Compensation of Navigation Uncertainty for Target Tracking on a Moving Platform

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***Abstract*—Established state-of-the art methods for target tracking assume perfect knowledge of the sensor position and orientation. This assumption is violated when the tracking sensor is mounted on a moving platform such as a ship. Two methods for solving this problem are compared. The Schmidt-Kalman filter maintains correlations between the ownship and the target, while a converted measurement approach merely translates the navigation uncertainty into the measurement model of the target. Simulation results indicate that the Schmidt-Kalman filter yields the best improvements with regard to consistency, while the converted measurement approach yields better improvements in root mean square error.**

I. Introduction

Target tracking from moving platforms has seen extensive research the last years [1]–[3]. This has applications such as in collision avoidance for autonomous surface vehicles. Increased autonomy in the maritime industry can lead to safer and more efficient operations, either as a decision support module or a completely autonomous vehicle.

One suggested approach is to extend methods for simultaneous localization and mapping (SLAM) to include moving objects [1]. Some approaches to this SLAM with moving objects tracking (SLAM-MOT) warns that care should be taken to let the estimate of a moving target influence the estimate of the sensor platform pose [2], while others allow measurements of the target to affect the position of the sensor platform [4]. This can be advantageous when the platform lack sensors for determining its own absolute position and attitude. However, one must take care not to let false tracks or erroneous data association affect the platform pose estimate. Furthermore, it may be difficult to readjust navigation estimates when the navigation is performed by "black-box" proprietary software, which is often the case.

If the sensor platform is equipped with a navigation system, it is possible to simply let the uncertainty in this system affect the target tracking, without letting information flow the other way around. This mindset leads into the Schmidt-Kalman filter [5], where the state vector of the system is augmented by nuisance parameters. These nuisance parameters are not estimated, but their correlations with the estimated states will affect the covariance of the target. The navigation state of the ownship can be considered to be a nuisance parameter in this formulation. Previous research has investigated the performance of such a filter in the case when the navigation error is zero mean and uncorrelated, and with constant bias [6]. This approach can also be formulated in a random finite set (RFS) framework such as in [3].

Another popular approach is to convert measurements into linear position and/or velocity measurements of the target before applying the Kalman filter framework. This approach is not possible with tracking sensors such as a single camera, since it only has bearing and no range measurement. A radar, however, provides measurements of the target in polar coordinates, which can be converted into Cartesian coordinates in the body frame. This means a linear Kalman Filter can be used. The conversion of measurement noise covariances from polar to Cartesian coordinates has previously been discussed in [7] and several other references. We can also convert a covariance representation of navigation uncertainty along similar lines. Previous research has addressed compensation of ownship heading uncertainty [8]. In this paper we address the more general problem of compensating for the entire generalized pose uncertainty of the ownship.

This article presents two methods for target tracking with navigation uncertainty, one based on the Schmidt-Kalman filter and one based on converted measurements. The former takes advantage of knowledge of the navigation error model and maintains the correlations between the navigation error and the target state. The second approach does not maintain any correlations, but proposes a Cartesian measurement with covariance inflated by the navigation covariance. Finally, note that this paper only concerns the filtering part of target tracking, and not data association.

The rest of the paper is organized as follows: In section II, we introduce the error state Kalman filter in a navigation system. In section III, we discuss the tracking model. In section IV, we describe the tracking architectures. In section V, we present the simulation results with focus on accuracy and filter consistency.

II. Navigation

Our work is in particular inspired by collision avoidance and navigation at sea, where it is common to use an inertial measurement unit (IMU) in combination with one or several global satellite system (GNSS) receivers, such as receivers for the American GPS system. Attitude is typically estimated by use of a magnetic compass or several GNSS antennas. All navigation quantities will be superscripted with o to denote the ownship system.

A. Model

Ownship navigation is done using the error-state formulation [9], [10]. In this form, one sensor is used to provide a nominal navigation estimate, and the other sensors are used to estimate the errors. The attitude error is estimated by a multiplicative extended Kalman filter (EKF). The velocity and position error are the difference between the true and nominal estimates. The conventional choice is to integrate the IMU output to orientation, velocity and position nominal estimates, and use GNSS and compass as complementary sensors.

The kinematics of the attitude, velocity and position are given by

$$\dot{\mathbf{q}}^o = \mathbf{T}_q(\mathbf{q}^o)\boldsymbol{\omega}^o \tag{1}$$
$$\dot{\boldsymbol{\nu}}^o = \mathbf{C}(\mathbf{q}^o)\mathbf{f}^o + \boldsymbol{\gamma} \tag{2}$$
$$\dot{\boldsymbol{\eta}}^o = \boldsymbol{\nu}^o \tag{3}$$

Where $\mathbf{q}^o$, $\boldsymbol{\nu}^o$ and $\boldsymbol{\eta}^o$ are the attitude, velocity and position of the ownship, respectively. The angular rate and specific force from the IMU are denoted $\boldsymbol{\omega}^o$ and $\mathbf{f}^o$, respectively. Gravity is denoted $\boldsymbol{\gamma}$, and $\mathbf{C}$ is a rotation matrix. The matrix $\mathbf{T}_q$ is determined by the choice of attitude representation.

We represent attitude as a unit quaternion, as opposed to Euler angles, to avoid confusion regarding the sequence of Euler angles, and because quaternions are a standard parametrization in inertial navigation systems. The unit norm constraint would induce singularities in the Kalman filter covariance matrix if a full-state parametrization was used in the filter, and for this reason the Kalman filter works on the three-parameter error angle $\delta\boldsymbol{\Psi}^o$. The relationship between the error angle $\delta\boldsymbol{\Psi}^o$, the estimated attitude $\hat{\mathbf{q}}^o$ and the true attitude $\mathbf{q}^o$ is

$$\mathbf{q}^o = \delta\mathbf{q}^o(\delta\boldsymbol{\Psi}^o) \otimes \hat{\mathbf{q}}^o \tag{4}$$

This formulation of the error angle and the use of an EKF for state estimation leads to the multiplicative EKF for the attitude estimate [11], as the attitude error is related to the true attitude by a quaternion multiplication. The error has been defined such that the error angle is represented in the global frame. The errors in position and velocity are additive, and are related to the true and nominal estimates by

$$\boldsymbol{\nu}^o = \hat{\boldsymbol{\nu}}^o + \delta\boldsymbol{\nu}^o \tag{5}$$
$$\boldsymbol{\eta}^o = \hat{\boldsymbol{\eta}}^o + \delta\boldsymbol{\eta}^o \tag{6}$$

Let $\hat{\mathbf{q}}^o$, $\hat{\boldsymbol{\nu}}^o$ and $\hat{\boldsymbol{\eta}}^o$ be the attitude, velocity and position obtained by integrating the (noisy) IMU measurements through equations (1)-(3). We consider only the IMU biases and additive white noise, such that the IMU measurements are on the form

$$\mathbf{f}^o_{imu} = \mathbf{f}^o + \mathbf{b}^o_a + \mathbf{w}^o_a \tag{7}$$
$$\boldsymbol{\omega}^o_{imu} = \mathbf{f}^o + \mathbf{b}^o_g + \mathbf{w}^o_g \tag{8}$$

where $\mathbf{b}^o_a$ and $\mathbf{b}^o_g$ are the accelerometer and gyro biases, respectively. This leads to an error state on the form

$$\delta\mathbf{x}^o = \begin{bmatrix} \delta\boldsymbol{\Psi}^o \\ \delta\boldsymbol{\nu}^o \\ \delta\boldsymbol{\eta}^o \\ \delta\mathbf{b}^o_a \\ \delta\mathbf{b}^o_g \end{bmatrix} \tag{9}$$

where $\delta\mathbf{b}^o_a$ and $\delta\mathbf{b}^o_g$ are the residual bias errors. A continuous system model can be derived by inserting the error expressions and IMU measurements. The system is discretized with a suitable discretization method, and the error state is given as

$$\delta\mathbf{x}^o_{k+1} = \mathbf{F}^o_k\delta\mathbf{x}^o_k + \mathbf{v}^o \tag{10}$$

where $\mathbf{v}^o$ is zero-mean gaussian white noise with covariance matrix $\mathbf{Q}^o$. Details of the modelling and discretization can be found in, for example, [9].

When the navigation filter has calculated an error estimate, the state estimates are corrected by this estimate. This resets the error estimate to zero.

B. Navigation measurements

We assume that the ownship is equipped with an IMU, measuring specific force and angular rates, and a GPS compass, which measures the position and heading of the ship. This heading measurement can be used along with roll and pitch estimates from the accelerometer to obtain a full measurement of the ship attitude. Given this, the measurement of the ownship is simply given as

$$\mathbf{z}^o = \mathbf{h}^o(\mathbf{x}^o) + \mathbf{w}^o = \begin{bmatrix} \boldsymbol{\eta}^o \\ \mathbf{q}^o \end{bmatrix} + \mathbf{w}^o \tag{11}$$

where $\mathbf{w}^o$ is zero-mean gaussian white noise with covariance matrix $\mathbf{R}^o$. Note that most measurements are of the full state rather than the error state, which necessitates evaluation of the Jacobian matrix

$$\mathbf{H}^o = \frac{\partial \mathbf{h}^o}{\partial \delta \mathbf{x}^o} = \frac{\partial \mathbf{h}^o}{\partial \mathbf{x}^o} \frac{\partial \mathbf{x}^o}{\partial \delta \mathbf{x}^o} \tag{12}$$

The first term of this is, in our simplified case, linear for position and attitude. However, as seen from equation (4), the second term will be nonlinear for the attitude. This is because of the error angle representation in the Kalman filter. For this reason, the navigation filter must use a nonlinear estimation tool such as the EKF.

III. Tracking

This section introduces the tracking models and measurements. All the parameters and states of the target will be superscripted by t.

A. Model

With the target state vector

$$\mathbf{x}^t = \begin{bmatrix} \eta_x^t \\ \nu_x^t \\ \eta_y^t \\ \nu_y^t \end{bmatrix} \tag{13}$$

we use a standard constant velocity (CV) kinematic model, which in discrete time is

$$\mathbf{x}_{k+1}^t = \mathbf{F}^t \mathbf{x}_k^t + \mathbf{v}_k^t \tag{14}$$

where $\mathbf{v}^t$ is assumed to be a zero-mean gaussian white noise with covariance $\mathbf{Q}^t$, and $\mathbf{F}^t$ is given by

$$\mathbf{F}^t = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

where T is the time update interval of the filter, assumed to be the radar sampling time.

B. Target measurements

The radar is the primary sensor of interest in a maritime environment, and it provides measurements of range r and bearing θ of the target in the ownship body frame. Given the target position and ownship pose, the world frame measurement model becomes

$$\mathbf{z}^t = \begin{bmatrix} \sqrt{(\eta_x^t - \eta_x^o)^2 + (\eta_y^t - \eta_y^o)^2} + w_r^t \\ \operatorname{atan}_2(\eta_y^t - \eta_y^o, \eta_x^t - \eta_x^o) - \psi^o + w_\theta^t \end{bmatrix} \tag{16}$$

$$= \mathbf{h}^t(\mathbf{x}^t, \mathbf{x}^o) + \mathbf{w}^t \tag{17}$$

where atan_2 is the four-quadrant arctangent function, as the radar is able to detect targets in all quadrants around the ownship. $\mathbf{w}^t$ is zero-mean gaussian white noise with covariance matrix $\mathbf{R}^t$, given as

$$\mathbf{R}^t = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix} \tag{18}$$

IV. Tracking architectures

A. Schmidt-Kalman filter

Schmidt [12], presented a method for dealing with parametric uncertainty, or bias, in the process and measurement model. Instead of estimating the bias, the filter only accounts for its uncertainty through a covariance update. This means that it considers the uncertainty of the bias when estimating the state, and the covariance added to the state from the bias is also called the consider covariance [6].

We include the navigation states as biases in the target state vector. The process model of the augmented system then takes the form

$$\begin{bmatrix} \mathbf{x}_{k+1}^t \\ \delta \mathbf{x}_{k+1}^o \end{bmatrix} = \begin{bmatrix} \mathbf{F}^t & 0 \\ 0 & \mathbf{F}^o \end{bmatrix} \begin{bmatrix} \mathbf{x}_k^t \\ \delta \mathbf{x}_k^o \end{bmatrix} + \begin{bmatrix} \mathbf{v}_k^t \\ \mathbf{v}_k^o \end{bmatrix} \tag{19}$$

The target measurement Jacobian and covariance matrix for this system can now be written

$$\mathbf{H}_S^t = \begin{bmatrix} \mathbf{H}^t & \mathbf{H}^{ot} \end{bmatrix} \tag{20}$$

$$\mathbf{P}_S^t = \begin{bmatrix} \mathbf{P}^t & \mathbf{P}^{to} \\ \mathbf{P}^{ot} & \mathbf{P}^o \end{bmatrix} \tag{21}$$

The matrix $\mathbf{P}^o$ is the covariance matrix of the ownship errors, which will influence the target covariance and cross-covariance terms. It is updated at every iteration with the current covariance from the navigation filter. The crosscovariance then changes through the measurement equation.

B. Converted measurements

Another alternative is to invert the measurement, Equation (16), and use a linear Kalman filter to estimate the target states. In the sequel, we assume that the error state consist of the errors in horizontal position and heading, as these are the primary variables of interest in a 2D tracking system:

$$\delta \mathbf{x}^o = \begin{bmatrix} \delta\eta_x^o \\ \delta\eta_y^o \\ \delta\psi^o \end{bmatrix} \tag{22}$$

The corresponding marginalization is straightforward, since the error angle is defined in the world frame.

Denote the inverse of $\mathbf{h}^t$ by $\mathbf{g}^t$ such that the Cartesian target position is given as

$$\mathbf{g}^t(\mathbf{x}^o, \mathbf{z}^t) = \begin{bmatrix} \eta_x^t \\ \eta_y^t \end{bmatrix} = \begin{bmatrix} \eta_x^o + r\cos(\theta + \psi^o) \\ \eta_y^o + r\sin(\theta + \psi^o) \end{bmatrix} \tag{23}$$

The inverted measurement function is a nonlinear transformation of the random, assumed Gaussian, variables $\mathbf{x}^o$ and $\mathbf{z}^t$. The statistics of $\mathbf{g}^t$ can thus be found by any nonlinear transformation method such as linearization. Linearization around the point $(\hat{\mathbf{x}}^o, \mathbf{z}^t)$ gives the expectation and covariance of the transformed measurement as

$$\mathbf{z}_\eta^t = \mathbf{g}^t(\hat{\mathbf{x}}^o, \mathbf{z}^t) \tag{24}$$

$$\mathbf{R}_\eta^t = \mathbf{G}_{\mathbf{x}^o}\mathbf{P}^o\mathbf{G}_{\mathbf{x}^o}^T + \mathbf{G}_{\mathbf{z}^t}\mathbf{R}^t\mathbf{G}_{\mathbf{z}^t}^T \tag{25}$$

where $\mathbf{z}_\eta^t$ is the cartesian position measurement, and the Jacobians $\mathbf{G}_{\mathbf{x}^o}$ and $\mathbf{G}_{\mathbf{z}^t}$ are given by

$$\mathbf{G}_{\mathbf{x}^o} = \frac{\partial \mathbf{g}^t}{\partial \mathbf{x}^o} = \begin{bmatrix} 1 & 0 & -r\sin(\theta + \psi^o) \\ 0 & 1 & r\cos(\theta + \psi^o) \end{bmatrix} \tag{26}$$

$$\mathbf{G}_{\mathbf{z}^t} = \frac{\partial \mathbf{g}^t}{\partial \mathbf{z}^t} = \begin{bmatrix} \cos(\theta + \psi^o) & -r\sin(\theta + \psi^o) \\ \sin(\theta + \psi^o) & r\cos(\theta + \psi^o) \end{bmatrix} \tag{27}$$

After this transformation, the estimates can be used in a linear Kalman filter.

As the ownship pose covariance only enters the tracking filter before the measurement update, this approach does not maintain any correlations between the ownship and the target.

V. Results

We compare the two described methods against two other tracking filters, neither of which compensates for navigation uncertainty. One of them will use the ground truth ownship pose in the filter such that the position and attitude of the sensor is perfectly known. The other will use the pose of the navigation system.

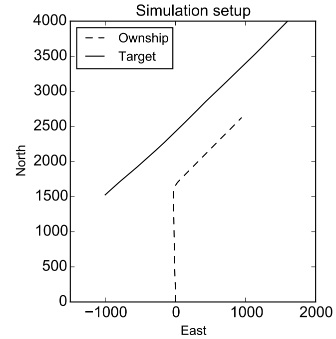

Fig. 1. The ground truth trajectory for the ownship and the target ship.

A. Simulation setup

The trajectory of the target and ownship are shown in Fig. 1. The ownship start in the origin and moves north, while the target ship starts at the upper right corner. At $t = 150$, the ownship makes a right turn to avoid collision. The simulation ends after 300 seconds.

To evaluate the methods, we use position and velocity root mean square error (RMSE) and normalized estimation error squared (NEES). The RMSE values are calculated as

$$\text{RMSE}_\eta = \sqrt{\frac{1}{N_{mc}} \sum_{i=1}^{N_{mc}} (\eta_{x,i}^t - \hat{\eta}_{x,i}^t)^2 + (\eta_{y,i}^t - \hat{\eta}_{y,i}^t)^2} \tag{28}$$

$$\text{RMSE}_\nu = \sqrt{\frac{1}{N_{mc}} \sum_{i=1}^{N_{mc}} (\nu_{x,i}^t - \hat{\nu}_{x,i}^t)^2 + (\nu_{y,i}^t - \hat{\nu}_{y,i}^t)^2} \tag{29}$$

where N_{mc} denotes the number of Monte-Carlo simulations. The NEES are given by

$$\text{NEES} = \frac{1}{N_{mc}} \sum_{i=1}^{N_{mc}} (\mathbf{x}_i^t - \hat{\mathbf{x}}_i^t)^T \left(\mathbf{P}_i^t\right)^{-1} (\mathbf{x}_i^t - \hat{\mathbf{x}}_i^t) \tag{30}$$

and provide a measure of consistency [13].

The navigation system has sample rates of 100Hz and 5Hz for the IMU and GPS, respectively. For simplicity, the biases in Equation (7) and Equation (8) are constant, while the covariances of the noise parameters are given as

$$\text{cov}(\mathbf{w}_a^o) = \sigma_a^2\mathbf{I} \qquad \sigma_a = 0.7m/s^2 \tag{31}$$

$$\text{cov}(\mathbf{w}_g^o) = \sigma_g^2\mathbf{I} \qquad \sigma_g = 0.4°/s \tag{32}$$

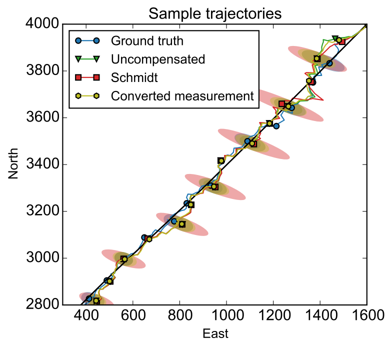

Fig. 2. Part of the position tracking results. The ellipses are 95% confidence-ellipses.

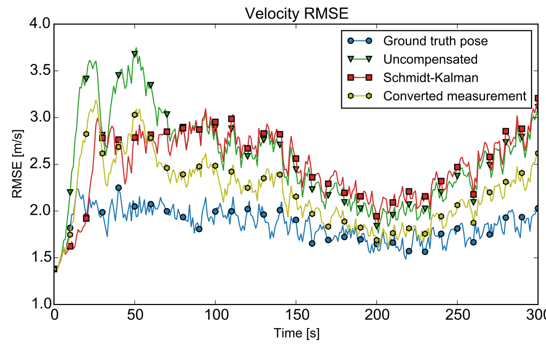

Fig. 4. Velocity RMSE for 100 Monte Carlo runs.

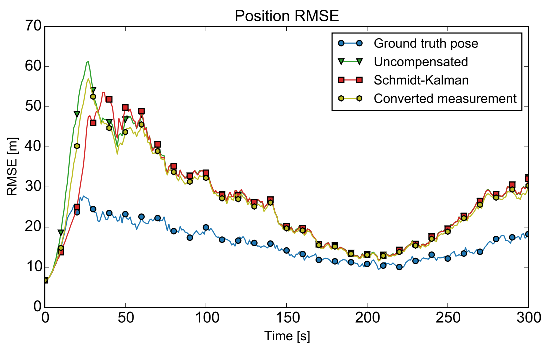

Fig. 3. Position RMSE for 100 Monte Carlo runs.

The radar has a sample rate of 1Hz, and the measurement covariances in Equation (18) are given as

$$\sigma_r = 20m \qquad\qquad \sigma_\theta = 1^\circ \tag{33}$$

B. Tracking results

Fig. 2 shows the start of the target trajectories for the different filters. The most notable difference between the filters in this setting is the stretched covariance ellipse from the Schmidt-Kalman filter. The ground truth filter have both the smallest errors and covariance ellipses.

The position RMSE is shown in Fig. 3. The difference between the uncompensated filter and the two compensated methods is very low. A slightly lower RMSE is seen for the converted measurements in the transient phase. However, as the distance between the ownship and target decreases, the difference becomes negligible, and at the closest point is even comparable to the ground truth tracking filter. The primary reason for the relatively small improvement is that the heading uncertainty from the navigation system is dominating the measurement noise. In practice, this means that the navigation covariance can be seen as an increase in the measurement covariance, common to all the filters.

The velocity RMSE is shown in Fig. 4. This time, the compensated filter has a more significant performance increase over the uncompensated filter. The transient phase in particular shows an improvement of factor 1.5. The RMSE also decreases as the distance between the ships decrease.

The notable improvement seen in velocity are the results of filtering. The increased uncertainty in the compensated filters leads to a lower Kalman gain. The biggest effect is seen in the converted measurement filter, as it has less information than the Schmidt-Kalman filter. As we approach steady-state, this effect seems to decrease, in particular for the Schmidt-Kalman filter which seems to become more erroneous. This effect could arise from the fact that it maintains cross-correlations between the target and ownship. As such, it has more information and is more inclined to increase the Kalman gain. Another possibility is that the effects of the navigation error reset have not been captured by the filter.

C. Consistency results

Fig. 5 shows the NEES for the filters. The ground truth filter and the Schmidt-Kalman filter is mostly within the 95% confidence interval, while the converted measurement approach is slightly above. The uncompensated filter suffers severely from inconsistency.

Although the Schmidt-Kalman filter were outperformed in the RMSE-analysis, it performs very well in the consistency analysis. The reason for this is, as previously mentioned, that it keeps track of the correlations between the ownship and the target. This information is discarded in the converted measurement filter. Finally, note that an inconsistent filter may have severe effects

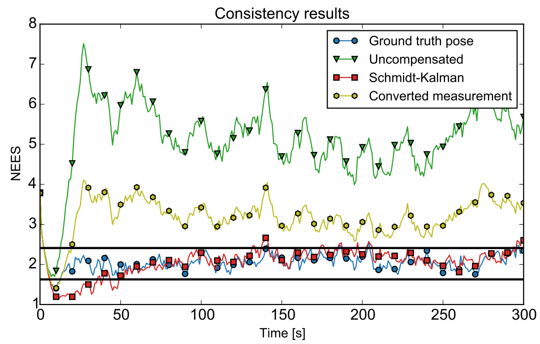

Fig. 5. NEES analysis for 100 Monte Carlo runs, with 95% confidence interval.

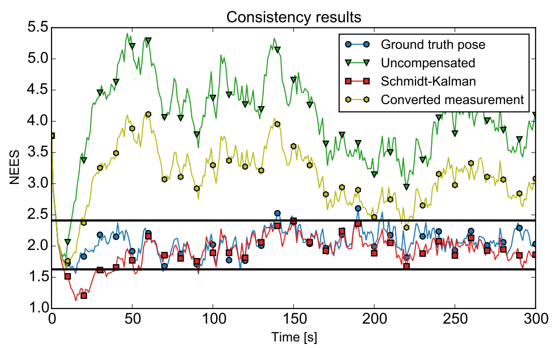

Fig. 6. NEES analysis for 100 Monte Carlo runs, with 95% confidence interval. The navigation standard deviation values are halved from the values used in Fig. 5.

on data association. If the filter covariance is not commensurate with the respective error, the validation region set up by some data association methods, such as the PDA [7], might be too small to capture the measurement. Inconsistency can also be a source of divergence in the EKF.

Reducing the navigation covariance parameters only reduced the NEES values for the uncompensated tracking system and slightly for the converted measurement filter, as seen in Fig. 6.

VI. Conclusion

Uncompensated navigation errors can cause significant inconsistency in a target tracking system mounted on a moving platform. This makes the estimator overconfident and can be critical for data association. This can be remedied, for example, by either inflating the measurement covariance in a converted measurement filter, or by designing a Schmidt-Kalman filter. While the RMSE of the position were not significantly affected by the compensation, the RMSE of the velocity saw an improvement, particularly in the initial transient.

Future research includes a more in-depth study of body-parametrized tracking filters, as well as comparisons of Schmidt-Kalman filters and SLAM-based methods. Testing the methods in clutter with data association is also an interesting topic.

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