Breaking wave interaction with tandem cylinders under different impact scenarios

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ABSTRACT

The interaction of plunging breaking waves with a pair of cylinders placed in tandem is investigated using the open-source computational fluid dynamics (CFD) model REEF3D. The model is validated using experimental data for total wave forces and free surface for breaking wave interaction with a single cylinder. Wave interaction with the tandem cylinders is investigated for four different wave impact scenarios on the first cylinder and six different distances between the cylinders in each scenario. The wave forces on the upstream cylinder are generally found to be less than the force on a single cylinder for the particular scenario. The force on the downstream cylinder is lower than the force on the upstream cylinder when the breaker tongue impacts the first cylinder. Under conditions where the breaker tongue impacts the downstream cylinder around the wave crest level, the wave force on the downstream cylinder is higher than the force on the upstream cylinder. The wave forces experienced by the tandem cylinders is highly influenced by the location of the breaking point with respect to the cylinders and the distance between the cylinders.

Keywords: breaking wave forces, vertical cylinder, tandem cylinders, CFD, computational fluid dynamics

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INTRODUCTION

The interaction of breaking wave forces on structures involves complex two-phase air-water interaction, rapid free surface deformations and an impulsive force. The short duration over which these interactions occur, pose several challenges to the evaluation of breaking wave forces. In shallow waters, the hydrodynamic loading on structures such as offshore wind turbine substructures is mostly governed by the loading due to plunging breaking waves (Alagan Chella et al., 2012). The theoretical description of breaking waves in shallow waters is rather limited up to the transition region close to breaking. The evolution of breaking process and the underlying flow physics can not be described theoretically. This is due to the simplifying assumptions of single-phase and two-dimensional flow, irrotational motion, no return flow and hydrostatic pressure made in obtaining analytical solutions (Cokelet, 1977).

The current knowledge on breaking wave kinematics are mainly based on experimental investigations. In current literature, studies for deep water breaking waves by Kjeldsen and Myrhaug (1978); Battjes and Sakai (1981); Bonmarin (1989); Rapp and Melville (1990) and Duncan (2001); studies for wave breaking on plane beaches by Stive and Wind (1982); Miller (1987); Nadaoka et al. (1989) and Ting and Kim (1994) and for wave breaking over submerged structures by Gourlay (1994); Smith and Kraus (1990) and Blenkinsopp and Chaplin (2008) are notable. While these studies focussed on the kinematics and dynamics of breaking waves, several other researchers experimentally investigated breaking wave forces on cylinders such as Goda et al. (1966); Watanbe and Horikawa (1974); Apelt and Piorewicz (1986); Chan and Melville (1988); Sawaragi and Nochino (1984); Chaplin et al. (1992); Wienke et al. (2000) and Arntsen et al. (2011). However, the measurement of the quantities related to the wave breaking and their interaction with structures are challenging.

Theoretically, the total breaking wave force on a vertical slender cylinder can be expressed in terms of a slowly varying quasi-static force and the impulsive wave impact force. Goda et al. (1966) proposed the use of an impact force term in addition to the quasi-static force predicted by the Morison formula (Morison et al., 1950) to evaluate breaking wave forces.

The impact force characteristics are mainly determined by the geometric properties and kinematics at breaking such as the shape of the wave and the distribution of water particle velocities under the wave crest (Goda et al., 1966).

Watanbe and Horikawa (1974) investigated breaking wave forces on a large cylinder and proposed a formula including the phase difference between the water particle acceleration and the inertia force. They also pointed out that empirical coefficients used to calculate the breaking wave forces are not universal and depend on the breaking wave characteristics. Apelt and Piorewicz (1986) carried out experiments to study the interference effects on breaking wave forces on a row of two and three vertical cylinders placed along and normal to the direction of wave propagation. Their results suggested that both the distance between the cylinders and incident wave steepness are important factors when in the row is arranged normal to the direction of wave propagation. They further concluded that the distance of separation does not have a significant influence on the wave forces when the row is along the direction of wave propagation. Sparboom et al. (2005) studied breaking wave forces on two and three cylinder arrays due to freak waves and found that the breaking wave forces are reduced significantly along the array due to the sheltering effect from the upstream cylinders.

Wienke et al. (2000) carried out large-scale studies on breaking wave impact on a single slender cylinder and presented different wave loading cases, considering the position of the cylinder with respect to the wave breaking point. Irschik et al. (2002) extended this work and presented the Empirical Mode Decomposition (EMD) method to separate the slowly varying quasi-static loading and the dynamic response of the cylinder from the measured breaking wave force history. Wienke and Oumeraci (2005) proposed a theoretical model to calculate breaking wave forces on a single slender cylinder using the wave celerity and the curling factor as inputs based on their large-scale investigations.

The curling factor (λ) is a parameter used to determine the contribution of the wave crest to the wave impact force during breaking wave impact. The values for λ are determined experimentally for different bottom slopes and water depths and these values depend on the breaker type. According to Wienke and Oumeraci (2005), the wave impact scenario for different distances of the cylinder surface from the breaking point is different. The assumption of instantaneous impact of the wave on the cylinder while calculating λ can also lead to overestimation of the breaking wave force. Hildebrandt and Schlurmann (2012) investigated breaking wave forces on a tripod structure in large-scale experiments to study the detailed temporal and spatial variation in the wave slamming loads. They concluded that the curling factors, vertical position of impact and the maximum slamming coefficients increase with decreasing distance between the cylinder from the point of wave breaking. Their results agreed with the theoretical slamming coefficients given by Goda et al. (1966).

Most of the current approaches to evaluate breaking wave forces strongly depend on the experimentally determined coefficients. However, the measurement of the various parameters such as velocity and acceleration during breaking is a challenging task (Arntsen et al., 2011). Also, these methods are not valid for cases which are not similar to the experiments used to obtain the coefficients and cannot be applied for multiple cylinders and different arrangements of the cylinders. In addition, the distance of the cylinder from the breaking point results in several breaking wave interaction scenarios that have to be studied in detail to gain useful insights into the breaking wave-structure interaction problem.

Numerical modeling of breaking waves requires the evaluation of the fluid physics with few assumptions as carried out using computational fluid dynamics (CFD) models (Christensen, 1998) in order to obtain detailed insights into the breaking wave-structure interaction. Many numerical studies have been carried out to investigate the breaking process in shallow waters with single-phase CFD models (Lin and Liu, 1998; Bradford, 2000; Christensen and Deigaard, 2001; Zhao et al., 2004). Hieu et al. (2004) showed that a two-phase CFD model better resolves the breaking wave kinematics. Thus, two-phase CFD models are generally used in recent literature to include the air-water interaction in the modeling (Chen et al., 1999; Christensen, 2006; Wang et al., 2009; Jacobsen et al., 2012; Xie, 2013; Alagan Chella et al., 2015b). In addition, results from Alagan Chella et al. (2015b) and Alagan Chella et al.

(2015a) show that higher order discretization schemes, a tight velocity-pressure coupling and a sharp representation of the free surface provide a more realistic description of the breaking waves. These studies have advanced the knowledge in current literature regarding breaking wave kinematics.

Bredmose and Jacobsen (2010) carried out simulations of focussed wave breaking forces on a slender cylinder using the open-source CFD model OpenFOAM, without an explicit turbulence model with half of the computational domain and assuming lateral symmetry in the flow field. Mo et al. (2013) investigated solitary wave breaking and its interaction with a slender cylinder over a slope with a CFD model assuming lateral symmetry and also with experiments. Good agreement between the experimental and numerical results is found for the free surface elevations and particle velocities. Choi et al. (2015) studied the free surface elevation and breaking wave forces on a vertical and inclined single cylinders using a CFD model. A good agreement was obtained between the computed results and the filtered experimental data. However, numerical investigation of breaking wave forces on tandem cylinders, the effect of neighboring cylinders on the breaking wave forces, along with the complex free surface deformations associated with the interaction has not been presented in current literature to the knowledge of the authors.

The interaction of breaking waves with a cylinder involves several important free surface features such as runup on the cylinder, the separation of the breaking wavefront around the cylinder, formation of a water jet behind the cylinder and the rejoining of the separated wavefront behind the cylinder. The scenario is further relevant in the presence of neighboring cylinders, as is the case in coastal and offshore constructions. In this study, the open-source CFD model REEF3D is used to evaluate breaking wave forces on tandem cylinders placed at different distances from each other in a three-dimensional numerical wave tank. The model has been previously used to investigate the breaking wave kinematics (Alagan Chella et al., 2015b) and to calculate non-breaking wave forces on tandem cylinders (Kamath et al., 2015). Several free surface free features and wave impact scenarios associated with breaking wave interaction with a single cylinder and the consequences on the wave forces acting on the cylinder have been discussed in current literature. This paper investigates the case of two cylinders placed in tandem with focus on the influence of the distance of separation between the cylinders on the wave forces along with the consequences of the flow features associated with breaking wave interaction with the cylinders. Four different wave impact scenarios on the first cylinder and six distances of separation between the cylinders are considered. The numerical model is validated using experimental results from the Large Wave Flume (GWK) (Irschik et al., 2002) for breaking wave interaction with a single cylinder.

NUMERICAL MODEL

Governing equations

The numerical wave tank REEF3D solves the incompressible three-dimensional Reynolds-Averaged Navier-Stokes (RANS) equations:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu + \nu_t)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + g_i \tag{2}$$

where u is the velocity, ρ is the density of the fluid, p is the pressure, ν is the kinematic viscosity, ν_t is the eddy viscosity and g the acceleration due to gravity.

The fifth-order conservative finite difference Weighted Essentially Non-Oscillatory (WENO) scheme proposed by Jiang and Shu (1996) is applied for the discretization of the convective terms of the RANS equation. Time advancement is carried out using a Total Variation Diminishing (TVD) third-order Runge-Kutta explicit time scheme (Shu and Osher, 1988). The time step size is controlled with adaptive time stepping based on the CFL criterion. This results in an optimal time step value for numerical stability and accuracy. The diffusion is treated with an implicit time scheme in order to exclude it from the CFL criterion. The pressure is treated with the projection method (Chorin, 1968). The Poisson equation for the pressure is solved with the preconditioned BiCGStab solver (van der Vorst, 1992). The domain decomposition strategy and MPI (Message Passing Interface) is used for parallelization. A Cartesian grid with a staggered arrangement is used in the numerical model. Complex geometries are taken into account with the ghost cell immersed boundary method (Berthelsen and Faltinsen, 2008).

The k-ω model is employed for turbulence closure (Wilcox, 1994) with transport equations for the turbulent kinetic energy k and the specific turbulence dissipation ω shown in Eq. (3) and (4) respectively. Wall functions are used for k and ω.

$$\frac{\partial k}{\partial t} + u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \beta_k k\omega \tag{3}$$

$$\frac{\partial \omega}{\partial t} + u_j \frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + \frac{\omega}{k}\alpha P_k - \beta\omega^2 \tag{4}$$

where, eddy viscosity $\nu_t = k/\omega$, P_k is the production rate and closure coefficients $\sigma_k = 2$, $\sigma_\omega = 2$, $\alpha = 5/9$, $\beta_k = 9/100$, $\beta = 3/40$. Eddy viscosity limiters (Durbin, 2009) are used to control the overproduction of turbulence, often occurring in highly unsteady free surface flows. In addition, the fact that the turbulence length scales cannot pass the interface between water and air is considered with a free surface turbulence damping scheme (Naot and Rodi, 1982).

Free Surface

The complex wave hydrodynamics are modeled with a two-phase flow approach, calculating the flow for water and air. The interface between the two fluids is captured with the level set method (Osher and Sethian, 1988). The zero level set of the signed distance function $\phi(\vec{x}, t)$ represents the location of the free surface. With its signed distance property, it gives the shortest distance from the interface to all the points in the flow domain. Based on the sign of the level set function, the phases can be distinguished as follows:

$$\phi(\vec{x},t)\begin{cases} >0 & if\ \vec{x}\ is\ in\ phase\ 1 \\ =0 & if\ \vec{x}\ is\ at\ the\ interface \\ <0 & if\ \vec{x}\ is\ in\ phase\ 2 \end{cases} \tag{5}$$

The flow velocities calculated from Eq. (2) are used to convect the level set function:

$$\frac{\partial \phi}{\partial t} + u_j \frac{\partial \phi}{\partial x_j} = 0 \tag{6}$$

During the computation, reinitialization is carried out after every iteration using a partial differential equation Peng et al. (1999) in order to maintain the signed distance property of the level set function. The level set function is discretized with the Hamilton-Jacobi formulation of the WENO scheme by Jiang and Peng (2000)

Wave generation and absorption

The numerical wave tank uses the relaxation method (Larsen and Dancy, 1983) for the wave generation. A relaxation function is used to moderate the velocity and the free surface using a wave theory in the relaxation zones with Eq. (7):

$$\begin{aligned} u_{relaxed} &= \Gamma(x)u_{analytical} + (1-\Gamma(x))u_{computational} \\ \phi_{relaxed} &= \Gamma(x)\phi_{analytical} + (1-\Gamma(x))\phi_{computational} \end{aligned} \tag{7}$$

where $\Gamma(x)$ is the relaxation function and $x \in [0,1]$ is the x-coordinate scaled to the length of the relaxation zone. The relaxation function shown in Eq. (8) is used in the current numerical model (Jacobsen et al., 2012):

$$\Gamma(x) = 1 - \frac{e^{(1-x)^{3.5}} - 1}{e - 1} \tag{8}$$

In order to avoid reflections from the downstream boundary, an active wave absorption method is employed. Here, waves opposite to the reflected ones are generated, canceling out the reflections. Based on shallow water theory (Schäffer and Klopman, 2000), the following horizontal velocity is prescribed on the downstream boundary:

$$u\left(t\right) = -\sqrt{\frac{g}{h}}\,\xi\left(t\right) \tag{9}$$

where

$$\xi\left(t\right) = \eta\left(t\right) - h \tag{10}$$

Here, $\eta\left(t\right)$ is the actual free surface location along the downstream boundary and h the still water level. The method is applied in vertical strips along the downstream boundary, which are one grid cell wide. This way, different free surface elevations along the boundary can be taken into account (Higuera et al., 2013). Also, the handling of oblique waves is also implemented in the current model.

Numerical evaluation of wave forces

The breaking wave forces on the cylinders is calculated by integrating the pressure p and the surface normal component of the viscous shear stress tensor τ on the surface of the solid objects:

$$F = \int_{\Omega} (-\mathbf{n}p + \mathbf{n}\cdot\tau)d\Omega \tag{11}$$

where $\mathbf{n}$ is the unit normal vector pointing into the fluid and Ω is the surface of the object.

RESULTS AND DISCUSSION

Validation of the numerical model

The breaking wave force on a single vertical cylinder is calculated numerically and compared to experimental data to validate the numerical model. The experiments were carried out at the Large Wave Flume (GWK), Hannover, Germany (Irschik et al., 2002) on a vertical cylinder of diameter $D = 0.7$ m in a water depth of 3.80 m with incident waves of period

$T = 4.0$ s. The cylinder is placed at the top of a 23 m long 1 : 10 slope, such that the still water depth at the cylinder is 1.50 m. In the numerical setup, the wave tank is 59 m long, 5 m wide and 7 m high with a grid size of $dx = 0.05$ m resulting in a total of 16.52 million cells. A cylinder with $D = 0.7$ m is placed with its center at 44.0 m and the incident waves of period $T = 4.0$ s break exactly on the front surface of the cylinder. The complete numerical setup is illustrated in Fig. (1a). The definition sketch for tandem cylinders in the wave tank showing the location of the wave gages and the separation distance is shown in Fig. (1b).

The numerically calculated wave force is compared to the EMD (Empirical Mode Decomposition) treated experimental data from Choi et al. (2015) to filter out the dynamic amplification of the wave forces due to the vibration of the cylinder in Fig. (2a). A good agreement is seen between the numerical and experimental wave forces. The numerical results are also similar over several wave periods, showing that the numerical model predicts the the wave breaking location and consequently the breaking wave forces consistently. The free surface elevation near the wall along the frontline of the cylinder provides a representation of the wave incident on the cylinder. The comparison between numerical and experimental free surface elevation shows a good agreement in Fig. (2b). The vertical wavefront in the figure shows that the wave breaks on the front surface of the cylinder.

Effect of wave impact scenario and distance between tandem cylinders on the wave forces

The wave forces on tandem cylinders placed at different distances from each other are studied for different wave breaking scenarios. The different scenarios are determined based on the location of the wave breaking point with respect to the front surface of the first cylinder. The scenarios considered in this study are:

- scenario A: overturning wave crest impacts cylinder 1 just below the wave crest level
- scenario B: overturning wave crest impacts cylinder 1 at the wave crest level
- scenario C: wave breaks exactly at cylinder 1 with a vertical wavefront

- scenario D: wave breaks just behind cylinder 1

The various scenarios are illustrated in Fig. (3). Simulations are carried out to determine the breaking wave force for a single cylinder, F_0 in each of the scenarios. Previous studies dealing with breaking wave interaction with a single slender cylinder have presented that the mode of wave impact on the cylinder due to the distance between the breaking point and the location of the cylinder have a significant impact on the wave forces acting on it. According to Irschik et al. (2002), scenario A and B result in the highest and the second highest total wave forces on a single cylinder respectively. The lowest wave forces on a single cylinder are obtained in scenario D. In the context of tandem cylinders, the wave impact on cylinder 1 and the separation distance between the two cylinders can play an important role in the wave forces experienced by both the cylinders. This is investigated in this study by placing the second cylinder at separation distances of $S = 1D$, $S = 2D$, $S = 3D$, $S = 4D$, $S = 5D$ and $S = 6D$ from the first cylinder. The resulting 24 different cases are listed in Table (1) along with the numerical force calculated for a single cylinder in each of the wave breaking scenarios, F_0, the maximum force on each cylinder with respect to F_0 in each case (F_1/F_0 and F_2/F_0) and the maximum wave crest elevations in front of the cylinders with respect to the incident wave crest elevation $\eta_0 = 0.789$ m (η_{cyl1}/η_0 and η_{cyl2}/η_0). In the following sections, results from selected cases are presented to obtained detailed insights into the breaking wave interaction, free surface features and the wave forces on the cylinders. The selected cases present the prominent breaking wave hydrodynamics for different separation distances in the different wave impact scenarios.

Scenario A1: overturning wave crest impacting cylinder 1 just below the wave crest level with $S = 1D$

The breaking wave force and the free surface elevations around the cylinders calculated for scenario A1 are presented in Fig. (4). The breaking wave force on a single cylinder in this wave impact scenario is $F_0 = 14000$ N. The breaking wave force (F) on cylinder 1 and 2 are calculated to be $0.92F_0$ and $0.59F_0$ respectively in Fig. (4a). In this case, the wave incident on the second cylinder is a broken wave that has dissipated most of its energy in the breaking process and during its interaction with the first cylinder. Thus, the breaking wave force on the second cylinder is significantly lower than that on the first cylinder. The free surface elevations (η) calculated in front (WG 1) and behind (WG 2) the first cylinder and in front of the second cylinder (WG 3) are presented in Fig. (4b). The free surface elevation in front of cylinder 2, placed $S = 1D$ away is $\eta/\eta_0 = 1.69$, higher than the free surface elevation in front of cylinder 1, $\eta/\eta_0 = 1.58$. The higher free surface elevation is attributed to the large runup on cylinder 2 due to the close placement of the cylinders.

Further insight into the wave interaction problem is obtained from the free surface around the cylinders for case A1, presented in Fig. (5) with horizontal velocity contours. The incident wave impacts cylinder 1 with the breaker tongue just below the wave crest level in Fig. (5a). The overturned wavefront is separated around cylinder 1 in Fig. (5b). This phenomenon of separation of the wave crest around the first cylinder and spreading of the water mass around the sides of the cylinder is also reported by Sparboom et al. (2006) in large-scale experiments investigating breaking wave interaction with slender cylinders. Figure (5c) shows the separated broken wavefront incident on cylinder 2 and reconnecting with the free surface. As the broken wave crest propagates past cylinder 2 in Fig. (5d), the high runup on the front surface of cylinder 2 is observed. This runup results in a higher free surface elevation in front of cylinder 2 compared to cylinder 1 seen for WG 3 in Fig. (4b). Figure (5) also shows that in scenario A, cylinder 2 is always exposed to an already broken wave, due to which the cylinder experiences lower wave forces.

Scenario B2: overturning wave crest impacting cylinder 1 at the wave crest level with $S = 2D$

Figure (6) shows the breaking wave forces on and the free surface elevations around the two cylinders in scenario B2. A single cylinder in the same impact scenario experiences a forces of $F_0 = 13400$ N. The breaking wave force on cylinder 1 is $0.93F_0$ and on cylinder 2 it is $0.85F_0$, shown in Fig.(6a). The free surface elevations in front of cylinders 1 and 2 in

Fig. (6b) are $\eta/\eta_0 = 1.70$ and $\eta/\eta_0 = 1.72$ respectively. The slopes of the wavefront at the moment of impact on the cylinders are similar and the wave forces on the two cylinders are comparably similar.

The free surface around the cylinders in scenario B2 with horizontal velocity contours is shown in Fig. (7). The overturning wave crest impacts cylinder 1 at wave crest level in Fig. (7a). The waves reflected by the cylinders from the previous wave impact is seen interacting with the incident wave crest. The separation of the overturning wave crest around cylinder 1 is seen in Fig. (7b). The overturning wave crest impacts cylinder 2 below wave crest level along with the water jet formed behind the first cylinder in Fig. (7c). The high runup on the second cylinder due to the water jet originating behind cylinder 1 and the small separation distance is seen in Fig. (7d). In this scenario, though cylinder 1 separates the wavefront, the sheltering effect on cylinder 2 is seen to be reduced. This is due to water jet formed behind cylinder 1 that impacts cylinder 2 along with the breaking wave. This results in comparably similar forces on the two cylinders in this scenario, with the upstream cylinder experiencing a slightly higher force.

Scenario C3: wave breaking exactly at cylinder 1 with $S = 3D$

The breaking wave forces on and the free surface elevations around the cylinders in scenario C3 are shown in Fig. (8). The breaking wave force on a single cylinder in this scenario is $F_0 = 11850$ N. Here, cylinder 1 experiences a force of $0.92F_0$ and cylinder 2 a force of $0.97F_0$. It is observed that the breaking wave force on the downstream cylinder 2 is slightly higher than the force on the upstream cylinder 1. The free surface elevation in front of cylinder 2 is $\eta/\eta_0 = 1.70$, slightly lower compared to $\eta/\eta_0 = 1.82$ in front of cylinder 1.

The wave interaction in scenario C3 is further studied using the free surface around the cylinders with horizontal velocity contours in Fig. (9). The incident wave impacting cylinder 1 with a vertical wavefront is seen in Fig. (9a). The incident wave is separated around cylinder 1 in Fig. (9b) and the wave crest also begins to overturn just behind the cylinder. The breaker tongue impacts cylinder 2 along with the water jet originating behind cylinder

1 in Fig. (9c). The breaking wave incident on cylinder 2 impacts the cylinder just below the wave crest level along with the water jet, justifying the higher forces on the cylinder. The runup of the trapped water between the cylinders is seen in Fig. (9d) and the overturning wave crest rejoins the preceding wave crest after passing cylinder 2.

Scenario D4: wave breaking just behind cylinder 1 with $S = 4D$

The waves force on a single cylinder in this wave impact scenario is calculated to be $F_0 = 9800$ N. In Fig. (10a), the calculated breaking wave forces on cylinder 1 and 2 are $0.88F_0$ and $1.04F_0$ respectively. In this scenario, the upstream cylinder 1 is exposed to very steep incident waves approaching the wave breaking point. Cylinder 2 is exposed to an overturning wave crest and the breaking wave impact force contributes to the total wave force on the cylinder, resulting in a higher wave force on the downstream cylinder compared to the upstream cylinder. The free surface elevations in Fig. (10b) show that $\eta/\eta_0 = 1.78$ in front of cylinder 1 (WG 1) and higher than $\eta/\eta_0 = 1.59$ in front of cylinder 2 (WG 3) in this case.

In order to further understand the wave interaction with the cylinders in scenario D4, the free surface around the cylinders is presented in Fig. (11) along with horizontal velocity contours. Figure (11a) shows the steep unbroken wave incident on cylinder 1. The wave breaks just behind cylinder 1 and the overturning wave crest along with the water jet originating behind cylinder 1 is seen in Fig. (11b). The overturning wave crest then impacts the second cylinder just below the wave crest level in Fig. (11c) along with the water jet. The breaker tongue reconnects with the preceding wave trough behind cylinder 2 in Fig. (11d). The higher forces on the second cylinder are justified by the mode of wave impact on each cylinder. Figure (11) clearly shows that the upstream cylinder 1 is exposed to a steep non-breaking wave, whereas the overturning wave crest impacts the downstream cylinder 2 just below the wave crest level.

Scenario D6: wave breaking just behind cylinder 1 with $S = 6D$

The wave forces on the cylinders 1 and 2 in this case are calculated to be $0.85F_0$ and $1.03F_0$ respectively in Fig. (12a). The wave force on the cylinder 2 is significantly higher than the force on cylinder 1 in this scenario. The free surface elevation in front of cylinder 1, $\eta/\eta_0 = 1.78$ is higher than the free surface in front of cylinder 2 which is $\eta/\eta_0 = 1.37$ in Fig. (12b). The breaking wave impact on cylinder 2 is represented by the steep front face of the wave in front of the cylinder during the same time as the peak force on the cylinder. The runup on cylinder 2 is seen to lesser in this scenario ($\eta/\eta_0 = 1.37$) compared to scenario D4 ($\eta/\eta_0 = 1.59$).

The free surface around the cylinders along with the horizontal velocity contours is presented in Fig. (13). The steep unbroken wave incident on cylinder 1 is seen in Fig. (13a), similar to that in Fig. (11a). Figure (13b) shows the overturning crest and the water jet originating behind cylinder 1 in between the two cylinders. The impact of the water jet on cylinder 2 after the overturning wave crest has impacted the cylinder is seen in Fig. (13c). The runup on cylinder 2 in this scenario is lower due to the longer separation distance between the cylinders. The overturning wave crest and the water jet impact the cylinder close to the point of reconnection of the breaking wave crest with the preceding wave trough. The broken wave and the water jet formed behind cylinder 2 are seen in Fig. (13d).

Variation of the breaking wave forces on the cylinders with separation distance in the different wave impact scenarios

The variation of the total breaking wave forces on each of the cylinders in the different wave impact scenarios for different separation distances is presented in Fig. (14). The following sections correlate the variation of the forces with the separation distance with the free surface features associated with the wave impact scenario.

Scenario A

The total wave force on cylinder varies over a small range between $0.95F_0$-$0.88F_0$ for scenario A in Fig. (14a). For cylinder 2, the total wave force varies significantly with a lowest of $0.45F_0$ for $S = 3D$ to a highest of $0.59F_0$ for $S = 1D$. Cylinder 2 is always exposed to a broken wave and the water jet originating behind cylinder 1, along with the free surface features behind cylinder 1 have a significant effect on the total wave force on cylinder 2. For small separation distances of $S = 1D$ and $2D$, a separated broken wave crest is incident on cylinder 2 as seen in Fig. (5c). The water jet originating behind cylinder 1, that develops in the small region between the cylinders is mainly responsible for the force on cylinder 2. The resulting forces for $S = 1D$ and $2D$ are seen to be around $0.58F_0$ in Fig. (14a). On increasing the distance to $S = 3D$, the force resulting from the impact of the water jet is reduced and the minimum force is calculated for this scenario. On further increasing the separation distance to $S = 4D$ and $5D$, the wave crest separated by cylinder 1 rejoins the preceding wave trough, undergoes secondary breaking and impacts cylinder 2 along with the water jet. This results in the slightly increasing trend in the force on cylinder 2. For $S = 6D$, cylinder 2 is mainly exposed to the post-breaking splash up and the force on cylinder 2 is lowered again. Further increase in the separation distance S would result in further reduction on the wave force on cylinder 2.

Scenario B

The total wave forces on both cylinders are significantly affected by the separation distance between the cylinders in this scenario as seen in Fig. (14b). The total wave force on cylinder 1 is highest for $S = 2D$ with $0.93F_0$ and lowest for $S = 3D$ with $0.61F_0$. For cylinder 2, the total wave force is a maximum of $0.85F_0$ for $S = 2D$ and a minimum of $0.57F_0$ for $S = 4D$. In this case, the waves reflected from the cylinders interact with the incident overturning wave crest as seen in Fig. (7a). This results in significant changes in wave forces on both cylinders as the separation distance between the cylinders is varied. When $S = 1D$, the incident wave is separated by cylinder 1 and cylinder 2 is impacted mainly by the water jet. This results in lower forces on cylinder 2. As S is increased to $2D$, the separated wave crest is rejoined just before impacting cylinder 2 and the force on the cylinder increases. The interaction between the incident wave crest and the reflected waves from the cylinders for $S = 3D$ result in reduced forces on cylinder 1. At the same time, the wave incident on cylinder 2 rejoins the preceding wave trough just in front of the cylinder and the force on the cylinder is reduced, but is higher than the force on cylinder 1. Further increase in S results in lower forces on cylinder 2, since the incident wave has rejoined the preceding wave trough and the cylinder is exposed the splash up. The forces on cylinder 1 increase and reach around the value calculated for $S = 1D$ following the interaction between the incident and reflected waves. Hildebrandt et al. (2008) found through large-scale experiments with non-breaking waves on groups of slender cylinders that for certain distances of separation, the forces on the upstream cylinder are influenced by the wave interaction between the cylinders and waves reflected by the cylinders. Their observations are applicable in this case with a strong interaction between the incident wave and the reflected waves from the cylinders when the overturning wave crest impacts cylinder 1 at wave crest level.

Scenario C

In scenario C the front surface of cylinder 1 is at the wave breaking point and the peak breaking wave force on the cylinder varies between $0.92F_0$ ($S = 3D$) and $0.83F_0$ ($S = 6D$) in Fig. (14c). The peak wave force on cylinder 2 varies significantly with the separation distance with a maximum of $0.97F_0$ for $S = 3D$ and a minimum of $S = 0.61F_0$ for $S = 6D$. It is seen that for $S = 3D$ and $4D$, the breaking wave force on the downstream cylinder 2 is slightly higher than the upstream cylinder 1. The variation of the forces on cylinder 2 can be justified by the wave breaking process in this scenario and the resulting free surface features seen between the cylinders. For $S = 1D$, the incident wave is separated by cylinder 1 and the water jet originating behind the cylinder impacts cylinder 2 leading to a lower force on the cylinder. The separated wave crest rejoins before impacting cylinder 2, along with the water jet when $S = 2D$ and the wave force is increased. On further increasing S to $3D$ and $4D$, the breaker tongue impacts the cylinder around the wave crest level along with the water jet as seen in Fig. (9c), resulting in a higher force on cylinder 2 than on cylinder 1. For $S = 5D$ and $6D$, cylinder 2 is exposed mainly to the splash up along with the water jet. The impact of the broken wave on cylinder 2 results in a lower force for $S = 5D$ and $6D$ and further increase in S would result in a lower force.

Scenario D

The total wave force on cylinder 1 in scenario D varies over a small range between $0.84F_0$-$0.90F_0$ in Fig. (14d). The peak wave force on cylinder 2 is the lowest for $S = 1D$ $(0.81F_0)$ and the highest for $S = 5D$ $(1.18F_0)$. Due to the wave breaking just behind the upstream cylinder 1, cylinder 2 is exposed to breaking wave impact and generally experiences higher forces than cylinder 1. Similar to the previous scenarios where cylinder 2 is placed at a distance of $S = 1D$, the incident wave crest is separated by cylinder 1, resulting in a lower wave force on cylinder 2. From $S = 2D$ to $S = 5D$, cylinder 2 is impacted by the overturning wave crest at and around the wave crest level as seen in Fig. (11c) for $S = 4D$, leading to higher wave forces. The maximum peak force is calculated for $S = 5D$ where the breaker tongue impacts cylinder 2 just below the wave crest level. On increasing S to $6D$, the overturning wave crest rejoins the preceding wave trough during impact with cylinder 2 as seen in Fig. (13c) and the wave force on cylinder 2 is reduced.

Discussion

The results show that the wave forces on both cylinders are generally less than the wave force on a single cylinder in the same wave impact scenario (F_0). The exception to this observation are the cases where the breaker tongue impacts the downstream cylinder 2 around the wave crest level. This is particularly the case in scenario D, where the wave breaks behind the upstream cylinder 1 and the overturning wave crest impacts cylinder 2 around or just below wave crest level depending on the separation distance between the cylinders. Another observation is that high runups are calculated on the second cylinder when the cylinders are placed close to each other $(S = 1, 2D)$, but the higher free surface elevations do not correspond to higher wave forces. In fact, for scenarios C3, D4 and D6 the free surface in front of cylinder 2 is lower than that in front of cylinder 1 whereas the wave force is higher on cylinder 2. The close placement of the cylinders leads to a high runup from the water jet developed in between the cylinders, but the second cylinder is shielded from breaking wave impact due to the separation of the incident wavefront by the first cylinder.

The trend of the breaking wave forces on cylinder 1 for scenario B is seen to majorly vary from the trend seen in the other scenarios for $S = 2D$ and $3D$. This is due to the strong interaction between the incident waves and the waves reflected from the cylinder as seen from previous studies by Hildebrandt et al. (2008) for cylinders placed close together. In addition, the superposition of the reflected waves on the overturning wave crest is the strongest as seen from Fig. (7a). This leads to a large increase followed by a large decrease in the breaking wave forces for $S = 2D$ and $S = 3D$ respectively in this scenario. On further increase in S, the breaking wave forces on cylinder 1 are around the values obtained for $S = 1D$, which is the general trend in all the other scenarios.

Some similarities can be drawn between the results for wave forces on tandem cylinders in this study and results for breaking wave forces on a single cylinder in previous studies. In the case of a single cylinder, the maximum wave forces are obtained when the breaker tongue impacts the cylinder just below the wave crest level (Irschik et al., 2002). In the present study, the upstream cylinder 1 also experiences the highest forces in scenario A2 ($F = 13300N$) when the breaking impacts the cylinder just below wave crest level. In scenario D4, cylinder 1 experiences one of the lowest forces ($F = 8330N$) when the wave breaks just behind the cylinder. However, the lowest force on cylinder 1 is calculated in scenario B3 ($F = 8174N$) due to the interaction between the incident and reflected waves when the overturning wave crest impacts cylinder 1 at wave crest level.

For cylinder 2, the highest forces are calculated in scenario D5 ($F = 11564N$), when the cylinder is placed at $S = 5D$ from cylinder 1 and the wave breaks just behind cylinder 1. The overturning wave crest impacts cylinder 2 just below the wave crest level along with the water jet. This is similar to the wave impact scenario leading to the highest breaking wave force on a single cylinder. The lowest force on cylinder 2 ($F = 6300N$) is calculated in scenario A3, where the overturning wave crest rejoins the preceding wave trough before impact with the cylinder. Thus, the results from Wienke et al. (2000) and Irschik et al. (2002) for breaking wave impact on a single slender cylinder are applicable in the case of tandem cylinder as well, though with a few changes due to the interaction between the two cylinders placed in proximity. The results in this study differ from the small-scale experimental results presented by Apelt and Piorewicz (1986), which concluded that the separation between the cylinders did not affect the wave forces on the cylinders when they are arranged in the direction of wave propagation.

CONCLUSIONS

The open-source CFD model REEF3D is used to simulate plunging breaking wave interaction with a pair of cylinders placed in tandem at different distances of separation for different wave impact scenarios. The model was validated by comparing the numerical results for wave force and the free surface elevation with the experiments carried out on a single cylinder at the Large Wave Flume, Hannover, Germany by Irschik et al. (2002). The free surface features associated with breaking wave interaction with a slender cylinder are presented and correlated to the wave forces on the cylinders and the following conclusions can be drawn from the results:

- Similar to the results from wave impact on a single slender cylinder, the maximum breaking wave forces in this study is calculated in cases where the breaker tongue impacts the cylinders just below the wave crest level.
- The free surface features behind the first cylinder such as the separation of the wavefront around the first cylinder, the formation of the water jet, the rejoining of the separated wavefront and reconnection of the overturning wave crest with the preceding wave trough have significant influence on the wave forces on the second cylinder. The distance between the cylinders also determines the development of the various free surface features.
- The wave forces on the first cylinder are lower than the force on a single cylinder for

the same wave impact scenario for all the cases studied. The highest force on the first cylinder is $0.95F_0$ when the wave impacts the cylinder just below the wave crest level and the second cylinder is at a distance of $2D$.

- The wave forces on the second cylinder are generally lower than the force on the first cylinder when the wave breaks in front or at the first cylinder and the separation distance is more than $4D$ with a highest force of $0.71F_0$ when the wave breaks exactly at the first cylinder.
- The wave force on the second cylinder is higher than the force on the first cylinder and the force on a single cylinder when the breaker tongue impacts the second cylinder around the wave crest level. The highest force on the second cylinder is $1.18F_0$ when the wave breaks just behind the first cylinder and the second cylinder is at a distance of $5D$.

This study provides insight into the challenging problem of plunging breaking wave interaction with two cylinders in tandem for different wave impact scenarios and distances of separation. Further studies can be carried out extended to investigate breaking wave interaction with three or more cylinders in tandem, oblique wave incidence and engineering problems including tripod substructures and coastal constructions with multiple cylinders in proximity.

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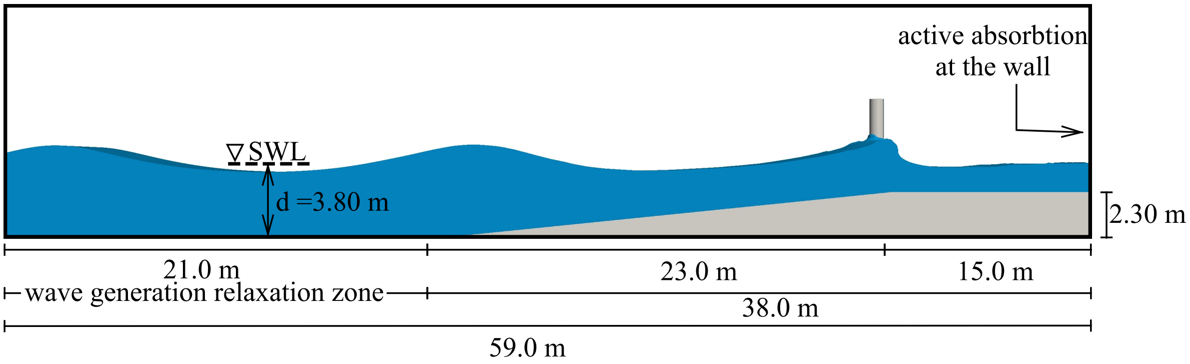
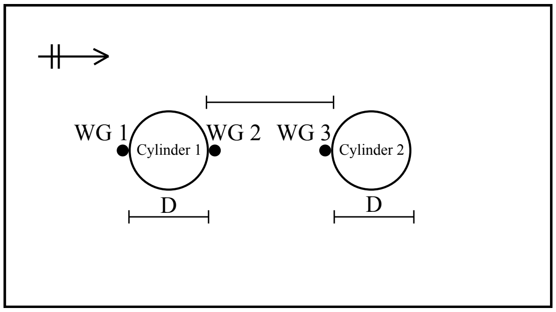
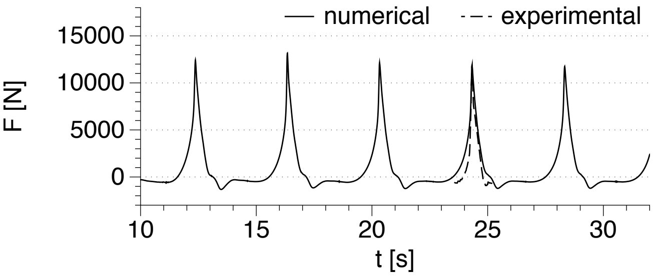


(a) numerical wave tank showing the dimensions of the tank and wave generation and absorption zones

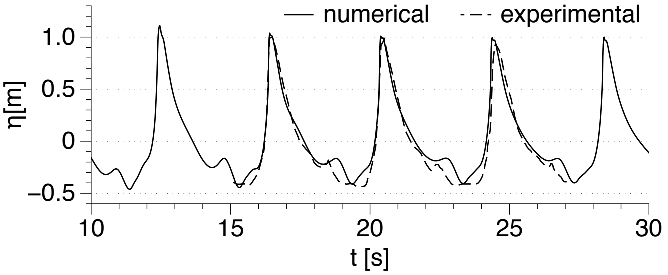


(b) schematic diagram showing the wave gage locations around the cylinders

Fig. 1. Numerical wave tank setup used in the study



(a) wave force on the cylinder



(b) free surface near the wall along the frontline of the cylinder

Fig. 2. Comparison of the numerical and experimental results

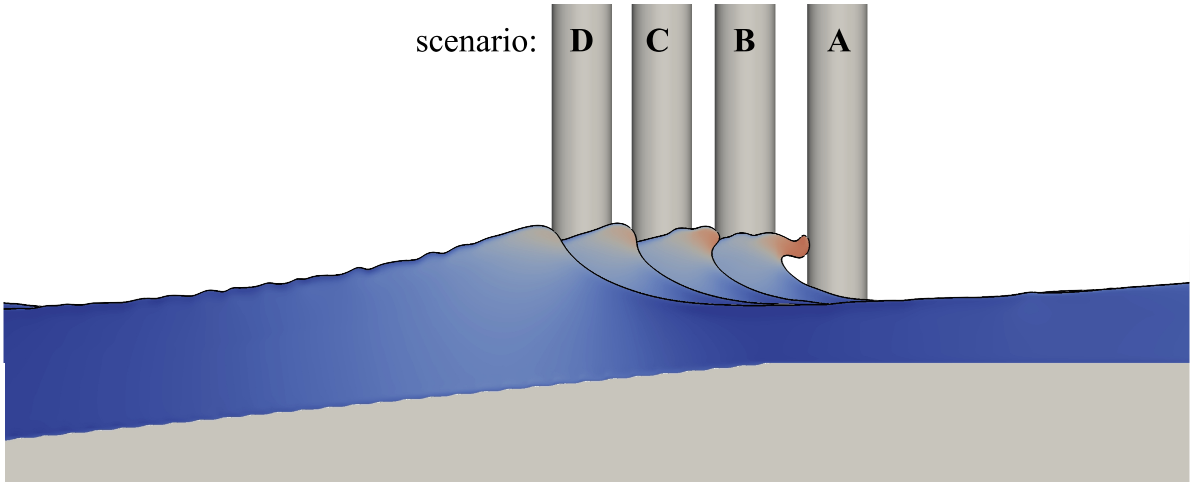
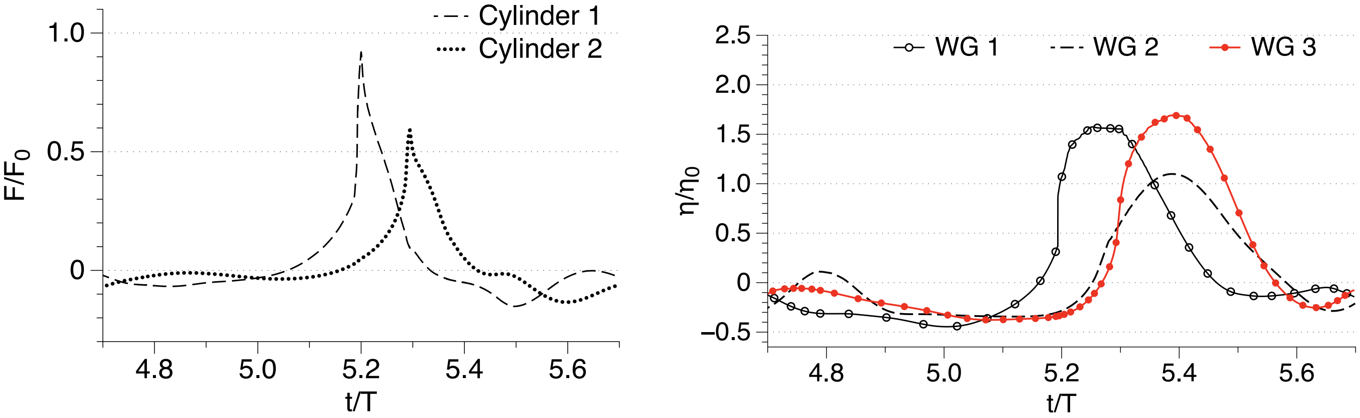


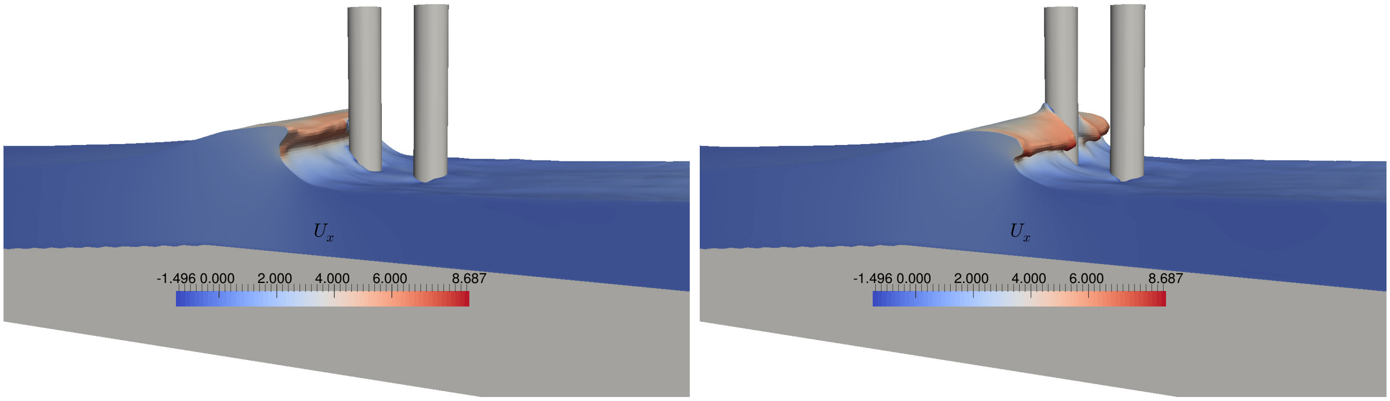
Fig. 3. Four different locations of cylinder 1 with respect to the wave breaking point considered in the study



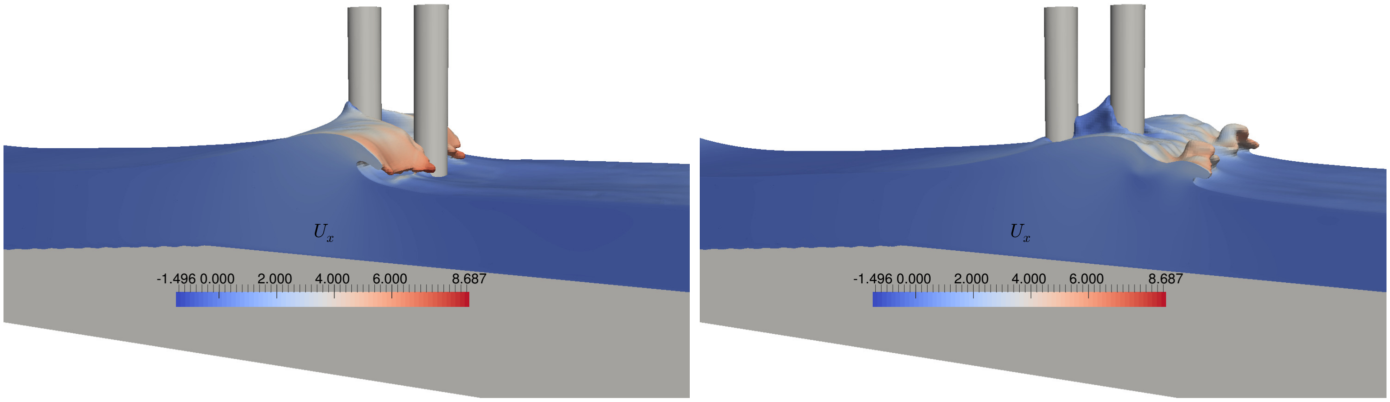
(a) breaking wave forces on the cylinders

(b) free surface elevations around the cylinders

Fig. 4. Wave forces on and free surface elevations around the cylinders for scenario A1: breaker tongue impacting cylinder just below wave crest level with $S = 1D$

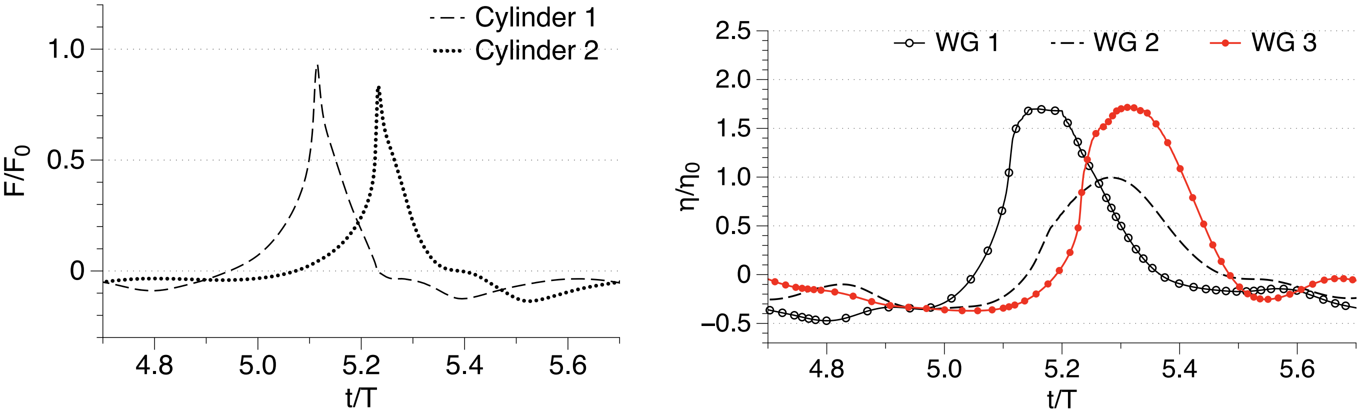


(a) $t/T = 4.20$ (b) $t/T = 4.24$



(c) $t/T = 4.29$ (d) $t/T = 4.40$

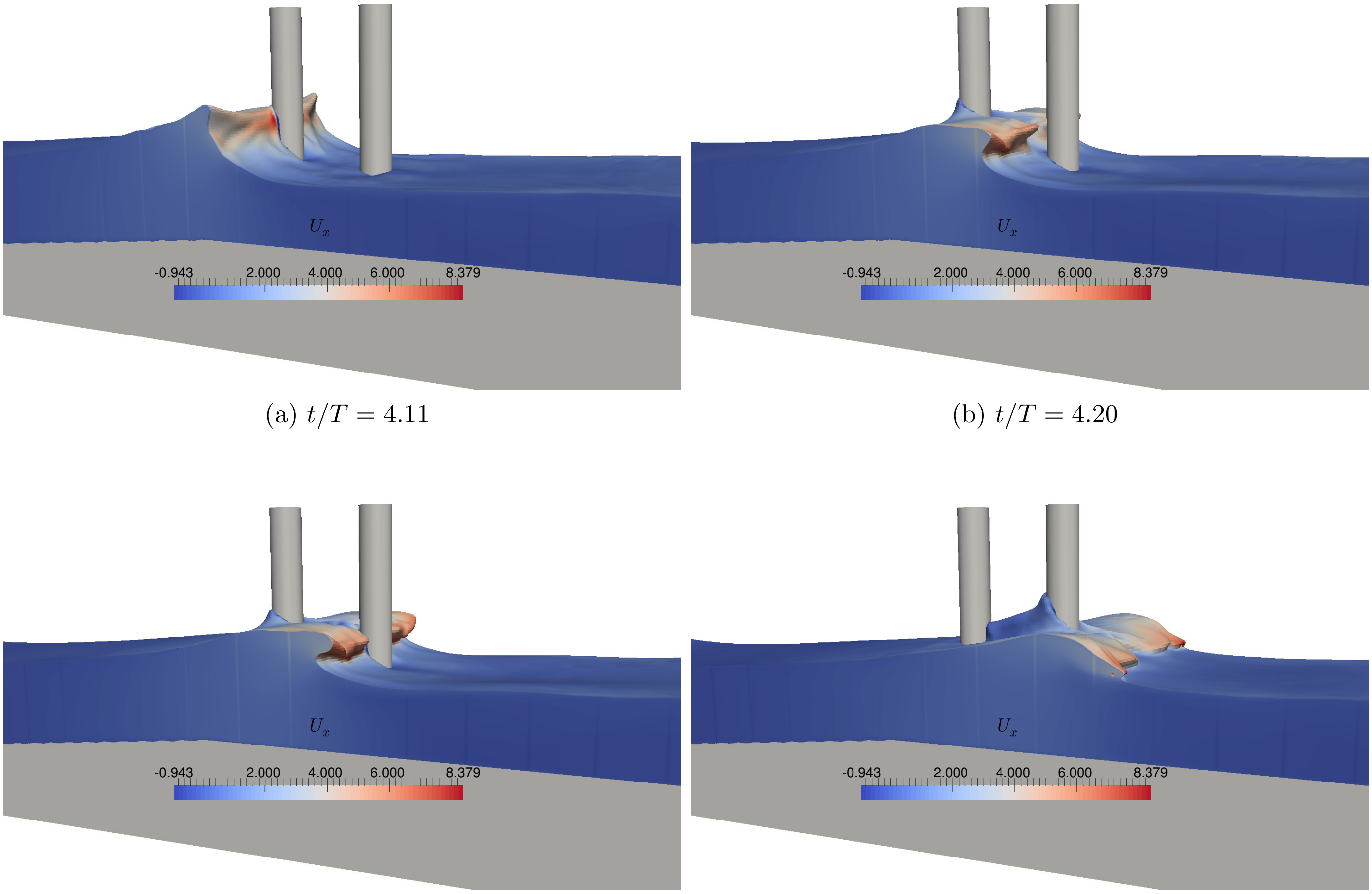
Fig. 5. Free surface around the cylinders in scenario A1 ($S = 1D$) with horizontal velocity contours



(a) breaking wave forces on the cylinders

(b) free surface elevations around the cylinders

Fig. 6. Wave forces on and free surface elevations around the cylinders for scenario B2: breaker tongue impacting cylinder at wave crest level with $S = 2D$



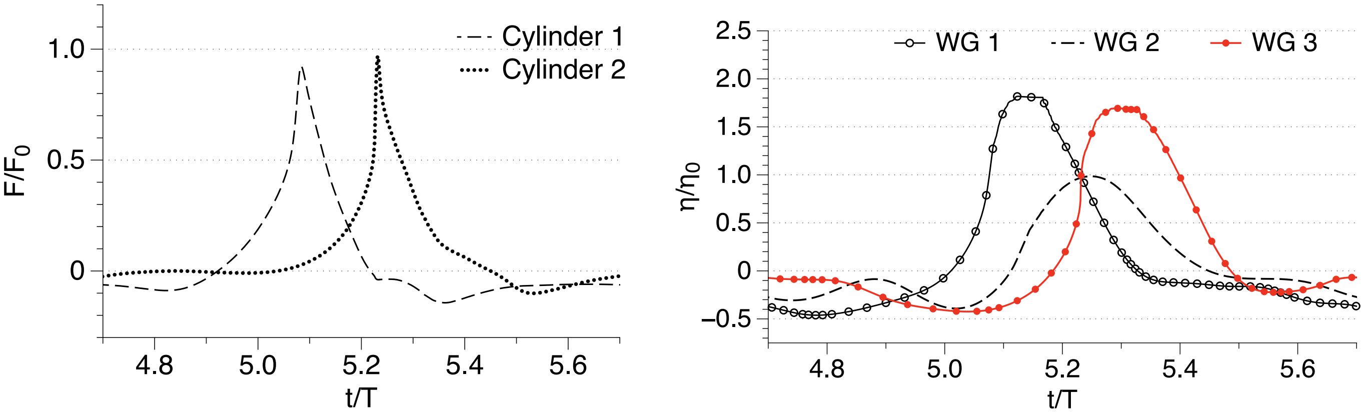
(a) $t/T = 4.11$

(b) $t/T = 4.20$

(c) $t/T = 4.23$

(d) $t/T = 4.31$

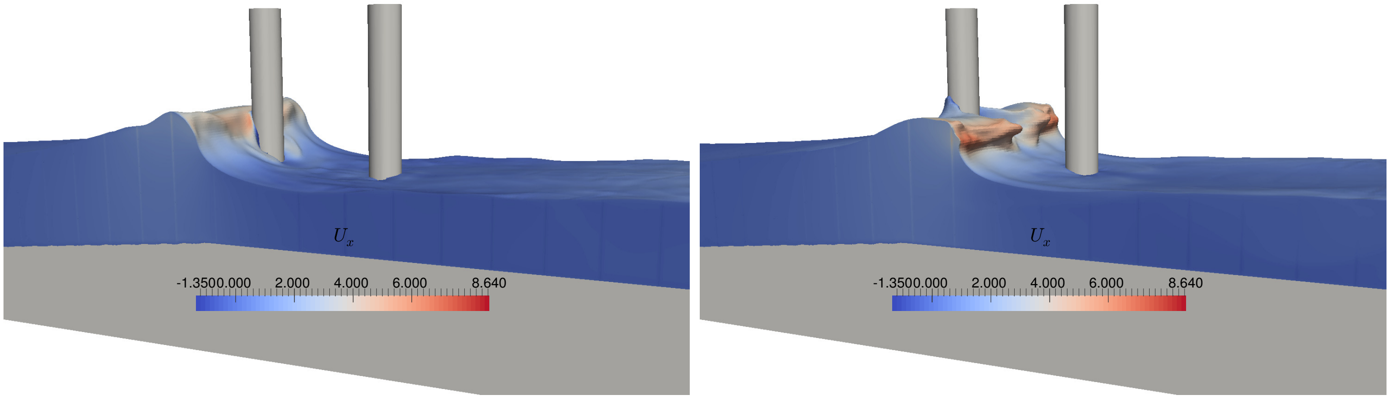
Fig. 7. Free surface around the cylinders in scenario B2 ($S = 2D$) with horizontal velocity contours



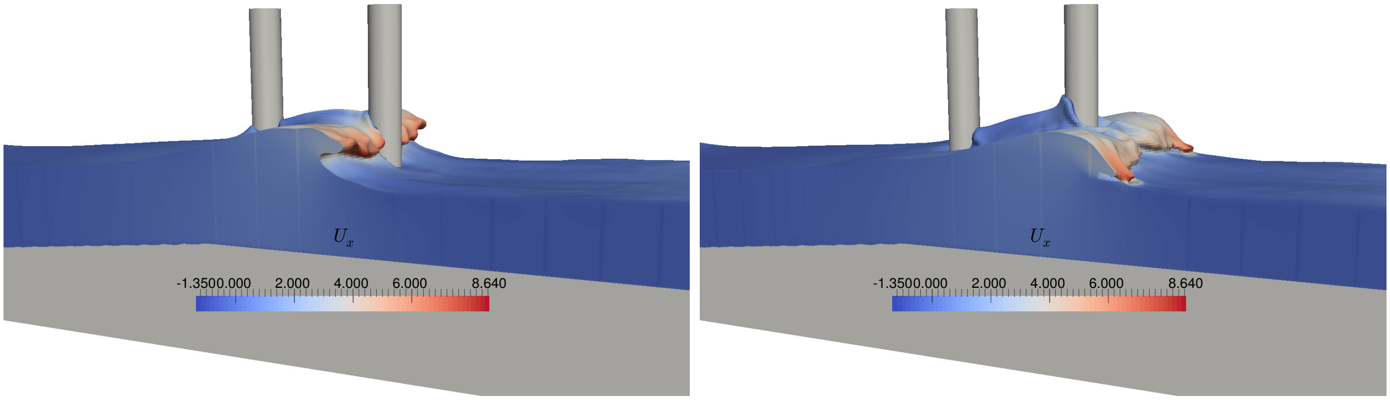
(a) breaking wave forces on the cylinders

(b) free surface elevations around the cylinders

Fig. 8. Wave forces on and free surface elevations around the cylinders for scenario C3: wave breaking exactly at the first cylinder with $S = 3D$

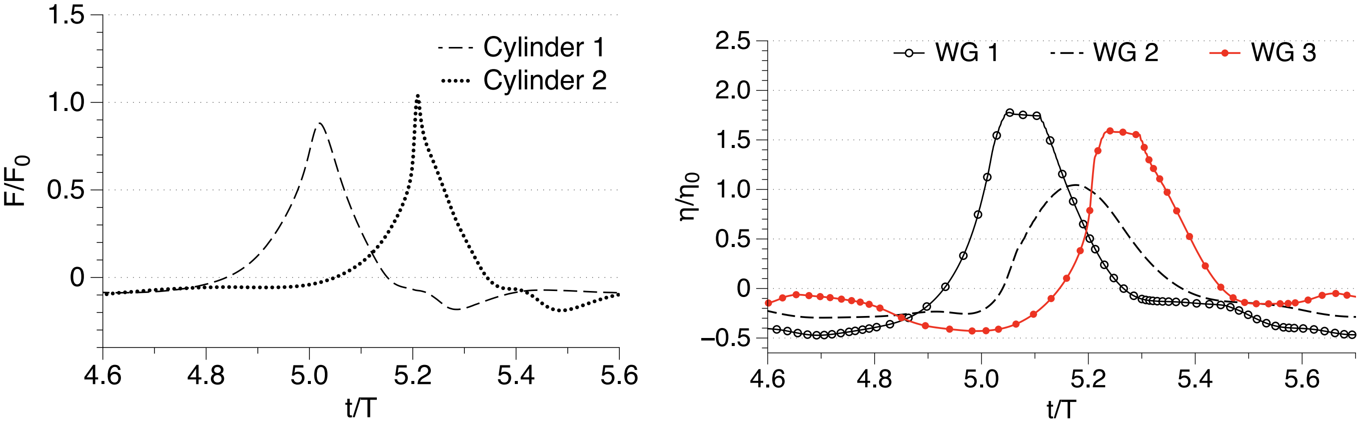


(a) $t/T = 4.08$ (b) $t/T = 4.18$



(c) $t/T = 4.25$ (d) $t/T = 4.32$

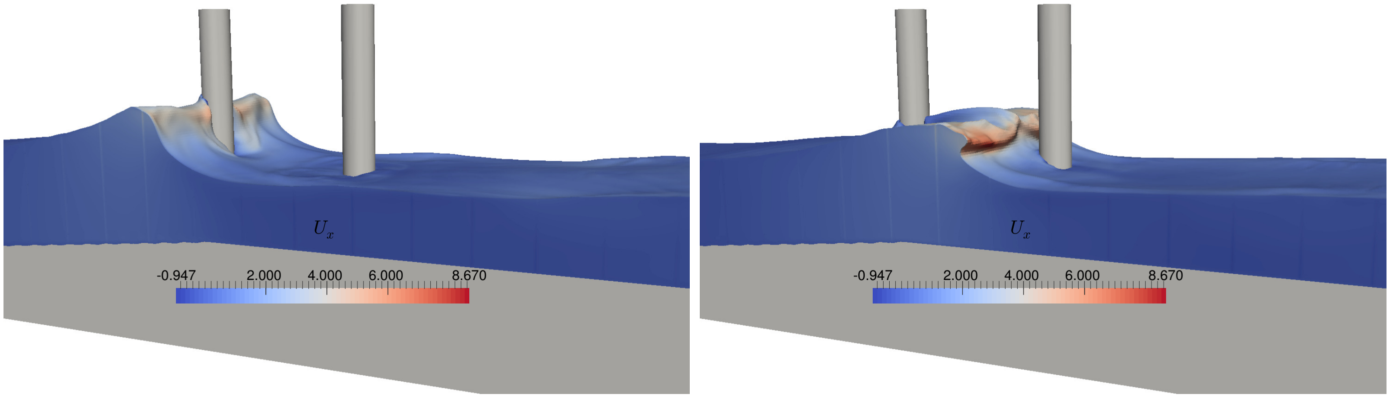
Fig. 9. Free surface around the cylinders in scenario C3 ($S = 3D$) with horizontal velocity contours



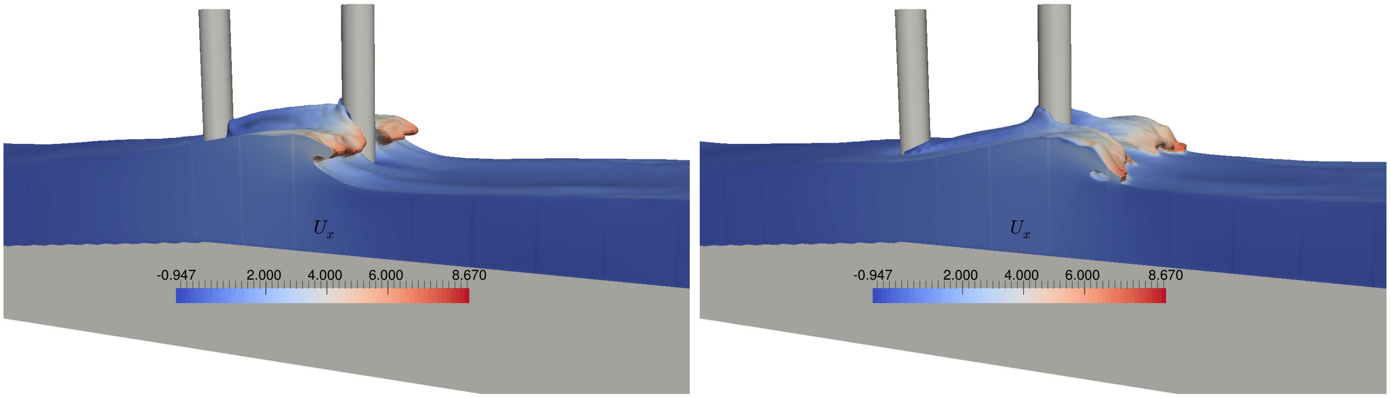
(a) breaking wave forces on the cylinders

(b) free surface elevations around the cylinders

Fig. 10. Wave forces on and free surface elevations around the cylinders for scenario D4: wave breaking just behind the first cylinder with $S = 4D$

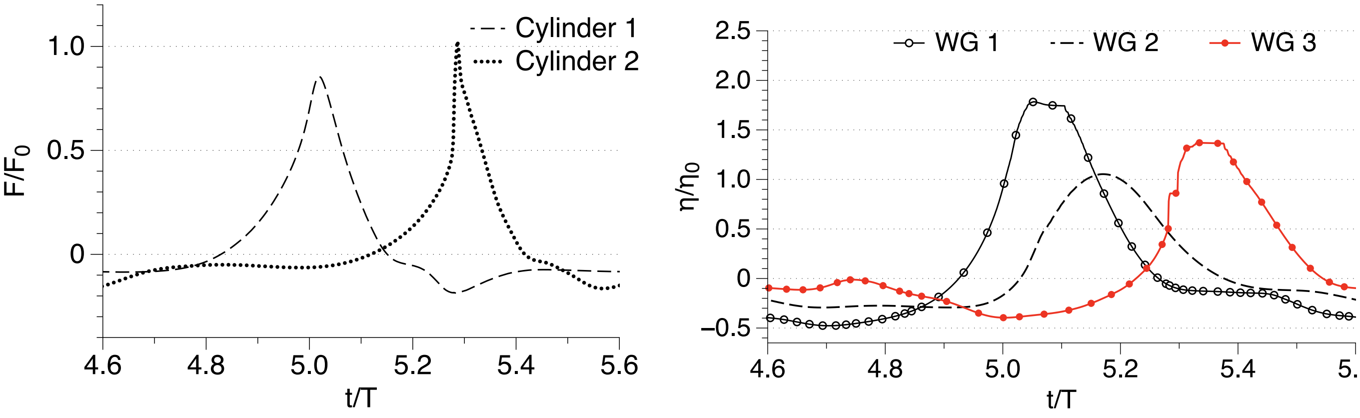


(a) $t/T = 4.03$ (b) $t/T = 4.16$



(c) $t/T = 4.21$ (d) $t/T = 4.30$

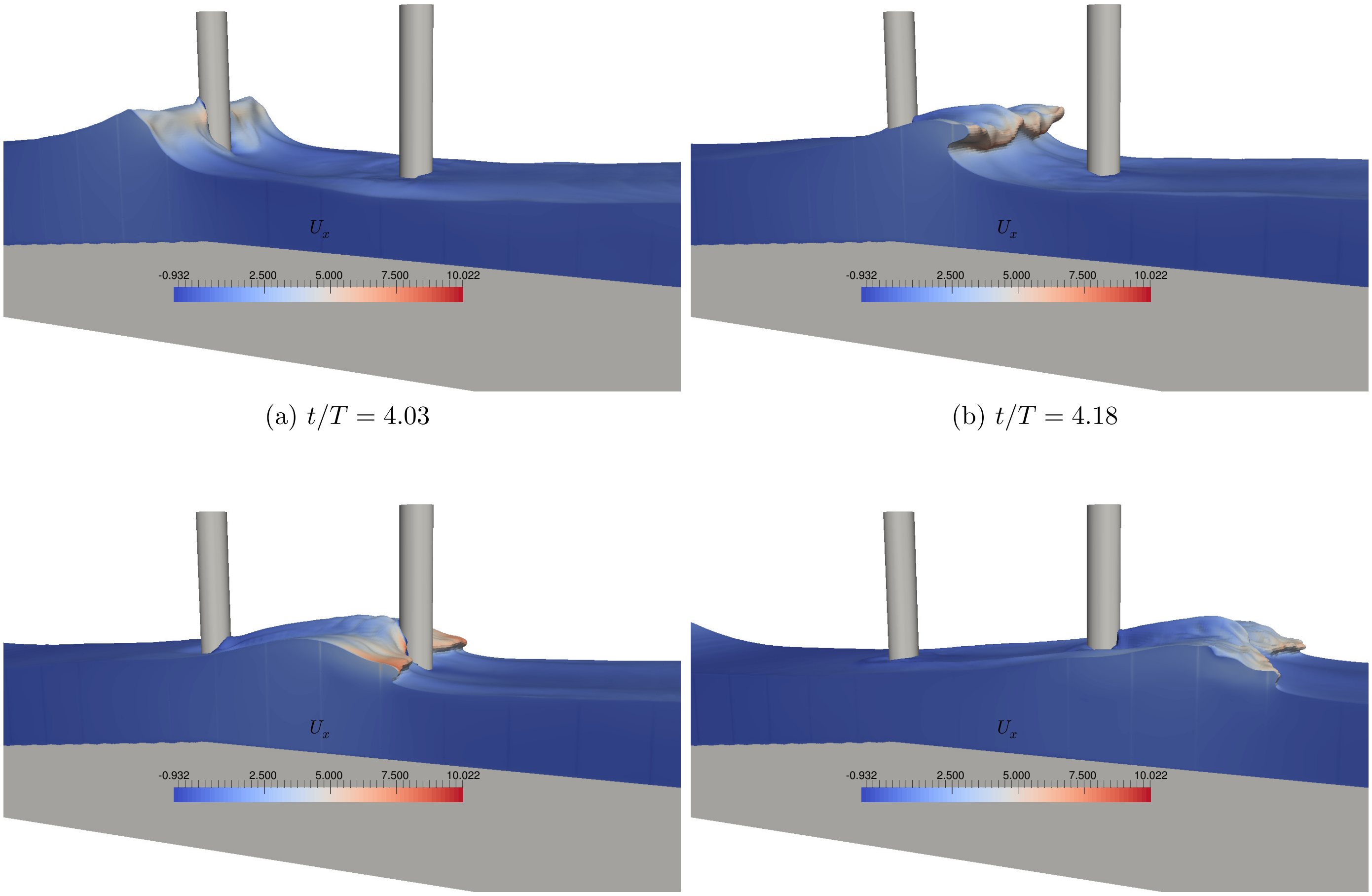
Fig. 11. Free surface around the cylinders in scenario D4 ($S = 4D$) with horizontal velocity contours



(a) breaking wave forces on the cylinders

(b) free surface elevations around the cylinders

Fig. 12. Wave forces on and free surface elevations around the cylinders for scenario D6: wave breaking just behind the first cylinder with $S = 6D$



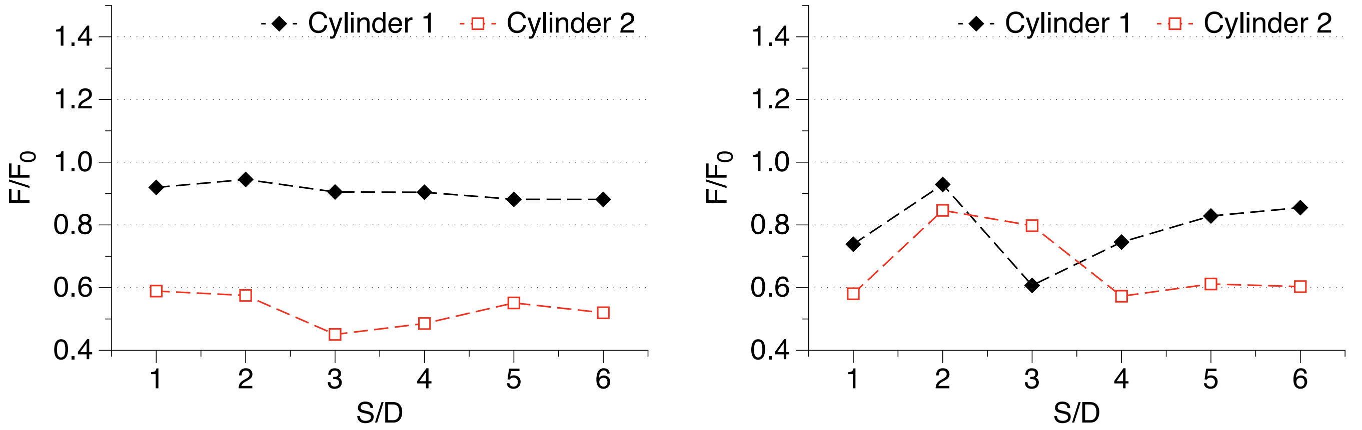
(a) $t/T = 4.03$

(b) $t/T = 4.18$

(c) $t/T = 4.28$

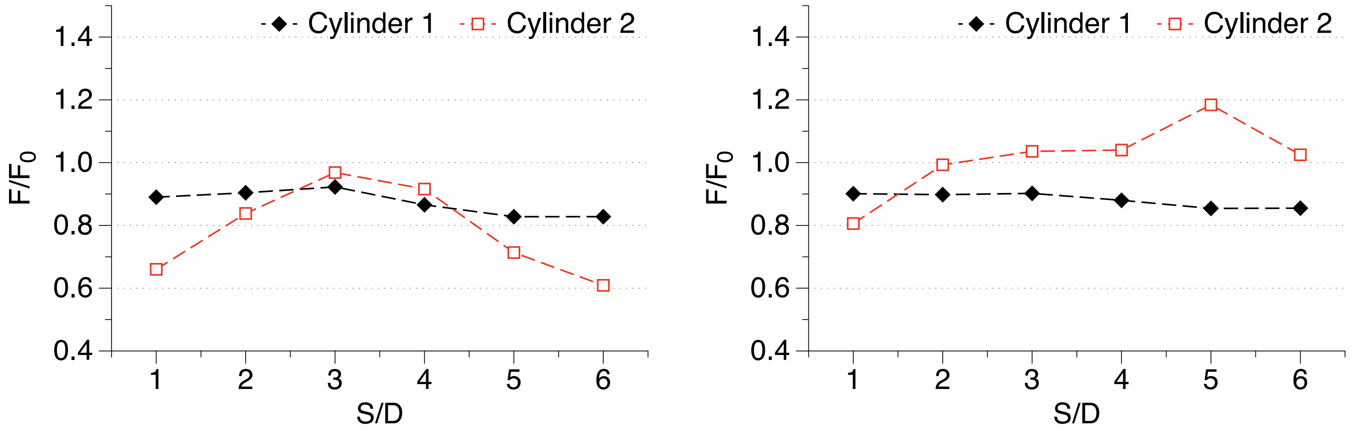
(d) $t/T = 4.46$

Fig. 13. Free surface around the cylinders in scenario D6 ($S = 6D$) with horizontal velocity contours



(a) scenario A: breaker tongue impact on cylinder 1 just below wave crest level

(b) scenario B: breaker tongue impact on cylinder 1 at wave crest level



(c) scenario C: wave breaking exactly at cylinder 1

(d) scenario D: wave breaking just behind cylinder 1

Fig. 14. Variation of the maximum wave force on the cylinders with distance of separation S in different wave impact scenarios

Table 1. Details of the setups used in the different simulations

Case	H (m)	T (s)	S (m)	scenario	F_0 (N)	F_1/F_0	F_2/F_0	η_{cyl1}/η_0	η_{cyl2}/η_0
A1	1.30	4.00	1D	overturning wave crest impact on cylinder 1 just below wave crest level	14000	0.92	0.59	1.58	1.69
A2			2D			0.95	0.58	1.64	1.75
A3			3D			0.91	0.45	1.57	1.58
A4			4D			0.90	0.48	1.56	1.62
A5			5D			0.88	0.55	1.59	1.70
A6			6D			0.88	0.52	1.68	1.58
B1	1.30	4.00	1D	overturning wave crest impact on cylinder 1 at wave crest level	13400	0.74	0.58	1.76	1.71
B2			2D			0.93	0.85	1.70	1.72
B3			3D			0.61	0.80	1.75	1.58
B4			4D			0.75	0.57	1.69	1.56
B5			5D			0.83	0.61	1.69	1.45
B6			6D			0.86	0.60	1.70	1.37
C1	1.30	4.00	1D	wave breaking exactly at cylinder 1	11850	0.89	0.66	1.82	1.77
C2			2D			0.90	0.84	1.70	1.84
C3			3D			0.92	0.97	1.82	1.70
C4			4D			0.86	0.92	1.76	1.63
C5			5D			0.83	0.71	1.70	1.44
C6			6D			0.83	0.61	1.76	1.32
D1	1.30	4.00	1D	wave breaking just behind cylinder 1	9800	0.90	0.81	1.83	1.79
D2			2D			0.89	0.99	1.94	1.89
D3			3D			0.90	1.03	1.70	1.76
D4			4D			0.88	1.04	1.78	1.59
D5			5D			0.85	1.18	1.83	1.45
D6			6D			0.85	1.02	1.78	1.37