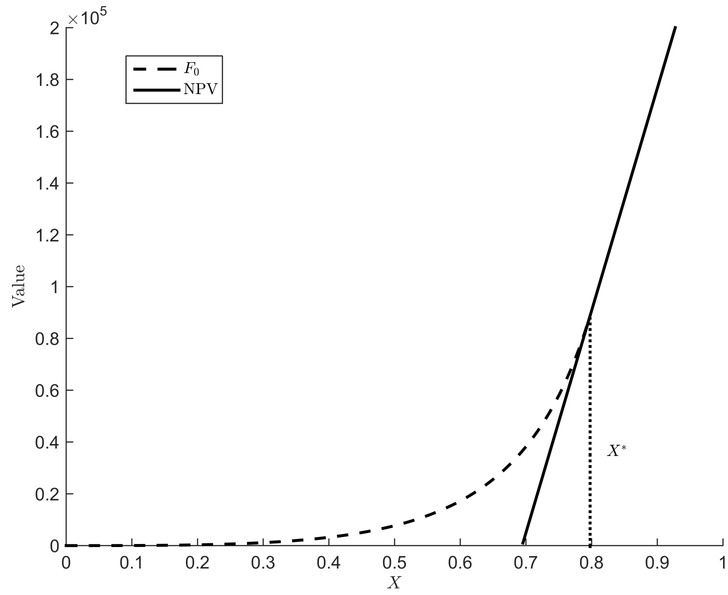


Figure 4: Option value and NPV vs belief of being in the Good state.

4.3 Sensitivity analysis

In this section we examine the sensitivity of the option value and the investment threshold to selected parameters, and discuss the implications for investors and policy makers.

4.3.1 Sensitivity in the signal strength

In this subsection we analyze how changes in the signal strength, σ_X, affects the investment threshold and the option value.

One important difference between our model and standard real option models, is that the option dynamics are governed by the evolution of the belief process and not by a process related to the value of the project. A change in σ_X does not affect the value of the project, but does affect the rate of learning. One can interpret σ_X as the amount of information received per signal. With a higher information arrival, the rate of learning increases, leading to a higher option value as illustrated in Figure 5a.

An increase in the signal strength results in a more volatile belief process, and the belief of being in the Good state can therefore change quickly. For high σ_X it is more likely that X reaches high values even when the true state of the world is Bad. The higher rate of learning, and the possibility of a quickly changing belief, leads to an increase in the investment threshold as shown in Figure 5b. When σ_X goes to zero, no information arrives and there is no value of learning. Since the investor's initial belief about the state of the world will not change, she faces a now-or-never scenario with investment according to the NPV rule.

Generally, in real option models, a higher investment threshold is associated with a lower investment rate (Dixit and Pindyck, 1994). In our model, the effects of a higher or lower investment threshold are not as straightforward. The timing of the investment decision depends on two effects, the rate of learning and the level of the investment threshold. An increase in the investment threshold may be counteracted by an increase in the rate of learning. We might therefore observe a higher investment rate at a higher investment threshold.

The optimal policy of the investor is characterized by a single threshold. The expected time to investment is infinite due to a positive probability that the belief process will never reach this threshold (Kwon and Lippman, 2011). To illustrate how the investment rate is affected by a change in the signal strength, we have run Monte Carlo simulations of the probability process. Since the expected time to investment is infinite, the results are relative, however suitable for our analysis. By discretizising X as given by Equation (3.3), we have generated 10 000 sample paths of the belief process in the base case (Table 1), with the initial belief, X_0, set to 0.4.

We find that the relative time to investment is decreasing in σ_X (see simulations in Figure 6). The increasing investment threshold is therefore offset by a higher rate of learning, and the result is a higher investment rate.

In practice, high information arrival can correspond to a transparent government, which clearly communicates the current and intended framework conditions to RE investors.

In the next sections, we will use that for constant σ_X, a lower investment threshold corresponds to a higher investment rate.

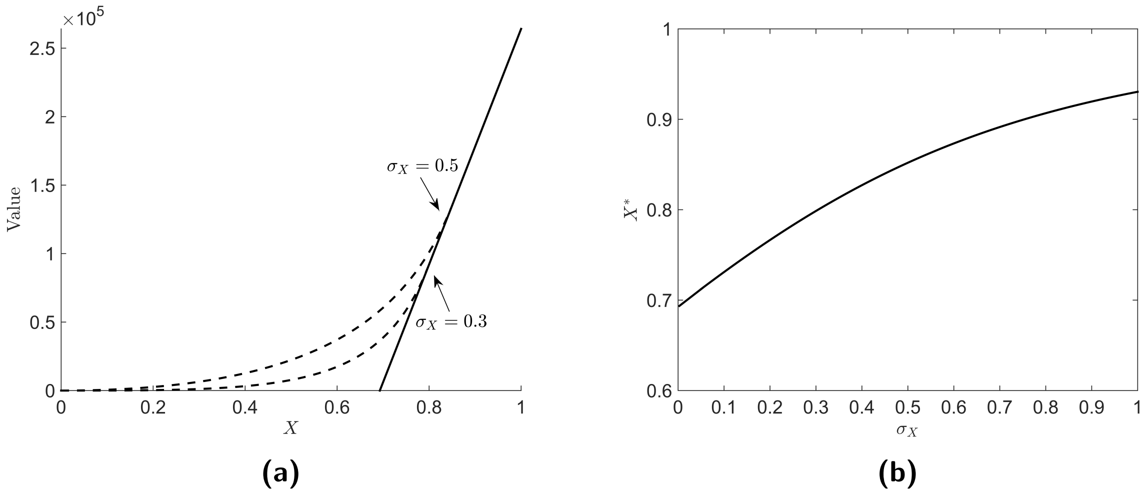


Figure 5: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the signal strength.

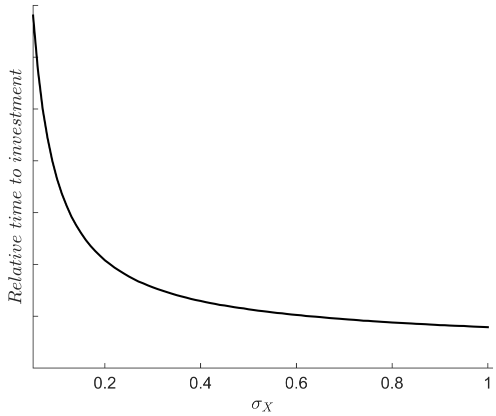


Figure 6: Relative time to investment as a function of the signal strength.

4.3.2 Sensitivity in the investment cost

In the following we will analyze the sensitivity of the investment threshold and the option value in the investment cost.

The investment threshold increases in the investment cost, see Figure 7b. This is an intuitive and standard result in real options analysis. As the expected gain from investment decreases, the investor must be more certain of the high FIT scheme being long-lasting before investing.

Similarly, the option value naturally decreases in the investment cost. However the optimal payoff is non-monotonic in the investment cost as seen in Figure 7a. For a standard option to invest the relationship is monotonically increasing (Dixit and Pindyck, 1994). When the value of the underlying project is derived from an unbounded stochastic variable, such as price, the optimal payoff can always increase to offset an increase in the investment cost. In our model, the stochastic variable is a probability measure and bounded between 0 and 1. Since the project value is static in both states of the world, the expected value can not exceed the value in the Good state. The combination of the bounded stochastic variable and the static value of the project leads to the non-monotonic relationship.

One can see from Figure 7a and 7b that the option to learn only has value for a limited range of investment costs. When the investment cost approaches the project value given the Bad state, the potential loss from investment decreases. When the potential loss is zero the investment threshold is $X^* = 0$, which means that the investor would invest immediately. The NPV in both the Good and the Bad state would be non-negative, and there would be no downside from investing. By postponing investment the investor will miss out on the higher revenues under regime 0. When the investment cost approaches the project value in the Good state, the potential upside from investing goes to zero and naturally the investment trigger goes to $X^* = 1$. Investment would never happen, since

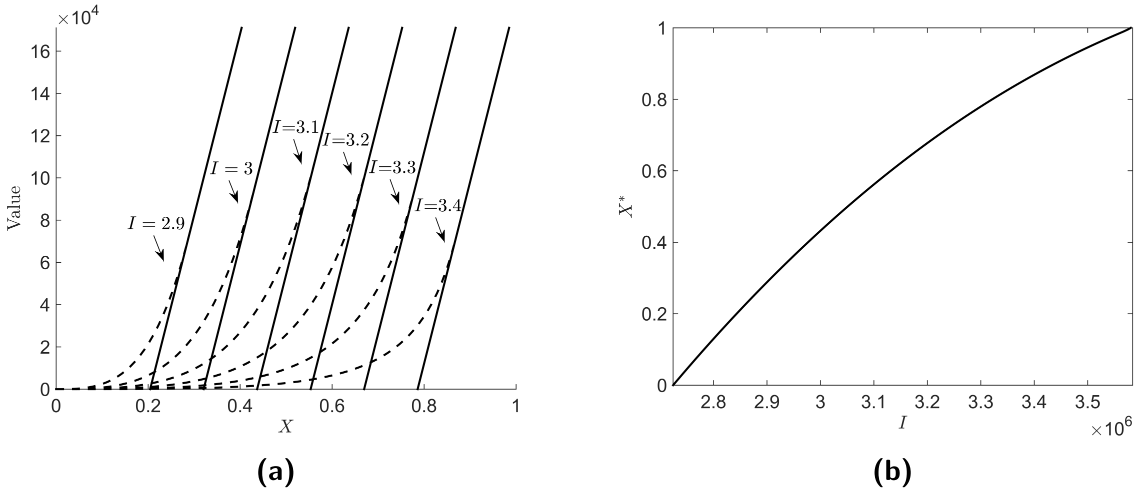


Figure 7: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the investment cost (in millions in (a)).

NPV in both states is less than or equal to zero.

From the perspective of an investor, uncertain payoff can be compensated by a reduction in investment cost. A lower total investment cost can either be achieved through lower upfront costs or lower O&M costs. The investor will therefore invest at a lower subjective belief if technology progress and/or additional subsidies reduce the investment cost. From the perspective of policy makers, the investment rate can be influenced through subsidizing the investment cost by introducing for example tax credits. In the United States, RE plants are subsidized through investment tax credits (ITC)[10] and production tax credits (PTC)[11] (US Department of Energy (2015a), US Department of Energy (2015b)). Our results indicate that reducing the total investment cost of investors by issuing ITC, lowers the investment threshold and increase the investment rate in RE plants.

4.3.3 Sensitivity in the transition rates

In this section we analyze the sensitivity of the investment threshold and the option value in the transition rates.

We start by examining the sensitivity in λ_G. There are two effects that cause the option value to decrease in λ_G (see Figure 8a). First, a higher transition rate means that a revision of the high FIT scheme is expected to arrive sooner. This effect makes it less attractive to delay the investment. Second, as λ_G increases, the expected value of the project goes to zero since the expected lifespan of regime 0 will go to zero.

If $\lambda_G = 0$, a revision of the subsidy scheme will never occur and the project would

[10]Investment tax credits allow eligible RE producers to subtract a percentage of the investment cost from the amount of tax owed to the government, indirectly reducing the investment cost.

[11]Production tax credits reduces the tax owed to the government for eligible RE producers, based on the amount of electricity produced.

receive the high FIT throughout its lifetime, given that the world is in the Good state. The difference between the NPV in the Good and the Bad state is largest, and the option value is at its maximum, all else equal. In addition, postponement has no negative effect and will eventually reveal which state the world is in. Therefore, the value of learning is at its highest.

The investment threshold is affected by two opposing effects when λ_G increases. First, the shorter expected time to a revision makes it less attractive to postpone investment. As a consequence, the investment threshold decreases. Second, the expected value of the project decreases, which causes the investment threshold to increase. The first effect is always dominated by the second effect, as illustrated by the monotonic relationship in Figure 8b.

In the following we examine the sensitivity in λ_B. As seen in Figure 9a and 10, the sensitivity in λ_B and λ_G is similar. However, a change in λ_B does not affect the value of the project in the Good state, and the value of waiting is non-monotonic in λ_B. As λ_B increases, the expected lifespan of the high FIT scheme decreases, therefore the option to postpone investment has less value. In effect, for large enough λ_B, it becomes costlier to wait instead of investing (see Figure 9b).

The investment threshold increases in λ_B, by the same reasoning as for λ_G. From equation (3.4) and (3.5), we see that the value in the Bad state approaches the value of the project under regime 1 for large λ_B. Since the potential downside has a lower bound and the NPV in the Good state is positive for all λ_B, the investment threshold is less sensitive for larger λ_B.

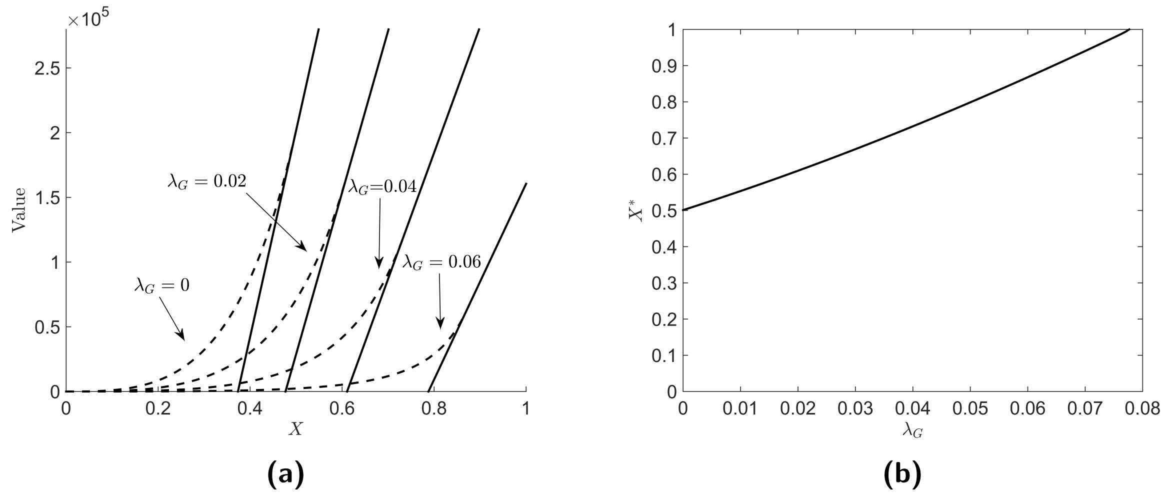
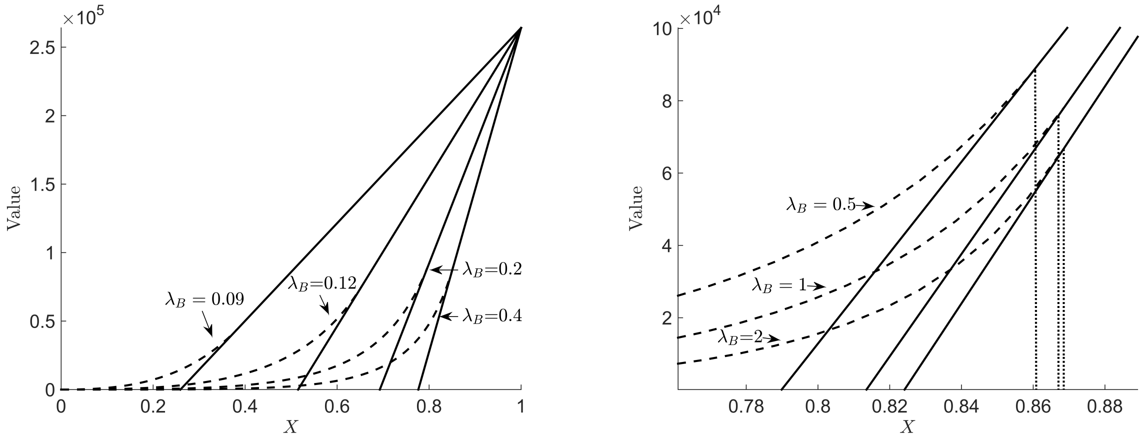


Figure 8: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the arrival rate of a policy change, given the Good state.



(a) **(b)**

Figure 9: The figures (a) and (b) show the sensitivity of the option value to the arrival rate of a policy change, given the Bad state. Plot (b) is zoomed in to illustrate the non-monotonic optimal NPV.

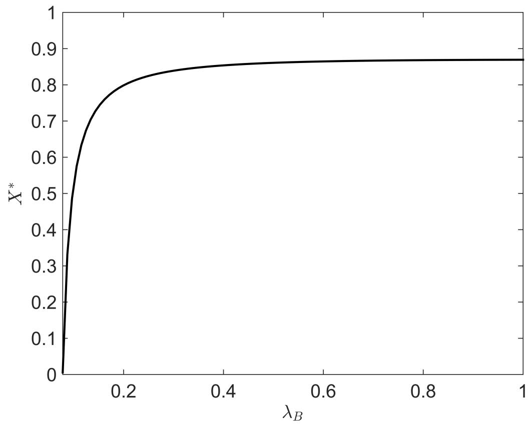


Figure 10: Sensitivity of the investment threshold to the arrival rate of a policy change, given the Bad state.

4.3.4 Sensitivity in the FIT

In this section we analyze how changes in the subsidy payments affect the option value and the investment threshold.

We start by looking at the FIT level in regime 0. As shown in Figure 11a, the option value increases in K_0. This result is intuitive, since a higher K_0 leads to a higher expected value of the project. However, the optimal payoff is non-monotonic in K_0, by the same reasoning as for I.

The investment threshold decreases in K_0 (see Figure 11b). As K_0 decreases, the NPV of the project in the Good state goes to zero, and the investor needs to be more certain of regime 0 being long-lasting before investment.

The sensitivity in investment threshold and option value and the non-monotonic optimal payoff is similar for K_1, as shown in Figure 12a and 12b. As K_1 increases, the expected value of the project increases, and naturally the option to invest becomes more valuable.

Since the expected project value in regime 1 increases in K_1, the investment decision is less dependent on the lifespan of the high FIT scheme. As a result, the investment threshold decreases in K_1.

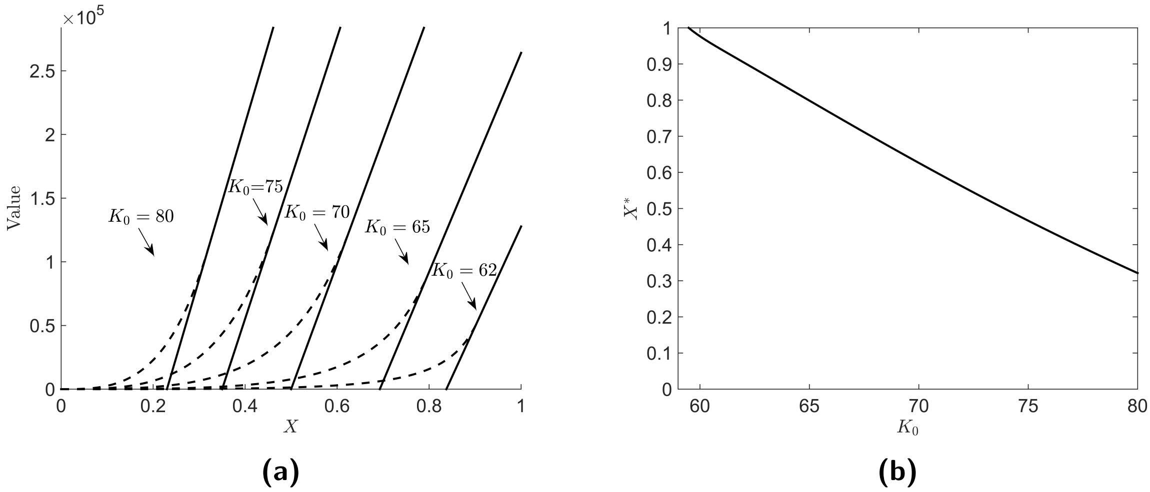


Figure 11: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the fixed feed-in tariff in regime 0.

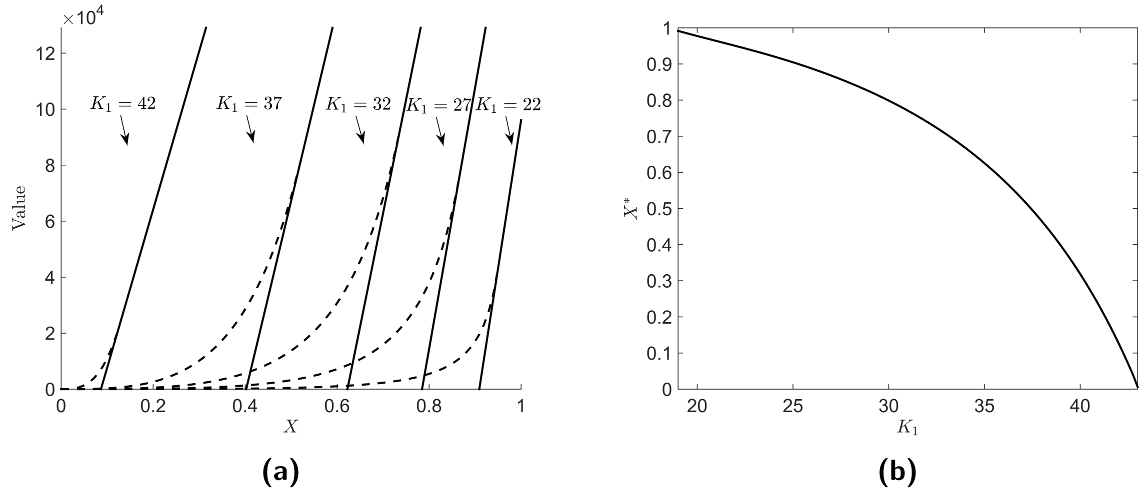


Figure 12: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the fixed feed-in tariff in regime 1.

4.3.5 Relationship between FIT and transition rate

The purpose of this subsection is to examine the relationship between the FIT payment and the transition rates, while keeping the investment rate constant. The fixed investment rate corresponds to the investment threshold for the parameters in Table 1.[12]

We show the FIT payment needed for a constant investment threshold for different transition rates in Figure 13, 14, 15a and 16a.

As previously stated, the investment trigger increases in the arrival rate and decreases in the FIT level. Thus, in order to keep the investment trigger and investment rate constant, an increase in λ must be offset by an increase in K, and vice versa. The marginal required subsidy level decreases in λ_B (see Figures 13b and 14b). This result follows from the fact that the investment trigger becomes less sensitive to changes in λ_B as λ_B increases. Similarly, a diminishing increase in K_1 for increasing λ_G is illustrated in Figure 14a.

The results indicate that a lower subsidy payment, which is expected to be sustainable in the long term, gives the same investment rate as a higher payment which is believed to be less sustainable.

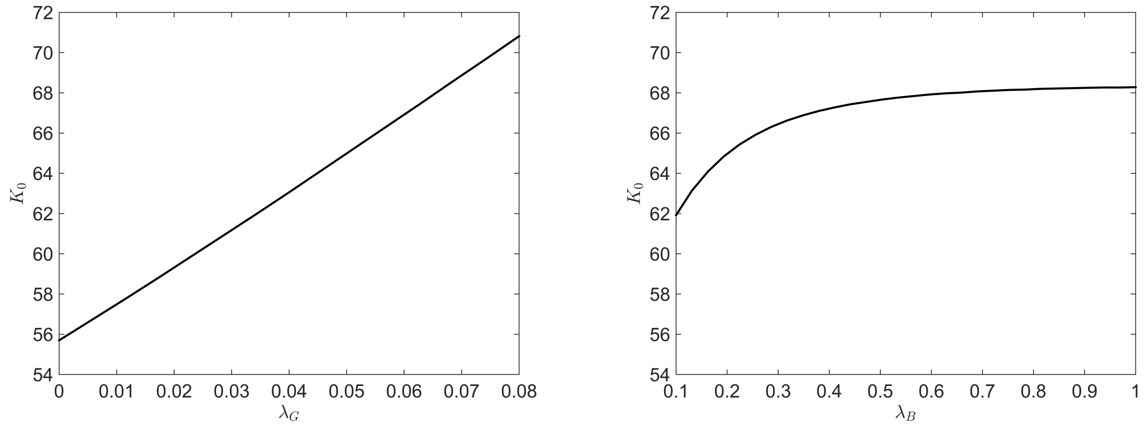
In figure 15b, we plot the expected NPV at the time of investment, $V_0(X^*) - I$, for the different combinations of K_0, λ_G and λ_B found in Figure 15a.[13] Interestingly, even though the expected NPV at investment varies greatly for the different combinations of subsidy payment and transition rates, the investment rate is the same. We find that the expected NPV is higher for a combination of lower K_0, λ_G and λ_B. This implies that an investor who chooses to invest will prefer a lower subsidy payment for a longer horizon.

The expected NPV at the time of investment is lowest for high λ_G and low λ_B (see 15b). This combination of transition rates and K_0, leads to the lowest difference between the value of the project in the two states of the world. The NPV in the Good state is positive and close to zero, and in the Bad state it is negative and close to zero. Therefore, the increase in expected NPV is small for increasing X, and the value of waiting is low. The combination of low NPVs and a low value of waiting leads to a low option value, and correspondingly a low $V_0(X^*) - I$. From the point of view of policy makers, a subsidy scheme with this combination of a high λ_G, low λ_B and corresponding K_0 gives the lowest amount of FIT paid to RE producers. If policy makers are consistent in their policies, the difference between λ_G and λ_B should be small. Hence, for policy makers it is in their best interest to be consistent.

The same conclusions can be made by examining the relationship between K_1 for a given combination of λ_G and λ_B and the corresponding expected NPV at the time of investment (see Figure 16a and 16b).

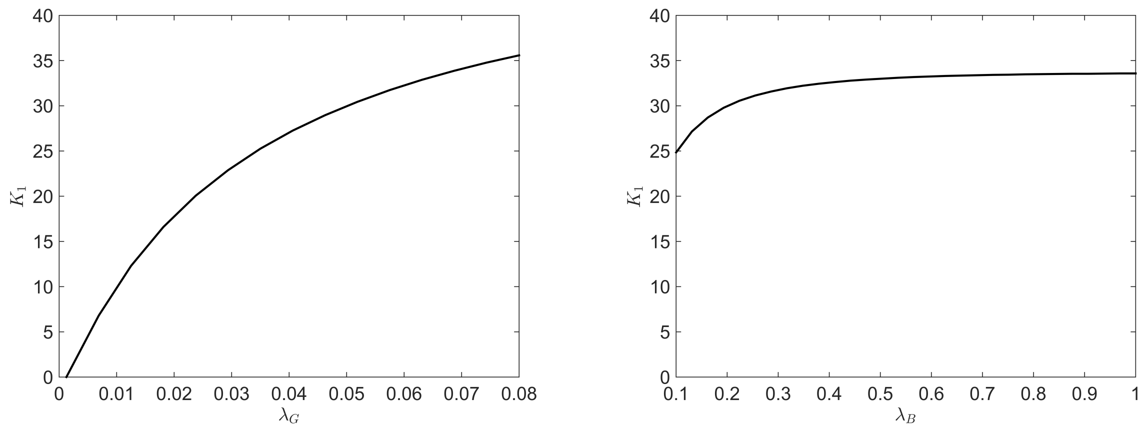
[12] $X^* = 0.799$

[13] For a given combination of λ_G and λ_B, we find the necessary K_0 for keeping the investment threshold constant. Based on the investment threshold, $X^* = 0.799$, we calculate $V_0(X^*) - I$ for this mix of K_0, λ_G and λ_B. The other parameters are given by Table 1.



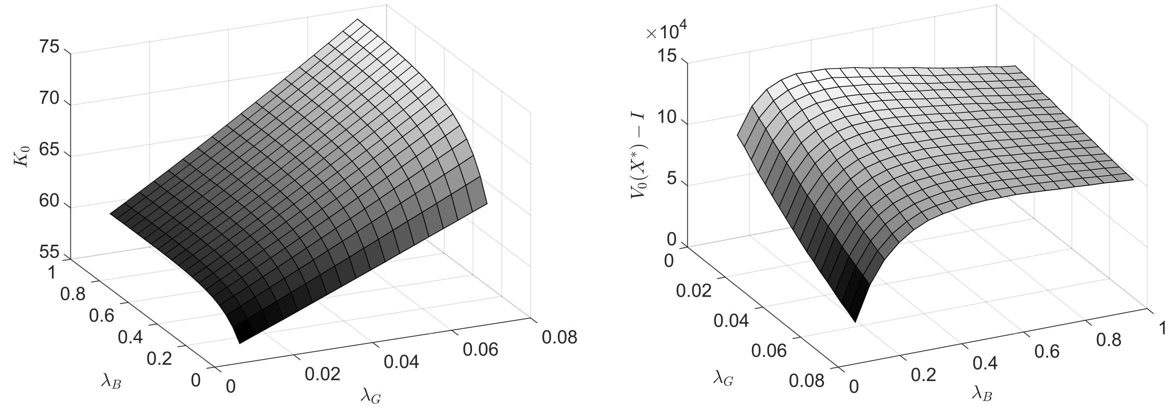
(a) **(b)**

Figure 13: The figures show the relationship between the fixed feed-in tariff in regime 0 and the arrival rate of a policy change for a constant investment rate, given (a) the Good state and (b) the Bad state.



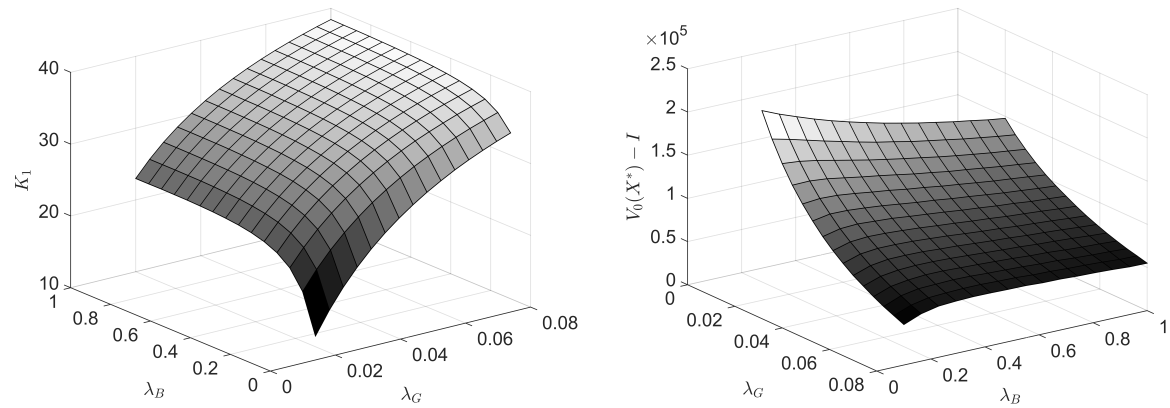
(a) **(b)**

Figure 14: The figures show the relationship between the fixed feed-in tariff in regime 1 and the arrival rate of a policy change for a constant investment rate, given (a) the Good state and (b) the Bad state.



(a) **(b)**

Figure 15: The figures show, for a constant investment rate, (a) the relationship between K_0, λ_G and λ_B and (b) the expected NPV at investment for different values of λ_G and λ_B (and implicitly K_0 as given in (a)).



(a) **(b)**

Figure 16: The figures show, for a constant investment rate, (a) the relationship between K_1, λ_G and λ_B and (b) the expected NPV at investment for different values of λ_G and λ_B (and implicitly K_1 as given in (a)).

5 Model extension

So far we have considered a policy change in the form of a retroactive downward adjustment of the FIT received by RE producers. In the following, we extend our model and examine a scenario where investors expect an adverse retroactive transition from a regime of FIT to a regime where electricity is sold in a free market. Investors are now exposed to both the policy uncertainty and fluctuating electricity prices.

5.1 Model formulation

We still take the perspective of a single RE investor, and consider two regimes

- Regime 0: the termination has not yet occurred, project value denoted by $V_0(X, S)$, option to invest denoted by $F_0(X, S)$ and instantaneous revenue denoted K,
- Regime 1: a termination of the subsidy scheme has occurred, project value denoted by $V_1(S)$, option to invest denoted by $F_1(S)$ and instantaneous revenue at time t denoted S_t.

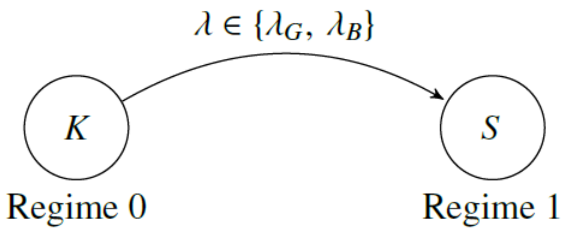


Figure 17: Illustration of the transition between the two regimes. Under regime 0, the FIT payment is K, and under regime 1 the electricity is sold on a free market at the spot price S. The transition rate, $\lambda \in \{\lambda_G, \lambda_B\}$, is unknown to the investor.

We assume that the electricity price $(S_t)_{t\geq 0}$ follows a geometric Brownian motion (GBM), such that

$$dS_t = \mu_S S_t\, dt + \sigma_S S_t\, dW_{St},$$

where μ_S and σ_S are constants that represent the drift and volatility of the electricity price, respectively, and dW_{St} is the increment of a Wiener process.[14]

While Lucia and Schwartz (2002) find that two factor models[15] provide a better fit than one factor models to the data of the Nordic electricity market, Nord Pool, Schwartz and Smith (2000) claim that the short-term variations can be neglected for long-term investments. Similarly, when considering long-term commodity related investments, Pindyck

[14]For ease of notation, we will drop the subscript t on S in the following.

[15]In two factor models of energy prices, short-term variations are often assumed to follow a mean reverting process and long-term variations are assumed to follow a GBM.

(2001) states that the assumption of energy prices following a GBM will not lead to large errors. Fleten et al. (2007) argue that an investment in an RE generation unit should be treated as a long-term investment. Correspondingly, Fleten et al. (2007) assumes that long-term electricity prices follow a GBM. Other research using a GBM to model electricity prices include Boomsma and Linnerud (2015), Boomsma et al. (2012), and Ritzenhofen and Spinler (2016).

The belief process is assumed to be independent of the electricity price, such that $\mathbb{E}\left[dW_X dW_S\right] = 0$. In addition, the policy change is independent of the electricity price.

With retroactive revision of the subsidy scheme and starting in regime 0, for a given λ, the project value, calculated as revenue per MWh of electricity produced, is given by

$$\begin{aligned} V_0(S) &= \mathbb{E}\left[\int_0^T K1_{\{\delta_t=0\}}e^{-rt}\,dt \,\middle|\, \delta_0 = 0\right] + \mathbb{E}\left[\int_0^T S_t 1_{\{\delta_t=1\}}e^{-rt}\,dt \,\middle|\, S_0 = S,\, \delta_0 = 0\right] \\ &= K\int_0^T e^{-rt}\,\mathbb{P}(\delta_t = 0)\,dt + S\int_0^T e^{-(r-\mu_S)t}\,\mathbb{P}(\delta_t = 1)\,dt \\ &= K\int_0^T e^{-rt}\left(1 - 1 + e^{-\lambda t}\right)\,dt + S\int_0^T e^{-(r-\mu_S)t}\left(1 - e^{-\lambda t}\right)\,dt \\ &= K\frac{1 - e^{-(r+\lambda)T}}{r+\lambda} + S\left(\frac{1 - e^{-(r-\mu_S)T}}{r-\mu_S} - \frac{1 - e^{-(r+\lambda-\mu_S)T}}{r+\lambda-\mu_S}\right). \end{aligned}$$

The third equality holds since the time to termination is exponentially distributed.

Starting in regime 0, and considering the two possible transition rates, the expected value of the project is equal to

$$\begin{aligned} V_0(X,S) = {} & X\left[K\frac{1 - e^{-(r+\lambda_G)T}}{r+\lambda_G} + S\left(\frac{1 - e^{-(r-\mu_S)T}}{r-\mu_S} - \frac{1 - e^{-(r+\lambda_G-\mu_S)T}}{r+\lambda_G-\mu_S}\right)\right] \\ & + (1-X)\left[K\frac{1 - e^{-(r+\lambda_B)T}}{r+\lambda_B} + S\left(\frac{1 - e^{-(r-\mu_S)T}}{r-\mu_S} - \frac{1 - e^{-(r+\lambda_B-\mu_S)T}}{r+\lambda_B-\mu_S}\right)\right]. \end{aligned}$$

Under regime 1, the project value is given by

$$\begin{aligned} V_1(S) &= \mathbb{E}\left[\int_0^T S_t e^{-rt}\,dt \,\middle|\, S_0 = S, \delta_t = 1\right] \\ &= S\left[\frac{1 - e^{-(r-\mu_S)T}}{r-\mu_S}\right]. \end{aligned}$$

The value of the option to invest in the two regimes must then satisfy the following Bellman equations

$$F_0(X,S) = max\left\{V_0(X,S) - I,\ \frac{\mathbb{E}[1-\lambda\,dt]}{1+r\,dt}\,\mathbb{E}[F_0(X+dX, S+dS)] + \frac{\mathbb{E}[\lambda\,dt]}{1+r\,dt}\,\mathbb{E}[F_1(S+dS)]\right\},$$

$$F_1(S) = max\left\{V_1(S) - I,\ \frac{1}{1 + r\,dt}\,\mathbb{E}[F_1(S + dS)]\right\}.$$

Since the electricity price is stochastic, the option to invest under regime 1 has positive value, in contrast to the model in Section 3.3.1.

Applying Itô's lemma and rearranging terms, we obtain the following system of second order partial differential equations (PDE), which holds when continuation is optimal

$$\frac{1}{2}\sigma_X^2 X^2(1-X)^2\frac{\partial^2 F_0}{\partial X^2} + \frac{1}{2}\sigma_S^2 S^2\frac{\partial^2 F_0}{\partial S^2} + \mu_S S\frac{\partial F_0}{\partial S} - \Big(X\lambda_G + (1-X)\lambda_B\Big)\Big(F_0 - F_1\Big) - rF_0 = 0, \quad (5.1)$$

$$\frac{1}{2}\sigma_S^2 S^2\frac{\partial^2 F_1}{\partial S^2} + \mu_S S\frac{\partial F_1}{\partial S} - rF_1 = 0. \quad (5.2)$$

Equation (5.2) can be solved analytically to obtain (see Appendix F for details)

$$F_1(S) = A_1 S^{\beta_1}. \quad (5.3)$$

Let $u = F_0$, $x = X$ and $s = S$, then substituting (5.3) into (5.1) we get

$$a(x)\frac{\partial^2 u}{\partial x^2} + b(s)\frac{\partial^2 u}{\partial s^2} + c(s)\frac{\partial u}{\partial s} + d(x)\,u + e(x,s) = 0, \quad (5.4)$$

with coefficients

$$\begin{aligned}
a(x) &= \frac{1}{2}\sigma_x^2 x^2(1-x)^2,\\
b(s) &= \frac{1}{2}\sigma_s^2 s^2,\\
c(s) &= \mu_s s,\\
d(x) &= (\lambda_B - \lambda_G)x - \lambda_B - r,\\
e(x,s) &= \Big((\lambda_G - \lambda_B)x + \lambda_B\Big)A_1 s^{\beta_1}.
\end{aligned}$$

The resulting PDE (Equation (5.4)) is solved using the finite element method (FEM). In order to use FEM, a bounded computational domain with appropriate boundary conditions must be defined. The variable x is naturally bounded. s however, is not bounded so we need to choose an upper bound, S^{max}, small enough to be computationally feasible, and large enough so that the free-boundary is fully contained in the domain. The boundary of the domain, $\partial\Omega$, is divided into non-overlapping segments so that $\partial\Omega = \cup_{i=1}^{6}\Gamma_i$. We impose Dirichlet type boundary conditions on all parts of the boundary (see Appendix F for details and derivation).

Next, FEM require the PDE to be expressed in its variational form. To arrive at the variational formulation we multiply Equation (5.4) with a test function $v(x,s)$, defined to

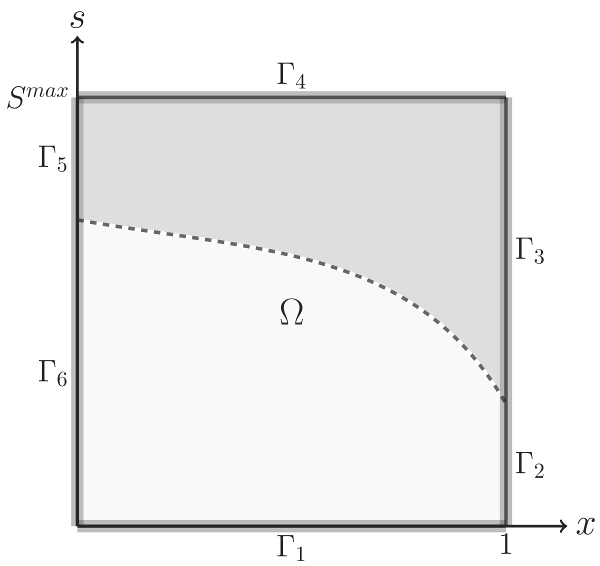


Figure 18: Domain $\Omega = (0,\,1) \times (0,\,S^{max})$. Dashed line illustrates the free-boundary/exercise boundary and separates the continuation region (light shade) and stopping region (dark shade).

be zero on the Dirichlet boundaries, and integrate over the domain.

$$\int_\Omega \left(a(x) \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} + b(s) \frac{\partial u}{\partial s} \frac{\partial v}{\partial s} \right) + \int_\Omega \frac{\partial a(x)}{\partial x} \frac{\partial u}{\partial x} v + \int_\Omega \left(\frac{\partial b(s)}{\partial s} - c(s) \right) \frac{\partial u}{\partial s} v = \int_\Omega d(x) u v + \int_\Omega e(x,s) v. \quad (5.5)$$

The PDE is solved on the domain and the free-boundary is recovered by a level-set method.

5.2 Numerical results

In this section, we solve the model presented in Section 5.1 numerically based on the case study presented in Section 4.1.[16] We also examine sensitivity in the investment threshold to the volatility in electricity prices and the FIT level.

A retroactive termination of the FIT scheme will happen at a random point in time. Following the termination, the electricity produced will be sold on a free market. The FIT is set equal to $K = 65$ EUR/MWh. For the electricity price, we set $\mu_S = 0$ and $\sigma_S = 0.06$, as Boomsma and Linnerud (2015).[17] Setting the drift term equal to zero implies that the

[16]The variational formulation (5.5) is solved by FEM using FreeFem++ (Hecht, 2012). Level-set and plots have been made using MATLAB R2015a.

[17]Boomsma and Linnerud (2015) estimate σ_S by the annual standard deviation of the log returns implied by average weekly prices of three-year forward contracts traded at NASDAQ OMX for the period 1 January 2005 to 30 April 2015.

electricity price will grow according to the inflation rate. All other values are as in Table 1.

The optimal investment threshold is characterized by both the electricity price and the investor's belief in the FIT scheme being long-lived. In effect, the threshold for undertaking investment, the free-boundary, is defined by a line that separates the continuation region from the stopping region (see Figure 19). At each point in time, the investor observes the electricity price, and must decide whether the combination of the expected lifespan of the FIT scheme and electricity price justifies investment. The first time this combination is at or above the free-boundary, the investor will choose to invest.

If the investor expects the lifespan of the FIT scheme to be short, a higher electricity price is needed before she is willing to invest. Hence, we can conclude that either a high electricity price or a high probabilistic belief of an attractive FIT scheme being long-lived, is needed in order to motivate investment.

The effect of the FIT scheme on the investment behavior is largely dependent on the perceived policy uncertainty. For high X, investors expect the lifespan of the FIT scheme to be long. Hence, a high X corresponds to a low perceived policy uncertainty.

In regime 1, where the FIT has been terminated, we find that the investor will choose to invest at an electricity price of 60 EUR/MWh (see Equation (F.10) in Appendix F). In Figure 19b we show that FITs with a low expected lifespan will accelerate investments. However, FIT schemes are most effective when the perceived risk of a revision is low.

Active learning, as modeled by an increasing X, can significantly decrease the electricity price at which it is optimal to invest.

5.3 Sensitivity analysis

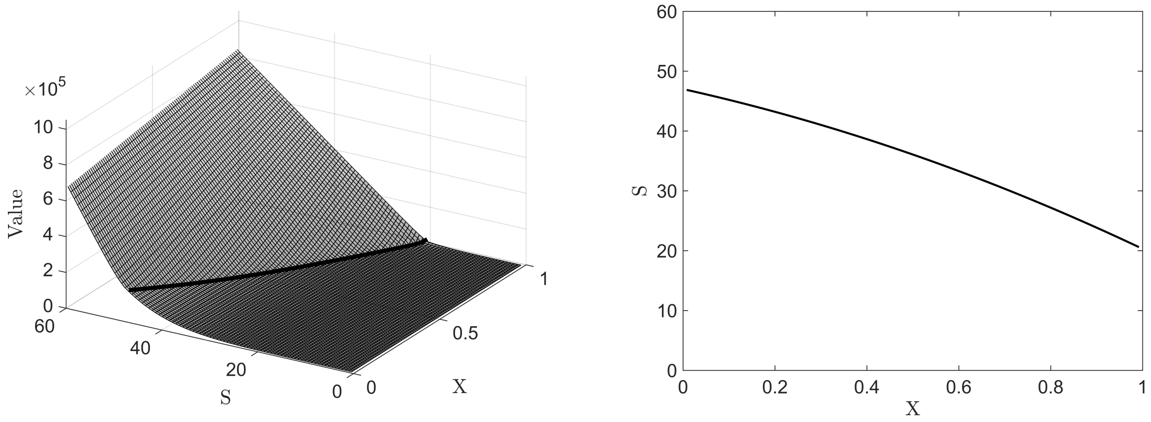
In this section we examine the sensitivity of the investment threshold to the volatility in electricity prices and the FIT, and discuss the implications for investors and policy makers.

5.3.1 Sensitivity in the volatility of electricity price

In the following we consider the sensitivity of the exercise boundary to changes in the volatility of electricity price.

For standard real option models, an increase in the volatility of an underlying price process will increase the value of the project (Dixit and Pindyck, 1994). Therefore, the value of the option to invest and the critical price at which it is optimal to invest increase. The critical price increases since the option value is more sensitive to changes in volatility than the project value. For a higher volatility the investment rate is expected to decrease, due to the higher investment threshold.

We conclude that the exercise boundary increases in the volatility of electricity prices



(a) **(b)**

Figure 19: The figures show (a) the option value as a function of X and S (free-boundary as solid line), and (b) the free-boundary that separates the continuation region from the stopping region in two dimensions. The area below the free-boundary is called the continuation region (postponing investment is optimal) and the area above the free-boundary is called the stopping region (investment is optimal). Investment is undertaken as soon as the combination of X and observed S is above the free-boundary.

(see Figure 20). For a given X, the required S at which it is optimal to invest, increases in σ_S. This effect is decreasing for larger values of X. As the investor becomes more confident in the FIT scheme being long-lived, a higher volatility in electricity prices has less effect on the investment decision.

For investors in more volatile electricity markets, the FIT scheme is less effective at accelerating RE investment when the perceived risk of a revision is high. When $X = 0$, the investor expects the revision to arrive in a relative short amount of time and the policy uncertainty is high. At this point, there is a large difference between the electricity price at which it is optimal to invest for a high and a low σ_S. Conversely, when $X = 1$, the policy uncertainty is low and and a larger σ_S has little effect on the electricity price needed for investment.

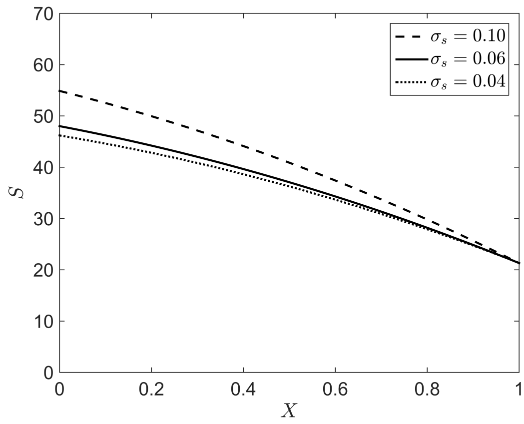


Figure 20: Sensitivity in free-boundary/investment threshold for different volatility of electricity prices.

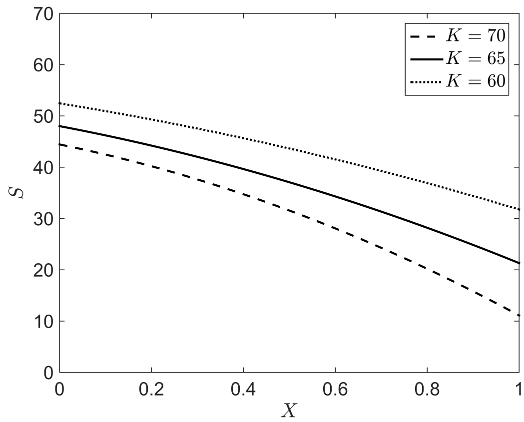


Figure 21: Sensitivity in free-boundary/investment threshold for different values of the fixed feed-in tariff.

5.3.2 Sensitivity in the FIT

In this section we examine the sensitivity of the exercise boundary to changes in the level of the fixed feed-in tariff.

We find that the exercise boundary decreases as the FIT level increases (see Figure 21). The effect is stronger when the belief in a long-lived FIT scheme increases, since the investor is increasingly eager to take advantage of the subsidies. When the perceived policy uncertainty is low (X close to 1), policy makers can have a large impact on the investment rate in RE capacity by a relatively small change in the FIT. The effect is significantly lower when the perceived policy uncertainty is high, so a more generous subsidy is required to achieve the same investment rate.

6 Conclusion

This paper extends standard real options models by including exogenous arrival of information in the decision making process through a Bayesian learning approach. We consider an investor with a perpetual option to invest in a renewable energy project. The profitability of the project is highly dependent on long-lasting government subsidies. Policy uncertainty in the form of adverse changes of a subsidy scheme have a large effect on the investment decision.

A support scheme of fixed feed-in tariff (FIT) is considered, where at some random point in time, investors expect a retroactive downward adjustment of the FIT. We extend our model and examine a situation where the subsidy scheme will be retroactively terminated and electricity must be sold on a free market where the market price is uncertain.

The arrival rate of a subsidy revision is unknown, but as time passes, the investor updates her belief of the expected lifespan of the support scheme. The aim of our paper is to examine how this learning affects investor behavior.

At every point in time, the investor must weigh the benefits from exercising the investment option, against continued observation and learning. We find that the optimal investment decision is characterized by a threshold on the subjective posterior belief of the current subsidy scheme being long-lived. In an extension of the model, the investor faces both policy uncertainty and uncertain electricity prices. The optimal investment threshold is a function of both electricity price and the subjective belief of the investor.

We find that policy uncertainty may introduce risk in the environment given by fixed FIT regimes, due to the likelihood of a revision. Our results have three important implications for the designers of FITs: i) The investment threshold increases in the arrival rate of a policy change, thereby reducing the investment rate in renewable energy plants. ii) We find that investors and policy makers prefer differing combinations of uncertainty and FITs. Investors who choose to invest will prefer a lower FIT with a long expected lifespan, while policy makers will prefer a higher FIT with shorter life span. The challenge for policy makers is to find the right mix of subsidy payment and risk that trigger the intended amount of investment. This mix should reflect the specific characteristics of a given RE project. iii) We conclude that policy makers can have a large impact on the investment rate by a relatively small change in the FIT, when the policy uncertainty is low. The effect is significantly lower when the policy uncertainty is high, so a more generous subsidy is required to achieve the same investment rate. Active learning can greatly reduce the perceived policy uncertainty, and thereby increase the effectiveness of subsidy schemes.

We can identify at least three potential directions for further research. One possibility is to examine different type subsidy schemes, e.g. feed-in premiums or green certificates, in a similar way to Boomsma and Linnerud (2015). Adding another stochastic process will however, increase the mathematical complexity of the model, which already requires advanced numerical methods for partial differential equations.

Information arrival is likely to vary. Some events might lead to a large amount of information in a short amount of time, and there might be periods of very little or no information arrival. This effect can be captured by modeling information arrival as a Poisson process or a jump-diffusion process.

Finally, it is reasonable to assume that investors do have some discretion over the magnitude of investment. Incorporating capacity choice will allow for an analysis of how policy uncertainty affects the investment rate and installed capacity at the same time.

References

Abadie, L. M. and Chamorro, J. M. (2014). Valuation of wind energy projects: A real options approach. *Energies*, 7:3218 – 3255.

Adkins, R. and Paxson, D. (2016). Subsidies for renewable energy facilities under uncertainty. *The Manchester School*, 84(2):222–250.

Andalaft-Chacur, A., Ali, M. M., and Salazar, J. G. (2011). Real options pricing by the finite element method. *Computers & Mathematics with Applications*, 61(9):2863–2873.

Boccard, N. (2009). Capacity factor of wind power realized values vs. estimates. *Energy Policy*, 37(7):2679 – 2688.

Boomsma, T. K. and Linnerud, K. (2015). Market and policy risk under different renewable electricity support schemes. *Energy*, 89:435–448.

Boomsma, T. K., Meade, N., and Fleten, S.-E. (2012). Renewable energy investments under different support schemes: a real options approach. *European Journal of Operations Research*, 220(1):225–237.

Couture, T. and Gagnon, Y. (2010). An analysis of feed-in tariff remuneration models: Implications for renewable energy investment. *Energy Policy*, 38(2):955 – 965.

CSPWorld (2014). The role of public finance in CSP. How Spain created a world-leading industry then shattered investor confidence. Retrieved May 2, 2016, from `http://www.cspworld.org/sites/default/files/csplibrary/SGG-Brief-%E2%80%93-The-Role-of-Public-Finance-in-CSP-How-Spain-created-a-world-leading-industry-then-shattered-investor-confidence.pdf`.

del Rio, P. and Mir-Artigues, P. (2012). Support for solar PV deployment in Spain: Some policy lessons. *Renewable and Sustainable Energy Reviews*, 16(8):5557–5566.

Dixit, A. K. (1993). *The art of smooth pasting*. Harwood Academic Publishers, Chur, Switzerland.

Dixit, A. K. and Pindyck, R. S. (1994). *Investment under Uncertainty*. Princeton University Press, Princeton, New Jersey.

European Commission (2015). Communication from the commission to the European parliament, the council, the European economic and social committee, the committee of the regions and the European investment bank, a framework strategy for a resilient energy union with a forward-looking climate change policy. Retrieved

May 5, 2016, from URL:http://eur-lex.europa.eu/resource.html?uri=cellar:1bd46c90-bdd4-11e4-bbe1-01aa75ed71a1.0001.03/DOC_1&format=PDF.

European Wind Energy Association (2009). The economics of wind power. Retrieved April 4, 2016, from URL:http://www.ewea.org/fileadmin/ewea_documents/documents/publications/reports/Economics_of_Wind_Main_Report_FINAL-lr.pdf.

Fleten, S.-E., Maribu, K., and Wangensteen, I. (2007). Optimal investment strategies in decentralized renewable power generation under uncertainty. *Energy,*, 32(5):803–815.

Fuss, S., Johansson, D. J., Szolgayova, J., and Obersteiner, M. (2009). Impact of climate policy uncertainty on the adoption of electricity generating technologies. *Energy Policy*, 37(2):733–743.

Fuss, S., Szolgayova, J., Obersteiner, M., and Gusti, M. (2008). Investment under market and climate policy uncertainty. *Applied Energy*, 85(8):708–721.

Harrison, J. M. and Sunar, N. (2015). Investment timing with incomplete information and multiple means of learning. *Operations Research*, 63(2):442–457.

Hecht, F. (2012). New development in freefem++. *J. Numer. Math.*, 20(3-4):251–265.

Helm, D., Hepburn, C., and Mash, R. (2003). Credible carbon policy. *Oxford Review of Economic Policy*, 19(3):438–450.

Jensen, R. A. (1982). Adoption and diffusion of an innovation of uncertain profitability. *Journal of Economic Theory*, 27(1):182–193.

Kelly, D. L. and Kolstad, C. D. (1999). Bayesian learning, growth and pollution. *Journal of Economic Dynamics and Control*, 23(4):491–518.

Klessmann, C., Rathmann, M., de Jager, D., Gazzoa, A., Resch, G., Busch, S., and Ragwitz, M. (2013). Policy options for reducing the costs of reaching the european renewables target. *Renewable Energy*, 57:390–403.

Kolstad, C. D. (1996). Learning and stock effects in environmental regulation: The case of greenhouse gas emissions. *Journal of Environmental Economics and Management*, 21(1):1–18.

Kwon, H. D. and Lippman, S. A. (2011). Acquisition of project-specific assets with Bayesian updating. *Operations Research*, 59(5):1119–1130.

Linnerud, K., Heggedal, A. M., and Fleten, S.-E. (2014). Uncertain policy decisions and investment timing: Evidence from small hydropower plants. *Energy,*, 78:154–164.

Lucia, J. J. and Schwartz, E. S. (2002). Electricity prices and power derivatives: Evidence from the Nordic power exchange. *Review of Derivatives Research*, 5(1):5–50.

Martzoukos, S. H. (2003). *Real R & D Options with Endogenous and Exogenous Learning.* In D.A. Paxson (ed.), Real R & D Options, Butterworth-Heinemann Quantitative Finance Series.

McKenna, R., Hollnaicher, S., and Fichtner, W. (2014). Cost-potential curves for onshore wind energy: A high-resolution analysis for germany. *Applied Energy*, 115:103 – 115.

Pawlina, G. and Kort, P. M. (2005). Investment under uncertainty and policy change. *Journal of Economic Dynamics & Control*, 29(7):1193–1209.

Peskir, G. and Shiryaev, A. N. (2006). *Optimal Stopping and Free-Boundary Problems.* Birkhäuser, Basel, Switzerland.

Pindyck, R. S. (2001). The dynamics of commodity spot and futures markets: A primer. *The Energy Journal*, 22(3):1–29.

Pinto, H., Howell, S., and Paxson, D. (2009). Modelling the number of customers as a birth and death process. *The European Journal of Finance*, 15(2):105 – 118.

REN21 (2015). Renewable Energy Policy Network for the 21st Century. Renewables 2015 Global Status Report. Retrieved February 10, 2016, from `http://www.ren21.net/wp-content/uploads/2015/07/REN12-GSR2015_Onlinebook_low1.pdf`.

Ritzenhofen, I. and Spinler, S. (2016). Optimal design of feed-in-tariffs to stimulate renewable energy investments under regulatory uncertainty — A real options analysis. *Energy Economics*, 53:76–89.

Ryan, R. and Lippman, S. A. (2003). Optimal exit from a project with noisy returns. *Probability in the Engineering and Informational Sciences*, 17(4):435–458.

Schwartz, E. and Smith, J. E. (2000). Short-term variations and long-term dynamics in commodity prices. *Management Science*, 46(7):893–911.

Shiryaev, A. N. (1967). Two problems of sequential analysis. *Cybernetics and Systems Analysis*, 3(2):63–69.

Statkraft (2014). Annual report. Retrieved March 5, 2016, from `http://hugin.info/133427/R/1903362/676851.pdf`.

The Institute of Energy for South East Europe (2014). Spain approves retroactive policy to replace feed-in tariff. Retrieved April 10, 2016, from `http://www.iene.eu/spain-approves-retroactive-policy-to-replace-feed-in-tariff-p692.html`.

Thijssen, J. J., Huisman, K. J., and Kort, P. M. (2004). The effect of information streams on capital budgeting decisions. *European Journal of Operational Research*, 157(3):759–774.

Trench, W. F. (2013). *Elementary Differential Equations with Boundary Value Problems*. Faculty Authored Books. Book 9.

US Department of Energy (2015a). Business energy investment tax credit. Retrieved May 10, 2016, from `http://energy.gov/savings/business-energy-investment-tax-credit-itc`.

US Department of Energy (2015b). Renewable electricity production tax credit. Retrieved May 10, 2016, from `http://energy.gov/savings/renewable-electricity-production-tax-credit-ptc`.

White, W., Lunnan, A., Nybakk, E., and Kulisic, B. (2013). The role of governments in renewable energy: The importance of policy consistency. *Biomass and Bioenergy*, 57:97–105.

Yang, M., Blyth, W., Bradley, R., Bunn, D., Clarke, C., and Wilson, T. (2008). Evaluating the power investment options with uncertainty in climate policy. *Energy Economics*, 30(4):1933–1950.

Appendix

A Expectation and variance of $Y_T - Y_0$

$$
\begin{aligned}
\mathbb{E}\left[Y_T - Y_0\right] &= \mathbb{E}\left[\Delta y \sum_{i=1}^{n} Z_i\right] \\
&= \frac{T}{\Delta t}\Delta y\, \mathbb{E}[Z] \\
&= \frac{T}{\Delta t}\Delta y \left(p - (1-p)\right) \\
&= \frac{T}{\Delta t}\Delta y \left(2p - 1\right),
\end{aligned}
$$

To find the variance, we note that since Z_i are independent random variables, their correlation is 0, and the variance of their sum is equal to the sum of their variances.

$$
\begin{aligned}
Var\left(Y_T - Y_0\right) &= Var\left(\Delta y \sum_{i=1}^{n} Z_i\right) \\
&= \left(\Delta y\right)^2 \sum_{i=1}^{n} Var\left(Z_i\right) \\
&= \left(\Delta y\right)^2 \sum_{i=1}^{n} \mathbb{E}\left[\left(Z_i\right)^2\right] - \Big(\mathbb{E}\left[Z_i\right]\Big)^2 \\
&= \left(\Delta y\right)^2 \sum_{i=1}^{n} \left(1 - \left(2p-1\right)^2\right) \\
&= \frac{T}{\Delta t}\left(\Delta y\right)^2 4p(1-p)
\end{aligned}
$$

B Derivation of dY

The expectation and the variance of Y over the time horizon are given by

$$\mathbb{E}[Y_T] = \frac{T}{\Delta t}\Delta y(2p-1), \tag{B.1}$$

$$Var(Y_T) = \frac{T}{\Delta t}(\Delta y)^2 4p(1-p). \tag{B.2}$$

While taking the limit as $\Delta t \to 0$ we want the variance (B.2) to stay finite and independent of Δt. Thus we must have

$$\frac{(\Delta y)^2}{\Delta t} = constant \qquad \Rightarrow \qquad (\Delta y)^2 = constant \cdot \Delta t$$

Setting the constant variance equal to σ^2 we get

$$\Delta y = \ln\left(\frac{p}{1-p}\right) = \sigma\sqrt{\Delta t} \quad \Rightarrow \quad p = \frac{e^{\sigma\sqrt{\Delta t}}}{1+e^{\sigma\sqrt{\Delta t}}} \tag{B.3}$$

Next, we want the mean, μ, to be independent of Δt. Substituting (B.3) into (B.1), we get

$$\frac{\sigma\sqrt{\Delta t}}{\Delta t}\left(\frac{2e^{\sigma\sqrt{\Delta t}}}{1+e^{\sigma\sqrt{\Delta t}}} - 1\right) = \frac{\sigma}{\sqrt{\Delta t}}\left(\frac{-1+e^{\sigma\sqrt{\Delta t}}}{1+e^{\sigma\sqrt{\Delta t}}}\right) = \mu$$

Now, taking the series expansion of e, we have

$$\frac{\sigma}{\sqrt{\Delta t}}\left(\frac{-1+1+\sigma\sqrt{\Delta t}+\frac{1}{2}\sigma^2\Delta t+\mathcal{O}\left((\Delta t)^{\frac{3}{2}}\right)}{1+1+\sigma\sqrt{\Delta t}+\frac{1}{2}\sigma^2\Delta t+\mathcal{O}\left((\Delta t)^{\frac{3}{2}}\right)}\right)$$
$$= \sigma^2\left(\frac{1+\frac{1}{2}\sigma\sqrt{\Delta t}+\frac{1}{6}\sigma^2\Delta t+\mathcal{O}\left((\Delta t)^{\frac{3}{2}}\right)}{2+\sigma\sqrt{\Delta t}+\frac{1}{2}\sigma^2\Delta t+\mathcal{O}\left((\Delta t)^{\frac{3}{2}}\right)}\right) = \mu$$

Finally, we take the limit as $\Delta t \to 0$, and obtain

$$\frac{\sigma^2}{2} = \mu$$

Then, in the limit,

$$dY = \mu dt + \sigma dW$$

C Derivation of dX

Consider a function $F(x,t)$ that is at least twice differentiable in x and once in t. Itô's Lemma gives the differential dF as (Dixit and Pindyck, 1994)

$$dF = \frac{\partial F}{\partial t}\,dt + \frac{\partial F}{\partial x}\,dx + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}\,(dx)^2$$

Our starting point is the arithmetic Brownian motion dY, given by

$$dY = \begin{cases} \mu\, dt + \sigma\, dW & \text{Good state} \\ -\mu\, dt + \sigma\, dW & \text{Bad state} \end{cases}$$

where $Y_t = \ln \frac{X_t}{1-X_t}$

Assuming the Good state and applying Itô's Lemma, we obtain

$$\begin{aligned} dX &= \frac{\partial X}{\partial t}\,dt + \frac{\partial X}{\partial Y}\,dY + \frac{1}{2}\frac{\partial^2 X}{\partial Y^2}\,(dY)^2 \\ &= \frac{\partial X}{\partial Y}\left[\mu\, dt + \sigma\, dW\right] + \frac{1}{2}\frac{\partial^2 X}{\partial Y^2}\left[\mu\, dt + \sigma\, dW\right]^2. \end{aligned}$$

Using that $X = \frac{e^Y}{e^Y+1}$, we get

$$\begin{aligned} dX &= \frac{e^Y}{(e^Y+1)^2}\left[\mu\, dt + \sigma\, dW\right] + \frac{1}{2}\sigma^2 \frac{e^Y(1-e^Y)}{(e^Y+1)^3}\,dt \\ &= \frac{\sigma^2}{2}\left[\frac{e^Y}{(e^Y+1)^2} + \frac{e^Y(1-e^Y)}{(e^Y+1)^3}\right] dt + \sigma \frac{e^Y}{(e^Y+1)^2}\,dW \\ &= \frac{\sigma^2}{2}\left[\frac{2e^Y}{e^Y+1}\left(1 - \frac{e^Y}{e^Y+1}\right)^2\right] dt + \sigma \frac{e^Y}{e^Y+1}\left[1 - \frac{e^Y}{e^Y+1}\right] dW \\ &= \sigma^2 X(1-X)^2\, dt + \sigma X(1-X)\, dW, \end{aligned}$$

which describes the evolution of X given the Good state.

Following the same procedure given the Bad state, we get that the process X evolves according to

$$dX = \begin{cases} \sigma^2 X(1-X)^2\, dt + \sigma X(1-X)\, dW & \text{Good state} \\ -\sigma^2 X^2(1-X)\, dt + \sigma X(1-X)\, dW & \text{Bad state} \end{cases}$$

D The Bellman equation

Starting in regime 0, the value of the option to invest must satisfy the Bellman equation

$$F_0(X) = max\Big\{ V_0(X) - I,\ \frac{\mathbb{E}[1-\lambda\, dt]}{1+r\, dt}\, \mathbb{E}\big[F_0(X+dX)\big] + \frac{\mathbb{E}[\lambda\, dt]}{1+r\, dt}\, \mathbb{E}\big[F_1\big] \Big\}.$$

In the continuation region, the following must hold

$$(1+r\, dt)F_0 = \mathbb{E}[1-\lambda\, dt]\, \mathbb{E}[F_0 + dF_0] + \mathbb{E}[\lambda\, dt]\, \mathbb{E}[F_1].$$

where

$$F_1 = max\,\{V_1 - I,\, 0\} = 0, \quad V_1 - I < 0 \quad \text{by assumption,}$$

and

$$\mathbb{E}[\lambda] = X\lambda_G + (1-X)\lambda_B.$$

Applying Itô's lemma and using that X is the probabilistic belief of being in the Good state, we get

$$\begin{aligned}(1+r\, dt)F_0 &= \Big(1 - X\lambda_G\, dt - (1-X)\lambda_B\, dt\Big)\, \mathbb{E}\Big[F_0 + X\left(\sigma^2 X(1-X)^2\, dt + \sigma X(1-X) dW\right) \tfrac{\partial F_0}{\partial X} \\ &\quad + (1-X)\left(\sigma^2 X^2 (1-X)\, dt + \sigma X(1-X) dW\right) \tfrac{\partial F_0}{\partial X} + \tfrac{1}{2}\sigma^2 X^2 (1-X)^2 \tfrac{\partial^2 F_0}{\partial X^2}\, dt\Big] \\ &= \Big(1 - X\lambda_G\, dt - (1-X)\lambda_B\, dt\Big)\Big[F_0 + \tfrac{1}{2}\sigma^2 X^2 (1-X)^2 \tfrac{\partial^2 F_0}{\partial X^2}\, dt\Big].\end{aligned}$$

We rearrange the terms, and obtain the following second order ordinary differential equation, which holds when continuation is optimal,

$$\frac{1}{2}\sigma^2 X^2 (1-X)^2 \frac{\partial^2 F_0}{\partial X^2} - \Big(X\lambda_G + (1-X)\lambda_B + r\Big)F_0 = 0.$$

Since the ODE is independent of the drift term in dX, we do not have to consider the two possible states of the world. Hence, we can reduce dX to the much simpler form

$$dX = \sigma X(1-X)\, dW,$$

regardless of the state of the world.

E Solving the ODE

We seek an analytical solution of the ODE

$$\frac{1}{2}\sigma^2 X^2(1-X)^2\frac{\partial^2 F_0}{\partial X^2} - \Big(X\lambda_G + (1-X)\lambda_B + r\Big)F_0 = 0, \tag{E.1}$$

for $X \in (0,1)$.

Assuming a solution on the form of a Frobenius series

$$F_0(X) = X^c \sum_{n=0}^{\infty} a_n(r)\, X^n. \tag{E.2}$$

We want to find the terms and coefficients of the series solution corresponding to the differential equation at hand. Differentiating (E.2) and substituting into (E.1), we get

$$\frac{1}{2}\sigma^2 X^2(1-X)^2 \sum_{n=0}^{\infty}(n+c)(n+c-1)\, a_n(r)\, x^{n+c-2} - \Big(X\lambda_G + (1-X)\lambda_B + r\Big)\sum_{n=0}^{\infty} a_n(r)\, x^{n+c} = 0.$$

Next, we examine the coefficients of different powers of X. For the first term of the series ($n = 0$), we get

$$\frac{1}{2}\sigma^2\left(1 - 2X + X^2\right)c(c-1)\, a_0\, X^c - (\lambda_B + r)\, a_0\, X^c - (\lambda_G - \lambda_B)\, a_0\, X^{c+1} = 0. \tag{E.3}$$

Equation (E.3) has two trivial solutions: $a_0 = 0 \,\vee\, X = 0$. We are however interested in finding a nontrivial solution, and must examine the three equations

$$\begin{aligned} p_0(c) &= \frac{1}{2}\sigma^2 c(c-1) - \lambda_B - r, \\ p_1(c) &= -\sigma^2 c(c-1) - \lambda_G + \lambda_B, \\ p_2(c) &= \frac{1}{2}\sigma^2 c(c-1), \end{aligned}$$

corresponding to the different powers of X.

The possible values of c are determined by $p_0(c)$, as we seek the non-trivial solution ($a_0 \neq 0$). Therefore, we get two possible values of c,

$$\begin{aligned} c_1 &= \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(\lambda_B + r)}{\sigma^2}}, \\ c_2 &= \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2(\lambda_B + r)}{\sigma^2}}. \end{aligned}$$

The Frobenius method states that the solution corresponding to c_2 does not exist if the difference between c_1 and c_2 is an integer (Theorem 7.5.3, Trench (2013)). Hence, if $\sqrt{\frac{1}{4}+\frac{2(\lambda_B+r)}{\sigma^2}}$ is an integer, only the solution corresponding to c_1 will be valid.

Assuming that the difference between c_1 and c_2 is not an integer, the general solution can be expressed as

$$F_0(X) = A_1 X^{c_1} \sum_{n=0}^{\infty} a_n(c_1)\, X^n + A_2 X^{c_2} \sum_{n=0}^{\infty} a_n(c_2)\, X^n.$$

The solution is valid and converges for $X \in (0,\, 1)$ (Trench, 2013).

The option to invest is worthless if $X = 0$, which is an absorbing state of the belief process. Therefore, $\lim_{X\to 0} F_0(X) = 0$ should hold. For $\lambda_B > 0$ and/or $r > 0$, we have $c_2 < 0$, and X^{c_2} goes to infinity as X goes to zero. This implies that we must have $A_2 = 0$.

We continue to examine the coefficients of different powers of X, in order to find the terms of the series. For the two first terms ($n = 0$ and $n = 1$), we get

$$\begin{aligned}
&\frac{1}{2}\sigma^2\left(1-2X+X^2\right)c(c-1)\,a_0\,X^c - (\lambda_B+r)a_0X^c - (\lambda_G-\lambda_B)\,a_0\,X^{c+1}\\
&\quad+\frac{1}{2}\sigma^2\left(1-2X+X^2\right)c(c+1)\,a_1\,X^{c+1} - (\lambda_B+r)\,a_0\,X^{c+1} - (\lambda_G-\lambda_B)\,a_0\,X^{c+2} = 0.
\end{aligned}$$

Collecting the coefficients of X^{c+1}, we get

$$\left(\frac{1}{2}\sigma^2 c(c+1) - \lambda_B - r\right)\, a_1 - \Bigg(\sigma^2 c(c-1) + \lambda_G - \lambda_B\Bigg)\, a_0 = 0.$$

Choosing $a_0 = 1$, gives

$$a_1(c) = \frac{\sigma^2 c(c-1) + \lambda_G - \lambda_B}{\frac{1}{2}\sigma^2 c(c+1) - \lambda_B - r} = -\frac{p_1(c)}{p_0(c+1)}.$$

For the three first terms ($n = 0$, $n = 1$ and $n = 2$), we have

$$\begin{aligned}
&\frac{1}{2}\sigma^2\left(1-2X+X^2\right)c(c-1)\,a_0\,X^c - (\lambda_B+r)\,a_0\,X^c - (\lambda_G-\lambda_B)\,a_0\,X^{c+1}\\
&\qquad+\frac{1}{2}\sigma^2\left(1-2X+X^2\right)c(c+1)\,a_1\,X^{c+1} - (\lambda_B+r)\,a_1\,X^{c+1} - (\lambda_G-\lambda_B)\,a_1\,X^{c+2}\\
&+\frac{1}{2}\sigma^2\left(1-2X+X^2\right)(c+2-1)(c+2)\,a_2\,X^{c+2} - (\lambda_B+r)\,a_2\,X^{c+2} - (\lambda_G-\lambda_B)\,a_2\,X^{c+3} = 0.
\end{aligned}$$

Collecting the coefficients of X^{c+2}, we get

$$\frac{1}{2}\sigma^2 c(c-1)\,a_0 - \left(\sigma^2 c(c+1)\,a_1 + \lambda_g - \lambda_B\right)\,a_1 + \left(\frac{1}{2}\sigma^2(c+2-1)(c+2) - \lambda_B - r\right)\,a_2 = 0.$$

Thus,

$$\begin{aligned} a_2(c) &= \frac{2\left[\sigma^2 c(c+1)a_1 + \lambda_g - \lambda_B\right]\,a_1 - \left[\sigma^2 c(c-1)\right]\,a_0}{\sigma^2(c+2-1)(c+2) - \lambda_B - r} \\ &= -\frac{p_1(c+2-1)\,a_1(c) + p_2(c+2-2)\,a_0(c)}{p_0(c+2)}. \end{aligned}$$

Examining the terms $n-2$, $n-1$ and n and collecting the coefficients of X^{c+2}, we get the general expression for the nth coefficient

$$a_n(c) = -\frac{p_1(n+c-1)\,a_{n-1}(c) + p_2(n+c-2)\,a_{n-2}(c)}{p_0(n+c)}, \qquad n \geq 2.$$

Thus, the solution of the ODE can be expressed as

$$F_0(X) = A_1 X^{c_1} \sum_{n=0}^{\infty} a_n(c_1)\,X^n,$$

where

$$c_1 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(\lambda_B + r)}{\sigma^2}},$$

and

$$\begin{aligned} a_0(c) &= 1, \\ a_1(c) &= \frac{\sigma^2 c(c-1) - \lambda_B + \lambda_G}{\frac{1}{2}\sigma^2 c(c+1) - \lambda_B - r}, \\ a_n(c) &= \frac{2\left[\sigma_X^2(n+c-1)(n+c-2) - \lambda_B + \lambda_G\right]a_{n-1}(c) - \left[\sigma_X^2(n+c-2)(n+c-3)\right]\,a_{n-2}(c)}{\sigma_X^2(n+c)(n+c-1) - 2(\lambda_B + r)}, \qquad n \geq 2. \end{aligned}$$

F Solving the system of PDEs

We want to solve the following system of PDEs

$$\frac{1}{2}\sigma_X^2 X^2(1-X)^2\frac{\partial^2 F_0}{\partial X^2}+\frac{1}{2}\sigma_S^2 S^2\frac{\partial^2 F_0}{\partial S^2}+\mu_S S\frac{\partial F_0}{\partial S}-\Big(X\lambda_G+(1-X)\lambda_B\Big)\Big(F_0-F_1\Big)-rF_0=0 \tag{F.1}$$

$$\frac{1}{2}\sigma_S^2 S^2\frac{\partial^2 F_1}{\partial S^2}+\mu_S S\frac{\partial F_1}{\partial S}-rF_1=0 \tag{F.2}$$

From the PDE in Equation (F.1), we observe that the value of the option to invest in Regime 0, F_0, depends on the option value in regime 1, F_1. Therefore, our starting point is to find an expression for F_1.

Solving Equation (F.2)

In regime 1, a revision has already occurred, and the option value depends only on the stochastic electricity price. We assume that the solution of (F.2) is on the form

$$F_1(S)=A_1S^{\beta_1}+A_2S^{\beta_2}. \tag{F.3}$$

By substitution, we see that (F.3) satisfies Equation (F.2) if $\beta_1>1$ and $\beta_2<0$ are the roots of the characteristic equation

$$Q_1(\beta)=\frac{1}{2}\sigma_S^2\beta(\beta-1)+\mu_S\beta-r.$$

Finally, $F_1(S)$ must satisfy the following boundary conditions

$$F_1(0)=0, \tag{F.4}$$

$$F_1(S^*)=S^*\left[\frac{1-e^{-(r-\mu_S)T}}{r-\mu_S}\right]-I, \tag{F.5}$$

$$\left.\frac{\partial F_1}{\partial S}\right|_{S=S^*}=\left[\frac{1-e^{-(r-\mu_S)T}}{r-\mu_S}\right]. \tag{F.6}$$

Condition (F.4) arises since $S=0$ is an absorbing state of a GBM, and the option is worthless for $S=0$. Since $S^{\beta_2}\to\infty$ when $S\to 0$, we must have $A_2=0$. Condition (F.5) is a value-matching condition and condition (F.6) is a smooth-pasting condition. Solving for A_1 and S^*, we get

$$F_1(S)=A_1S^{\beta_1}, \tag{F.7}$$

where

$$A_1 = \left(\frac{\beta_1 - 1}{I}\right)^{\beta_1 - 1} \left(\frac{e^{(\mu_S - r)T} - 1}{\beta_1(\mu_S - r)}\right)^{\beta_1}, \tag{F.8}$$

$$\beta_1 = \frac{1}{2} - \frac{\mu_S}{\sigma_S^2} + \sqrt{\left(\frac{\mu_S}{\sigma_S^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{F.9}$$

$$S^* = \frac{\beta_1}{\beta_1 - 1} I \frac{\mu_S - r}{e^{-(r - \mu_S)T} - 1}. \tag{F.10}$$

Rewriting Equation (F.1)

Next, we substitute (F.7) into (F.1) and let $u = F_0$, $x = X$ and $s = S$. Equation (F.1) can then be written more compactly as

$$a(x)\frac{\partial^2 u}{\partial x^2} + b(s)\frac{\partial^2 u}{\partial s^2} + c(s)\frac{\partial u}{\partial s} + d(x)\,u + e(x, s) = 0, \tag{F.11}$$

where

$$a(x) = \frac{1}{2}\sigma_x^2 x^2 (1 - x)^2, \qquad b(s) = \frac{1}{2}\sigma_s^2 s^2, \qquad c(s) = \mu_s s,$$

$$d(x) = (\lambda_B - \lambda_G)x - \lambda_B - r, \qquad e(x, s) = \Big((\lambda_G - \lambda_B)x + \lambda_B\Big) A_1 s^{\beta_1}.$$

Boundary conditions

On the bottom boundary, the electricity price is 0, and the option value must be 0. On the top boundary we are in the stopping region, and the option value must equal the payoff. We get

$$u(x, 0) = 0 \qquad \text{on } \Gamma_1,$$

$$u(x, s) = V_0(x, s) \qquad \text{on } \Gamma_4.$$

On the left boundary we must solve Equation (F.11) for $x = 0$ and on the right boundary for $x = 1$ (see below for derivation), which gives

$$u(0, s) = C_1 s^{\gamma_1} + A_1 s^{\beta_1} \qquad \text{on } \Gamma_6,$$

$$u(0, s) = V_0(0, s) \qquad \text{on } \Gamma_5,$$

$$u(1, s) = D_1 s^{\eta_1} + A_1 s^{\beta_1} \qquad \text{on } \Gamma_2,$$

$$u(1, s) = V_0(1, s) \qquad \text{on } \Gamma_3.$$

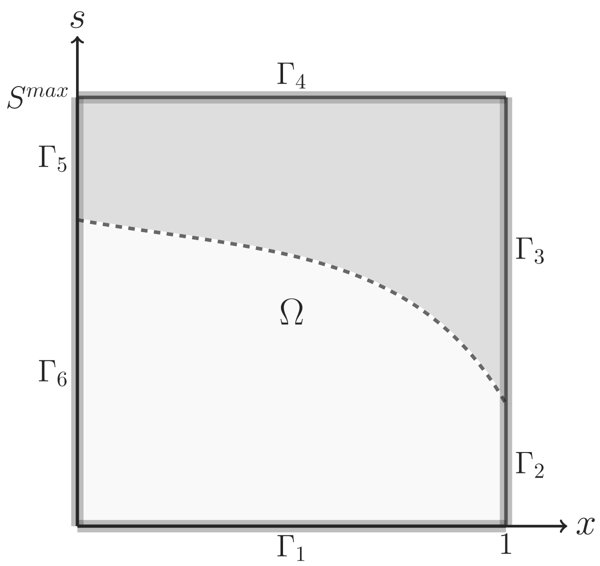


Figure 22: Domain $\Omega = (0,\, 1) \times (0,\, S^{max})$. Dashed line illustrates the free-boundary and separates the continuation region (light shade) and stopping region (dark shade).

Boundary conditions at $X = 0$

When $X = 0$, the transition rate is λ_B, and the differential equations that must be satisfied by F_0 and F_1, is reduced to

$$\frac{1}{2}\sigma_S^2 S^2 F_{0SS} + \mu_S S F_{0S} - \lambda_B(F_0 - F_1) - rF_0 = 0 \tag{F.12}$$

$$\frac{1}{2}\sigma_S^2 S^2 F_{1SS} + \mu_S S F_{1S} - rF_1 = 0 \tag{F.13}$$

The solution of (F.13) is given by equation (F.7). The solution to equation (F.12) takes the form

$$F_0(0, S) = C_1 S^{\gamma_1} + C_2 S^{\gamma_2} + A_1 S^{\beta_1}$$

where A_1 and β_1 are specified by equation (F.8) and (F.9), respectively, and $\gamma_1 > 1$ and $\gamma_2 < 0$ are the roots of the characteristic equation

$$Q_2(\gamma) = \frac{1}{2}\sigma_S^2 \gamma(\gamma - 1) + \mu_S \gamma - (r + \lambda_B)$$

Finally, $F_0(0, S)$ must satisfy the following boundary conditions

$$\begin{aligned}
F_0(0, 0) &= 0, \\
F_0(0, S^*) &= K\frac{1 - e^{-(r+\lambda_B)T}}{r + \lambda_B} + S^*\left[\frac{\left(1 - e^{-(r-\mu_S)T}\right)}{r - \mu_S} + \frac{\left(e^{-(r+\lambda_B-\mu_S)T} - 1\right)}{r + \lambda_B - \mu_S}\right] - I, \\
\left.\frac{\partial F_0}{\partial S}\right|_{S=S^*} &= \frac{1}{r - \mu_S}\left(1 - e^{-(r-\mu_S)T}\right) + \frac{1}{r + \lambda_B - \mu_S}\left(e^{-(r+\lambda_B-\mu_S)T} - 1\right),
\end{aligned}$$

Since the option is worthless for $S = 0$, we must have $C_2 = 0$. We therefore have

$$F_0(0, S) = C_1 S^{\gamma_1} + A_1 S^{\beta_1}$$

where

$$\gamma_1 = \frac{1}{2} - \frac{\mu_S}{\sigma_S^2} + \sqrt{\left(\frac{\mu_S}{\sigma_S^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda_B + r)}{\sigma_S^2}},$$

and C_1 and S^* are solved for numerically. Note that S^* defines the left endpoint of the free-boundary, separating Γ_5 from Γ_6.

Boundary conditions at $X = 1$

When $X = 1$ the transition rate is λ_G and the same system of PDEs as for the case with $X = 0$ must be solved, only with λ_G in stead of λ_G. We get

$$F_0(1, S) = D_1 S^{\eta_1} + A_1 S^{\beta_1}$$

where

$$\eta_1 = \frac{1}{2} - \frac{\mu_S}{\sigma_S^2} + \sqrt{\left(\frac{\mu_S}{\sigma_S^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda_G + r)}{\sigma_S^2}}.$$

and D_1 and S^{**} are solved for numerically from

$$F_0(1, S^{**}) = K\frac{1 - e^{-(r+\lambda_G)T}}{r + \lambda_G} + S^{**}\left[\frac{\left(1 - e^{-(r-\mu_S)T}\right)}{r - \mu_S} + \frac{\left(e^{-(r+\lambda_G-\mu_S)T} - 1\right)}{r + \lambda_G - \mu_S}\right] - I,$$

$$\left.\frac{\partial F_0}{\partial S}\right|_{S=S^{**}} = \frac{1}{r - \mu_S}\left(1 - e^{-(r-\mu_S)T}\right) + \frac{1}{r + \lambda_G - \mu_S}\left(e^{-(r+\lambda_G-\mu_S)T} - 1\right),$$

Note that S^{**} defines the right endpoint of the free-boundary, separating Γ_2 from Γ_3.

Variational formulation

FEM require the PDE to be expressed in its variational form. To arrive at the variational formulation we multiply Equation (F.11) with a test function $v(x, s) \in H_0^1(\Omega) = \{v \in H^1(\Omega) : v = 0 \text{ on } \partial\Omega\}$, to get

$$a(x)\frac{\partial^2 u}{\partial x^2}v + b(s)\frac{\partial^2 u}{\partial s^2}v + c(s)\frac{\partial u}{\partial s}v + d(x)\,u\,v + e(x, s)\,v = 0.$$

Then integrating over the domain yields

$$\int_\Omega a(x)\frac{\partial^2 u}{\partial x^2}v + \int_\Omega b(s)\frac{\partial^2 u}{\partial s^2}v + \int_\Omega \frac{\partial u}{\partial s}v + \int_\Omega d(x)\,u\,v + \int_\Omega e(x, s)\,v = 0 = 0 \qquad \text{(F.14)}$$

Applying Green's Theorem to the first integral gives

$$\int_{\Omega} a(x)\frac{\partial^2 u}{\partial x^2}v = \int_{\partial\Omega} a(x)\frac{\partial u}{\partial x}n_x v - \int_{\Omega} \frac{\partial u}{\partial x}\frac{\partial}{\partial x}\Bigg(a(x)\, v\Bigg),$$
$$= \int_{\partial\Omega} a(x)\frac{\partial u}{\partial x}n_x v - \int_{\Omega} \frac{\partial u}{\partial x}\left(\frac{\partial a(x)}{\partial x}v + a(x)\frac{\partial v}{\partial x}\right),$$
$$= -\int_{\Omega} a(x)\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} - \int_{\Omega} \frac{\partial u}{\partial x}\frac{\partial a(x)}{\partial x}v. \tag{F.15}$$

The last equality follows from v being defined to be zero on Dirichlet boundaries. Applying Green's Theorem to the second integral gives

$$\int_{\Omega} b(s)\frac{\partial^2 u}{\partial s^2}v = \int_{\partial\Omega} b(s)\frac{\partial u}{\partial s}n_s v - \int_{\Omega} \frac{\partial u}{\partial s}\frac{\partial}{\partial s}\Bigg(b(s)\, v\Bigg)$$
$$= \int_{\partial\Omega} b(s)\frac{\partial u}{\partial s}n_s v - \int_{\Omega} \frac{\partial u}{\partial s}\left(\frac{\partial b(s)}{\partial s}v + b(s)\frac{\partial v}{\partial s}\right)$$
$$= -\int_{\Omega} b(s)\frac{\partial u}{\partial s}\frac{\partial v}{\partial s} - \int_{\Omega} \frac{\partial u}{\partial s}\frac{\partial b(s)}{\partial s}v \tag{F.16}$$

Now, substitute (F.15) and (F.16) back into (F.14) to get

$$-\int_{\Omega} a(x)\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} - \int_{\Omega} \frac{\partial u}{\partial x}\frac{\partial a(x)}{\partial x}v$$
$$-\int_{\Omega} b(s)\frac{\partial u}{\partial s}\frac{\partial v}{\partial s} - \int_{\Omega} \frac{\partial u}{\partial s}\frac{\partial b(s)}{\partial s}v + \int_{\Omega} c(s)\frac{\partial u}{\partial s}v + \int_{\Omega} d(x)\, u\, v + \int_{\Omega} e(x,s)\, v = 0.$$

Rearranging, gives

$$\int_{\Omega}\left(a(x)\frac{\partial v}{\partial x}\frac{\partial u}{\partial x} + b(s)\frac{\partial v}{\partial s}\frac{\partial u}{\partial s}\right) +$$
$$\int_{\Omega} \frac{\partial a(x)}{\partial x}\frac{\partial u}{\partial x}v + \int_{\Omega}\left(\frac{\partial\, b(s)}{\partial s} - c(s)\right)\frac{\partial u}{\partial s}v = \int_{\Omega} d(x)uv + \int_{\Omega} e(x,s)v \tag{F.17}$$

The variational formulation can then be written as,

$$\begin{aligned}
&\text{Find } u \text{ such that} \\
&u = g_{\Gamma_i} \text{ on } \partial\Omega \quad \text{for } i = 1, ..., 6 \\
&\text{Equation (F.17) holds for all } v, \text{ such that } v = 0 \text{ on } \partial\Omega
\end{aligned}$$

where g_{Γ_i} is a given function on the Dirichlet boundary Γ_i.