Estimation of actuator multiplicative faults for discrete-time LPV systems using a switched observer

Damiano Rotondo[a,*], Francisco-Ronay López-Estrada[b], Fatiha Nejjari[a], Jean-Christophe Ponsart[c,d], Didier Theilliol[c,d], Vicenç Puig[a,e]

[a] *Department of Automatic Control (ESAII), Technical University of Catalonia (UPC), Rambla de Sant Nebridi 10, 08222 - Terrassa (Spain). Tel: +34 93 739 89 73*
[b] *Electronic Department, Technological Institute of Tuxtla Gutierrez (ITTG), Tecnológico Nacional de México, Carretera Panamericana km 1080, Col. Terán, CP 29050, Tuxtla Gutiérrez, Chiapas, Mexico*
[c] *Université de Lorraine, CRAN, UMR 7039, Campus Sciences, B.P. 70239, Vandoeuvre-les-Nancy Cedex 54506*
[d] *CNRS, CRAN, UMR 7039.*
[e] *Institut de Robòtica i Informàtica Industrial (IRI), UPC-CSIC Carrer de Llorens i Artigas, 4-6, 08028 Barcelona, Spain*

Abstract

This paper proposes an observer for the joint state and fault estimation devoted to discrete-time linear parameter varying (LPV) systems subject to actuator faults. The major contribution of this work is that the proposed observer is able to estimate multiplicative faults, contrarily to the existing approaches, that consider additive faults. The main characteristic of the proposed observer is that it is scheduled not only by means of the endogenous varying parameters of the faulty model, but also by the input vector. Another contribution of this paper consists in adding a switching component in order to guarantee the feasibility of the conditions for designing the observer gains. It is proved that, as long as the input sequence satisfies some characteristics, the convergence of the observer error dynamics to zero is assured. A numerical example is used to demonstrate the effectiveness of the proposed strategy.

*Corresponding author

Email addresses: damiano.rotondo@yahoo.it (Damiano Rotondo), frlopez@ittg.edu.mx (Francisco-Ronay López-Estrada), fatiha.nejjari@upc.edu (Fatiha Nejjari), Jean-Christophe.Ponsart@univ-lorraine.fr (Jean-Christophe Ponsart), didier.theilliol@univ-lorraine.fr (Didier Theilliol), vicenc.puig@upc.edu (Vicenç Puig)

Keywords: Linear parametrically varying (LPV) methodologies; Multiplicative actuator faults; Fault estimation; Switched observers.

1. Introduction

Fault detection and isolation (FDI) systems have been a very active area of research in the last decades, and many schemes have been developed (see [1], [2] and [3] for recent surveys of the most relevant results). The FDI approaches are generally classified into model-based/data-based and quantitative/qualitative techniques [1]. A quantitative model-based FDI scheme utilizes a mathematical model, often known as analytical redundancy, to carry out FDI in real-time.

Among the proposed solutions for fault diagnosis systems, several methods are based on observers. These *fault estimation* methods attempt to reconstruct the fault rather than to detect its presence, and provide a direct estimate of its magnitude and severity, which is important in many applications, especially when an active fault tolerant control (FTC) strategy is implemented. In [4], a sliding mode observer that decouples the effects of the fault signals from the response of the system estimated outputs has been developed. The observer is designed to maintain a sliding motion even in the presence of faults, that are explicitly reconstructed through the manipulation of the equivalent output injection signal. Further improvements of this technique can be found in [5] and [6]. [7] provided a robust state-space observer for Lipschitz nonlinear descriptor systems with bounded input disturbances. The proposed method can simultaneously estimate descriptor system states, actuator faults, their finite time derivatives, and attenuate input disturbances in any desired accuracy. In [8], a method for state estimation of Takagi-Sugeno (TS) descriptor systems affected by unknown inputs has been presented. Sufficient existence conditions of the unknown input observers (UIOs) are given and strict linear matrix inequalities are solved to determine the gain of the observers. In particular, in [8], it has been shown that, by designing a bank of observers and using a simple decision logic and thresholds, robust fault diagnosis can be performed. In [9], the authors have further developed the use of UIOs for fault detection/isolation in overactuated systems. In [10], a robust residual generator in order to achieve the tasks of fault detection, isolation and estimation for nonlinear systems described by a TS model has been proposed. The main result consisted in extending the method of fault diagnosis based on $\mathcal{H}_\infty$ control framework including a reference model corresponding to the desired response of the residual to the fault.

The provided list of references is not exhaustive, and many other observer-based FDI methods have been developed for actuator faults, e.g. [11] and [12]. However, most of the proposed approaches consider the case of additive faults, and there is a lack of results concerning observer-based estimators for actuator multiplicative faults. The main difference between an additive and a multiplicative fault is that, as a result of the additive faults, the mean value of the output changes, while if the fault is multiplicative, it generates changes on the system parameters [13]. The design of observers for multiplicative fault estimation is not as straightforward as the case of additive fault estimation, because the effect of the input and the fault are mixed. To the best of the author's knowledge, the only observer-based solution for the estimation of multiplicative faults has been provided by [14], where the multiplicative faults have been reshaped into additive faults, such that a sliding mode observer is used. Hence, finding solutions to this problem remains an open research issue.

On the other hand, the concept of linear parameter varying (LPV) systems was introduced by [15] to distinguish such systems from linear time invariant (LTI) and linear time varying (LTV) ones [16]. Since then, the LPV paradigm has become a standard formalism in systems and control, for analysis, controller synthesis and even system identification. In some cases, due to the loss of feasibility of the LMIs or the inherent switching modes of the system, it may be needed to split the parameter region into subregions, and switch among them during the LPV system operation. Thus, the LPV system is transformed into a new class of system, referred to as *switched LPV system* [17].

The main contribution of this paper is to propose an observer for the joint estimation of the state and multiplicative faults in discrete-time LPV systems. The proposed observer is scheduled not only by the endogenous varying parameters of the faulty model, but also by the input vector. Its design is performed solving matrix inequalities, and it is shown that if any of the system inputs can take a value equal to zero, a problem of feasibility of the matrix inequalities would appear if a non-switching structure is used for the LPV observer. However, the addition of a switching component allows to overcome this issue by considering different feasibility regions generated by the scheduling parameters. It is worth highlighting that the proposed method can also be applied to LTI systems, by considering that the switched LPV observer is scheduled only by the input vector.

The paper is structured as follows. Section 2 presents the problem of joint state and actuator multiplicative fault estimation. Section 3 provides the solution proposed for this problem. Section 4 illustrates the proposed approach using a numerical example. Finally, Section 5 outlines the conclusions.

2. Problem Statement

Let us consider a discrete-time LPV system subject to actuator faults

$$\bar{x}(k+1) = \bar{A}\left(\bar{\theta}(k)\right)\bar{x}(k) + \bar{B}\left(\bar{\theta}(k)\right)\Gamma(k)u(k) \tag{1}$$

$$y(k) = \bar{C}\bar{x}(k) \tag{2}$$

where $\bar{x}(k) \in \mathbb{R}^{n_x}$ and $y(k) \in \mathbb{R}^{n_y}$ are the state and output vector, respectively. The input vector, denoted by $u(k)$, takes values in a subset $\Upsilon \subset \mathbb{R}^{n_u}$, defined as follows

$$\Upsilon = \left[u_1^{\min}, u_1^{\max}\right] \times \ldots \times \left[u_{n_u}^{\min}, u_{n_u}^{\max}\right] \tag{3}$$

where $u_j^{min} < 0$ and $u_j^{max} > 0$ for all $j = 1, \ldots, n_u$.

The matrices $\bar{A}\left(\bar{\theta}(k)\right) \in \mathbb{R}^{n_x \times n_x}$, $\bar{B}\left(\bar{\theta}(k)\right) \in \mathbb{R}^{n_x \times n_u}$ are known and scheduled by the vector of varying parameters $\bar{\theta}(k) \in \Theta \subset \mathbb{R}^{n_\theta}$. The matrix $\bar{C} \in \mathbb{R}^{n_y \times n_x}$ is a known constant matrix. On the other hand, the matrix $\Gamma(k)$ is unknown and describes the multiplicative faults, as follows

$$\Gamma(k) = \begin{bmatrix} \gamma_1(k) & 0 & \cdots & 0 \\ 0 & \gamma_2(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_{n_u}(k) \end{bmatrix} \tag{4}$$

where each $\gamma_j(k)$, $j = 1, \ldots, n_u$, represents the loss of effectiveness of the j-th actuator, such that its extreme values 0 and 1 correspond to the total loss of the actuator and to the healthy actuator, respectively.

Remark 1. *The assumption of constant output matrix is quite common in the literature, and could be relaxed either by increasing the mathematical complexity of the solution proposed in the following, or by post-filtering the output vector $y(k)$, as proposed by [18].*

The problem addressed in this paper is the joint estimation of the system states and the multiplicative faults using the model (1) and the available measurements (2). In order to achieve this objective, let us notice that, thanks to the diagonal structure of $\Gamma(k)$, which implies

$$\Gamma(k)u(k) = U\left(u(k)\right)\gamma(k) \tag{5}$$

$$U\left(u(k)\right) = \mathrm{diag}\left(u_1(k), \ldots, u_{n_u}(k)\right) \tag{6}$$

$$\gamma(k) = \begin{bmatrix} \gamma_1(k) & \gamma_2(k) & \ldots & \gamma_{n_u}(k) \end{bmatrix}^T \tag{7}$$

it is possible to rewrite the system state equation (1) as

$$\bar{x}(k+1) = \bar{A}\left(\bar{\theta}(k)\right)\bar{x}(k) + \bar{B}\left(\bar{\theta}(k)\right)U\left(u(k)\right)\gamma(k) \tag{8}$$

Under the assumption of slow-varying faults, i.e. $\gamma(k+1) = \gamma(k)$, and by considering the augmented state vector $x(k) \triangleq [\bar{x}(k)^T \, \gamma(k)^T]^T$ and the scheduling vector $\theta(k) \triangleq [\bar{\theta}(k)^T \, u(k)^T]^T$, the following augmented system is obtained

$$x(k+1) = A(\theta(k))x(k) \tag{9}$$

$$y(k) = Cx(k) \tag{10}$$

with

$$A(\theta(k)) = \begin{bmatrix} \bar{A}(\bar{\theta}(k)) & \bar{B}(\bar{\theta}(k))U\left(u(k)\right) \\ 0 & I \end{bmatrix}, \; C = \begin{bmatrix} \bar{C} & 0 \end{bmatrix}$$

Remark 2. *The assumption of slow variation of the faults could appear very restrictive. Nevertheless, this assumption could be relaxed from a practical point of view, as stated by [19] and [20].*

Under the assumption that the augmented system (9)-(10) is observable, the following observer for the joint state and fault estimation could be proposed

$$\hat{x}(k+1) = A(\theta(k))\hat{x}(k) + L(\theta(k))\left(\hat{y}(k) - y(k)\right) \tag{11}$$

$$\hat{y}(k) = C\hat{x}(k) \tag{12}$$

In this case, the problem would reduce to find the observer gain $L(\theta(k))$ such that $\lim_{k\to\infty} e(k) = \lim_{k\to\infty} \left(\hat{x}(k) - x(k)\right) = 0$.

Taking into account the augmented system (9)-(10) and the state/fault observer (11)-(12), the dynamics of the estimation error $e(k)$ is given as follows

$$e(k+1) = \left(A\left(\theta(k)\right) + L\left(\theta(k)\right)C\right)e(k) \tag{13}$$

Let us recall the following lemma.

Lemma 1. (Lyapunov condition for the stability of discrete-time LPV systems) *Consider an autonomous discrete-time LPV system*

$$x(k+1) = A\left(\theta(k)\right)x(k), \quad \theta \in \Theta \subset \mathbb{R}^{n_\theta} \tag{14}$$

If there exists a matrix $P = P^T \succ 0$ such that $\forall\theta \in \Theta$ the following holds

$$\begin{bmatrix} P & PA(\theta) \\ A(\theta)^T P & P \end{bmatrix} \succ 0 \tag{15}$$

then the system (14) *is stable in the sense of Lyapunov.*

Proof: *The proof is straightforward by considering the Lyapunov function $V(k) = x(k)^T Px(k)$ and imposing that the difference $V(k+1) - V(k)$ is negative.* $\square$

Then, sufficient conditions for guaranteeing the stability of (13) are obtained by applying Lemma 1, that leads to the following matrix inequalities

$$\begin{bmatrix} P & PA(\theta) + \Xi(\theta) C \\ \star & P \end{bmatrix} \succ 0 \quad \forall\theta \in \Theta \times \Upsilon \tag{16}$$

However, due to the structure of the matrix $A(\theta)$ and C, if any of the inputs can take a value equal to zero, then a problem of feasibility of the matrix inequalities appears due to the loss of observability of the pairs $(A(\theta), C)$. For example, if $u(k) = 0$ were an admissible input, the observability matrix for this value, defined as

$$\mathcal{O} = \begin{bmatrix} C \\ CA(\theta) \\ \vdots \\ CA(\theta)^{n_x+n_u-1} \end{bmatrix} = \begin{bmatrix} \bar{C} & 0 \\ \bar{C}\bar{A}(\bar{\theta}) & 0 \\ \vdots & \vdots \\ \bar{C}\bar{A}(\bar{\theta})^{n_x+n_u-1} & 0 \end{bmatrix} \tag{17}$$

is such that $\mathsf{rank}(\mathcal{O}) < n_x + n_u$.

In order to guarantee the feasibility of the conditions for designing the observer gains, a switched LPV state/fault observer is proposed in the next section, which is the main contribution of this work.

3. Main result

Let us define the following subsets of the input space

$$\mathcal{R}_{s_1 s_2 \ldots s_{n_u}} = \left\{ u : u_1 \overset{s_1}{\gtreqless} (-1)s_1\varepsilon_1, \ldots, u_{n_u} \overset{s_{n_u}}{\gtreqless} (-1)s_{n_u}\varepsilon_{n_u} \right\} \tag{18}$$

$$\mathcal{Q}_{s_1 s_2 \ldots s_{n_u}} = \left\{ u : u_1 \overset{s_1}{\gtreqless} s_1\varepsilon_1, \ldots, u_{n_u} \overset{s_{n_u}}{\gtreqless} s_{n_u}\varepsilon_{n_u} \right\} \tag{19}$$

$$\mathcal{R}\backslash\mathcal{Q}_{s_1 s_2 \ldots s_{n_u}} = \left\{u : u \in \mathcal{R}_{s_1 s_2 \ldots s_{n_u}}, u \notin \mathcal{Q}_{s_1 s_2 \ldots s_{n_u}}\right\} \tag{20}$$

where the operators $\overset{s_j}{\gtreqless}$, $j = 1, \ldots, n_u$, are a shorthand notation for

$$\overset{s_j}{\gtreqless} = \begin{cases} \geq & if\ s_j = + \\ \leq & if\ s_j = - \end{cases} \tag{21}$$

and ε_j, $j = 1, \ldots, n_u$, are given small scalars. The piecewise constant switching signal

$$\sigma(k) = s_1(k) \ldots s_{n_u}(k) \tag{22}$$

defines at each time sample whether the index s_j of the active subsets $\mathcal{R}_{s_1 s_2 \ldots s_{n_u}}$ and $\mathcal{Q}_{s_1 s_2 \ldots s_{n_u}}$ is + or −, as given in (21). In particular, the switching rule that provides the switching signal is chosen to be dependent on the values of the inputs, as follows

$$s_j(k) = \begin{cases} s_j(k-1) & if\ u_j(k) \overset{s_j(k-1)}{\gtreqless} (-1)s_j(k-1)\varepsilon_j \\ -s_j(k-1) & if\ u_j(k) \overset{s_j(k-1)}{\lessgtr} (-1)s_j(k-1)\varepsilon_j \end{cases} \tag{23}$$

where the operators $\overset{s_j}{\lessgtr}$ are a shorthand notation for

$$\overset{s_j}{\lessgtr} = \begin{cases} < & if\ s_j = + \\ > & if\ s_j = - \end{cases} \tag{24}$$

Example: Let us consider the case where $n_u = 2$, that is, a system with two available inputs $u_1(k)$ and $u_2(k)$. Eq. (18)-(20) define the following subsets of the input space

$$\begin{array}{cc} \mathcal{R}_{++} = \{u : u_1 \geq -\varepsilon_1, u_2 \geq -\varepsilon_2\} & \mathcal{R}_{+-} = \{u : u_1 \geq -\varepsilon_1, u_2 \leq \varepsilon_2\} \\ \mathcal{R}_{-+} = \{u : u_1 \leq \varepsilon_1, u_2 \geq -\varepsilon_2\} & \mathcal{R}_{--} = \{u : u_1 \leq \varepsilon_1, u_2 \leq \varepsilon_2\} \end{array} \tag{25}$$

$$\begin{array}{cc} \mathcal{Q}_{++} = \{u : u_1 \geq \varepsilon_1, u_2 \geq \varepsilon_2\} & \mathcal{Q}_{+-} = \{u : u_1 \geq \varepsilon_1, u_2 \leq -\varepsilon_2\} \\ \mathcal{Q}_{-+} = \{u : u_1 \leq -\varepsilon_1, u_2 \geq \varepsilon_2\} & \mathcal{Q}_{--} = \{u : u_1 \leq -\varepsilon_1, u_2 \leq -\varepsilon_2\} \end{array} \tag{26}$$

$$\begin{array}{cc} \mathcal{R}\backslash\mathcal{Q}_{++} = \{u : u \in \mathcal{R}_{++}, u \notin \mathcal{Q}_{++}\} & \mathcal{R}\backslash\mathcal{Q}_{+-} = \{u : u \in \mathcal{R}_{+-}, u \notin \mathcal{Q}_{+-}\} \\ \mathcal{R}\backslash\mathcal{Q}_{-+} = \{u : u \in \mathcal{R}_{-+}, u \notin \mathcal{Q}_{-+}\} & \mathcal{R}\backslash\mathcal{Q}_{--} = \{u : u \in \mathcal{R}_{--}, u \notin \mathcal{Q}_{--}\} \end{array} \tag{27}$$

Then, $\sigma(k-1) = ++$ would indicate that the active subset at time sample $k-1$ is $\mathcal{R}_{++}$, while $\sigma(k-1) = +-$, $\sigma(k-1) = -+$ and $\sigma(k-1) = --$ would indicate

$\mathcal{R}_{+-}$, $\mathcal{R}_{-+}$ and $\mathcal{R}_{--}$, respectively. Then, if the active subset at time sample $k-1$ were $\mathcal{R}_{++}$, the switching rule (23) would be as follows

$$\sigma(k) = \begin{cases} ++ & if\ u_1(k) \geq -\varepsilon_1, u_2(k) \geq -\varepsilon_2 \\ +- & if\ u_1(k) \geq -\varepsilon_1, u_2(k) < -\varepsilon_2 \\ -+ & if\ u_1(k) < -\varepsilon_1, u_2(k) \geq -\varepsilon_2 \\ -- & if\ u_1(k) < -\varepsilon_1, u_2(k) < -\varepsilon_2 \end{cases}$$

On the other hand, if the active subset at time sample $k-1$ were $\mathcal{R}_{+-}$, the switching rule would be

$$\sigma(k) = \begin{cases} ++ & if\ u_1(k) \geq -\varepsilon_1, u_2(k) > \varepsilon_2 \\ +- & if\ u_1(k) \geq -\varepsilon_1, u_2(k) \leq \varepsilon_2 \\ -+ & if\ u_1(k) < -\varepsilon_1, u_2(k) > \varepsilon_2 \\ -- & if\ u_1(k) < -\varepsilon_1, u_2(k) \leq \varepsilon_2 \end{cases}$$

Similar switching rules are obtained for the remaining active subsets $\mathcal{R}_{-+}$ or $\mathcal{R}_{--}$.
■

Taking into account the definition of the switching signal provided in (22)-(23), the following switched LPV state/fault observer is proposed in order to guarantee that in each subset of the input space $\mathcal{R}_{s_1 s_2 \ldots s_{n_u}}$, the design conditions for the gains are feasible

$$\hat{x}(k+1) = A(\theta(k))\hat{x}(k) + L_{\sigma(k)}(\theta(k))\left(\hat{y}(k) - y(k)\right) \tag{28}$$

$$\hat{y}(k) = C\hat{x}(k) \tag{29}$$

where $L_{\sigma(k)}(\theta(k))$ corresponds to the active LPV observer gain, that is defined by the value of the switching signal $\sigma(k)$. The design problem becomes to find the possible LPV observer gains $L_{s_1 \ldots s_{n_u}}(\theta(k))$ for all the subsets defined in (18), such that the dynamics of the estimation error

$$e(k+1) = \big(A\left(\theta(k)\right) + L_{\sigma(k)}\left(\theta(k)\right)C\big)\, e(k) \tag{30}$$

satisfies some stability condition.

The following theorem provides a sufficient condition for the stability of the estimation error dynamics (30). This proof resembles the reasoning used to prove the stability of switched LPV systems with average dwell time [17].

Theorem 1. *If there exist 2^{n_u} positive definite matrices $P_l = P_l^T \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_u)}$, 2^{n_u} matrices $\Xi_l(\theta) \in \mathbb{R}^{(n_x+n_u)\times n_y}$, scalars $0 \leq a \leq 1$, $b \geq 0$ and $\mu > 1$ such that the following conditions hold*

$$\begin{bmatrix} P_l & P_l A(\theta) + \Xi_l(\theta) C \\ \star & a P_l \end{bmatrix} > 0, \quad \forall \theta \in \Theta \times (\Upsilon \cap \mathcal{Q}_l) \tag{31}$$

$$\begin{bmatrix} P_l & P_l A(\theta) + \Xi_l(\theta) C \\ \star & b P_l \end{bmatrix} > 0, \quad \forall \theta \in \Theta \times (\Upsilon \cap \mathcal{R}_l) \tag{32}$$

$$\frac{1}{\mu} P_m \leq P_l \leq \mu P_m \tag{33}$$

with l and m equal to all the possible combinations of indices $s_1 \dots s_{n_u}$ as defined in (18)-(19)*, then the error dynamics* (30) *converges asymptotically to zero as long as the LPV observer gains are calculated as*

$$L_l(\theta(k)) = P_l^{-1} \Xi_l(\theta(k)) \tag{34}$$

and the input sequence $u(k)$ is such that for any $k_0 \geq 0$, it is possible to find a $k_f > k_0$ such that

$$\left(\mu^N\right)\left(b^{N\bar{k}_{\mathcal{R}\setminus\mathcal{Q}}}\right)\left(a^{N\bar{k}_{\mathcal{Q}}}\right) < 1 \tag{35}$$

where N is the number of switches in $[k_0, k_f]$ given by (23)*, $\bar{k}_{\mathcal{Q}}$ is the average number of samples per switch during which $u(k)$ belongs to one of the subsets $\mathcal{Q}_{s_1 \dots s_{n_u}}$, and $\bar{k}_{\mathcal{R}\setminus\mathcal{Q}}$ is the average number of samples per switch during which $u(k)$ belongs to the regions that belong to a subset $\mathcal{R}_{s_1 \dots s_{n_u}}$ but do not belong to any subset $\mathcal{Q}_{s_1 \dots s_{n_u}}$.*

Proof: *First of all, for each possible combination of indices $s_1 \dots s_{n_u}$ in* (18)-(19)*, let us define the corresponding Lyapunov function*

$$V_l(e(k)) = e(k)^T P_l e(k) \tag{36}$$

Also, it is assumed that over an interval $[k_0, k_f]$, the input $u(k)$ changes the active subset as $\mathcal{Q}_0 \to \mathcal{R}_0\setminus\mathcal{Q}_0, \dots, \mathcal{Q}_l \to \mathcal{R}_l\setminus\mathcal{Q}_l, \dots, \mathcal{Q}_{N-1} \to \mathcal{R}_{N-1}\setminus\mathcal{Q}_{N-1}$ at time samples $k_{\mathcal{Q}}^0, \dots, k_{\mathcal{Q}}^l, \dots, k_{\mathcal{Q}}^{N-1}$, and as $\mathcal{R}_0\setminus\mathcal{Q}_0 \to \mathcal{Q}_1, \dots, \mathcal{R}_l\setminus\mathcal{Q}_l \to \mathcal{Q}_{l+1}, \dots, \mathcal{R}_{N-1}\setminus\mathcal{Q}_{N-1} \to \mathcal{Q}_N$ at time samples $k_{\mathcal{R}\setminus\mathcal{Q}}^0, \dots, k_{\mathcal{R}\setminus\mathcal{Q}}^l, \dots, k_{\mathcal{R}\setminus\mathcal{Q}}^{N-1}$.

Then, due to the conditions (31)*, at* $k = k_{\mathcal{Q}}^0, \ldots, k_{\mathcal{Q}}^l, \ldots, k_{\mathcal{Q}}^{N-1}$*, the Lyapunov functions* $V_0, \ldots, V_l, \ldots, V_N$ *satisfy*

$$V_0\left(e(k_{\mathcal{Q}}^0)\right) < \left(a^{k_{\mathcal{Q}}^0 - k_0}\right) V_0\left(e(k_0)\right) \tag{37}$$
$$V_1\left(e(k_{\mathcal{Q}}^1)\right) < \left(a^{k_{\mathcal{Q}}^1 - k_{\mathcal{R}\setminus\mathcal{Q}}^0}\right) V_1\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^0)\right) \tag{38}$$
$$\vdots$$
$$V_l\left(e(k_{\mathcal{Q}}^l)\right) < \left(a^{k_{\mathcal{Q}}^l - k_{\mathcal{R}\setminus\mathcal{Q}}^{l-1}}\right) V_l\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^{l-1})\right) \tag{39}$$
$$\vdots$$
$$V_N\left(e(k_f)\right) < \left(a^{k_f - k_{\mathcal{R}\setminus\mathcal{Q}}^{N-1}}\right) V_N\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^{N-1})\right) \tag{40}$$

On the other hand, the conditions (32) *guarantee that at* $k = k_{\mathcal{R}\setminus\mathcal{Q}}^0, \ldots, k_{\mathcal{R}\setminus\mathcal{Q}}^l, \ldots, k_{\mathcal{R}\setminus\mathcal{Q}}^{N-1}$*, the Lyapunov functions* $V_0, \ldots, V_l, \ldots, V_{N-1}$ *satisfy*

$$V_0\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^0)\right) < \left(b^{k_{\mathcal{R}\setminus\mathcal{Q}}^0 - k_{\mathcal{Q}}^0}\right) V_0\left(e(k_{\mathcal{Q}}^0)\right) \tag{41}$$
$$V_1\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^1)\right) < \left(b^{k_{\mathcal{R}\setminus\mathcal{Q}}^1 - k_{\mathcal{Q}}^1}\right) V_1\left(e(k_{\mathcal{Q}}^1)\right) \tag{42}$$
$$\vdots$$
$$V_l\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^l)\right) < \left(b^{k_{\mathcal{R}\setminus\mathcal{Q}}^l - k_{\mathcal{Q}}^l}\right) V_l\left(e(k_{\mathcal{Q}}^l)\right) \tag{43}$$
$$\vdots$$
$$V_{N-1}\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^{N-1})\right) < \left(b^{k_{\mathcal{R}\setminus\mathcal{Q}}^{N-1} - k_{\mathcal{Q}}^{N-1}}\right) V_{N-1}\left(e(k_{\mathcal{Q}}^{N-1})\right) \tag{44}$$

In addition, the conditions (33) *guarantee that at* $k = k_{\mathcal{R}\setminus\mathcal{Q}}^0, \ldots, k_{\mathcal{R}\setminus\mathcal{Q}}^l, \ldots, k_{\mathcal{R}\setminus\mathcal{Q}}^{N-1}$

$$V_1\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^0)\right) < \mu V_0\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^0)\right) \tag{45}$$
$$\vdots$$
$$V_{l+1}\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^l)\right) < \mu V_l\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^l)\right) \tag{46}$$
$$\vdots$$
$$V_N\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^{N-1})\right) < \mu V_{N-1}\left(e(k_{\mathcal{R}\setminus\mathcal{Q}}^{N-1})\right) \tag{47}$$

Therefore, linking all the inequalities (37)-(47)*, the following is true*

$$V_N\left(e(k_f)\right) < \left(\mu^N\right)\left(b^{\sum_{l=0}^{N-1}\left(k_{R\setminus Q}^l - k_Q^l\right)}\right)\left(a^{k_f - k_{R\setminus Q}^{N-1} + \sum_{l=1}^{N-1}\left(k_Q^l - k_{R/Q}^{l-1}\right) + k_Q^0 - k_0}\right) V_0\left(e(k_0)\right) \tag{48}$$

By considering

$$\bar{k}_{\mathcal{Q}} = \frac{k_f - k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1} + \sum_{l=1}^{N-1}\left(k_{\mathcal{Q}}^{l} - k_{\mathcal{R}/\mathcal{Q}}^{l-1}\right) + k_{\mathcal{Q}}^{0} - k_0}{N} \tag{49}$$

$$\bar{k}_{\mathcal{R}\backslash\mathcal{Q}} = \frac{\sum_{l=0}^{N-1}\left(k_{\mathcal{R}\backslash\mathcal{Q}}^{l} - k_{\mathcal{Q}}^{l}\right)}{N} \tag{50}$$

the inequality (48) *can be rewritten as*

$$V_N\left(e(k_f)\right) < \left(\mu^N\right)\left(b^{N\bar{k}_{\mathcal{R}\backslash\mathcal{Q}}}\right)\left(a^{N\bar{k}_{\mathcal{Q}}}V_0\left(e(k_0)\right)\right) \tag{51}$$

Then, if the input $u(k)$ is such that for each $k_0 \geq 0$ it is possible to find $k_f > k_0$ that satisfies (35)*, the Lyapunov functions will be decreasing to zero, and therefore the estimation error $e(k)$ will converge asymptotically to zero, completing the proof.* □

From a practical point of view, Theorem 1 cannot be used because it relies on the satisfaction of infinite constraints. However, the number of constraints can be reduced to a finite number by choosing the observer gain L to depend only on $\bar{\theta}(k)$, and by considering a polytopic representation of $A\left(\theta(k)\right)$ and $L\left(\bar{\theta}(k)\right)$, as follows

$$A\left(\theta(k)\right) = \sum_{i=1}^{N_{\bar{\theta}}}\alpha_i\left(\bar{\theta}(k)\right)\sum_{j=1}^{N_u}\beta_j^l\left(u(k)\right)A_{ij}^{\mathcal{Q}_l} \quad \forall\theta \in \Theta \times \left(\Upsilon \cap \mathcal{Q}_l\right) \tag{52}$$

$$A\left(\theta(k)\right) = \sum_{i=1}^{N_{\bar{\theta}}}\alpha_i\left(\bar{\theta}(k)\right)\sum_{j=1}^{N_u}\chi_j^l\left(u(k)\right)A_{ij}^{\mathcal{R}_l} \quad \forall\theta \in \Theta \times \left(\Upsilon \cap \mathcal{R}_l\right) \tag{53}$$

$$L_l\left(\bar{\theta}(k)\right) = \sum_{i=1}^{N_{\bar{\theta}}}\alpha_i\left(\bar{\theta}(k)\right)L_i^l \quad \forall\theta \in \Theta \times \left(\Upsilon \cap \mathcal{R}_l\right) \tag{54}$$

with

$$\sum_{i=1}^{N_{\bar{\theta}}}\alpha_i\left(\bar{\theta}(k)\right) = \sum_{j=1}^{N_u}\beta_j^l\left(u(k)\right) = \sum_{j=1}^{N_u}\chi_j^l\left(u(k)\right) = 1 \tag{55}$$

and $\alpha_i \geq 0\ \forall i = 1,\ldots,N_{\bar{\theta}}, \beta_j \geq 0\ \forall j = 1,\ldots,N_u, \chi_j \geq 0\ \forall j = 1,\ldots,N_u$.

Then, the following corollary can be obtained easily from Theorem 1.

Corollary 1. *Choose scalars* $0 \leq a \leq 1$, $b \geq 0$, *and find* 2^{n_u} *positive definite matrices* $P_l = P_l^T \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_u)}$ *and* $2^{n_u} N_{\bar{\theta}}$ *matrices* $\Xi_i^l \in \mathbb{R}^{(n_x+n_u)\times n_y}$ *such that*

$$\left[\begin{array}{cc} P_l & P_l A_{ij}^{\mathcal{Q}_l} + \Xi_i^l C \\ \star & a P_l \end{array}\right] > 0 \tag{56}$$

$$\left[\begin{array}{cc} P_l & P_l A_{ij}^{\mathcal{R}_l} + \Xi_i^l C \\ \star & b P_l \end{array}\right] > 0 \tag{57}$$

and such that there exists $\mu > 1$ *for which*

$$\frac{1}{\mu} P_m \leq P_l \leq \mu P_m \tag{58}$$

with l *and* m *equal to all the possible combinations* $s_1, \ldots, s_{n_u}$ *as defined in* (18)-(19), $i = 1, \ldots, N_{\bar{\theta}}$ *and* $j = 1, \ldots, N_u$.

Then, the error dynamics (30) *converges asymptotically to zero as long as the LPV observer gain is given by* (54) *with*

$$L_i^l = P_l^{-1} \Xi_i^l \tag{59}$$

and the input sequence $u(k)$ *is such that for any* $k_0 \geq 0$, *it is possible to find a* $k_f > k_0$ *such that* (35) *holds.*

Proof: *The proof is based on a basic property of matrices [21], which establishes that any linear combinations of* (56) *and* (57) *with non-negative coefficients (of which at least one different from zero) is definite positive. Using the coefficients* $\alpha_i\left(\bar{\theta}(k)\right)$ *and* $\beta_j^l\left(u(k)\right)$, (56) *becomes*

$$\sum_{i=1}^{N_{\bar{\theta}}} \alpha_i\left(\bar{\theta}(k)\right) \sum_{j=1}^{N_u} \beta_j^l\left(u(k)\right) \left[\begin{array}{cc} P_l & P_l A_{ij}^{\mathcal{Q}_l} + \Xi_i^l C \\ \star & a P_l \end{array}\right] > 0 \tag{60}$$

that, taking into account $\Xi_i^l = P_l L_i^l$, *and* (52), (54) *and* (55), *becomes* (31).

If the same process is applied to (57) *using the coefficients* $\alpha_i\left(\bar{\theta}(k)\right)$ *and* $\chi_j^l\left(u(k)\right)$ *and taking into account* (55)-(57), (32) *would be obtained.*

Since (58) *corresponds to* (33), *the conditions provided by Theorem 1 are recovered, that completes the proof.* □

4. Illustrative Example

Consider the discrete-time LPV system subject to multiplicative actuator faults (1)-(2) with state and input matrices described by

$$\bar{A}(\bar{\theta}(k)) = \begin{bmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & \bar{\theta}(k) & 0.1 \\ 0.1 & 0.3 & 0.5 \end{bmatrix}, \bar{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

with the varying parameter $\bar{\theta}(k) \in [0.1, 0.3]$ for all k, and with the inputs $u_1(k)$ and $u_2(k)$ taking values in $[-10, 10]$. By considering (5)-(8), under the assumption of slow-varying faults, the augmented system (9)-(10) is obtained as follows

$$A(\theta) = \begin{bmatrix} 0.3 & 0.2 & 0.1 & u_1(k) & 0 \\ 0.6 & \bar{\theta}(k) & 0.1 & 0 & u_2(k) \\ 0.1 & 0.3 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The subsets $\mathcal{R}_{...}$, $\mathcal{Q}_{...}$ and $\mathcal{R} \backslash \mathcal{Q}_{...}$ are defined as in Example 1 with a choice of $\varepsilon_1 = \varepsilon_2 = 1$. By taking into account the limits of $\bar{\theta}(k)$ and $u(k)$, 8 vertex matrices are obtained for each subset.

By choosing $a = 0.9$ and $b = 2$, the LMIs (56)-(57) have been solved using the YALMIP toolbox [22] with SeDuMi solver [23], and it has been verified that, for the obtained Lyapunov matrices, the condition (58) holds with $\mu = 5.1$. According to Corollary 1, if the LPV observer vertex gains are calculated as in (59), then the estimation error would converge to zero as long as the input sequence satisfies condition (35). It is worth remarking that, since the Lyapunov-based conditions are always sufficient for convergence, and not necessary, it is possible that the estimation error would still converge to zero even though the input sequence does not satisfy (35).

The results shown in the following refer to a stepwise profile of the multiplicative faults, that are assumed to occur in both inputs. The initial conditions and the scheduling parameter trajectory are as follows:

$$x(0) = \begin{bmatrix} 0.2 & 0.5 & 1 \end{bmatrix}^T \tag{61}$$

$$\hat{x}(0) = \begin{bmatrix} -0.2 & 0 & -0.5 & 1 & 1 \end{bmatrix}^T \tag{62}$$

$$\bar{\theta}(k) = 0.2 + 0.1 \sin(0.05k) \tag{63}$$

The input sequences are chosen as follows (see Fig. 1)

$$u_1(k) = \begin{cases} 0 & \text{if } k \in [500, 1500] \\ 5\sin(0.001k) & \text{else} \end{cases} \tag{64}$$

$$u_2(k) = -5\cos(0.0046k) \tag{65}$$

As shown in Fig. 2 and Fig. 3, at the time instant $k = 500$, $u_1(k)$ becomes zero. As a result, condition (35) is not satisfied anymore. However, the estimation errors have already converged to zero, and no convergence problems arise. On the other hand, when the fault appears at sample $k = 1000$, the lack of excitation of the input causes the multiplicative fault in the first actuator γ_1 not to be correctly estimated. Consequently, the state estimation errors and the Lyapunov functions do not converge to zero (see Fig. 4). However, when satisfactory input sequences enter into the system, the switched LPV observer behaves as expected, such that both the faults and the states are correctly and rapidly estimated.

Notice that, although in the proposed scenario $\gamma_i(k+1) = \gamma_i(k)$, $i = 1, 2$, does not strictly hold at some samples ($k = 1000$, $k = 2000$, $k = 3000$ and $k = 4000$), the proposed approach is able to correctly achieve its goal of jointly estimating the states and faults in the system.

5. Conclusions

This paper has proposed a method for estimating simultaneously the states and the actuator faults in discrete-time LPV systems, using a switched LPV observer. Differently from the existing approaches that consider additive faults, the proposed observer is able to estimate multiplicative faults. The proposed approach considers switching rules between different regions and design LMIs that take into account the properties of observability and non-observability of different regions. Sufficient conditions to design the observer gains were provided in the form of a set of LMIs. Moreover, it has been shown that if the input sequence has some characteristics in terms of numbers of switching in an interval, and average number of samples per switch in every switching region, the convergence of the error dynamics to zero is assured. Simulation results have shown the relevant characteristics of the proposed method in terms of fault estimations and state estimation errors, validating the proposed methodology.

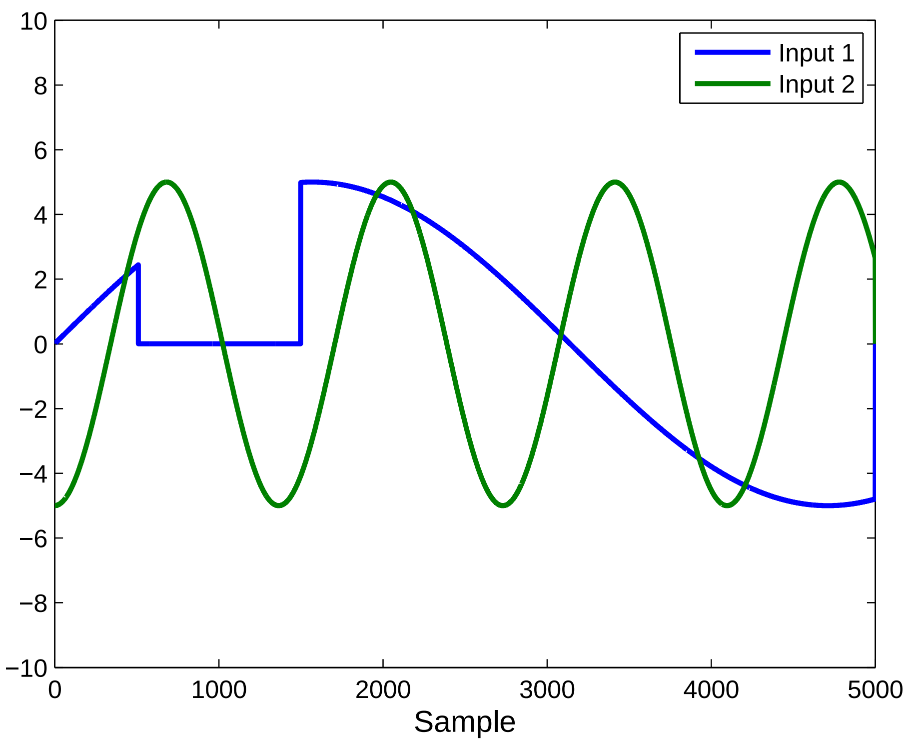

Figure 1: Applied inputs $u_1(k)$ and $u_2(k)$.

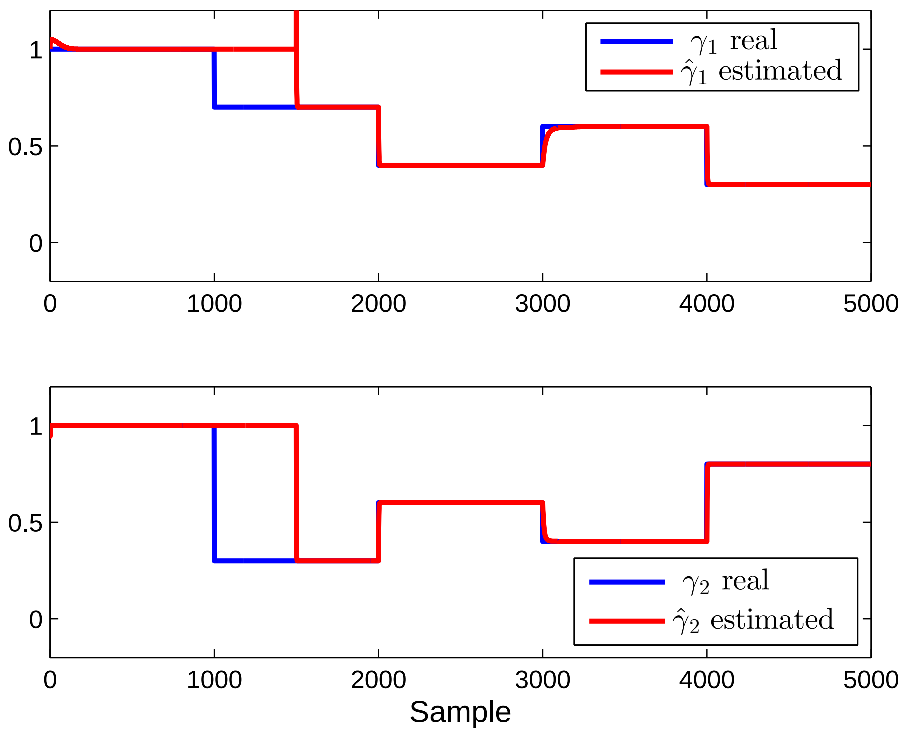

Figure 2: Faults and their estimation.

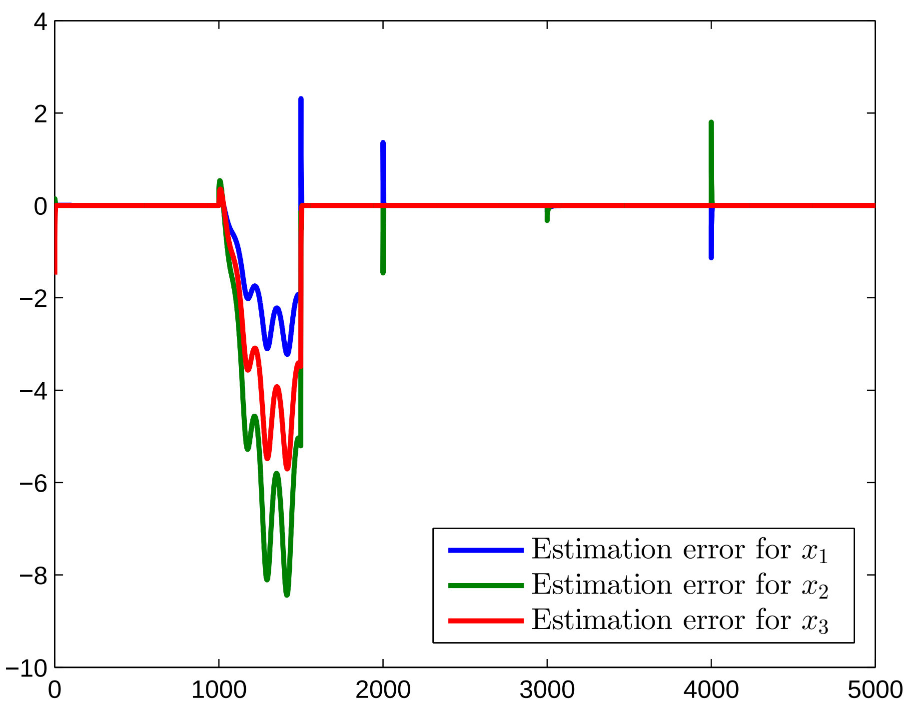

Figure 3: State estimation errors.

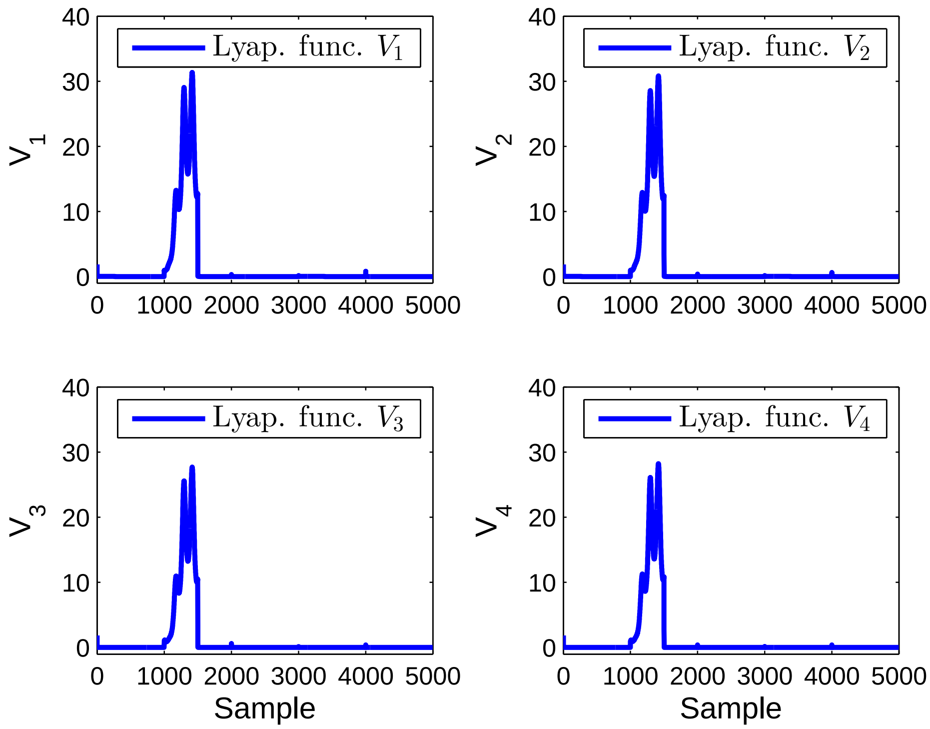

Figure 4: Lyapunov functions.

Acknowledgements

This work has been funded by the Spanish Ministry of Science and Technology through the projects CICYT SHERECS (ref. DPI2011-26243) and CICYT ECOCIS (ref. DPI2013-48243-C2-1-R), by AGAUR through the contracts FI-DGR 2014 (ref. 2014FI_B1 00172) and FI-DGR 2015 (ref. 2015FI_B2 00171), by the DGR of Generalitat de Catalunya (SAC group Ref. 2014/SGR/374), and by the National Council of Science and Technology (CONACyT) of Mexico. The supports are gratefully acknowledged.

[1] Y. Zhang, J. Jiang, Bibliographical review on reconfigurable fault-tolerant control systems, Annual Reviews in Control 32 (2008) 229–252.

[2] I. Hwang, S. Kim, Y. Kim, C. Seah, A survey of fault detection, isolation, and reconfiguration methods, IEEE Transactions on Control Systems Technology 18 (2010) 636–653.

[3] I. Samy, I. Postlethwaite, D.-W. Gu, Survey and application of sensor fault detection and isolation schemes, Control Engineering Practice 19 (2011) 658–674.

[4] C. Edwards, S. K. Spurgeon, R. J. Patton, Sliding mode observers for fault detection and isolation, Automatica 36 (2000) 541 – 553.

[5] C. P. Tan, C. Edwards, Sliding mode observers for detection and reconstruction of sensor faults, Automatica 38 (2002) 1815 – 1821.

[6] X.-G. Yan, C. Edwards, Nonlinear robust fault reconstruction and estimation using a sliding mode observer, Automatica 43 (2007) 1605 – 1614.

[7] Z. Gao, S. X. Ding, Actuator fault robust estimation and fault-tolerant control for a class of nonlinear descriptor systems, Automatica 43 (2007) 912 – 920.

[8] B. Marx, D. Koenig, J. Ragot, Design of observers for Takagi-Sugeno descriptor systems with unknown inputs and application to fault diagnosis, IET Control Theory And Applications 1 (2007) 1487–1495.

[9] A. Cristofaro, T. A. Johansen, Fault tolerant control allocation using unknown input observers, Automatica 50 (2014) 1891 – 1897.

[10] D. Ichalal, B. Marx, J. Ragot, D. Maquin, Fault detection, isolation and estimation for Takagi-Sugeno nonlinear systems , Journal of the Franklin Institute 351 (2014) 3651 – 3676.

[11] S. K. Nguang, P. Shi, S. X. Ding, Fault detection for uncertain fuzzy systems: an LMI approach, IEEE Transactions on Fuzzy Systems 15 (2007) 1251–1262.

[12] Y. Zhao, J. Lam, H. Gao, Fault detection for fuzzy systems with intermittent measurements, IEEE Transactions on Fuzzy Systems 17 (2009) 398–410.

[13] J. Chen, R. J. Patton, Robust Model-Fault Diagnosis for Dynamic Systems, Springer, 1999.

[14] C. P. Tan, C. Edwards, Multiplicative fault reconstruction using sliding mode observers, in: 5th Asian Control Conference, volume 2, pp. 957–962.

[15] J. S. Shamma, Analysis and design of gain scheduled control systems, Ph.D. thesis, Massachussets Institute of Technology, Department of Mechanical Engineering, advised by M. Athans, 1988.

[16] J. S. Shamma, An overview of LPV systems, in: J. Mohammadpour, C. Scherer (Eds.), Control of Linear Parameter Varying Systems with Applications, Springer, 2012, pp. 3–26.

[17] B. Lu, F. Wu, Switching LPV control designs using multiple parameter-dependent Lyapunov functions, Automatica (2004) 1973–1980.

[18] P. Apkarian, P. Gahinet, G. Becker, Self-scheduled H_∞ Control of Linear Parameter-Varying Systems: A Design Example, Automatica 31 (1995) 1251 – 1261.

[19] H. Habib, M. Rodrigues, C. Mechmeche, D. Theilliol, Fault detection and isolation for linear parameter varying descriptor systems via proportional integral observer, International Journal of Adaptive Control and Signal Processing 26 (2012) 224–240.

[20] M. Chadli, S. Aouaouda, H. Karimi, P. Shi, Robust fault tolerant tracking controller design for a VTOL aircraft, Journal of the Franklin Institute 350 (2013) 2627–2645.

[21] R. A. Horn, C. R. Johnson, Matrix Analysis, Cambridge University Press, 1990.

[22] J. Löfberg, YALMIP : a toolbox for modeling and optimization in MATLAB, in: Proceedings of the CACSD Conference, Taipei, Taiwan.

[23] J. F. Sturm, Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones, Optimization Methods and Software 11-12 (1999) 625–653.