An Output Feedback Controller with Improved Transient Response of Marine Vessels in Dynamic Positioning *

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Abstract: An output feedback controller for dynamic positioning (DP) of marine surface vessels is developed. The proposed algorithm has good performance during transients as well as good steady state performance. The method achieves this by a flexible injection gain in the bias estimation dynamics in the observer. In addition, the traditional integral action is replaced by a filtered bias estimate from the observer. Both these elements combined provide good DP performance in transients, as well as calm behavior in steady state. A simulation study is performed showing the benefit of the proposed output feedback controller, and a stability analysis is performed to show uniform asymptotic stability.

Keywords: Dynamic positioning; Observers; Output feedback; Integral action

1. INTRODUCTION

A surface vessel performing dynamic positioning (DP) has to keep position and orientation (stationkeeping) or do low speed tracking while compensating for the slowly-varying loads that affect the vessel. These loads are typically due to current, mean wind loads, and second order wave loads. The sum of these loads together with unmodeled dynamics, is lumped into the bias load vector. For modelbased observer designs it is important to estimate this bias in order to achieve good estimation of the velocity, and thereby the position of the vessel. In addition, this bias load needs to be compensated in the controller to keep the desired position. This is typically achieved through integral action in the control law.

In standard model-based observer designs (Fossen, 2011), the tuning of the bias observer is set low to ensure good performance of the observer in steady state. Since the bias is typically slowly-varying, low tuning will lead to less oscillations in the bias estimate, and therefore also less oscillations in the velocity and position estimates. However, when there is a significant transient in the bias force, for instance by a heading change, a wave train, or a mooring line that breaks (for position mooring), the bias estimate will take some time to converge to the new value. This is problematic for transient performance of the DP system, since the velocities will not be estimated correctly over the course of the transient.

The objective of this paper is to construct a model-based observer and controller with good performance in both transients as well as in steady state. This will be achieved by two changes from the standard model-based design. The first is to allow for a flexible bias estimation in the observer. The injection gain in the bias dynamics will be allowed to take values ranging from a nominal gain matrix to higher gains and a more aggressive tuning. The second contribution is to add a lowpass-filtered bias estimate which has a less oscillatory and smoother characteristics than the direct bias estimate. This filtered estimate will be used to compensate for the bias in the controller. There are two reasons for this implementation. From the literature, the two existing options for compensating the bias is to either use the bias estimate from the observer (Loría and Panteley, 1999), or to add integral action in the controller (Sørensen, 2011). The integral action in the controller finds the bias estimate based on the tracking errors. Since the control performance depends on the convergence of the observer, it is reasonable to believe that the bias estimate in the observer will always be faster than the integral action based on tracking errors (with reasonable tuning).

However, if we use a filtered version of the bias estimate, we allow for fast bias convergence in the observer, without having to send this noisy estimate directly to the controller. At the same time the bias compensation term in the controller is oscillating less than the direct bias estimate itself, and this is most likely faster than integral action based on tracking errors. This is a similar idea as used in L1 adaptive control (Hovakimyan and Cao, 2010).

In addition, there is a tuning benefit of using the bias estimate from the observer, both because tuning an ob-

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server does not require the system to be in closed loop, and because tuning of integral action (on tracking errors) heavily depends on how fast the observer estimates converge. This is especially beneficial in the current design, since the proposed observer have time-varying gains.

Similar use of time-varying gains is present in the literature. See for instance Tutturen and Skjetne (2015) where hybrid integral action for DP of marine vessels is proposed, and Lekkas and Fossen (2014) where the authors propose to use a time-varying lookahead distance as a function of the cross track error in a line-of-sight algorithm. In Belleter et al. (2013, 2015) a wave encounter frequency estimator is proposed, where the frequency adaption law has a timevarying gain. In Bryne et al. (2014) time-varying gains are proposed for an inertial observer (aided by GNSS) for DP, in order to improve convergence and suppress sensor noise.

2. PROBLEM FORMULATION

In the following we will separate between a simulation model and a control design model. The simulation model has higher fidelity and is used for simulation and verification of observer and control designs. Because of the low-speed nature of the dynamic positioning operations, the control design models typically neglect centripetal and Coriolis terms, as well as nonlinear damping; see (Sørensen, 2005, 2011), and (Fossen, 2011). The control design model considered here is a horizontal motion 3 degree of freedom (DOF) model, with the dynamics

$$\xi = A_w \xi + E_w w_w \tag{1a}$$

$$\dot{\eta} = R(\psi)\nu \tag{1b}$$

$$b = w_b \tag{1c}$$

$$M\dot{\nu} = -D\nu + R(\psi)^{\dagger}b + u \tag{1d}$$

$$y = \eta + C_w \xi + v_y, \tag{1e}$$

where $\xi \in \mathbb{R}^6$ is the state of a synthetic white noise-driven model of the vessel motion due to the 1st order wave loads. In normal operating conditions it is beneficial to counteract the low frequency part of the wave motion only, and the model therefore consists of a wave model (1a) and a low frequency part (1b) - (1d), which consists of the low frequency position in north and east, as well as the heading angle, $\eta := [N, E, \psi]^\top \in \mathbb{R}^3$, the velocities in surge, sway, and the yaw rate, $\nu := [u, v, r]^\top \in \mathbb{R}^3$, the slowly varying NED-fixed bias force $b \in \mathbb{R}^3$ that constitutes the sum of all slowly-varying perturbation loads, such as current, mean wind, 2nd order waves, and unmodeled dynamics. In (1b) the kinematic relation is described by the 3 DOF rotation matrix from the body to the NED frame $R(\psi) \in \mathbb{R}^{3\times 3}$,

$$R(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0\\ \sin(\psi) & \cos(\psi) & 0\\ 0 & 0 & 1 \end{bmatrix},$$
 (2)

and the time derivative of $R(\psi)$ is given by $\dot{R} = rS$, where

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
 (3)

and $r = \dot{\psi} \in \mathbb{R}$ is the yaw rate. In (1d), $M \in \mathbb{R}^{3\times 3}$ is the inertia matrix including added mass, $D \in \mathbb{D}^{3\times 3}$ is the linear damping matrix, and $u \in \mathbb{R}^3$ is the control input vector. The measurements $y \in \mathbb{R}^3$ in (1e) measure the actual position of the vessel, that is, the sum of the low frequency and wave frequency position, where $C_w = [0 \ I] \in \mathbb{R}^{3 \times 6}$, and $v_y \in \mathbb{R}^3$ is the measurement noise.

The control objective of the paper is to construct an output feedback tracking controller for DP, that has good performance in both steady state as well as in transients. This output feedback controller will track a reference trajectory given by an open-loop reference system (Sørensen, 2011).

Below are some assumptions relevant for the observer and control design.

Assumption 1. Starboard/port symmetry, $M = M^{\top} > 0$, and $\dot{M} = 0$. The damping matrix satisfies $D + D^{\top} > 0$. Assumption 2. Because of physical limitations of the thrusters, the yaw rate is bounded, by $|r| \leq r_{max} < \infty$.

3. OUTPUT FEEDBACK DESIGN

3.1 Model-based observer

The model-based observer considered is similar to the traditional "nonlinear passive observer" presented in Fossen and Strand (1999) with an additional state \hat{b}_f , which is a lowpass-filtered version of \hat{b} . By copying the dynamics of (1), neglecting the noise terms, and adding injection terms we get the observer dynamics as

$$\hat{\xi} = A_w \hat{\xi} + K_{1,\omega} \bar{y} \tag{4a}$$

$$\dot{\hat{\eta}} = R(\psi)\hat{\nu} + K_2\bar{y} \tag{4b}$$

$$b = K_3 \bar{y}$$
 (4c)

$$\hat{b}_f = -T_f^{-1}[\hat{b}_f - \hat{b}]$$
 (4d)

$$M\dot{\hat{\nu}} = -D\hat{\nu} + R(\psi)^{\top}\hat{b} + u + K_4 R(\psi)^{\top}\bar{y} \qquad (4e)$$

$$\hat{y} = \hat{\eta} + C_w \hat{\xi}, \tag{4f}$$

where $\hat{\xi} \in \mathbb{R}^6$, $\hat{\eta}, \hat{b}, \hat{b}_f, \hat{\nu} \in \mathbb{R}^3$ are the state estimates, $K_{1,\omega} \in \mathbb{R}^{6\times 3}, K_2, K_3, K_4 \in \mathbb{R}^{3\times 3}$ are non-negative gain matrices, and $\bar{y} = y - \hat{y}$ is the measurement error. The underlying assumptions for the observer are:

Assumption 3. (a) $R(\psi + \psi_w) \approx R(\psi)$. That is, the heading angle due to wave-induced motion is small.

(b) The frequency used in the wave filter does not change. It corresponds to the peak frequency of the wave spectra of the incoming sea state.

By defining the estimation error states $\bar{\eta} := \eta - \hat{\eta}$, $\bar{\nu} := \nu - \hat{\nu}$, $\bar{b} := b - \hat{b}$, $\bar{b}_f := b - \hat{b}_f$, and subtracting the observer equations (4) from the control design model (1), we get the observer error system,

$$\bar{\xi} = A_w \bar{\xi} - K_{1,\omega} \bar{y} \tag{5a}$$

$$\dot{\bar{\eta}} = R(\psi)\bar{\nu} - K_2\bar{y} \tag{5b}$$

$$\bar{b} = -K_3 \bar{y} \tag{5c}$$

$$\bar{b}_f = -T_f^{-1}[\bar{b}_f - \bar{b}] \tag{5d}$$

$$M\dot{\bar{\nu}} = -D\bar{\nu} + R(\psi)^{\top}\bar{b} - K_4 R(\psi)^{\top}\bar{y}.$$
 (5e)

3.2 Varying bias gain

To improve the transient response, we want the injection gain K_3 in (4c) to vary. In steady-state it is desired that K_3 stays close to a nominal gain such that the bias estimate is calm. Whenever the vessel experiences transients, K_3 should increase to make the bias estimate more reactive, and when the vessel again reaches steady state, the gain should return to the nominal gain. To solve this, K_3 is allowed to take a range of values within $K_3(t) \in$ $[K_{3,min}, K_{3,max}], \forall t \geq 0$. We let $K_3(t) := \kappa(t)K_{3,min}$, where $\kappa(t) \in [\kappa_{min}, \kappa_{max}], \forall t \geq 0$. The update law for κ is given by

$$\kappa = \max\{1, \beta\},\tag{6a}$$

$$\beta = \min\{\varepsilon_{r_d}|r_d(t)| + \varepsilon_\eta |\bar{\eta}_f| + \kappa_{max} e^{-\varepsilon_t t}, \kappa_{max}\}, \quad (6b)$$

$$\dot{\bar{\eta}}_f = -T_{\eta_f}^{-1} \{ \bar{\eta}_f - \bar{y} \}.$$
 (6c)

The first term in (6b) contains a constant $\varepsilon_{r_d} \geq 0$ and the desired yaw rate $r_d(t) \in \mathbb{R}$, related to a heading change. The second term is a performance term that triggers a higher gain when the observer error is large, and the third term only makes κ large during the initial transient. In (6c) T_{η_f} is a positive definite diagonal matrix with filter time constants, and these time constants and the size of $\varepsilon_{\eta} \geq 0$ are tuned such that κ approach κ_{min} at steady state.

In order to have a convenient expression for K_3 in the further analysis we introduce $\lambda \in [0, 1]$ and write $K_3 := K_{3,\lambda}$ as

$$K_{3\lambda} := \lambda K_{3,min} + (1-\lambda)K_{3,max}.$$
(7)

3.3 Output feedback tracking control

The control law consists of a reference feedforward term and a feedback term. The feedback part consists of a nonlinear PD-term, and a bias rejection term, which is the filtered bias estimate from (4d),

$$u = u_{FB} + u_{FF} \tag{8}$$
$$u_{FF} = M\dot{\nu}_d(t) + D\nu_d(t) \tag{9}$$

$$u_{FF} = M \nu_d(t) + D \nu_d(t)$$
(9)

$$u_{FB} = -K_p R(\psi)^{\top} (\hat{\eta} - \eta_d(t)) - K_d(\hat{\nu} - \nu_d(t)) - R(\psi)^{\top} \hat{b}_f$$

$$= -K_p R(\psi)^{\top} (\tilde{\eta} - \bar{\eta}) - K_d(\tilde{\nu} - \bar{\nu}) - R(\psi)^{\top} (b - \bar{b}_f).$$
(10)

where $\eta_d(t)$, $\nu_d(t)$, $\dot{\nu}_d(t)$ are the desired references generated by a reference generator. By defining the tracking error states $\tilde{\eta} := \eta - \eta_d(t)$, $\tilde{\nu} := \nu - \nu_d(t)$, the kinematics in (1b) along with the kinetics in (1d) inserted for (8) gives the tracking error system,

$$\dot{\tilde{\eta}} = R(\psi)\tilde{\nu}$$
 (11a)

$$M\dot{\tilde{\nu}} = -(D + K_d)\tilde{\nu} - K_p R(\psi)^{\top}\tilde{\eta}$$
(11b)

$$+ K_d \bar{\nu} + K_p R(\psi)^\top \bar{\eta} + R(\psi)^\top \bar{b}_f \qquad (11c)$$

4. STABILITY ANALYSIS

We collect all error states in $x := col(x_c, x_o)$, where $x_c := col(\tilde{\eta}, \tilde{\nu}), x_o := col(\bar{\xi}, \bar{\eta}, \bar{b}, \bar{b}_f, \bar{\nu})$ and combining (5), (7), and (11) the total error dynamic becomes

$$\dot{x} = A_{\lambda}(\psi)x \tag{12}$$

where

$$A_{\lambda}(\psi) = \begin{bmatrix} A_c(\psi) & B_{co}(\psi) \\ 0_{18 \times 6} & A_{o,\lambda}(\psi) \end{bmatrix},$$
(13)

and

$$A_{c} := \begin{bmatrix} 0 & R(\psi) \\ -M^{-1}K_{p}R(\psi)^{\top} & -M^{-1}(D+K_{d}) \end{bmatrix},$$
(14a)
$$B_{co} := \begin{bmatrix} 0 & 0 & 0 \\ 0_{3\times 6} & M^{-1}K_{p}R(\psi)^{\top} & 0_{3\times 3} & M^{-1}R(\psi)^{\top} & M^{-1}K_{d} \end{bmatrix},$$
(14b)
$$\begin{bmatrix} A_{w} - K_{1,\omega}C_{w} & -K_{1,\omega} & 0 & 0 & 0 \\ -K_{0}C_{w} & -K_{0} & 0 & 0 & R(\psi) \end{bmatrix}$$

$$A_{o,\lambda} := \begin{bmatrix} -K_2 C_w & -K_2 & 0 & 0 & R(\psi) \\ -K_{3,\lambda} C_w & -K_{3,\lambda} & 0 & 0 & 0 \\ 0 & 0 & T_f^{-1} & -T_f^{-1} & 0 \\ -M^{-1} K_4 R(\psi)^\top C_w & -M^{-1} K_4 R(\psi)^\top & M^{-1} & 0 & -M^{-1} D \end{bmatrix}.$$
(14c)

The dynamics (12) can be written (Lindegaard, 2003),

$$\dot{x} = T(\psi)^{\top} A_{\lambda}(0) T(\psi) x, \qquad (15)$$

if the matrices $K_{1,\omega}$, K_2 , $K_{3,\lambda}$, and T_f^{-1} commute with the rotation matrix $R(\psi)$. The transformation matrix $T(\psi)$ is given as

$$T(\psi) = \operatorname{diag}\{T_c(\psi), T_o(\psi)\}$$
(16a)

$$T_c(\psi) = \operatorname{diag}\{R(\psi)^{\top}, I\}$$
(16b)

$$T_o(\psi) = \operatorname{diag}\{R(\psi)^{\top}, \dots, R(\psi)^{\top}, I\}.$$
 (16c)

By inserting (7) we can write

$$A_{\lambda}(0) = \lambda A_{min} + (1 - \lambda)A_{max}, \qquad (17)$$

where A_{min} contains $K_{3,min}$ and A_{max} contains $K_{3,max}$. *Proposition 1.* The equilibrium x = 0 of (12,) where $K_{3,\lambda}$ can arbitrarily take any value in $[K_{3,min}, K_{3,max}]$, is uniformly asymptotically stable under the following conditions:

- The matrices $K_{1,\omega}$, K_2 , $K_{3,\lambda}$, and T_f^{-1} commute with the rotation matrix $R(\psi)$.
- The following LMI's are satisfied,

$$A_{min}^{\dagger}P + PA_{min} + r_{max}(S_TP - PS_T) < -Q \qquad (18a)$$

$$A_{min}^{\dagger}P + PA_{min} - r_{max}(S_TP - PS_T) < -Q \qquad (18b)$$

$$A_{max}^{+}P + PA_{max} + r_{max}(S_TP - PS_T) < -Q \qquad (18c)$$

$$A_{max}^{\top}P + PA_{max} - r_{max}(S_TP - PS_T) < -Q, \quad (18d)$$

where $S_T = \text{diag}\{S, 0, S, \dots, S, 0\}$, and P and Q are symmetric positive definite matrices.

Proof. Consider the transformation $z = T(\psi)x$ given by (16), and notice that $T(\psi)^{-1} = T(\psi)^{\top}$. From (15) we get

$$\dot{z} = T(\psi)T(\psi)^{\top}A_{\lambda}(0)z + \dot{T}(\psi)T(\psi)^{\top}z$$

= $A_{\lambda}(0)z - rS_{T}z$ (19)

where r is the yaw rate. We introduce a quadratic Lyapunov function $V(z) = z^{\top}Pz$, and from (19) we define $f(z) := A_{\lambda}(0)z$ and $g_r(z) := -rS_Tz$ such that (19) becomes

$$\dot{z} = f(z) + g_r(z), \tag{20}$$

where $f(z) := \lambda f_{min}(z) + (1-\lambda)f_{max}(z)$. From (18a)-(18d) and $r \in [-r_{max}, r_{max}]$ we have

$$\langle \nabla V(z), f_{min}(z) + g_r(z) \rangle \le -\alpha(|z|) \tag{21a}$$

$$\langle \nabla V(z), f_{max}(z) + g_r(z) \rangle \le -\alpha(|z|),$$
 (21b)

where $\alpha(|z|)$ is a positive definite function. Finally, we get $\langle \nabla V(z), \lambda f_{min}(z) + (1-\lambda) f_{max}(z) + g_r(z) \rangle \leq$

 $\begin{aligned} \lambda \langle \nabla V(z), f_{min}(z) + g_r(z) \rangle + (1 - \lambda) \langle \nabla V(z), f_{max}(z) + g_r(z) \rangle \\ \leq -\lambda \alpha(z) - (1 - \lambda) \alpha(z) \leq -\alpha(z), \end{aligned}$ (22) and this concludes the proof.

Table 1. Supply vessel, main parameters

Parameters	Value
Length between perp.	80 m
Breadth	$17.4 \mathrm{m}$
Draft	$5.6 \mathrm{m}$
Displacment	6150 tons

If the observer and controller gains are set such that A_{min} and A_{max} are Hurwitz, and if the ratio of $\kappa_{max}/\kappa_{min}$ is not very large (in practice, up to 5), it is easy to satisfy (18) for a maximum yaw rate far above "normal" yaw rates.

5. SIMULATION RESULTS AND DISCUSSION

The simulations are performed in MATLAB/Simulink on a high fidelity model based on building blocks from the MSS Toolbox (MSS, 2010). The case simulated is a platform supply vessel in an environment consisting of waves, wind, and current. See Table 1 for the main parameters of the vessel. The sea state is very rough with significant wave height of 6 meters, and a peak frequency of 0.53 rad/s taken from the JONSWAP¹ spectrum. The mean incident wave heading is 190° in the north-east frame (Price and Bishop, 1974). The current has a speed of 0.5 m/s and direction of 180° , and the wind has a mean velocity of 5 m/s with a direction of 160°. A first order model for the thrust dynamics is included, and the time constants for thrust force is set to 5 seconds. The GPS measurements have realistic noise properties, and are sampled at 1 Hz, and the measurements are processed by a zero-order hold element before they are sent to the observer.

Three different output feedback controllers are compared to illustrate the benefit of changing the gain K_3 in (4c). The only difference between the three setups is a variation of allowed values for κ from (6a). For two of the output feedback controllers the κ -value is fixed, where the "nominal" controller has $\kappa = \kappa_{min}$ for steady conditions, while the "aggressive" controller has $\kappa = \kappa_{max}$ for transient conditions. The last controller is our proposed algorithm in (6) where $\kappa \in [\kappa_{min}, \kappa_{max}]$, called the "flexible" controller. Even though the difference between these three systems is in the observer we often just write "controller" to describe the system. However, when just the observer performance is discussed, "observer" is used.

At the beginning of the simulation, the position and orientation of the vessel is at $\eta = [0, 0, 0]^{\top}$. At 1000 seconds there is a setpoint change 20 meters north, 20 meters east, and to heading -90°. Due to the ship hull shape this maneuver will change the bias force experienced by the vessel in the body frame, as well as in the NED frame. After 3000 seconds the direction of the current changes to 90°, to see how the vessel responds to a sudden change in bias force that is not known in advance. The current direction changes as a first order filtered step with time constant 30 seconds.

In Figure 1 the cumulative low-frequency position tracking error of the vessel is shown for the three controllers. The left part starts from the instance of the heading change, and the right part is a zoom-in on the steady period 2000-3000 seconds. The top plots show the combined error in north and east, and the bottom plots show the error in yaw. From the left part it can be observed that the aggressive and flexible controller perform much better than the nominal controller in the transient regime, that is, just after 1000 seconds, and just after 3000 seconds. From the right part of Figure 1 it can be observed that after the system reaches steady state, the flexible and nominal controller perform better than the aggressive controller, and this is due to lower oscillations of the bias and velocity estimates from the observer. This implies that since the flexible and aggressive controllers have similar performance in transients, the flexible controller will eventually perform better.

From the left part of Figure 1 it is observed that already around 2000 seconds the flexible controller has a lower cumulative position deviation. This is because the heading change is a transient known in advance, and the flexible controller can react fast, and go to a higher value for κ quickly. This is observed from Figure 4, where κ for the flexible controller is shown ($\kappa_{max} = 2.5$). In addition, we can observe from Figure 4 that at 3000 seconds it takes a bit more time for κ to go to κ_{max} than at 1000 seconds. This is natural since this increase is based on the estimation error in the observer, and not a command in the reference system as with the heading change. Even though κ will be slower for the "unknown" transients, we see from Figure 1 that the flexible controller has a similar performance to the aggressive controller, and will eventually outperform the aggressive controller if the steady state conditions persist.

In Figure 2 the cumulative bias estimation error (in the body frame) from the observer is plotted for the entire simulated case study. The combined error of surge and sway is shown in the top plot, and the yaw error is shown in the bottom plot. Here we see the same trend as in Figure 1, but the trend is even clearer. The flexible observer is superior to both the aggressive and nominal observer. Even the nominal observer performs better than the aggressive observer after 5000 seconds for the error in surge and sway.

In Figure 3 the bias in surge is plotted, along with the observer estimate, and the filtered bias estimate for the flexible controller. It is observed that the bias estimate (and the filtered estimate) converge to their new bias values quite fast, and within 200 seconds after a transient steady state conditions are reached.

6. CONCLUSION

The proposed output feedback controller was shown to have good closed-loop properties in both transients and steady state. Both the flexible bias estimation, and the filtering of the bias estimate used in the control law, contributed to a good overall performance for the system.

For the flexible bias estimation, the lowest tuning should be quite responsive to ensure good overall responsiveness. There are a couple of reasons for this. If we failed to detect a transient, or the detection was slow, a moderate nominal tuning vastly improved the performance in the transient compared to a very low nominal tuning. That is, if excellent positioning capabilities is the goal, the tuning should have a fairly high minimum. In the presented

¹ Joint North Sea Wave Project



Fig. 1. Cumulative low frequency position tracking error in north and east combined (top plots), and yaw (bottom plots). The right plot is a zoom-in on the steady period 2000-3000s (the flexible and nominal controllers overlap because of steady state conditions).



Fig. 2. Cumulative bias estimation error in surge and sway combined (top), and yaw (bottom).

simulation case study all the bias estimate tunings were quite fast, and they all converged within 300 seconds.

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Fig. 3. Bias estimate (blue, solid), actual bias (green, dashed), and filtered bias estimate (red, dotted), all in surge. For the entire simulation (top) and zoom-in (bottom).

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Fig. 4. Plot of κ (top) and desired yaw rate (bottom).

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