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## **FORESTRY AND THE CLIMATE PROBLEM. JOINT TIMBER AND BIOENERGY PRODUCTION**

by

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### **Abstract**

This paper discusses and contrasts two main roles of forestry in light of the debate on the global climate. As the main problem is related to the increases of the CO<sub>2</sub>-concentration in the atmosphere, forests may be viewed as part of the alleviation of the problem through their function as (i) carbon storage, since a growing forest extracts atmospheric CO<sub>2</sub> and fixes it as carbon in biomass, and as (ii) a source of biomass for energy production, which may replace fossil fuels and thus reduce CO<sub>2</sub>-emissions. The addition of the carbon storage value to the traditional focus on the forest's timber value is well known in the economics' literature, and its main effect is to increase the rotation length of the forestry. In this paper we also add the value of forests as source of bioenergy and discuss the effect of this addition on the optimal rotation length within the traditional Faustmann model. Formulated as a joint production model, and based on data for spruce, the dominant species in the Scandinavian forest, it is demonstrated that the rotation length is shortened compared to the standard Faustmann model. Utilisation of forest biomass for energy production thus comes with a cost relative to the climate problem, in that the resulting shortening of the optimal rotation length implies less carbon sequestration. The empirical consequences of this model expansion are discussed.

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## 1. BACKGROUND

The forest is a fundamental natural resource that represents values for many aspects of human development. Traditionally, forests have been a provider of energy for heating and cooking, building materials, protection and cooling, habitat for huntable and harvestable species, to mention just a few examples. In addition to these more direct uses, there has been a growing recognition of the more indirect services supplied by forests. These range from the aesthetical and recreational values of a living forest to the role of the forests in the question of biodiversity and general life support functions. We further know the forest may contribute to the stabilisation of the local “climate”, both as a moisture retainer and as a wind breaker, in addition to the provision of flood prevention services. For a general overview, see, e.g., van Kooten and Bulte (2000).

On the grand scale, the forest is an important part of the carbon cycle. Relative to the observed increase in the atmospheric concentration of CO<sub>2</sub> and the global climate question, one may identify two main functions of forests. First, by expanding forest biomass, one achieves an increased volume of carbon stored in other forms than CO<sub>2</sub>. This may be achieved through an expansion of forests, either by planting currently unforested land, or by allowing the existing forests to accumulate greater amounts of biomass. The second main approach is to utilise the forest directly as a source of raw materials for energy production, usually referred to as bioenergy, which is considered a carbon neutral energy source. Use of bioenergy represents a positive contribution towards the climate problem if it replaces fossil fuels like coal, oil, or gas (again, see van Kooten and Bulte 2000).

These two different approaches to forests in the climate question will be analysed in this paper. A financially motivated optimal forestry rotation model traditionally focuses on the timber value of the forest, like in the classical Faustmann model (Samuelson 1976). The inclusion of the value of carbon storage or/and supply of energy raw materials represents an extension of this traditional forestry model. A body of literature exists that discusses aspects of the carbon sequestration problem. These models are basically a variation of the multiple use forestry modelling suggested by Hartman (1976). One main effect of the inclusion of the value of carbon sequestration, is an extension of the optimal rotation length (see, e.g., van Kooten *et. al* 1995).

The alternative to carbon sequestration, direct use of forest biomass for energy production, can take many forms. Sedjo (1997) describes one possible approach suitable for the developed countries. Forest land previously occupied by a traditional forestry operation could be replanted with an energy crop. The economic viability of such an operation rests on two assumptions. First, the energy crop must generate economic returns to the land that are at least as high as the returns from the traditional use of the land (timber production). Second, the energy crop, or it's derived energy product, must sell in the energy market at a price low enough to compete with the relevant competing energy sources (like coal). This approach represents a clear break between the traditional forestry and the new energy production operation.

In the Scandinavian countries, particularly in Sweden and Finland, and in a more initial phase in Norway, however, the introduction of wood based bioenergy seems to be in accord with a different logic. Instead of switching the production from traditional forestry to energy crops, there has been a gradual increase in the bioenergy production using as raw materials *secondary products* from the traditional forestry operations. This has taken place as harvesting

residues like limbs and tree tops, possible also low quality timber, are gathered and stored in the forest in such a way that this biomass may conveniently be picked up and transported to further processing for energy production purposes. Such production systems are currently developing in the Scandinavian countries, and in 1999 energy from biomass accounted for 25 % of Finland's total domestic energy consumption while the shares in Sweden and Norway were 16 % and 6 %, respectively (Statistics Finland 2000, Statistics Sweden 2000, Statistics Norway 2000)<sup>1</sup>. This Scandinavian perspective will be at the focus here.

A fundamental postulate for this emerging production system for bioenergy in the Scandinavian countries, in which forestry is important, is that timber and energy biomass are produced *jointly*. In a situation where the forest owner faces a demand for woody bioenergy raw materials, and the net stumpage price for the energy part of the tree is positive, then it will be economically rational for the forest owner to engage in the joint timber and energy production. In this paper we will add the value of the energy biomass of the tree to the timber value. The specific question, not analysed before, is how the presence of joint production affects the timing of the forest harvest within the Faustmann framework, and what the consequences of such an adjustment of the rotation length might be in light of the climate problem. If the rotation length increases, the inclusion of bioenergy in the harvesting decision means a 'double dividend' effect within the climate context as the increased standing volume of the forest fixes more carbon in biomass while, at the same time, the advantage of the potentials of forest biomass to replace fossil energy sources is present. However, as will be shown, the optimal rotation length will be somewhat shorter compared to the standard model of no joint production for spruce (*Picea abies*), the dominant Scandinavian species. Increased use of forest biomass for energy production therefore comes with a cost in terms of the CO<sub>2</sub> balance.

The rest of the paper is organised as follows. First, we sketch the classical approach for addressing the problem of optimal harvesting time in forestry. Next, in section three, we take a closer look at the biology of forest production in order to expand the classical forestry (timber) production function to also account for the energy part of the tree. In section four we then analyse how the optimal timing within the Faustmann model is affected by this inclusion of bioenergy. In section five the theoretical results are illustrated by some numerical examples. In section six these findings are related to the role of forests in the climate debate. A benefit function for carbon sequestration is introduced, and the harvesting decision balancing the private value of timber and bioenergy together with the social value of carbon sequestration is analysed.

## 2. THE TRADITIONAL APPROACH

As a background for the subsequent analysis, we first briefly review the classical forestry problem where an even aged stand of trees grows on a given piece of land. A standard s-shaped biological growth function for timber is assumed, where  $s(t)$  is the volume (or mass) of the stem, measured in some physical unit, and where  $t \geq 0$  is the age of the stand (or the time in years after planting). General properties of this production function are discussed, among others, by Clark (1990).

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<sup>1</sup> However, not all of this energy biomass originates in the forest; some is peat and other biomass.

When assuming the timber price to be fixed over time at unity, so that  $s(t)$  also yields the stumpage value of the forest, the timing problem of the forest owner is to find the age of the forest that maximises the present-value profit  $V(t) = -c + s(t) \cdot e^{-rt}$ .  $r \geq 0$  is here the rate of discount, i.e., the opportunity cost of rival assets, and  $c$  is the planting costs, measured in terms of the timber price. The first order condition for this problem yields  $\dot{s} = r \cdot s(t)$ , where  $\dot{s} = ds/dt$ , and states that the forest should be harvested when the increment in value of the forest due to biological growth equals the opportunity cost of the value of the forest.

This simple model is valid only when there is one harvest of the forest, which is equivalent to saying that forest land is in abundant supply with a zero land rent. If this is not the case, which is usually true, the forest owner must consider the cost of postponing future revenues from the forest land in addition to the interest on the timber capital. If we assume a forest operation with an infinite number of harvesting rotations, the problem becomes one of maximising

$V(t) = \sum_{m=0}^{\infty} [-c + s(t) \cdot e^{-rt}] \cdot e^{-mrt}$  where  $m$  is the rotation number, and hence,  $t$  is the rotation

length, assumed to be equal for all rotations. The objective function can be rewritten as

$V(t) = \frac{-c + s(t) \cdot e^{-rt}}{1 - e^{-rt}}$ , and the first order condition for maximum yields

$$\dot{s}(t) = r \cdot s(t) + r \cdot \frac{-c + s(t) \cdot e^{-rt}}{1 - e^{-rt}}.$$

As in the single stand model, the left hand side of the optimum condition,  $\dot{s}(t)$ , represents the natural value increment of the forest. On the right hand side,  $r \cdot s(t)$  is the interest that could have been earned if the forest had been harvested, sold, and invested at time  $t$  while  $r[-c + s(t) \cdot e^{-rt}] / (1 - e^{-rt})$  is the interest of the value of all future rotations of the forest. The right hand side of the optimum condition therefore represents the alternative cost of not harvesting *and* replanting the forest at a given point in time. This is the so-called Faustmann model for optimal rotation length of a forestry operation.

This analysis of the optimal rotation period of a forest has been source of much dispute, both within the discipline of economics and between economists and biologists, even though the correct solution, as described above, was found by Martin Faustmann in the mid 1800's. Samuelson (1976) gives a clear account of the various problems and simplifications related to the model. See also the overview by Reed (1986). Applications of the Faustmann model in empirical work have not proven overly successful. As a result, the strict assumptions of the model have been relaxed, for example by allowing for variations in prices and introducing functional and/or stochastic characteristics of the parameters in the model. In addition, there have also been attempts to add the non-timber values of the forest to the model, such as flood control services, wildlife habitat, recreational services, etc. More recently, as already mentioned, the role of forestry in climate policy has also been analysed, see, e.g., van Kooten and Bulte (2000) for a discussion. In spite of all these shortcomings, however, we choose to develop our joint timber and bioenergy harvesting model within the Faustmann framework. One important reason for this is that the Faustmann model is well known in the literature and thus serves well as a benchmark against which to compare our results.

### 3. THE BIOLOGY OF THE JOINT ENERGY/TIMBER PRODUCTION

The introduction of energy biomass into the production function for the forestry operation complicates matters somewhat. A fundamental question is how to handle the production of the biomass part of the tree in a meaningful way, a problem that apparently has not received much attention in the literature. One way to approach it is suggested by Clarke and Shrestha (1986) in their development of a model for the management of heterogeneous biomass resources. Within a rotation period, the biomass of a single species develops a “high grade” type and a “low grade” type of biomass. For our purposes these could be timber and branches (energy), respectively. The relative composition of the two grades depends, among other factors, on the stocking density of the biomass. Since the “high grade” and the “low grade” biomass sell at different prices, one important management objective is to choose an initial stocking and subsequent restocking and harvesting regime that yields the highest return over time of the biomass. Harvesting parts of the tree for bioenergy at a younger age (or multiple younger ages) and the whole tree for both timber and energy biomass at the end of the rotation could be one solution, see Li and Löfgren (2000) for an example of this approach applied to seed and timber production on red pine. A closely related problem is the development of optimal thinning practices, see Näslund (1969). In considering the different deviations from the standard practices and analyses of forestry, together with the increasing complexity indicated by these examples, the possible existence of nonconvexities in the benefit function must be kept in mind, as pointed out by Swallow *et. al* (1990).

In spite of the wide range of theoretically possible practices for bioenergy production, we choose to base our analyses on the observed current practices in Scandinavian forestry. This development indicates that it is reasonable to view timber and bioenergy production closely linked, as it is the traditional harvesting residues (branches and tops) that are used as raw materials for bioenergy production. The bioenergy part of the tree is an inevitable result of harvesting timber since all trees have branches and tops not suitable for timber production. We can therefore view the production of bioenergy as a function of the biological composition of the tree. Moreover, as this energy part of the tree, for obvious reasons, must be harvested simultaneously with the timber part, bioenergy and timber represent a type of *joint production*. However, just like lamb and wool, this joint production is not of the fixed coefficient type since the composition of bioenergy mass and timber changes over time. This functional relationship, and the changing composition of the tree, is fundamental in the following model.

In the above discussion of the standard Faustmann model,  $s(t)$  denoted the production function for the stem (timber) of a forest, measured in some physical unit, like kg or  $\text{m}^3$ . Let now  $f$  represent the mass of the energy part of the tree. In order to incorporate the energy part of the tree into the standard model, we search some function  $f = f(s(t))$  to represent this relationship; that is, the biological composition of the tree, or the even stand. A study performed by Marklund (1988) is of great help for specifying this relationship. Based on data obtained from 1286 single trees in 131 stands from various parts of Sweden, Marklund estimated biomass functions for spruce, pine, and birch using characteristics of the tree and the biotope in which the tree grows as explanatory variables. It turned out that the composition of a tree is well explained by just one independent variable, namely the breast height diameter of the tree.

Spruce (*Picea abies*) is the dominant species in the Scandinavian forest, and will be used as the basic reference in the subsequent analysis. The dry mass of the stem, or timber part of the

tree  $s$  measured in kg, is estimated by Marklund as

$$(1) \quad s(d) = e^{11.3341 \frac{d}{d+14} - 2.0571}$$

for spruce where  $d$  is the diameter in cm at breast height<sup>2</sup>.

The bioenergy, or fuel, part of the tree is defined to be the dry mass of the living branches and the dead branches less the dry mass of the needles, also measured in kg. Again as a function of the breast height diameter, this functional form was estimated as<sup>3</sup>

$$(2) \quad f(d) = e^{8.5242 \frac{d}{d+13} - 1.2804} + e^{9.9550 \frac{d}{d+18} - 4.3308} - e^{7.8171 \frac{d}{d+12} - 1.9602}.$$

The first of the three terms of (2) represents the dry mass of the branches including needles, the second represents the mass of the dry (or dead) branches, while the last, which is being subtracted, is the dry mass of the needles. This formula implies that the needles are not used for energy production. Figure 1 graphs the timber and the bioenergy functions. It reveals that as the tree grows in terms of increasing diameter at breast height, the timber mass  $s$  increases. The mass of the energy part of the tree  $f$  increases as well, but at a lower rate than the timber part.

### Figure 1 about here

In equations (1) and (2) both  $s$  and  $f$  are functions of  $d$ . By combining these relationships the mass of the bioenergy part can be expressed as a function of the mass of the stem. This function is plotted in Figure 2. Hence, according to the empirical observations by Marklund, we infer that as the stem grows, the mass of the energy part of the tree also grows, but to a smaller extent. Thus, according to the Marklund study, the energy parts of the Scandinavian spruce forest can be functionally attributed to the stem as

$$(3) \quad f = f(s) = f(s(t))$$

with  $f(0) = 0$ ,  $f'(s) > 0$ , and  $f''(s) < 0$ , so that  $f$  is a strictly concave function. By approximating it with the Cobb-Douglas type production function  $f = Bs^g$  and estimating the coefficients, we obtain  $B = 0.805$  and  $g = 0.800$  (for details see the Appendix). This specific functional form is used in the numerical simulations below.

### Figure 2 about here

## 4. OPTIMAL HARVESTING WHEN JOINT PRODUCTION

The production function for timber  $s(t)$  together with the relationship between timber and energy growth of the forest given by equation (3) define the joint timber and bioenergy

<sup>2</sup> Using OLS, the t-value for  $d$  was 207.4 while the t-value for the constant term was 76.8. The overall degree of fitness was  $R^2 = 0.988$ .

<sup>3</sup> This relationship was estimated in the same manner as equation (1), again with high t-values and a high degree of overall fitness.

production function. Based on these biological growth relations we proceed to analyse the economic implications of the joint production within the Faustmann framework. The situation for the forest owner is therefore that he now finds the forest producing two products, timber and raw materials for bioenergy production. Moreover, the forest owner knows that it is necessary to harvest the two products at the same point in time. Let  $p$  represent the per unit market stumpage value of energy mass *relative* to the stumpage value of timber. Using the stumpage value for timber as the numeraire,  $f$  and  $s$  are to be interpreted as the value of the energy and timber, respectively, evaluated at the price for timber. The relative price of biomass production is restricted to  $0 < p \leq 1$  as the timber is more valuable than bioenergy raw materials<sup>4</sup>. The question is then how the inclusion of energy mass in the production function of the forest will influence the forest owner's optimal harvesting decision. We first analyse the timing problem within the context of a single harvest.

### Single harvest

The forest owner's objective is now to find the time  $t$  that maximises the present-value joint production

$$(4) \quad VJ(t) = -c + [pf(s(t)) + s(t)]e^{-rt}$$

where the planting, or the regeneration cost,  $c$ , is assumed to be the same as in the production of timber only. The first order condition for this problem reads

$$(5) \quad [pf'(s(t)) + 1]\dot{s}(t) = r(pf(s(t)) + s(t))$$

which implicitly determines the optimal harvesting time  $t^*$ . The interpretation of this optimality condition is just as in the Faustmann model, saying that it is optimal to harvest when the marginal value of the stand due to biological growth, now comprising timber mass as well as raw material for energy production, i.e.  $d(pf + s)/dt$ , is equal the opportunity cost of the standing forest capital. The second order condition says that  $[(pf' + 1)\ddot{s} + (pf''\dot{s} - rpf' - r)\dot{s}] < 0$  should hold at the optimal harvesting time. Hence, concavity of  $f(s)$  contributes in the direction of securing that (5) represents a maximum.

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<sup>4</sup> For an individual forest owner  $p$  is determined by two main factors. One is the costs of collecting, transporting and preparing the logging residues for sale. These costs depend on the internal characteristics of the specific forestry operation, such as topography, transportation lengths, mechanical solutions, etc. The other factor is the market price facing the forest owner as a supplier of energy raw materials (Bjørnstad 1999). Such a market price is a function of the general energy price level and the existence of and closeness to a district heating plant or some other purchaser of bioenergy raw materials. Obviously, if the forest owner is located in an area with no demand for logging residues,  $p$  equals zero. This has traditionally been the situation in most of Scandinavia. However, the increasing share of bioenergy in the heating market indicates that there is an increasing proportion of forestry operations facing a positive  $p$ . Still, even with increasing demand for bioenergy, energy bio-mass from forestry must be considered a secondary product in terms of stumpage value, i.e. we have the restriction  $0 < p \leq 1$ . If  $p$  should be above 1, the forest owner would simply adjust to the new market situation by selling the whole tree, including the stem, as energy mass. That is, the economic problem would revert to a single product optimisation problem, but with  $f + s$  instead of  $s$  as the production function.

The central issue now is how the optimal harvesting time is influenced by the biomass joint production; that is, by the price parameter  $p$ . Differentiating the optimality condition (5) yields

$$(6) \quad \frac{dt^*}{dp} = \frac{(rf - f's)}{[(pf' + 1)\dot{s} + (pf''s - rpf' - r)\dot{s}]}$$

after some rearrangements. The denominator is equal to the second order condition for the maximization of (4), and is thus negative.  $dt^*/dp$  is therefore negative if  $rf > f's$  holds. Hence, the optimal harvesting time will be lower compared to the standard model of no joint production if the opportunity cost of the energy mass part of the stand is greater than the value increase of the energy mass due to biological growth, evaluated at the timber price. Moreover, under this condition, when joint production, the harvesting time will decrease if  $p$  shifts permanently up.

When using the optimality equation (5), the condition of  $dt^*/dp < 0$  can be given another interpretation as we then obtain  $rf > f's = f' \frac{r(pf + s)}{(pf' + 1)}$ . After some rearrangements this yields  $(f/s) > f'$ , or  $(f(s)/s) > f'(s)$ . The condition is therefore straightforward as it says that if the *average* energy mass to stem ratio is greater than the *marginal* value, the optimal harvesting time will decrease as the value of the energy mass  $p$  increases. We therefore have that  $dt^*/dp < 0$  if  $f(s)$  is a strictly concave function. As analysed above, this is just what the functional form of the Marklund (1988) study suggests. The conclusion is therefore that the optimal harvesting time will be lower compared to the standard model of no joint production for spruce, the dominant Scandinavian species. Moreover, when joint production of spruce, the harvesting time will be driven down if the biomass part of the stand becomes permanently more valuable. This holds for all values of  $p$  in the interval  $0 < p \leq 1$ .

The above finding may be given an intuitive interpretation. The addition of the value of the biomass to the value function of the timber causes the total value function to shift upwards. A concave bioenergy function means that the most of the growth of the energy mass is achieved "early" relative to the growth of the timber function. The implication of this is that the point in time where the value of the natural growth of the total value function becomes smaller than the opportunity cost of the standing forest, appears earlier than in the case of only timber production. A linear bioenergy production function would be neutral relative to the optimal harvesting time, while a convex function would delay harvesting in the single harvest case.

### Optimal rotation length

Assuming an infinite time horizon for the forestry operation, as briefly discussed above, the problem becomes one of finding the optimal rotation length for the infinitely many identical planting/harvesting rotations. Hence, the problem is now to maximise

$$(7) \quad VJ(t) = \frac{-c + [pf(s(t)) + s(t)]e^{-nt}}{1 - e^{-n}}$$

The optimal rotation time  $t^*$  is given implicitly by the first order condition



$$(8) \quad (pf'(s(t)) + 1)\dot{s}(t) = r(pf(s(t)) + s(t)) + r \left[ \frac{-c + (pf(s(t)) + s(t))e^{-rt}}{1 - e^{-rt}} \right].$$

The interpretation of this condition is analogous to the solution of the standard Faustmann model. The left hand side, just as in the single harvest model, represents the growth of the value of the stand, now comprising biomass and timber production. At the optimal harvesting time this must be equal to the opportunity cost of the standing forest plus the opportunity cost of all future rotations of the forest. As in the standard Faustmann model,  $t^*$  is reduced compared to the single harvest case.

Again we are interested in analysing how the optimal harvesting time, or the optimal rotation length, is influenced by the value of the biomass. Differentiating with respect to  $p$  now yields

$$(9) \quad \frac{dt^*}{dp} = \frac{rf - f'\dot{s} \cdot (1 - e^{-rt})}{(1 - e^{-rt}) [pf''(\dot{s})^2 + (pf' + 1)\ddot{s} - r(pf' + 1)\dot{s}]}$$

after some manipulations and rearranging. As the denominator in (9) is negative due to the second order condition for maximum (see the Appendix),  $dt^*/dp$  is negative if  $rf > f'\dot{s} \cdot (1 - e^{-rt})$  holds. This condition is similar to the single harvest model except for the term  $(1 - e^{-rt})$ . Substituting for the first order condition (8) and rearranging, it reads  $\frac{f}{s} > f' - \frac{c \cdot f'}{s}$ .

This condition relates directly to the properties of  $f(s)$ , and it obviously holds when the bioenergy production function is concave. Hence, based on the empirical observations by Marklund (1988), the conclusion is again that the optimal rotation length will be shorter compared to the standard model of no joint production for spruce, the dominant Scandinavian species. The condition for obtaining this result is now, however, somewhat more relaxed compared to the single stand model as  $f(s)$  no longer needs to be strictly concave. Hence, if linear, we still have that  $dt^*/dp < 0$  holds, and this will be true even if the function is weakly convex. It is also seen that the regenerating costs now influence the outcome directly, and a higher  $c$ , *ceteris paribus*, works in the direction of a lower rotation length as the energy biomass becomes more valuable.

## 5. NUMERICAL ILLUSTRATIONS

The above theoretical reasoning will now be illustrated by some numerical examples. The simulations will indicate the significance of the analytical results, and demonstrate the robustness of the findings against variations in the parameters in the theoretical model. First, we present the estimates of the stem- and bioenergy functions and calculate the optimal rotation based on those estimates. Next, we allow for changes in the biological parameters and assess the effect of these changes on the optimal rotation.

van Kooten *et al.* (1995) suggest that the functional form of the natural growth function of the timber may be specified as

$$(10) \quad s(t) = At^h e^{-kt}$$

where  $A$ ,  $h$  and  $k$  all are positive parameters. Applying this functional form and using forestry data from Norway for spruce (*Picea abies*), the parameters are estimated as  $A = 1.21 \cdot 10^{-4}$ ,  $k = 0.0228$ , and  $h = 3.824$  when  $s(t)$  is measured in  $\text{m}^3$  per ha and  $t$  in years. This function displays the three stages of the s-shaped growth function with a maximum value of  $849 \text{ m}^3$  at  $t = h/k = 168$  years, while the inflection point is at  $t = 82$  years (for more details, see the Appendix).

Next, we have the biomass function. Based on the analysis of the Marklund (1988) data, this function is, as already mentioned, approximated by a Cobb-Douglas type function

$$(11) \quad f = Bs^g$$

with values  $B = 0.805$  and  $g = 0.800$  (again, more details given in the Appendix)<sup>5</sup>. Based on the estimated functional forms of  $s$  and  $f$  we then have, as an example, that a stand at age 70 years will consist of  $279 \text{ m}^3$  of timber per ha and  $73 \text{ m}^3$  of energy biomass.

In addition to the biological data, we need cost and price data. The planting cost of a stand is assumed to be approximately NOK 7000 per ha (in 1999-prices) while the timber price, net of cutting costs, i.e., the stumpage price, is set to NOK 350 per  $\text{m}^3$  (NILF 2000). As the stumpage price is used as a numeraire, the corresponding planting cost  $c$  used in the calculations is  $7000/350 = 20$ . As discussed above, the biomass value  $p$  is likely to vary substantially among the different forestry operations. As a rough illustration, sampled market prices indicate that wood chips are sold at NOK 0.12 – 0.16 per kWh (also in 1999-prices) (NoBio 2001). Assume costs per kWh of producing chips from NOK 0.05 and up (Bjørnstad 1999). As  $1 \text{ m}^3$  of spruce biomass is equivalent to around 2000 kWh of energy (Norsk Treteknisk Institutt 1991), these prices and costs imply a  $p$  in the range between 0 and 0.63.  $p$  together with the discount rate  $r$  are crucial for the numerical results, they will therefore be varied throughout the simulations. In the baseline calculations we use  $p = 0.5$  and 5 % rate of discount,  $r = 0.05$ .

Table 1 gives the optimal rotation length for various values of  $p$  and  $r$ . According to the theoretical reasoning,  $t^*$  decreases when  $p$  shifts up when  $f(s)$ , as assumed here, is concave. The results reported in Table 1 confirm this, but the effect is quite modest and is just somewhat between 2 and 3 years when  $p$  varies between 0 and 1. Hence, when the real rate of discount is 5 %, a reduction from 55.1 years in the standard Faustmann model,  $p = 0.0$ , to 53.6 years when  $p = 0.5$ , takes place.

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<sup>5</sup> As mentioned, the Marklund study measures the various components of the tree in kg. In the numerical analysis,  $s(t)$  as well as  $f(s)$  are given in  $\text{m}^3$ . Even though there might be slight systematic differences in terms of densities of the different parts of the tree, this possibility is neglected and a one-to-one relationship between measures of dry mass and volume of the different parts of the tree is postulated. Hence, we assume the same functional relationship between the stem and the bioenergy part of the tree measured in terms of mass as well as in volume.

Table 1: Optimal rotation length  $t^*$  (years) for different combinations of the price parameter  $p$  and the real interest rate  $r$  ( $g = 0.8$ ).

		$p$						
		0.0	0.1	0.3	0.5	0.7	0.9	1.0
$r$	0.01	100.7	100.4	99.9	99.4	98.9	98.5	98.3
	0.02	82.6	82.3	81.8	81.2	80.8	80.3	80.1
	0.03	70.1	69.8	69.2	68.7	68.2	67.7	67.5
	0.04	61.4	61.1	60.5	59.9	59.4	58.9	58.7
	0.05	55.1	54.8	54.2	53.6	53.1	52.6	52.4
	0.06	50.5	50.2	49.5	49.0	48.4	47.9	47.7
	0.07	47.0	46.7	46.0	45.5	44.9	44.4	44.2
	0.08	44.3	44.0	43.3	42.8	42.2	41.7	41.5

The above specification of the biomass function  $f(s)$  is based on data from the Scandinavian spruce. However, other tree species with different biological characteristics might be of interest for joint timber and energy production. We therefore take a closer look at variations in the parameters of the specific functional form (11). We first find what happens when the function shifts through the power parameter  $g$ . Table 2 gives the results.

Table 2: Optimal rotation length  $t^*$  (years) for different combinations of the price parameter  $p$  and the biological parameter  $g$  ( $r = 0.05$ ).

		$p$						
		0.0	0.1	0.3	0.5	0.7	0.9	1.0
$g$	0.4	55.1	55.0	54.8	54.6	54.5	54.3	54.2
	0.6	55.1	54.9	54.6	54.2	53.8	53.5	53.3
	0.8	55.1	54.8	54.2	53.6	53.1	52.6	52.4
	1.0	55.1	54.8	54.2	53.7	53.3	53.0	52.8
	1.2	55.1	55.4	55.9	56.1	56.3	56.4	56.4
	1.4	55.1	58.5	60.8	61.7	62.2	62.5	62.6

According to the theoretical reasoning, the optimal rotation length should be reduced compared to the standard Faustmann model even if the biomass function is linear, i.e.  $g = 1$ . The numerical results confirm this. It is also seen that, when the energy mass function is somewhat convex, the rotation length will increase compared to the Faustmann model. However, again the effects are quite modest.

We have also studied how changes in the scale parameter  $B$  of the bioenergy function (11) influence the optimal timing. We find that  $t^*$  decreases when  $B$  increases for a given value of the biomass price  $p$ . Hence, a larger  $B$  reinforces the shortening of the optimal rotation length when  $f$  is concave or linear in  $s$ , which is the normal situation in the Scandinavian forestry. This result hinges on the fact that a higher value of  $B$  shifts the total value function of the stand upwards, and under the presence of a concave biomass function most of the growth of the energy mass will be achieved earlier compared to the growth of the timber. However, also this effect is quite modest, and when  $p = 0.5$ , shifting  $B$  from 0.4 to 1.4 reduces  $t^*$  with somewhat below two years.

## 6. JOINT PRODUCTION AND CLIMATE

The problem of joint timber and bioenergy production analysed above was set up within the framework of the role of forestry in the climate problem; that is, as carbon storage or as a source of renewable energy. Within this perspective, increased use of forest biomass for energy production comes with a cost as our extended Faustmann model indicates that it is optimal for the forest owner to shorten the rotation length when timber and raw materials for energy utilisation are produced jointly. Hence, as a consequence, the volume of CO<sub>2</sub> uptake in the forest decreases. Using a stylised framework, we will now take a closer look at this balance by introducing the social benefit of carbon sequestration.

We follow van Kooten *et. al* (1995) and assume that the benefit from carbon sequestration can be modelled as a function of the *change* in the amount of carbon stored in the forest<sup>6</sup>. Hence, according to our joint production model, the carbon uptake at a given point of time is governed by  $(f's + \dot{s})$ . Like van Kooten *et al.*, we also assume a *fixed* parameter  $\mathbf{a} > 0$  to represent the translation from increases in forest biomass into sequestered tonnes of CO<sub>2</sub>. Let further  $q_a > 0$  be the shadow price, or *social value*, of carbon removed from the atmosphere in terms of the stumpage value of timber. Then  $q_a \mathbf{a} (f's + \dot{s}) = q(f's + \dot{s})$  represents the social value of removal of CO<sub>2</sub> from the atmosphere at a given point of time. Hence,

$q \int_0^t [f'(s(\mathbf{t}))\dot{s}(\mathbf{t}) + \dot{s}(\mathbf{t})] e^{-r\mathbf{t}} d\mathbf{t}$  gives the social value of reduced future costs, in terms of the

timber price, of removal of CO<sub>2</sub> within a single rotation. This benefit, however, has to be corrected by the value loss  $q[f(s(t)) + (1 - \mathbf{b})s(t)]e^{-r}$  when harvesting takes place.

$0 \leq \mathbf{b} \leq 1$  represents the fraction of timber that goes into long-term storage in structures and land-fills, the "pickling" factor according to van Kooten *et. al* (1995). The net social value from carbon sequestration of a single rotation in terms of the stumpage value of the timber is

accordingly  $q \int_0^t [f'(s(\mathbf{t}))\dot{s}(\mathbf{t}) + \dot{s}(\mathbf{t})] e^{-r\mathbf{t}} d\mathbf{t} - q[f(s(t)) + (1 - \mathbf{b})s(t)]e^{-r}$ . Integrating the

first term by parts and incorporating the resulting expression in the joint product model (7), we obtain the full rotation joint product model where both the private value of the forest as a source of renewable energy and timber and the social value as carbon storage are accounted for as

$$(12) \quad VJC(t) = \frac{-c + (pf(s(t)) + s(t))e^{-rt} + q[r \int_0^t (f(s(\mathbf{t})) + s(\mathbf{t}))e^{-r\mathbf{t}} d\mathbf{t} + \mathbf{b}s(t)e^{-r}]}{1 - e^{-r}}$$

When the forest owner faces the price  $q > 0$ , imposed as a carbon *subsidy*, the socially efficient harvesting time in our extended joint production model is found by maximising (12). Also within this full model, with a concave energy mass function  $f(s)$  and a non-negative

discount rate  $r$ , we find analytically that  $\frac{dt^*}{dp} < 0$  holds (see the Appendix for details). Hence,

the gain in terms of CO<sub>2</sub>-balance from increasing the share of bioenergy in the energy mix is

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<sup>6</sup> See, among others, Englin and Callaway (1993), Plantinga and Birdsey (1994) and van Kooten (2000) for alternative approaches to the carbon sequestration problem.

being diminished by the reduced level of carbon sequestration resulting from the shortened rotation length. This is confirmed by the numerical simulations shown in Table 3 which builds on the same baseline biological and economical parameters as in section 5 above. In addition, in Table 4, we have imposed a value  $\mathbf{b} = 0.5$  to illustrate the effect of the pickling factor.<sup>7</sup>

Table 3: Optimal rotation length  $t^*$  (years) for different combinations of the bioenergy price parameter  $p$  and the carbon value parameter  $q$  with pickling factor  $\mathbf{b} = 0.0$  ( $r = 0.05$ ).

		$p$						
		0.0	0.1	0.3	0.5	0.7	0.9	1.0
$q$	0.0	55.1	54.8	54.2	53.6	53.1	52.6	52.4
	0.2	65.0	64.4	63.2	62.1	61.1	60.2	59.8
	0.4	79.8	78.7	76.4	74.5	72.7	71.1	70.3
	0.6	104.3	101.9	97.3	93.6	90.3	87.5	86.1
	0.8	150.8	145.2	135.5	127.3	120.4	114.4	111.7
	1.0	279.6	256.6	221.7	197.6	179.5	165.1	159.0

Table 4: Optimal rotation length  $t^*$  (years) for different combinations of the bioenergy price parameter  $p$  and the carbon value parameter  $q$  with pickling factor  $\mathbf{b} = 0.5$  ( $r = 0.05$ ).

		$p$						
		0.0	0.1	0.3	0.5	0.7	0.9	1.0
$q$	0.0	55.1	54.8	54.2	53.6	53.1	52.6	52.4
	0.2	63.5	63.0	62.0	61.0	60.2	59.4	59.0
	0.4	73.4	72.7	71.2	69.8	68.5	67.3	66.8
	0.6	85.0	83.9	81.8	79.9	78.1	76.5	75.7
	0.8	98.6	97.0	94.2	91.6	89.2	87.0	86.0
	1.0	114.4	112.4	108.7	105.2	102.1	99.2	97.8

We demonstrated in the previous section that the use of tree tops and branches as energy raw materials implied a shortened rotation length in the amount of not more than 2 to 3 years when the forestry operation is modelled without the value of carbon sequestration (cf. also line 1 Table 3). The hypothetical case of applying such a shortening of rotation length to the entire Scandinavian forestry would probably represent a significant negative contribution to the carbon balance, but at the current rates of bioenergy utilisation this effect is no cause of concern, if at all significant. Including the value of the forest biomass as carbon storage, that is a positive  $q$ , however, causes far greater changes in the optimal rotation time. Hence, the numerical response in  $t^*$  to changes in  $p$  increases as  $q$  increases. As demonstrated in Table 3, while an increase in  $p$  from 0 to 1 causes a reduction in  $t^*$  of just 2.7 years when  $q = 0$ , a

<sup>7</sup> As mentioned, the pickling factor  $\beta$  represents the proportion of the timber harvest that is converted to products in which carbon remains "permanently" fixed in biomass. Different wood products represent different speeds through the carbon cycle. These range from bioenergy (pellets, firewood) which recycle carbon quickly, via pulp products to construction materials which recycle carbon slowly. However, almost all wood products sooner or later degrade and release the carbon content. Estimating  $\beta$  empirically therefore rises many principal questions that space limitations prevent us from discussing. Our  $\beta$  value must therefore be viewed just as an example.

similar change in  $p$  implies a decrease in the optimal rotation length of 18.2 years if  $q = 0.6$ . This clearly demonstrates that inclusion of the social value of the forest as carbon storage drives a considerable wedge between the optimal harvest time of the traditional timber only case and the case of joint timber and bioenergy production. Hence, in contrast to what follows from private optimisation with no value imposed on carbon sequestration, there is a significant difference between the socially optimal harvest time of the traditional timber only case and the case of having joint production.

The numerical results also confirm the expected result that the efficient rotation length increases when carbon becomes more valuable,  $\frac{dt^*}{dq} > 0$ <sup>8</sup>. Indeed, increasing the value of carbon sequestration quickly extends the optimal rotation length. It is shown in Table 3 that if the value of carbon is added to the forest in the classical Faustmann model (i.e.  $p = 0$ ), the rotation length would increase from 55.1 for  $q = 0$  to 279.6 years when the carbon storage value of the forest equals that of the timber,  $q = 1$ . Consequently, if the social carbon benefits of the forest are valued sufficiently high, it will be optimal to extend the rotation length very much. On the other hand, the effect of the pickling factor  $b$  is to shorten the rotation length. Comparing Tables 3 and 4 we observe that increasing  $b$  from 0 to 0.5 reduces the rotation length for all combinations of  $p$  and positive values of  $q$ .

## 7. CONCLUDING REMARKS

A sustainable future energy system will be one that is much less dependent on fossil fuels than the current one. In this paper we have taken a closer look at one element of such an energy system, namely the forest. The forest may play two main roles in the development towards a sustainable energy system. One is as carbon sink, that is the storage of carbon in the form of biomass, a strategy that will reduce the concentration of atmospheric CO<sub>2</sub>. A consequence of such a strategy will be to expand the world's forests, either in terms of increasing the area occupied by forests or by increasing the mass of the existing forest. Applied to commercial forests, the implication is an extension of the harvesting rotation length.

The other role of the forest relative to the energy/climate problem, is as direct source of energy and thereby as a substitute for fossil fuels. Such a use of the forest may take many forms, however we use as reference the current forestry practices of the Scandinavian countries. Our analysis is based on the management of a normal commercial spruce forest, which in addition to the traditional timber production also includes production of bioenergy, represented by the branches and tops of the trees. Working out this problem in the format of the Faustmann model, we demonstrate that it is profitable for the forest owner to shorten the rotation length compared to the timber only model. However, numerical simulations performed on empirically based production functions show that the magnitude of the shortening of the optimal rotation is only 2-3 years. Our results therefore indicate that there will be no dramatic effects on forestry practices in terms of rotation length as a result of jointly producing timber and bioenergy raw materials in the Scandinavian spruce forests.

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<sup>8</sup> However, it seems difficult to prove analytically that this result holds generally.

When including also the benefit from carbon sequestration, we obtain a full rotation joint product model where both the private value of the forest as a source of renewable energy and timber and the social value as carbon storage are accounted for. The analysis based on this model specification shows that an increasing carbon value contributes to rapidly increasing rotation lengths. The analysis further shows that the sensitivity of the optimal rotation length to changes in the value of energy biomass also increases, and a significant wedge appears between the optimal harvest time of the traditional timber only case and the case of joint production of timber and bioenergy.

Since accounting for CO<sub>2</sub> sequestration increases the socially optimal rotation length while accounting for the forest as a source of bioenergy shortens the rotation length, it becomes a challenge to design an appropriate forestry policy in response to the climate problem. First, in order to achieve both carbon sequestration and energy biomass supply from the Scandinavian forestry, a situation which implies extended rotation lengths, it is necessary to establish a regime of carbon payments to the forest owner. Second, due to the increasing sensitivity of the rotation length, the relative value of carbon sequestration to the value of energy biomass is important. The design of the policies and mechanisms that determine these values is therefore crucial for the actual outcome of an optimal forestry management.

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## APPENDIX

### Analytical results

Differentiating equation (7) with respect to  $t$  yields after some small rearrangements

$$(A1) \quad \frac{dVJ(t)}{dt} = \frac{e^{-nt} \left\{ [(pf' + 1)\dot{s} - r(pf + s)](1 - e^{-nt}) - r[-c + (pf + s)e^{-nt}] \right\}}{(1 - e^{-nt})^2}.$$

When using (A1), the second order condition reads

$$(A2) \quad \frac{d^2VJ(t)}{dt^2} = (1 - e^{-nt}) [pf''(\dot{s})^2 + (pf' + 1)\ddot{s} - r(pf' + 1)\dot{s}] < 0.$$

Differentiation of the FOC for maximization of the model including carbon sequestration (12) gives the result that  $\frac{dt^*}{dp} < 0$  is assured if

$$(A3) \quad \frac{f}{f'}(1 + qb) + q(f + s) + c > s(1 + qb) + q \int_{t=0}^t (f'\dot{s} + \dot{s})e^{-rt} dt.$$

The inequality (A3) holds for all non-negative discount rates.

### Numerical results

Based on the biomass functions described by Marklund (1988), predicted values for  $f(d)$  and  $s(d)$  are calculated. Given  $d$ , these functions represent "data" pairs representing  $f(s)$  plotted in Figure 2. Assuming the functional form  $f = Bs^g$ , OLS was used to estimate the parameters. This yields  $g = 0.800$  ( $t = 1396$ ) and  $\ln(B) = -0.217393$  ( $t = -70.4$ ), which implies  $B = 0.805$ .

The forestry management software BESTPROG (short for "forest stand development prognosis") was used to generate a timber growth function  $s(t)$ . This software, developed at the Agricultural University of Norway, integrates the most important biological and managerial parameters necessary for prediction of the development of the standing timber volume of a typical Norwegian stand of spruce forest. A set of parameters representative of the Trøndelag area, assuming no thinning of the forest, results in a predicted time series that is used to fit the forestry production function,  $s(t) = At^h e^{-kt}$ , cf. the main text. Based on the generated data, the estimated parameter values are as reported in the main text. This function fits the generated data well,  $R^2 = 0.997$ , with t-values for the parameter estimates ranging from 16.3 to 34.

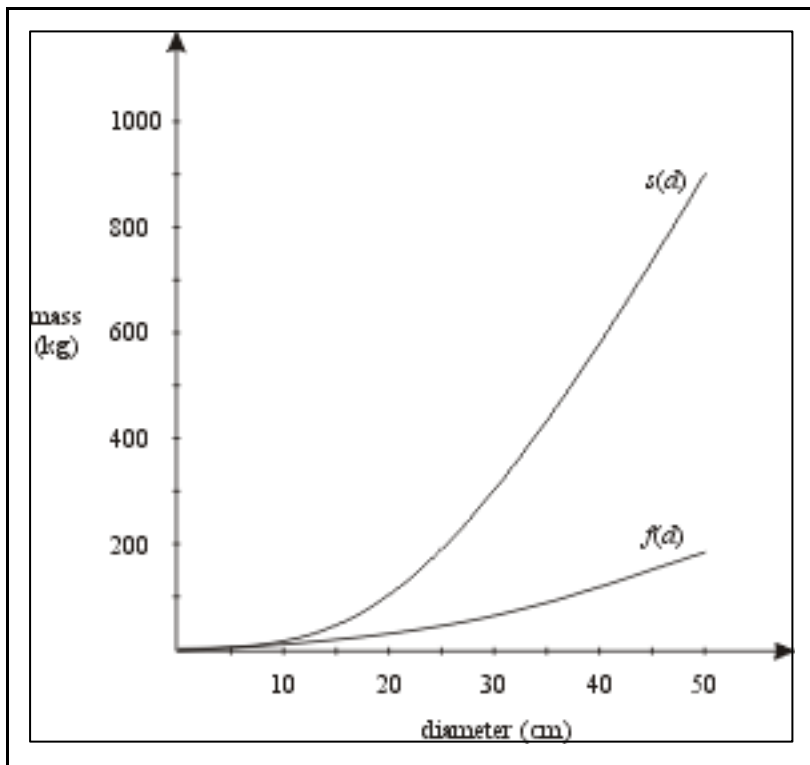


Figure 1: Dry mass of stem ( $s$ ) and bioenergy part ( $f$ ) of spruce as a function of diameter at breast height

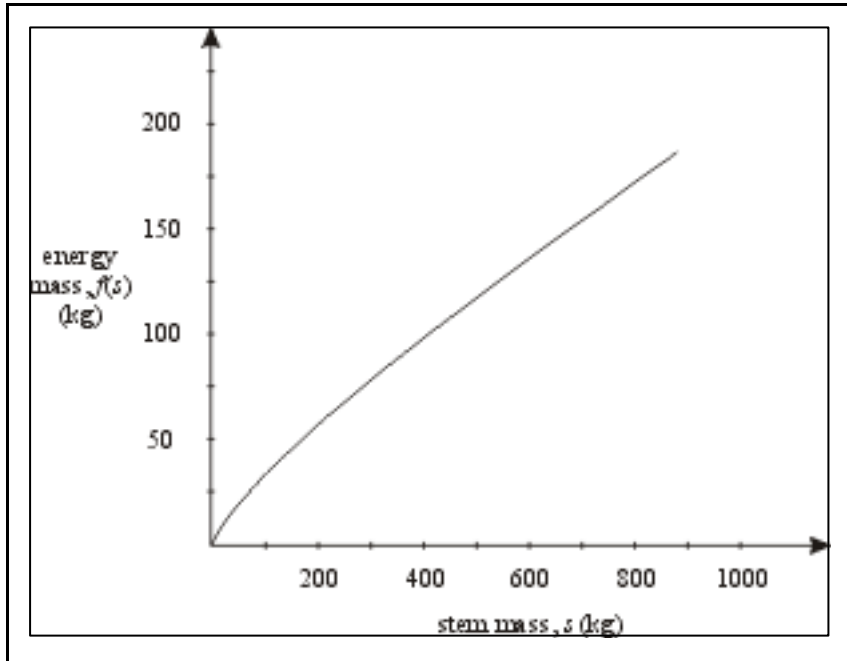


Figure 2: Relationship between the dry mass of timber ( $s$ ) and the dry mass of the energy part ( $f(s)$ ) of spruce trees.