ON THE FINANCIAL BALANCE OF INPUT–OUTPUT CONSTRUCTS: REVISITING AN AXIOMATIC EVALUATION

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Abstract

Financial balance is fundamental to input–output analysis (IO), and consequently the respect of this balance is one of the dominant criteria in evaluating IO constructs. Kop Jansen and ten Raa (1990) proved that the byproduct-technology construct (BTC) and the industry-technology construct (ITC) do not generally conserve financial balance. In contrast, Majeau-Bettez et al. (2016) demonstrated that the BTC necessarily respects financial balance and that the ITC is always financially balanced when applied to data recorded in monetary units. The present article resolves this paradox.

1 Introduction

1.1 Background

Input–output analysis (IO) is based on the survey of all economic activities within a given region and a given time period (United Nations, 1999; Miller and Blair, 1984). These activities are described in terms of their use and supply of products, and also in terms of their use of factors of production, such as labour, capital, or mineral resources (Weisz and Duchin, 2006; European Commission, 2008). In monetary surveys, the value of these factors of production is often simply aggregated as *value added* (Duchin, 2009).

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If these surveys are complete, some balances must hold. First, for accounting purposes, the different markets must balance; that is, the total use of a product must be matched by an equal supply, and all supply must be assigned to a user. Then, the different industries must be financially balanced; the value of products that they supply must equal their product costs and their value added. These balances are such central quality checks that they have become standard survey requirements (SNA 2008). They are conveniently assessed in supply and use tables (SUTs), which are the reporting standard since SNA 1968. Multi-layered SUTs, in which each flow is described simultaneously in terms of different properties, allow for the assessment of more balances, such as conservation of mass, energy, and the various chemical elements (Schmidt et al., 2010; Merciai et al., 2013; Majeau-Bettez et al., 2016; Pauliuk et al., 2015).

Because of the presence of coproduction — i.e., situations where more than one commodity is produced by the same process or industry (International Organization for Standardization, 1998) —, some modelling steps are necessary in order for SUT survey data to be used in IO or lifecycle assessment (LCA). In other words, because multifunctional activities generate more than one product, some assumptions must be introduced to distinguish the requirements of one coproduct[1] from that of another. Whereas in the LCA community this has given rise to a vast body of literature on *allocation* issues (as reviewed by Guinée, 2001), the IO community has defined these modeling steps in terms of *constructs*: models that generate symmetric system descriptions from asymmetric SUT data. Some constructs remove industries from SUT descriptions, leading to symmetric systems in which products are directly required in the production of other products, as represented in product-by-product technology matrices. Other constructs remove products from SUT descriptions, leading to systems where industries depend directly on other industries, in industry-by-industry transaction matrices (United Nations, 1999; European Commission, 2008).

Because of the fundamental character of SUT balances, the respect or violation of these balances by construct assumptions has received a lot of attention. Do the constructed symmetric representations still present *realistic markets*, or do they lead to a mismatch between supply and demand of products? Are the modelled *technical coefficients* still financially credible, or does the construct introduce a mismatch between costs of inputs and the value of the output? Kop Jansen and ten Raa (1990) assessed these two balances for the different product-by-product constructs, along with two other axiomatic requirements. Special cases were further assessed by ten Raa and Rueda-Cantuche (2003), and Rueda-Cantuche and ten Raa (2009) extended this axiomatic analysis to industry-by-industry constructs.

Kop Jansen and ten Raa (1990) found that only the commodity-technology construct (CTC) respected all four axiomatic criteria, contrary to the byproduct-technology construct (BTC) and industry-technology construct (ITC), which were notably both found to violate financial balance. The axiomatic superiority of

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[1] We use the term *coproduct* as the most general term to designate any commodity that originates from a coproduction, i.e., that is produced alongside with different commodities in the same industry (International Organization for Standardization, 1998). The literature further classifies coproducts based on their importance for the industry (primary versus secondary products), their dependence and technological connection to the primary product (e.g., byproduct, subsidiary product, etc.), and their exclusive or ordinary character (United Nations, 1999; Londero, 1999; European Commission, 2008).

CTC is now widely acknowledged in practitioner guides (United Nations, 1999; European Commission, 2008), textbooks (Raa, 2006; ten Raa, 2009), and the scientific literature (e.g., Viet, 1994; Bohlin and Widell, 2006; Mariolis and Soklis, 2010; Rueda-Cantuche and ten Raa, 2009; Lenzen and Rueda-Cantuche, 2012; Sulaiman and Fadzil, 2013; ten Raa and Rueda-Cantuche, 2013). Continued use of the ITC —despite its "economically unacceptable results" (Viet, 1994; United Nations, 1999)— is typically justified on practical rather than theoretical grounds: this construct can be directly applied to rectangular SUTs and never leads to negative coefficients, which may be difficult to interpret and distinguish from statistical discrepancies (Viet, 1994; Lenzen and Rueda-Cantuche, 2012; ten Raa and Rueda-Cantuche, 2013).

Recently, Suh et al. (2010) proved that lifecycle environmental impacts calculated with an environmentally extended input–output analysis (EEIO) are independent of whether it is based on BTC or CTC. This finding would indicate that the imbalances of BTC do not affect all scientific findings equally.

1.2 Aim and structure

The recent harmonization of IO constructs and LCA allocations (ten Raa and Rueda-Cantuche, 2007; Suh et al., 2010; Kagawa and Suh, 2009; Majeau-Bettez et al., 2014), along with the recent development of multi-layered SUT inventories in both fields (e.g., Schmidt et al., 2010; ?), has revived the question of the respect of balances in coproduction models (Weidema and Schmidt, 2010; Merciai and Heijungs, 2014). Majeau-Bettez et al. (2016) assessed which models can lead to technical recipes that simultaneously conserve financial value, mass, energy and the various chemical elements. They found that BTC necessarily does respect balances across all layers, including financial balance. Similarly, they demonstrate that ITC is necessarily financially balanced when applied to a SUT recorded in monetary units.

The proofs of Majeau-Bettez et al. (2016) are then in direct contradiction with the proofs of Kop Jansen and ten Raa (1990). The BTC cannot, on the one hand, respect balances across all physical and monetary layers and, on the other hand, be in violation of financial balance. As the two sets of proofs cannot simultaneously be valid, the formalism of both approaches should allow for the unambiguous identification of an error or an unnoticed assumption in either proof. This constitutes the objective of the present article.

In section 2, we present the construct and balance equations that define the current issue. In section 3, we revisit the counterexample used by Kop Jansen and ten Raa (1990) and the derivation of their financial balance test. Then, in section 4, we briefly present the approach of Majeau-Bettez et al. (2014). This allows for clear conclusions.

2 Problem definition

2.1 Variable and construct definitions

Whereas bold uppercase and lowercase letters respectively designate matrices and vectors, italic uppercase and lowercase letters respectively represent individual industries and products, such as industry J or product k. The symbol ($'$) indicates transposition, and the summation vector $\mathbf{e}$ is taken to be a column vector of ones with the appropriate length. The symbol ($\hat{}$) denotes diagonalization when applied to a vector, or the removal of off-diagonal elements when applied to a matrix. Conversely, the filter ($\check{}$) indicates that only off-diagonal elements are preserved in a given matrix.

Let the use and supply of products by industries be recorded in product-by-industry matrices $\mathbf{U}$ and $\mathbf{V}$, respectively. Let value added by each industry be recorded in a row vector $\mathbf{w}$. The column vector $\mathbf{q}$ records the total supply of each product ($\mathbf{q} = \mathbf{V}\mathbf{e}$), whereas the row vector $\mathbf{g}$ presents the total supply by each industry ($\mathbf{g} = \mathbf{e}'\mathbf{V}$). Prices of products are recorded in the column vector $\mathbf{p}$ and are expressed in terms of the units in which their use and supply are accounted.

From such a SUT representation, various constructs exist to obtain symmetric (product-by-product) input–output tables of technical coefficients ($\mathbf{A}$) and per-product normalized value added ($\mathbf{v}$). We briefly present the defining equations for the ITC and BTC.

With the ITC, coproducts of a given industry are assumed to have identical production technologies, that is, identical requirements and value added per unit of output. Consequently, the technical recipe for the production of each product is assumed to be the weighted average of the technologies of the industries that supply it. This construct is thus calculated with the normalized product requirements of each industry (i.e., $\mathbf{U}\hat{\mathbf{g}}^{-1}$) and the market shares of the different industries in the supply of each product (i.e., $\mathbf{V}'\hat{\mathbf{q}}^{-1}$) (Kagawa and Suh, 2009).

$$\mathbf{A}^{itc} = \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{V}'\hat{\mathbf{q}}^{-1} \tag{1}$$

Similarly, the value added per unit of each product ($\mathbf{v}$) is calculated with the normalized valued added in each industry ($\mathbf{w}\hat{\mathbf{g}}^{-1}$) averaged-out and weighted based on their market shares for each product ($\mathbf{V}'\hat{\mathbf{q}}^{-1}$) (European Commission, 2008).

$$\mathbf{v}^{itc} = \mathbf{w}\hat{\mathbf{g}}^{-1}\mathbf{V}'\hat{\mathbf{q}}^{-1} \tag{2}$$

With the BTC, it is assumed that all secondary coproducts must be produced in a fixed ratio relative to the primary product, i.e., that they are all byproducts (Miller and Blair, 1984). Secondary coproduction flows are thus subtracted from both the supply and the use tables. The ensuing product flows are then normalized relative to primary production, as represented by equation 3, which assumes that rows and columns are organized such that the supply matrix is square and the primary productions are recorded on its diagonal.

$$\mathbf{A}^{btc} = \left(\mathbf{U} - \check{\mathbf{V}}\right)\hat{\mathbf{V}}^{-1} \tag{3}$$

Similarly, the value added per unit of product is calculated by normalizing the value added of each industry relative to its primary production, as in equation 4 (Suh et al., 2010).

$$\mathbf{v}^{btc} = \mathbf{w}\hat{\mathbf{V}}^{-1} \tag{4}$$

2.2 Financial balance definition and test

If a survey accurately records all product flows and value added of each industry, the resulting SUT should respect financial balance. The monetary value of the outputs of each industry ($\mathbf{p'V}$) should equal the combination of its product costs ($\mathbf{p'U}$) and value added ($\mathbf{w}$).

$$\mathbf{p'V} = \mathbf{p'U} + \mathbf{w} \tag{5}$$

From such a balanced SUT system description, constructs model symmetric product systems that may or may not be financially balanced. A symmetric IO system is financially balanced if the value of the product requirements ($\mathbf{p'A}$) and the value added ($\mathbf{v}$) for the modelled recipe of each product add up to its price, as in equation 6 (Kop Jansen and ten Raa, 1990).

$$\mathbf{p'} = \mathbf{p'A} + \mathbf{v} \tag{6}$$

From this definition of financial balance, Kop Jansen and ten Raa (1990) then derived a *financial balance test*, as expressed by equation 7, which was then used to prove that CTC is financially balanced whereas BTC and ITC are not.

$$\mathbf{e'AV} = \mathbf{e'U} \tag{7}$$

3 Revisiting the financial balance test

3.1 A first counterexample with BTC

Kop Jansen and ten Raa (1990) originally used a counterexample, reproduced in equation 8, to prove that BTC and ITC are not financially balanced. As the vector of prices ($\mathbf{p}$) is not a vector of ones, we can deduce that this SUT is not inventoried in terms of a single monetary unit but rather in some physical unit or in mixed-units. For illustrative purposes, and without loss of generality, we added units to the original example.

$$\mathbf{U} = \begin{array}{ccc} & I & J \\ i\ [m^3] & 1/2 & 0 \\ j\ [\$] & 1 & 1/2 \end{array} \quad \mathbf{V} = \begin{array}{ccc} & I & J \\ i\ [m^3] & 1 & 0 \\ j\ [\$] & 1 & 1 \end{array} \quad \mathbf{p} = \begin{array}{cc} i[\$/m^3] & 2 \\ j[\$/\$] & 1 \end{array} \tag{8}$$

Since the SUT is given as initially balanced, the value added generated in industries I and J is calculated based on equation 5 and reported in equation 9.

$$\mathbf{w} = \begin{array}{ccc} & I & J \\ [\$] & 1 & 1/2 \end{array} \tag{9}$$

Applying the BTC to this data calculates normalized recipes (in product requirement matrix $\mathbf{A}^{btc}$) for commodities i and j, along with a normalized per-commodity value-added vector ($\mathbf{v}^{btc}$).

$$\mathbf{A}^{btc} = \begin{array}{c} \\ i\ [m^3] \\ j\ [\$] \end{array} \begin{array}{c} \begin{array}{cc} i & j \end{array} \\ \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \end{array} \tag{10}$$

$$\mathbf{v}^{btc} = \begin{array}{c} \\ [\$] \end{array} \begin{array}{c} \begin{array}{cc} i & j \end{array} \\ \begin{pmatrix} 1 & 1/2 \end{pmatrix} \end{array} \tag{11}$$

This system description indeed violates the financial balance test proposed by Kop Jansen and ten Raa (1990) (equation 7), as shown in equation 12.

$$\mathbf{e}'\mathbf{AV} = \begin{pmatrix} 1 & 1/2 \end{pmatrix} \neq \begin{pmatrix} 3/2 & 1/2 \end{pmatrix} = \mathbf{e}'\mathbf{U} \tag{12}$$

And yet, it *does* respect financial balance (equation 6), as shown in equation 13.

$$\mathbf{p}' = \begin{pmatrix} 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1/2 \end{pmatrix} + \begin{pmatrix} 1 & 1/2 \end{pmatrix} = \mathbf{p}'\mathbf{A} + \mathbf{v} \tag{13}$$

Since it respects financial balance, the system description in equations (8) to (11) cannot be used to prove the financial imbalance of BTC. On the contrary, as it respects financial balance without respecting equation 7, *this counterexample proves that equation 7 is not a valid test for assessing financial balance.*

3.2 Embedded assumptions in the financial balance test

In this section, we revisit the derivation of the financial balance test by Kop Jansen and ten Raa (1990) to understand the source of this discrepancy.

To derive equation 7 from equation 6, the latter must first be multiplied on both sides by supply matrix $\mathbf{V}$, as in equation 14.

$$\mathbf{p}'\mathbf{V} = \mathbf{p}'\mathbf{AV} + \mathbf{vV} \tag{14}$$

Kop Jansen and ten Raa (1990) must then assume that the value added within each industry ($\mathbf{w}$) can be calculated by multiplying per-commodity value added ($\mathbf{v}$) and commodity production within each industry ($\mathbf{V}$).[2] As we will later explain, this introduces a first assumption that reduces the applicability of the financial test.

$$\mathbf{w} \overset{?}{=} \mathbf{vV} \tag{15}$$

This assumption simplifies equation 14 to equation 16,

$$\mathbf{p}'\mathbf{V} = \mathbf{p}'\mathbf{AV} + \mathbf{w} \tag{16}$$

in which $\mathbf{w}$ may then be reformulated, based on SUT financial balance (equation 5), as in equation 17,

$$\mathbf{p}'\mathbf{V} = \mathbf{p}'\mathbf{AV} + \mathbf{p}'\mathbf{V} - \mathbf{p}'\mathbf{U} \tag{17}$$

which then simplified to equation 18

$$\mathbf{p}'\mathbf{A}\mathbf{V} = \mathbf{p}'\mathbf{U} \tag{18}$$

Kop Jansen and ten Raa (1990) then replace the price vector by a vector of ones ($\mathbf{e}$), which introduces the assumption that prices are *relative* prices and that the inventory is recorded in monetary units. This thus embeds a second assumption in the financial test.

$$\mathbf{p} \overset{?}{=} \mathbf{e} \tag{19}$$

Equation 19 then simplifies equation 18 to the financial balance test of equation 7.

$$\mathbf{e}'\mathbf{A}\mathbf{V} = \mathbf{e}'\mathbf{U} \tag{rep. 7}$$

The applicability of the financial balance test therefore depends on the validity of two simplifying assumptions, equations (15) and (19).

Starting with the second and simplest assumption, it is clear that the SUT example in equation 8 does not respect the assumption of relative prices. For such physical or mixed-unit inventories, equation 18 would be more appropriate as a balance test than equation 7.

Even equation 18, however, proves unreliable in predicting financial balance. The technical coefficient matrix in equation 10 fails this test, despite its financially balanced character.

$$\mathbf{p}'\mathbf{A}\mathbf{V} = \begin{pmatrix} 3/2 & 1/2 \end{pmatrix} \neq \begin{pmatrix} 2 & 1/2 \end{pmatrix} = \mathbf{p}'\mathbf{U} \tag{20}$$

This failure of the test is explained by the discrepancy introduced by the first embedded assumption (equation 15). Indeed, constructs take industry-specific requirements ($\mathbf{U}$ and $\mathbf{w}$) to produce a “general” aggregated recipe for each product, independently of their industry of origin. It is therefore in no way a given that the value-added coefficients that are constructed for each product can be used to calculate the value added within specific industries, as would be required by equation 15.

As a mental experiment, let us have a rectangular SUT in which three single-output industries (hydropower, solar, and nuclear power plants) produce the same product (electricity), with different profit margins. In going from a SUT to a product-by-product **A**-matrix, the different industries are removed from the representation and a unique recipe for electricity is devised from the three productions. As part of this recipe, an aggregated value added is calculated for this product. Clearly, this aggregated value added for electricity could not be used to directly recalculate the profits margins specific to each industry (e.g. the nuclear industry), as would be required by equation 15. In fact, equation 15 embeds a commodity-technology assumption in the financial balance test of Kop Jansen and ten Raa (1990), and it then follows that only the CTC construct could satisfy this test.

We therefore find that the financial balance test put forth by Kop Jansen and ten Raa (1990) embeds two assumptions, which cause this test to exclude constructs based on criteria other than financial balance.

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[2] Kop Jansen and ten Raa (1990), page 217, fourth equation

3.3 A second counterexample with ITC

It is clear that the ITC can lead to representations that are not financially balanced. Applying this construct directly to the mixed-unit SUT in equation 8 leads to a system that respects neither the definition of financial balance (equation 6) nor the test of Kop Jansen and ten Raa (whether equation 7 or equation 18).

Through unit conversion, however, this SUT can be fully expressed in monetary units (equations (21) and (22)),

$$\mathbf{U} = \begin{array}{ccc} & I & J \\ i\ [\$] & 1 & 0 \\ j\ [\$] & 1 & 1/2 \end{array} \qquad \mathbf{V} = \begin{array}{ccc} & I & J \\ i\ [\$] & 2 & 0 \\ j\ [\$] & 1 & 1 \end{array} \qquad \mathbf{p} = \begin{array}{cc} i\ [\$/\$] & 1 \\ j\ [\$/\$] & 1 \end{array} \tag{21}$$

$$\mathbf{w} = \begin{array}{ccc} & I & J \\ [\$] & 1 & 1/2 \end{array} \tag{22}$$

and applying the ITC to this monetary SUT yields a symmetric representation (equations (23) and (24))

$$\mathbf{A}^{itc} = \begin{array}{ccc} & i & j \\ i\ [\$] & 1/3 & 1/6 \\ j\ [\$] & 1/3 & 5/12 \end{array} \tag{23}$$

$$\mathbf{v}^{itc} = \begin{array}{ccc} & i & j \\ [\$] & 1/3 & 5/12 \end{array} \tag{24}$$

that *is* financially balanced (equation 25).

$$\mathbf{p}' = \begin{pmatrix} 1 & 1 \end{pmatrix} = \begin{pmatrix} 2/3 & 7/12 \end{pmatrix} + \begin{pmatrix} 1/3 & 5/12 \end{pmatrix} = \mathbf{p}'\mathbf{A} + \mathbf{v} \tag{25}$$

Just as with the BTC counterexample in section 3.1, this financially balanced representation fails the financial balance test of Kop Jansen and ten Raa (1990), this time solely because of the first embedded assumption.

$$\mathbf{e}'\mathbf{AV} = \begin{pmatrix} 23/12 & 7/12 \end{pmatrix} \neq \begin{pmatrix} 2 & 1/2 \end{pmatrix} = \mathbf{e}'\mathbf{U} \tag{26}$$

4 An alternative approach based on LCA allocation

The convergence of EEIO and LCA modelling offers a different perspective on constructs and their balance (Suh et al., 2010; Majeau-Bettez et al., 2014). All product-by-product IO constructs can be presented as the combination of two models: [1] an allocation model that ascribes requirements specifically to each coproduct in each industry; and [2] an aggregation model that aggregates the different technologies for producing the same product in different industries, thus removing industries from the system description (Majeau-Bettez et al., 2014). The BTC and ITC constructs can therefore be analyzed in terms of the allocation models on which they are based.

The BTC is based on *substitution allocation*, which models secondary coproducts as avoiding some other, primary supply of products. More specifically, BTC assumes an *ideal* substitution, in which each product from secondary production perfectly displaces an identical product from primary production in a 1:1 ratio (Majeau-Bettez et al., 2014). As such ideal substitutions do not introduce any imbalances in the modelled production functions[3], the BTC necessarily respects financial balance (Majeau-Bettez et al., 2016).

The ITC is based on *partition allocation*, which splits the requirements of each industry proportionately to a selected property (e.g. mass, value, etc.) of its supply flows. The units with which the SUT is recorded affects the structure and the balances of the symmetric representation generated by the ITC. In fact, applying ITC to a SUT recorded in terms of a given property is equivalent to partitioning the whole system based on this property, and then aggregating across industries (Majeau-Bettez et al., 2014). In other words, an ITC applied to a monetary SUT is underpinned by an economic partition allocation model. This makes it intuitively clear why ITC must be financially balanced when applied to a monetary SUT: if joint requirements (costs) are split proportionately to the value of each coproduced supply (revenues), this automatically ensures that costs and revenues are well matched.

Analyzing constructs as combinations of allocation models and aggregation models may therefore allow for a more intuitive perspective on balance issues, relative to the traditional “technology-assumption” description. It should also facilitate both the application and the assessment of mixed-technology constructs (cf. Bohlin and Widell, 2006; Smith and McDonald, 2011) and hybrid EEIO-LCAs (Nakamura and Kondo, 2002; Suh et al., 2004).

5 Conclusion

To resolve the contradicting findings concerning the financial balance of BTC and ITC, we reviewed the financial balance test established in the seminal publication by Kop Jansen and ten Raa (1990). Based on the original publication’s counterexample, we proved that this test leads to false negatives; constructs can fail the financial balance test while still respecting financial balance. Revisiting the derivation of this test, we found that it embeds two assumptions that cannot be expected to hold true in general.

Therefore, and contrary to the longstanding perception, we find that the CTC is not the only construct that can systematically lead to IO representations that are financially balanced. The BTC necessarily respects the financial balance of an initial SUT inventory, whereas ITC is always financially balanced when applied to a SUT recorded in monetary units.

To complement the formal proofs, we presented a two-step approach to constructs, based on an LCA allocation step and an aggregation step, which allows for a more intuitive understanding of the reasons behind their respect or not of financial balance. This is but one example of the gains in modelling clarity and transparency that we can expect from the current convergence of EEIO and LCA

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[3] Mathematically, ideal substitutions subtract the same values from the supply and the use accounts, preserving the balance.

practice.

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