NTNU – Trondheim
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Statistical analysis of failures and failure propagation in railway track

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Reliability, Availability, Maintainability and Safety (RAMS)
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Supervisor: Jørn Vatn, IPK

Norwegian University of Science and Technology
Department of Production and Quality Engineering

RAMS
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Maintainability, and Safety

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MASTER THESIS

Department of Production and Quality Engineering
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MASTER THESIS
2013
for
Erik Sagvolden

Statistical analysis of failures and failure propagation in railway track
(Statistisk analyse av svikt og feilutvikling i jernbanespor)

The Norwegian Railway Administration (JBV) uses a measuring car to measure track performance. From a safety point of view, special attention needs to be paid to spots on the line where failure propagation is out of control, and critical failures could develop in between measurements typically carried out twice a year. A challenge in the modelling is that we are dealing with so-called line objects, where there are an almost infinite number of places a failure can occur. This is complicated by the fact that the measuring car reports the position of failures with some uncertainty, making it difficult to compare results across different measurement series. The main objective of the master thesis is to propose and apply statistical methods related to the available data, and the parameters needed in the decision models related to inspection and operational restrictions. In this work the candidate shall especially:

1. Become familiar with the available data in JBV, relevant maintenance models, and related technical rules.
2. Perform a literature study on statistical methods used for similar situations.
3. Discuss relevant existing statistical methods, and if required propose new methods that could be used when analyzing failures and failure propagation in railway track with a special focus on "spot" failures.
4. Review existing data in JBV, and discuss to what extent the data is suitable for the proposed statistical methods. If necessary, propose ways to deal with any mismatch.
5. Apply the proposed statistical methods on data from JBV.
6. Discuss improvements in the way JBV collects data for track failures

Within three weeks after the date of the task handout, a pre-study report shall be prepared. The report shall cover the following:

- An analysis of the work task's content with specific emphasis of the areas where new knowledge has to be gained.

- A description of the work packages that shall be performed. This description shall lead to a clear definition of the scope and extent of the total task to be performed.
- A time schedule for the project. The plan shall comprise a Gantt diagram with specification of the individual work packages, their scheduled start and end dates and a specification of project milestones.

The pre-study report is a part of the total task reporting. It shall be included in the final report. Progress reports made during the project period shall also be included in the final report.

The report should be edited as a research report with a summary, table of contents, conclusion, list of reference, list of literature etc. The text should be clear and concise, and include the necessary references to figures, tables, and diagrams. It is also important that exact references are given to any external source used in the text.

Equipment and software developed during the project is a part of the fulfilment of the task. Unless outside parties have exclusive property rights or the equipment is physically non-moveable, it should be handed in along with the final report. Suitable documentation for the correct use of such material is also required as part of the final report.

The candidate shall follow the work regulations at the company's plant. The candidate may not intervene in the production process in any way. All orders for specific intervention of this kind should be channelled through company's plant management.

The student must cover travel expenses, telecommunication, and copying unless otherwise agreed.

If the candidate encounters unforeseen difficulties in the work, and if these difficulties warrant a reformation of the task, these problems should immediately be addressed to the Department.

The assignment text shall be enclosed and be placed immediately after the title page.

Deadline: June 10th 2013.

Two bound copies of the final report and one electronic (pdf-format) version are required.

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Preface

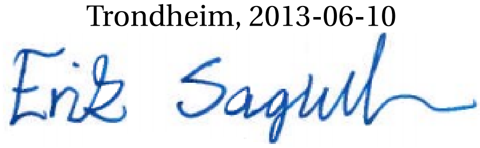
This master thesis is written at the Norwegian University of Science and Technology (NTNU). It is the last part of a MSc in Reliability, Availability, Maintainability and Safety. The project was conducted between 14th of January and 10th of June 2013.

The project is performed in cooperation with the Norwegian Railway Administration (Norwegian: Jernbaneverket). Jørn Vatn, is a Professor at NTNU, and the main supervisor for this thesis. He have played an important role with his guidance during the whole project. There have also been several people at the Norwegian Railway Administration that have contributed to this project with their time and knowledge.

It is adviced that the reader have some basic knowledge about railways and some understanding of statistics in order to fully understand the methods and result of this thesis.

The pre-study report and progress reports are not including in the final report by agreement with Jørn Vatn.

Trondheim, 2013-06-10



Erik Sagvolden

Acknowledgment

There is many people that have contributed to this project in different ways. First of all I would like to thank my supervisor at NTNU, Jørn Vatn, for his guidance and support while working with this thesis. He have given advice and corrections which have been crucial for the result.

The project is performed in cooperation with the Norwegian Railway Administration and there are several people here that deserves some credit for their contribution to the project. Terje Vasset and Knut Bjarne Green have given an introduction to different tools used to obtain data. Frode Teigen and Geir Ingvaldsen have helped out with their expert knowledge on track geometry and Hans Svee have been a great help as a contact person and organiser.

I would also like to thank Narve Lyngby who gave advice and shared his experiences, working with a similar research, in the startup phase of this research.

Last but not least I would like to thank Trond Storrønning for being a great office mate and discussion partner. Both while writing this thesis and through the whole master program.

E.S.

Summary and Conclusions

The Norwegian Railway Administration (JBV) uses a measuring car to measure track performance. From a safety point of view, special attention needs to be paid to spots on the line where failure propagation is out of control, and critical failures could develop in between measurements typically carried out twice a year. A challenge in the modelling is that we are dealing with so-called line objects, where there are an almost infinite number of places a failure can occur. This is complicated by the fact that the measuring car reports the position of failures with some uncertainty, making it difficult to compare results across different measurements series.

This report presents the result of an analysis of propagation of spot failures on track geometry. The analysis is based on ten inspections performed on the railway line between Eidsvoll and Hamar in the period 2006 to 2012. The data for the analysis is obtained from JBV's databases. Follow up of track performance is regulated by laws and regulations. JBV uses maintenance and renewal to improve track performance.

To analyse deterioration of the track performance it is applied statistical methods. There exist several methods that deals with trend modelling, and a literature survey is performed to cover relevant methods for this project.

Because of uncertainties in the obtained data a comprehensive work is performed to format data. This is done to understand what is included in the data and how the data can be used to analyse track performance. Two important parts of this work are to adjust the position of measurements and use the measurements to create time series for individual spots on the line.

The adapted statistical methods are used to analyse the time series. The result of the analysis is a model that can predict the probability of failure development. The accuracy of the model is related to the accuracy of the obtained data and the methods used in the analysis. The intention of this model is to adapt inspection intervals and maintenance strategies that can reduce the probability of critical failures. This may in addition be used to increase safety on the railway.

The accuracy of future data series can be considerably improved by using a better system to accurately position the location of measurements. This is also in accordance with plans in JBV to implement GPS as part of the measurements.

Contents

Acronyms and List of Symbols

Acronyms

CDF Cumulative Distribution Function

IAL Immediate Action Limit

IL Intervention Limit

JBV Norwegian Railway Administration (Norwegian: Jernbaneverket)

LS Least Square

MLE Maximum Likelihood Estimation

MLR Multiple Linear Regression

NMTL Newly Maintained Track Limit

PDF Probability Density Function

List of Symbols

y Matrix of observations

y_{ij} Observation j in time series i

$\hat{y}_{ij}$ Fitted value for observation ij

$\bar{y}$ Mean of y

t Time unit

n Number of time series

m Length of a time series

k Number of covariates in model

α Trend function

ψ Cycle function

τ Season function

ε Statistical error

λ_z Function of covariates

β_z Function of covariates

z Covariates

γ Coefficients for λ_z

δ Coefficients for β_z

ls Sum of least square

w Parameter vector in MLE

X Covariate matrix

b Coefficient vector

R^2 Coefficient of determination

R^2_{adj} Adjusted coefficient of determination

e Residual

λ_i Parameter

β_i Parameter

$\lambda_{test,i}$ Tested parameter for time series i

$\beta_{test,i}$ Tested parameter for time series i

$\hat{\gamma}$ Fitted coefficient

$\hat{\delta}$ Fitted coefficient

$\hat{\lambda}_z$ Function based on fitted $\hat{\gamma}$

$\hat{\beta}_z$ Function based on fitted $\hat{\delta}$

s^2 Variance

μ Mean

σ Standard deviation

F(e) Cumulative Distribution Function

f(e) Probability Density Function

e_λ Error for $\hat{\lambda}_z$

e_β Error for $\hat{\beta}_z$

$a - b$ Integration interval

D Deviation

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Chapter 1

Introduction

1.1 Background

Failures on railway tracks are critical errors that can have fatal consequences. There are several different failures that can occur on a railway track and several ways to measure track performance before failure occurs. Track geometry is a term that describes the geometrical position of tracks position in all three-dimensions and an important measurement for track performance. There have been several occasions where failure on track geometry was the primary cause of an accident (RAIB, 2011; SHT, 2011).

In Norway it is the Norwegian Railway Administration (JBV) that is responsible for both the construction and maintenance of the railway infrastructure. An inspection car called *Roger 1000* is used to measure deviations on different track geometry variables, typically twice a year on most lines.

Deterioration of the track geometry is a continuous process. Several researchers have looked into this problem to analyses the speed of deterioration. It is suggested several different models to describe the deterioration of track geometry. It is important to understand how failures develops in order to minimise the risk of critical failures in the period between inspections and to set the right criteria for when to implement maintenance.

However, most of the researchers have divided the track in sections and only looked at the average deterioration or key performance indicators of each section. To estimate deterioration the quality of each section is based on several spots. This means that there can be one or more critical spots on the track, even though the general condition of the section is within the limits.

The Norwegian Railway Authority (Norwegian: Statens Jernbane Tilsyn), which is the control and supervisory authority for rail traffic in Norway, also want some documentation regarding this issue (SJT, 2013). They are responsible for ensuring that JBV meet the conditions and requirements that govern the traffic through rail legislation. Thus it is important that JBV can document that the inspection interval is sufficient for the authority (Ministery of Transport, 2013). A good model to describe propagation of failures can also be used for maintenance optimisation and renewal strategies.

1.2 Literature Survey

1.2.1 Deterioration Models

In Norway there has been performed a research on track deterioration on several lines in the Norwegian railway network. It is done by dividing the track into sections with similar characteristics and using key performance indicators to measure track performance. The resulting model is a combination of two different exponential functions, where it is assumed that the deterioration can be divided into two different phases. The first phase starts straight after the maintenance activity *tamping* is performed, and the second phase is found to start when between 100 000 to 200 000 tons of train have passed on the track (Lyngby, 2007)

In the Netherlands a similar research has been performed, where a linear model is found to describe the deterioration (Westgeest et al., 2012). There have also been several other researchers that have investigated models to describe deterioration of track geometry. Dahlberg (2001) has performed a research where several different models are included. It gives a description on what these models are based on and how they can be applied in practice. This report gives several examples of models that is used to describe deterioration of track geometry, (e.g. logarithmic, exponential, linear or polynomial models). In most models deterioration accelerates over time, if maintenance is not implemented. Most models expresses deterioration as a function of loading cycles or tonnage.

1.2.2 Maintenance

Maintenance is performed to increase the lifetime and improve the condition of a track. Most of the research performed on this area suggests that track deterioration with maintenance can be described as a Lévy process with gamma distributed increments (Quiroga and Schnieder, 2011; Meier-Hirmer et al., 2009a). When the maintenance *tamping* is performed, the condition of the track is restored to a reachable quality (figure 1.1). The reachable quality decreases with age of the track until a point where *ballast cleaning* or renewal is necessary. This is also in accordance with maintenance philosophies in other industries (Ghosh and Sandip).

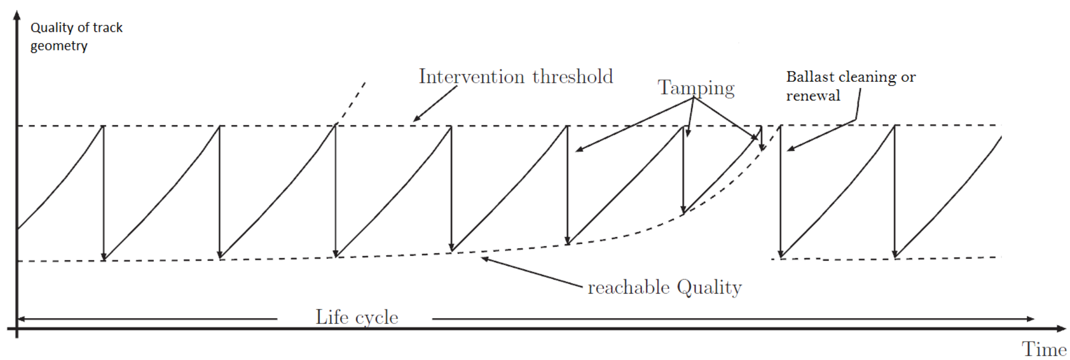

Figure 1.1: Gamma process for ageing (adapted from Quiroga and Schnieder, 2011)

1.2.3 Explanatory Variables

The deterioration process and the lifetime of a track can be affected by different explanatory variables. Lyngby (2007) found that several variables influence the deterioration of track geometry (e.g. steel used in rails, axle load and sleeper type). Other researcher that have looked into models describing deterioration of track geometry have found other significant explanatory variables. These variables can vary from type of rail and subgrade to usage and climate (Westgeest et al., 2012).

What Remains to be Done?

The literature study have described different models to predict track deterioration and maintenance activities to cope with it. It is suggested several different models that look into different track geometry variables or functions to calculate key performance indicators based on track geometry variables. Despite this, very limited information on deterioration of spot failures is found.

An analysis that looks at deterioration of spot failures over time, can give more information regarding this issue and be used to find out if there is necessary to look further into the subject. Thus, it remains to find a model that can describe how spot failures develop for all different track geometry variables. Spot failures might follow different deterioration models than deterioration of sections, but the information found in the literature on deterioration of sections, might be used as a basis to start the analysis.

In order to do this type of analysis, track geometry measurements must be obtained together with factors that can affect the deterioration rate. Depending on the obtained data, different statistical methods can be used for analysis. A study of different methods is necessary in order to implement a method that can predict how different explanatory variables are affecting deterioration of spot failures.

1.3 Objectives

The main objectives of this project are

1. Choose a railway line and collect data for a case study
2. Calibrate and format data to be used in analysing spot failures
3. Perform a study on statistical methods used for trend modelling
4. Perform a statistical analysis on the formatted data and find a deterioration model based on explanatory variables
5. Interpret the model and illustrate how the model can be used inspection purposes

1.4 Limitations

This report will be based on data gathered from a line in the Norwegian railway network. The quality of the result will be limited to the accuracy of the obtained data and how much data that can be obtained.

It will be assumed that the chosen lines is representative for finding a model that can be used to predict deterioration on several other railway lines.

Because of limitations in time and resources the finished model will not be tested in practice. The assumptions used to adapt the statistical methods to the data, might cause some uncertainties in the result and limitations in implementing the result.

1.5 Approach

A literature survey is performed to evaluate the available research within this subject and to find relevant statistical methods and tools. Data will be obtained manually from JBV's databases. There will also be performed manual inspections of the data in order to understand how this data can be calibrated and formatted to a useful format. Railway experts will be used to make the right assumptions regarding the data in order to adapt the right statistical methods. The statistical analysis is based on known methods, and methods obtained through a limited literature survey. To adapt the methods to the data, it might necessary to have a pragmatic approach to keep up progress of the research. This is done in order to have a steady progress. If there is some uncertainties related to using this approach, assumptions will be made that states what the research is based on. MATLAB and Excel will be used as tools to format and analyse the obtained data.

1.6 Structure of the Report

Chapter 1 gives an introduction to the report and the result of the literature study. It also includes the objectives of this project and the approach used to reach the objective. Chapter 2 explains what track deterioration is, and how this can be analysed. Chapter 3 describes how all the data used in the analysis is obtained. Chapter 4 explains how the data is calibrated and formatted. Chapter 5 introduces statistical methods and tools that is relevant to analyse the data. How the analysis is performed is described in chapter 6. Chapter 7 will explain how the model can be used to estimate deterioration of spot failures. Chapter 8 is the summary, conclusion, discussion and recommendations for further work.

Chapter 2

Track Deterioration

Deterioration of railway track geometry is a continuous process. To be able to model this process it is helpful to understand what is happening in practical terms. This chapter explains what track geometry is and what is done to prevent track geometry failures. It also describes how track geometry is measured in Norway, and how these measurements can be used in analysis.

2.1 Track Geometry Variables

Five different variables are used to measure track geometry. These five are vertical levelling (a), cant (b), gauge (c) and horizontal levelling (d), which can be seen in figure 2.1, and the last one is twist. In the figure variable a and b is viewed from the side and c and d is viewed from above. Twist can be explained as the vertical difference in height between two cross sections for the track measured with a specified distance apart (CEN, 2008). In practical terms twist is a measure of the amplitude for longitudinal *waves* on the track.

2.2 Deviation and Maintenance

Track geometry is deteriorating when train passes on a track. This causes vibrations in the track and ballast and can make the track geometry get out of position. After some loading cycles this might lead to some gaps in the ballast which makes the sleepers have less support, leading to heavier vibrations. Each track geometry variable have different limits specified to indicate

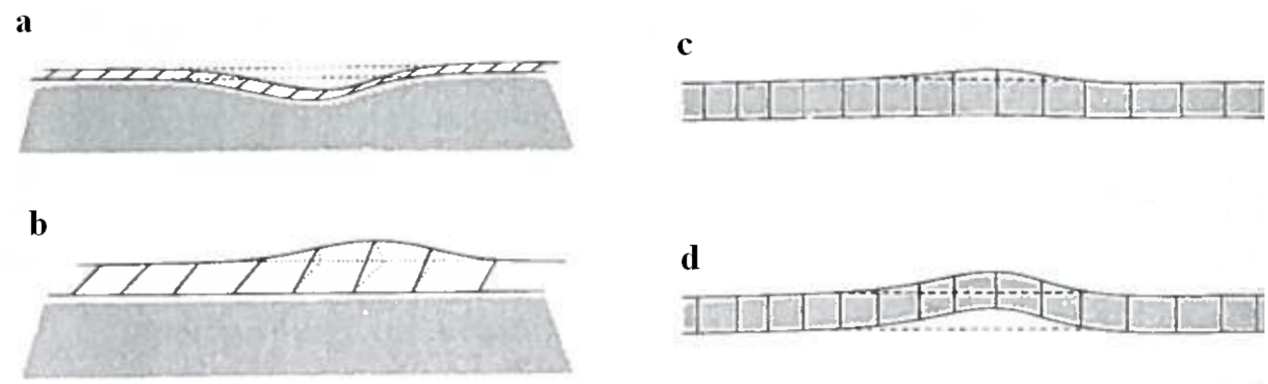

Figure 2.1: Track geometry variables; vertical levelling (a), cant (b), gauge (c), horizontal levelling (d) (Meier-Hirmer et al., 2009b)

when it is necessary to implement maintenance. These limits are specified in JBV's technical regulations, and varies with the speed limit on the track (JBV, 2013). The different limits are Newly Maintained Track Limit (NMTL; Measurements should not exceed this limit right after maintenance), Intervention Limit (IL; Limit for when maintenance should be planned and implemented) and an Immediate Action Limit (IAL; Immediate action must be implemented when this limit is exceeded). A measurement that exceeds the NMTL is called a deviation. Figure 2.2 shows six hypothetical measurements of twist 2 meter. It includes all the different limits, explanations to how different measurements are defined and which action that is required when different limits are exceeded. It is important to notice that the limits can differ depending on speed limit and radius. The IL and IAL is based on the standard EN 13848 (Railway applications level-Track geometry quality). These limits are there to reduce the risk of derailment to an acceptable level based on both theoretical data and experience (CEN, 2008).

A deviation is not necessarily a sign of failure, but if it develops further it might lead to an unacceptable condition. When this happen, *tamping* is the main maintenance activity that is implemented to restore the track to an acceptable condition. Tamping is performed with a tamping machine that uses claws to vibrate the ballast under the sleepers and restore the support of the sleepers (Lander and Petterson, 2012).

If the interval for when tamping is reduced a lot, this might be a sign that there is a problem with the track or subgrade. Two factors that can affect this is; worn out rails, which can be fixed by grinding or milling the rails, or worn out and polluted ballast. For the latter, *ballast cleaning* can be used to restore the condition of the track. In some cases renewal of different

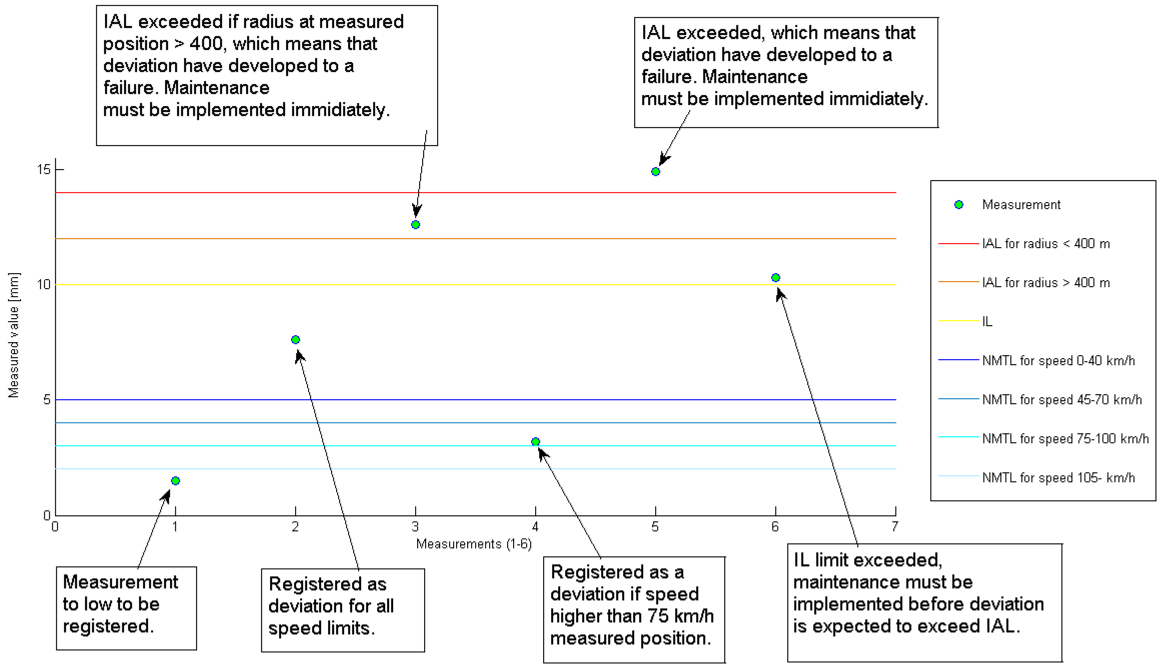

Figure 2.2: Hypothetical measurements of twist 2 meter, with explanations

track elements can also be necessary. Ballast cleaning changes the worn ballast and removes impurities before laying new ballast back around the sleepers. It requires more resources than tamping and is performed only when tamping is insufficient and a test shows worn out and/or polluted ballast. Ballast cleaning can increase the lifetime of a track significantly. (Teigen, 2013)

The mentioned maintenance activities above are used to deal with deviations related to vertical levelling, cant, horizontal levelling and twist. Deviations related to gauge is most commonly connected to worn out sleepers and rail-fastening, which usually is fixed by changing the components. (Teigen, 2013)

Explanatory variables are factors that can affect the deterioration process for track geometry. In statistics explanatory variables can be called independent variables or covariates. Further in the report the term covariates is used to describe explanatory variables. To model deterioration of track geometry these covariates can be connected to both the track and usage, but also to external conditions like climate, weather, geography and soil. To reduce the influence of covariates that have a negative impact on deterioration, different measures can be used, both when tracks are built and during maintenance. Still, it is difficult to reduce the influence by these covariates totally.

2.2.1 Measuring Track Geometry

Nowadays inspection cars are used to measure track geometry variables. In Norway, JBV uses an inspection car called *Roger 1000* to perform all the measurements. It is done using a combination of laser technology, cameras and special sensors. The car can also measure several other parameters that are not related to track geometry (e.g. radius of track or failure in the overhead line). Measurements are registered every half meter. (Mermec, 2013b)

Snow and water affect the accuracy of the sensors on the equipment and can cause big errors in the measurements. The first inspections is done in the spring when the snow has melted. Before the first inspection every year the equipment is calibrated to make the measurements as accurate as possible (Ingvaldsen, 2013). This causes and error in the reproducibility. Some uncertainty is also caused by the resolution of the equipment, but this is a much smaller fraction than the error caused by reproducibility. (Mermec, 2013a)

Another type of error is linked to the position of the measured deviation. The position of the inspection vehicle is set by manually entering starting position and a trip meter is used to estimate the position. Wear of the wheel, wheel spinning and manual adjustment of position in the inspection car are causes of displacement (Lyngby, 2007). Because of this, the position can also be adjusted manually when the vehicle is passing known positions.

2.3 Time Series

To be able to follow the deterioration of spot failure, a deviation must be followed over a period of time. If the same deviations is found in several inspections the deviations are *matched*. The best parameter that can be used to match deviations is the measured position. Several deviations that are matched together represents a time series. A time series shows how a deviation is developing over time and can be used to find the deterioration trend.

Since there is some uncertainty in the position of deviations, a match made just by comparing position can cause a high possibility of mismatch. To cope with this, other parameters like size and length of the deviation, can also be used to find similarities between deviations that confirms a correct match.

Chapter 3

Obtaining Data

For the analysis in this report the line between Eidsvoll and Hamar is chosen. This line is a part of "Dovrebanen" in Norway. The line have km markings from 68.9 to 126.9, measured from Oslo Central-station. This is a single track line and is chosen because some parts runs along water, and because it has a steady traffic flow per time unit. This chapter explains how relevant data for the Eidsvoll to Hamar line is obtained. The data that is needed for the analysis is stored in different databases and some of the information might be difficult to get hold of. This chapter also describes which data that is applied in the study.

3.1 Inspection Data

Measurements from ten inspections performed between October 2006 and September 2012 is used in the analysis. Track geometrical variables between Eidsvoll and Hamar is measured two times a year. In two of the inspections from this period, the data was collected differently and the data from these two are not a part of the analysis. The inspections performed in May 2001 was done during rain, which might have caused a higher error rate, but since there was not found any signs that indicated a problem with the measurements, these data are also used.

Data from all inspections are stored in a database and the program InOffice is used to export data sets. The exported files contains information on length, size, start position and position of maximum impact on all measured deviations. The twist variable is measured for both two and nine meter *waves*, which means that in total there is six different track geometry variables. All deviations are measured in millimetre.

In addition to information about all deviations, the radius measured every half meter is also exported from InOffice. The radius is used to adjust the position of deviations.

3.2 Theoretical Curvature

Theoretical curvature is a measure of the radius on a railway line. The theoretical curvature is based on how the railway line should have been built and it may differ from the true curvature of the line. This information is stored in the data base BaneData. BaneData is a internal data base for JBV and contains information regarding all tracks in the Norwegian railway network. The theoretical curvature is used as a reference to improve the position of measurements by comparing the theoretical curvature to the measured radius.

3.3 Covariates

Earlier research have suggested different covariates that affect deterioration of track geometry. Based on this research and suggestions from railway experts, a selection of different covariates is tested to see if they affect the deterioration rate of spot failures. Table 3.1 gives a short description of the covariates that is tested in this report and how they are obtained. In total there are ten covariates that each can affect the development of spot failures differently. What is important to notice is that the covariate super elevation and the track geometry variable cant is connected to each other. super elevation is a theoretical measure, while cant is measures the deviation from a the super elevation.

3.4 Maintenance

There is a wide range of maintenance activities that are used on the railway. All these activities are registered in BaneData. Tamping and ballast cleaning is assumed to be the only maintenance that affects track geometry. Data about these activities is obtained from BaneData. The data is used in the analysis to describe deterioration as a function of days since maintenance. It

Table 3.1: Covariates included in the analysis

Covariate	Explanation	Numerical value	Source
Curvature	Radius of track	- 3630 m to 5730 m	BaneData
Culvert	Culvert within 10 meters	0 or 1	BaneData
Super elevation	Intended super elevation	0 to 150 mm	BaneData
Renewal of track	Days since renewal of rail	0 to 39 000 days	BaneData
Steel quality	Type of steel used for rails	0-4	BaneData
Speed limit	Classes defined for different speed	2 to 4 (105 to 120, 75 to 100 and 45 to 70 km/h)	BaneData
Sleepers	Wooden or Concrete sleepers	0 to 2	BaneData
Track switch	Track switch within 20 meters	0 or 1	BaneData
Water	Evaluating if lake/river is close to the track	0 or 1	Finn.no

is possible that other maintenance activities can affect track geometry in different ways, but this is disregarded in this project.

Chapter 4

Formatting Data

Before the data obtained in chapter 3 can be analysed, deviations must be matched into time series and connected to the correct covariates. The data must also be formatted into a suitable format for analysis. This is done stepwise by using algorithms written in MATLAB.

4.1 Calibrating Position

The position of a deviation is used to match data from different inspections. To match as many deviations as possible, the error in position must be as small as possible. When the uncertainty in position increases, the probability of mismatch also increases.

Radius is a continuous variable that have been measured in all inspections. The position of a deviation is calibrated by adjusting the measured curve to coincide with the theoretical, each position in an inspection is moved to a common reference point. By doing this it is assumed that the optimised position is found. Figure 4.1 illustrates the difference between the theoretical curvature and the radius for one section of the line and for two different inspections. It can be seen how the difference between the theoretical and measured curve varies. It clearly shows the manual adjustment of position in the September 2007 inspection. By looking at the inspection from October 2008 it can be seen that both the position is lagging behind the theoretical curve and also that the maximum measured radius is lower than the maximum theoretical radius. The calibration improves the position by minimising these differences.

A customised algorithm is used to adjust the position of each deviation. It is made by man-

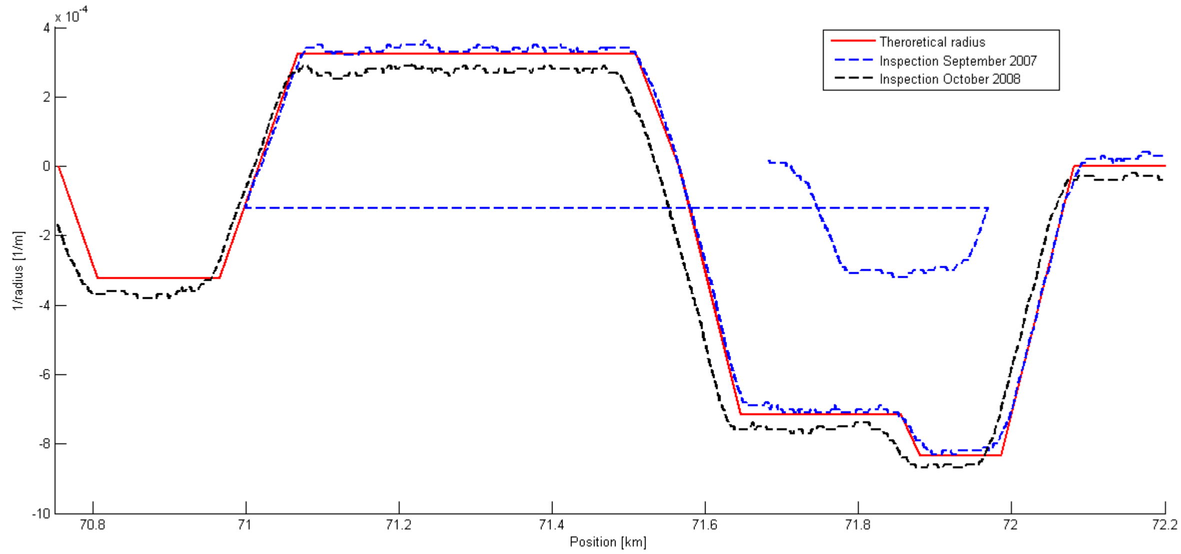

Figure 4.1: Measured radius compared to theoretical radius

ually investigating inspection data to find out exactly how accurate the measurements are and how the manual adjustments are performed. This have made the algorithm optimised to adjust inspection data based on the theoretical curvature.

The most important steps of the algorithm is listed as:

- Step 1
  - Remove manual adjustments
- Step 2
  - Divide in line into sections
  - Adjust each section by minimising the squared distance between measured and theoretical radius
- Step 3
  - Find local min/max on measured radius
  - Find local min/max on theoretical radius
  - Adjust measured radius between local min/max to coincide with theoretical radius

- Step 4
  - Repeat step 2 with adjusted radius

For more details describing how this calibration is performed a recap of the most important parts of the algorithm that can be found in algorithm 1.

Data: Data set from Inspections (=Inspect) and theoretical curvature

Do all steps for one inspection at the time;

Step 1: Remove manual adjustment;

$n = length(Inspect)$; $xmin = 10^5$;

for *i=1* → *n* **do**

    if |*Position(Inspect(i)-Inspect(i-1))*|>*0.5 meter* **then**

        Position(Inspect(i))=Position(Inspect (i-1)) + 0.5 meter;

    end

end

Step 2: Adjust position by dividing in *i* sections;

for $i = \frac{Position(Inspect(1))}{500 meter} \rightarrow \frac{Position(Inspect(n))}{500 meter}$ **do**

    for *k=-50 meter* → *50 meter* **do**

        Optimise fit for section(i) by testing if position + k is a better fit;

        Best fit found by minimising difference between theoretical and measured radius ;

        $x = \sum$(Radius Section(i) + k) - Theoretical radius)^2;

        if *x<xmin* **then**

            xmin=x;

            Optimal position for Section i = position + k;

        end

    end

end

Step 3: Adjust radius;

k=0;

for *i=1* → *n* **do**

    if *Radius(Inspect(i)) is a local min/max* **then**

        Radius(Inspect(i))=Theoretical radius;

        Adjust curvature between Inspect(i) and Inspect(k) linearly to the theoretical radius;

        $k = 1$;

    end

end

Step 4: Repeat step 2 with adjusted radius;

Algorithm 1: Improve position of inspections

Table 4.1 shows the sum of squares between the measured and theoretical radius. It demonstrates how the sum is reduced both after step 2 in the algorithm and after the whole adjustment.

Table 4.1: Change in sum of squares during adjustment

	September 2007	**October 2008**
Sum of squares before adjustment	2.84×10^{-3} $[1/m^2]$	3.95×10^{-3} $[1/m^2]$
Sum of squares after step 2	6.23×10^{-4} $[1/m^2]$	8.18×10^{-4} $[1/m^2]$
Sum of squares after complete adjustment	2.84×10^{-4} $[1/m^2]$	3.10×10^{-4} $[1/m^2]$

Figure 4.2 gives a graphical view of the improvement for the same inspections as earlier, and gives a strong indication that the position is improved. Because of the uncertainties in the measurements and because the theoretical curvature is not a perfect representation of the reality, it is difficult to determine exactly how good the fit is. Thus there are used some biased indicators that illustrate the improvement. After the adjustment all deviations are updated with a new position.

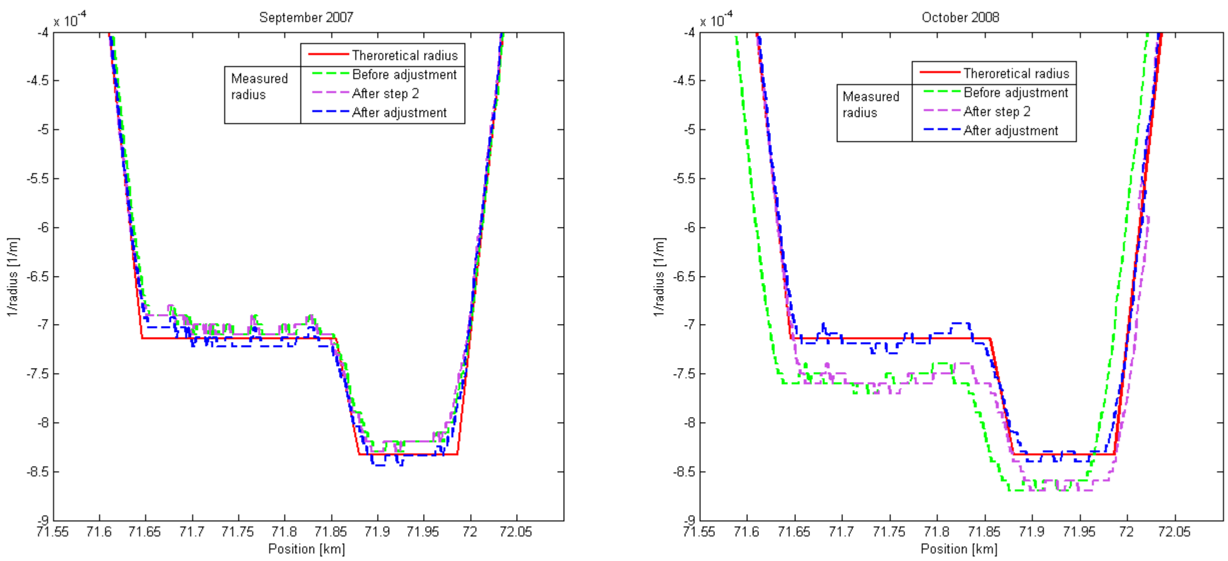

Figure 4.2: Improved position and radius during during calibration

4.2 Matching Deviations

Deviations is measured with a length, size and position. These three factors are used to match from different inspections. Different boundaries are set for each variable and for each type of deviation to assure that only the correct deviations are matched. These boundaries are based on information from JBV's technical regulations and manual inspection of the data (JBV, 2013). Because it is better to remove a correct matched time series than to include time series with mismatch, a filter is used to remove matches with a too high possibility of mismatch (time series are too close to each other).

The most important steps in matching deviations is listed below, for more algorithm 2 shows this process in more detail.

- Step 1
  - Set boundaries that two deviations must be within in order to be matched
  - For one deviation at the time, search through data from all inspection and check for deviations within the boundaries
- Step 2
  - If the average position of two time series are too close, both time series are removed
  - Store all time series in a matrix

Data: Data set from all ten inspections with adjusted position (= $Inspect$)

Type = type of deviation; Position = position of deviation; Length = length of deviation;

Size = size of deviation; ;

typ=Input(Type);

Step 1: Find matches;

pos =Boundary Position ; len =Boundary Length; siz =Boundary Size ;

```
for x=1 → 9 do
    for i=1 → length(Inspect_x) do
        for y=x+1 → 10 do
            for j=1 → length(Inspect_y) do
                if typ=Type(Inspect_x(i)) AND typ=Type(Inspect_y(j)) AND
                pos>|Position(Inspect_x(i) − Inspect_y(j))| AND
                len>|Length(Inspect_x(i) − Inspect_y(j))| AND
                siz>|Size(Inspect_x(i) − Inspect_y(j))| then
                    Inspect_x(i) and Inspect_y(j) is a match;
                end
            end
        end
    end
end
```

Step 2: Create time series;

All matches are created into time series;

Time series with a high probability of mismatch deleted;

```
if Difference in position for two time series < pos then
    Delete both time series;
end
```

Save time series in a matrix;

Algorithm 2: Optimisation of position

For the variable cant, the measured deviations is measured relative to a horizontal level, and does also include super elevation. This means that the measure is not the *true* deviation and that deviations can be difficult to compare. The intended super elevation is known at every position and can be subtracted from the deviation cant. Because of some inaccuracies in the position this might be insufficient for some deviations.

To deal with this problem it is assumed that the super elevation is correct, if the subtracted cant is between the NMTL and the IAL. In the situations where this is not the case, the average cant for that time series is said to be the average of all subtracted deviations. The rest of the deviations in that time series are reduced with the same amount. This is only a problem for around 10 % of the time series, and it is assumed that adjusting by using the average deviation is not biased.

4.3 Time Unit

Deterioration of track geometry can best be described as a function of load cycles or tonnage. This is confidential information that is difficult to obtain. Since only one line is used in this analysis, it is assumed that the tonnage is constant over the period the inspection data is gathered. This means that days are proportional to load cycles or tonnage, and can be used as a consistent parameter. If the model is used on a different line, or the tonnage changes on the analysed line, the model should be adapted to this new traffic flow.

When tamping or ballast cleaning is implemented on the track, the averaged condition of the track is restored to a reachable quality. The reachable quality when tamping or cleaning the ballast can be different, as seen in figure 1.1. The analysis looks only at what happens between maintenance intervals. This means that some time series must be divided into more time series if maintenance is implemented on that part of the track. If some of the *new* time series only consist of one measurements, the time series is removed. After this filtering the data set consist of time series with between two and ten measurements.

4.4 Formatted Data

Matched deviations are linked with the correct covariates according to the principles described in section 3.3. This gives a matrix that contains matched deviations, position, and covariates in a format that can be used for statistical analysis. Table 4.2 shows a part of this matrix.

Table 4.2: Example from part of data set used for analysis

Deviation type	**Position**	**Radius**	...	**October 2006**			...
				Deviation	**Maintenance**	**Rain**	
Twist 2 m	70.7673 km	-767.91 m	...	-3.1 mm	51 days	35.12 mm	...
Twist 2 m	71.0326 km	945.16 m	...	3.8 mm	51 days	35.05 mm	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

4.4.1 Matched Deviations

Out of a total of 115 076 deviations 92 735 is matched with a deviation in a different inspection. Figure 4.3 shows how many percentage of the measured deviations that is matched with a deviation in a later inspection, by using algorithm 2. It also shows how many percentage that have found a match in the consecutive inspection. It can be seen that it only is a small variation between the deviation types and in average around 60 % have found a match. Even though some time series are removed because of a high possibility of mismatch, there are a big amount of data left to be used in the analysis. Table 4.3 shows how many time series there are for the different deviation types.

Table 4.3: Number of time series for different variables

Track geometry variable	Vertical levelling	Cant	Gauge	Horizontal levelling	Twist 2 m	Twist 9 m	Total
Number of time series	10642	2466	3006	2692	1323	799	20928

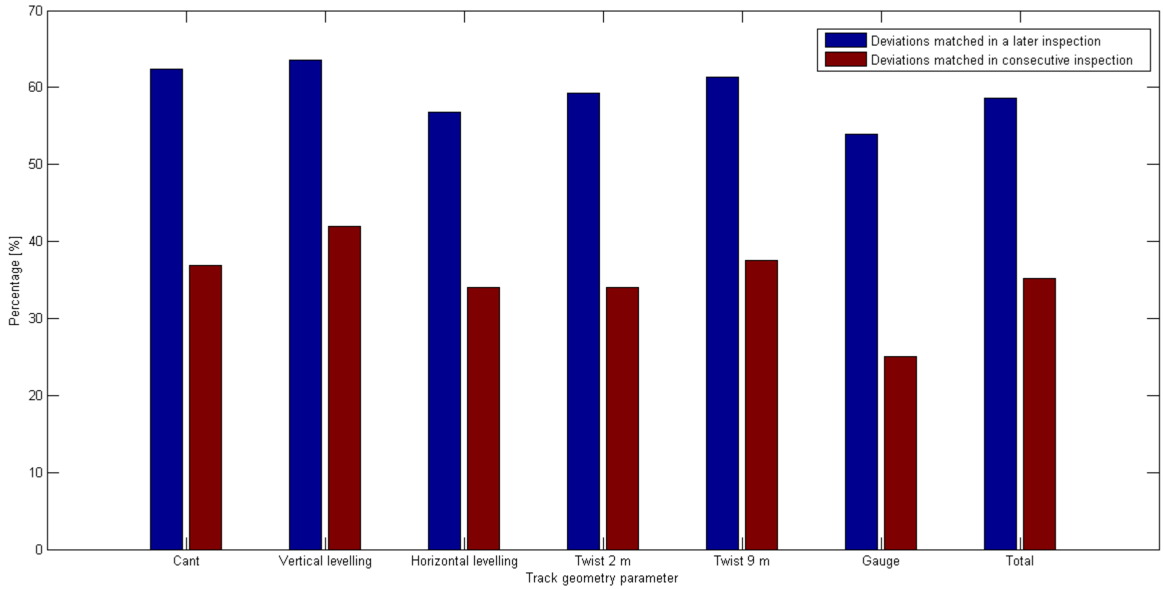

Figure 4.3: Measurement with match

4.4.2 Covariates

The covariates that are linked to time series are either continuous, binary or categorical. Curvature, super elevation and renewal of track are continuous variables. How the other six covariates are distributed among the deviations can be seen in figure 4.4. This figure illustrates the distribution of different outcomes for each covariate. For other tracks, different variables and other outcomes can be relevant.

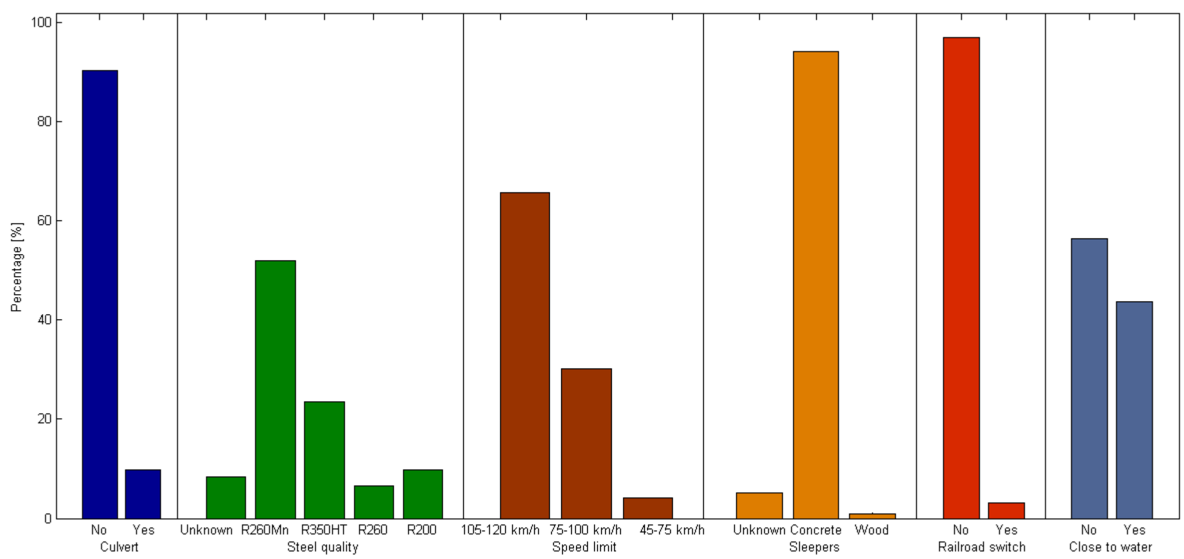

Figure 4.4: Different states for categorical covariates

Chapter 5

Statistical Models

Trend modelling is a statistical technique to analyse how time series are developing. There exists several different tools and methods that for trend modelling. To find the underlying trend these methods must be adapted to the specific data in the analysis. For complex data several methods or combination of methods may be used. Based on the assumptions used in the analysis this can lead to different results. Because of this it is important to understand what the data represents, what the methods do and which assumptions that are valid. This chapter presents different methods that can be used in trend modelling and how these can be used to analyse the formatted data. The methods are found in different statistical literature.

5.1 Time series

The data set consist of multiple time series with different covariates. A structural time series are a time series that consist of a combination of trend $\alpha(t)$, cycle $\psi(t)$, seasonal $\tau(t)$ and error components $\varepsilon(t)$ for a set of observations $y(t)$, see equation 5.1 (Harvey and Shephard). j and i meaning deviation j in time series i.

$$y_{ij}(t) = \alpha_{ij}(t) + \psi_{ij}(t) + \tau_{ij}(t) + \varepsilon_{ij}(t) \tag{5.1}$$

Figure 5.1 illustrates how trend, cycle and seasonal changes can occur for track geometry. This figure can be compared with figure 1.1 to see which effect maintenance may have. For this analysis both cyclic and seasonal effects are disregarded. Cyclic trends can occur after ballast cleaning or renewal, where the quality of the track can be improved significantly. Other researchers have looked into how maintenance affect the cycle (Quiroga and Schnieder, 2011), but this is not looked into in this analysis. Thus, it is assumed that the trend in a cycle is constant. As illustrated in figure 5.1 this can be a valid assumption as long as the the trend is not in the end of a cycle. It is also possible that seasonal changes occur, but with the limited amount of inspections this is not looked into, and this should not affect the overall trend.

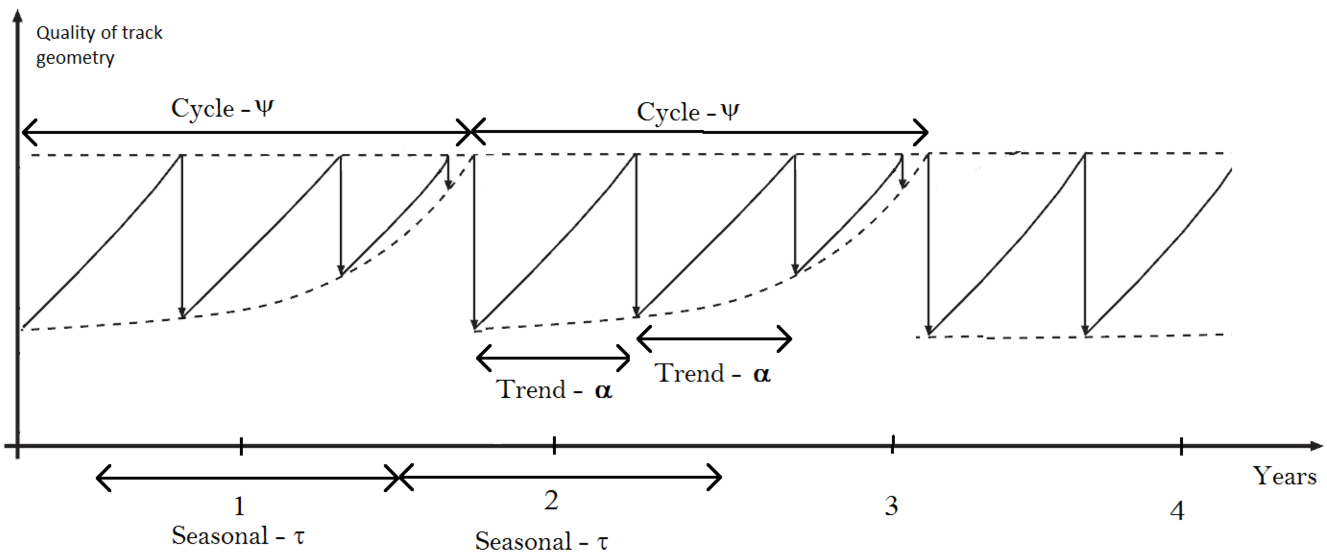

Figure 5.1: Trends, cycles and seasonal effects on track geometry (adapted from Quiroga and Schnieder, 2011)

5.2 Trend

The trend $\alpha(t)$ can have several different functional forms. Before analysing the data it can be difficult to know what kind of function that gives the best fit. Degradation trends for track geometry have in earlier research been suggested to follow different variations of linear, exponential and logarithmic models, and also combination of these (Lyngby, 2007; Dahlberg, 2001). Based on this, the five models in table 5.1 is suggested.

λ_z and β_z can further be a function of the covariates, where z_1 to z_n are numerical values of covariate 1 to k and γ_1 to γ_k and δ_1 to δ_k are coefficients. ε is the error term.

Table 5.1: Suggested models to describe deterioration of track geometry

Model	**Function**
Linear	$\alpha(t) = \lambda_z + \beta_z t$
Exponential	$\alpha(t) = \lambda_z exp^{\beta_z t}$
Logarithmic	$\alpha(t) = \lambda_z + \lambda_z ln(\beta_z t)$
Exp-log	For t<x Exponential For t>x Logarithmic
Log-exp	For t<x Logarithmic For t>x Exponential

x = time when deterioration changes from first to second phase

$$\lambda(z) = \gamma_0 + \gamma_1 \times z_1 \ldots \gamma_k \times z_k + \varepsilon_\lambda \tag{5.2}$$

$$\beta(z) = \delta_0 + \delta_1 \times z_1 \ldots \delta_k \times z_k + \varepsilon_\beta \tag{5.3}$$

To estimate the coefficients in a suggested model, either Maximum Likelihood Estimation (MLE) or Least Square (LS) estimation could be used. MLE is based on maximising the likelihood function, see equation 5.4. $f(y|w)$ is the probability density function for the observed data y, given parameters w. Which in practical terms means to find the parameters w, which gives the highest likelihood of having observed the data y. (Myung, 2004)

$$L(w|y) = f(y|w) \tag{5.4}$$

LS estimation is based on minimising the squared difference between observed and expected data. y_i is the observed value and $\hat{y}_i$ the fitted value. From this ls_1 is estimated with equation 5.5. For a linear model parameter estimation can be performed analytically. For non-linear models a analytical solution to parameter estimation is in some cases not possible, or be very time consuming. The model can in some cases be transformed into a linear model, but it is important to understand what happens with the parameters in the transformation. In cases where this is not beneficial or possible, the parameters can be found numerically. (Van De Geer, 2005)

$$ls_1 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \tag{5.5}$$

5.3 Multiple Regression

For each time series, both a λ_z and β_z is estimated. These functions depends on the covariates. Multiple regression can be used to estimate the coefficients in equation 5.2 and equation 5.3. When using this it is assumed that both the covariates and the covariates are independent of each other. This means that the value of one covariate does not affect the probability of different outcomes for other covariates. Related to time series it means that the deterioration of a deviation in one time series does not affect the deterioration of a deviation in a different time series. It is possible that two deviations close to each other on the track can be affected by factors that not are included in this model, and that this in practice causes a correlation, but this is disregarded in the study.

Multiple regression models can be based on different functions and the parameters in the models can be estimated by using both MLE or LS. Like in trend analysis. In Multiple Linear Regression (MLR) the coefficients can be estimated with matrix operations. This is done by minimising equation 5.6. Here y is the response vector (size $n \times 1$), X the covariate matrix (size $n \times (k+1)$) with first column being for the constant term (vector of ones), b the coefficient vector (size $1 \times k$), and ls_2 is the resulting least square sum. The LS solution for the coefficient vector b is given in equation 5.7 (Walpole et al.).

$$ls_2 = (y - Xb)' \times (y - Xb) \tag{5.6}$$

$$b = (X'X)^{-1} \times X'y \tag{5.7}$$

5.4 Stepwise Regression

In cases with many covariates, it is not always preferable to include all covariates in the model because this can include covariates that not really are significant for the model. In these cases stepwise regression can be used. Stepwise regression is a method to find the best combination of covariates to describe the model. This can be done by all the time including that covariate which makes the model able to explain as much of the variation in the data a possible. Every time a covariate is included in model it is tested if some of the covariates that already were included have become insignificant. This is done until no more covariates increases the fit of the model significantly. (Walpole et al.)

R^2 is a measure that is used to explain how many percentage of the variation in the data that the model can explain, and is called coefficient of determination. R^2 is estimated by equation 5.8, which in practical terms can be described as the ratio between explained variation and total variation. R^2_{adj} is a variation of R^2 where it is adjusted for the degrees of freedom in the data, see equation 5.9. Here, n is the number of observations and k is the number of covariates. R^2 is always increasing as covariates is included in the model, but this is not necessarily true for R^2_{adj}, which then can be used to analysed how many covariates that should be in the optimised model. $\hat{y}_i$ is the fitted value to observation y_i and $\bar{y}$ the mean value of all observation. (Walpole et al.)

$$R^2 = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{5.8}$$

$$R^2_{adj} = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2/(n-k-1)}{\sum_{i=1}^{n}(y_i - \bar{y})^2/(n-1)} \tag{5.9}$$

5.5 Residuals

Residuals is the difference between an observed value and a predicted value, see equation 5.10. It is comparable with the error, ε, in equation 5.1, but differs in the fact that ε is the difference between the true model and an observed measurement, while residuals are the error (e_i) between an expected value based on the fitted model ($\hat{y}_i$) and an observed measurement (y_i)

$$e_i = y_i - \hat{y}_i \tag{5.10}$$

Residuals can be used to analyse how appropriate the estimated model is for the data. To use the model for prediction the residuals for the fitted model should be randomly and normally distributed (Walpole et al.). That the residuals are randomly and normally distributed can be checked by plotting residuals chronologically with time and look for a pattern and with a normal probability plot. A normal probability plot is by sorting the residuals in ascending order and plotting this against a theoretical normal distributed residuals which is $N(0,\sigma)$. The points in the plot should form a straight line if they are normally distributed (Walpole et al.).

Normality can also be tested analytically with different tests. The Anderson-Darling test can be used to test for normality and give a p-value that represents the probability of having observed at least as extreme values as the one actually observed, given that the data is normally distributed(NIST/SEMATECH, 2013). If the data is not randomly and normally distributed this can mean several things and there is several ways that possibly can fix this. Various transformation of both the response variable, covariates and time variable is one method that can be used (Walpole et al.).

Chapter 6

Statistical Analysis

Based on the methods described in chapter 5, this chapter explains the process of analysing the formatted data to find a model that can describe the trend of deterioration. To illustrate how the analysis is performed, the variable twist 2 meter and the exponential model are used as examples. The same calculations is performed on combinations of all the other track geometry variables and models to find the best fitted model for each variable. Track gauge have not been regarded, because there is not the same deterioration processes that affect the degradation of this variable. MATLAB is used to perform the analysis.

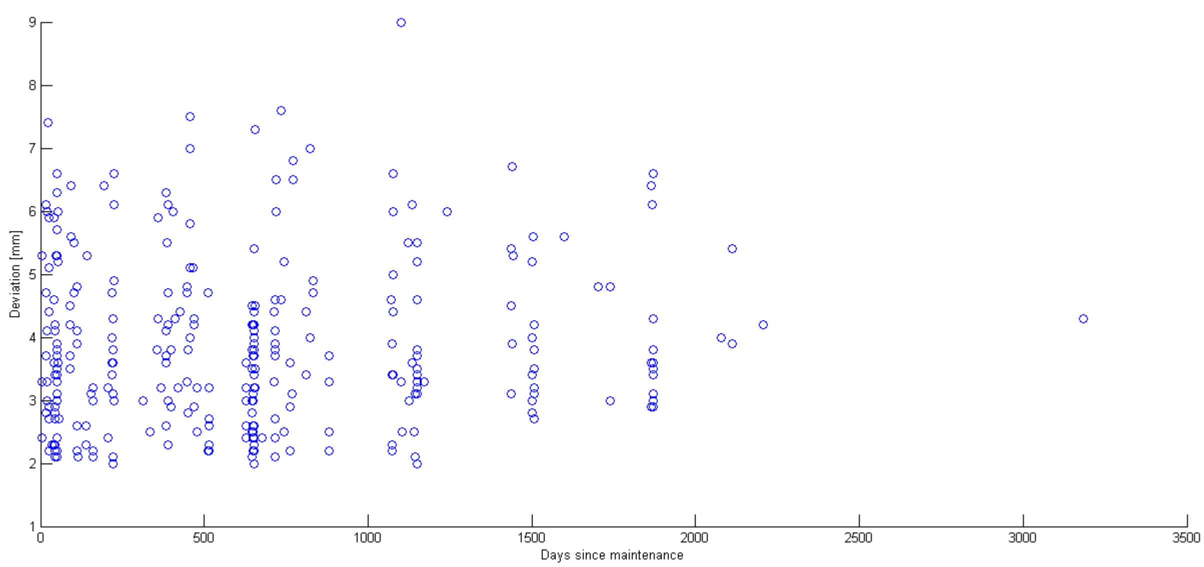

Figure 6.1: 1000 random deviations for twist 2 meter

6.1 Time Series Parameters

The data set consists of observations y_{ij}, i indicates a time series and j refers to which observation in time series i. Each time series consists of between two and ten observations. Deviations can be both positive and negative, and on the right and left side of the track. It is assumed that for one track geometry variable these deviations follow the same trend and are fitted to the same model. Figure 6.1 shows 1000 random observations of twist 2 meter. From this figure it is difficult to see any clear overall trend, but it can be seen that there is not any observations under two millimetre, which is the lowest limit to register deviations for twist 2 meter.

To find the overall trend explained by covariates, every time series should fit the same model. Table 5.1 shows the proposed models that the data is fitted to. Figure 6.2 illustrates a possible forms for all the models. Each model are a possible fit for the trend of each track geometry variable.

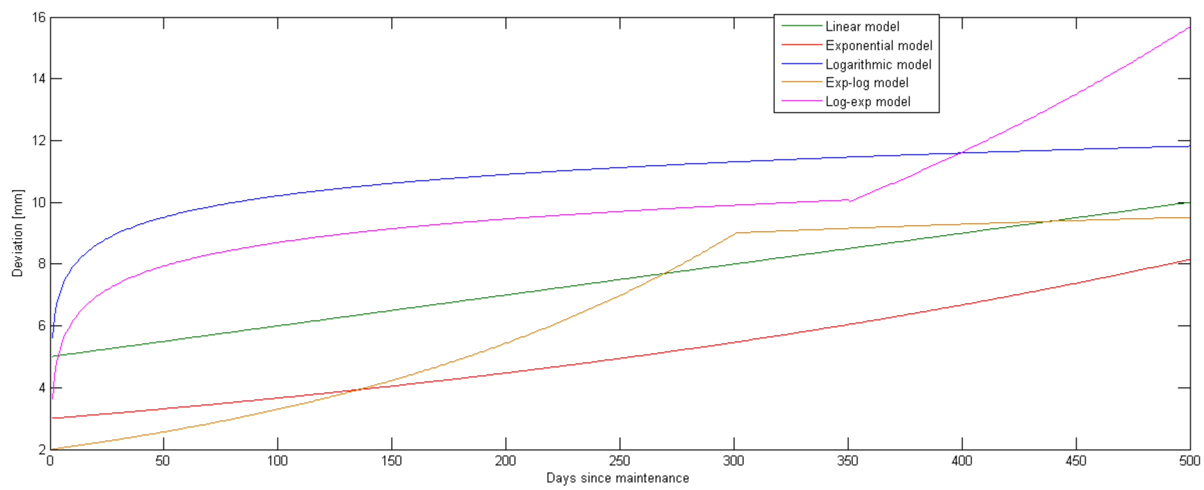

Figure 6.2: Possible forms for the five proposed models

The data is fitted to a model with the parameters λ_i and β_i. The parameters for each time series are estimated separately. All the parameters are estimated numerically. This is done by testing different values for $\lambda_{test,i}$ and $\beta_{test,i}$, and then increasing the accuracy of the parameters in an iteration process. LS is used as a principal to find the best fit. Equation 6.1 shows how ls_3 is estimated for the exponential model.

$$ls_3 = \sum_{i=1}^{n}\sum_{j=1}^{m}(y_{ij} - \lambda_{test,i} exp^{\beta_{test,i} t})^2 \tag{6.1}$$

For time series with few measurements the parameters for each model can differ a lot because of the variance in the data. Figure 6.3 illustrates five random time series from the data set and the fitted exponential model to each time series. Deviations are only included in the data set if they are above a specified limit, which is between two to four millimetre for twist 2 meter, depending on speed limit at the given position. The measurements that are included in the time series as a zero, illustrates deviations that have disappeared in an inspection. These are not included when the fit of the model is found, making it a conservative fit.

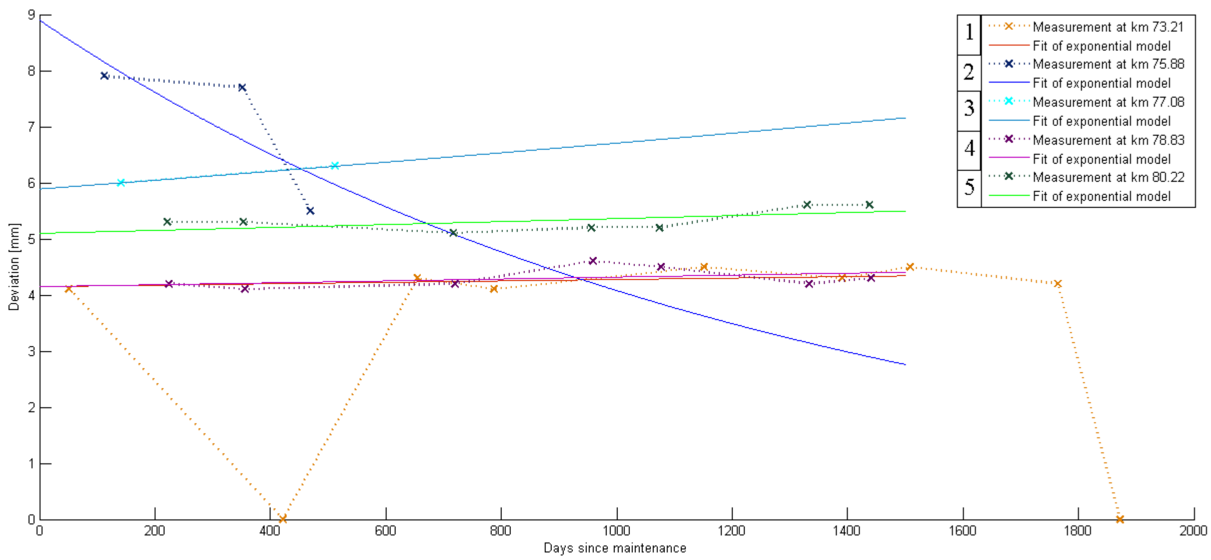

Figure 6.3: Exponential fit to five random time series

For the exp-log and log-exp model it is necessary to find the transition time between first and second phase. This is done numerically by taking the daily average deviation and optimising the models by using LS. It is assumed that there is a generic point for all time series and for every model, where the deterioration process changes from first to second phase. Figure 6.4 shows this change for twist 2 meter. Table 6.1 shows the time where the deterioration process changes from the first to second phase for all track geometry variables. From the table it can be seen that the exp-log model in general have a longer time before the deterioration changes. It can also be seen that the log-exp model have a change from phase one to phase two after 0 days for some of

Table 6.1: Transition from first to second phase

Exp-log		Log-exp	
Deviation type	**Time [days]**	**Deviation type**	**Time [days]**
Vertical levelling	540	Vertical levelling	0
Cant	625	Cant	0
Horizontal levelling	605	Horizontal levelling	0
Twist 2 meter	580	Twist 2 meter	310
Twist 9 meter	535	Twist 9 meter	225

the models, which means that a exponential model definitely is a better fit.

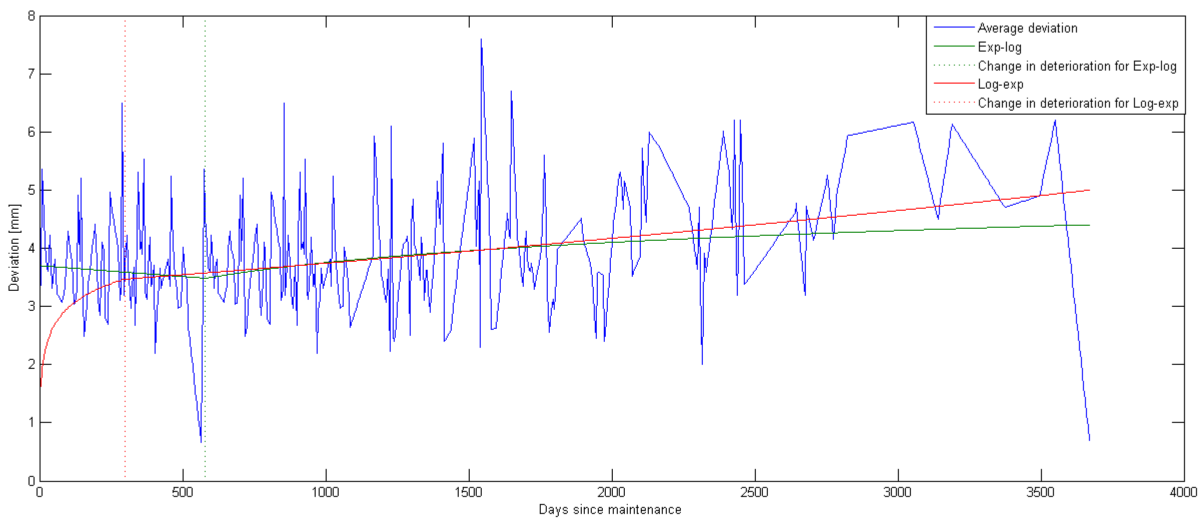

Figure 6.4: Transition from first to second phase

6.2 Stepwise Regression

To include covariates in the model, stepwise regression is used. Linear regression is used to estimate the coefficients for λ_z and β_z. The equation to estimate the coefficient of determination is also based on linear regression (Walpole et al.). But since it is more important that the model accurately can predict the observations j in each time series i, rather than the λ_i and β_i, R^2 and R^2_{adj} is not necessarily a good estimate of the fit as it is stated in equation 5.8 and equation 5.9.

These equations can be adjusted to include each observation, see equation 6.2 and equation 6.3.

$$R^2 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{y}_{ij} - \bar{y}_i)^2}{\sum_{i=1}^{n}\sum_{j=1}^{m}(y_{ij} - \bar{y}_i)^2} \tag{6.2}$$

$$R^2_{adj} = \frac{\left(\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{y}_{ij}-\bar{y}_i)^2}{(\sum_{i=1}^{n} m_i)-k-1}\right)}{\left(\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}(y_{ij}-\bar{y}_i)^2}{(\sum_{i=1}^{n} m_i)-1}\right)} \tag{6.3}$$

$\hat{y}_{ij}$ is estimated by estimating with the coefficients $\hat{\gamma}$ and $\hat{\delta}$ and using these coefficients to estimate $\hat{\lambda}_z$ and $\hat{\beta}_z$ for each model in table 5.1, see equation 6.4 and equation 6.5, where z_1 to z_k is the significant covariates in the model. These equation are the estimated coefficients to equation 5.2 and equation 5.3. For the exponential model $\hat{\lambda}_z$ and $\hat{\beta}_z$ is estimated as in equation 6.6. $\hat{\lambda}_z$ and $\hat{\beta}_z$ is estimated with stepwise regression. By estimating $\hat{y}_{ij}$ with this method, it is possible that the estimated value can be far of the observed value y_{ij}. Figure 6.5 illustrates how the differences between y_{ij} and $\bar{y}_i$, $\hat{y}_{ij}$ and y_{ij} is estimated. If $\hat{y}_{ij}$ is *overestimated* it means that the difference between $\hat{y}_{ij}$ and y_{ij} is be large compared to the difference between y_{ij} and $\bar{y}_i$. This can cause R^2 to be above 100 %. Because that the method used is adapted from normal linear regression this is possible. This can also cause R^2 to be incorrect even though it is between 0 and 100 %.

$$\hat{\lambda} = \hat{\gamma}_0 + \hat{\gamma}_1 \times z_1 \ldots \hat{\gamma}_k \times z_k \tag{6.4}$$

$$\hat{\beta}_z = \hat{\delta}_0 + \hat{\delta}_1 \times z_1 \ldots \hat{\delta}_k \times z_k \tag{6.5}$$

$$\hat{y}_{ij} = \hat{\lambda}_{zi} exp^{\hat{\beta}_{zi} \times t_{ij}} \tag{6.6}$$

To avoid that R^2 gives a wrong of how much of the variation in the data the model can explain, LS is used to estimate the fit. A reduction in ls_4, with $\hat{\lambda}$ indicates that the fitted model have improved and is used to determine how many covariates that should be included in the model. Equation 6.7 shows how ls_4 is estimated. Since ls_4 should decrease for every included covariate

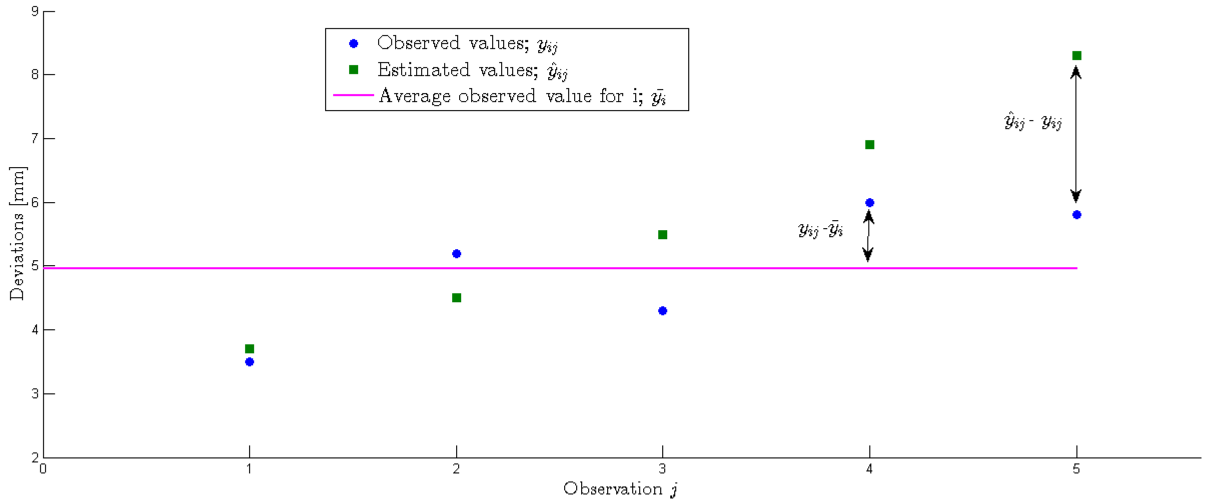

Figure 6.5: Observed and estimated values for a hypothetical time series i

it is set as a limit that ls_4 must decrease with at least 2 % in every step for the new covariate to be significant. To perform the stepwise regression all categorical covariates is changed into one or several new variables with only two possible states (0-1). Speed limit is the exception, because it is assumed that deterioration is proportional to the speed limit on the track. All covariates with numerical values can be found in table 6.2.

$$ls_4 = \sum_{i=1}^{n} \sum_{j=1}^{m} (y_{ij} - \hat{y}_{ij})^2 \tag{6.7}$$

The coefficients $\hat{\gamma}$ and $\hat{\delta}$ are estimated with the matrix operations in equation 5.7, where y is vector of λ_i or β_i (for $i = 1, \cdot, n$), X are the included covariates z and b are the estimated coefficients $\hat{\gamma}$ and $\hat{\delta}$. The adapted stepwise regression is then performed by using a algorithm. The main steps of the algorithm that performs this can also be seen as a pseudo-code in appendix A. A description of the main steps can also be listed as:

- Include one covariate at the time
- Estimate $\hat{\gamma}$ and $\hat{\delta}$ with matrix operations as in equation 5.7
- Estimate $\hat{\lambda}_z$ and $\hat{\beta}_z$ with equation 6.4 and equation 6.5
- Estimate $\hat{y}_{ij}$ using 6.6

Table 6.2: Covariates with numerical values

Covariate	Description	Numerical value
z_1	Culvert	0 or 1
z_2	Curvature	-3630 to 5730
z_3	Super elevation	0 to 150
z_4	Renewal of rail	0 to 39 000
z_5	Steel type R260Mn	0 or 1
z_6	Steel type R260	0 or 1
z_7	Steel type R200	0 or 1
z_8	Speed limit	2 to 4
z_9	Concrete sleepers	0 or 1
z_{10}	Wooden sleepers	0 or 1
z_{11}	Railroad switch	0 or 1
z_{12}	Water	0 or 1

- Estimate ls_4 from equation 6.7
- Compare ls_4 for all covariates and chose the covariate that gives the lowest ls_4.
- Include another covariate and to the same steps.
- If ls_4 is reduced with more than 2 % the covariate is significant.
- Continue until there is no more significant covariates

This algorithm is used on all track geometry variables and all models. The result finds the type of model that best fits each track geometry variable.

6.3 Fitted Model

For each model, coefficients are estimated and the significant covariates are found together with coefficients. A comparison of each model for twist 2 meter and a list of which covariates that are significant can be found in table 6.3. The table also shows the variance, R^2 and R^2_{adj} for each model. The variance is estimated with equation 6.8, where k is number of significant covariates included in the model. For comparison, the variance of all the observations is 1.64.

$$s^2 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}(y_{ij}-\bar{y}_i)^2}{(\sum_{i=1}^{n} m_i)-1} \tag{6.8}$$

Table 6.3: Results for twist 2 meters

Model	**Significant covariates**	**ls$_4$**	**Variance (s^2)**	**R^2**	**R^2_{adj}**
Linear	z_9	4.30×10^3	1.50	0.16	0.16
Exponential	$z_8$and z_3	3.67×10^3	1.28	0.26	0.26
Logarithmic	z_8 and z_4	9.08×10^3	3.16	1.30	1.31
Exp-log	z_7, z_5 and z_6	20.38×10^3	7.09	43.96	43.98
Log-exp	z_{11}	16.21×10^3	5.64	15.78	15.79

From this the best model is found to be the exponential model with an ls_4 of 3.67×10^3 and a R^2_{adj} of 0.26. It can be seen from the table that all the other models except the linear have a R^2_{adj} that is above 100 %. Which indicates that by using this method R^2_{adj} is a bad estimate of fit.

The different models have also found different significant covariates but most are related to the elements on the track (e.g. steel quality and type of sleepers). The significant covariates for the exponential model to twist 2 meter are speed limit and super elevation. This model can be seen in table 6.4.

Table 6.4: Fitted model for twist 2 meters

Fitted model	**Function**	**Parameters**
Exponential	$\hat{\lambda}_z exp^{t\hat{\beta}_z}$	$\hat{\lambda}_z = 0.55 + 1.12z_8 + 3.6\times10^{-3}z_3$
		$\hat{\beta}_z = 2.61\times10^{-4} - 7.21\times10^{-5}z_8 + 5.17\times10^{-7}z_3$

6.4 Adjusting Model

The stepwise regression is based on λ_i and β_i and not directly on each observation in a time series. It is possible that the regression model is not the best fit for each time series. To deal with this and improving the final model, a method that estimates the coefficients $\hat{\gamma}$ and $\hat{\delta}$ based on the time series and not based on λ and β. It is assumed that the same covariates are significant for this improved model. This is performed numerically and is done with an algorithm. The algorithm is time consuming if no initial values are known, this is the reason why this method is not used to begin with, and the reason why this method is not used to estimate the fit of all models.

The main steps in the algorithm that performs this improvement can be seen in algorithm 3.

Data: Data set and covariates for optimised fitted model

n=Number of time series;

$opt\gamma$=optimised coefficient vector for γ ;

$opt\delta$=optimised coefficient vector for δ ;

m=length($opt\gamma$)= length($opt\delta$);

$lsopt$= Least square estimate based on optγ and optδ

Step 1: Adjust coefficient vector;

Try to adjust all values in $opt\gamma$ and $opt\delta$ up and down simultaneously;

improved=false;

for *i=1* → *n* **do**

  for *l=1* → *length(time series* i **do**

    $ls = ls + (observation_{ij} - observ\hat{a}tion_{ij})^2$ (Based on adjusted coefficient vector for γ and δ;

  end

end

if $ls < lsopt$ **then**

  $lsopt = ls$;

  $opt = s$;

  $opt\gamma$=adjusted coefficient vector for γ ;

  $opt\delta$=adjusted coefficient vector for δ ;

  improved=true;

end

if $improved = true$ **then**

  Jump to step 1 and adjust more;

end

Algorithm 3: Improving model

After this improvement the fitted model for twist 2 meter can be seen in table 6.5. The ls_4, variance, and R^2 for the improved model is respectively 3.39×10^3, 1.18 and 33%. Which indicates the that the model is slightly improved.

Table 6.5: Improved model for twist 2 meters

Fitted model	Function	Parameters
Exponential	$\hat{\lambda}_z exp^{t\hat{\beta}_z}$	$\hat{\lambda}_z = 0.62 + 1.14z_8 + 3.2 \times 10^{-3} z_3$
		$\hat{\beta}_z = 2.4 \times 10^{-4} - 7.13 \times 10^{-5} z_8 + 5.17 \times 10^{-7} z_3$

6.5 Residual Plots

The method used to fit the models are based on MLR of both λ_i and β_i. By doing this it is also assumed that the residuals are normally and randomly distributed. Even though the method used is adapted from MLR and improved with a numerical method afterwards, this is important to test in order to use the model for prediction. Residual plots are used to test this and analyse the residuals. Even though the model have been improved numerically after the regression, these assumptions are assumed to be valid.

Figure 6.6 and figure 6.7 shows a residual plot and a normal probability plot for the residuals for respectively $\hat{\lambda}_z$ and $\hat{\beta}_z$ for the fitted exponential model to the track geometry variable twist 2 meter. In the residual plot in both figures it can not be seen any clear trend, but the residuals seem to be skewed a bit to the positive side which indicates that the mean of the residuals is not 0. That the residuals is skewed means that the Cumulative Density Function (CDF),$F(e)$ is not symmetrical (Skymark (2013)). The normal probability plot for $\hat{\lambda}_z$ shows almost a straight line, but the observed residuals slightly higher than the normality line, which confirms that the mean could be higher than 0.

In the normal probability plot for $\hat{\beta}_z$ it can be seen that the residuals are not normally distributed because of the s-shaped form with long tails in both ends. It is skewed on both sides but especially skewed on the right. This indicates that *extreme* values are more common than what would be expected for normally distributed residuals.

The residuals are also estimated based on each observation. These residuals can indicate

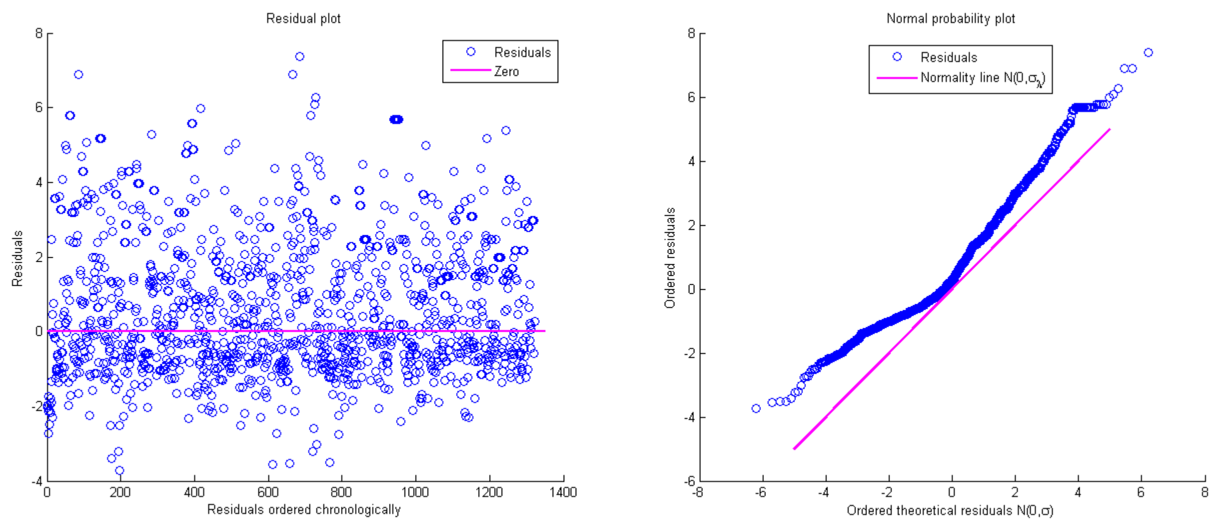

Figure 6.6: Residual plots for $\hat{\lambda}_z$

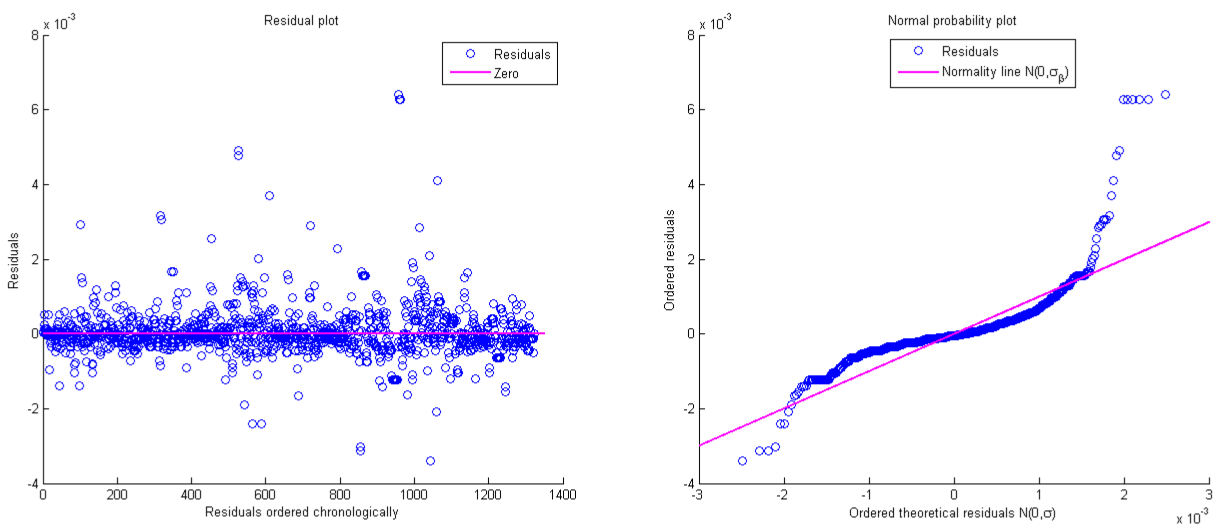

Figure 6.7: Residual plots for $\hat{\beta}_z$

if there is any trend in the residuals over time. By looking at figure 6.8 it can be seen from the residual plot that there is not any clear trend, which confirms the assumption of randomly distributed residuals. The normal probability plot does however seem to be skewed to the right which indicates that the residuals is not normally distributed.

There are different ways to fix the residuals to be in accordance with the assumptions and still using MLR. One way is to do different transformations of both the parameters λ_i and β_i and response variable y. For all three the transformations in table 6.6 was tested independently to

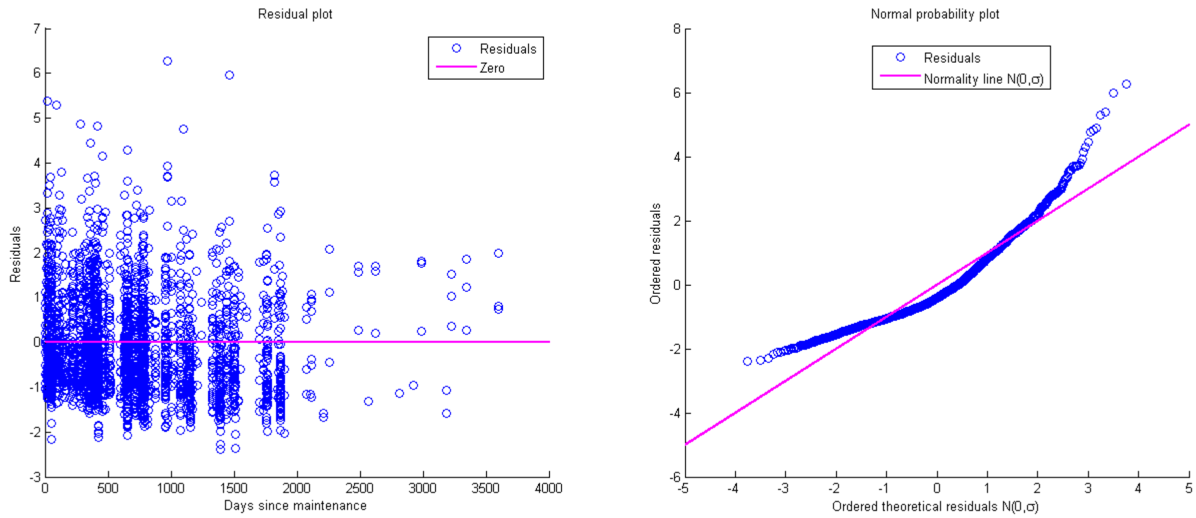

Figure 6.8: Residual plots for all observations

Table 6.6: Transformation of response variable and parameters

Variable/parameter	y	λ_i	β_i
Transformation	$y_{tran} = ln(y)$	$\lambda_{tran} = ln(\lambda_i)$	$\beta_{tran} = ln(\beta_i)$
	$y_{tran} = \frac{1}{y}$	$\lambda_{tran} = \frac{1}{\lambda_i}$	$\beta_{tran} = \frac{1}{\beta_i}$
	$y_{tran} = \sqrt{y}$	$\lambda_{tran} = \sqrt{\lambda_i}$	$\beta_{tran} = \sqrt{\beta_i}$
	$y_{tran} = y^2$	$\lambda_{tran} = \lambda_i^2$	$\beta_{tran} = \beta_i^2$

see if the residuals changed. To include additional covariates in the fitted model was also tested to see if that made the residuals closer to a normal distribution. Neither of the methods did however make it seem probable that the residuals where normally distributed.

6.6 Model Uncertainty

Even though it is difficult to know the distribution of the residuals, it is necessary to state some assumptions in order to predict the deterioration rate of the deviations. Thus, it is still assumed that both the residuals for $\hat{\lambda}_z$ and $\hat{\beta}_z$ are normally and randomly distributed, even though the residual plots indicated different. It is also assumed that the residuals are independent of the covariates included in the model. An estimated normal Probability Density Function (PDF), $f(e)$, is used to find the uncertainty e_λ and e_β to $\hat{\lambda}_z$ and $\hat{\beta}_z$. The PDF is estimated from the mean

(μ) and standard deviation ($\sigma = \sqrt{variance}$) for the residuals, see equation 6.9. The PDF is then transformed into a discrete probability distribution by dividing the PDF into five intervals with limits a and b. A mean value for each interval (a-b is estimated from equation 6.10 and by solving equation 6.9 with respect to e. This gives a discrete probability distribution for the uncertainty to $\hat{\lambda}_z$ and $\hat{\beta}_z$. Figure 6.9 illustrates the estimated mean values for each interval and the probability to get this value. Table 6.7 shows this numerically.

$$f(e) = \frac{1}{\sqrt{2\pi}\sigma} exp(-\frac{(e-\mu)^2}{2\sigma^2}) \tag{6.9}$$

$$f(\bar{e}) = \int_a^b f(e)\frac{1}{b-a} \tag{6.10}$$

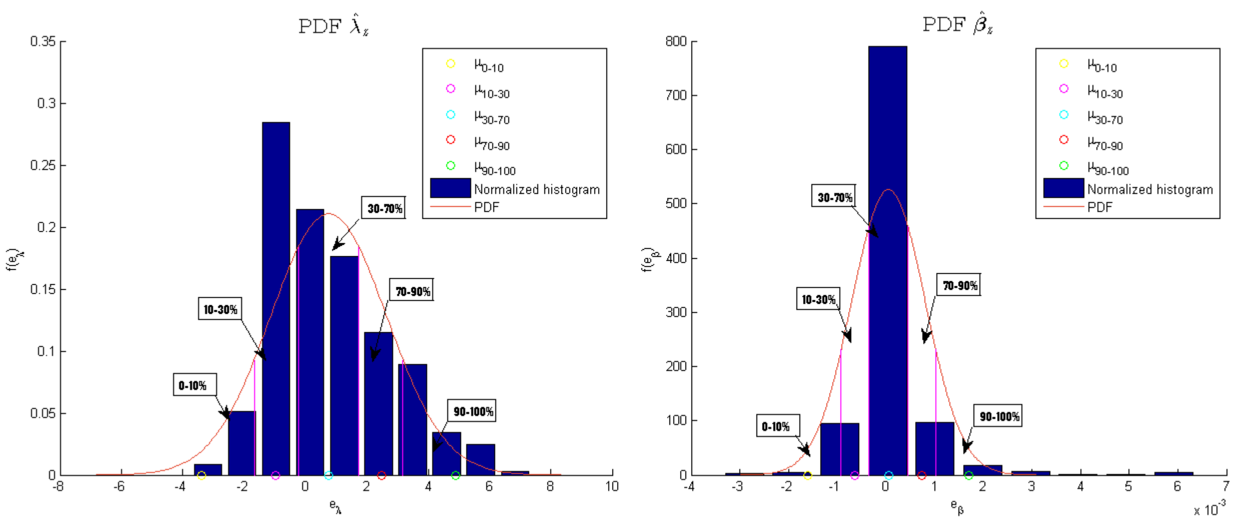

Figure 6.9: PDF for residuals to $\hat{\beta}_z$ and $\hat{\beta}_z$

Table 6.7: Distributed probability of model uncertainty

Uncertainty	**Probability**				
	10 %	**20%**	**40%**	**20%**	**10 %**
$\mathbf{e}_\lambda$	-3.38	-0.96	0.76	2.48	4.89
$\mathbf{e}_\beta$	-1.60×10^{-3}	-6.34×10^{-4}	0.54×10^{-4}	7.43×10^{-4}	1.70×10^{-3}

Based on the estimated uncertainty and the fitted model in equation 6.5, deterioration of deviations can be estimated for twist 2 meters. This is explained in detail in chapter 7.

6.7 Fitted Model and Model Uncertainty for All Variables

For the other track geometry variables the best fitted model and the uncertainty to each function are estimated with the same method as for twist 2 meter. The same assumptions is also valid for these estimations. Table 6.8 shows the best fitted model for the track geometry variables vertical levelling, cant, horizontal levelling and twist 9 meter. For all variables it was the exponential model that gave the best fit.

Table 6.8: Proposed model for twist 2 meters

Track geometry variable	**Function**	**Parameters**
Vertical levelling	$\hat{\lambda}_z exp^{t\hat{\beta}_z}$	$\hat{\lambda}_z = 3.33 - 1.90z_9 + 1.07z_8 + 0.60 \times 10^{-3} z_3$
		$\hat{\beta}_z = 4.80 \times 10^{-4} - 2.48 \times 10^{-5} z_9 - 5.20 \times 10^{-5} z_8 + 9.02 \times 10^{-7} z_3$
Cant	$\hat{\lambda}_z exp^{t\hat{\beta}_z}$	$\hat{\lambda}_z = 5.01 - 0.25z_9 + 1.05 \times 10^{-2} z_3$
		$\hat{\beta}_z = 3.40 \times 10^{-4} - 1.45 \times 10^{-4} z_9 - 3.11 \times 10^{-7} z_3$
Horizontal levelling	$\hat{\lambda}_z exp^{t\hat{\beta}_z}$	$\hat{\lambda}_z = 2.87 + 0.69z_8 - 1.13z_9 - 0.35z_5 - 0.17z_6$
		$\hat{\beta}_z = 2.29 \times 10^{-5} - 1.37 \times 10^{-5} z_8 + 5.16 \times 10^{-5} z_9 - 2.21 \times 10^{-5} z_5 - 3.45 \times 10^{-5} z_6$
Twist 9 meter	$\hat{\lambda}_z exp^{t\hat{\beta}_z}$	$\hat{\lambda}_z = 10.48 - 4.10 \times 10^{-2} z_3 + 2.70z_8 - 6.36z_9 - 3.35z_{11}$
		$\hat{\beta}_z = 1.54 \times 10^{-4} - 2.56 \times 10^{-7} z_3 - 8.58 \times 10^{-5} z_8 + 1.78 \times 10^{-4} z_9 + 2.42 \times 10^{-4} z_{11}$

The estimated uncertainty for the $\hat{\lambda}_z$ and $\hat{\beta}_z$ functions combined with the probability of each uncertainty are found in table 6.9. Appendix B shows the residual plots and PDF plots for the fitted model for the variables vertical levelling, cant, horizontal levelling and twist 9 meter.

Table 6.9: Model uncertainty

Track geometry variable	**Uncert-ainty**	**Probability**				
		10 %	**20%**	**40%**	**20%**	**10 %**
Vertical levelling	e_λ	-6.41	-2.66	4.5×10^{-3}	2.67	6.42
	e_β	-2.30×10^{-3}	-0.90×10^{-3}	7.01×10^{-5}	1.01×10^{-3}	2.40×10^{-3}
Cant	e_λ	-7.77	-3.32	-0.16	3.00	7.45
	e_β	-3.50×10^{-3}	-1.50×10^{-3}	-3.45×10^{-5}	1.40×10^{-3}	3.50×10^{-3}
Horizontal levelling	e_λ	-3.79	-1.42	0.13	1.81	4.18
	e_β	-1.60×10^{-3}	-6.74×10^{-3}	-1.02×10^{-5}	-6.74×10^{-4}	-1.60×10^{-3}
Twist 9 meter	e_λ	-12.74	-5.10	0.34	5.78	13.42
	e_β	-1.40×10^{-3}	-5.60×10^{-4}	4.52×10^{-6}	5.69×10^{-4}	1.40×10^{-3}

Chapter 7

Deterioration Model

This chapter proposes methods to predict deterioration of track geometry based on the fitted model and model uncertainty. This is illustrated by using laws of probability. Twist 2 meter is used as an example to illustrate the methods. Deterioration of other track geometry variables can be predicted using the same methods.

7.1 Probability of Failure

When a deviation deteriorates and exceed the IAL it is defined as a failure. A failure can increase the risk of derailment. When a inspection of the track geometry is performed all deviation is assumed to be detected. Thus, it is important that a deviation not develops into a failure between two inspections. This may cause a situation with an increased risk of derailment without knowing it. To predict deterioration of deviations it is assumed that two situations are possible between inspections. Either a deviation has deteriorated above (event A) or below (event B) the IAL. In the fitted model the probability of each event ($P(A)$ and $P(B)$) depends on number of days since maintenance, the covariates at the position of the deviation and the uncertainty related to $\hat{\lambda}_z$ and $\hat{\beta}_z$.

It is also possible that the probability of each event depends on the size of the measured deviation. This correlation is not looked into in the analysis and is disregarded when estimating the probability of event A.

7.1.1 Estimated Probability of Failure

The numbers found in table 7.1 is related to a twist 2 meter deviation (D). The uncertainty for $\hat{\lambda}_z$ and $\hat{\beta}_z$ is distributed with a discrete probability distribution with five possible outcomes each. The sample space to D, consist of all possible combinations of $\hat{\lambda}_z$ and $\hat{\beta}_z$, which means that there are 25 possible outcomes. Each of these outcomes are estimated with the coefficient to $\hat{\lambda}_z$ and $\hat{\beta}_z$ from the fitted model in table 6.5 and adding the uncertainty from table 6.7. The probability of each outcome is estimated based on the probability of each uncertainty. All values for $\hat{\lambda}_z$ and $\hat{\beta}_z$ and the probability of each combination can be found in table 7.2. The predicted size of deviation for all outcomes, at the time of next inspection can also be found in this table. The deviations are predicted by using the exponential model, $\hat{\lambda}_z$ and $\hat{\beta}_z$ and adding time since maintenance and time to next inspection. It can be seen in the table that that the sum of probability for all outcomes equals to 1, which is in accordance with laws of probability (Walpole et al.).

Table 7.1: Deviation D

Covariates	$z_8 = 3$
	$z_3 = 76mm$
Radius	$800m$
Days since maintenance	300
Days to next inspection	200
Size of deviation	$4mm$

Table 7.2: Probability of all outcomes for deviation D

	Probability	10 %		20 %		40 %		20 %		10 %	
	$\hat{\lambda}_{\mathbf{z}}$	0.90		3.32		5.04		6.76		9.17	
Probability	$\hat{\beta}_{\mathbf{z}}$	Dev. [mm]	Prob.	Dev. [mm]	Prob.	Dev. [mm]	Prob.	Dev. [mm]	Prob.	Dev. [mm]	Prob.
10 %	-1.53×10^{-3}	0.42	0.01	1.54	0.02	2.35	0.04	3.15	0.02	4.27	0.01
20 %	-5.69×10^{-4}	0.68	0.02	2.50	0.04	3.79	0.08	5.09	0.04	6.90	0.02
40 %	1.19×10^{-4}	0.95	0.04	3.52	0.08	5.35	0.16	7.17	0.08	9.73	0.04
20 %	8.08×10^{-4}	1.35	0.02	4.97	0.04	7.55	0.08	10.13	0.04	**13.73**	**0.02**
10 %	1.77×10^{-3}	1.62	0.01	5.98	0.02	9.08	0.04	**12.18**	**0.02**	**16.52**	**0.01**

The IAL for twist 2 meter with a radius larger than 400 meters are 12 millimetres. From table

7.2 it can be seen that there are three outcomes that exceeds this limit (cells in blue). The total probability for these three outcomes is 5 % (cells in green). This means that it is a 5 % probability of event A or a 95 % probability event B.

The probability of event A can be estimated the same way for other track geometry variables. If the probability of event A is known for n deviations, the probability that one of these deviations exceeds the IAL, $P(A_{tot})$ can be estimated with equation 7.1.

$$P(A_{tot}) = 1 - \prod_{i=1}^{n} P(B_i) \tag{7.1}$$

Chapter 8

Summary and Recommendations for Further Work

8.1 Summary and Conclusions

The Norwegian Railway Administration uses a measuring car to measure track performance. From a safety point of view, special attention needs to be paid to spots on the line where failure propagation is out of control, and critical failures could develop in between measurements typically carried out twice a year. A challenge in the modelling is that we are dealing with so-called line objects, where there are an almost infinite number of places a failure can occur. This is complicated by the fact that the measuring car reports the position of failures with some uncertainty, making it difficult to compare results across different measurements series.

This report presents the result of an analysis of propagation of spot failures on track geometry. The analysis is based on ten inspections performed on the railway line between Eidsvoll and Hamar in the period 2006 to 2012. The data for the analysis is obtained from JBV's databases. Follow up of track performance is regulated by laws and regulations. JBV uses maintenance and renewal to improve track performance.

To analyse deterioration of the track performance it is applied statistical methods. There exist several methods that deals with trend modelling, and a literature survey is performed to cover relevant methods for this project.

Because of uncertainties in the obtained data a comprehensive work is performed to format data. This is done to understand what is included in the data and how the data can be used to analyse track performance. Two important parts of this work are to adjust the position of measurements and use the measurements to create time series for individual spots on the line.

The adapted statistical methods are used to analyse the time series. The result of the analysis is a model that can predict the probability of failure development. The accuracy of the model is related to the accuracy of the obtained data and the methods used in the analysis.

The proposed model for all track geometry parameters is found in table 6.5 and 6.8 with uncertainties found in table 6.7 and 6.9. The most common significant covariates is super elevation, speed limit and concrete sleepers. The model can be used to predict the probability that failures occur on the track geometry within a given time. This is showed in chapter 7. Predicting the probability of failure can be used to set the right inspection interval, or be used to plan maintenance and renewal. It is also possible to use this information to estimate the risk of derailment on a specific railway line.

The accuracy of future data series can be considerably improved by using a better system to accurately position the location of measurements. This is also in accordance with plans in JBV to implement GPS as part of the measurements.

8.2 Discussion

The measured position for each deviation can cause the wrong deviations to be *matched* together. Even though it is used an algorithm to adjust the position, the accuracy of this method is limited. Equipment that accurately can find the position of a deviation would improve the accuracy of the result and make it unnecessary to adjust the position of deviations. This will also improve the efficiency of data analysis.

The proposed model is chosen from five selected models that have been used to describe deterioration of track geometry in other studies. These studies are based on deterioration of the average quality of sections, and not for individual spots. All track geometry variables had the best fit with the exponential model. This indicates that the exponential model is suitable to predict deterioration of track geometry. It is still possible that a different model would fit spot failures even better.

The statistical analysis is performed with a combination of different methods. The MLR that is used to find the coefficients in the model assumes that the underlying data can be fitted to a linear model and that the residuals are normally and randomly distributed. It is shown with residual plots that this assumption probably is not valid. In this case it is possible that a non linear regression method should be used to fit the model.

The model uses time since maintenance to predict the probability of failure. Both tamping and ballast cleaning is included as maintenance activities. In the model it is assumed that both these activities have the same effect on the condition of the track. If the effect of cycle would be included in the model these activities could be separated. Ballast cleaning could also be included as a covariate to differentiate between maintenance activities. It could also be possible that other covariates that are not tested in this study can be significant and could improved the model.

There are several steps in the process to fit a model that is based on assumptions that can make the result inaccurate. The probability of failure, estimated with the proposed model, still seems to be reasonable under the given conditions. To determine the accuracy of the model, it should be tested in practice.

8.3 Recommendations for Further Work

Based on the discussion there is suggested areas where more research could lead to an improved model. It is also possible to use the result from this report as a basis for new analysis. Some recommendations for further work is listed as:

- Perform the same analysis on data from other railway lines
- Use non linear regression for trend modelling
- Suggest other models to estimate deterioration
- Take cycle effects into account when fitting a model
- Test the model in practice

Appendix A

Algorithm for Stepwise Regression

Data: Data set with time series for all track geometry variables

For one variable at the time;

n=Number of time series;

m=Number of models;

k=Number of covariates;

v=vector of ones (size $n \times 1$) ;

Step 1: Estimate parameters;

for $i = 1 \rightarrow n$ **do**

    for $j = 1 \rightarrow m$ **do**

        Estimate λ_i and β_i numerically for each time series;

    end

end

Step 2: Multiple linear regression;

For one model at the time;

$lsopt = 10^5$ (set $lsopt$ to a high number);

Continue on next page

for $r = 1 \rightarrow k$ **do**
  $lsopt = lsopt \times 0.98$;
  improve=false;
  Test all covariates to find the optimal;
  for $s = 1 \rightarrow k$ **do**
    s_r=vector of covariate s (size $n \times 1$);
    To get coefficient vector for γ: Solve equation 5.7, $y = \hat{\lambda}_z$ (size $1 \times n$),
    $X = v + s_1 + \cdots + s_r$ (size $1 + r \times n$);
    To get coefficient vector for δ: Solve equation 5.7, $y = \hat{\beta}_z$ (size $1 \times n$),
    $X = v + s_1 + \cdots + s_r$ (size $1 + r \times n$);
    $ls = 0$;
    for *i=1* $\rightarrow n$ **do**
      for *j=1* $\rightarrow$ *length(time series* i*)* **do**
        $ls = ls + (observation_{ij} - obser\hat{v}ation_{ij})^2$ (Based on coefficient vector for γ and δ);
      end
    end
    If this covariate decrease ls;
    if $ls < lsopt$ **then**
      $lsopt = ls$;
      $opt = s$;
      opt γ=coefficient vector for γ ;
      opt δ=coefficient vector for δ ;
      improved=true;
    end
  end
  $s_r = opt$ **if** *improved=true* **then**
    Continue;
  else
    Significant covariates = $r - 1$;
    Jump to step 3;
  end
end

Algorithm 4: Stepwise regression

Appendix B

Residual and PDF Plots for All Track Geometry Variables

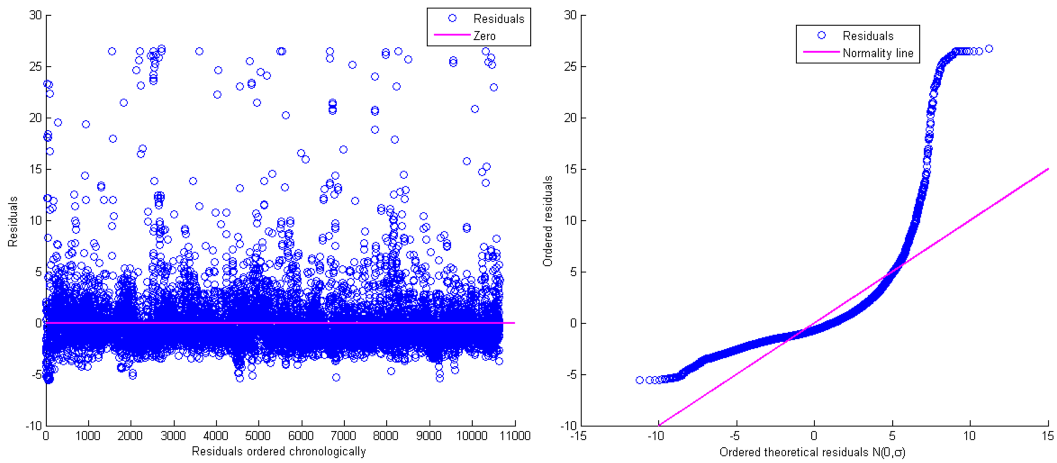

Figure B.1: Residual plots for $\hat{\lambda}_z$ for vertical levelling

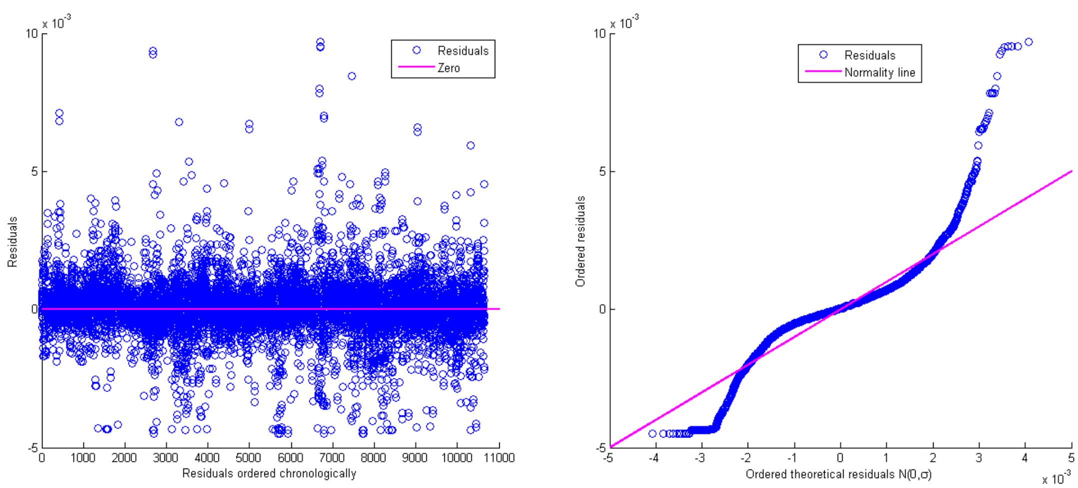

Figure B.2: Residual plots for $\hat{\beta}_z$ for vertical levelling

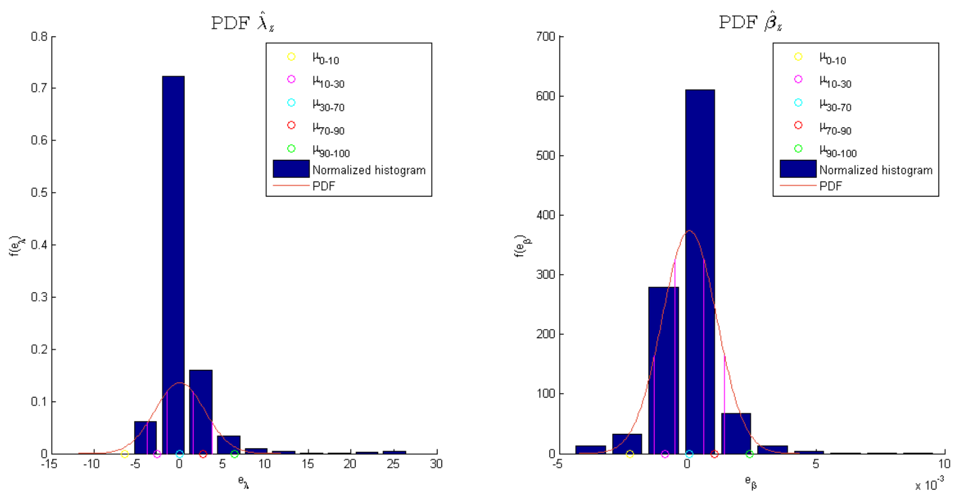

Figure B.3: PDF for $\hat{\lambda}_z$ and $\hat{\beta}_z$ for vertical levelling

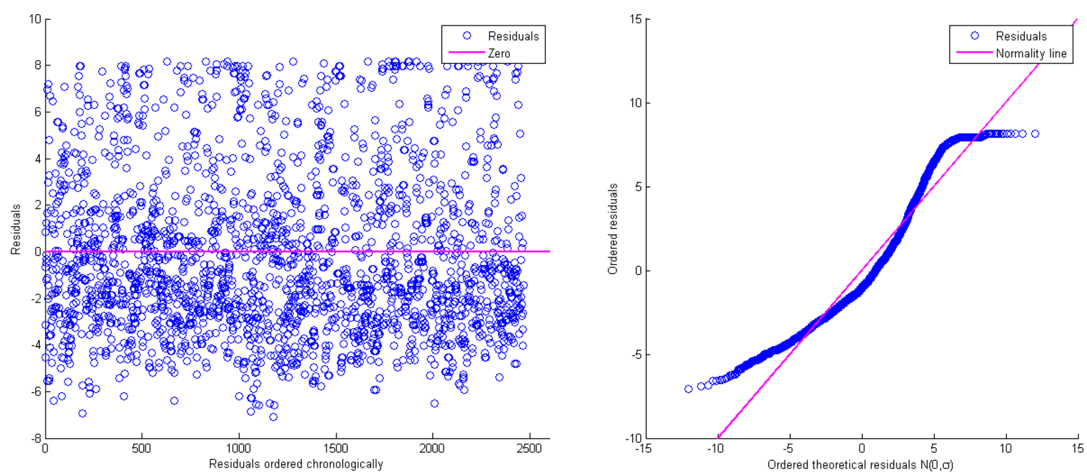

Figure B.4: Residual plots for $\hat{\lambda}_z$ for cant

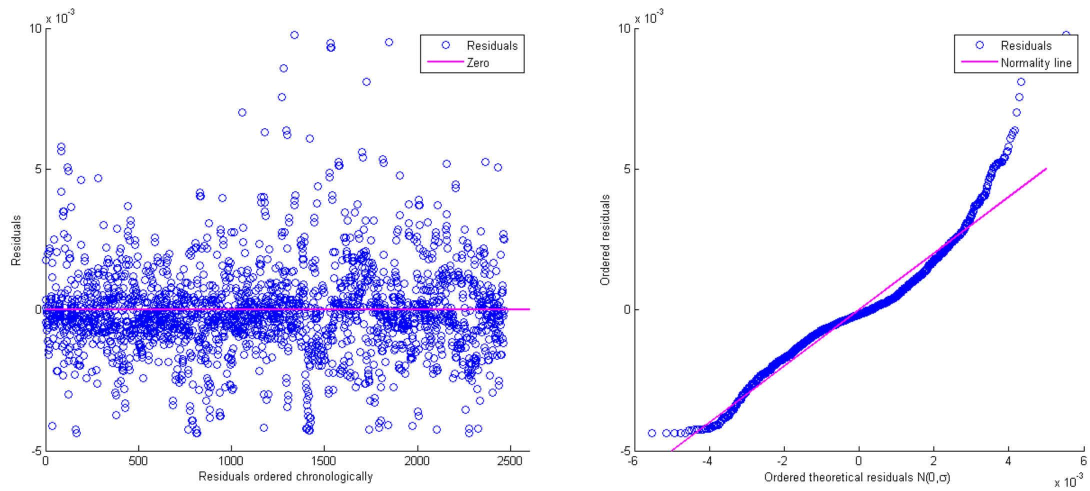

Figure B.5: Residual plots for $\hat{\beta}_z$ for cant

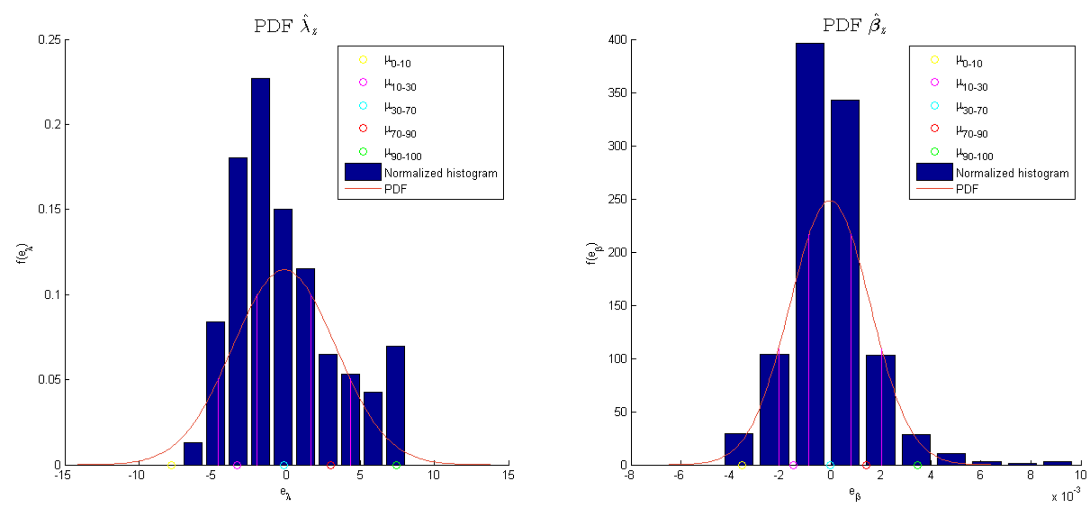

Figure B.6: PDF for $\hat{\lambda}_z$ and $\hat{\beta}_z$ for cant

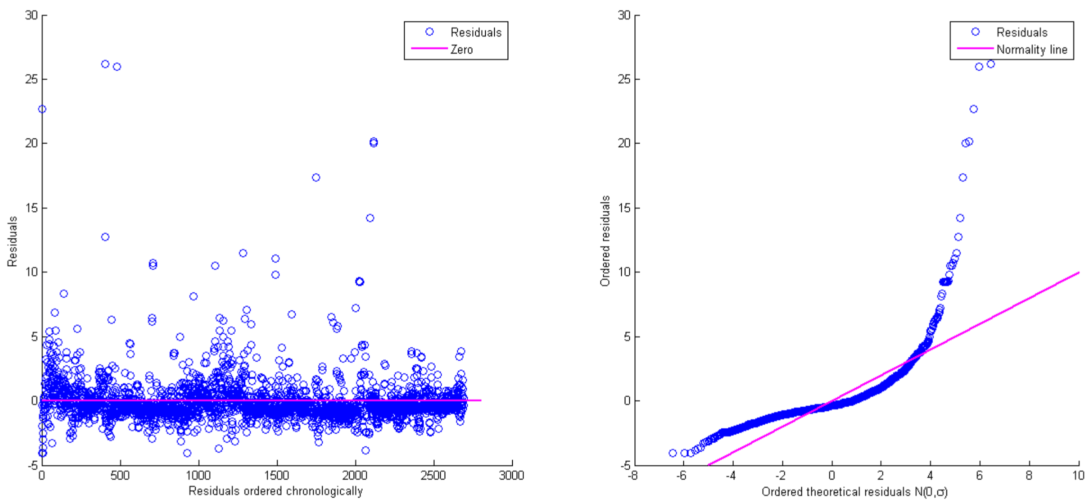

Figure B.7: Residual plots for $\hat{\lambda}_z$ for horizontal levelling

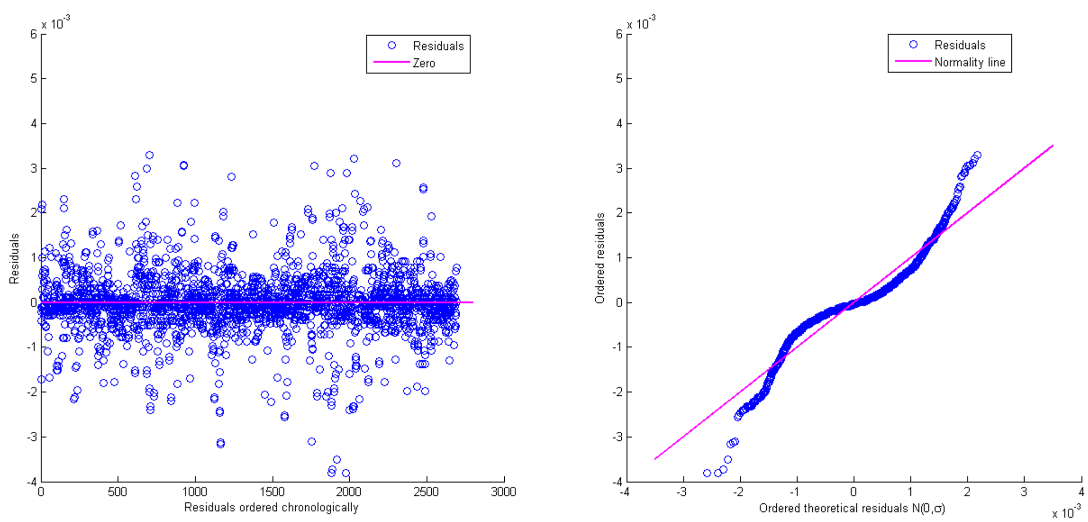

Figure B.8: Residual plots for $\hat{\beta}_z$ for horizontal levelling

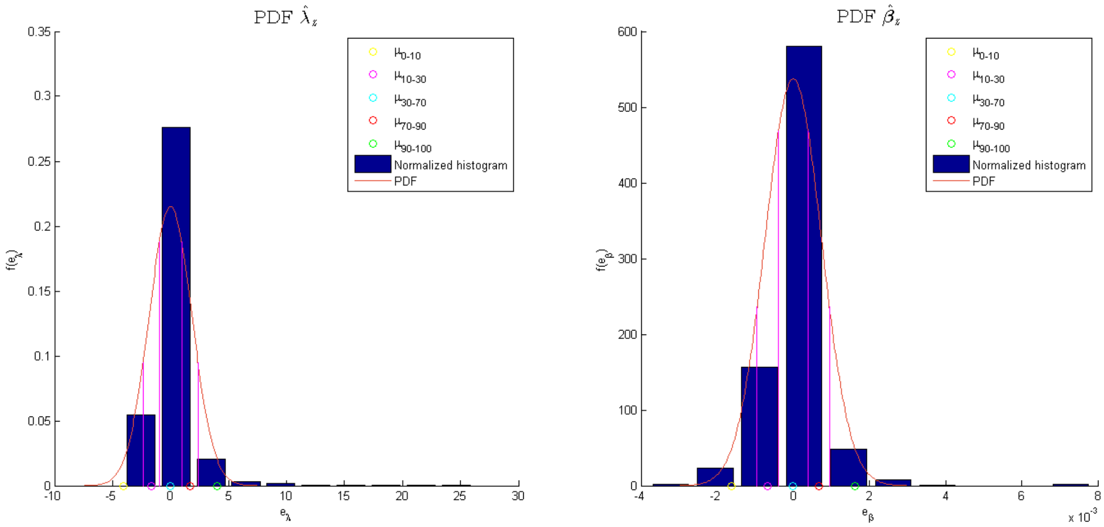

Figure B.9: PDF for $\hat{\lambda}_z$ and $\hat{\beta}_z$ for horizontal levelling

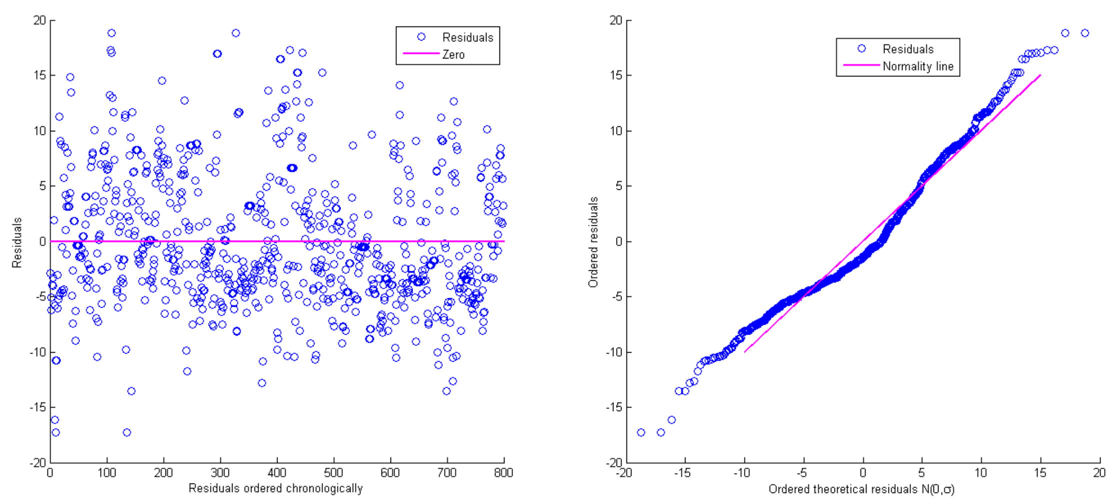

Figure B.10: Residual plots for $\hat{\lambda}_z$ for twist 9 meter

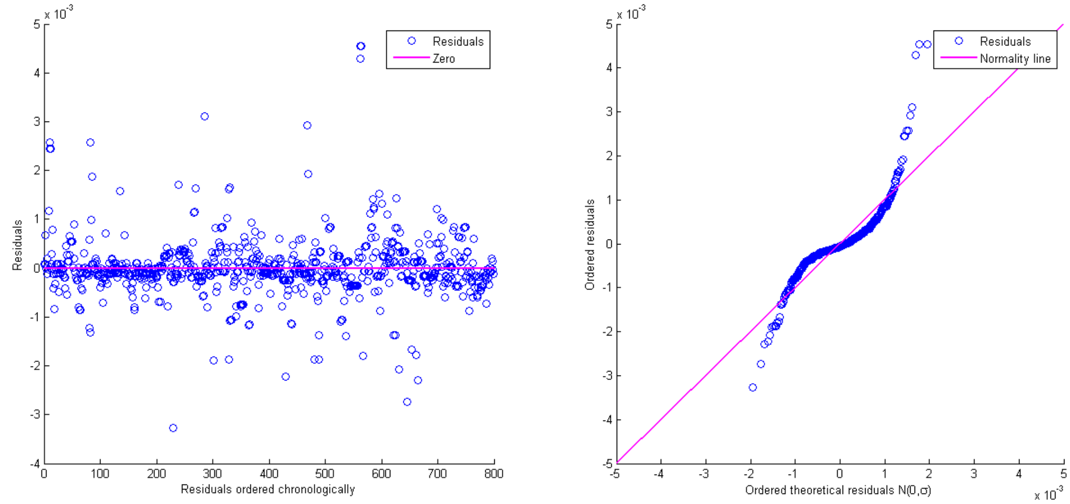

Figure B.11: Residual plots for $\hat{\beta}_z$ for twist 9 meter

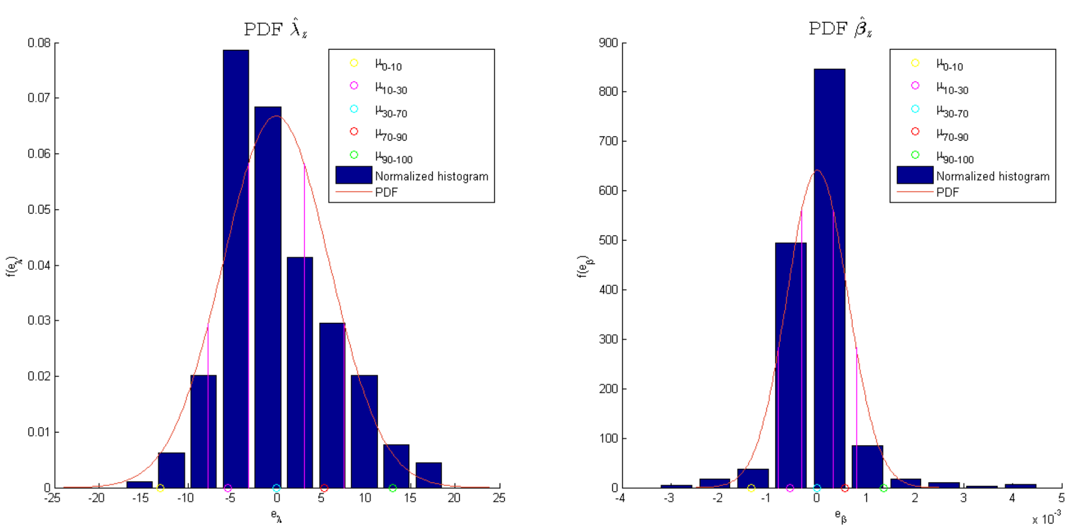

Figure B.12: PDF for $\hat{\lambda}_z$ and $\hat{\beta}_z$ for twist 9 meter

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August 2011-June 2013: Master in RAMS (reliability, availability, maintainability and safety) at Norwegian University of Science and Technology (NTNU).

August 2008- May 2011: Bachelor in mechanical engineering, with specialization in machine design at Chalmers University of Technology in Gothenburg.

August 2007-June 2008: Economical statistics and business economy at Gothenburg School of Business, Economics and Law.

August 2004-June 2007: Lillestrøm Upper Secondary School

Work experience
June-August 2012: Had a vacation job for Scandpower Risk Management at their Kjeller office. The work task was mainly as a RAMS advisor for the Norwegian Railway Administration. This consisted in documenting RAMS demands and evaluate risk related to constructing new railway stations.

2005-2012: Part time job at Interflora Strømmen. The job involved delivery of flowers, write bills and other occasional task.

June-August 2011: Work as a sales consultant for Ringnes Brewery. Ordered products and sat them in the right place in grocery stores.

January-May 2011: Part time job for Posten Sweden in Gothenburg. Responsible for sorting, delivering and forwarding mail 1-2 times a week.

June-August 2010: Had a vacation job for Byggmakker Lillestrøm, a building and hardware chain in Norway. My job was mainly as a salesperson in the hardware department.

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