0.1 PATTERNS AND PROGRAMS IN PREMODERN ROME

Shape, form and message systems: an open-source approach

(Staale Sinding-Larsen)

(Internet/CD version)

To Liv Erstad S.-L. A little in return for what I owe you.

ABSTRACT

An abstract in the usual sense but also a statement of intent preceding the *Introduction.*

(1) *The main thrust of my venture is to develop theory and methodology about problems surrounding the deceptively plain task of object and design description, focusing on the shape and the form aspects (for this distinction, see below). The up-front issue is the question of linkage between the "differential curves" and manifolds in Borromini's architecture and the jumpy and non-linear growth of the budding calculus (here labelled "protocalculus"), roughly 1580 - 1680. The idea is that this architecture, as well as some cases in sculpture, to a critical degree evolved in response to tendencies, interests and attitudes comparable to those that informed the math ventures. With this in view, the argumentation will draw on resources from the cognitive sciences, information, management and the social sciences.*
(2) *In an experimental turn it is assumed that tendencies can be attested here that may be analysed under a common denominator of variability, uncertainty, instability and approximation; movements of a critical or cutting-edge nature. Similar characteristics (not necessarily dominating) show up in contemporary literature and music, subjects therefore discussed using available sources.*
(3) *Integrated in the analysis procedure, several strands of historical matter will be running through the book. The whole procedure being a stage-like sequence of arguments gradually propagating increased information synthetized in verbal and graphic models; the passage through these stages is the only "conclusion". It is this characteristic that may alienate some readers in the humanities - but there is no alternative. The interrelations between the "strands", that is, the correlated functioning of suggested systems (see below, - g), is the critical theoretical and methodological issue of the book, which is proposed for debate***.**
(4) *Searching for a (not "the") workable theory and methodology, the focus has centred on cultural phenomena that are in themselves of a systemic character, with math and physics in a spearhead position. The Roman Church, centrally positioned in matters of culture and science, provided a mandatory, complex but fully accessible system (here, called the Canonical System) which constituted a fundamental basis for all the relevant visual message programs and imagery. This invariable rule system (with lower-level variations over time) has had to be epitomized. An analysis of the Sixtine vault in the Vatican and of some sculptures (Bernini) has been developed for instantiation.*
(5) *The above points should make it clear that hard as well as soft data or objects will be in the focus, requiring a systems view for their handling, integrating distinctions such as between form and shape, and between system and elaboration, while it is accepted that explanation is a by-product of systemization, avoiding causal chains and linearity. With this is connected the operational modality, asking How instead of What, that is, methodology being not just a tool but an integrated part of description and analysis. Consequently, the object under examination is considered as a product of the analysis process. This series of tenets*

make up the spearhead theory of the work, and a scenario inviting to a sustained debate, which the present work does not pretend to conclude (condensed overview in 4.7).
(6) *The open-source approach (a term from computer language) is being taken as a consequence of the cited perspectives. It means that theories, models and insights are freely used as resources, irrespective of "period" or the academic fields to which they may originally belong. The only criterion is that these resources can be used as models for analytical purposes. Theories and models are tools, while academic disciplines containing them are not; hence I avoid the term "interdisciplinarity", which, on account of its ambiguity, too often masks theoretical confusion and causes budgetary troubles in research organizations***.**
(7) *For a preliminary assessment of the book: perusal of the following chapters with graphic models will convey an idea of the theoretical and methodological drive: 1.7, Configuration spaces, Fig. 1.7 - 1 --- 1.11, Structured argumentation, Fig. 1.11 - 1 ---- 3.3, Testing models: the Spire's conceptual position, Fig. 3.3 - 1 --- 4.5, Filling out the framework, Fig. 4.4 -1.*

NOTES. The present digital version may be freely copied and distributed. The author gratefully welcomes comments, suggestions and criticisms which will be useful for a final paper version. - A paper copy for consultation at the Norwegian Institute in Rome. - Depending on storage devices, digital figures will often disintegrate after some time. A FigsStorage file is appended after the Bibliography, with copies of those figures that are essential for the argumentation. It is strongly recommended that this file be printed out separately.

0.2 TABLE OF CONTENTS

0.3 PREFACE

This is the *internet/CD version* of a complex discourse which may be followed, after a while, by a shorter and more readable paper version. Some readers may find the text unwieldy and repetitious. Apart from the requirements arising from the internet access, and the need often to use one and the same subject or argument under different headings, as is obvious in cases of even relatively simple systems approaches, the factor of *openness* has influenced the presentation. My book is *tentatively* a pioneering effort, and I can claim as my masters - obviously with due modesty - some authors of critically framed books. Norbert Wiener's *Cybernetics* was declared unreadable; Wittgenstein never wrote pleasantly; working through Hofstadter's *Gödel Escher, Bach* requires a constant supply of mature *Médoc,* however much he invites us to *have fun*; none of the books by Giordano Bruno, hero in my book (and mine), is readable by modern standards. I am aiming at a full disclosure of my argumentation. Smooth, well-written discourses, like Gombrich's *The sense of order*, only too easily cover up flawed observational or argumentation procedures. I may take a cue from Robert Graves. According to his *I Claudius*, Augustus *always sacrificed elegance to clarity... instead of hunting about for a synonym or periphrasis (which is the common literary practice).*

Criticizing my presentation, a colleague nevertheless noted that generally my writings are *unconventional, based on unusually wide reading, and open ways of thinking prevalent among scientists, mathematicians and philosophers, but rare in the humanities*, noting that he could *imagine students of methodology, and in general more philosophical readers, savouring precisely this feature of the book (see* also the section below: *The book as a tool).*

To reiterate the point, *openness* is what I am aiming at, presenting probabilities and variations without pretending to offer any *definite* statements or *results.* This attitude is not prevalent in the humanities in which I started my working life. A great light in New York told me, when I hinted that someone Very Big should revise his conclusions at specific points, *he doesn't need to revise anything.* The passage from deterministic to open, probabilist, even statistical, science, seems to have started with Charles Darwin and James Clark Maxwell in the nineteenth century (Fischer, 2007, 91ff.). So we are latecomers.

Methodology debate is not popular in some humanist circles. As someone wrote, *The only reason to care about a methodological innovation is if it bears fruit - a 'contribution to knowledge', as all PhD programmes phrase it*. But the moment method means going beyond cooking recipes (*Add sugar and the hash gets sweeter*), the debate spills over into complex problems of theory and science philosophy. How can one do scientific work without continuously focusing on the operative aspect? Nor is systems thinking popular, even though some sort of a system underlies each and every research thematics of some complexity - only waiting to be called forth and discussed. Brutal and destructive urban planning in the US, justified by reference to systems analysis (Robert Moses in New York City), has contributed to the belief, that systems cannot be dynamic and flexible.

The role of theory as a tool is often not recognized. Thus, for instance, the application of planning and organization theory has been called *a generalised speculation on group decision-making* as a substitute for dealing with “primary sources”. We have to have recourse to modern theories of planning, management, organization and information (3.1 *Idea generation and planning process*). As a substitute for factual information, whenever this is not available, such theories have a crucial role in warning against simplification and against attributing everything to the artist or architect. Most importantly, theory, in the humanities as well as in physics, is necessary in order to point up directions for one’s research.

My book aims at a spray of probabilities rather than some “definite” conclusion, its subject being such as never allowing a final word. I note with pleasure that P. E. Russell in his monumental edition of Fernando de Rojas’ *Celestina*, "concludes" on the subject of ideological anchorage by saying that he can come up with nothing more than possibilities (*Puede concluirse que la crítica celestinesca debe resignarse a que, en el plano ideólogico, no puede haber soluciones definitivas, sólo posibilidades* (Russell, 165).

My approach has not been to *interpret*, that is to say, find one or several meanings or significance of Borromini’s Spire(as appears pro toto for the larger group of cases), certainly not some The Meaning. I have studied and assembled in a systems view the conditions and circumstances from which the idea of the Spire and its construction arose; in other words, a functional pattern. Some people hope to come up with *a genuine understanding*, apparently unaware that such a thing does not exist, as they would have discovered had they looked a little beyond the boundaries of their academic tradition.

I have noted already that I shall use Borromini’s Spire and other works for *methodologica*l purposes. The exercise has a twofold origin. Evaluation of the helical Spire in a context of mathematics was suggested in my *Pratt Institute* lectures in 1989. Further inquiry of relevant themes followed in my *The Burden of the ceremony master* in 2000 (referred to, simply, as *Burden;* let me emphasis that the book, like all my Norway-based publications, was financed by the Norwegian Research Council, has always been and still is sold without royalties for myself).

The arrows-vector model expressing a probability picture was launched in my lecture at a Strasbourg Conference in 1988 (with a different subject), published in 1991; and I used it again in my *Burden* for *attention arrows.* It is sad to reflect that my gratitude to Richard C. Trexler, late of SUNY, NY, who encouraged me at that conference, now has to go to his memory. Such books of his as *Public life in Renaissance Florence* and *The Christian in prayer*, should be required reading in most humanities. I believe it was Wittgenstein who claimed that social anthropology is the fundamental (non-physics) science, and Trexler's work bears him out.

The distinction between *systems level* and *elaboration level* has been with me always, such as in SL, 1969 and in Gisolfi and SL (*see below*, 1.6); and in SL, *Working with pictures.* This essay was developed under the stimulating influence of Gerhard Jaritz, of the *Institut für mittealterliche Realienkunde,* at Krems a/d Donau*;* my warmest thanks for his support.

The question of mathematical dimensions in architecture, here treated in an historical context, is discussed in general terms in my forthcoming *Operational determination*, to be published at the Norwegian Institute of Technology (NTH/NTNU).

The contributions on Borromini by the art historian *Joseph Connors* at Columbia, New York, with new documents on the Sant'Ivo story, have been indispensable reading for me; even though my focus is different from his. I take this opportunity to thank him for interesting exchange of opinions.

As always, I have a lot to be thankful for. I will express my gratitude to *Herbert Lindenberger*, Emeritus at Stanford. I always feel stimulating contact (and also silent criticism: what would Herbert say to this?). He kindly read some of the c*hapters* in the present work.

I will most emphatically express my indebtedness to Senior Engineer *Knut Rø* at our technical university for his great helpfulness and encouragement. It is he who, besides support and advice, has introduced me to the various *Adobe* products; the most important of them *FrameMaker*'s book program. My thanks also to the university itself (now the NTNU, formerly the *Norwegian Institute of Technology*; NTH), at Trondheim, for allowing me, through the good offices of Knut, to use their system and digital resources; and to the people at the Institute where I worked for thirty years for their continuing support, particularly Knut Einar Larsen.

My sincere thanks also to my colleague Åse Ødegaard, formerly NTNU, for her stimulating encouragement; she is the author of an exceptional account of how a major pictorial

cycle can be planned and carried out (a fresco cycle in Oslo Townhall). And to Donatella Fiorani and Carlo Cacace in Rome for stimulating exchange of opinions and experiences.

I also will thank my family in Oslo (Charlotte and her people) and in Rome/Tuscany (Camilla and her people) for their patience with me over the years and their manifold support.

The readers may forgive a personal note to conclude this Preface. Liv Erstad (later Sinding-Larsen) left her teaching job at the University of Göttingen (which they wanted her to keep) to come to Rome and stay with me. She had already started teaching me things about argumentation and writing that I never learnt at the university, and now my manuscripts for my first book, *Christ in the Council Hall,* became blackened by her corrections of my English and of my writing generally. Ever since she has actively participated in my handling of problems. Liv has always been the most articulate supporter of my view on scholarship. Briefly stated: *in the humanities they try to convince you; in the sciences they take you through an argumentation. I have opted for the latter tack, not wanting to demonstrate anything but attempting to structure an argumentation.*

Homepage (bibliographical not biographical):
http://folk.ntnu.no/staalesl/staalesinding/iconography.html
E-mail: staale.sinding-larsen@ark.ntnu.no
Cell phones: +47 95034683 - +39 3337204345

Rome, Quartiere Appio-Latino - Trondheim, Kalvskinnet, 2010.

0.4 *COLOPHON*

- The book is written in the *Adobe FrameMaker,* 7.2, book program. The picture material was processed in *Adobe Photoshop* and *CorelDraw,* 10.
- The text is subdivided in three levels, *Part (I to IV), Chapter* and *Section*. Thus a reference (3.2.11) means *Part* III, *Chapter* 2 and *Section* 11. Generally in referring to such units, only the numbers in parenthesis will be used: (3.2.11).
- To avoid troublesome footnotes, I adopt the modern science fashion, giving author(s), year (if there are more than one of them), and page(s), in a parenthesis. Thus a statement such that *Bacchus (8) affirms that...,* will mean that just one of this prolific author’s works is cited in the bibliography, and the page number is 8. To myself I refer with a modest SL, and to my *The Burden of the ceremony master,* with just *Burden.* I do not use the now obsolete *op.cit* and *ibidem*, etc.; but I do use *passim* sometimes, meaning "here and there", since there is no better format.
- *Figure* (abbreviated *Fig*.) refers to the graphic and black-and-whites in the text body. They are numbered for *Part* and *Chapter* separately (and easy to locate); thus *Fig. 2.3 - 4* means *Fig*. 4 in *Chapter* 3 of *Part* II; *Section* numbering is not given for the figures.
- Readers may resent my inclusion of some rather long *quotations* from scholarly and other literature; why don’t I say it in my own words? In an interdisciplinary venture, which involves communicating opinions and insights culled from experts in different

fields, having the writers explain their views directly will convey the message and the flavor much better than I could do. What is lost in cost of digits is regained in terms of understanding and contact with the origin of a statement.

- In a venture going across several fields, one must consider those readers for whom some of the stuff is not familiar. For similar reasons I have deliberately committed the grave sin of repeating things now and then. Repetitions of an argument in different contexts can also be useful for testing the argument itself and one's understanding of the context.
- The role of the Catholic Church will be more or less constantly referred to, and here "tradition" is spelled with capital T: Tradition (details in SL, 1984, 18f.); some familiarity with this issue is indispensable for studies in art and culture in the contexts considered here.
- Contrary to common practice today, I shall call Galileo Galilei by his family name, *Galilei* (it being clear I am not speaking of his father Vincenzio), adopting the practice in the Paolo Rossi volumes. I can see no reason to refer to him by his first name as the only one among his contemporaries. We are no longer in the days of Leonardo and Michelangelo. Alternatively, should we speak of Galilei's contemporaries as Francesco, Johannes, Gianlorenzo, Antonio, Roberto, Marin and René?

0.5STRUCTURE OF THE BOOK

The trail to follow can be briefly summarized.

Part I contains some case histories, in particular concerning Borromini's Spire and its closest context linkages (the papal university church). While the entire discourse in the book is considered as the model, a summing-up model is presented and commented at the end (1.11), in terms of *structured argumentation*.

Part II conveys critical aspects in a wider historical view and concludes with some observations on science in the late sixteenth and first half of the seventeenth century. A crude picture of the problems is drawn up.

Part III is dedicated to considerations on the planning process concerning the Spire, its larger context linkages, and its possible relations to contemporary science, especially the mathematics developing around the ideas leading on to the calculus. In a summing-up the helical Spire is decomposed in relevant contextual and mathematical aspects.

Part IV is dedicated to capitalizing on the insights acquired concerning the Spire project, in the light of the use of models and of the idea of probabilities. Spanning chapters 4.1 to 4.3.7, model and analysis theory will be brought under a single compass, which will materialize in the rest of *Part* IV, in which the analysis of the Spire will be implemented.

The *Supplements* take care of material that is indispensable but too heavy to include in the main text body.

Readers desiring to concentrate on the *historical material* rather than the theoretical handling of it, can read through the *Abstract, Introduction* and *Parts* I and II only; doing so, they will get a grasp on the essential ideas also of the remaining two *Parts*.

0.6 THE BOOK AS A TOOL

Today's world of scholarly literature, greatly enriched in the digital format, is cluttered to such a degree that only very exceptional books can be *read* as they used to be. The present one is structured to be used as a *tool.* Readers can scan it as a source of references, without reading it. The book is designed to meet such demands, the reference system being focused on objects, events, terms and concepts, rather than on persons and places; and it consists of the *Absrtract*, the *Introduction*, the *Table of contents*; the list of special or new terms (*Introduction, - f*), and the intensively calibrated *Cross-references*, making an *Alphabetic index* which would redouble the references superfluous. Let me cite an example of such a tool-styled book: FitzGerald and FitzGerald's *Fundamentals of systems analysis,* 3. ed., New York 1987, published by the prestigious company John Wiley & Sons. The *Table of Contents* lists 19 Chapters with 259 (unnumbered) *Sections*.

0.7 INTRODUCTION: THE WHAT AND THE HOW

The aim and purpose of the venture to be presented in the book are set out in the *Abstract.* The following points from *a* to *g* are used for specifying the principal tenets and procedures.

-a The Spire as a model

Why choose *Borromini's Spire* (*see the Fig. next page*) for my exercise? The motive is twofold. In terms of shape and position, a *helix* surmounting Rome's Papal university church was unique and visually as well as conceptually striking, not to say provocative. Secondly, we are going to discuss rather complex forms and shapes, and the Spire makes a good point of departure; for, while being relatively simple, it represents a conspicuous deviation from the canonical helix form in that its torsion increases conspicuously toward the top, making it comparable to the cited formal complexities.

To handle the complexities adequately requires *calculus* and other modern mathematical techniques that were not available in the seventeenth century. But concern with them seems to have interacted with the *protocalculus* efforts (my name for the pre-calculus development), stimulating them. When discussing Borromini's peculiar shapes, I do disregard the fact that other modern mathematics are also required, since in the seventeenth century only techniques and themes that were more or less firmly connected with the protocalculus efforts arose in the intellectual baggage of the scholars. When I refer to *manifolds,* a term that has significance today but not in the specific historical context, it is because it is an effective device in my discussion of some of the space curves we see in seventeenth-century design and architecture.

Strictly speaking a *helix*, in modern terminology, has constant curvature and constant torsion; but for my purposes I shall stretch the point a bit and include irregular deviations, such as the configuration of the Spire with the torsion increasing toward the top (for torsion calculations, *see* Harris and Stocker, 527f.).

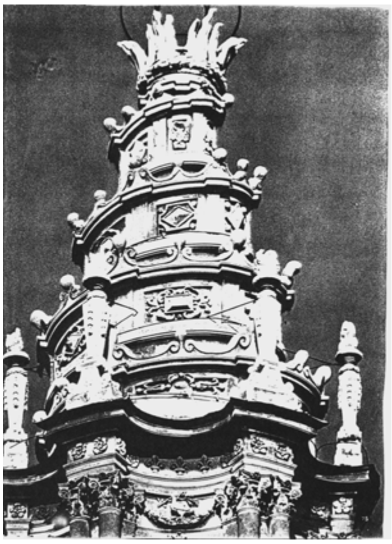


Fig. 1, Introduction. Closeup of Borromini's Spire on Sant'Ivo. For an overview, see Fig. 1.1 - 1).

The Spire, even when we consider its increasing torsion or pitch, is relatively simple. For my choice of it as a test case, I modestly have adopted Descartes' strategy of using a simple model, in his case the parabola, for the investigation of more complex curves. In addition, I let myself be influenced by Herbert Simon:

Research in problem solving has shown that the efficiency of problem-solving efforts can often be greatly increased by carrying out the search for a solution, not in the original problem space with all of its cluttering detail, but in an abstracted space, from which much of the detail has been removed, leaving the essential skeleton of the problem more clearly visible; and further: *'Simple' theories are generally thought preferable to 'complex' theories. A number of reasons have been put forward for preferring simplicity, but the most convincing is that a simple theory is not as easily bent, twisted, or moulded into fitting data as is a complex theory* (Simon, 1979, 31, 234 resp.)

My pivotal idea is to use the helical curve as an anchorage for studying mathematical issues characteristic of the time, taking it for granted that such conceptual or cognitive connections were made by *some* contemporary people and communicated among them. The emergence and development of the Spire idea, its planning and implementation can hardly have occurred in a vacuum. Some kind of pattern of conceptualization concerning the Spire must have been generated. Vague as this issue really is, it is incumbent to come to terms with it at least in an abstract probability view.

There must have been talk all around at the Sapienza, the normal community grapevine taking care of that. People directly or indirectly involved probably came mainly in the following categories: the mathematicians and other math-oriented scholars, the higher clergy, the university scholars in general and, connected with these various categories, circles of men with a higher education who would, to varying degrees, share acquaintance with the *scientific aspects of the helical form.* In this manner a distribution of competence across the groups would arise. This can be indicated, not measured, in terms *not* of people's abilities, *but* on account of *issues understood* by varying degrees of knowledge, interest and grasp of the issues: a *distribution of issues that were being perceived and conceptually handled according to varying (descending) degrees of competence.* The distribution is of primary relevance in the planning process (3.1). We can only propose probabilities here, but their potentiality is strong enough to confer to them the status of a primary source. The idea of such a distribution can be visualized in a model using a graphic of a *damped oscillatory motion* (*B*, the slackening wave curve) expressed in a sine curve (*A*); the diminution of the amplitudes, which represents reduction of issues being handled conceptually, is described by the converging curve *C*.

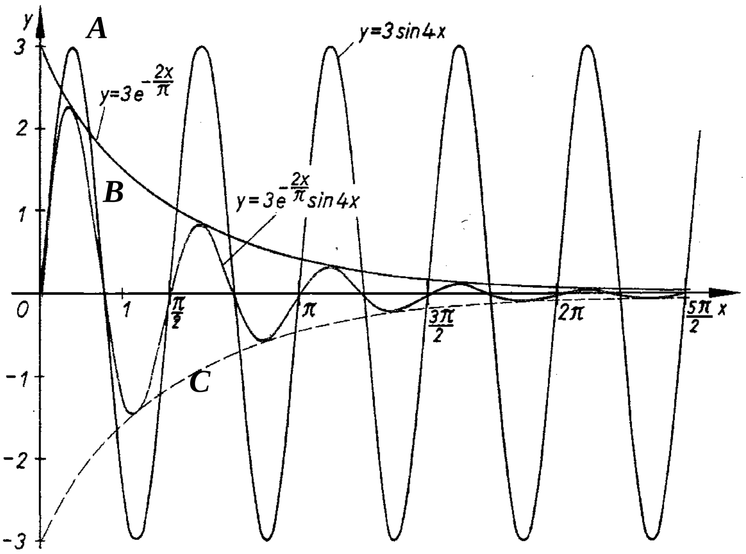


Fig. 0.7 - 1 Oscillatory motion, normal and damped; loss of energy causes the amplitude to decrease, illustrating a concept pattern.

The concept *pattern* does not denote people's capacities but the *amount of and depth of the issues that were being understood,* along a scale from high competence to that of a nonprofessional. in the different groups listed above. The indicated categorization among people does not permit our stipulating precise boundaries between one and the next along the curves. Thus the model shares with many of those in the social sciences the dependence on non-discrete values.

The material involved here is evasive because social categories usually tend to overlap and dovetail (a problem even with present-day material). Let the regular sine curve (*A*, with constant amplitude) represent some series of high-powered modern mathematical historians writing retrospectively (say, Baron, Boyer, Mancosu or Møller Pedersen). The protocalculus issues involved would hardly have been grasped to a comparable degree by seventeenth-century mathematicians, so the corresponding amplitudes (extreme left) fall short of the full one (*A*). The amount of and degree of issues understood gradually diminishes towards the right, as the material so to speak passes through the groups just mentioned.

-b Focus, subject and scope.

The main perspective of the book is an *open-source* approach, a term borrowed from data language. This is preferable to *interdisciplinarity*, because I borrow theory and model wherever I find them useful, without regard for academic disciplines. Theory and model are (usually) *tools*, disciplines are not; so it seems more straightforward to focus on whatever tools they may contain, rather than operating with academic frameworks. What is often missing, is integrated use of theory and model and of system-oriented argumentation structure premised on consistent, at least stated, validity criteria. *People must get away from the idea that serious work is restricted to beating to death a well-defined problem in a narrow discipline, while broadly integrative thinking is relegated to cocktail parties. In academic life, in bureaucracies, and elsewhere, the task of integration is insufficiently respected* (Murray Gell-Mann, Nobel Prize in Physics).

The historical *substantive material* is not primarily in my focus. It represents test cases for trying out *two interconnected theoretical functions:*
one, possible connections, even interactions, between *soft* (natural-language) and *hard* (quantifiable) models; and, *two,* surfacing of instantiations of the models in *systemic terms or pictures*; the entire venture not aimed at "showing the way" but merely at assessing (non-math) *limits* to what we can do (as in my *Burden* of 2000).

The experiment, as I have said, has consisted largely in bringing this on to *a common denominator, interrelated changing entities.* The functionality of this concept in the present context seems less generic and more to the point by being connected with seventeenth-century physics and, especially, mathematics.

The duo, *shape* and *form*, is borrowed from that indispensable book by E. A. Lord and C. B. Wilson, *The mathematical description of shape and form* (1986). Their employment of the two terms is useful for my purpose, as should became evident after a while. Let me quote them on this point. *We have chosen the word shape to indicate those aspects of geometrical form which have to do with the external aspect that an object presents to the world. The word form has been reserved to indicate that some aspects of internal structure is also under consideration* (from the *Preface*).

Three themes will be discussed from the vantage point just stated, as they were instantiated mainly between the late sixteenth century and the middle of the seventeenth, and will be evaluated together under the single compass of *object design description. T*hey are *design structure, scientific thematics* and *the framework of official religion,* features that will be tentatively evaluated in a *systems* view. In addition, my aim being development of theory and methodology, I am using some *modern* cases *in terms of models* for illuminating situations from the seventeenth century. This may disturb some people. But we always reconstruct earlier history under the influence of our own experiences, so we may just as well be clear about it. It also should be taken into consideration that we freely use graphic and mathematical models without claiming any specific historical connection for them; alien stories and scenarios can also be used as abstract models.

Briefly expressed, my line of inquiry is to *factor the shape and form of Borromini's Spire into the networks of two international institutions* in the late sixteenth and first half of the seventeenth century which interacted on many levels: science and the program of the Roman Church.. The subject, in other words, is the problems that, to attentive eyes, at least, were built into the fabric of the Spire.

The book does not aim at general research theory (à la Schulkin in his fascinating *The pursuit of inquiry* of 1992). The only field in which my work in this book might make some claim to being general, would be that of *cognitive and conceptual handling of complex objects.* I can endorse a recent business advice: *Putting in key frameworks rather than point solutions* (Bruce A. Stewart, *Opinion: Essential To-dos for Belt-tightening Times,* in *Computerworld management,* July 2007: http://www.computerworld.com/action/article.do?/command=view/ArtcleBasic&article).

There are two more points to make concerning the general perspective.

My *first* point concerns the acceptance and exploitation of *complexity*. In an important contribution concerning *Alessandro Scarlatti*'s *music* (modestly termed *Vorwort*), Max Lütolf (whom I want to thank for giving me this publication at a memorable encounter at Regensburg) offers some observations on the requirements and research conditions attending the task of approaching *the net of relations, so hard to penetrate, between disparate fields* (Lütolf, 8); *Es wurde jedoch wenig Sinn machen, sich auf Spekulationen im schwer durchsichtebaren Beziehungsnetz disparater Bereiche einzulassen, ohne zunächst den Grundlagen...* etc We are, as I have discussed more carefully in my *Burden* (*Part* I), *facing complexity,* and we have to accept this as a resource while recognizing the restrictions it imposes on our work (*see* also SL, *Categorization*)

Secondly, working with complexity makes is unavoidable to use operators and features from *quantifiable* and *hard sciences like mathematics and physics* as prototypes or moulds for the templates I am using for my argument, which concerns issues of *intuitive and* *observational nature*; adopting Immanuel Kant's distinction between operating in the two modalities of *rein formale Logik* and *inhaltliche Logik* (Geier, 171). The idea of *physicalism*, reducing cognitive facts back to those of physical law, will be a later concern (4.3.1). I am using this principle not in the literal sense, to find *What There Is*, but as reflected into procedural models, called *picture_matrices* (*p_matrices*) (4.5). My purpose is to obtain a clearer picture with which to work.

-c How-issues

My *impresa*, then, revolves around two arguments, one concerned with the structural buildup and system of objects, with the *curvature in Borromini's Spire* (*Fig. 1, here*) as a test case; the other one about how to develop theory and methodology from the resulting picture; this in itself being, needless to say, an experimental one; not a *The interpretation*. This is not such a plain going as this statement would seem to imply, for, as stated already, the leading tenet of my argument is that *the structure is a product of our analysis*. The distinction just posited between the object and my handling of it is valid in procedural terms, *heuristically,* let us say (4.4 and 4.4.1). For the analysis process takes possession of the object and outputs it as part of itself (see below, - *d;* also 4.7), a case of *operational determination.*

Unwilling to be swallowed up in an infinite regress (Lakatos, 2 - 23) by delving into the question *What is design*? I merely ask *How to describe it* (what we normally take as design features)*?* an issue that lies across a number of fields of research and - healthily, as I see it - invites focusing on the *How* rather than the *What*, action rather than state of things.

The main thrust of the book is, as hinted earlier, to develop models for describing *physical objects* in contexts of networks in conceptual and cognitive interaction, which means *systemization*. Explanation is a by-product of systemization (Radnitzky, II, 102). The same wisdom is expressed also by Morris Kline when he speaks of an hypothesis that will coordinate the facts (Kline, 1981, 123). In the intuitive probability framework of analysis proposed in the present book, this idea holds not only for physics, which was what he had in mind, but also for historical studies. There is no alternative. Historical causality-arguments leave us sinking ever deeper into the quagmire of infinite regress where one station is as good as the next. Trying to quantify a *causality* relation, Heisenberg notes, would take us to the end of the Universe (Selleri, 31).

The causality-effect chain could be monitored quantitatively only by counting in the entire universe, but then physics disappears and we wit back with a purely mathematical scheme (Die Kette von Ursache und Wirkung könnte man nur dann quantitativ verfolgen, wenn man das ganze Universum in das System einbezöge - dann ist aber die Physik verschwunden und nur ein mathematisches Schema geblieben).

The same applies to similar pretensions in our "soft" fields, only we would never discover that we were approaching some "end of the universe", there being no boundary markings.

The models and observations construed above trivia-levels are *artificial constructs*. Chapter 1.11 with Fig 1.11 -1, gives an overview of the main graphic models and their interconnections. This notion will accompany us through the entire book; it certainly represents no original insight (*Burden*, 10f., 16f., 22ff.). In the present *Introduction,* just a crude picture can be laid out. Articulating it is the thrust of the entire process that constitutes the backbone of the present book.

*A*n *objec*t cannot be "defined" for what it is or was meant to be or symbolize in some dictonary-like definition, for an *object is what we make it to be* or what our historical protagonists made it to be; a claim (certainly not an original one) that will be substantiated as we proceed. So once again the question is not *What* but *How,* recalling Ryle's distinction. The object, in this perspective of *operational determination,* consists of the operations we subject it to, technical ones, cognitive etc. (*Burden,* 184 - 187).

In physics we have heard that *Die Bahn entsteht erst dadurch, daß wir sie beobachten (Werner Heisenberg).* Yes, the path comes into existence only as we observe it; also in a non-physical environment like the one the present book invites us to move around in. For the course we are going to follow in the analysis process concerning the primary object, depends on our handling it cognitively and conceptually, and it thus comes into accessible and manageable existence.

The analysis absorbs and takes possession of the object, sending it back to us in a workable condition or format. This is "the" moral of the book (*see* SL, *Burden,* 184ff., with references; and 4.7, here, for further support.). This principle may be labelled *operational determination.* Obviously, such a "determination" is a *process* and rarely a final decision.

For this theme one might go back to Kant, namely that *our object, whatever we find in our way, beyond mere trivia, comes as we make it ourselves by our conceptual elaboration*. In the quotation from the physicist Werner Heisenberg the intended *path* is of course that of an electron. It does reflect, however, a more general statement by Albert Einstein: *It is fundamentally the theory that determines what we are able to observe* (*Erst die Theorie entscheidet darüber, was man beobachten kann*) (quoted several times by Heisenberg, among others in his *Der Teil und das Ganze*).

Thus an object is an *event* or a *process.* Regarding the topic of *causality* (an idea irrelevant to cases when we have systems in motion), Salmon discusses the relations between processes and events, deploring the reduction of causal processes to *chains of events,* which he finds seriously misleading (Salmon, 1984, 139ff., 156). In my modest use, a process is not simply a linear chain or sequence of events, but motions and interactions in some system; for an event is itself a process, perhaps eventuated on another (lower?) level. In the region of computer programming a process *is* treated as a succession of events (Krakowiak, 109ff.). But then event is taken to mean *activity* in serial or parallel processing of *discrete elements*; and the "atoms" come in *discrete* format (a serious obstacle to "strong" artificial intelligence; cf. the graphic models in Truss).

-d *Methodology and general model use*

The assignment to investigate such interlocking networks as contemplated in the present book is not primarily to carry out a historical investigation, even though this to some extent will have to be attempted in order to fill out the framework with substance. The purpose, as hinted already, is to develop, at least discuss, *analysis methodology*, especially with regard to *connecting hard and soft evidence*, using the abstract structures in the former to handle the latter. This means that empirical observation, as always in serious work, is subordinated to the theoretical standpoint. What this platform amounts to, should, I hope, become clearer as we go on.

Let me state at once that my use of *models* primarily regards visual or graphic models (*see* also 1.7). In most cases, some kind of abstract graphic model (like a lattice) is the most efficient in *displaying the variable structural properties of a concept,* thereby offering us relevant visual control and problem definition. This procedure is more constructive and reliable than a linear procedure in normal prose. A graphic model *shows structure*, to quote Richard Skemp.(SL. *Burden*, 158f. for Boltzmann and Skemp; 30f. for visualization versus verbal accounts). In the procedure I shall go through I shall be weighing against one another possibilities (later on elevated to *probabilities*) without any illusion that my pictures will be definite or confirmative. Nor clear in the usual meaning of the word, for, taking the lesson from calculus seriously, the only manageable exactitude is a product of *approximation.* Bertrand Russell was definite about this: *All exact science is dominated by the idea of approximation;* and Georges Bidault, French prime minister some decades ago, said the same more directly: *Méfiez-vous des idées claires, elles sont rarement exactes.* (for the idea of *approximation*, *see* 2.2.2 (§8),.3.4.2, 4.3)

There are *three procedural tacks* for the progress of the present discussion. *First,* as I said in the opening of my *Burden* of 2000, I am trying to discuss the *limitations* of what we can do with such cases as have just been cited. How far can we go? I am a little less optimistic now than formerly, as we shall see, as to be possibility of establishing recordable limits. When the ruling parameter all through is approximation, this is not to be expected. The related problem of *objectivity* will be addressed later (4.3.3).

How is the object related to the relevant technical, practical, ideological and cognitive interlocking networks? Even though one might say that such relations are thinning out and almost vanishing when the distance becomes great enough, it is not possible to say exactly *where* they cease to be relevant. For such networks are not static; they are dynamic and usually active on several levels in various manners of interaction. Thus, also from this viewpoint we have to accept that there *are no limits we can define better than in terms of approximation,* starting out, as I said, centrifugally from internal positions.

Secondly, let me repeat that I have tried all methodological devices out on *one and only one* object. Only a single object of primary focusing can offer consistent reactions to our attempts, resist some of them and force us to clarify and revise, while a cluster of test objects drawn in sequentially will easily at any stage contain one that *does not* resist, and the tendency will then usually be to pick one's way wherever there is an opening. Analysis has to start out from *one object* and stay with it to the bitter end; and *work its way centrifugally out into the environment or the network(s) in which it takes part.* The circumstance that realities in nature have no clear boundaries or outlines and must be studied starting from inside them working one's way outwardly, was realized by the French impressionists, as John Rewald has emphasized in his book *The ordeal of Paul Cézanne*, with reference also to similar ideas in Honoré de Balzac's *Le chef d'oeuvre inconnu.* They also were aware of the absence of absolute reality and that realities are being created by the human mind, as noted by Rewald, who quotes Guy de Maupassant in this connection.

Finally now to point *three.* I shall use *one model* (with relevant submodels) all through my inquiry. Strictly speaking, the entire course of my argument is a model, indeed, the main-level model, summarized in 1.11 and with *Fig. 1.11 - 1.* As stated in the *Abstract*, most of the material discussed in *Part*s I and II represents strings of tendencies under the common denominator of change and instability; features in design, general culture and science, that were managed as resources the handling of which demanded specific, in part developing new tools - the incipient *calculus* being the most outstanding among them. At the end, the features will reappear in the format of a kind of pseudo-vectors denoting probable tendencies (4.5 and 4.6). It is this model application that is my answer to the challenge of *object description.*

Sticking to one dominant model is the only way in which to *discuss theory* and to obviate the usual emergency solution that consists in picking up ever new devises whenever new issues arise and one's bag of tricks does not deliver any longer. Some works in management and the social sciences shilly-shally between models with too much of an ad-hoc character, without referring them back to a dominant or central one, so that one does not know where one is at any moment. Proceeding analytically with intuitively accessed evidence usually faces the problem of *where to find the principle behind the efforts.*

The only option we have to compensate for the lack of precision in a field where, above the layer of *trivia*, only few and inessential things are amenable to formalization, is to *turn a model into a principle*, which, to reiterate the point, requires us to focus on *one object* and employ *one model* (possibly with a submodel or two attached to it).

Deferring till later to specify at an operational level what I mean by a *model* (*Part* IV, with reference to *Burden*), let me endorse, *mutatis mutandis,* the "moral" expressed in the following terms by Herbert Simon in his autobiography (*Models of my life*, 73*):*

> *Organizations, it appeared, could be understood by applying to them what you know of human behavior generally. Where specific experience was lacking, metaphors and analogies might fill the gap.... But this reliance of administrative theory on common sense was not entirely acceptable to me. Systematic observation and experimentation were badly needed if this field was ever to become scientific. But until someone built a satisfactory theoretical framework, it would not be clear what kinds of empirical studies were called for.*

For my employment of the stated kind of models I appeal to Heisenberg's comment about Niels Bohr's atom model with its planetary orbits. Bohr, Hheisenberg claims,

> *believed in his pictures of the atom, less in his own hypothesis about the atomic reality behind them (Es ist also gar nicht so sicher, daß Bohr selbst an die Elektronenbahnen im Atom glaubt. Aber er ist von der Richtigkeit seiner Bilder überzeugt)* (Heisenberg, 1996, 49).

A small digression with reference to Miller's comment on this subject. To obviate the problem that Bohr's picture was *encumbered with meanings inappropriate for the atomic realm,* Heisenberg, according to Miller, substitutes *Visualizability* for *Anschauligkeit, reserving "intuition" or Anschauung to denote the prior state of affairs in atomic physics* (Miller,1996, 69; a subject treated at great length in his earlier work, *Imagery in scientific thought,* see the Index under *Visualizabilty,* also on the role of geometry). Bohr's view of pictorial models, mental or graphical, or better, Heisenberg's interpretation of this, provides a lead for analysis of the sort undertaken in the present book. The picture modality (verbal, graphical, whatever) is the only "reality" available for our analytical (and in everyday emotional involvement and vagaries) management of it. *Each and every stage in a process of analysis is tentative, and pictures representing theories lay out road maps for further possible directions.* For such models I shall use the label *picture_matrices* (*p_matrices*), since this format, even in the figurative sense applied here, may be a set of *pseudo vectors:* tendencies..

- e *History of design*

From the vantage point just set down, a main line in the parallel courses in the book becomes an exercise in the study of *history*. A physicist tells us that

History is the most fundamental of all sciences because there is no field of human knowledge [or *awareness*] *which does not lose its scholarly character if we forget the conditions under which it came about, the questions that it answered and the functions it was meant to implement* (Erwin Schrödinger quoted in Selleri, 1: *Die Geschichte ist die grundlegendste aller Wissenschaften, da es kein menschliches Wissen gibt, das nicht seinen wissenschaftlichen Charakter verliert, wenn der Mensch die Bedingungen vergißt, unter welchen es entstand, die Fragen, die es beantwortete, und die Funktion, der es dienen sollte*).

At any rate the *historical concepts* to be briefly examined in the pages ahead will be presented in a way that university historians might find "skewed"; which is the way things must be, since the story must respond to requirements of a theory-premised framework. Where, moreover, do we come across "professional" historical narratives that do not share the same debatable property, even when a fiction of objectivity and sufficient coverage is being upheld?

To repeat, the main focus is on Borromini's Spire. Facets in its build-up as this emerges in the analysis process will arise at relevant stages. It will be accompanied by considerations of other works which find their place in the flow of the investigation: works by Caravaggio, Bernini and Gherardi, as well as other productions by Borromini. *Design, along with architecture has gained some respectability in engineering and in the relatively new digital world; as the concept of design is worked out in Winograd's (and others') Bringing design to software, of 1996, at its 11th printing in 2006. A more comprehensive, functionally structured and dynamic sense of design is given by Herbert Simon in his The sciences of the artificial (Chapter 5, 111ff.). I take it that he has arrived at this concept under the impact of two other of his important theories, on the one side, problem solving, and on the other, planning. In this perspective, most human products, mental and physical, can be subsumed and analysed under the model of design procedures.*

-f *Special terms*

Thinking in the modality of *open-source* as just explained, leads me to use *new or modified known operators*; here are some of them.

- The object under analysis is a product of the analysis process: a principle here called *operational determination* - *d*, above, and 4.7).
- The sociological issues in the virtually undocumented context in focus here, cannot even be approached. The most one can do, is to indicate with an abstraction the relevant probable *competence distribution* (above, - *a).*
- There are two basic terms involved; syntactically, *structured argumentation* (1.11), and, thematically, *systems level and elaboration level* (1.6, 1.6.1). The latter is especially relevant in the contexts of science (not in the main focus here) and religion in the Roman Church; in which contexts some operators may be listed here:
- *relevance pattern:* are selected in accordance with whatever can be acted upon, can be handled, can provoke *action* (4.1.2).
- *iconic interface,* a term for the area that stays closest to the core of the system itself (1.7);
- *canonical system* (1.6.1); the cluster of actions and doctrinal directives in the liturgy, theology, ecclesiology and Tradition in the Roman Church; and
- the *diametrical conjunction* (1.2); the fundamental notion that heaven and earth (the Church celebrating at the altar) are conjoined in the Mass.
- *Positional recall* concerns the role a subject or theme plays in a program on account of its position in a space with definite functions (1.6.2). The relevant distinction concerns *altar area* (including apses, altar-pieces etc.) and the *support areas* (congregation space; walls and vaults not spatially close to the altar):
- *Dictionary attitude* is a reason why systems issues are not understood (1.6.1).
- On the syntactic level, again, there are the operators *shape and form* (below, here),
- *Picture_matrix* (4.6, and 0.7.5 - d)*;* and on the model level:
- *solid_state* and *fluid_state* (4.4)*;* and
- probability matrix and *pseudo-vectors* (4.5.1).
- *Hardframing* (4.3.1), not a linguistic beauty but chosen to indicate a central interdisciplinary perspective: using structure from quantifiable subjects (info, for instance) for handling non-quantifiable subjects ("soft" values and categories).
- *Configuration space* (1.6, 1.6.1 1.7 and 4.1) refers to the limited area of conceptualization, model-building and argumentation in which one specific object or a cluster of them, is being handled by means of a *structured argumentation* (1.11), convertible into a system (pt. *g*).
- *Planners* in a public venture include authorities, experts, consultants and the artist(s): (3.1)

-g *Order, structure and the systems issue*

A *system*? A mono- or plurilevel pattern of interconnected units in which the weight of the units and of the links between them may be groupwise or entirely constant or variable. So far, in this model of a system, the descriptive structure of the pattern. The systemic properties arise conceptually (or physically, as in a machine) when activization by some use or functioning occurs (I would reserve the adjective systematic for any descriptive order and nothing more). The idea is illustrated in Setion 3.3 and Figs. 1.7 -1, 3.3 - 1 and 4.4 - 1. Classical accounts by Von Bertalanffy, West Churchman and, more recently, Anthony Wilden's System and structure, second edition 1980);

The idea can be developed a little further. *A sequence is a set of numbers u_1, u_2, u_3, in a definite order of arrangement (i. e. a correspondence with the natural numbers) and formed according to a definite rule* (Wrede and Spiegel). This descriptive *structure* acquires *systems properties* when used or otherwise activized, for example in operations pertaining to calculus integration.

Now we look at the interrelations between order, structure and system.

I would use *order* for any relationship between two entities, while a resulting *structure* would be the overall or partial pattern of the arrangement of them in a larger image. Thus the order between any two members of the number line (1, 2...), when repeated infinitely to constitute the number line, illustrates the structure. When put to use, for example for developing a sequence for use in integration, the structure becomes a system, since some kind of dynamics is being introduced into the image - causing variability.

The flowchart-like process image in 1.11 is called a *structure*, just to look at and study for consistency etc. When put to use, it is turned into a system. The systemic figures referred to above are so labelled because their employment is in part taken for granted and in part actually integrated in them (for *object orientation, see* 4.3.4).

We have a good example in the Canonical System (1.6.1), in which a dynamical system acquires increased or extended dynamism and confirmation by the ritual factor (the liturgy and ceremonials; for rituals, *see* SL, *Burden, passim*).

A system of soft (not purely quantifiable) values, however, presupposes that the underlying conceptual apparatus (from, say, the cognitive sciences, "soft" information, and others among the resources cited in 4.3 with *subsections*) will a) work together, and b) support the system and its dynamics. Systemization is the only dimension we have for what we use to call *explanation: e*xplanation is a by-product of systemization (Radnitzky, II, 102), ruling out the Why (survey in Silverman), substituting the How. (*see* also 3.3).

The interrelations between the four Parts that make up the present book should be evaluated for their systems status or potentials. In 4.3 with subsections, resources related to this assignment are being discussed.

1PART I BORROMINI’S CURVES AND SURFACES

1.1 Critical patterns

In this *Part*, some specific cases will be discussed which on certain levels display a common denominator. These are in part specifically determinate levels, but include also the level of *emergent properties,* the abstraction of a sum of features in a system that is, however, more than and hence less graspable than the sum of the specific features as evaluated separately (4.3.4). The instantiations of the common denominator more than the separate events represented a *critical pattern*, since this is what people often seem to care most about emotionally and hence intellectually, and because a factor of imbalance or instability is usually involved.

Around 1600, a highly interactive international scholarship in intense exchange of letters and books on physics and mathematics, including Copernicus’ *De revolutionibus celestium orbium* (widely read: Gingerich), found itself frequently and under varying conditions on a *critical edge* beyond which lay something vaguely perceived but only fragmentarily amenable to technical and conceptual handling and manipulation.

This "something" we today might call *interrelated changing entities* (Tenenbaum and Pollard, 1). I want to use the term *denominator* for this, since the substantive material is going to be related to it. The term is mathematical and denotes the principal dimension of the then inchoate mathematics of the *calculus.* In addition, there were many phenomena in life and nature for which the term and the perspective just mentioned do seem to apply metaphorically, or better, may be made to work in an analytical framework.

Observations on some aspects of artistic works by three or four personalities active around the year 1600 and further on, set the strategy for pursuing the central questions of the present book:

Within each their respective fields, the painter *Michelangelo Merisi da Caravaggio*, the theologian, philosopher and science theorist *Giordano Bruno* and, as we shall see later on, the architect *Francesco Borromini* and others, like *Bernini* and *Gherardi*, developed concepts and visions that hovered on a critical edge between what was more or less certain and what was vaguely perceived, seemed possible and approximately manageable.

Their view of the world seems to accept change, variability, motion and to some extent even imbalance between conditions of reality, on one side of an edge, things we know and can control conceptually, and on the other, fathomable worlds not as yet charted. In this outlook they were exponents of prominent tendencies of the culture of their time, also in science.

Fig. 1.1 - 1, Rome, Sant'Ivo alla sapienza (Borromini). Courtyard view.

While the painter and the philosopher evaluated their world directly through an optics of oscillating visions, the architect, specifically Borromini, experimented with shapes that literally had left their moorings in the stable world of Euclidean geometry - he must have been aware of this. His design grew into shapes that challenged, even defied, the mathematics of the day and called for novel techniques, thus contributing to new networks that stimulated and justified intensification of the drive toward the mature calculus. Others created spaces in architecture, design and painting of a new kind of complexity; this too hovering between the presumably opposite modalities of what is real and graspable and what is to be approached in an seemingly non-factual but attractive and challenging world. Works by *Caravaggio, Gianlorenzo Bernini* and *Antonio Gherardi* will be considered in this connection.

This system revision came very gradually and certainly not linearly or continuously, but it does seem to have meant *accepting approximation* in one's knowledge and measuring of the world not as a necessary evil but, at least in many regards, as a *promising path to pursue*. At the same time, physical problems arising in connection with moving objects involved *indetermination* in a manner that Drake has compared to the uncertainties arising in quantum dynamics (Drake, 1989, 77). The comparison can be carried further. The debates in the protocalculus context, such as between Cavalieri and Guldino (3.5.1 and *Supplement,* 5.2.1), remind one strikingly of those in the 1920s about quantum theory as reported in the listed publications by Fischer (2002 and 2007), Heisenberg and Selleri (*see* the *Bibliography*). The controversies took hold of everything: issues of calculation techniques, of the conceptual bases for this (what sort of units or elements are involved?), as well as fundamental philosophical questions. Both "movements" coincided with novel experiments in the visual arts (as noted by Busche in the *Nachwort* to Heisenberg, 2006); but I am not going to enlarge upon this analogy.

Something remains to be said about the *time interval* in focus. For the sake of brevity, the term around 1600 will occasionally be given in the notation ±*1600*, which in the present book will mean more of the plus than of the minus. For the mathematics development, which is to a very great extent recordable, it has seemed convenient to count the time from 1629, when Cavalieri became professor at Bologna and announced his idea of *indivisibilia*, down to 1656, the year in which André Taquet published his *Arithmeticae theoria et praxis*, thus to include 1655, the year in which John Wallis published his *Arithmethica infinitorum.* These chronological termini serve merely to set the start and end for the choice of historical material for a somewhat close consideration; nothing more; no meaningful periodization is being proposed.

In general, time limits will be merely indicative. For most historical material, no delimitation from "before" and "after" is constructive when getting down to essentials, periodization here being analytically useless. *In analysis one focuses on some crucial issue and expands one's view in centrifugal directions*. The idea of period is often being supported by the one about some kind of *the spirit of the time/day*, for which the Germans have their *Zeitgeist.* Such concepts may be useful for the budgetary subdivision of teaching and personnel policy at schools. Klaus Eidam, in his fundamental and newly researched biography of Johann Sebastian Bach, very efficiently dispels ideas like these, calling the attempt to put thing in drawers (*Schubkasten*) an appealing to *space without coordinates* (*Raum ohne Koordinaten*) (Eidam, 329).

My choice is to focus Borromini's *Spire* and things probably close to it in terms of shape and form. In my case, I would rather say, we are facing coordinates without space, since we have handles on coordinates (for configuration space, *see* 1.7).

In the pages closely ahead, I shall focus on some works by two historical protagonists who seem to represent a common denominator, not in substance but in *method of handling it*: Caravaggio and Giordano Bruno. The intention is not to write a piece of history but to abstract some critical features that seem pertinent to the specific scenario proposed in the present book and which can serve as a platform for advancing further into the subject

1.2 Comparing methods: Caravaggio and Giordano Bruno

As noted already, a striking feature in many works of the Italian painter *Michelangelo Merisi da Caravaggio* (1573 - 1610), and of some of his followers, is the *use of light and shadow in such a way that the scenery seems to be on the brink of changing from one to the other* (*Fig. 1.2 - 1,* in the next *Section*). Much is in a half-light that could turn the next moment to a Monteverdian *notte bruna,* or to full light - something that reminds me of *Giordano Bruno's* (1548 - 1600) approximately contemporary use of fluctuating light and shadow as a methodological operator in his book *On the shades of ideas* (*De umbris idearum*, 1582). No substantive comparison is intended; on that level it would be incommensurable.

We are facing comparisons between between rules or principles for *the use of or appeal to* visual imagery and for philosophical arguments. Then the question obviously arises about the *cognitive and operational levels* on which such a correlating venture might make sense.

Bruno uses *shade* to define both the presence of light and the vanishing of light in combination (details in 1.3). The *verbal* description of alternation between light and shadow and of the one emerging within the other, involves a time dimension between states that is not reproducible in a painting. On the other hand, marked pictorial realism that involves visual signs of change, even suddenness, can compensate for this by having the onlooker impute the time dimension. The common denominator will then arise in terms of *method; using a play on contrasts to approach and come to terms with whatever is in focus.*

In the two cases the employment of the idea in one medium or another works differently but has a common source. Light and darkness is a fundamental image in Catholic theology and liturgy (and Islamic, for that matter). Light emanates from the Holy Spirit - this is illustrated almost in every church and chapel, also Sant'Ivo (on the pseudo-drum) (*Burden,* 78f.). The notion of a protective shadow is Biblical (and as such is quoted by Bruno; *see* below), giving rise to a theological commonplace. In Bruno's book, *De umbris idearum,* the shadow is *not* a physical or conceptually contemplated object in the focus, it is a *model for a cognitive process,* which has a time dimension, and which shall be *used on objects.* Caravaggio's use of light and shadow in his pictorial spaces means that for *objects* in his realistic world, which had Catholic endorsement, he introduces an element of oscillation between conditions or states which arises from real-world experience and religious engagement.

In both cases, we witness expressions of rapid shifts between conditions, but these expressions arise on levels that allow of no analogy further than this. The levels concern the

How not the *What*, and the relevant distinction is between the use of *cognitive shadows* and of *object shadows*. Yet, the latter case, too, concerns a way of approaching and conceptualizing a subject or an idea.

While for Bruno a solid body of information as to his relations to the world, the scholarly life and to the Church is available, in Caravaggio's case we fare less well. Indeed, his fascinating and reputedly modern attitude has tempted some scholars to simplify by eliminating from their discourse the fundamnental system in his work, which was in full consonance with the teaching and practice of the Roman Church, while fastening on the elaboration level of style. It is incumbent to bring in a corrective note. This can best be done by citing one specific but typical case. Discussing it will also furnish us with material for a more general framework that will come in usefully for further argument development, especially with regard to the distinction between *systems level* and *elaboration level* (1.6).

1.2.1 *Caravaggio and the diametrical conjunction*

I am referring particularly to H. W. Janson's *History of Art,* any one of the many editions. My purpose with this rather detailed discussion is not to prove a great man wrong on some specific point; no one is immune against making mistakes. But Caravaggio is an important protagonist in my argument, and his position would be shaky if he were stored away in a niche as an unorthodox "genius". There is no trace of unorthodox attitudes in his works; painted representations of dirty feet and prostitutes are not theological issues. Here again we are faced with two levels, a canonical religious system to which Caravaggio adhered and an elaboration level at which he made new proposals, not all of them authorized.

In Janson's universally accepted account (ploughed through by generations of art-history students), we read about Caravaggio's *Christ calling St. Matthew* in San Luigi dei francesi, Rome (1590s). (*Fig. 1.2 - 1*).

> *...Never have we seen a sacred subject depicted so entirely in terms of contemporary low life. Matthew, the tax-gatherer, sits with some armed men - evidently his agents - in what is a common Roman tavern; he points questioningly at himself as two figures approach from the right. The arrivals are poor people... Why do we sense a religious quality in this scene? Why do we not mistake it for an everyday event? What identifies one of the figures as Christ? Surely it is not the Saviour's halo (the only supernatural feature in the picture), an inconspicuous gold band that we might well overlook. Our eyes fasten in stead upon his commanding gesture, borrowed from Michelangelo's Creation of Adam...*

The author even claims that Caravaggios picture shows a *lay Christianity, untouched by theological dogma*; without saying what he means by "lay Christianity".

Fig. 1.2 - 1 Calling of St. Matthew, San Luigi dei francesi, Rome (Caravaggio).

It is incumbent to correct these misconceptions, lest Caravaggio should disappear, historically speaking, inside a niche all by himself (with a few followers), and no longer remain amenable to being evaluated integrated in the larger cultural picture. In serious analysis, the genius has to go unemployed.

First some details. Well-attired and armed gentlemen, in fact, customs officers, hardly represented low life by the standards of the time, whatever the cited author might have in mind with such a classification. St. Peter does not appear as a "poor" man, wearing as he does a manifestly “biblical” - and hence "religious" - garment with which no contemporary Roman, rich or poor, would be seen on the street except perhaps at Carnival.

The entering figures of Jesus and St. Peter *both* point toward Matthew, Jesus with the normal gesture always in use in Italy for summoning someone, a friend across the street, one’s kids; you see the gesture constantly on the street (not perhaps at international conferences and cocktail parties). Paul Theroux, with no pretensions to teach us about things Italian, knows better than Janson:*’pray’, Father Cruciani said. He then beckoned, Italian fashion, dog-paddling with one hand.’Come’* (*Dark Star Safari,* 2002).

To compare Christ’s hand gesture with the one with which God infuses the spirit into Adam in the history of Creation, is entirely superfluous and, if one should take it seriously, would create theological confusion. Calling one of twelve apostles and infusing the spirit of life into the First Man during the act of Creation are two fundamentally different things, as elementary theology will teach us.

There *is one dogmatic reference* in the painting, one which the cited author tells us we do not need to note at all: *Christ’s golden halo,* denoting his divinity. If there are no further explicit dogmatic references in the picture, it is because the relevant Biblical text reused for the liturgy, would not require one; the dogmatic context being obvious to all contemporaries with some education who saw the picture in a chapel in a church: a *positional recall* (1.6.2).

Now to essentials. The cited description misinterprets the depicted scene on two crucial points. The *first* is the erroneous but popular belief that *depicting in a church a Biblical event in the realistic appearance of an everyday urban scenario* would reduce or even nullify the religious guidance of the message. Turning to music, it may be noted that Emilio de' Cavalieri's *Rappresentazione di anima e corpo* of 1600 (performed at the Filippine Oratory in Rome) also is noted for its programmatic simplicity and realism. The *second* notion is not stated explicitly but lies implicit the quoted account. It is that painters working for patron families in a public Roman Catholic church were allowed to indulge freely in their individual inclinations and turn the commission into one that was customized for personal satisfaction; that the attitudes of artists determined not only formal and stylistic values on the elaboration level, but also the message system itself.

There is no room for doubt that Caravaggio in all his work acted as a completely orthodox Catholic. To affirm this is not to say anything about his personality; it is an information about his working conditions and the criteria for official acceptance of paintings in a public context, especially in a church with its strict rule system.

Whenever we come across documentary evidence about the planning of pictures for liturgical environments such as churches, even for family chapels paid for by wealthy patricians, and political ones such as government and municipal buildings, we are confronted with theological or political or administrative *experts*, who were employed for setting up the message programs, usually under the control of superior authorities (Gisolfi and SL, 73ff; but the matter is too frequently documented to require specific references). The Church, who needed imagery for liturgical and instructive (such as the Catechism) purposes, did not want to confuse or alienate the congregations, nor to be suspected or accused of theological and liturgical incompetence or worse. State, to consider them, would customarily make a point of displaying pictorial decorations for the purpose of propaganda, celebrating their more or less doctored history and political morals on diplomatic occasions. Isabella d'Este took the Emperor to the *Camera Picta* with Mantegna's frescoes, which celebrated the dynastic takeover of the church of Sant'Andrea (rebuilding it) with the relic of the Holy Blood (my blog in *Staalesangiovanni*). In Venice, all important representatives of foreign States were dragged along on guided tours of the Doge's Palace to be impressed by that great show of historical myth and celebrative rhetorics. The State would not want to be exposed to diplomatic ridicule by accepting non-professional ideological programs and frivolous expressions of them; political theory had became increasingly complex since the fourteenth century (SL, 1974, 134 - 155).

No artist was equipped nor authorized to handle that, nor were there any budgets for paying them for it. Occasionally, the message programs look very simple; for instance in the library decorations of about 1570 at the Benedictine monastery of Praglia (Gisolfi and SL). It takes experts to distil a simple message system out of conditions that were, without exceptions, highly complex.

Caravaggio, to return to him, was working in the Saint Matthew Chapel after the conclusion of the Council of Trent (1563), a synod that had set down severe regulations for religious art and demanded orthodox and social commitment all around.

In theology as well as liturgy, the subject of *daily life* and *ordinary living conditions* were *explicitly integrated with* and not opposed to religious and celestial matters. A perusal of many of the publications of the *Institut für mittelalterliche Realienkunde* in Krems a/d Donau (*www.imareal.oeaw.ac.at*) will substantiate this with regard to general religious culture. On the theological and liturgical level the doctrine of a direct relationship between heaven and earth is stated again and again in the recital of the Mass Preface and Canon. *We pray you, bid our voices to be admitted with theirs [in Heaven]* (*Cum quibus et nostras voces, ut admitti iubeas, deprecamur;* from the Common Preface). The teaching is carefully explained and commented in medieval writings from Gregory the Great (died 604) to Cardinal Guillaume Durand (died 1296). This is the formulation several times repeated: at Mass: *Here we sing the hymn of the angels because we have no doubt that through the sacrifice earthly things are joined together with heavenly things...* (*Postremo hic cantica Angelorum canimus, quia per hoc sacrificium terrena iungi caelestibus non dubitamus*) (details in SL, 1984, 24). There is an intimate, interacting and dynamical relationship between the celebration at the church altar during Mass and the celestial liturgy at the *altar in Heaven* (SL, 1984, 20 - 26; *Burden*, 62 - 69).

The contrapuntal interrelation between earth and heaven effected by the sacrament of the Mass - for want of an authorized name, let me call it the d*iametrical conjunction* - is often represented in painting, showing a unification of the space of the architectural church and the celestial world (Santa Francesca Romana, Rome, 1161), or between the wooden frame and the pictorial interior with divinity and saints. Bernini's Santa Teresa Chapel in Rome is one outstanding example (2.2.9). Andrea Pozzo's vault frescoes (1680s) in Sant'Ignazio, Rome, is a seventeenth-century rendering of the same kind of harmonious interplay. In two chapels in Rome, Caravaggio's realist paintings were combined in a single commission with more academic, idealized and "beautified" representations (by Cavaliere d'Arpino and Annibale Carracci) which the cited author and many with him believe are "more religious" (Santa Maria del Popolo; Sant'Agostino).

We have to accept that the *everyday* character of a depicted scene perfectly matches its *sacredness;* as is implied in the diametrical conjunction. This is the crucial point. Of course contemporary people would never fail to notice the significant halo over Christ's head in Caravaggiio's painting as marking him out for his divine nature; and they would note St. Peter's cloak. But they would hardly need such cues in order to capture the gist of the story. They would know that the scene was religious because of the subject and because they saw it in a chapel where they would participate in the Mass celebration. People had not yet learnt to look at a painting in isolation from its context.

No contemporary Roman, nor any Catholic for that matter, would find anything unfamiliar in facing the conjunction of Heaven and Earth in the liturgical environment nor misconstrue the everyday scenes as anything but religious and orthodox. Otherwise the Church would not have sanctioned the realist imagery for churches and chapels. It is quite another thing to note that usually the rich enjoy pictures of poor people more than do the poor themselves. Caravaggio worked *with* but not *for* the latter category.

Furthermore, the narrative in Caravaggio's Calling of St. Matthew is definitely liturgical in that it reproduces accurately the sacred event as it is described in the *Breviary* in use in Caravaggio's time. Here, the depicted room is not a tavern but *a customs house*, and the men sitting at the table are customs officers. This story about Jesus calling Matthew was recited every year on the saint's feast day on 21 September and familiar to all.

The first version of the *altarpiece* for the same chapel also conformed to the description in the *Breviary*: *St. Matthew writing the gospel under the guidance of an angel* (the work was destroyed in Berlin in the war), by showing the open book of the Gospel with the text written *in Hebrew*, rather than Greek. *Evangelium hebraice scripsit, he wrote his Gospel in Hebrew*, we, and Caravaggio's contemporaries, read for September 21.

Caravaggio's realism enjoyed official, but as was always the case, conditional approval by the Church. When the first version for the St. Matthew altarpiece, the one with the Hebrew gospel, was rejected, this was a trivial occurrence and implied no reaction against everyday imagery. The clergy refused to let the congregation assembled at Mass have a dirty foot right up under their nose. Caravaggio had been going too far in a realist direction which in general terms was not only accepted but cherished by powerful circles within the Church.

Our observations have shown that the case of Caravaggio cannot be pulled out of a cultural and religious context that was traditional and established, in Rome more than any other place. It is above all on account of this embedment that his *method* stands out as a critical factor in his time and place. Caravaggio was no outsider but accepted on the religious level while disputed on the artistic level; cherished by some and tolerated by others as an exponent of Catholic culture.

The story of Giordano Bruno's life is relatively well known but his many books make up a tangled image of his thinking.

1.3 Bruno's De umbris idearum and the model of fluctuation

The debate on Giordano Bruno's philosophy has been riddled by an either/or attitude; he is primarily a magician (Yates, Rossi); no! primarily a science theorist (Sturlese, Gatti). Paul Richard Blum steers a middle course, compounding the two strands. His book is useful also in that it gives an understandable account of Renaissance *magic* (Blum, 28f., 129ff.).

I am not going to involve myself in this *querelle*, limiting myself to discussing those factors that clearly relate to Bruno's *method of cognition,* as they are worked out in his book, *De umbris idearum* (The shades of ideas/concepts/notions/thoughts) (detailed repertory in *Supplement* I). In this *Section,* the subject is the foundations of his theory; for the working of his *umbra, see* 1.3.1.

In terms of what kind of methodology did he approach and treat whatever interested him? this is my question; an enquiry concentrated on his *De umbris,* since he is unpredictably shifting focus, accent and perspective from one book to the other (at times from a chapter to the next). He seems to be groping in many directions in order to *catch a falling star,* to adopt an expressions of John Donne's.

In the present discourse, I am not primarily concerned with the use of light, shade and darkness on the *interpretive* level, which in this connection is accidental (as the Aristotelian *Peripatetici* would probably say); nor the philosophical *content* in his writings, but *how* he develops his themes. It is the burden of his thoughts on the *methodological* level that I am trying to get the sense of, the *circumstance that he does use such a configuration for a cognitive framework,* which is a methodological one. It is so because he uses it as a model of the reality he is out to explore, as a handle it offers with which to grasp essentials of the world and the universe.

What is this *umbra* and how does it work, technically speaking? - *technical* then taken in the model sense not the mechanical. And how to visualize it? Let us first ask *Tradition* (with a capital T, i.e. the tradition of the Church; S L, 1984, 18f.).

Summing up the various connotations of the term *umbra* in the Tradition of the Roman Church, Francesconi essentially says that the shadow is a provisional, passing thing dependent on and constantly related to whatever concrete object might produce it. *La caratteristica che accomuna tutti questi usi è la dimensione di provvisorietà e di relatività insita nel significato originario: l'ombra esiste solo in dipendenza e in costante riferimento all'oggetto che la produce* (Francesconi, 199, for the following, 199 - 216). The tradition in Christian literature is primarily using the term *umbra* with direct reference to places in the Old Testament, which, as cited by Francesconi, focus on situations, events or happenings involving man (such as death).

St. Paul's letters somewhat shift the ground, laying the foundations for another tradtion. Put briefly, the *umbra* here represents the image of what is to come, as the world *sub lege* foreshadows (!) the world *sub gratia*, and thus becomes a typological figure (*umbra futurorum*). This notion is very much in use in St. Ambrose's writings. In early writers, on whom St. Ambrose's use of the term in part depends, *umbra* becomes an *archetype-image* of Platonic-Alexandrian stamp. To simplify the matter, all the cited connotations are to do with human conditions in worldly, salvational and celestial perspectives, including the world of the Old Testament in its relation to Christ and the Church (again typologically), and with Christ himself in focus in related roles; *never* with cognitive or conceptual managing on the part of man.

Applying this particular modality to the term of *umbra*, Giordano Bruno appears to break with a section of the established Tradition - once again, one might say.

Vocabulary senses of *umbra* in classical Latin that seem relevant are at least the following ones (but we should have to consult scholastic literature as well: Francesconi): 1. shadow as from a tree; 2. shades and half darkness in a painting or other visual pattern; 3. night darkness (*umbra noctis*); 4. protective (*sub umbrae romanae amicitiae...*); 5. in which to be peacefully enclosed or ensconced; also in buildings etc.; 6. a companion, for example in social fellowship; 7. shadow(y) image, apparition, ghost - (cf. also *umbraculum* with comparable meanings).

If the *umbra* is a tool for accessing real things, then meaning nr. 1 above, the shadow of a tree, would be an effective image since here the internal content of the thing is not visible in the shadow, only its *outline*; a geometrical simplification of the essential characteristics of the thing, which allows us - and this is crucial - to encompass it and thus describe it: a methodological tool not alien to a scenario in which geometry still at Bruno's time and later, presented a cherished medium for appreciating objects cognitively. But later we learn that such a shade is not completely effective, in which case Bruno's *umbra* must derive from a combination of the listed concepts.

In the first *intentio* concerning the shadows, Bruno takes as his point of departure a quotation from *The Song of Solomon*: *SVB VMBRA ILLIUS QVEM DESIDERAVERAM SEDI*, in which the shadow from him whom I desired, must be of the same kind (Supplement I, *Intentio* 1): his shadow determining the terms of approach and, given human limitations, of cognizance concerning the One desired. In default of precise text references from relevant theology or philosophy (which the cited studies do not supply), I have to admit that the status of the quotation is not clear. It cannot be *used* for anything at all, but the general context of Bruno's book may at least be invoked in the hope of understanding the Solomonic *shadow* as something more than just an image of protection.

The geometric outline model presented in the next *section* (1.4) may turn out to work. Our normal mental operations seem not to be accustomed to separating an object out from its application as well as its usual context. But in many research programs, we need such a separation. Here, we have recourse to heuristics, in the present kind of case, we can invoke *object-orientation* (4.3.4). I suspect that one reason for the impenetrable nature of Bruno's philosophy was that heuristics were not available, or at least not commonly accepted, as a technical device. Bellarmino's entirely pragmatic solution concerning the cosmological issues would hardly have been serviceable in his domain (2.2.10).

The *De umbris*, irrespective of the difficulty just identified, does not seem to concern primarily objects as such. Rather Bruno prowl around for methods for approaching and knowing them. The title of the book is *De umbris idearum,* shadows of something through which he perceives or takes cognitively hold of whatever interests him; that is, shadows carrying implicitly light, of some object whose universal ingredients are called forth through the process of oscillation between shadow and light.

Taken at face value, the idea of illustrating sight and understanding in terms of *light* by contrasting it to darkness and *shadow* is as natural and commonplace as the underlying notion of accentuating one thing by contrasting it to another. Indeed, a similar idea is briefly hinted at in Thomas Mann's *Buddenbrooks* (10 Teil, Chapter 5), the alternation of light and darkness denoting the protagonist's alternating vague adumbrations, sudden insights and lack of understanding, But Bruno does not let us get away that easily.

First, let us hear how the gist of Bruno's approach has been synthesized in two relatively recent comments concerning his shadow/light imagery (Blum, Gatti).

In his *De umbris idearum* Bruno evokes the notion of contrasts (to cite *Paul Richard Blum* in one of the most satisfactory studies on Bruno) in order to explain

the undivided principle of thought and faculty of understanding and perceiving... with the focus on'illuminating the shade', with its double function of light and of loss of light (*... Aufweis des einheitlichen Prinzips des Denkens und Erkennens... im Hinblick auf die 'Klärung der Schatten', ... in ihrer zweifachen Funktion als Zeichen für das Licht und das Verlust des Lichtes...*) (Blum, 59f., 131f.).

In certain aspects relying, more less directly, upon Cusanus' *Coincidentia oppositorum*, Bruno's argumentation is grounded upon contrasts that presuppose one another, and hence also upon the idea of motion (Frigerio, 67ff. and *passim*). Blum also focuses on this play on contrasting and mutually activizing entities (*... ein welchselseitiges Verhältnis*) (Blum, 131ff., cf. 61f.).

Blum's observations on this subject, as I understand them, may probably be summed up as follows. Bruno's ideas were *in some regards* culled from Marsilio Ficono's philosophy of levels of existence. And the English university *pedanti* - Bruno's term - could not see the difference and accused him of plagiarism. Specifically for Bruno is the notion of an alternation between light and shadow, in which the one is a sign of the other, such that the relationship consists in the concept that absence of one of the elements is at the same time a proof of its presence.

Ficinos Lehre von den Stufungen des Seiendes, die in der Schönheit zum Ausdruck kommt, wird hier [in the *De vinculis in genere*] *nicht nur rhetorisch ausgeschmuckt, sondern mit den spezifisischen Denkform Brunos gefüllt, vor allem der gleichförmigen Analogien der Ebenen und dem Spiel von Licht und Schatten, in dem das eine Spur des anderen ist, so daß die Verbinding gerade darin besteht, daß die Abwesenheit des einen immer zugleich Indiz für seiner Anwesenheit ist*) (Blum, 132).

In this manner the notion of universal unity is established. But this works so to speak bottom-up: starting out from real-world conditions.

For Bruno the discourse on the unity of the universe acquires a sense only when it can be referred to for illuminating the structure of the world as this is accessible on the level of human senses. The point on which he criticized the Platonist-Pythagorean tradition is this: finite things are not only figures and metaphors of the Unity, for this Unity is retrievable in the things themselves, such that appreciation of the unique leads to knowledge about of the manifold which in turn leads back to an understanding of the real Unity.

Seine Kritik der Umwandlung der pythagoreischen Zahlenlehre durch Platon zielt ... darauf, daß die endlichen Dinge nicht nur Figuren und Abbildungen des Einen sind, sondern daß sich dieses Eine in den Dingen findet wie die Eins in jeder Zahl, so daß die Erkenntnis des Einen zur Erkenntnis des Vielen und die Erkenntnis des Vielen umgekehrt zur Erkenntnis des wirklichen Einen zurückführt) (Blum, 59).

The idea, directly or indirectly, is connected with Bruno's construct of *motion* in an *infinite cosmos* (Blum, 65f.).

Hilary Gatti gives a slightly different account of the *light-shadow image* (Gatti, 185ff.). Her account arises under the heading of *Picture logic*, for she (and others since Yates's days) considers Bruno's use of pictures as logical elements in *scientific argumentation* in fields in which pure abstract mathematics fail. Thus the contrast light-shadow is a thing that can be *visualized*; set up in model format, we might say.

There is a similarity between shadows and ideas; a central point in the entire model; indeed, the book is about *Umbra idearum*. Do not disregard the similarity between the shadows and the ideas, Bruno advises us (*Intentio* 21*, see Supplement* I); for neither shadows nor ideas are the extremes in a relation of opposites. For, in fact, one may proceed through both of them to further understanding and discernment, but the comparison should not let one confuse essentials with causality.

Attempting to translate his argument into modern idiom, we should first note *what he did not have in mind.*

According to Gatti (Gatti, 185ff), *the problem is to comprehend the order of things in space and time, a question on which Bruno feels the need to differentiate himself from both Plato and Aristotle*; and she quotes a piece from the *De umbris* in which the following distinctions are made:

1. Ideas of single things versus ideas of species of things (Plato stuck to the latter); 2. such ideas relate to forms not to matter; 3. cognitive intention is concerned with forms, not genus or individuals. 4. Bruno wants ideas of single things, whether they are apriori or otherwise related. Gatti concludes from this: *The shadows of the ideas, then, are the shadows or imprints in the mind of the ideas of single things, subject to space and time.* And, I might add: the mental handling of things.

Gatti appends a note of such an importance that one might have wished it to appear in the text body and more carefully commented. She refers to a book by Vere Chappell in which a distinction is drawn between *two kinds of ideas. Ideas*$_o$ are ideas as objects of thought, while *ideas*$_m$ are ideas as mental acts or events.

Gatti: *According to this distinction, Bruno's ideas of single things would be Ideas*$_o$ *and his shadows of ideas would be ideas*$_m$. *Shadows of ideas of single things, then, are the stuff of which the new science will be made.* The last statement is elaborated in the following terms: rejecting Plato's ideas and also Aristotles' basic chain of logical reasoning - a *process of logical-linguistical abstractions - over-subtle analytical logic,* as Gatti synthesizes how Bruno considered this tradition.

Bruno opted, along with Bacon and Galilei, for a more direct approach and observation-rooted appraisal. Bruno often rails against the *pedanti* who hide themselves in linguistic formulations and grammar, not caring about content, in which they would have been more vulnerable to attacks. After some references to possible sources for Bruno, Gatti goes on to what, in her opinion, was his problem, *how to define a thing.* Rejecting some facets in Plato and accepting others, Bruno would hold that *all things in the universe are composed of two interrelated elements, the active idea (or form, or soul or light), which acts from within like yeast forming the single object of the phenomenological world, and the passive, material element composed of agglomerations of atoms...*, both aspects being infinite in spatial and temporal extension. Bruno recognizes an ordinary, everyday kind of knowledge of things and also a higher and more difficult form of knowledge available only to the inquiring mind.

In synthesis, Bruno's art of memory, which is brought into play in this context, *although finding its sphere of inquiry in an infinite universe of being composed of an infinite aggregate of constantly moving atoms, is not concerned with knowing or defining either the infinity of the whole or the atomic minimum, both of which Bruno considers certain but unknowable.*

He applies a distinction here, 1. a *common ground* of the multiple subjects to be ordered by his art of memory; and 2. those multiple subjects themselves, what the ground contains - and the subject for his art of memory, which cannot fathom the ground itself, but *imposes order on finite things.* So far Gatti.

If one should try to combine Blum's and Gatti's accounts as just cited, then one might, at this stage, propose such a synthesis as the following one. *The function of the umbra is that of a dynamic model through which cognitive distinctions are operated. The umbrae are the area of mental elaboration or of being mentally offered for elaboration (action, event), with* *a view to handling cognitively and conceptually* (as we would say today) *the things that are observably accessible and thus available for the new science.*

Let me, so to speak under the leadership of the two cited modern authors, emphasize certain statements in the *De umbris idearum* (*see* also *Supplement* I).

He is concerned with *motion* and *change* (*Intentio VI,* for example; for these units, *see* below), chains or sequences between things and spiritual phenomena, connected with the light/shade model (*Intentio VII*). In *Intentio II*, we learn that we should distinguish between shade and the properties of darkness. For various combinations of them, such as discerning one of them *through* the other, is connected with truth and goodness:

> *Non est umbra tenebrae: sed vel tenebrarum vestigium in lumine. Vel luminis vestigium in tenebris. Vel particeps lucis & tenebrae. Vel compositum ex luce & tenebris. Vel mixtum ex luce & tenebris. Vel neutrum a luce & tenebris, & ab vtrisque seiunctum. Et haec vel indé quia non sit plena lucis veritas, vel quia sit falsa lux.*

In *Intentio V* we are told that the entity of shade is the *pivot of the aspects and faculties of cognition* (*Vmbras eas in proposito maximé consideramus quae sunt appetituum, & cognoscitiuae facultatis obiecta, sub specie veri bonique concepta*). This is the crucial point.

A scenario of *unity* is being conveyed in which *fluctuation* and *approximation* rule the game, instead of clear distinctions and definite states of things, symptomatic also for contemporary, and, especially future, engagements in the pre-calculus direction. It is perhaps not altogether out of place to remind ourselves of the fourteenth-century use of the terms *fluxus* and *fluens* for pre-*calculus* work and, subsequently, of Newton's term *fluxions* and his *fluxionary calculus*.

In the beginning section of the First Dialogue of the *Spaccio de la bestia trionfante*, of 1584, Bruno enlarges upon the idea of *contrariety.* All things, feelings, concepts are binary and have their *contrary* or *opposite* parts and there is continuous *motion* between them. *There is no concord without discord, what is concave belongs where there is convexity. For something to come into being, is coincidence between contraries...*; this is how Frigerio presents his synopsis of a text which in the original is not too easily accessible (Frigerio, 69ff.). Obviously, as is also pointed out elsewhere in Frigerio's account, Nikolaus von Kues' (*Cusanus'*) idea of *coincidentia oppositorum* hovers in the background.

In line with established tradition, Bruno uses *imagery* for important cognitive operations; as noted by Sturlese, Gatti, Saiber and many others; the point has always been considered a central one in G.B. You *see* light, darkness (by *not* seeing anything) and half-shade. Here he was in good contemporary company, namely that of his later opponent, Cardinal

Roberto Bellarmino: humans, whatever they know through senses or reason, they know in the shape of images (*Homo quidquid cognoscit sive sensu, sive intellectu, per imagines cognoscit*; 1583); something that was well known to medieval and later teaching institutions, in which explanation and learning to some extent were explicated in graphic models (for the use of graphic models, see the books by Miller and Gisolfi-SL). Of course an illustrative picture - of an horse, say, or of St. Peter - has a far larger range of interpretations and conceptualizations than the matching verbal expression and correspondingly a higher complexity (SL, 1984, 148ff.).

The use of imagery as an operative tool and explanatory model dovetailed with the paradigms and contemporary practice of *geometrization*. The latter, when brought to its full consequence, leads to *the introduction of the infinite* (Blay, *Introduction;* or perhaps: a further articulation of the notion of infinity?), and Giordano Bruno's preoccupation with this (*Dell'infinito, universo e mondi,* 1584) can be seen in this perspective (Saiber).

As I see the matter, reality for Bruno is Pythagorean (often affirmed) and this reality can be grasped only in terms of what we would call abstract models. This is the *How* rather than the *What* concerning his sources and his goals.

By perceiving the chains of similarities (*similitudines*) and noting contrasts - accessible in an ordered progress ascending or descending between specifics and generalia - the intellect can grasp anything. This is a central idea in Bruno's theory of cognition. The chain provides the matter for memory to work on, and the art of memory consists in setting the mechanism in action. - Who knows/recognizes the connections between opposites/comparison-terms, can distinguish anything from anything else, both intuitively and rationally.

Things in this world would not have been beautiful if they were all completely similar/ alike. Hence the <natural> shadow-vision is the most imperfect mode of seeing/recognizing, for it does not allow us to discern among the differences, for the shadow, as in a picture, presents the mere outlines and even this not <always> correctly. (*Hinc rei vmbratilis visio est visionum imperfectissima: quia quod imago cum varietate demonstrat, vmbra quod est infra extrinsecae figurae terminos vt plurimum etiam ementitos, quasi sine varietate profert* (*Intentio* 1; 5.1 for the *intentiones*).

The <cognitive> shadow is a *scientific concept*, not fundamentally a magical one, even though supported by some appendages of that kind. In operative in terms, our mental faculties have their origin here. One should not, because of the homonymity - shadow in two senses, natural and ideal - evaluate and indiscriminately compound the meanings of <the word> *shadow*. For that shadow which is protected by other shadows, should be contrasted with that shadow which raises up above the reality of things up to the borderline of what is intelligible (this seems to be the correct reading of *Intentio* 15; *see Supplement* I).

Bruno further elaborates the distinction of real shadows from cognitive shadows. Among physical shadows, such as those from a tree, there are some that serve to protect weak animals; and others with the opposite effect.:

<Amongst> the ideal shadows, however, there are none that do not guide to <the ideas> perfectly, if you ascend <to the world of ideas> through them, since they all are connected with our intellect and our internal purified cognition (*Intentio* 17; *see Supplement* I).

The normal shadow of a tree is *static* and shows just its outline. The true *umbra* is *dynamic*, involving, as if along a wave curve, alternating states revealing its counterpart, light. All the proceedings described are *dynamic*, characterized by motion among similar, diverse or contrasting things. In this way, they organize the passage from one entity to another, in a chain of connections. From now on, he says,

try to recognize how the shadow, at the motion of the light <itself>, <also> moves, as if it escaped from it, while at the motion of a body, it seems to pursue this. And this (these occurrences) do not seem to imply/involve contrariety, much rather concordance in leaving one thing and pursuing another, <whether it is> opposed or contrary (*Intentio* 20; *see Supplement* I).

1.3.1 *How Bruno's shadows work*

This seemingly pretentious "explanatory" subtitle means nothing more than presenting an *assumption* in the normal manner of logics: a provisional platform from which to start out.

Having attempted to sound out the foundations of Bruno's *shadow ideas*, let us have him explain how they work. Bruno's ideas were not always new (obviously), but he seems to have combined them in a fashion that was not common. On the other hand, it is almost as hard to be sure one reads him correctly as one would be if trying one's hand on a combination of Wittgenstein's *Notebooks* and Thomas Aquinas' *Summa*; if there really *is* one correct reading in an author who seems to vacillate to such an unusual degree.

Authors like Spampanato, Ciliberto, Della Porta and Frigerio discuss Bruno's conclusions in the *De umbris* and the historical context in which he works this out, *without examining closely the special methodological means by which he believes to achieve his results,* the tools of light, shade and darkness in fluctuation and interaction (variables discussed by Spampanato, 323; La Porta, 169; Frigerio, 127). This, however, is what Blum emphasizes and it will become the main subject in this *Section.* Gatti too refers to the methodological issue. I shall try to develop the subject a little further.

Switching between opposites and production of motion between them, is a crucial cognitive operation. It is important to recognize Bruno's philosophical contributions for what they are and not hide them in a magical hypothesis. Gatti, with the support of Sturlese's comments in her edition of the *De umbris,* goes a long way to invalidate Yates' view on this matter:

... her appreciation of Bruno's relation to the new science has been largely negative and misleading. For Yates limited his contribution to the occult and Hermetic elements in his thought... The result has been to deny him any contribution to the emergence of the new scientific discourse that was at the center of his attention throughout his philosophical life (Gatti, 8, 208).

It runs as a red thread through Gatti's book that Giordano Bruno's philosophy is a theory of science rather than one of magic (see also *Intentio* 15 below). To steer a middle course here, or better, operate with two distinct but interacting levels, let us say that whatever balance or imbalance one might attribute to Bruno's attitude, there is no doubt that his *theory of science* is sufficiently marked-out to merit, and require, a focused treatment.

Another point is his integration of the idea of human conception and knowledge acquisition as something *not regular and "Euclidean" but highly variable, in other words, of human nature.* In this sense his work seems to support the main perspective of *interrelated changing entities* adopted in the present book (1.1).

Rossi, however, in his *Il tempo dei maghi*, rejected some of Sturlese's and Gatti's arguments and reopens some of Yates' claims (Rossi, 2006, 81f., 141f., and *passim*). For example, he accuses Gatti of not having understood that science consists of more than just a pack of theories. He also refuses to endorse Sturlese's claim that Bruno system is effective only and exclusively in the way of operating with manipulating symbols (*si realizza solo e proprio in virtù del fatto che essa opera con segni manipolabili*) (Rossi, 2006, 83ff.). On the other hand, Bruno himself does presuppose a certain automatism in the operation of the art of memory. This idea is present as early as in the *De umbris.* By means of signs, symbols, etc., *the art assumes a power so great that it seems to operate independently of nature, above <the level of> nature* (Rossi, 2006, 98).

Quaedam vero adeo arti videntur appropriata, ut in eisdem videatur naturalibus omnino suffragari; haec sunt signa, notae, characteres, et sygilli, in quibus tantum potest, ut videatur agere praeter naturam, supra naturam, et si negotium requirat, contra naturam (cited edition, p. 72): let me supply: even, Bruno affirms, *against nature.*

Rossi is probably right in finding Bruno's system much more complex and many-levelled than is acknowledged by Sturlese, especially concerning the idea of *segni manipolabili*. My more or less unaided reading of the *De umbis* leaves me with an impression of several sketched-out systems superimposed rather loosely upon each other, and that Bruno, like most philosophers, to tell the truth, does not always know what he might be *really thinking* (granted there is such a thing as that).

My concern is not that he developed a kind of automated procedure but that his conception not *of reality* but of *handling reality,* approaches our manner of thinking in terms of *abstract models equipped with dynamic powers*. Bruno seems to be very much focused on *action*, even if it is purely conceptual; that he harbors an inner need to handle things, operate with them, in order to master them. It is symptomatic that perhaps his most crucial point of criticism against the Lutheran reform regarded the *sola fides* dogma. Bruno emphatically and repeatedly stresses the necessity of *work,* adopting - or taking for granted - the arch-Catholic dogma of the *opus bonum;* the good works centering around and issuing forth from the supreme Good Work, which consists in celebrating and participating in the Mass (2.2.12). He hardly insisted so much on this merely to sound safely orthodox on at least one point. He stresses the importance of sentiment and feeling over against words: *With his otiose and pedantical literalism Luther substitutes names for thing, words for sentiments* (*Con il suo "letteralismo" ozioso e pedantesco, Lutero sostituisce i nomi alle cose, le "parole" ai "sentimenti",* Ciliberto, 54, synthesizing Bruno). Quite apart from his reaction against the - equally pedantic! - Oxford professors at the celebrated evening meal, the *Cena de le ceneri,* Bruno on the scientific level classifies them in the same category as the Northern Reformers, accusing them of verbalizing pedantry:

> *All of them Doctors of grammar... in whom, in that happy country, dominates a combination of pedantic and stubborn ignorance and pretentiousness mixed with a rustic incivility* (*.. tutti dottori di gramatica... ne' quali in la felice patria regna una costellazione di pedantesca ostinatissima ignoranza e presunzione mista con una rustica inciviltà...*;) (*Cena le le ceneri,* 4. dialogue, p. 84 of the cited edition).

Bruno evidently was not familiar with universities; Galilei would not have been surprised.

Bruno's *Umbra,* then, is not concerned with categories in or out of Aristotle, nor Platonic ideas, nor, finally, Aristotelian laws.

His method handles *objects,* simply *things*. They have structure of a universal kind in a unified, infinite universe. This is not directly approachable. But potentially a cognitive connection with it, by a scale of levels or stages, can be obtained through the mechanism of the play of opposites in the Shadow. Thus, *being in the shadow of the One I desire* (*svb vmbra illius qvem desideraveram sedi,* from the Song of Solomon; *see above*), I am being enabled to approach him, even though, as with God, he cannot be seen directly in my natural life, explained or comprehended. This kind of relationship also applies to *things.* Through the dynamical *Umbra* mechanism of oscillation (the shadow of a tree being static), they acquire meaning in universal principles: the shadow implying its opposite, light, in alternation (shadows being unthinkable without light).

There are degrees or stages in such processes, tending to ever closer view of essentials. This is not a Plotinian ascent by degrees from the world of realities to that of ideas. It is a *cognitive mechanism* designed to penetrate things in the world with a view to find there the universe. When Bruno, to the consternation of the Church, imagined an infinity of universes, he thought just as much of interior penetration into the infinitely small, as of expanding out into open space. This is the time in which penetration into the *minusculae* of nature became a big theme (Catherine Wilson, *The invisible world*). This reference is not intended to suggest that Bruno was up in such matters as this, merely that there may have been a common tendency to look inside things more closely than had been usual. It did make a sensation when a cardinal declared himself shocked to see, for the first time in history, the living creatures crawling about in what had been supposed to be clean water; no wonder people drank wine.

1.4 Bruno, science and mathematics

Bruno's concern with methods of cognition is definitely a scientific one. Some of his ideas are reputedly related to mathematics.

Regarding the relations between religion, philosophy and general culture, on one hand, and mathematics on the other, it is important, as has been often affirmed, to take into account cases in which mathematical thinking modalities may be operative without being tried out in calculation and other technical formats. Bruno appears to represent such a contingency.

He has one section that seems to support this view.

Bruno's attitude is dynamic, geared on movement and action and his discourse rather than a philosopical treatise is *a syntax and a program for action with philosophical backing.* We are being taken into a *Zwischenzustand,* a state of instability and of intermediacy under the condition of oscillation or fluctuation between states. These are, however, each of them *separately* open to description, at least in pragmatic terms. The nut that remained to be cracked concerned the description of the *oscillation* and *fluctuation* as such. Of course the notion was not a new one; Aristotle having stated that *change* (*kínesis*) was the essence of nature. But the subject now became a target for what we may call a concerted action. The *second point* is that this kind of fluctuation and ensuing vagueness of form, is what the *new developing mathematics,* which I call *protocalculus,* was devised for handling or, at least, approaching.

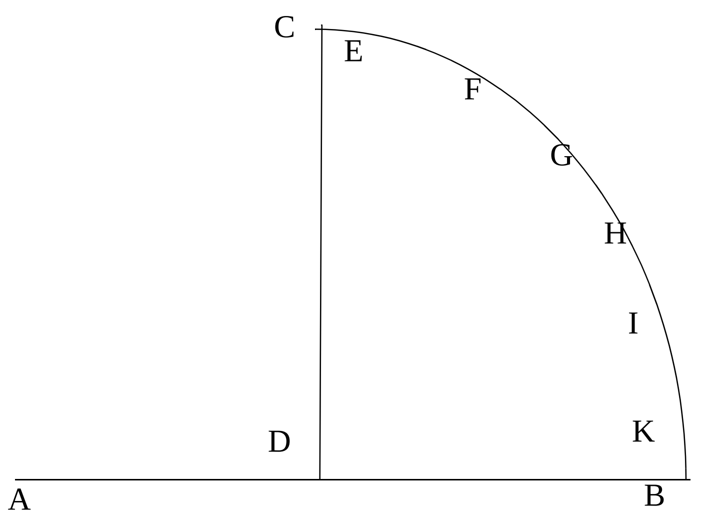
In the midst of a host of ambiguities, Bruno's light/darkness idea as set out in his *De umbris idearum,* seems relatively clear and strikingly close to the cognitive tendencies that pervaded some of the *protocaculus* efforts. The crucial *methodological device*, as already hinted, was dedicated to *handling approximation.* If this does not show clearly in Bruno's writings, it seems to be due to the fact that he wrote prose not math. It was gradually realized, since the fourteenth century especially, that you cannot describe an irregular shape, water moving down a hill, or a fall from high up under increasing gravity, in short, any complex motion, by any other method than by working out *increasingly close approximations* to it, and this indeed in some sort of formalism, geometrical or numerical. I believe it was Sharratt who noted that Galilei used the word *almost* exceptionally often. Bruno, it seems, glorified the idea of approximation.

Let me return to the last paragraph of Bruno's 30. *intentio,* in the *De umbris,* where he presents his graphic model*;* he writes:

> *Here is the model* [*paradigma*] *of that unique idea that to all intents and purposes contains/embraces the infinite differences between <real> things, and of that unique shadow, <which is evaluated> in its capacity of assuming/taking on infinite differences* (*Adest paradigma unius ideae actu infinitas rerum differentias habentis, et unius umbrae in facultate infinitarum differentiarum*).

The model is a graphical one which I am redrawing here (with no substantial changes), and with the accompanying explanation synthesized after Bruno's text. Consider, he says, the segment D - C, and let D be the pivotal point and let C move towards B. Then at D the two angles [on either side of C - D] vary from being right to assuming various degrees of obtuseness and acuteness.

Fig. 1.3 - 1 Bruno: diagram



This capacity of the segment D - C basically means that the point D in which the two lines [A - B and C - D] cross, represent, both an actual state and potential states [*esse et posse*]; so that at this point unity and infinite quantities become identical (between which the putative intervals become infinitely small. This is a synopsis of how I read his formulation, which sounds like this:

Ita patet quomodo in facultate duarum illarum rectarum linearum sint infinitae acutorum, obtusorumque angulorum differentiae. In prima causa haec facultas non differt, quae et in qua quidquid [anything] *esse potest, quandoquidem esse et posse identificantur in ea...*

In this manner, at the *point D,* the difference between the [infinite number of] angles are one and infinitely many at the same time. *Ideoque in ipso D infinitae simul, et unum sunt angulorum differentiae.*

I call this model *vectorial* with some hesitation, since length from tail to head is not given. Nevertheless, the infinite variations in *angle* between the two lines is an immediate product of the degree of revolution of the segment D - C when this is considered as an arrow heading downwards, so that the vectorial dimension of *direction* is determinant. On this account I would regard the model as illustrative of an attitude on the part of Bruno that is attestable elsewhere (see above): namely a dynamic mode of *approaching*, focusing on directions rather than on fixed *termini* or points. Engrossed as he was in the idea of infinity, both the endlessly great and the endlessly small, he must have considered the segment CD, in its oscillation between $\pi = 0$ and $\pi/2$, as a vectorial element that was directed upon a *point ever diminishing towards infinity.* The idea seems to me to come closely to that of approaching a *limit* (leaving the verdict to the experts). In this respect, I believe, Bruno's attitude comes close to conceptions that informed the *protocalculus* efforts.

For a man, however, so geared on using images, and at a time when geometry was still the main tool, the question remains how much *mathematics* there is in his pseudo-vectorial model and how much naive configuring.

Once a relationship has been suggested to hold between Bruno's operative or methodological principle and the mathematical development into what, with Newton and Leibniz, was to become the *Calculus*, then we have a question to ask ourselves.

What kind of operational level could serve to make this connection workable? Bruno seems never to have taken up this new branch of mathematics technically. Gatti's comment on the problem of assessing Bruno's conception and use of mathematics is worth quoting. There have been some attempts around to connect his approach with that of quantum mechanics. I find such attempts interesting if they limit themselves to assuming that he cherished conceptions that in a very general and not precise manner could be projected on to the *ideological foundations* of the modern view, as set out by Heisenberg in his philosophical writings; but nothing more than that. Gatti, however, comments:

I am not claiming that what Bruno had in mind already was quantum mechanics. I am nevertheless convinced that the difficulties in correctly reading and interpreting his *mathematical works derive largely from a fundamental discrepancy between his mathematical doctrine and the classical mathematics emerging in his time, based on consecutive processes of reasoning seen as creating a mirror image of the order in the natural world. Bruno had in mind, although in a very embryonic form, a quite different idea of what mathematical doctrine was and how it related to the mathematics of his atomic and infinite universe. It is an idea... leading from higher forms of rational certainty into the chaos of the phenomenological world. It can also be closely related to the modern breakdown in the concept of an object and to the proposal in quantum mechanics of measurement as a grid through which a shadowy and "virtual" world of objects can (possibly) be indirectly grasped* (Gatti, 149).

When someone among our historical protagonists approached the subject of handling complexities mathematically in the way of what I have dubbed *protocalculus* or, indeed, took up important features in its epistemological setup, this may have happened on one of three levels (or two or all of them):

* Working with and/or within the *technique* at issue;
* working on problems, issues, or terms that *belonged to the subject or touched it*, directly or tangentially;
* working with other topics (say, astronomy, infinity and the extremely minute, limits in some non-*calculus* sense) from a *cognitive perspective* that were to prove crucial for the development of the *calculus*.

Giordano Bruno appears to have been most active on this *third level*, even though the second and indirectly the first may be relevant. The pseudo-vectorial model (*Fig. 1.3 - 1*) could be a sign of an intuitive and non-technical and yet profoundly mathematical manner of thinking. Richard W. Pogge is probably right when claiming that Bruno's science was rather weak, unaccompanied by a good grasp of Copernican astronomy (reputed *mathematically murderous*) (Pogge). Thus in his case we should not expect any productive engagement in contemporary *protocalculus* efforts. And yet, in a philosophical sense and a geometrical modality, *infinity* and intuitively understood *limits* were uppermost in his thinking, as we know from his writings. To approach these issues and the related ones in cosmology and for the development of his memory program, he used *image models*, set forth in his *De umbris idearum* (and elsewhere). Here the operative principle was construed on the configuration of *interchanging* light, shade and darkness. Bruno clearly thinks in terms of *approximation* (2.2.2 (§8),.3.4.2, 4.3), a facet of increasing role as a *resource* in the protocalculus story (3.4.2, pt. 4.3).

In this connection I must note that Rossi, in his *Il tempo dei maghi*, sarcastically reports how some *journalists* (such bad guys!) have tried to connect Bruno's work with computer science and informatics. The comparison is untenable, he insists (*Ma quell' accostamento non è sostenibile*). Unfortunately Rossi omits to specify *what sort of connection* those journalists have had in mind. For there are at least two alternatives. That Bruno did not think in terms of computer science and informatics, we are in no need of being told. What is not obvious is that we should not be allowed to *use modern models for our* description of Bruno's system. We *have to* express our understanding, for what it might be, in *our language, imagery* or *experience.* The only way of describing the Bruno system *in his own terms*, would be to copy out his own text exactly and completely. The Research Council would hardly sponsor a publication of that.

The analytical trade-off from this provisional and much too brief perusal would seem to be that Bruno at essential points of his thinking makes his reasoning (and that of humanity, as he seems to imply) be pivoted by the faculty of exploiting change, oscillation and *interchanging entities,* with a view to acquiring knowledge and understanding by playing them against each other.

Now, finally, over to our main substantival topic.

1.5 Ces lignes plus composées: Borromini's complex design

Equipped by the considerations so far, we shall, in the remainder of this *Part*, reflect on the material that forms, substantivally speaking, the core subject of my inquiry.

1.5.1 *Differential curves and manifolds*

Complex shapes and curves, we have noted, such as those studied by Descartes and referred to as *ces lignes plus composées,* represented a challenge to mathematicians, physicists, philosophers and architects. To handle them requires the calculus and other modern mathematical techniques not available in the seventeenth century. Mathematically, a straight line is also a curve, a special case of this class, but in the present connection we shall generally focus on curves that are somehow *bent.* Several of the curves to be considered here (on the Spire. in San Carlino, in San Giovanni) are in fact *space curves*, for which I shall use the modern term*, manifolds;* further below)

Apparently the concern with such forms was symptomatic of the mindset that provoked, or at least lent force to, the drive towards the *calculus*. This amplification of scope in turn must have stimulated the search for and attempts at handling the complexities of nature and human creativity and research.

Descartes used the term *Ces lignes plus composées* for members of this class of forms (5.2.10); and, surely, they are more complex (in terms of the mathematics of those days) than the usual Euclidean shapes (*Burden,* 79ff.) or the elementary conical derivations (parabola, ellipse etc.). Only in the eighteenth century the real hassle connected with Euclidean innocence became evident; resulting, ultimately, in non-Euclidean geometry. With the development of analytical geometry started by Fermat and Descartes, algebra gradually took priority over geometry, which had so far dominated the field (Derbyshire, *passim,* and 3.5.11 and 5.2.12).

These curves *and manifolds* are worth further inquiry under at least four headings:

- on account of their resemblance to shapes in nature (water flow, for instance, and the non-circular motion of a comet) and technology (ballistic);
- by virtue of the challenges they posed to mathematical treatment and, through this, the incentive they contributed to bridging over to a vast area of scientific activities in the years around and after 1600 in which *protocalculus* efforts were crucial but not isolated;
- since their unpredictable and often dramatic shapes were probably felt as being vaguely relevant to or mirroring conditions of life at a time of preoccupation with fluctuations and uncertainty;
- on the strength of their capability, by virtue of their extreme flexibility, of tying together the features of new architectural spaces that arose from similar considerations and, especially in churches, from the new freedom for space construction resulting, paradoxically though it may seem, from a more rigorous liturgical discipline. (2.2.9; SL, 1965).

The *complex curves* and manifolds can be seen as transitional forms between what is graspable and calculable and what was not so, until a sufficient refinement of calculus techniques was achieved at a much later date, with Leibniz and Newton.

For space surfaces and curves it is preferable for us to think of them in terms of *manifolds.* They are handled in *differential geometry.* Some readers, and the autor!, will need an explanation of this term, and I borrow it from a dictionary (*The VNR,* 561): *In differential geometry the concepts and methods of analysis, particularly of differential calculus and the theory of differential equations, are applied to the study of geometric figures. The underlying "geometrical spaces" or "manifolds" must, as in analytical geometry, be referred to coordinates. Other geometrical figures are embedded in these spaces, for example, general curves or curved surfaces....* So when I speak of *differential curves,* they should be understood as being embedded in manifold configurations, as the "ovary" shapes in San Carlino, which must be understood in connection with the complex wall structure. Glenn and Littler's *Dictionary of mathematics* gives the following explanation (rather than definition, since the term to be defined is included in the premise) of *manifolds: Any entity constructed from a number of entities usually infinite, as a three-dimensional manifold constructed of all points with coordinates x, y, z.* Let this be supplied by some specifications in the *VNR Dictionary* 573): *Al-*

though a geometry can be developed for differential manifolds, it is in comparison with Euclidean geometry, very meagre, because concepts such as length, angle, area parallel displacement and curvature are completely missing. Not to worry: describing the Borromini forms noted above, we need none of these elements, and can ignore boundaries, just focusing on surface shape and its corresponding form.

This is thus an *operational* procedure rather than a definition of meaning or significance, telling us how a manifold comes into existence. Taken at the outline level, without entering into the math details, it should be applicable to all architectural forms in curved surfaces regardless of their physical boundaries (serving better than topological ideas). So this is an *idea application* rather than a technical one.

Anthony Blunt coined the name *indefinite shapes* (speaking of Borromini) for the forms now in focus.The denomination is not entirely adequate. For gradually mathematical formula, however unfamiliar, were found for the a form, then this is no longer *indefinite,* it will stand out as definite indeed. By regarding them as *differential curves*, I take care of that, for the implication is then that *some kind of differential, as was being developed at the time*, was required for handling them (3.4 and s*ubsections*). In fact, some of the curves and manifolds to be fully manageable, would require *differential geometry*, a method not available at the time (on differential geometry used on a helix, *see* Lord and Wilson, pp. 20f., Gellert a.o., pp. 561f., Harris and Stocker, pp. 527f, and Bronstein, p. 249 [*Schraube*]. For the relevant geometric interpretation of a vector derivative, *see* Spiegel and Wrede, pp. 157f, 181).

The idea of differential curves and manifolds is to be taken seriously. Borromini unwittingly (it seems) contributed to concretizing and publicly displaying the foundations of a new math, here labelled *protocalculus,* the central feature of which was the *direction of a point on a bent curve*; furthermore, the tangent angles of a curvature (3.4.2, 3.6).

Of course the interest in *curves* is as old as geometry, but in the late Middle Ages some consolidation of issues took place that laid a firmer basis for the development that was to culminate in the seventeenth century. Leaving the latter subject till later (*Part* III), let me offer a short introduction by quoting Margaret E. Baron’s superb summary.

> *When the idea of the Merton Calculators* [Merton College, Oxford, 1328 - 1350: concerning ideas of time, speed and instantaneous velocity] *were accorded geometrical representation by Giovanni di Casali, Nicole Oresme and others, the concepts of time, speed, distance and instantaneous velocity became associated with the study of curves. The shape of the curve defines the motion and the motion in turn defines the curve. The instantaneous velocity becomes associated with the tangent to the curve and the total motion or distance*

covered is represented by the area under the curve. Thus, a quadrature in the form of a space bounded by abscissa, two parallel ordinates and curve [in the Cartesian coordinate system], *becomes a general integration model for handling any continuously varying quantity; the construction of a tangent to the curve becomes a means of determining the instantaneous velocity or rate of increase of the same type of variable...* (Baron, 5).

In several connections geometrical figures functioned as *models*. Typically, when Descartes discusses the “more composite lines” in his *La géometrie* (appended to his *Discours de la méthode* of 1637), he takes the modern line of using a simpler *model*, namely the ordinary parabola (a well-known curve), to analyse and clarify more complex ones. Argumentation over models, which means abstraction and simplification for analytical purpose, now becomes an established method, while hardly an innovation in itself; as we have seen also in the case of Giordano Bruno.

1.5.2 *Borromini’s curves and surfaces*

When I say "Borromini’s" curvatures, many of them manifolds, this is just a semantic shortcut (*see* the "topological" drum of Sant’Ivo [*Fig. 1.1 - 1*]; the interiors of Sant’Ivo [*Fig. 1.5 - 1*] and San Carlino [*Fig. 1.5 - 2*], and the monument for Cardinal Acquaviva, rebuilt by Borromini [*Fig. 1.5 - 3*]). For even though the architect will probably have been the master of the design itself, its being approved of and used on public buildings takes more than a craftsman’s or architect’s initiative, creativity and control (3.1).

At the time, in want of adequate mathematical methods with which to asses them, such shapes as in works by Borromini’s, Bernini’s and Gherardi’s will necessarily have been perceived and evaluated, more or less directly, in the framework of traditional architectural and space-representing grammar and syntax; the “ovary” shapes in San Carlino thus connected to the oval or ellipse; the curved pseudo-architrave in San Giovanni in the framework of traditional horizontal elements. This means that these shapes are hovering on the verge between being normal and describable (mathematically and verbally), and being abstracted out of any perceivable linkage to its surroundings. In this sense, these *lignes plus composées* represent instability and change and hence involve a time dimension. The point to be reiterated is that several among the curves and surfaces in Borromini’s architecture *defy* the mathematical techniques available in his time.

Let us look at some specific cases, leaving the Spire till later (1.9).

The most notable architectural case is the transition from wall to cupola in the interior of Sant’Ivo (*Fig. 1.5 - 1*).

Fig. 1.5 - 1 Sant'Ivo, interior; transition from main body up to the cupola, from convex to concave (in Norwegian but hardly difficult).

Here, there is a passage from *convex* wall sections upwards into the cupola, where the surface necessarily assumes *concavity*. In this fashion, a convex surface is turning *continuously,* that is, *differentially*, into a concave one; a remarkable case of a manifold being used in the very construction of a building. The principle can be illustrated in the appended figure that I borrow from Lord and Wilson, *fig. 62,* my *Fig. 1.5 - 1Comments* (below), and today mathematically determined by $u(u^2 + v^2)$ (Lord and Wilson, 84). At the time, such a surface configuration would have been considered as consisting of an infinity of parallel lines, in default of calculus methods.

There can be no doubt that contemporary experts would interpret the transformation surface, not as a curved plane, but as a vertically running sequence of horizontally directed lines, starting from below with crests and gradually flattening upwards to reach a straight horizontal, followed upwards with troughs - à la Cavalieri. In his mathematics, a plane was made up of an infinite number of lines (3.5.1). Quoting Møller Pedersen (32f.): *They imagined an area to be filled up... by an infinite number of parallel lines.... But the approaches differed. Roberval's approach was founded rather on the division of surfaces (or solids) into strips (or slices) with unit bases. In this way, by increasing the number of divisions indefinitely the measure of the ordinate in linear units became the measure of the surface in square* *units. A surface, he says, is divided into an infinite number of small surfaces which are equal, or have equal differences, or maintain some regular progression such as from squares to squares, from cubes to cubes, and so on. And, since the surfaces are enclosed by lines, instead of comparing the surfaces one compares the lines* (cf. Baron, 124ff., 153ff.). But back to Cavalieri's method.

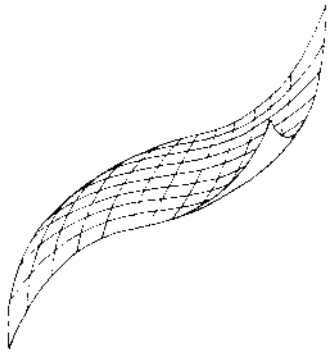


Fig. 1.5 - 1Comment, Manifold (Lord and Wilson).

The surface illustrated on *Fig. 1.5 - 1Comment* represents a case that was both problematic *and* in the focus of wide interest in the seventeenth century. If we remove all the longitudinal lines, while keeping the transversal ones and multiplying the latter up towards infinity, then we have an adequate image of how the double-bulge surface in the Sant'Ivo interior would have been explained mathematically by Cavalieri and his followers. Borromini employed and his contemporaries accepted a shape much in view in contemporary science.

The corresponding *exterior* shape of Sant'Ivo also eludes description in classical architectural terms (*Fig. 1.1 - 1*). The main body bulges out between vertical elements arranged in an irregular ring. At a loss for adequate description, I use to propose to students that the ring of verticals was constructed first and then a balloon of some flexible material placed inside and then inflated so as to bulge out where there was room enough; absurd, but it seems the closest one can come in verbal terms (but *see* 1.5.1).

Let us consider some further relevant shapes in Borromini's works. In the squinches under the cupola of *San Carlo alle quattro fontane* (*San Carlino*), there are framed oculi that would normally have been circular or, exceptionally, elliptical (*Fig. 1.5. - 2*). They are of an unusual "ovary" outline, as if well-rounded ovals had their lower half pressed somewhat inward.

This *somewhat* could not at the time be described mathematically, but must have represented a challenge. Borromini was no better equipped than Palladio (*see* 3.4.2) when it came to handling mathematically such shapes as these; but of course he was under no obligation to try.

Fig. 1.5 - 2 San Carlo alle quattro fontane, interior (Borromini) (Phot. Liv S.-L.).

To stay in San Carlino for a while. The bands over the lateral arches in the same church are twisted, starting from the horizontal ends at a diagonal angle with the length axis of the room (exactly parallel to the straight entablatures between the end and middle niches) and at the summit end up parallel to the floor - almost like a Möbius band (just take a strip of paper, twist it once and glue together the ends: you sit back with a flat tape with one surface).

The pseudo architraves in San Carlino, the elements supported by the columns, also elude classical geometry, appearing to have been designed on the pattern of a prolate cycloid; a form, however, that was well known in contemporary mathematics.

There are many other challenging shapes in Borromini's renovation and partial re-building of the Cathedral of Rome, San Giovanni in Laterano; let me cite one of them, On the frame that Borromini designed for an earlier funerary monument to be reinstalled in the partly renovated church (also the work of Borromini), the *monument for Cardinal Giulio Acquaviva* (*Fig. 15 - 3*), the horizontal element that classically would have been a straight or curved architrave, is bent inwardly so as to convey the impression of an inverted perspective.

In fact, the pseudo architrave may be described as a relatively close approximation to a projection onto a vertical plane of a cosine curve between 0 and 2π. The element, however, is *not* closed in one plane, for the trough is gradually bent inwards starting from the end points, the lateral crests, so that it deflects from the vertical front plane to a plane at some 15 degrees to that. Thus the architrave represents a spatial shape of high complexity.

Fig 1.5 - 3, San Giovanni in Laterano, Funerary monument for Cardinal Acquaviva.

Borromini lets us go away without a usable vocabulary or nomenclature except mathematics; even that problematic, seeing that many of his forms were at best *approachable* in terms of contemporary mathematics. Of course, since the shapes were on public display and accepted by high-level officialdom, they must have stimulated involvement in the new mathemaitcs.

This is exactly the crossroads at which this entire book is situated.

The architect's tampering with classical element - vaults, arches lintels and so on - is all the more striking because he does *keep them where they belong*, classically speaking, and as separately identifiable, numerable units. He does *not* let one element start out classically only to slide or merge into the next, as Giulio Romano does on his *Palazzo Maccarani* (1523) across the street behind Sant'Ivo. Here the situation is the opposite: classical elements connected unclassically.

Let me go briefly back to the *drum* of Borromini's Sant'Ivo (*Fig. 1.1 - 1*). Its basic *form* undoubtedly is a *cylinder*, while its *shape* is - what is it? It strikes me we may use the modern term of *topology* to describe the relationship between the two, so that the latter is a topological transformation of the former. Topology, according to the brief but concise description by Derbyshire (2; he elaborates the issue later in his book), *is dealing with the properties of figures that are unchanged by stretching and squeezing but not cutting, the figure... Topology tells us the difference between a plain loop of string and one that is knotted, between the surface of a sphere and the surface of a doughnut.*

This comparison can be extended to other *differential* curves and manifolds in Borromoini's work, which are based on *continuity* and not discrete values, for which, to repeat, another modern term may apply in a pictorial manner: they are *manifolds.*

Can we say, correspondingly, that the *Sant'Ivo Spire*, now to turn to that, with its increasing torsion represents a similar transformation from the *form* of a regular conical helix into the actual pig snout *shape*? This question can probably not be answered with a clear Yes or a No *today,* since we have mathematically *general* means by which to define both of them, thus setting both on one and the same level. At the time, the latter shape could be obtained in an ad hoc fashion, either by setting up a selective range of verticals, as Dürer did when he determined the relations between spiral and helix (*see Fig. 1.9 - 2*), or by determining a specific *geometrical procession* or *series*; series were in use at least since 1619, the year Napier published his book on logarithms (Struik, 11f.). Of course such an argumentation as the one I have ventured here can never be conclusive in any definite sense, only provisional. But it may serve as an incentive for my approach to a methodology of description (4.4 and 4.7).

In architecture, the curves and surfaces, despite the differential and manifold formats, necessarily have to follow the tectonic and functional structure of the building elements; one cannot live in a building as designed by Konrad Escher. In sculpture, on the contrary, some shapes allow free rein to a creative play with shapes. A few cases will be considered that, in addition to providing relevant examples of shapes, offer an opportunity of instantiating some methodological principles regarding what I call *systems level* and *elaboration level*.

1.5.3 *Complex spaces*

The introduction of the *differential curves and manifolds* in Borromini's work (to stay with our main protagonist), contributed to breaking up space conditions known from classical and classicizing architecture. Neither in San Carlino nor Sant'Ivo can the architectural space be scanned in a straightforward order respecting classical grammar and syntax. Comparable space complexities turn up in Bernini's and Gherardi's works (2.2.9). In the main space of a church, the introduction of new plans and new complexities was facilitated by new regulations in Mass celebration (2.2.9, again).

In order to develop further the design theme discussed in the present *chapter*, further on, in *Part* II, shall discuss some cases that involve turbulent surfaces - rather than linear curves - draperies or curtains that are waving and billowing as in strong winds and one with a collapsed carpet assuming similar shapes, all of them as challenging to contemporary mathematics as the widely studied motions of water freely running, a cherished subject at the time (2.2.9).

1.6 System and elaboration

At this point, however, we shall take a break in order to set up some necessary premises for going on; mainly regarding the relation *system-elaboration, configuration space,* and *structural argumentation,* while supplying underway some indispensable substantive material. The problem of linearity! With a graphical model, the items could have been organized in space constructs showing their mutual relations.

So far, I have commented at some length on a group of architectural cases with the purpose to approach our main test object, Borromini's Spire, which shares comparable characteristics in curvature and surface. They are highly relevant also regarding the other main subject in the present book: the issue of *systems*. There is a systems level and an elaboration level in the case of the Spire, too.

Unhampered by academic "field" thinking, we can use a liturgical space for which the conceptualizations are not highly complex and probabilistic as in the case of the Spire but simple and recordable and definable on account of functional and elaborated features arising from a functional system. We have an adequate test case for this in a liturgical space with a clear and consistent message system which is being communicated through *imagery* reflecting fundamental doctrines and dogmas in the Roman Church: the frescoes in the vault of the Sixtine Chapel in the Vatican.

1.6.1 *Systems level and elaboration level: the Sixtine Vault as a canonical model*

Functionality in complex environments should be tested on simple - or simpler - ones. The *Sixtine Chapel in the Vatican* (from now just *the Sistina*, to distinguish from the Sixtine Chapel at Santa Maria Maggiore) was intended for being used by a select set of people consisting of the pope himself, the cardinals and a few others among men at the summit of the hierarchy. The case offers the best imaginable occasion for developing a *theoretical and methodological distinction between systems level and elaboration level*.

This was the main papal chapel in the Vatican palace complex (also used for papal elections), functioning for *experts on ecclesiology, theology, Tradition (with a capital T) and liturgy* (themes introduced with bibliographies in SL, 1984). The "clients" for whom the chapel was designed and functioned would always be made up of competing and conflicting factions always on the look-out for positions, power, prestige and money - with their attention permanently on the next papal election. Not a scenario made for taking chances with adventures that could be censured for unorthodoxy.

The Roman Catholic confession is a systemic one, in which the system is no mere reading matter but is acted out in the sacraments. The interpretation of the Bible is subjected to the system. No one should be allowed to read it differently, an anathema repeated at the synods, and it is directly linked up with the actions of the liturgy. Even translations were outlawed, a rule to be toughened at the Council of Trent. In Spain, the Inquisition, referring to the Council, which here followed earlier council canons (Denzinger-Schönmetzer, A7bd), condemned Fray Luis de León to five years in prison for having translated Solomon's Canticle into Castillian.

There are *two contexts* in which to consider the pictorial series in the Sistina; the functional and, obviously connected with it, the political.

Reversing this order, let us note that as a pope, Julius II, who commissioned Michelangelo's cycle in the *vault*, would be obliged to adhere entirely to Tradition on the systems level and so also on the elaboration level. Having, in addition, personally to face the threat from the impending Council of Pisa, which undoubtedly would represent a challenge to his position, he must be expected to make a show of total orthodoxy; or, at the very least, avoid anything that might be used against him by his numerous and powerful enemies. Even a "Renaissance Pope" could not tamper with the essentials of the teaching and action of the Church. *The pope could not flout the very Tradition that defined his position and prerogatives.* No ideas about the artistic genius of Michelangelo can affect this. If a genius he was, then he should be perfectly able, as he seems to have been, to create a supreme artistic and emotionally catching work at *elaboration level,* without interfering with the essential system. The idea that the artist did what he liked not only on the elaboration level but also on the system one, and sprung the result upon the top hierarchy of the Church, is not to be taken seriously. Contemporary artist biographies are no good sources if taken at face value. It was the age for the *genius* to become an emblematic figure. When an art writer like Condivi credited Michelangelo with the entire work, no one among contemporary people who read such things at all, would misunderstand this, the basic system being taken for granted.

Now to the functional issue.

The visual manifestations of the Roman Church were anchored in complex interlocked *systems* of functional patterns, text traditions and rituals, together constituting an orderly, systemic and officially sanctioned grid or structure; here labelled the *Canonical System* (survey with extensive bibliographies in SL, 198; the cited terms not in use here). This structure consists of the following intimately interrelated factors basic for the teaching and the action of the Church: *ecclesiology, theology, Tradition (with a capital T) and liturgy (especially the*

Mass liturgy). This system, developed over the centuries, is perhaps one of the most complex, consistent and well-running virtual machines ever created by man.

What makes such systems meaningful and *functional*, is the fact that underneath their dynamics complexities such as we see them with our eyes and experience them in organizational terms, there is an orderly, coherent and officially sanctioned grid or structure, which was to a variable extent and depth accessible to and understood by all who were directly and often indirectly involved - as they were meant to.

Music, song or recital, and visual media were used to *elaborate* such configurations for the purpose of supporting declaration, teaching, promulgation, public and political appeals well as Catechism instruction. Elaboration of a system can occur in various modalities; in the present discussion, I have in mind *visual* or *visualizing elaboration.* We have just seen this most clearly in the cases of Caravaggio's St. Matthew and we shall see relevant cases in Bernini's Ludovica and Teresa programs (2.2.9). The distinction is crucial for Diana Gisolfi's and mine treatment of the library decorations at the Praglia Benedictne monastery. In all these cases regarding the *Roman Church*, the structure underlying the *artistic elaboration* and situation-conditioned complexities, consists in a dogmatic, functional and conceptual topology that tradition had preserved because it was considered mandatory, adequate and also perceivable in terms of the teaching of the Church, as well as being in general - and this is a crucial point - accessible to the congregations and to the general public.

Most importantly, the *liturgy* in which the people participated regularly, spelled out the principal teachings, doctrines and notions in Tradition and theology. It is the liturgy and the Tradition that teaches *how to understand the Bible*; a point missed out by some modern scholars. Sixtus V was to profess that the liturgy provides the proof of the theology, again following the Tradition: the liturgy is *verae fidei protestatio* (cf. a synod canon of ca. AD 431: *fons cognitionis theologiae*, to cite an early example).

It must be taken into consideration that institutionally established traditions and rules, which constitute the system, when *applied to pictorial programs*, are not usually recorded in contemporary documents, since they were taken for granted. In order for the modern scholar to understand the pictorial messages, it is necessary for her or him to gain some familiarity with the body of teachings and ritual practices. The authorities of the Church, as managers, could and did reckon with normal *recall* mechanisms at work in variable measure among the population, depending of catechismal and other instructions.

Now a brief survey of the program on the main axis of the Sixtine vault. The Catholic historian Guido Gerosa notes (in is biography of Charles V) that the program is *molto semplice*, and so it is.

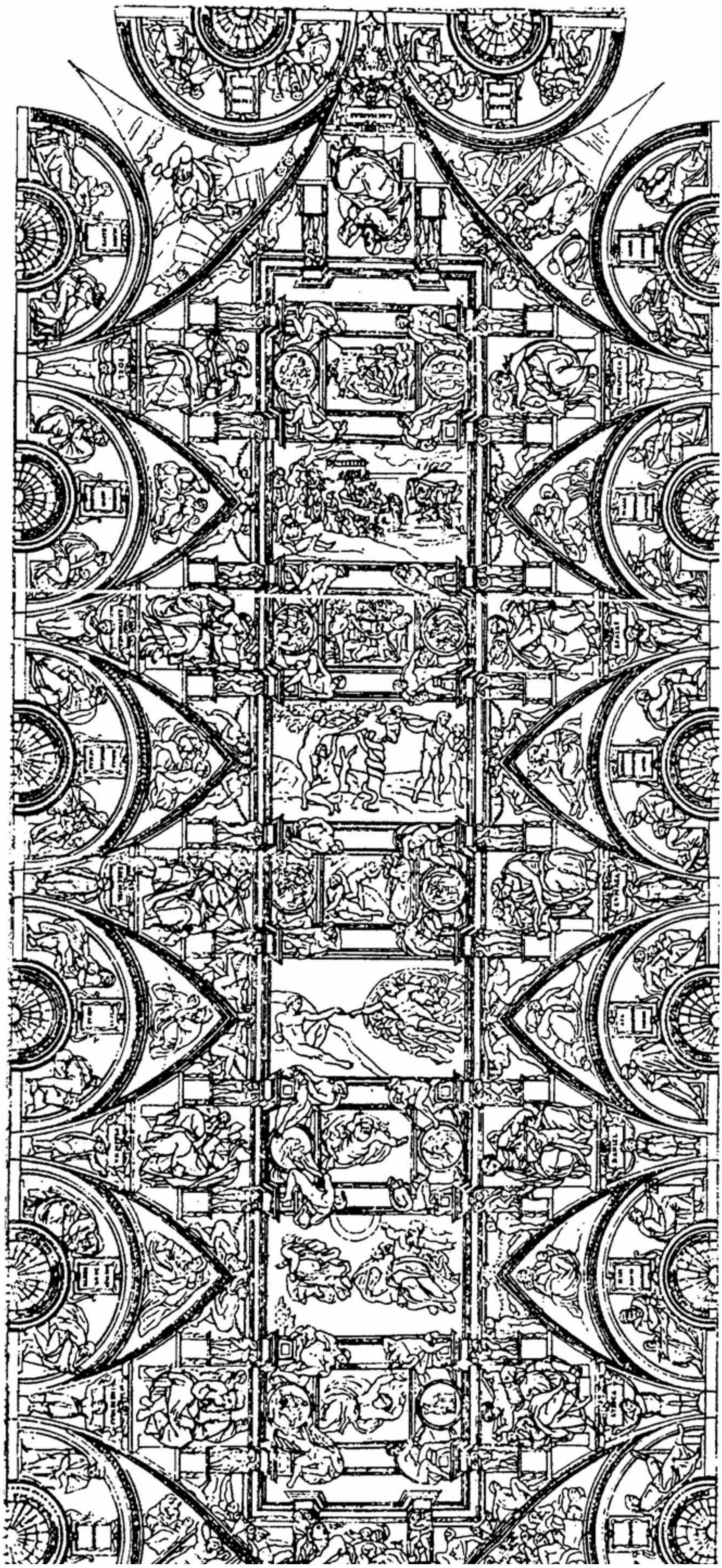
Joining the wall frescoes supervised by Sixtus IV (and most competently studied by Ettlinger in his *The Sixtine Chapel before Michelangelo)*, Michelangelo's vault frescoes were ordered by Sixtus' nephew, Julius II. To see the latter thematically *disconnected* from the former, would violate on of the most basic principle for program functions in the Roman Church: space disposition and use (as in most cases of public buildings).

We have *two solid handles* by which to get a grasp on the essentials in the program. It comes as no surprise that it is very simple, dealing with fundamentals familiar to all involved in the chapel rites. One is the wall series focusing, as Ettlinger has shown, on the *Primacy of the Pope*, which is an *ecclesiological* theme.Ettlinger's exposition of the papal and ecclesiological thematics in the fifteenth-century wall frescoes, provides the basis from which to start out when considering the message program in the vault.

The other handle is this. The vault frescoes along the main axis consist of three times three scenes, and the *last three* ones, near the main entrance, show scenes from the patriarch Noah's history. The central one shows the Ark in the Flood, a standard parable of the Church as saving mankind and receiving them into her fold. Then the Church becomes effective for mankind through the Mass sacrifice, and the first scene shows Noah's sacrifice, a prefiguration of the mass sacrifice. The third scene, with Noah's drunkenness, shows mankind, now under the aegis of the Church, being subdivided into lords and servants the way God had "programmed" (as we might say today) right since the Creation; a common theme in political theory; and used, for example, on the *Palatium iustitiae* - the Doge's Palace - in Venice (SL, 1974, 167ff.; 1965, with references). The thirteenth-century corner sculptures represent the three laws governing the world: divine law (the archangel Michael with Adam and Eve; present also in the Sistina vault); natural law (the drunkenness of Noah), and positive law (the Judgement of Solomon).

To stay at the fundamental level, it is adequate to say that the whole series of pictures along the axis of the vault has the *Church* as main theme, leading up to and supporting the thematics in the earlier wall frescoes. Dogmatically, *the Church started at the Creation*; this is the way the Biblical account is officially interpreted in agreement with the Catholic Tradition (relevant inscription of ca. 1116 in San Clemente, Rome, quoted in SL, 1978; and *Working*).

SIXTINE CHAPEL - VAULT PROGRAM



Noah's drunken-ness

The Flood

Noah's sacrifice

Expulsion from Paradise

Creation of Eve

Creation of Adam

God hovering over the water

God creating the universe and earth

God dividing light and darkness

altar-wall with Jonah above

Prec. page: a plan of the frescoes in the Sistine vault

The first scenes show God dividing light and shadow, his creation of the universe and soaring over the waters like the Holy Spirit, as we read in the Bible. Again dogmatically, however, the *Trinity as undivided and whole* enacted the Creation. All actions *ad extra* are the work of the Triune God, all three Persons acting in unity (Denzinger, 1856, 401f. giving the most important dogmatic references; further in Denzinger-Schönmetzer, B 2e/2ea). In Denzinger's repertory, we read as follows (synopsis in my translation): God is not triple - as in Tolstoy's *Three hermits,* let me submit - but triune. The work/action towards the outside is common to the three persons, there being one principle for what has been created *(Opera ad extra sunt tribus personis communis, sunt unum principium creaturarum.* Mart. 1, can. 1; Lat. IV. A, Flor. C. Tolet. XI, p. 84, 87, 88), See also Ott, 100, Council of Florence, 1441: *Pater et Filius et Spiritus Sanctus non tria principia creaturae, sed unum principium*). The dogma of consubstantiality in the Trinity was illustrated in the three identical-looking males in the Santissima Trinità cave at Vallepietra in the Subiaco Valley (an iconography common in the area until at least the 1950s).

The liturgy in particular, but also other parts of Tradition determines how thee Bible is interpreted by the Church, how it is *read and consequently also functions* (differently from the various "direct" readings in use, especially, since the Reformation*; see* also Gisolfi and SL, *Index: Bible,* with some of the most important synod canons.

In the middle three scenes of the Sistina vault, that of the Fall of Adam and Eve is the critical one, for it was this "happy failing" (*felix culpa*) which made salvation necessary and made the Church necessary. So here we see God, with the female personification of Ecclesia (an old subject in paintings and mosaics) in his arms, creating Adam and, in the second scene, Eve, represented in the improbably soaring woman. Tradition can fortunately tell us that Eve was to be renewed through the action of the Church (St. Zeno of Verona, for example: *Adam per Christum, Eva per Ecclesiam renovaretur*).

The series is connected with the liturgy on the altar below, a connection illustrated with the figure of *Jonah and the whale*, just above it. He is, according to the Mass Ordinal, a prefiguration of Christ's death and resurrection and is the *typus dominicae passionis* (details in SL, 1984; *see* 1.6.2). This text is centrally positioned in the Mass liturgy and its message would impose itself on every educated Catholic, to say nothing about the clergy. No art-student's fancy can change that.

A feature that has caused any amount of speculations is the fact that the angels, including the fallen one, Satan, and Michael with the sword, are wingless; and that so are the so-called *ignudi.* Angels would be a "must" in a program like this, in whichever way we interpret it; and no other role can be found for the *ignudi.* Wings are a pictorial device, utterly superfluous in the present case intended for experts (SL, *Working*, 2010).

The entire program is basic and simple, omitting many pictorially attractive and educationally adequate trappings: no wings, no attributes; no Sibylline inscriptions; didactic extras being superfluous in this environment of experts. The later vault program provides the *logical* start for the earlier wall program that was there already from Sixtus IV's days, thus completing it in ecclesiological terms. Later came the Last Judgement, focusing - as on the original altarpiece by Perugino - on the Virgin as the chief intercessor.

In summary terms: The Sixtine program as a whole and in every detail corresponds to the established practice and Tradition of the Roman Church (SL, 1969), no reference beyond this is necessary for its description and explanation. A program in writing could very well have filled just two pages, so there was enough to do for the genius. For example, he might have been told to make the Trinity appear as Creator, in accordance with the dogma, and, after some consultations, elaborated the idea using the traditional triple-male model.

The vault program can have been devised by almost anyone among the higher clergy. On the systems level, the programs in the Chapel, developed under two Franciscan popes, when taken together, seem to be a simplified and spatially adapted replica of the one in another, but much larger, papal basilica, the Basilica Superiore of San Francesco at Assisi (SL, *Working*). It is on account of the *stability* of the Canonical System as determined by the *Tradition* (2.3.1), that a user interface can vary considerably on the elaboration level, and that the basic level remains unreported in contemporary written documentation.

At the same time, in the Sistina, the system could be presented in a summary manner, emphasizing essential features only. Another thing is that since Michelangelo was tied to this professional program, he was forced to introduce a striking visual imbalance between the two halves of the series along the vault axis. Close to the altar: open spaces with huge human-like figures (the Trinity and Satan, the latter heavily bolting, all buttocks and feet); at the other end, there is the triple series of more ordinary rectangular pictures with many small features and human figures.

Diana Gisolfi justly reminds me that often, especially in medieval contexts, pictures may denote *layers* of significance; in the Sistina case, possibly on top of the essential one.

Of course. The problem is that we do no longer have access to the minds of sixteenth-century people. The only cues remaining would be special features in the pictures themselves and/or in the arrangement of them in some sort of compositional coordination. Since in the Sistina both parameters are entirely satisfied by the Canonical System, I would wait for some new observations that have escaped me before going any further.

But people do go further. Many modern contributions on Michelangelo's work have started at the wrong end, rather than with the functional basis, having fallen prey to traditional teaching. The hunt for an *author* of the program has dominated recent publications. The fact that someone was invited to lecture on the Jewish Kabbala in St. Peter's has led some writers to elaborate the absurd notion that the Sistine program might have been borrowed from the Kabbala. But the invitation was not necessarily positive, regardless of polite compliments paid to the lecturer. Adhering to its established tradition, the Church, if she wanted to debunk presumed heresies, would let the exponent for these teachings expose them publicly, thereupon to discredit them all the more effectively; the technique of all Church councils: leave the positive statements to your adversary, including the traps he is apt to build for himself, then push him into them (*see* also 2.2.12, end of the *chapter*).

To search for an *author* of the program is a somewhat futile operation because the message program in the vault of the Sistina is so simple and to-the-core that *almost any text with Old Testament subjects can be made to fit.*

Connected with the mistakes just cited is the lack of understanding of the functional issues. This is in part due to what I would call the *dictionary attitude* inaugurated mainly by Erwin Panofsky. The wisdom here would seem to be that if one term is listed (or depicted) already, you do not repeat it; for it would be a strange dictionary that listed a word twice. This attitude is the normal one among Protestant and others with typically bookish approach to religion, but it is utterly out of place in a Roman context. It is a direct consequence of the Canonical System and is an ingrained attitude in Catholic usage, that one and the same fundamental subject (in dogma or doctrine, among liturgical elements, etc.) - let me label it a *canonical sign*, usually conveyed through the liturgy - can be *expressed verbally and/or pictorially in a number of ways*; in a *tree-like process*, the mechanism supporting the process in the iconic interface in the model in *Fig. 1.7 - 1.* Important examples are the various verbal and pictorial renderings of the Mass sacrifice, of God, the Trinity. Chicharro's observation concerning Teresa de Cepeda y Ahumada (since 1622, St. Teresa de Jesús) illustrates the tradition perfectly: because of the integration of body and soul, the most varied (kinds of) pictures and themes denote very few kernels of significance (*Las más variadas imágines y los más distintos temas se expresan entorno a muy pocos núcleos de significado, como resultado de esa integración, vitalmente sentida y comunicada, de cuerpo y alma;* Chicharro, 48). It is this multiplicity of display options for any one fundamental datum that has left Panofsky and followers in a state of confusion (unnoticed by themselves); believing, for example, that *All Saints Iconography* is just one type of picture, while in fact there are heaps of them. There *are* cases in which dictionary models come usefully in, but in a different sense, namely concerning *synonyms.*

Then there is sheer ignorance about the subject - not very complimentary for our teaching institutions. The idea, arising from artistic “motifs” rather than functions, has been in circulation that Perugino’s altarpiece with the *Assumption of the Virgin*, originally on the altar wall, showed a "resurrection like Jonah’s" (in the vault above the altar); mixing up functions and expressions. But the Virgin’s being *assumpta* in her *dormition* or *koimesis* is no resurrection from death, and there is nothing *sacramental* about the Assumption (in Jungmann’s *Missarum Sollemnia* of two big volumes, the Assumption of the Virgin is mentioned once and then as a feast day along with the rest). The Virgin did not sacrifice herself for mankind, while the Jonah figure is used in the liturgy of the sacrament of the Eucharist, as a symbol for exactly this.

1.6.2 *Positional recall and space distribution (Michelangelo’s Jonah)*

In spaces in which a rule system determines and distributes functions, such as in a liturgical space in the Roman Church, the *spatial position* of a subject can acquire additional significance from that very position: a case of *positional recall.* This situation of course depends directly on the functional distribution within the same space, to which we now turn.

Liturgical practice is tied to the Canonical System of which it is an integrated and crucial component and this is acted out in an *architectural space subdivided in an altar area and one or several support areas*. Confusion has for example arisen over the circumstance that there are *two Eucharistic configurations* in the Sixtine Chapel seen as a whole, the Jonah figure above the altar and Cosimo Rosselli’s *Last Supper* in Sixtus IV’s wall cycle.

We have to take into account, then, *in which position and close to what, in a liturgical space, an image is located.* Working with larger chapel and church spaces, which include a congregation area in addition to the altar area, it is necessary to apply a functional distinction between the *altar area* and the rest, which can be called the *support area,* not only in terms of congregational activities there, but also to account for visual message programs (painting or sculpture) that may occupy walls, vaults, columns, etc. Very often, more or less similar pictorial subjects are used in both areas but with different message focus and even text origin (the Basilica Superiore at Assisi: a crucifix over the high altar, a Crucifixion on a side wall of the nave; the basilica at Pomposa: Eucharistic themes in the apse fresco and on a side wall; the Sixtine Chapel: as we have just noted; St. Peter's in the Vatican: St. Peter enthroned in the nave, his throne behind the apse altar, or the common cases with a crucifixion in the nave and near the high altar, or in both transept, as in Assisi). It is immaterial in terms of the principle on hand whether both items are contemporary or not; this is so because of the permanence of the Canonical System., which ensures consistency.

The pictorial contents or program in the imagery spatially connected with the altar (apse mosaics, antemensalia, "altar-pieces" etc.) therefore often display two themes, one of them: illustrating in one of the many different idioms the *Mass Sacrifice,* the Eucharistic offering; secondly a *saint* or several saints, that is, the *intercessors* cited in the Mass ritual, evoking the relics of intercessors "buried" in the *sepulchrum* of the altar mensa (2.2.9). In this manner, the altar area is set apart from the rest, so that pictorial programs on the walls outside this area are free to contain narrative stories that may not primarily reflect the Ordinal or Canon of the Missal as much as the Breviary (or earlier corresponding collections) and Biblical texts.

Let me return for a moment to the figure of the *prophet Jonah in the Sixtine vault* (1.6.1). It is part of the series of prophets and sibyls on the four sides of the vault. So far, so good. But it is positioned vertically above the altar. Irrespective of the chance that some scholars may fail to see the point, contemporary viewers, and indeed those who were admitted to this special chapel, would never fail to be alerted to the prominent role of the figure of Jonah in the liturgy of the Mass. Being swallowed up by the whale and then ejected, he became a typological image of Christ's descent into Limbo and his resurrection, with a text repeating *Matthew* 12.40: *Sicut enim fuit Jonas in ventre cetis tribus diebus, et tribus noctibus* (in this manner Jonah was in the belly of the whale three days and three nights)*;* and his figure is traditionally referred to as *Jonae signum, ut typus Dominiae passionis* (the figure of Jonah, as an image of the Lord's passion) (above, and SL, 1984, 63).

By a mechanism of *positional recall,* the Jonah figure's place right over the altar activates the Passion motif beyond any doubt, whereas, if he were placed somewhere else in the chapel, the figure might have a significance just as part of a Biblical narrative. When we find

Cosimo Rosselli's *Last Supper* on the wall in the chapel, then this belongs to a narrative cycle with Breviary references, whereas the Jonah figure is, by virtue of its *position* in addition to the citation in the Missal, a place marker for the function at the altar below. So when Jonah is concerned, the fact that a Eucharistic picture is already in the chapel, is irrelevant. The two images belong to different functional contexts, the *altar area* and the *support areas (walls and/or vaults).*

Adequately schooled (and who else used this chapel?) would connect the Jonah figure with the altar underneath it. And the authorities, whatever their ideas, would know that this was what was going to happen; in a so-called *explorative prevision.* This principle of positional recall accounts for the various specific roles of separate parts of a pictorial program in a church or a chapel. And it is applicable, as we shall see, also to architectural shapes like Borromini's Spire (1.10.1).

1.7 Configuration spaces

The perceptive reader will not have failed to note that the proposed distinction between *system* and *elaboration* suffers from a certain vagueness with regard to *that area of the latter level which stays closest to the borderline of the system itself.* This zone should be singled out from the rest of the elaboration area and represent the *first step* across the boundary from the systems level. This means that some measure of dynamics and hence approximation is introduced into the model. For if we could not postulate an area in the *elaboration zone* that stays closer to the system than the rest of the elaborated material, a matter of degrees, then the entire distinction would vanish into one between the system and the large and loose category of "pictures" or even "iconography". Let me use the shorthand term of system-elaboration *iconic interface* for the area that stays closest to the system itself (F*ig. 1.7 - 1*).

The function of the iconic interface can be metaphorically compared to a modem, switching between modalities. It is supported by the function that one and the same fundamental *canonical sign* can be *expressed verbally and/or pictorially in a number of ways*; in a *tree-like algorithm* (*see* 1.6.1, end of *section*). The fundamentals of a *canonical sign* or notion as expressed in dogma or doctrine, are subjected to a process, usually fuzzy and not surveyable, of sorting out *potentials* for elaboration (for a model with data migration and potentials, *see* Levine and Rheingold, 127). For a canonical sign, its specific form is immaterial; the only things of importance is that whichever benchmark pattern is chosen, it can be recognized as such; its *basic form* thus is pure abstraction, a kind of average: a ball with two triangles attached or a white feather standing for the Dove. In other words, evaluation takes place as to the denotations and connotations of the sign, marshalling the verbal and/or visual expressions with which it can be elaborated and communicated (alternatively: SL, my *Working*).

An example: at one stage in history (in reality, gradually), people felt the need of a more accessible figure of God, and the old, bearded man coming out of Daniel's vision (*Dan.* 9), substituted in many connections the earlier configurations with a Christ-like appearance. The Church adopted this configuration (in terms of our model, into the iconic interface) along with the traditional one, and, again in terms of our model, the sign moved towards the right, being further elaborated there.

The overall structure just indicated is illustrated in the process model below. Canonical signs arising in the iconic interface, are (usually) enriched by various elaboration alternatives (arrows fanning out). Passing over relevant specific subjects (SL, *Working)*, we may focus on the following general scenario. Closest to the Canonical System, *basic configuration* types are accepted, with the *signs* here, while elaboration is developed in the elaboration zone. This is accessible for inputs from society including local churches, commissioners and artists or craftsmen. The acceptance is mainly grounded in four considerations: liturgical focus, conceptualization (making people grasp essential ideas), veneration, and attraction (making the themas visually interesting and relevant). I use the term *sign(s)* in order to distinguish from Gerald Holton's *thematics* used in the context of science theory (3.2).

From the managerial point of view, the two main requirements regarding the iconic interface, let us assume, were *correctness* (which is to say, containing no flawed configurations) *in dogmatic and liturgical terms in imagery that was in itself considered as a necessary item*, and, secondly, *sufficient clarity and efficiency of communication of the relevant canonical structure* (of the Mass, for example). *Fig. 1.7 - 1* is a *system model* (for this type of models, and other examples, *see* 3.3., *figs.* 3.3 - *1* and *4.4 - 1*).

Further out in the area of elaboration, in our move to the right in the model, came concessions to situation-dependent pastoral needs and opportunities, and popular demands, as well as artistic availability; all variable factors, as we know.

For the communication requirement, speaking of the Church, one might call in Herbert Simon's *satisficing* terminology: *Reconciling alternative points of view and different weightings of values becomes somewhat easier if we adopt a "satisficing" point of view: if we look for "good enough" solutions rather than insisting that only the best solutions will do* (Simon, 1979, 11 - 14; and 1983, 85).

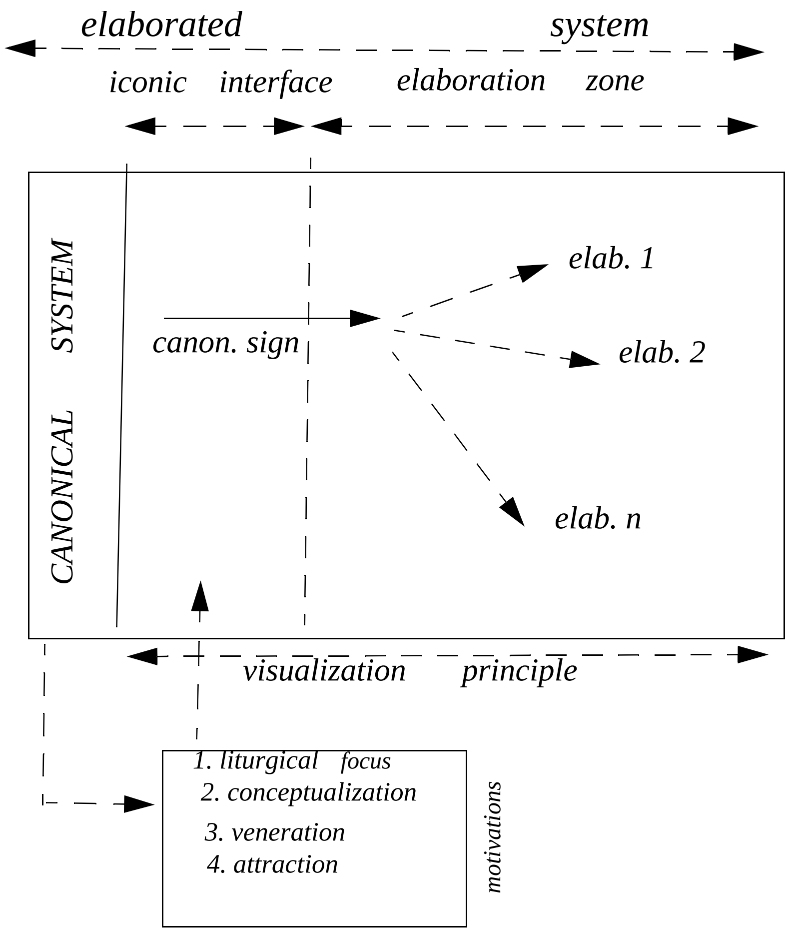


Fig. 1.7 - 1 Process model with inputs- output (no info model, i. e., no feedbacks.)

Having tested the system/elaboration distinction on a simple case (the Sistina) and on a model that seems to account for the essential features in the concept, we must admit that the issue theoretically speaking is very far from being simple. We sit back with a configuration space which we can, by way of transfer from "hard" paradigms (Part IV, *passim*), describe in terms of multiple (above 3D) coordinates defining vectors that are expressible in (pseudo) matrices (4.4 - 4.6): *n*-spaces with vectorial structure. Needless to say, the order of the coordinates has to be taken intuitively, there being no metric for them.

This configuration does not make things much easier for us. For it seems to follow that we cannot work out an entire configuration space or even a restricted liturgical space in one go. We have to consider one or a set of coordinates at a time. These reservations do not imply that the distinction system/elaboration does not work as a pragmatic tool helping us to sort things from one another, especially at the outset of a project.

The only option left us, as already noted several times, is to consider *the analysed object (thing or concept) as not only a product of the analysis process but as an instantiation of it* (1.11). And of course the countable number of possible analysis processes is *n* with no predictable limit to it.

On the other hand, methodologically speaking, we shall have construed a heuristically reliable platform if the notion of *system/elaboration* and the distinction *form/shape* (Lord and Wilson) admit of being seen both of them as reflecting a more general level structure, that of a *basic level* and *dependent level*, or, mathematically speaking, domain and range.

A characteristic of configuration spaces is that they can be described for *what there is* only partially, if at all, while nothing can be said about their conceptual space dimensions. They have to be described in terms of *operations* we perform in them, some among which, as on the model just considered, can be visualized. We may perhaps compare with the space developed by Joseph Louis de Lagrange (died 1813) (as distinct from the Euclidean space used by Newton). This *Darstellungsraum, dessen Dimension der Anzahl der Freiheitsgrade entspricht* (configuration space whose dimension is expressed through its number of degrees of freedom) *ist eine recht unanschauliche Sache* (is not easily visualized).

But it is highly usable for performing operations, specifically with differential equations (Linhard, 91ff.). For the benefit of non-physics readers (like myself), let me quote from an earlier edition of the *Encyclopaedia britannica*: *Degre of freedom: any of the number of independent quantities necessary to express the values of all the variable properties of a system. A system composed of a point moving in space, for example, has three degrees of freedom because three coordinates are needed to determine the position of the point...*

We use a configuration space to analyse *functions*, that is, relations that give us handles by which to perform operations such as identifying structural features, perform calculations, etc. But, as we see in a machine, while we can isolate for treatment some patterns of functions, we usually are hard put to say *where* in the machine the functions arise or are based or "found". For this, I refer to three sections in Gregory (83 - 95.): *How are functions found in machines? - Recognizing functions in machines - Emergent properties: ghosts in machines*?

1.8 And from now on...

Having collected some specific cases under a common denominator (as I see it) and derived some principles from them (1.6 and 1.7), we shall *concentrate the rest of the discourse on Borromini's Spire* and on whatever method-relevant insight we might draw from its structure and context.

I have noted that analysis has to start out from *one object* and stay with it to the bitter end; and *work one's way centrifugally out into the context or the network(s) in which it is integrated*. The rest of this *Part*, therefore, is mainly concerned with the make and structure of the Spire and its closest environment linkages, that is, to the papal university church. In *Part* III, the perspective will be enlarged beyond this rather narrow scope.

1.9 Borromini’s helical Spire

To return to Borromini, the most famous of all the “strange” shapes in his work, is the *helical spire on Sant’Ivo alla Sapienza* (*Burden*, 75ff.). For the rotating curve we have a nomenclature; it is a *conical helix with variable torsion*; today we even have mathematical means to describe it. But this method, *differential geometry*, was, as already mentioned, not available in Borromini’s time.

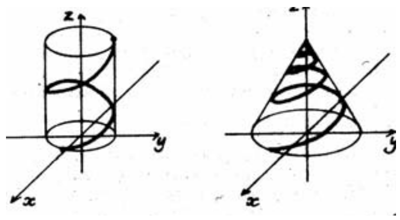


Fig. 1.9 - 1. Cylindrical and conic helices (Lord and Wilson), both with constant torsion or pitch.

Even the simpler *cylindrical* helix with constant torsion happened to be seen in a problematic perspective. In an oft-quoted letter to Mersenne of 1629, Descartes discusses this specific configuration (Mancosu, 78, French original, note, p. 225; quoted in full below in 5.2.10). The following observations by Mancosu may be highlighted, for they reveal that the very *status* even with regard to the normal helix could be subjected to doubt. Descartes to Mersenne:

> *... it is not the cylinder which is the cause of the effect* [that of the helix curve]... *The effect depends on the helix..., which is a line that is not accepted in geometry* [as Descartes saw it] *any more than that which is called <a> quadratix... For although one could find an infinity of points through which the helix or the quadratix must pass by, however, one cannot find geometrically any one of those points which are necessary for the desired effects of the former as well as of the latter...* And so on. (Mancosu, 78).

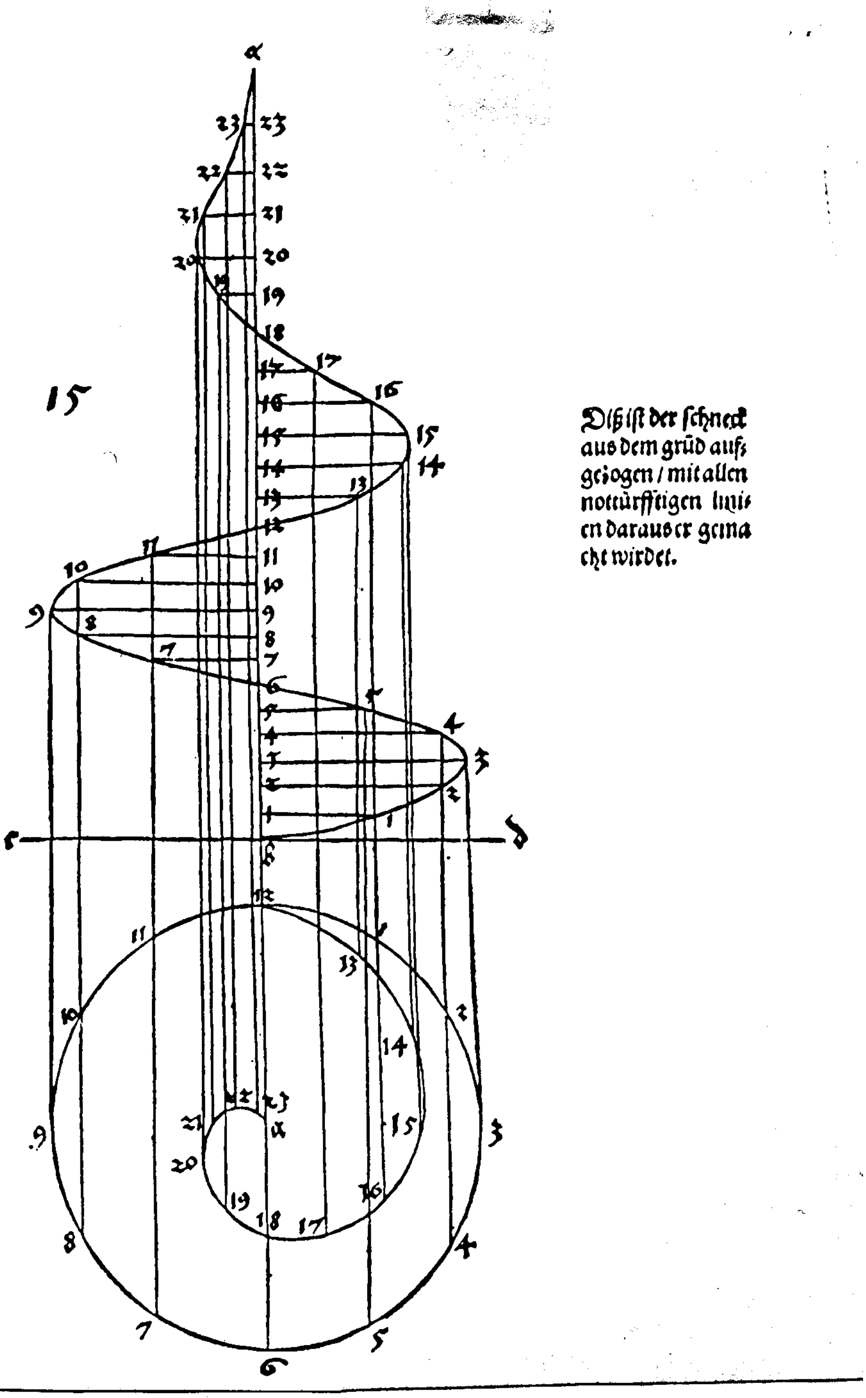
The curvature of Borromini’s Spire rises up from the base by a slowly increasing torsion, but the upper part suddenly becomes much steeper, making the whole thing look rather like a pig’s snout.

Torsion according to Webster, is *the degree of departure of a curve from a plane, a number measuring this.* Applied to a helix: *the displacement parallel to the axis of a point as it makes one revolution about the axis.* The *plane* in the Webster definition is the *horizontal section,* usually taken as the base surface of the helix. So that with a *variable torsion,* the helical curve along any generator (line parallel to the vertical axis of a cylinder; correspondingly for a cone), would not show identical intervals.

Let me note here that I shall take Descartes’ reference to the infinity of points as a support for my splitting up the helix in point with tangents (3.4).

At the time a helical shape as a whole was often referred to as a *cochlea* (as we shall see in Sixtus V’s inscription: 1.10.2), from Greek, κοκλιας, snail, and correspondingly in Dürer’s German, *Schnecke*. One *helix* was taken to mean *one* rotation, while a system of rotations, two or more of them, would amount to a *cochlea.* Guidubaldo speaks of a *cochlea duas habens helices; a cochlea with two helices.* In this manner, Borromini’s helix (in modern parlance) or *cochlea*, with three rotations, in contemporary parlance consisted of three *helices* (*Burden,* 78), or, as stated in some documents, of *tre corone*; a *corona* being the top piece of a building (*Burden,* 78; a fact that has been misunderstood).

Fig. 1.9 - 2



Prec. page: Fig. 1.9 - 2 Albrecht Dürer, Construction of a conical helix from a plane spiral (Underweysung der messung mit dem zirckel und richtscheyt in linien, ebnen, vnnd gantzen corporen, Nürnberg 1525).

The abstract helical design was probably premised on a completely traditional use of *proportions*; exactly as is the case with Albrecht Dürer's *"snail"* or *Schnecke* (*see* below, *Fig. 1.9 - 2*).

Planning to discuss the *helical theme* in more general terms later (3.2), I shall nevertheless at this point consider Albrecht Dürer's *Schnecke.* I do this because of the close affinity between his construction and the one behind Borromini's Spire.

The *increasing torsion* is characteristic also in Albrecht Dürer's *Schnecke* (*Fig. 1.9 - 2*). This is a conical helix constructed over a proportional system of numbered verticals rising up from a spiral (*aus dem grund aufgezogen*) and ending at numbered points that determine specific coordinates on the helical curve itself.

The construction is explained on the drawing *Diß ist der schneck aus dem grund aufgezogen / mit allen nottürfftigen linien daraus er gemacht wirdet*: the *snail shell drawn up from the plain spiral on the ground with all the lines that are required for its construction.* Dürer's helix shows a roughly constant torsion up to a certain level. From the top end of vertical Nr. 17, there is a gradual increment resulting in an increasing torsion, much in the same manner as in the Sant'Ivo spire, which, as a brick-and-mortar construction, will obviously be much less precise than a measured drawing.

Comparing Borromin's *helix* with Dürer's *Schnecke,* we may note one important difference between them. Normally, a conic helix would terminate in a *point,* which is the case also in Dürer's design. Having the torsion increase sharply towards the summit makes it possible to end the whole structure in straight-linear vertical axis. This increase is not consistent with the spiral that was probably used as its base. The Spire, on the other hand, had to terminate in a *vertical shaft or column* supporting a small ring shaped platform (open in the middle)*;* this should serve as a support for the triple flames in travertine and the iron structure carrying the globe and the indispensable *cross*. In view of this arrangement, a torsion increase may well have been implied from the outset of the building process, so as to smoothen the transition between the helix and the ring shaped platform. But there is another alternative (*see* below).

In order to gain support for his general idea (whatever the details may have been), Borromini seems to have prepared *presentation drawings* where things looked more conventional: in fact, a conical helix with constant torsion, as on *Fig. 1.9 - 3,* the section drawing in the *Opus architectonicum*. In some drawings connected with Borromini's Spire - but who can tell how they were connected? - the torsion is approximately constant all the way up, in some, the torsion increases by steps. In others, there is no regular helix, rather a construction made out of semicircles. If the regular representations relied on some measured construction drawing, why then was not this design reflected in the masonry structure? For a shape with a constant torsion would have been easier to build. Attempts at sorting out the interrelationship between all these studio experiments of Borromini's will remain inconclusive. Experience from architectural practice makes them seem familiar. Some orientation in planning, management and organization theory should serve as a warning against the naive belief, that the drawings can be taken at face value. It is simply not possible to determine their positions within such complex but impenetrable processes.

At any rate, I am resting my argumentation on the *finished product, which is a conical helix with increasing torsion.* My point of departure concerning the Spire has been, and will remain so, that it is the *conical-helical curve* which is substantiated in the *fillet* or *thread* that is in our focus of interest, especially on account of the increasing torsion, an unbroken band winding up the Spire (or down) (Descartes used the term in a comparable connection: *filet;* cf. *Le petit Robert: Ce qui ressemble à un fil fin - saillie en hélice (d'une vis)*).

I would not take it for granted that the Spire drawings known to us today were correct preparatory designs or an accurate representation of the construction work itself. Rather, as I noted, some of them may have been presentation pieces, *demos,* intended to impress high-ranking members of the Church and the university who could have been predicted to prefer regular shapes and hardly notice that the finished product deviated somewhat from what they had been shown; then it was too late anyway. The Spire was (and is) visible from several quarters, closely and at distance (*see* for example *Figs. 1.10 - 1* and *2*).
It is quite another question to what extent the peculiarity of the shape was noticed at all. People not trained to evaluate design, especially not in built structures seen at some special perspective, would be likely not to notice the torsion increase when looking up at the Spire surmounting the cupola of Sant'Ivo.

Everyone with some experience from architectural practice, historical as well as recent ones, will be familiar with enterprises in which presentation drawings for a project make things look nicer than real conditions would allow or make convenient; have them look conventional to render the project more acceptable to authorities that perhaps were not particularly fond of experimentation and worried about extra cost.

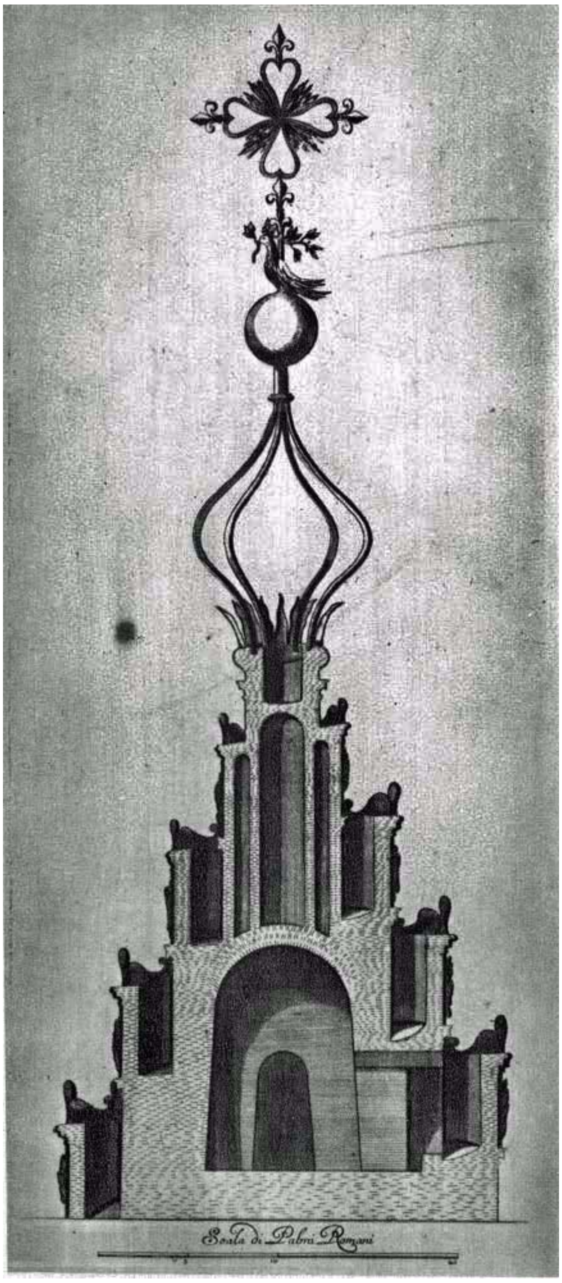


Fig. 1.9 - 3, Borromini, Sant'Ivo demo picture (Opus architectionicum)

Architectural treatises and drawings should never be taken on their face value, since usually they are intended for display in a market that will almost always include groups of people who are not directly involved in the planning and not mentally keyed in to related aspects. Participation in an "expert" group is often a question of rank and political or financial standing and authority rather than artistic or technical competence (3.1). Some disorder in the following up of a project on the part of the Church in seventeenth-century Italy should not come as a surprise (further in 3.1).

The Vatican was as yet not the biggest bank and real estate operator in Italy. The scandal of the mess over Borromini's project for the nave of the very cathedral of Rome, San Giovanni, should not engender undue confidence in public planning. The project was discontinued when half of it was carried out because at that late stage the pope "discovered" that Constantine's big columns would have to be removed (details in Blunt). A later pope angrily but too late discovered that Ferdinando Fuga had turned the nave of Santa Maria Maggiore into a *ball room.*

We can safely draw the conclusion that the Church would be inclined to prefer a *regular conical helix shape with constant torsion.* Apparently also Torricelli's *cylindrical helix* (3.2) would, if carried out, come out with a constant torsion. It was possibly, as an alternative to what I said earlier, in the production process supervised by Borromini that an *increasing torsion as the helix was nearing the summit* was introduced. This could have been the outcome of a rather simple consideration of convenience. Since the Spire was to end with a column supporting a globe and a cross, it may have seemed preferable to thin it out at the top part so as to fit in this crowning feature. This is a contingency that typically might have arisen during the execution of a project that did not originally contemplate such a feature, or rather, which had been approved by the authorities without thinking too much about the likely consequences, it being taken for granted that a cross would be raised atop the structure. We know from modern practice that an architect may not tell his commissioners more than is necessary to get the project accepted. Non-professionals do not usually foresee more than the most obvious consequences of a project. Finally, any amount of drawings and projects leaves us in a blank with regard to the sequential planning and implementation of the work.

At the same time the Spire, a *cochlea* or helix (a conical one), surmounted by a laurel wreath, in a university context, could hardly escape being compared to the Column of the philosopher emperor Marcus Aurelius (dedicated to his stepfather Antoninus Pius) a few blocks away, on which another (cylindrical) helix rotates upwards, described in the inscription set up by Sixtus V as a *cochlea,* and with a laurel wreath (1.10.2 for details).

To sum up: the Spire was welcome and even attractive in the scenario envisaged by the Church and her university as an image of the elevation of the Cross, of a Roman-inspired Triumph, and, finally, as a generalized image of mathematical science alluding to the role of this discipline in science and teaching. The last-mentioned ingredient would have met a reasonable desire to emphasise the papal university's involvement in science in comparison with the Jesuit *Collegio romano* a couple of blocks away; Joseph Connors has suggested such a competitive note in the entire business, but his idea was grounded on some iconographical interpretations that I find somewhat overdone (*Burden*, 76). Whatever the ecclesiastical and university concern for the helix itself, the image of a helical curve combined with a laurel wreath must have seemed attractive, also since it could hardly escape being associated with *rotational motion*; and consequently with corresponding terms in *traditional* astronomy, such as sphere, epicycle, and so on; which could be attested in the cosmological literature, most importantly in the case of the Collegio Romano Jesuit Christoph Clavius' astronomical commentaries (Lattis, 38ff.).

For people in sciences such as mathematics and physics, the Spire must have seemed striking as a conical helix with or without variable torsion, and a concentrated expression of a series of interrelated issues in contemporary mathematics, along with others like the "Möbius" band in San Carlino and the bent cosine arch in San Giovanni.

All these factors make the Spire a good candidate for becoming the center-piece in our further investigation. But along the way we have to attend to some other design issues that seem relevant to the argumentation I am trying to pursue. The story of how the Spire was constructed in masonry is not supported by any documents, but for one's peace of mind some guesses have to be made.

Another likely side of the story is that on account of *its increasing torsion,* even if this feature was a masonry-related product rather than the outcome of mathematical calculation (this can be excluded), the *Spire did function as a feedback* in the sense that people engaged in mathematics would notice the torsion and feel challenged by it.

1.9.1 *The Spire as a building job*

The Spire is constructed as a roughly outlined conical helix in bricks and mortar with details in travertine. It is provided with a narrow winding ramp, indispensable for setting up and later keeping in order the iron frames supporting the globe with the cross. Considering the complexity of the helix with its "snout", this was perhaps not the technique most inviting for copying from exact measured drawings. Borromini's pig-snouted helix must have made demands on the craftsmen who executed the work. In all likelihood a clay or stucco small-size model was employed for this, probably on the basis of some drawing with similar make-up as Dürer's helix (*Fig.1.9 - 2*).

A reconstruction of the technical procedures going into the physical construction in brick, mortar and travertine of Borromini's Spire has to rely on hypothesis (for the building history, see Connors, 1996 and 1996). The new documents dug out by Connors concern the iron works, not the masonry. This is understandable, because the former required competence and craftsman jobs outside the range of the architect's competence and hence probably paid from another budgetary posting. When the masonry work is not documented, this can be explained by assuming an overall agreement between the commissioners and Borromini concerning the latter's work, including, as usual, his workshop personnel and hired-in craftsmen, so that the detailed implementation of the architect's proper project did not figure specifically in the budget, because this most likely was a work *by contract* (*lavoro di appalto*).

An attempt to develop a hypothesis concerning the physical building up of the Spire might highlight the problems surrounding the construction.

Three groups of information may help us.

First, the only extant *construction drawing* for the Spire, or rather a late reproduction of one, to distinguish from perspective views (which clearly are mere demos), appears to be a plan of a *spiral* (a flat curve, not a helix), complete with the correct three rotations, dated 1652, that is the year when the cross and globe surmounting the spire were gilded (Connors, *Fig.* 46). *This spiral does not support the torsion increase in the final product, the way Dürer's drawing shows.*

The *second* point to make is that helical figures, both cylindrical and conical, had been in the focus of common interest for a very long time (3.2).

The *third* consideration relates to the point just made. We know from a vast literature that, besides high-competence artistic mathematicians such as Piero della Francesca, traditions of a simpler proportional geometry was current among builders (Paul Booz, *Die Baumeister der Gothik*, Berlin/Munich 1956; and SL, 1975, appendix on mostly fourteenth-century Florentine arch construction for doors and windows). We also seem to know that Borromini must have carried with him from North Italy some knowledge from such traditions, current among stonemasons.

Dürer's *Schnecke*, we have seen, gives the guidelines for the construction of a conical helix (*Fig. 1.9 - 2*). The simple idea behind it must have been widely known, probably also to Borromini. He could design a spiral on the ring shaped platform designed to support the Spire, erect verticals the way Dürer does it, but now by straight wooden poles or yardsticks of heights calculated to define selected points on the course of the curve.

A possible procedure goes like this; here simplified down to the basic principles (*Fig. 1.9 - 4*). On the circular platform, being the top of the Balbeek tempietto, the correct spiral will be drawn up in 1 : 1. Then set up a relatively skinny cylindrical column right on the center of the circle, for example like the column marking the vertical axis of Borromini's Spire at its summit. In reality, as is seen from some drawings, the column consists of cylindrical sections with upwards decreasing diameter; but this does not affect the building principle proposed here. The cylindrical-helical staircase in the Palazzo Barberini, Rome, also probably by Borromini, is constructed with such a column up the main axis. This is built on a circular ground plan, is hollow and has systematically arranged openings all the way up.

But back to our Spire. Let the flat *spiral* start at the circle periphery, rotate it inwards and, after the desired amount of rotations (in the specific case, three of them), end up where it touches the column. The column intended as a kernel for the Spire must provide surface enough for inscribing numbers on it. Now erect the poles or yardsticks, such as *x* and *y* on

Fig.1.9 - 4, equidistantly (the shorter the interval, the more precise will the guide result) on the spiral the same way as on Dürer's drawing, with lengths corresponding to the heights of the respective points on the planned *helix*. For a constant torsion, use an arithmetic series, for an increasing torsion, as high up on the Spire, use a geometric one. Then measure the distances horizontally and orthogonally from the pole top points to the column (arrows on *Figure, 1.9 - 4*). For each point thus derived from the summits of the equidistant poles we now have *two numbers,* one for height and one for the distance between the column and the future helix. Mark off on the column these two numbers on all the selected points. A frame has been constructed that can determine the necessary number pairs.

Then pull down the ring of poles (now superfluous) so as to free the access and start the masonry work. Measure by the two numbers on each crossing point of height and distance from the column to the top points across which the helical curve shall run, to obtain the line of the helix curve. In this manner, the column complete with the measures can be used to build the conical helix around it.

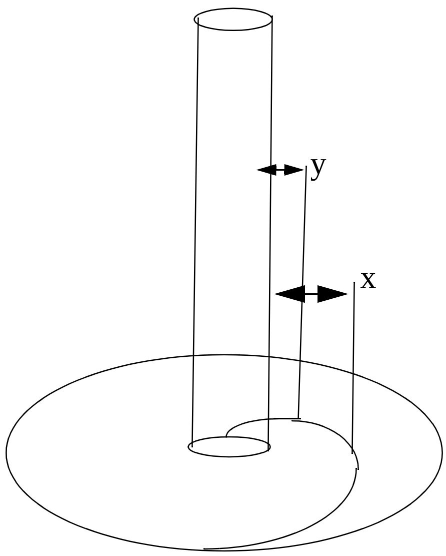


Fig. 1.9 - 4 Diagram for Spire building, much simplified, with only a half rotation, and the central column reduced to an elementary form.

During construction work, a line following the curve will be useful for controlling the operation; such as the broad and flat thread running the full helical course up the Spire (*Fig.s 0.1 - 1* and *1.1- 1*). This is a relatively broad unbroken band, strikingly visible even from a distance. It brings the *helical curve* to full view.

Now let us have some reflections on the Spire's functions in the environment of the Church.

1.10 The Church, the university and the Spire

In those peace-less days of protest and refutation, even disbelief, the Church had to think of her own dogmatic, doctrinal, socio-religious and political safety and role. The Ceremony Master of San Marco in Venice, by 1564 complained that people even in that arch-Catholic (dogmatically speaking) Republic, under the influence of Lutheranism, tended to stay away from the crucial and spectacular, and politically important indulgence rites (*Burden*, 98).

The Church had just accomplished (1563) the colossal achievement of the Council of Trent and also had works such as Bellarmino's enormous *summa* (2.2.12) to fall back on. But the problem remained to get the substance accepted (if not understood) and the injunctions respected and obeyed. She also had to think of her reputation in view of the changes, some of them genuinely dangerous, in the understanding of the world and the universe that surrounded her (2.2 with *subsections*).

Missionary programs - this in a wide sense of the term - have to use whatever may be accessible and understandable - in one way or another - to the regular congregations but also to people at the fringe of the inner circle, and here visual symbols will always be used, in politics as in religion (SL, 1974), the more spectacularly the better.

The Spire's primary functional role on the *symbolic* or *message level* - this much can be affirmed with confidence - was to present the image before the world of the papal university, employing triumphal connotations like those at Marc Aurelius' column a few blocks away, and to support the fundamental, supreme and universally understood sign, the *cross,* which surmounts the Spire; a deeply meaningful image also for a papal university church. Not only so, the Spire, surmounting a *liturgical* edifice, would have been unthinkable without the cross, so it does not matter when it is not illustrated on preparatory drawings or models; but it does appear on the *demo* figure in the *Opus architectonicum* (*Fig. 1.9 - 3*). At the same time, the conical-helical shape undoubtedly was appreciated also as an innocuous sign of mathematical engagement (1.10.1).

Now I propose to discuss two interrelated subjects: *the Spire's scientific connotation*, and *the Spire as a primarily religious sign.* I am doing this for now in a rather narrow perspective, intending to widen the scope later (2.2.11 and 12).

At a papal university, mathematics that were in those days considered "modern" but dogmatically innocuous, might be well worth a public display. *For the dangerous issues of cosmology and physics belonged, in the classification current in the period, to philosophy, while mathematics was something apart* (*Burden*, 81). This distinction would be accepted before the more complete mathematization of physics in more recent times. In those days, physics and cosmology belonged to natural philosophy, whereas mathematics was science properly, less affected by the post-Copernican conflicts and less suspect in the view of the Church (for Clavius, *see* 2.2.11). Characteristically, according to Drake, in 1550 the Dominican friar Domingo de Soto, when presenting an improved explanation of *free fall* seems to have omitted the relevant mathematical abstractions, *because his superiors in the Dominican* *order disliked mathematical abstractions. They wanted physical examples for things taught in natural philosophy* (Drake, 1989, 26). Things might look as going quite the opposite way in an extremely important case cited by Owen Gingerich.

Kepler's teacher Michael Maestlin in Tübingen, one of the top astronomers of the time, criticized him for integrating physics in his study of the motion of the stars, *scolding his student Kepler for dragging physics into astronomy, which he believed was inappropriate* in a context where geometry provided the adequate method. He wrote to Kepler:

> *I think that one should leave physical causes out of the account, and should explain astronomical matters only according to the astronomical method with the aid of astronomical, not physical, causes and hypotheses. That is, the calculation demands astronomical bases in the field of geometry and arithmetic* (Gingerich, 157, 168).

Hypotheses were now considered as *the geometric devices or arrangements used to explain celestial motions* (Gingerich).

This all together would seem to mean that essentially mathematical calculations and physical argumentation were considered two different things that did not necessarily even meet at crucial points.

In such a framework, the papal university, to return to the Roman *Sapienza*, might make a show of its modernity in mathematics, as representing science in general, without involving itself in issues related to cosmology and the embarrassing *Galilei Affair* and the scandal of the official condemnation in 1616 of whoever rejected the Biblical view of the universe and the centrality and immovability of the earth. The affair was highly problematic, since well-established scholarship concerning the universe now had to face direct observation and relatively precise measurements in it. It is important to note - with many modern authors, including Feyerabend - that the Church did not outlaw the proclaiming of Copernicanism as an *hypothesis*, merely as a *truth*. Feyerabend, with his usual acumen, and probably rightly, has argued that Bellarmino and the other representatives of the Church *acted rationally*, in fact, within their conceptual framework tied to dogma, *wissenschaftlich wohlbegründet*, and that their stand on the issue was the only possible option open for them (Feyerabend, 206 - 220) (details in 2.2.10).

The helical Spire, even though its conical shape was dictated by its position (see below), must have been foreseen to be met with large response in the world of science and among highly educated people of the day, and this in a configuration space characterized by a lively mathematical debate (2.2 and 3.5) which was relatively - from the Church's viewpoint - "uncontaminated" by cosmological novelties and non-Scholastic and non-Aristotelian natural philosophy.

Having so far proposed a general framework for the functioning and conceptual evaluation of the position and role of the Spire, we shall now take a closer look at some aspects of the Spire's functional role in the context of the papal university.

Borromini's church of Sant'Ivo, dedicated to the *scholar saint*, with its helical spire, was erected for the Papal University of *La Sapienza*. Borromini was appointed architect to the university (originally *Archiginnasio*) in 1632. The foundation stone for the church was laid in january 1643 and the structure finished by 1648; the cross and globe surmounting the spire was gilded in 1652.

I shall disregard for the moment the presence of a standard-type program of symbols and images on the Spire and lower down which belong to *normal liturgical repertoire* (triple flames, other symbols, such as jewels and torches, of light emanating from the Holy Spirit, the dove of the Holy Spirit, the Lamb, including the cross, elements listed in *Burden*, 79), reserving this catalog of *iconographical features* for later use. The figurative motifs on drum and guglia were standard features in churches, but in their position at Sant'Ivo at the same time striking, because of the position, and a reassuring demonstration that the Church with her university might well aspire to science eminence and show it with the Spire's helical shape; still she stuck to safety in religious matters, as was for all to see in the collection of liturgical figures and symbols as well as papal emblems.

I shall now concentrate on the *helical feature* by virtue of which it stands out as something unique, while at the same time offering an opportunity to search for links to contemporary mathematics.

Borromini's Spire, we have noted, represents *a conical helix of increasing torsion, with a laurel wreath and surmounted by the cross*.

Evangelista Torricelli, as an amanuensis in mathematics at the Sapienza during part of Borromini's activity there, was elaborating the configuration of a cylindrical helix, to which there is an affinity in terms of curve shape, in the conical helix that Borromini used for his spire at the Sapienza of Rome in the 1630s. Torricelli, next door to Sant'Ivo, in or before 1641 developed a method for calculating the volume of an helix and was internationally praised for the feat.

We have no information in support of the contingency that Borromini's rotating pig snout had anything directly to do with Torricelli's helix calculations. But Borromini had probably heard about them, directly or from men in the university circles, since in fact they were worked out at the University of *La Sapienza* and caused some stir. The architect would not have understood Torricelli's discourse, as we will readily appreciate when confronting Torricelli's graphs with his Latin text. This presupposes expertise in science Latin as well as in seventeenth-century mathematics language and practice.

Borromini's work on Sant'Ivo was in full swing while *Torricelli* was still an amanuensis at the Sapienza (until his transfer to Florence in October 1641). His mathematical studies of helix problems (for his *De cochlea*, *see* 3.2.1), would not in themselves authorize us to conclude that he was involved, directly or even by way of informal consultation, in the deliberations leading up to the final decision concerning the helical Spire.

The point is rather that, even when we find a written contract binding the artist to some specific project solution, there will usually have been some shuttling of ideas and concepts back and forth between those involved (3.1). Torricelli's struggle with the *cochlea* issue must have been known to quite a few of them; the normal grapevine would see to that; and this may have prompted *someone* to launch the idea (I hope that I shall not be cited as attributing the idea to Torricelli).

The upwards narrowing conical helix quite obviously was evaluated for its *motion towards the summit cross*. The cross is the goal and end term of a common drive upwards of whatever one might find at the lower levels. In the considerations of the Church and the papal university, therefore, there seem to be two perspectives to the shape, one concerning the type of motion, the other the direction of the motion, that is, toward the terminating cross itself.

The *type of motion*, to consider that first, would naturally be associated with mathematics *tout court*, and by some even with the efforts current in contemporary approaches in *protocalculus*. At the same time, the dramatically increased torsion at the top seems to speed up the ascending movement. The *calculus, in the early stages of its development, was bound up with concepts of geometry of motion, and with explanations of indivisibles and the infinitely small; for these ideas are suggested by naïve intuition and experience of* (Boyer, 1959, 11).

The Church accepted, respected and practised mathematics and at the Jesuit Collegio romano, the famous mathematician - and interlocutor with Galilei - Christoph Clavius was charged with drafting the mathematical component of the *Ratio Studiorum* and formulating a course of studies to train the teachers of mathematics in the Jesuit schools. One thing that strikes one about Clavius' approach to mathematical education was his insistence on the importance of mathematics as giving philosophers examples of *certain and solid demonstration* (Garber. 1992, 7). We shall hear more from him on this issue (2.2.11).

A conical helix when considered for its motion towards its summit, moves *ad infinitum* towards the interior; it is only design or building pragmatics that makes the process discontinue. Galilei, with Archimedes to support him, was aware of this. Many of those involved in the Sapienza planning and building would be more or less familiar with Archimedes' work on *spirals* and would appreciate the points made by Galilei in his *Dialogo dei massimi sistemi,* published in 1632 (cited ed., 173), about the *spiral*. Here he connects the idea of infinity with its rotating movement when narrowing towards a center it never reaches. Galilei emphasises the quantities involved in the ever diminishing spiral resulting from motion as being, as he says, (in the singular) *piccolo, anzi minimo, anzi pur minissimo,* and *gradi... infiniti,* and that *tali discostamenti* [towards the center] *sieno minori e minori in infinito.* Such statements as these, even by someone as suspect as Galilei, could hardly fail to appeal to the mathematicians of the two Roman universities, one papal and one Jesuit. The passage from a (flat) spiral to a (volume of a) helix is not long, and these scholars could hardly escape connecting the Sant'Ivo Spire with these concepts.

The *second* consideration concerning the Sant'Ivo spire is this. As for the *summit cross*, the helix arrangement brings it into view more dramatically, because it is unusual, than in the case of the crosses on the normal Roman cupolas, either semispheres or paraboloids. The inescapable - however loosely conceived - mathematical and geometrical connotations of a helix would eminently suit the *cosmic connotations and significance* of the Cross, as attested in liturgy and theology (*see* below).

It is superfluous to try to be specific about the fact that a cross was and is the standard image of the True Cross and normal for any liturgical site. It is the instrument of victory and triumph: though his death on the cross, the wooden object (*lignum*), Christ achieved victory over death to the benefit of mankind (Mass *Preface of the Cross*: *Qui salutem humani generis in ligno Crucis constituisti; ut unde mors oriebatur, inde vita resurgeret: et qui in ligno vincebat, in ligno quoque vinceretur.* Votive mass of the Cross: *Dicite in Gentibus, quia Dominus regnavit a ligno* [Ps. 95:10 - used elsewhere, too]. *O crucis victoria* ... [Pascher, 447]). There are Italian painted crucifixes with the dead, bleeding Christ and inscriptions glorifying him as the victor of death and restorer of life (SL, 1978). The *Cross* itself was (and is) no mere logo and much more than just one among many symbols; it is an embodiment of the True

Cross at Golgotha and the instrument by which the central part of the liturgy was enacted; and an object of a cult and the ultimate instrument of salvation.

The Council of Trent, concluded in 1563, set afresh the doctrinal and dogmatic stage for approaching the sacraments, among them that of the Eucharist, effected in the Holy Mass (Duval, 61 - 150; Denzinger-Schönmetzer, no. 1750, 410f.). The tradition of debates and, of course, the final statements at Trent, formed a central part of the conceptual system of the men of the Church and obviously also of those involved with the Papal university of *La Sapienza.*

The *cross* had been a sign of victory over the forces of darkness and evil right since Constantine the Great:. In the liturgy, there are a number of places where appeal is made to the Cross as the sign of protection against enemies and victory over them; thus in the *Communio* of the Mass at the feast of the Elevation of the Cross, September 14: *Per signum Crucis de inimcis nostris libera nos, Deus noster (By the sign of the Cross, free us, our God, from our enemies*). In the late sixteenth and the seventeenth century, with the wars against the Protestants and with the Turks threatening at the door, it was natural to set music to a text like this one:*...elevatum est signum in nationibus, Crux sancta salvatoris nostri, et dispersa sunt castra tenebrarum (A sign is erected throughout the land, the holy Cross of the Lord, our Saviour, scattering the fortresses of darkness)* (Battiferri, 1). The motet, *Vola de Libano*, from which the quotation is taken, with Battiferri's music, was published in Bologna in 1669 but used liturgically, in or near Loreto, in the first half of the seventeenth century (see the CD booklet, *Sacro e profano*, di Marco Mencoboni: *Musica nelle Marche al tempo di Ridolfi* [born 1570]) (my thanks to Marco Mencoboni for sending me this material). Loreto in religious terms was no provincial town in those days; it was an important center with very active connections to papal Rome.

In the late sixteenth century, no less of an authority than Cardinal Roberto Bellarmino presented a careful analysis of the significance the Cross as a metaphor for the *structure of the universe* in the salvational perspective, in Vol. II of his *De controversiis*, published in 1583 (Bellarimno, II, cols. 743, and 744, D-E). The world had been accustomed to symbolic interpretations of objects that somehow were used in the service of religion; church architecture, for example had always been exposed to *architectural iconography*. And yet it is noteworthy that such a theological rationalist as Bellarmino (rationalist even over the Galilei affair, according to Feyerabend) indulges, at great length, in such ruminations on a symbolism that might seem rather unnecessary because the Holy Cross was already so heavily loaded with profound meaning that nothing else could compete with it for fundamentality in human, religious and social life. Still, Bellarmino dwells upon the cosmic aspects.

This is what he tells his readers; and it is included in a prominent cardinal's *magnum opus*. The summit of the Cross signifies Heaven as it was opened through Christ's death (*passio*); the transversal beams denote East and West; and the bottom end dug into the ground, Hell opened up and the Devil vanquished. The Cross as a whole signified the entire world redeemed (*totum orbem terrarum redemptum*). The alleged text basis for all this was St. Pauls's Letter to the *Ephesians*, 3, but it was patristic theologians - Irenaeus, Augustine and others, followed by Scholastic writers - who had connected this text, in which no cross is mentioned, to the True Cross; another example of Bible usage in the Church. So the idea was firmly established and well-known, at least to the theologians.

1.10.1 *The Spire: university logo*

For contemporary people with a certain educational level, Borromini's Spire, on account of its helical curve ascending towards the Cross, and the Cross itself (plus other religious symbols), would acquire additional significance from its visual domination of the church of a papal university, a liturgical site serving a place dedicated to unifying science and religion. Thus the mechanism of *positional recall* (1.6.2) would apply.

The helical shape was (and to some extent still is) visible from close up and also from far away; strikingly so from the Capitol hill, from the Monte d'Oro at the Gianicolo and from the Castel Sant'Angelo. The Palazzo Madama was not the Senate building then, and the enormous Palazzo Pamphili in Piazza Navona was not built. The Sapienza complex was visually dominant. A close view is obtained not only from the courtyard but also from the square today named Piazza Sant'Eustachio and the street now named for Via del Teatro Valle (*Fig.s 1.10 - 1* and *2*). It was conspicuously unusual.

In a comprehensive appreciation of the Spire, not only the helical shape, but also the iconographical trappings reflecting the liturgy, have to be taking into account (*see* the models in *Fig.s 3.3 - 1* and *4.4 - 1*). In terms of public awareness, however, there is an overarching configuration notable regardless of the details just cited: that this is *a guglia rather than a cupola.*

The choice of some sort of *guglia* or spire, instead of a normal cupola, was *dictated by space considerations* ("space" now in its architectural sense). The Roman quasi-paraboloid domes, due principally to Giacomo della Porta & Co, that had been introduced on the skyline since the late sixteenth century, were too numerous and too conspicuous from close up as well as from a long distance for yet another specimen of such a shape being chosen to stand out so as to mark out the very special papal university complex.

Fig. 1.10 - 1 The Spire seen from Via del Teatro Valle (the summit cross, clearly visible from the street, does not show on the photo).

Moreover, given the restricted area available for Sant'Ivo (bounded by the sixteenth-century courtyard and the street at the back of the church), any regular *dome* or cupola would have to be made *much too low* not to exceed acceptable proportions in relation to the building as a whole, and then it would *not* have stood out on the Roman skyline. In the actual case, some kind of *spire*, or *guglia*, like the chosen one, provided the only option for a display of some prominence. So in fact there was no choice, and Borromini and his patrons must have understood this.

The spire or guglia shape would stand out as a place marker for the Sapienza irrespective of any interest in or active perception of the details (helix, imagery). In this sense the Spire served as a *logo* for the papal university, prominently and conspicuously rasing above the neighbouring buildings.

Such was the urban *logo* of the papal university of La Sapienza. Webster has this to say on a *logo*: *Also called logotype. a graphic representation or symbol of a company name, trademark, abbreviation, etc., often uniquely designed for ready recognition.* But, let me add, very often also designed to make visually conspicuous the particular features for which the firm or whatever hopes to be recognized and valued, so as to attract customers or clients and to be considered a valuable factor in the world around.

In the liturgy, but really in most types of visual communication, we can distinguish between *direct attention* and *conditional attention* (SL, 1984, 100f, 201f.). Primarily, the thing is there and we can see it clearly. Then we evaluate, even interpret it, according to some general and some specific criteria. The mechanism is the same as using a *logo* in marketing. Of importance for the functioning of a logo is its capacity to make people interested. In this respect it seems clear that the helix must have drawn attention to the science (and math) as well as religious and ecclesiastical themes that *might* be evoked by it; if not actually *designed* for this, then at least officially approved with this view.

Fig. 1.10 -2 The Spire seen from Piazza Sant'Eustachio

At any rate, the framework I am suggesting can serve heuristically to start off an analysis process relevant for a methodological venture, especially as this regards not only the Spire but also, approaching through it, the other cases of "differential" shapes we have noted (1.5, 1.5.1 and 1.5.2). The chosen object has no one significance in itself but is a *place marker* for context-dependent variations. Closer than this we cannot come to objects that are not predefined for significance (liturgy, traditions and conventions, pattern books like Cesare Ripa's). So there is no question of finding *The* Interpretation but of stipulating some domain of frameworks in relation to which various ranges of significance or connotations can be discussed.

1.10.2 *Sixtus V and the triumphal helix*

One monumental helix had always been in the neighbourhood, on the Column of Macus Aurelius in today's Piazza Colonna (*Burden,* 75f. It is called a *cochlea in* Sixtus V's inscriptions of 1589, which show how he so to speak took ideological possession of this historic column with its helical surface and laurel wreath. He specifically characterizes the winding band (with historical scenes from the emperor's wars) as a *cochlea* (precisely *cochlide*): SIXTVS V PONT MAX / COLVMNAM HANC / COCHLIDEM IMP<eratori> / ANTONINO DICATAM / MISERE LACERAM / RVIONOSAMQ<ue> PRIMAE / FORMAE RESITVIT / A . MDLXXXIX . PONT IV (*south side*). (Sixtus V Pope restored this helix dedicated to the Emperor Antonius, from its ruinous state to its original shape. In the year 1589, 4th of his pontificate).

It was under this pope that the *Archiginnasio*, later *Sapienza* palace was completed (inscription on the portal), with its facade on the present Corso Rinascimento (a street enlarged under Mussolini). When I am spending some time on the pope's reuse of the column, it is not to claim that the Sapienza Spire was "influenced" by it. My purpose is merely to suggest that certain ideas were ready for developing, and that this was one such case, our Spire another.

There are three more inscriptions, and they give us the information we need about the message that Sixtus' initiative was intended to convey.

Usually a person would not dedicate a thing to himself, but an adoptive son, Marcus Aurelius, could dedicate the column with a pictorial record of his military and other achievements to his adoptive father, Antoninus Pius, who had "created" him. In fact, the *east side* inscription reads:

M. AVRELIVS IMP<erator> ARMENIS PARTHIS GERMANISQ<ue> BELLO MAXIMO DEVICTIS TRIVMPHALEM HANC COLVMNAM REBVS GESTIS INSIGNEM IMP<eratori> ANTONIO PIO PATRI DEDICAVIT (Emperor M. A, after having subjected the cited three peoples, raised this triumphal column illustrating the history and dedicated it to his [step]father Antoninus Pius).

On the *west side* we learn that Sixtus V purged this column of any impiety, and set the gilt bronze statue of St. Paul the Apostle on top of it, dedicated the year 1589, the 4th year of his papacy:

SIXTVS V PONT<ifex> MAX<imus> COLVMNAM HANC AB OMNI IMPIETATE EXPVRGATAM S<ancto> PAVLO APOSTOLO AENEA EIVS STATVA INAVRATA IN SVMMO VERTICE POSITA DD (dedicatio) A<nno> MDLXXXIX PONT<ificati> IV.

In fact**,** Sixtus V set up the present statue of *St. Paul* atop the column. This is interesting in view of the fact that the Northern Reformers had selected, and, by Roman criteria, *misused* Pauline theology as the main basis for driving a wedge into the heart of the Church of Rome.

Even worse, an attack *against* Pauline theology came from Catholic, even if not entirely orthodox quarters. Just a few years before, in 1585, Giordano Bruno had published his *Cabala del cavallo pegaseo* in London, a book which contains a series of ruthless assaults on the same Pauline theology. So Sixtus V with his statue tried to guard Roman Tradition on both sides.

The culminating statement on the *north side*, puts on record the fundamental reason for setting up the *statue of St. Paul:* the column now triumphal and sacred supports/bears the true and pious disciple of Christ, who through <his> teaching about the cross triumphed over the Romans and the barbarians, referring, of course, to the *modern* barbarians:

TRIVMPHALIS ET SACRA NVNC SVM CHRISTI VERE PIVM DISCIPVLVM FERENS QVI PER CRVCIS PRAEDICATIONEM DE ROMANIS BARBARISQ>ue> TRIVMPHAVIT.

In this manner, the idea of an ascending helix culminating in the triumph of the Church, now in the emblematic figure of St. Paul, had been established by Sixtus V and conveyed conspicuously in the urban setting. The ground was prepared for developing and making understandable the functional framework of the Sant'Ivo Spire.

1.11 Structured argumentation

The book, as I have emphasized, aims at *theory and methodology* development, rather than substantive research, and consequently has had to start out from real-world occurrences; as has been well argued by Minsky and Papert (3):

Good theories rarely develop outside the context of a background of well-understood real problems and special cases. Without such a foundation, one gets either the vacuous generality of a theory with more definitions than theorems - or a mathematically elegant theory with no application to reality (cf. the comparable statement by Heisenberg, 0.7.c - 3).

Even the arch-theorist Max Planck was of the same opinion (*Ihm ist - als Physiker - bei aller Lust am Prinzipiellen wichtig, sich an der Erfahrung zu orientieren...*) (Fscher, 2007, 17). The same attitude is reflected in Einstein's dislike of the quantum theories.

The aim of this *chapter* is to restate my objective in an operationally clearer fashion, setting the stage for successive choices along a pre-established path.

When I say I am using *open-source* approach instead, what I mean goes as follows.

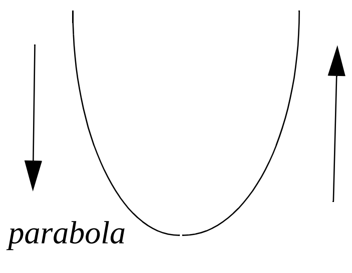
The term *open sources* is well known from computer language and is useful for its four main characteristics (for points 2, 3 and 4, *see* Winkler, 614f.; substituting "source" for "software"):

1) The sources are accessible singly or in combination, and will be accessed whenever the user finds them convenient or helpful or clarifying;
2) any chosen source is accessible for whoever wants it and in such a way as being generally understandable; that is, across adjacent fields and disciplines;
3) the source can be freely used and distributed;
4) the source can be modified and in this form distributed (copyrights permitting, as the rule goes; in my sort of cases: after duly having recorded the original state and derivation).

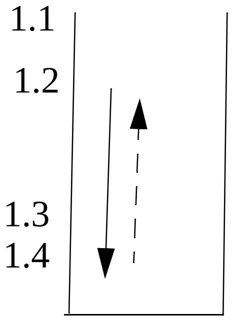
Facing a process of creating a systemic picture of objects of any type, means operating on several levels. Matters become especially complicated when the objects *defy quantification* in what we might regard as their crucial features, or in what must be considered collateral features that are integrated in the picture. In the present case, the object mainly in focus is a piece of architecture - *Borromini's Spire*. This consists of a physical nucleus that *is* quantifiable (a helix) but which is embedded in one or several social and *cognitive* networks of various nature, instantiated through the architectural and sculptural elaboration: historical, cultural, institutional, sociological, religious and artistic. I shall insist on not trying to *define* them but use them to indicate not what things might really *be*, but how we make them work in a process of *description.*

It is well known that most research processes do not follow a linear course; that the eventual "progress" occurs by random evidence collection, intuitive leaps and stages or steps that are not always clearly interconnected. Artistic intuition has played a role even in mathematical research. Order is worked out as the process moves along.

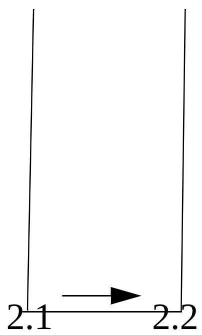
Underneath such processes, or running roughly parallel with their main course, and thus extractable from the general disorder, there must be an accountable, describable, adjustable and correctible *structure*, so as to ensure coherence and stability of general principle and theory. The rest of the present c*hapter* is dedicated to outlining the main features of the relevant structure in the present book. The issue will come to light again in *Part* IV.



Roughly, my argumentation follows the curve of a parabola, first top-down and then bottom-up. I shall proceed step by step and include some submodels; *see the diagram in three sections on this and the next couple of pages*: 1 Top-down, 2. Ground level; 3. Bottom-up (*Fig. 1.11 - 1*). The comments here are summary, the emphasis being placed on the significance in the present three-steps model. Only specifications strictly necessary for identifying some elements of the model contents will be appended. The numbers in parenthesis (...) refer to the *Sections* elsewhere in the book where the relevant subjects are discussed more carefully.

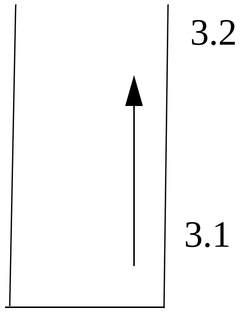


1. TOP-DOWN *Fig. 1.11 - 1* (in three sections)
1.1 *General theory*
-conceptions of truth, reality, objectivity
- scope, validity, coverage by analysis tools
- relations obtaining between observation and models/pictures/description
The upwards heading arrow indicates normal shuttle between observation and theory development
1.2 *Assignment theory*
- purpose, aims
- argumentation issues, models, analysis methods and tools (such as using a *p_matrix; see below*).
1.3 *General framework*
- design and mathematics under one compass.
1.4. *Assignment framework*
interrelations design - science - general culture - religion (the Church)
2**.** GROUND LEVEL
development of an *information model* (4.3.5) from the above. further developing toward an *output model* (next).



2.1 *Info model (see Fig. 4.3 -3)*
- Spire hardware with some obvious, traditional operations on it, put in *Storage.*
- *input* from top-down.
- *output* -> cited figure.
2.2. *Conceptual structure* (4.4.1) *(see Fig. 4.4 - 1)*
- Distinction between Spire *solid_state, Fluid_state*, and *sh*ape and form, location and interrelations between them.
1. *Spire_hardware*: the physical, built object itself;
2. *Spire_solid_state*: selected hardware, abstraction of its shape and quantification of it into a mathematical form; that is, subdivided into shape and form;
3. *Spire_fluid_state*: cognitive, conceptual and ideological attributions, regarding the object itself and its use**.**
- Articulating Spire structure in preparation for context evaluation.

The relevant graphic (4.4 and 4.5) subdivides the Spire structure in the two main states of *solid* and *fluid*, and subordinated here, in what is formally definable and what must be considered as informal; under these two categories, *form* and *shape*. At this point, the graphic under 2.2, i. e., *Fig. 4.4 - 1,* can only be considered as a rough summary of the concept structure (and the inserted numbers from 1 to 6 are for later reference).



3. BOTTOM-UP

3.1 *Object-in -context* (3.3, *see Fig. 3.3 - 1)* The graph in 3.3 presents a survey of the Spire's main conceptual linkages. Its main features are arranged on a diagonal, and this is set into its historical context in three interrelated field: the Church, Science and "lore" in general.

- Articulating interrelations in Spire's configuration space.

3.2 *Probability matrix (p_matrix)*

Discussion in 4.5.1.

End Fig. 1.1 1 - 1.

2PART II FRAMEWORKS IN FERMENTATION

The picture obtained in *Part* I is built on specific cases. Their selection and treatment have been guided by the notion of *interrelated changing entities* (Tenenbaum and Pollard, 1), detailed as follows: *acceptance, application and utilization of ideas that can be subsumed under the headings of change and chance, oscillation between alternatives that may be diametrically opposed to one another, and resulting conditions of instability; hence acceptance and utilization of approximation as a tool for handling nature, society and creative works: attestable at least in leading-edge people.*

To apply this model to the material presented in the present *Part* does not imply a claim that world realities were very much different from earlier epochs; only that political, social and general cultural and scientific conditions brought changes in the *frameworks* some people applied to their contemporary world. *I do not know how we want to live, for everything is so uncertain* (*No sé cómo queremos vivir, pues es todo tan incierto*); this is how Teresa de Ávila introduces her argument about ultimate dependence on God (Teresa, 155).

To subsume these changes under the term of *fermentation* amounts to suggesting that in a network some wirings got disconnected, were reconnected in new topologies, that some nodes melted, diminished or disappeared, while some of them resurfaced in changed form or weightage. Like Descartes' famous wax lump (*melting, does it remain the same?*), the scenario changed but probably remained fundamentally the same. He did not have access to modern chemistry to help him settle the issue. There is truth value in the French expression. *Plus cela change, plus c'est la même chose.*

The subject of this *Part* II is a search, in roughly the same time interval, for corresponding factors in science, philosophy, literature, music and in the Roman Church.

The argument can be introduced by noting that signs of change in world outlook and fermentation of frameworks could be attested even in the reputedly conservative circles of the Church and over such an allegedly rational field as *mathematical astronomy*. Lattis gives us some very acute observations concerning such an "established" person as Christoph Schlüssel, from Bamberg, or *Clavius*, at the Collegio Romano, commenting as follows.

> *In every edition of Clavius' commentary on Sacrobosco* [i. e., his *De sphaera*] *there is a tone in the cosmological exposition that is distinctly different from the medieval Sphere commentaries - less confident and more conscious of a world of alternatives...* (Lattis, 61ff.).

Clavius, a Jesuit professor of mathematics at the Collegio Romano (2.2.11), started publishing his commentary in 1570, which was quickly followed by new editions. Re-examining the Ptolemaic models of the universe was nothing new; it is his attitude, as recorded by Lattis, that appears significant.

Religion was an important ingredient in the scholarly preoccupations of the day and a stabilizing factor except on certain specific matters The ensuing controversies had very little effect on the general adherence to the Church, even among avantgarde scholars. Most scholars were actively Catholic (or Protestant), Descartes, representing the norm (2.2.10). No subject here can be studied without keeping religion and the Church of Rome in mind all the time.

That Nature right from the Creation produces from conflicts - *conflictividad como característica del universo;* that *di doman non c'è certezza*, and ideas that in part could be subsumed under the term of *coincidentia oppositorum,* all this was known in the earlier centuries. With due remarks on Herakleitos and to Francesco Petrarca, Fernando de Rojas expressed these conceptions with great clarity in the *Prólogo* to the *Celestina* (1499/1505; with Russell's comments: Russell, 213 - 220). In the seventeenth century, the idea of man's position in this universe and his capability to understand it, became a critical issue, resulting, often, in a sense of *Ziellosigkeit*; and the Scholastic image of Aristotle, if not entirely rejected, at least was severely reinterpreted.

2.1 Galilei looking at Rembrandt's *Night Watch*

Beyond official Catholic Traditions (1.6.1), the historical material is rather chaotic. Reflecting on all these great lights in science, from Kepler, Galilei and Bruno over Torricelli and Descartes to Clavius, Mersenne and Bellarmino, I try to see them as they are moving within one main intellectual framework of engagements and conflict-resolving goals and thus to some extent in unison; this much emerges also from the patterns of mutual communication. But the various individual or group processes are nevertheless moving in diversified manners, between which the interface was often not at all clear, and focused inwardly on their special issues, the protagonists now and then looking up from their work and engaging themselves in some commonly shared issue, some of them actively engaged in contemporary society, others sitting, as the historian in Anatole France's account, sitting isolated in a tower writing the history of the city while the enemy is busy demolishing it.

Let us invite *Galileo Galilei* to Amsterdam to take an anachronistic look at Rembrandt's *Nightwatch,* assuming that, with his well-known leanings, he would fasten on the dynamic features in the painting. He was fated in the long run to look at the contemporary world of science from some distance, from an observer's vantage point, and he would possibly recognize the dynamics in Rembrandt's painting as one that was typical of the contemporary world of scholarship What kind of *pattern of motions* would our notional Galilei identify?.

In Rembrandt's *Night Watch* in the Rijksmuseum at Amsterdam (1642), a painting that broke the Dutch *Gruppenporträt* tradition of statically posing fraternity members, factors like suddenness, almost-motion and diversified space directions and occupations dominate the picture. Unintentionally, I presume, but perhaps with a fine perceptivity for contemporary cultural tendencies, the painter has captured the dynamics inherent in them.

Rembrandt's enormous painting is traditionally dubbed *The Night Watch,* irrespective of the fact that the ceremony whose initial stage we see there, took place in full daylight. The men belonging to this burgher shooting fraternity (at the time more ritual than martial) are about to start moving out at the command of their leader and with one common but vague and complex ceremonial and social purpose, that of being seen in this institutional capacity and thus be confirmed in their positions in Amsterdam, a town in which many of them played important commercial and political roles. So the ritual skeleton so to speak is embedded in a larger picture of social and ideological conditions and events

Common goal maybe, but very different and individual behavior; one man in fact shooting with his gun, another one cleaning it (a thing you would hardly do at this stage of the proceedings) and, judging also from their face expressions, almost distractedly focusing inward on their own selves, not seeming unidirectedly concentrated on the processional order and the closed march out in the city. The space in which and from which they are moving is rather unsurveyable, being blurred even more by the complex movements of the men, and was so still more before a vertical strip at the left edge was cut off (there is a small contemporary copy of the original in the same museum showing the entire original composition).

Science in ±1600 seems to have been operating in the same manner, and probably had always been conducted on such a model as this. It is another thing to see that the dynamics as just noted finds its way into a conceptual recognition and a pictorial representation. This may be a sign of a special awareness of this kind of dynamical pattern. Attention to complex actions and spaces characterizes scientific and artistic behavior around 1600 and after.

2.2 Challenge and response

The *reasons and promptings* behind the attitudes and concerns just recorded can be analysed under four very roughly bounded headings:

- challenges from Nature;
- challenges from physics and cosmology;
- anxiety about general life uncertainty and instability (world worry);
- new design practice and new openings here (and in music).

Crisis had become a *literary subject* in well-established and relatively well-established and unified nations: England, France and Spain; less so, for political reasons, in the fragmented German and Italian areas. .

2.2.1 *Communication*

Universal diffusion of similar or related feelings, interests and attitudes in the world presupposes some measure of communication and international exchange. Business people, craftsmen and monks had created international patterns all through the preceding centuries; the Benedictine abbots, leaders of great centers of learning, were obliged to circulate at regular intervals. *Wandernde Gesellen,* as in Hermann Hesse's *Narziß und Goldmund,* also created information networks. Now science including mathematics was becoming increasingly international. Scholars and other *prominent* people travelled a lot; and more on behalf of academic roles than earlier, even the notoriously homeless and ever-wandering Giordano Bruno. In addition, a widespread international communication by letters in the field of mathematics, physics and astronomy, and the sending of books - now published in printed editions - across the continent and to England, ensured a sustained debate between the "schools" of various countries. The *Frankfurter Buchmesse* was in full swing by the sixteenth century. Even *the book nobody read,* Copernicus' *De revolutionibus,* was widely known, as Owen Gingerich has amply documented (Gingerich in the *Bibliography*).

Letters were exchanged at an impressive rate (Baron, 149ff.). Marin Mersenne, OFM, was perhaps no great exception when he actively circulated scientific letters between himself and other scientists; but he certainly contributed substantially. In published form his letters fill eight volumes. He was not a very original scientist; for the *Mersenne primes*, standard items in number theory even today, had been *known since antiquity and occur in almost every medieval numerological speculation* (Ore, 71, citing Mersenne's relevant publications). He nevertheless became a catalysator for events in the incipient *Calculus* and has preserved an important body of mathematical history for us in terms well described by Baron (Baron, 149ff.; cf. Struik, 27).

Mersenne corresponded with mathematicians and scientists throughout Europe: Galileo, Cavalieri and Torricelli in Italy were kept in touch with Roberval in Paris, Fermat in Toulouse and Descartes in Holland. In 1644, Mersenne visited Italy and was able to establish personal contacts with the Italian mathematicians. Besides raising problems for considerations arising out of his own interests and distributing the queries of others, Mersenne made himself responsible for the circulation of manuscripts for comment. Although this may at times have created difficulties, through his good offices conflicting points of view were often resolved and greater clarity achieved.

Mersenne may be counted as a mathematical equivalent to Sébastien de Brossard in seventeenth-century music: collecting and communicating scores, theory and data, showing himself as a good but not brilliant example of a creative scientist. The fervid collecting and elaboration of data, for which the Encyclopaedists are famous, started now. The respect for *The Data* went hand in hand with gradual liberation from utter dependence on books, Aristotle, the Bible or the Christian Fathers.

In order to substantiate further the claim about the intensity of letter correspondence among the scientists in focus in this book, Evangelista Torricelli's correspondence may be cited. Extant letters listed in the national Italian edition of his works show him in extensive communication with mathematicians and physicists in Italy and in France, including, among the latter, Mersenne, Niceron, Roberval, Du Verdu. Among the Italians there were, in addition to Castelli, Cavalieri and Galilei:, also Ricci - with whom he corresponded on spirals; and Carlo Dati - on *the true story of the cycloid*.

Despite this big volume of written communications, it is often hard to say which route across the land an idea really took, so that typically Torricelli's position was not all that clear:

The controversies regarding the use Torricelli made of suggestions received through Mersenne and other intermediaries from Roberval and Fermat in France resounded through France and Italy for centuries after his death. The unpublished papers which he left behind him, however, indicate that in his thinking he had progressed far beyond Cavalieri in the sophisticated use he was making of rectangular elements and in the scope of the tangent methods he was able to develop by relating them to the integration process. Although unquestionably Fermat, Roberval and Torricelli covered a great deal of common ground, the work of each had its own distinctive character and in consequence provides new insight into the discovery process (Baron, 193f.; also Toscano).

What about the masses of educated and influential people who did *not* send and receive conspicuous amounts of letters and books? As has been observed by Maxime Rodinson, admittedly in a different context, and quoted in substantially the following terms by Thierry Hentsch (13): *Scholarly studies influence the general area of interests much less than the general setup influences the scholars. Scientific results are often less the result of research than of influence from the surroundings.* Encircling the intensively informed and focused core of thinkers and scientists, grey zones have formed in which there were at play vaguer versions, fragments of, or uncontrolled, even inadequate, translations of the intellectual products at the core. Some of the ideas arising here found their way into books, much like today's often highly informative science journalism (James Gleick's and John Horgan's books are excellent examples of this).

Exchange of ideas or techniques did not always work smoothly. One problem was the absence of *standards* (Cajori has numerous examples). At times communication met with obstacles from the very interior of mathematics. A third-degree equation could be written as *a cubis + b in a quadr. 3 + a in b quadr. 3 + b cubo aequalia* <u>*a + b*</u> *cubo* by one and differently by another. Often one could not develop other people's math by simply continuing where they left off; the complexities of translation having to be faced. Witttgenstein's dichotomy was given full rein. Geometry, *showing* rather that *saying* dominated even in Galilei's and Torricelli's work. As in the early phase of *protocalculus,* geometry and proportions dominated where *algebraic numbering* would have served better, in which case a standardized automatism would, as Whitehead said, relieve the higher faculties of the brain of a burden. Even as late as the eighteenth century, Johann Bernoulli the Elder (died in 1748) complained that he could not make sense of Newton's *Principia*; Newton, who, to quote Mania, *wrapped up his trailblazing discoveries - as if he wanted to hide them - in a non-conclusive geometry, facing which most colleagues had to surrender* (Mania, 69f.). And yet - there is a *yet* here, it seems: maybe the integrated opaqueness forced people to think over scenarios which, had they been as plain as they are today, might have lead them to go unreflectively on.

2.2.2 *Challenge from Nature and Physics (Kepler and Galilei)*

Mathematics development, with which the architectural shapes and embedded forms are connected, will be our main concern (*Part* III). The following report om problems in physics, which interacted with the mathematical ones, should be adequate for illustrating certain tendencies in a scenario in which the math debates and efforts evolved.

Challenges came from nature's uncharted domains and unresolved problems, and also from physics itself, from attempted solutions that often alerted scientists by opening unexplored, even unsuspected vistas.

I am aware that the concept of *nature* as discussed in the pages ahead is somewhat skewed by being related primarily to the specific framework under consideration. The perspective on seventeenth-century ideas concerning nature, would be advantageously enriched by a consultation of Magne Malmanger's contribution, *Between Renaissance and Baroque: Attitudes to Nature and the Concept of Nature.*

The first thing to note is that *the overall picture is not simple nor consistent*. The pioneers of the new cosmology, Copernicus and Kepler, still clung to the Aristotelean notion of what is *natural.* There were ideas abroad about inherent dynamical potentialities (*vis viva*) in things like a stone, the air. Then, when, as everyone had learnt, the earth is heavy and imperfect while the celestial bodies are perfect and (almost) weightless, so what? And since it was believed by most, as Aristotle ha claimed, that the motion of an object is proportional to its weight, how could these things stay together? To increase scope but also complicate matters, came the notion of the earth as a magnet. Furthermore, the new heliocentrism, offered more problems that solutions.

Speaking of Galilei, Stevin and Kepler, Carl B. Boyer (1980, 370), notes that physics and astronomy had by now reached a level of development that made it more and more incumbent to have recourse to considerations concerning *the infinitely large and the infinitely small*, in other words, one of the crucial issues of modern infinitesimal analysis

At any rate, here is a brief and, as I understand it, concise and essentially acceptable abstract of scientific attitudes toward nature and physics in the early seventeenth century. The account emerges in an argument about *artificial intelligence.*

Accordimg to Montaigne, the sciences had established no real knowledge. But to Francis Bacon, this lack of past success called not for despair but for a new approach: "Things which have never yet been done can be done [only] by methods which have never yet been tried"... Using the Cartesian method of "analysis" - studying macroscopic phenomena by resolving them into their component parts of bits of matter in motion - men like Pascal, Torricelli, and Boyle were able to explain, by mechanical means, a whole range of phenomena previously requiring notions like the abhorrence of a vacuum [reference to Westfall: Descartes' appreciation of Harvey's method]. .. Cartesians saw the method as opening all of nature to being reduced to Descartes' principles... (Grabiner, 332).

In physics an impressive amount of observations were made, insights acquired and theories promoted. It is necessary to consider the circumstance that many single of them were not directly connected with what in the present book goes under the label of *protocalculus.* Challenges and needs were being met whose technical and also mental integration into the incipient calculus had to wait till the latter was ready for it. This was no linear nor a consistent drive but rather more like Baron's symphony picture referred to below or the pattern in Rembrandt's *Night Watch* (2.1). Some of the subjects and themes will be listed later in the present *Section*. First, however, some better clarification may be achieved through citing some points of comparison between modern and classical physics.

In a general evaluation of physics, Niels Bohr affirmed that his science did not concern nature but human knowledge about nature. The scientist would never know how nature is in reality. They can know only how nature appears to them... (*Physik... handelt nicht von der Natur, sondern vom Wissen, das wir als Menschen von der Natur haben. Niemals können Wissenschaftler erfassen, wie die Natur wirklich ist. Sie können nur erfassen, wie die Natur erscheint* (Fischer, 2002, 82). According to Ernst Peter Fischer, in the quantum context,

> *Physics concerned human knowledge about the atoms rather than about these in themselves, and Heisenberg and Pauli are the first physisists to understand this...* (*Die Physik handelt eben mehr vom menschlichen Wissen von Atomen und weniger von diesen selbst, und Heisenberg und Pauli sind die ersten Physiker, die dies verstehen und sich gegenseitig klarzumachen versuchen*) (Fischer, 2002, 22 and a few pages later): *From this moment the rational explanation of the world starts out from an irrational point of departure* (*Seit diesem Augenblick beginnt die rationale Erklärung der Welt an einem irrationalen Ausgangspunkt*).

In the seventeenth century the outlook was different. Up to the 1920s, it was firmly believed, really taken for granted, that physics whenever successfully conducted revealed bits of *The Reality* in the world. This was so, regardless of the circumstance that not only measurement and "objective" observation ruled the game, but in many cases also transcendental factors, such as belief in the role of celestial harmony (Kepler, for one; *see* also Holton, 1988). This attitude of innocence predominated among the pioneers as long as *classical physics* reigned uncontested, that is, from its beginnings with Galilei and some predecessors, followed by Newton's laws and up to and including Einstein's theories of relativity.

It is when *quantum physics* enters the scene that this conviction is revealed as untenable *as a principle* in the science of physics. Up to that time, it was possible to believe that physics could, at least in theory, be *successful and yield correct and staying results.* Quantum theory put an end to that. Heisenberg as a young student was told by one of the great lights in contemporary physics that he should devote his life to something else, since in physics all essential work had by then been accomplished and only marginally interesting new knowledge would be acquired.

With classical beliefs still reigning, scholars in the seventeenth century had, now to simplify the issue perhaps unduly, a twofold level on which to base their approaches, the Aristotelian outlook and the Archimedean method. This sounds like issues on two different levels, and so they were; not directly commensurable, rather accessible by cognitive jumps from the one to the other. To sum up on a complex issue which is treated in detail all through modern literature, the following comment must suffice in the present context. While the Aristotelian approach, and that of his followers (the *Peripatetici* derided by Galilei and colorfully by Bruno) consisted in a kind of logical-intuitive evaluation of the forces inherent in nature, only naively based on real observation (a heavy body falls faster than a light one because so it looks when air resistance is not taken into account), Archimedes and his followers performed experiments and measured them. Galileo Galilei was to become the major follower of the latter course.

Some *challenges from nature* had always been there, but now, in addition came opportunities and problems connected with new technology, discoveries and new ideas, and we know that opportunities create challenges as well. It was *physics* (*natural science*) that sought them out and tried to describe them. Physical facts that had always been known in one form or another now came dramatically into view. New tools made new approaches to them worth trying, and this in turn led to new discoveries. The *new cosmology* not only pushed mankind from a central position to one far out in a possibly endless universe (2.2.4), but left them on a globe that rotated on its own axis at incredible speed and spun around the sun at great velocity.

A newly developed concern over the importance of *relativization* and *uncertainty* in the physical world stimulated the study of such elusive quantities as became the focus of the incipient *calculus*, here dubbed *protocalculus.*

Morris Kline gives a useful summary of *the historical considerations behind the development of the calculus* with reference to four sets of the most *urgent physical and technological problems* in the time-span that I have labelled ±*1600* (Kline's points I, II, III and IV to follow in a commented and slightly abbreviated report) (Kline, 1977, *Introduction*).

- I. Seventeenth-century scientists were concerned with the problems of *motion*, hence with variable velocities, on account of astronomic and other gravitational and, indeed, ballistic, issues.
- II. Consequently, *the determination of tangents to various curves* stood in the focus of research, since the *deeper significance is that the tangent to a curve at a point represents the direction of the curve at the point*. Closely related to the notion of curve tangents was that of directed motion (velocity), a problem studied in numerous different contexts such as those just mentioned and, furthermore, optics and hydrography. The concept of *vectors* had been used in a pragmatical-geometrical manner since the sixteenth century (but the term and the mathematical elaboration had to wait for Hamilton, who died in 1865).
- III. A third set of problems concerned the determination of *maxima* and *minima*: what is the relation between elevation and range of the trajectory of a cannon ball; what are the maximum and minimum distances of the planet from the sun; and so on.

- IV. Finally, it was essential to determine exactly *areas bounded by complex curves and volumes bounded by complex surfaces*. This, so to speak, was the basic challenge and one for which geometry still provided the principal tool. So far Kline.

In a larger perspective, charting the development up to the late eighteenth century, Eves provides an additional list (Eves, 45).

Let me synthesize very rapidly the overall picture of *physics* that made the need evident for a mathematics that was capable of handling *interrelated changing entities* and make *approximation* a virtue. Typical cases are forces under varying conditions, the effects of gravity; free fall and the acceleration of a falling body and of one thrown up in the air and subjected to desceleration; the *curves* of the trajectories of cannon balls, the weight, heat and density of objects, the center of gravity of objects, changes in materials exposed to heat, strength of materials, and so on. Some of the subjects attacked within physics were encumbered by the virtual inaccessibility for exact description. The principal items are *centre of gravity, motion, velocity and acceleration, inertia, moment, impetus.*

With the assignment now facing us to bring some more flesh to Kline's sturdy skeleton, the question arises, how to do this in an orderly manner. There seems to be no metric for bringing the relevant thematics into some sort of a system that might be directly relatable to seventeenth-century ideas; there being no consistent usage with respect to them. I shall discuss the material under the following headings: 1. *Change in form, shape or state;* 2. *Motion (kinematic, dynamic) "essentially" (phenomenon of things moving, being moved);*.3. *Motion from place to place;* 4. *Motion, differential (such as in hydraulics)*; 5. *Motional force, inertia, gravity;* 6. *Acceleration, Intensity*; 7.*Motion, composite*; 8. *Approximation.*

It goes without saying, that such categories will often dovetail into each other or even overlap (motion generally, force - for example).

§ 1 *Change in form, shape or state*

Descartes' famous reflections of what happens to a lump of wax under changing conditions (heat increase, pressure, etc.), is symptomatic for an interest that arose occasionally but with not much hope of a constructive formulation. The problem would have required developed chemistry and thermodynamics.

§ 2 *Motion (kinematic, dynamic) "essentially" (phenomenon of things moving, being moved)*

Approaches of course varied. While Bruno in his complex ruminations daydreamed about the universe and Descartes philosophized over the *real nature* of his melting lump of wax, Galilei *observed, measured and experimented* (2.2.3) with his inclined planes and free-falling bodies, but their interest and engagement coincided on many levels. Eves (174f.) offers an instructive catalog of the problems the new technique was finally - from Leibniz and Newton and on - developed to handle; which, to the extent that they were considered at all, *had been baffling and quite unassailable in earlier days.* An important problem concerned the *conservation* of momentum and impetus, ancestors to today's conservation laws (Alonso and Finn, 1080ff; less technical in Fischer, 2007). *The concept of motion simply* ***conserved****, and not caused by a force somehow contained within a body thrown, was of enormous importance to Galileo's mature physics* (Drake, 1990, 75 - 78).

In the scenario pictured in the present book, the issues relating to *motion* and *change* are obviously crucial, so that we need some further comments on the subject.

Galilei we know, and others with him, started new research. A new *theory of motion and of free fall of objects was needed* and was sought, leading to better *mathematical techniques and* intensified experimenting in *protocalculus* (for more details, 3.5.7 and 5.2 8). Very gradually and jumpily, perceptional reality-intuition now gradually became supplanted by mathematical calculation, in parallel with new techniques of *algebra* gradually taking hold; approaching modern views on science (Kline, 1981, Chapter 8). Another motive for reflection could be expressed by such a question as this one: can what is abstract but measurable be more real than what people see, hear and feel?

§ 3 *Motion from place to place*

The mathematical pioneer Niccolò Tartaglia, who died in 1557, is emblematic. He is reputedly the first to have applied mathematics to the art of artillery (in a book with the resounding title *Nuova scienza, cioè Invenzione nuovamente trovata, utile per ciascuno speculativo matematico bombardiero...,* Venice 1537: *New knowledge, that is solutions found recently, useful for every reasoning mathematical bombardier*). Figuring out the trajectory was a problem even for Galilei.

Two cases may be singled out as symptomatic for the role of geometry - while geometry and thus also proportionality continued to dominate generally right through the first half of the seventeenth century. Issues of *force* and *gravity* unavoidably arise here (*see* §5).

Kepler's work furnishes us with a major example of how the physics of *motion* was applied to cosmology. While working on his *Harmonics,* and trying to fit physical data into his predicted ideal structure, one notion became crucial for further development: *that there is a moving force in the sun that sets the rest of the system moving and keeps it in orbit.* To follow up this idea for the building up of a celestial mechanics, geometry remained his only really workable tool (Caspar, 129ff.; Kepler, ed Hawking, *passim*). It seems likely, however, that, in view of the contemporary progress in the *protocalculus,* he must have fathomed some operative potentiality in this direction; without, however, having at his disposal, as we have since Newton and Leibniz, a reliable mathematical analysis. But the case must have been felt as another challenge that made the new mathematics potentially useful. The next stage in this work of Kepler's came when he found himself confronting *the rate at which the earth moves in the orbit, as a consequence of the force issuing from the sun,* [which] *is inversely proportional to its distance from the sun.* Such a case requires a so-called *elliptic integral,* which lies on an advanced level even for *modern* analysis, and was elaborated in the early eighteenth century (an example in Kline, 1977, 677; see also Struik, 374, on Fagnano's publication of 1714-15 in which the concept is being handled formally).

The *second* story regards Galilei. It is not entirely clear but does illustrate important parameters in the way of thinking in those days. Boyer (1980, 376f.; cf. Drake, 1990, 198) notes that Galilei, through Salviati in his *Dialoghi dei massimi sistemi* (cited edition, 206ff, not the *Giornata terza*, as in Boyer, but the *seconda*) argues against Simplicio's attempted demonstration, that the earth could not possibly *move*. The latter did so by saying that the rotation of the earth would have thrown objects off it and out into space in a straight line, tangentially to the surface (recognized as a problem even by Copernicus).

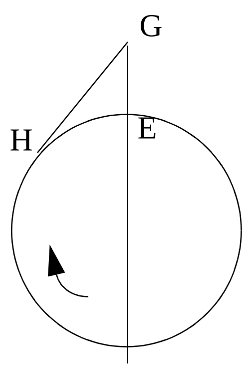


Fig. 2.1 - 1 Galilei's diagram simplified (see next figure)

Let me copy Galilei's diagram, simplifying it and paring it down to the essentials, with an arrow added to show the direction of the rotation, and report this telling story. For visibility purposes I have made the distance E - G much too big (it should be infinitesimally small). This is another example of how the crucial idea of the incipient calculus, namely that of *infinitesimals,* was managed with *geometrical* tools. In fact, Salviati claims that Simplicio is wrong, not from a logical, physical or metaphysical point of view, but from a purely Euclidean-geometrical one (*Voi dite così, e dite falso, solo per difetto non di logica o di fisica o di metafisica, ma di geometria*). An object thrown off the globe, as Simplicio imagined, along the tangent from H to G, would instead be drawn towards the centre of the globe along the *segante* (as they called it), at the distance G - E. In reality, this would be of infinitesimally short, so that the amount of force required to keep to object on the surface would be too small to count in this respect. Salviati follows up with a lengthy geometrical demonstration, which is endorsed by Sagredo.

Drake, in his summary of Salviati's argument, first notes that physics in the *Giornata Seconda* is totally cinematic without trace of dynamics: conservation of velocity without force (inertia) being calculated in; and Galilei took his point of departure in the law of free fall and the notion that the velocity of fall is proportional to the time measured from rest. Here is Drake's summary (with references to my *Fig. 2.1 - 1* added).

Euclidean geometry sufficed for Galileo's proof, in which a body was assumed to leave a rotating sphere along the tangent (T)*, at uniform speed and in the direction of rotation. A segment* (H - I) *was cut off from the point of tangency, and secants* (two of them on the *Figure*) *were drawn from points along the tangent, equally spaced and hence representing equal times. These right triangles were those of the "triangle of speed" in fall from rest, placed with the time-line along the horizontal instead of the usual vertical. Since the surface of the sphere always intersected the secants, the body must always have time to reach the surface - if, indeed, the body could ever leave that along the tangent, as Galileo duly questioned* (Drake, 1990, 198).

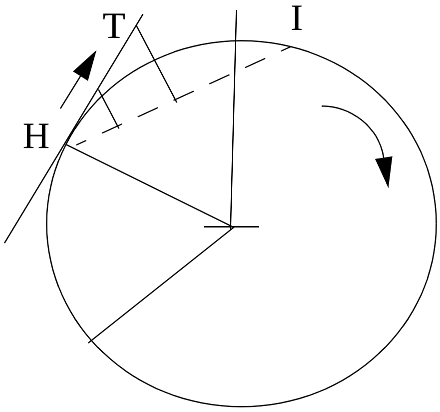


Fig. 2.1 - 2 See Drake's comment, above.
The circle rotates along the tangent (T, fixed in space) in the indicated direction (arrow). An object at H was supposed to be thrown off along T as indicated by the arrow at H.

Drake reports that for a long time he, like many before him, tried to fault this argument of Galilei's, but failed. Anyway, the issue is too complex for the present context. It is the general idea of relying upon *classical* geometry that merits attention

This particular case is emblematic for the depth it allows us to sound in seventeenth-century handling of *physics* in terms of *geometry* exactly at the moment when new *algebraic* laws of physics were being sorted out. The case is typical also in that *infinitesimal* values are being put to use. Galilei-Salviati's insistence on the *geometrical* basis for the argumentation does not seem entirely clear, unless we read the statement as ruling out physics only in the common understanding of the term, while thinking of *physical forces* in terms corresponding to the modern notion of *vectors*. In consideration of Galilei's concern with the interrelation between forces and geometry, it is unlikely that his thinking should not have approached closely, albeit not explicitly and formally, the *idea of vectors* (cf. also Bruno's model, 1.4).

It is also a noteworthy consequence of a purely geometrical reasoning, that his tangent argument presupposes, at least in model terms, *a perfectly smooth spherical surface for the earth.* This was something the Church attributed to the celestial bodies (sun, moon and all), while of course knowing that the earth is not smooth; while the Galileans claimed that the known celestial bodies were all of them rough. Geometry could hold the ground because the idea of universal *gravitation* was not recognized by Galilei (or Huygens), having to wait for Newton to be sufficiently formalized (summary in Drake, 1990, 227). The idea that two *masses* attract one another was not clear.

§ 4 *Motion, differential (such as in hydraulics)*

The subject of *motion in water and floating of objects in it* may be chosen as a typical issue in the intensified search for ways of describing "capricious" changes stimulating the search for new mathematics for their handling. The subject had become increasingly urgent especially because of the necessity and the relatively novel opportunity to undertake great regulation works in channelling and redirecting rivers and draining huge stretches of landscape - work at which the Republic of Venice had been especially active and innovating. Today's landscape between Ferrara and the coast still bears visible marks of this.

The stand-out case of Archimedes' *De his quae vehuntur in aqua...* (as the work was known to the Latin West; *On these* [*things*] *that float in water*) hovered in the background. But now the subject becomes what today we might call *topical.* In 1612 Giorgio Coresio published *Operette intorno al galeggiare* (*Minor contributions on* [*things*] *floating*). Federico Commandino (1509 - 1575), a mathematician from Urbino, wrote a book on objects floating in water, using Archimedes' title almost verbatim: *De iis quae vehuntur in aqua libri duo,* Bologna 1565. Galilei's talk about the issue and his *Discorso sopra le cose che stanno in acqua e che in quella si muovono,* Florence 1612 (*Discourse about the things that are in water and move around in it*), opened up for a flood of writings, many among them hostile (a number of them cited by Basile in Viviani, notes 127 and 128). Galilei's friend, Benedetto Castelli, followed up with his *Delle misure dell'Acque correnti,* Rome 1628 (*On measuring flowing water*; also later revised and augmented editions). Raffaello Magiotti came up with a book, *Renitenza certissima dell'acqua alla compressione dichiarata con varij scherzi in occasion d'altri problemi curiosi,* Rome 1648 (*Resistance of water against a given compression - with various tricks connected with other curious puzzles*); a title with an appeal that reveals the popularity of the subject outside expert circles. Such naturalist-literary efforts are a characteristic outcome of the preoccupation with natural phenomena that defied description in classical terms and symptomatic for the atmosphere of publicity in which the *protocalculus* developed. Typically, one of the mathematicians who succeeded in coming quite close to insights that were to become central in the calculus, also did one of the few calculated experiments with liquids. Evangelista Torricelli worked out a method for quantifying the amount of water flowing out of a recipient, laid down in his famous *effluence formula.*

§5 *Motional force, inertia, gravity*

Many tasks, we have seen, require the determination of a body's *gravitational center,* geometrically expressed and studied. This today is handled by means of *position vectors* which were not available at the time. The issue came up with the Greeks and was being pursued up to modern times. Still in the seventeenth century, finding the gravitational center was attempted using geometry, and through this determining so to speak the *ideal midpoint* of a body or also of two bodies connected as if making up a single one (Baron, 90ff., Boyer, 1959, 99ff.).

To start out from today's physics (Alonso and Finn; Mansfield and O'Sullivan; Dugas), let us note that there are four fundamental mutually independent quantities: *length, time, mass* and *electric charge.* A body's mass is measured in its *inertia,* that is, its resistance to acceleration, or the *gravitational force* it produces. In modern formulation, *the inertial mass of a particle is a property that determines how its velocity changes when it interacts with other bodies* (noting that velocity is directed speed).

An especially demanding task was to account for the quantity of *force*. Before Newton, even with Galilei (Drake, 10090, 221), this was not accessible in any strict format. *Impetus* (classically *onset, attack, force, impulse*) covered the knowledge that some kind of force was in action while of unknown origin and not admitting as yet of precise description or masurement. In late Scholasticism *impetus* was used for both straight and curvilinear motion; or alternatively only for the former. The Scholastic view of *impetus*

> *gave to motion a so-called intensive characteristic, for it centered attention upon the act of moving rather than on change of position and extension.* <The later> *shift of emphasis made acceptable the notion of motion at a point, an idea which Aristotle had specifically rejected* (Boyer, 1959, 177).

Motion at a point: here they came close to *protocalculus* issues.

The notion of *mass* was not developed, nor was the related idea of *inertia.*

Newton and others with him studied what we would call *force,* at the time called *impetus,* acting for instance on a ball rolling down an inclined plane. Without possessing a clear notion of what a force might be in quantitative terms, at least they were coming close to the modern *vector* concept. Motion in different formats also occurs in *changing volume* of a mass under varying temperature, the resistance of materials, the *complexities* of flowing water - the whole subject so to speak squeezed into conflicting ideas of the infintesimally small, vaguely felt limits, and infinity.

Jean Buridan (died in Paris after 1358) put forward a carefully argued and detailed thesis concerning impetus, of which we can give only the flavor here (Dugas, 49ff; 105, for what follows). *When some agency sets a body in motion, it imparts to it a certain impetu, a certain power which is able to move the body along in the direction imposed on it from the outset* [!]*... The greater the velocity that the body is given by the motive agency, the more powerful will be the impetus which is given by the motive power. It is this impetus which moves a stone....* Generally, in the principle of *impetus* were embedded the two principles of *inertia* and *energy*, which were not clearly grasped but confused with each other.

Galilei used *moment of speed* interchangeably with *impeto, after he had freed the latter word from its medieval implication of an impressed force, and redefined it in the "Dialogue" to mean simply speed of a heavy body, however acquired* (Drake, 199, 21), the last clause being typical for his refusal to go beyond the facts and search for deeper reasons (2.2.3). Drake in his *Galileo: Pioneer scientist* (1990) has, as he himself details, modified and to some extent corrected what he had found and published in his *Galileo at work* (1978), but I believe the following observation from his earlier book is still valid. Galilei defined

> "*moment"... as the tendency to move downwards caused not just by the weight, but also by the arrangement of heavy bodies, as when we place a given weight far from the center along the arm of a balance. Later... Galileo named speed as a factor in moment. "Center of gravity" was defined as that point in a body around which parts of equal moments are arranged, a definition previously given by Commandino* (1558) (Drake, 1978, 56f.).

Fermat, according to Baron (177f.), approached the issue of gravitation center in a fashion that led to approaching closely other crucial issues. *One of the earliest links between differential and integral processes* [3.5.10 and 5.2.11] *was provided by Fermat who extended the method of small variation, not only to the construction of tangents to curved lines* [3.5.3 and 5.2.4]*, but also to the determination of the centre of gravity of a conchoid.*

A theory or scientific practice can often be better understood if, from viewing how they came into being, one extends the vision to what happened to it *afterwards,* when it was about to be supplanted by a revised, modified or new version. Therefore, the Chapter *From Galileo to Newton* (Ch. 15) which Drake wisely has appended to his main text body in his book, *Galileo: pioneer scientist,* has much to say (too much for the present context) about Galliei himself (Drake, 1990, 221 - 234).

Evangelista Torricelli was a pupil of him. He developed a "principle" described by Dugas (145f.). Galilei had found that

an ensemble of heavy bodies could only start to move spontaneously if its centre of gravity came nearer to the common centre of heavy things; again *nearer, almost - quasi,* Torricelli, according to Dugas, *made this remark precise, and raised it to the status of a principle* (1644, in Florence), *that two bodies connected together cannot move spontaneously unless their common center of gravity descends. Indeed, when two bodies are connected together in such a way that the motion of one determines that of the other, this connection being produced by means of a balance, a pulley or any other mechanism, the two bodies will behave as a single one formed by two parts. But such a body will never set itself in motion unless its center of gravity falls. But if it is made in such a way that its center of gravity cannot fall, the body will certainly remain at rest in the position that it occupies. From another point of view, it would move in vain because it would take a horizontal motion which did not tend downwards in any way* (Torricelli, translation Dugas).

§ *6 Acceleration, Intensity*

Speed increase in connection with curvature will be a later concern; also the issues of heat increase and volume expansion crop up occasionally.

§ *7 Composite motion*

So-called *compound motion* was elevated to a special topic at the time. The problem was discussed by Giordano Bruno and Roberval (Dugas, 105, 150 resp.). With Galilei this was a recurring theme (Drake, 1978 and 1990), connected with the issue of *moment* (of speed, not the mathematical term: 5.2.13), and I shall quote some of Drake's account from the years 1607/8 concerning Galilei's method for tackling the problem, which is very illuminating on a number of counts (Drake, 1978, 125, 133). Here we are to do with the fundamental difference between total speed as a sum of moments of speeds and *continuous speed.*

Concerning the term *momenta velocitatis* (Drake, 1978, 113ff. 125f.; 1990, 86ff.), at one time synonymous with *gradus velocitatis* (moment or degree of velocity), Drake notes in an illuminating comment that Galilei's

bold identification of an overall speed with the totality or aggregate of an infinitude of separate "degrees" or "moments" introduced the novelty of dealing *with speeds in acceleration as not separate and additive, but as literally continuously changing:...* "[quoting Galilei:] *The moments of velocità of things falling from a height are to one another as the square roots of the distance transversed"* [his famous Square Law]. [Drake continuing]... *the inherent difficulty in reaching the decision to treat speeds in natural acceleration as truly mathematically continuously changing is hard for us to appreciate. Fundamentally, the difficulty arose from the concept of cause, which had to be abandoned for continuous change of speed. Newton himself was unable to find a cause behind his law of gravitation, and that distressed him. Medieval thinkers had accounted for changing speed in fall by assigning heaviness as the cause of the "first" motion from rest, and increments of inputs as the causes of successive changes in speed of motion while the original cause continued to act uniformly.... Galileo... tried to ground the law he knew to be true on some kind of addition of separate speeds, always unsuccessfully in terms of mathematically rigorous derivation. The* <cited>*... decision*

put an end to such efforts and made possible a rigorous mathematics of accelerated motion by a fundamental redefinition of velocità which permitted speed, like space and time, to undergo truly continuous change (Drake uses the Italian word *velocità* because in that language the word covers both speed and velocity) (Drake, 1978, 125f.).
§ *8 Approximation* (*see* also 3.4.2, 4.3)

The reported efforts and debates drew attention to *approximate quantities* both as a fact of life and as something that might turn out to be useful, at least in a heuristic perspective. What I mean by saying this goes as follows. The variously named and understood parameters of motion and change were handled in philosophical, that is, *verbal* terms. The only available tool of quantification, apart from direct measurements of time of fall etc. (to the extent that this was considered sufficient) was *geometry*, and this was used for the determination - right back to Archimedes - of the *center of gravity*. In this way one had to handle what we would call interrelated changing entities with two different tools - measurement and geometrical abstraction - whose output and conceptual foundations, were not entirely commensurable. Intuitions about the inherent potentialities of the mathematics that are subsumed here under the term *protocalculus*, provided a driving force toward the more active grappling with physical and cosmological problems. But the solutions strictly speaking were - and still are - grounded in *approximation*.

The calculus is dominated by the criterion of approximation, and this is a critical point on which gradually physics and the protocalculus converged. The approximative factor shows its head not only in the handling of physical issues; including written expressions. Here are two examples. In mathematics, Cardano explains *subtilitas* in terms of approximation, as will be noticed in the next *Section.* Galilei himself realized the importance of *approximation*, basic to the new approach in contemporary mathematics (and the future calculus) and was fond of using the adverb *almost to cover the gap between the observed and the ideal case* (of course in Italian or Latin) (pointed out by MacLachlan, cited by Sharratt, 192).

2.2.3 *Measuring the world*

So far, rules and procedures somehow related to specific developments in design, mathematics and physics have been discussed. The many ventures come under the umbrella of *Measuring the World*, to borrow a term from Daniel Kehlmann’s book *Die Vermessung der Welt* (dealing with the much later scientists Alexander von Humboldt and Carl Friedrich Gauß; the latter now more deeply portrayed in a book, *Gauß. Eine Biographie*, Hamburg 2008, by Hubert Mania).

In the seventeenth century, geometry still furnished the method, scope *and* limitations, but simultaneously there was a strong drive toward accepting and handling approximation values in a way that surpassed static geometry and gradually led toward the calculus. Descartes, in his *Second Meditation*, tries to get a conceptual grip on an evasive process, an object subjected to change in substance, color and smell under varying conditions such as temperature, while retaining its mass constant: his famous *lump of wax*; and he asks: *La même cire demeure-t-elle après ce changement?* Of interest for us at present is not the much-debated question of its utility for discussing *substance theory* and its importance for the *self* and the *Cogito* (thus Dicker, 52), but the very circumstance of Descartes' having chosen an object *in order to study change*, whatever conclusion that might lead to. The purpose may well have been philosophical, but the test object was used on account of its *physical* properties.

The process of measurement has never been an "objective" procedure with absolute standards. The history of architectural description provides a good example. "Exact" drawings from the same Egyptian monuments under Napoleon and in the nineteenth century are dramatically different. Differences in mathematical notions also are influential.

The fundamental ideological perspective is that mathematics more and more became the instrument for observation and evaluation of the surrounding world and the universe. Among the arts and crafts, architecture naturally - partly because it had always involved issues of proportions - came within its range in many connections. It is worth recording here that some technical device that we, today, take for granted, were not available, to wit, *the function concept* while *the idea of symbols as representing variables does not seem to enter into the work of any mathematician of the time* (Boyer, 1959, 156).

The following comment on Galilei is worth noting, for of course measurement was a pivotal instrument for surveying the world and conceptually a comparison-standard for discussing infinitesimals. He invented systems of measurement that were ever more exact, but he shifted his attention from ideal precision to an idea that was necessary for his purpose and workable with the instruments at his disposal.

The view into the stellar universe was a question of discovery and of ideas of cosmology, fraught with religious problems. At the same time the activity involved "objective" observation and measuring. The gradual crumbling of the Aristotelian division of the universe in worlds above and under the moon opened up for metric exploration. The gaze into the interior of the world and of mankind was intensified. The anatomy of Vesalius (*De corporis humani fabrica*, 1543) continued to be controversial well into the seventeenth century. Harvey's description of the blood circulation came later: *Exercitatio anatomica de motu cordis* *et sanguinis in animalibus*, 1628. In these cases as in the investigation inwards in nature, a subject of the important studies by Goodfield and Toulmin and Wilson, there was always an issue of *approaching limits and boundaries of how far one could reach into matter and out into space.* At the same time new geographical worlds at the fringes of what could be known, were being discovered and registered (and plundered, then as now).

Catherine Wilson in her extraordinary book, *The invisible world*, of 1995, besides the many and penetrating observations on how the smallest things were being approached, notes that this process was not one of clear linear development. With reference to Girolamo Cardano's *De subtilitate* of 1560, she comments that

... the corpuscularian philosophy established itself in the first half of the 1600s as the product of a progressive refinement of the Renaissance notion of "subtlety", and a materialization of hidden resident spirits....; some of them will be listed later in the present *Section*. Wilson continues: *What exactly is Cardano talking about? What he says here amounts to a declaration - against the spirit of Aristotle - that the truth about things is not reached by ordinary processes of reasoning and observing. Like Nicholas Cusanus*, Cardano explains *subtilitas in these terms* (Wilson's translation): *a certain intellectual process whereby sensible things are perceived with the senses and intellectual things are comprehended by the intellect, but with difficulty. Cardano pairs intellectual and visual subtlety* (Wilson, 40f.).

The question of *mathematical notation* has a direct bearing upon the general issue raised here. To reiterate the point with more details a typical example of the "rhetorical" notation (thus labelled by Kline), the equation $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$ could be rendered as *a cubis + b in a quadr. 3 + a in b quadr. 3 + b cubo aequalia* <u>*a + b*</u> *cubo.* The example is given by Clericuzio, 158. Cajori's *History of mathematical notation* is useful for being so "factual", but since the book classifies the subject in addition and subtraction, division and ratio, proportion etc., we remain hard put to gain a more comprehensive view from it.

While the latter (pre-modern) notation offers the possibility of arguing one's way through the string, using one's linguistic competences along with the numerical ones, the modern notation admits of context-less calculatory automatism. While the modern notational system allows of easy operations in division and with fractions, the former, to permit this, must be elaborated linguistically. Not as bad as multiplying with Roman numerals (multiplication of MCLVII x LXIV!), but almost so. We seem to have two frameworks, one verbally and numerically argumentative, the other one operating routinely with pure quantities. D. J. Struik warns against taking things for granted (worth a long quotation).

We end with a word of caution. Despite the fact that, in order to understand these seventeenth-century mathematicians, we are inclined to translate their reasoning into the notation and language with which we are familiar, we must constantly be aware that our point of view is not equivalent to theirs. They saw geometric theorems in the sense of Euclid, where *we see operations and calculating processes. At the same time, just because these mathematicians applied their geometric notions in an attempt to transcend the static character of classical mathematics, their geometric thought has a richness that may easily escape observation in the modern transcription. If we were to rewrite Euclid in the notation of analytic geometry we would obtain a body of knowledge with a character different from that of Euclid and, despite all the advantages that the algebraic computations would bring, we would lose some of the more subtle and aesthetic qualities of Euclid* (Struik, 263).

Møller Pedersen follows up: *An important reason why mathematicians failed to see the general perspectives inherent in their various methods was probably the fact that to a great extent they expressed themselves in ordinary language without any special notation and so found it difficult to formulate the connections between the problems they dealt with* (Møller Pedersen, 47).

Now let us turn to Galilei for a moment. As already mentioned, Galilei used *moment of speed* interchangeably with *impeto, after he had freed the latter word from its medieval implication of an impressed force, and redefined it in the "Dialogue" to mean simply speed of a heavy body, however acquired* (Drake), 1990, 21). It is the last clause that is relevant here, the fact that he observes and *takes measurements without asking for the reason or motive* behind the protocols; precisely what Decartes blamed him for in his famous letter to Mersenne.

In the Introduction to his book *Galileo: pioneer scientist* (1990), Stillman Drake records facts from Galilei’s working method that I shall synthesize here.

He was careful with his units: the *punto* was set at 0,94 millimeters; his time unit or *tempo* at 1/92n sec.s. It is remarkable, however, that *Galileo did not use algebra and he never wrote an equation in his life, not even in his private papers. Algebra in equation form was just beginning to be adopted during his early years as professor at Peso.* Fully aware of this, he nevertheless stuck to the Decline theory of ratios and proportionality. Nor did he use decimal fractions, introduced in 1585. His declared scope was to observe, measure and draw conclusions, not to do philosophy: in 1605 he asked what measuring has to do with philosophy (much like Orchard Finnan in modern times).

Careful measurements of distances and times are made without conscious appeal to philosophy. The data yield information upon mathematical analysis - the same information to Aristotelian as to Platinised. These *actual pioneer processes* belong to the history of physical science, and were *legitimately employed according to the tenets of every philosophical school.*
So far Drake.

2.2.4 *Man receding into the infinite spaces (Copernicus and Pascal)*

Man’s central position in a universe created by God for his benefit now began to appear somewhat unsettled. Feeling uprooted, man searches something firm and stable to which to attach himself - but where? Infinity and various kinds of end terms had formed part of general lore since the Middle Ages, as has been discussed in Gerhard Jaritz' important book, *Zwischen Augenblick und Ewigkeit* (Jaritz is a leading figure at the *Institut für mittelalterliche Realienkunde,* of the Austrian Academy of Sciences, at Krems an der Donau).

In his book *Kants Welt,* Manfred Geier notes - not being the first to do so - that Copernicus' heliocentric image of the universe forced man out of his former central position and into faraway and unknown spaces. He cites cases from the second half of the seventeenth century and the next, with Kant present, in which this sense of expulsion from the inherited central position and role became a real problem. *With the downfall of the geocentric world image, man, too, seemed to lose in importance* (*schien auch der Mensch unbedeutend geworden zu sein*) (Geier, 89ff.). Obviously he also refers to Pascal's comment (written before 1670) on reactions in the world around him. *Combien de royaumes nous ignorent! Le silence eternel de ces espaces infinis m'effraye* (Pascal, 41; he is speaking not especially of himself but of people at large).

Galilei's *Simplicio* voiced perhaps a more general worry than just his own (2.2.7). At any rate, not only had Copernicus launched a heliocentric picture of the universe, thus extending the world beyond measure; while Giordano Bruno alarmed the Church and alienated many with his idea of innumerable and infinite universes. Disturbing prospects were abroad.

2.2.5 *Catch a falling star (Donne, Góngora, Calderón, Cervantes, Lope de Vega)*

We may probably say that some such situation as the one just sketched out became increasingly felt as common and progressively threatening, but also challenging, in the first half of the seventeenth century in Europe (for *world worry, see* 2.3). The well-documented overview by Souiller, *La litérature baroque en Europe,* mostly concerning France and England, offers a survey, with an ample array of references, of some of the critical emotional issues of the time. The same can be said of De la Flor's *Barroco. Representación e ideología en el mundo hispánico (1580 - 1680).* Concerning the so-called *edad conflictiva* (the era of conflicts; the title of a book by Américo Castro), he refers to Popkin's *The third force* - which I have not seen, a book which argues in terms of a "third force" or ideological tendency of scepticism and nihilism, at work between the two forces of Aristotelism and the new science (Flor, 56).

All "periods" were certainly conflict-ridden. But now features which we can subsume under the term of "conflict", such as internal contrasts, insecurity and vacillations, scepticism and pessimism, become a central literary *topos* understood and accepted or rejected by intellectual Europe, albeit with variations due to national traditions and characteristics, particularly in England, France and Spain (this is noted by most modern commentators). The substance behind the *topos* can be indicated by noting that there are degrees of intensity, depth and penetration and spread to a crisis. The matter is too complex to gauge by some metric beyond recording events and literary expressions.

In Souiller's and De la Flor's works, unfortunately, as is the case in almost all publications on the subject, acute and useful *observations* are connected, with the purpose of explanation or depth underpinning, to a concept, *The Baroque*, which has no identifiable systemic and hence no explanatory status.

The political and social conditions in the fragmented lands of Italy and Germany provided a less fertile soil for exploiting such tendencies as a literary theme. It takes institutional stability and consolidated authority as in France, England and Spain to allow writers to pass beyond conventional truths. Pidal speaks of *un momento coincidente en España, Francia e Inglaterra* (Pidal, 124 - 126). The Spanish Inquisition certainly did not tolerate deviations, but those people were genuinely professional and stuck to a limited set of issues. Vacillating, unstable or contested authoritarian regimes, on the other hand, do not enjoy cultural liberty (in the twentieth century we knew that all too well). Even so, there can be no doubt that in fragmented Italy, too, the epithet *frameworks in fermentation* is justified for the scenarios of intellectual activities, while corresponding literary pronouncement were reserved, generally, at least, for the top scientists with a limited audience.

The scholars to be cited shortly tell us *what* Donne and others wrote, but do not offer much of an analysis of the whereabouts. A literary statement, in prose or verse, is and remains - a literary statement. All professions and relevantly focused communities develop and respect certain rituals, so it seems always to be an issue whether some dramatic statement on the part of Donne or whoever should be taken at face value or as, at lest in part, an adherence to actual social attitudes. Secondly, on a more specific level, sometimes crisis or worry and instability, real or ideological, seem to have found expression on formal levels, typical of ritualization patterns, such as by disrupting conventional style or relativizing norms, just as much as in dramatized content. It is hard to distinguish between such various factors. I shall be looking briefly at these factors under the more comprehensive, operational perspective of *thematics* in Holton's sense of the term (Holton, 1988).

One case has been discussed already, that of Giordano Bruno, who started out as a poet and later wrote a comedy, *Il candelaio.* Another case, to be briefly considered in the next *Section* (2.2.6), is Corneille's letting the Aristotelian *unities* go by the board.

The boundary between "artistic" literature and philosophical-literary and contemplative genres being none too neat, Bruno’s writings should be considered also in the present connection (on his philosophy, 1.2, 1.3 and 1.3.1; *Supplement* I). He pictures an unstable, oscillating and boundless world in an inwardly and outwardly boundless universe containing an infinite number of universes; and does so in an idiom that has prominent literary and poetic components. His literary and poetical inclinations have been noted by many, from the classic work by De Sanctis to the guide to Italian literature by Doris and Arnold Maurer. In recent “in-depth” scholarship, however, having taking an either-or attitude to Bruno - magics or science, his literary qualities have not been pursued with equal fervor. To try to fill this gap is beyond my competences and aim.

I shall limit myself to venture a few suggestions. One is that Donne’s phrase *Catch a falling star* could have been, if not written by, then at least taken to epitomize central tenets in Bruno’s very varied and multifarious literary output. At the same time, of course, he does come up with a kind of *system* - which Donne, for example, does not, at least not explicitly. Still, Bruno’s systems are, if not supported by, at least expressed in literary and poetic idioms.

He implicitly depicts a *cognitive crisis* (Ciliberto, ed. Bruno, *Spaccio,* 19f.: *crisi del sapere*), and occasionally localizes the crisis, especially at Oxford (with a liberal spray of invective: *asini et pedanti...*). Not only did Bruno complain about the crisis, his own writings to no little extent must have contributed to creating one, since awareness of crisis tends to sharpen it.

The literary character of his metaphorical, vague and roundabout treatment stands out clearly if we compare his handling of his *shadows* (1.3.1 and *Supplement* I; see, for example, Blum, 122f.) with Descartes’ conception of his *vortices* (Garber, 1992, 210f., 227 f.), which is as speculative as Kant’s *Theorie des Himmels*, but nevertheless *is* a cosmologically conceived piece of physical geometry rooted in contemporary natural philosophy, embedded in a model of the world as geometry and man as a machine (Garber, 1992, 19, and elsewhere).

Let us hear what some literature scholars have to say about the larger picture.

Souiller and *Morel*, to consider them at first, make the following points, amply supporting them with references to French and English literature, less so from Spain. In the years between 1580 and 1640 an extraordinary *confusion concerning spiritual values and doctrines* can be attested (Souiller, 43).

Looking back, he says, *we are in danger of misinterpreting how the new visions* [new physics, new cosmology] *were received* [when they were so at all], *which were not accepted as a whole as the only and indisputable expression of reality.*
(*Notre regard rétrospectif,... risque de fausser l'interpretation de la réception de cette vision nouvelle, qui ne fut pas acceptée d'emblée comme la seule et indiscutible expression de la verité. Le bilan de l'évolution des idées entre 1580 at 1640 révèle une extraordinaire confusion des esprits et des doctrines: l'héliocentrisme a été surtout perçue comme un hypothèse parmi d'autres*).

I am not going to discuss or evaluate Souiller's contribution, beyond what I have noted already. If his book in places suffers from a certain lack in precision, at least it points up some critical features such as they may look to scholars working in literature. For it is undeniable that many works of seventeenth-century writers lend substance to Souiller's summary.

Souiller's punchline is this: *the change did not origin in the generation of Lorenzo il magnifico nor in that of Erasmus or Luther: it started only at the end of the sixteenth century and the beginning of the next* (Souiller, 37; *La mutation n'a pas été le fait de la géneration de Laurent le Magnifique ni de celle d'Érasme ou de Luther: elle s'est opéré seulement à la fin du xvi^e^ siècle et au début du suivant*). Furthermore, citing M. Lever: *A profound political and social change characterized the generation of 1630.* (Souiller, 58). *The revival of mental conflicts and reopening of the campaign against the Protestants...* (*Une profonde mutation politique et sociale a marqué la génération de 1630. La résurgence des conflits intérieurs, la reprise des luttes contre ls protestants...*).

The heading to this section, C*atch a falling star*, comes from John Donne (*dunne*) and is the first verse of a strophe continuing with: *Get with child a mandrake root/ Tell me where all past years are/ Or who cleft the Devil's foot*, and contains these lines, too: *If thou be'st born to strange sights/ Things invisible to see/ Ride ten thousand days and nights*.

Morel speaks of attempts to arrive at an *exposition of rules that might control the apparent world disorder* (*une exposée des lois qui dominent l'apparent désordre du monde* and *une philosophie de l'ordre du monde*) (Morel, 84ff.). *Never has English metre, the heroic metre, suffered as at his hand*, Émile Legouis wrote concerning Donne (Legonis and Cazaman, 321; Souiller, 33ff), and after some further comments in the same vein, quoted the following lines and concluded that *he violated the iambic rhythm over and over again and many of his lines cannot be scanned*. Thus John Donne: *If all things be in all/ As I think, since all which were, are and shall/ Be, be made of the same elements/ Each thing each thing implies or represents.* Legouis continues: rather than being a sign of archaism, the verses show that Donne *aimed at modernism and a reproduction of the inflections of everyday speech. It is* *rather that he despised the laws of versification* (*see* 2.2.8 about Heinrich Schütz's conception of "rules" in music).

De la Flor's book, now to turn to that, has the telling subtitle: *Representación e ideología en el mundo hispanico (1580 - 1680).* and his first chapter has the rather sweeping heading, *Emblema de melancolia, Nihilismo y desconstrucción de la idea de mundo* (not in need of translation). Spanish-writing authors, starting out from traditional ideas, involved themselves in paradoxical antitheses and internal conflicts, making the entire picture a matter of doubts (*se pusieron a trabajar en la dirección epistemologica contraria*: *aquella justamente en que se desautoriza y se arroja una sombrada duda sobre la autosuficiente afirmación cartesiana que sostiene el orden todo de la razón instrumental*) (Flor, 45f.). Wisdom (*sabiduría*) fails to emerge and to be sufficiently communicable (esp. 61), realities eluding our observation (*Y no hallé cosa en que poner los ojos*) (63f.). The works of "the Baroque" (his term, not mine) are marked by unusable and negative energies (*se convierte vicariamente en el vehículo impensado de un movimiento súbitamente vuelto "entrópico", encarnando una "energía nihilificadora"[una fuerza radicalmente escéptica], en esencia contradictoria con los verdaderos intereses que la animan*) (13). Flor's account is genuinely Spanish in its clarity and limpidity; my brief references fail to convey an idea of the thoroughness with which he examines his vast and well-documented material.

Seventeenth-century literature in Spain rested on a twofold tradition: accentuation of action rather than emotions explicitly expressed; and a harsh realism. *Lazarillo de Tormes* (*see* the Bibliography) is a fictional (?) autobiography of an anonymous author. The ups and downs in his life are described so briefly and tersely that I am reminded of Stendhal's *Le rouge et le noir.* Ferdinando de Roja's *Comedia o tragicomedia de Calisto y Melibea* (end of the fifteenth century), usually referred to as *Celestina*, realistically elaborates avarice, fraud, violence, and sexual marketing, with an impenitent flourish that leaves nothing to the confessor: a touching love story against the background of an *alcahuete* (procuress), and former prostitute, with her following of harlots. Perhaps this background made later writers and the public especially sensitive to the emotional sublevels of real events.

We seem to have a relevant case in the second-phase poetry of Luis de Góngora (1516 - 1627) (González, 86f.), disrupting as he did the traditional standards (*Soledades* of 1613). Contemporary criticism - one might speak of protests, sounds strikingly similar to Artusi's invective against Monteverdi (summarized in http://www.biografiasyvidas.com/biografia/g/gongora.htm). There is a voluminous and mainly consonant literature on this subject, so let me just cite a few of the more "emblematic" accounts. *Mas luego cambió de rumbo, abandonó el arte menor y se lanzó a las mayores "extravagancias", como se llanaba a su arte, porque rompía con todos los moldes y se abría nuevos caminos por terrenos inesplorados, absolutamente desconocios* (Rafael Mesa y López, *Antologia de los mejores poetas castellanas*, London 1912): changing models in his "extravagances", seeking new and unexplored ways. In 1610, Góngora *cambia rotundamente para volverse culterano, haciendo uso de metáforas difficiles, empleando mucha griega, utilizando para ello muchos neologismos, hiperbatones, etc., haciendo. a veces, muy dificil su lectura* (Justo Alarcón, http://www.los-poetas.com/h/bigongo.htm; see also Manuel Gahete Jurado, with a more detailed account: http://www.cervantesvirtual).com/bib_autor/gongora/pcquarttonivel.jsp...): using difficult mertaphors, many Greek expressions, neologisms and linguistic artifices.

Two works seem to stand out by representing modern-science attitudes, Cervantes' *El ingenioso hidalgo Don Quijote de la Mancha*, and, particularly Calderón de la Barca's *La vida es sueño* (carefully presented, respectively, in Cervantes, and Calderón I and II and by Morón; *see* the *Bibliography*). In both works the authors take a scientific attitude by manipulating the accounts so as to produce an artificial platform allowing them to concentrate undisturbed on what interests them especially. In *Don Quijote*, all the hero's *actions* are absurd, nullifying themselves (fighting single-handed against big armies - foes that turn out to be sheep, etc.), while his verbally expressed ideas were often acute and valued also by protagonists in the book. The story provides a path in which to develop a study of human intellectual, moral and emotional behavior. The hero, *rematado ya su juicio* (his capacity of judgement failing; probably affirmed in order to keep the Inquisition away), as we are told, nevertheless is not a patient from a medical bulletin but a typical if highly strung representative of humanity at large (and being highly strung is also a sign of a general tendency), and it is in this context the work is a glorification of insecurity and even fear; characteristic of an atmosphere in Spain noted for its *sentimenti d'angoscia, scontentezza e disillusione* (Iván Fernández González).

At the same time, the multilayered story of *Don Quijote* contains a deeply philosophical undercurrent, and a deadly mockery, the hero functioning as the mouthpiece of Cervantes himself, demolishing once and for all the traditional ideas of there being a "*real reality"*. The poetic nature of *the sciences* is accordingly recognized. The *hidalgo del Verde Gabán* complains that his son, instead of studying the sciences at Salamanca, prefers to do poetry. Don Quijote's corrective reply (Cervantes, 822fF.) contains the following proposition (824); that poetry is like a beautiful young woman whose beauty serves to embellish the other sciences (recalling a medieval allegory now put to novel use): *La poesia,señor hidalgo, a mi parecer, es como una doncella tierna y de poca edad, y en todo estremo hermosa, a quien tienen cuidado de enriquecer, pulir y adornar otras muchas doncellas, que son todas las otras ciencias* (Cervantes, *Introducción* by Alberto Blecua, cxix-cxx). The sciences, the statement seems to say, are expressed through poetry or even turn out in the end to be poetry.

It is interesting to note the definition in the Spanish Academy dictionary of the term *quijote*: *A man who puts his ideals before his advantages and acts disinterestedly and dangerously in the defence of causes he considers just, without achieving it* (*Hombre que antepone sus ideales a su conveniencia y obra desinteresada y comprometidamente en defensa de causas que considera justas, sin conseguirlo*) (DRAE, *ad vocem*). The *Aventura de los leones,* in which Don Quijote seems to make himself ridiculous by provoking an enormous lion, offers the occasion for a crucial discussion. Don Diego de Miranda declares that Don Quijote in his *actions* is insane (*loco*), while his talk is consistent, elegant and well expressed (*concertado, elegante y bien dicho, y lo que hacía, disparatado, temerario y tonto*) *(*Cervantes, 836f.).

The *hidalgo's* long reply induces his interlocutor to a polite but reserved acceptance. The important thing about the story is that the conflict between *doing* and *thinking* (and saying) has been elevated to a major literary topic. Summing up, one may say that Don Quijote gives a picture of the human condition, oscillating between ideal mental constructions and hard facts, a forerunner to our cognitive sciences.

In Calderón de la Barca's *La Vida es sueño,* to turn to that *comedy* (more in the Alighieri sense than the modern one, I believe), a point zero is established by the trick of having the main protagonist, Segismundo, closed up since the earliest age in isolation in a tower until the moment when the story starts developing, with the hero appearing as a *tabula rasa;* the; a reinterpretation of previous - Lope de Vega - and medieval material (survey in Rull, *see* Calderón I in the *Bibliography*). Having isolated Segismundo in the tower since he was very young, and releasing him and having him face society from a zero position, Calderón creates a scenario very much like a modern science model (not De la Haza's words but the idea sits right under the surface of his discussion: Calderón II, ed. De la Haza39ff.).

As with Shakespeare, life is "real" on the level of imagination (*sueño,* lit. dream); a strikingly "modern" concept. The idea is developed in images reflecting concepts in Seneca, and especially in *Hamlet* and *Don Quijote,* and to some extent, Góngora. The king, Don Basilio, facing the hopeless task of ruling in an unmanageable world, fails his social and political position, debasing himself by hiding himself in *matemáticas sutiles* (*verses* 612 - 639), probably the budding *calculus*) (Morón, *passim*); not an occupation for the upper classes.

The *theatre* is being renewed, reducing the "rules", dear to Boileau (*L'art poétique*) , and communicating with the public, including common people, and appealing to what is natural and what engages them (Pidal, 102f.). Lope de Vega, in his *Arte nueva de hacer comediad en este tiempo* (1609), a drama leaving Aristotle behind (Pidal, 102f., Castro, 194f.) (*se* also 2.2.6, about *Phasing out Aristotle*), put the idea epigrammatically: *Los casos de la honra son mejores, / porque mueven con fuerza a toda gente* (Pidal, 198); *honra* and *honor* playing both of them a crucial social and political role in Spanish society (overview with references in Marín's chapters, *La dinámica social de la España del siglo XVII* and *El tema del honor*, in Lope de Vega, 17 - 29).

Comparisons with Shakespeare and other European authors are obvious, *lo mismo en Madrid que en París o que en Londres* (Pidal, 127), but while the British dramatist elevates historical reality to dramatic experience, Lope de Vega exploits historical subjects in order to create a new, autonomous reality, that of the theatre (Gilman, 185f.).

Spanish literature in the seventeenth century offers complexities unheard-of earlier (Lope de Vega, Calderón de la Barca, Luis de Góngora, to cite three of the most outstanding), and my untutored report on the subject culled from recent studies at least seems to have captured the essence of what Rull says concerning the work of Calderón as an "emblematic" exponent of the tendencies of the time; leaving behind the harsh realism of earlier days (*Lazarillo, Celestina*), the new works offer a *rica polivalencia de sus multiples sugerencias de significado* (Rull, 42, in *Calderón I*). The sharpest critic of Góngora's work, Juan de Jáuregui (1614). claimed that his *Soledades*, a complex poem in two parts with a "pilgrim" as its main protagonist, suffered from *disintegration by its internal conflicts.* To which a recent editor, John Beverley retorts, noting that these contradictions work within the protagonist himself, as an expression of a "new sensibility" developed in the midst of an intensified feeling of crisis and decadence in Spain (Góngora, *Soledades.* 35f.) - *un poeta heterodoxo que escribe en medio de un sentimiento creciente de crisis y decadencia en España...*

2.2.6 *Phasing out Aristotle (the Professors, Galilei - and Corneille)*

The *decline of Aristotelism* entailed an increasing sense of cognitive insecurity. Abstractly, but disturbingly, even crucially, to groups of some standing in the society of the time, came the very gradual and non-collinear but evident decline of Aristotelism. With this went by the board, again in a piecemeal fashion, a good many truths that had made life worth living and protected the academe and well-thinking people behind thick walls of conventional truth and university teaching. The importance of Aristotle, as his writings had been in different ways absorbed by Catholic tradition, may be hard to gauge, since the adoption was in no way a smooth and uniform process (see for instance Edward Grant's book cited in the *Bibliography*).

But it has been pointed out by several authors, such as Lattis in his study on Clavius at the Collegio Romano (Lattis, 6f.), that many convinced and dedicated Aristotelians nevertheless redefined features within the philosopher's writings in order to meet the challenges that were coming up (for *Clavius*, *see* also the introduction to the present *Part* II, and below). Even Thomas Aquinas, in the midst of his definitive implanting of *Aristotelism* into *Scholasticism*, considered it mandatory to correct Aristotle on certain points.

Giordano Bruno, philosophically as well as (as we might call it) politically, rejected a fundamental principle in Aristotle. In his *Intentio 22* (*see* Supplement I), he writes, in an anti-Aristotelian vein, that the distinctions operated between *substance* and *accidentals*, if applied to the shadow, do not apply.The shadows are not substance nor accidentals but a certain conception of both. By a careful consideration, <we see that> substance and accidentals do not subdivide everything so much as is usually being claimed [with Aristotle]. This understanding helps to get a rational grasp of <the concept of the> shadows. By a correct evaluation, substance and accidentals do not divide everything that we consider as belonging to the universe in such a manner that is currently believed. This consideration is of no little value for a rational understanding of the shadows

The famous Aristotelian distinction was crucial for the Church since it provided the basis for the explanation of the Eucharistic species; to state it roughly, the bread substantially transformed into Christ's body (*transsubstantiation*) still tasted bread (*see* below for Bellarmino on the subject).

From 1624 onwards Pierre Gassendi published his *Exercitationes adversus Aristoteleos*, one of the documents in which the rupture between faith and human reason appears most sharply. So it did also in Giordano Bruno's philosophy, with his claim that there are two kinds of truth, one scientific and the other one religious.

Galilei, to quote Sharratt, *knew that he had more than the beginnings of the new science of motion, which would be necessary to defend a moving earth. More generally he knew that any campaign for Copernicanism would also be a campaign against Aristotelianism*

(Sharratt, 95). Scientists, however, are humans (nowhere better documented than in Kai Bird and Martin J. Sherwin's colossal biography of Robert Oppenheimer), and Galilei in his eagerness to disprove some point in Tycho Brahe's astronomy, *found himself in the necessity of playing the role of a conservative Aristotelian,* involving himself in a wilderness of contradictions (*in una selva di incoerenze*) (Rossi, 1997, 122f.).

Even more telling is the case of Clavius, who represented no liberal thinking and sat in the very center of Roman Catholic scholarship (*see* 2.2.11).

He was responsible for the teaching of mathematics at the Collegio romano, and Gilson, in his comment (Descartes, 1637, ed. Gilson, 128), notes the great difference in attitudes to Aristotelism between the mathematician Clavius and contemporary physicists, citing the former's *attaque vigoureuse... contre la dialectique aristotélicienne,* in his *Operum mathematicorum <liber>***,** published at Mainz in 1611. Here one might submit a comment, saying that in the seventeenth century there was this a difference between *physics* and *mathematics*, that whereas mathematics dealt with quantities, physics, as one among the natural sciences in seventeenth-century categorization, concerned motion, weight, speed and velocity and a number of issues that do not carry much meaning, to say nothing of instructional utility, if they are not applied to things, bodies such as stones or water or air. So it was much harder for physics, not having yet attained the level of mathematical abstraction familiar today, to stay aloof from unpleasant realities.

Now let us hear (in my translation) what Professor Clavius has to say on the difference (the Latin text is in Gilson's comments to Descartes, *Discours de la méthode* (128f.)*;* partly translated in Lattis, 35); a long quotation but worth the cost, since the comment, coming from such a source, is illuminating for its argument, tone and scholarly style.

The discipline of Mathematics, in fact, establishes everything about which there is debate with solid reason, and corroborates it, so that true science takes root in the students, really removing all perplexity. This much we can hardly attribute to the other sciences, since here repeatedly any number of evaluations and varieties of assertions that are up to judgement about their truth remain undetermined and uncertain. But the Aristotelian sects (not to mention others) put great faith in them [the cited sciences], *who arise from <the teachings of> Aristotles like the branches from the trunk, while at the same time they disagree among themselves and even with the source of this same Aristotle, because they certainly do not understand whether his discourse is mainly about names or about things. For this reason, some interpreters among Greeks, Latins, Arabs and those, finally, who are nominalists or realists - while they all of them praise themselves by the name of Peripatetics - choose their leaders accordingly. Whatever consistently (quam longe) stays away from the demonstrations of mathematics, I should not count in as belonging to my company (latere existimo). For the theorems of Euclid and the other mathematicians for so long time in the schools have been considered the purity and certitude of things, the strength and solidity of demonstrations...*

The truth of the disciplines of mathematics has then for so long been sought after, truly loved and cultivated, because not only nothing that is false but even nothing that is probable [!] *has reality, therefore in fact admits of nothing that is not verified and corroborated by absolutely certain demonstrations, and there can be no doubt that they* [*the disciplines of mathematics*] *should be allocated the first place among the sciences.*
(*Mathematicae disciplinae sic demonstrant omnia, de quibus suscipiunt disputationem, firmissimis rationibus, confirmantque, ita ut vere scientiam in auditoris animo gignant, omnemque prorsus dubitationem tollant: id quod aliis scientiis vix tribuere possumus, cum in eis saepenumero intellectus multitudine opinionum ac sententiarum varietate in veritate conclusionum judicanda suspensus haereat atque incertus. Hujus rei fidem faciunt tot Peripateticorum* [Aristotelians] *sectae (ut alios interim Philosophos silentio involvam) quae ab Aristotele, veluti rami et trunco aliquo, exortae, adeo et inter se, et nonnunquam a fonte ipso Aristotele dissident, ut prorsus ignores, quidnam sibi velit Aristoteles, num de nominibus, an de rebus potius disputationem instituat. Hinc fit, ut pars interpretes graecos, pars latinos, alii arabes, alii nominales, alii denique reales quos vocant (qui omnes tamen Peripateticos se esse glorientur) tanquam ductores sequantur. Quod quam longe a mathematicis demonstrationibus absit, neminem latere existimo. Theoremata enim, Euclidis caeterorumque Mathematicorum, eamdem hodie quam ante tot annos in scholis veritatis retinent puritatem, rerum certitudinem, demonstrationum robur, ac firmitatem*...[*caesura* by Gilson] *Cum igitur disciplinae mathematicae veritatem adeo expetant, adament, excolantque, ut non solum nihil quod sit falsum, verum etiam nihil quod tantum probabile existat, nihil denique admittant quod certissimis demonstrationibus non confirment, corroborentque, dubium esse non potest quin eis primus locus inter alias scientias omnes sit concedendus*).

This long passage to my mind has merited to be quoted almost *in extenso,* for it makes an important assertion about critical distinctions in contemporary science and mathematics.

A scholar trained, as they were all of them, in the Aristotelian tradition (albeit with some modifications), here relegates the entire body of Aristotelian physics and teachings on the world and universe to a field of only relative truth-value, while it is *mathematics* that harbors absolute certainty and truth. So here traditional scholarship was assailed from within, if admittedly in an academic manner and through channels inaccessible to the public at large.

Opposition to Aristotles coincided with an even more massive censure applied to the so-called *Peripatetici*, the academic world still relying more or less entirely on that philosopher's world image and applying it to almost everything; while refusing to perform direct observation in favor of studying texts. To some extent the usual thing happened. The master's reputation suffered from the shortcomings of his pupils.

Salviati, in Galilei's *Dialogo* (cited ed.,p. 333), makes the point notably in a response to Simplicio:

I am not asking about the Peripatetics, I am asking about Aristotle himself. For I know perfectly well how those people would respond. They... would deny all experiences and all observations in the world, and would back away from seeing these things, lest they would have to take them into account, and they would claim that the world is as described by Aristotle, not the way nature has decided... (*... negherebbero tutte l'esperienze e tutte l'osservazi-*

oni del mondo, e recuserebbero anco di vederle, per non le avere a confessare, e direbbero che il mondo sta come scrisse Aristotele, e non come vuol la natura...).

Nevertheless, the core of Catholic teaching was traditionally linked up with Aristotelian philosophy. Rejecting Aristotle altogether, or almost so, was a serious offence. Tommaso Campanella's *First Point*, in his *Apology for Galileo*, provides a terse but principally complete summary of the Church's accusations against Galilei for the latter's disregard of the Greek philosopher:

They accuse him, first of all, for this reason: whoever tries to introduce new teachings in opposition to Aristotles' physics and metaphysics, on which St. Thomas <Aquinas> and all the Scholastics built the doctrines of theology, seems to be trying to overthrow completely the foundation of theology (Arguitur primo contra Galilaeum: videri omnino theologica everti dogmata ab eo, qui contra Aristotelis physiologiam et metaphysicam, in quibus D. Thoma et omnibus scholasticis theologica doctrina fundatur, novitates introducere studet).

Indeed, rejecting the Aristotelian kernel of Scholasticism also often meant rejecting its methodology, which has been briefly characterized in the following terms, by Eizenhöfer (*Liber sacramentorum,* XXX): starting out from preconceived, defined theses, with the purpose of proving them (... *eine von den scholastischen Methoden der Philosophie und Theologie stark belastete Arbeitsweise, von festen Thesen, in unseren Falle von vorgefassten Ideen zu beweisen schreitende Untersuchung...*) - quite the opposite procedure that was to be introduced by Bacon, Galilei and others.

Attacking Aristotle in any fundamental way would concomitantly lay open to doubt his belief in an unchangeable universe and his separation of the sublunar world from the cosmic world, as well as his distinction between *substance* and *accidence.* The latter couplet represented a system of absolute values that was indispensable for the formulation of the dogma of the transsubstantiation of Wine to Blood and Bread to Body in the Mass sacrifice.

In the third volume of his *Controversiae,* of 1592, in full accord with Catholic Tradition, Cardinal Roberto Bellarmino used the familiar and fundamental Aristotelian categories of *substance* and *accidence* to define the realities of the species of bread and wine as subjected to transsubstantiation; the process working by the principle *ex opere operato* (the sacraments become effective through the ritually conducted sacramental action: SL, 1984, note 96) that converted the elements of bread and wine into the substance of Christ's body and blood, while retaining the accidental properties of the species.

There is no better way of explaining the mode of existence of the Sacrament [of the Eucharist] *and at the same time its truth and reality than with the adverb "substantialiter". For this means that Christ totally exists in the Sacrament, in the substantial modus and not in terms of quantity or quality or any other of the accidental modes...*
(*Porrò modus existendi Sacramentalis, & tamen simul verus, & realis, non poterat melius explicari, quàm illo aduerbio, "Substantialiter". Id enim significat, totum Christum in Sac-*

ramento existere ad modum substantiæ, non quantitatis, vel qualitatis, vel aliorú [aliorum] *accidentium*...) (Bellarmino, III, col. 350 A).

Of course, after the emergence of Protestantism as a fact of life, particularly Lutheranism with its *sola fide* principle (that faith alone is sufficient), this became an especially sensitive issue. Now the notion of the necessary Good Work (the *Opus bonum*; 2.2.12) was rejected, thus making the supreme Good Work, that is, the Catholic enactment of and participation in the Mass Sacrifice, superfluous, not to say blasphemous.

The conflict invaded even the elegant world. In Pierre Corneille's *Le Cid*, the Aristotelian *règles* concerning the three units (one day, one place, one action), *les regles*, glorified by Boileau (*L'art poétique*), were disregarded (also in the Spanish theatre, in works by Lope de Vega; 2.2.5) - a break provoking the famous and genuinely French *querelle du Cid* and serious enough to have Corneille use much of his later *examen* in defending it (Rat, 12ff.). To be honest, however, and not take for granted that what was valid for one moment was so also for another, let me note that this *lateness* coincided with what Niderst calls *l'époque des polémiques* (Niderst, 243). Such a general picture of course is never clean or consistent. Lope de Vega, for one, breaks with the "rules" in some works, while sticking to them in others.

In this uncertain world with its unpredictable events, the best option was to stick to what had been accepted so far. Montaigne, Descartes and Pascal are quoted à-propos by Souiller: let us have interior free and penetrating judgement, but outwardly follow *entièrement les façons et les formes reçues* (Michel de Montaigne). Bruno's mistake, if it was a mistake, was precisely not to adopt this healthy principle. Also from the point of view of this perspective of hiding behind walls of accepted paradigms, the total picture of *The Science* is fated to remain blurred. Therefore it is best not to pretend to be setting up some total picture of it.

2.2.7 *Mettere in conquasso cielo e terra (Galilei's Simplicio & Salviati*)

While some people thought it possible to control the understanding and cognitive handling of nature, their attempts caused ideological upheaval in others. So the very drive towards quantification and order had the opposite effect in some quarters. *Simplicio* complains about the harassment.

In Galilei's *Dialogo dei massimi sistemi* (39, in the cited edition), the traditionalist *Simplicio* accuses his "Galilean" interlocutors, *Salviati* and *Sagredo*, of trying to *disrupt the order of heaven and earth and the entire universe* (d*isordinare e mettere in conquasso il cielo e la terra e tutto l'universo*). The implied order as he had learnt it at school was the one established by Aristotle more than anyone else and in Galilei's time endorsed by the Aristotelian *Peripatetici* (who dominated the universities). Tycho Brahe in a letter of 1589 took a similar view and expressed it in equal terms:

If you prefer to make the earth and the seas, together with the moon, revolve; if you wish that the earth, however ill-suited to motion and far below the stars it may be, behave like a star in the ethereal regions, you are certainly the master... But are not earthly things being confused with celestial things? is not the whole order of nature being turned upside-down? (quoted in translation by Dugas, 109).

Blaise Pascal was aware of the discrepancy between scientific models of the world and the real one such as it is accessible to us. This awareness, combined with other symptoms some of which have been noted here, amounted to something like the *Verlust der Mitte* (loss of a central and fixed point of reference) that Hans Sedlmayr believed to have diagnosed in the Western world of the 1950s. Speaking of reality and pictures of it, Pascal says:

In this manner, the pictures have to remain <at the same time> *at a too long distance and too close, while there is but one indivisible point that represents the true place, the others being too close, too far, too high and too low. Perspective* [*construction*] *allocates it in the art of painting. But who can allocate it in truth and in moral* [*= in epistemology*]*?*
(*Ainsy le tableaux veux de trop loin & de trop prés, & il n'y a qu'un point indivisible qui soit le veritable lieu, les autres sont trop prés, trop loin, trop haut ou trop bas. La perspective l'assigne dans l'art de la peinture. Mais dans la verité & dans la morale qui l'assignera?*) (Pascal, 41).

Kant, perhaps following Pascal, distinguished between *mathematisch* and *moralisch* understanding (Geier, 85).

The feeling of having incurred the loss of central and reliable anchors in the world had become a philosophical and literary subject, and hardly less serious for that. Of course the increasing relativization in society's relations to the Church and to religion must have worked in the same direction. This was due not only to the Reformation but also to an alternative cosmology that might make the Bible look like a piece of literature. There were serious cracks in the cohesion between the Church and the people, right in the center of the Catholic world. In sixteenth-century Venice, arch-orthodox on all fundamental issues, the ceremony master of the government church of San Marco complained, in his Ceremony Book of 1564, of dwindling attention to the important and spectacular distribution of Indulgences, even absence from it, on account of the evil influences from the Protestant North (*Burden*, 98, 307).

2.2.8 *La Seconda Prattica in music (Monteverdi and Carlo di Venosa)*

On many levels science and music had always, since the Greeks, been considered as closely entwined. Music was claimed to be grounded on mathematics, especially proportions. This tradition now, around 1600, was being upset. Galilei's father, the lutenist and musical theorist Vincenzio, protested against the "geometrization" of music - a sign that it was taken seriously. Parallel to the tendencies to be recorded here, the Palestrina tradition was kept alive indeed, for example in the music by *Tomás Luis de Victoria* **(**Filippi in the *bibliography***)**

Two composers of music, *Carlo Gesualdo principe di Venosa* and *Claudio Monteverdi da Cremona* (in fact, also a third one, *Luca Marenzio*, and, to some extent *John Dowland*) in some of their music disrupted the tradition of counterpoint polyphonic style, much to the annoyance of the angry representative of tradition, the Rev. Giovanni Maria Artusi. He aired his consternation in a book published in Venice in 1600. His book bears a title that merits being quoted here: *L'Artusi Ouero Delle imperfettioni della musica moderna. Ragionamenti dui. Ne' quali si ragiona di molte cose vtili et necessarie alli Moderni compositori,* Venice 1600 (*The Artusi or About the imperfections in modern music. Two discourses. In which are discussed many things useful and necessary for modern composers*).

Monteverdi, too, wrote *on* music, and launched the term for his alternative to the *prima prattica*, represented above all by Orlando di Lasso and Giovanni Pierluigi da Palestrina, and in an almost caricatural form, by the Florentine *Camerata* movement. The new manner became the *seconda prattica - with his norm of unconventional dissonance* (Abraham, 267). Monreverdi called his new style *stile concitato* (excited, agitated, impassioned). Don Carlo Gesualdo, *principe di Venosa*, does not seem to have written on music, but he did write music which is commonly compared, somewhat obliquely, to Monteverdi's seconda prattica. It breaks even more radically with the established order.

Monteverdi's and Carlo Gesualdo's field was mostly music with song, such as *frottole, madrigali* and variations on these very vaguely classified categories (De' Paoli has much to say on the often unclear classification criteria) of social entertainment and meditation, as well as some *motets*; and, in Monteverdi's case, the opera (especially the short *La battaglia di Tancredi e Clorinda*; less so the *Orfeo*).

Liturgical music, of course, a field in which Monteverdi was very active because of his official positions, set different requirements. There is thus a vast difference between his more traditional *Vesperi della Beata Vergine* of 1610 and his ferocious *Battaglia di Tancredi e Clorinda* (obviously stimulated by the subject, not merely by musical principles) or his slightly more easygoing *Sestina* (Whenham). In Gesualdo's work, religious subjects tend to call forth the most extreme expressions, but his are not liturgical pieces.

Monteverdi himself stressed the dramatic aspect: he *dates the introduction of the'concitato' style to the battle music in his miniopera 'Il combattimento di Tancredi e Clorinda'*, *and he sees his achievement as a way of putting into music the two contrary passions of war, namely prayer and death'* (Herbert Lindenberger, 81).

For an assessment of Monteverdi's new practice in relation to tradition, it may be illustrative to listen to contemporary criticism. The most aggressive one was due to the musician and theoretician we have met already, Artusi. Invited one evening (as he sets the scene) to listen to *certain new madrigals*, but not being told the name of the composer, thus avoiding to name Monteverdi, he found that the structure (*tessitura*) was not graceless, even though new rules, new modes and new phraseology were introduced.

They were, however, grating [*aspri*] *and not pleasant to listen to; and it could not be otherwise, seeing that they transgressed good rules, which in part are based on experience, mother of all things, partly thought by nature and partly tried out* [*dalle demonstratione demonstrate*]. *One must believe that <the madrigals> were distorted in relation to nature and to what is essential in true harmony and far away from what the musician should pursue, namely pleasure* [*la dilettatione*]. *The new music was grating for the ear and more offensive than pleasing*.

In general Artusi complained about the dissonancy and irregular harmonies (*dissonanze* and *irregolarità armoniche*) (De' Paoli, 110ff.). Artusi published a second part of his work (1603), *in which is treated many abuses introduced by modern writers and composers* (*Nella quale si tratta de' molti abusi introdotti da i moderni Scrittori & Compositori*). Monteverdi later replied explaining his *Seconda Prattica ovvero Perfettione della Moderna musica,* invoking his creative right and powers. He insisted on thinking freely in music (as Giordano Bruno did in philosophy, we might submit).

Don Gesualdo supplied exacting songs-with-music that to many contemporaries must have sounded as Arnold Schönberg's *Die Jakobsleiter* did to many people in his day. This applies to much of Gesualdo's *Quarto libro di madrigali a cinque voci,* published at Ferrara in 1596, where he had to stay in involuntary exile for some years, having murdered his wife and her lover, conveniently marrying into the Este family, a fine compensation (Judica).

In such pieces as *Moro, e mentre sospiro,* he uses five voices for madrigals and his style of chromatic scales of widely varying intensity, combined to yield what the Italians called a *languore,* much befitting the often desperate tone of the poetry involved. Often the music tends to dwell on short phrases or even single words, as in *Sparge la morte*, a madrigal, or better, motet, in which Christ's agony is followed from stage to stage (Cera). Some speak of his use of *dissonancy,* while the present Principe Francesco d'Avalos in Naples (involved in terms of family heritage and music) corrects this, calling it *not dissonance but unusual distance or gaps between the accords.*

Charles Burney (1726 - 1814) evaluated Carlo Gesualdo in almost exactly the same terms as Artusi did Monteverdi. Recently Gerald Abraham, apart from describing some of the music as being *charged with chromatic emotionalism*, does not see much of a "progress" from earlier music, having mainly this to add: *Gesualdo's later madrigals, however fascinating their scent of decadence, are an evolutionary dead end* (Abraham, 253f., 267 resp.). The Sixth book of madrigals is dated 1611, two years before his death, and it contains the most notable (and famous) pieces, especially *Moro lasso.* Abraham's pseudo-biological evaluation is itself a dead end and the product of thinking in terms of periods and "progress". There are, however, more constructive evaluations, even if they may be a little beside the point. Some people today make a jump from Don Carlo over to Richard Wagner, even to Anton Bruckner; while it may be recorded that Igor Stravinskij arranged Don Carlo's *Moro lasso* and two other madrigals for orchestra, and made "pilgrimage" to Gesualdo twice (he was everything but uncritical, detesting Vivaldi).

In Monteverdi the ruling principle was that the music should be a direct expression of the text content; so that textual expressions of emotions entered the music and not the other way round. In the context relevant for Monteverdi this was expressed by the slogan, *l'Horattione* [orazione = recital] *signora della Harmonia et non ancella* (*the recital patroness of the harmony and not its servant*), a concept that led to new dramatic effects (De' Paoli, 124ff.). In terms of philosophical statement, this was akin to what was claimed by the Florentine *Camerata*, with Vincenzo Galilei as a figurehead and theoretician; only that in Florence theory took precedence over feeling, so that the new music drew criticism not so much for novelty as for being mostly a dry academic exercise.

The case of Don Carlo is comparable but different. Giovanni Iudica, law professor in Milan, offers a characterization of Don Carlo's music, the essence of which can be conveyed as follows (Judica, 78f.). He did *not* let the word take command; on the contrary, he subordinated the texts to such a degree as to split single words musically whenever the *musical* logic required it. Iudica claims that within the structure of accepted music, in the practice of the *Seconda prattica,* he insinuated *dissonancy so far unheard of* (*pace* D'Avalos), unstable tones and irregular breaks or pauses, oscillating and *mysteriously ambiguous* tones. So far Iudica.

Traditional academic attitudes were effectively overcome also in another field that arose at exactly the same time, around the year 1600, namely with the *opera*: the musical art that fused everything into a whole: prose and poetry, song and recitative, solo singing and chorus singing, obbligato use, *concerto grosso* use and other varieties of instrumentalization, action played out, stage-effects oscillating between the fabulous, the real and the abstract.

The very first notable case of reshuffling established order among genera was Emilio de' Cavalieri's *Rappresentazione di anima e di corpo... per recitar cantando,* not all of it sung, however, long sections being *spoken dialogue*, as in the later German *Singspiel*. The opera was first produced at the Roman *Oratorio* of San Filippo Neri in 1602. Let me refer to Paul Henry Lang's judgement: that this is not a typical *oratory*, despite the fact of its religious site of production; it is a fullfledged *opera* (Lang, 79f.). Sadie refers to the *Rappresentazione* as the *first dramatic work set entirely to music* (Sadie, 45). With the date 1602 of the production of the *Rappresentazione* in Rome, we have but five years to wait for the production in 1607 at Mantua of Claudio Monteverdi's *Orfeo* at the Gonzaga court. In contemporary letters the work was referred to as *la fauola cantata*, which was *singolare posciaché tutti li interlocutori parleranno musicalmente* (Sadie, 45).

The dilemma that arose - starting with the tradition from Petrarch - with the new insistence upon the text and the emotive values in the madrigal, led to no longer keeping musical composition within traditional and absolute bounds but making the expressive values and thus the musician's subjectivity the primary issue.:Concerning *Heinrich Schütz* and his relation to Monteverdi, Marenzio and Don Gesualdo, Michael Heinemann is clear on this point: *The question now arose - specifically for Schütz - whether anything at all could be restricted by rules* (*Es stellte sich die Frage, was noch durch Regeln überhaupt beschränkt werden konnte*) (Heinemann, 22). Perhaps one might see John Dowland's dramatic, dark and troubled madrigal, *In darkness let me dwell* of 1610 in the same light.

2.2.9 *Design (Trent, Bernini, Gherardi, Barbo and Teresa customized*

Comparable tendencies arose in visual design. The cases discussed in this section are all instantiations of the distinction between *system* and *elaboration* (1.6, 1.6.1 and 1.7).

In the fifteenth century, Leon Battista Alberti complained about the disorder reigning in the churches of his time, indulging in a nostalgic look back to the earliest days, when, as he claimed, there was just one altar and one daily Mass. The number of *altars* in the churches had increased exponentially through the centuries. The practical ritual consequences of this structural growth was - chaos; some groups attending Masses at side altars while High Mass was being celebrated at the main altar; groups disturbing each other, being mixed up.

The Council of Trent put a stop to that, decreeing that, while Mass at the main altar was being celebrated, no rituals could be performed at other altars (SL, 1965, 204 - 207, with a catalog of buildings roughly between 1400 and 1600). When Leon Battista Alberti complained about the liturgical chaos in his time, the statement must be seen in the cited context, not as an isolated “literary” topos (SL, 1965, 210f.). Architects’ comments do not tell us much unless seen in their functional, social and urban setting.

The cited ecclesiastical restriction opened up for greater freedom in modelling architectural space in Roman Catholic churches; made it possible to create another type of small spaces with side altars close to the main one; for no disturbance of ritual participation would now occur on account of spatial closeness between the main and the lateral altars. Borromini’s San Carlino is one example of the new planning, Bernini’s Sant’Andrea al Quirinale another, and the *Klosterkirche Banz* in Bavaria a third (SL, 1965). It was this ritual reform that made the difference, not, as has been suggested, the veneration of the exposed Sacrament; this started much earlier, the first known case of permanent exposition being the tabernacle in San Clemente, Rome, form Bonifacius VIII’s time (ca. 1300).

One of the very first churches to be planned under the new conditions, was San Giacomo in the Roman hospital of San Giacomo degli Incurabili of ca. 1590. The groundplan is an oval, with the longitudinal axis between the entrance and the high altar. (as B on *Fig. 2.1 - 3*). There are no less than three chapels on either side (taking care of hospital needs).

It must be noted that it was not the architects, but the ecclesiastical authorities who decided on the *number* of altars/chapels; the determinant factors being the ones I have just listed (*see Fig. 2.1 - 3*). Many altar services used to be, and still to some degree are, important sources of income for the churches. These factors belong to the systems level; the architect could never tamper with the resulting number of altars.

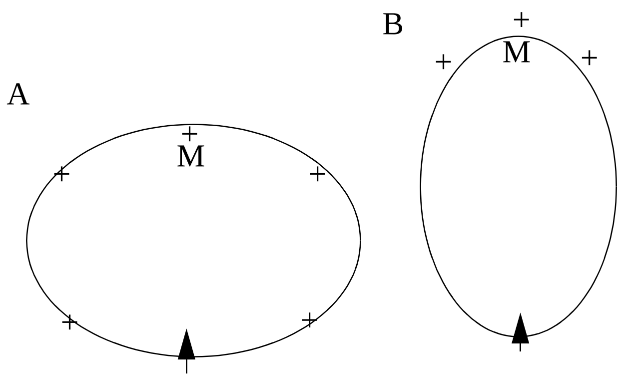


Fig. 2.1 - 3. A. Sant’Andrea del Quirinale (Bernini): B. The San Carlino/Banz type plan. + = chapels / altars. M = main chapel / altar.

It has been claimed that in placing the four side chapels as we see on *Diagram A*, Bernini designed a Saint-Andrew cross, and also that it was a stroke of genius of him *not* to place two of them at the ends of the major axis of the ellipse (Wittkower). But if you are commissioned to design an oval or elliptical plan with one main and four lateral chapels, what options do you have beyond the present solution?

The new flexible scope of church design made unheard-of shapes possible, as in these three cases, to say nothing about Sant'Ivo. With greater freedom in space disposition, not in choice of basic elements like altars, new possibilities opened up for revising the entire architectural bodies, including separate parts of a building, such as chapels and sacristies. Borromini and his patrons exploited the new scopes intensively, as others after them, in Rome, Turin and Bavaria.

In *sculptural works* we are witnessing the consequences of the development just mentioned; a more dynamical, let us say, dramatic, elaboration of spaces - where this is possible, such as in family "occupation" of the entire chapels (Bernini and Gherardi: *Fig.s 2.1 - 4* to *8*). Since Émile Mâle's days it has been fashionable to refer to "the theatre" when speaking of visual arts since the Middle Ages. Theatrical performances, however, and their predecessors such as religous plays and *laudi*, arise in parallel with iconographical programs, both being rooted in liturgy or classical canons, on the systems level. So one of them cannot be used to *explain* the other, only as support in terms of comparison.

Sculptures in the seventeenth century are so to speak spatially free enough (there are patron economics here, too) to involve great expanses of *turbulent surfaces, as if a manifold had been multiplied up to the extreme or variations of it packed together in a continuous network*, such as in draperies and curtains that are waving and billowing as in strong winds and one, as we shall see, with a collapsed carpet assuming similar shapes. All of them must have seemed as challenging to contemporary mathematics as the widely studied motions of water freely running. Bernini and Gherardi, in fact, created surfaces with an unheard-of complexity, almost cultivating disorder in a fashion not seen in earlier design. This characteristic becomes all the more striking - and must have seemed so to contemporaries - because the *message system* itself in the cases at issue is *entirely traditional in religious and ritual principles*.

The curtains and draperies are represented in strong and billowing motions, rather than the relatively "logical" sweep in customary representations of draperies and curtains that more or less fit the bodies they cover or the spaces they reveal by being drawn aside. They are more striking and occupy more of the available space than one might think structurally and space-economically adequate.

In Bernini's *funerary monument for Maria Raggi* (1643) in Santa Maria sopra Minerva in Rome (Bernini, 349f.), the theme of the *differential surfaces* or *super-manifolds* is represented in a concentrated form, occupying a little space on a column (*Fig. 2.1 - 4*). The epitaph consists of a marble sheet in black-and-gold, illustrating a waving curtain, on which we find a picture with the Madonna and Child (in a roundel or *clipeus* surmounted by a cross) carried by angels and provided with a long funerary inscription. The cloth is represented waving as if subjected to the impact of the toughest winds. Its movements defy verbal description entirely; and also mathematics within reach in Bernini's time. They nevertheless belong to an almost ubiquitous category of shapes that must have been felt as challenges to contemporary mathematicians and similarly focused people.

Fig. 2.1 - 4 Rome, Santa Maria sopra Minerva, Monument for Maria Raggi (Bernini)

The idea behind this curious monument is obviously the well-established one of *revelation,* in the sense of unveiling and opening up into Heaven with the Virgin and Child, angels and cross. This theologically and liturgically established and employed mechanism works even more dramatically, being allowed to do so by ampler space conditions, in the next two cases to be considered, another one by Bernini, and a chapel by Antonio Gherardi.

In San Francesco a ripa, also in Rome, Bernini's *Beata Ludovica Albertoni* (1671-75) (Bernini, 374f.) is reclining on a coach or bed just behind the altar, which of course is the liturgical site of death and resurrection (*Fig. 2.1 - 5* and *6*). She is lying with angels above her; in liturgical terms, there is no difference between the various alternatives of angel images, whether they show just some heads with wings or entire figures (even wingless, for that matter). On the back wall is the altarpiece; this is not the original one but any regular altar image would amount to the same, as long as the tradition of heavenly presences is respected. The Beata Ludovica is reclining in a tense and taut attitude (originally with lateral daylight directly on her figure), evocative of the moment of passing away and rising up to the heavenly world.

The revelation in the guise of a curtain being drawn aside or being caused to fall down, was a customary theme concerning the *dead, saints as well as social individuals*.

Obviously the Beata Ludovica's personal garments reflect her agitated bodily movements. But in addition, between her figure and ourselves as we are standing (or kneeling) in front of the altar, there is a big heap of *collapsed curtain* in an unruly pattern: again a *differential surface* or *manifold.* This disorderly heap is close to us and so big as to almost compete with the saint herself for our attention.

The curtain is in brown-and-white marble with a golden fringe along the lower edge. Like any normal *portiera* or *portière* covering a door opening (as was usual) it is trimmed with such a fringe on the nether border. A curtain, then, has been caused to fall down, revealing to us the scene of a person's direct communication with God, while clearly implying the notion that *the celestial vision is being revealed to the protagonist herself*, in this case the Beata Ludovica.

Curtains had always been used in liturgical functions as a covering to conceal or, alternatively, reveal, as the rites evolve or the ecclesiastical year proceeds, holy objects (crosses, holy images) or indeed the Mass celebration itself (SL, 1978, 1984). The configuration is a restatement of the commonplace funerary iconography current in Italy at least since the thirteenth century. Bernini's reclining figure is nothing but a dramatization of the standard reclining figure of the defunct at funerary wall monuments in Italy through three centuries, with representations above the couch of the heavenly throne and angels or saints accompanying the kneeling defunct. It is a spatially extended and modernized monument in the medieval tradition.

In fact, one would normally interpret the reclining figure as that of a dead person present in the heavenly sphere, enjoying the *supranatural experience*. Traditionally the *gisant* figure is often, but not always, accompanied by an image higher up on the monument with *the same person kneeling before the celestial throne,* usually being presented by some patron saints. This represents, the thirteenth-century Bolognese canonist, Buoncompagno, tells us, *how angels and saints lead the souls of the defunct into the presence of the Divine Majesty* (in his *Candelabrum eloquentiae,* chapter on *De tumulorum ornamentis*) - *dipinguntur etiam quomodo angeli vel sancti mortuorum animas divine majestati presentant* (SL, 1984, 35, note 10).

The notion of the revelation opening up for the supranatural experience is obviously the essential subject here too, the supranatural experience the defunct or her or his family devoutly hope will ensue. The subject is almost ubiquitous in medieval and later art. Titian's *La gloria,* with Charles V, the queen and son kneeling before the Trinity (the Prado, Madrid), is one among many. When *all the saints* are more spectacularly represented, this only means that Charles was in the position of ordering a bigger picture, requiring more figures than did a simpler painting or a wall monument.

Fig. 2.1 - 5, Rome, San Francesco a ripa, Ludovica Albertoni chapel (Bernini)

I have to expound the revelatory theme more in detail, for it instantiates the distinction between system level and elaboration level set out more carefully in *Chapter* 1.6. The *velum* or often *vela* in the plural, was a traditional motif connected with *revelation of Heaven*, possibly even the *Beatific Vision* in use since way back into the "Middle Ages" (Benedict XII's bulla *Benedictus Deus* on the *beatific vision*, 1336, SL, 1984, 123; cf. also the *Commendatio animae:* SL, 1974, 47, 69f., 73, 77f.). It would take us too far afield were I, in the present connection, to specify further the liturgical and theological basis for the application of the *velum* imagery. Angels carrying an image of the Virgin and Child (or some saint) is another common idiom denoting the idea of a *revelation.*

In the cited wall monuments God's angels or priestly figures or deacons (to posit a specifically liturgical level, as on the De Braye monument at Orvieto) draw aside curtains to open for the view of heaven to the dead reclining on a a pseudo-sarcophagus (the burial place being in the floor or elsewhere); or else, we are invited to see, between hangings already drawn aside, the defunct kneeling before the throne.

The scenario of vela drawn aside to open up the heavenly world *for the defunct* cannot technically be represented in a spatially strict space-logical manner, by having the carpets interposed *between* the defunct and the heavenly figures. The combined revelation to us of the saint as principal iconographical protagonist and liturgical focal point, and of heaven opening up to the saint, conveys the image of a chain of intercession for the people. The rationale for this intercession is the saint's direct contact with heaven, which we now see revealed to the saint. Any congregation practising the liturgy would take the point, even when the space structure of the architectural and sculptural shapes cannot be entirely logical, which would have involved the awkward arrangement of having the curtain to be placed between the portrayed protagonist and the depicted heaven, rather than between the former and ourselves.

The normal solution on funerary monuments is using carpets the ordinary way as lateral hangings that are being drawn aside. Anyway, the presence of the collapsed curtain in Bermnini's narrow little chapel would have been sufficient for those familiar with the tradition to get the message.

When the personal reaction in the heavenly perspective is rendered by such dramatic means as in Bernini's *Beata Ludovica* (Bernini, 374f.), it is merely a more explicit spatial rendering of a notion that keeps close to the foundations also as regards prayer and meditation among those still alive. For this, we may consult Ludovico Barbo's (OSB) descriptions in his *Modus orandi et meditando* of 1443 (Gisolfi and SL, 21, 76f.).

The actions of the body assists the spiritual elevation (*gesta corporis mentis ellevationem adiuvant*). Thus prepared, Barbo continues, with our imagination we should see Heaven opening up. Then we are led to having visions, and our *soul completely "liquifies" at the touch of <these> divine and supratranscendent illistrations/visions* (*anima vestra tota liquefacta sentiens divinas et supertranscendentes illustrationes*) (Barbo, 104). Domenico Cavalca's *Specchio di croce* (1325 - 1340) also insists on seeing with one's imagination. Bodily attitudes were considered essential for effective prayer (Trexler).

Fig. 2.1 - 6, Rome, San Francesco a ripa, Ludovica Albertone chapel, detail (Bernini).

These observations lead us on to consider another chapel by Bernini, with St. Teresa d'Ávila. The two Bernini complexes, with the *Beata Ludovica* and *Saint Teresa,* in Santa Maria della Vittoria, Rome, are not directly comparable. In the first case, the reclining figure is on her deathbed and at the moment of her passage, heaven is revealed to her. In the second, an experience from the protagonist's terrestrial life is conveyed by her being elevated into the heavenly sphere, a very common, almost standard, type of image reflecting an important and liturgically stated circumstance for her sainthood.

The image of Teresa receiving the arrow shot in her heart, refers to experiences when she was alive, and as usual, such biographical events so to speak accompany her into heaven (and into the Breviary), as part of the *treasury* (*see* below) on account of which she *is re-*

ceived into Heaven. Being hit by the arrow of Christ or his angel is a property attaching to her and one of her merits for which she is received into heaven, a merit vested in the canonically defined *thesaurus* (SL, 1984, 21; Denzinger-Schönmetzer, J 10ba: *thesaurus meritorum*) This particular event is taken, as is usual, from the *Breviarum romanum*. On each 15 October the congregation would hear that she was hit *Divinae amoris cuspide*, by the lance of divine love. She is hit in her *heart*, not, as Sigmund Freud optimistically believed, lower down (he saw photographs taken at an orthogonal angle; but the scene is intended to be seen by people standing or kneeling at floor level).

The Breviary contains the *official biography* of Saint Teresa, in which her dubious (by the then accepted norms) social lineage is ennobled, suitably enough for the Cornaro family kneeling on either side of the chapel (and in agreement with ecclesiastical falsification of her genelogy) (SL, *Teresa*, with ref. to Chicharro). The Breviary material, giving her a faked social uplift, is crucial for understanding the project for the Bernini chapel, sponsored by the Cornaro family of ancient Venetian nobility. A number of recent investigations (carefully cited by Chicharro, 20 - 32) have identified her family as belonging to the low-rated category of *judío-convversos*, and have noted how her own *Vida* reflects her worry about this; apparently her insistence on strict *clausura* in the nunneries (Teresa, Chapter VII, esp. 158f.; before the new rules issued by the Council of Trent) reflects her desire to make social distinctions less definite. A Constitution of 1597, five years after her death, made *limpieza de sangre* (bluntly stated: non-Jewish blood) a requirement for being accepted into religious convents, rendering usual practice canonical. To meet this problem in the case of Teresa, the Vatican simply used the Breviary to claim a respectable lineage for her, again, as in other Breviary readings, establishing a reassuring and stabilizing social and ecclesiological image for public consumption. Take it or leave it.

Often in relevant publications such dramatized representations as Bernini's Beata Ludovica and his Santa Teresa Chapel are lumped together under the headings of *ecstacy* and *mysticism.* The canonically performed Mass sacrifice itself enacts a *mysterium tremendum,* so the phenomenon should not be uncritically applied to human psychology. Regarding the action of the *angel*, let us not forget that one of the functions of the angels is, precisely, to establish contact between God and mankind or single human beings (references in SL, 1984; important texts in Lubac).

The buildup of the Teresa chapel relies on another century-old usage. A saint hovering in midair, in heaven, being assisted by angels, we find, e. g., in Pinturicchio's *San Bernardi-*

no fresco in the Araceli in Rome (1486). Here, angels place a crown upon his head. From the structural point of view, Bernini's work differs because the configuration has been dramatized dynamically and lifted out of the flat surface of a fresco and spacially extended so as to occupy an entire chapel. In both cases, God intervenes through his angels, using them as they are intended to. Indeed, in the Teresa chapel, an *angel* had to act as in-between, since the configuration of *Christ* hurling down arrows was a traditional image of the plague hitting mankind.

The cited Breviary reading does not follow the autobiography, *Libro de la vida*, of Teresa de Cepeda y Ahumada (1515 - 82), in which she describes her heart being hit *twice* by the divine arrow (Teresa, 351, 353), in the first case by one that seemed like a poisoned one (*la saeta parece traía yerba = hierba*); in the other by one with its point flaming. Here as elsewhere in her autobiography, the pain is not physical but spiritual, even though it affects the body, too (*No es dolor corporal sino espiritual, aunque no deja de participar el cuerpo algo, y aun harto*; Teresa, 353).

In her famously "emblematic" poem about seeing Jesus with her eyes, she does so not directly and physically but in the *subjunctive* (*Véante mis ojos, / dulce Jesús bueno*) (Gonzáles, 66: Ital. transl.: *Ti ved**a**no i miei occhi...*). Does "mysticism" operate in the subjunctive? No.

The myth of "Baroque mysticism" is in need of invalidation. In the Roman world the fundamental *mysterium (tremendum*) is enacted every Sunday (at least), mankind enjoying direct contact with divinity on the strength of the incarnation, God becoming Man. Teresa could not, while alive, *see directly*, she would have to wait until, eventually, she might enjoy the *beatific vision in heaven* (*see above*).Divinity could intervene in human life, and in the New Testament the earliest event is when Saul is transformed into Paul, the *protomarttyr*.

Inspired imagination that makes man feel closer to divinity and heavenly things were common; Barbo's writings (1443) are especially articulate in this sense. Teresa in her splendidly succinct language describes in precise terms what happened: she felt the arrow (twice, in fact), and this caused a spiritual pain which spilled over into her body; a common kind of experience. The s*upernatural* quality does not concern her experience as described but the notion of *Christ's intervention*; usually through angels. This, however, had been recognized for all the saints, for example St. Francis with the *stigmata*

The fact that this sort of event in the seventeenth century became a dramatized literary topic should not lead us into believing that something new was afoot. Even if Ludovico Barbo stays at the functional level (2.2.9), not feeling it relevant, in terms of contemporary practice, in a literary report, to dwell upon the psychological level, emotionally his experience may have been more or less similar to Teresa's; a question of literary levels in different contexts or periods. González (63) rightly speaks of the development of a literary genre: *mysticism* (*lo sviluppo di una corrente letteraria: il Misticismo*). Of course there are no limits to emotional experiences. One of the important cases from Teresa's time concerned St. Ignatius of Loyola, as expounded in a splendid book, published with ecclesiastical approbation (and arguing accordingly) by Hugo Rahner, SJ: *Ignatius the theologian*. Ignatius' *spirit was hovering in the heights of the Trinity whilst his eyes were fixed on the text of the missal* (Rahner, 3). A central factor in setting up altar or chapel imagery - next to the Sacrifice itself - as those discussed in the present *Section,* is the *intercession* of the saints for mankind before God. In the *Ordinal of the Mass* appeal is made to God that he, *by the merits of the saints, whose relics are here, and of all the saints,* would be pleased to forgive me all my sins (*per merita sanctorum tuorum, quorum reliquae hic sunt, et omnium sanctorum: ut indulgere digneris omnia peccata mea*). By the merits of the saints a *treasury* (*thesaurus*) is constituted from which mankind can draw for salvation; this is the core idea of the intercession (SL, 1984, 21, 197f.). The presence of relics in the altar was fundamental (codified by Gratian, *ca.* 1140, referring to the V Council of Carthage, of 401) (Godley, 32f.). The altar iconography of saints had always referred to this.

Bernini's chapel gives us the visually most dramatic example of the tradition of showing the earth, with the church altar, in a *diametrical conjunction* with heaven, an illustration of the contrapuntal interrelation between earth and heaven effected by the sacrament of the Mass which is, we noted (1.2.1), often represented in painting.

Now back to our main subject.

One of Borrominii's followers in Rome was Antonio Gherardi. His *Cappella Ávalos* in Santa Maria in Trastevere (1680) is packed with tricky shapes for which history has left us no vocabulary at all. His sculptural elaboration of the walls and the cupola in the *Chapel of St. Cecily* in San Carlo ai catinari, also in Rome (1692 - 1700), represents a case comparable to Bernini's Ludovica (*Figs. 2.1 - 7* and *8*).

Here two angels, one on each side of the upper part of the chapel, draw aside wide and wildly fluttering curtains hanging down over them, and downwards so as to cover elements of the internal chapel architecture. These curtains too are provided with the golden trimming at the lower edge. They are drawn aside to reveal the view up into heaven with angels that are playing music on various instruments (normal expression of the celestial liturgy).

Fig. 2.1 - 7, San Carlo ai catinari, Rome, Santa Cecilia chapel, detail (Gherardi)

The entire configuration is focused upon the figure of Saint Cecily being revealed in her celestial existence. There is a double matrix here; one refers to Cecily as the patroness of music par excellence; the other and basic factor being an extension to the fundamental notion of any saint's participation. This is another a case of enjoying the supranatural experience, as is stated in the Breviary for 22 November, the feast of St. Cecily: *Elegit eam Deus, et prææelegit eam. In tabernaculo suo habitare facit eam*: *God elected her and preferred her and made her an abode in heaven*.

Fig. 2.1 - 8, San Carlo ai catinari, Rome, Santa Cecilia chapel, detail (Gherardi)

She was connected with music only relatively late, but from the sixteenth century she was universally glorified with this attribution. John Dryden (1631 - 1700), probably a Roman Catholic himself, wrote an ode to her, set to music by Georg Friederich Händel, a testimony to the persistence of the involved ideas. Here her role in music and her vision of celestial harmony is celebrated. In the Church (even the Protestant Churches), the notion of celestial harmony was an obvious, essential and celebrated notion (from which Kepler could draw for his planetary system). Dryden's ode begins with these verses: *From harmony, from heav'nly harmony / This universal frame began,* etc.

The missionary use of visual (and sound) effects in the Roman oratories from the early seventeenth century and in the incipient opera probably contributed to the dramatization of ancient image traditions. But to compare the draperies directly with theatre curtains, we would miss out the essential functions. The subjects of arts and the theatre/opera/oratorium may be used to illuminate one another; but they are parallel phenomena and one should look for their common basis; for the oratorium, and the religious arts: the liturgy, directly or indirectly, and the canonical system.

In the cited cases the *planning outfit*, including the artists and architects (3.1) operated within the restricting boundaries set by the Tradition and activized anew at the Council of Trent (concluded 1563), whose dogmas and teaching they had to respect.

2.2.10 *Rationalistic attitudes?*

Asking for *rationalist* attitudes is strictly anachronistic, since the idea belongs to the eighteenth century. So the question must be understood as asking for attitudes comparable to the scenario as we know it.

Most scholars were actively Catholic (or Protestant). An example is the Catholic, former pupil of the Jesuit school at *La Fleck*, René Descartes, who published his famous and much-discussed formula, *Cogito, ergo sum,* usually read as a profound philosophical novelty. I had worked out a detailed argument about this proposition for the present book (with due references to a great number of scholars, from Gilson to Garber), but had to let it go for space considerations. In the briefest possible terms, my argument goes as follows. The *Cogito* is not a profound philosophical statement about existence, but merely a reminder of the fact that at the Creation, man was endowed by the capacity of *cogitatio,* of discerning true from false (T or F), and good from evil. Descartes simply begins his *Discours* by reminding us of man's being endowed with the necessary qualification for reasoning. Catholic-Traditional use of the term *Cogitatio* would seem to confirm this. I disagree with De la Flor (Flor, 48f.) when he posits a *contrast* between Juan de Borja's *hominem te esse cogita* (*consider you are [nothing more than] a human*) and the Cartesian *Cogito ergo sum.* Both statements are arch-Catholic, but the first on the level of morals and existence, the other on the intellectual and cognitive plane.

Descartes includes this reminder to make it clear that it is in view of *how God created mankind* that he now sets out to develop his ideas. Once it has been established that man's faculty of *distinguishing* through his *cogitatio* is ensured, his existence as a human being is proved, and then the next step is to deduce that he is enabled (and God would never fool him, as Descartes claimed) to ascertain the existence of *things* in the world. Is it really that simple? So I should be inclined to think, seeing, for example, that with Bellarmino *cogitatio* can be *inanis* or futile, applying *cognosco* to something, referring to the knowledge of truth and falsehood, good and evil etc. This faculty *cannot* be relativized, at least not in the thinking of his day. For the use of *cognosco* implies just this: either you know or you don't.

Giordano Bruno seems to have operated in such a cognitive scenario as suggested here, when, in a pamphlet (*Articuli adversus hujus tempestatis mathematicos*, 1587), dedicated to Rudolph II, he rejects the standards of contemporary mathematicians, most of them classified as *Peripatetici*, whom he considered pedantical, revealing a petrified understanding of nature (Foa, 41, Spampanato, II, 431). Bruno's accusation against the mathematicians is remarkable indeed, since this was a time in which at least numerous *avantgarde* mathematicians did everything but practising pedantry.

At the same time, a contrast can be detected between Bruno's philosophy and ideas in the general focus around 1600; it has been synthesized in the following terms by Anna Foa in her excellent little study on Giordano Bruno:

> *Bruno's thinking contained elements, such as the magical and Platonicizing, that were considered completely alien to the new outlook, and also his rigorous stand on philosophy and mathematics as being leftovers from a vanished past* (*... aspetti... che erano visti come del tutto estranei alla nuova cultura, e al suo rigore filosofico e matematico, e considerati residui di un passato ormai sepolto*) (Foa, 25f..).

If Bruno may be seen as a pioneer of modern science philosophy, he also may be seen as the exact opposite. Anna Foa asks whether it would not be more to the point to interpret his *ecclesiastical inquisitors* as representing *modern thinking*, read: rationalistic thinking, more consistently than Bruno did himself (Foa, 88: whether *gli inquisitor*i*... fossero partecipi ben più di Bruno, in quell'inizio del secolo XVII, di una mentalità "moderna", e ... rientrassero ormai nell'ambito di un pensiero razionalistico e "geometrico" che non offriva più né spazio né credito alla cultura magica del Rinascimento*).

While Foa's proposition goes a long way to restore the figures of the churchmen to their proper scientific role (she is not alone here; another one is Feyerabend), more than that cannot be concluded from this kind of observation. After all, the incipient *modern science* was larded with religious appeal and transcendentalism (Holton, 1988). On the other hand, learned Jesuits, for example Christopher Clavius at the Collegio Romano (2.2.11), did their scientific work without making appeal to such anti-rationalistic universes, and the same applies to Bellarmino's system of theology in his *De controversiis* (2.2.12). The scholars within the Catholic Church relied on truths that for them were dogmatic, amenable to strict formalization (within the linguistic idiom at their disposal) and hence rational within the given framework. It was the *scientists* who relied on what Herbert Simon has called *bounded rationality* (Simon, 1983, 17ff., 19ff.; see *Models of thought,* 1979, 20f.; *here*: 3.1).

We are witnessing a merge between the science of quantification and the science of natural observation, an encounter rich in perspectives but also fraught with problems that in part arose out of the gradually intensified insistence upon sheer numerical quantification of all nature, a loosely structured program that was hampered by the insufficiency of available mathematics and by the disparities in the makeup of the program itself.

On several occasions we have noted and will note a "modern" commitment to using *models, instrumentalism* and *heuristics* (2.2.12); nothing new in itself but now advocated as a theory basis, apparently more consistently than earlier. The same applies to the principle of *simplicity* in problem solution (for Simon, *see* my *Introduction,* no. 1). Simplicity not in the sense of superficiality but as a program, can be considered as part of a *larger program of rationalistic and heuristic procedures, intellectual experimentation*. Descartes insisted on using *simple models,* trying out a number of problems on the ordinary parabola (Book Three of his *La géometrie*). He writes

We should always choose with care the simplest curve that can be used in the solution of a problem, but it should be noted that the <term> simplest means not merely the one most easily described, nor the one that leads to the easiest demonstration or construction of the problem, but rather the one of the simplest class that can be used to determine the required quantity (transl, Smith and Latham) (Descartes, *La géometrie,* 152ff.:)
(*... mais il faut auoir soin de choisir tousiours la plus simple, par laquelle il soit possible de le resoudre. Et mesme il est à remarquer, que par les plus simples on ne doit pas seulement entendre celles, qui peuuent le plus aysement estre descrites, ny celles qui rendent la construction, ou la demonstration du Problesme proposé plus facile, mais principalement celles, qui sont du plus simple genre, qui puisse seruir a determiner la quantité qui est cherchée*).

Newton also used a simple model when he substituted a circle for Kepler’s ellipses (which, however, came graphically very close to a circle, the foci staying closely together).

2.2.11 *A visit to the Collegio romano (Christoph Schlüssel/Clavius)*

In the midst of turbulence and insecurity, some institutions and States represented stabilizing forces (2.3.1). Great potentialities for stability, but also for obstructing "progress" laid within the Roman Church. Cardinal Bellarmino’s intellectual activities and his great published system can be cited as one of the most conspicuous Latish of the times (2.2.12).

When on the following pages I am going to consider the role of the *theologians*, my intention is to follow up on earlier writers who accentuate their role as advanced scientists in their own right. There are several weighty reasons for taking them seriously in this role. First of all they actively contributed - partly by conditional support, partly by severe opposition (as stressed by Lattis, *see* below) - to Galilei’s and partners’ struggle through many years, thereby contributing definitely to setting high scholarly standards. Secondly, they delivered original works, partly in science and partly in dogma, theology and general world outlook, that should not be relegated to any *background* but should be integrated together with and on the same level as the rest.

While for science some of the fundamental means to attain the preconceived goals were confessedly premised on the idea of *approximation*, it was with a view to fixed values that the Roman Church concentrated her efforts, and her *means*, Tradition (with cap), theology and liturgy, were already in place and certainly did not allow space for approximations. The great work of Roberto Bellarmino, to say nothing of the products of the Council of Trent (1646 - 1563), should be seen in this light (2.2.12). Before approaching Bellarmino's enormous accomplishment, some introductory attention to the institutional framework is called for (already briefly touched in *Chapter* l.6). Theology, Tradition and liturgical theory and practice (SL, 1984, 16 - 209) established a world of rationality and semantical logic by internal criteria and in this manner represented a firm stand both against hermetic traditions and the new calculated realities and also philosophically a corrective to the idea of approximation.

There is, as hinted already, one scholarly entity that is especially relevant in the context of science discussed in the preceding *Chapters*. It was *the Jesuits at the Collegio romano*, among whom the most notable scholars were Cardinal Roberto Bellarmino and Christoph Clavius. We have noted the latter's role in the curriculum development at the Collegio romano. The *fields* were dogmas, doctrines and traditions concerning the Roman Church and its beliefs, assignment and prerogatives, the whole complex rooted in mainly three sources, the Bible as interpreted by Tradition (not as read by unauthorized people), the liturgical and written Tradition of the Church and the patristic writers. Around this core of the Collegio romano curriculum, scientific disciplines such as mathematics developed, and this combination gave the entire institution an edge that was really felt at the time, not least by Galilei himself, who was now confronted with opponents on a comparable scientific footing and who tried to dislodge him from his scientific position.

In his 1976 edition of Descartes' *Discours de la Méthode*, Étienne Gilson comments upon the philosopher's claim of a backing from mathematicians for his thinking, *à cause de la certitude et de l'évidence de leurs raisons...* (Descartes, *Discours*, 7). Thus formulated, affirms Gilson, the dictum, validated by the purely formal character of the operation, would be endorsed also by Aristotle and his followers, including Thomas Aquinas and Descartes' own teacher at *La Flèche*, the Jesuit Jean François. This also applies to Clavius, whose long discourse on mathematics Gilson quotes almost in extenso (2.2.6).

Christoph Clavius was professor of mathematics and astronomy at the *Collegio romano.* He also became an important interlocutor for Galilei. Being a mathematician and not a physicist, he felt free to take a relatively sober view of the new cosmological observations, keeping them at safe distance. He considered math as the holder of the only absolute scientific truth. But he did strongly oppose the Copernican view of the universe. Lattis' comment on the role of the Church authorities in the cosmological debate is worth recording.

The opponents of the new astronomy were just as important as the innovators because the authoritative opponents, like Clavius, were the ones who determined the criteria and the tests by which the new theories and observations would be judged... (Lattis, 11).

Clavius and some others among the Jesuits might have accepted Copernicanism less reluctantly as an interesting *hypothesis*, had not the issues hardened, in part due to Galilei's strangely un-Italian inflexibility and undiplomatic behavior.

Carl Friedrich von Weizsäcker in his *Aufbau der Physik* enlarges upon the heuristic factor in seventeenth-century science, also among the clergy. As a protagonist in the early debates on the quantum theory, he would be open for such contingencies. Historically, he notes, there were *two distinct conceptions of motion*, one absolutist: Ptolemy, Copernicus, Kepler, Galilei, Newton; and one relativist: Nicolaus von Kues (*Cusanus)* and Bellarmino (to stay with the old people, not counting Leibniz, Mach and others) (may I add Giordano Bruno here?).

Bellarmino since some time had been able to accept the idea of relative motion. One can then at will describe the same motions geocentrically or heliocentrically (*Bellarmin vermochte längst den Gedanken der Relativität der Bewegung zu denken. Man kann dieselben Bewegungen nach Belieben geozentrisch oder heliozentrisch beschreiben*) (Von Weizsäcker, 255ff.).

The cardinal accepted instrumentalism and heuristics as long as the dogmas remained unaffected (2.2.12).

Instrumentalism was a way of escape from conflict areas (well described by Sharratt, 118f.). Von Weizsäcker in fact emphasizes Cardinal Bellarmino's willingness to accept the Copernican system as a mathematical hypothesis, *heuristically*, we might say, but not as a truth. Many authors have made much the same comment. But the German physicist goes a step or two further by stressing that *hypothesis* here, as also later in Newton, does not mean presumption or supposition (*Vermutung*), but assumption (*Unterstellung*); this in the sense current in predicate logics; and that thus *we come close to what Bellarmino wanted to say with our term "model"* (*wir haben uns mit dem Wort 'Modell' nahe an die von Bellarmin* *gewünschte Sprechweise angeschlossen*) (Von Weizsäcker, 256f.). Bellarmino *combined systems thinking with model managing*, thus coming closer to his opponents in science than might appear at first sight.

Paul Feyerabend on his side, after a penetrating perusal of the documents, can report that at a certain point, the cardinal did go as far as stating that under certain conditions he might accept Copernicanism *as a truth.* But he demanded *proofs*, not plausibility (*Beweise*, not merely *Plausibilitätsargumente*) (Feyerabend, Chapter 14, esp. 213ff.). This no one was able to deliver; the cardinal evidently realized that and felt safe. At the same time he had made a show of respect for the standards of science.

When explaining the Tradition of the Church, however, nothing was relative in Bellarmino's thinking. Here, he entered a world that for him was real in an absolute sense, which he faced with what we might call a rationalistic mind.

2.2.12 *Bellarmino system-builder*

There had been many reform movements within the Church and much earlier than the sixteenth century, but the appearance on the scene of the Reformers, and the dramatic expansion of the anti-Roman confessions in the center of Europe, in the kernel area of the Holy Roman Empire, made some issues seem more crucial than earlier. Among these there were, in basic principles, two sets of what today we might call institutional issues: first, the *teaching authority of the Church*, including her role and privilege as depositor of faith and truth and interpretation of the Scriptures; secondly, and directly concomitant to these factors but also with political relevance, the *primacy of the pope*. Internally, of course, other issues were at stake that concerned the very raison d'être of the Catholic Church: the dogmas on the sacraments, particularly on the Mass sacrifice, the dogmas on the *opus bonum* (the good work) and justification. The Mass was (and is) the supreme *opus bonum*, the source and prototype of all good works in Christianity (even to the point of doing one's regular job well). The Council of Trent had stated: *No one must have the temerity of deny that the Mass is <the> good work* (*nemo audeat negari missam... esse opus bonum*) (Gisolfi and SL, 119, note 54, citing Duval).

Cardinal Roberto Bellarmino and his work represented a strong force supporting stability and resistance against innovation, acting as a bulwark of a Church that had consolidated her Tradition and teachings at the Council of Trent (1545 - 1563). His enormous *Controversia* system was meant to render the Church's position unassailable. And in this he was successful since the premises were woven into the conclusions.

Bellarmino's *De controversiis christianae fidei adversus hujus temporis hæreticos* (*On the controversies in Christian faith against the present-day heretics of our time*), focuses on just these subjects as a compass for virtually the entire system of the Catholic faith and practice, including ritual usages such as the veneration of holy images.

Bellarmino's *summa* was rooted in Tradition's interpretation of the *Biblical books*. It therefore had a twofold polemical, or should I say, didactic, address, against the Northern Reformers with their *direct* readings of the Scriptures, and against the new physics and cosmology against which the scriptural authority had to be upheld. Through the entire work a number of heretical ideas are refuted, and not only those of the recent Northern Reformers. Bellarmino takes no chances, covering himself completely, and going all the way back to Arius (whose teachings on divine nature were condemned in 325).

But Bellarmino's *De controversiis* was not only directed against the Northern Reformers, specifically and against what was considered false innovations of any kind. It was also a positive effort intended to explain and illustrate the structure and role of the Church as God had created her; with the foundations also of cosmology, without elaborating its distinct features. The colossal work was a genuine contribution to systems thinking and as such it provided an especially hard nut to crack for modern-minded scholars. His system was created by God - and Descartes claimed the same for the foundations of his system.

Thus Cardinal Roberto Bellarmino played an important role in the readjustment of the Church after the Council of Trent (concluded 1563), in her confrontation with the new cosmology, and, in a narrower sense, in the *Galilei Affair*.

My subject here is mainly his use of the Tradition of the Church (with capital *T*) (SL, 1984, 18f.), including the achievement of the Council for the building of a compact, systematic and definitive account of the *structure of Catholic faith and obligations and the Church as embodiment of that structure*. Some introductory comments on the *De controversiis* are called for.

From the vantage point of Rome, the work constituted as solid a body of knowledge as was conceivable, with which the new cosmological ideas (Copernicus, etc.) if accepted as representing *truths*, would be massively ruled out. The rebellious scientists would have to disprove the content of the four thick volumes in their entirety, since "everything depends on everything" in this vast but coherent and, by the given standards, completely documented edifice. Refuting a small part of it would affect the rest, the teaching authority of the Church and the authority of the Pope would go by the board, and the whole edifice would crumble.

Systems, we know, are fine things, but unfortunately vulnerable. But Bellarmino's Traditionally established criteria of documentation made his work unassailable except from alien positions, which, however, were considered incompatible.

A paradoxical situation arose. To secure one's position on the side of the Copernicans, one would really have to disprove Bellarmino. But when Galilei tried to call up scriptural material, he was severely censured for exceeding his competence, and with excellent motivation, for his argumentation would emerge as invalid unless he could tear down the entire building; which he was not allowed to try, since he was not a theological scholar. The edifice was closed to all except those who were already inside. Being inside assured sufficient competence also in cosmological affairs, because the Scriptures were the depository of truth even on these subjects, the Scriptures, then, as the Church had interpreted them.

On this background it is clear that Feyerabend was right when he claimed that Bellarmino's argument against the new cosmology was *rational*: he had no option (Feyerabend, 1976). It would have been simply and profoundly unimaginable for Bellarmino to engage himself in any argumentation that might have seemed to violate the teachings of the just-concluded Council of Trent (1563):

> *No one is allowed to distort the <significance of> Holy Scripture in his own terms, and against the teaching that has been given and is given by the holy Mother our Church, whose* <*privi*lege it is to> *judge about the true significance and import of the Holy Scriptures* (*ut nemo... sacram Scripturam ad suos sensus contorquens, contra eum sensum, quem tenuit, et tenet sancta mater Ecclesia, cujus est judicare de vero sensu et interpretatione Scripturarum sanctarum...*). (Denzinger, 1856, 176, from Sessio IV, *Decretum de canonicis Scripturis,* 1546).

It was a question of Yes or No, for once excluding the middle absolutely.

Presenting Bellarmino's great work, I shall do four things, not always in the stated order; indicate its dimensions; offer a synopsis of the contents by chapter/section titles; very briefly sketch out the typical *algorithm of argumentation*; and, finally, indicate the main outline of the chapters that treat of the *position and role of the pope*. This latter subject may not sound very exhilarating, but it is fundamental in the sense that, the way the *Primatus Papae* is affirmed in Bellarmino's *Summa*, the whole building construed in the four volumes would become considerably more shaky, perhaps even crumble, if the position of the papacy should even begin to vacillate; and in consequence also the position of the cardinal himself and his co-militants.

I am going to present the main headings of the gigantic table of contents of Bellarmino's four volumes, *De controversiis.* I know one could just tell the reader that there were so and so many of them and present an outline of their contents. I would not find this adequate, and for two reasons. For most modern readers an abbreviated report would not give a meaningful representation, since some of my readers (whom, optimistically, I cite in plural) would probably not be much attuned to such a subject. Secondly, *seeing* before one's eyes the entire *schema* will convey an impressive and informative picture that is required for an adequate appreciation of this monumental work, an achievement that is no less of a system-building than Galilei's *Dialogues*, but considerably more firmly grounded (on the given criteria).

Quantification is not out of place here, for it will show the bulk of the achievement, and, one may assume, also give an idea its significance for the author himself. My own copy of the *De Controversiis,* in the Milan 1721 edition (which is complete and includes some other minor writings by Bellarmino), consists of *four volumes* (each a *tomus*) with 39 x 24 cm printed pages. Each page has two columns and they are paginated by column numbers (each page thus having two numbers). Each column has on average 72 lines with approximately 50 points each. *Volume* I has 1044 columns text body and 48 columns alphabetic index and 96 columns Supplement. *Volume* II is a bit skinnier, but this "reduction" is vastly compensated for by *Volume* III with its 1427 columns text body and 84 columns alphabetical index. *Volume* IV is roughly as large as *Volume* I.

The extremely copious references to works by the writers of the Greek, Syrian and Roman Churches do not come in footnotes but in abbreviated summaries in the text body. Here is an example from Vol. I, col. 337: *Vide* [*See*] *Athanasium serm. 1.2. & 3. contra Arianos. Basilium lib. 4. in Eunomium. Nazianzeum lib. 3. de Theolog. Cyrillum lib. I. thesau. c. 5. & lib. 12. c.7. Ambrosium lib. 4 de fide cap. 4 & Augustinum lib. 6. Trinitat. c. 1. & lib. 7. cap. 4.*; references for people who knew the texts.

The work is written in Latin with quotations in Hebrew, Greek, Syriac and Italian.

Now a *synopsis of the headings.* I shall cite the main headings only, not all the subheadings, of which an impression may be gained from my account of the *argumentation algorithm,* a few pages ahead. In my sectionwise translations (in Italics) I have "normalized" some terms a bit, by choosing equivalents for Latin/ecclesiastical terms that should render them immediately graspable for normal modern readers, even though an authorized translation might look slightly different.

INDEX TOTIUS OPERIS

Index of the complete work

P r i m i t o m i

Volume I

Controversiæ generales tres

Three general subjects for debate

I. DE VERBO DEI SCRIPTO, & NON SCRIPTO

On God's Word, written and unwritten

II. DE CHRISTO CAPITE TOTIUS ECCLESIÆ

On Christ, head of the total/universal Church

III. DE SUMMO PONTIFICE CAPITE MILITANTIS ECCLESIÆ

On the Most Holy Pope, head of the Miltant Church

T o m i s e c u n d i

Volume II

Controversiæ generales quatuor

Four general subjects for debate

I. DE ECCLESIA MILITANTE, TUM IN CONCILIJS CONGREGATA, TUM SPARSA TOTO ORBE TERRARUM

On the Militant Church, assembled in councils as well as spread over the whole world.

II. DE MEMBRIS ECCLESIÆ MILITANTIS, CLERICIS, MONACHIS, LAICIS

On the members of the Militant Church: the clergy, the monks, layfolk/congregations

III. DE ECCLESIA, QUÆ EST IN PURGATORIO

On the Church in Purgatory

IV. DE ECCLESIA, QUÆ TRIUMPHAT IN COELIS

On the Church triumphant in Heaven

T o m i t e r t i i

Volume III

Controversiæ generales quinque

Five general subjects for debate

I. DE SACRAMENTIS IN GENERE

On the sacraments generally

II. DE BAPTISMO, & CONFIRMATIONE

On Baptism and Confirmation

III. DE EUCHARISTIA, & SACRIFICIO MISSÆ

On the Eucharist and Mass Sacrifice

IV. DE POENITENTIA

On Penance

V. DE SACRAMENTIS CÆTERIS

On the other sacraments (baptism, marriage, Ordination etc.)

T o m i q u a r t i

Volume IV

Controversiæ generales tres

Three general subjects for debate

I. DE GRATIA PRIMI HOMINIS, & STATU INNOCENTIÆ

On the (state of) Grace of the First Man, and on the state of innocence

II. DE GRATIÆ AMISSIONE, & STATU PECCATI

On the state of receiving Grace, and the state of sin

III. DE GRATIÆ REPARATIONE, & STATU IUSTIFICATORUM PER CHRISTUM

On the state of being forgiven and on Justification.

Bellarmino's *Book on the Pope* (Book III in Vol. I) covers 94 columns (47 pages) and is in reality, though this does not appear from the headings, subdivided into *three main groups* of arguments.

Chapters 1. to 4. are devoted to a controversy with a long history behind it, about which one among the known *categories of government* would turn out to be the ideal one, monarchy, aristocracy or democracy, also considering so-called *mixed government* (a combination of the three). Mixed government was claimed in the political theory of the Republic of Venice; politically a very good reason not to apply it in the Church (SL, 1974, 134ff.). The conclusion is that, particular circumstances apart, *monarchy,* absolute and pure (*simplex, absolutè & simpliciter*), that is, papal government *tout court,* tops the list.

In these chapters one notes a significant deficiency of references to medieval and modern writers on *political theory*, even in connection with such a debated subject as that of *mixed government,* and also concerning the great medieval debates on the validity of *general councils* of the Church, sustained very actively by important contributions from secular writers but left unrecorded here. Bellarmino cites ancient Greek and Roman authors and from them passes straight on to the Fathers of the Church, right from the beginning and up, making a big jump over writings that might, it has to be admitted, detract too much from his mandatory considerations; and weaken his argumentation not a little. He was justified in allowing himself such "omissions": Marsilius of Padua, for example, was no ordained ecclesiastical authority, nor was, for that matter, Tommaso Campanella, with his *Città del sole* (he was an ordinary monk, and so was Bruno).

Chapters 5. to 9. concern the *Church herself*: should her government be monarchical? (yes, of course); is her government democratic? (of course not); do secular princes have any prerogatives here? (the answer is no). This section concludes with the crucial principle that the government belongs to *the bishops* (not comprising the lower clergy: this against the Reformers, but also against the Republic of Venice). The affirmation is in accordance with the Apostolic Tradition and confirmed by the General Church Councils, in which ideally all bishops of the universe sit in assembly. Bellarmino certainly had to take into account the *Council of Trent* that had concluded its work a few years earlier and which laid heavy obligations on everybody inside the Church as well as on the congregations of the faithful.

From this affirmation it follows that the Church really should be monarchical, since the Pope is the head among the bishops (the principle of *primus inter pares*).

The Church is one body and thus with one head. Its head is Christ, and by his and the Apostles' heritage, the bishops are governed by the first among them, the bishop of Rome, the Pope. An anthropomorphical model that is consonant with the dogma of God having created man in his image.

After these standard ideas, Bellarmino, from Chapter 10 onwards, transfers the argumentation (he would have said: documentation) on to the *Biblical* level, citing all the normal sources about Christ choosing St. Peter (*Matthew* 16); Peter as shepherd (*John* 21); the significance of *Pasca;* and that the shepherd mandate was addressed to the Universal Church, which on earth is headed by the Church of Rome. It is well known that in the contest over Copernicanism, Bellarmino read the Biblical stories in their Roman interpretation as the indubitable truth, and so he does here. The idea of the Bible being open to variant interpretations (something with which we are familiar in Protestant cultures) was irrelevant, since, as noted already (1.6.1), the Church claimed the exclusive competence, right and privilege in understanding and interpreting it.

In the remaining eleven chapters of Book III, Volume I, these themes are being elaborated.

So this, on the doctrinal level, is what people like Bruno and Galilei had to take into account when facing ecclesiastical authority. We must not forget that most of the disputation in Bellarmino's colossal compendium would be accepted and taken for granted by Galileo Galilei and practically everyone among the contemporary scholars. This was, again on the doctrinal plan, what the cardinal himself could never relativize. Bellarmino was, as Feyerabend perceptively saw, as tied up as Galilei, or even more so, since he represented a stringent and well-defined system established by one and half millennium of accumulated traditions. Galilei had made scattered observations that seemed to be consistent with one another but which nevertheless did nothing more than pointing up new possibilities. These observations, indeed, had been made by a human being and with human-crafted tools; they did not emanate from divine revelation.

Generally, Bellarmino avoids stating the truth at the outset of his paragraphs. Instead he lets the Reformers and other contestant speak, whereupon he disproves their teachings. This was the usual way of managing polemic business; and it is by far the most efficient. The Church used it in the cosmological controversies, too.

In this typical *argumentation algorithm,* the first thing to note is the role of *negation*; next to the method of relying on written documentation in the *Tradition of the Church,* which *included* the Bible in its Roman interpretation. The Church of Rome in her logical wisdom has always realized that trying to state something in positive terms (*this is so-and-so, anathema who does not agree*), easily can draw you into a never-ending process of definitions and syllogisms. *Negation,* on the contrary, lands you where you intend to end up. *Whoever claims so-and-so, is anathema.* Therefore such formulations are appended as conclusion - *canon* - to introductory formulations that indeed may take positive form. Then you are safe, having stated nothing in the definitional *canon* that they could throw back on you. You want to avoid that and leave to the opposition the responsibility and peril of stating his case, so that you can limit yourself to invalidate it. Invalidating something does not so easily bring in unforeseen consequences and implications.

An example can illustrate this millennial technique in the Church of Rome (going back, I believe, to Roman law). One of the *canons* on the sacraments at the Council of Trent (*Sessio VII*, *Can. No. 5*) in 1547 reads: *Si quis dixerit, haec sacramenta propter solam fidem nutriendam instituta fuisse: anathema sit* (*whoever should claim that these sacraments were instituted for the sake of nourishing* [*supporting*] *belief only, is rejected*)*;* against Luther's *sola fide* principle. I have quoted another one concerning the Mass as the good work earlier in this *Section*.

This model of thought was intellectual baggage in minds like Bellarmino's. Such an ingrained mode of reasoning made it all the harder, in the conflicts about Copernicanism and related subjects, to steer a middle course between full rejection and half acceptation; the former tack had a millennium and a half behind it, providing a safe barrier for inside people. As Feyerabend concluded: *Bellarmino had no choice*. It would have been formally, logically and professionally impossible for him to accept ideas that were declared untrue by the Council of Trent; and that ran counter to the Bible as interpreted by Roman Traditions and his own *summa*.

But at the same time Bellarmino's efforts, and those of his *Parteigänger,* should not be accepted as an unpleasant historical necessity or considered under a negative heading. The great work of Roberto Bellarmino contributed to *keeping systems thinking* alive, indeed, invigorated it; an indirect or even direct support also for non-ecclesiastics, such as for instance Descartes, when he *ceased to be a problem-solver and became a system-builder* (Garber, 1992, 50)*.* Taking their respective premises in consideration, we must admit that Bellarmino's system was considerably less shaky than either Galilei's or Descartes'. Whoever wants to insist on the *scientific revolution*, should take into full account the fact that Bellarmino's work as a *system building* was a very constructive contribution in this context, and that this prepared him for appreciating the new cosmology more than he could allow himself to admit.

2.3 World worry (Donne, Maynard, D'Aubigné)

It appears that the intensified focusing on *change, variability and approximation* that I have outlined, was associated with a *more general world worry*. Nothing absolutely new in that, but, as noted a few pages back (2.2.5 and 2.2.7), now this became a literary subject and a matter of universal worry. There were valid motivating reasons for this: a dramatic increase in the cost of living; economic, agricultural and political crisis; terrible wars, destroying entire cities; and, of course, what was felt as religious disintegration; the Turks expanding menacingly. Christian unity manifested itself mainly when the parties involved had to face a common enemy; as at Speyer, when Catholics and Protestants in touching agreement determined the fate of the Anabaptists. Big events like the war that started in 1618 (to last for thirty years), the emergence of Protestant states and the Turkish presence in the depth of Europe stimulated international consciousness and highlighted political networks.

The disasters and what we may call their emergent properties (4.3.4) found their place prominently among the thematics of contemporary literature and philosophy. A general feeling of insecurity and uncertainty is documented; and *an extraordinary spiritual and doctrinal confusion.*(Souiller, 43f.). We have noted the corresponding effects of the spectacle of an infinite universe. A few examples should indicate the tendency.

Comparing the Spanish picaresque novel and the religious autobiographies and "confessions" like Teresa's De Cepeda y Ahumada's *Libro de la vida,* Dámaso Chicarro notes that, paradoxically, the pessimism in the former genre and the providentialist optimism in the latter, both end up in the definite conviction that the world needs a change (*la última convicción en una y otra de que el mundo necesita una trasformación*) (Teresa, 81).

John Donne in 1611, quoted by Souiller, complains: *And New Philosophy calls all in doubt, / The Element of fire is quite put out / ... 'T'is all in pieces, all coherence gone; / all just supply, and all Relation.* It was probably Donne who expressed the philosophical complexities of the theme in the most drastic terms. He was a *clergyman* but seems to have lost some of the confidence that his vocation would seem to have demanded. *Disruption,* by traditional or classical standards, entered his poetry as it did in music (2.2.8). Flor (93f.) while referring to Donne, cites strikingly similar complaints in Spain (Bartolomé Leonardo de Argensola: *nada parece seguro...* and *el enigma y el desconcierto rodean la condición humana*; - nothing is certain - enigma and confusion surrounds human life; Francisco Sánchez: *nada se puede saber*, nothing can be known with certainty).

Morel speaks of attempts to arrive at an *exposition of rules that might control the apparent world disorder* (*une exposée des lois qui dominent l'apparent désordre du monde* and *une philosophie de l'ordre du monde*) (Morel, 84ff.). Shapin, too, cites Donne as a witness to the awareness of a general instability (Shapin, 27f.).

In 1643, just before the death of Louis XIII of France, François Maynard, a protégé of Marguerite de Valois and thus no backwoodsman, wrote poems that expressed, according to Morel, *the fragility of man, of society, of the universe itself* (*la fragilité des hommes, des cités et des civilisations: l'universe lui-même*); and about the *universe that on its long voyage beholds so many oceans flowing and flourishing so many worlds, without knowing where to* *fall; it will fall some day* (*qui dans son large tour / Voit courir tant de mers et fleurir tant de terres, Sans savoir où tomber, tombera quelque jour*) (Morel, 98). The ardent Calvinist Agrippa d'Aubigné (with his *Tragiques*, 1616) is quoted for his vision of a retreating God who regrets having created the Earth: *Dieu voulut en voir plus, mais de regret et d'ire/ Tout son sang écuma: il fuit, il se retire... /Il se repent donc d'avoir formé la terre.../ Il sauta de la terre en l'obscur de la nue...* (*God did not want to see more of this, but in regret and anger, his blood boiling, he flees, he retires, deploring that he had created the earth, he escapes it into the dark clouds*).

Souiller refers to studies by Michel Vovelle (Souiller, 27ff.). The latter - on the cue of *le frisson du baroque* - speaks of a *crise de sensibilité collective* and of the idea of death invading life (Souiller, 27ff.). This may have started at the most explicit with Michel de Montaigne, *dont le point de départ a consisté en une incessante méditation de la mort*.

Death thus becomes the first entry in a catalog over the predominant themes. Observations on this and on the allegedly sharply increasing number of witchcraft trials and burnings, led another French scholar cited by Souiller, Jean Delumeau, to write a thesis under the striking title *La Peur en Occident* (Paris 1978; which I have not seen).

Closely related is the literary theme of *the corruption of our time* (expressed by Donne, among others), the belief in a barbaric time of injustice and corruption. The *angoisse commune* leads to the craving for *divertissement* - in Marlowe as well as in Calderón and others.

Shapin concludes:.. *the sixteenth and seventeenth centuries were the first periods in European culture when cosmic infinity seriously challenged the more comfortable dimensions of common experience... The new philosophy assaulted common sense at a mundane as well as a cosmic level* (Shapin, 27f.). Indeed, but *was* it really the first time? and more "serious" than previously? Or was it now more perceptively activized in literature on account of general framework transformations and thus mentally suffered more than earlier; and fashionably elaborated?

The attitudes outlined so far, according to Souiller, implied also *scepticism* (a factor prominent also in Spanish literature) clearing the way for new approaches in science (Copernicus, Galilei, Kepler and others).

Anyway, the Thirty Years War *was* serious. The Peace of Westphalia of 1648 brought military peace but also political unrest, as reported by Fabio Chigi (later Pope Alexander VII, 1655 - 67). In the quality of Apostolic envoy at Münster, he was active there during the decisive period of the proceedings (from 1644). He wrote a series of long poems about the place itself (rather uncomplimentary, to tell the truth), the social and cultural conditions there and, in 1649, about the outcome of the peace conference. This is the subject of the last poem, entitled *Iter Monasterio Westphaliae Aquisgranum anno MDCXLIX, in ipso itinere scriptum in pugillaribus* (*Travel from Münster to Aachen in the year 1649, written down with pencil during the voyage*) (Chigi, 30; my thanks to Stefanie and Alfons Schröder at Münster for sending me this offprint). Here he concludes with a bitter comment on the outcome of the big peace conference, noting *en passant* that two royal representatives had left early in protest. Having worked hard to establish better relations among the nations, Chigi has to report that the participants

> *finally left each other in discord. When disunited Germany saw her hopes coming to nothing, and yet respected her oath, then Bavaria* [*Norica*] *threw suffering mankind anew into misery,* etc. (*Nam votis frustrata suis Germania discors, / Iurato quamquuam strinxisset foedere dextras, / Norica tecta novis complebat maesta querelis...*).

2.3.1 *Stability and pr obabilities: institutions, traditions and rituals*

World worry, yes. But there were compensations. Belonging to something that had a stable *look* whatever the realities, was one of them. Another consisted in being positioned in one of the sides in the contests; having a footing somewhere however shaky is often reassuring.

There were certain factors that contributed to some amount of stability. This applies to international institutions such as the Church and the Empire and traditions (in the Church, the Tradition, too) arising from and within them. The case of the *Roman Empire of German Nation* is a more unruly one. It has been claimed that around the time in which Florence was erecting her new government building, the Palazzo Vecchio (1299 - 1313), the empire was already *a dead issue* (in an internet blog by one Brenda Bayer I happened to come across but have since lost). What did *The Empire* mean? 1. The active political institution (which certainly had its ups and downs); 2. the idea of permanence of the idea of Rome (Kantorowicz); 3. the suprapersonal institution (Beumann); 4. dynastic traditions, claims and policies; 5. the idea of supranational unity and confirmation of inherited values. As an institution, the Empire did not collapse until 1806; in one shape or another, the idea of the Empire did not die definitively until 1918. A few states also offered images of a more or less reliable stability (France, Spain, England, the Republic of Venice and a few others).

Perhaps the most prominent case of *systems stability* was represented by the *Tradition* of the Roman Church (*see* 1.6); the various manifestations of her *canonical system,* as they were understood in different terms on different levels of social standing and preparation. Undoubtedly, the daily rites in the churches contributed to a feeling of permanence and stability, also when experiencing marginal variations in terms of period and place. The Breviary hymns, recitals and readings, explained to the people whenever necessary, created and sustained a groundwork of fixed or even absolute values - as Stephen Spender said concerning rituals in general - in people's everyday life. We have noted how Rome embellished the biography of Teresa d'Ávila customizing her figure for public use (2.2.9). The breviary readings usually present the congregations with a completely reassuring image, regardless of realities

Most of the institutions, Church, state, municipality, guilds, monastic and other orders, expressed themselves and found support in *rituals* (SL, 1974, *passim*; 1984, *passim; Burden, passim*). When the state or municipal rites are studied in the religious context in which they without exception arose and were conducted, it will emerge clearly that the complex institutional rituals represented *stabilizing factors* in religious, social and political life. They were all rooted in religion and structured in connection with liturgy.

One example: the spectacular *Wedding to the Sea* in Venice had its basis, let me note, in the government procession each Wednesday to the government chapel of St. Clement Pope and Martyr, in San Marco (*De sancto clemente papa et Martyre*; thus in the 1564 *Rituum cerimoniale*: *Burden*, 269). His martyrdom had provoked a miracle at the bottom of the Black Sea, qualifying him as a patron of a sea republic. At any rate, long before 1480, two individuals named Clement, a martyr and the pope, were knocked into one, and "Pope" Clement's martyrdom in the sea was recorded in the liturgy (*Dum iter ad mare coepisset...*) (Lodi, 891). In Venice, Bibl. Correr, *Orazionale Cicogn*a, 1602, fols. 277-8, we read: *Dedisti Domine habitaculum martyri tuo Clementi in mari in modum templi marmorei angelicis manibus preparatum...* : an event illustrated in a twelfth-century fresco in San Clemente, Rome. Thus, in the earlier Roman Catholic world, in which most states defined themselves in religious terms and in which all important internal political functions were connected with *altars*, there are no "civic rites" that are not religious, almost always extensions to liturgy; a point that has escaped some modern writers on rituals, in Venice and elsewhere.

Human interrelations are *normally* acted out through rituals, in which category we must count the entire scale between everyday regularities such as saying Good morning to Mrs. Giordano Bruno when meeting her in Park Slope in Brooklyn, and highly complex ceremonies of Church and State.

Thus the great institutions and their ritual expressions to some extent represented a comforting if not always reliable matrix of stability for contemporary people.

2.4 Systems in motion and scientific convergence

So far, I have been trying to establish methodologies and cite cases for describing interrelated changing entities in some specific design areas and in a more general cultural context. Here, one parameter is *change over time.* In current literature, this change, when specifically concerning *science,* is often interpreted as a *revolution* - speaking of The Scientific Revolution headed by Galilei and a few others. Scientific knowledge, we are told, depends on what David Bloor calls *social imagery* (Bloor, also on the ideas of Kuhn and Popper).

2.4.1 *Handling novelties*

Whenever we come across novelties, it is hard to decide in what cognitive and conceptual framework to position them. Facing complex, sometimes even fuzzy, phenomena like "a culture" or "the science", we have no chance of conjuring up anything more consistent in response than *literature,* fiction, recording a *flow- image* of history. If we need an account that we can *use*, that has an operative status and may turn out functional, then we have to reduce our scope, paring it down to some selected *denominator,* to which to relate our findings; such as the *interrelated changing entities* proposed as the metric for the present book.

With Galilei and some others at the time, the image of things in the world and the universe changed because of improved observation media and a more systematic and theoretically active elaboration of the findings; breaking or disregarding many Aristotelian and generally "peripathetic" rules and norms.Galilei has been cited for his uniquely “modern” attitude to science, *its method of inquiry and its criterion of truth* and to practicing research (Drake, 1957, 3). He was “modern” in not asking Why, concentrating on the What and How. Did that mean that *science* as such, as a whole (whatever that might mean) underwent radical or fundamental changes?

2.4.2 *Ambiguities*

Concepts of fields or areas in science were even more uncertain than they are today (perhaps because today we have media smoothing things out for us). Or rather: today they are judged as being shifty but this shiftiness has been elevated to philosophical standards, leaving academic structures unaffected. At the time there were several alternatives, each of which was usually considered more or less absolute, while, paradoxically as it may seem, their interfaces were everything but manifest.

Let me explain by citing an example. The history of the protocalculus was not a well-ordered process; nor was that of physics. Thomas Hobbes’ beautiful image of the development as a tree growing after the biological pattern *is altogether too orderly a picture to convey anything of the stress and strain, the conflict and the tumult which have been associated* *with every stage in its long growth to maturity* (Baron, 2). Furthermore, it cannot be claimed that things, *anything,* happened in one go. Scanning the recent literature on calculus-related topics, it becomes clear that we are faced with complex and hazy systems in now continuous, now discontinuous and *non-linear* motion.

Furthermore, the relevant notions as we know them, with mathematics at the core, did not exist. In those days, physics and cosmology belonged to philosophy, whereas mathematics was science properly, less affected by the post-Copernican conflict and less suspect in the eyes of the Church. Characteristically, in 1550 the Dominican Domingo de Soto was criticized for relying on mathematics instead of natural science (1.9).

A statement attributed to Kepler seems to give a contemporary expression for the unstable boundaries between the sciences. With his ellipse proposition *he had overthrown for all time the two-thousand-year-old axiom, according to which every motion retrograde in itself must of necessity be a uniform circular motion... When he was reproached for having a passion for innovation because he wanted to mix together such heterogeneous sciences as astronomy and physics, he explained "I believe that both sciences are so closely bound with one another that neither can achieve perfection without the other"* (Caspar, 135).

Nor was the divide between the more or less professional scientists and men of the Church, as we have seen, definitely sharp (2.2.10). Not only were many of the former in fact priests or belonged to some order (Copernicus, Kepler, Bruno, as well as Cavalieri and Mersenne), but some men held positions within the Church expressly to forward science. Clavius used his position to advance and establish the role and place on the curriculum of mathematics at the Collegio Romano; and there were numerous similar cases all over scientific Europe. On top of all this, we have the large *turba peripatetica* (as Vincenzo Viviani sarcastically calls them in his biography of Galilei), the university people, many of whom belonged to the Church, that clung to Aristotelian doctrines, at least felt they had to make a show of it.

2.4.3 *Do we need a Scientific Revolution?*

We now consider the tendency to apply the concept of *revolution* to express and, in fact, interpret, the *character and the achievements of seventeenth-century science as such.* Thus, according to Boyer (82.f),

> *there is a widespread belief that the science of dynamics... was almost entirely the product of the genius of Galileo, who "had to create... for us" the entirely new notion. That such a view is a gross misconception will be clear to anyone who makes even a cursory examination of the fourteenth-century doctrine of the latitude of forms.*

And Boyer follows up with specific references.

Speaking of *revolution of science* calls for elucidation of two terms: "science" and "revolution". Whereas we can elaborate things in the context of the first of these terms, since we more or less agree about the contents of the subject, we are less well equipped with regard to the second. We can always define and redefine the term *revolution* to fit our needs; but what is the use? It comes as no surprise that scepticism about the *analytical usefulness* of the notion of *revolution* has been voiced in many quarters. Concerning the *law of fall*, that the distances from rest are as the squares of the elapsed time, Drake notes that

... the law of fall was still in dispute among scholars in 1650. The notion that it produced a sudden shift of attention among 17th-century natural philosophers, away from causal inquiry and toward measurements and mathematics, is more a product of our customary terminology than a demonstrable truth... When the old theories and rules are studied in [chronological] order, they show a certain pattern of recurrence at each epoch in the history of theories of fall. In turn, some events of the 17th century then appear less surprising, and less revolutionary in the usual sense, than they now seem to have been, by becoming more easily understandable in this chronological framework (Drake, 1989, 3f.).

Gerald Holton has a most pertinent comment on the subject concerning the issue of "revolutions" generally (and Einstein specifically).

If one has studied the development of scientific theories, one notes here a familiar theme: the so-called revolution which Einstein is commonly said to have introduced into the physics in 1905 turns out to be at bottom an effort to return to classical purity. Not only is this a key to a new evaluation of Einstein's contribution, it indicates a fairly general characteristic of great scientific "revolutions",.. (Holton, 1988, 195).

A "real revolution" may have come with the break with classical physics (which ruled the game, roughly, from Newton to Einstein) and the emergence of quantum theory. But here we witness new factual and theoretical discoveries that evoked and led to further elaboration of old themes in philosophy. Whether Holton is "right" in his views on the *thematic origins of scientific thought* or not, is a futile question; it its usefulness and applicability to analysis that counts. He defines (Holton, 1988, 12ff.)

the contingent plane as the plane in which a scientific concept or a scientific proposition has both empirical and analytical relevance. Contingency analysis is the study of the relevance of concepts and propositions in the x- and y-dimensions. It is a term equivalent to operational analysis in its widest sense. If, he continues, *we want to make it* [the history and philosophy of science] *part of our business to understand how new discoveries are made and how scientific ideas meet with acceptance or rejection, it is necessary at this point... to define a third dimension, or z-axis, perpendicular to the x- and y-axes of the contingent plane. It is the dimension of themata, of those fundamental preconceptions of a stable and widely diffused kind that are not resolvable into or derivable from observation and analytic ratiocination. They are often found in the initial or continuing motivation of the scientist's actual work, and also in the end product to which his work reaches out* (see also 3.2).

In Thomas Kuhn's work, the idea of revolution is premised on the idea of paradigm shift. A number of writers have found that the *shift of paradigm* imagined by Kuhn is unclear, at the best too sweeping. The term *paradigm,* at least in some of the twenty-one meanings Kuhn, somewhat inattentively, attributed to it (Masterman, in Lakatos-Musgrave, 61;*see* Gillies, 270) is operationally a catchword and ideologically burdened by some ambiguous components. There is also an articulate and generally harsh - though well-mannered! - criticism also in Feyerabend's and Lakatos' closely interrelated contributions in the same Lakatos-Musgrave volume.

Lattis, in his book on Clavius, is severe when it comes to Kuhn's dramatization of the passage from Ptolemaic to Copernican astronomy by describing it, historically speaking, as a question of *logical coherence* (Lattis, 62f.). A balanced synthesis of Kuhn's view in Ben-David, 5f.; but this author, with his roots in sociology, concludes that a careful examination of Kuhn's ideas in a social perspective, *implies that paradigmatic behavior is a limiting state, which scientific communities tend to approach but which they never actually attain.*

So we see that Kuhn's notions of *paradigm* and *revolution* have been criticized in the contributions to science theory just referred to, and so it has, in the context of mathematics, by Philip Kitcher. The latter's evaluation is highly articulate, structured and clear, especially with regard to the concept of paradigm, *which is as suggestive as it is unclear* (Kitcher, 162ff.). Kitcher himself wishes *to salvage the notion of practice and jettison the concept of paradigm which Kuhn generates from it,* and focus on the former. On this basis he builds a model that seems much more workable. I could not have done him justice merely by giving a brief summary, so I refer the reader directly to his work.

With the proviso that I may have misunderstood Kitcher on the general subject of *mathematics,* it seems to be in line with the general drive of the present book, to wish to descend a level or two, from mathematics generally, when discussing relevant *models.* No one ever did *mathematics*. They did calculus, or even better, yet another level down, they did *limit theory, philosophy behind the fundamental theorem of the calculus,* or *Fermat's tangent solutions.* It is with such issues as these that a *bottom-up* model-building must start. Top-downing from *Science* or *Mathematics tout court* does not seem to land us on a firm and reliable ground*;* or, better, on a workable platform.

For me, then, the upshot is that I find, not being the first to do so, the concept of *revolution of science* working badly; *in science,* somewhat better. For it really means attributing one thing to another one thing, both of which are flawed by being too comprehensive and unstable, historically speaking. It probably was in a reaction against such an implicit negation of the *complexity and plurality of things* that made Feyerabend write his stimulating book, *Wider den Methodenzwang* (English edition with the title *Against method*).

In order to ask *Was there a revolution*, at least one must be able to give a starting date for it and a location for it (for the latter aspect, see 2.4). But no start or end terminus, however approximate, can be working under the circumstances we are to do with here, nor indeed analytically adequate or defensible, even on abstract model levels. For usually ideas in someone's head and only fragmentarily recorded on paper, precede its documented codification in verbal or numerical formalism, and there are too many *someones* for us to keep a reliable book-keeping.

Instead of asking *What was and what is science?* we might rest contented with asking: *What was or is being done and what could they do*? Less heroic but more constructive. Clericuzio, commenting upon the sociological part-explanation of the so-called *new science* in the seventeenth century (in the studies by authors like Shapin and Schaffer), makes two important observations; here is how I understand him.

First, he claims that the new sociology of science simplifies too much in holding the view, that whatever new there was, represented a unity of well-defined insights and practices; *secondly*, that in the theory, developments underlying this hypothetical body of knowledge and paradigms, or supporting it, the cognitive interactions and the ideas in philosophy and religion that went along with the social ones, were of such a complexity that one cannot, with claims of generality, set up any clear order of priority among the factors (Clericuzio, 18f.).

We can create meaningful networks of interrelated and interacting factors and stipulate for them some *typical* direction of drive or motion. But there are surely alternative networks also, that may be applicable to the same documentary material, and the one or the specific ones we have chosen, is *our creation*. Such a *human* basis for our models have since long been recognized, even in physics.

In applying this understanding of research, does it give good meaning to speak of scientific r*evolution*? How *do* we speak of revolution?

There are two steps here, one regarding *events*, the other, *actions*; two manners of asking. *Was there or was there not a Revolution? Did someone perform a revolutionary action?*

The second question has been discussed already; it is really a matter of taste, for evaluating some single action, we have to link it up with a larger picture, and then we are back where we started. The first question, *Was there a revolution?* dumps us in a quagmire. How many more like Galilei (to stay with this *example)* does it take to make a development worthy of such a comprehensive denomination? Is there an upper or nether limit? As a minimal requirement for the term to be meaningful, at least usable, there must have been some sort of concerted action with some sort of common goal or focus *and* the action must bring about some changes that provoked some amount of further changes *and* these changes must have been of importance according to some systems-external criterion. No one can seriously claim that this was what happened in seventeenth-century physics, cosmology or mathematics. Nevertheless it has been asked: *Are there revolutions in mathematics?* (the title of a contribution in Gillies).

2.4.4 *Working conditions*

Nothing is lost in clarity if we say that *specific actions or programs* of Galilei were revolutionary in the context of contemporary *university* scholarship. There is, however, another cluster of activities to take into account here: craftsmens' and builders' practice and technologies.

When Galilei started observing physical objects (mountains on the moon, sun-spots, satelites for Jupiter) or experimenting with them (objects in free fall or moving down inclined planes), this is usually evaluated in *contrast* to the Aristotelian *Peripatetici* (and other scholars), who preferred to read books, primarily by Aristotle, to find the answers there.

The apparent contrast is not altogether clear, because it only seems to work diagonally. Throwing things around or climbing up somewhere to drop them, did not belong to the job or suit the dignity of university professors. Considerations of prestige and reputation easily will crowd out rational scientific engagement. In order to secure your position and your science and uphold your professional name, you will normally do what you are paid for and what your scholarly tradition implies, not wanting to be reputed a frivolous scholar. And you want to stay where your profession and prestige put you, for you earn more money and other advantages by seeming reliable and solid; when you say Yes, then so it is (in Europe, it took the revolution [!] of 1968/69 to change some of this; but now we seem to be back again). From modern times we know that the passage from good to bad science is often a seamless one; and we know about political corruption (Nobel Prize in physics, Philipp Lenard's *Deutsche Physik* - or "Aryan physics" - opposed to the "Jewish physics" of Einstein & Co; the Nazi "studies" of race and ethnic origins; for which see Heather Pringle's frightening book, *The Master Plan*).

Galilei broke with the tradition of scholars (even to the extent of removing his academic cloak to feel free to visit the brothels) - but only in order to take up another one equally old and "historical" but less highbrow tradition. He put what craftsmen, such as surveyors, builders, carpenters, smiths and so on, had always done, on a more elaborate systems level, analyzing the material in a way that his forerunners did not need. Observing and measuring natural conditions and recording them verbally and graphically, this surveyors had done for some millennia. Observing the universe and drawing conclusions, however silly, from what they saw, astrologers had done also for some millennia. For his part, *Galileo did not use algebra and he never wrote an equation in his life, not even in his private papers* (Drake, *see* 2.2.3). As far as I can see, his mathematics, as they emerge from Drake's studies, do not seem *in principle* different from the computations on which sophisticated traditional craftsmen used to rely. Let me canvass this issue a little further, risking some repetitions, for it seems to me crucial for a grasp of science history in the seventeenth century.

Free from university rituals and verbiage, Galilei introduced the *craftsman's approach into scholarship*. But this link has hardly been noticed because his discoveries were so sensational and because he developed theories, a thing craftsmen had never found any motivation or salary to try. Experimentation was not new, but it was practiced under different circumstances; normally by craftsmen, to say nothing about the so-called cathedral builders. Some of the mathematical arguments, uncolored by metaphysics, developing around the middle of the fourteenth century and in the works of Niccolò Tartaglia in the 15540-50s, are obviously based on experimentation and they are closely comparable to the achievements around 1600, provided the different technical conditions are taken into consideration (Drake, 1989, 17, 19, 26f.). Galilei's work as I have understood it, was the same in procedural principles and practice as those that had been always typical of *traditional craftsmen*. He worked in different environments from theirs - and just this craftsman factor caused him difficulties with the authorities and university people.

The working conditions began to change with some personalities like Galilei, who liberated himself from the closed world of the universities, supported in this by his being called to work for statesmen. In politics and diplomacy, results you can *see and show* are more useful than written theory. The Medici Stars at the Tuscan court became cosmic symbols of archducal splendor and universality. The *cannocchiale* that impressed the Venetian Senators with its power to scan the lagoon and make it possible to oppose the enemy in time (like radar in the Battle of Britain), could conveniently be credited with lending credibility to the Republic's professed power and modernity in the face of its too well known maritime reverse and economic and political decline.

Yet we cannot get around historical facts, that Galilei for instance, did something new; but we may discuss the context in which to investigate this innovation: linear foreground-background history *or* integrated systems in interaction related to some denominator. Rossi gives a list in five points concerning the differences between "medieval"and "modern" science (*see* also 3.3) (Rossi, 1997, XVIf.).

2.4.5 *Organizational issues*

Science does not work in a void, we know, and organizational issues will condition the structures and the theory implementation.

Hoping to alert the reader to the dynamics of the scientific interrelation patterns around 1600, I used Galilei's fictional observations on Rembrandt's *Night Watch* as a model. Examining approximately contemporary mathematics, Baron uses, rather re-uses, the picture of a musical symphony:

> *a number of movements of varying degree of complexity are successively developed: at each stage we have the introduction of a theme, sometimes by a single instrument, the development and steady unfolding of significant statements, a process of recapitulation and consolidation and finally integration of the whole with the central "underlying mood" which subsists throughout* (Baron, 2).

Herbert Simon's experiences from Chicago provides another model that can to some extent be applicable also to historical contexts. He writes in his autobiography, *Models of my life*, about what Chicago University taught him about *the march of ideas, and how the interplay of scientific research with the social organization of the disciplines determines its direction and pace. It helped me understand that new ideas do not fly solely on their own wings; the scientist is a communicator as well as a discoverer - sometimes even a missionary.*

Pretended attributions to *science* or sections of it presupposes ideas about *where to place it,* in physics, mathematics or, preferably, in more specific operational procedures and models. Thus Fischer claims that the quantum idea of *uncertainty* (Heisenberg) was in reality revolutionary, philosophically rather than physically (*Die Unbestimmtheit war tatsächlich besonders revolutionär, und zwar mehr aus philosophischen als aus physikalischen Gründen*) (Fischer, 2002, 91; for Max Plankc's fundamental role, see Fischer, 2007). But *place* is also an *organizational* issue.

The most adequate reality model for a description of general history of science in the late sixteenth and first half of the seventeenth century that I can think of, is the history of the development of the nuclear project in Nazi Germany, with the common but not always stated aim of constructing a bomb (not a centralized bomb program like the Manhattan Project in the US). The complex story is admirably recounted by Rainer Karlsch in his book *Hitlers Bombe*. A number of institutions and teams of scientists worked partly independently of one another, and partly in competition or even conflict, on roughly the same topic (mastering U235 technology), but with variously conceived goals, some of them explicitly that of creating a nuclear bomb in order to knock out the Allies, others, more vaguely, developing a so-called *Uranmaschine*; even these things, however, not uniformly defined. The very *model-nation of organizational perfection*, centrally and hierarchically controlled by a strong dictatorship, did not manage to organize this vital effort efficiently; which the loosely-constituted "democracy" of the US did successfully (assisted by many Europeans). The Germans also had to face continuous resource problems they had not counted in from the outset, surely a flaw in their planning, a major one concerning the supply of heavy water (D_2O),. This was produced in sufficient quantity only in Norway, but destroyed by the Norwegian resistance, assisted by British aircraft, when the Germans tried to fetch it home.

At the same time, even those who insisted on the *Grundlagenforschung* character of these efforts (Werner Heisenberg among them; who seems to have tried to delay the entire program), were well aware of the war potential. At any rate, in October 1944, a medium-sized bomb was successfully tested at Peenemünde on the Rügen and in early 1945 another one at Ohrdruf, both tried out on some prisoners. The entire story can be described as a set of partly interactive networks mainly aimed at one overarching goal, concerned with a group of interrelated topics but with disparate subgoals, from basic physics to practical technologies. If a contrasting rational and systemic *model of science planning* is needed, Ramon, 94f. offers the best possible example.

2.4.6 *Protagonists*

There seems to be at least three sets of parameters to account for in construing a model of research organization (really an enormous subject, here pared to the bones).

Firstly, do we consider if something new was *done?* Galilei *measured* with a high degree of exactitude, as craftsmen and builders had always done, and like them he scoffed at the notion that measuring had anything to do with philosophy (Bohr and Heisenberg would not like that). Descartes performed what must be regarded as experiments in geometry. Or do we consider also, or rather, that new feelings, conceptions, *ideas* were around? Each alternative, as long as we want to treat them separately, subsumes a set of factors. Of course they interact, but it may be preferable to let that wait till we have considered the next set.

The *second* set regards *observation.* There is scale here, going from left to right (one might say). Let it start out with pure *retinal vision*, and. corresponding to this, *measurement*; then develop through a series of differently conditioned and mentally underpinned and directed *reflective observations* and *experiments;* and further, *explanations* or *systems evaluations* and *equations*; and at the right end of the scale, we have *theory and hypothesis* (or even *belief*).

Thirdly, where to locate, for us to understand, Galilei's or Descartes' *thinking* (presuming this was *one* kind of process) in the framework? We are back again to the so-called "genuine understanding", a quagmire.

What did the people involved in contemporary science *really mean*? I would never have asked such a question, were it not for the fact that Daniel Garber in his recent and important book on Descartes does so. We must *reach a genuine understanding of what he thinks*, we are told (Garber, 2001, 5). I do not believe that such a thing ever existed, except on trivial levels. The protagonist himself may have thought that he had arrived, after having struggled around in a network of alternatives and different motivations, at a good approximation to what, in most cases, was only incompletely expressible in words and only imperfectly in equations. I wonder how even the most articulate modern historical appraisal of the traditional kind could improve on that. For the originator of an idea often fails to see the implications and surrounding context pattern of such a thing and also its flaws and its consequences, finding at a later state that he meant something, in the best of cases, a nuance or two removed. Most philosophers have been exposed to experiences such as these.

Let us take a look at some other cutting-edge cases of innovation, or "modernity" studied in the literature.

One case is when Descartes, to examine complex curves in his *Géometrie*, uses the method, which we would recognize as “modern”, of investigating intricate problems on a very *simple model*, in his case the ordinary parabola. This was a technical device with no philosophy necessarily attached to it. Does it say anything about his thinking processes around it? What kind of mechanisms and ideas might lie behind or at the base of even simple devices? - Gregory gives a rich picture of the complexities involved (*see* also 4.3.4 for *emergent properties*). Giordano Bruno's distinction between different realities and between models and the reality they refer to (Blum, 48f.), is another case. “Modern” thinking penetrated even the lion's den. *Instrumentalism* was a way of escape from conflict areas (Sharratt, 118f.); even Bellarmino had recourse to that (2.2.11). Next, we can note how Torricelli in his attempts to handle *approximation*, came very close to handling *limits*, while his method remained purely geometrical. But he didn't fully understand the issue, we are told (Boyer); which seems to mean he did not see the all the implications and consequences; but who ever did?

Kepler collected observations and used borrowed information (from Tycho Brahe), but this happened with reference to notions of cosmological harmonics, in part geometrical, in part religious (Kepler, with Hawkins' comments; and Holton, 1988, 18f., 51 - 74). He, as well as Copernicus, about whom something similar may be said, had recourse to Aristotle whenever necessary (*natural* place and properties of things, etc.). While Galilei sought the truth in the world around him, including God in the picture without operational reliance on him, Descartes sought it inside himself, and with appeal directly to God in support of his thinking (rather than to the Church, which had educated him). His objects were *grosso modo* the same as Galilei's, but the mathematical thought experiment distinguished him from the Italian. Mathematics was more of a goal in itself for him, while Galilei looked upon it both as a question of the laws of nature and as a tool. Fermat, not being himself a mathematician by profession, was still the one who more than the biggest lights conducted his mathematical studies in a professional manner, attacking crucial issues such as the tangent problem or the confines of algebra without any declared philosophical underpinnings or links. Giordano Bruno, we have seen, was a pure theoretician (as was Campanella), and, while expressing views on mathematics, substituted a picture logic (Hilary Gatti) for it. His scope was a grand philosophy of the universe, rejecting the machine image, powerfully involving infinity, something that might be compared to what Immanuel Kant had in mind with his *Theorie des Himmels* (Geier, 62ff.). Bruno broke both with the tradition of the Church (and her reading of Scripture) and with Aristotle, more decisively than the persons just mentioned (and had to suffer for it). He was *out of step* with current development, we are told. But this precisely seems to me to mean that he was actively inside the same development (there is a Catch 22 here).

How much these people *understood* cannot be known, because *understanding* and *knowledge*, a vast recent literature teaches us, are very complex mechanisms operating at a number of levels. For example, is to see a thing and categorize it and use it, sufficient, or does understanding require the formalism of equations? Galilei himself realized the importance of *approximation* and was fond of using the adverb *almost to cover the gap between the observed and the ideal case* (of course in Italian or Latin), as already noted. *The nature of continuous quantities*, a notion that was to emerge as fundamental to *calculus,* also occupied him (Drake, 1957, 63). Continuity is visualizable in image format (you *see* an unbroken line); but if one requires formalism as a condition for real understanding, then the closest one could come was to symbolize with *discrete* numbers; the conundrum artificial intelligence seems to have to live with.

Reaching some understanding of what today we understand as a *limit* or what we call *The Fundamental Theorem of the Calculus,* for example, occurred stepwise and not along a continuous line, rather in bits and pieces and coming to the fore in connection with different people's grappling with very different problems. Concerning the operator *limit,* a suggestion offered by D'Alembert in 1754, deriving from seventeenth-century ideas, was turned into a workable theory only in 1821 by Augustin-Louis Cauchy.

It would be difficult to find in these protagonists any dominant, goal-directed and unified action of the kind we know from political upheaval, for which the term *revolution* does seem useful and well communicating.

2.4.7 *Convergence*

Even on the level of simple language usage, *revolution in science* is inadequate, since in all other contexts the qualifying noun has never been clearly defined (how could it?) and because it is too heavily loaded with drama and violence, meaning *upheaval, Umwälzung, capovolgimento.* What we witness in the field presently under observation, is a great amount of more or less interacting or interrelated ventures with a few cases of methodological jumps in-between. The world has always witnessed such jumps. Nevertheless. the world may very well have moved towards some measure of *convergence.*

Rather than trying to squeeze historical material into a preconceived notion, I prefer a discussion of *thoughts concerning subjects that at one time and place or another were considered as being relevant to explorative theory or practice, again as the two terms were accepted at the given time and place.* I believe that this configuration approximately covers what is at the bottom of arguments such as those in Gerald Holton's *Thematic origins of scientific thought.* In the *Preface* to his later *Science and anti-science* he declares his aim *to understand these words* [such as proper ends, legitimation of good science] *and actions not in the abstract, but in the natural setting of specific historical cases*. The force of his argumentation seems to stem in part from the fact that he distils "philosophy" directly from real, even laboratory, cases and histories, rather than starting in some more or less celestial sphere of academic ideas. A physicist who has been recorded as "doing philosophy" by practicing definite ideological precepts and codifying them in formalisms (in his case as matrices), was Werner Heisenberg (Fischer, 2002, 86; later of course he did write philosophy).

It is a characteristic of highly complex dynamic systems like the processes just consider, that *chaos is productive*. The sharp increase in the number of different or alternative ways of doing some specific task, the conditions under which they occur, and of arranging items in specific configurations; in other words, a *combinatorial increment*, even *explosion,* will almost automatically actualize and render attractive new and partly untried programs. In other words, along the way some *cutting-edge* insights and programs may arise.

A concept indicating more precisely what took place in physics, cosmology and mathematics in the first half of the seventeenth century would be *convergence; a gradual, not always linear, gathering together of needs, interests, inclinations, ideas, discoveries and opportunities which almost randomly was channelled towards a complex of methods, notions and techniques that became gradually, but by no means linearly, sufficiently compact to resemble what today we have in mind when speaking of a relatively coherent scenario in science.* The system in which this convergence took place, was (and will always be) such a complex pattern of technical, sociological, economical and political (even religious) elements, that any attempt to come up with a readable and workable account would have to end up with much over-simplified models.

As our focus we may select the central issues of calculus - as I am going to do, but let there be no illusion that the historical process *converged* uniformly toward one focus; rather toward a complex of foci that were intertwined and also interacted on various levels. And yet for model purposes we must stipulate a common target for this convergence. One has, after all, to talk about this thing and must make some decisions about essential features.

I would substitute a picture of *scientific convergence* for *scientific revolution,* attributing, however, only sectionwise validity to it. Let us say that, substantively speaking, the main strand in the scientific convergence consisted in measurable observations of physical entities in the world and in the cosmos, experimental (factual and argumentative) testing of them, and some degree of mathematical formalization of the results, with some parameters of prediction as the scope; all of it with the drive, or convergence, towards a goal-set that was common for most of the active protagonists, namely that of *measuring the world* - which, however, meant a lot of different specific things. On different levels, with different backing and competence and different ideologies (from rationality to a mixture of this and faith), the goal was to establish methods for the intellectual control of physical realities.

The outcome of this was some idea of the world as a machine running its course under the laws of nature. The title of Clericuzio's book, *La macchina del mondo*, is appropriate.

2.4.8 *Charting science revolution*

Let me close this *Chapter* with a matrix of the average pattern of application of the name *revolution* with some comments appended.

1. The name *revolution* is heavily loaded with the idea of dramatic and goal-driven actions arising with a certain suddenness; so, if we can find a better term, then we should use that; provided there is any need for it.
2. The implication is commonly to pick out from an historical context *certain specific features* and take the rest as the background. This is in violence of the systems idea.
3. Thereby we are involved in chains of *cause and effect*, not indeed necessarily but this is what mostly happens. Also in violence of the systems idea (avoiding infinite regress).
4. This mode of arguing implies attributing actions or notions to historical protagonists in terms of *what happened in the aftermath*. But actions do not point forward, they take up resources from the past, making them contemporary. A development or time-dimensioned process involving the *future* makes sense only in terms of systems change and prediction, and in this model, the notion of *revolution* is no longer working in the sense usually attributed to it.
5. Using a notion in analysis, description and communication that involves an *either-or* mechanism leads one to discuss whether the term applies or not, rather than examining a system under an integrated perspective.

2.5 What to do next?

Some material has been discussed in this *Part* and *Part* I, philosophical, artistic and architectural cases, especially the Spire, that can be related to developments in contemporary mathematics and science in general. Parallel with these critical processes, there are the religious ones managed primarily by the Roman Church. Next we need to prepare the Spire construct for more in-depth investigation. Before doing so, a closer consideration of three areas may further support our principal assignment, *form and shape description*: *one,* helix thematics related to contemporary mathematics; *two* planning and production problems, and, *three*, some resources available in recent literature.

33 PART III THE SPIRE IN PLACE

In the present *Part* III, some of the observations in *Part* I will be reiterated in more articulate contexts.

The expression, *the Spire in place,* denotes its position, not on the building, but, analytically speaking, in the scenarios described in *Parts* I and II. There are two main groups of relationships. *One* concerns the Spire *helix* and contemporary *mathematics;* let me say, *the metric connection.* The *second* linkage, a configurative one, regards relations holding between the cited issue and the historical, non-quantifiable, features sketched out in *Part* II. In this model, the ideas of *change, variability* and *approximation* has been, in the present book, considered as representing prominent, if not dominating, tendencies and properties that may have supported operative tools in the protocalculus development conditioned in terms of a denominator designated as *interrelated changing entities*. The only available tool for handling the *configurative connection*, is to abstract out some probably critical tendencies attestable in highly complex networks of relationships.

In the present *Part* III, the main issue is the relations probably holding between the Spire helix and contemporary mathematics.

With the Spire in focus, when I bring in seventeenth-century involvement with mathematics, I am going to concentrate on *curves*, rather than surfaces and manifolds, since they are directly relevant. That is why I will look at the early stages of the infinitesimal calculus from the vantage point of *differentiation* rather than *integration,* without excluding this from the purview. I am aware that this strategy is somewhat skewed in a historical perspective, that the two branches dovetail or integrate on numerous points and levels, are, in fact, in an inverse relation to one another (3.5.10. and 5.2.11). Still, the chosen strategy may prove to be adequate for an exercise in methodology.

3.1 Idea generation, planning process and networks

The emergence and development of the Spire idea, its planning and implementation can hardly have occurred in a vacuum. To repeat myself, there must have been talk all around at the Sapienza, the normal community grapevine taking care of that.

In what kind of framework did the unprecedented and for many probably provocative idea arise of placing a *conical helix on the university church*? As already noted, some kind of *guglia* or spire, thinning out upwards in the normal manner of church spires, was *technically* required, for the particular position (1.10.1). The idea would have to be accepted and defended also on the ideological and message-political level. So we can concentrate on the specific helical shape.

For a case of such an obviously complex and multi-levelled planning and implementation, it is indispensable to adopt some kind of *systems view*, inspired, for example, by classical accounts such as those by Von Bertalanffy and West Churchman, with some insights gleaned from Anthony Wilden's *System and structure*, second edition 1980; without pretending to argue with such formal concepts as demanded by Herbert Simon's systems requirements: *that a formal concept of causal ordering among variables in a system could be constructed, and that the causal ordering was uniquely defined when the system was fully identified* (summary in Simon's *Autobiography*).

I want to have in mind the contingency, not to say the likelihood, that the actual groups involved were anything but homogeneous. Everyone with some experience from inter-group handling of even apparently very simple tasks, will have been alerted to the entangled workings of the organizations and, indeed, the human mind when the process carries with it some prestige interest.

In their *Social identifications*, Hogg and Abrams note that their

emphasis is on the fusing of cognitive and social (sometimes societal) processes in accounting for the relationship between individual psychological functioning and the social nature of identity, behaviour, and cognition. Its application is broad: behaviour both within and between groups is explicable largely in terms of the social identifications of their individual members... The interrelationships between a diverse range of hitherto unintegrated topics, from discrimination, prejudice, stereotyping, attraction, groupthink, social facilities and self-preservation to conformity, collective behaviour, language and communication, ca be understood within the framework of social identity perspective (Hogg and Abrams, 208).

In public enterprises (directed by Church, State, municipality, etc.), a project was normally carried through by special outfits, often more than one and usually acting in stages through idea conception, planning and implementation; but, as Jakobsen notes (*see* below), often not clearly and sequentially. These planning bodies normally numbered all or several among the following categories: local authorities, direct users of the projective work, specialists called in for consultation, and also normally the artist or architect, all of which *together* will henceforth, as noted already, be labelled simply *the Planners.*

They would have to answer for their doings to some superior authority, such as a king, a duke, the doge in Venice, and in the Church, the pope or a bishop, through the agency of some local authority such as the head of the clergy at the actual site or the family head in the case of a private chapel commission (some references in Gisolfi and SL, 74f., in part based on documentation in SL, 1974). For public programs, experts were called in (*see* also 1.2.1). We have a typical example in Paolo Veronese's ceiling decorations in the Sala del Collgio in the Doge's Palace, Venice. The program is explained in a series of inscriptions that can only have been formulated by an expert in central themes at the Council of Trent or the professional debates surrounding it (SL, 1990, 36 - 41).

Documentation has relatively meagre informative value, unless there are a day-by-day records as often in modern times. Written contracts or dated and documented drawings do not tell us very much either about the planning process or about the specific influence on the final result by the different participants, including the artist or architect. For we seldom are told *at what stage* and after how many rounds of consultations and with or between whom the contract was stipulated and the drawings circulated. Usually we do not know if such a process had been reiterated and how many times. Contracts are often final, conclusive things, at the end of processes that remain unknown to us; but they can equally well have been provisional stipulations at some point in a long non-linear process.

Nor should written statements by artists or architects about their work be taken at face value and be read directly as they come, the philologist way, without being integrated in the larger functional context (SL, 1965, 1975). They may express constructive thinking, marketing interests (as in most architectural treatises), justification for peculiar products or for stylistic attitudes and choices, reflect contemporary fashion, or they may simply serve to promoting the status of the writer. Occasionally they reflect specific and problematic conditions in the working scenario (as when Alberti complained about the disorder in the liturgical practice in his time). Examining them means at best to concern oneself with an indirect and fragmentary aspect of the architecture itself.

Experience from modern administration studies teaches us, Herbert Simon has shown, that people tend to only a *bounded rationality* and are prone to *identify with subgoals* (Herbert Simon: the principal wisdom of his *Administrative behavior*). His rationality concept provoked other scholars to complex argumentation over the issues; March offering a useful survey of the reactions (March, 143 - 170). But I do not find it adequate to treat the issue as one concerning a complete theory, rather as a corrective to rationalist optimism. People, especially authorities, resent having to admit that their assessments of reality and their planning is anything but rational (my lesson from growing up in the mildly socialist country of Norway). While almost any chosen case seems to be managed according to Simon's criteria, to pick out cases that definitely do *not* involve "boundedness", is a hopeless task.

Planning was hardly much more rational, goal-determinated and consistent in seventeenth-century Italy than in modern times. Experiences from modern design processes can at least indicate what may have happened and aspects to which we should want to have but do not have access. Jakobsen (41 - 53), speaking of *recursive* product development, concludes in the following terms (his "five steps" not to be taken literally in our context):

The process of product development is a recursive process in the sense that a five-step procedure calls upon itself as the different levels of the product to be designed are uncovered as "chinese boxes". Thus the functional tree cannot be completely specified at the beginning of the product development process, but must be developed gradually as each level is uncovered, and the process of determination of functional requirements (or rather requirement specifications) and of creative thinking (solution generation) are progressing stepwise parallel as a dualism, rather than as a two-step sequence. (Jakobsen, 53).

Perhaps the decision procedure concerning Borromini's Spire was much simpler than assumed here. But a responsible analysis, respecting the rule of *maximation* (4.1, 4.7), must take into consideration such complexities as are, indeed, quite normal in public enterprises.

It can never be taken for granted that the idea was first conceived by the architect or artist, though in the case of the Spire this would be a reasonable conjecture on account of the characteristics of Borromini's work. But others, too, were familiar with the architect's inclinations and capacities and might very well have come up with the new idea, knowing he would respond adequately to it. Even so, the discrepancy we have noted between the extant drawings and the completed construction (1.9) does suggest the usual participation of other people than just the architect.

It happened that the process went haywire, as in the case of Borromini's modernization of San Giovanni, but that did not necessarily mean that the practitioner had been able to act more freely than usual; the blame seems often to have to be placed high up in the hierarchy.

It is hardly likely that an explicit mathematical motivation for Borromini's Spire was included in the commission to the Planners. In the context of a papal university church this might have been an attractive notion, but hardly sufficient to overpower the fundamental ecclesiological issues. Scientific connotations may simply have arisen as a strikingly adequate by-product *or* by the normal mechanism of attribution in the way of a so-called *imputed significance* (SL, 1984, 36, 58). The thing is up there, and you attach whatever associations you want to it; one or two of them gaining preponderance in public communication and interpretations and turning out in the long run to stick in the public mind and challenge math-oriented people. The point is that the Planners would have been aware of what sort of associations the project would be likely to call up. Let me dwell a little further upon the likely intricacies in order to warn against positivist claims founded upon the few hard data at our disposal.

It is not so sure it *was a The idea*, for a cluster of associations can have emerged, again as by-products of the need for some sort of crowning *guglia* carrying the cross, perhaps because of some specific notions or considerations that seemed relevant to someone in the Planner group or outside it. Or the search may have started at the other end, looking out for some specific and suitable *content or message*.

In most cases, particularly in those that concern public display, some predicting will take place: how is this or that group among the public going to react to this? The Planners, as always in public cases, would have to evaluate in more or less specific terms the reactions of the *direct and indirect users*: the members of the Church and the Sapienza, and anyone living or working in the neighborhood or locals and foreigners passing by in the street and somehow having to come to terms with the spire and hopefully be impressed by it. If the Spire shall work well as a company logo for the papal university, how do we want the public to take it (*normative prediction* or *prevision*) and how do we believe they *will* react (*explorative prediction* or *prevision*)? In the planning and implementation process, the following question will be continuously present: how will people be likely to evaluate the Spire, and *who are these people*? This is all a question of a complex situation that we today can only come to terms with in abstract modalities, such as competence distribution (*Introduction*, -*a*).

The Church, whose chief client was and is the people making up the congregation, would have to take the factor *people* seriously. Large sections of the liturgical texts, at least back to the *Gelasianum* of the seventh to eighth century (*Liber sacramentorum*, in the *Bibliography*), are dedicated to the care for the *familia* of Christians.

Some people inside or outside of the congregations, that may have counted politically and economically (always the two main parameters for the Vatican, beside the religious one), would more or less expertly, and more or less dependently on professional suggestions, have connected the helix with things that were happening in *contemporary science and mathematics*. The entire issue depends on what sort of cues the Spire might *alert* in the authorities and the direct and indirect users. Often, it seems, it may take just some vague cues to alert one to chains of conceptual connections. Distribution of cues contributes to creating a potentially rich array of socially distributed terms of knowledge, notions and concepts, effected by people in the know or simply taken care of by the community grapevine. In addition to the themes familiar from conversance with the history of the Church and with general organization and management theory, much of the material presented in the publications of the *Institut für mittelalterliche Realienkunde* at Krems a. d. Donau, and several of the contributions in Frost, et al., *Reframing organizational culture*, deserve a closer inspection than is possible in the present publication.

The paths along which a visual or mental idea arose in an artist's mind is inscrutable for anyone including the artist; whether it comes about by the equally impenetrable mechanism called artistic imagination, by a more or less explicit assignment or commission, by being fired by some event, by experimentation (as for instance in the École de Paris),

I shall limit myself to a few rather elementary comments on the highly intricate subject of the *interaction of dynamical networks and systems behind design solutions*. The most evident consequence to be drawn from the vast but relatively incoherent available literature, is that so-called *creative achievements* cannot be evaluated linearily and without consideration of the process of idea generation and planning processes. How can *creativity* surface except through the actions it elicits? "Creativity" seems to be a useful term for bringing something into some specific network and change its constitution there. This "something" appears to consist in a *theme* in Gerald Holton's understanding of the term.

Basically, a network, whether, consists of a certain number of *nodes* representing things, abilities, concepts, experiences or whatever, and *connections* or *paths* between them that, with variable strength, weight (professional people speak of *weightages*) and intensity, represent linkages of reference, retrieval and interaction, often allowing a two-way movement (classical descriptions in Laumann-Pappi, 17ff., and Abler, *passim*, esp. 255f. - Connectionism: Bechtel and Abrahamsen, some notes on *connectionism* in SL, *Burden*; interesting controversy over *connectionism* in Smolensky, 202ff.).

Networks come in many shapes and functions, from the box models (excluded from the category by some theorists) in use in management, the social sciences, and so on, over the many types of grids typical of geography, area planning and brain research (neural nets), computer science and of "classical" writings on the subject of artificial intelligence (connectivity, for example), and, finally, to pure mathematical abstraction (*see* the publications with often widely different perspectives around a central core, listed in the *Bibliography:* Beam, Bechtel, Blair, Booch, Bouzghoub, Bratko, Coyne, Davis, Firesmith, FitzGerald, Ganter, Greenly, Hitchins, Hussey, Italiani, Kirsch, Olle, Parker, Scott, Silverman, Sowa, Wilden and Wilson). This long but selective list (from my own shelves) is all I can squeeze into the present text. Most of the contributions examine sections of the field here subsumed under the name of *creativity.*

The subject of *creativity* has been studied by Herbert A. Simon and Margaret Boden, for example, in the structured frameworks of computer science and artificial intelligence (Simon, 1969, 1979; Boden, 1990 and 1991). The accepted wisdom is that some amount of formalism is required. From this point of view, the issue of creativity to some extent seems to dovetail with *problem solving* as this kind of process has been studied by Simon, his many collaborators and others. Enlarging upon this theme would take us too far afield. I would recognize Simon's observations and models in his *Models of thought* I (*see* his *Index*) as a structural platform from which to go ahead (for instance, Simon, 1979, 144ff.).

Even a crude variety of a *network* to some extent takes care of the question of creativity at least in the initial phases of research. The artist or whoever "creates" something will do this within a network of resources distributed over the available workspace, by activating specific nodes and connections in it. The resources can be earlier works by someone or in some field; Borromini having features from Roman architecture somewhere in his *relevance grid*, technical and craftsmanship devices and methods, and in public tasks the requirements the work is meant to meet, issues related to the authorities and the direct and indirect users.

The moral is, I would say, that the mechanism of creativity (whatever it may turn out to be inside our as yet undiscovered mind, to adopt John Horgan's well-founded epithet), cannot be pinned down in a predominantly *biographical* perspective. It has to be treated as a network and integrated with the larger planning and implementation issues, premised not so much on biographical individuals as on *social individuals* analyzed in terms of *a multiplicity of selves constituted in communicative interaction* (references in SL, 1984, 171f., and *Burden*, 187 - 190).

The great advantage with *network models* is that, in a dynamical reconstructive process, they can be organized as structures with known but also hypothetical nodes and linkages as meaningful place markers.

Let me take the highly disputable, possible and, at the most indirect relationship between Borromini's helical project and Torricelli's approximately contemporary helix studies as a test case (details in 3.2.1). I will *assume* (to give myself a zero starting point) that there was no direct contact between the two, either personally or in terms of knowledge about the respective work programs.

A few of Evangelista Torricelli's works will be briefly recorded below (3.2.1), since he was closely connected with La Sapienza.

Networks, usually force things their way. Some people in authority will have been aware of both projects. There must have been some around of an observant turn. A new scenario thus arose. In relatively closed communities (even Rome as a whole was *small*) such scenarios will rise up to some prominence in the communicative patterns, kicking the matter up to a level for all to see.

Such a *network imposition* most likely will, if not directly influence the production (planning and implementation) process, at least it will highlight important features, let us say, the scientific and mathematical aspects of the relevant work or works, in this case by B. and T.; thus to enhance consciousness about the aspects in the local community. To suggest such a network role is not to "reduce the importance of the artist" (a fear voiced among people who prefer a belletristic simplification of the issue). On the contrary. Furthermore, especially if we extending the network effect to comprise also Borromini's other differential curves and manifolds, we can understand *one* reason why his contributional potential was appreciated and exploited. Yes, one. But his model of a network imposition can be applied to other aspects, too.

3.2 Thematics: a conceptual network

To reiterate a central tenet in the present book; there is nothing new in assuming that concepts and notions more or less in the focus of attention among groups of people, are subjected to being embedded into each other or to being linked up with one another across networks of varying formats. It should be sufficient to refer to information paradigms, the cognitive and social sciences and normal everyday experience.

The historical material examined in *Parts* I and II would seem to justify suggesting a *totality of variously interrelated factors* in what has been observed in philosophy, general culture, literature, music and design. In design, for example, contemporaries with a certain level of information and interest, hardly would evaluate Borromini's Spire without seeing it in connection with his other *differential curves and manifolds,* a comprehensive vision may have been reinforced in a similar direction by the works of Bernini and Gherardi. These tendencies, which manifested themselves over time, correspond, as far as I can see, to the mental operations on *thematics* such as those explored in Gerald Holton's *Thematic origins of scientific thought.*

Holton offers, and discusses far more deeply than will emerge from the present account, some examples of themes, such as *the efficiency of geometry and other branches of mathematics as explanatory tools, the conscious and unconscious preoccupation with symmetries;* and *evolution and devolution* in life cycles used as models in scientific research. He appends the following synopsis:

> *It is the interdisciplinary spread or sharing of such fundamental themata that produced something like a scientific imagination shared by all scientists, and making possible the interdisciplinary approach that characterizes so many of the new developments* (Holton, 1988, *Introduction; see* also above, 2.4.3).

Let me risk the suggestion that this model applies to a much wider field than the one contemplated by Holton.

3.2.1 *A thematic view of the helix: Borromini and Torricelli*

After the general overview we now shall extract features in the helical theme from the relevant networks. My point is that the helix, *including* the basic mathematical operations on it, formed part of a scientific theme more or less in Holton's sense of the term.

To this basic theme some adequately tuned people might, as I have insisted several times already, attach a more specific one, that of the *techniques, methods, apparatus and operations* that more or less openly characterized the incipient calculus, the *protocalculus,* or that, at least, belonged to a relatively coherent pattern of *geometrical and mathematical* topics; structures which were eventually pooled into one large scientific experience.

Staying with the helical form generally, we see that it was quite well known from constructions and images.

There are the helical staircases all over Europe, including the one in the Palazzo Barberino, Rome (by Borromini?); some among them "angular"; that is, with interconnected chords of equal length substituting the arcs subtending them (town hall in Rottweil). The construction idea was applied beyond the merely utilitarian scope, at least since the early fourteenth century. A miniature illustration accompanying the text *In dedicatione ecclesiae,* Santa Croce in Gerusalemme, Rome, *Codex D. c. 289,* shows Jacob's Dream (*Fig. 3.2 - 1*). The traditional ladder with twelve rungs, as described in earlier comments (Smaragdus and others: *see* Gisolfi and SL, 34f.), had been invested with meaning, including the significance of the number 12. In the present case, the angels are climbing up a *helical* ladder with unorthodox fourteen steps.

Representations of Dante Alighieri's Paradise were often conical-helical, and so is the Pharos at Alexandria in Heemskerck's etching, and in Pieter Brueghel's of Babel; there is also a painting of the same by an unidentified Flemish master in the Galleria Nazionale at Siena. The Gonzaga family in Mantua used a little conical helix as one of their emblems.

Fig. 3.2 - 1. Santa Croce in Gerusalemme, Rome, Cod. D. c. 289, Jacob's Dream

In the world of Galilei, *orologi solari*, sun clocks, were manufactured in the shape of a cylinder with a double helix engraved on the surface, one with constant torsion, the other with upwards reduction (1.9) (Miniati, *Strumenti scientifici toscani,* Cat. No. III, 56, dedicated to Francesco I de' Medici [1541 - 1587]; III, 56 on p. 54; no illustration; I saw it exhibited at the Palazzo Castellani, Museo di storia della scienza, Florence, in October 1987). Helical springs, screws and clock mechanisms had been in use for a long time (the Florentine museum has a large collection of gadgets and also a great number of relevant publications).

Some of the big lights in mathematics studied the cylindrical helix. Continuous fascination with spirals such as the Archimedes screw or Galilei's spiral (Baron, 151ff. Fig. 5.1), the fact that on a helix the torsion need not always be constant, and, thirdly, the tricky business of searching for and determining volumetrics in a cylindrical or a conical helix, all these factors seem to have contributed to a persistent interest in helical structures.

The *general helix*, meaning that its torsion is *variable* (constant or variable), did present a tricky problem of calculation, which became amenable to successful determination only with modern-age differential geometry (see below). Descartes, in fact, looked upon the spiral, the quadratrix *and the like*, including the cylindrical helix, as problematic, and, following Pappus and, ultimately, Clavius, with whom he quarrelled, dedicated a sizeable space to the theme (Mancosu, 1992, 94ff.).

Mancosu comments on *the idea that there is no proportion between curved and straight lines* [these are connected in the quadratrix], *or motion*, an idea going back to Aristotle; and, according to Boyer, the foundation of Descartes' distinction between geometrical and mechanical curves. In a letter to Mersenne, quoted in translation by Mancosu, Descartes refers to his excluding from geometrical curves of *the spiral, quadratruix and the like* (because, ultimately, the issue was connected with the unsolvable squaring of the circle: Mancosu). The crux of the matter here lies not in the cylinder around which one can rotate a helix but in the helical curve itself: *which*, says Descartes, *is a line that is not accepted in geometry any more than that which is called quadratrix.* The helix is excluded because *it is generated by a fillet (thread).* Galilei considered the *normal curve* (*linee semplici*) as the crucial element of a helix. Typically (it appears) he focused directly on the *linear curve*, while for Guidubaldo and Torricelli this became a by-product of manipulation of plane figures, that is, plane-geometrical ones. Galilei, speaking through Sagredo, in his *Dialogo sopra i due massimi sistemi del mondo* (*The two great systems*...), put the cylindrical helix among simple lines (in Drake's translation):

... simple lines, these being the straight and circular only,... nor do I care to quibble about the cylindrical helix, of which all parts are similar and therefore seems to belong among the simple lines.
(*... chiamando semplici quelli <movimenti> che si fanno per linee semplici, che tali sono la circolare e la retta solamente, lo ricevo quietamente, né mi curo di sottilizargli l'instanza della elica intorno al cilindro, che, per esser in ogni sua parte simile a se stessa, par che si potesse annoverar tra le linee semplici)* (Galilei, *Dialogo dei massimi sistemi*, 16; Drake's translation, Galilei, *Two New Sciences*, Madison 1974; cited in Gray, *Ideas of Space*, 140).

Galilei seems to take a view different from that of his pupil, Torricelli, who spent much energy on constructing a helix, and not by a proportional system like Dürer's ad hoc solution arising from a specifically chosen geometric series (1.9). Torricelli's approach was radically different from Dürer's proposal in that he formalized the upwards (or indeed downwards) rotation that characterizes a helical curve (below) in terms of generalization, admittedly by purely geometrical means.

Seen in the light of the scenario just laid out, Borromini's preoccupation, noted by Joseph Connors (lecture at the British School at Rome in the late 1980s), with what Th. A. Cook has called *The curves of life*, snail shells among them, falls into place. Moreover, interest in natural phenomena like these was widespread, a trend.

Galilei's pupil, Evangelista Torricelli, dedicated an entire "book" to the regular cylindrical helix or *cochlea.* In 1641, in Rome, while still engaged at the university as a stand-in teacher in mathematics for Bernardo Castelli (a Benedictine called to the chair of mathematics by Urban VIII in 1626, a job he kept till his death in 1643), Evangelista Torricelli completed his work *Opera geometrica*, which was to be published at Florence in 1644, two years before his death there. In October 1641 he had transferred to Florence after an appointment - due to Galilei's recommendation - as court mathematician to Archduke Ferdinand. He elaborated Bernardo Cavalieri's pre-calculus notion of *indivisibilia* and presented solutions to numerous problems concerning conic sections, as is indicated also by the titles of some of his contributions: *De cycloide, De infinitis spiralibus, De motu gravium naturaliter descendentium et proiectuorum, De dimensione parabolae* and *De infinitis hyperbolicis* [On the cycloid, On infinite spirals, On natural free fall and forced fall, On the dimensions of the parabola, On the infinite hyperbolas], as well as the long Supplement to his above-mentioned book of 1644, *De dimensione Cochleae*, which is discussed above).

Already in 1615, in his *Mechanicorum liber,* Guidubaldo del Monte, one of the really important supporters of Galilei (as a nobleman, enjoying some immunity), and more or less the father of modern mechanics, had published studies on a *cochlea* that consisted of two helices or full helical rotations, but these studies were focused on the practical properties of an *Archimedes screw: De cochlea* in his *Mechanicorum liber* of 1615 (pp. 97 - 104). Applying triangular wedges (a wedge being called a *cuneus*) to a cylindrical surface, he constructs a *cochlea duas habens helices:* a helix with two rotations, such that *each rotation is nothing but a flat surface revolving around the cylinder* (*Dico has <helices> nihil aliud esse præter planum horizonti inclinatum circa cylindrum revolutum*).

Let me recall of the following points. The Latinized Greek term, *cochlea,* is used in Torricelli's writings and also in Sixtus V's inscription of 1589 on the Column of Marcus Aurelius in today's Piazza Colonna: (1.10.2). In the seventeenth century a helical shape was often referred to as a *cochlea,* from Greek, κοκλιας, snail. *One revolution* was counted as *one helix,* thus the Borromini helix or *cochlea*, with three rotations, consists of three helices, or, as stated in some documents, of *tre corone* (a *corona* being the top piece of a building) (*Burden*, 78, with ref. to Tommaseo's Italian dictionary).

A few of *Evangelista Torricelli's* works will be briefly recorded here.

Today, a general helical curve, *in which the torsion and curvature may be variable,* is measured in terms of a so-called position vector (Lord and Wilson, 19ff.): *Fig. 3.2 - 2*. This configuration and the geometry required for its treatment were not available at the time, requiring as it does differentiation and the Frenet-Serret formulas. A cylindrical helix with constant torsion requires only elementary mathematics that was well known at the time (Lord and Wilson, 20; Wrede and Spiegel, 181). Torricelli studied various approaches for the determination of the *volume* of the cylindrical helix with constant torsion in his treatise *On the cochlea*. He calculated the volume swept out by an oblique rectangle moving along the course of a helix curve. He works with manipulations of plain geometrical figures, as did also Guidubaldo del Monte (*Mecanicorum liber,* pp. 97 - 104, *De cochlea*). I shall briefly present the two most directly relevant *Figures* from Torricelli, namely Nos.*T1* and *T4* (redrawn by Camilla Sinding-Larsen, Rome).

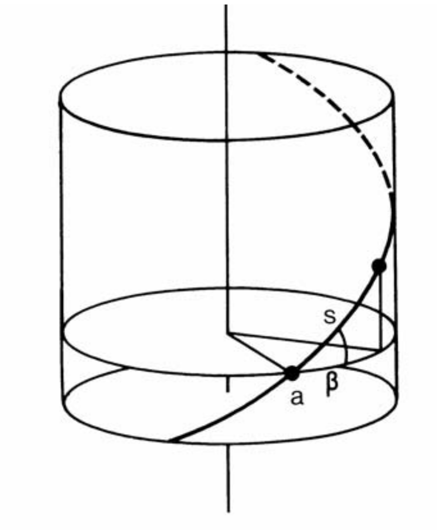


Fig.3.2 - 2 Cylindrical helix: a model for applying differential geometry (Lord and Wilson)

Torricelli does refer to his own hyperbola studies (the hyperbola comes in concerning the cases illustrated in Figures *T5* and *T6*, which are discussed by, among others, Baron and Toscano. Torricelli's *Opera geometrica* is available in http://archimedes.mpiwg-berlin.mpg.de/cvs-web/read/cvswebread.cgi/~checkout~/texts/archimedes/xml/torri_opere_090_la.xml, but without the illustrations to which the texts refer.s in almost every paragraph.

Here follow excerpts from the *Opera geometrica Evangelisti Torricelli,* Florence 1644, pp. 136, 145f. (due to mispagination, page numbers 137 - 144 are passed over) with comments

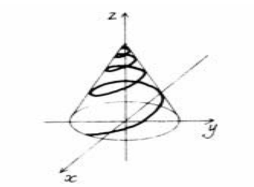


Fig.3.2 - 3. Conical helix (Lord and Wilson)

.

My presentation here of his helix program does not cover the entire course, for two reasons. The program is a complex one and a full account would take us too far afield; secondly, I am not equipped for such an undertaking, since it would involve me in hyperbola problems with which I am not sufficiently familiar. The important thing in the present context is to note the active interest in helix problems at a time when Borromini's Spire was being planned. Torricelli uses a definition of the cylindrical helix that is *purely geometrical*, depending on the rotation of surfaces on an axis.

Let us have a somewhat extended translations and his original Latin texts concerning the two cited configuration.

Definition

The definition of a cylindrical cochlea or helix refers to Fig. 3.2 - 4, T1. The rectangle ABCD and the arbitrarily chosen polygon (which he indicates with just two letters, D and E) *are attached to one another as on the drawing and always in the same plane. They rotate together around the vertical axis AB. At the same time, the polygon DE moves upwards along DC. The polygon thus becomes the genetrix of a figure rotating in an upwards direction. This is the* (geometrical) *definition of the helix or cochlea.*

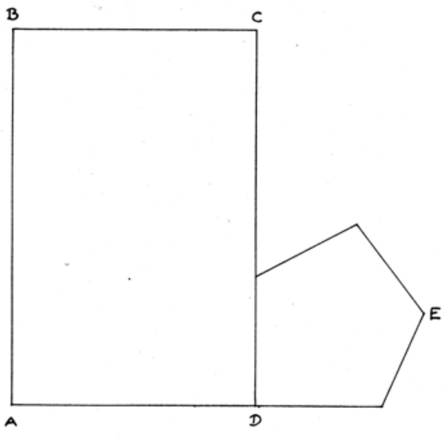


Fig. 3.2 - 4 Torricelli 1(redrawn by Camilla Sinding-Larsen)

Definitio.
Si eodem tempore moveantur duae planae figurae, quae semper in eodem plano consistant, nempe rectangulum ABCD, circa axem AB motu circulari aequabili, et figura quaecunque DE motu progressivo super latere DC. Solidum quod à figura genitrice DE describitur, Cochleam appello.

Lemma III (*see the next figure: 3.2 - 5*).

With the *next figure*, setting up an upright triangle and attaching a triangle to it at ground level (Fig. 3.2 - 5), then rotating A-E-B-F, with the triangle moving upwards as indicated by D, G and H, at constant speed, then Torricelli has constructed a helical volume, a screw (a cochlea, as he says). The extremum, that is, the outward apex of the triangle, traces a helical curve.

The points are here is, first, that the upwards rotation of the triangle BCD on the axis AE, passing from D through G -> H represents the first helical revolution, and, secondly, that Torricelli compares a stretch of the helix with a belt or ring, both on the same cylindrical surface. This he does by erecting the arbitrary vertical IO, stating that a complete, closed and horizontal ring or belt starting from IO will be equal to the helix surface with the same length, also starting from IO (the argumentation we are used to from measuring the area of a parallelogram). As long as such rings are equal, so will the corresponding first helix revolutions.

Si rectangulum AB, et figura quacunque genitrix BCD moveantur, ut in definitione positum est donec peracta integra revolutione ad idem planum redeant unde ceperant moveri. Dico factam cochleam primae revolutionis DGH, aequalem esse annulo circulari, qui ab eadem figura genitrice describetur circa axem AE.
Concipiatur enim figura BCD describere primum cochleam primae revolutionis DGH, quae initium habeat à figura BCD, et finem in figura LFH. Deinde intelligatur describere annulum circularem in se redeuntem, qui habeat initium, et finem in figura eadem BCD. Accipiatur in figura BCD quaelibet recta IO parallela axi AE, quae quidem recta IO in revolutione duas zonas cylindricas, et aequales (per lemma praecedens) describet, in una eademque cylindrica superficie, alteram quidem in cochlea, alteram verò in annulo. Et aequales semper erunt, ubicunque sumatur recta IO, ergo omnes simul Zonae cylindricae quae sunt in cochlea, aequales erunt omnibus simul Zonis cylindricis quae sunt in annulo, propterea et ipsa cochlea aequalis erit ipsi annulo. Quod <erat demonstrandum> *etc.*
Corollarium. Hinc manifestum est omnes cochleas primae revolutionis esse inter se aequales, quandoquidem singulae eidem annulo circulari aequales sunt.

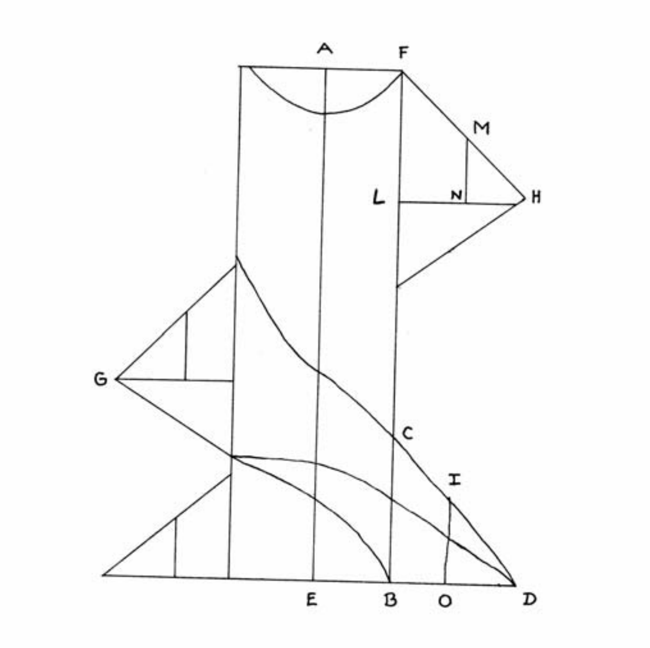


Fig. 3.2 - 5 Torricelli 4 redrawn by Camilla Sinding-Larsen)

Torricelli, then, *starts out from thinking in terms of a volume and ends up with a curve*; a fact he does not state explicitly. Perhaps one may put it this way: that the helix is conceived of as the extremum of a volume. We must realize, with Toscano and others, that the procedure is *geometrical and proportional*, not involving, as was usual, numbers.

The next drawing *T5* (not reproduced here) Torricelli uses for demonstrating certain properties of hyperbolas, and these are exploited for the final demonstration of, again, the cylindrical helix, now with further demonstrations, again purely geometrical, on *T6* (not reproduced here).

This procedure is a reflection of the difficulty of handling mathematically the curve as such, which becomes more tractable when evaluated as a by-product of a stereometric exercise; as long as one does not prefer Dürer's ad-hoc proportional device. This did not take care of helical surfaces in general, just one specific helical curve; but of course a rule for handling other specific cases can be derived from Dürer's figure.

Torricelli pointed out the novelty of his results concerning the helix, and even Gilles Personnier de Roberval, who was sceptical and, apparently with good reason, hostile to the rather arrogant Torricelli, praised this achievement in a letter of 1643 to Marin Mersenne (5.3.3): *because of this alone, I have no reluctance to count the author among the outstanding mathematicians of this century*; a reluctance easily dispelled, one is led to believe, since after all the helix solution was a minor achievement and because the French mathematicians stood their ground firmly against the Italians on issues of far greater importance. Fine diplomats, the French.

Torricelli was also celebrated for the determination, in 1641, that the volume of an infinitely long solid, obtained by revolving about its asymptote a portion of the equilateral hyperbola, was finite. Boyer affirms, however, that the notion had been known earlier (Boyer, 1959, 125). But perhaps there is more to it.

The matter may turn out as considerably more complex than conjectured so far. Torricelli uses *T5* for demonstrating certain properties of hyperbolas, and these are exploited for the final demonstration of, again, the cylindrical helix, now with further demonstrations, again purely geometrical, on T6. Furthermore, in the introduction to his *De dimensione Cochleae*, he notes that the argumentation here will be seen to be connected with that of his preceding work on hyperbolas (*Non enim aliena erit à praecedenti libelli [De solido hyperbolico acuto] praesens speculatio*...).

Torricelli's helix studies as briefly reported here focused on *volume.* My point of departure concerning the Spire has been, and will remain so, that it is the *conical-helical curve* substantiated in the *fillet* or *thread*, an unbroken band winding up the Spire (or down) that is in our focus of interest, (Descartes used the term in a comparable connection: *filet;* cf. *Le petit Robert: Ce qui ressemble à un fil fin - saillie en hélice d'une vis*), especially on account of the increasing torsion. Questions concerning the *volume* of the Spire will hardly arise.

Baron notes repeatedly that in general Torricelli's approach was wholly geometrical. But this principle, it would seem, may be active in two different frameworks. *One* would come in the way of a purely design-based conception, the *other one* with algebraic, thus even *protocalculus*, ideas (especially with regard to his *De infinitis hyperbolis*.) (Baron, 185ff.), so to speak lying underneath the design apparatus. If I have understood Baron correctly, her analysis (and those of other experts) would seem to consist partly in just this: to distil the *implied* algebraic notions from the seventeenth-century geometrical demonstrations and set them out in modern notation. This kind of operation probably was what Roberval prospectively had in mind when writing his flattering comment on Torricelli's purely geometric demonstration in his work on the *cochlea*.

The importance of Torricelli's work should not be overrated. It is relevant for our subject to be clear that he was in no way a trail-blazer; that he, ideologically speaking, belonged to mainstream tradition, in spite of his coming quite close to the idea of limit which is fundamental for the calculus. Torricelli also has been thought to exceed - in a "modern" direction - the limits of Galilei's concept of science, but this claim has been challenged by Galluzzi (Arrighi, 35f.). Nevertheless, now to quote Boyer,

Among the results achieved by Torricelli through the application of the methods of exhaustion, of indivisibles [after Cavalieri]*, and of the composition of motions are to be found a number of remarkable anticipations of those found in the calculus. They include, as well as theorems on quadratures and tangents, some of the earliest results on rectification* [of curves] (3.5.8 and 5.2.9) (Boyer, 1959, 132).

3.3 Testing models: the Spire's conceptual position

Now back to Borromini's Spire.

Graphical models as I use them, are something more than just illustrations accompanying texts. They are tools in themselves (*see Burden,* for details). They have to be tested for functionality and consistency by applying them to substantive and theoretical material, such as in the present *section.* These *system models* (below*, see* also *Fig. 1.7 - 1*) are not conclusive, not affirming or "freezing" anything, just positing probabilities that can make up platforms for further research and argumentation. Finding shortcomings in them is a productive process. A model is good provided it can be faulted *in parts,* can stand such a treatment or is amenable to being restored or repaired with its kernel structure remaining (*Burden*, for details). The models are potentially dynamical in that they contain variable entities (further on models in 4.5).

After a short preamble, a model will be tested out on the Spire's position in "reality" contexts. Further cognitive and conceptual operations on this system are charted in another system model: *fig. 4.4 - 1*. Both graphics can be regarded as system models, corresponding to the systems framework proposed in the *Introduction, g, The systems issue.*

I do take it for granted that all Borromini's differential curves and manifolds would have been looked upon as one characteristic and easily identifiable group of works; so that, observing the Spire, the rest would come to light conceptually with it, constituting a theme in a *configuration space* (1.7) within which a rich array of elaborative processes can evolve.

Before proceeding to discussing the Spire's internal structure and the functions probably imputable to it (4.4, 4.5), we need a picture of the greater scenario, to set the Spire as a whole into a larger configuration space, the main components of which have already been tentatively described in *Part* II: world worry.

There are the following components in this overall picture regarding the Spire, which is graphically surveyed in *Fig. 3.3 - 1 (Spire diagona*l): mathematics development, especially the protocalculus, the cosmological novelties and controversies, the Tradition of the Church (reinforced at the Council of Trent), general cultural activities (literature, the arts, music). Beside the vaguely definable cultural institutions (*lore* on the model), there is one that is well-defined and actively involved: the Church. In the conflicts between science and the Church, there were heavy concerns on both sides; scientists generally religious and accepting the religious obligations while their science demanded independence and innovation, the whole process resulting in an attitude on the scientists' side that for model briefness' sake may be labelled a *yes, but!* attitude. As for the Spire itself, I consider it sufficient at this stage simply to articulate it in three components, *one,* its all-over shape, *two,* its *iconographical program,* and *three,* its helical curve.

Let me propose a graphic model for the mutual relationships in the thematic configuration of the Spire (*Fig. 3.3 - 1*). Such a model is simplifying but shows structural order, in this case in terms of three institutional and cultural forces, Church, Science and general "Lore", and relating the Spire-internal structure (the *diagonal*) to levels in the three groups. Elsewhere in the present discussion, I use the terms *mirroring* dimension (*Introduction,* `tithe` *systems issue*) and *hardframing* (4.3.1) in relating the quantifiable and the "soft" components ("bridging" in SL, *Burden*) to one another. To render the notion more digestible, we might subsume the two terms under the heading *metaphorical configuration.* On the other hand, the model, for all its possible deficiencies, does illustrate a system. Systemization is the only dimension we have for what we use to call explanation, a by-product of systemization (Radnitzky, II, 102), ruling out the Why (survey in Silverman), substituting the How.

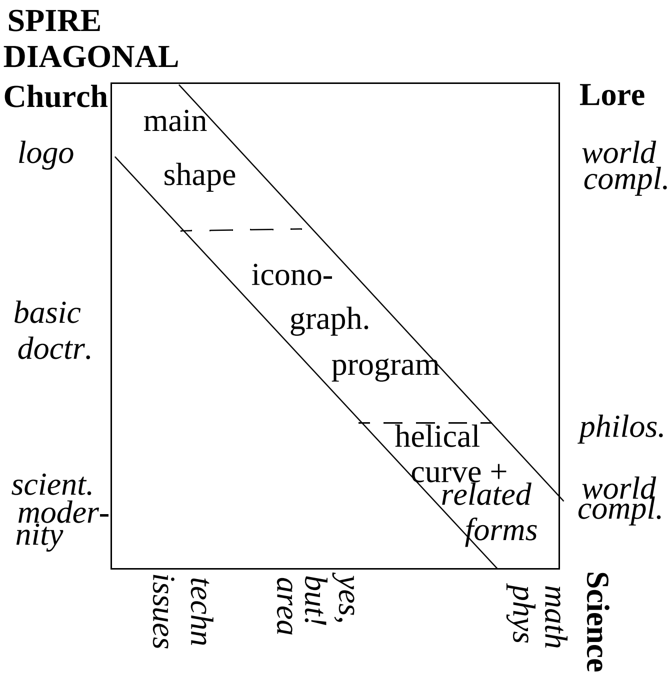


Fig. 3.3 - 1 Spire-in-context. A system model.

The model on *Fig. 3.3 - 1* lays out a crude and much simplified picture of the system in focus in the present book., setting down the subjects and notions subjected to mental, argumentative, analytical, mathematical, *etc.* operations: a configuration space (1.7). It is amenable to being extended and also being broken down into subsystems. The model brings out hard and soft evidence and concepts and in this way highlights the areas in which connecting these entities is problematic. A functional model should uncover or, at least, indicate, relevant problems. A good systems picture should do this also.

The obvious feature for mathematical abstraction on the Spire is the *broad unbroken band running up around it* (or down), *a solid manifestation of a helical curve with variable torsion.* The band is clearly visible on *Fig.s 0.1 - 1, 1.1 -1, 1.10* - and *2.* At the time in question, just as today, it was in full close-up view both from the Via del Teatro Valle and the Piazza I Sant'Eustachio (*Fig.s 1.10 - 1* and *2*).

Using the helical thread or fillet, really a broad unbroken band, as a test object, I am going to break it down conceptually into infinitesimal bits and use one of them, by necessity arbitrarily selected (thus remaining nameless here) for locating mathematics-related issues as a whole (3.3.1).

There were, as indicated earlier (3.2.1) at the time two different approaches by which a conical helix curvature itself could be cognitively construed and studied (disregarding now Torricelli's volumetric determination of the cylindrical one).

One method we have already considered. Dürer's *Schnecke*, we recall, (*Fig. 1.9 - 2)* is constructed in terms of proportions, by building up from specific points on a plane spiral on the ground a grid of verticals whose upper end-points determine points on the resulting helix, points selected with the purpose of creating a rising helix.

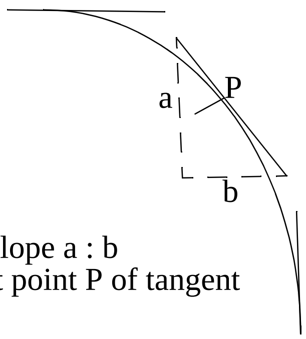


Fig. 3.3 - 2 Three tangents to the same curve, one passing the point P marked off with an orthogonal

This structure is a static grid and seems fully compatible with, and an expression of, the preoccupation with static systems of proportions in Dürer's days. The helix is created by establishing a specifically chosen geometrical series and in this way it may be said to be followed along its course continuously.

The *second method*, making up an alternative framework of conception and cognitive handling, consists in focusing *not* on a proportional system that pointwise determines the space positions of curvature, but on the helix curvature itself, as it is winding upwards (or indeed downwards), by considering it in terms of tangents of varying slope. The appended *Figure 3.3 - 2* illustrates the point. The helix in this conception is produced by a dynamical process of steady changing one and the same entity: the tangent slope determining the veering off in the curvature (as in the Bézier curves of today). This veer, as we have seen, is not constant in the case of Borromini's helix, since its torsion increases steeply in the upper part.

3.3.1 *Breaking up the thread*

Borromini's Spire will now be evaluated in the perspective of probable mathematical associations evoked by its curvature among those contemporaries who were competent and active in the relevant field. Technically, in what follows, the helix is considered in its entirety and, as I have said, also for its curve at any arbitrary point or infinitesimal stretch. Descartes, we have heard, excluded the helix from the class of geometrical curves just because *it is generated by a filet (thread)* (3.2) (Mancosu, 1992, 97).

In a model perspective, we can break the broad fillet or thread up into infinitesimally short pieces and select anyone of them, abstracting it, in order to visualize everyone and all of the crucial differentiation issues in contemporary protocalculus activities; and some of those regarding integration. This arbitrary choice among curve pieces rather than circumscribed volumes in the helix, is dictated by the assignment to concentrate on differentiation and derivation issues, leaving questions of areas and volumes, and hence integration, as a side-issue. An infinitesimal subdivision of units is crucial in Cavalieri's method, we have seen (comment on *Fig. 1.5 - 1*); in his case specifically surfaces are analyzed in terms of a set of infinitesimal parallel curves or lines.

Galilei appears to give some further justification for such an abstracting down to infinitesimal stretches of the helical curve from a helical shape of any specific variant, and focus primarily on one of them. Galilei, we have seen (3.2.1) counted the cylindric helix among *simple lines.* Speaking of the *Wheel of Aristotle,* he considered *an infinite number of infinitely small and indivisible parts with an infinite number of infinitely small indivisible spaces interposed between the parts* (Baron, 116ff.). From a general mathematical point of view, any piece of curve extending in three dimensions would have met the requirement. We are however considering the crowning piece of a building, and have to stick to our conical helix. Baron writes that there was no widespread interest at the time for curves, and she probably has in mind the vast gallery of classical types of curvatures, rather than the nature of curvature (for such a collection, see *Famous Curves Index,* http://turnbull.mcs.st-and.ac.uk/~history/Curves/Curves.html.

3.4 The Spire and the calculus

First, a general observation on the relations holding between architecture and mathematics. My observations touch on a so-called *meta-perspective,* concerning the alternatives of *use* or *mention.* But - are they really alternatives? Concerning the insistence on the importance of the divide *between* use and *mention,* the *meta* issue, in Nagel and Newman's stimulating booklet on *Gödel's Proof,* I seem to remember that Douglas Hofstadter somewhere (in his *Gödel, Escher, Bach,* or his more recent book about himself as a *Strange loop*), claimed that a *statement about number theory is number theory,* and that in Gödel, *numbers <are> being put to use in reasoning about mathematics.* It may seem tempting, then, to say that any statement about the relations between math and design *is* an integrated part of that relation.

Can there be a math expression that does not say something about something? Would it not tell us that certain rules are being employed and that they are considered valid, in other words state something about them? So-called *aboutness* means handling the things the expression ranges over. Handling involves a process of some kind. Looking at what we are doing from such a point of view, we can remain optimistic, for we participate in a never-ending process. What more can one expect? (see SL, *Operational determination. Math in buildings and math statements about them;* forthcoming).

My further step-by-step program unpacks as follows in the *subsections* to follow (3.5.1 - 3.5.11), preceded, however, by some historical notes and some observations on basic math principles (3.4.1 - 3.4.4).

3.4.1 *Protocalculus issues (Cavalieri to Newton and Leibniz)*

Before evaluating Borromini's helical Spire in relation to contemporary mathematics, we need an introductory survey about the most relevant issues in the protocalculus development. I shall keep this presentation on the high ground, leaving details to *Supplement* II, while wanting to make it clear that my account is no original contribution, merely an attempt at exploiting for my own uses a rich and penetrating modern scholarly literature which focused on philosophical matters as well as on the technical ones. This introduction is indispensable precisely because technical matters are intertwined with far-reaching issues concerning concept-formation and attitudes and ideas of science generally speaking.

The term *protocalculus* is here taken to mean the long-winding, non-linear and jumpy processes in mathematics from the sixteenth century and well into the next, up to the time of Newton and Leibniz, which converged upon what today is called *the infinitesimal calculus*, or just *calculus* or, with a larger compass, *analysis*. The two principal methods are *differentiation* and *integration*, and they stand in an inverse relation to one another.

Let me quote from Baron's illuminating overview (Baron, 4f.).

The geometric model... has operated not only in illuminating problems but also in determining the language and vocabulary in which the structure of mathematics has been described and the results presented. The infinitesimal calculus accordingly emerged in the form of two major classes of problem, the first [1] *concerned with the determination of arc, area, surface, volume and centre of gravity, the second* [2] *with angle, chord, tangent, curvature, turning point and inflexion... The relation between tangent and arc ultimately became one of the most significant links between differential and integral processes and, for this and other reasons, the problem of rectification* [calculation of the straight-linear length of a "bent" curve] *became crucial in the seventeenth century. The inverse nature of the two classes of problem* [differentiation and integration] *was approached in terms of a geometric model by Torricelli, Gregory and Barrow but only with Newton did the relation emerge as central and general* (Baron, 4f.).

There were two further major problems, the first being involved when the matter is seen in a top-down view regarding overarching choices of theory, the second rather regarding the internal machinery, connected with the second point in the quoted overview by Baron.

One was to get out of pure geometrical over to algebraic/arithmetic reasoning. The drive, prompted more by problems arising than by programming, was to compound algebra and geometry (3.5.11 and 5.2.12), and from this process proceed to a complete algebraization. Even Galilei thought in geometric terms, never using equations: as we have heard (Drake, 1990, 7)). On the other hand, the sense of proceeding geometrically was being discussed, as is shown, for example, in Descartes' criticism of Clavius regarding the classification of curves in geometrical and mechanical: is the quadratrix best handled by pushing straight lines over the area or by using infinitesimal points (*pointwise construction:* Mancosu, 1992, 91ff.)? Of course the increasing familiarity with Archimedes' work stimulated geometrical approaches. Galilei's pupil and collaborator, Evangelista Torricelli, discussing Cavalieri's theory of indivisibles (3.5.1), believed that the latter had only rediscovered a method that Archimedes must have known (Blanck and Krantz, 415). Torricelli is credited by Baron with an almost direct hit on the limit concept in his argumentation concerning the measurement of irregular surfaces. His procedure, however, was still purely *geometrical* (Baron, 185ff., Toscano). Being tied up with rigid geometry delayed progress in many directions. Boyer claims that Torricelli *had no more idea of defining the notion of instantaneous velocity than had his master, Galileo* (Boyer, 1995, 134).

The *second problem* of the protocalculus story, now to specify more closely Baron's second point just cited, and which arose irregularly and in different problem environments, turned out in the end to represent the core of the entire venture. The issue was, as intimated already, to understand the difference between 1) *an infinitely small point* (infinitesimal, ultimately, the differential) and 2) a zero-approaching limit for approaching a point (ultimately the derivative); and thus to grasp and formulate the notion of *increment near an infinitesimal point.* Grappling with these problems led gradually towards the concept of a *limit* towards which a function or quantity is tending so as to *approximate instantaneous velocity or change*. In fact, concerning the inversely related process named *integration*, Boyer feels justified in speaking of *the admitted approximate nature of scientific measurements* (Boyer, 1959, 5), following Russell's observation about *approximation* being the fundamental property of exactitude.

3.4.2 *Curve, point and direction - continuity*

Before considering historical issues, we shall take a look at a matter that may intuitively be considered as the kernel of the entire subject; the location, determination and interaction of three factors: *curve, point and direction at a point*. I am going to introduce two fictional historical phases, *Phase One* and *Two.* It should once more be emphasized that there was no clear and definite transitions in terms of passing a boundary from Phase One to Phase Two, which represent, roughly, two succeeding scenarios. Ideas from both often dovetailed, so rather than thinking in terms of *from-to*, we might think of a scenario of transition.

When we ask for the rate by which *direction* at some specific point on a curve changes - *p* on the *Figure 3.4 -1*, this is today defined by a *limit value*, being approached from some chosen dummy value (*see* below), where the tangent indicates the instant changing value. The rate at which the curve (a function) changes at such a point, is today called the *derivative.*

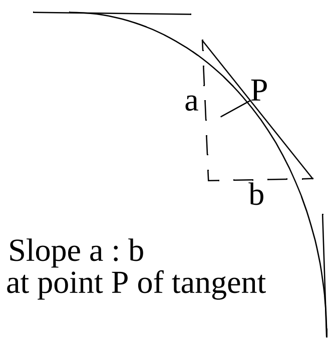
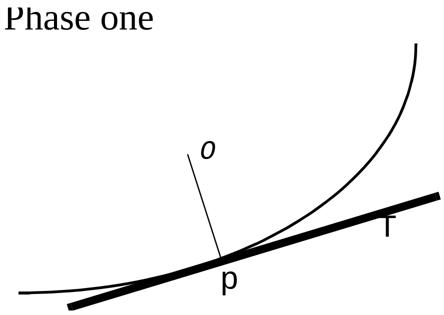


Fig. 3.4 - 1. Three tangents to a curve, one of them setting off a point P. The quotient a: b defines the slope of the tangent at P. In this fashion, the change of direction - or velocity - at some specific point is determined. Obviously, there is an infinite number of such points on the curve.

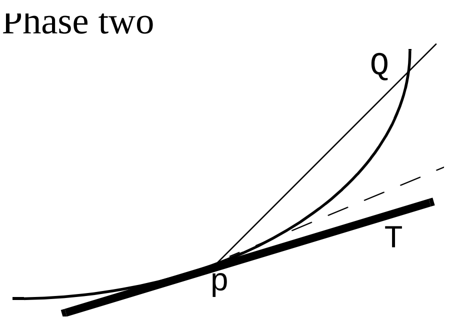
Things looked differently in the seventeenth century. Historically, there are two main alternatives which we shall discuss with the support of two further figures (*Fig.s 3.4 - 2* and *3*), the upper one for the earlier (*Phase One*, a very roughly bounded entity) and the next for the modern alternatives. Before facing the details, here is a synopsis of the intuitive ideas underlying the efforts aimed at handling the problems involved. It was realized that such a curve is made up of an infinite number of points, that the curve must be continuous (3.5.5), and that the points must be considered as diminishing *towards, without ever reaching*, sizes infinitely small, in fact, -> 0.



Figs. 3.4 - 2 and 3 (below): The same curve, first with a tangent T and an orthogonal O setting off the point P touched by it; next, with an increment Q - P, Phase One and Two respectively.

But then, how can one determine visually a *direction* at any point of the curve, since a point cannot have direction? A straight line is also a *curve*, but now we look at a *bent curve*. That is a curve with continuously changing directions, without any sharp angles or *cusps* (*see* 3.5.5 for continuity). One directional path is indicated by the tangent T.

The solution in *Phase One* (*Fig. 3.4 - 2*) was to establish a tangent T at the point P, and this could be done crudely by determining an orthogonal O to the point. This is a *static* procedure and it was accepted because it yielded acceptable results. One understands why so much work was invested into tangent solutions (3.5.3 and 5.2.4).



In *Phase Two*, illustrated on *Fig. 3.4 - 3*, it was understood that you cannot define a point by simply pointing to it; only by *approaching it according to some rule*. The point P is not found with the static tangent method but by using an *increment*: Q - P on the graphic. Moving Q along the curve, the secant (*increment*) will slide downwards, passing an infinite amount of positions, among them the broken line (as an example). The line Q -P has a *slope,* an inclination. This can be expressed as the quotient of height to width ($\Delta y/\Delta x$ on *Fig. 3.4. - 4*). By reducing the value *towards zero* (never make it *like* zero, for that would mean division by zero), the point Q approaches P closer and closer towards an ideal number called the *limit* (see below). Approaching zero ever so closely, at some point Q approaching towards the infinitely small has *conceptually* to stop; this is the limit (intuitively spoken).

The idea is to have Q *approach* P without coinciding with it. In order to make Q approach P, Q has to *start somewhere*. That is where the *increment* Q - P comes in. It has no "truth-value", being merely an auxiliary device or dummy variable. But it was a crucial one, for on this basis some mathematics were devised that took care of the measure of Q's approach to P, in terms of a *limit* (see below). In this manner a method was developed by which approximation of Q to P could be determined strictly and philosophically much more satisfactorily than was the case previously. And yet, still *during the eighteenth century... the inherent difficulty of formulating the underlying concepts became increasingly evident, and then it became customary to speak of "the metaphysics of the calculus"* (Boyer, 1959, 5).

Some terms will be indicated that will come in usefully later on; refer to *Fig. 3.4 - 4*.

The first is the *derivative*, a post-seventeenth-century notion. *The derivative is the mathematical device used to represent point properties of a curve or function* (Boyer, 1959, 6). The derivative at any value x_0 of x is the *slope* of the tangent at the chosen point; that is, not the tangent itself but the *degree of its inclination.* This is indicated by the ratio *a:b*, as we saw on *Fig. 3.4 - 1*; that is, the *ratio of the differential dy to the independent variable dx*; which means the difference quotient. The *differentials* are the *dy* and *dx* on *Fig. 3.4- 4* The technique of handling them is called *differentiation.* They are the operators that determine, not the variable slope's changing inclination, as we saw, but the "definitive" slope, depending on the limit, that defines the tangent to the point *P* itself and which defines the rate of change at the same point. The quotient of the differentials *dy* and *dx* at a value of *x* is the *derivative* of the given function at that value of *x* (Morris Kline).

Let us have Boyer specify further the nature of the crucial operator, the *derivative*, since it will then be easier to understand why *Phase One* (*protocalculus*) mathematicians had problems digging it out.

The derivative is thus found not in terms of the ordinary algebra, but by an extension of these to include the concept of the limit of an infinite sequence (Boyer, *1959,* 8f., a historical exposition of the development of the *limit* concept).

Finally, *continuity,* what about it? The operations performed on a curve consisted in enabling us to work with any *point* and the corresponding *tangent*. This can only mean that, whatever point we are elaborating, we shall always be able to do the same on the point most closely preceding and succeeding it. This is a very crude way of describing things, but it may provisionally alert us to the fact that all the operations in the present discussion demand that the curve be *continuous.* Continuity of a curve can seem counterintuitive as long as one considers *points* on the curve, for a line without breaks would seem to mean that there are no break between any two points on it. This requires heuristic thinking: the claim is valid whenever we posit it and operational whenever it works: we use it, therefore it exists. We can impose continuity in a real case hoping for the best. If it works, then so it does.

Graphically we can of course illustrate continuity if the curve *can be drawn with one uninterrupted motion of a pencil* (Kline). Then the *graph* is continuous. What does this tell us about the curve of which the graph is an illustration?

$$y = \frac{1}{x-2}$$

We want to probe a little deeper into the problem; as did our protagonists in the seventeenth century. Things are different today. Continuity and discontinuity are analyzed under several different categories with their corresponding and complex math operations (which I shall pass over here; to get the flavor of it, one might consult Kitcher, 246f.and 261ff. for Cauchy's five definitions). *Just one simple example. The continuous property of a curve can be proved with equations such as the one shown here (from Kline). The curve is broken at x = 2 since division with zero yields no value. For the curve at x > 2 and x < 2, we can set in whatever values we might wish, and this means that the curve is continuous at every point expect when x = s.*

Now some more details.

NOTE: the next few paragraphs, to the end of this Section, can be skipped without losing the thread, but can be used for reference whenever convenient.

The above argument will now be reiterated with more details in order to make certain matters clearer and better applicable to the subject in focus. First, more on Phase One. Instantaneous change is to be located at some infinitely small point on a curve (in speed, for example). Geometric thinking predominated. Subdivide some geometric shape indefinitely towards the infinitely small, and you have sums of them to apply to whatever you want to measure (for Cavalieri, s*ee* 3.5.1 and 5.2.1).

The infinitesimal point - such as *P* - at which the *rate of change* shall be determined, is selected for attention. The selection of it is guided by the main purpose of the operation, for in fact there will be an infinitude of such points, since a curve that is "bent" and not having edges, is so *continuously*; today we say that it is *differentiable*, amenable to the operations we are considering. One factor in determining such a point is the *tangent passing the point.*

For the purpose of determining *direction* of a curve at a specific point, we need a tangent to the curve at that point, and this is constructed by setting up a straight line perpendicular to it, *P* on *Fig. 3.4 - 1*, so that the *tangent* runs orthogonally, at 90 degrees, to it. This obviously is a tricky assignment, since it really concerns finding a perpendicular to a *point* (easier with a circle with defined center and radius). The trouble: *Tangent and orthogonal depend on one another, but for neither of them is there a reliable technique available which guarantees exactitude.* This in turn presupposes that we artificially imagine we can *diminish* the stretch of curve where *P* appears, towards infinity, so that we can treat the curve at this point as if it were an *infinitesimally small straight segment.* We can imagine now some of the difficulties posed by the so-called *tangent problem* (3.5.3).

In *Phase One*, how did one *define* the point *P* illustrated on the *figures* above? By imagining that the point had to be approached across an interval where the point would be found, *from both sides on the curve*, steadily getting closer and closer by gradually diminishing the interval which as a consequence reached some incalculable but assumed infinitely small value here, an *infinitesimal point*. The point itself is static, has a definite position, and must be found by a double approach.

This is the *Phase One static* view, whose central notion is the idea of Cavalieri’s *indivisibles* (points too small to admit of further subdivision). It was the starting point for a drive towards the next and alternative proposition (*Phase Two*), substituting the use of *limits* for handling differentiation as the concluding operation, substituting *derivatives* for differentials as the final operators (4. below).

It is the process just drawn up sketchily and labelled Phase One that I have in mind with the term protocalculus, in many respects and on many levels foreshadowing the achievement typical of the next phase.

Now some further details from Phase Two

The development of *mature, if not modern, calculus* occurred primarily by adopting the following ideas and methods.

Here the idea, not of reaching some fixed point by "infinitely" diminishing from both sides along the curve, but by *dynamically following the course of the curve in one direction towards the desired point*, in the following manner.

See Fig. 3.4 - 4. Without for the present bothering to understand all the symbols, the main idea can be grasped by observing the big graphic itself (Fig. 3.4 - 4). We may look at change as change in speed. We can take the speeds at points P", P and P', sum them up and divide by 3, then we have the roughly the arithmetic mean or average speed from P" to P'. But this is not what we want; we wish to know the speed at P, the instantaneous speed there.

This is the crux of the calculus treatment of curves. If the police calls me aside because of speeding at Orvieto, they do so on account of my speed *there*, not the average one from Rome to Florence (though they now seem able to do that also).

But let it be noted that the *infinitesimal point* is *in principle* an *average quantity.* There is no "quantum jump" from this to the infinitesimal point. By introducing the dummy variable increment, the process starts out with a *visible* average (*Fig. 3.4 - 4*) that admits of being given in natural numbers, from Q to P on the figure above. When this is being reduced down to approaching *P* at almost zero distance or interval, it still is an average, but one over an infinitesimally small stretch.

We proceed by two tricks, *one*, to establish a diminishable *increment (above, Fig. 3.4. 3* and *4*; *see* 3.5 and 5.2.3), *two* by searching for a *limit* (also 3.5.2 and 5.2.2). The increment is $\Delta x = dx$, measured from the arbitrarily selected point *P*; thereby we obtain a starting point for the slope at *Q*.

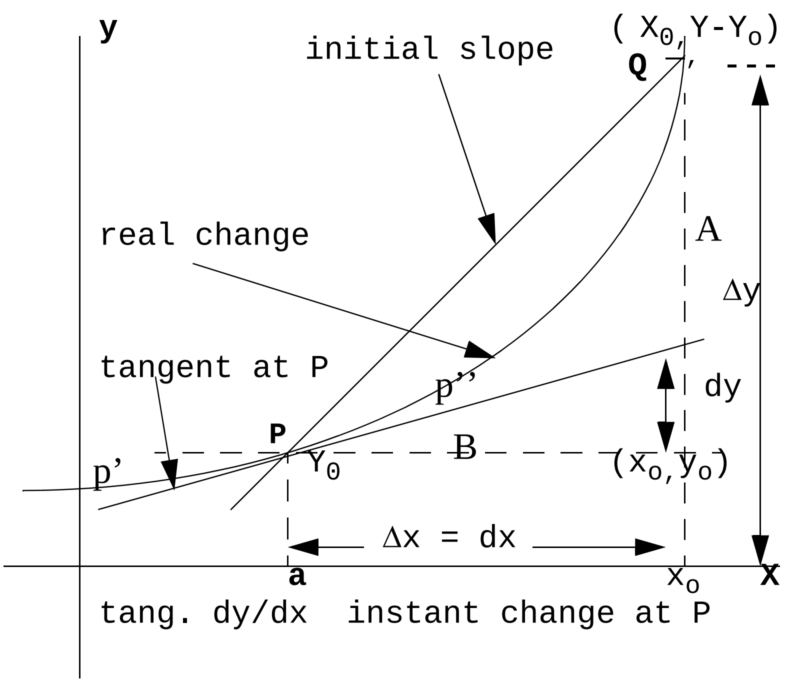


Fig. 3.4 - 4. Slope initially determined by the increment, Q - P gradually approaching P as the increment dwindles toward (not to) zero; then the resulting slope, degree of inclination, is the derivative.

By gradually reducing the increment, *Q* moves along the curve toward *P and* the slope line slides downwards to coincide with the tangent at *P*.

This tells us what the instantaneous change is at this point.

Why not simply set Δx = dx *like zero* and have done with it more quickly? That would have meant to set the line segment B in the figure to zero. But then A/B could not have any significance left, for it would mean to divide by zero, which is not defined in mathematics.

The idea will be appreciated when we look at the equation on *Figure 3.4 -5 and compare* it with *Fig. 3.4 -4,* in which E stands for the increment. *E* can diminish infinitely close to zero but cannot *become* zero, for division with zero is not accepted.

$$\lim_{E \to 0} \frac{f(x+E) - f(x)}{E}$$

Fig. 3.4 - 5. Equation defining a limit. E: the increment added to the original function f(x): Δx = dx. Diminishing this, by letting dx approach, never reach zero, the slope becomes the tangent to the critical point.

This is why we need an extension, namely the *increment*, to provide us with "something" that can be diminished infinitely and let the sloping line approach, with infinite closeness, the tangent to *P*. In the equation reported here, the *E* corresponds to the Δx increment. It is easy to see what would happen if *E = 0*; a disaster, making us subtract *f(x)* from *f(x)* and divide the resulting nothing by zero**.**

How to define the "minimal distance" to zero? This is the most tricky business of the entire procedure and one that took a long time to being understood. A *limit* must be somehow defined; and the equation in *Fig. 3.4 - 5* determines it with respect to the increment *E.* The procedure is relatively complex and will be passed over here. But we shall encounter the subject again, for, as indicated already, its discovery and use marked a distinctive stage in the development of the calculus, the passage from *protocalculus* to c*alculus, Phase One* to *Phase Two* in my much simplified relational picture.

Yet another theme we shall meet without going into detail about it, is *integration*, the inverse process of *differentiation.* By working with sums of series of points like the *P*, one could determine areas under curves and one could extend the method to be valid also for evaluation of volumes.

A special pair of concepts much in view in the seventeenth century, was the *maximum* and *minimum,* meaning points on top or bottom of a curve, area or volume where the tangent would be parallel to the *x*-axis. No slope there and hence no derivative: p_1 on *Fig. 3.4 - 6.* But the notion had a wider use than just this, as will transpire later. The entire venture was a play with approximate and elusive quantities.

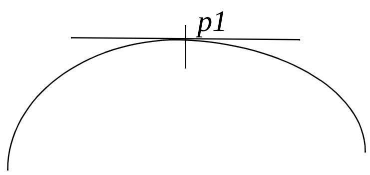


Fig. 3.4 - 6 Curve with a point P in the maximum position: no slope here, so there can be no differential or derivative.

Pierre de Fermat, a civil servant doing mathematics as a side line, homed in on a crucial factor in the way he employed *variables.* He got the opportunity to see the proofs of Descartes' *La géometrie,* and this provoked him to make public his own tangent method (3.5.3), which *occupies a significant place in the history of the calculus for in it occurs, apparently for the first time, the idea of a slight change of variable (it is primarily on the strength of this method and the applications which he developed for it that Fermat has been declared the inventor of the calculus by Lagrange)* (Baron, 166f.). With this comment, we enter the story of the *protocalculus.*

3.4.3 *Terms of transition*

The fundamental operating principle in protocalculus and in mature calculus is, to reiterate a crucial idea, *approximation* (*see* also 2.2.2, $8)., making the increment approach but never become zero. Accepting approximation not as an unavoidable handicap but as a positive and constructive idea, came gradually in the seventeenth century, the awareness becoming fundamental in many environments. Here we see why Bertrand Russell could claim that all exact science is dominated by the idea of approximation. Paolo Rossi, in his *La nascita della scienza moderna in Europa,* quotes Bianchi's statement about the new acceptance of approximation, in a passage I am going to translate with the original appended; for it is an appropriate comment (Rossi, 1997, XVII).

Galilei developed increasingly exact systems of measuring of nature, but

he shifted his attention from ideal precision over to one that proved indispensable in relation to the areas <in focus> and <which were> manageable with the available tools [an undefined but working kind of heuristics]... *the paralyzing myth of absolute exactitude was one of the factors that hindered the thinkers of the fourteenth century in passing from the abstract "calulationes" to an efficiently quantitative study of natural phenomena* (L. Bianchi, quoted by Rossi, 1997, XVII). (*Galilei inventava sistemi di misurazione sempre piú accurati, ma spostava l'attenzione dalla precisione ideale a quella necessaria in relazione agli scopi e raggiungibile con gli strumenti disponibili ... il mito paralizzante della esattezza assoluta fu tra i fattori che impedirono ai pensatori del Trecento di passare dalle astratte "calculationes" a uno studio effettivamente quantitativo del fenomeni naturali*).

Palladio's *Quattro libri* of 1570 may be cited as referring to at least one case in which the *approximation* issue had arisen earlier, even though he does not say so explicitly, and had no tools with which to handle it, the contingence not yet being uppermost in the minds of people and hence not marketable among his clients. In his book, which is tightly packed with measurements - there are numbers all over his drawings, one critical case is treated verbally in default of a numerical method. One of the ancient Roman vaults in the *Libro Primo* is a *volto tondo*. Here, we are told,... *the curve of the vault, the closer it approaches the corners, the rounder it becomes* (*quanto più s'approssima à gli angoli; tanto più diuenta ritondo;* in SL, *Operation*, I use this case for developing a distinction between *implicit* and *explicit mathematics*). One may risk the guess that Palladio would have desired to express this increasing "roundness" in a more precise, numerical format. His clients, the majority most of whom were business men, would most likely have felt even safer if he had been able to do that; but no one was at the time.

It seems typical of transitional scenarios that potentials of an initiating but not yet operative technique are being surmised, while one still have to have recourse to verbal handling of the issue (*see* also 3.4.4). The idea of *limits* was linked up with the notion of the infinitely small, in later terminology, *infinitesimal.* Giordano Bruno's preoccupation with what he subsumed under the term *limits* had nothing directly to do with this, but it reveals an attitude with comparable conceptual foundations. Bruno - let us recall that he died in 1600 - comes very close to one central aspect of the issue. Consider a circle and draw a chord across it so close to the perimeter that the chord so to speak "melts" into the curve - the arc - of the perimeter. Push it still closer at infinitely small distances, and go on with this: at a certain point, Bruno asks, will the chord and the curve become identical, and the arch a straight line? (Gatti, 153f.). Perhaps without being aware of it, he came very close to the idea of tangents to curves in the significance of the notion that was to become crucial to the calculus. Giordano Bruno further *overcomes the contrast between designed geometry and arithmetical computation* and uses this to prove the existence of *minima* or "atoms" (Blum, 18f.).

Ideas from both *Phase One* and *Two,* I have noted, intermingled here and there, and while it would be hard to define a phase of transition, one may probably speak of *terms of transition.* The present summary could convey the impression that everything centered around calculus issues. This was not so.

Let us have Mancosu, in his book on mathematics in the seventeenth century, remind us of the fact that something more was involved than just the *protocalculus* (not his term).

...not only were new mathematical results added to the preceding body of mathematics, but new techniques were grafted onto the old ones and several new areas of mathematics emerged or were consolidated. Reaching full maturity within a hundred years, algebra, analytic geometry, the geometry of indivisibles, the arithmetic and geometry of infinites, and the calculus are among the most significant. Each new area led inevitably to foundational discussions about its status (Mancosu, 1996, 6).

He also submits penetrating observations on the relations between mathematicians and philosophers facing each other: 4f.).

Øystein Ore's evaluation of Fermat's achievement illustrates the bredth of scope that could be reached in those days.

He made considerable contributions to the foundation of the theory of probability in his correspondence with Pascal and introduced coordinates [3.5. 11] *independent of Descartes. The French, when too exasperated over the eternal priority squabble between the supporters of Newton and Leibniz, often interject the name of Fermat as cofounder of the calculus. There is considerable justification for this point of view. Fermat did not reduce his procedures to rule-of-thumb methods, but he did perform a great number of differentiations by tangent determinations and integrations by computations of numerous areas, and he actually gave methods for finding maxima and minima corresponding to those at present used in the differential calculus. Nevertheless, his greatest passion was number theory, a field in which he became a pioneer* (Ore, 55).

The entire process just outlined is aptly summarized by Møller Pedersen (48): *Briefly, it may be said that the mathematicians in the period preceding the invention of the calculus* [she refers to the "invention" by Leibniz and Newton] *blazed the trail for its invention. They did so by employing heuristic methods, by making geometry analytical, and by seeking methods for solving problems of quadratures and tangents.*

Two problem areas may, as intimated earlier, be considered as emblematic for the whole subject, the *limit idea versus the indivisibles method* (*Phases One* and *Two*), and that of *approximation.*

There were three problem areas that lay closer to the ground of mathematical practice:
1. the questions of the techniques aiming at handling *tangent determination for various curves*;
2. the localization of *maxima and minima on curves,* and
3. the much more complex and evasive subject of the *relationship between differentiation and integration.*

With regard to the *philosophical* issues involved in the protocalculus story, Boyer comments that *the derivative has throughout its development been... precariously situated between the scientific phenomenon of velocity and the philosophical noumenon of motion* (one can understand motion but not measure it unless it is defined as speed or velocity) (Boyer, 1959, 5). This "conflict between incommensurables" may be proclaimed, if not *the*, so at least *a* crucial issue in the whole process.

From a larger perspective and with a barely permissible simplification, it may be said that there were three roughly sequentially emerging philosophical leanings, towards Aristotelian top-down or Archimedes' bottom-up mathematical teachings, and finally, the modern experimentation with techniques under development. According to Baron (108), Kepler was *the first to leave the Archimedean proof structure, and introduced free use of infinitesimals in the determination of areas and volumes* (further on Kepler's methods, see Baron, 108ff., to some extent to be traced to medieval sources, Baron, 116). Kepler's *Stereometria* became a spur, both to Galilei and to the latter's pupil Cavalieri. On the other hand, facing astronomical problems which would have required trigonometrical integrals, a technique which was not mastered at the time, Keppler applied a method for summing up infinitesimal trigonometric quantities. Baron has a detailed account of *the arithmetisation of integration methods* (Baron, 151 - 162).

I should take Mancosu's words about the *philosophy of mathematics* as a warning against simplification. *By "philosophy of mathematics" I mean the specific set of concepts, categories, and theories employed, implicitly or explicitly, by philosophers and mathematicians in their discourse on mathematics,* with the accent upon *mathematics as it is done, not as it should be dome...* (Mancosu, 3f.). In this way Fermat *might* have seen how close he came to the Fundamental Theorem - formalizing the inverse relation between differentiation and integration (3.5.10), even though he appears to have understood in an intuitive way the inverse relationship (Boyer, 1959, 164). Or how Descartes implied the notion of *limit* without being aware of it (3.5.3.on tangents).

There were other themes with a definite philosophical slant.

Torricelli's conception of science accepted, like Cavalieri's, the principle of *heuristics*, differently from their teacher Galilei. The question is of some consequence, for it concerns the adoption of experimentation also in mathematics; the idea, however, was far from new.

In a letter to Michelangelo Ricci of 1646 Torricelli wrote:

I don't care much whether the rules of motion are true or false, let us image they are true as we have been used <to believe>, and then make all the deductions one can derive from these principles... (*Che i principi della dottrina de motu siano veri o falsi a me importa pochissimo, fingansi che siano veri conforme habbiamo supposto, e poi prendansi tutte le altre specolazioni derivate da essi principij, non come così miste, ma pure* (= *puramente*) *Geometriche. Io fingo o suppongo che qualche corpo o punto muova all'ingiù et all'insû con la nota proporzione et horizontalmente con moto equabile*) (Torricelli, *Opere*, p. 357). The method of heuristics was used also by Giordano Bruno (*see Supplement* I, *Comment* 5).

Not that attitudes had been massively on the side of realism in earlier days. In the second half of the fourteenth century, three views were abroad: that mathematical forms like circle, line and points were real and true entities; that they were formally correct by definitions, but having no other validity than in terms of usable method tools (*definizioni formalmente corrette, ma non aventi altra validità che quella delle regole di ragionamento inferenziale vero ex suis terminis*); or thirdly, that mathematical notions are fictive objects, images created by the intellect (Vescovini, 284).

The deepest split in the attitudes to science laid between the alternatives of *measuring and recording* and *causal explanation of observables.* While Descartes tried to see the world through mathematics, Galilei used mathematics to describe nature as this was studied by physics and used by craftsmen, regardless of quality and causality considerations (in the Aristotelian tradition of *natural properties*) over to quantitative description focusing on *what and how, not on why*. Descartes, in a famous letter to Mersenne, criticized Galilei for not trying the explain the *nature of weight* and disregarding the *why*. But Galilei was not always consistent. Thought experiment counted for as much as real experiments, as when, in his *Dialoghi*, Simplicio asks Salviati (Galilei's mouthpiece) whether he has experimented in reality with what he claims (concerning motion), and the latter replies that he has not conducted the experiment, for it must necessarily be as he says (*Io senza esperienza sono sicuro che l'effetto seguirà come vi dico, perché cosí è necessario che segua*). Kline notes how mathematical abstraction removes the view away from complex nature, but, paradoxically, just because of that, brings nature stronger into view.

Of course studies over area and volume, destined to become *integration,* necessarily were tied to geometry.

Johannes Kepler, in his work on calculation of volume in wine casks (*Nova stereometria doliorum vinariorum,* published in Linz in Latin in 1615, a simplified German version the year after) applied the ancient (Greek, in fact) method of *exhaustion* in his determination volumes in wine casks. Because of new developments in vineyard economics, new and bigger casks were required, and the volumetrics became an issue. Kepler focused on what was to become *integration* (measuring infinitesimally bounded areas and volumes), and his search for the best method for calculating the volume of wine casks that, because they are not cylindrical but bulge out around the middle, required a mathematics of integration. The commission led Kepler to consider *a number of problems on maxima and minima,* in some respects developing methods used by Oresme in 1360 (Boyer, 1959, 110f.). The *latter had noticed that for a form which was presented graphically by a semicircle, the rate of change was least at the maximum point* [*see Fig. 3.4 - 7*]. *The thought appeared again... in the methods of. .Fermat*. In this connection, Kepler also touched on the subject of *increments* and *decrements* (Boyer, 1959, 111) - a technique used to approaching closely approximate values.

In these and comparable cases, ideas approaching the notion of *limit* (itself an approach!) arose with varying clarity and recognition on the part of the protagonists.

Protocalculus development was not linear nor consistent, sequential or "logical", so the word *development* should be left independent of biological metaphor. Speaking of the calculus development in terms of the *limit processes of the calculus,* Eves notes that the driving force behind it was no "revolutionary" discoveries but the *gradual emergence of contradictions within the discipline itself...*. Mancosu, 1996, too, repeatedly emphasizes the *continuity* from earlier times; and the main problem was that here one *had to come to serious grips with the old and difficult concept of infinity* (Eves, 174). It took a long time before the *unsatisfactory foundations* (Eves' term) were investigated, *for, after all, the processes employed justified themselves for the researchers in view of the fact that they worked* - but that, of course, was long after the time contemplated in the present book.

The central technical quantity of full-fledged calculus - that of a *limit* - took even longer to be made fully workable (for the following informations, see Eves, 177f. and 263). In 1754, Jean-le-Rond d'Alembert noted that a theory of limits was needed, but only in the nineteenth century, *Cauchy took the first steps toward resolving the crisis* [in mathematical handling of approximation] *by replacing the hazy method of infinitesimals by the precise method of limits* (Eves). *The limit concept is certainly indispensable for the development of analysis, for convergence and divergence of infinite series also depends on this concept* (Eves). Boyer's characterization of the historical context for the emergence of the limit concept should be quoted in part here; for the passage or transition *from the geometrical visualizable to the thinking in pure numbers* occurred gradually from *Phase One* to *Phase Two*. He notes: *In order to free the limiting process... from the geometrical intuition inherent in the notion of an area, mathematics was constrained to give formal definition of a concept which should not refer to the sense experience from which it had arisen* (Boyer, 1959, 9).

It was a highly relevant shortcoming that *seventeenth- and eighteenth-century mathematicians had little understanding of infinite series* (Eves) - number strings indispensable for definite integration (assessment of area under curve or volume in terms of series of infinitesimal "strips" added up).

When in 1797 Joseph Louis Lagrange developed rigorization of calculus further, still he did so without considering the values of *convergence* and *divergence*, without which handling and using infinite series cannot function properly.

3.4.4 *Forerunners*

The importance of looking at what the so-called forerunners did and thought lies in the following consideration, apart from the obvious fact that they set up platforms for later development and use. What I have in mind, is the question of *need*. They may have been conscious of certain requirements they could not meet in a satisfactory fashion, but attempted responses to which, nevertheless, might lead them to visualize, however vaguely and in fuzzy outlines, some handles or tools which were later to be connected with features that could result in a solution. Such "prefigurations" often came in purely verbal terms - or, as we see often enough, in geometrical configurations later to be codified algebraically: verbalization followed by technical processing.

A digression about "medieval" traditions - to supply the notes already submitted - is especially justified by the necessity to make clear two things. *First*, that there was no abrupt, rather a continuous, though by no means linear, passage from the mathematics of the fourteenth century to what followed in the seventeenth. *Second*ly, and consequently, that the Spire in the conceptualization proposed here, did not mirror something completely new. Perhaps we can say that in the "medieval" world, with less developed and hence less usable techniques, philosophical issues were in the position of claiming more attention; *motion* rather than *velocity* and *speed*, as we have just seen. When things elude description or explanation, philosophy tries to fill the gap.Building theoretical models is quite another matter.

Pre- and *protocalculus* ideas and techniques were wrapped up in general science images, and the medieval heritage should have been referred to more amply that is possible in the present account. So a few points will have to do, saying only that much more "modern" stuff than is often recognized, was already there some centuries earlier, as demonstrated by Boyer, Baron and, above all and more recently, by Grant in his book, *The foundations of modern science in the Middle Ages*, for which I have been using the Italian translation, *Le origini medievali della scienza moderna.*

The years between Leonardo da Pisa's (Fibonacci) *Liber abaci* of 1202 and Luca Pacioli's *Summa de arithmetica* of 1494 have been considered a barren period in European mathematics; a view Boyer and especially Grant achieve to modify substantially. Instead of isolating technical progress in the purview, they regard the broader aspects of mathematics - the speculations and investigations which led up to the propositions which are in the end deductively demonstrated - it will appear that this so-called barren period furnished points of view of significance in the development of the calculus. We know that math ideas and attitudes can be active previously to being set down in formalisms.

Aristotle, whose work was prohibited at the University of Paris in 1210 and then, in 1255, prescribed for use there, had, in his *Physics*, which by now began to be accessible, considered in some detail the infinite, the infinitesimal, continuity, and other topics related to mathematical analysis. In the fourteenth century his work was studied assiduously; in Peripatetic (Aristotelian) philosophical tradition: rather than postulational thought. This nevertheless meant sustained interest in such conceptions. Generally, Boyer (1959, 67f.) notes, the Scholastics centered upon the

metaphysical question of the reality of indivisibles, rather than upon the search for a representation which should be consistent with the premises of mathematics. There was in the medieval views no conception of the rigorous foundation of arithmetic which has characterized modern thought upon the subject. Nevertheless, the fourteenth-century disputations on the indivisible represent a keen appreciation of the difficulties involved and a clarity of thought which was, several centuries later, to lend respectability to the infinitesimal method leading to the calculus

Another comment by Boyer provides an excellent summary - and another argument against the simplification of assuming a "revolution".

The blending of theological, philosophical, mathematical, and scientific considerations which has so far been evident in Scholastic thought is seen to even better advantage in *a study of what was perhaps the most significant contribution of the fourteenth century to the development of mathematical physics...* The novelty: *which in the end led to the concept of the derivative... <a> dialectical rather than mathematical method - of studying change quantitatively, and thus admitting into mathematics the concept of variation* (Boyer, 1959, 70f.).

Nevertheless, the Scholastics found themselves trapped by their insistence on *exactitude*. The infinitesimal calculus, strictly speaking, is a branch of mathematics that handle *approximation*, even though this occurs on an infinitesimal level. Hence the example of Greek, especially Euclidean *exactitude,* would not be very encouraging for its development. But medieval mathematicians tended to adhere to Greek theory in this respect.

The *overpreciseness* [of early Scholasticism]... *which was anathema to the growing Humanism, was gradually replaced by Platonic and Pythagorean ideas and allowed to geometry what Aristotle's philosophy and Greek mathematical rigor had denied - the free use of the concept of infinity and the infinitesimal which Platonic and Scholastic philosophy had fostered* (Boyer, 1959, 88ff., concerning Cardinal Nikolaus von Kues, *Cusanus*, 1401 - 64).

Even such elusive notion as *continuity* was on the agenda (for continuity *see* 3.4.2, end of section, and 3.5.5). The larger subject of *infinitesimals* was discussed in many connections, among others by Bradwardine (1328. *Liber de proportionalibus*), in the midst of Peripatetic opposition to the idea of *atomism* (infinitesimal, indivisible parts, but still parts). He studied the *nature of continuous magnitude*, and claimed that, although including an infinite number of indivisibles, such a magnitude is not made up of atoms (*Nullum continuum ex indivisibilibus infiniti integrari vel componi*): using perhaps for the first time the term *integration* (Boyer, 1959, 66f.). For Aristotle infinitesimals had only potential existence; many scholars now discussed them in terms of real existence.

Baron, showing a circular arc (with O and H at the extremities and E at the summit), notes (citing M. Clagett, *The science of mechanics,* pp. 392 - 401, which I have not seen) that in the fourteenth-century,

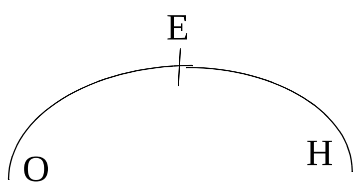


Fig. 3.4 - 7 Arc with summit

Jacobus de Sancto Martino in discussing a curvilinear form (a circular segment) makes some useful comments... He notes that the velocity is greatest at the highest point E, that acceleration (intension) ceases at E and that deceleration (remission) begins at E, that acceleration decreases from O to E ...

etc. (Baron, 87). Whether he handled this case in a manner that proved correct or not is immaterial in the present connection. What is important is that this kind of thinking is attestable that early, which linked up geometrical shapes with acceleration (changes of velocity); important in itself but also since it leads to the immediate supposition that similar or comparable ideas were entertained while not necessarily being documented.

The medieval scholars also sought to *come to serious grips with the old and difficult concept of infinity* (Eves, 174) Concerning Richard Suiseth, nicknamed *Calculator*, whose *Liber calculationum* appeared after 1328, Boyer (1959, 69f.) cites the following important comment:

> *... all sophisms regarding the infinite could be easily resolved by recognizing that a finite part can have no ratio to an infinite whole* [*Infinita quasi sophismata possunt fieri de infinito que omnia si diligenter inspexeris quod nullius partis ad totum infinitum est aliqua proportio faciliter dissoluere poteris per predicta.*]. - And Boyer continues: *This conclusion, he said, would be conceded by the imagination, for the contrary would imply that any part, when added to the whole, would not change in magnitude. Arguments with respect to the infinite do not proceed, therefore, as do those concerning finite quantities*
>
> (*Que potest concedi de imaginatione et causa est quia nulla pars finita finite intensa respecta tocius infiniti aliquid confert quia nullam habet proportionem ad illud infinitum si tamen subiectum esset finitum conclusio non foret immaginabilis quia tunc conclusio immediate repugnaret illi positioni quia tunc quelibet pars in comparatione ad totum conferet aliquantum et sic non est nunc ideo nullum argumentum proceditur de infinito sicut faceret de finito*). Both quotations are from the *Liber calculationum.*

Boyer here notes the striking difference brought about in Galilei's conception of relevant issues.

Thus the Scholastics in many respects contributed to ideas and techniques that surfaced later on. To a certain extent following Aristotle, they developed the quantitative form of expression which was so successfully developed later in the seventeenth century. Some, like already Bacon, brought in the concept of impetus to the *idea of motion*; the notion that a body, once set in motion, will continue to move because of an internal tendency which it then possesses, rather than, as Peripatetic doctrine had taught, because of the application of some external force, such as that of the air, which continues to impel it. Here, some of the ideas of Galilei were foreshadowed.

Then also the intuitive notion of *instantaneous velocity*, crucial to the calculus, was implied by the quantitative study of variation of the fourteenth century, an idea excluded by Aristotle from his science. At this stage no precise definition of *instantaneous rate of change* could, of course, be given - nor was one given by Galilei - but there appeared at the time a large number of works, more philosophical than mathematical, all voicing the intuition of this concept.

These works were devoted to an investigation of the *latitude of forms,* that is, of the variability of qualities, such as degrees of intensity, density, velocity, intensity of illumination; phenomena that require access to a calculable notion of limit (3.5.2). The concept of form, inherent variable qualities, laid a basis for *verbally* treated problem of variability, to be replaced later in mathematics and physics by algebra and the differential calculus; which latter subject is now to be considered.

3.5 Curvature features

My main assumption, a hypothetical platform and no "claim", concerning the relationship between the Spire helix and contemporary mathematics, is that *awareness of the curve as an image of continuity* (for continuity *see* 3.4.2, end of section, and 3.5.5) *among initiated people, would come readily and would lead to the centre of contemporary debates in mathematics and physics.*

This indeed *was* the dimension most directly linked up with two problems in focus in all debates on protocalculus subjects, how to handle *tangents* (3.5.3) and *infinity* (3.5.6). In addition, both tasks required acceptance and use of the fundamental operator in calculus-related work, that of *approximation.* Any helix would seem potentially to advance in both directions, beyond its registered end points. At the same time, since it is bent not in a plane but in a 3D space, its curvature must have seemed challenging, even if Galilei counted it among the simple curves.

Before starting the process of examining the mathematical parameters that may more or less directly be conceptually connected with the helix of the Spire, there is one thing I want to emphasize by repeating it. *The Spire helix serves as a simple model for handling this relationship also regarding the rest of Borromini's differerential curves and manifolds.*

As stated already, I have chosen an *order of presentation* by following a course that seems the optimal one for approaching these relatively complex matters. From the point of view of those who did consider the architectural problems, Cavalieri's *indivisibles,* which is the first subject in this *Chapter*, were hardly uppermost in anyone's appreciation of it. This subject, however, offers the preferable theme of introduction to the more closely relevant ones. Therefore, topics that probably were much more in the focus, the themes of *general curve issues* and of *infinity*, are presented later on, in fact after having cited some crucial aspects of the relevant mathematics.

Some attention to mathematical practice is unavoidable since, as Mancosu points out, emphasis on mathematical practice is justified by the fact that the *foundational reflections* *arise closely from the development of mathematical techniques in the seventeenth century* (Mancosu, 4).

I have said I want to break the Spire helix down into small pieces and concentrate on any one of them, since it is here that calculus scenarios arise critically. In two interconnected regards, however, the entire curve has to come into view. Visually and conceptually, the immediate contact with the Spire would encompass the helix as a whole. This vantage point will be taken when it comes to the items of *change, variation*, etc. (3.5.7), and *infinity* (3.5.6).

The following paragraphs bring short summaries of essential points to be presented more in detail in *Supplement* II, which also is designed to extend the overall image. Under each heading, I shall first briefly discuss possible relations between the abstracted curve and the theme at issue, namely the Spire (*helix connections*); thereupon elaborate this theme.

I have found it impossible to posit a *definite distinction* between what the following paragraphs (3.5.1 to, and including, 3.5.11) should contain and what should go into the corresponding *Supplement* (5.2.1 to, and including, 5.2.12). This is so because technical, procedural and philosophical matters are so tightly intertwined; which is what will happen especially in transitional phases in which solutions were tried out that would later turn out to be fundamental or, at least, important, based on operations that were still very much subjected to surmise, *inchoate processes* and *sidetracking* (4.3.7). Generally speaking, however, historical survey material will more typically be presented in this *Part* III, while in the *Supplement*, the balance is tilted more towards technical matters.

NOTE: In the following, the numbers in parenthesis after the Section heading refer to the relevant sections in Supplement II.

3.5.1 *Indivisibles and infinitesimals (5.2.1)*

•*Helix connections*

The shape of the Spire, extraordinary as it was in its chosen position, would have called attention to its peculiarities among people familiar with the *thematics* (3.2) with which it would be connected and thereby be situated in the general compass of mathematically interesting curves. The Spire would stand out as a *cognitively activized object*, an *undirected attractor*.

Seen thus as a thing by itself, the noticeable feature is that the curve is a helix in the general sense of the term, and specifically one with increasing torsion or pitch; and it is three-dimensional and bounding a rather complex space. To scientists and mathematicians, the configuration may have seemed challenging enough to draw attention to the whole curve as well as infinitesimal significance attached to it, so that interest might focus on any small section of the curve and thus give rise to protocalculus speculations.

Cavalieri, chief protagonist in the present *Section*, was one of the most prominent and productive among those who turned such "speculations" into usable procedures. It must be emphasized that his and related efforts were mostly dedicated to determining *areas and volumes bounded by curves and surfaces*. A number of pivotal formalisms concerning curvature did require consideration of infinitely small points. Evaluated under this perspective, our piece of curve would be one outstanding case of a curve that required consideration of *infinitely small points* or approaching them by imagining infinitely small distances (as in *Phase One* and *Two*, 3.4.2). Things could easily work both ways, from general curve-related concerns conceptually applied to the Spire helix - so to speak by imputation (3.1) - *or* from observing the helical curve viewed directly with associative calling-forth of such features. This would probably be a an actual consideration also for the other differential curves and manifolds in Borromini's work (1.5 with *subheadings*) and goes for all the items discussed in the present *chapter*.

•*Problem outline*

Some notes concerning Bonaventura Cavalieri's achievement may serve for introducing the rather complex topic.

Bonaventura Cavalieri's *Geometria indivisibilibus* of 1635, launching ideas that were, however, shared by many, became fundamental in the calculus development. Cavalieri built his theory partly on Kepler's book *Nova stereometria doliorum vinariorum,* Linz 1615. Baron notes his troubles in distinguishing between mathematical indivisibles and physical atoms but regards modern criticism of this uncertainty as historically unsound. *Cavalieri did explain his indivisibles in the only terms he knew - the language of Oresme and the mediaeval Calculators* (Baron, 123). He also had problems with the *continuum* (Baron, 125), a feature arising from concern with the infinitely small. It is interesting to note that he (like others, such as Torricelli) took a *heuristic* view of his methodological work, using the device of an *artificium which works, rather than any definite or dogmatic view as to the nature of indivisibles and the spaces which they occupy* (Baron, 125; she offers a thorough description of his method: pp. 122 - 135).

Cavalieri's idea of *indivisibilia* sits right in the centre of the early *protcalculus* development in the crucial years from his announcement of the theory in 1629 to 1656, when André Taquet published his *Arithmeticae theoria et praxis*, thus to include 1655, the year in which John Wallis published his *Arithmethica infinitorum.*

The printing history of Cavalieri's book must mean that the theory was considered useful or interesting enough to a large audience for it to appear in another edition some forty-three years after the first edition; eighteen years after it had been announced and obviously made available, then probably to a more narrow circle of scholars. In fact, *these two works of Cavalieri rapidly became the most quoted sources (save for Archimedes) on geometric integration in the seventeenth century, and Cavalieri himself is frequently named as the founder of the new analysis* (Baron, 123).

The title of Cavalieri's first edition in fact contains two terms that were emblematic in the protocalculus context, and I shall return to them in due time: the idea itself of *indivisibles*, and the concomitant one of *continuities* (3,4,2). The argumentation, therefore, also introduces the notion of *infinity*. Even though his efforts are directed toward a practicable computation of line, surface and volume, and the three notions so to speak emerge automatically, the notions they convey are decisive for the development of *differentiation*, and for working with *derivatives*, as well.

In his attempt to obtain the chair of mathematics at the *Studium* of Bologna, which he did receive in 1629, Cavalieri wrote his first work on the subject, published later, in 1635, as the *Geometria indivisibilibus continuorom nova quadam ratione promota,* in 1647 followed by his *Excercitationes geometriae sex*. The *Geometria*, to quote Baron (122),

> *was the first complete work entirely dedicated to indivisible methods and it received wide publicity, was extensively read and its contents hotly debated.* The term *indivisible,* however, *is medieval in origin, had been familiar to Bradwardine.... and was used extensively by Galileo.* Cavalieri *for the most part... presents his results entirely in verbal and geometrical form* (Baron, 123f.).

Cavalieri was a pupil of Galilei. His ideas appear to be reflected in Galilei's *Two sciences* (for a careful historical evaluation, see Boyer, 1959, 111ff.). Generally, the idea of indivisibles is linked up with those of *infinity* (in Galilei, for instance) and *continuity* (for continuity *see* 3.4.2, end of section, and 3.5.5). Much of the effort was directed upon *integration*, regarding the *definite integral* (this applies to Torricelli, see Boyer, 1959, 123ff.; see also the debate between Cavalieri and Gulding, focused on the nature of infinite elements, in Boyer, 1959, 121f.).

Cavalieri, it should be pointed out here (as does Toscano, 77), worked not with *numbers* but with *proportional relations between surfaces and volumes*. This observation is crucial for the understanding of the transitional state of relevant mathematics at this time.

Let me quote from D. E. Smith's *History* (I, 362f.),.

Cavallieri's theory is based upon the assertion that a line is made up of an infinite number of points, a plane of an infinite number of lines [see also 1.5 and *Fig. 1.5 - I comment,* for references arising with the interior of Sant'Ivo], *and a solid of an infinite number of planes. The theory thus forms the basis of a crude kind of calculus, and by its aid Cavalieri found it possible to solve many problems in mensuration that would now be solved by the more scientific methods of integration. The term'indivisible' is ancient, and Cavalieri made no claim to originality in its use*.

Here one might note by way of an aside that instead of "more scientific" one could say "more modern", for certainly Cavalieri's work was eminently scientific by the standards of his time and must have been so regarded, despite, or better: just because of the fact that it drew attacks from such a scholar as Paolo Guldino (Mancosu, 50ff.) and other contemporaries.

Cavalieri launched a systematic use of summation (today expressed by the symbol Σ), a collective and a distributive version that complement one another (details in Baron, 125 - 135), and further developed, among others, in the work of Grégoire de Saint-Vincent (1584 - 1667) (Baron, 135 - 148).

We have seen that in this evolution the main interest, or should we say, the *stated* preoccupation, concerned *areas and volumes*, and that the technical procedures employed methods representing a preparatory stage of *integration*. *Geometry*, then, provided the principal scope and also generally the testing ground. The idea of points on a curve (3.4.2), which is the crux of differentiation, so to speak hovered behind the stage, while lines and surfaces came directly into view.

Operating with infinitesimal entities for these elements, Cavalieri met with difficulties when trying to define the notion of *all the lines* and *all the planes.* This debate conveys a very illuminating picture of the kind of problems facing the scholars and of the terms in which the issues were cast. The Jesuit mathematician Paolo Guldino (1577 - 1643), among others, criticized these "alls" as vague and unreliable (Mancosu offers a penetrating review of the *querelle* between the two). One big problem here arose from Cavalieri's stipulation of *ratios* between such entities: how can you determine ratios (a : b, a/b, etc.) between anything else than finite quantities? Most particularly, how can one establish ratios between one finite and one infinite number? Finally, Guldino criticized Cavalieri for confusing issues in his attempt to determine the apex of volumes. What he found became a kind of *tangents*, while the complexities of the tangent issue (3.5.3) were not as yet being recognized; and his proofs only showed the *possibilities* involved rather than establishing certainty.

The much-criticized notion of "all the" infinite lines or planes also necessarily implied the *composition of a continuum* out of infinite quantities of discrete (numbers of lines) elements. Since the notion of a continuum was unavoidable in order to create lines or surfaces or volumes, Cavalieri was blamed for not making it clear how such a continuum could come into being. In response he launched a *second method* aimed at obviating the reliance on *comparison of infinite aggregates.* Here he tried to evade the dependence on *infinity* in his technical procedure (if not in the basic idea). Drawing parallel lines across geometrical figures of given height (so this *had* to be given), he proceeded to measure the figures i terms of line segment and developing an infinitesimal overall image from them.

Finite and infinite entities were to be compared, continua had to be construed out of units that had to be regarded as units without being in any way understandable as discrete units.

These are deeply philosophical problems even though they were hardly elaborated as such. Other philosophically fundamental (Mancosu calls them *foundational*) issues arose from the experimentation and debates. One concerned the difference between producing *certainty* (in technical proofs) and conveying *evidence.* Many mathematicians voiced dissatisfaction with the Archimedean tradition of construing proofs, protesting that it was never shown *how the results had been found*. Another cluster of issues regarded the very idea of *causality*.

Extra-mathematical considerations further muddied the issue. Jesuit mathematicians refused to accept the very idea of indivisibles, probably because in this notion, once it was applied to reality, they saw a potential conflict with the dogma of the absolute reality of the transsubstaniation of the Eucharistic species of bread and wine into Christ's body and blood at the Sacrament (Mancosu, note 18, pp. 219f., with references). Cavalieri himself belonged to a religious order, the *Gesuati* (not the one of the *Gesuiti* - Jesuits).

3.5.2 *Limit (5.2.2)*

•*Helix connections*

Having established *probable* protocalculus relevance of Borromini's Spire in terms of a focusing on infinitesimal sections of a curve, the following question arises.

•Problem outline

If our view is going to be focused on an ever diminishing portion of the curve, right down to what Cavalieri and others with him called *indivisibilia*, at which point is this process going to stop, or better, how can we determine an end terminus sufficiently close to zero to be honored with the name of *a point*?

The idea of *limits* may perhaps be thought of as *the* crucial operator in full-fledged infinitesimal calculus and some space must therefore be devoted to the subject. It can only be introduced here in a very crude format, saying that in differentiation it is necessary that the increment (5.2.3), which is a dummy variable used to reduce a given quantity to infinitesimal size, does not become equal to zero, but that it approaches zero within a margin that is defined as amenable to infinite diminution; or it approaches infinity (for this even more complex notion, see 3.5.6 and 5.2.7).

In many connections, both medieval and early modern, the need for some kind of limit concept seems to have been vaguely felt. Such feelings can be productive in that they animate ventures that would achieve something more graspable if the hunch could be turned into knowledge, however provisional, and operative capacity. In Roberval's work with infinitesimals, for instance, *the idea of limit is implied, but is considered under the terminology of Roberval's method of indivisibles* (Boyer). He touched on the idea without being fully aware of it. One of his lines of argument, in which he possibly followed Luca Valerio, is equivalent to a statement that the limit of a quotient of variables is equal to the quotient of their respective limits (Boyer, 1959, 145f.).

The idea of *infinite diminution* was entertained by many, but would this yield a solution to the issue? It must be emphasized once more that there are two historical alternatives, *either* using *indivisibila* or *infinitesimals,* going *inwardly* to a point, subdividing the quantity in ever smaller units until a certain precision is achieved; *or rather* working with *limits,* making a calculation of a curve *approach across an increment, a limiting value that was, however, never reached.* John Wallis' pioneering achievement consisted partly in opting for the latter tack (Baron, 205ff.). The slippery subject is intuitively introduced in *section* 3.4.2, sketching out the alternative (often overlapping) *Phases one* and *two.*

A technique then in its initial phase and somewhat scattered attention and study concerned the use of *increments,* the adding of small sections that were subsequently shrunk to gain control over the limit determination (3.4.2 and 5.2.3).

3.5.3 *The tangent problem (5.2.4)*

•Helix connections

Once we have shifted our attention, and imagining scholars among our historical protagonists to have shifted theirs correspondingly, from the entire helix down to minute bits of curve, in fact *points* on it, then a number of questions arose, among them the *problem of the tangent,* an entity intensely in the focus of seventeenth-century mathematics and physics.

Tangents became more and more a crucial issue in handling bent curvature, and a helix is all bends, nothing but, as Wodehouse would say. This problem was in the focus of professional interest so intensely that it would seem inconceivable should it not have been evoked on beholding the Spire.

•Problem outline
Tangent issues have been briefly introduced above (3.4.2).

The question arose, how to account mathematically for the *bend* of the curve at any such arbitrary but "almost infinitely small" stretch, in fact an infinitesimal point - and how to tackle the task of positioning a tangent to a curve which is *continuous*, offering no single definite *point-like handle* for grasping the subject and no directly observable or measurable *direction* to our view?

To establish the infinitesimal *spot* at which we want to study the curve's behavior - how crooked *is* the bend? - and to exploit whatever fact we can derive here for further calculations, then we have - and *they* had - recourse to the *tangent* at this point. How to construct the normal (or orthogonal) on to an infinitesimal point on the curve?, i. e., *at 90 degrees to the imaginary tangent*. It was precisely this way the problem, a major one at the time (3.4.1), was posed and tackled (cases and details in 5.2.4). One may pick out a tiny bit of curve and ask this question. Equally critical it becomes when one makes the curve rise up to form a crest above the straight line of the x-axis. Where is the top point, the *maximum* (or the opposite alternative, the *minimum*) (*Fig.s 3.4 - 6* and *7*), and what is going to happen there, where the tangent must necessarily be horizontal? It has to be remembered that for a long time the Cartesian coordinate system belonged to the future.

The theory of indivisibles took care of part of the problem, but only as regards any curve as a whole, as a specific geometrical shape or form.

A tangent at a point of a curve must be symmetrically placed in relation to equal stretches - but how to measure them? - of the curve on either side of the point and come at 90 degrees to a line, like the radius of a circle, through the point. Obviously, such a verbal description as this is far from being clear, but it does reflect the trickiness of the whole issue, since several floating variables have to be set in a fixed relationship while having no obvious metric as a point of departure or anchorage (which one does have in a circle).

A comment by Descartes highlights the importance of the issue. Claiming that he had developed a *general method of drawing a straight line making right angles with a curve at an arbitrary point upon it,* he affirms that *this is not only the most useful and most general* *problem in geometry that I know, but even that I ever desired to know* (quoted by Møller Pedersen, 16).

Fermat's tangent method (Baron, 165f.) *occupies a significant place in the history of the calculus for in it occurs, apparently for the first time, the idea of a slight change of variable. By manipulating equations, he obtained extreme values in the variables, approaching the idea of limit* (3.5.2 and 5.5.2); a crucial step even if he was not at the time entirely aware of its real novelty.

Let the curve on *Fig. 5.2 - 2* be y = f(x). The task now is to find the subtangent TQ and from this find the tangent TP. In the following synopsis of Fermat's tangent *approach* I shall adopt Thorvaldsen's figure (Thorvaldsen, 76), which is, however, more or less identical to the one used by Møller Pedersen (Møller Pedersen, 27).

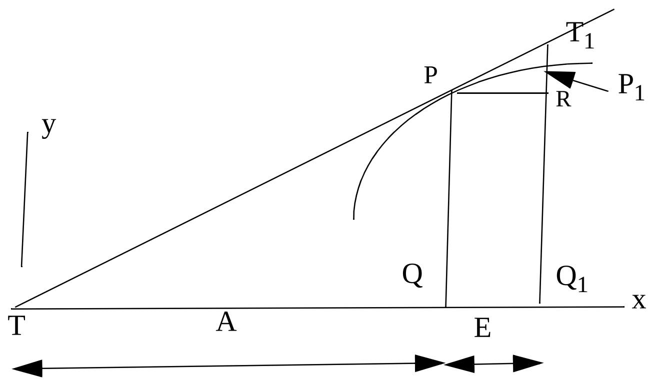


Fig. 5.2 - 2 Tangent construction

The increment (3.4.2) QQ_1 = E. The triangle TQP is congruous with PRT_1. Hence TQ : E = QP : T_1R. Fermat makes $T_1R = P_1R$. That means shifting P_1 toward P along the curve. *The increment then approaches zero*. This makes TQ : E = PQ : (P_1Q_1- QP). Setting PQ like the y-value, namely PQ = f(x), we obtain TQ : E = f(x) : (f(x + E) - f(x). Multiplying on both sides with E, we obtain: TQ = f(x)E/(f(x + E) - f(x)). Setting - illegally but usefully! - E = 0, Fermat obtains TQ, and the tangent is there. Having found the tangent, the critical point of the infinitesimal value of change of direction, or of speed, is also found. This was the closest approach to modern calculus at the time - the closest approach to the modern *science of approximation*.

3.5.4 *Maxima and minima (5.2.5)*

•*Helix connections*

Since the question about maxima and minima concerns extreme points on bent curves, the problem is directly linked up with the tangent issues (again 3.4.2); while the helix itself, being, one might say, *all maxima*, must have alerted inside people to the subject - which preoccupied them anyway.

The subject may have been considered directly in connection with the helix when any stretch of its curvature is mathematically translated onto a plane one and thus considered as a system of *maxima*, of the extremes of the curve (or *minima*). Such an experimental approach is attributable to the thinking of the time. Then the subject does arise in the general preoccupation with the nature of curves and the methods for handling them, to repeat, in connection with the tangent issue, since horizontal tangents indicate maxima or minima and identify points at which the derivative is zero (f'(x) = 0). Further uses of the max and mini concern the so-called Mean Value Theorem and other techniques devised subsequently.

•*Problem outline*

Intuitively, the issue of *maxima and minima*, or *extrema*, concerns finding upper and nether bulge of a curve, or the crest(s) or the trough(s) on it (*Fig. 3.4 - 6*). For the development of the calculus, the points identified here are central business. Finding them and working with them is also a question of locating the corresponding tangents.

In the general pursuit of curve issues, the mini-max problem would arise with the rest. Finding *extrema* had occupied Kepler (1615), who noted that there are imperceptible changes to a curve *close to* an *extremum.* The modern expression for this is that this holds for a function that is *differentiable*.

The common awareness that one had to face the problem of *maxima and minima* had a tremendous impact. For not only did the issue touch directly on calculus methods but it also brought in its wake, at least indirectly, questions of boundaries and limits and hence also ideas about infinity. In some respects, the idea of maxima and minima is related to mathematical approach processes that at the time in question came quite close to a recognition of the notion of a limit (5.2.2); thus in Fermat's tangent method (5.2.4), the maximum of a variable (*extrème*) (Baron, 167f.). Touching informally on the notion, Fermat discusses extreme values in his tangent method, and here he shows *how to distinguish between a maximum and a minimum, using algebraic manipulations*.

3.5.5 *Continuity (5.2.6)*

•*Helix connections*

Just a look at the Spire helix would conjure up the notion of continuity: the curve winds upwards toward the cross without any break until it is cut off at top (and bottom); at the top an idea of its ideal or notional continuing upward *ad infinitum*, getting smaller and smaller, might easily arise. The upwards perspective would be taken up by the cross at the summit, with its cosmic connotations (Bellarmino cited in 1.10).

• Problem outline
General considerations on the subject are ventured in *section* 3.4.2.

The question of *continuity* - too complex for an adequate treatment in the present book - had for a long time been an obsession among mathematicians, in part because the nature of *points* was subjected to both philosophical and technical debate and also because continuity was an ingredient in the crucial notion of *infinity* (3.5.6). The subject is central in Cavalieri's work on the indivisibles (for continuity *see* also 3.4.2, end of section.

Continuity *at a point* is what was sought but which was straining the imagination as well as the available tools. A point can be determined by means of a *tangent* (3.5.3), and, with good will, one might even attribute *direction* to it by this method. But *continuity*? What is a point, mathematically speaking (or philosophically)? A point is no *stretch* and can be subdivided infinitely into ever smaller points; and so can each of them, while at the same time they are not countable.

The issue was linked up with ideas concerning infinity and *point structure of the continuum* (Baron, 74f.) and had been discussed since Aristotle and through the Middle Ages, alternating between qualitative and numerical conceptions. The modern treatment is based on the notion of *limit* (3.5.2), and in this sense an early approach was tried out by Bernhard Bolzano in 1817. In the *protocalculus* programs, the subject is very complex, defying any reasonably brief and succinct treatment, due to ideological, technical and philosophical differences in the methods applied to the matter (for continuity, *see* Baron, Boyer, *passim*; the best modern explanations, *not-too-technical*, are in Kline, 1977, and Blank and Kranz, *Chapter* 2,3).

Problems pertaining to physics also arose.

Did an object in free fall follow its course in an unbroken, continuous curve, or was the motion determined according to *the impetus-theory or fall by quantum-jumps*, this was a crucial issue, for *like modern quantum theory, the impetus theory of fall entailed an indeterminacy. Each uniform speed, however brief, corresponded with neither a unique point in space nor a unique instant of time* (Drake, 1989, 76f.).

Boyer notes Kepler's vagueness regarding the issue and recapitulates:

This striving for an expression of the idea of continuity constantly reappears throughout the period of some fifty years preceding the formulation of the methods of the calculus. Leibniz himself, like Kepler, frequently falls back upon his so-called law of continuity when called upon to justify the differential calculus (Boyer, 1959, 11).

The idea of *continuum* presented one of the most intractable problems in early ventures into the infinitesimal universe and has been discussed already in a previous *section* (3.5.1). Descartes was one among several who grappled with it, but *His algebra... was still grounded in the geometry of lines, and the idea of continuously varying quantity was not really established in analysis until the time of Euler* (Boyer, 1959, 167; Leonhard Euler, 1707 - 1783).

It became clear that continuity was a necessary condition for handling infinitesimals and erecting tangents. Today we say that a curve is not *differentiable* at a point at which it is not continuous, when, for example, it forms a corner or a *cusp.* There are three types of *discontinuity.* Some awareness of them is useful for understanding the role of inverse: continuity (Blank-Kranz give all of them in an easily accessible format).

3.5.6 *Infinity (5.2.7)*

•*Helix connections*

As noted above, a helix may be considered as moving infinitely in two main directions, by winding downwards and upwards in the same curve mode without end, if it is *cylindrical*; if *conic*al, by winding endlessly upwards and infinitely diminishing, or downwards widening endlessly. This feature may have been noted and been found to tally with the cosmic properties of the cross surmounting the Spire (1.10).

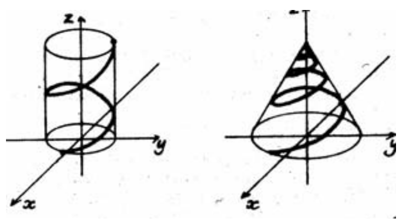


Fig.3.5 - 1 cylindrical and conical helix, both with constant torsion or pitch.

The Spire itself, surmounted by a cross, could be seen as a symbol of universality. The Cross, we have seen, was interpreted as reflecting the *structure of the universe* in a salvational perspective by no less an authority than Cardinal Roberto Bellarmino (2.2.12), of course with traditional backing. This idea represented the nearest the Church would approach spatial, rather than transcendental infinity (Lattis, *passim,* especially pp. 65ff., 77f., 84, on Clavius' cosmology), and it evaded dangerous cosmological traps. It is almost unthinkable that people at the time, who were engrossed in the issues of curves and related issues of infinity, should not have reflected on these alternatives regarding the Spire project. Thus the conical helix expresses graphically what Pascal was to call the *double infinity* (infinite largeness, infinite smallness) (Blay, 6ff.). In his *De l'esprit géometrique*, Pascal maintained that *in the sphere of number the infinitely great and small are complemetary* (Boyer, 1959, 151).

- Problem outline

As we have noted, the idea of infinity, even an infinite number of universes, is almost ubiquitous in Giordano Bruno's work. He was not alone in considering the subject worthy of investigation, nor in arriving at it through chains of argumentation started off in widely different scenarios.

Infinity is in fact a crucial concept in almost all calculus issues, and the term arises in connection with Cavalieri's method of *indivisiblia* as well as with continuity, limit and elsewhere in the present series of items. Again, as noted above, work on areas and volumes stimulated the development of the infinitesimal dimension and hence infinity. Galilei considered *an infinite number of infinitely small and indivisible parts with an infinite number of infinitely small indivisible spaces interposed between the parts* (Baron, 116ff.). To avoid obvious pitfalls, however, he clearly distinguished between the *potentially infinite* and the *actually infinite*. According to Christoph Clavius and other Church commentators, in Lattis' summary, *there can be no non-functional space or unnecessary mechanisms in the universe* (Lattis, 65), so that if cosmos is infinite, this must have a meaning. Even Descartes *always denied the possibility of an absolute void* (while this was accepted by Galilei and a basis for his *atomism*); and developed his peculiar cosmological *vortices* theory (Garber, 1992, *Index*; Goodfield and Toulmin, 158f.); a tentative metaphysical cosmology in a sense anticipating Kant's *fractal universe* (Geier, 72f.).

There are two alternative modes (apart from the directions). For infinitesimals, an almost-endless diminution towards a limit at an indivisible *átomos*; or, in both directions, pure infinity. The latter, like the number zero, is not pictorially understandable and the Church for a long time rejected both. For the Church, infinity in cosmology was a touchy issue, less so in pure mathematics. Contemporary computational operations could not get around handling infinity in one manner or another. Here, too, the Church might make an escape from the cosmological aspect to the mathematical one, which could be evaluated as a technique without cosmological implications or, at least, she could accept infinity *heuristically*, as a procedure of *instrumentalism*, the way Bellarmino was willing to consider (accept is too strong) Copernicanism (2.2.12).

3.5.7 *Change/variation of direction and intensity (5.2.8)*

•*Helix connections*

Again, the Spire's convoluted curve could easily be taken as an image of *motion* and generally changing conditions or a *modality of alteration, approximation and indefiniteness,* especially so, in fact, because the *torsion* undergoes a notable change which was and is clearly perceivable to a close-up views (*Fig.s 1.1 - 1*, *1.10 - 1* and *2*) as well as from great distance, from the Campo de' fiori and the Campidoglio.

• *Problem outline*

Since variation and change are fundamental properties of the mechanics of the math leading up to the mature calculus itself, as its focal issue, it must be stressed at once that the *present section* does not discuss this particular subject; instead, in the seventeenth century, how protocalculus tried to cope with *motion* and *change and variation in physics and nature.* These phenomena held a pivotal position in the world view of science (2.2.2).

Seventeenth-century scientists approached the modern concept of *vector* (from Latin, "a moving thing", from *vehi, move*), and some pertinent notes may be convenient (*see* 5.2.8). The concept was directly connected with that of tangent construction (3.5.3).

The study of *motion and change/variation of intensity* (in temperature, for example) had always been central to physics, from the infintesimal level to the cosmological one, and now new methods for studying it were being developed. In the context of the problem of motion, a critical point in protocalculus, as well as in mature calculus, was *change of direction* (of course critical in physics, too).

On a curve, the point of change, its *inflection* was studied, as noted above, in connection with maxima and minima and the localization of tangents. Change of direction had since long been associated with change in *velocity* and acceleration. Together with natural phenomena of change, variation and graduation of intensity or weight (which depends on height above the earth's surface), these subjects were a main concern at the time under consideration.

3.5.8 *Rectification of curves (5.2.9)*

•*Helix connections*

Torricelli tried to establish the volume of a helix and also, to assess the length of a *bent* curve if it were stretched out to make a straight line (*rectification*). With such interests in the air, the Spire might easily be called into attention: *what is the true length of the winding curve* such as it has been constructed?

•*Problem outline*

A particular problem concerning *bent curves*, was to compute their length when notionally stretched out along a straight line, a process called *rectification*, which had been rejected by Aristotle who claimed that an *arc cannot be expressed as a finite ratio of a given straight line* (Baron, 163). The study of *motion* required rectification. For instance, trying to measure the flow of water (a very popular subject at the time), you are faced with complex curvatures. How to measure their length? Desecrates, in *La géometrie*, attempted this without success on *algebraic* curves, which was to be expected, and he declared bluntly that the ratios between curved and straight lines *cannot be known*; whereas Torricelli in 1645 succeeded on *mechanical* curves, that is, graphically constructed ones (Boyer, 1980, 393f.). He resolved the problem for a logarithmic spiral. Roberval, in 1642, had established the length of an Archimedean spiral and of the parabola. The subject stimulated the discussion, even controversy, over the nature of respectively mechanical and algebraic curves.

3.5.9 *Curves, 2D and 3D, including some special cases (5.2.10)*

•*Helix connections*

Curves can be "flat" (2D) or occupy space (*space curves*: or *manifolds* - 3D). Of course space curves are crucial in architecture.

On the Roman skyline Borromini's helical Spire, especially when compared to the predominant more or less paraboloid cupolas, must have been considered as a most remarkable specimen of *curvature*, sufficient to provoke scholarly interest and markedly enough also to identify the Papal university and hence the Church as avant-garde institutions when it came to the important but, unlike cosmology, innocuous field of modern mathematics. A harmless sign on a troubled sky.

• *Problem outline*

The expression used for a heading (see 1.5), *ces lignes plus composées*, is taken from Descartes' *La géometrie* (II, pp. 40-43, in the cited parallel edition; Kline, 1999, 363). He rejects the Greek distinction between *mechanical* and other curves, saying that even a straight line and a circle requires some tool for their construction. He then censures the tendency to leave out of consideration curves that cannot be thus constructed, *on n'en doit pas plutost exclure les lignes plus composées que les simples, pourvû qu'on les puisse imaginer estre descrites par vn mouuement continu, ou par plusieurs qui s'entresuiuent & dont les derniers soient entierement reglés par ceux qui les precedent*. And he recommends focusing on those that are amenable to being expressed in a single algebraic expression (corresponding to using our *x* and *y*): *qui peut estre exprimé par quelque equation, en tous par vne mesme* (p. 49). These he classifies as *geometrical curves*; the rest, such as the spiral, he labels *mechanical*. Here, he would probably also have included the *helix*.

Descartes went further and insisted on accepting all curves that could be imagined and were to prove constructible in terms of analytic geometry, in our usage, with *x*, *y*, and *z*. Kline notes that his attitude initiates the abandonment of the traditional *credo*, that only what was provably constructible had the right of existence. Mathematics underwent a radical change, not so much by virtue of the technical advantages provided by analytic geometry, as through Descartes' principle, that a *curve is any geometrical locus that can be described in an algebraic equation* (Kline, 1999, 375; further in 5.2.12). At a later stage, through the work of Wallis and Newton, the possibility of creating *any* curve, and new curves, by using algebra, the principle led to a unification of geometrical cases that formerly had to be treated separately (Kline, 1999, 376). The new development reversed the respective roles of algebra and geometry, giving now the former priority over the latter. The incipient analytical geometry launched by Fermat and Descartes came at the right moment, just as *physics and cosmology* needed more flexibility and a better control over geometry.

The growing awareness of the productive value of *approximation*, not only as an necessary evil, became a crucial facet in the calculus development. Baron's comment at this point is very illuminating and a recapitulation of it follows here (Baron, 163):

If change and variation were such crucial concerns of the day, and scientists were used to express such phenomena by geometrical means, it should come as no surprise that almost any sort of *curvature* would be under persistent and extended debate in mathematics.

By curves is here meant "bent lines", with crest and trough, whereas strictly, in mathematics, a straight line is a special case of a curve, but it is not suitable for studying variation and thus is not of course relevant for tangent determination. From the narrowed-down point of view adopted here, there are three principal aspects of curves that need to occupy us; the activized critical interest in curves as such, the *rectification* of them, and the construction and positioning of *tangents* to them (see 3.5.3).

In the years under consideration here, even though well-known curves, such as the parabola, the different kinds of cycloids, and the conchoid, were being subjected to investigation, the scope was definitely *any curve, curves generally*. Baron is probably right in saying that *there was little general interest in the special forms of curves* (Baron, 171, Fig. 5.13). Nevertheless, in contemporary architecture the interest was sufficiently keen to justify our focusing on some special curves, too. In addition to the Spire helix, we have the 3D (third dimension) "möbius band" in San Carlino and the 3D "bent cosine curve" and manifolds in San Giovanni, to say nothing of the surface passage from concave to convex in the interior of Sant'Ivo (1.5; *Fig. 1.5 - 1*). Such things can hardly have gone unheeded by the scientists. And *general curves* certainly did not fail to command interest, even causing worry.

One of the kinds of case that did attract attention and serious concern was the *ballistic* curve of the trajectory taken by the cannon ball on leaving the cannon. Artillery had played an increasing role since it proved decisive in the battle at Governolo near Mantua in 1526 between Giovanni de' Medici *dalle bande nere* and general Georg Frunsberg (a Protestant general fighting for the Most Catholic Emperor, while carrying on the campaign a gold noose with which, as he hoped, to hang the Pope). Alfonso d'Este had delivered the artillery and generally promoted technical expertise in cannon construction (Titian portrayed him with a cannon).

Dynamics, then, literally and aggressively entered the game. Torricelli studied a large number of curves generated by a point which *moves along a uniformly rotating line.* An ellipse may seem a very simple form; it had been studied since the Greeks. Nevertheless, even as late (for our context) a scientist as Newton, familiar though he was with Kepler's elliptical planet orbits, chose to calculate them out as circles because the mathematics of elliptic motion would have raised unsurmountable problems (White, 128, note). It must be understood that Kepler's ellipses mathematically remained ellipses with two foci; but that graphically they came as near to a regular circle as almost to look like one, because of the relatively short distance between the foci.

Phenomena of *oscillatory motion*, such as is attested in a pendulum or in the strings of a lute, belong to the present area of subjects. Galilei and his father were accomplished musicians (the father Vincenzo also in music theory), and music had always been considered to be a phenomenon of oscillations in proportional systems. Robert Hooke's law for *simple harmonic motion* (in a sine or a cosine curve) came toward the end of the century.

The *helix*, to return to that, counted among curves under attentive observation, making up a conspicuous *theme* (3.2). In a letter to Mersenne of 1629 published by Mancosu (Mancosu, p. 78, French original, note 28, p. 225), Descartes discusses the *cylindrical helix*, which he does not mention in his book on geometry. The following formulations may be highlighted as especially relevant for us, the difficulties involved being expressed with great clarity.

... it is not the cylinder which is the cause of the effect [that of the helix curve],... The effect depends on the helix..., which is a line that is not accepted in geometry [as Descartes

saw it] *any more than that which is called quadratix. For although one could find an infinity of points through which the helix or the quadratix must pass by, however, one cannot find geometrically any one of those points which are necessary for the desired effects of the former as well as of the latter. Moreover, they cannot be traced completely except by the intersection of two movements which do not depend on each other, or better the helix by means of a thread* [filet] *for revolving a thread obliquely around the cylinder it describes exactly this line...*

3.5.10 *Relation between differentiation and integration (the fundamental theorem of the calculus) (5.2.11)*

•*Helix connections*

The inverse relationships between differentiation and integration was only indirectly imagined by some; alternatively, they came close to it without being fully aware of it. The idea could hardly be associated with the Spire unless we have in mind the vague feeling that here is a curve one can handle with *indivisibila* and in addition here is a shape that has circumscribes a certain volume. But the subject must be faced at this stage of our inquiry simply on account of its pivotal role in the calculus story and because in fact something like it was surmised by a few among the protagonists involved in the protocalculus venture. Torricelli came close to an understanding of this important issue, and so did Fermat (5.2.11). Thus the subject contributes to conveying the flavor of the mathematicians' concerns and their debates.

•*Problem outline*

Realizing that differentiation and integration are two closely related processes, the one the inverse of the other, took some time. In modern usage the relation is expressed in two versions of the so-called *fundamental theorem of the calculus.* That is, *differentiation* by going from finding change-points on a curve, from distance to speed to acceleration - and *integration* by going the opposite way, and finding the curve from the point, speed from acceleration and distance from speed, and so the equation, and then finding the surface enclosed or partly-covered by a curve - these procedures are inverse to one another.

We have seen that geometric methods predominated, focusing principally on *integration* methods for measuring areas and volumes, a work that concomitantly produced insights and methods also for differential problems. However, the gradual *arithmetization of integration methods* (Baron, 151 - 162), forced contemporary mathematicians to recognize in an indirect way the notion of an inverse relation between the two classes of problems, not so much technically as in terms of vague suspicion. Besides, several scientists and mathematicians came up with specific matters that led them to incipient realization of the inverse condition. Galilei's work on the trajectory of a cannon ball, proved *ultimately* <useful> *in establishing the inverse nature of differentiation and integration* (Baron, 120f.) The contribution of Isaac

Barrow (1630 - 77) and the investigations leading up to that, are discussed with appropriate attention to historical details by Struik (Struik, 253ff.).

Fermat's handling of the fundamental-theorem issue is worth noting, since it almost emblematically illustrates the merging of creative intuition and inconsistent mathematical procedures typical of the handling of complexities in the protocalculus era that were imagined but not entirely understood. Baron discusses the case, but her account is technically demanding, and I refer to a synopsis in *Supplement* II, 5.2.11.

In his work with the quadrature of the hyperbola, Torricelli was able to formulate *the tangent construction from the integral* (Baron, 185). The method he developed was *not only important as a means of integration, but also a significant transition from an integral to a differerential operation... it is not difficult to conclude, with Bortolotti, that Torricelli understood clearly the inverse relation between integration and differentiation...* (Baron, 188f.). It should be noted also that, according to Baron (183), Torricelli *recognizes the supreme importance of continuous proportions, i. e. geometric series*. Mathematical series were of crucial importance to the development of the techniques of *integration.*

3.5.11 "Cartesians": coordinate systems and geometrical loci (Descartes and Fermat) (5.2.12)

•*Helix connections*

In almost any kind of mathematical ideas surrounding the helix issue the question of the relationship between numerical (algebraic) and geometrical handling would arise, and after the publication of Descartes' *La géometrie* in 1637, the perspective was opened up almost dramatically; and especially so in connection with the *cylindrical helix* (in a letter of 1629 to Mersenne: Mancosu, 78f.; and our conical helix would naturally get a share of the interest.

•*Problem outline*

The subject for this section is algebraization of geometry and its inverse, geometrization of algebra, or, in Kline's fitting epithet, the *wedding of curve and equation*. The honor for this crucial step, admittedly a gradual and not linear one, goes to Fermat, Descartes and several others, also from earlier times. For the idea was widely known. Descartes' *coordinate system* and Fermat's *geometrical loci*, represent two subjects that conflate in many respects and which, when taken together, touch on central protocalculus problems, even if - and this must be emphasized - the main scope of Descartes' geometrical studies were intended to solving *equations* of higher degrees (Blay, *Introduction*).

Descartes, we recall, wrote a long Supplement, *La géometrie*, to his *Discours de la Méthode* of 1637. The Cartesians (as they are called today) consisted in establishing in any configuration space two or three coordinates (today usually the *x*, *y* and *z* axes), thus making it possible to express geometry by arithmetic units - and the other way around. In Descartes'work, however, the *baselines corresponding to the x and y axes of today, were not at right angles to each other*. This was, as Körner points out, more than a mere technical affair. The issue also was a philosophical one and foreshadowed later work on the general theory of *consistency*. Under the heading of *The science of formal systems*, Körner writes:

Among indirect proofs of the consistency of any geometrical or physical theory the most common are based on arithmetization, i. e. on representing the objects of these theories by real numbers or systems of such. This is by no means surprising. For on the one hand the creative work of mathematicians, at least since Descartes, has been characterized by the demand that all mathematics should be capable of being embedded in arithmetic, and, on the other hand, the creative work of physicists, at least since Galileo, has been characterized by the demand that all physics should be mathematized. These are philosophical demands... etc (Körner, 75).

In the sixteenth century, there was an increasing or intensified concern with ever more complex *equations* that defied any attempts at visualization. This experience must have helped to unseat *geometry* from its habitual dominating position; ever more pressing requirements for complex notation may have intensified the search for image expression wherever this was possible. Descartes held out the promise that every algebraic formula could be expressed in curve format; so there was hope at least for the future.

3.6 Curves in differential geometry

Having indicated various attempts at handling *tangents* (3.4.2 and 3.53) and differential curves and surfaces (3.4.2 and - for curves: 3.5.9), let me enlarge the picture historically. We noticed that for an overall view, they did not present a clear and consistent picture in the first half of the seventeenth century.

For two reasons it may be illuminating to compare them to the solution in modern mathematics of the problem of determining the seemingly absurd idea of determining the direction of a point on a bent curve. First, this would give us a stable model against which to compare the various attempts we have noted in previous chapters. Secondly, by inspecting the modern solution, one might discover tendencies and part-solutions in the seventeenth century that come more or less close to it; as several scholars, like Baron, Boyer and others have in fact done (*see* also text to *Fig. 3.2 - 2*).

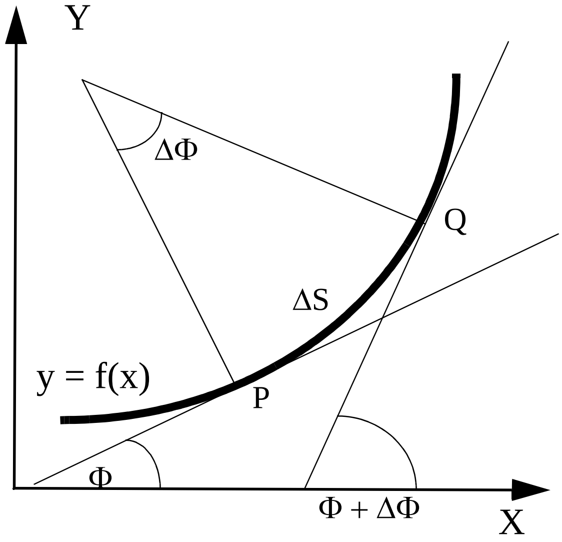


Fig. 3.5 -3 Direction on a bent curve

Let us have one example, turning to our much simplified *figure* 3.5 - 3 (redesigned after James). Omitting the rather complex calculatory operations needed to make sense of the stated principle (Kline, 1977, 459ff.), let us just note that the problem is to determine the relationship between Δϕ and ΔS as a limit when ΔS tends to zero (details in the following books, with descending amount of calculus technicalities: Finney and Thomas, 796f.; Gellert, 564f., James, 4414ff., and Kline, 1977, 457ff. - Kline's is as usual the intuitively easiest presentation). *Fig. 3.5 - 3* shows a much simplified example of differential definition of curvature. One vector tangent passing Q and one passing P (proportions not exact).

This is where we see that a straight line is a curve. If with an irrational move we let the tangent angle ϕ stand for the angles at *all* points on the curve, and then let it tend seamlessly towards zero, we see that the curve gets flattened out until both tangent and angle disappear and the curve becomes a perfect straight line. There is no quantifiable jump involved in this process.

Before proceeding with the issue on hand, the argumentation may stand out more clearly if we make a comparison with the *differentiation* issue discussed above. While this regarded how to calculate *instantaneous speed or change at some point on a curve* evaluated in terms of the domain, the *x*-axis, and the range, the *y*-axis, the present problems turns on the variable angular values of curve tangents related to the domain axis (or any axis, for that matter). The issue is a stretch of curvature, not some point on it.

The argument goes as follows, concerning an arbitrary - *any* - curve. An intuitive way of looking at a curvature, as noted by Kline, is to see how rapidly a curve changes direction. The direction of a curve at a point *P* is the direction of the tangent (on our figure, the one with ϕ as the angle with the *x*-axis). *Then the rapidity of change of direction is really a matter of how much* ϕ *changes from point to point along the curve or rather how much* ϕ *changes compared to distance moved along the curve. What this suggests is that the curvature of a curve at any point be taken to be the rate of change of* ϕ *with respect to arch length* - ΔS on *Fig. 3.5 - 3*; that is the angle divided by the arc length. Kline again: *Suppose that an object moves* *from Q to P* (as on our figure) along a curve. During the motion the change in direction is Δϕ and the distance moved is ΔS. *Then the average change in direction per unit* [of arc length] *is* Δϕ/ ΔS. *The curvature at P should be the limit of the average as* ΔS *approaches 0, that is, the curvature at P is dϕ/ds*. So far Kline. Now let James take over. *The curvature of a plane curve* [not a space curve] *having equation y = f(x), at any point is the rate at which the curve is bending or curving away from the tangent at that point. In other words, the curvature measures the rate at which the tangent to the curve changes <its direction> as it moves along the curve.* We do not try to perform these operations on a mathematically defined point, since we could subdivide that infinitely. We use an *average* which *approaches* zero (3.4.2); lending substance to Russell's observation about *approximation* being the fundamental operator to achieve exactitude.

The algorithm we have followed justifies the following equation (rewritten after James) containing elements already familiar, with *k* as the curvature - the "point direction" - at the point *P, that is, the absolute value of the average curvature* [indicated by the verticals on the equation] *as Q approaches P. That is,*

The equation may look simple enough but it requires further calculations which are rather complex.

$$\boldsymbol{K} = \left| \lim_{\Delta S \to 0} \frac{\Delta\phi}{\Delta S} \right| = \left| \frac{d\phi}{dS} \right|$$

The procedure illustrated in this *chapter* belongs to *differential geometry*, which was not yet available to Borromini's contemporaries. Some mathematicians, however, came close to the model along different ways of reasoning. This process, which was at the time even less surveyable than it is today, was hardly a baggage in the minds of Borromini and the planning people. But he, probably unwittingly, mirrored some of its ingredients in his design, sharing in the cultural tendencies of his day.

3.7 *Looking ahead*

Before the above catalog of probable readings of the Spire's helical thread can be exploited further, a grounding in more articulate observations has to be established. Much of *Part* IV is dedicated to this attempt.

4PART IV WORKING WITH SHAPE AND FORM

In this *Part*, I am going to try out a thought-experiment intended to bring the material discussed in the previous *Parts* on to a more articulatedly analyzable format. For this, I have to discuss some crucial issues concerning the handling of objects (4.1), and I shall need a survey of the principal available resources from the cognitive sciences and related paradigms (4.2 and 4.3 with subsections). This requires some further elaboration of the subject of *frameworks* (SL, *Burden* (22ff., 239ff.).

4.1 Framework heuristics

Borromini's Spire will have been, like any other public work, subjected to conceptual elaboration on several levels. The initial phase concerns what may happen in the idea generation and planning process (3.1). At the same time, the university connection made special demands on the display policy (1.10.1).

No illusion should be harbored that analysis can ever cover an entire configuration space like the one on hand. We shall never know which specific sections of it we do encompass with our hypotheses. The only solution available is to *maximize* (also 4.7) in the sense of drawing up a systematic account of as many probable parameters and coordinates as seem to work in an analytic sense of the term (4.1.1).

We build up a heuristic framework within which our specifics are located, calling for further information so as to provide us with a platform for going on.

Let me illustrate general framework issues using *Fig. 4.1 -1*, for the present concentrating on interaction and intercommunication generally among social participants. My use of a framework model involving *rats* is motivated by the wish to simplify. With human protagonists, the experiment would have imposed such complexity that the scope would shift from that of framework theory to social psychology. Happily we (I, at least) know almost nothing about rat mindsets, so we can keep closely to the ground and the picture free of what Herbert Simon called *cluttering details.*

Analyst rat (AR) in his framework is looking at Object rat (OR) in his framework. Each of them in his conditioning framework, which will be situation-dependent, evaluates the other one; the OR being conscious, let us assume, of being under observation and evaluation and thereby placing a particular interpretation on AR and probably also on his framework. The only thing we can get an analytic grip on, is the fact that there are two frameworks, each with its rat, one embedded in the other as will happen when OR in his frame is being observed and evaluated by AR, thus included in the latter's framework.

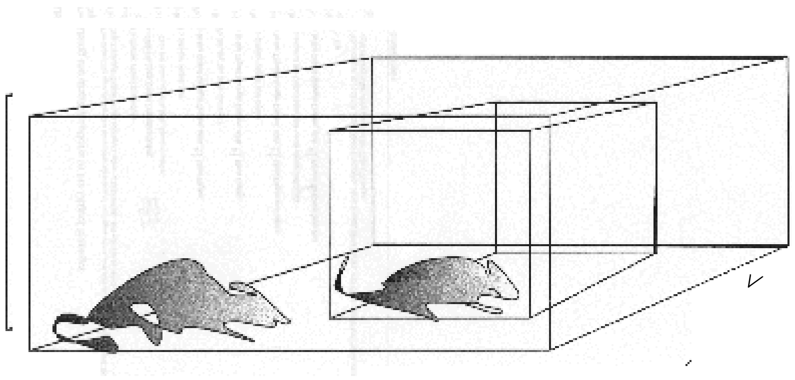


Fig. 4.1 - 1 Two rats in their respective cages or frameworks. AR in the big, OR in the small cage.

We *know* nothing about their mental or conceptual behavior and position in relation to one another from time to time. Deferring further examination of this complex situation (4.3.3), I will just note that this is one example of a situation model. We can fill in specifics for general parameters and arrive at some hypothesis. It may be realistic, but we never can tell. In order to work with the case, we have to stipulate one or several specific situation descriptions.

4.1.1 *Using models*

In principle, there is nothing new or "modern" in using abstracting graphic (or math) models. Belletristic storytelling of the kind we are used to encounter in the humanities have always used abstracting mode, without, it seems, always being aware of it. The difference is only that they are not brought out to view but lay hidden in prose accounts, permitting the scholar to believe the narrative is more real than scientific abstractions like our models in math or graphs, in which the abstract character is at full display.

The main advantage of abstract models, which are tools and not truth statements, is that they show structure, making it possible to decide *where* to operate modifications and corrections, if necessary, rejection. This means that the model becomes a scenario for us to develop argumentation and thus to govern our thinking.

The model with the cages denoting conceptual frameworks can be used to disclose some of the intricacies attending situation analysis. In most cases some kind of abstract graphic model (like a lattice) is more efficient in displaying the variable structural properties of a concept, thereby offering us relevant visual control and problem definition. This procedure is more constructive and reliable than a linear procedure in normal prose. A graphic model *shows structure,* to quote Richard Skemp.(SL. *Burden,* 158f. for Boltzmann and Skemp; 30f., for visualization versus verbal accounts).

How to use a *model*? No definition is available or useful, but the query can be answered with a catalog of features that represent or accommodate the most salient needs and functions in situation and object analysis (a more careful discussion in SL, *Burden, see* the Index).

1. A list of subjects considered pertinent to problem description; this is the place in which to apply the *maximation principle* as set out above (4.1) and below (4.7);
2. a configuration (verbal, mathematical, graphic) reflecting a structure that accommodates the items in the list;
3. features in the configuration emphasized for interacting with one another, mutually or sequentially or across networks;
4. a set of instructions (or just one) regulating the issues under pts. 1,2 and 3, and defining and explaining the position of the structure in relation to surroundings or context (model criteria in *Burden,* 22ff.).

No model, such as for example the *picture_ matrix* to be presented later on and to some extent tested at the end of the present *Part* IV, does in any fashion represent a conclusion. It is merely a simplified summing-up of the progress of analysis. The methodology is the sequence of decisions over this process; a synopsis in 4.7: *Theory and methodology for description.*

In the chapter on *structured argumentation* (1.11), the parabola model development started out from a top down view, working from theory down to realities and specific occurrences (then to ascend through an abstracting process). Opting for this course certainly represents no novelty. Einstein, for example, asserted that theory cannot be grounded exclusively on factual observation, nor can the latter be sufficient, for things work the opposite way, since it is the theory that determines the scope of observation (... *vom prinzipiellen Standpunkt aus ist es ganz falsch eine Theorie nur auf beobachtbare Größen gründen zu wollen. Denn es ist ja in Wirklichkeit genau umgekehrt. Erst die Theorie entscheidet darüber, was man beobachten kann*) (quoted in Heisenberg, 2006, 37). Nevertheless, Einstein affirmed that the goal of physics is to find out how nature works in reality. At the same time, a view voiced by Bohr brought theory and pictures closely together, more in agreement with modern conceptions of models. We have heard Heisenberg note that Bohr believed in his picture of the atom rather more than in the reality behind his pictures. Furthermore, let me repeat the following comments. In a general evaluation of physics, Niels Bohr affirmed that his science did not concern nature but human knowledge about nature. The scientist would never know how nature is in reality. They can know only how nature appears to them... (*Physik... handelt nicht von der Natur, sondern vom Wissen, das wir als Menschen von der Natur haben. Niemals können Wissenschaftler erfassen, wie die Natur wirklich ist. Sie können nur erfassen, wie die Natur erscheint* (Fischer, 2002, 82).

A model is *workable* if it can serve as a platform on which and from the point of view of which we can create a a *system* that incorporates whatever we consider important in a manner that seems consistent and not showing tendencies to partial disintegration; submitting once more that explanation means systemization. It is generally unavoidable, however, as I commented at some length in *Burden*, Part V, that models not designed to run automated processes (handling nuts and bolt, etc.), will involve some amount of ad hoc approach and circular argumentation.

A model must depict processes and provide space (configuration space, 1.7) for changing conditions without being tied up with fixed definitions. Banesh Hoffamann's realistic comments upon the nature of definitions are very much to the point.

> *To define a vector we have to add to the above definition* [here skipped] *something analogous to "having broad nails", and even then we shall find ourselves not wholly satisfied with the definition. But it will let us start, and we can try patching up the definition further as we proceed - and we may even find ourselves replacing it by a quite different sort of definition later on. If, in the end, we have the uneasy feeling that we have still not found a completely satisfactory definition of a vector, we need not be dismayed, for it is the nature of definitions not to be completely satisfactory...* (Hoffmann, 1f.).

Nevertheless, he reminds us, we can *use* the vectors. Fundamentally, a model is generally usable if it locates *operations* and *objects subjected to them,*
regardless of more or less sloppy descriptions of *what there is.*

4.1.2 *Description and representation as tools*

A program for handling the issues just recorded requires decision on the alternatives of *description and representation* versus *causal explanation*; the two former terms taken in their garden-variety versions: somehow depicting, giving a system-oriented image of something (the idea of representation concerning "our Mind" in the cognitive literature will be a later concern: 4.2.1).

Whereas *description* is a process by which the features of an object and the structure they form, with the purpose of achieving tractable pictures of the object, are rendered by any verbal or visual or numerical means, *representation,* a sub-category of description, I take to mean transposing or transferring a description of the *structure* or *system*, with the purpose of making its buildup clearer, over to a different verbal or visual format such as, for example, a graphic model. This is in agreement with my idea that abstraction does not work one way only (from the concrete to the less so), but should be taken to mean transfer from any one format to another (SL, *Burden,* 69ff.). Description, to specify further, is the process of applying theory and model to the object in focus, which then as a construct becomes a product of the process. I am aware that there are no neat distinctions here, but I do not need one, for the important things is what is being tried, worked out and achieved under these headings, the operational attitude.

As a provisional platform, I take representation to be directly or in a metaphorical sense a question of handling and elaborating topologies (Coyne, 123) of things, concepts, chunks of knowledge; more closely, abstraction in verbal (semantic), graphic or both modalities of structural properties of objects such as they may have been perceived, understood or handled by our protagonists; without trying to fathom any mental depths that might support such an imagery; or, to go a step further, patterns of objects of this category (category then in the Rosch significance: *see* 4.4); abstractions that occasionally but not regularly admit of calculation or logical handling in accordance with preset rules.

Our representations presuppose that there is something to represent or replicate or metaphorize mentally. This will often be a thing we do not know or about which we do not have sufficient knowledge. It is our probably workable *description* that makes the thing manageable and amenable to processing according to some principle. Our description *absorbs the "thing"*, turning it into a tractable entity. Regardless of whether we operate with pictures or other abstract models, we will, on that basis, be manipulating *representations* rather than things or other features in the pizza-corner variant of "reality". Simulation, whether verbal or graphical, as in the present book, or on the machine, replicates operations or mechanisms in terms of which our abstractions (i. e., transfer to some other level: *Burden,* 69ff.) become more real than the evasive things that have been subjected to the abstracting procedures. Nut and bolt or other object in focus, have attached to them, in the object-oriented manner (4.3.4), attributions in terms of metaphors, cues or codes, that turn them into something that can be acted upon. *Relevance patterns* are selected in accordance with whatever can be acted upon, can be mentally handled, can provoke *action.*

Scholarly comments on the mechanism of *representation* often include *metaphors.* The idea comes from the cognitive sciences (4.2), concerned with people’s mental elaborations. For my use of it for model construction in the present book, I may perhaps claim support from Richard L. Gregory:

There is nothing new in this idea of turning a perception model into a research model: The status of perception may be very like that of scientific hypotheses. What we see is affected by what is likely; and we can be driven into error by following assumptions which are not appropriate for the available sensory data (Gregory, 395ff.; and his article on *perception* in the *Fontana dictionary of modern thought*).

The analyst or researcher formulating a scenario, appeals to and marshals the cognitive apparatus available to her or him, so that technical and operational insights or theories from the cognitive sciences give guidelines for evaluating the process; in other words, to estimating one's method and approach Physical laws cannot be *used* in this process but images reflecting them, can indeed (4.3.1).

4.2 Cognitive sciences (our undiscovered mind)

My exposure to the cognitive sciences, artificial intelligence and related paradigms has been purely literary, since I have had neither the competence for nor the opportunity of following or performing experiments on data. I feel indebted to Zenon Pylyshyn but have to do without referencing here his numerous central - and lasting, I believe, after a book published back in 1985 - observations on cognitive science. Perhaps his book and Trehub's are among the most penetrating texts available on this difficult theme (*see* the *Bibliography*).

We are discussing how people conceive, construe, perceive or evaluate things they plan, implement or encounter, or their reactions to other people's conceptions, beliefs or preferences, in some corresponding configuration space, in short, how they *handle and elaborate them.* This is the field of a number of academic fields, today with the cognitive sciences in the central position; we have social anthropology, social and organizational psychology and management and business information, to name the principal perspectives and resources. The cognitive perspectives require a more careful consideration, while the rest in a general fashion to some extent informs my way of thinking all through the book.

Pylyshyn, in his *Computation and cognition*, takes a pragmatic view of the general issue of *cognitive science* as a branch of enquiry. The field

> *may have no single foundation; it may just be an umbrella title for a number of different sciences, all of which... are attempting to understand the workings of the mind;* but, he continues, *there may well exist a natural domain corresponding roughly to what has been called "cognition", which may admit of such a uniform set of principles* (Pylyshyn, *Preface*).

An even more sceptical outlook has been summarized by Baumgartner and Payr in their *Speaking minds*, building upon interviews with a number of prominent cognitive scientists and philosophers (quoted in SL, *Burden*, 35).

In other words, in the cognitive sciences, the general model of *representation* (4.2.1) is proposed and used for sounding the depth of human mental mechanisms with the aim to explain their functional patterns.

A comment from 1991 on Jerry Fodor's work illustrates a central tenet in the field: ... *he has articulated and defended... a computational theory of intentional causation that is* *central to the emerging cognitive sciences* (Loewer and Rey, xi). There have been (and still is) disagreement about the deeper principles, as noted by Anderson back in 1983 (1ff.); he himself believing in *the unity of human cognition, that is, that all the higher cognitive processes, such as memory, language, problem solving, imagery, deduction and induction, are different manifestations of the same underlying system.* Further, after quoting Chomsky, who launched the alternative, allegedly faculty view: *This faculty approach holds that distinct cognitive principles underlie the operation of distinct cognitive functions. The unitary approach holds that all higher-level cognitive functions can be explained by one set of principles.* So far Anderson (numerous references to his views in Braisby and Gellatly's *Cognitive Science* of 2005; see the *Index*).

From a cognitive point of view, many human activities can be subsumed under the term *stimulation*, impulses and various forms of excitation coming from the outside or from interior processes. In the (relatively) early days of the cognitive sciences, in 1979, the difficulties attending this subject were already being recognized ... *histories of stimulation may be very long and complicated, or unavailable for inspection,* and *a given output may occur as a function of many different histories of stimulation* (Whitehurst and Zimmermann, 2).

These speculations serve as a kind of shell around internal models in the discipline; along with general theories on language and symbols (a crucial question continuously raised: do these tools obey computable rules?) and on the seat of human cognition: the head or the perceived surroundings?- a vast array of ideas. Here are some of them.

Evans (*see* 4.3.6), in his *Hypothetical thinking* of 2007 (186), tabulates what he calls *some of the major dual-process theories of cognition,* which represent, after all, just one smallish section of the field. The table displaying *the major* examples, has 11 rows counting so many different authors or couples of them, and the classifications of their respective theories run to (of course) 11 rows, each classified by a double set of criteria distributed over two columns: *input modules/higher cognition* - *heuristic/systematic* - *automatic/controlled* - *intuitive/analytic* etc. The product is a semantic matrix with 33 elements.

Even such a "hard" field as mathematics can be studied in the perspective of cognitive science. This is the subject of a reputedly ground-breaking book by Lakoff and Núñez, *Where mathematics comes from.* The book is constructed as an semantic algorithm gradually exposing the subject in mental notions. The general claim is, as the subtitle says: *How the embodied mind brings mathematics into being.* Many years ago, the chemist Max Thürkauf brought a related message, but in the language of the mid-seventies, in his book *Sackgasse*

Wissenschaftsgläubigkeit (the cul-de-sac of naive faith in science as an absolute, read: non-human, concern).

Ever since the 1960s the world has seen a vast amount of publications aiming to explain the nature and constitution of *The Mind* and *Consciousness*, each book more or less fundamentally corrective of the preceding one (and some of lasting interest, methodologically as well as substantively, such as Margaret Boden's *Computer models of mind*, R.I.Gregory's *Mind in science*). Trying to work my way through some of them, say, Roger Penrose's *The emperor's new mind* or *Shadows of the mind*, to say nothing of D. C. Dennett's *Consciousness explained*, I end up with the unprofessional understanding that you cannot discuss *The Mind* or *The Consciousness* any more than you can with *The Love* or *The Pleasure of being retired with a well-supplied wine cellar*: all of them are *emergent properties* that defy analysis (4.3.4). Their underpinnings are too elusive to get a grip on from some general perspective like the cited ones. Some contributions seem to rest on firmer ground, mostly arguing in terms of *semantics*, such as John R. Searle's *Intentionality. An essay in the philosophy of mind*, but then the scope seems more limited.

As I claimed in my *Burden* of 2000, the cognitive sciences represent a huge idea bank, a highly useful and fascinating elaboration of terminology and analysis criteria, and also a resourceful incitement to going ahead with theoretical work. The disparate activities that go under the name of cognitive science, a kind of *concept machine*, are especially useful for clearing up terminology and articulating model concepts and design; this of course partly because of close relations to computer science and artificial intelligence. Covering a number of fields belonging to different academic disciplines, the cognitive program may contribute to render the boundaries between them meaningless, thus contributing to bridging over between them.

Herbert Lindenberger, Emeritus at Stanford, has recently exploited new findings and insights in *neuroscience* concerning the brain mechanisms that seem to rule our access to various phenomena in arts, music and literature (Lindenberger, 2009). This pioneering study is, Lindenberger notes, not at the moment sufficiently developed to afford adequate means for tackling our problems. At any rate the program is a serious incentive to take interdisciplinarity serious and to puncture academic borders whenever the tools of one's own field fail.

The richness of ideas but at the same time the corresponding instability of the overall picture, is mainly due to the fact that the focus is on something as vague, manifold and evasive as the human *mind*. Baumgartner and Payr, *Speaking minds*, and Horgan, *The undiscov-*

ered mind, provide sobering reading. Horgan's book is based upon published literature and, above all, inspection of ongoing research projects, including laboratory work. This survey allowed him to study projects and research methods as well as aims and goals. Here we obtain a rather discouraging view into the field as it stands (or stood a few years ago, in 1999), justifying the title of his book.

However one looks at the entire issue, a very important asset remains; that of alerting us to problems that range from human mental machinery to techniques, methods, ideas and models with which to face them, thus affording us a rich repository from which to draw for a wide range of assignments.

4.2.1 *Cognitive representation*

In the cognitive sciences, we have noted, the idea of representation is being carried deep down (one may believe) to allegedly kernel levels in the human mind. The idea, if workable instantiations are available, is highly relevant to the argument of the present book, for it turns on how design experiences may stick in the mind and be elaborated. The constructive role of the observer, is elaborated in technical language by Hernández, developing further ideas from Winograd and Flores (1986): *representation is in the mind of the beholder.*

Pylyshyn suggests that *one of the main things cognizers* [people being subjected to being studied in the cognitive science and neighboring fields] *have in common is, they act on the basis of representations... Knowing the representations they possess, together with the assumption that much of their behavior is connected with their representations by certain general principles, we can explain an important segment of the regularities in behavior of these cognizers* (Pylyshyn, xii); we are facing *the representational theory of mind.... Even assuming that this general picture is correct, it raises important and deep puzzles. How is it possible for a physical system (and I assume that cognizers are physical systems) to act on the basis of "knowledge" of objects and relations to which the system is not causally connected in the correct way?* The correct way?

The idea is widely debated in the cognitive sciences. Here, representation is not a tool in our methodological setup (as in 4.1.2), but, optimistically, *the* mechanism by which our protagonists - or their "mind" - conceptualize or construe the world. Pylyshyn argues against using *metaphors* in order to gain access to the human mind because it fails to work on certain levels (Pylyshyn, xiv, 251f.). This need not detain us, seeing that I am not after probing those unmeasured depths in myself [!] or in others, but attempting to discuss *representation as a* *model or set of models for bringing probable structures in people's handling of what they see and perceive, as we try to describe them, on to a model format*.

Perner offers illuminating insights on the issue of how cognitive psychology and *our commonsense theory of mind* make use of the notion of representation, investigating some of the preceding contributions from Gottlob Frege to Nelson Goodman (1ff.). As an aid to penetrating current concepts of the murky regions called the human mind, Perner's contribution seems efficient enough, but I am not letting that take me farther afield than the present project justifies.

Sterelny, in his *The representational theory of mind,* discusses what the title of his book would lead us to expect. He starts out with a query which conveys the flavor typical of this almost vacuous debate.

What is the function [! just one?] *of our mental states? According to the representational theory of the mind, while mental states differ, one from another, mental states are representational states, and mental activity is the acquisition, transformation and use of information and misinformation. The perceptual system's causal role is not memory's causal role, but both make available information to other psychological systems.*

Further elaboration on this leads him to asking three questions that express the gist of his enquiry: *1) What are beliefs, desires and other propositional attitudes? 2) What is representational or semantic content; in virtue of what do psychological states have it? 3) In virtue of which of their properties do the attitudes play the role they do in the causation of behavior?*

Leaving to the reader to evaluate this program, which is apparently driving toward definite answers, I go on to Winograd and Flores, hoping for the best. In their book *Understanding computers and cognition,* they spend much effort on using the allegedly more or less symmetrically positioned systems of *mind and computer* to probe the depth of both.

I am not trying to explore any depths, and will just quote the formulation of the *representation hypothesis* (as Winograd and Flores call it; 74, having first given a synopsis of some philosophical views relevant to their discourse): the principal idea is *the assumption that cognition rests on the manipulation of symbolic representations that can be understood as referring to objects and properties in the world* (so far we seem to stay safely on an operational level). In their fascinating chapter on *Computers and representation,* they list (85) three points that seem to me relevant not only to depth research but also to a functional argument like the present one. The passage is a long one, but a synopsis of my own making could hardly compensate adequately. They start out from AI:

In general, artificial intelligence researchers make use of formal logical systems (such as predicate calculus) for which the available operations and their consequences are well understood. They set up correspondences between formulas in such a system and the thing being represented in such a way that the operations achieve the desired veridicity. There is *a great del of argument as to the most important properties of such a formal system, but the assumptions that underlie all of the standard approaches can be summarized as follows: 1. There is structure of formal symbols that can be manipulated according to a precisely defined and well-understood system of rules. 2. There is a mapping through which the relevant properties of the domain can be represented by symbolic structures. This mapping is systemic in that a community of programmers can agree as to what a given structure represents* [?]. *3. There are operations that manipulate the symbols in such a way as to produce veridical results - to derive new structures that represent the domain in such a way that the programmers would find them accurate representations. Programs can be written that combine these operations to produce desired results.*

The authors' crucial comment (further elaborated later in their book) is worth quoting: *The problem is that representation is in the mind of the beholder. There is nothing in the design of the machine or the operation of the program that depends in any way on the fact that the symbol structures are viewed as representing anything at all* (86), and they refer to similar views in works by Fodor and Searle. I modestly subscribe to the general idea.

Sowa's book *Conceptual structures* delivers an analysis of formal-technical sides of computer-and-logic-relevant cognitive issues, including also the standard linguistic ones. Knowledge management is a central theme. *Knowledge is more than a static encoding of facts; it also includes the ability to use those facts in interacting with the world* [cf. Putnam's view on concepts; 4.3.7]. *A basic premise in AI* [artificial intelligence] *is that knowledge of something is the ability to form a mental model that accurately represents the thing as well as the actions that can be performed by it and on it* [object orientation: 4.3.4]. *Then by testing action on the model, a person (or a robot) can predict what is likely to happen in the real world* (2).

Sowa later discusses conceptual graphs (68, 76ff.), which is one important class of representation media. *Conceptual graphs form a knowledge representation language based on linguistics, psychology and philosophy. In the graphs, concept nodes are interconnected.* Furthermore: *A conceptual graph has no meaning in isolation. Only through the semantic network are its concepts and relations linked to context, language, emotion and perception.* Intuitive charts and mental maps of topographical areas are typical (Canter has numerous examples).

The literature on cognition, artificial intelligence and related programs has more to offer that may be of use in my inquiry, or, on the other hand but equally important, insights and theories with which any attempt like mine must be evaluated for fits and misses. The next *Chapter* brings a survey of the *resources* they propose; in addition come some "moral" teachings from modern physics.

4.3 Resources

NOTE: Readers should skip over to 4.4 if they prefer the following as a reference.

As I have reminded the reader earlier, in order to hold my venture on course, but more importantly, to put my argument to a serious test, the present book has its *main focus on one object* - Borromini’s *Spire,* but draws from resources in theory and methodology wherever I have found them useful or invalidating of my suggestions. Scope and perspectives are very wide indeed but so are the difficulties attending such an approach. My use of the notion of *resource* does not contemplate passive items in storage but active entities; it is adopted from Anthony Giddens, for whom *resources* are capabilities of making things happen (SL, 1984, note 64); available and exploitable entities that contain incentives and indicate directions, one might say.

Let me also remind the reader and myself of a statement by Minsky and Papert: *Good theories rarely develop outside the context of a background of well-understood real problems and special cases.* Development of theory and methodology has to start out from concrete things in an approachable world; the *bottom-up* modality. Which means that Einstein’s rule about theory directing the entire business holds, because we need theory at least to tell us what is *up* and what is *down*, and which objects and themes to focus on and in what kind of general scenario to locate them. Whatever we do, such a process will always, to emphasize now the obvious and well-known, be embedded in a larger *top-down* process, since notoriously we cannot avoid having pre-construed conceptions, glorified as *theories* or not, of what we are about to do (1.11: *Structured argumentation*). It is with this kind of flow in mind, from theory-based choice of specifics to a larger scope, that the typically theoretical resources visited on the pages ahead may come in usefully or as a potential corrective.

The most important paradigms to be introduced in this chapter are: *physical base-models, processing concepts, emergent properties* and *object orientation; personal construct theory, hypothetical thinking* and other *network features* [4.3.8] and parallel and pseudo-vectorial patterns.

4.3.1 *Physicalism and "Hardframing"*

With the neologism (in German one might say *Notname*) *hardframing* (not very elegant, I must admit), I intend *using hard-science models for heuristic modelling in intuitive and "soft" cases and issues.* Put another way, it means using structures from models of quantifiable objects or thematics for handling non-quantifiable issues. The principle is not, we shall see, a novel one; it is closely related to one version, the weaker one, of *physicalism*.

This is not to try to derive intuitive concepts and processes from physical ones. Nor is it an attempt at improving the current concept of *physicalism* (also called *materialism*) (a very thorough investigation of the idea: Poland, *Physicalism*); it is rather an expression of the view that such a program concept, while seeming tangentially workable, claims too much.

"Soft" approaches in concept evaluation can, tentatively, be related to "hard" ones in the following terms. In Ganter and Wille's *Formal concept analysis. Mathematical foundations* (*see* the Bibliography; and 4.3.7), a subject which, to quote the authors, *can be regarded chiefly as a branch of applied lattice theory* (*Preface*); so we are *dealing with mathematical notions, which only reflect some aspects of the meaning of "context" and "concept" in standard language* (17). The issue is of interest for the present debate, since the cited venture is an attempt at formalizing pictures that for the ordinary citizen are rather vague and at any rate soft and not open to formalization. Fig. 1.4 (p. 24) illustrates a lattice for the concept of an educational film, "Living beings and water", at least, one may think, a relatviely factual affair (the authors provide a number of cases; anyone can go in the present discussion). If Ganter and Wille take the high road, formalizing things that work vaguely in our mind, I shall take the low road and suggest that such lattice structures can help us - not to formalize for being run on a computer, but merely to structure pictures we everybody have and had, imagining that *whatever appears to be open to formalization can be used as a model for structuring our softer pictures*. Math and other hard approaches can serve as structure modelling. They do represent the normal way of thinking anyway, but offer in addition a manageable representation of it. The method is more efficient and broad-spanning than the usual linguistic approach. This, instead of working with structures, follows linear propositions, then to be linked up with a so-called context, which is *not* linear.

I am less tempted by the theory called *physicalism.* To convey the flavor of this variously conceived configuration, let me spend some time over the subject, quoting from Loewer and Rey's *Introduction* (xif.), concerning Fodor's special version, the reference to which conveys an idea of the kind of problem that is involved.

Physicalism is the view that all genuine phenomena (properties, event, states, processes, laws, causal relations) supervene on physical phenomena. Fodor's version of this doctrine involves three basic commitments: (1) all events (and objects) are identical to or composed of physical events, (2) for every exemplified property there are physical conditions which are sufficient and explanatory of the exemplification of that property, (3) all basic laws are laws of physics.

The two editors comment: *notoriously, however, psychology had trouble complying with these constraints. Such properties as rationality, intentionality, and consciousness, and* *the processes involving them, have persistently resisted incorporation into physical theory* - and, let me add, also the closely related (as is often believed) computer operations (artificial intelligence).Hatfield notes: *Belief in phsyicalism generally is of a piece with rejection of Cartesian dualism* [mind and body]*; and, because dualism has been traditionally associated with any realistic theory of the mind, this rejection has led many philosophers to view such theories with suspicion. Thus Ryle (1949) railed against "the ghost in the machine", Wittgenstein (1953) against believing in some "yet uncomprehended process in [a} yet unexplored medium", and Quine (1960) argued for an "indeterminacy of translation", for relegating mental talk to a "second-grade status", a "dramatic idiom" applied with "charity" but lacking any serious factual or scientific status* (Hatfield, *255*ff.).

Under the heading, *Theories of meaning*, Loewe and Rey give a short historical overview that is useful for our understanding of the problem. They note Franz Brentano's (1838 - 1917) pointing up

> *a problem with which twentieth-century philosophy has been grappling ever since: how can the fact that a propositional attitude and/or representation is "about" something be explained in terms of natural science? Mentalese sentences, then, as a physicalist, Fodor must find a way of naturalising the semantics of Mentalese. Some philosophers are skeptical that this is possible, and this has led some of them* (e.g. Quine [1960]; Paul Churchland [1981]) *to conclude that there are no semantic properties at all, and others to draw the more cautious conclusion that intentional properties simply do not appear in scientific laws.*

Quine, concerning the debate on the proposed dualism of *mind-body*, notes that it should be obvious that if the two are to interact, *we are at a loss for a plausible mechanism for the purpose;* and the law of conservation, fundamental in physics, goes by the board (*see* Quine, *Theories and things,* p. 18., and Jennifer Hornsby's discussion of *Physicalist thinking and conceptions of behaviour*, in Pettit and McDowell, 95ff. Trehub, *The cognitive brain* provides an instructive and challenging view of the neurobiological bases for cognition).

Having one's perception and observation anchored in physical laws is certainly no straightforward affair. Correct understanding according to Galilei or Newton can easily have been under certain circumstances commingled with psychologically elaborated conceptions, as described by Bozzi under the title of *Fisica ingenua*. This book seems to elaborate ideas that come close to Holton's *themata* (3.2), but they are less precisely outlined. If our mental setup can to a significant extent be reduced to physical laws, then perhaps Bozzi's physics, which should also be included, is not so naive as he seems to admit. Schoolbook physics may be blamed for distorting things (though probably not in the sense contemplated by Cartwright, *How the laws of physics lie*).

Physicalism to me seems to be open up for approach under two alternative headings: that its claims have reality value; or that it provides us with precise, well-structured and consistent patterns, which can be used as models for the softer occurrences.

4.3.2 *Uncertainty*

Physics is not exclusively a "hard" science, and that such an experience as the development of quantum theory and the related debates as reported in Selleri's *Die Debatte um die Quantentheorie*, may encourage me to accept probabilities, even possibilities, as the final outcome, rather than hunting around for definite conclusions; and also a vectorial mode of seeing things, dealing with directions, tendencies and propensities, rather than hoping to come up with something converging upon a centre or orbiting around it (this should not be confused with *possible world* logics; for which, see Grayling, 67ff.) (a highly recommendable publication on the basic - statistical - ideas of probability, also for its accessibility for the general reader: Howson and Urbach). Furthermore, and this seems the most useful aspect of "interdisciplinarity", Niels Bohr's often faltering approaches and Werner Heisenberg's mixture of "hard" and inspirational discovery of the nucleus of the uncertainty principle (Heisenberg, 1969, 96f.), must have a sobering effect on claims to be *wissenschaftlich* in any definite sense.

The debates in the 1920s about quantum mechanics produced a fundamental and very challenging body of philosophy of science and of general philosophy as well. This story, as presented by Polkinghorne, Selleri, E. P. Fischer and Camejo, as well as by Heisenberg himself (*see* the *Bibliography*) and others, is perhaps the best model one can find for science development as evaluated in an almost-totality of levels and possible contexts and ideologies. Fundamentals such as causality, reality, excluded middle and objective observation and measurement were, if not rejected, at least taken as contingencies not necessarily always reliable or indispensable.

Not many years later, in 1934, Bruno de Finetti wrote, among many other things, the little book recorded here, *The invention of reality* (*L'invenzione della verità*), published posthumously with comments in October 2006. An internationally acclaimed mathematician specialized in probability calculus, economics and decision theory, De Finetti adopted the Italian dramatist and prose writer Luigi Pirandello's version of relativism, succinctly expressed in his book title, *cosí è - (se vi pare). this is how it is - if that is what you think*). Characteristically, De Finetti quotes from Pirandello's novel *One, nobody, hundred thousand* (*Uno nessuno centomila*) the passus stating that each of us have a proper inherent probability setup and from *Six characters in search of an author,* 1922 (*Sei personaggi in cerca d'autore*): *We have to invent the world in order to find place for our awareness* (*Dobbiamo inventare il mondo per inquadrarvi le nostre sensazioni* ...).

De Finetti also avails himself of Giovanni Papini's view of *pragmatism,* as it attracts *all who think in order to act, and thus prefer provisional truths rather than the inebriation from hyper-abstract words* (*...tutti quelli che pensano per agire, cioè che preferiscono verità provvisorie ma operanti, all'ebbrezza delle parole iperastratte*) (De Finetti, 13).

The world and our lives, and thus also in historical contexts, are complex because cognition, conceptualization, idea, goals, interests, as well as unsurveyable social, political and economic factors, enter into the picture. The recognition of this is reflected in art. Whereas earlier painters tried to establish a firm and recordable world, structured by the rules of the central perspective and laws of proportion, more recent artists have discovered that the probabilities are almost endless (in music noted already by Jean Philppe Rameau) and that you can only pick out a few of them. And what you do pick out, is unstable: *The eye of the painter lends to the objects a human value and reproduces them as they are seen by a human eye, and this vision is mobile, it is changeable,* this was the way Pierre Bonnard expressed the idea. A modern physicist (the crystallography physicist Cyril Smith, active at Los Alamos) could suggest, some decades ago, that artists intuitively, not precisely but somehow essentially, can perceive *systems of great complexity* in a way that could teach scientists something. This was written before *complexity* became a commonplace concern all around (quoted in SL, 1984, 180). The *artistic approach* has attracted wide interest in writings on science history (books by, e.g., Arthur I. Miller and Gerald Holton), and in Ernst Peter Fischer's *Werner Heisenberg*, this issue is discussed at length. Holton's *Thematic origins of scientific thought,* has numerous references to Heisenberg and the quantum debates.

Apparently Werner Heisenberg was one of the first in modern times to notice this kind of relativism and to elaborate the subject - in his *Der Teil und das Ganze* (1969) and other books. Ernst Peter Fischer quotes from this autobiography Heisenberg's review of the great Niels Bohr's presentation of his atom model at a seminar in Göttingen in June 1922 (the so-called *Bohr-Festspiele*). The actual model did not hold against criticism but proved seminal nevertheless in that it started thought processes in new directions. Bohr may have been aware of this, for *It became immediately clear that Bohr had based his results not on calculations and proofs but on his feel <of the subject> and by guessing* (*Einfühlen und Erraten*) *and that he now was hard put to defend his views before the "High School of Physics" in Göttingen*

(Heisenberg); and Fischer supplies: *One is lead to believe that Bohr better than anyone else knew how weak the basis was for his intuition concerning the atom* (Fischer, 52, 54, Heisenberg, 1969, 50f.).

Another illustration may be culled from quantum physics. Probability physics (applied first to wave forms) meant two things: reliance on a spray of probable facts (in the scientific context, statistically determined), and *something like a tendency towards a specific event. It meant the quantitative conceptualization of the old term of* dunamis *or "potentia" in the philosophy of Aristotle. This lead to a strange kind of physical reality, which hovered between possibility and reality*. (*Sie bedeutete so etwa wie eine Tendenz zu einem bestimmten Geschehen. Sie bedeutete die quantitative Fassung des alten Begriffs der "dunamis" oder "Potentia" in der Philosophie des Aristoteles, Sie führte eine merkwürdige Art von physikalischer Realität ein, die etwa in der Mitte zwischen Möglichkeit und Wirklichkeit steht*) (Heisenberg, 2006, 17ff.).

Fired by the ideas just referred to, the present book may be seen as a tentative process towards the recognition that sprays of probabilities are the most we can expect in the way of result and conclusions. Thus the process becomes the only reliable factor, which means methodology.

Working with probabilities in intuitive research requires, as noted earlier, *maximizing* (4.1 and 4.7) the scope in order to cover at least a minimum section of the spectrum, since we cannot construe any kind of statistics on the material discussed in the present book, but must assume that this or that factor might have been present and active, taking into account objects over a scale from cognitive abstractions to "real" things. A program for description of objects must, in principle, embrace the entire domain.

4.3.3 *Handling objects*

Bob Hale, in his extensive and deep-searching book, *Abstract objects,* asks the leading questions of his book (modestly labelled *essay*): *Are there abstract objects? If there are, do they, or at least some of them, enjoy a mind-independent existence?... what sort of knowledge can we have of them?* (Hale, *Introduction*). To forestall blame for having missed out a popular and relevant theme, let me submit an extra note about content assessment. The debate has been going on for a long time about meanings being "in the head?" (Putnam's polemical question) or in the context; and whether there are two categories of truth they can rely on: the categories being conceptual truth and empirical truth (overview and discussion in Pettit and McDowell, *Subject, thought and context, Introduction*).

Not being inclined to venture into this murky landscape, let me refer to discussions such as Chalmers' (1996). Theories of *content*, for instance Fodor's (Fodor, 1990), usually seem to embrace theories of semantics, language, and versions of formal logics and the propositional calculus; highly appealing corrective notes in Hörmann's *Meaning and context.*

On observing a concrete object, a *Gegenstand*, we elaborate it mentally so as to *output it as our own product* (*see* the *Introduction, - b,* and *4.7,* for o*perational determination*). To claim the same for abstract objects such as numbers and logical sets seems an entirely different matter. The current acceptations of the term *abstraction* are far from being so clear as to provide us with a usable metric; adopting her term for any shift of level (SL, *Burden,* 69ff.), rids us of a tangle while preserving the crucial idea, that some representation now appears in another shape than previously or that it represents some reality object in another format. Borromini's *Spire,* to return to that. certainly is a thing with weight and dimension, but regarding the mental elaborations that we or the contemporaries might apply to it, what about the terms and concepts churned out of such processes? The ± 1600 handling of the *shapes* and *shape concepts* and the series of scientific events (real or conceptual) in which this takes place, is a lengthy, multi-level and partially discontinuous, recursive and retrogressive process (2.4).

Generalizations neither on *development* (in some deeper sense than the everyday one) nor on *revolution* make sense here. Only analytical models featuring specific historical or other remote processes and interactions among these and between the various levels on which such events arise, can to some extent be hoped to capture significant elements in such a material and make them available to argumentation. They can fixate and formalize the chunks of the world that we apprehend and conceptualize in one of the several modes of rationality with which we operate, such as Hebert Simon's bounded rationality or other forms described by such authors as Bruno de Finetti or László Méró (De Finetti, Méró).

The notion of conceptual frameworks is too well known to need much comment here. Socially, my family and neighbors, and my personal history and situation and career (for what it may be worth) contribute to my framework. Scenarios of this kind are attestable and relatively readily inspected in ritual processes. In fact, rituals usually enact framework-sharing mechanisms. The study of rituals, therefore, offer efficient models for using in scrutiny reaching into recordable everyday patterns.

We are facing the classic antithesis between appearance and reality, being subjective or objective (carefully considered in Hacker). It is my ambition that the argument in the present book may contribute to making this dichotomy seem less fascinating.

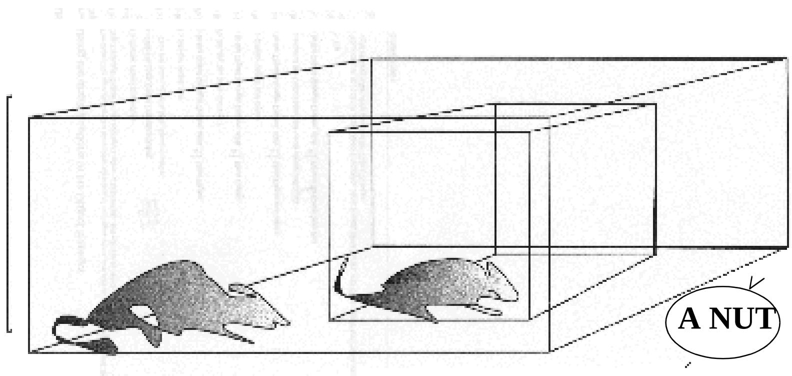


Fig.4.3 - 1, Analyst Rat in her/his framework looking at Object Rat in her/his framework, but as I sadly suspect, also on the appetizing big nut (the nut picture inspired, as everyone can see, by Cézanne).

The figure above (*4.3 - 1*; a *pluricagecage interrattion*), already introduced, without the nut, just to instantiate *frameworks* (4.1), is a little less obvious than one might imagine at first sight. The Observer Rat, to the left, within her/his conditioning framework, beholds an object, a nut, but at the same time has to consider another perceiving entity, a second rat, in *his*/*her* conditioning framework. The latter, let us say, is conscious both about the nut and about being under observation by a possible competotor. To some extent he evaluates the other rat and his framework, which then becomes, fragmentarily, a part of *his* framework. The physically idle *nut* is being mentally elaborated, we may guess, by both rats in the complex scenario just noted. The nut is conceptually activized, being turned into a pattern of information, to adopt Penrose's notion. Complexities arise that are essential in inter-human communication but not readily penetrable. We can study and by models describe such intercommunication patterns without attempting to seek into the depths beneath them.

The *Analyst Rat* must consider at least two frameworks, of which the hardest one to evaluate is often the one surrounding or embedding oneself (this also applies to the Object Rat). This is so particularly because one cannot properly evaluate a system from its inside, a predicament reflecting Gödel's incompleteness theorem, briefly stated in Webster in the following terms: *the theorem that states that in a formal logical system incorporating the properties of the natural numbers, there exists at least one formula that can be neither proved nor disproved within the system; the corollary* <is> *that the consistency of such a system cannot be proved within the system* (carefully popularized by Nagel and Newman, *Gödel's Proof*; a classic, published numerous times. Gödel's Proof was used, misused according some writers, by Roger Penrose in his *The emperor's new mind*: *see* Baumgartner and Payr for such an *outrageous idea*).

In order to assess how she/he is being evaluated, the *Object Rat* must try to grasp not only her/his own framework but also the observer's framework, since AR's observation involves OR in it; this will affect her/his attitudes and behavior, making the task even more hazardous for the *Analyst Rat*. It does not make much sense for us to speak of any *The reality* in the face of such a situation, which is nothing more than a simplified model of any commonplace one. Even if OR were not a live animal but a dead one or an inorganic entity, we would sit back with approximately the same picture. For AR again, in an operation of cognitive access and elaboration, will attribute frameworks to the object, with many of the variables that mark them off against the surroundings and against the object itself.

What about myself, who crafted this superior piece of drawing? Counting myself in, we can deepen our wisdom (my friends would wonder about this). For I too sit within a framework. The upshot is, of course, that we can see the outlines of an extremely complex system of relationships involving the researcher in person (not so long ago, we were taught that science is objective). The rats are hardly aware of this, but it works on them all the same, for they are nilly-willy involved in an artificial situation of analysis that forces them *virtually* to evaluate one another, including their respective frameworks (without being able to or even interested in knowing that relevant circumstances may be called frameworks). So far we have observed the scenario without much taking into account the well-tasting *nut*. Now let us place it just outside the cage system.

Neither of our friends can reach it while sitting inside their respective frameworks; for my argument condemns them to remain there. But they can look at it and, probably, try to evaluate it from any viewpoint that might arise. Let us hope that, since they have the honor of appearing in my book, they do not exclusively think of eating it. Anyway, each has to take into account possible intentions and actions on the part of the other. At the same time, the situation may arise that each rat will its have attention divided by two targets (the other rat and the nut), thus acting out central notions in Herbert Simon's *Administrative behavior*. So here we go again.

The upshot is that the idea of an objective world is analytically useless (it turns out to have something to say for itself the moment we are bitten by a rat). Relevance is reserved for the one we create by observing it or otherwise handling it cognitively. There is a healthy lesson from physics; and a philosophy that does not arise within physics or can hold its position when facing it, may have literary value but hardly much more than that.

The notion of *dependence on observation*, the insight that a thing is affected by our observation of it, became crucial in the context of quantum theory, under which the idea of *observation-independence,* according to Wolfgang Pauli, was

> *too narrow an abstraction*; for *material or common physical things whose nature is independent of the way in which they are being observed, are metaphysical extrapolations. We have seen that modern physics has been forced, by recordable facts, to relinquish such abstractions as being too narrow.* (*Materielle oder allgemein physikalische Objekte, deren Beschaffenheit unabhängig sein soll von der Art, in welcher sie beobachtet werden, sind metaphysische Extrapolationen. Wir haben gesehen, dass die moderne Physik, durch Tatsachen gezwungen, diese Abstraktion als zu eng aufgeben mußte* (Wolfgang Pauli, quoted in Selleri, 29).

Verbally descriptive reconstructions and interpretations of complex historical situations, processes or events can be seen as reversed predictions, and if we grant that given published accounts on the same topic amount to a statistics *in nuce* (being non-quantifiable), then Pauli may be quoted once more: *predictions can be assessed only statistically, whereas for single observations there are no laws, so the case remains a-causal.* It all depends on our choice of framework. (*Bei gegebenem Zustand eines Systems [Objektes] lassen sich über die Resultate künftiger Beobachtungen im allgemeinen nur statistische Voraussagen machen (primäre Wahrscheinlichkeit), während das Resultat der Einzelbeobachtung nicht durch Gesetze bestimmt, also letzte Tatsache ohne Ursache ist* (Selleri, 30).

Analytically speaking, all objects can, and should, be seen in an information perspective (*objects as patterns of information,* Penrose) (1994, 13) and also in one of *management*, because of the logistic factor in dealing with objects. This is so because analysis requires a knowledge-building and cognitive process that is open for articulate description and goes by distinct stages, so that we always know, or hope so, where we are and where we are heading (*Burden*, 152f. on analysis by distinct stages in management theory).

4.3.4 *Emergent properties and object orientation*

Dealing with objects is further complicated, but also enriched, by an ordinary human routine, now formalized in computer languages under the label of *object-oriented analysis* or *programming* (*OO*R for short): that of perceiving an object while *including* what the object may serve for, how it can be handled and what it requires of ourselves or our environment; and including notions concerning it position in the same surroundings (*see* a couple of pages ahead). My rats would hardly be looking at the delicious nut just as a solid-state thing, but as something embodying the idea of action.

The paradigm is not a simple one and merits some further attention, for the general idea, that a thing is evaluated in connection with that we will do with it or about it, is relevant almost in any normal case, even in everyday occurrences. Furthermore, if an object, a system items or a process like a ritual, is highly complex, there are the so-called *emergent properties* (*EP*). Most systems are conceptually endowed with significance that means more than the sum of the parts. For such occurrences the somewhat disputable term, *emergent properties* is widely in use (*Burden,* V, 2). It seems preferable to comment on this kind of feature characterizing most objects of some importance, before approaching the question of the operational mechanisms concerning the objects.

Take any complex object with visual appeal; the chapel with Beata Ludovica Albertone by Bernini, for example (2.2.9; *Fig. 2.1 - 5*).

We can endeavour to analyse all the factors making up the totality one by one, considering relevant levels, and try to bring them together by means of comparisons and grouping them integrated. Still, *as a totality,* the cited chapel seems to involve a kind of *comprehensive effect* that is more than and different from what we achieve when we try to bring all the cited features together one by one in a synthesis. At one level, notions are graspable if not localizable: the emotional intensity, for example. At a higher (?) level, the work may be considered an attractive piece of art, a notion that remains ungraspable regardless of optimism among aesthetes. Such a kind of comprehensive effect is what recent technological and cognitive sciences attempt to capture with the concept of *emergent properties* (EP). The Ludovica figure itself is part of a larger system of imagery and symbols. Each of them has its own set of emergent properties, each brings forth, dependent on the onlooker's framework, appreciated (or negatively retrieved but vaguely assessed values.

But so must also the *totality of features* have (to the extent that such a thing is numerable). In this way, in model terms, we obtain a hierarchy of levels with each their *EP.*

The figure below, adapted from Hitchins, illustrates this, one level producing its *EP* and encompassed by the next one with its proper *EP,* and so on like a Chinese box. The graph is useful for illustrating the concept, but it should not inspire too much confidence (*see* below). The model lets one emergent level follow upon the lower one. The matter becomes considerably more complex if we adopt the idea of systems level and superimposed elaboration level(s) (1.6).

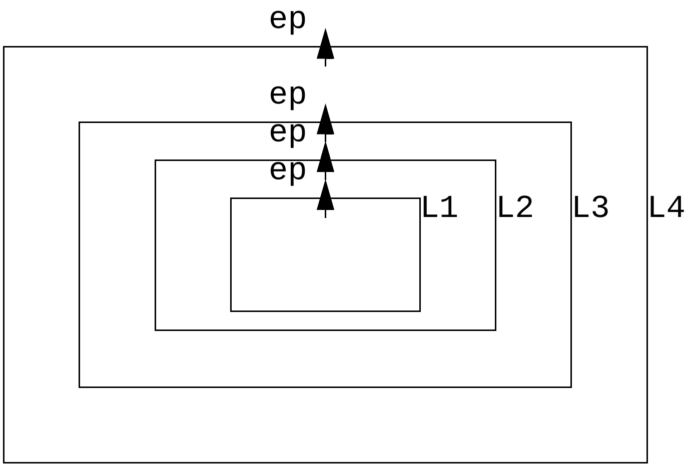


Fig. 4.3 - 2. System of Emergent Properties (after Hitchins). Four levels from each of which properties are "emerging"

The general issue of emergent properties is evidently important but so evasive that level gradation and interfaces elude our grasp. That is why I am spending a little time over it.

Any complex system, and there are few that aren't, can be described as consisting of a number of sub-systems. Herbert Simon may be quoted by way of introduction.

In a weaker interpretation, emergence simply means that the parts of complex systems, have mutual relations that do not exist for the parts in isolation. Thus, there can be gravitational attractions among bodies only when two bodies interact with each other. We can learn something about the (relative) acceleration of binary stars, but not of isolated stars. (Simon, 1996, 170ff.).

Concerning *agent-oriented and emergent view of intelligence*, Luger writes: *Finally, the phenomenon of intelligence in this environment is "emergent". Although individual agents are seen as possessing sets of skills and responsibilities, the overall cooperative result can be viewed as greater than the sum of its individual contributors. Intelligence is seen as a phenomenon resident in and emerging from a society and not just a property of an individual agent* (Luger, 18ff.). Hitchins (104f.) offers the following examples of how a "containing" system is related to the emergent properties of some subsystem; aircraft or ship related to engine; company to division: *For example, an aircraft or ship 'sees' the thrust, weight, fuel consumption, heat dissipation, noise, etc. of its engine - these are engine systems* [a subsystem's] *emergent properties. A Company or organization 'sees' divisional profitability, operational costs, work-in-progress, enthusiasm, resilience, etc. - these are divisional emergent properties.* Steve Weinberg (29ff.) gives another similarly construed definition of

the buzzword 'emergence'. As we look at nature at levels of greater and greater complexity, we see phenomena emerging that have no counterpart at the simpler levels, least of all at the level of the elementary particles [in physics]. *For instance, there is nothing like the intelligence on the level of individual living cells, nothing like life on the level of atoms and molecules... The emergence of new phenomena at high levels of complexity is most obvious in biology and the behavioral sciences... <but> it also happens within physics itself...*

Under the heading *Cognitive modelling and cognitive architectures,* Braisby and Gellatly ((2005) in their discussion of *parallel distributed processing* (*PDP*), show a *PDP* network model (also called *connectionist or neural network model*) which, as they claim,

can exhibit emergent properties. An emergent property is a behaviour that the model comes to exhibit over time, through training that was not explicitly programmed into the model. This feature also makes PDP models interesting from a psychological viewpoint as much of human learning derives from accumulated experience rather than explicit instruction (Braisby and Gellatly, 582f.; for related networks, *see* also Zornetzer et al., *Neural and electronic networks*).

We have here an example of a physical model that can be used for *hardframing* (4.3.1) specific intuitive issues, thereby making them ready for analysis.

In my *Burden*, I have noted, in a tone of reservation, several cases of emergent properties in liturgical systems; one of them concerning the overall *objective value* vaguely but importantly attributed to it.

Gregory, straightforward as always, reduces the notion to the vagueness it has to live with. As noted above (2.3.1) and also in the *Introduction* (*Burden*, 15, 244f; Gregory, 86ff.), while certainly not representing any truth value (and also being rather loosely conceived), *EP*s are useful as *recycling bins* for phenomena that are attestable, important and active but so far defy scientific and precise operational grip; and because of their almost-unsurveyable complexity of the same phenomenon, and their very flimsy and elusive boundaries (*function* in a clock, for instance; for the predicament of localizing functions in machines, *see* Gregory, 82ff.).

When we are talking about the mind, intelligence, consciousness, science, mankind, the clock, the weather, art, style (in pictures, music etc., not of course style in a computer program), then we are referring to emergent properties below the level of which there are measurable substructures. We are evaluating entities that elude and defy comprehensive analysis. Take a game of chess. What one can see is the process of moving the pieces by distinct steps. Above this, or underneath if you like, there are the patterns of goal-setting, intuitive prediction of probabilities, in short, the mental logistics; the ideological attributions concerning oneself and the opponents as players - emergent properties and emergent behavior. The computer can handle such steps and plan them efficiently, but it cannot handle or simulate the rest, being hampered by the *discrete* nature of its building blocks. It might seem that anything a computer cannot handle or simulate can be allocated to the category of emergence (for the limitations imposed by the "hard-wiring" of machines, *see* Sterelny, 38f. with reference to Fodor).

Looking at two of the pseudo-vectors (**pv**) proposed later on (4.6), the usefulness of the idea of emergence may be discussed with some relevance to our specific subject. Both the following notions are canonically and formally defined by the *Tradition* [with a capital T] *of the Roman Church*: **pv** *Heavenwards to cross - triumphus crucis* (1.9, 1.9.1 in **pv** numbering); **pv** *Cosmos divinely created* (1.9). Whereas the former can be read literally off the Spire because of the officially accepted image of the Cross (with a laurel wreath, too), the latter is an ideological overlay, obvious enough but certainly not formally connected with the Spire's configuration. Some humanities mainly work, regarding what is then considered essentials, with *EPs* without being aware of it;.

The "emergence" graph just presented (*Fig. 4.3 - 2*) serves, as I noted, as an illustration of the idea under observation. But we should not let it send us away with the illusion that *levels* are as clearly determined as the graph might seem to imply.

The big problem with emergent properties is that the idea only very rarely can be subjected to formalization that distinguish well among interconnected levels. This probably is a very useful general idea that is, however, hardly usable unless it can be connected up with some other model that has a firmer anchorage. The operational device called *Object Orientation* (*OOR* to friends) appears to be such a model. To this model mechanism we now turn our attention.

It happens regularly in life that an object is conceived or understood *together with* notions concerning its use and the procedures and means for technically or mentally (or both) using or elaborating or it. The Spire, rising up in the Roman skyline (at that time less crowded), carried with it its utility concept, that of being a place marker for the Papal university, thus inviting appropriate reflections; obvious but not unimportant in principle. Such an operational modality, which can usually be sufficiently well determined, can carry with it, imply, systems of vaguer issues - vague, but not less important. So that in many cases emergent properties, barely describable in themselves (Borromini's style), become to a sufficient degree concretized in terms of *operations on the object* that appears to call them forth. Such operations can be mental and not visible, and they can belong to a pattern of community interchange. Lest these comments should make the notion seem utterly banal, it has to be noted that in serious analysis, no attention, reaction or perception is to be taken for granted.

This normal human mechanism has been developed in computer usage under the name of the *Object-Oriented* models. The related models are worth discussing here, since the articulation they offer seems applicable to the material discussed in this book.

Technically speaking, *OOR* operations handle modules in a digital format (with its remote origins in data programming in Oslo), and so can be seen as a syntactical model for the theory that *the frameworks of emotional judgments are essentially modular* (Schulkin, 50, 134), and for Fodor's insistence on *the modularity of thought* (summary and references in Braisby and Gellatly, *Cognitive Science*, 632ff; but the subject is discussed in a great number of publications since 1983, the year of the publication of Fodor's *The modularity of thought*).

This program is complicated, but I shall try convey the flavor of it, for the idea supporting it imposes itself in most contexts of "reality" analysis. Here a survey of relevant literature: generally: *Microsoft Computer Dictionary*; Winkler, *Computer-Leksikon 2007*, see items under *Objekt* (numerous entries); Blair, Gallagher, and Shepherd, *Object-oriented languages*; a corresponding distinction between a data base (accumulation of facts) and a knowledge base: data base plus rules for how to handle them: Coyne, Rosemann, Radford, Balachandrian and Gero, *Knowledge-based design systems*.

In my *Burden* of 2000, (IV, 5.3), I introduced the *OOR* program type into "our" field, noting that

my appeal to analytical patterns culled from computer science should not be taken as a sign of any belief in their mechanical utility: my empirical material and the models I have developed for handling it cannot be programmed to be run on a machine. So when adopting an idea from the so-called object-oriented paradigm, it is the moral and the methodological perspectives behind it I do find useful, considering the ideas behind the programs as a bank or store of ideas, not as a program on a software level.

The crux of the approach is to ask, not what a thing is, but for a list of some of its characteristic features or attributes, and then ask for the methods considered adequate for *handling* them. I do not ask, what is a ritual? (*Burden, passim*), knowing only too well that attempts at definition here would either be superficial or take me for a ride. I am asking for the system's ingredients *and* characteristic functions and active factors in a ritual process and how to handle them analytically in a way that highlights the apparently important features in their functional interplay. I wouldn't "cover" anything, but I might pick out significant features in the dynamic relevance pattern. Thus, quoting from the publications just listed (details: *Burden*, IV, 5.2).

An object consists of a set of attributes and methods. Methods are groups of instructions with reference to the attributes; or even: *Object. A variable comprising both routines and data that is treated as a discrete entity.* Furthermore,... *what is an object at the conceptual level (the user view) and how is an object realised in practical systems (the implementor's view). At the conceptual level, an object is any perceived entity in the system being developed... In more detail, an object is defined as follows: - An object is an encapsulation [joined together in a packet or module] of a set of operations or methods which can be in-*

voked externally and of a state which remembers the effect of the methods... The methods are the set of operations which we are allowed to perform within the context of the object.

More recently, Luger (2005, 791ff.) has a description that is not very different from the one of 1994; a sign that the program type has shown staying capacity but also that it is flexible.

Object-oriented languages support an approach to problem solving that lets us decompose a problem into interacting objects. These objects have a state that can change over time, and a set of functions or methods that define the object's behaviors. Essentially, object-oriented programming lets us solve problems by constructing a model of the problem domain as we understand it. This model-based approach to problem solving is a natural fit for artificial intelligence, an effective programming methodology in its own right, and a powerful tool for thinking about complex problem domains.

Three operators are needed in the full-blown digital program: *1) encapsulation, 2) polymorphism, 3) inheritance.* They are useful not only to help us grasp the main idea; in addition, they articulate prominent features in human conceptualization processes. Here is Luger's exposition of the three key terms.

§1) *Encapsulation*. All modern programming languages allow us to create complex data structures that combine atomic data items into a single entity. Object-oriented encapsulation is unique in that it combines both *data items* and the *procedures used for their manipulation into a single structure*, called a class.... CLOS [the name of a data application with which I am not familiar] implements method as generic functions. These functions check the type of their parameters to guarantee that they can only be applied in instances of a certain object class. This gives us a *logical binding of methods to their objects*.

§2) *Polymorphism*. A function is polymorphic if it has many different behaviors, depending on the types of its arguments... An example from a drawing program, where drawing a square, a line etc., are each of them under command of one function: "draw". CLOS supports polymorphism through generic functions. A generic function is one whose behavior is determined by the types of its arguments. In our drawing example, CLOS enables us to define a generic function, "draw", that *includes code for drawing each of the shapes defined in the program.* On evaluation, it checks the type of its argument and automatically executes the appropriate code.

§3) *Inheritance*. A mechanism for supporting class abstraction in a programming language, defining general classes that specify the *structure and behavior of their specializations* (tree -> pine, poplar, oak...). In intuitive approaches, let me submit, this digital operational feature remains outside the range of what is workable. Instead, cateorization along the lines set forth by Rosch is more relevant (4.4).

The idea of object orientation can help making the slippery notion of emergent properties better workable.

As noted above, the idea of object orientation and emergence are interrelated - or can be made into being so. In the *Introduction* to his book on object-oriented analysis and design, Booch discuses complexity, and in this connection also emergent behavior:

In the photosynthesis of a plant there are no centralized parts that directly coordinate the activities of lower level ones. Instead, we find separate parts that act as independent agents, each of which exhibits some fairly complex behavior, and each of which contributes to many higher-level functions. Only through the mutual cooperation of meaningful collections of these agents do we see the higher-level functionality of a plant. The science of complexity calls this "emergent behavior" (Booch, 20ff., and Chapter 4).

In whatever way we perceive and mentally handle an object, we extract some bits of information from it, by attribution or otherwise, which seems to be what Roger Penrose has in mind when he speaks of objects as *patterns of information*.

4.3.5 *Information*

Most of the processes contemplated in the present book require some variant of information, to people, between people, coming from authorities or other human agents or from retrieved objects. All sorts of collective activities too are dependent on information (Laumann and Pappi, *Networks of collective action*).

The spectrum is a wide one, ranging from explicit, delivered and understood data via a number of intermediate forms over to procedures that have been called *equivocal communication* (Bavelas and others). The latter perspective, reminding me of Evans' hypothetical thinking (4.3.6), concerns vague, often unstable idioms intended to cover embarrassing or indefinite things which can be applied to much of the intercourse and exchange between humans and between humans and things, as in the case of my rats.

Data and information should not be confused; indeed, the illustrated model drives the point home. To quote Parker on this crucial issue (Parker, 12): Data

refer to facts. When data are filtered through one or more processes so that they take on both meaning and value to a person, they become information. Information, rather than data, is what people use to make decisions .. both the computer and the human mind act as processors that select the data and transform them into meaningful information. As information is generated from data, it, too, eventually becomes part of the store of data (Parker); and: *Information is data that has been processed into a form that is meaningful to the recipient* *and is of real or perceived value in current or prospective actions or decisions* (Davis and Olson, 200).

Information, to be a useful construct, cannot be static, as a datum can be; it must be a process, usually a set of interconnected processes. The object is a place-marker for an event (or several of them) in a process. Our (or rather the rats') *nut* thus is a cognitive event, going through the phases of attention, interest, appreciation, decision and action (being eaten or discarded), as noted above (I take *event* pragmatically, regardless of such contributions as Edgar Morin's collection volume on *Theories of the event*; restricting his conception of an event as, 1. change of status - in math, physics, conceptually etc.- and, 2. passing a node in a network).

The *information model* now to be presented (culled from Davis and Olson in their *Management information systems*), can be used for all the variants just listed, also in the processes of long-term memory. I present the original version, which I have developed with the purpose of distinguishing further between various sections of the relevant processes and thus between *data* and *information.*

We cannot speak of a relatively complex planning and implementation process focused on display in a public place or in a ritual as a means of displaying and exchanging messages, without looking at it as an information system. The simplest information system consists of three "boxes": inputs (of data and instructions) - processing (of data according to the instructions) - output (results, information).

These inputs are *external inputs*. But here as elsewhere, the information processing function frequently needs data collected and stored previously [and data and acquired information, too, may be stored here, as we just noted]. *When data storage is added, the information processing function includes not only the transformation of data into information but also the storing of data for subsequent use* (Davis and Olson, 288f.). Thus I have added a fourth "box" to the system, one for "storage". Here data and data processed into information, are stored.

Modern formal info models, like the one just shown, or in the so-called *information paradigm,* can be used as a hardframe (4.3.1) basis for handling softer versions (Inmon, *Data architecture,* brings a survey of data versions and procedures). In fact, there is no shortage of literature in this field. The *contents* entered into the sections of such a model should be maximized far beyond what at the actual moment *seems* historically plausible, yielding an artificial totality from which any specific concept or cluster of concepts may be called up, evoked or applied in the period (Geen, *Human motivation,* brings a good survey).

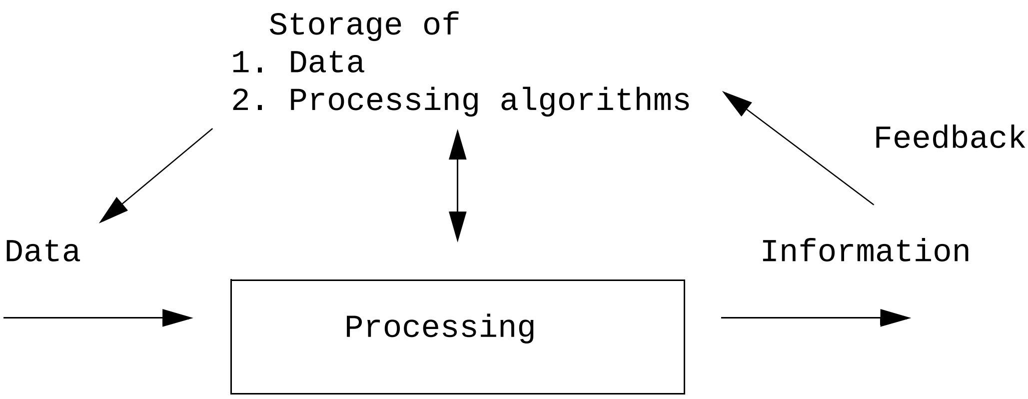


Fig. 4.3 - 3 Information model. To the original model by Davis and Olson I have added feedback mechanisms and specified the contents of storage, adding processing algorithms, since I believe methodology also would be stored for repeated use. Indeed, a cache should also really have been included. Still, the model is much simplified, omitting entropy (for a survey of this, see Fischer, 2007, passim).

This would serve as a graphically striking warning against simplifications; moreover, it allows us to focus on essential historical interests and preoccupations without having to pick out, and thereby prefer, specific cases, something that tends to happen in linear modeling, in which it is almost impossible not to imply predefined priorities. *Motivation* in individuals or groups authorizing us to extract and select some specific specimens in the box will often be highly complex. Here again, under the *probability perspective* concerning *types* of people, social groups or authorities, *approximation* has to be accepted as a principle. The most useful aspect of an info model is perhaps that it illustrates graphically the complexity and uncertainty of attributing meanings to things and people.

At a musicologist conference at Regensburg in 2002, to which I was most surprisingly invited to talk about interdisciplinarity, I was told that info models were useless on the relevant material. I can easily image this if one tries to run Johann Sebastian Bach on a computer model of information. The point of using such a model, which is originally designed for computer, on "soft" material, is its articulate structure, which - and this is crucial - has resisted the tests in its original use. Such a structure comes handy in for bringing order into our material and acting as a warning against simplification by semantical untidiness.

In most cases we will have to set up a *black box* (*processing* on the model*).* This will be an information elaboration (*see Fig. 4.3 - 3*), for example concerning what may be considered the crucial concepts in ±1600 mathematics, or we might work out a set of hypothetical data concerning the planning, purpose attribution and implementation of the Spire as evaluated by the Church through the model.

With reference to the debates about the complex *spiral-helix-hyperbola* (3.2) as well as one of the pseudo-vectors presented below (4.6), namely (**pv** *Thematic views of the helix form*), we might imagine an anachronistic visit to Rome by *Fermat*, one of the chief protagonists in the controversies and debates over central questions regarding the relations between geometry and algebra (another one was Torricelli). Let the information model in *Fig. 4.3 - 3* represent Fermat's brain. The Spire as it looks, its shape, is the input data. A processing algorithm in storage turns it into a pure abstraction in the form of a conical helix with variable torsion. By the feedback process, another processing algorithm connects the naked helix curve to the complex cluster *spiral-helix-hyperbola.* At a next round of output-feedback, larger issues of contemporary mathematics, in this book dubbed *protocalculus*, are being called up and sent to output as information to be shared and elaborated by the community in the relevant network. Under such circumstances as these, which represent normal scenarios, it would not make much sense to ask about *The Meaning* of the Spire.

4.3.6 *Hypothetical thinking and dual processes*

The idea of *hypothetical thinking* appears to inject more vitality, but therefore also some amount of irregularity and vagueness, into the processes sketched out in the above *Sections*. There are several versions of the idea. Jonathan Evans (2007) elaborates his version of so-called *dual-process theory of thinking* in a program that he presents in the following terms.

Human intellectual setup implies, to quote from Evans' (1f.) highly acclaimed book, *an extraordinary ability to reason, entertain hypotheses and make decisions based on complex mental simulations of future possibilities. I will use the term "hypothetical thinking"* [here: *HT*] *as a catch-all phrase for thought of this kind.*

It would appear that *simulation* is a key word here. There are two distinct kinds of thought that, by way of an introduction, he calls *intuitive* and *deliberative.* Enlarging somewhat upon the former, he notes the factors of suddenness etc., as a frame for rapid problem solutions of the intuitive kind.

Here I would refer to Hubert Dreyfus' theory concerning *expertise* in his *Misrepresenting human intelligence.* According to this view, expert solution of problems makes big jumps from one step in the flow of reasoning to the conclusive one, while skipping a number of intermediate steps that are stored in the brain, working subconsciously (whatever that may mean) and in no need of being retrieved. This, Dreyfus contends, is exactly the thing that a computer even in parallel programs of artificial intelligence cannot replicate. Perhaps some of the steps passed over are summed up in our system in terms of emergent properties.

The central theme of Evans' book, to return to that, concerns how to address the problem of *the relative influence of intuitive and deliberative thinking and the interaction between the two systems*; an issue that should be attacked, he says, by experimental psychology. He criticizes current evaluations and discussions of rationality and irrational cognitive behavior, for being too tied up with semantics.

It would pay to dwell a little longer on Evans' account. The *HT*, Evans (2007) affirms,

involves the imagination of possibilities and the exploration of their consequences by a process of mental simulation (*Introduction*)*;* in fact, *thought that requires imagination of possible states of the world. Examples include hypothesis testing, forecasting, decision making, counterfactual thinking, deductive reasoning and suppositional reasoning.*

I am not sure I understand what is new and what is reformulation of old stuff here, but my failure most likely is due to my own shortcomings. Anyway, the paradigm gives rise to a slightly different view of the famous *mental models* propounded particularly by Johnson-Laird in 1983. Evans continues:

Readers may wonder at my use of the term "mental models". Is this some variant of the famous theory of Johnson-Laird and colleagues? Not so, in fact... his models are largely described as semantic devices. This means that they can only represent possible states of the world (does it?).

Hypothetical-thinking theory needs a processing model, Evans affirms; *in decision making, for example, when we consider a possible action we may construct a mental model by mental simulations the consequences of that action* (Evans, 17ff.). So, to repeat, simulation is a keyword here. The general idea reminds me of the theory of anticipation of events in *personal construct theory* (*next section*), with some of the flavor in Herbert Simon's theories as an extra ingredient in the recipe.

The idea of hypothetical thinking invites reflection on *probability* also, which is why it becomes especially interesting for my venture. There are two major academic schools of thought on probability, Evans informs us (a systematic account, from logic to philosophy, in Howson and Urbach, *Scientific reasoning*).

The classical or objectivist position is that probability is a relative frequency and can only be considered when there is a frequency distribution of some kind... [in other words, statistical], *The Bayesian or subjective view is that a probability is no more no less than a degree of belief in something.*

By the criteria set out thus far, Evans builds up the two complementary (as I understand them) theories of *dual-process* and *dual-system*; the technical details of which need not occupy us here; it is the "moral" that is relevant at present. Heuristic analysis too comes into play here (Evans, 15ff.).

4.3.7 *Processing concepts*

The *concepts* of historical relevance discussed in the present book are intuitively approached and verbally and graphically represented, allowing of no algorithmic handling. That is, they are not constructive and formally manipulable in the sense of *formal concept analysis based on lattices*, as studied in terms of the mathematical foundations by Ganter and Wille. As a guide, but nothing more, lattice patterns can turn out to be useful for discussing concept structures (*Introduction,* 7.3 - *d*, 4.1.1 and 4.6.1).

A good illustration of how conceptual networks may be thought to work formally is to be found in Minsky and Papert's *Perceptrons*, p. 5, Fig. 0.1, developed in parallel computing. Various values in the domain of *x* in the function *Y(x)* are elaborated in parallel and separately, but in the next step assembled through a function labelled *W*, thus yielding the total value of *Y.* I cite this as an example of how some sort of exactitude is being sought for the handling of this rather evasive material.

An unexpected shape or an unusual position, both applying to the Spire, experience tells us, can alert people's observational acuity. Bernini was aware of this and used striking shapes to attract people's attention (Sant'Andrea al Quirinale: portal decoration "preparing" for the configuration of the main chapel) - as he explained to his *cicerone*, Sieur de Chantelou, in Paris in 1665. When something starts showing unusual or problematic features, it tends to attract attention to its character and makeup. Herbert Simon has a neat illustration of a comparable contingency. *A bridge, under its usual conditions of service, behaves simply as a relatively smooth surface on which vehicles can move. Only when it has been overloaded do we learn the physical properties of the materials from which it is built* (Simon, 1996, 13).

It is a fact of life, perhaps not welcome to some, that articulate analytical models make it impossible to try to pin down what was the case concerning this-or-that individual or group-wise conceptualization-pattern. Only *type*s of persons and groups, or categories of them, can be elaborated here (*Burden,* 187f. on social categories); using *Rosch's categorization criteria* (towards the end of the present *Section*; *Burden,* 257f.). Out go *The* commissioner and *The* architect, as individuals loaded with biographical matter, only to turn up again in another format: as participants in the planning outfit, while bringing with them some specific features into the model (3.1). In the humanities, a mix-up between general categories and biographical individuals often makes it hard for us to be sure where we are at any one time.

Conceptualization processes have been studied from widely different vantage points, Herbert Simon using artificial models, Putnam philosophical and mainly semantic ones; to cite just two outstanding contributions.

The notion of *concept* that seems most constructive for my purpose, is the *operative*, not definitional, format developed by Hilary Putnam (1975, 271, details in SL, *Burden*, 166ff., 251ff.; see also N. U. Salmon, 31 f.; 93 - 157; Schulkin, 6ff.). Concepts are *abilities to handle whatever may be at hand, in terms of knowledge or other mental processes*, particular subjects, objects, themes or similar.

Thus Putnam:

... possessing a concept is not a matter of possessing images... since one could possess any system of images you please and not possess the ***ability*** *to use the sentences in situationally appropriate ways... A man may have all the images you please, and still be completely at a loss when one says to him 'point to a tree'... For the image, if not accompanied by the ability to act in a certain way, is just a* ***picture****, and acting in accordance with a picture is itself an ability that one may or may not have .. He would still not know that he was supposed to point to a tree, and he would still not* understand *'point to a tree'.*

So the ability to use certain sentences... *<is> the criterion for possessing a full-blown concept*; and in conclusion:

... no matter what sort of inner phenomena we allow as possible ***expressions*** *of thought, arguments exactly similar to the foregoing will show that it is not the phenomena themselves that constitute understanding, but rather the ability of the thinker to* ***employ*** *these phenomena, to produce the right phenomena in the right circumstances.* And: *the understanding of <an>... expression,,, is not an occurrence but an ability.* We have to look at realities of *usage and practice.*

The *operative* status of concepts arose already with Einstein, albeit in another categorization and with a different name (SL, *Operational determination).*

Personal Construct Theory (*PCT*) comes in usefully (Downs, 77; Moore and Golledge with other contributions on the same theme), for it lays out a *general scenario* in which conceptualizing abilities can be eventuated. The *Fundamental Postulate* in its simplest terms reads like this: *A person's processes* [mental, conceptual] *are psychologically channelized by the ways in which he anticipates events.*The scenario depicts possible future states in addition to previous and present ones. The basic idea is *anticipation of situations or events about to be met with*; and it was introduced in the 1920s by the British neurologist Henry Head (a vast literature followed) (Canter, 13ff, also on Head and on Bartlett's *remembering* studies).

A series of *Corollaries* follow so as to substantiate this claim and how the channelling works; such as:

Construction corollary. A person anticipates events by construing their replications -... Organization corollary. Each person characteristically evolves, for his convenience in anticipating events, a construction system embracing ordinal [Webster: denoting order] *relationships between constructs. -... Choice corollary. A person chooses for himself that alternative in a dichotomized construct* [among a person's finite number of them] *through which he anticipates the greater possibility for the elaboration of his system.*

The PCT introduces us to a kind of *sociology of cognition*; so that Callero's claim to being the first in not absolutely accurate (Callero, 44 - 54). The latter has, in his *Toward a sociology of cognition,* a very illuminating introduction to the subject; while it is less uplifting to see that he entertains hope for *a complete explanation of the social world.*

Knowledge of things and concepts of them hardly ever arise in purity (we all seems to know). One disturbing factor, a noise in the system, is *emotions*, and much recent writing in cognitive fields has been dedicated to this slippery subject (Geen, Benjafield). I shall let Schulkin give us a synopsis that seems to cover the most workable facets of the issue, noting as particularly relevant his experience from medicine and neurobiology.

Emotion is supposed to be unmediated and pure, thought is mediated. Emotion is supposed to be receptive, thought is constructive. But the discourse is wrong and misleading, not because some of us are not detached from our emotions, or embedded in them. It is often said that there is an act of just seeing or experiencing unmediated. But again to see something is to have a perspective; that is, a framework is presupposed that structures and is necessary for seeing. While the emotion is not reduced to thought, one cannot feel without thinking. The question is, how should one think while feeling, and how does it play a role in inquiry? This is its "cash value" (Schulkin, 49):

Reputedly, the frameworks of emotional judgements are essentially modular, as Schulkin assumes (for *modularity*, see 4.3.4, towards the close of the *Section*); that is, coming not in homogeneous format, but in highly specialized clusters embedding other psychological and cognitive factors. Emotions will typically be connected with perceptions, and *there are no noninterpretive perceptions... At best one can speak of degrees of interpretation or "theoreticity"... The split between qualia and thought, or emotions and reasons seems false on a number of fronts* (Schulkin, 48f.; for a case of defending the realism behind conceptualization and truth-evaluation, see Vision, *Modern anti-realism*).

Let me, with no claim to originality, introduce *two modalities* that make working with concepts, notions, ideas even more complex than seem to arise in the thematics of Schulkin's book. Both may occur in a state of more or less unreflected conceptualization of something.

If this something is important to the Planners (3.1), then we have to take the subject seriously. The modes can be classified under the labels of *inchoate processes* and *sidetracking* and seems to be related to Evans' ideas about hypothetical thinking (4.3.6), as well as, indirectly, to Simon's ideas about administrative behavior and bounded rationality (3.1).

The *first* is relevant under ordinary observer conditions when the actor is not particularly attentive in his reflection over what he is seeing. This is a modality instantiated in everyday life and often contributes to shaping attitudes towards things in the surroundings, creating a sort of atmosphere: an object or image or logo that the local authorities such as ecclesiastics would have to take into account concerning their display policy. They would be used to such cases from their pastoral practice. Even when not taking into account phenomena like Simon's *Bounded Rationality,* conceptualization, acquiring bits of knowledge and understanding, and perceptions, are very often *incomplete processes*, which can nevertheless have manifest directions, tendencies and scopes, but which literally *stop* in middle course, at some floating point; tending towards, approaching something without getting there. It is a well known circumstance, elaborated by Evans (4.3.6), that humans often rely on and make decisions on the basis of incomplete experiences, surmise, loosely underpinned explorative previsions (an argument against so-called strong artificial intelligence; Born, *The case against artificial intelligence*, contains several contributions on this subject).

I need a term for this type of contingency. The usual sense in English of something *inchoate* is *not yet completed or fully developed; rudimentary*. I am using the adjectival form as a *noun*, *inchoate*, for a loosely directed or focused process of conceptualizing, appreciation, understanding, comprehension that has not been carried through to some definitely conceived terminal point but has been left floating underneath something else of a more definite character.

Such phenomena can leave their marks on a local society. The same can be said of the next type of cases.

The s*econd* model of concept access and use, which I label *sidetracking,* takes stock of the conditions arising when conceptualization occasionally fails to make a direct hit. Even at an advanced stage in elaborating a notion, if this is relatively complex, one can be sidetracked and *believe* one is on the right track; one can easily be diverted by subgoals (Simon, *Administrative behavior*). Many of the "mistakes" that later generations have laid at the door of some big light, bear witness to this contingency. This kind of occurrence accompany many of our mental processes and is one of the reasons why it does not make sense to ask *what* *someone really meant.* We must *reach a genuine understanding of what he thinks,* Garber writers concerning Descartes, to which one has to reply that there is no such thing (1.4). The history of mathematics contains plenty of examples of people believing they think something about something, only to have a later generation discover that they did "in reality" not focus on what they thought they had in mind, nor did their contemporaries.

The case of Descartes as considered by Garber is one of *verbal expressions.* A word string does not have to be very long before the possible implications fan out in so many directions that it becomes almost impossible to account for the intentions behind it and at the same time take into account all the branches of possible significances. We see occasionally attempts at explaining architecture directly in terms of what architects have written. The upshot is that one never can tell in any definite manner. Only equations can do the job for us. Typically, when Herbert Simon was confronted with John Maynard Keynes' famous book on economics (1936), just verbal and no equations, he did not understand it. Then, some well-meaning souls transposed the arguments into math, and Simon did get the message (that it still did not seem to make sense, is another story).

4.4 Spire structures

The ideas reviewed in the foregoing *sections* all are somehow related to our task of describing Borromini's Spire, as a pars pro toto for the other curves and manifolds. Personal construct theory supports the notion that people come to a situation or an object of some consequence with partly preconceived ideas about them, thus configuring them in terms that are meaningful for themselves. They would regard the Spire in the light of such factors. The emergent properties imply meaning attributions beyond, or "above" if you like, what is literally or protocol assignable. The object orientation mechanism makes it, when applied to humans rather than to machines, natural when seeing or contemplating something, to include specific kinds of (mental) operations on the thing. This, I want to say, is what mathematicians might tend to do when beholding or reflecting on the Spire: the strikingly irregular helical *shape* would automatically imply the idea of math operations on its *form.* The dual processing idea supports the notion of oscillations among probabilities in people's conceptualizing of the Spire. The information paradigm supports several of the cited mechanisms *and* it dispels any hope of any static and determinate interpretation, allowing us to envision only a flow of outputs that is conditioned by situations, in themselves dynamical or unstable (my rats!) and resources (mental and others).

Evans' ideas about *hypothetical thinking* and his two categories (however loosely defined) of *intuitive* and *deliberative* thinking, as well as ideas from the cognitive sciences in general, can contribute to articulating our idea of how the Planners might have handled the Spire mentally. Forecasting and mental simulation of the prospective situation the planned Spire might contribute to produce, or provoke, would contain important, perhaps crucial, irrational or, at least, very loosely devised factors in the explorative prevision of the planning outfit. There will always be irrational factors at work, and Evans contributes to giving this fact a name.

Along with the irrational factors enter into the project also chunks that are *not* irrational. Experience from architecture suggests that good and working ideas often enter as appendages to highly emotional or irrational ones. The upshot? That rationality not only is often *bounded*, as Simon has it, but also that it may play a constructive role by subverting some of its own rules.

It is quite another thing to ask about *professional attributions* regarding the Spire's links to contemporary mathematics. Somebody in the planning outfit would most likely have such a competence and certainly also many people at the university and in its entourage.

By now we should be sufficiently armed for attacking the problem of *how to structure* Borromini's Spire, find a usable image of the "hard" and the "soft" aspects and issues that I have associated with it. I cannot discuss interaction and interdependence among elements unless they are localized and identified within a system. We have collected a few attributes that characterize Borromini's Spire both from the hardware and the software or conceptual side.

It is my task now to propose an experimental structure that might accommodate these data. Above the levels of trivia, there exists no one correct way for doing this. Thus to realize that the picture is not pre-determined of course amounts to acknowledging that the selection of the pieces and the assemblage of them are identical with the analysis procedure; coming into existence as relevant and manageable entities in operational terms (cfr. SL, *Burden.* 184f.)

4.5 Filling out the framework

In *Chapter* 3.3, I presented and discussed a "diagonal" graphic model (*Fig. 3.3 - 1*) intended to convey a general survey of the Spire's position in a larger confguration space. I shall now let it be followed by a graphic model designed for charting the categories making up the Spire's internal configuration (*Fig. 4.5 - 1*).

Intuitively accessed, the Spire consists of, *one*, a masonry helical *guglia* or *spire* (*1, 2 and 3 in the items list below) and, modelled or placed upon it, *two,* an *iconographical program* (1.9, 1.6.1) of traditional liturgical symbols, namely, light, in the shape of jewels, torches and flames, a triumphal laurel wreath and the cross (*4). Under category *one,* my focus makes it convenient to subdivide the spire into two elements, the guglia as a whole (*1, 3) and the helical thread or fillet rotating upon it (*2).

I shall be putting these distinctions to use, and naming them, respectively, *informal* and *formal imagery.* The distinction between formal and informal is justified by the fact already stated, that No. *one* (*1, 3) is non-definite and open to various conceptualizations, while Nos. *two* (*2, 4) certainly is also that, but nevertheless supported by mathematics and, for the imagery, in all essentials by the canonical system of the Church.

Applying the term *solid_state* to the hardware and its aspects and *fluid_state* to its being subjected to mental operations, must be regarded with due reservation; but the distinction seems pragmatically workable. This idea owes something to the object orientation paradigm (4.3.4), since the visual features are here evaluated for the mental (or other) operations by virtue of which they function as effective display.

The model is construed on the maximation principle; not all items are necessarily relevant, but they should figure in a model to forestall oversights (4.1 and 4.7).

Since strictly speaking any observed object, even the solid_state ones, is what we make it, being cognitively activized by ourselves, the distinction between solid and fluid is not a neat one, but it seems heuristically workable in so far as specific cases are being analyzed. We can use the argumentation ventured in connection with the *iconic interface* in the model in *Fig. 1.7 - 1* for a configuration space, concerning an area between a systems space and an elaboration space.

Is there a place for Lord and Wilson's *distinction* between *form* and *shape* in all this? Applied to the Spire, we might say that the helical masonry thread or fillet running up the whole surface, and which can be directly seen, is a *shape*. It has been constructed - produced, at least in its overall make - by certain rules having been applied, explicitly or by implication. Thus the resulting curve, implied or interior, if you prefer, would be the corresponding *form.* The same goes for the *iconographical program* (flames, cross etc.) (1.9, 1.6.1); visible shapes with canonically established, significant, doctrinal form determining them.

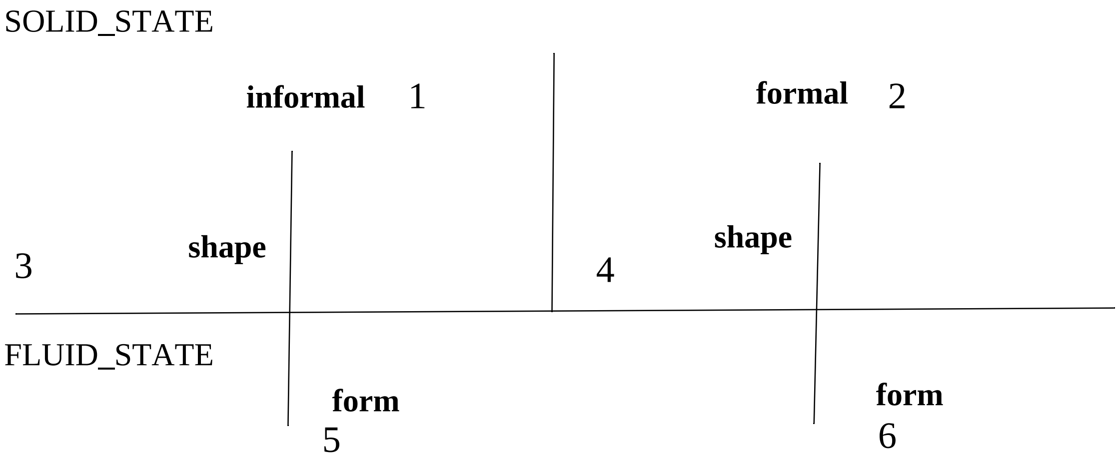


Fig. 4.4 -1. Model of Spire structure; model structure and elements assessable on a probability pattern; informal: non-definite attributions; formal: definite by law (Cath. Church) or rule set (mathematics).
*The helical thread (*2) is in the primary focus in this book, since it is on this curve that connections to contemporary mathematics can be discussed* (*chapter* 3.4).

References in the model.
- items
- *1 Spire body*
- *2 helical thread*
- *3 external appearance of Spire body (conical helix with upwards increasing torsion)*
- *4 imagery configurations established in canonical signs (see comments on Fig. 1.7 - 1) (cross, flames etc.)*

- operations
- *5 conical helix is described verbally or in elementary geometry for its outline features*
- *6 conical-helical curvature with increasing torsion, handled in advanced math, such as some protocalculus or geometrical technique.*
- *6 systems-level (chapter 1.6) handling of iconography*

Having discussed the internal working of the model, I shall ask a more generic question. What is the general status of this kind of model? Returning to an argument introduced above (4.1, with *subsections*), let us remember that an object functions in society not merely by virtue of its physical components (rather obvious), but in human perception and communication crucially and in terms of its being cognitively and conceptually elaborated.

The model represents an attempt to develop a structure from functions and levels in such a scenario. It thus represents the Spire as constituted by cognitive and conceptual operations on it; that is to say, to picture a structural framework within which to accommodate specific historical probabilities.

Managed the way I have indicated, the Spire model in *4.4 -1* assumes the role of a general model of design in Herbert Simon’s application of this term, but now extended to encompass *objets d'art* (Simon, 1996, 111ff). It is not complete under any perspective; it is not definite by any criterion. But it may prove useful as a platform for theory development; and also, for a further underpinning of the models presented earlier, as well; and for accommodating historical evidence with some manageable order; which is the most one can expect of a model of this kind. It is a platform with programming features, in the present case designed to produce a *picture of probabilities.*

4.5.1 *Real and metaphorical matrices*

Surveying the protocalculus story, and relating the Spire helix to it, it would be unwise to insist on *one* solution. This is what a *matrix model* comes usefully in. Its main characteristics as relevant in the present context, come as follows. Structurally, the model requires a probability choice among alternatives represented metaphorically as *vectors*, and this because we have to do with focusing *directions* or *tendencies*, not definite solutions.Within any one of them, we load some choice of *probable relevances*, since, to repeat, we do not face fixed values but tendencies or *potentiae* (Heisenberg uses this Aristotelian term, *dunamis*).

What I need, then, is a picture of an energy-laden grid in which directed tendencies (pictured like vectors) are at work, illustrating so many probabilities of visual and conceptual attention and attribution or clusters of them. Using for this the relatively modest name of picture_matrix, should exonerate me from being blamed for misusing mathematics; shorthand p_matrix; with their p_vectors.

$$\begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix}$$

Fig. 4.4 -2 A regular matrix

To be clear about distinctions between my *p_matrices* and the mathematical ones, let me note that a *probability matrix*, speaking now of a *real* one, from a particular subset, gives a set or more sets of probable events indicated by the numbers contained in it or resulting from further operations on it, also in connection with other matrices. Such a calculational precision is beyond my pseudo matrices. To quote Kolman,

in many applications we know the present state of the system and wish to predict the state at the next observation period, or at some future observation period. We often can predict the probability of the system's being in a particular state at a future observation period from its past history (Kolman, 365).

A brief reference to the basis for the idea of metaphorical matrices (a *picture_matrix*) may be worthwhile. A *matrix*, in mathematics and so also in physics, sometimes called a register (Heisenberg), is a rectangular array of real or complex numbers arranged in *m* numbered horizontal rows and *n* lettered vertical columns deriving from systems of equations and being subjected to particular operations such as addition and multiplication. An extremely simplified matrix is shown in *Fig. 4.4 - 2*. In many cases single rows or columns function as *vectors*, directed quantities (cf. the difference between speed and velocity) and are thus named; a2-b2-c2 might be one. Particular values are allocated to the so-called main diagonal, a1-b2-c3, which illustrates the fact that a matrix is regarded as a structural unit of interrelated elements, not just a set of separate rows and columns.

The *p_matrix* model is being used for expressing *probabilities,* and the vectors represented figuratively in parallel will indicate concepts and notions or chains of them. These strings can combine into new ones, making up interlocking grids or networks. Possible combinations here are products of cognitive, conceptual and situation factors and will expand far beyond our control.

The many possibilities of some of the vectors merging into some new concept, are not taken explicitly into account in the model presented below (4.6); doing so would have meant a choice at random, rendering the model unsurveyable and made an exercise in methodology spread out beyond our grasp.

4.6 Uploading Spire data into a p_matrix

A *p_matrix* as introduced in the foregoing *chapter,* will now be loaded with contents. It would not be consistent with the idea of the entire argumentation in the present book to expect from such an operation, a *grande finale* or dramatic *dénouement.* The terminal matrix model will appear as just a listing of items and operations on them that have been discussed at so many points in the book. To my mind, this is the most appropriate, not to say, the only, functionally consistent conclusion to my exercise in object description.

A **pv** anteposed a line in the list below indicates that this line concerns a *probable conceptual direction or tendency* on the part of whoever or whichever category among the historical protagonists we have in mind: denoting visual and conceptual attention and attribution or clusters of them. We cannot *generally* hypothesize about the degree of elaboration that characterizes, more or less seamlessly, someone’s being interested in an issue. It would be futile to try to set up a kind of scalar, distinguishing between whatever kind of mental operations accompany one’s simply having one’s attention directed on something and, on the other hand, one’s conducting more elaborate conceptual operations on the same thing.

The simplest way of setting up the *p_matrix model*, seems to be to list examples of horizontal *p_vectors.* Taking care of some among the many possible cases of interference or linkages between them by noting that this could be achieved by using the elementary operators of logics for inclusion, belonging, implication and so on: $\subset$. $\Rightarrow$, $\in$, and the rest. Alternatively, two or more concepts can be linked together linearily after a model I have labelled *consecutive approach processes* (SL, *Burden,* 182ff.). Were I to propose adequate associative networks for my material, I should have to postulate a considerable number of them, in probable accordance with a diversity of scenarios. So an open list is the only viable option. The list should make it clear that an exclusively literary account would be insufficient.

The following list of *p_vectors* indicated with **pv,** is left unnumbered, to forestall any impression of ranking by importance or priority. They are categorized with Roman numbers. General themes under which the single parameters may be thought to have been relevant, are marked with Arabic numbers and capital letters. In the nature of models of this kind, repetitions in places are unavoidable.Such a list model can be set up in various manners, both as regards choice of items and their arrangement. The one proposed here is merely meant to point up the main idea.

I. *Spire_hardware*: the physical, built object itself
1. SPIRE AS BUILT ELEMENT
(considered as construction in masonry, travertine etc.)
pv "Pigsnout" conical helix, construction in bricks, travertine and iron
pv administration of planning and work
pv building construction (for example as proposed in 1.9.1)
pv idea conception, planning, financing, implementation (3.1)
pv cost-benefit evaluation
pv practical considerations, such as steps for ascending to surmounting cross
2. SPIRE CONSIDERED AS TOP PIECE *COCHLEA*
pv science image suited for a university
pv no cosmological relevance, except as regards the cross (*see below*, III).
3. SPIRE CONSIDERED AS TOP PIECE *COCHLEA* WITH SURMOUNTING CROSS
(considered more for its generical helical *shape* than for its curvature or *form*; for *shape* versus *form*, *see* the *Introduction*, pt. 1)
pv Heavenwards to cross - *triumphus crucis*
pv Cosmos divinely created
(Comment: the two above are canonical, formal authority-defined notions).

II. *Spire_solid_state*: selected hardware, abstraction of its shape and quantification of it into a mathematical form; that is, subdivided into shape and form
4. SPIRE CONSIDERED AS HELICAL CURVE
(considered more for its total mathematical curvature, its *form,* than typological feature)
Relevance to *physics* is implicitly involved where it is appropriate.
pv Spiral, helix, conical helix, conic sections; earlier traditions, back to the Greeks
pv Proportions as in Dürer
pv Volumetrics (Torricelli, 3.2).
pv Thematic views of the helix form (3.2)

pv Because of increasing torsion, no algebraic solution until *differential geometry*; attempts at solution via curve techniques (3.2).
5. HELIX CONSIDERED AS CONSISTING OF GENERAL CURVE FEATURES
(considered as consisting of smaller units of “bent curves”). Relevance to *physics* is tacitly implied whenever it is appropriate. This part needs a comment; *see* below.
pv Indivisibles and infinitesimals
pvThe tangent problem
pv Limit
pv Maxima and minima
pv Change/variation of direction and intensity
pv Velocity, acceleration
pv Rectification of curves
pv Curves, 2D and 3D, including some special cases
pv Increments

III. *Spire_fluid_state*: cognitive, conceptual and ideological attributions, regarding the object itself and its use
6. SPIRE CONSIDERED AS CHURCH TOP PIECE
(considered as a spire on the church with its ecclesiastical connotations)
pv spire merged with cupola, a unique feature
pv "radicalism" in the Church: choosing an unconventional shape by an unconventional architect (Borromini): a sign disclaiming presumed old-fashion attitudes.
pv marking off Papal university site, against usual domes
pv marked out on skyline
pv *Helix (cochlea), laurel*- triumph Christian-Roman
pv connection to, continuation of triumphal Roman- Pauline monument of Sixtus V (the Marc Aurelius columns)
pv uniqueness of papal university
pv evaluation of public response
pv papal and papal-university patronage policy, art policy of Urban VIII and Innocent X.
pv cosmic, rather than cosmological image in the surmounting cross (1.9.1)
pv world image expressed in absolute-value (not considering protocalculus) mathematics

7. SPIRE IMAGERY BETWEEN SOLID AND FLUID STATES
PV Imagery considered in its sign value (basic forms) and elaboration alternatives (*see* comments on *Fig. 1.7 - 1*).

Comment on the "vectors" under 5.
The lines here indicate just the summary notions. Nevertheless, even so the setup remains a point of departure and nothing more. All sorts of combinations, or "vector addition", to stay in the somewhat dubious metaphor, will have occurred, depending on the situation. But we have no way of tracing that, for want of precise information about single actors in the landscape, to say nothing about their attention and understanding of themselves. From which perspective and with which scope would this or that "vector" have been followed to some consequence or outcome - and which ones? To fill the picture with further content, we would have to have a visit from, say, Fermat. Let him come to Rome, and look at the Spire at a mo-

ment when he is busy with connecting tangent and maximum/minimum issues. In that case he might have something to tell us; but unfortunately, he died in 1665.

4.6.1 Concept formation

The pseudo-matrix with the tendencies expressed in the pseudo-vectors represents *our* (or, not to drag the reader into it, *mine*) rich but *indeterminate* picture of the probabilities regarding the information material to be gathered from the Spire by people in the seventeenth century. Out of this web more specific concepts must have been formed.

Such concepts can be expressed by combining some of the "vectors" into meaningful connections, forming thus a *lattice* of the kind already referred to (4.3.1) of items in its nodes such as the following five ones (to give just one example): *The canonical notion of the triumph of the Cross - emphasized in terms of a comment by the case of the column of MarcAurelius and Sixtus V, - anchored in a world order - which is confirmed by the exact and absolute mathematical science* (Clavius et al), - *which is cultivated at the Sapienza* (for the use of graphic models versus prose accounts, *see Introduction,* 7.3 - d, and *Section* 4.1.1).

The model just presented is useful beyond its role in a methodological exercise, by demonstrating the complexity, and non-factual nature, hence elusive character of human mental elaboration of objects that they deem important.

4.7 Theory and methodology for description.

Looking back, the reader has the right to ask: what is *theory* and what is *method* in all this? The first part of my *fundamental theory* is that the analysis process, the evolution of which according to some rule-set, which is the *method,* the formulation and implementation of smaller or bigger relevant projects, produces the object (historical or contemporary) (SL, *Burden,* 184ff.).

The issue I have raised, here and in my *Burden*, about somehow bundling hard and soft formats together, without implying any direct mapping, comes down to the question of the nature and use of *models* and *operational definitions, which we can actually apply.* Thus the fundamental problem is one of methodology and procedure, which must be evaluated from perspectives of heuristics and/or science philosophy.

In his pioneering book, *Mental models* of 1983, Johnson-Laird assumed that human beings construct mental models of the world, and that they do so by employing tacit mental processes. The paradigm may be labelled *operational determination* (*Introduction, - d*). This program is evolved by attributing probable values to possible actions and elaboration of the situations and contexts considered relevant, attributions in their turn supported by sub-theories (such as the "resources" listed in *Chapter* 4.3 and the related *Sections*). For further support, I shall cite Philip Kitcher in his *The nature of mathematical knowledge.* Kitcher discusses the case of a small child playing with blocks on the floor, thus learning *the meanings of 'set', 'number', 'addition' and to accept basic truths of arithmetic by engaging in **activities***.

Rather than interpreting these activities as an avenue to knowledge of abstract objects, we can think of the rudimentary arithmetical truths as true in virtue of the operations themselves. On this basis, Kitcher argues that *arithmetic describes those structural features of the world in virtue of which we are able to segregate and recombine objects: the operations of segregation and recombination bring about the manifestation of underlying dispositional traits* (Kitcher, 107f.).

The kernel of a chosen *method*ology consists mainly in the construing and application of models (graphical, verbal or mathematical). No specific choice is the right one; there can be several working equally well. I trust not to be misunderstood when I bring in a lofty example: Heisenberg's matrices and Schrödinger's equations settling (rather unsettling) the same matter.

The *methodological procedure,* to specify a little further, employs a number of devices, many of them borrowed from "alien" fields, such as structured argumentation, open-source methods, positional recall and resources from the cognitive sciences. The process as a whole incorporates a sub-theory *of tools,* mainly models. The entire process is furthermore accompanied by a sub-theory *of limitation*. This distinguishes between hypotheses of probabilities that are amenable to description and hypotheses of mere possibilities that defy description, leaving us with an impression of opacity, for while we usually can say when something *cannot* be described, it is not always evident when it *can.*

Many people in various sciences have been devising projects on the basis of two points that have been fundamental also for my attempt. The first is the principle - or better, the practice - of searching for resources wherever one may find them; from a storage that I have named *open_source.* The other, to elaborate this a little more, is the recognition that results beyond the strictly quantifiable will never be definite or "conclusive". Morris Kline's book, *Mathematics. The loss of certainty,* should suffice to drive home the point. But even more so do the teachings of Herbert A. Simon. His "open-source" (not his term) perspectives and wide-ranging scope emerge even from the titles of some of his works: *Administrative behavior, The new science of management decision, Models of bounded rationality, Reason in hu-*

man affairs, Models of thought, The sciences of the artificial. In his autobiography, *Models of my life,* of 1991, he writes in his recollections about the Chicago School of Political Science: with its head and chief stimulator, Charles E. Merriam:

To attract disciples, one must provide certainty, and a catechism from which there can be no deviations and which can be recited to solve nearly all problems. Neoclassical economics provides that kind of certainty. So do Skinnerian psychology, Chomskyian linguistics, Piagetian psychology. There is no Merriam political science. Political science of the Chicago School provided a goal - to understand political behavior and political processes - and some directions from which to approach it: data and theory in psychology, economics, and other social sciences and modern techniques of experimentation, statistical analysis, and mathematical modelling. There were plenty of problems to which these data, theory and techniques could be applied, but no simple templates for applying them and no guarantee of the form the results would take. Hence "followers", but certainly not "disciples".

The present work is less "interdisciplinary" than it might look. It is to be expected that most of the basic ideas informing it are familiar to scientists, whether they accept them or not. But so they should be among people in the humanities as well, seeing that once upon a time they were central topics of debate among them. I am referring to the protagonists of the German *Aufklärung* in the late eighteenth and early nineteenth century (Kant, Alexander and Wilhelm von Humboldt, Lichtenberg, Goethe, Schiller and others). I have in mind particularly the *systems idea* (Kant's frequently quoted key formulation: *Ein organisiertes Produkt der Natur is das, in welchem alles Zweck und wechselseitig auch Mittel ist*), the evocation of *pictures* or *visualizable models* as an operational substitute for "reality" and a tool for grasping *the whole* of complex matters, and the *productive role of observation* (largely based on Kant's *regulatives Prinzip*). Even the notion of *emergence* was abroad, at least implicitly.

Our scope in theory and methodology should be opened horizontally toward our neighbors but also vertically across "periods" to free us from what may be considered fashionable at the moment - counteracting academic limitations in both directions.

And the future for ventures like the present one? I have referred to networks such as the connectionist ones. They are aimed at understanding our (undiscovered) mind. Forgetting about this elusive entity, we sit back with a more graspable issue: how can we use it to improve our faculties of arguing, to handle our problems? The network studies can be turned into a superb tool, be used as a model not run on the machine, for structuring and articulating our scientific awareness, preferences and reasoning - never mind our soul or mind.

The main challenge (as far as I can see; but others will tell me more) of my venture can be described using a wisdom from *connectionist* network studies; for I have, respecting the powers but also the limitations of model thinking, had to flatten out and simplify my subjects, highlighting the regularities there. And *if there is a predominant regularity in a set of patterns, this can swamp exceptional patterns until the set of connections has been acquired that captures the predominant regularity. Then, further gradual tuning can occur that adjusts these connections to accommodate both the regular patterns and the exceptions* (Rumelhart and McClelland cited by Bechtel and Abrahamsen, 188). So the most I can hope for, is to have approached my problems in a workable direction, while being aware that it all has been but a first-stage approach: pointing up regularities; and choosing material that *is* regular; and connecting with it, through a common denominator, material that is not obviously systemic: literature, sculpture, music.

Several times, particularly in *section* 1.11 on *structured argumentation,* I have insisted, as I have done in this *Chapter,* on my *open-source approach* (also in *Abstract,* pt. 6, *Introduction,* pt. b). There would seem to hold a direct mapping between this model and that of my perspective option, preferring argumentation display rather than delivery of a (tentatively) tightly knit proposal intended to convince (*Preface, section n* 1.11, again). The latter set of alternatives is strikingly parallel to choices in the data universe. In his stimulating and important book, *Breaking windows,* about Microsoft (with deep undercurrents of organizational, managerial and production theory), David Bank is concerned with an issue which he condenses in these terms: *the transformation... in the software industry from packaged products... toward a service-delivery model* (Bank, 211).

5SUPPLEMENTS

SUPPLEMENTS I and II

5.1 Supplement I: How Bruno's *shadows* function

In the following discussion of the thirty intentions in Bruno's *De umbris idearum*, the capital letters attached to them in the original have been omitted, since the present purpose is to study the methodological principles rather than using the method. I shall begin with my sequential comments in eight sections on the thirty *intentiones*, since this will tell the critical reader what I consider the gist of the entire piece; after which the salient formulations in the *intentiones* (as I see it) will be presented in my English translation with appended quotations from the Latin original (*The thirty intentiones in the De umbris*), giving the reader a ready opportunity to evaluate my comments.

In some cases statements discussed in Part I (1.3 and 1.4) will be further elaborated.

If there is one idea that pervades the entire discourse, it is *motion* and *dynamical relationships.*

COMMENT 1. The shadows are not magical nor mystic things but cognitive operators (int. 15 = *intentio* 15), related to ideas (int. 21). Distinction between real/object shadows and cognitive shadows (int. 17 and elsewhere), but under certain conditions one may pass from the one to the other (int. 18, 19). To the Court of inquisition he claimed: *non era per arte magica ma per scientia* (quoted in the Meroi ed. of the *Cabala*, note 66, p. 59).

Here I make an explanatory digression from the texts of the *intentiones.* Blum, 28f., clarifies the most important aspects of the *idea of shadow* in a comment, the gist of which I am going to synthesize, relying on my reading of his German text (*see* also 1.3).

For Bruno the *shadows* are the image of the difference(s) and also the links/connections [*Verschiedenheit, Verbundenheit*] between the different levels of existence [*verschiedene Stufen des Seins*], to which production of knowledge/realization [*Erkenntnis*] is entrusted. Shadows (not merely darkness etc.) thus are signs/ symptoms [*Anzeichen*] of light.... The title of the book (*De umbris idearum*) promises [Blum considers it a program] that the *ideas* <as presented> in the book, being pure(ly) spiritual/mental shapes/forms/manifestations [*reinen geisteigen Formen*], are recognizable/discernable [*erkennbar*] in the manifestation of shadows. Thus perceiving the shadows leads on to light and to clarifying the conception of existence and truth [*die Schatten, die niedrigeren Grade von Sein und Wahrheit, in ihrem relativen Recht zu erkennen und zu "klären"*]. - Blum takes the process as a *justification of the art of memory* by relying on analogies between mnemotechnical imagery and facts of nature and he brings a lucid explanation of the workings of *magic*. But one may reverse the case, believing that the memory operations serve the understanding, rather than the opposite (see int. 15)

COMMENT 2. Shadow provides the field of truth, in which man can seek shelter (int. 1); shadow of tree of Knowledge provides wisdom (int. 23). Since the shadow is grounded in real things, therefore the wise man (Solomon) placed the personification of acquired knowledge in the shadow of the tree (of knowledge).

COMMENT 3. Shadows are the *obiecta* of search for (knowledge) and cognition. *Obiecta - that with which something must be connected in order to function in the correct or efficient manner* (int. 5). Bruno expresses here as elsewhere the need to reduce everything back to some principle, for which the number theory attributed to Pythagoras was available and also well known at the time. In his *La cena de le ceneri, 1. Dialogue*, he makes this explicit by referring to the Pythagorean reduction of everything back to numerical fundamentals: *the finite* and *the infinite* (*Perché due sono le prime coordinazioni, come dice Pitagora, finito ed infinito, curvo e retto, destro e sinistro, e via discorrendo. Due sono le spezie di numeri, pare ed impare*...etc.; *La cena de le ceneri,* p. 14 in the cited edition). Here is the root of the concept of a *passage* from one thing to the next (*ordo et connexio*) (Rossi. 2006, 85f.), especially between opposites, that is attested in many of his *intentiones* (*see* int. 7, especially).

COMMENT 4. Human intellect can grasp things/ideas; and shadows and memory have their effect because things are linked/connected together in unity (several *intentiones*: esp. int. 9). Almost a Science of the Artificial!

COMMENT 5. Memory, i. e., acquisition and storing of knowledge/understanding, acts with an *artificial mechanism,* which is what makes the linking/connecting cited in int. 9 (cf. int.13) work.

My impression is that Bruno realizes that the only reality to which we have access, is the one that today we would express or represent in terms of *abstract models.* Von Weisszäcker attributes model thinking to Bellarmino (2.2.12, *Bellarmino as system builder*). It seems natural to connect Bruno's artificial mechanism with what he says about the *pure* [Maddamma translates *puro* as *naturale*] *architect of immagination* in his *Ars memoriae,* XVI, a feeling that I believe finds support in Rossi's comment on this passage. *We cannot have sufficient access to the complexities of a describable reality,* is what Bruno seems to say in Section XVI, and thus we have to take recourse to the *architect of imagination* (*sed puro phantasiae architecto innixi, ordini rerum memorandarum locorum ordinem adligavimus*) (*De umbris,* Sturlese, 74).This method corresponds to Cavalieri's and Torricelli's method of heuristics (3.4.3). In fact, Blum makes the point that Bruno relies on Osiander's method of artifice: *die Daten der Himmelsbewegungen zu sammeln und geometrische Prinzipen zu "finieren", mit deren Hilfe Konstellationen berechnet werden können.* (Blum, 48).

Blum makes a related point concerning Bruno's conception of the universe: ..

evidently, what counts for Bruno is not a geometric structure of the Cosmos, but rather he uses the geometrical description to represent the "true" nature of the world (*Dennoch geht es Bruno offensichtlich nicht um eine geometrische Struktur des Kosmos, vielmehr nimmt er diese geometrische Beschreibung, um die "wirkliche" Natur der Welt darzustellen*).

Apparently this is so at a deeper level than what is directly observable.

It was an important step, the same author notes, when Galilei assumed that the mathematical structure of the universe was a reality (Blum, 47; *see* also Blum, 135 - 141 on Bruno's geometry).

Rossi (2006, 91f.) notes that Classically, memory was connected with *spatial realities,* such as a room with a table and people seated at it. This direct connection with physical reality is abandoned by Bruno (as follows in my selective translation). *In Bruno's view, this <kind of> structure disappears. There was and there isn't any table <anymore>. There is absolutely nothing stable or fixed*...[*Nella prospettiva di Bruno questa struttura scompare. Non c'è e non c'era nessuna tavola. Non c'è assolutamente nulla di stabile e fisso*]. *Bruno - and this is the crucial point - does not want to "bind/tie up* [Ital. *vincolare*] *the order of things that we <want to> remember, to the order of places or spaces* [Ital. *ordine dei luoghi*]; *on the contrary, he wants to "bind the order of place/space to the order of the things to remember". He believes he does not need any more material places/spaces (that are amenable to being verified by our external senses). He evaluates his radical modification <of the art of memory> a discovery which means a perfection/completion* [Ital. *perfezionamento*]. [Here comes the crucial formulation:] *But how is it possible to connect the order of places/spaces to the order of* *things to remember?* [This smacks of modern model thinking] *The answer is typical and shows how Bruno has completely abandoned the art of memory as a transferable technique* [in the Classical sense]. *There are no prescriptions* [Ital. *ricette*] *for such a connection, and one must "rely on the inborn* [as I translate the Latin *purus* as used here - with support from Maddamma] *architect of imagination* [once more: *puro phantasiae architecto innixi*]. [Rossi concludes a little later:] *A transferable technique* [the Classical one] *has become a complicated machine of indefinite structure, that can function only theoretically but is not employable for improving the memory. One would almost be tempted to affirm that in Bruno's art <of memory> there is in reality images/ pictures only* [*Si sarebbe quasi tentati di affermare che nell'arte di Bruno ci sono in realtà solo immagini*]. - As I see the matter, reality for Bruno is Pythagorean (often affirmed) and this reality can be grasped only in terms of what we would call abstract models. This is the *How* rather than the *What* concerning his sources and his goals.

COMMENT 6. The shadows lead to the light (truth, good) through the chain of similarities (int. 14).

COMMENT 7. The shadows appear in various degrees and are *dynamic* (int. 2, 5 and 23). It is crucial that there is a continuous motion and alternating between contrasting things (inf. 7).

COMMENT 8. Implicit in this syntax but also elsewhere in Bruno, that man is in and out of the various shadows and their condition.

Now over to

The thirty intentiones in the De umbris: a choice of those that seem fundamental (but who knows?)

Codes: (...) explanatory completion of the text; and the corresponding Latin text; <...> completion of missing or incomplete words; or completing by supplying words that seem implied in the Latin original; [...] modern comments. References to two frequently cited authors: *Sturlese, S*, and *Maddamma*, *M* (if otherwise, the case is specified) are inserted when considered incumbent from the point of view of the methodological/operational perspective of the present discussion, not aiming at text criticism (quite obviously). Shadow is spelled *vmbra* and *umbra* according to the two Latin versions used.

1. intentio

The wisest (man) among the Hebrews [Solomon] in order to instill (*insinuare = in animam inducere, M*) in the mind (the idea of) human perfection and the pursuit of all that is the best in this world, recommends his Friend (*amica*), who exclaims: (*Hominis perfectionem, et melioris quod in hoc mundo haberi possit adeptionem insinuans Hebraeorum sapientissimus, amicam suam ita loquentem introducit*), SUB UMBRA ILLIUS QUEM DESIDERAVERAM SEDI (Song of Solomon, 2,3) [see int. 6]. Our nature is not great enough to be able to take abode, by virtue of personal ability, in the field of truth itself (*Non enim est tanta haec nostra natura ut pro sua capacitate ipsum veritatis campum incolat*) - for him [man] it is enough, in fact, much indeed, to be seated in the shadow good and truth (*Sufficiens ergo est illi atque multum, ut sub umbra boni, verique sedeat*). - Bruno is not speaking here of <sitting> under the shadow of the good and natural and rational truth - (*Non inquam sub umbra veri bonique naturalis atque rationalis - hinc enim falsum diceretur atque malum*) - but of the metaphysical, ideal [at the level of ideas] and above the level of <natural> substance (*sed methaphysici, idealis, & supersubstantialis.*) - through which the mind, according to its capacity, is enabled to have a share in what is good and true, even if it is not sufficiently endowed to become the image of it, at least reflects the image [Greek reading of *Genesis,* 1,26: man continues to reflect God's image even after the Fall; *M*] (*Unde boni & veri pro sua* *facultate particeps efficitur animus, qui & si tantum non habeat vt eius imago sit*; *ad eius tamen est imaginem*) - in that case, the soul in its transparency [being open to sight and influences?] [*M:* Marsilio Ficino], which is bounded by the density that is proper to the body, becomes accessible to something from the image by approaching it (*dum ipsius animae diaphanum, corporis ipsius opacitate terminatum, experitur in hominis mente imaginis aliquid quatenus ad eam appulsum habet*) - <namely/that is to say> the shadow itself, by intuition and cognition, to which we in our animal life are appealing (*in sensibus autem internis & ratione, in quibus animaliter viuendo versamur: vmbram ipsam*).

2. intentio

[Systems issues] Remember to distinguish the shadow from the category of darkness and obscurity (*vt a tenebrarum ratione seiungas vmbram*) - the shadow is not darkness and obscurity (*non est vmbra tenebrae*) - but either a sign/trace of this in the light (*sed vel tenebrarum vestigium in lumine*) - or a sign/trace of light in the darkness (*vel luminis vestigium in tenebris*) - or having something from both (*Vel particeps lucis & tenebrae*) - or being a compound of light and darkness (*Vel compositum ex luce & tenebris*) - or an <indeterminate> mixture of light and darkness (*Vel mixtum ex luce & tenebris*) - or nothing of any of them, being unrelated to light and darkness or separate from both - (*Vel neutrum a luce & tenebris, & ab vtrisque seiunctum*) - And this is so because, either truth is not entirely filled with light, or it is nor <entirely> true nor false (*Et haec vel indé quia non sit plena lucis veritas, vel quia sit falsa lux. Vel quia nec vera nec falsa, sed eius quod vere est aut falsé, vestigium, &c*) - And concerning the traces/signs etc., it follows from the discourse, that the shadow is trace/sign of light, or appears within the light, but <is> never full light. (*Et haec vestigium, &c. Habeatur autem in proposito, vt lucis vestigium, lucis particeps, lux non plena*) [*M*: *lucis vestigium*: Bruno, in *Lampas triginta statuarum*: beholding, not light <itself>, but the sign/trace of light, not the categories <of reality> and the ideas <themselves>, but the shadows of both, sine the mirror consists of an opaque body in which it would not have been possible].

3. intentio

Whenever one perceives light in a double condition, either in the field of substance [*in regione substantiae*], or in the field of everything that is present together with substance or in substance <itself>, such that the shadow can take on double nature/character - on this background one must remember that the light which encloses [*est circa*] the substance, derives/emanates, as an extreme trace/residue from that which is named an act of creation [*quae primus actus dicitur*]*;* and also that the shadow (No. 1, to distinguish!) close to the substance emanates from the shadow (No. 2) that is thus named when it derives from the substance. This <shadow>.. is what is called First Matter [or: First-Order object] by Aristotle and everything that has a share in it, while not receiving any pure light, we will say subsists under the shadow of the <natural?> light and operates in it. (Full quote: *Porro cum bifariam accidat intelligere lucem, et in regione substantiae, et in regione eorum quae circa substantiam, vel in substantia consistunt - unde secundum duplicem sumitur umbra oppositionem -, illud te meminisse oportet, lucem quae circa substantiam est, tanquam ultimum eius vestigium a luce quae primus actus dicitur, proficisci; umbram quoque quea est circa substantiam, ab umbra quae ex substantia dicitur, emanare*. *Ipsa est primum subiectum quod et materiam primam appellant phisici nostri* [*M*: ref. to Aristotle]; *eius omnia participia cum puram non recipiant lucem, sub umbra lucis esse et operari dicuntur*).

4. intentio

The shadow has something from light and something from darkness - (*quod cum vmbra habeat quid de luce, & quid de tenebris*): [the relation is examined in connection with moral questions (*bonum*, *malum*)]. <What all this means is the following:> On the horizon [*orizonte* = *horizon, -tis*; probably *at the edge of our sight range, our a limit*] of light and darkness (*tenebrarum*) we cannot discern anything but the shadow [*quam umbram*]. (*In orizonte quidem lucis et tenebrarum nil aliud intelligere possumus quam umbram*). - This occurs at the limit [*in orizonte*] of Good and Evil, of True and False (*Haec in orizonte boni et mali, veri et falsi*). - In it subsists the same <entity> which can be made beneficial or harmful, falsified or become truth (*Hic est ipsum quod potest bonificari, et maleficari, falsari, et veritate formari*) - and we say it is a shadow in either direction (*istorsum, illorsum*)(*quodque istorsum tendens sub istius, illorsum vero sub illius umbra esse dicitur*).

5. intentio

[As already mentioned, this paragraph appears to bring the crucial methodological statement: how the model works] Here the main concern is with those shadows that are the objects of the "appetites" and *faculty of cognition*, considered in terms of what is good and truth. (*Vmbras eas in proposito maximé consideramus quae sunt appetituum, & cogniscituae facultatis obiecta* [*obiecta* - late Latin: *obiectum:* I would translate this as *that with which something must be connected in order to function in the correct or efficient manner*] *sub specie veri bonique concepta*) - These shadows move in relation to the supernatural unity via/by virtue of [?] an increasing multitude, to <attain> infinite multitude (as the Pythagoreans would say) (*quae sensim ab vnitate illa supersubstantiali decedentia, per crescentem multitudinem, in infinitam multitudinem, vt Pythagoreorum more loquar, progrediuntur*) - the more they (the shadows) move beyond unity, the more they distance themselves from truth (*quae quantum ab vnitate recedunt, tantum ab ipsa quoque veritate elongantur*). - Here there is a <gradual> process [*excursus*] from the supra-essential properly speaking [pure idea?] to the essentials, that is, of the things themselves, and from here to the signs or traces of them, and images or effigies and the shadows, <that is,> towards matter, since the objects emerge from it, or towards understanding and reason, because it is by virtue of them that they are recognized/understood (*Fit enim ab ipso superessentiali ad essentias ab essentiis ad ipsa quae sunt, ab iis ad eorum vestigia, imagines, simulachra, & vmbras excursus: tum versus materiam vt in eius sinu producantur, tum versus sensum, atque rationem vt per eorum facultatem dinoscantur*).

6. intentio

The shadow which is grounded in matter and in nature, in the natural things themselves, in their interior and exterior, and <in both cases, the shadow> is in motion and in change (*Vmbra in materia seu natura, naturalibus ipsis, in sensu interno atque externo, vt in motu & alteratione consistit*). - On the contrary/contrariwise, the shadow in the intellect or in memory is in a condition of stasis (in difference from *things*, *see* above). (*In intellectu vero, intellectumque consequente memoria est ut in statu* <stabili>). Therefore the wise man presents the supernatural and suprasensual - sexless - heroine as <an image of> acquired knowledge, who is seated in the shadow [Solomon, int. 1; but cf. also the shadow of the Tree of knowledge, int. 23] of what is the first true and virtuous good that can be desired (*Ideo sapiens ille viraginem supranaturalem & suprasensualem quasi notitiam consequtam: sub illius primi veri bonique desiderabilis vmbra sedentem inducit*). - But since being seated thus in condition of stasis does not last long for whoever lives a natural life, this way of being seated should be linguistically expressed in a short-term manner [strictly corresp. to Italian *passato remoto* or *imperfetto*]. (*Quae* *sessio seu status <quietis> quia in naturaliter degentibus non multum perseverat ... sessio illa potius praeterito absoluto vel inchoato, quam praesenti tempore designatur...*).

7. intentio

[Summary: *Things* are chained together in a certain order, everything under a unifying principle; and through this system there is a continuous movement from the light to the shadows... focused on *cognition*. There is a continuous motion with alternation between contrasting things] (*Ita generaliter videmus in iis quae mutantur, motum statu, & statum motu semper terminari*). ...- The classical authors were aware of and taught the principle according to which in man the actively contemplating thought [Italian: *pensiero discursivo*, with a long appendage by Maddamma, note 22], ascends from many specific things up to the class/group and up to a single category... etc. (*Novit quidem et docuit antiquitas quomodo proficiat discursus hominis a multis individuis ad speciem, a multis speciebus ad unum genus ascendens...*). - However the antique <authors>, whether they knew or not how memory proceeds, progressing from numerous memorable species to a single one with [encompassing] many memorable things, certainly did not teach this. (*Porro si antiquitas novit quomodo proficiat memoria, a multis speciebus memorabilibus ad unam multorum memorabilium speciem se promovendo, ipsum certe non docit*).

8. intentio

(Here a general reference to Blum, pp. 44 - 73, on Bruno's concept of infinity).

[Key term here: *Contractus* - *contrazione, M*, Introd., 9; *compression*, Gatti, 184. - Accommodate a multiplicity of things in one scheme or unity; the thing gradually sheds its characteristics and more and more assumes generalized features or modes. A graded process of the shadows through similarities and differences].

<Consider two things comparable in terms of similarity at two levels; then>) A thing at an inferior level raises up to the closest level through a gradual process of compression of similarities <until differences are eliminated> (*Ad proximius quidem superius proximum inferius per aliquos gradus contracta similitudine promovetur, quos certe gradus cum nactum fuerit omnes, iam non simile, sed idem cum illo dicendum erit*) - The outcome is that a passage takes place, via a mutual similarity, from the shadow to the traces/signs <of it> and <on>) to reflected replicas and from them further on to other <things> (*Per communem igitur similitudinem ab umbris datur accessus ad vestigia, a vestigiis ad speculares imagines, ab istis ad alia*).

9. intentio

By perceiving the chains of similarities (*similitudines*) - accessible [as according to *intentio* 13] in an ordered progress ascending or descending between specifics and generalia - *the intellect can grasp anything:* [a central statement in Bruno's theory of cognition] (*... hinc accidit ut - infra suos limites - natura facere possit omnia ex omnibus, et intellectus, seu ratio cognoscere omnia ex omnibus*). The chain makes memory work, and the art of memory consists in setting the mechanism in action. It is this chain that makes memory active also with regard to present things. (*Sicut inquam materia formis omnibus informatur ex omnibus, et passivus, quem vocant, intellectur formis omnibus informari potest ex omnibus, et memoria memorabilibus omnibus ex omnibus, quia omne simile fit, omne simile simili cognoscitur, omne simile simili continetur. Porro simile remotum ad suum distans per simile medium sibique proximum tendit*) - Who knows/recognizes the connections between opposites/comparison-terms, can grasp anything from anything both intuitively and rationally (*Hinc qui nouerit apta extremorum media, & naturaliter & rationaliter omnia poterit ex omnibus elicere*). This may seem a weak conclusion for the foregoing statements, but Bruno's (often somewhat confusing) technique is to work by waves; crest following upon trough and trough again upon crest, and so on.

10. intentio

Subject further elaborated. General advise.

11. intentio

[Further distinction between *object shadow* and *cognitive shadow*]. Things in this world would not have been beautiful if they were all completely similar/alike. Hence the <natural> shadow-vision is the most imperfect mode of seeing/recognizing, for it does not allow us to discern among the differences, for the shadow, as in a picture, presents the mere outlines and even this not <always> correctly (*Hinc rei vmbratilis visio est visionum imperfectissima: quia quod imago cum varietate demonstrat, vmbra quod est infra extrinsecae figurae terminos vt plurimum etiam ementitos, quasi sine varietate profert*). - I am saying this concerning the shadow as a <physical> shape, certainly *not* with regard to the shadows as it is considered in this investigation (*De vmbra dixerim quatenus vmbra est: non autem quia talis quam in proposito recepimus*).

12. intentio

No order without diversity.

From now on, the central themes are being further elaborated and explained in view of experienced, possible and likely opposition. Bruno, with some obvious justification, felt always under attack. This intensifies his hang to over-explicitness, to the point where the reader, perhaps even he himself, loses track of the flow of argument and, facing ever new variants of seemingly similar statements or propositions, one starts doubting about what at the outset may have seemed relatively clear. Did he read his texts for someone before having them published?

13. intentio

<Premisisng the chain or conjoined order of things at different levels, we may say:> The memory acts in terms of an highly articulate mechanism [as I translate *artificiosa connexio*, avoiding the sense of *artful, artificial*], which also brings an order into the things perceived, even if they are not at all so ordered in our <natural> memory (*Per hanc artificiosam connexionem magnum experiri possumus memoriae releuamen, quae valet etiam nullam ad inuicem per se retinentia consequentiam memoriae ordinata presentare*).

14. intentio

[Model concept] With regard to the shadow at the level of ideas, the passage is not <always smooth and continuous>, but may occur in various <other> ways that are going to be explained shortly [Int. 15] (*Ascensus quidem qui fit per connexa atque concathenata, in proposito vmbrarum idealium, non est per cathenam similibus constantem annulis, ratione quae concipitur ex proxime dictis, atque deinceps enunciandis*). - Through the chain of similarities [here called *annuli*] the shadows lead to the light, and even though the light is not equivalent to truth [*veritas*], it derives from light and leads to it... The light entails truth [and is, as we would day today, a model for it] (*....non inquam vmbra abducens á luce, sed conducens ad lucem, quae etiam si non sit veritas, est tamen á veritate, & ad veritatem, ideóque in ipsa non credas esse errorem sed veri latentiam).*

15. intentio

[*The shadow is a scientific concept*, not fundamentally a magical one; our mental faculties have their origin here]. One should not, because of the homonymity [shadow in two senses, natural and ideal] evaluate, understand and discern the meaning of <the term> shadows indiscriminately (*Non igitur confundens umbrarum sig-*

nificatum per occultam omonimyam, .. ut sine delectu de umbris sentias, intelligas, et decernas). - For that shadow which is protected by other shadows, should be contrasted with that shadow which raise up above the reality of things up to the borderline of what is intelligible (*Opponitur enim ea quam protegunt aliae umbrae - pro qua dicitur*: *Protegunt vmbrae vmbram eius - ei quae eleuatur super corporum altitudinem in confinio intelligentiarum*). <From this shadow of divinity, which "covers the mountains"> emanates/derives those things that create in us intelligence and memory, and in it finally terminate those things that arise towards the light (... *a qua ea quae producunt in nobis intelligantiam & memoriam, deducuntur & emanant, et in quam tandem scandentia versus lucem terminantur*). [Some difficult passages ahead:] Hence the shadow arranges for the ability to see/discern the light. The shadow softens/makes more accessible the light. By means of the shadow, divinity [God] disposes and distributes those entities that make <all> things perceptible/knowable, so that they become amenable to being seen by the eye <which is> dimmed/obscured by the hungry and thirsting soul (*M: Ps.* 106:5) (*Umbra igitur visum preparat ad lucem. Umbra lucem temperat. Per umbram divinitas oculo esurientis, sitientisque animae caliganti nuncias rerum species temperat, atque propinat*). - You should recognize the shadows that... preserve and contain the light in us, through which we are being started off toward and led to understanding and memory (*Eas igitur vmbras, quae non extingunt, sed servant, atque custodiunt lucem in nobis, & per quas ad intellectum, atque memoriam promovemur, atque perducimur, recognosce*).

16. intentio

[A potted history of science!] The theologian said: "If you do not believe, you will not understand" (... *nisi crederitis, non intelligetis*). And <thus> in their manner the theologians affirm that one must possess oneself of the sciences by using those hypotheses in which one expresses confidence: namely in this manner: 1. among the Pythagoeans: in the things not <yet> demonstrated; 2. among the Peripathetics [Aristotelians of Bruno's time]: in the things not amenable to demonstration; 3. among the Platonists: in the things of both kinds.- Those who find it difficult to deal <conceptually> with the shadows, or [trying to do so] believe it futile, in the case that light should not become accessible through them, <they> should know that this failure is not due to the shadows <themselves> (*Si quibus vero arduum videtur in umbris exerceri, et vanitatis suspectum si per ea ad lucem non pateat accessus, norint talem defectum non esse ab umbris*)

17. intentio

[Further distinction of real shadows from cognitive shadows] Among physical shadows, such as those from a tree, there are some that serve to protect weak animals; and others with the opposite effect. <Amongst> the ideal shadows, however, there are none that do not guide to <the ideas> perfectly, if you ascend <to the world of ideas> through them, since they all are connected with our intellect and our internal purified cognition (*De vmbris physicis sunt ex arboribus & herbis quae fugant serpentes, & mitiora fovent animantia, sunt & contrariae iis. De vmbris autem idealibus, si vere sint ideales, cum omnes referantur ad intellectum & ad purgatum sensum interiorem, non sunt quae maxime non conducant si per eas fiat ascensus, et non dormiatur sub eisdem*).

18. intentio

[Keeping the distinction <between ideal and real/object shadows>, one may pass from the real shadow to the cognitive one]. You are not asleep [= you observe correctly] if from the observation of physical shadows you go further to consider in terms of proportion [order of importance?] the shadows of ideas (*Non dormies si ab umbris physicis insoectiss ad proportionalem vmbrarum idealium considerationem promoueris*...).

19. intentio

[The shadow is perceived and evaluated by way of observation of real things]. The shadow becomes more transparent to being scrutinized by a stronger intensity of the light and density of the body and then the shadow becomes clearer outlined and better defined (*Maiora* [*S*: *maiori*] *intensione lucis & densitate corporis vmbra perspicacior efficitur*) - and this comes from the fact that it reflects the body in density and specificity, continuity and discontinuity (*quod inde est, quia in densitate, & raritate, continuitate & discontinuitate corpus imitatur).* - This imitation, however, is revealed by means of the body <itself> (*At vero talis imitatio per corpus detegitur*).

20. intentio

[All the proceedings described are dynamic, characterized by motion among diverse and contrasting things. In this way, they arrange for the passage from one entity to another, in the chain of connections (*see Intentio* 18)]. From now on, try to recognize how the shadow, at the motion of the light <itself>, <also> moves, as if it escaped from it, while at the motion of a body, it seems to pursue this. And this (these occurrences) does not seem to imply/involve contrariety, much rather concordance (Cusanus!) in leaving one thing and pursuing another, <whether it is> opposed or contrary (*Porro adverte quemadmodum ad lucis motum movetur umbra quasi fugiens, ad corporis autem motum quasi sequens; unde non videtur implicari contrarietas, sed concordantia in fuga unius, et prosequtione alterius oppositi, atque contrarii*).

21. intentio

[Similarity between shadows and ideas. A central point in the entire model; *M*: note 43]. Do not disregard the similarity between the shadows and the ideas (*Non te praetereat tandem vmbrarum cum ideas similitudo),* - for neither shadows nor ideas are the extremes in a relationship of opposites (*tum enim vmbrae, tum & ideae non sunt contrarie contrariorum*). - In fact, one may proceed through them to further understanding and discernment, but the comparison should not entail confusing essentials with effects (*Illud enim quod est eis proprium, est non ens in ente, vel, vt apertius dicam, defectus in effecto).*

22. intentio

[In an *anti-Aristotelian - and anti-Catholic - vein*, a distinction is here operated between *substance* and *accidence* and applied to the shadow, for which, as Bruno claims, the distinction does however not apply]. The shadows are not substance nor accidentals but a certain conception of both. By a careful consideration, <we see that> substance and accidentals do not subdivide everything so much as is usually being claimed [with Aristotle]. This understanding helps to get a rational grasp of <the concept of the> shadows. [Some further considerations are included that are not necessary to report here. The long paragraph ends with the following two statements:] By a correct evaluation, substance and accidentals do not divide everything that we consider as belonging to the universe in such a manner that is *currently believed*. This consideration is of no little value for a rational understanding of the shadows (*Recte enim speculantibus substantia & accidens non diuidunt quidquid esse per vniuersum dicitur, vt modo supponimus*. *Consideratio ista non modicum valet ad vmbrarum rationem habendam*).

23. intentio

The shadow does not depend on time in a general sense, but on the time of this [*M* supplies: *corpo* - body], nor on space location in a general sense, but on its <specific> locality, nor on motion in a general physical sense, but on its <specific> motion. The same applies to opposites (*Vmbra non subest tempori, sed istius tempori, non loco sed istius loco, non motui sed istius motui*. *Similiter de oppositis est inteligendum*) [this should mean: its time, place and dynamics are not scalable on a general scale but are dependent on physical realities]. - Therefore it [the shadow] is unconnected with any truth [probably: in a general sense], but is certainly not destitute of it [truth: probably specifically speaking] (*Abstrahitur ergo ab omni veritate, sed non est sine illa*) - And to the extent that it is a shadow on the level of ideas, it does not hinder <us in> arriving at the truth; for really, since it is a unity, it allows <us to> perceiving things that are opposites (to one another) and different. (*Et non reddit ineptos ad illam, si idealis sit umbra, concipere enim facit contraria et diversa, cum sit unum*). - Nothing, in fact, is contrary to the shadow, and indeed neither is darkness nor light (*Umbrae enim nihil est contrarium, preciseque nec tenebra, nec lux*). - This is why Man took refuge in the shadow of the Tree of Knowledge, because of knowledge of darkness and light, truth and falsehood, good and evil, as when God asked him: *Adam, where are you?* (*Gen.* 3.9) (*Ad vmbram ergo arboris scientiae confugit homo pro cognitione tenebrae, & lucis, veri, & falsi, boni; & mali, cum quaereret ab illo Deus ADAM VBI ES?)*

24. intentio to *29. intentio*

A single opaque body may throw several shadows is places against two or more sources of light. These intentions examine various modalities of projection from bodies to shadows.

30. intentio

[Central in Bruno, the idea of reference back to unity, ultimately to infinity] All the different <kinds of> shadow can be referred back to six principal ones, which ultimately refer back to a single one, with a rich fertility, and <which is>) the source of all the rest (*Ut vero intelligis omnes umbrarum differentias ad sex cardinales tandem referri, non minus scire debes quod omnes tandem ad unam foecundissimam, aliarumque fontem generalissimum reduci debeant*). - In the framework of our investigation/discussion [*in proposito*], I would say, there may/can be one single shadow for all <possible> ideas, and this modulates, evaluates and presents <them>, by means of what [the mechanism] we define aggregation, disgregation and alteration [*M*: Aristotle], all the others... (*In proposito - inquam - nostro una potest esse omnium idearum umbra, additione, subtractione, et alteratione generaliter dictis omnes alias conflans, iudicans, atque praesentans, ... quae recipiunt in se ipsis alterantia, transponentia et universaliter diversificantia*).

5.2 *Supplement* II: Protocalculus problems

The subjects discussed in this *Supplement* are the same as have been introduced in *Part* III; the subheadings correspond more or less exactly. Some further historical details are supplied with the purpose of lending more substance to the issues. Some repetitions have been found unavoidable.

The cited parameters *as applied to our notional helix section* - a little bit of curve we broke off - represent a set of *probabilities,* reflecting in a non-technical sense a set of vectors, as already emphasized.

5.2.1 *Indivisibles and infinitesimals*

To appreciate the instability but also the richness of Cavalieri’s program, we must consider that the *very idea of indivisibles* was hotly debated and many versions emerged from the works of single scholars and from criticisms of their contributions, some closer to, others some steps away from Medieval work. Galilei, Baron notes (116ff.), was influenced by the Scholastics, making *no distinction between physical atoms and line and surface elements; in fact he finds support for his belief in the efficacy of mathematical indivisibles from his desire to* *demonstrate the possibility of the existence of vacuum* (*Due nuove scienze*, see also Rossi, 1997, 141). Galilei used the so-called *wheel of Aristotle* to approach infinitesimals by rolling polygons with ever increasing number of sides, thus making them approach a circle (Baron, 177f, Figs. 4.12, 4.13 and 4.14). *Thus Galileo lent his authority to the concept of lines, solids and surfaces, made up of infinitely many indivisible elements variously distributed (Due nuove science), while warning against treating them by assigning proportions <to them> as we employ for the finite* (Baron, 118) (5.1.7).

Smith (II, 686ff.) remarks that Cavalieri did nowhere clearly define his *indivisibles* and *proceeded to find lengths, areas and volumes by the summation of these "indivisibles", that is, by the summation of an infinite number of infinitesimals*; hardly satisfactory but stimulating Leibniz and others in their drive toward the development of full-fledged calculus (*cf.* Smith, I, 687 with an example of his technique). What really was missing to make such a summation work properly, was the idea of *limit*.

Besides some algebraic notations intended to generalize the concepts, Cavalieri *presents his results entirely in verbal and geometrical form*. Two or more configurations are tied together in the form of *ratios*. Ratios are easily conceivable and amenable to visualization when applied to geometric structures (*cf. Fig. 5.2 - 1*). His approach, to return to Baron, was heuristic, operated through an *artificium*. He discusses at some length the problem of comparison among his indivisibles, reaching an advanced technical development for the evaluation of *integrals*; an argument, however, that reaches no definite conclusion with regard to their position so to speak within a single line or curve, in other words, terms bearing directly upon the development of *differentiation*.

Here follows a much simplified illustration of Cavalier's *integration* method, culled from Struik (214f.). It has been noted several times already that *differentiation* rather than integration is in our focus. The appended example, however, while dealing with areas, does illustrate the idea behind Cavalieri's use of infinitesimals or *indivisibles*, which are the building blocks of the *protocalculus* edifice.

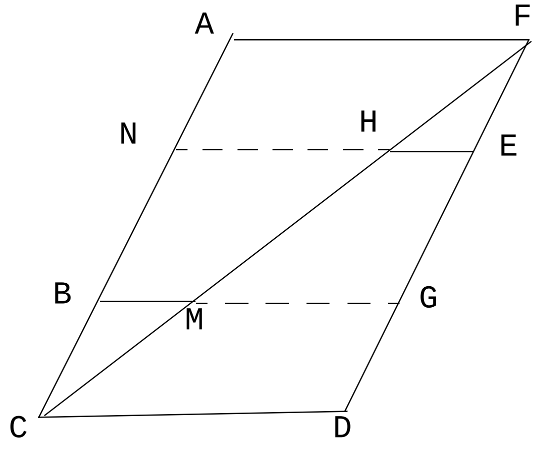


Fig. 5.2 - 1 An example of Cavalieri's integration

By comparing (cf. *Fig. 5.2 - 1*) the *parallelogram* denoted by the diagonal C- F, the *segments* N - E and B - G, and the *triangles* resulting from these subdivisions, Cavalieri could state three sets of propositions in the sums on the different elements. In this manner, Cavalieri developed a system of summing up *proportional* relations of lines and areas and thus to achieve a crude integration method: subdividing down towards infinite smallness, he could reconstruct areas by summing up these indivisibles fitting them into an approximate evaluation of the area at hand (*see* Smith, quoted in 3.5.1). Cavalieri's calculations, not being based on some simple formulas, are too complex to summarize here (*see* Baron, 130f.).

The idea of breaking a line, a surface or a volume up into indivisbles, literally atoms (*átomos* - not divisble), contains or presupposes those of *infinitesimal point*, hence *infinity*, and *continuity*, *tangent to curve* and *change of direction or intensity*. Let it be noted that the *point structure of a continuum* had been acknowledged by medieval mathematicians (Baron, 73; Boyer, 1980, 378ff.). Cavalieri used the notion *heuristically*, nowhere did he explain precisely what he understood by the word *indivisible* (Smith, II, 686f.), which he employed to capture the infinitesimal elements used in his method. Nor did he follow his master Galilei in speculating about the *nature of the infinite* (Boyer, 1959, 116).

Baron, 124, offers the following comment on the main issue: The elements, that is areas or volumes, are based on *ratios* (a thing rejected by Galilei, see below), and *In moving from a relation between the sums of the indivisibles and thus to a relation between the spaces the infinite is employed, but purely in an auxiliary role* (Baron, 125). The auxiliary idea is based upon *the idea of a device or artificium which works rather than on any definite or dogmatic view as to the nature of indivisibles and the spaces which they occupy*, in which process the problem of the *continuum* imposed itself upon him (Baron, 125). Cavalieri launched a systematic use of *summation* (today expressed by the symbol

Σ), a collective and distributive version that complement one another (details in Baron, 125 - 135), and further developed, among others, in the work of Grégoire de Saint-Vincent (Baron, 135 - 148). Summation techniques were to become fundamental in the subsequent development of *definite integration* (and Pascal came up with important solutions here; Baron, 196ff.).

Again, we see that the ideas of infinitesimals were developed in an *integration* context, concerning area and volume. This seems natural enough for us today, since an inverse relationship obtains between integration and differentiation, even if this was not entirely clear to the mathematicians of that time.

Baron offers useful beginnings for a somewhat deeper penetration into the subject (Baron, 122ff.). *Cavalieri's method was mainly focused on what we would regard as integration problems: on him fell the task of shaping the vague concept of indivisibles into a serviceable tool in the determination of areas and volumes*. Work of this kind evidently stimulated the development of differentiation methods, too.

The principal subjects in this connection are indivisible or line infinitesimals (Baron, 163) and instantaneous centre of rotation (Baron, 163ff.).

In order to supplement this necessarily compressed picture of the issue of the indivisibles, reference must be made to further comments on Cavalieri himself and his geometry of indivisibles (Baron, 122ff.; Mancosu, 1996, 39 - 47 on his *First method*, 47 - 50 on his *Second*).

It is typical for the drive towards the infinitely small that the issue of *general infinity* faces us almost steadily. Boyer (1980, 376) notes that the infinitely small was more important for Galilei than the infinitely large (his dynamics owed something to Oresme's *De proportionibus proportionum,* of ca. 1360 (Boyer. 1980, 308f.). We have noted already Galilei's concern for what is *piccolo, anzi minimo, anzi pur minimissimo,* and having *gradi...infiniti* in his work on spirals (1.10). Blaise Pascal should be cited in this connection.

> *Pascal had two predominating interests in mathematics: geometry and the theory of numbers... In this connection he enunciated in the "Potestatum numericorum summa" of 1654, the theorem on the integral of* x^n*, which we have met with in the work of Cavalieri, Torricelli, and Roberval. <But> Pascal's demonstration of this is derived not from classical geometrical propositions alone, but from an examination of the figurate numbers represented in the arithmetic triangle...* (Boyer, 148).

What is important in the present context is that, even when elaborating integration issues, *number theory*, tangentially to the theory of infinity, begins to gain ascendancy among the mathematicians.

Speaking of another related facet of Pascal's work, Boyer, 150f. gives an important comment that throws light on essential concerns in the *protocalculus* efforts and in the development towards the calculus:

The essential point in Pascal's demonstration is the omission of terms of lower dimension [i.e., dropping quantities too small to make a relevant difference].... *The method of dropping terms seems to have entered in the work of Roberval and Pascal through the association of the indivisible of geometry with arithmetic and the theory of numbers. The geometrical intuition of indivisibles of lower dimension was, in their work, carried over into arithmetic to justify the neglect of certain terms of lower degree. Pascal went so far as to compare the indivisible of geometry with the zero of arithmetic, much as Euler later regarded the differentials of the calculus as nothing but zeros. - This neglect of quantities, as found in Pascal, has been characterized as the basic principle of the differential calculus. Such a designation is indeed misleading, for the subject is no longer explained in terms of the omission of fixed infinitesimals... Newton also occasionally <adopted the practice of> dropping out of the calculation "moments" which did not add significantly to the result. For almost two centuries mathematicians tried to justify such procedures, but in the end the basis of analysis was found, not in these, but rather in the method of limits toward which the geometrical method of exhaustion and arithmetical modifications of this by Stevin, Tacquet, Roberval, and others had pointed out* (Boyer, 190f.).

In modern mathematics, these ideas have been modified and connected with the notion of *neighborhoods* and *regions* (Wrede-Spiegel, 117).

Estimating instantaneous velocity or change was linked up with the idea of infinitesimals. Descartes, in his *La Géometrie* of 1637, in determining the tangent to the cycloid (Baron, 163ff., Fig. 5.10), *developed the idea of an instantaneous centre of rotation*, and since this is a very remarkable and illustrating case, let me quote Baron's comment, to the effect that:

if a polygon rolls on a given straight line the curve described by each vertex will be made up of a succession of circular arcs and the tangents to the curve at each point on it will be at right angle to the radius of the arc. If a circle rolls on a straight line then the circle can be considered as a polygon made up of "cent mil milions" of sides and the tangent at each point will be perpendicular to the line joining that point to the point of contact of the generating line with the base line.

It may be mentioned here, to indicate the wide scope that was opened up by the use of indivisibles, that Descartes used a version of the method for studying the law of falling bodies (Baron, 163).

5.2.2 *Limit*

As we have noted, the introduction of *limits* came gradually and not linearily.

To gain an insight in the wider historical context for the emergence of the limit concept, a quotation from Boyer (1959, 79) will do. *Our definitions of uniform and non-uniform rate of change are, as Suiseth* [also Calculator, mid-fourteenth century; see also 3.5.7 on intensity] *had anticipated, numerically expressed; but their rigorous definition could be given only after the development, to which Newton contributed, of the limit concept. This latter arose out of the notions of the calculus, which, in their turn, had evolved from the intuition of geometry.*

Boyer also notes (3.4.2) that Torrcelli's method represents a marked advance over tradition in that it employs *the idea of instantaneous direction and implying, therefore, the limit concept.* This is probably what some writers, Baron, for example, have in mind when they say that Torricelli came very close to a conception of the limit. But Torricelli, in his work with *indivisibles* did not realize that this concept was associated with the structure of *instantaneous velocity* through the limit concept (Boyer, 1959, 134). Thus an adequate basis for indivisibles was lacking and instantaneous velocity could not as yet be properly defined.

Extreme value of a variable in Fermat's elaboration approaches the idea of limit (Baron, 167): *In all these considerations, although the notion of limit is not in any formal sense present, the idea is emerging of two converging values so that, as y -> x, e -> 0, and the extreme value is reached* (5.2.5). In fact, *The formation of* *an approximate equation, discarding of terms, and so on, is at this stage* [1638] *Fermat's only conception of a limit*... (Baron, 169). The *discarding of terms* is crucial (5.2.1, see also the note on "moments", 5.2.13).

5.2.3 *Increments*

The use of increments is a technical device "inside" the differentiation or derivation process, basic for determining a workable limit (*see* 3.4.2).

Pierre de Fermat was one of the very few, if not the only one, who introduced the *method of increments* in the differentiation process, but there were important precedents both in the fourteenth century and in his own time. In a work that was begun in 1629, written out by 1637 but published only in 1679, *On maxima and minima,* Fermat introduced an increment *E* and let it diminish as we have seen above. For want of any exact *idea of limit* his process failed to reach the precision with which modern calculus tackles the problem (Boyer, 1959, 157f.; calculation and diagram in Thorvaldsen, 76f.). Fermat therefore had to have recourse to the mathematically unacceptable device of making his increment *equal to zero* (instead of making it *approach* zero) at a certain point in the process; and for this division by zero he was severely criticized. But, once more, apparently we have a case of heuristic thinking, using models. In Fermat's work seems to emerge for the first time *the idea which has become basic in such problems, that of changing the variable slightly and then letting this change vanish* (Boyer, 1959, 156). But, to continue citing the same author, *Fermat therefore formed the pseudo-equality* [labelled *adaequalitas*] *which became equality on letting E* [Fermat's increment, we recall] *be zero. From this it is clear that he was thinking in terms of equations and the infinitely small, rather than of functions and the limit concept. Fermat's procedure resembles most closely the method of limiting values* (Boyer, 1959, 156).

If I am not mistaken, here is implied a distinction between operating with *static* (a spot homing in on an ideally fixed infinitesimal point) or with *linearily dynamical* values (see also 3.4.2), two distinctly different conceptions of mathematics. *As a result, infinitesimals were uncritically introduced into analysis, to become firmly entrenched as the basis of the subject for about two centuries before giving way, as the fundamental concept of the calculus, to the rigorously defined notion of the derivative* (Boyer, 1959, 157).

5.2.4 *The tangent problem*

The relation of *tangents to curves has been our subject for some time* (3.4.2, 3.5.3 and 3.6).

Among the Greeks, Archimedes may have been the only one who seems to have had a clear idea about the tangent problem. The Greeks had worked on the problem and recognized that finding the normal to a point at a curve, the tangent at the same point was easily established. The introduction of analytical geometry in the 1630s (3.5.11 and 5.2.12) revived the tangent problem within the more articulate scenario that had emerged in the meantime.

Descartes considered this as the most important puzzle in geometry and developed his *celebrated method of tangents in terms of the equality of roots* [making the curves of two equations cross each other or coincide]. Descartes' *method consisted in passing through two points of the curve a circle with the center on the x axis through which the normal to the curve passes, and the tangent is consequently known* (Boyer, 1959, 166). He also insisted on the importance of the tangent issue. But his method was cumbersome and it befell to Fermat to develop the concept formally (ca. 1637, more or less contemporarily with Descartes' *La géometrie*) (Boyer, 1980, 395f.; for Descartes' handling of the problem, see also Mancosu, 79f.). A quarrel arose between them about what really had been found, a rule for maxima and minima or just the normal to the curve at the given point (Boyer, 1|959, 158f.). The famous *tangent controversy* arose, especially between Descartes and Fermat (Baron, 168ff.), from which it becomes clear that Fermat's (and that of his contemporaries') concept of limits was insufficient. Fermat's argumentation works along two lines that were deeply rooted in tradition, use of *proportions* and of *geometry*. A problem area the study of which resulted in improved approach to tangent determination, was the *composition of motions* (5.2.8).

Substituting the Greek concept of a tangent as a straight line that *touches* the curve, there arose now the notion of the tangent as the straight line that indicates the *resulting velocity*. With this concept, pure geometry and dynamics were compounded into a whole. Before Galilei's work, these were considered apart from one another. On the other hand, as noted by Kline (1999, 402), the notion came to light from physical conditions, whereas many curves arose that had nothing to do with motion.

Descartes in his *La géometrie* of 1637 argues as follows (here, in a simplified summary of a rather complicated method). In order to find the tangent at, say, the point *P* of a parabola, he found a circle the periphery of which touched the parabola at *P* without crossing it.

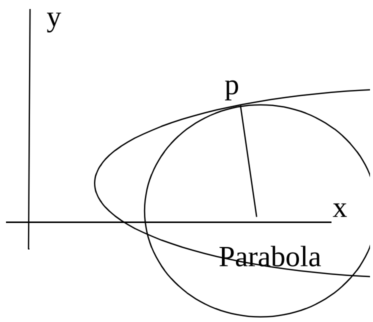


FIG. 5.2 - 3 Simplified demonstration of Descartes' approach (Blank and Krantz, 245; see also Baron's Fig. 5.11). Just one common point, P, is considered here.

Here he applied his *celebrated method of tangent in terms of the equality of roots* (Boyer, 1959, 166), by letting the increment be so determined that the resulting equation has equal roots. The method is cumbersome, even though Descartes does not admit so. For finding the *tangent circle* was not an easy task.

Mathematically speaking, Torricelli was more radical than his teacher Galilei. He employed the idea *of instantaneous direction* (seeing that the motion is following the parabola curve). He studied a large number of curves generated by a point which moves along a uniformly rotating line with a velocity that is not necessarily uniform. It is clear that the idea would be evoked also by such *helical curves* as those noted above (3.2); the helix curve is however three-dimensional, except on the infinitesimal level (which is why they require modern *differential geometry*), whereas Torricelli's method as just recorded was based on plane curves. The subject bears on a wide range of perspectives in the focus of the time and requires some further attention.

Torricelli's used tangent geometry *dynamically*, which is of course important at a time when motion was studied from so many points of view. He developed, as in *Figure 5.2 - 5*, a dynamical conception of tangents from a notion of virtual velocities (Boyer, 1959, 129f.), using *parallelograms of velocities* (Baron, 190ff. for Torricelli's method). Already with his master, Galilei, the notion of *composition of motions* began to gain ground (*see* below). Imagining a projectile moving with a composite motion made up of a uniform horizontal velocity (B - D, *Fig. 5.2 - 5*) and simultaneously a vertical velocity (corresponding to D - E) which varies with the square of time, the curve traversed will be a cubical parabola.

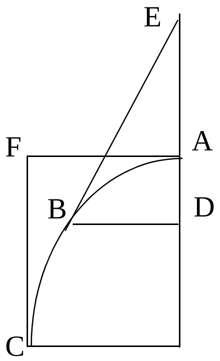


Fig. 5.2 - 5. Torricelli, after Boyer, 1959, Fig. 15.

To determine the tangent to the parabola ABC in the figure, the procedure went as follows. With EB as the tangent, ED = 3AD (not visually evident on this simplified diagram). The moving point will have double impetus at B, one horizontal (BD) and one vertical (given by 3AD; on the figure, this velocity is indicated in parallel from D instead of from B, which amounts to the same). Hence, in consideration of the parallelogram, of which we see one half: BDE, the composite velocity at B will be that of the line BE, which is the tangent.

Torricelli's method as illustrated in the above figure employs the *idea of instantaneous direction* (the motion is following the parabola curve). He studied a large number of curves generated by a point which moves along a uniformly rotating line with a velocity that is not necessarily uniform. According to Boyer (Boyer, 1959, 132), Torrcelli's method represents a marked advance over tradition in that it employs

> *the idea of instantaneous direction and implying, therefore, the limit concept* and the notion of *instantaneous velocity into geometrical demonstrations.... The Scholastic philosophers of the fourteenth century... had given quantitative representations of dynamics, work which was elaborated by Galileo. Torricelli now employed these in pure geometry in the determination of tangents to parabolas.* So he did with a number of curves. He considered the *curves generated by a point which moves along a uniformly rotating line with a velocity which is not necessarily uniform, but is, instead, a function of its distance from the fixed point about which the line rotates* (Buyer, 1959, 133).

Boyer also finds that Torricelli grasped the idea that the tangent problem is the inverse of the quadrature problem. He did not, however, take up the concomitant analytic problem, did not develop these ideas from the ad hoc case to a general principle. *Only with the analytic methods of Fermat, Descartes, and Barrow, or with the calculus of Newton and Leibniz, did it become possible to determine in general, from the equation of a curve, the motions by which the curve could be regarded as traced, or the instantaneous direction* (Boyer, 1959, 134).

5.2.5 *Maxima and minima*

Fermat discovered that at an *extremum*, the tops or crests of a curve and the bottoms the troughs, there are no derivatives, so that f''(x) = 0. This he did before his involvement with *analytic geometry*. At the summit (crest) and bottom (trough) of a curve the extreme point and some point very close to it have *approximately* the same value. For y = f(x), Fermat compared f(x) and f(x + E), where E is the *increment*. Setting f(x) = f(x + E), and then reducing E to zero, but *probably thinking of E as approaching zero*, he obtained the value of the point. The notion of *limit* (3.4.3) was not yet available. Today we would write his equation as

$$\lim_{E \to 0} \frac{f(x+E) - f(x)}{E}$$

and normally write E as h.

Fermat, in his *Methodus ad disquirendum maximam et minimam*, followed up, partly basing his work on Viète's project concerning Greek solutions, and published his work on the subject in 1637, the year of Descartes' *La géometrie*. The so-called point of inflection (or inflexion) was an especially tricky issue. There is a change of direction in a curve not only at an extremum, it may be one also at a so-called *point of inflection;* at which the curve becomes steeper or starts flattening out, or at on point turns from upwards concavity to a downwards one, inversely. Fermat's work led him, it seems, close to discovering that at such a point, translated into modern terms: *the second derivative* must be zero, f ''(x) = 0 (in the simplest terms, if the first derivative represents velocity, the second represents acceleration, the amount by which velocity increases). Since such a point means a challenge to establish the related *tangent*, these two subjects were compounded (also Gilles Personnes de Roberval in correspondence with Fermat, in 1636 to 1638). Apparently Fermat thought of his tangent method as a by-product of this work on the extrema. Ideas concerning maxima and minima problems led him also to searching for the center of gravity on paraboloids, a context in which he made contributions to using summation to achieve definite integrals (Boyer, 1959, 158f.) The point was formalized in Leibniz's *Nova methodus* of 1684, in which he used *d* for *differentiae* and demonstrated that the second derivative at inflection point was zero: ddv = 0 in his notation (Struik, 271).

5.2.6 *Continuity*

The efforts that here go under the epithet of *protocalculus* presupposed attention to the *continuity* of a curve for it to be valid as a testing ground.

The concepts of *continuity* and *infinity, we have seen* (3.5.5), are so closely connected with one another as to be almost integrated and fused into one idea. Therefore, continuity, even more difficult to grasp intuitively, was one of the most tricky issues, and it was satisfactorily treated very late, mainly by Leibniz and Euler. Boyer notes Kepler's vagueness regarding the issue and recapitulates:

> *This striving for an expression of the idea of continuity constantly reappears throughout the period of some fifty years preceding the formulation of the methods of the calculus. Leibniz himself, like Kepler, frequently fell back upon his so-called law of continuity when called upon to justify the differential calculus* (Boyer, 1959, 11).

In 1604 Kepler, perhaps as the first one, had examined the *continuity* property of conic sections, circle, ellipse, parabola and hyperbola (Kline, 1999, 348ff.). A conic section as a type is c*ontinuously reducible* to any one of those mentioned by moving one of the foci along straight lines. Blaise Pascal, too, worked on the notion of continuous change among geometric figures.

The notion of a *point structure of a continuum* had been entertained by medieval mathematicians (Baron, 73). The problem of infinity in the diminishing direction is of course related to this issue. In 1647 Gregory of St. Vincent published his *Opus geometricum* at Antwerp. His work is mainly concerned with integration, but of course this subject requires attention to things relevant also in differentiation (see about that in section 3.5.10 about the *Fundamental Theorem*). The issue that interests us at present is the distinction between two different methods for determining *continuity* of a curve. Gregory *connected the question* [of infinitesimals and hence continuity] *with the Scholastic discussions on the nature of the continuum and the result of infinite division. Archimedes, Stevin and Valerio had subdivided only until the error was less than a certain amount.* - For continuity in Cavalieri's methods, see Baron, 125.

5.2.7 *Infinity*

Eves notes that a major issue in the protocalculus drive was that here one *had to come to serious grips with the old and difficult concept of infinity* (Eves, 174). Infinity, either in a diminishing direction or the opposite, is , there being symbols with which to note it down. But it is not understandable, because there is nothing to compare it with and no context in which to embed it. As late as in the 1820, Niels Henrik Abel used 1/0 to indicate infinity on his integrals. A mathematical absurdity unless one thinks of it as equivalent to ∞, which of course he did. But Fermat used the same pseudo fraction where later mathematicians would use a *limit* (3.4.3. and

3.5.2); he probably had an inkling about this rather elusive entity. It appears that conceptualization concerning infinity came very close to the limit idea though not to its later formalization. In fact, Bernouilli and De L'Hôpital showed, ca. 1700, that infinity can be used as a limit just as well as zero (Kline, 1977, 404; Harris-Stocker, 510; Blank and Krantz, 301).

Infinity, closely related to continuity, is in fact a crucial concept in almost all calculus issues. Again, as noted above, work on areas and volumes stimulated concern about the development of the infinitesimal dimension and hence infinity. Galilei considered *an infinite number of infinitely small and indivisible parts with an infinite number of infinitely small indivisible spaces interposed between the parts* (Baron, 117f.). To avoid obvious pitfalls, however, he clearly distinguished between the *potentially infinite* and the *actually infinite*. Galilei (Baron, 118) developed mathematical infinity of the figure called *Aristotle's wheel*. Here, however, he warns against treating the infinite *by assigning to it properties which we employ for the finite. He is careful to distinguish between the potentially infinite, exemplified in the process of subdivision without end, and the actual infinity, arrived at by some direct process.*

The idea of infinity, even an infinite number of universes, is almost ubiquitous in Giordano Bruno's work. He was not alone in considering the subject worthy of investigation, nor in arriving at it through argumentation.

Boyer (1959, 70) notes how between the medieval conception and that of Galilei, there is an essential difference between the rules for the finite and those for the infinite.The Italian focuses *on the correspondence between infinite aggregates, rather than upon the ratio of finite to infinite magnitude - a change of view which led to the final formulation of the calculus in the nineteenth century*.

Gregory of St. Vincent, in his *Opus geometricum* of 1647, brought the issue of infinity a step further than his predecessors by using the classical method of *exhaustion* (filling out a curve area with ever smaller measurable geometrical shapes such as rectangles until this construction almost fills out the area, thus making it measurable). Gregory's idea was to continue the filling-up process *to infinity* - instead of thinking of static indivisibles, he reasoned in terms of a varying *subdivision, thus approximating the method of limits* (Boyer, 1050, 137). His work led him to study *infinite series*, essential in the development of the definite integral (but he was not alone in this). Boyer continues:*...but Gregory interpreted this as meaning an actually infinite subdivision* (Boyer, 1959, 135: see further next section).

5.2.8 *Change/variation of velocity, direction and intensity*

The general idea behind what we call *vectors*, was imagined, but technically not as yet rendered mathematically usable. Borrowing from Archimedes, Roberval interpreted a curve as a the locus of a point that moved in the direction resulting from a composite of two distinct velocities, that is, in a parallelogram in which the diagonal indicated the tangent (Kline, 1999, 401. Boyer, 1959, 57f.). The method, which was studied also by Galilei and Torricelli, relied entirely on geometry.

We saw under the heading concerning limits (3.5.2), that medieval contributions foreshadowed in important aspects early modern achievements. This holds also for change in *intensity*. I am using this abbreviating term to cover all sorts of change *within* something, such as temperature, density, color change, and so on.

Again, to forestall the impression that unheard of things were now emerging, medieval precedents should be taken into consideration, at least as much as to convey the flavor of the activities in earlier centuries.

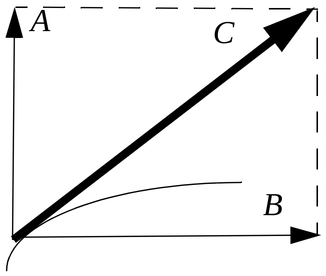


Fig.5.2-4 A vector parallelogram. C is the resultant of A and B.

Richard Suiseth, called *Calculator*, publishing around mid-fourteenth century, discussed changes in intensity at length (Boyer, 1959, 74f.f.). He strove towards the idea of average intensity, *an idea which could not be made precise without the use of the concept of the calculus... The rigorous proof of this requires the use of the limit concept, but Calculator had resort to dialectical reasoning, based on physical experience of rate of change*.

According to Boyer (Boyer, 1959, 82), Oresme's work (late fourteenth century)

marks a notable advance in mathematical analysis in that it associated the study of variables with the representation by coordinates. Although Aristotle had denied the existence of an instantaneous velocity, the notion had continued to be invoked implicitly, upon occasion, by Greek geometers and Scholastic philosophers. Oresme, however, was apparently the first to take the significant step of representing an instantaneous rate of change by a straight line. He could not, of course, give a satisfactory definition of instantaneous velocity, but he strove to clarify this idea by remarking that the greater this velocity is, the greater would be the distance covered if the motion were to continue uniformly at this rate. Also the science of *dynamics* was approached in an intuitive manner in the fourteenth century (Baron, 1959, 82f.; see 1.16.1).

The passage from "Medieval" to "Early-Modern" mathematics was mostly seamless. Even Galilei found support in early achievements. His view on the *Mean speed theorem* was, according to Baron (120f., Fig. 4.17), *very similar to that of Oresme (.....). His concept of the sum of lines as a measure of the space contained within a plane figure was to become the foundation for a great many seventeenth-century integration methods*, notably in Cavalieri's work.

Studying *uniformly accelerated bodies*, Galilei imagined a sum of lines in a triangle representing the acceleration geometrically, since line length increased gradually at a constant rate, and translating this into a sum of lines in a rectangle representing constant velocity, with all the lines of equal length. This approach is geometrical and the calculation is developed on a what we would call a graphic model. Descartes on the other hand, used indivisibles (5.2.1) to determine the laws of falling bodies (Drake, 1989). For his *instant centre of rotation*, see Baron, 163ff.).

A special subject was *The composition of motions* (thus a heading in Baron, 174ff). This represented a cluster of problems relevant for tangent determination (5.2.4). Roberval, in correspondence with Fermat from 1638 onwards, *had no general method for determining the true motion of a point on a curve and the fact that he arrived at correct results was sometimes due rather to a happy instinct than to any real understanding of the nature of the motion* (Baron). *He may have been in possession of an analytic method*, but no trace remains of it. Described in intuitive terms, the approach consisted in considering every curve as the path of a moving point and the tangent to the curve at a point, so that *the resultant of two or more motions impressed upon it* [the curve] *simultaneously*. The validity of the approach, according to Baron, depends on the physical or mathematical mutual *independence* of the involved motions. The idea originated with the Greeks and was further developed by Galilei and his pupil Torricelli.

5.2.9 *Rectification of curves*

What is the length of a bent curve; the length of the Spire helix?

Descartes, in *La géometrie*, attempted to answer the general question without success on *algebraic* curves, which was to be expected, and declared that the ratios between curved and straight lines *cannot be known*; whereas Torricelli in 1645 succeeded on *mechanical* curves, that is, graphically constructed ones (Boyer, 1980, 393f.). He resolved the problem for a logarithmic spiral. Roberval, in 1642, had established the length of an Archimedean spiral and of the parabola. For Descartes' handling of the problem, see Mancosu, 79. For the distinction between *algebraic curves and mechanical curves*, see above, 3.5.9. and below.

Torricelli used Cavalieri's *indivisibles* and infinitesimals due to Galilei to achieve his rectification of the logarithmic spiral. Of importance in the present context, is the fact that Torricelli used infinitesimal methods to achieve his result.

5.2.10 *Curves, 2D and 3D, including some special cases*

With regard to curves in general, let it be noted that in the years indicated, that is, before 1655, the technique of using *position vectors* to describe a curve, especially one in 3D, was not as yet available. A complete treatment of general curvature with all possible variations had to await the emergence of *differential geometry* (3.6). Hence, elaboration of a curve had to start out so to speak from its interior, from any chosen point on it, with the corresponding tangent. The *helix* was constructed over a spiral using proportions (Dürer: 1.6) by rotating geometrical figures.

In Descartes' *La géometrie* of 1637, *algebraization of curves*, or inversely, construction of curves from functions, laid the foundations for the so-called *Cartesian coordinate system* (5.2.12). He studied algebraic curves (as distinct from mechanical ones), where the image is controlled by manipulation of the equations, thus changing and eventually unifying the roots. He was concerned with the determination of the normal to a curve, thereby to find the tangent at the given point, thus avoiding the use of infinitely small quantities (Baron, 165).

If, says Descartes, a circle cut a curve in two distinct points, then it is possible to form an equation (in x or y) with two roots corresponding to the points. As these points approach each other the differences between the roots of the equation diminishes; and when the points become coincident the roots become equal and in this case the circle will touch the curve (Baron, Fig. 5.11; *see* our *Fig. 5.2 - 3* after Blank and Krantz).

Using as he did the parabola as standard testing model, the method served to tangent determination via the establishment of the normal at any chosen point on the curve.

Finite algebraic processes constituted the precision tool through which he hoped finally to resolve all mathematical problems capable of resolution. The classification of curves which he adopted in La Géometrie (1637) enabled him to accept the use of solutions involving motion in the case of the so-called mechanical curves [curves not derived from functions but from design procedures] *whilst rejecting any other than finite algebraic processes in determining g the tangents to algebraic curves. The problem of rectification he dealt with by reiterating Aristotle's assertion that no curve is capable of rectification, i. e. the arc cannot be expressed as a finite ratio of a given straight line. At the same time he seems to have accepted that mechanical curves, such as the spirals could, by mechanical means, be expressed in such terms* (Baron, 163).

The problem of handling curves was intimately connected with those concerning areas and volumes, subjects for *integration*. The case of Grégoire de Saint-Vincent as presented by Baron illustrates the continuing dependence on classical geometry.

Although, after the extension of algebraic methods to the study of curves, it became possible to create an infinite variety of curves and solids by the mere manipulation of symbols, as long as methods remained purely geometrical the range of known curves and solids was limited to those which could, in some sense, be imaginatively constructed or perceived in the external world... The weakness, however, of the whole structure lies in the fact that the power of the techniques developed is entirely dependent on the availability, or otherwise, of appropriate Euclidean properties (Baron, 144).

A special subject of some importance with a view to discussing curve mathematics was, as hinted already, how to cope with their *classification*.

Curves are a central subject in Descartes' *La géometrie* of 1637 (an appendix to his *Discours de la Méthode*), and Mancosu's exploration brings out some issues that are directly relevant to the present record (Mancosu, Chapter 3, pp. 65 - 91; also in Gilles, pp. 83ff.).

The classical conception of geometric problems forms the background of Descartes' approach: Pappus' distinction between plane, solid and linear problems. A special category is *lines and other than those mentioned* <which> *are used in the construction, which have a varied and more intricate genesis, such as the spirals, the quadratics, the conchoids and the cissoids, which have many marvellous properties.* Descartes wants to operate distinctions between different degrees of these more complex curves (*ces lignes plus comporées*) and he wonders why they are not categorized as mechanical curves rather than geometrical curves (Descartes, *Géometrie,* p. 315). At any rate, he wants to have the four curves just cited in one class. For what is most characteristic of them, is that they *must be conceived of as described by two separate movements whose relation does not admit of exact determination* (*... a cause qu'on les imagine descrites par deux mouements separés, & qui n'ont entre eux aucun rapport qu'on puisse mesurer exactement...* ; *Géometrie,* p. 317).

There are two paradigms here that are of compelling interest in the present study. The fundamental one is the admittance of *approximation* as an absolute value and not only heuristically. The second is the involvement of *motion* in the operations on curves, thus also what is called *point-wise construction*, a subject examined also by Christoph Clavius in Rome (Mancosu, pp. 73f.). For example, the *quadratix* is a curve generated by the intersection of two segments, one moving with uniform rectilinear motion and the other with uniform circular motion, as Mancosu explains.

One special type of curve is the so-called *catenary*, illustrated by a chain or rope suspended between two poles, as in a suspension bridge. The curve requires calculus (Kline, 1977, 487). The equation for it was found as late as about 1700 (Bernouilly). Galilei believed it to be a *parabola*, but it was shown later not be a parabola and not even an algebraic curve (Boyer, 1980, 376). It represented one of the challenges that animated the Protocalculus efforts. Galilei treated the curve as a parabola, and *introduced some techniques which proved particularly fruitful in the study of curves* (Baron, 121f, Fig. 4.18). He thought in metaphors of what we today handle with the use of *vectors*, considering two motions, one horizontal and one vertical, and a constant acceleration, i. e. the distance varies as the square of the time. The curve circumscribes an area, the "inside" of the parabola, and the technique was to prove useful for Torricelli and Barrow. From Descartes' work (and that of others) arose a clearer recognition of the value of *approximation*: this led him to distinguish between *precision* methods and *approximate* methods.

Another curve to be cited here, the *cycloid*, was studied by Charles de Bouelles in 1501, if not earlier. The curve, which does not require calculus, was studied by Galilei (1599), Mersenne (1628) and Roberval (1634), and it was successfully formalized by Pascal in 1659, under the name of *la roulette* (Smith, 1953, II, 327f.). I am mentioning this curve because one of the cycloid versions possibly is at the base of the *pseudo architraves in Borromini's San Carlino*; this was at a time when this class of curves was still problematical.

5.2.11 *Relation between differentiation and integration (the Fundamental theorem of the Calculus)*

Realizing that differentiation and integration are two closely related processes, the one the inverse of the other, took some time. That is, differentiation by going from equation to finding change-points on a curve, from distance to speed to acceleration - and integration by going the opposite way, and finding the curve from the point, speed from acceleration and distance from speed, finding the surface enclosed or partly-covered by a curve - these procedures are inverse to one another.

The idea of the *inverse relationship between differentiation and integration* is complex and, one might say. emblematic for decisive steps from the protocalculus to the full-fledged calculus and requires some more attention, even though the subject certainly does not arise directly from considerations of the Spire. But the subject occupied a central position of things under debate, even though it was only, if at all, a vaguely formulated hunch.

We have the equation of a curve, say $y = x^2$, a parabola. By differentiating, finding a formula for the instantaneous change, we have $y' = 2x$, so that, where y has the value of $2x$, the x then to be exactly given in the specific case, we have the point itself. Now, if we have the formula for a given point, and this is $y = 2x$, then by *antidifferentiation*, or *indefinite integration* (reversing the process), we learn that the equation of which this is the derivative or differential value, is $y = x^2$. Here we see clearly the inverse relationship between the two operations. Technically, however, we should have to go step further, to set down the rules for the relationship between the derivative and *definite integration.*

Here I take the liberty of quoting a particularly simple but illuminating account (Reade, 121).

There are two essentially different ways of looking at integration. One can either regard it as the inverse process of differentiation, or one cam regard it as a kind of continuous summation.

The first view gives rise to the'indefinite integral' or'primitive' of a function f(x) as any function F(x) whose derivative F'(x) = f(x). The second view gives rise to the'definite integral' of a function f(x) which is defined as the limiting value of sums of the form $\Sigma f(x)dx$ as $dx \to 0$. *The definite integral, where F(x) is any indefinite integral of f(x). On the other hand. the definite integral of f(x) can be used to define*

$$\int_a^b f(x)dx = F(b) - F(a)$$

an indefinite integral F(x) of f(x) by writing [with the upper value an indefinite *x,* no longer the definite *b*]

$$F(x) = \int_a^x f(x)dt$$

These two results are collectively known as the fundamental theorem of calculus... (so far Reade).

Behind this assumption of clarity, however, lies a technical procedure that determines the step from $y = x^2$ to $y' = 2x$, and this presupposes the use of the idea of *limit, which was not technically available at the time of our concern* (5.2.2).

The idea of the *definite integral*, which is *the limiting value of sums* (Reade, see 3.7), is different. In fact, *the indefinite integral is a function with variables*, and it is on this level that the *inverse* relation to differentiation (like B -> A related to A -> B) is directly observable. A *definite integral*, on the other hand, is a *number*, since it gives the value determined by an interval on a number scale between two limit numbers on this scale, the upper (greatest) one corresponding to the *indefinite integral*. Since the derivative excludes constants (that are not differentiable), the antiderivative may include constants (usually written C), because these may have belonged to the original function, back to which the antiderivative leads.

For an account of how at the time in consideration here, the idea of inverse relation was, if at all, suspected or imagined, one should consult Section 5.6 in Baron (177 - 182), entitled *The Link between differential and Integral Processes*. This account takes us right into the middle of the protocalculus problems.

Gradually the point was being taken. Quoting Kirsti Møller Pedersen on the relevant point:

Many methods were developed to solve calculus problems; common to most was their ad hoc character. It is possible to find examples from the time before Newton and Leibniz which, when translated into modern mathematical language, show that differentiation and integration are inverse procedures; however, these examples are all related to specific problems and not to general theories. The special merit of Newton and Leibniz was that they both worked out a general theory of the infinitesimal calculus. However, it cannot be said that either Newton or Leibniz gave to his calculus a higher degree of mathematical rigour than their predecessors had done (Møller Pedersen, 10).

Torricelli worked with *continued proportions*, i. e., *geometric series* - among the basic features in later integration methods, but his *approach was wholly geometrical* (his calculations are reported in modern notation in Baron, 186ff.). The way was opened up for further development when Wallis *extended the index notation to include negative and fractional exponents* (such as a^{-n} and $a^{n/m}$). It may possibly be held that the main immediate effect of the Cartesian *wedding of curve and equation* (Kline's term) (3.5.11 and5.2.12) was to tilt the balance even more over to geometry; a non-expert conjecture on my part, but perhaps justified by a comment in Baron, 195: *The spread of Cartesian notation... meant that an increasing volume of geometrical work received the benefits of algebraic symbolism*. At any rate, we are being constantly told that most approaches to protocalculus issues were geometrical. Here Torricelli made integration workable on infinite ordinates and furthermore achieved a *significant transition from an integral to a differential operation.* Baron finds it *difficult not to conclude, with* <Ettore> *Bortolotti* [an Italian historian of math]*, that Torricelli understood clearly the inverse relation between integration and differentiation in the specific case* (Baron, 189; Bortolotti, 718ff., esp. 727).

With reference to Fermat, Baron (178) notes *Although the application of a differential method to a problem previously resolved by summation indicates the appreciation of the fundamental nature of the differential process and its link with integration* [based on summation]*, the particular example is not so handled as to bring out clearly the inverse nature of these operations*. Roberval, on the other hand, established centres of gravity in the traditional way though a summation process. By geometrical considerations he was *able to relate the tangent and area properties of curves* (affinity to solutions by Guldin, Saint-Vincent and, above all, Torricelli). Working with the quadrature of some specific hyperbolas, and to some extent taking hints from Fermat, he developed a general solution, causing Baron to conclude as follows with reference to Torricelli:

In the history of the calculus these investigations of Torricelli are supremely important, for the manner in which he expressed his results enabled him to perceive, and to formulate, the tangent construction from the integral. Amongst all those who contributed to the development of infinitesimal processes before Newton and Leibniz, Torricelli exhibited most clearly the link between the two operations now known as differentiation and integration. Torricelli's work was published only in 1918 and *the importance of his contribution has scarcely, even now, been fully recognized* (Baron, 185).

5.2.12 *Cartesians: Coordinate systems and geometrical loci (Descartes and Fermat)*

The introduction of such an *analytical geometry*, or rather the preludes to one, is mainly due to Descartes and Fermat. And so also the concomitant study of the geometrical *locus*.

This is the position in the Cartesian plane or space in which the expression of an equation, for example a geometrical figure like the conchoid, is *localized, the set of all points on a curve satisfying a given condition*. Fermat later claimed historical precedence for this discovery, but Descartes' work is more renowned, so it is a matter of taste under which heading the subject should be considered.

The use of abscissa and ordinate (usually *x* and *y*) for working with plane curves took some time to develop, but the potentials inherent in the method inception had a bearing upon the protocalculus development. The idea in itself did not originate with Descartes or Fermat, but was known much earlier.

Already *Nicholas Oresme (1323 - 1382) identified the real numbers with a geometric continuum. After another long hiatus, François Viète (1540 - 1603) further demonstrated the power of geometrically interpreting the real numbers... The crucial idea that the solution set of an equation such as F(x, y) = 0 can be identified with a curve in the plane appeared in the Géometrie...* (Blank-Krantz, 72). The idea is attestable in earlier centuries, such as in Raffaele Bombelli's *algebra linearia* of 1572. There were other forerunners, too, among them, as Prof. Razi Naqvi informs me, Umar al-Khayyam in the twelfth century (5.2.12). Boyer (Boyer, 1959, 80f.) notes how in the Oxford *Liber calculationum* from early in the century, there is *no diagram or reference to geometrical intuition, the reasoning being purely verbal and arithmetical.* With Nicholas Oresme, bishop of Lisieux (ca. 1323 - 1382), things start changing.

The work of Oresme... makes most effective use of geometrical diagrams and intuition, and of a coordinate system... This graphical representation... marked a step toward the development of the calculus, for although the logical bases of modern analysis have recently been divorced as far as possible from the intuitions of geometry, it was the study of geometrical problems and the attempt to express these in terms of number which suggested the derivative and the integral and made the elaboration of these concepts possible. Already Nicholas Oresme (1323 - 1382) *identified the real numbers with a geometric continuum. After another long hiatus, François Viète (1540 - 1603) further demonstrated the power of geometrically interpreting the real numbers... The crucial idea that the solution set of an equation such as F(x, y) = 0 can be identified with a curve in the plane appeared in the Géometrie...*(Blank-Krantz, 72).

Mancosu (82) takes a penetrating look at the *relationship between analysis objects (equations) and geometrical objects (curves),* which he considers crucial for Descartes' *Géometrie,* discussing two different positions regarding the role of algebra in his geometric constructions. This debate need not occupy us.

The subject on hand is in fact a double one, Descartes' *coordinate system* and Fermat's *geometrical loci*, two issues that overlap in many respects and which, when taken together, touch on central protocalculus problems, even if - and this must be emphasized - the main aim of Descartes' geometrical studies regards the *solution of equations of higher degrees*. Blay, *Introduction,* introduces a terminology of *geometrization* etc. that I refer to here for the sake of completeness; I have some problems following him on some of this (but I have not had access to the French original). Blay claims that Descartes' and Galilei's project was geometrization. Most mathematics, however, since the Greeks had been dominated by geometrical thought. Even the early approaches to the Calculus issues were mainly geometrical.

The ventures that were to initiate *analytic geometry* proved crucial for the general *mathematization of science*. But let us not forget that the Church, too, played a very active role in preparing the ground for this development (see 2.2.11, on Christoph Clavius at the Collegio Romano).

Pierre de Fermat, as we have seen, had a similar intuition earlier than Descartes, most likely already by 1629, but documented in 1636, supported by earlier work by François Viète, (Boyer, 1980, 398). This is one year before Descartes' publication. In fact, Fermat's idea, which in certain respect came closer to the modern usage (with ordinate orthogonally to the abscissas; development of 3D), was published only in 1679 but had been circulated widely long before that. Characteristically, Fermat's work bore title *Ad locos planos et solidos isagoge* (*Introduction to plane and solid loci*). *With this work were connected two others of fundamental importance: his study of maxima and minima* [5.1.5] *and introduction of the increment in his approach to differentiation* [5.1.3]. *Fermat wrote that whenever two unknowns* [conceived as axes corresponding to our x and y, abscissa and ordinate] *appear in an equation, there is a locus such that the maxima of one of them describe a straight line or a curve. In fact, the varying values of what we would note as f(x) or the y yield this line or curve when the top points are connected by a line drawn through them. In this way, an equation is geometrically expressed* (transl. from Boyer).

John Wallis - now to cite the person publishing at the end of our time interval, 1655 -*went far beyond earlier mathematicians in that he sought to free arithmetic completely from geometric representation... Wallis came nearer to the limit concept than did any other of Newton's predecessors. It is clear that this notion is implicit in the work of most of his French and Italian contemporaries, but it was not expressed by them. Instead the concept of the infinitesimal was employed* (Boyer, 1959, 168f.; 170f.).

A parameter of a certain consequence was *homogeneity* (that units are of the same type or dimension). For example, Viète in his handling of geometric problems by their reduction to the solution of algebraic equations, used equations that *betrayed their origin in geometry, in that he was always careful to have them all homogeneous...* (Boyer, 1959, 1549.

Fermat's and Descartes' work in this connection went further than those cited above, *for it associated with each curve an equation in which are implicit all the properties of the curve... This recognition... constitutes the basic discovery of analytic geometry* (Boyer, 1959, 154).

As we have noted, the main purpose in Descartes' *La géometrie* was the *solution of equations*. This also was the principal understanding among most mathematicians at the time, as Kline has observed. By stating that a curve is *any locus describable with an algebraic equation*, Descartes opened up a new field of mathematics. Kline also comments that Fermat's centering his attention on the problem of *geometrical loci* came closer to modern perspectives, because it s a higher degree of generality (Kline, 1999, 369f.). The new *analytic geometry* (thus named at a later date) to a remarkable degree met the growing need to apply quantification to natural phenomena (see 2.2.2).

According to Boyer (1980, 390ff.), Descartes' purpose was twofold. *One*, to free geometry from having to have recourse to designed figures (graphics), through the application of *algebra*; *two*, to make available interpretation of algebra operations with the use of geometry. The effort was primarily applied to the solution of second- and third-degree equations with an ensuing classification of geometrical problems. The outcome, of course, was the early version of what today we call the *Cartesian system* of coordinates, with *abscissa* and *ordinate*, usually *x* and *y*, extendable into 3D and indicated by *z.* But many features which are indispensable for the functioning of the *Cartesian* system in modern mathematics, are absent from his account (survey in Boyer, 1980, 394f.). And the baselines that Descartes used were not at right angles to each other. (Derbyshire, 91).

5.2.13 *The use of "moments"*

This section brings a general comment upon some of the issues discussed in the above sections.

Most mathematicians engaged in the drive towards the calculus realized that they had to relinquish the hope of achieving absolute results, being aware of the necessity to make do with approximation, either in the form of *indivisibilia, towards extremes of diminution* (5.2.1) or *limits, towards extreme closeness to an imagined point or value* (5.2.2). *Some terms considered inessential for the outcome should be discarded*. But which ones? could they be given some general formal status? Or was the issue fated to remain at an *ad hoc* level? The question regarded the method of *dropping out of the calculation "moments" which did not add significantly to the result* (Boyer). Following Boyer (1959, 149ff., 178f., 191, 113f. concerning Galilei), it seems that the method of dropping terms had been initiated, not with Cavalieri (as some have believed), but with Roberval and Pascal, a method, however, encumbered by some confusion right up to the time of Newton (Boyer, 224). The long-winding debate concerned not only the difficulty of establishing a generally valid status for the terms, but also the very usefulness of the method against claims that *the omission of infinitely small quantities constituted a violation of common sense.* Some confusion continued up to modern times, *even though modern mathematics have shown that the basis of the calculus is to be found in the derivative* [using the limit] *rather than the differential* (Boyer, 179).

5.3BIBLIOGRAPHY

Please note: All the cited publications, except Torricelli, *Opere*, Guidobaldi, and Saiber, have been acquired over the years in four different countries; in Europe driving yearly up and down between Trondheim and Rome, and in New York and Cambridge, MA. As a result, some of them are cited in translation; but with reference to the original.

Abler, R., Adams, J. S. and Gould, P., *Spatial organization. The geographer's view of the world*, London 1972.

Abraham, Gerald, *History of music,* Oxford 1979 (paperback of 1985 cited here).

Alonso, M., and Finn, E. J., *Physics,* Harlow 1992, reprints 1995, 1996.

Anderson, J. R., *The architecture of cognition, Cambridge (MA) 1983.*

Arrighi, G., et al., *La scuola galileiana. Prospettive di ricerca*, Florence 1979. Articles/comments in the order n here by Arrighi, Galluzzi, Torrinui, De Angeli, Baldini, Belloni, Garin, Altieri Biagi, Bulferetti, Vasoli, De Franco, Brancaforte, Salemi, Dollo.

Bank, D., *Breaking Windows. How Bill Gates fumbled the future of Microsoft,* New York 2001.

Barbo, Lodovico, *Metodo di pregare e di meditare,* parallel Italian and Latin version, published by the Benedictines at Pragila (PD); no date, but the *imprimatur* is dated 1924.

Baron, M. E., *The Origins of the Infinitesimal Calculus,* New York 1987 (Dover Publications; orig. 1969).

Bartocci, C., *Raconti matematici,* Turin 2006.

Battiferri, L., *Ricercari a quattro, a sei, con 1, 2, 3, 4, 5, 6 soggetti sonabili*, Bologna 1669.

Baumgartner, P., and Payr, S., *Speaking minds. Interviews with twenty eminent cognitive scientists,* Princeton (NJ), 1996.

Bavelas, J. B. , Black, A., Chovil, N. and Mullett, J., *Equivocal communication,* Newbury Park 1990.

Beam, W. R., *Systems engineering. Architecture and design,* New York 1990.

Bechtel, W., and Abrahamsen, A., *Connectionism and the mind. An introduction to parallel processing in networks*, Oxford 1991.

Bellarmino, Roberto, *Disputationum Roberti Bellarmini ... De controversiis christianae fidei adversus hujus temporis hæreticos*, II, Milan 1721 [1583], III, Milan 1721 [1592].

Ben-David, J., *The scientist's role in society. A comparative study*, Englewood Cliffs (NJ) 1971.

Benjafield, J. G., *Cognition,* Englewood Cliffs (NJ) 1992..

Berenson, W., Harris, J. W., Stocker, H., and **Lutz, H.,** *Handbook of physics,* New York 2000.

Berlin, Isaiah, *The originality of Machiavelli,* in Gilmore, M. P., *Studies on Machiavelli,* Florence 1972, pp, 149ff.

Bernini, mostra del = Gian Lorenzo Bernini, Regista del barocco, exhib. catalog, Maria Grazia Bernardini and Maurizio Fagiiolo dellArco, Milan/Rome 1999.

Beumann, H., *Zur Entwicklung transpersonaler Staatsvorstellungen,* in *Das Königtum. Seine geistigen und rechtlichen Grundlagen, Mainvorträge 1954,* Lindau 1956.

Blair, G., Gallagher, J., Hutchison, D., Shepherd, D. (eds.), *Object-oriented languages, systems and applications*, London 1991.

Blank, B. E., and Krantz, S. G., *Calculus. Single variable,* New York 2006.

Blay, M., *Reasoning with the Infinite. From the Closed World to the Mathematical Universe,* Chicago 1998 (transl. from *Les raisons de l'infini. Du monde clos à l'univers mathématique*, Paris 1993).

Bloor, D., *Knowledge and social imagery,* 2nd ed., Chicago 1991.

Blum, P.R., *Giordano Bruno,* Munich 1999.

Blunt, A., *Guide to Baroque Rome,* London 1982.

Boden, M. A., *The creative mind. Myths and mechanisms,* New York 1991 (orig. London 1990).

Boden, M. A. (ed.), *The philosophy of artficlal intelligence,* Oxford 1990.

Booch, G., *Object-oriented analysis and design. With applications*, 2. ed., Redwood City (CA) 1994.

Booz*,* **P.,** *Die Baumeister der Gothik*, Berlin/Munich 1956

Born, R.(ed), *The case against artificial intelligence*, London 1987, republished 1989.

Bortolotti, E., *Storia della matematica elementare* [including the calculus!], in Berzolari, L. (ed.), *Enciclopedia delle matematiche elementari*, Vol. III, 2nd. Part, Milan 1983 (reprint), 539 - 750.

Bouzeghoub, M., Gadarin, G., and Valduriez, P., *Object Technology. Concepts and Methods*, Boston (MA) 1997.

Boyer, C. B., *The History of the Calculus and its Conceptual Development*, New York 1959 Dover Publications (orig. 1949).

Boyer, C. B., *Storia della matematica,* Verona 1980, transl from *A history of mathematics,* New York 1968.

Bozzi, Paolo, *Fisica ingenua. Studi di psicologia della percezione,* Milan 1998.

Braisby, N., and Gellatly, A. (eds.), *Cognitive psychology,* Oxford 2005.

Bratko, I., PROLOG. *Programming for artificial intelligence*, 2. ed., Wokingham 1990.

Bronstein, I. N., Semendjajew, K. A., Musiol, G., and Mühlig, H., *Taschenbuch der Mathematik,* 6. ed., Frankfurt a/M 2005.

Brooks, R. A., *Intelligence without representation*, in Kirsh (ed.), *Foundations of Artificial Intelligence,* pp. 139ff.

Bruno, Giordano, *La cena de le ceneri* [London 1584]*, ed. Augusto Guzzo a.o.,* Milan-Naples 1956, republ. 1995.

Bruno, Giordano. *Spaccio de la bestia trionfante,* ed. and com. by Michele Ciliberto, Milan 1985, reprint ibid. 2001.

Bruno, Giordano, *De umbris idearum*: 1. Latin original: ed. Rita Sturlese, *Le opere latine*, Florence 1991 (= *De umbris/Sturlese*)*, and* http://www.fatalibelli.it/Alez/Giordano%20Bruno%20De%20-Umbris%20-idearum.htm 2. Italian translation by Diego Fusano: www.fatalibelli.it/Alez/de-umbris.htm.

Bruno, Giordano, *Le ombre delle idee*, ed. Manuela Maddamma (= *De umbris*/*Maddamma*), Milan 2003 (orig. 1996).

Bruno, Giordano, *Eroici furori*, ed. S. Bassi, Introd. M. Ciliberto, 3. ed., Bari 2004.

Bruno, Giordano, *Cabala del cavallo pegaseo* (London 1585), ed. F. Meroi, Milan 2004.

Burden, *see* Sinding-Larsen, *The burden of the ceremony master.*

Cajori, F., *A History of Mathematical Notations, I, Notations in Elementary Mathematics; II, Notations mainly in Higher Mathematics,* Chicago (IL) 1928/9, republished in one volume by Dover Publications, Mineola (NY) 1993.

Calderón I = **Pedro Calderón de la Barca**, *La vida es sueño*, ed. and annot. by Enrique Rull, Madrid 1992.; *see also* Morón.

Calderón II = **Pedro Calderón de la Barca**, *La vida es sueño*, ed. and annot. by José Maria Ruano de la Haza, Madrid 1994.

Callero, P. L., *Toward a sociology of cognition,* in Howard, J. A., and Callero, P. L. (eds), *The self-society dynamic,* Cambridge 1991, 43 - 54.

Camejo, S. Arroyo, *Skurrile Quantenwelt,* Berlin 2006.

Campanella, Tommaso, *Apologia per Galileo,*[orig. Latin version Frankfurt a. M, 1622: *Apologia pro Galilaeo, mathematico florentino*] ed. Paolo Ponzio, Milan 1997.

Canter, D., *The Psychology of Place,* London 1977.

Cartwright, Nancy, *How the laws of physics lie,* Oxford 1983.

Caspar, M., *Kepler,* transl. from German (1948) by C. Doris Hellman; new ed. by O. Gingerich and A. Segonds, New York 1993.

Castro, A., *El drama de la honra en la literatura dramática,* in Romeralo, 193 - 225.

Cavalca, Domenico, *Specchio di croce* (1325 - 1340), Rome 1738.

Cera, Francesco, *Carlo Gesualdo da Venosa. Stile e poetica del Quarto Libro,* text to CD *Tactus,* TC560701.

Cervantes, Miguel de, *Don Quijote de la Mancha,* ed., notes and introduction by Alberto Blecua, 2.ed., Madrid 2008.

Chalmers, David J., *The conscious mind. In search of a fundamental theory,* New York 1996

Chigi, Fabio, *Gedichte aus seinem Aufenthalt in Münster von 1644 - 1649,* offprint from Westphalische Zeitschrift für Vaterländische Geschichte und Altertumskunde, Vol. 108, Latin and German translation in parallel texts. Thanking Stefanie and Alfons Schröder, Münster i/W for sending me this.

Ciliberto, M., *Giordano Bruno, Bari 2005,* with 50 pp. adjourned bibliography (orig. 1990). = Ciliberto, *G.B. I.*

Clericuzio, A., *La macchina del mondo. Teorie e pratiche scientifiche dal Rinascimento a Newton,* Rome 2005.

Connors, J., *Borromini's S. Ivo alla Sapienza: the spiral,* in *The Burlingtom Magazine,* October 1996, pp. 668 - 682.

Connors, J., *Sant'Ivo alla Sapienza. The rfirst tree minutes,* Journal of the Soc. of Arch. Hist., March 1996, 38 - 57.

Cook, Th. A., *The curves of life, being an accont of spiral formations...*, Dover Publications, New York 1979 (orig. ed., London 1914).

Corneille, Pierre, *Théatre chiosi,* ed. Maurice Rat, Paris n. d. (1950s).

Coyne, R. D., Rosemann, M. A., Radford, A. D., Balachandrian, M., and **Gero, J. S**., *Knowledge-based design systems,* Reading (MA) 1990..

Cresci, L., *Le curve celebri. Invito alla storia della matematica attraverso le curve piane piú affascinanti,* Padua 1998.

Cusano, see **Kues**.

Damasio, Antonio R., *Descartes' error. Emotion, reason and the human brain,* New York 1994.

Davis, G. G., and **Olson, M. H.,** *Management information systems. Conceptual foundations,* 2 ed., New York 1985.

De Finetti, Bruno, *L'invenzione della verità* (1934), ed. and Introduction by Giordano Bruno and Giulio Giorello, Milan 2006.

Del Monte, Guidubaldo = *Guidi Ubaldi... Meccanicorum liber...,* Venice 1615.

Dennett, D. C., *Brainstorms. Philosophical essaays on mind ond psychology,* Brighton 1981..

Dennett, D. C., *Consciousness explained,* Boston 1991.

Denzinger, H., *Enchiridion symbolorum et definitionum quae in rebus fidei et morum a conciliis oecumenicis et summis pontificibus emanarunt,* Naples 1856 (1.st Italian ed.)

Denzinger, H., Schönmetzer, A., *Enchiridion symbolorum definitionum et declarationum de rebus fdei et morum,* Freiburg i. Br. 1965.

De' Paoli, *Monteverdi,* Milan 1979.

Derbyshire, J., *Unknown quantity. A real and imaginary history of algebra,* London 2006.

Descartes, René, *Discours de la méthode pour bien conduire sa raison et chercher la verité dans les sciences,* ed. and commentaries by Étienne Gilson, 5th ed., Paris 1976.

Descartes, René, *La Géometrie*, Supplement to *Discours de la Méthode*, see *The Geometry of René Descartes translated from the French and Latin by David Eugene Smith and Marcia L. Latham*, Dover Publications, New York 1954 (originally 1925). Uncommented.

Descartes, René, *Méditations métaphysiques. Objections et réponses suivies de quatre lettres,* ed. J.-M. and M. Bryssade, Paris 1979.

Descartes, R., *Regulae ad directionem ingenii*, ed., com. and parallel Ital. transl. by Lucia Urbani Ulivi, Milan 2000, reprint ibid. 2001.

Dicker, G., *Descartes. An analytical and historical introduction,* New York 1993.

Downs, R. M., *Personal constructions of personal construct theory,* in Moore and Gollege, *Environmental knowing,* pp. 72 - 87.

DRAE, Real Academia española, Diccionario de la lengua española, 22 ed., CD version, Madrid 2003.

Drake, S., *Discoveries and opinions of Galileo,* Toronto 1957.

Drake, S., *Galileo at work. His scientific biography,* Chicago 1978, reprints.

Drake, S., *History of free fall. Aristotle to Galileo,* Toronto 1989.

Drake, S., *Galileo Galilei pionere della scienza. La fisica moderna di Galileo,* Padua 1992 (orig. *Galileo. Pioneer scientist,* Toronto 1990; further elaboration and revison based on later documentation, of Galileo I). .

Drake, S., *Galileo. Pioneer scientist,* Toronto 1990, a further development and also correction of the material presented in the 1978 book.

Dreyfus, H. L., *Misrepresenting human intelligence***,** in Born, *The case*, pp. 41 - 54.

Dugas, R., *A history of mechanics*, New York 1988, transl. from French ed. Neuchâtel (Switz.) 1955.

Dunand, F., Spieser, J.-M., and Wirth, J., (eds.), *L'image et la production de la sacré,* Paris 1991.

Duval, A., *Des sacrements au Concile de Trente,* Paris 1985.

Eidam, Klaus, *Das wahre Leben des Johann Sebastian Bach,* Munich 1999.

Eriksen, R. (ed.), *Contexts of Baroque: theatre, metamorphosis, and design,* Oslo 1997.

Evans, J. St. B. T., *Hypothetical thinking. Dual processes in reasoning and judgement,* Hove (Sussex) 2007.

Eves, H., *Foundations and fundamental concepts of mathematic s,* 3. ed., Mineola (NY) 1990.

Fetzer, J. H., *Artificial intelligence: its scope and limts*, Dordrecht 1990.

Feyerabend, P., *Wider den Methodenzwang,* Frankfurt a. M. 1983 (thoroughly revised edition of the orig. 1976 publication in English, *Against Method*).

Filippi, D. V., *Tomás Luis de* ***Victoria****,* Palermo 2008 (thanking Camilla and Paolo for this).

Fischer, E. P., *Werner Heisenberg. Das selbstvergessene Genie,* Munich 2001, paperback Munich 2002. His evaluations of Heisenberg's role in the war efforts must be compared to the new findings in Karlsch' book cited below.

Fischer, E. P., *Der Physiker. Max Planck und das Zerfallen der Welt,* Munich 2007.

Firesmith. D. G., *Object-oriented requirements. Analysis and design. A software engineering approach*, New York 1993.

FitzGerald, J., and **FitzGerald, A.,** *Fundamentals of systems analysis. Using structural analysis and design techniques,* 3 ed., New York 1987.

Flor, Fernando R. de la, *Barroco. Representación e ideología en el mundo hispánico (1580 - 1680),* Madrid 2002.

Foa, A., *Giordano Bruno,* Bologna 1998.

Fodor, J. A., *A theory of content and other essays,* Cambridge (MA) 1990.

Francesconi, G., *Storia e simbolo. "Mysterium in figura": la simbolica storico-sacramentale nel linguaggio e nella teologia di [St.] Abrogio di Milano,* Brescia 1981.

Frigerio, M., *Invito al pensiero di Bruno,* Milan 1991.

Frost, P. J., Moore, L. F., Louis, M. R., Lundberg, C. C., and ***Martin, J.,*** (eds), *Reframing organzational culture,* Newbury Park (CA) 1991.

Galilei, G., *Dialogo dei Massimi Sistemi*, ed. F. Flora, Verona 1996 (orig. part of *Opere*, same editor, Milan 1953).

Galilei, G., *Sidereus Nuncius*, ed. and introduction by Andrea Battistini, with parallel translation by Maria Timpanaro Cadini, Venice 1993.

Ganter, B., and Wille, R., *Formal concept analysis. Mathematical foundations,* Berlin 1999.

Garber, D., *Descartes' Metaphysical Physics,* Chicago (ILL) 1992.

Garber, D., *Descartes' Physics,* in Cottingham, *The Cambridge Companion to Descartes,* 286 - 334.

Garber, D., *Descartes embodied. Reading Cartesian philosophy through Cartesian science,* Cambridge (MS) 2001.

Gatti, H, *Giordano Bruno and Renaissance Science,* Ithaca/London 1999.

Geen, R. G., *Human motivation. A social psychological approach,* Belmont (CA) 1995.

Geier, M., *Kants Welt. Eine Biographie,* Hamburg 2003.

Gellert, W., and others**,** *The VNR concise encyclopedia of mathematics*, 2. ed., New York 1989.(MA), 1990.

Gillies, D., ed., *Revolutions in Mathematics,* Oxford 1992.

Gilman, S, *Lope, dramaturgo de la historia,* in Romeralo, 181 - 191.

Gingerich, Owen, *The book nobody read,* New York 2004 (early publication/copying history of Copernicus' *De revolutionibus*).

Gisolfi, D., and **Sinding-Larsen, S**., *The Rule, the Bible and the Council. The Library of the Benedictine Abbey at Praglia,* Seattle and London 1998, College Art Association, Monographs, LV, ed. **Robert Nelson**.

Glenn, J., and Littler, G. (eds.), *A dictionary of mathematics,* London 1984.

Glymour, C., *Theory and evidence,* Princeton (NJ) 1980.

Godley, J. JCL, *Time and place for the celebration of Mass. Historical synopsis and commentary,* Washington DC 1948.

Góngora, Luis de, *Soledades,* ed. J. Beverley, Madrid 2002 (?).

González, Iván Fernández, *Letteratura spagnola,* Milan 2003.

Goodfield, J., and **Toulmin, S.**, *The Architecture of Matter,* Chicago 1962 (later reprints).

Grabiner, J. V., *Partisans and critics of a New Science: the case of Artifical Intelligence and some historical parallels,* in Asprey, W., and Kitcher, Ph.(eds.), *History and philosophy of modern mathematics, = Minnesota Studies in the Philosophy of Sceince,* Vol. XI, Minneapolis (MN) 1988, pp. 329 - 345.

Grant, Edward, *Le origini medievali della scienza moderna,* Turin 2001, transl. from *The foundations of modern science in the Middle Ages. Their religious, institutional and intellectual contexts,* Cambridge 1996.

Grattan-Guinness, I. (ed.), *From Calculus to Set Theory, 1630 - 1910,* Princeton (NJ) 2000.

Gray, J., *Ideas of Space. Euclidean, Non-Euclidean and Relativistic, 2nd. ed.,* Oxford 1989.

Grayling, A. C., *An introduction to philosophical logic,* London 1982.

Greenley, G. E., *Strategic management,* New York 1989.

Gregory, R. L., *Mind in Science. A History of Explanations in Psychology and Physics*, Harmondsworth, 1984 (originally London 1981, reprint 1988).

Grünbaum, B., and **Shephard**; **G. C.,** *Tilings and patterns,* New York 1987.

Guidubaldo del Monte, *Guidi Ubaldi e' marchionbus Montis Mecanicorum liber,* Venice 1615

Hacker, P. M. S., *Appearance and reality. A philosophical investigation into perception and perceptual qualities,* Oxford 1987.

Hale, B., *Abstract objects,* Oxf9ord 1987.

Hall, A. R., *From Galileo to Newton,* New York, 1981.

Harris, J. W., and Stocker, H., *Handbook of mathematics and computational science, New York 1998.*

Hatfield, G., *The natural and the normative. Theories of spatial perception from Kant to Helmholtz,* Cambridge (MA) 1990.

Heinemann, M., *Heinrich Schütz,* 2 ed., Hamburg 2005.

Heisenberg, W., *Der Teil und das Ganze. Gespräche im Umkreis der Atomphysik,* Munich 1969.

Heisenberg, W., *Quantentheorie und Philosophie,* ed. J. Busche, Stuttgart 2006 (A collection of articles by Heisenberg originally published in various fora).

Hentsch, T., *L'orient imaginaire. La vision politique occidentale de l'Est méditerranéen,* Paris 1988**.**

Hernández, D., ***Qualitative representation of spatial knowledge*** (*Lecture notes in artificial intelligence, 804),* Berlin 1994.

Hitchins, Derek K., *Putting systems to work,* Chichester 1992.

Hoffmann, B., *About vectors,* New York 1966 (orig. 1975).

Hogg, M. A., and Abrams, D., *Social ideintifications,.* A social psuchology of intergroup relations and group processes, London 1990 (reprint; orig. London 1988).

Holton, G., *Thematic origins of scientific thought: Kepler to Einstein,* rev. ed., Cambridge (MA), 1988.

Holton, G., *Science and anti-science,* Cambridge (MS) 1993.

Horgan, J., *The Undiscovered Mind. How the human brain defies replication, medication, and explanation,* New York 1999.

Hörmann, H., *Meaning and context. An introduction to the psychology of language,* transl. from German, ed. and intro. by R. E. Innis, New York 1986. (*Einführung in die Psycholinguistik).*

Howson, C., and Urbach, P., *Scientific reasoning. The Bayesian approach,* 2. ed., Peru (Il) 1993.

Hussey, M., *Pensare per modelli. Schemi logici e strumenti di calcolo,* ital. transl. of *Open University, The Man-made world,* Milan 1979.

Inmon, W. H., *Data architecture. The information paradigm,* 2nd. ed., Boston 1992.

Italiani, M., and **Serazzi, G.,** *Elementi di informatica. Algoritmi, architetture, strutture dati, linguaggi di programmazione, Pascal, sistemi operativi, Unix,* 2 ed., Milan 1989.

Iudica, G., *Il principe dei musici. Carlo Gesualdo di Venosa,* Palermo 1997.

Jaritz, G., *Zwischen Augenblick und Ewigkeit. Einführung in die Alltagsgeschichte des Mittelalters,* Cologne 1989.

Jakobsen, K., *Functional requirements in the design process,* in Jakobsen, K. (ed.), *Modern design principles in view of infrmation throry; with refference to aluminium in competition with other materials,* Trondheim 1988, pp. 41 - 53.

James, G. *et al.*, *Modern engineering mathematics* Wokingham, 1092.,

Jaritz, G., *Zwischen Augenblick und Ewigkeit. Einführung in die Alltagsgeschichte des Mittelalters,* Cologne 1989.

Johnson-Laird, P. N. *Mental models, towards a cognitive science of language, inference and consciousness,* Cambridge 1983.

Jungmann, J. A., *Missaruøm sollemnia. Eine genetische Erklärung der römischen Messe,* Vol. I/II, Freiburg i/Br, 1962.

Karlsch, R., *Hitlers Bombe. Die geheime Geschichte der deutschen Kernwaffenversuche,* Munich 2005.

Kenny, A., *Descartes. A study of his philsophy,* Bristol 1993.

Kepler, J., *Harmonices mundi,* Book Five, ed. and comment St. Hawking, Philadelphia 2002.

Kirsh, D., *Foundations of AI: the big issues,* in Kirsh (ed.), Foundations of Artificial Intelligence, Cambridge (MA), 1992, pp. 3ff.

Kitcher, Ph., *The nature of mathematical knwledge,* New York 1984.

Kline, M., *Storia del pensiero matematico, I. Dall'antichità al Settecento,* Turin (1991, 1996), 1999 (orig. *Mathematical thought from Ancient to Modern Times,* New York 1972).

Kline, M., *Calculus. An Intuitive and Physical Approach*, 2. ed., New York 1977.

Kline, M., *Mathematics. The loss of certainty*, Oxford 1980.

Kline, M., *Mathematics and the physical world*, New York 1981 (1959).

Kline, M., *Mathematics and the search of knowledge*, Oxford 1985.

Kolman, B., *Introductory linear algebra with applications,* 3. ed., New York 1984.

Körner, S., *The Philosophy of Mathematics. An Introductory Essay*, London 1968, Dover Publications, Mineola (NY) 1986.

Krakowiak, S., *Principles of operatng systems,* Cambridge (MA) 2nd printing 1989.

Lakatos, I., *Mathematics, science and epistemology,* in Worral and Currie (eds.), *Imre Lakatos. Philosophical papers*, Vol. 2, 1978 (later reprints).

Lakoff, G., and **Núñez, R. E.**, *Where mathematics comes from. How the embodied mind brings mathematics into being,* New York 2000.

Lang, P. H.. *George Frideric Handel*, New York 1966; edition cited here, Mineola (NY) 1996.

La Porta, G., *Giordano Bruno. Vita e avventure di un pericoloso maestro del pensiero*, Milan 1991 (later reprints).

Lattis, J. M., *Between Copernicus and Galileo. Christoph Clavius and the collapse of Ptolemaic cosmology,* Chicago 1994.

Laumann, E. O., and Pappi, F. U., *Networks of collective action. A perspective on community influence systems,* New York 1976.

Legouis, É, and **Cazamian, L.,** *A history of English literature, rev. ed.,* London 1930 (later reprints).

Levine, Howard, and **Rheingold, Howard,** *The cognitive connection. Thought and language in man and machine,* New York 1987.

Liber sacramentorum romanae aeclesiae ordinis anni circuli, Sacramentarium gelasianum (sevemth to eighth century), ed. Mohlberg, L. C., and Eizenhöfer, L., Rome, 1981.

Lindenberger, H. *Opera. The extravagant art,* London 1984.

Lindenberger, H., *Arts in the Brain: Or What Might Neuroscience Tell Us?, in Toward a Cognitive Theory of Literary Arts*, ed. Frederick Aldama (Austin: University of Texas Press (to be published in 2009).

Linhard, F., *Klassische Mekanik,* Frankfurt a/M 2002.

Lodi, E., *Enchiridion euchologicum fontium liturgicorum,* Rome 1979.

Lord, E. A. and Wilson, C. B., *The mathematical description of shape and form,* repr. New York 1986.

Loewer, B., and Rey, G.(eds.)**,** *Meaning in mind. Fodor and his critics,* Oxford 1993 (orig. 1991).

Lope de Vega, *Peribañez y el Comendadror de Ocaña,*, ed. Juan María Marín, Madrid 2003 (19th ed., first ed. 1979).

Lubac, H. de, *Corpus mysticum. L'Eucharistie et l'Eglise au Moye Age,* Paris 1949.

Luger, G. F., *Artificial intelligence. Structures and strategies for complex problem solving,* 5th ed., Harlow 2005.

Lütolf, Max, *Vorwort,* in Lütolf (ed.), *Alessandro Scarlatti und seine Zeit,* Bern 1995.

Malmanger, M., *Between Renaissance and Baroque: Attitudes to Nature and the Concept of Nature* (in Eriksen , 26 - 51)

Mancosu, P., *Descartes' Géometrie and revolutions in mathematics,* 1992, in Gillies, *Revolutions in Mathematics,* 83ff.

Mancosu, P., *Philosophy of mathematics and mathematical practice in the seventeenth century,* New York 1996.

Mania, H., *Gauβ*, Reinbek/Hamburg 2008.

Mansfield, M. and **O'Sullivan, C.**, *Understanding physics,* 2. rev. ed., Chichester 1999 **.**

March, J. G., *Bounded rationality, ambiguity, and the engineering of choice,* in Elster, J (ed.)., *Rational choice,* Oxford 1986, 142 - 170.

Massey, L., *Review* of J. V. Field, *The invention of infinity: Mathematics and art in the Renaissance,* in *The Art Bulletin,* 2001, No. LXXXII, pp. 564ff. Field's book is devoid of references and not considered in the present discusion. Massey's review worth reading.

Masterman, M., *The nature of a paradigm,* in Lakatos, I., and Musgrave, A. (eds.), *Criticism and the growth of knowledge,* Cambridge 1970, 59 - 89.

Méró, L., *I limiti della razionalità. Intuizione, logica e trance-logica,* Bari 2005, orig. *Új eszjárások. A racionális ereje és korlátai,* Budapest 2001.

Microsoft Computer Dictionary, 5th ed., Redmond (WA), 2002.

Miller, A. I., *Imagery in scientific thought,* Cambridge (MA) 1986, repr. 1987

Miller, A. I., *Insights of genius. Imagery and creativity in Science and art*, New York 1996.

Miniati, M., *Strumenti scientifi toscani,* in *Istituto e moseo di storia della scienza di Firenze,* exhibition *L'età di Galileo. Il s ecolo d'oro della scienza on Toscana,* Palazzo Castellani, 1. July 1987 - 9 January 1988, pp. 41ff..

Minsky, M. L., and Papert, S. A., *Perceptrons. An introduction to computational geometry,* expanded edition, Cambridge (MA) 1988.

Moore, G. T., and Gollege, R. G., *Environmental knowing, Theories, research and methods*, Stroudsburg (PA) 1976.

Morel, J., *Histoire de la literature française. De Montaigne à Corneille,* rev. edition, Paris 1997.

Morón, C., (ed.), Calderón de la Barca, *La vida es sueño,* 31 ed. (1977), Madrid 2008.

Niderst, A., *Pierre Corneille,* Paris 2006.

Nye, Mary Joe, *Before big science. The pursuit of modern chemistry and physics 1800 - 1940,* Cambridge (MA) 1996, reprint 1999.

Olle, T. W., and others, *Information systems methodologies. A framework for understanding*, 2 ed., Wokingham (England) 1991.

Ore, Øystein, *Number theory and its history,* Dover Publ., New York 1988 (orig. ed., New York 1948).

Ott, L-, *Grundriss der katholischen Dogmatik,* 8th. ed., Freiburg i. Br., 1970.

Parker, Ch. S., *Management information systems. Strategy and action,* New York 1989.

Pascal, B., *Pensées,* ed. Molinier (original French), Paris 1879.

Pascher, J., *Das liturgische Jahr,* Munich 1963.

Pedersen, K. Møller, *Techniques of the calculus,1630-1660,* in Grattan-Guinness, *From the calculus...,* pp. 10 - 48.

Penrose, R., *The emperor's new mind. Concerning computers, minds, and the laws of physics,* Oxford 1989.

Penrose, R., *Shadows of the mind. A search for the mssing science of consciuousness,* Oxford 1994.

Perner, J., *Understanding the represeentational mind,* Cambridge (MA) 1993 (orig. 1991).

Pettit, Ph., and McDowell, J., *Subject, thought and context,* Oxford 1986.

Pidal, R. M., *Lope de Vega. El arte nuevo y la nueva biografia,* in Romeralo, 89 - 144.

Pflieger, A., *Liturgicae orationis concordantia verbalia. Prima pars: Missale romanum,* Freiburg i/Br 1964.

Pogge, R. W., *The folly of Giordano Bruno,* www. giordano.bruno.info/folly.htm, no date.

Poland, J., *Physicalism. The philosophical foundations,* Oxford 1994.

Polkinghorne, J. C., *The Quantum world,* London 1984, paperback 1986.

Putnam, H., *Mathematics, matter and method,* 2nd. ed., Cambridge 1979 (orig. ed. 1975).

Putnam, H. *The meaning of meaning,* in *Mind, language and reality.* Cambridge 1975, pp. 223 - 227.

Putnam, H., *Reason, truth and history,* Cambridge 1981.

Pylyshyn, Z. W., *Coputation and cognition. Toward a foundation for cognitive science,* Cambridge (MA, 2nd. ed. 1985.

Quine, W. V., *From a logical point of view, Nine logico-philosohical essays,* 2nd. revised ed., Cambridge (MA) 1980.

Quine, W. V., *Theories and things,* Cambridge (MA) 1981.

Quine, W. V., *Philosophy of logic,* 2. ed., Cambridge 1986.

Radnitzky, G., *Contemporary schools of metascience,*I, II, Gøteborg 1968, reprint 1970.

Rahner, H., SJ, *Ignatius the theologian,* New York 1968, transl. from German, Freiburg i. Br., 1964.

Ramon, S., *The business of science. Winning and losing in the high-tech age,* New York 1988.

Rat, M. (ed. and introduction), *Théatre choisi de Corneille* (Le Cid, Horace, Polyeucte, a. o.), Paris 1950**.**

Reade J. B., *An introduction to mathematical analysis,* Oxford 1986.

Romeralo, A. S. (introd. and ed.) *Lope de Vega: el teatro - el escritor y la critica,* Madrid 1989 (an anthology of republished contributions)

Rossi, Paolo (ed.), *Dalla rivoluzione scientifica all'età dei lumi, 1, (Storia della scienza moderna e contemporanea,* ed. P. R.), Turin 2000 (orig. 1988). Contributions by Ferd. Abbri, Enr. Bellone, Walt. Bernardi, Umb. Bottazzini and Paolo Rossi.

Rossi, Paolo, *La nascita della scienza moderna in Europa,* Bari 1997.

Rossi, Paolo, *Il tempo dei maghi. Rinascimento e modernità, M*ilan 2006.

Russell P. E., (introd. and ed.), Fernando de Rojas: *La Celestina. Comedia o tragicomedia de Calisto y Melibea,* 3 ed., Madrid 2008.

Ryle, G., *The concept of mind.* London 1949, numerous late reprints, also in Penguin 1963 and later reprints there.

Sadie, J. A. (ed.), *Companion to Baroque Music,* Oxford 1998.

Saiber, A., *Giordano Bruno and the geometry of language*, London 2005.

Salmon, W. C., *Space, time and motion. A philosophical introduction,* 2. ed., Minneapolis 1982.

Salmon, W. C., *Scientiific explanation and the causal structure of the world,* Princeton (NJ) 1984.

Schneider, H., and Barker, G. Ph., *Matrices and linear algebra, New York 1973.*

Schröder, W., and Hans-Juergen Treder, H.-J., *Abschied von der Weltformel, www.verplant.or/history-geophysics/heisenberg.htm*

Schulkin, J., *The pursuit of inquiry,* Albany 1992.

Scott, W. R., *Organizations. Rational, natural and open systems,* 3 ed., Englewood Cliffs (NJ) 1992.

Seiffert, H., and **Radnitzky, G.** (eds.), *Handlexikon zur Wissenschaftstheorie,* Munich 1992.

Selleri, Franco, *Die Debatte um die Quantentheorie*, 3rd. revised ed., Braunschweig 1990, a German edition of the original manuscript in English, reworkd by Selleri and Roman U. Sexl.

Shapin, S., *The Scientific Revolution*, Chicago 1996.

Shilov, G. E., *Linear algebra, New <york 1977 (transl. from Russian).*

Sharratt, M., *Galileo. Decisive Innovator,* Cambridge 1994.

Silverman, D., *The theory of organizations. A sociological framework*, London 1970: (later reprints)

Simon, H. A., *Models of thought*, New Haven (CT) 1979.

Simon, H. A., *Reason in Human Affairs,* Stanford (CA) 1983.

Simon, H. A., *The Sciences of the Artificial*, 3rd. ed., Cambridge (MA) 1996.

Sinding-Larsen, S., *Some finctional and iconographical aspects of the centralized church in the Italian Renaissance*, Ist. rom. Nor., Acta, II, 1965, pp. 203 - 252.

Sinding-Larsen, S., *A re-reading of the Sixtine Ceiling*, Ist. rom. Nor., Acta, IV, 1969, pp. 143 - 157. Reprint in Wallace, William E, *Michelangelo. Selected scholarship*, 2, The Sistine Chapel, New York 1998, and in the later one-volume edition.

Sinding-Larsen, S., *Christ in the council hall. Studies in the religious icnography of the Venetian Republic,* Rome 1974.

Sinding-Larsen, S., *A tale of two cities. Florentine and Roman visual context for fifteenth-century palaces,* in *Institutum romanum Norvegiae, Acta,* Vol.VI, 1975, pp. 163 - 212.

Sinding-Larsen, S, *Some observations on liturgical imagey of the twelfth centuy,* Inst. rom. Nor., Acta, VIII, Rome 1978, pp. 193 - 212.

Sinding-Larsen, S., *Le tecniche antiche e palladiane nel trattamento della pietra secondo i Quattro Libri,* in *Centro Internazionale di Studi di Architettura "Andrea Palladio", Quaderni***,** 1979, pp. 211 - 221**.**

Sinding-Larsen, S., *Iconography and ritual,* Oslo/Oxford 1984.

Sinding-Larsen, S., *Art history and the role of the artist. Three case histories,* Pratt Institute, New York 1989 (1. *Borromini's Sant'Ivo spire: must a work of art mean something?*).

Sinding-Larsen, S., Paolo Veronese tra rituale e narrativo. Note a proposito di un disegno per il Palazzo Ducale, **Gemin, M.** (ed.), *Nuovi studi su Paolo Veronese* (Venice conference, 1988), Venice 1990, 36 - 41).

Sinding-Larsen, S., *Créer des images. Remarques pour une théorie de l'image, in* Dunand, F., Spieser, J.-M., and Wirth, J. (eds.), *L'image et la production du sacré. Actes du colloque de Strasbourg (20-21 janvier 1988),* Paris 1991, pp. 35 - 76. Translated from English without my knowledge; my transscriptions in Arabic have become unreadable.

Sinding-Lartsen, S., *Categorization of images in ritual and liturgical contexts, Cahiers du Léopard d'Or,* Vol. 5, 1996, 109 - 128.

Sinding-Larsen, **S**., *The burden of the ceremony master. Image and action in San Marco, Venice, and an Islamic mosque,* Rome 2000. Review by **Ruth Simon/Schilling**, Institut für Geschichtswissenschaften, Humboldt-Universität, Berlin: http://www.h-net.org/reviews/showrev.cgi?path=91381022723569.

Sinding-Larsen, S., Teori og praksis for avhandlinger, Trondheim 2001 (in Norwegian: *Theory and practice for dissertations*).

Sinding-Larsen, S., *Between light and darkness. Caravaggio, Bruno and scientific convergence*, in C. L. Guest (ed.), *Rhetoric, theatre and the arts of design, Essays presented to Roy Eriksen* , Oslo 2008, 172 - 192. The article was subjected to incompetent editing. No proofs were sent me, so only too late did I discover that the reference system had been messed up (CA tuned into Calif, etc; titles added from the Author Presentation list; an Italian book title "corrected"; my honorary membership in the Ateneo Veneto makes me "a citicen of Venice"; more seriously, a book title that I never had seen (Donne, 1971) was added in the text - but left out in the bibliography.

Sinding-Larsen, S., WORKING **=** *Working with pictures in elaborated systems,* in **Jarits, G. (ed.),** *Image, ritual and daily life,, Institut für Realienkunde des Mittelalters und der frühen Neuzeit, Austrian Academy of Sciences,* forthcoming.

Sinding-Larsen, S., *Inspecting math in historical buildings,* forthcoming.

Smith, D. E., *History of mathematics,* Vols. I/II, New York resp. 1951/53 (orig. 1923/25).

Smith, E. E., and Medin, D. L., *Categories and concepts,* Cambridge (MA) 1981.

Smolensky, P., *Connectionism, constituency, and the language of thought***,** pp. 201ff. in Loewer, B. and Rey, G., eds., *Meaning in mind. Fodor and his critics, Oxfoird* 1991.

Souiller, D., *La litérature baroque en Europe,* Paris 1988.

Sowa, J. F., *Conceptual structures. Information processing in mind and machine,* Reading (MA) 1984.

Spampanato, V., *Vita di Giordano Bruno,* two volumes in one, Messina 1921, reprint Rome 1988.

Spiegel, M. R., and **Wrede, R.**, *Theory and problems of advanced calculus,* 2nd. ed., New Yok 2002.

St. Andrews, University of, Mathematical information: http://www-history.mcs.st-andrews.ac.uk/history/curves/

Sterelny, K. *The representational theory of mind. An introduction,* Oxford 1990.

Sturm, Günter, *<Die> heisenbergsche Unschärferelation, www.quanten.de/einung_schroedinger_katze.html*

Struik, D. J. (ed.), *A source book in mathematics 1200-1800,* Princeton (NJ), 1969 and later editions.

Tenenbaum, M., and **Pollard, H.**, *Ordinary differential equations,* Dover Publications, New York 1985 (original ed. New York 1963).

Teresa de Jesús (de Ávila), *Libro de la vida*, ed. Dámaso Chicharro, 14th ed., Madrid 2006.
Authorized Italian translation: *S. Teresa di Gsú, Opere*, ed. Postulazione Generale O. C. D., 5. ed., Rome 1069

Thürkauf, M., *Sackgasse Wissenscvhaftsgläubigkeit. Zur Überwertung der exact-naturwissenschaftlichen Betrachtungsweise durch die Errfolge der Technik,* 2.ed., Zürich 1976 (orig. 1975).

Finney, R. L., and **Thomas, G. B.,** *Calculus,* Reading (MA), 1990..

Torricelli, Evangelista, *Opera geometrica Evangelistae Torricelli,* Florence 1644. I, i. 223ff.: *De dimensione cochleae* (*see Supplement to this book*, 6.1).

Torricelli, Evangelista, *Opere,* ed. G. Loria and G. Vassura, 3 vols., Faenza 1919.

Toscano, F., *L'erede di Galileo. Vita breve e mirabile di Evangelista Torricelli,* Milan 2008 (thanking Camilla S.-L. for finding this book, Xmas 2008).

Trehub, A., *The cognitive brain,* Cambrdge (MA) 1991.

Trexler, R. C., *The Christian at prayer. An illustrated manual attributed to Peter the Chanter (d. 1197),* Binghamton (NY), 1987.

Truss, J. K., *Discrete matgematics for computer scientists,* Wokingham 1991.

Vescovini, G. F., "Arti" e filosofia nel secolo XIV. Studi sulla tradizione aristotelica e i "moderni", Florence **1983.**

Vision, G., *Modern abti-realism and manufactured truth,* London 1988.

Viviani, Vincenzio, Vita di Galileo [= *Racconto istorico della vita del sig.r Galileo Galilei academico linceo, nobil fiorentino, primo filosofo e matematico dell'altezze ser.me di Toscana,* (1654)], ed., comments and references by **Bruno Basile**, Rome 2001 (this small-sized book is extraordinarily useful because of Basile's highly informed, careful and extensive references in the 266 notes appended to the text).

VNR, *see* Gellert.

Weinberg, S., *Dreams of a final theory,* London 1992 (orig. ed. 1988).

Weizsäcker, C. F. von, *Aufbau der Physik,* Munich 1988.

Whenham, John, *Monteverdi. Vespers (1610),* Cambridge 1997.

White, M., *Newton. L'ultimo mago,* Milan 2001, trasnl. of *Newton. The last sorcerer,* 1997.

Whitehurst, G. J., and Zimmermann, B. J. (eds), *The functions of language and cognition,* New York 1979.

Wilden, A., *System and structure. Essays in communication and knowledge***,** 2 ed.**,** New York 1980.

Williams, Bernard, *Descartes. The project of pure enquiry,* Harmondsworth 1978.

Wilson, Cath., *The invisible world. Early modern philosophy and the invention of the microscope,* Princeton (NJ) 1995.

Wilson, B., *Systems: Concepts, Methodologies and Applications,* 2nd. ed., New York 1990.

Winkler, P., *Computer-Lexikon 2007. Die ganze digitale Welt zum Nachschlagen,* Munich 2007 (1050 pages). *Note:* This is an exceptionally well-structured and rich dictionary, by far surpassing the Microsoft one (635 pages). In consideration of the poverty of Germnan use in the English-speaking world (I even have come across art historians not familiar with the language: how is that possible?), someone should translate it or the subsequent edition.

Winograd, T. (ed.)**,** *Bringing design to software,* Boston (MA) 2006 (orig. 1996).

Winograd, T., and Flores, F., *Understanding computers and cognition. A new foundation for design,* Reading (MA) 1986.

Wrede, R. C., and **Spiegel, M.**, *Advanced calculus,* 2. ed., New York 2002.

Zornetzer, S. F., Davis, J. L, and Lau, C. (eds.), *An introduction to neural and electronic networks,* New York 1990.

FigsStorage

Sinding-Larsen, *Patterns*.

Illustrations essential for the argumentation replicated here. It is recommended that the file is printed out.

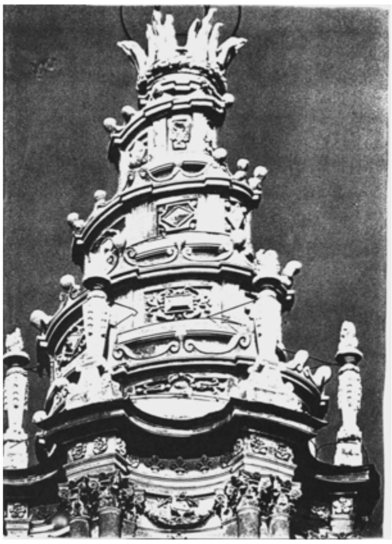


Fig. 1, Introduction. Closeup of Borromini's Spire on Sant'Ivo. For an overview, see Fig. 1.1 - 1).

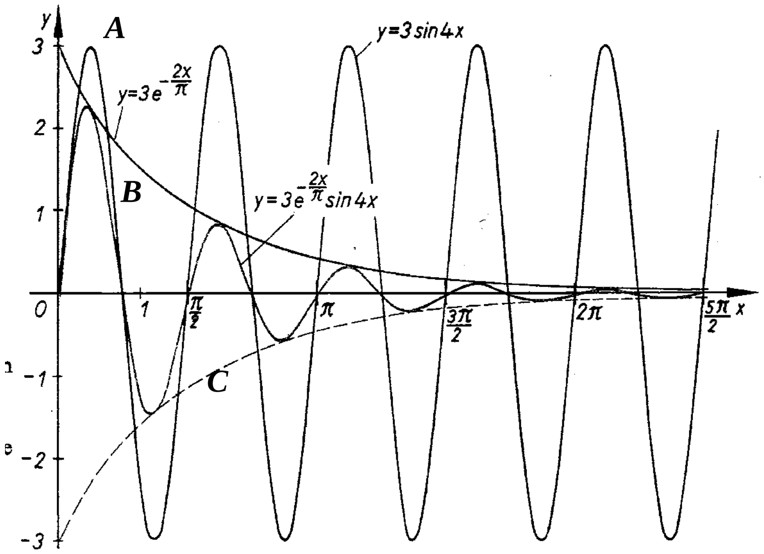


Fig. 0.7 - 1 Oscillatory motion, normal and damped; loss of energy causes the amplitude to decrease, illustrating a concept pattern.

Fig. 1.2 - 1 Calling of St. Matthew, San Luigi dei francesi, Rome (Caravaggio).

Fig. 1.3 - 1 Bruno: diagram

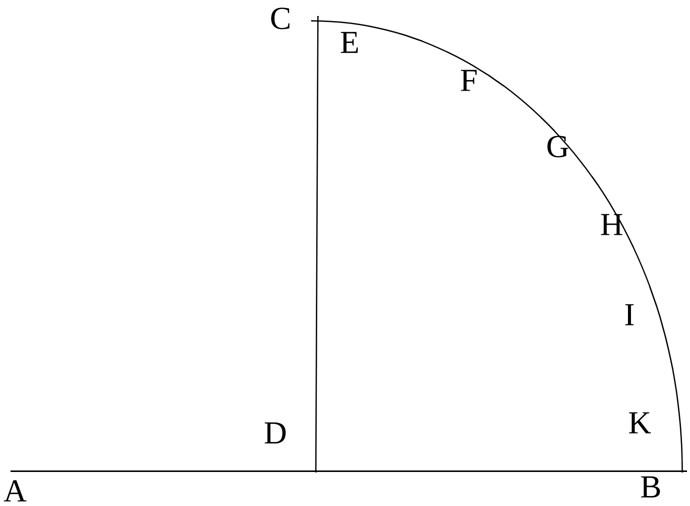
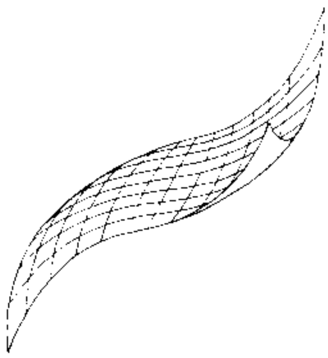


Fig. 1.5 - 1 Sant'Ivo, interior; transition from main body up to the cupola, from convex to concave (in Norwegian but hardly difficult).

Fig. 1.5 - 1Comment, Manifold (Lord and Wilson).



e

d
r
u
m

a

F

F

F*ig. 1.5 - 2 San Carlo alle quattro fontane, interior (Borromini) (Phot. Liv S.-L.).*

Fig 1.5 - 3, San Giovanni in Laterano, Funerary monument for Cardinal Acquaviva.

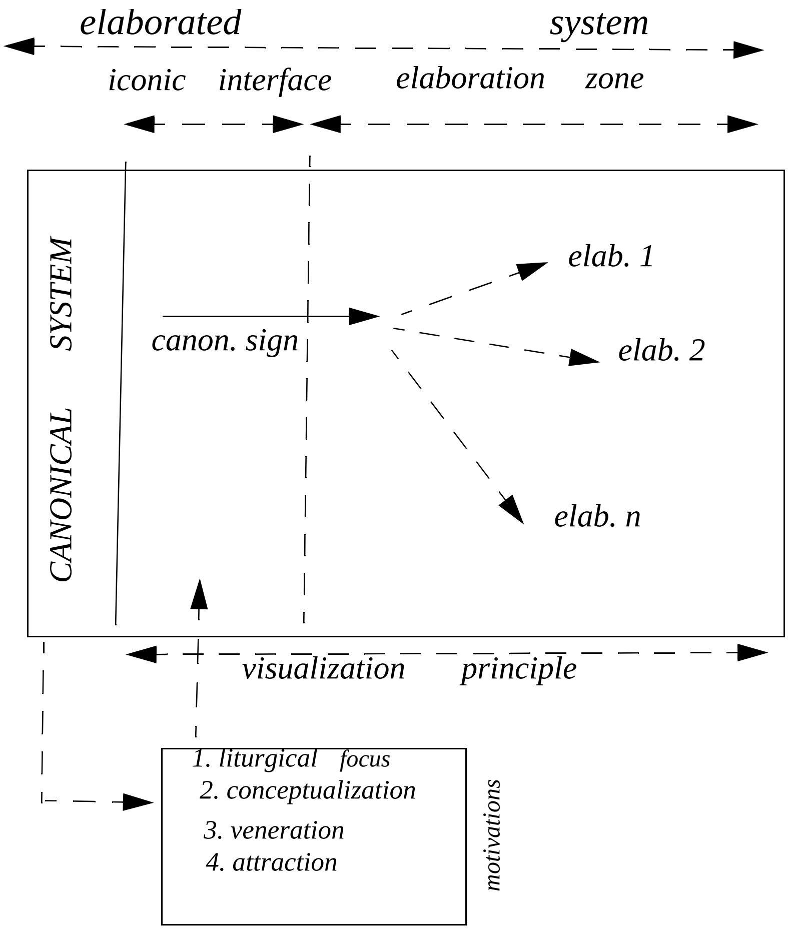
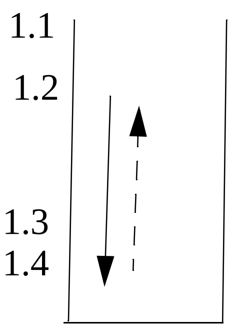


Fig. 1.7 - 1 Process model with inputs- output (no info model, i. e., no feedbacks.)



1. TOP-DOWN *Fig. 1.11 - 1* (in three sections)

1.1 *General theory*

-conceptions of truth, reality, objectivity

- scope, validity, coverage by analysis tools

- relations obtaining between observation and models/pictures/description

The upwards heading arrow indicates normal shuttle between observation and theory development

1.2 *Assignment theory*

- purpose, aims

- argumentation issues, models, analysis methods and tools (such as using a *p_matrix; see below*).

1.3 *General framework*

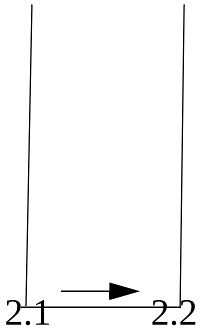
- design and mathematics under one compass.

1.4. *Assignment framework*

interrelations design - science - general culture - religion (the Church)

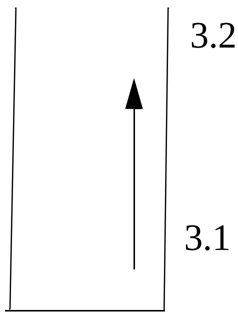
2**.** GROUND LEVEL

development of an *information model* (4.3.5) from the above. further developing toward an *output model* (next).



2.1 *Info model (see Fig. 4.3 -3)*
- Spire hardware with some obvious, traditional operations on it, put in *Storage.*
- *input* from top-down.
- *output* -> cited figure.
2.2. *Conceptual structure* (4.4.1) *(see Fig. 4.4 - 1)*
- Distinction between Spire *solid_state, Fluid_state*, and *sh*ape and form, location and interrelations between them.
1. *Spire_hardware*: the physical, built object itself;
2. *Spire_solid_state*: selected hardware, abstraction of its shape and quantification of it into a mathematical form; that is, subdivided into shape and form;
3. *Spire_fluid_state*: cognitive, conceptual and ideological attributions, regarding the object itself and its use**.**
- Articulating Spire structure in preparation for context evaluation.

The relevant graphic (4.4 and 4.5) subdivides the Spire structure in the two main states of *solid* and *fluid,* and subordinated here, in what is formally definable and what must be considered as informal; under these two categories, *form* and *shape*. At this point, the graphic under 2.2, i. e., *Fig. 4.4 - 1,* can only be considered as a rough summary of the concept structure (and the inserted numbers from 1 to 6 are for later reference).



3. BOTTOM-UP
3.1 *Object-in -context* (3.3, *see Fig. 3.3 - 1)* The graph in 3.3 presents a survey of the Spire's main conceptual linkages. Its main features are arranged on a diagonal, and this is set into its historical context in three interrelated field: the Church, Science and "lore" in general.
- Articulating interrelations in Spire's configuration space.
3.2 *Probability matrix (p_matrix)*
Discussion in 4.5.1.

End Fig. 1.1 1 - 1.

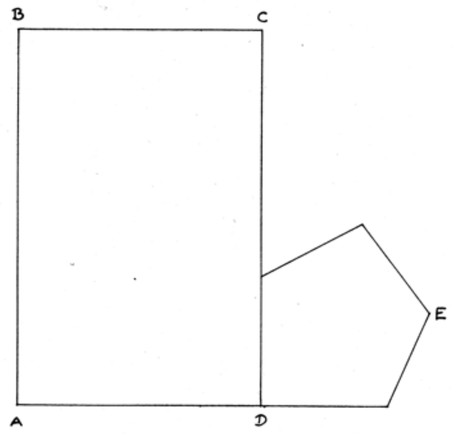


Fig. 3.2 - 4 Torricelli 1(redrawn by Camilla Sinding-Larsen)

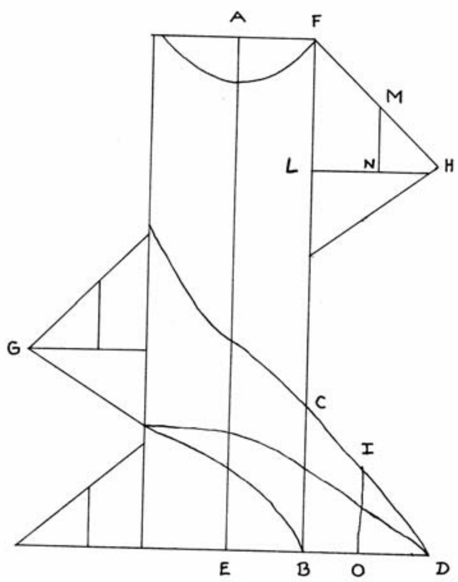


Fig. 3.2 - 5 Torricelli 4 redrawn by Camilla Sinding-Larsen)

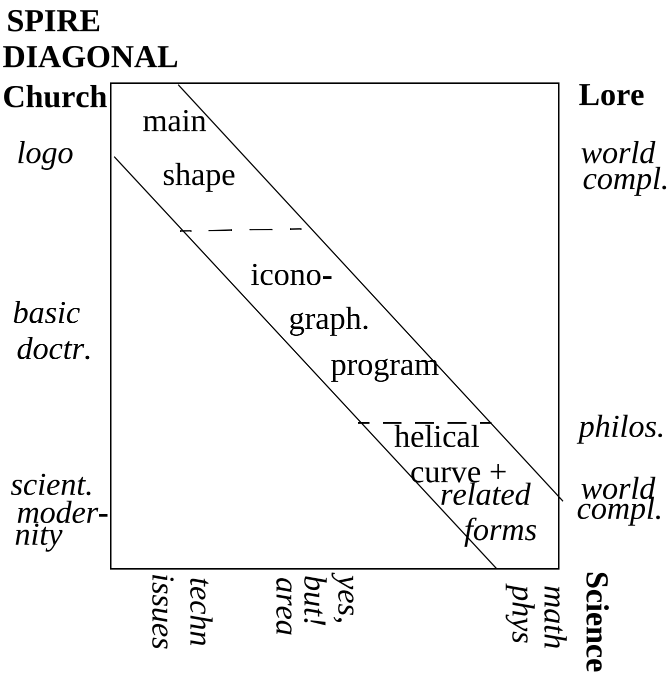


Fig. 3.3 - 1 Spire-in-context. A system model.

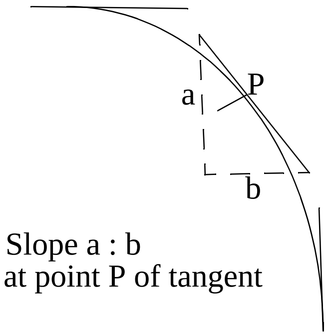
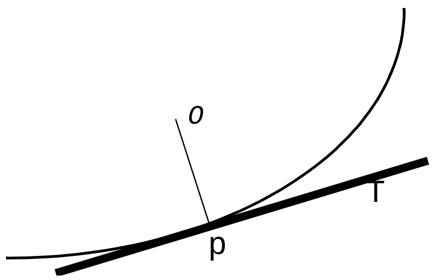


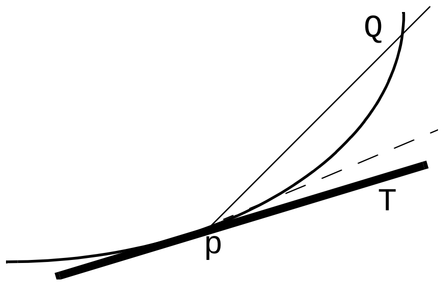
Fig. 3.3 - 2 Three tangents to the same curve, one passing the point P marked off with an orthogonal

Phase one



Figs. 3.4 - 2 and 3 (below): The same curve, first with a tangent T and an orthogonal O setting off the point P touched by it; next, with an increment Q - P, Phase One and Two respectively.

Phase two



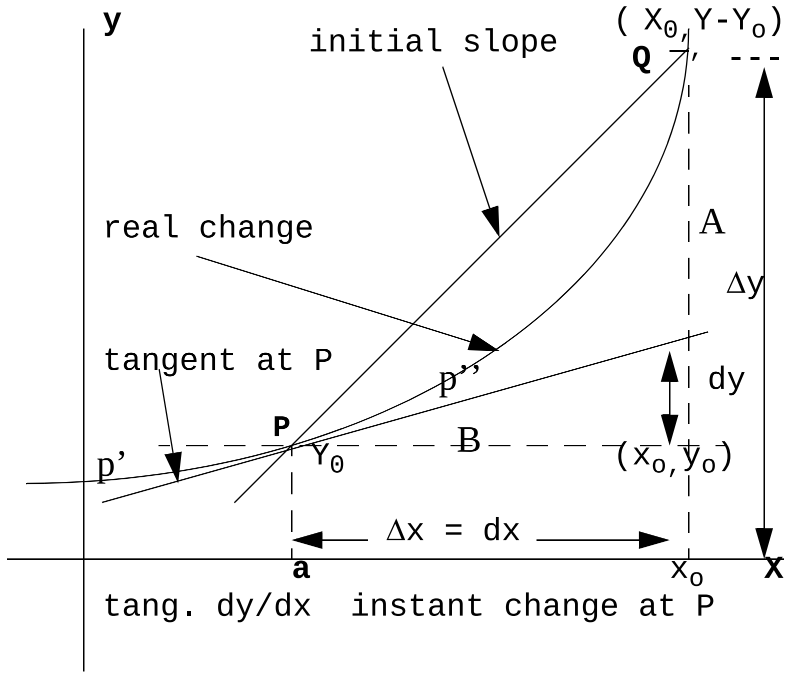


Fig. 3.4 - 4. Slope initially determined by the increment, Q - P gradually approaching P as the increment dwindles toward (not to) zero; then the resulting slope, degree of inclination, is the derivative.

$$\lim_{E \to 0} \frac{f(x+E) - f(x)}{E}$$

Fig. 3.4 - 5. Equation defining a limit. E: the increment added to the original function f(x): Δx = dx. Diminishing this, by letting dx approach, never reach zero, the slope becomes the tangent to the critical point.

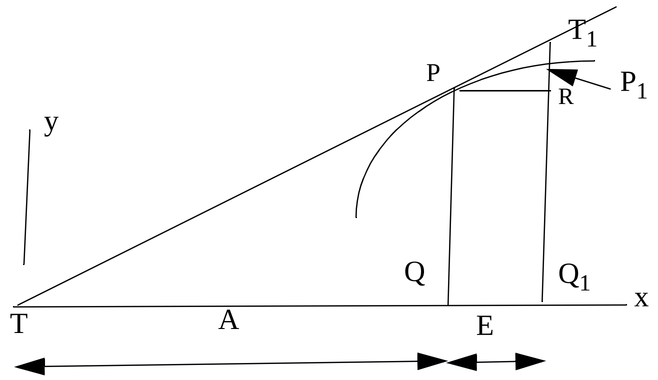


Fig. 5.2 - 2 Tangent construction

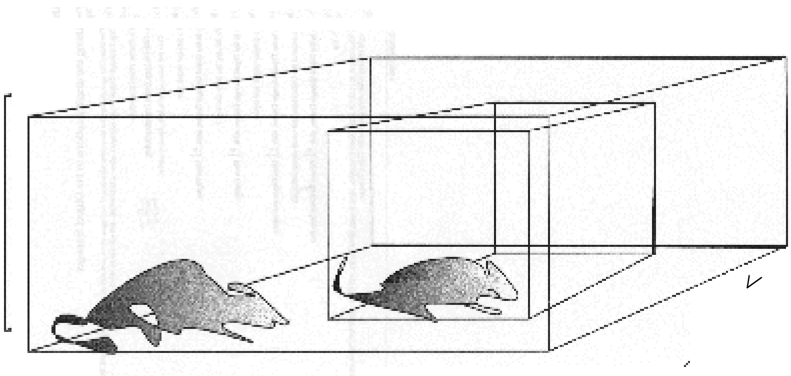


Fig. 4.1 - 1 Two rats in their respective cages or frameworks. AR in the big, OR in the small cage.

(Next Fig. with nut not copied her)

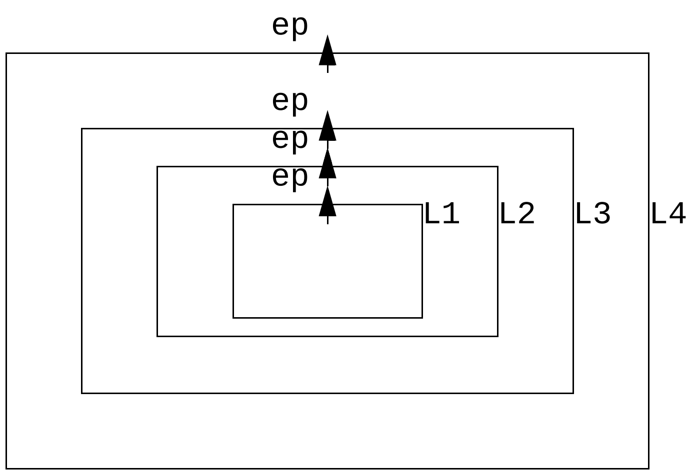


Fig. 4.3 - 2. System of Emergent Properties (after Hitchins). Four levels from each of which properties are "emerging"

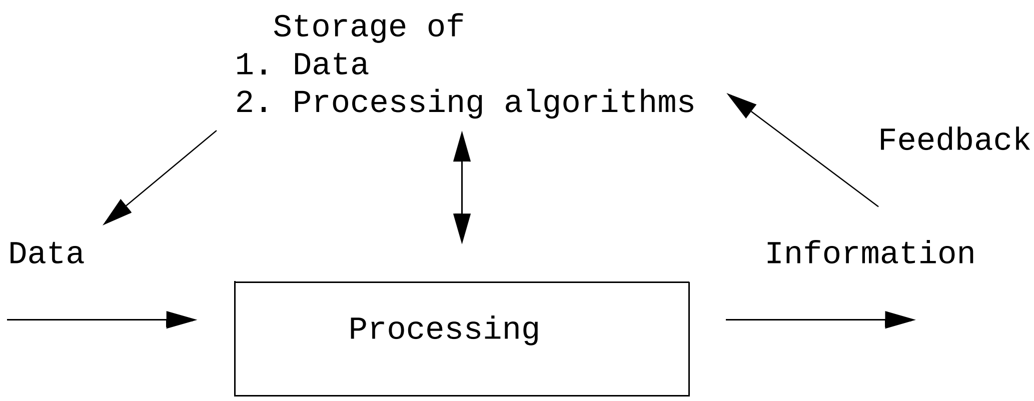


Fig. 4.3 - 3 Information model. To the original model by Davis and Olson I have added feedback mechanisms and specified the contents of storage, adding processing algorithms, since I believe methodology also would be stored for repeated use. Indeed, a cache should also really have been included. Still, the model is much simplified, omitting entropy (for a survey of this, see Fischer, 2007, passim).

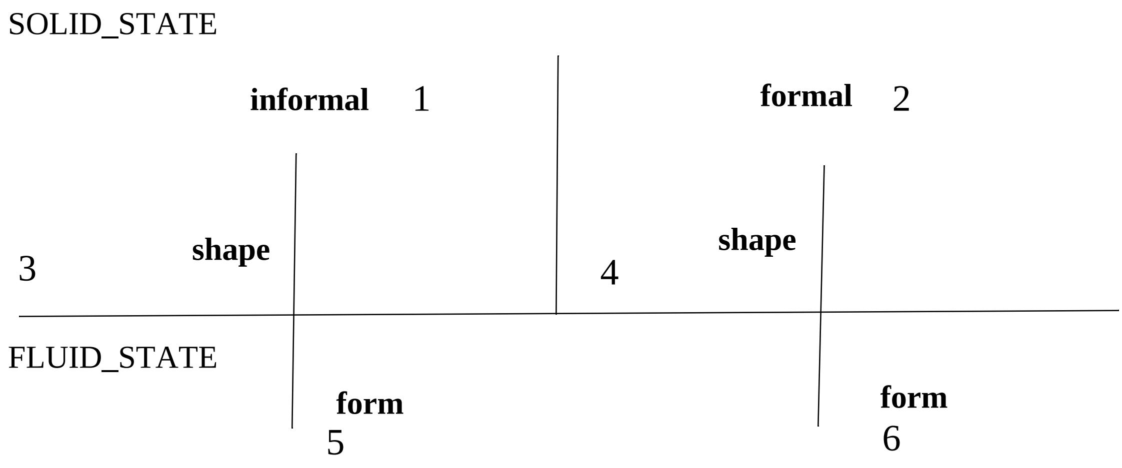


Fig. 4.4 -1. Model of Spire structure; model structure and elements assessable on a probability pattern; informal: non-definite attributions; formal: definite by law (Cath. Church) or rule set (mathematics).
*The helical thread (*2) is in the primary focus in this book, since it is on this curve that connections to contemporary mathematics can be discussed* (*chapter* 3.4).

References in the model.
- **items**
- *1 Spire body*
- *2 helical thread*
- *3 external appearance of Spire body (conical helix with upwards increasing torsion)*
- *4 imagery configurations established in canonical signs (see comments on Fig. 1.7 - 1) (cross, flames etc.)*

- **operations**
- *5 conical helix is described verbally or in elementary geometry for its outline features*
- *6 conical-helical curvature with increasing torsion, handled in advanced math, such as some protocalculus or geometrical technique.*
- *6 systems-level (chapter 1.6) handling of iconography*

$$\begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix}$$

Fig. 4.4 -2 A regular matrix
