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Deterioration and Maintenance Models for Components in Hydropower Plants

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Preface

This thesis is the result of a doctoral project at the Department of Production and Quality Engineering (IPK) at the Norwegian University of Science and Technology (NTNU). The work was carried out from November 2004 until April 2008.

This PhD position has been funded through the research project "Maintenance and refurbishment in hydropower" (Vedlikehold og rehabilitering innen vannkraft), which lasted five years and which was commissioned by the Norwegian Electricity Industry Association (EBL - Energibedriftenes landsforening). The research work was carried out at SINTEF Energy Research, Department of Energy Systems, and the PhD work was conducted in collaboration with research activities at SINTEF.

One of the objectives of the research project was the development of a deterioration and maintenance model for components in hydropower plants. A PhD position was announced with the goal of supporting research in the field of failure and maintenance modelling. The research project had already started in 2001. However, when I started to work as researcher at SINTEF Energy Research in August 2004, the position was still vacant. It turned out that filling the gap of the vacant PhD position was my mission and hence I started as PhD student at IPK in November of 2004. Unfortunately, the research project had already ended before my work was completed. In 2006, however, the follow-up project "Value adding maintenance in power production" (Verdiskapende vedlikehold innen kraftproduksjon) was launched, which will last until 2010.

Trondheim, April 2008   *Thomas Welte*

Acknowledgements

Contents

Part II Papers

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Summary

High reliability is an indispensable requirement for the operation of technical systems and infrastructure, such as power plants, oil platforms, aircrafts, railway lines and bridges. Failures can result in high costs and hazards to humans and the environment. Practically all technical systems are subject to deterioration, and a failure is often the consequence of excessive deterioration. Thus, inspections and maintenance are undertaken to uncover deterioration and to prevent failures and damage.

The improvement and the optimization of maintenance has great potential for cost savings. In order to exploit this potential, we need a systematic and structured approach. Furthermore, mathematical models are required to quantify the influence of maintenance decisions on reliability and costs.

The main objective of this thesis has been the development of a general deterioration and maintenance model for components in hydropower plants. The model was designed to serve as basis for maintenance planning and maintenance optimization. It is intended help to answer questions, such as:

- What is the probability of failure in a given time interval?
- How often should inspections be carried out?
- Is it better to carry out a maintenance action now or in x years?
- Is it advisable to postpone the action?
- What are the costs if the action is postponed?
- If we can choose between alternatives A and B: Which alternative should be performed first?

The aim has been to develop a general model that can be applied to different components and failure modes. The model was designed to utilize existing methods, processes and perspectives in the Norwegian electricity industry.

The general maintenance model presented in this thesis is based on a deterioration model consisting of a semi-Markov process with discrete state space. The model was built on an existing state definition established by the industry. This state definition is based on observable and thus physical properties of the deteriorating component. These states are therefore denoted

physical states. It is assumed that the sojourn time in a physical state may be modelled using a gamma distribution. A numerical solution procedure is suggested that requires states with exponentially distributed sojourn times. Thus, it is suggested transforming the gamma distributed sojourn times in the physical states into virtual states with exponentially distributed sojourn times, that is, transforming the gamma distributions into a Markov process. The thesis discusses different approaches on how to establish the virtual states.

A challenge in maintenance modelling is to provide a time-dependent model solution and to incorporate different maintenance strategies in the model, such as non-periodic inspections. An analytical solution for this case is difficult to obtain. Thus, a numerical solution is presented in this thesis for computing the expected number of inspections and maintenance actions in a given time interval. It is shown how deterioration, inspections and maintenance can be mathematically treated by simple numerical procedures. Furthermore, imperfect inspection and imperfect repair may also be realized. The numerical procedure presented serves the requirements of the Norwegian electricity industry.

A Bayesian framework is suggested for estimating the parameters of the sojourn time distributions. Both expert judgement and condition monitoring data may be used as sources of information for the parameter estimation. The thesis also provides suggestions on how to carry out expert judgement.

The thesis also discusses two other popular models: First, a maintenance model that also uses a Markov processes and that is frequently applied to modelling maintenance of components in electric power systems, and second, a maintenance model that treats the deterioration as a gamma process. It is shown that the former yields an error when it is used to analyse maintenance strategies with non-periodic inspections. The results presented make clear that the incorporation of a non-periodic inspection strategy is not as easy as suggested in some research papers. The latter yields similar results in maintenance optimization as the model presented if a maintenance strategy is analysed that is defined for discrete states. Differences and similarities between the models are described and advantages and disadvantages are discussed.

This thesis discusses all relevant modelling steps; from describing existing concepts and perspectives in the electricity industry, building the model, estimating parameters and calculating results, to the presentation of relevant examples. Solutions for all modelling steps are outlined. Thus, an operational model is provided that is ready to be implemented and applied by the electricity industry. Close corporation with the ongoing research activities at SINTEF Energy Research allowed successful testing of some of the thesis results at electricity companies participating in the research project.

The model presented provides a general framework for deterioration modelling. In some cases, there is the need for a more specialized deterioration model. One example is presented in this thesis where the influence of different operating conditions on the life of Francis turbines is analysed by a deterministic crack growth model. This model is based on the empirical Paris' law.

Crack growth simulations are used to investigate the influence of different load patterns on crack propagation and residual lifetime. The model may be used to quantify the relationship between turbine operation and damage progression, and consequently, to assess the lifetime reduction caused by changes of the operating conditions.

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Structure of thesis

This thesis is divided into two parts:

- Part I
  The first part starts with an introduction where the background for the thesis is described, where some perspectives and concepts used in the Norwegian electricity industry are described, and where the objectives and the scope of the thesis are defined. Maintenance and deterioration modelling is briefly discussed in the subsequent chapter. A description of the scientific approach is given in Chapter 5 and the main results are presented in Chapter 6. Some important topics are discussed in Chapter 7 and the conclusions, including suggestions for further work, follow in Chapter 8. The first part also comprises two appendices where the numerical solution procedure is described and where the influence of parameter uncertainty on the modelling results is discussed. This first part of the thesis combines the content of the papers found in Part II into a totality that serves to fulfil the thesis objectives.

- Part II
  The second part consists of a collection of papers constituting the major work that was carried out.

List of papers and publications

This thesis includes the following papers:

- Paper 1:
  T. M. Welte, J. Vatn and J. Heggset
  Markov state model for optimization of maintenance and renewal of hydro power components

- Paper 2:
  T. M. Welte
  A theoretical study of the impact of different distribution classes in a
  Markov model

- Paper 3:
  T. M. Welte
  Using state diagrams for modelling maintenance of deteriorating systems

- Paper 4:
  T. M. Welte
  Comparison of a gamma process and a state space model applied to main-
  tenance optimization

- Paper 5:
  T. M. Welte and A. O. Eggen
  Estimation of sojourn time distribution parameters based on expert opin-
  ion and condition monitoring data

- Paper 6:
  T. M. Welte
  A rule-based approach for establishing states in a Markov process applied
  to maintenance modelling

- Paper 7:
  T. M. Welte, A. Wormsen and G. Härkegård
  Influence of different operating patterns on the life of Francis turbine run-
  ners

Paper [1] and reports [2, 3] are based on the results presented in this work,
but are not included in this thesis.

Part I

Main Report

3

Introduction

This chapter provides a description of the background for the thesis, as a means to provide a broader context for the work. Some concepts and perspectives that are frequently used in the Norwegian electricity industry are described. These concepts and perspectives define a frame within which this work must be integrated. The thesis objectives are defined at the end of this chapter, along with its scope and the delimitations.

The title of this thesis "Deterioration and maintenance models for components in hydropower plants" describes in few words the contents of this thesis. The objective has been to develop a suitable deterioration and maintenance model for components in hydropower plants that can provide a basis for maintenance planning.

3.1 Background

Hydropower plant downtime, or damage to a power production unit, may result in substantial economic consequences. The failure costs comprise not only the repair costs for the failed equipment, but also loss of production. Each unit in a plant produces dozens of MWh of electrical energy, and for a major production unit or a big power plant, production losses due to an unexpected failure may add up to 100 000 EUR[1] per day. In addition, threats to human life and health may arise when personnel or other individuals are present at severe failure events. Other factors are environmental damage and loss of reputation, which ultimately might result in a loss of customers.

In order to avoid failures and unscheduled plant downtime, plant operators regularly carry out inspections and maintenance of their equipment. Typical questions the engineers responsible for maintenance planning must consider are (see also [4]):

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[1] Example: Run-of-river hydropower plant. Output: 100 MW. Electricity price: 40 EUR/MWh → 100 MW · 24 h · 40 EUR/MWh = 96 000 EUR

- What is the probability of failure within a given time interval?
- How often should inspections be carried out?
- Is it better to carry out a maintenance action now or in x years?
- Is it advisable to postpone the action?
- What are the costs if the action is postponed?
- If there must be chosen between two maintenance actions, which maintenance action should be performed first?

In order to answer these questions, maintenance engineers must rely on either experience or models and tools for decision support.

Traditionally, maintenance decisions have been based on experience. In the 20th century, most plants were manned by maintenance personnel. The personnel got to know 'their' plant on their daily tours of the plant, and they knew what 'their' plant's weaknesses and typical problems were. Irregularities and deviations were detected at an early stage and were corrected immediately.

A new situation arose after the deregulation of the Norwegian electricity market at the beginning of the 1990s. A national exchange for trading of electrical power was established in 1993. Sweden joined the exchange in 1996 and the world's first multinational exchange for trade of electrical power, named NordPool ASA, was established. The Nordic power market became fully integrated when Finland and Denmark joined few years later [5]. Apart from physical limitations in transmission capacity, this market allows for free trading of electrical power between all Nordic countries.

This new situation has led to two main trends:

1. Changing operating conditions, since the prices can vary significantly over short periods, driven by supply and demand. Companies try to produce electricity during periods when the prices are highest to maximize their profit. This results in more market-driven operating strategies, such as peak load production, an increased number of starts and stops, and faster start-ups and run-downs [6, 7].
2. An increased focus on cost reduction due to fierce competition on the deregulated market. This has led to a shift from technical to economic driving factors [8]. Thus, maintenance optimization and a reduction of operating costs have become a priority area for companies.

The former has led to extraordinary and increased loads, which may stress the components beyond their design limits and which may reduce their lifetime. Thus, previously used inspection and maintenance intervals are no longer valid and companies must reconsider and optimize their maintenance programs.

The latter resulted in activities such as staff reduction and in a need for suitable decision support systems. Mathematical models for maintenance planning and optimization hardly exist in the electricity industry [9]; and when they exist, they are seldom applied. Most of the maintenance models that are available have been developed in other industrial branches and must

be adapted to an application in the electricity industry. Research activities were therefore launched [10–12] in order to develop models and tools that take into account the requirements of the Norwegian electricity industry.

A model for decision support for maintenance and upgrading of hydropower stations had been developed in the 1990s [13, 14]. The model is denoted 'VTG' (water path, turbine, generator; in Norwegian: vannvei, turbin, generator), and helps to find the optimal point in time for upgrading hydropower plants. The VTG-model contains a simple failure model based on a lifetime distribution. However, the main focus of the model is not on failure modelling, but on production planning and on changes of the plant efficiency due to equipment upgrades. A deterioration model is not included in the VTG-model. Hence, it does not link deterioration with the residual lifetime and the failure probability, and it cannot use results from condition monitoring. Thus, a deterioration and maintenance model must be developed if a more detailed analysis of failures, deterioration and the influence of maintenance on the lifetime is desired.

3.2 Existing concepts and perspectives

This section describes concepts and perspectives that are used in the Norwegian electricity industry. An understanding of these concepts and perspectives is very important because they provide the background for some of the choices made during the modelling process.

One of the objectives in this thesis has been to use relevant concepts and perspectives existing in the industry. It is therefore important to start with a description of these concepts and perspectives. It is easier to apply a new model when it is based on existing foundations, because they are accepted and known by the practitioners, whereas the introduction of something that is completely new may cause opposition and refusal. The existing concepts and perspectives must be extended and supplemented when necessary.

3.2.1 Condition monitoring

In some cases, it is possible to measure physical deterioration directly (e.g. tire wear). In many other cases, however, physical deterioration cannot be measured directly [15]. Thus, one or several other quantities must be used as a deterioration indicator (e.g., power output of a generator, oil temperature in a gear). Based on these quantities, the deterioration (technical condition) can be judged. In an ideal case, the condition of a component can be related to one measurable quantity. If continuous monitoring of this quantity is possible, deterioration can be monitored and maintenance actions can be performed before a failure occurs.

In many situations, the technical condition of a component cannot be described by a continuous, measurable quantity. Component deterioration is

often assessed by visual inspections or by other methods that lead to qualitative results only. The appearance of a surface or the colour of a lubricant, for example, are properties that are difficult to quantify. As an alternative to continuous measures, discrete measures, such as states, grades and classes have been introduced. This results in the definition of several discrete deterioration states. The examples in the literature show that the number of states is usually set in the range of three to six.

A classification system with six states was used by van Winden and Dekker [16] to describe the condition of building elements such as masonry, window frames and painting. Kallen et al. [17–19] presented maintenance models where the condition of bridges was rated by six and seven states. Endrenyi et al. [4, 20, 21] presented maintenance models for electric power system components that are based on a classification system with four states. A similar classification system is used by the Norwegian hydropower industry where five states are defined, as described in the following section.

3.2.2 The EBL classification system

In the 1990s, the Norwegian Electricity Industry Association (EBL) introduced a classification system with five states for the description of the deterioration of hydropower plant components. A general description of the states is given in Table 3.1. EBL published handbooks [22] where a more detailed description of the states was provided for all major components and failure mechanisms. Examples of such descriptions are given in Tables 3.2 and 3.3 for the Francis turbine runner and 'material fatigue' as the failure mechanism. The examples show that it is sometimes necessary to combine two or several inspection methods to get an idea of the component's actual deterioration state. The handbooks and the classification system are widely used by plant operators, which means that the maintenance personnel are accustomed to these state definitions.

In 2002, a survey of condition indicators that are registered or measured by the Norwegian electricity industry was conducted by SINTEF Energy Research [23]. The survey lists 229 indicators and parameters for different plant components. The results show that for mechanical equipment in a hydropower plant, such as the turbine, approximately 50 % of the indicators are judged by means of the EBL classification system. Thus, if a maintenance model is developed for the hydropower industry it should be adapted to this system.

Table 3.1. General description of deterioration states according to the Norwegian Electricity Industry Association (EBL) [22].

State	Description
1	No indication of deterioration.
2	Some indication of deterioration. The condition is noticeably worse than 'as good as new'. ('Minor deterioration')
3	Serious deterioration. The condition is considerably worse than 'as good as new'. ('Major deterioration')
4	The condition is critical.

Table 3.2. Description of deterioration states. Component: Turbine runner. Failure mechanism: Fatigue. Inspection method: Visual inspection [24].

State	Description
1	Surface is plane and bright. No sign of damage.
2	Minor areas of the runner have a dull surface.
3	Surface is rough. Pitting. Small cracks evaluated as uncritical.
4	Critical cracks in the turbine runner.

Table 3.3. Description of deterioration states. Component: Turbine runner. Failure mechanism: Fatigue. Inspection method: Liquid penetration [24].

State	Description
1	No crack detection.
2	No crack detection.
3	No crack detection.
4	Cracks.

3.2.3 Inspection strategy

The operator of the plant must conduct inspections and maintenance to detect deterioration and to prevent failures. Preventive maintenance is the most beneficial strategy for many components in a hydropower plant. Condition-based maintenance is used for the major equipment such as the generator and the turbine. Recurrent inspections play a decisive role in this strategy. When condition information about the equipment condition is available, it is moreover advantageous to increase the inspection frequency, if the component is in an advanced state of deterioration. Thus, a so-called non-periodic inspection strategy is employed, instead of a periodic strategy with constant time intervals between each inspection. Three reasons support this strategy:

- It is more cost effective to reduce the inspection frequency for new components and to increase it when signs of deterioration become visible.
- The deterioration often accelerates as the deterioration itself progresses, and thus, the expected sojourn time in a deterioration state becomes shorter state by state.
- The likelihood of detecting a potential failure and progressed deterioration increases when the inspection interval length is reduced.

Some components have a high failure rate during the initial phase of their lifetime (known as 'infant mortality'). This behaviour is often illustrated by the decreasing part of the bathtub curve. If infant mortality is not caused by a shock, but by rapid component deterioration, frequent inspections during the early phase would help detect infant mortality and thus prevent these failures. Information about this type of failure is scarce in the electricity industry, and it is not clear to what extent this is of importance for hydropower plants. The deterioration model presented in this thesis does not take infant mortality into account. An extension of the model to include infant mortality is a potential topic for future work.

3.2.4 Economic framework

Many maintenance actions in the electricity industry are economically motivated. A common application of maintenance and deterioration models is in maintenance and refurbishment planning. In this case, the maintenance and deterioration models are part of a broader modelling framework, which consists of an economic analysis, a cost-benefit analysis, a failure consequence analysis, and so on. The overall objective is to provide decision support for maintenance and reinvestments; typical questions to be answered are listed in Section 3.1, and some typical application examples are presented in [1, 20, 21].

An electricity company will invest in maintenance or new equipment when it can be proven that the investment is economically profitable. Cost factors that influence profitability are, among others, the investment costs (new equipment, manpower etc.), production losses during scheduled downtime and

failure costs (replacement or repair of failed equipment, manpower, production loss during unscheduled downtime etc.).

It is common practice to discount all costs [1, 20, 21, 25]. This means that costs that occur far in the future are not of interest because the present value of these costs is approximately zero (see Paper 1). Furthermore, the analysis period is usually restricted to 30 to 70 years and costs in the far future occur outside of the analysis period. From this it follows that if we consider a component that has a long lifetime, we are rather interested in a time-dependent model solution describing the component's behaviour within the analysis period, rather than in an asymptotic solution that is only valid in a steady state situation.

3.2.5 Implications for maintenance modelling

The topics addressed suggest that a maintenance model for the Norwegian hydropower industry should meet the following needs:

- The model should be capable of handling inspection results expressed by means of discrete condition states.
- State dependent inspection interval lengths should be taken into account (non-periodic inspections).
- The model solution should yield a time-dependent solution to enable cost discounting.

This thesis thus presents a maintenance model that fulfils these requirements.

3.3 Objectives

The main objective of this work has been to develop a deterioration and maintenance model for components in hydropower plants. The model serves as basis for further modelling steps such as maintenance scheduling, maintenance optimization, investment planning and profitability analyses. The focus should not only be on modelling theory, but also on demonstrating how the model can be applied in practice.

The following more specific objectives have been identified:

- To identify a deterioration and maintenance model that is suitable for an application in the Norwegian hydropower industry. The model shall utilize and support commonly used concepts, perspectives, inspection methods and maintenance routines.
- To develop a general model that may be used for different types of equipment, deterioration mechanisms and failure modes.
- To compare the suggested model with other models to verify and validate the model and to discuss differences, similarities, advantages and disadvantages.

- To show how the input parameters in the model can be estimated based on different types of available sources of information.
- To provide realistic examples that show the application and use of the model and the results.

3.4 Delimitations and scope

This thesis is written from an engineer's point of view. Thus, it must be considered to be applied science and not pure science. Some parts of this thesis deal with mathematical models and statistical methods. Nevertheless, the main contributions of this thesis are not in statistics and mathematics, but in applied reliability engineering.

The work in this thesis is adapted to conditions in the Norwegian electricity industry; particularly those that apply to hydropower plants. Nevertheless, the results and models presented here are general enough so that they may also be applied to components in other types of power generating plants. Certain components in the electricity transmission and distribution grid might also be potential application areas. The results may be extended to applications in other industrial sectors, in and outside of Norway, as well.

One may classify all components in hydropower plants and electrical installations as either 'point objects' (also denoted 'discrete objects') or 'line objects' [8]. Many components in a hydropower plant are classified in the former category (e.g., most of the parts and components in a turbine or a generator). Some other components, especially in the electricity grid, are line objects, such as cables and transmission lines. The results from this thesis are primarily intended for point objects. If deterioration and failures of line objects are modelled, they may be treated as single point objects by, for example, dividing the line objects into several short segments. Then, the thesis results may be applied to each segment. Methods are required for aggregating the results from each segment to create valid results for the whole line object. This may be a challenging task and requires techniques that are not covered by this thesis.

The thesis results can serve as the basis for maintenance planning. One aspect of maintenance planning is maintenance optimization. The development of planning tools or optimization routines is, however, not the objective of this work. Nevertheless, some examples of maintenance planning and maintenance optimization are presented to show how the models can be used.

Other limitations are set by the modelling assumptions. These are, for example, sojourn times that are statistically independent and sojourn times that follow a given parametric distribution. These assumptions are discussed in more detail in Chapters 6 and 7.

4

Maintenance and deterioration modelling

Maintenance is carried out to avoid failures and excessive deterioration. Thus, maintenance cannot be modelled without a suitable failure and deterioration model. The failure and deterioration model is therefore the 'core' of any maintenance model. This chapter discusses some important topics related to maintenance and deterioration modelling.

4.1 Maintenance modelling

4.1.1 Classification of maintenance and maintenance models

The simplest classification of maintenance is given by the two types 'preventive maintenance' and 'corrective maintenance'. EN 13306:2001 [26] and IEC 60050-191 [27] define preventive maintenance as "the maintenance carried out at predetermined intervals or according to prescribed criteria and intended to reduce the probability of failure or the degradation of the functioning of an item"; corrective maintenance is defined as "the maintenance carried out after fault recognition and intended to put an item into a state in which it can perform a required function". During maintenance, different activities are performed, such as inspections/monitoring, overhaul, repair, and so on [26].

This thesis also uses the terms preventive maintenance, corrective maintenance and inspections. In maintenance modelling it is common to define the terms 'inspection', 'preventive maintenance' and 'corrective maintenance' as three different cost drivers. This is not in full accordance with the definition given in the standards, since inspections are defined as a part of the maintenance in the standards.

Not all inspections result in an 'active' maintenance action that changes the component condition. An inspection is therefore a 'passive' maintenance action to determine the component condition, whereas preventive maintenance includes all 'active' maintenance actions to change (improve) the component condition. Corrective maintenance is also an 'active' maintenance action to

change (improve) the condition after a failure. Each inspection, each preventive maintenance action and each corrective maintenance action results in different costs that must be quantified before economic analysis and maintenance optimization can be carried out.

A number of surveys of maintenance modelling and the application of models have been presented in the past decades. One of the most recent reviews has been provided by Wang [28], who lists dozens of older reviews and may therefore serve as a starting point for an overview of this broad topic. Wang [28] classified maintenance models into numerous categories based on the maintenance policies that were applied. Another survey conducted by Valdez-Florez and Feldman [29], introduced a classification scheme that categorized preventive maintenance models into inspection models, minimal repair models, shock models and miscellaneous models. The model presented in this thesis falls into the category of inspection models. Valdez-Florez and Feldman [29] defined an inspection model as follows:

"Inspection models usually assume that the state of the system is completely unknown unless an inspection is performed. Every system is normally assumed to be perfect in the sense that it reveals the true state of the system without error[1]. In the absence of repair or replacement actions, the system evolves as a non-decreasing stochastic process. In general, at every decision epoch there are two decisions that have to be made. One decision is to determine what action to take, whether the system should be replaced or repaired to a certain state or whether the system should be left as it is. The other decision is to determine when the next inspection epoch is to occur. Thus, the decision space of a maintenance inspection problem is two dimensional."

Figure 4.1 illustrates the principles provided in the definition. The deterioration and the maintenance/inspection strategy can be regarded as two parallel 'processes' that are connected to each other only in situations when inspections (I) or maintenance (M) are carried out. Then, information about the component condition can be gathered and decisions (D) about maintenance actions and the next inspection interval (τ) are made. After having set the component into operation, the deterioration process is hidden until the next inspection or a failure reveals the new component state.

4.1.2 Application of maintenance models

In 1995, the IEEE subcommittee on Application of Probability Methods established a task force to investigate the present status of maintenance strategies in the power industry. A summary of the results of this investigation can be found in [9]. From a review of current maintenance policies in electric utilities it was concluded that maintenance at fixed intervals is the most frequently used approach. Methods, such as reliability-centered maintenance (RCM), are increasingly being considered and used, but, methods based on mathematical

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[1] Note that this assumption may be relaxed (comment Thomas Welte)
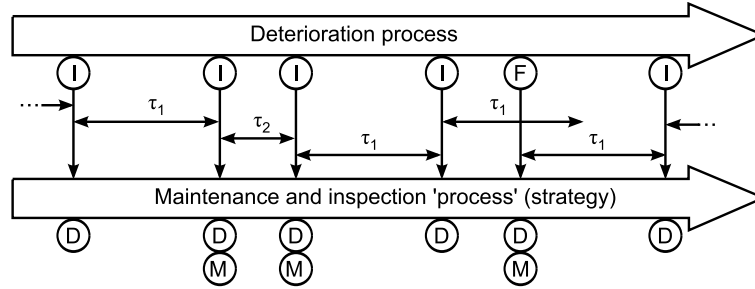
Figure 4.1. The deterioration and the maintenance/inspection strategy represented as two parallel 'processes'. The figure is taken from Paper 3.
D: decision, F: failure, I: inspection, M: maintenance, τ: inspection interval.

models are hardly ever used or even considered. What is the reason for this mismatch between available model theory and application?

Other authors have also pointed out that there is a gap between theory and practice [30–32]. They claim that many models are difficult to understand and to interpret and that some work has put too much focus on mathematical analysis and techniques than on solutions to real problems [30]. Thus, there is the appeal to work on real problems [31].

Dekker and Scarf [32] described the state of the art at the end of the 1990s in the applications of optimization models in maintenance decision making. They review, among other things, decision support systems. They pointed out that the success of one of the first commercial PC-based decision support systems for maintenance optimization was the embedding of mathematical models in a user-friendly environment so that the input for the models could easily be formulated by the maintenance engineer. This aspect, which turned out to be crucial, had completely been ignored in the mathematical analysis.

In my opinion, many of the statements and observations mentioned above are still valid. As discussed in Section 3.1, no deterioration and maintenance model is applied in the Norwegian electricity industry today. This work may contribute to changing this situation. The implementation of the models in user-friendly software tools remains a challenging task, and the work is therefore not completed by the development of a maintenance model. The development of such tools is a goal of the research projects that are being carried out at SINTEF Energy Research. Prototypes are available and under development [2, 3], and are currently in a testing phase at several power companies.

4.1.3 Multi-component maintenance models

Most maintenance models have been prepared for single-component systems. Optimal maintenance policies for multi-component systems are identical to those for systems with a single component only if there exists no dependence. In this situation, maintenance decisions are also independent and the optimal

maintenance policy is to employ an optimal single-component strategy for each component separately [28].

Some few papers also consider maintenance models for optimization of multi-component maintenance. The review paper by Wang [28], for example, contains a section that is devoted to maintenance policies for multi-component systems. However, Wang primarily considers models with economic dependence. A more comprehensive review of multi-component maintenance models is given by Nicolai and Dekker [33, 34].

Virtually all real systems consist of two or several components. Thus, a modeller should work for a multi-component maintenance model. However, a multi-component model can hardly be developed if we have no understanding of the degradation and failure behaviour of each single component. Since no suitable model exists for single components in the Norwegian hydropower industry, we have to start with this type of modelling. This is the starting point for this thesis. Consequently, the focus of this thesis is on single-component systems. Multi-component systems are not considered here.

4.2 Deterioration modelling

Up to the early nineties, most mathematical maintenance models were based on describing the uncertainty in ageing using a lifetime distribution. One disadvantage of a lifetime distribution is that it only quantifies whether a component is functioning or not [35]. Many failure mechanisms can be traced to an underlying deterioration process. Deterioration eventually leads to a weakness that can cause failure. When it is possible to measure deterioration, such measures often provide more information than failure-time data [15]. Hence, for engineering structures and infrastructures, it is generally more attractive to base a failure model on the physics of failure and the characteristics of the operating environment [35]. It is therefore recommended using a deterioration model for components in hydropower plants.

Deterioration can be represented as a curve in a diagram showing deterioration on the y-axis and the time (or another measure of usage, for example, load cycles, revolutions, odometer reading etc.) on the x-axis (Figure 4.2). This requires that the deterioration is measurable and one-dimensional, that is, that deterioration can be described, merged and represented by one measurable quantity. The curve in Figure 4.2 is called a 'degradation/deterioration curve', 'degradation/deterioration path', or 'life curve' [15, 21]. In the following, the term 'deterioration curve' is used to denote such a curve. Figure 4.2 shows examples of three general shapes of deterioration curves: linear, convex and concave. The examples represent three general types of deterioration rates: constant, increasing and degreasing.

The rate of many deterioration mechanisms is of the increasing type, that is, the deterioration curve has a convex shape. A classic example is fatigue

crack growth. The results of expert judgements on sojourn time durations (Paper 1, [36]) confirmed the assumption that the deterioration curves of many components in hydropower plants have a convex shape. Thus, a deterioration curve in this thesis is usually represented by a convex-shaped curve, as shown in Figure 4.3(a). The figure illustrates the deterioration process in a idealized way as smooth curves, which is not very realistic for real deterioration. Deterioration is not a deterministic process and hence the curves may be considered to represent the mean deterioration.

Many practitioners and maintenance engineers prefer to draw deterioration curves in a reverse way (Figure 4.3(b)). Increasing deterioration means a worsening situation, which is usually associated with a curve that runs 'downwards'. Close collaboration with maintenance personnel during the thesis research has shown that deterioration curves are confused with failure rate curves if deterioration curves are drawn as shown in Figure 4.3(a). A representation similar to the one in Figure 4.3(b) helps to avoid this mistake.

The terms 'technical condition', 'deterioration' and 'degradation' are used in the literature to describe the same measure. Equipment that is in a good technical condition means it has suffered less deterioration/degradation. Serious deterioration/degradation is usually a sign of poor technical condition. Thus, the terms 'condition', 'deterioration' and 'degradation' are often interchangeable, because they have the same interpretation.
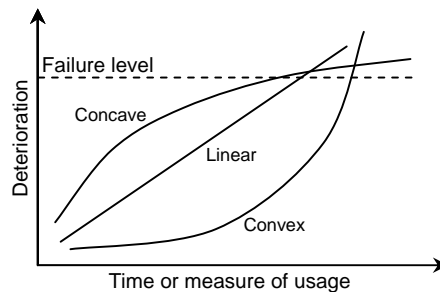
Figure 4.2. Possible shapes for deterioration curves (adapted from [15, p.319, Figure 13.2]).

4.2.1 Classes of deterioration models

Based on the degree of understanding of the problem, deterioration models may be classified as either [37]:

- Black-box models

- Grey-box models
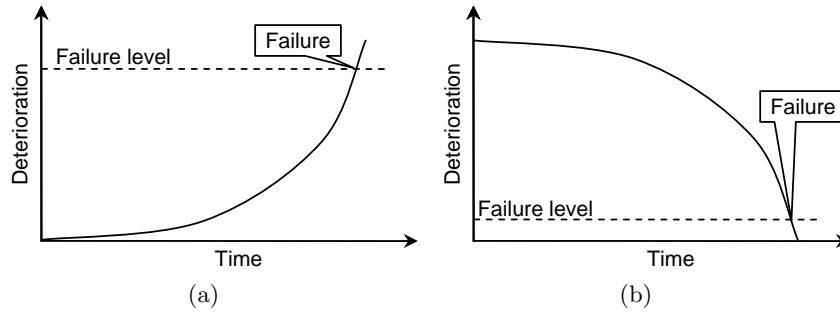
Figure 4.3. Two different representations of a deterioration curve.

- White-box models

A lifetime distribution describing the random time to failure is a typical example for a black-box model. One disadvantage of a lifetime distribution is that it only quantifies whether a component is functioning or not [35], and deterioration as a function of time is not modelled. A white-box model is obtained through the knowledge and modelling of the deterioration mechanisms. Typical examples of grey-box models are stochastic processes, which are based on a measurable quantity indicating time-dependent deterioration and failure [37].

In this thesis, the main focus is on grey-box deterioration models. A continuous-time semi-Markov process with discrete state space is suggested as a general deterioration model for the Norwegian hydropower industry. For one special application, a white-box deterioration model based on the empirical Paris law, is also considered in this thesis (Section 6.8 and Paper 7).

5

Scientific approach

This research work belongs to the field of applied science, that is, the research has direct practical applications in industry. Applied science is an activity of an original nature to gain new knowledge and insight, primarily to solve specific, practical problems. The quality of the research must be considered not only from a scientific point of view, but also from a user's point of view.

The general basis for this thesis and the topics it addresses have been established through literature surveys. These surveys represent the starting point for the research and support all subsequent activities. In addition, the professional experience from my supervisors Jørn Vatn and Gunnar Härkegård and from colleagues at NTNU and at SINTEF have contributed valuable input in the identification and solution of problems. Furthermore, useful insight into the maintenance challenges facing the electricity industry has been gained through discussions and cooperation with engineers working in the electricity industry and at equipment manufacturers.

The objective in this thesis is to develop deterioration and maintenance models for components in hydropower plants. From a classical point of view, the usefulness of models should be empirically verified, for example, by experiments or by collecting field data. Empirical verification may be impossible in the reliability and safety engineering field, where we deal with analysing and modelling of undesired events such as failures, accidents and catastrophes. These events occur infrequently. In addition, the objects analysed are often unique and expensive constructions with a long lifetime. It would be extremely costly or time consuming to carry out experiments and collect data to confirm the models and modelling results. Thus, the evaluation and verification of the scientific work and the models must be done by procedures other than empirical or experimental methods. This is discussed in the following section.

5.1 Model evaluation

Much of the scientific work in the field of reliability and safety engineering is related to the development of models for reliability and safety analysis. As discussed above, verification and evaluation can be problematic because empirical or experimental verification is often not possible.

An attempt to overcome this problem was made by the European Union with its launch of the Model Evaluation Group in 1992 [38]. The group's objective was to improve the culture in which models were developed, particularly by encouraging voluntary model evaluation procedures based on a formalized consensus protocol. The group suggested a model evaluation process, consisting of the following three main elements:

- Scientific assessment
- Verification
- Validation

The scientific assessment should comprise a comprehensive description of the model, an assessment of the scientific content, limits of applicability, and limitations and advantages of the model. Verification was defined by the group as "the process showing that a model has a sound scientific basis, that any assumptions are reasonable, that equations are being solved correctly, and more generally, that the model presented to the user actually does what the the document claims" and validation as "the process of assessing a model so that its accuracy and usefulness can be determined." The latter often involves comparison with other models [39].

This thesis is a contribution to the scientific assessment of the presented work by a comprehensive description of the model, an assessment of the scientific content, and a description of the limitations and advantages of the model. According to the definition given above, one aspect of validation is a comparison with other models. Comparison with other models may also be useful for model verification. If other models exist for the same application, a comparison of the results provides evidence that the model has a sound scientific basis, that the assumptions are reasonable and that the equations are solved correctly.

Another aspect of validation is closely related to the proof of usefulness of a model in practical applications. Close collaboration with an ongoing research project at SINTEF Energy Research provided the opportunity to test some of the results presented in this thesis. Prototypes of software tools have been established in the project and the project participants were able to test some of the tools.

It was very important for me to create models and methods that are applied by maintenance engineers in the hydropower industry to make good decisions. Many readers of this thesis probably know the following statement by J. Frank Dobie [40]: "The average PhD thesis is nothing but a transference of bones from one graveyard to another." It is actually impossible to do a PhD

work without dragging bones around. Literature research and the reading and understanding of previous work are indispensable elements of research. During this process the researcher often digs up old research results (bones). However, the aim must be to bring life to the bones, that is, the bones should not be buried again. Perhaps, I have dug some bones up during my PhD work. However, if the results originating from this work are used by practitioners for solving everyday problems in maintenance planning, or if parts of this work help other scientists to solve their problems, this thesis may then be more than transferring bones between graveyards.

5.2 Scientific quality

The term 'scientific quality' is difficult to define. According to the Research Council of Norway [41], quality in science may be related to the following three aspects:

- Originality, or to what extent the research is novel and features the innovative use of theory and methods.
- Solidness, or to what extent the statements and conclusions in the research are well supported.
- Relevance, or to what extent the research is linked to professional development or is practical and useful to society.

It has been pointed out that these three criteria may be contradictory in some cases. Strong solidity due to thoroughness may restrain creativity and originality, and research of little originality still may be very useful [41]. Thus, scientific work, as this thesis, may be seen as a kind of balance act between solidness, originality and relevance.

From my point of view, this thesis is a balanced compromise between originality, solidness and relevance. The work combines and advances existing theory and methods in a new way and proposes new solutions. The work is based on established standards for scientific work and research in the disciplines of reliability engineering and maintenance modelling. Furthermore, the results presented are useful in solving the problems with which they are concerned. Thus, the work has been carried out in accordance with the criteria given above.

5.3 Research approach

This work has used an analytical approach, which involves applying known and proven methods, techniques and frameworks. These were combined, modified and extended in order to adapt them for the intended purpose. The following methods, techniques and frameworks have been employed in this work:

- Stochastic modelling
- Numerical methods
- Expert judgement
- Lifetime data analysis
- Bayesian methods
- Fracture mechanics (empirical model)

5.3.1 In retrospect

When I started to write this chapter of my thesis, I took some old presentations of the very early phase of my PhD studies to review my original plans. In my first presentation made for one of the companies financing this PhD thesis, I presented a figure (Figure 5.1) in which I tried to describe my research approach. When I saw this figure three years later, I was really surprised how well it still describes the structure of the research approach used for my PhD work. I described the PhD task as 'building a house'. In the figure, the roof of the house represents the deterioration and maintenance model. The model input is based on (at that time unspecified) data about failures and plant operation. The roof (model) rests on pillars representing the various methods, techniques and frameworks used during the research work. Sure, the 'pillars' of my house (PhD work) changed over time and came to have other names, that is, I have used other methods, techniques and frameworks than I originally thought; nevertheless, the basic structure of the research approach has remained the same.
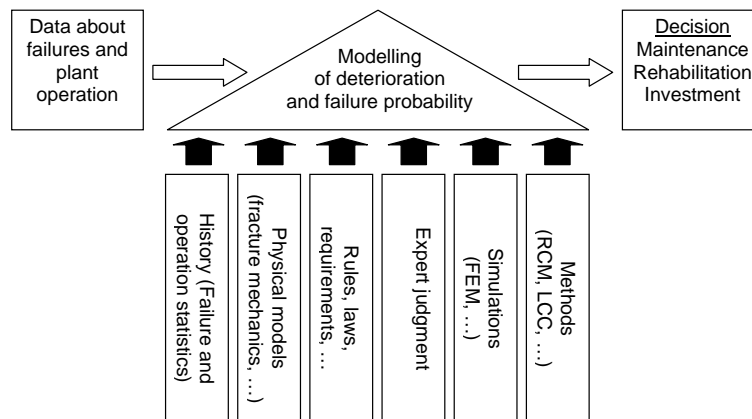
Figure 5.1. 'Building a house': A vision at a very early stage of my PhD work.

6

Main results

This chapter gives an overview of the main results of this work. The various publications associated with this research are referred to where they relate to specific topics. The main part of this work deals with a generic deterioration and maintenance model for components in hydropower plants. The model, the model solution, parameter estimation and some examples are presented in Sections 6.1-6.6 and in Papers 1, 2, 5 and 6. One result of this thesis is the discussion and comparison of different maintenance models; see Section 6.7 and Papers 3 and 4. The last part of this chapter (Section 6.8) and Paper 7 describe an approach for analysing the influence of different operating patterns on the life of Francis turbine runners. The approach is denoted the 'turbine model' and is based on the empirical Paris' law [42].

The main results from this thesis are summarized as follows:

- A general deterioration and maintenance model for components in hydropower plants has been developed.
- A numerical procedure, which provides a time-dependent model solution for a maintenance strategy with non-periodic inspections, has been developed.
- Recommendations on how best to carry out expert judgement have been provided.
- An approach for parameter estimation allowing the use of expert judgement, the combination of several expert judgements and data has been suggested.
- Different maintenance models have been compared. This includes a critical discussion of a model that is frequently applied by the electricity industry.
- An approach for analysing the influence of different operating patterns on the life of Francis turbine runners has been developed.

6.1 Deterioration model for components in hydropower plants

It is clearly more efficient and effective to use the existing classification system (as described in Section 3.2.2) as the basis for a deterioration model instead of introducing a new system or measure to describe deterioration. The advantage is that the maintenance personnel and the engineers responsible for maintenance planning can continue using a classification system with which they are familiar.

The deterioration process of a new component that is put into operation at time $t = 0$ will run through all four states, provided that no maintenance is carried out (Figure 6.1). Assume a population of equal components under equal operating conditions; some components will have a shorter lifetime than others. The time the process spends in state j (the sojourn time in state j) will hence vary as well. Thus, the sojourn time T_j in state j is a random variable. This is represented by the probability distributions in Figure 6.1. If we allow T_j to have a general 'lifetime' distribution (sojourn time distribution) and if we assume independence between the T_j, the deterioration process can be mathematically described as a semi-Markov process [43].
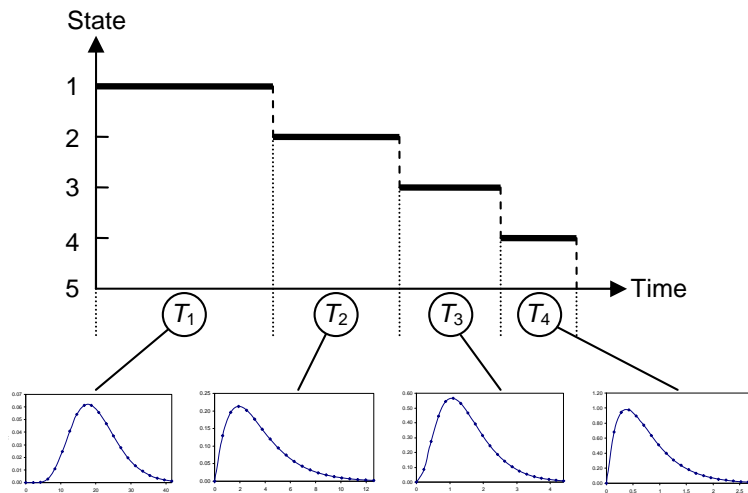

Figure 6.1. Modelling of deterioration by a semi-Markov process with discrete states.

6.2 The concept of physical states and virtual states

The states that are defined in Table 3.1 are so-called 'physical states' or 'main states'. They are defined by physical, and thus observable, characteristics of

the deteriorating component. It is not possible to monitor the states continuously. The exact time of state transition is hence not known. Even if it was possible to conduct continuous monitoring, it would be difficult to specify the exact transition time, because the state definition is somewhat vague. Nevertheless, the states are observable, and based on a sequence of several state observations, censored observations of the sojourn times may be extracted as described in Paper 5. Such observations may be used for parameter estimation.

In some applications, the sojourn times in the physical states are modelled by exponential distributions [4, 44]; see also Paper 3. The choice of the exponential distribution is mainly based on practical reasons, because in this case, the physical states can directly be used as states in the mathematical deterioration model. The resulting model is an 'ordinary' Markov process. Expert judgement results collected during this research and by SINTEF Energy Research have shown that the sojourn times are often not exponentially distributed; see Paper 1 and [36]. Thus, the choice of exponentially distributed sojourn times is in many cases not a realistic solution.

In this thesis, it is therefore suggested that the sojourn times in the physical states be modelled using a gamma distribution. The gamma distribution may be approximated by a sequence of exponentially distributed 'virtual states' (called sub-states in Paper 1) so that a chain of exponentially distributed virtual states represents a gamma distributed physical state [1]. From this it follows that the semi-Markov process is approximated by a Markov process. Figure 6.2 illustrates this approach, where the physical states are indexed by j, $j = 1, ..., 5$, and the virtual states are indexed by i, $i = 1, ..., I_j$. The sojourn time in the ith virtual state of the jth physical state is denoted $T_{j,i}$ in the figure.

The probability density function of the gamma distribution is

$$f(t) = \frac{1}{\beta^{\alpha} \Gamma(\alpha)} \cdot t^{\alpha-1} \cdot e^{-t/\beta} \tag{6.1}$$

where α is a shape parameter and β a scale parameter [15].

There are several reasons, which support the choice of the gamma distribution in the application presented. First of all, the gamma distribution has a more flexible shape than simply assuming exponentially distributed sojourn times. For a shape parameter $\alpha > 1$, the distribution has an increasing failure rate, which is a realistic property in most cases. However, the rate flattens for $t \to \infty$. Some other distributions may provide a more realistic choice. Nevertheless, the gamma distribution may be chosen, because it is closely related to the sum of exponential distributions. Thus, the gamma distribution is a natural choice when we divide the physical states into virtual states. It is known that a gamma distribution with an integer shape parameter n (which

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[1] Note that 'exponentially distributed virtual state' and 'gamma distributed physical state' means that the sojourn time is exponentially distributed and gamma distributed, respectively, and does not refer to the state itself.
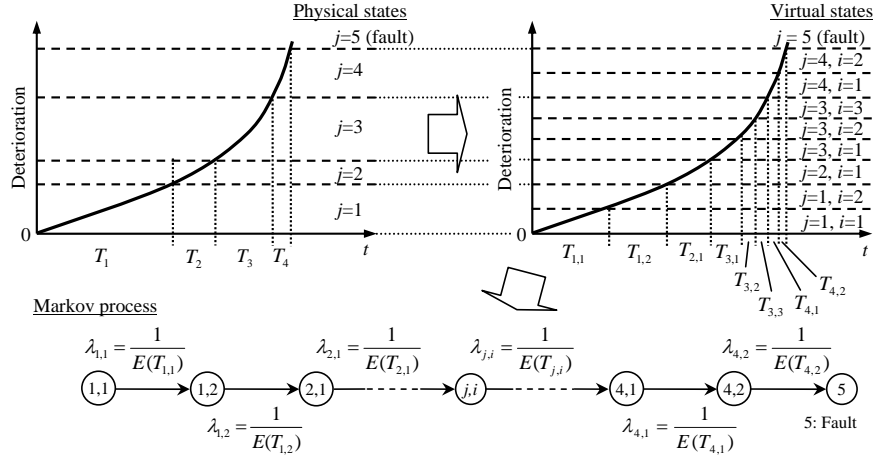
Figure 6.2. Physical states, virtual states and resulting Markov process.

is an Erlang distribution) is the distribution of the sum of n identically exponentially distributed variables. Even though the shape parameter is not an integer, the gamma distribution may be approximated by a chain of exponential distributions. Two simple and straightforward approximation approaches are discussed in Papers 2 and 6. From this it follows that the gamma distribution is a means of generating the states in the Markov process. A gamma distribution provides a good model when deterioration is caused by a sequence of shocks. This may be realistic for a number of failure mechanisms, but obviously not for all failure mechanisms. Whatever distribution is chosen, when applied in in a general maintenance model, it must result at the end in a good compromise of properties that fulfils different situations. It is assumed that the gamma distribution provides a balanced compromise between different properties.

6.3 Maintenance and inspection strategy

The following general maintenance and inspection strategy is considered in this thesis (see Figure 6.3):

- The component is regularly inspected. However, the inspection interval, τ_j, is not constant since the time of the next inspection depends on the deterioration state j revealed by the previous inspection (non-periodic inspections).
- If an inspection reveals that the deterioration exceeds an intervention threshold (maintenance limit), preventive maintenance (PM) will be carried out.

- If the component has failed, corrective maintenance (CM) will be carried out.
- Preventive maintenance and corrective maintenance improve the component's condition.

Unless otherwise noted, a failure is usually assumed to be self-announcing, that is, failure is immediately revealed, and a corrective maintenance action is conducted; inspection and maintenance durations are assumed to be negligible. Note that the definition of the inspection strategy is based on the definition of the physical states. Only physical states are observable, and only they can serve as a basis for maintenance decisions in a condition based maintenance strategy.

Inspections are usually assumed to be perfect, that is, the real deterioration state can be detected without error. Contrary to this assumption, inspections may be considered to be imperfect, that is, the real deterioration state is not always revealed and it remains a probability of erroneous classifications.

Maintenance may also be modelled as perfect, as shown in Figure 6.3. Perfect maintenance, also denoted perfect repair, is usually defined as a maintenance action that restores the system to a condition 'as good as new', whereas imperfect maintenance, also denoted imperfect repair, is defined as a maintenance action that improves the system condition, but not to a state of 'as good as new' [45, 46].

In addition to the definition provided, one may also associate with the term 'perfect' in connection with the term 'maintenance' that maintenance always fulfils its intended purpose, which means that the maintenance action always restores the component to a predetermined (planned) state. Consequently, the maintenance action may be considered to be 'imperfect' if it fails to fulfil its intended purpose. An example is a maintenance action that is intended to restore the component condition to a state of 'as good as new' (which is state 1 according to the EBL classification system), but in reality results in state 2 or 3.

Imperfect maintenance is the general, but more complicate, situation [46]. Since the assumption of perfect inspections and perfect maintenance is not always realistic, a model that covers imperfect maintenance (and in an ideal case also imperfect inspections) is desirable. Thus, a model solution is presented in Section 6.4 that provides the possibility to include imperfect inspection (see Paper 1) and imperfect maintenance. Nevertheless, the application of these extensions is not advisable in all situations. A simple model assuming perfect repair and perfect maintenance will often be a good approximation of the real situation. Furthermore, the extension of the model to imperfect inspections and imperfect maintenance requires the specification of additional model parameters that may be difficult to estimate. Thus, increased model precision will in many situations not justify the additional modelling effort.
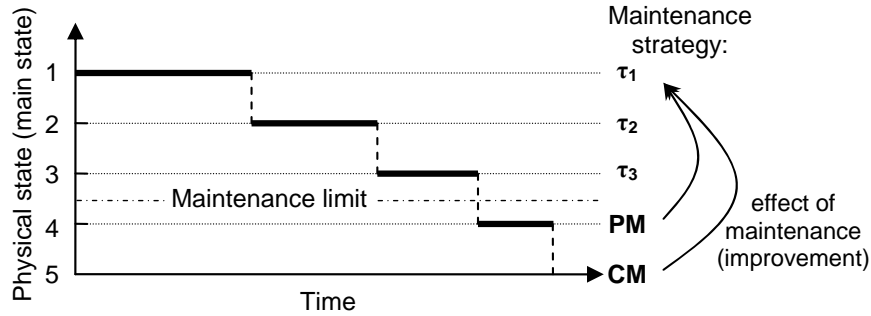
Figure 6.3. Illustration of the general inspection and maintenance strategy.

6.4 Model solution

A main result in this thesis is a numerical procedure that can be used to solve the maintenance model described in the previous section. Solving the maintenance model means to calculate

- the expected number of corrective maintenance actions (failures),
- the expected number of preventive maintenance actions, and
- the expected number of inspections

for a given time interval, and as a function of time and the inspection strategy. For example, if the time interval is chosen 1 year, the expected number of corrective/preventive maintenance actions and inspections is calculated for the first year, the second year, and so on. This approach allows for cost discounting.

A major challenge is to incorporate a maintenance strategy with non-periodic inspections. Such a strategy is difficult to treat analytically. An alternative is to use Monte Carlo simulation. In principle, this approach allows for the computation of results from all types of maintenance models representing all types of maintenance strategies (including non-periodic inspections, imperfect inspections and imperfect repair). The disadvantage is that the computation time strongly depends on the accuracy required. Monte Carlo simulation may become very slow when accurate results are required. The procedure suggested in this thesis may provide an alternative to a Monte Carlo simulation. If accurate results are required, the procedure suggested here is usually faster than Monte Carlo simulation.

The procedure satisfies the requirements described in Section 3.2.5, which are discrete condition states, non-periodic inspections and a time-dependent solution. The numerical procedure covers:

- Deterioration
- Inspections
- Preventive maintenance

- Corrective maintenance

The procedure is described in more detail in Appendix A and is outlined in Paper 1. Paper 1 also presents an example with imperfect inspections. The procedure can easily be extended to imperfect maintenance.

6.5 Parameter estimation

A main challenge in maintenance modelling is the estimation of model parameters. The maintenance model presented in this thesis requires estimates for the following parameters:

- Sojourn time distribution parameters
- Costs for a single inspection, preventive maintenance and corrective maintenance action
- Inspection interval lengths (if the inspection interval length is not a decision variable)
- Maintenance limit (if the maintenance limit is not a decision variable)

Inspection interval lengths and maintenance limits are usually decision variables that are given by the maintenance strategy. Inspections are frequently carried out and the hydropower companies record inspections and maintenance actions in the Computerized Maintenance Management System (CMMS) where they can be retrieved for statistical and other analyses. The average cost of an inspection is therefore a well-known quantity.

The estimation of the average cost for a preventive maintenance or corrective maintenance action is more challenging, especially for maintenance actions that are seldom carried out. Corrective maintenance actions are rare for long-lived products, such as many of the components in a hydropower plant. Statistics about historical events barely exist. Economic losses from the outage time as a consequence of maintenance are difficult to assess. In addition, the evaluation of failure consequences is a challenging task. Thus, other methods may be used to estimate costs, for example:

- A maintenance service provider can be asked to provide a quotation for a preventive or corrective maintenance action.
- Systematic approaches, such as event tree analyses [47], can be applied to carry out a more detailed analysis of undesired events.
- Simulation based methods are available for the estimation of production losses and market prices (e.g. EOPS and EMPS [48, 49], the one-area and multiple-area power simulator software developed by SINTEF Energy Research. This software is designed for generation scheduling and forecasting of market prices. The models treat uncertainties both in future inflow and market prices.)

Last but not least, good estimates for the sojourn time distribution parameters must be found. If sufficient reliability data is available, classical methods, such as least square or maximum likelihood methods [15, 50], can be used. However, reliability data is often scarce. Components in hydropower plants are often designed to such high reliability standards that deterioration is slow and that failures seldom occur. Furthermore, there are often missing or bad routines for the collection of failure and inspection data.

Beginning at the end of the 1990s, some operators had started to register inspection data in their maintenance management systems. Until enough data is available for statistical analysis, however, we must rely on alternative sources of information. Thus, Paper 5 suggests an approach for the estimation of sojourn time distribution parameters that allows for expert judgement as source of information. The approach provides a starting point until a suitable database is available. The suggested approach can easily be extended such that data can be incorporated.

6.5.1 Expert judgement on sojourn time distributions

The meaning of many probability distribution parameters is rather abstract for non-statisticians. Thus, it is generally difficult for experts to estimate probability distribution parameters directly. For example, asking directly for a Weibull shape parameter is not a good question; instead, an assessment of the failure probabilities should be requested [32]. We must therefore ask the experts for quantities that are easier to assess, such as probabilities, percentiles, or the mean, median or mode of a random event.

For the elicitation procedure in this thesis, it is suggested to request an estimate for the 10th percentile and the mean sojourn time. The 10th percentile is situated in the left part of the distribution and corresponds to early failures. The experts may have experienced such early failure events and it is therefore assumed that they can provide a good assessment for this percentile. A second reason to use a percentile in the left part of the distribution is that this percentile is close to the inspection interval. Paper 2 shows that the error in the maintenance model can be minimized when the percentile is in the same order of magnitude as the inspection interval, even though the 'correct' distribution class is unknown.

Expert answers are usually subject to errors. Sources of errors can be of a random or systematic nature. If experts are called upon to assess probabilities or determine degrees of belief, they rely often on various rules of thumb. Such rules are called heuristics [51–53], and if they lead to systematic errors, we talk about biases [53]. Typical biases are over-/underestimation and over-/underconfidence [53–55]. If biases are revealed and if the magnitude of the errors is known, one can try to remove the errors, that is, one can try to calibrate the expert estimates.

It is known that experts often tend to be overconfident, especially with respect to highly difficult tasks. Unfortunately, this is exactly the type of

situation in which expert opinion is most likely to be used and needed [56]. Thus, a broadening of the distributions can be carried out as suggested in [54, 57]. However, the broadening is a controversial matter [58–60] and there is no final recipe for the best way to do this.

Rules of thumb for the interpretation of expert opinion are given by Meyer and Booker [54]:

- When experts provide 5th and 95th percentiles, they are really only providing the 30-40th and 70-60th percentiles.
- When experts provide maximum and minimum values, they are really providing the 5-10th and 95-90th percentiles.
- When experts provide their best central estimate, they are really giving a value that corresponds to a median (50th percentile) rather than a mean.
- When experts provide a variance, they are really representing less than half of the variance.

It is suggested to apply some of the rules of thumb in the interpretation of the received expert responses. The recommendation is to use the expert's assessment of the mean sojourn time as the median (50th percentile) in subsequent analyses, and the 10th percentile as the quartile (25th percentile).

6.5.2 Combining expert responses and data

The use of expert judgement implies that we must aggregate the responses obtained by some means or another. In principle we can distinguish between two aggregation problems [54]:

1. Aggregation estimates, in which a single summary value (estimate) is calculated based on a set of answers.
2. Aggregation distributions, in which several probability distributions are aggregated or a distribution is calculated for multiple values from many experts.

The latter problem is relevant for this work because we must combine probability distributions obtained from several expert judgements (and perhaps from data). The combination of probability distributions is discussed in [53, 54, 61, 62]. Several researches have concluded that Bayesian approaches are the most appropriate aggregation method [62].

A Bayesian approach allows the incorporation of any type of information as long as it can be represented by a likelihood function. In this thesis, two types of information sources are considered; expert opinion and inspection data. The likelihood function for the expert opinion is calculated according to an approach described by Mosleh and Apostolakis [63]. The incorporation of the expert judgement in the likelihood function has not been without controversy [64], since expert judgement is usually represented by the prior distribution. However, as pointed out by Apostolakis and Mosleh [65, 66], the presented approach is one possible application of Bayes' theorem, because the theorem is

a "fundamental tool that allows us to coherently incorporate in our knowledge new evidence, which does not have to be statistical [65]". The approach was originally developed for the lognormal distribution and an analytical solution is presented for this case in [63]. The approach was generalized for this thesis by using numerical methods (see Paper 5). Then, it is possible to apply the approach to other distribution classes as well. Paper 5 shows furthermore how an additional parameter required for the approach can be estimated and how data can be combined with expert judgement. The paper presents a numerical example for a typical dataset, which will hopefully be available in the Norwegian electricity industry in the near future.

6.6 Model application

This section presents two applications of the proposed deterioration and maintenance model. The first example concerns maintenance and investment planning, while the second is about maintenance optimization. The examples show that the application of the maintenance model may help to provide answers on the questions raised in Section 3.1.

6.6.1 Maintenance and investment planning

This example represents a typical application in the hydropower industry. The example is similar to the case considered in [1]. Two maintenance alternatives are analysed and a decision must be made between these alternatives.

Assume the following situation: A company has a old component in one of their hydropower plants. The component has been inspected and the deterioration is classified in state 3. The following two maintenance alternatives are considered:

1. Immediate refurbishment in 2008
2. Carry out 'normal' preventive maintenance (PM) in 2008 and postpone the refurbishment by four years, that is, refurbishment in 2012

The refurbishment is a costly action. The effect of this action is that the condition is 'as good as new' afterwards and that the deterioration is slowed down. An example for such a refurbishment is the surface treatment of the turbine blades by a hard coating. This coating makes the turbine more resistant again wear, such as erosion and cavitation. Thus, the refurbishment will result in a component with other deterioration properties than the component before refurbishment.

The 'normal' PM action is a cheap maintenance action. The effect of this maintenance action is that the condition is improved to state 2 (the beginning of state 2). For the turbine example and the failure mechanism surface wear, the company may consider a repair of the damaged areas by build-up welding. The surface will not be 'as good as new' after this PM action. The

deterioration properties of the component are as before, because there is no principle changing or upgrading of the component.

It is obvious that an immediate refurbishment will reduce the probability of failures in the first four years. However, this advantage has to be paid through the investment costs in the year 2008. Thus, the question is if the early investment is profitable from an economic point of view.

The deterioration properties are given in terms of mean sojourn times and gamma distribution parameters in Table 6.1. In addition to the planned maintenance actions in 2008 and 2012, the company schedules inspections according to the inspection strategy given by the inspection intervals in Table 6.2. A PM action is carried out when the component deterioration is classified in state 3. Corrective maintenance (CM) is carried out after a failure. Both PM and CM improve the system condition to state 2. The costs for a single maintenance action are given in Table 6.3. Note that PM and refurbishment are planned actions, whereas CM is a unplanned action after a failure which may result in high costs.

The numerical procedure presented in this thesis can now be used to calculate the expected number of inspections, PM actions and CM actions for the following years. The analysis period is chosen 30 years in this example. Figure 6.4 shows the various costs for the different actions for both alternatives. All costs are discounted (discount rate: 8 %) with year 2008 as reference. The net present value (NPV) of the costs for the first alternative is 57.4, whereas the NPV of the costs for the second alternative is 68.5. Thus, it is profitable to carry out the refurbishment already in 2008 instead of postponing it by four years.

A more general question is: When is the optimal time for the refurbishment? Figure 6.5 shows a plot of the NPV of the costs as a function of the year of refurbishment. The plot shows that the most profitable alternative is to perform the refurbishment already in 2008. The NPV increases when the refurbishment is postponed 1 to 5 years and decreases afterwards. Thus, it would be advisable to do refurbishment immediately. However, the operator of the plant may have restrictions regarding the budget or the availability of personnel and decides that refurbishment is not performed in 2008, because other projects are more urgent and profitable than alternative 1. Once the company has performed normal PM in 2008, it would not be wise to schedule refurbishment in one of the following years. The normal PM action has improved the condition to state 2. It can be expected that the equipment and the plant operator may benefit from this action some years. Thus the refurbishment may be postponed several years.

state j	Before refurbishment			After refurbishment		
	mean	α_j	β_j	mean	α_j	β_j
1	10	3.4	2.9	20	5.3	3.8
2	8	3.1	2.6	10	2.3	4.4
3	4	1.5	2.7	7	3.9	1.8
4	2	1.9	1.1	3	2.6	1.1

Table 6.1. Sojourn time distribution parameters.

state j	Before refurbishment	After refurbishment
1	4	5
2	2	3
3	1	1

Table 6.2. Inspection interval lengths τ_j.

	Before refurbishment	After refurbishment
Inspection	3	3
PM	10	10
CM	100	100
Refurbishment	50	

Table 6.3. Costs per maintenance action.

Alternative 1: Refurbishment in 2008
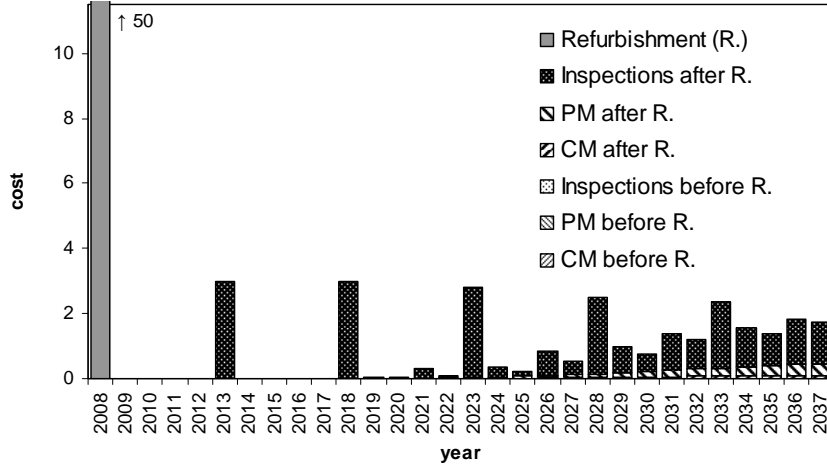
Alternative 2: PM in 2008 and refurbishment in 2012
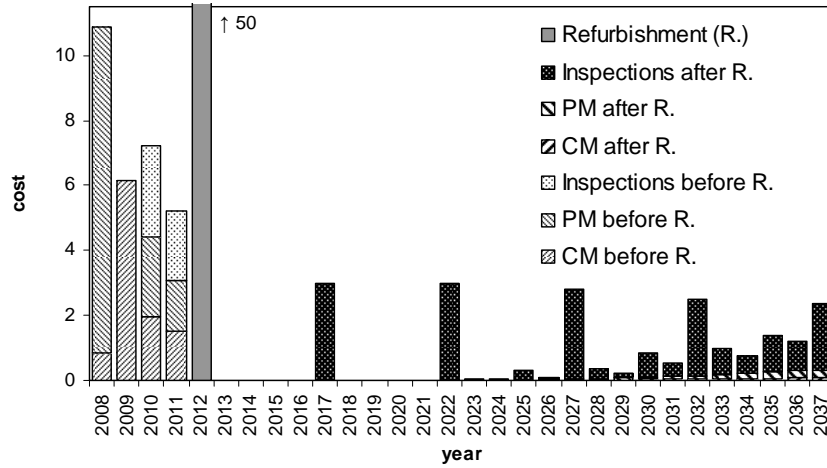
Figure 6.4. Various costs for the two alternatives.
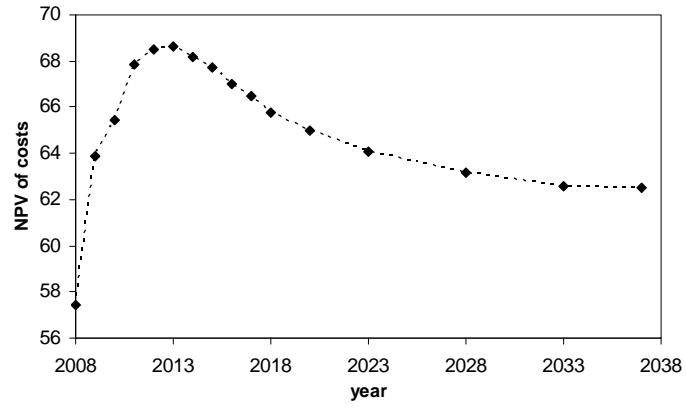
Figure 6.5. Optimal timing for refurbishment.

6.6.2 Inspection interval optimization

A typical example of an inspection interval optimization is presented in [36]. This example is very similar to the example in Paper 1. The purpose of the analysis was to find out whether Statnett (the Norwegian transmission system operator) is carrying out optimal maintenance on their circuit breakers. The analysis was restricted to a special sub-component, the driving mechanism. Statnett currently uses a strategy according to which they replace the driving mechanism when the condition is classified as state 3 (serious or major deterioration). The objective of the analysis was to find the optimal inspection interval lengths, τ_1 and τ_2, when the mechanism is classified in state 1 and state 2, respectively. The type of driving mechanism and the operating conditions were defined by means of the scheme shown in Figure 7.1 (see Section 7.1, page 51). Estimates for the sojourn time distributions, failure consequences, and costs for inspections, preventive maintenance and corrective maintenance are given in [36]. The maintenance model described in this thesis was used to calculate the net present value of the overall maintenance costs (costs for corrective maintenance, preventive maintenance and inspections) over a time horizon of 100 years.

A contour plot of the overall costs as a function of τ_1 and τ_2 is shown in Figure 6.6. The contour lines have some breaks, for example, at $\tau_1 = 48$, $\tau_1 = 60$ and $\tau_1 = 72$. This may be explained that the net present value is calculated year after year, that is, discounting is done in discrete time steps and not by using a continuous time measure. This means that cost drivers may occur in one year when the inspection interval is, say 59 months, but in the following year when the inspection interval is 60 months. This will result in different net present values. The minimal costs are obtained for $\tau_1 \approx 54$ months and $\tau_2 \approx 14$ months. The results confirmed that Statnett's present inspection strategy ($\tau_1 = 48$ months and $\tau_2 = 12$ months) is close to optimal. One could consider increasing the inspection interval length in state 1 slightly. However, the cost function is quite flat in the τ_1-direction, and the choice of $\tau_1 = 48$ months instead of, for example, $\tau_1 = 54$ months, would only result in minor savings.

6.7 Comparisons with other models

6.7.1 Markov processes based on state diagrams

Figure 6.7 shows an example of a state diagram that illustrates a simple maintenance strategy, where maintenance improves the condition by one state. Some researchers have suggested such state diagrams for analysing maintenance of electric power system components [4, 9, 20, 21, 44, 67–69]. For example, the model is quite popular in the IEEE Power Engineering Society. Deterioration and failure can be represented by the nodes S1-S3 and F in
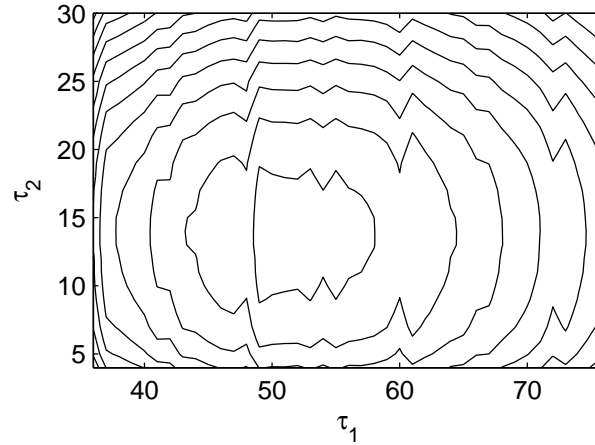
Figure 6.6. Inspection interval optimization for the circuit breaker driving mechanism. The figure shows a contour plot of the cost surface as a function of τ_1 and τ_2.

Figure 6.7. It has been suggested incorporating maintenance (including inspections) into the state diagram as shown by the additional nodes M1-M3 [9]. It is claimed that the model can be solved by standard Markov techniques [4].

During my PhD work, I have taken a closer look at this model; see Paper 3. The paper compares properties of this model with the situation in the real world. It is shown that discrepancies between the model and reality may occur when Markov processes are based on such state diagrams and when they are used for analysing maintenance strategies with non-periodic inspections. An example is presented that shows that this kind of model may yield erroneous results. In my opinion, the reason for this is that the model incorporates maintenance and inspections in a way that is not in accordance with reality; as discussed in more detail in Paper 3.

Figure 6.7. Example of a state diagram for maintenance modelling; adapted from [9].

6.7.2 Gamma process

A quite popular deterioration model that is used in many maintenance models is the gamma process (see [35] for an overview). Many different maintenance and inspection strategies have been modelled using the gamma process.

Some of the strategies are similar or equal to the strategy in this thesis (Section 6.3), for example, in situations where thresholds are introduced that define deterioration states and where maintenance d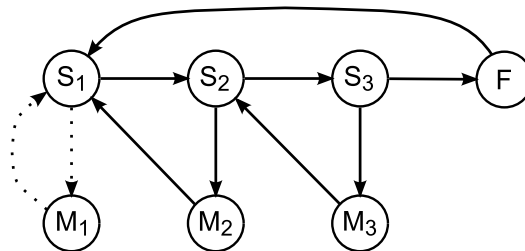ecisions depend on these states. Thus, results, which are presented in research papers using the gamma process for a maintenance and inspection strategy that is similar to the strategy analysed in this thesis, can be compared with results calculated by a maintenance model using a Markov process with discrete state space (state space model) as deterioration model. Both models should yield the same or at least similar results. Paper 4 compares a maintenance model based on a gamma process with a state space model that is similar to the model presented in this thesis. The two examples presented in this paper show that both models yield the same or similar results in maintenance optimization, but results such as costs may differ significantly, dependent on the model chosen. Paper 4 discusses advantages and disadvantages of the models and their possible field of application.

The work in Paper 4 is also a contribution to the verification and validation of the results in this thesis. Obtaining the same results with both models is a step towards verification and, furthermore, comparing the model results with the results of another model that is accepted and proven is a step towards validation.

6.8 Turbine model

While the main part of this thesis is about the generic maintenance and deterioration model (grey-box model) presented in Sections 6.1-6.6, the last part of the thesis (this section and Paper 7) is about a deterministic deterioration model (white-box model).

There is general agreement in the electricity industry that changed operating conditions that resulted from the the deregulation of the Nordic power market reduces the lifetime of the components (see Section 3.1), but there is little knowledge about the extent of the lifetime reduction. This means that we have a situation in the electricity industry where everybody complains about changed operating conditions and the consequences for equipment, but where nobody can quantify the extent of the problem. One or both of the following two questions should be answered when we want to specify to what extent these changed operating conditions are really a problem:

1. What is the lifetime reduction/increase due to changed operating conditions?

2. How much is reliability reduced/increased due to changed operating conditions?

Material fatigue is a typical failure mechanism in turbine runners in hydropower plants. Fatigue means failure due to repeated loading [70]. Fatigue starts with crack initiation, continues with crack propagation and terminates in the fracture of the component. However, a component failure can occur before a fracture occurs, for example, due to excessive deformation caused by large cracks.

In the case of fatigue cracks, an obvious and direct measure of deterioration is the crack size a. An idea at the beginning of this work was to use the generic model presented here with discrete deterioration states. In theory, we could define deterioration states by assigning each state a specific crack size (for example, state 1: 0.5 mm $\leq a <$ 1 mm, state 2: 1 mm $\leq a <$ 3 mm, and so on). However, it turned out that it is difficult to define states in practice. It was therefore decided to use the empirical Paris' law [42] as the basis for a deterioration model.

There are numerous factors that influence the formation and the growth of fatigue cracks. The most important factors are shown in Figure 6.8. Obviously, it is very difficult to cover all these factors in a model. The focus of this work was on operating conditions (denoted operational characteristics in Figure 6.8). The aim was to provide an answer to the first question raised above. Paper 7 presents an approach that may help answer this question. The factors influencing fatigue crack growth that are included in the approach are shown in Figure 6.9. Factors such as turbine design, runner geometry, rotational speed, and so on, are not explicitly modelled. However, they are covered indirectly by this approach, because the quantification of the stress cycles is based on measurements (dashed box and arrow in Figure 6.9). The influence of factors such as material properties may be analysed by the approach, but they were assumed to be constant in the application presented here.

From previous work, strain gauge measurements on a specific Francis turbine runner have been available. They have been analysed [71] by means of a procedure called rainflow cycle counting [70]. The analysis has yield the mean stress, and the amplitude and frequency of the stress cycles in the blade of a Francis turbine runner under different operating conditions. These results were used as input for a fatigue crack growth analysis in the approach presented in Paper 7.

The contribution of this work is the development of an approach where the measurements available are used for an analysis of the influence of the operating conditions on the turbine lifetime. Paper 7 describes how the operating condition may be specified by the number of start and stops per year and the magnitude of the load. The novelty of the approach is that it relates the results from the strain gauge measurements to the operating patterns. Then crack growth can be simulated for various operating conditions. The paper presents an example which shows that for the turbine runner analysed here,

an increase of the number of starts and stops may reduce the lifetime considerably, whereas an increase in overload operation may hardly change the turbine life. The results may support the plant operator in assessing the consequences of different operating conditions and establishing recommendations for plant operation.
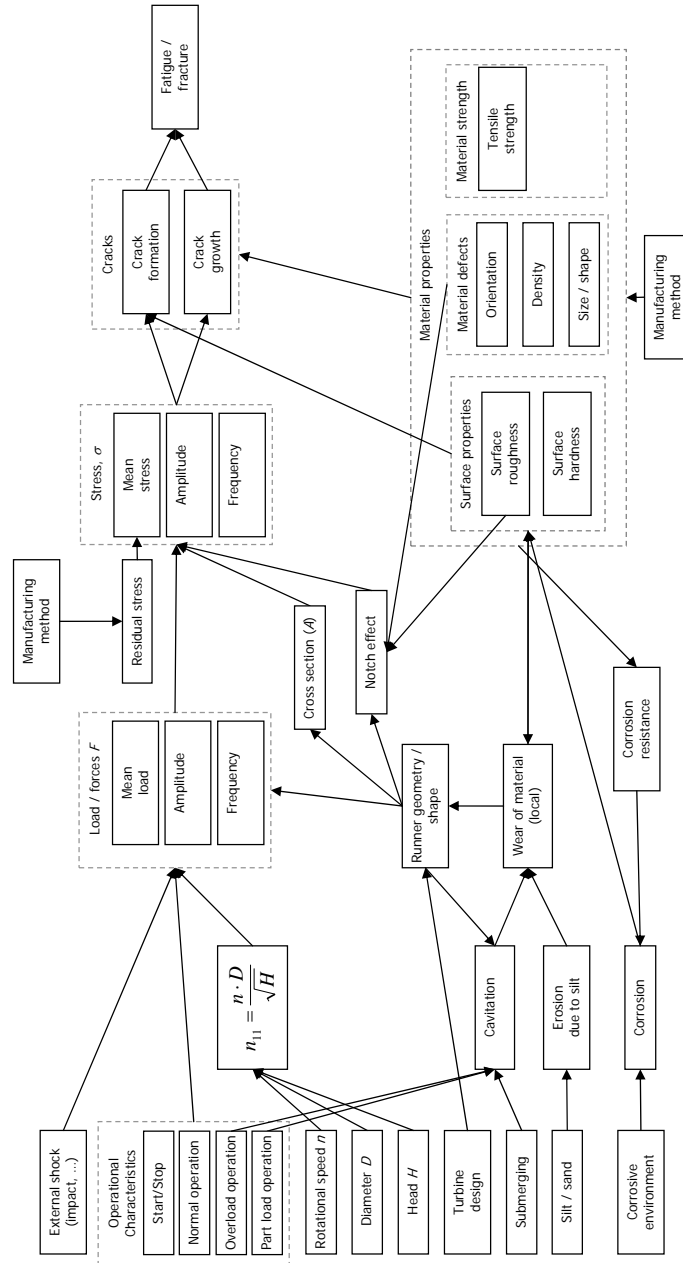
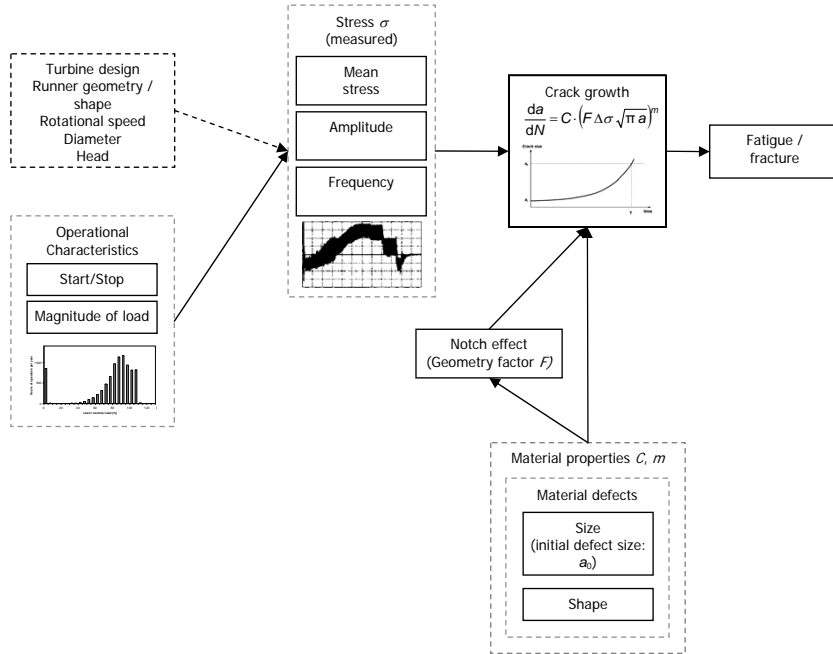**Figure 6.8.** Qualitative influence diagram showing causes of the failure mode 'fatigue of turbine runner'.

Figure 6.9. Causes covered by the approach presented in Paper 7.

7

Discussion

7.1 Independence between states

The model properties must be equal or similar to the properties of the real component, otherwise the model is not a valid representation of the reality. One model property is independence between deterioration states. This assumption is not always realistic. There are statistical tests to verify independence between states. However, it has already been pointed out by other researchers that testing of the hypothesis of independence is not practically feasible (see discussion in [19]).

Instead of simply assuming state independence, one may try to ensure independence as far as possible by providing a not too general definition of the component and its operating conditions. If the the component and its operating conditions are thoroughly defined, we may define a population that has states with approximately independent sojourn times. Obviously, and in contrast to this line of reasoning, the state sojourn times are not independent if the component definition is too general (e.g., all types of hydraulic turbines under unspecified operating conditions). Some turbines will then have a shorter lifetime and consistently shorter state sojourn times than others, dependent on factors, such as

- Type and exact design of the equipment
- Year of construction and manufacturer
- Material properties (type of material, strength etc.)
- Manufacturing method (cast, forged, welded, hardened etc.)
- Operating conditions (operating hours, number of start stops, machine load etc.)
- Ambient conditions (salt water or freshwater, silt and sand in the water etc.)

These factors may be modelled as 'covariates'. In the ideal case, a regression model may be established, which allows expressing the sojourn time distribution parameters as a function of the covariate(s) z_k, that is,

$$\alpha_i = \alpha_i(z_0, z_1, z_2, ...) \ \text{ and } \ \beta_i = \beta_i(z_0, z_1, z_2, ...) \tag{7.1}$$

This approach usually requires a large amount of data, which has not been available for components in hydropower plants yet.

Since there is too few data to establish a covariate model, other approaches are required to assess the most important factors that influence the state sojourn times. It is therefore suggested that the experts provide an thorough definition of the component and the operating conditions before expert judgement is carried out. A scheme was developed that supports the maintenance experts in defining the most important influencing factors (Figure 7.1).

The definition of the influencing factors can be regarded as an iterative process, as shown in Figure 7.2. The aim is to define the most important influencing factors such that it can be accepted that the sojourn times in the various states are approximately independent. If the expert feels that the component deteriorates rapidly in the state j if the component has deteriorated rapidly in the state $j-1$, or in other words, if the sojourn time in state j is assumed to be short (below average) if the sojourn time has been short (below average) in state $j-1$, this is a sign for a correlation between states. Then, the states are dependent. However, this dependence must have a reason. It can hardly be explained through pure intuition. Thus, we can reply "Why? What are the reasons for the dependence?" From an experienced engineer we will probably obtain reasons that are founded in one or several factors that have not been considered yet. By accepting and defining this (these) factor(s), the sojourn time in state j is assumed to be (approximately) independent from the sojourn time in state $j-1$.

The process described can be very challenging and it is obviously not possible to cover all influencing factors. Nevertheless, one may conduct judgements for relevant combinations of the most important influencing factors. which may be the better approach instead of ignoring the problem discussed completely. It is therefore suggested to carry out the steps illustrated in Figure 7.2 with reasonable care.

As an alternative to the described approach, models may be used that allow dependence between states. Adaptive approaches may be considered where the sojourn time in the following states may be adjusted to the knowledge gained during the early phase of deterioration. For example, one might consider a reduction of the mean time in states 2, 3 and 4, if the sojourn time in the first state was short. Such an approach has not been considered in this thesis, but is a potential topic for further research.

			Comment
1. Unit / component description			
Unit:	Turbine /Runner (Francis)		
Type:	High Head (300m) Medium runner (D=2500 mm)		Information about the unit that has influence on the deterioration process (type, design, operation, etc.)
2. Description of deterioration mechanism			
Deterioration mechanism:	Fatigue		
Reason:	Cavitation due to wrong shape of the blade combined with welding and or cast faults		
Damage development:	Crack growth		
3. Normal operating condition (design criteria)			
Name	Description		Depends on the operating pattern of all units in the power plant. No turbine is exactly equal, but most Francis turbines should avoid 45 – 55 % due to turbulence and pulsation.
Part load (30-80 %)	40 % of time		*Operating pattern varies much from machine to machine, i.e. it is difficult to define a 'nomal' operating condition*
Full load (80-100 %)	60 % of time		
Variation in head	6 %		

Figure 7.1. Screenshot of the scheme for define factors that influence the sojourn times.
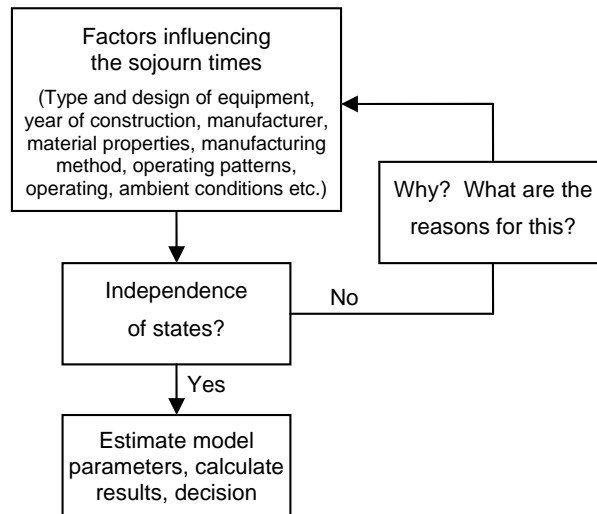

Figure 7.2. The process of defining factors that influence the sojourn times.

7.2 Approximating the gamma distribution

The approximation of a general probability distribution by a Markov process leads to the field of phase-type distributions [72]; see also Paper 6. The phase type distribution is a natural extension of the exponential distribution [73]. The time to reach an absorbing state in a finite-state Markov process has a phase-type distribution. The Erlang distribution is a special case of a phase-type distribution with a structure where all states are arranged sequentially. By allowing the Markov process to have a more general structure, the phase-type distribution becomes a very flexible distribution class. The disadvantage is that the number of parameters increases with the number of states.

The phase-type distribution may be fitted to a probability distribution or data. There are numerical tools [74–78] that can be used for fitting a phase-type distribution to a parametric probability distribution or a dataset. However, these tools require decisions about the Markov process structure, the number of states and other parameters that are required for starting the fitting algorithm. The results obtained must be judged, because the fitting tools use numerical optimization approaches, which can get stuck at a local maximum or at a saddle point [79]. Thus, a good outcome for the fitting is uncertain. This does not provide a good foundation for a practical application. Thus, simple and straightforward approaches, as suggested and discussed in Papers 2 and 6, seem more suitable for the intended purpose.

A reason for choosing a chain-formed Markov process structure is that the suggested model solution requires this structure (see Appendix A). Furthermore, we can give the virtual states an unspecified, but obvious, meaning. This is usually not possible with a general phase-type distribution [74]. The virtual state $i+1$ represents a stage of increased deterioration compared with virtual state i. During our work with maintenance engineers, we found that the condition of the component may sometimes be judged, for example, as 'good state 2', or 'close to state 3, but still state 2'. The appropriate deterioration state may be defined as the first virtual state (state 2,1 using the notation in Figure 6.2) in the former case, whereas the deterioration state may be defined as the last virtual state (state $2,I_2$) in the latter case. Thus, the additional information provided by maintenance engineers can be used when the Markov process is restricted to a simple, chain-formed structure.

7.3 Model solution

It is well-known that non-periodic inspections generally result in a policy with lower costs [80]. Jia and Christer [81] present a maintenance optimization model for a periodic inspection strategy, where the first inspection interval is allowed to be different from all subsequent inspection intervals. An example is presented where the savings of this strategy compared with a pure non-periodic strategy is 11.3 %.

As already discussed in Section 3.2.3, the general policy in the Norwegian electricity industry is reducing the inspection interval length as the deterioration state increases. Section B.2 presents an example where the influence of the lengths of the inspection intervals in the first two states, τ_1 and τ_2, on the total costs $C(\tau_1, \tau_2)$ is analysed. Figure 7.3 shows a contour plot of $C(\tau_1, \tau_2)$. The optimal solution is given by $\tau_1 \approx 44$ and $\tau_2 \approx 12$. This strategy yields annual long-term costs of $C_{\min} \approx 4.21$. In Figure 7.3, the location of the cost function for the case when $\tau_1 = \tau_2 = \tau$ is indicated by the bold, straight line in the upper left part of the contour plot. Figure 7.4 shows the cost function $C(\tau)$. The optimal inspection strategy is $\tau \approx 18$ and the total costs are $C_{\min} \approx 6.15$ in that case. The savings in applying the non-periodic inspection strategy compared to a periodic inspection strategy are 31 %.
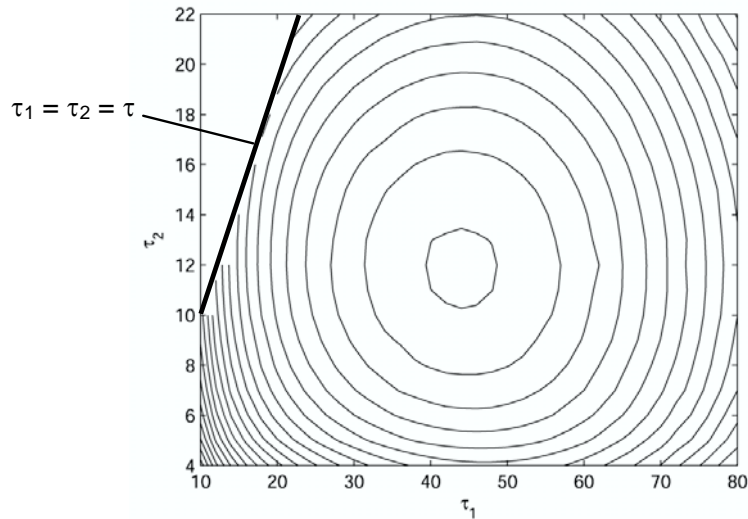
Figure 7.3. Optimization of total costs, non-periodic inspection strategy: Contour plot of the cost surface. $C_{\min}(\tau_1, \tau_2) \approx 4.21$ at $\tau_1 \approx 44$, $\tau_2 \approx 12$.

The maintenance model presented in this thesis is solved by a numerical procedure. It is possible, for example, to calculate the total costs for a given time interval and a given inspection and maintenance strategy. It is, however, not possible to find the lowest costs as a function of decision variables such as inspection intervals or the maintenance threshold by analytical methods. An approximate optimal solution may be determined by creating plots of the cost surface (as presented above, in Paper 1 or in Section 6.6), but the computation of this solution requires some time (minutes or hours). Numerical optimization methods might be considered. Since the objective function (total costs) is also calculated numerically, an optimization run might be time-consuming as well. Nevertheless, the combination of the numerical procedure proposed in this
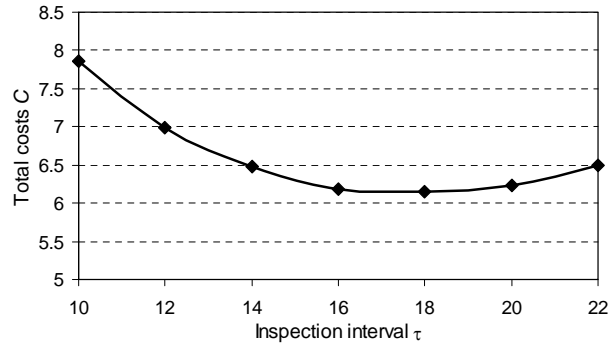
Figure 7.4. Optimization of total costs, constant inspection interval: Plot of the cost function. $C_{\min}(\tau) \approx 6.15$ at $\tau \approx 18$.

thesis with numerical optimization methods could be useful, even though it was not considered in this thesis.

Analytical solutions are available for periodic inspection strategies. Kallen [19], for example, has recently presented a model for the optimization of bridge maintenance. Similar to the model presented in this thesis, the model is based on discrete deterioration states. A analytical solution for the long-term average maintenance costs as a function of the inspection interval is provided. The solution may be used for the optimization of the inspection interval length. This model might be applied in the Norwegian hydropower industry as well. However, it only treats the periodic inspection strategy. As shown by the example above, this strategy can be far from optimal. Nevertheless, such a solution may be useful in combination with numerical models. It is clear that τ_1 is somewhat longer than the optimal solution for the periodic strategy τ, and that τ_2 is shorter than τ. This may help to identify the order of magnitude of the optimal non-periodic inspection strategy. Afterwards, the numerical procedure suggested in this thesis may be used to search for a non-periodic strategy that furthermore reduces the costs.

The great potential in cost savings may provide enough motivation for determining an optimal (or close to optimal) non-periodic inspection strategy, even though it must be obtained by computational effort. A computation time of several minutes or hours will not constitute a substantial problem since maintenance optimization requires no immediate solution. Furthermore, these computations can easily be carried out on a standard PC and require no special calculating capacity. Nevertheless, this solution is not completely satisfying and thus there is a need for further work. It is always useful to establish analytical solutions and to reduce computing time so that the results can easily be implemented in other models and analyses.

Note that some interesting solutions for non-periodic inspection strategies exist when deterioration can be described by a continuous variable (e.g. by a gamma process); see references [80, 82–85]. However, inspection results ex-

pressed by discrete deterioration states cannot be transformed to a continuous scale; see also discussion in Paper 4. Thus, these models cannot be applied as general deterioration and maintenance models for components in hydropower plants.

7.4 Expert judgement

The use of expert judgement in reliability analyses is not new. Much research on this topic has been carried out in past decades, and many techniques and methods were developed to provide the required input for reliability models; see [53–55, 86] for a survey. However, the use of expert judgement is not without controversy. Scarf [31] raised the criticism that the validity of such subjective data is sometimes suspect, particularly in maintenance applications where the interviewed experts are those responsible for the current maintenance strategy. He concluded therefore that their judgement must surely reflect current practices rather than the true underlying engineering phenomena. A second point that is criticized is that one might state cynically that data collected subjectively is mainly useful for fitting the complex models proposed, that is, the subjective data may be used for the benefit of the modeller rather than for the benefit of the decision maker.

One can oppose [87] that those types of analyses are undertaken because a decision must be made. If the analysis cannot be based on field data then the knowledge has to be found elsewhere, and the engineers with experience in the relevant field are the most obvious source [87]. Thus, quantification can normally not be carried out without employing expert judgement [88]. The experience with expert judgement during this research is that the experts does not understand the impact of their assessments on the modelling results and the decisions. Thus, the first argument raised against expert judgement is not valid for the applications discussed in this thesis.

The lack of suitable inspection data for parameter estimation is also a problem in the Norwegian electricity industry. In recent years, however, a classification system with five deterioration states has been employed by the Norwegian electricity companies. Several companies started to enter inspection results in their computerized maintenance management system. However, currently there are too few observations available for statistical analyses. A further problem is that the electricity industry is split into many independent companies. There is neither an authority nor an industry institution (as in the Norwegian oil industry [89]) that manages the collection of inspection and failure data. Thus, a national database should be established for effective data collection. Until a larger database is established, we must rely on expert judgement. As soon as data is available, the Bayesian framework presented in Paper 5 may be used to update expert judgements.

A general problem is that there is limited information about the sojourn time in state 4. The plant operator usually does not wait with maintenance

actions until a critical deterioration state is reached. If data collection is intensified, there will be many records of states 1-3 in the maintenance management system, but there will be few records of the last deterioration state. Expert judgement may therefore always remain an important contribution to the assessment of the last sojourn time. With regard to this fact, it might be advantageous to request expert judgements of the sojourn time in the last state as an assessment relative to the previous sojourn time(s). Then the assessment of the last sojourn time can still be used when the expert judgements of the sojourn times in the previous states are replaced by estimates based on data.

7.5 Modelling frameworks

A framework as described in [86] is used for modelling of the sojourn time distribution. One of the components in this framework is a mathematical model with one or several parameters. Both deterministic and stochastic models can be used in this framework. In this thesis, the model is of the stochastic type because it is given by the sojourn time distribution, which is assumed to have a gamma distribution. Note that this framework is used to estimate a proper distribution for the sojourn time T only, not for the whole maintenance model. The model parameters are the distribution parameters, denoted $\boldsymbol{\theta} = (\alpha, \beta)$. The parameters are represented by the so called state-of-knowledge distribution $\pi(\boldsymbol{\theta})$. The sojourn time distribution is denoted $f(t)$. Thus, the distribution $f(t)$ represents aleatory uncertainty (also called randomness, variability, or objective probability), whereas the distribution $\pi(\boldsymbol{\theta})$ represents epistemic uncertainty (subjective probability). The framework described here is also known as the probability of frequency framework [90, 91], where the word 'probability' is used for the 'subjective probability' and the word 'frequency' for the 'objective probability'.

The separation of uncertainties in 'aleatory' and 'epistemic' was (and is still) much discussed. The interested reader is referred to [90, 92–94]. A general consensus exists that basically all uncertainties are epistemic [95–97]. Nevertheless, distinctions are often made between aleatory and epistemic uncertainties. Mosleh [95] gives the following reason for this: "Randomness is nothing but a reflection of the level at which we choose to model the phenomenon of interest. It is a characterization we use for system behavior when we are unable or unwilling to model the system or phenomenon in enough detail such that for a given scope and set of boundary conditions the behavior can be predicted (deterministically)." Thus, the distinctions should be thought of in terms of a separation that make it easier to deal with the uncertainties effectively [96] and it is merely for our convenience in investigating complex phenomena [97].

The classical framework described has been criticized by others. Aven [91] argued that much of the existing classical thinking puts probability first, which

would give the wrong focus. Attention should be placed on observable quantities and the use of probability as a subjective measure of uncertainty. First comes the world, the reality (observable quantities), then uncertainties and finally probability. Thus, no distribution classes with unknown parameters should be introduced when not required because there is no point in in talking about uncertainties in parameters unless they are observable, i.e. not fictional [91]. This particularly applies to types of applications where the component is rather unique in the sense that we cannot find reasonably similar components without doing thought experiments. However, in situations where a real population can be defined (e.g. in experiments), the relative frequency probability, that is, the portion of failed units, is an observable quantity [98].

Despite the critique raised in the previous paragraph, a classical modelling framework was used in this thesis. In my opinion, the classical framework, which allows for stochastic models (aleatory uncertainty), is an approach that is well suited to the solution of the problem at hand. In the case of components in a hydropower plant, a reasonable population can be defined. A reasonable population might be, for example, all components of this type operated under comparable conditions in an electricity company or in Norway. For the specified component, the deterioration process cannot be predicted deterministically. Some components will deteriorate faster, and some others slower. Thus, we may use a probability distribution to model the variability (aleatory uncertainty) of the sojourn time in the deterioration states.

7.6 Approach to uncertainty

In the previous section, the state-of-knowledge distribution $\pi(\boldsymbol{\theta})$ was introduced. Consequently, the probability distribution describing the sojourn time T must be calculated by applying the following equation [86]:

$$\overline{f}(t) = \int_{\boldsymbol{\theta}} f(t; \boldsymbol{\theta}) \cdot \pi(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{7.2}$$

Note the special notation in this equation. $f(t; \boldsymbol{\theta})$ is the model for the sojourn time, which is a gamma distribution in this thesis, and which is conditioned on $\boldsymbol{\theta}$, whereas $\overline{f}(t)$ is the unconditional model solution, which is not a gamma distribution. This solution is denoted a 'probabilistic solution' [99].

In Section 6.2, it was assumed that the sojourn time is gamma distributed, under the assumption that the sojourn time distribution parameters are known. Estimates for the parameters may be obtained by the approach presented in Paper 5. Then, the following relation is used:

$$f(t) = f(t; \hat{\boldsymbol{\theta}}) \tag{7.3}$$

where $\hat{\boldsymbol{\theta}}$ is the best estimate for the model parameters. Consequently, the resulting distribution is still a gamma distribution. However, the approach is

not properly finalized in this case, since the state-of-knowledge distribution is not propagated through equation 7.2. This approach is denoted a 'deterministic solution' [99], or a 'best estimate approach' [91], since a deterministic, best estimate is used for the model parameters.

In Appendix B, it is analysed to what extent the use of a probabilistic solution and a deterministic solution, respectively, influences the model results.

7.7 Turbine model

This model assumes that cracking always starts at a material defect of a given shape and size, and at a given location. The three factors of size, shape and location of defect, will greatly influence the life of a turbine runner. In practice, however, these factors are often unknown because there is only limited knowledge about the size, shape and location of defects.

Some fairly realistic values, representing a kind of worst case (a large defect of a given size and shape at the location with the highest stress), have been assessed for the crack growth simulations in Paper 7. The resulting lifetime should not be considered a prediction for the real lifetime of a runner, since the real size, shape and location of the defect is usually unknown. However, the lifetime can be used as reference value. We can compare this value with the results when we repeat crack growth simulations with the same starting conditions but with changed operating patterns. By applying this procedure, is possible to judge the influence of the changed operating patterns on the lifetime of the equipment. It is recommended that the results be normalized as shown in Figure 7.5. 'A' denotes the reference alternative for which the normalized lifetime is one. If we change the operating patterns from 'A' to 'B' (see Paper 7 for a definition of operating pattern A, B and C), we expect that this will hardly influence the life of the turbine runner. However, if we change from 'A' to 'C', this will likely reduce the lifetime considerably.

In order to provide a better solution for the problem and to remove some of the limitations discussed, we need more sophisticated approaches. This requires an advanced treatment of the problem by fracture mechanics. This is far beyond the scope of this thesis. Thus, the approach was not further developed in this work. However, the co-authors of Paper 7 have worked intensively with a probabilistic fatigue assessment tool that post-processes results from finite element stress analyses. The PhD thesis by Wormsen [100] provides an extensive description of the latest activities and results.

Regardless of the limitations of the turbine model, the model is a straightforward and simple approach to assess the influence of different operating conditions. Several strain gauge measurements are available for other turbines. They may provide the basis for similar analyses of other turbines. A useful method is also the definition of the operation conditions by the number of start stops and the operating profile (see Paper 7). This method may be used in other applications as well.
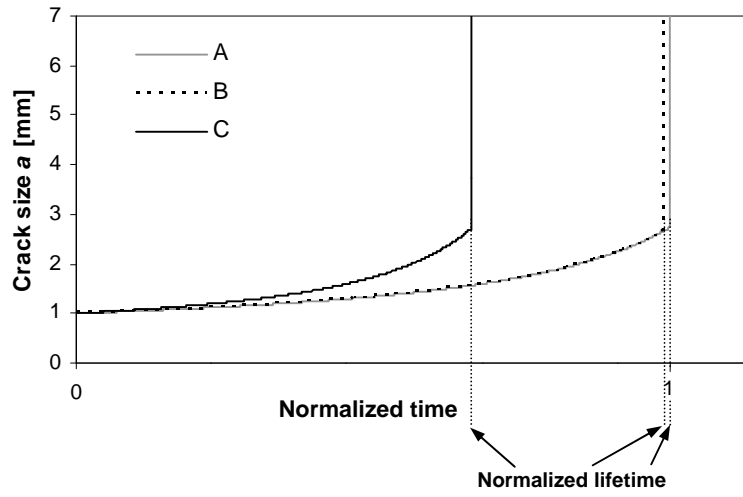
Figure 7.5. Crack propagation and normalized lifetime for three different operating conditions.

8

Conclusions and further work

8.1 Conclusions

The results presented in this thesis may mainly be used by maintenance engineers that are responsible for planning and scheduling of maintenance and renewal. The general deterioration and maintenance model may be applied to:

- Assess the condition development of a component.
- Calculate the failure probability of a component.
- Calculate the expect number of inspections, preventive maintenance, and failures/corrective maintenance in a given time interval and for a given maintenance strategy.
- Calculate the overall maintenance costs and the present value of these costs.

The intended field of application is hydropower plant components. At the moment, the Norwegian electricity industry is also endeavouring to introduce the EBL classification system (Section 3.2.2) for installations other than hydropower plants. Working groups have been launched [101–103] to establish specific descriptions of the deterioration states for components in wind turbines and in the power grid, as has been done for components in hydropower plants. Hydropower is the backbone of the Norwegian electricity production and most of the maintenance engineers working in the electricity industry know the perspectives and practices that are applied in the maintenance of hydropower plants. It therefore clearly makes sense to extend practices that have been successfully used for hydropower plants to other types of installation as well. This will establish a common foundation for the electricity industry.

The turbine model may be used to

- Simulate fatigue crack growth in a Francis turbine runner under various loading.

- Analyse the influence of different operating conditions on the life of Francis turbine runners.

It has been shown that the general maintenance and deterioration model presented in this thesis provides a suitable analysis tool for the Norwegian electricity industry. The model is general in such a way that it can be applied to different types of equipment, deterioration mechanisms and failure modes. This thesis discusses all relevant modelling steps: Description of existing concepts and perspectives, the model itself, the parameter estimation, a numerical solution procedure and relevant examples of use. Thus, this thesis provides an operational model. An advantage of the model is that it is built on an already existing state definition. The maintenance engineers are familiar with the systematics and no new and unknown concept had to be introduced.

The estimation of model parameters is always a great challenge. On the one hand, the modelling of the sojourn time in a (physical) state using a probability distribution is a very intuitive approach. On the other hand, the estimation of sojourn time distribution parameters is a demanding task because the parameters are abstract quantities for non-statisticians since they are not observable. The suggested parameter estimation process, however, is based on potentially observable quantities, which are the sojourn times. It is clear that the sojourn times are only observable when the deterioration is continuously monitored. This is seldom possible in practice. However, the deterioration states are observable by inspections and seen over a longer time period, the sojourn time in a state is an interval censored observation that is recorded in terms of condition monitoring data in the companies' maintenance management systems.

Condition monitoring data is still rare, and until now, no maintenance models existed in the Norwegian electricity industry that provided an incentive for data collection. However, missing data is a reason why no maintenance models were developed, because it was argued that the development of models is potentially useless without data for parameter estimation. This attitude creates a vicious circle where missing data cause a lack of models, and missing models cause a lack of data (Figure 8.1). This thesis is an attempt to break this vicious circle. A maintenance model has been developed. This will hopefully provide motivation for data collection.

Expert judgement provides an alternative starting point until a sufficient amount of condition monitoring data is available. The discussions and interviews with maintenance engineers and experienced plant personnel have shown that experts are able to judge the length of a sojourn time. They can provide useful information about the sojourn time in terms of percentiles and a best estimate. The model can immediately be applied even though data are scarce. It is important to note that the possibility of using expert judgement must not stall attempts to collect condition monitoring data. It is important to intensify the collection of condition monitoring data in the future; and it must be avoided that the possibility for using expert judgement does not cre-
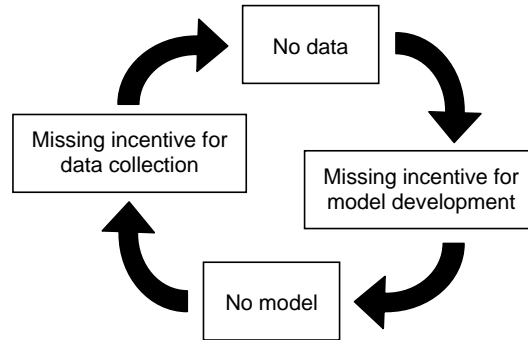
Figure 8.1. Vicious circle of data collection and model development.

ate a new vicious circle, where expert judgement causes a missing incentive for data collection, and the lack of data is an 'excuse'/reason for using expert judgement.

The comparison with other maintenance models has shown that it is important to check that the model is a realistic representation of the real maintenance situation. The model yields proper results only if the real maintenance situation is realized in the model. Provided that the same maintenance situation is analysed, and provided that this situation is also realized in the model, different models should yield similar results. The comparison of the model in this thesis with a model applying a gamma process has shown that the results of these two models are similar. The reason for this is that both models realize the real maintenance situation in the same (and correct) way.

The deterioration model presented in this thesis is still very rough. Only transitions from one state to the next state are modelled. The deterioration inside the states is not explicitly modelled. A more refined model could be developed by increasing the number of (physical) states. However, this would require a more detailed state definition, which is probably not possible in many cases. Alternatively, a continuous deterioration measure could be introduced. This allows the use of other solutions for modelling deterioration (e.g. the gamma process). In many cases, however, it would be difficult to define such a measure. Regardless of the roughness of the model used here, the examples presented in this thesis show that the model serves its intended purpose and that the main objective of the thesis, to "develop a deterioration and maintenance model for components in hydropower plants", is fulfilled.

The turbine model provides useful insight into the relationship between operating conditions, crack growth and residual lifetime. The disadvantage of the model is that it requires input that is based on stress measurements. Some of these measurements have already been carried out in the past by Norwegian companies and they may be utilized. It may be difficult and expensive to provide additional measurements. Nevertheless, the high costs that are related

to fatigue failures of turbine runners should justify a somewhat more expensive approach.

8.2 Further work

This thesis may be the basis for further research in several areas.

It was discussed earlier that the component and the operating conditions must be thoroughly defined to ensure the independence between the sojourn times in each state. The component and its operating conditions could be mathematically represented by covariates. A topic for further research could be to incorporate covariates into the model. Instead of carrying out expert judgement or data analysis for each specific component under given operating conditions, one could evaluate the covariates. In the longer term and when data collection is intensified, enough data might be available so that the sojourn time distribution parameters could be modelled as a function of the most important covariates. Once a covariate relation is established, it would simplify the estimation of the sojourn time distributions considerably, because it is easier for a user of the model to collect information about the covariates than information about the sojourn time. Alternatively, an adaptive approach could be considered where the sojourn time distribution parameters of subsequent states are updated based on the knowledge of the previous states.

The approach presented here uses the gamma distribution as a model for the sojourn time in a deterioration state. Further research could focus on methods that allow the use of other distribution classes as well. If the numerical model solution presented is applied, it will require methods to transfer the distributions into a sequence of exponentially distributed virtual states. The further application of the theory of phase-type distributions might be useful for this purpose. Another approach might be to replace the chain-formed Markov process by a more general type of phase-type distribution. The numerical solution procedure might be extended such that general phase-type distributions can be applied.

Further work should also focus on the handling of uncertainty. It would be desirable to propagate all uncertainties through the model such that the parameter uncertainty represented by the state-of-knowledge distributions is reflected in an uncertainty measure related to consequences, costs and optimal results.

It was discussed elsewhere in this thesis that virtually all systems consist of several components that depend on each other. Interactions offer the opportunity to pool maintenance activities, which may save costs. Thus, optimization of the maintenance of multi-component systems has great potential for costs savings and is an interesting future research topic.

Another topic for further work is the modelling of infant mortality. In a first step, one should assess to what extent infant mortality is of importance in hydropower plants. If this phenomenon is relevant, a second step is to

develop an approach to include infant mortality into the deterioration and maintenance model.

9

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A

Numerical model solution

This appendix describes a numerical procedure, which can be used to solve the general maintenance model introduced in Chapter 6. The challenge is the incorporation of a non-periodic inspection strategy where the next inspection interval depends on the condition (deterioration state) revealed by the previous inspection. Thus, the inspection interval is a decision made after an inspection or a maintenance action; see discussion in Paper 3.

The following nomenclature is used in this appendix (see also Figures A.1 and A.2):

i	Virtual state (sub-state); exponentially distributed, $i = 1, ..., r$
j	Physical state (main state); gamma distributed, $j = 1, ..., 5$
i_j^1	First virtual state in physical state j
m	Maintenance limit (preventive maintenance if $i \geq m$)
$P_{z,i}(t)$	Probability that the system is in virtual state i at time t, when the next inspection is scheduled after z time units
t	Time variable
$\widetilde{t}$	Time of the previous inspection
Δt	Small time interval
r	Fault state $(r = i_5^1)$
z	Remaining time until next inspection (discrete variable)
Δz	Step length of z
λ_i	Transition rate from virtual state i to virtual state $i + 1$
τ_j	Inspection interval length

The problem to be solved is 'multidimensional'. One 'dimension' is the condition of the system, which is described by discrete deterioration states. A second 'dimension' is the time, since the condition of the component changes, various decisions are made and different maintenance actions are performed over time. Another 'dimension' is the time until the next inspection. These

'dimensions' are represented by a time-dependent array in the numerical procedure suggested.

The probability that the system is in virtual state i at time t, when the next inspection is scheduled after z time units, is denoted $P_{z,i}(t)$. The remaining time until the next inspection is approximated by a discrete quantity z with step length Δz. In the applications presented in this thesis, an adequate value for Δz is usually 1 month. Thus, the time to the next inspection is covered by the 'z-dimension' and the condition by the 'i-dimension'.

The elements of $P_{z,i}(t)$ are arranged in an array where the elements change over time, as illustrated in Figure A.2. The dimension of the array may be chosen ($z_{\max} \cdot i_{\max}$), where $z_{\max} = \tau_1$ (τ_1 given in months) and $i_{\max} = r$. Deterioration, inspection and maintenance can be simulated by simple numerical algorithms that change the array elements according to some predefined rules.

Consider a simple example where the sojourn time in each of the four physical states is exponentially distributed. Thus, the number of physical states is equal the number of virtual states. Preventive maintenance (PM) is carried out when the system is classified in state 4. Corrective maintenance (CM) is performed after failure. PM and CM restore the system to a state of 'as good as new', that is, $i = 1$. It follows that $i_1^1 = 1$, $i_2^1 = 2$, $i_3^1 = 3$, $m = i_4^1 = 4$ and $r = i_5^1 = 5$. Assume that the inspection interval lengths are $\tau_1 = 6$ months, $\tau_2 = 3$ months and $\tau_3 = 1$ month. Δz may be chosen 1 month in this case.
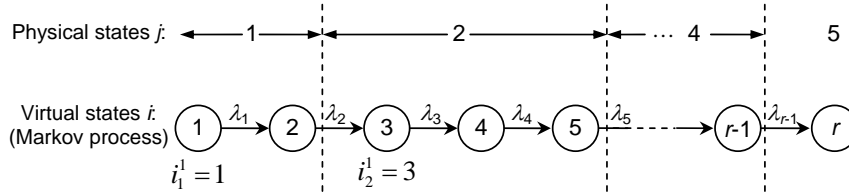

Figure A.1. Physical states and virtual states.

A.1 Deterioration

Deterioration within a small time interval Δt can be modelled by applying the approximations

$$P_{z,r}(t + \Delta t) \approx P_{z,r}(t) + P_{z,r-1}(t) \cdot \lambda_{r-1} \cdot \Delta t \qquad (A.1)$$

for all $P_{z,i}$ where $i = r$,

$$P_{z,i}(t + \Delta t) \approx P_{z,i}(t) \cdot (1 - \lambda_i \cdot \Delta t) + P_{z,i-1}(t) \cdot \lambda_{i-1} \cdot \Delta t \qquad (A.2)$$
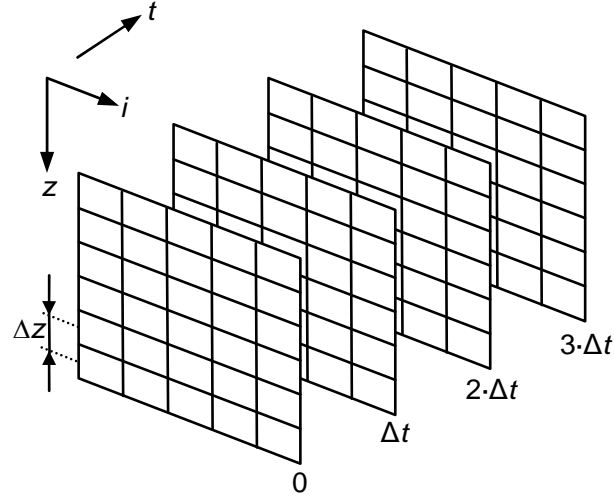
for all $P_{z,i}$ where $1 < i < r$, and

Figure A.2. Illustration of the basic principle of the framework for the numerical model solution.

$$P_{z,1}(t + \Delta t) \approx P_{z,1}(t) \cdot (1 - \lambda_1 \cdot \Delta t) \tag{A.3}$$

for all $P_{z,i}$ where $i = 1$.

If we have a new system that is set into operation at $t = 0$ and that is inspected the first time after 6 months, we can denote this $P_{6,1}(0) = 1$. This situation is shown in the upper left part of Figure A.3 where the 'probability mass' of $P_{6,1}(0) = 1$ is represented by a grey bar.

When time goes, the system will deteriorate, which means that the probability mass will spread 'rightwards' in the array. This can numerically be treated by equations (A.1)-(A.3). If no maintenance is carried out, the system will definitely fail sooner or later. If we wait long enough, say $n \cdot dt$ time intervals (n is large), the probability that the system is in the last state, which is the fault state, approaches 1. Then, the probability mass is concentrated in the last state, and $P_{6,5}(n \cdot dt) \rightarrow 1$. This is represented in the lower right part of Figure A.3. Note that simulating deterioration increases t by Δt. It does not move the probability in z-direction, but in i-direction.

A source code (here: Visual Basic) for simulating deterioration is presented in the following example:

```
For z = 1 To z_max
   P(z, r) = P(z, r) + P(z, r - 1) * lambda(r - 1) * dt
   For i = (r - 1) To 2 Step - 1
      P(z, i) = P(z, i) * (1 - lambda(i) * dt) + _
                P(z, i - 1) * lambda(i - 1) * dt
   Next i
   P(z, 1) = P(z, 1) * (1 - lambda(1) * dt)
```

```
    Next z
    t = t + dt
```

A.2 Corrective and preventive maintenance

It has been shown that deterioration spreads the probability mass in the z-i-array in i-direction. Let us consider a situation as illustrated in the upper left part of Figure A.4 (references to the appropriate equations are shown in the figure). For the given example where $\Delta z = 1$ month, inspections and PM are simulated each time when the numerical procedure has run through 1 month simulated time. Inspections and PM may be numerically realized by applying the following procedures; equations (A.4)-(A.10):

$$\widetilde{t} = t \tag{A.4}$$

$$P_{z-1,i}(t) = P_{z,i}(t) \quad \text{for all } z,i \tag{A.5}$$

$$P_{\tau_1,i}(t) = P_{\tau_1,i}(t) + P_{0,i}(t) \quad \text{for all } i < i_2^1 \tag{A.6}$$

$$P_{\tau_2,i}(t) = P_{\tau_2,i}(t) + P_{0,i}(t) \quad \text{for all } i_2^1 \le i < i_3^1 \tag{A.7}$$

$$P_{\tau_3,i}(t) = P_{\tau_3,i}(t) + P_{0,i}(t) \quad \text{for all } i_3^1 \le i < m \tag{A.8}$$

$$P_{\tau_1,1}(t) = P_{\tau_1,1}(t) + P_{0,i}(t) \quad \text{for all } m \le i < r \tag{A.9}$$

$$P_{0,i}(t) = 0 \quad \text{for all } i < r \tag{A.10}$$

Equation (A.5) reduces the remaining time until inspection by one month. As an alternative, one may use a circular list as illustrated in Figure A.5 and as realized by the following source code example:

```
    'Check if inspections must be simulated:
    If t >= (NextInspectionNo * (1 / 12)) Then
       'Find actual index in the circular list:
       inspIndx = ((NextInspectionNo - 1) Mod tau(1)) + 1

       'Inspection and re-scheduling of next inspection
       'PM when state 4 is detected
       For j = 1 To 3
          'Find new index in the circular list:
          newIndx = ((inspIndx + tau(j) - 1) Mod tau(1)) + 1

          For i = i_1_j(j) To (i_1_j(j+1) - 1)
             P(newIndx, i) = P(newIndx, i) + P(inspIndx, i)
             P(inspIndx, i) = 0
          Next i
       Next k

       'PM resulting in state 'as good as new'
       'Find new index in the circular list:
       newIndx = ((inspIndx + tau(1) - 1) Mod tau(1)) + 1
```

```
    For i = i_1_j(4) To (r - 1)
        P(newIndx, 1) = P(newIndx, 1) + P(inspIndx, i)
        P(inspIndx, i) = 0
    Next i

    NextInspectionNo = NextInspectionNo + 1
  End If
```

If a component failure is not self-announcing, that is, if a component failure is only detected by inspections, failures and CM may be treated by the numerical procedure each time when inspections are simulated by (see also Figure A.4)

$$P_{\tau_1,1}(t) = P_{\tau_1,1}(t) + P_{0,r}(t) \tag{A.11}$$

and

$$P_{0,r}(t) = 0 \tag{A.12}$$

If a component failure is self-announcing, the procedure must be modified; see Figure A.6. The array must be extended in z-direction by one row, that is, $z_{\max} = \tau_1 + 1$. The following procedure may then be used at each time step Δt:

$$P_{\tau_1,1}(t) = P_{\tau_1,1}(t) + P_{z,r}(t) \quad \text{for all } z \tag{A.13}$$

if $(t - \widetilde{t}) < \Delta z/2$, or

$$P_{\tau_1+1,1}(t) = P_{\tau_1+1,1}(t) + P_{z,r}(t) \quad \text{for all } z \tag{A.14}$$

if $(t - \widetilde{t}) \geq \Delta z/2$. Set afterwards

$$P_{z,r}(t) = 0 \quad \text{for all } z \tag{A.15}$$

The additional row becomes the new 6. row in the 2. step in Figure A.4 when PM is simulated.

The expected number of inspections, PM and CM in a given time interval can be 'calculated' by summing up $P_{0,i}(t)$, $P_{0,r}(t)$ or $P_{z,r}(t)$ in equations (A.6)-(A.9), (A.11) and (A.13)/(A.14), respectively, in counter variables (not shown in the script code).

A.3 Possible model extensions

The procedure can be extended to imperfect inspections as proposed in Paper 1. It is also possible to cover imperfect maintenance when the probability mass $P_{0,i}(t)$ in equation (A.9) is not solely added to $P_{\tau_1,1}(t)$ but to $P_{z,i}(t)$ where $z \neq \tau_1$ and $i \neq 1$. Furthermore, it is possible to treat situations where the properties of the component and the inspection strategy changes after a maintenance action, for example after a replacement with a new component that has other properties than the old one. Assume that this replacement is carried out at $t = t_M$. The situation may hence be illustrated as in Figure A.7. When $t = t_M$, the array elements $P_{z,i}(t_M)$ are rearranged in a new array; denoted $P'_{z',i'}(t_M)$ in Figure A.7.

A.4 Summary

The presented relations represent numerical routines and cannot be simply implemented one-to-one in a computer code. However, they may help to understand the proposed numerical procedures and to create the computer code, and in combination with the figures they illustrate the basic principle of the procedure.

The procedure can be characterized as 'moving of probability masses in a time dependent array'. Note that $\sum_z \sum_i P_{z,i}(t) = 1$ for all t. Referring to the illustration of $P_{z,i}$ in Figure A.3, the simulation of deterioration, inspections, PM and CM may be summarized as follows:

- Deterioration moves the probability mass 'rightwards' ($+i$-direction) and one time-step (dt) 'forwards' ($+t$-direction).
- CM and PM move the probability mass 'leftwards' ($-i$-direction).
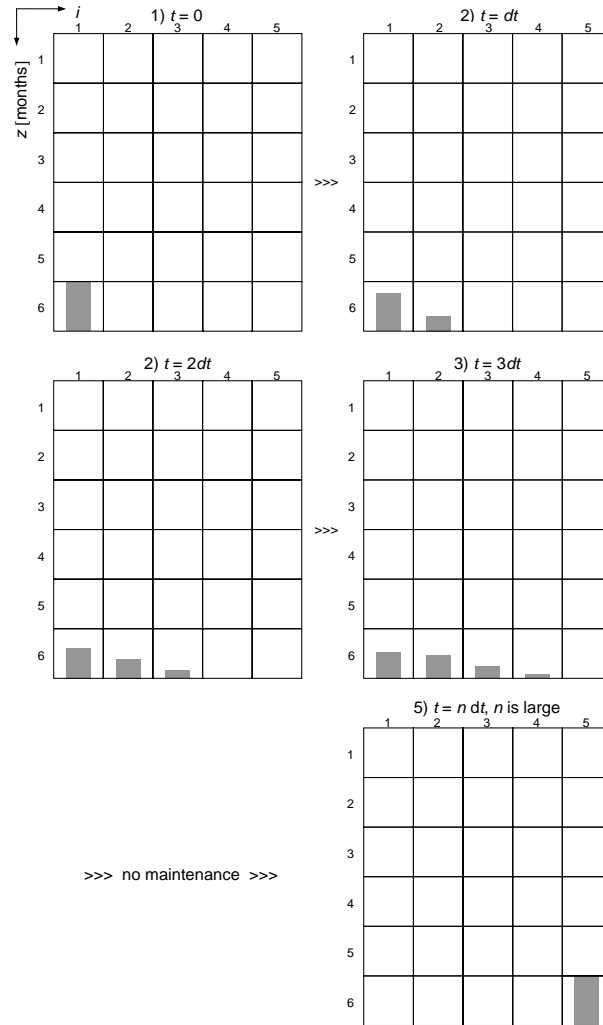- Rescheduling of inspection moves the probability mass 'downwards' ($+z$-direction).
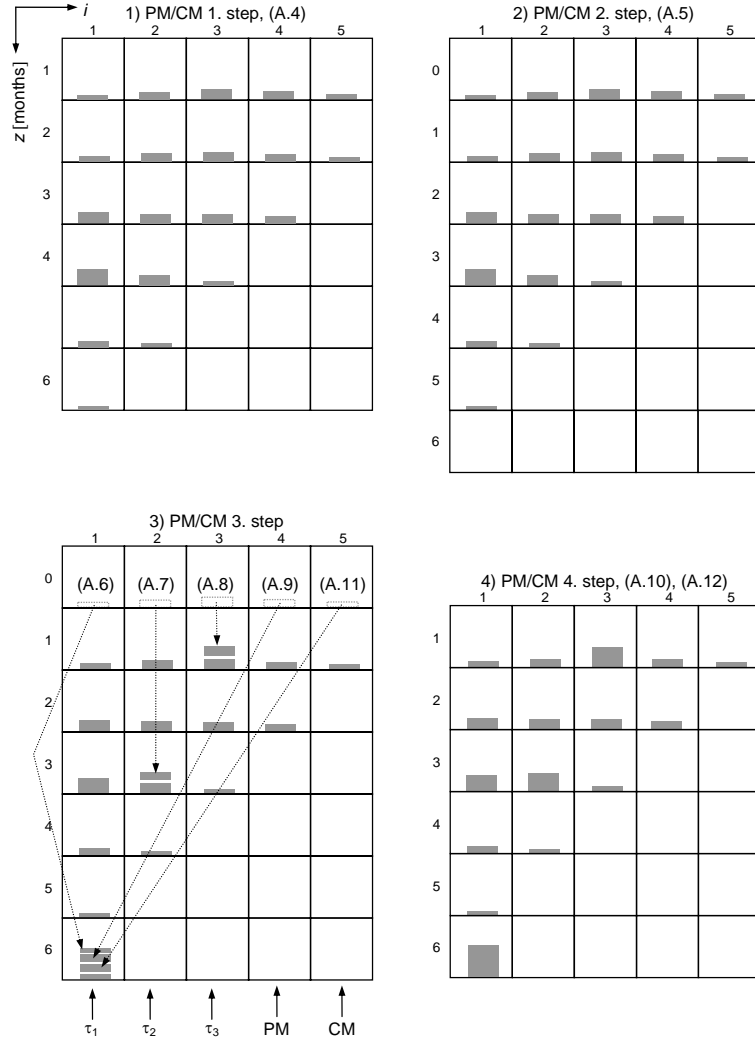
Figure A.3. Deterioration.

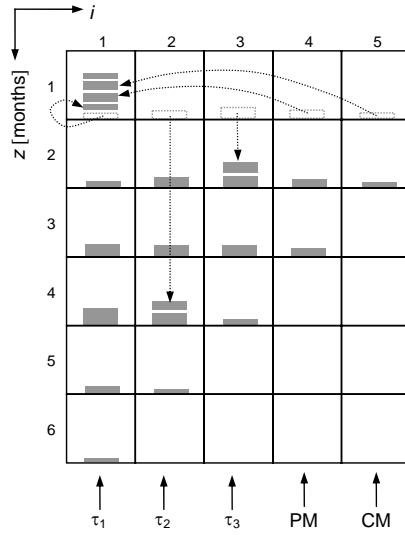**Figure A.4.** Preventive maintenance (PM) and corrective maintenance (CM).

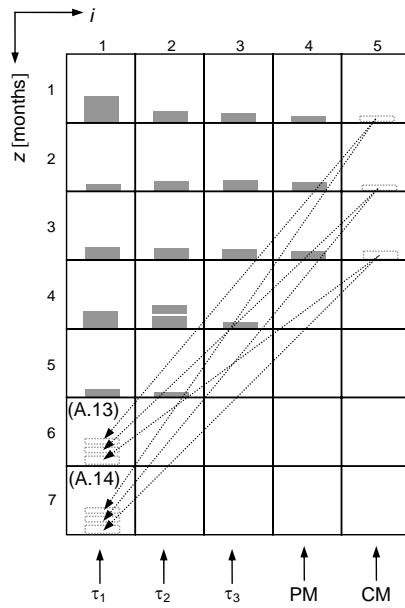**Figure A.5.** PM and CM; realized by a circular list.
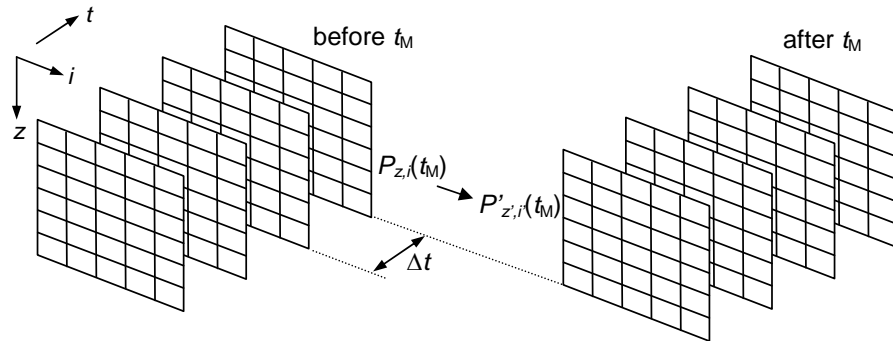

Figure A.6. CM is self-announcing.

Figure A.7. Schematic illustration of a maintenance action that is carried out at t_M and that changes the component properties.

B

Treatment of parameter uncertainty

The influence of a 'probabilistic solution' and a 'deterministic solution' on the modelling results is analysed in this appendix. This means that it is analysed to what extent the use of a best estimate (point estimate) for the sojourn time distribution parameters influences the results, compared with a solution where the parameter uncertainty is propagated through the whole maintenance model. It has been argued in Section 7.6 that the parameter uncertainty must actually be propagated through the whole maintenance model. This is usually not done in complex maintenance models, because handling parameter uncertainty is often too complicate. Thus, it is simply assumed that the model parameters are known and that they can be treated as fixed parameters in the model.

Vatn [99] presents a numerical example where the lifetime of a component is modelled by a Weibull distribution and where the shape parameter is either assumed to be a known, constant parameter (deterministic model) or to be gamma distributed (probabilistic model). The latter means that is is uncertain what the value of this parameter is. The uncertainty about the parameter value is therefore descried by a gamma distribution. A deterministic and a probabilistic solution is calculated for an example where a minimal repair strategy is analysed. The probabilistic model shows significant deviation from the deterministic model. Thus, the parameter uncertainty should be propagated through the whole maintenance model, because the uncertainty influences the model results and the decisions.

In the following two sections, it is analysed to what extent the use of a probabilistic model and a deterministic model influences the results of the maintenance model presented in this thesis.

B.1 Sojourn time distribution

Figure B.1 shows two probability density functions of the same sojourn time distribution; the deterministic solution $(f(t; \hat{\boldsymbol{\theta}})$, a gamma distribution; see

equation 7.3)) and the probabilistic solution ($\overline{f}(t)$; see equation 7.2)). The example is based on the dataset and the expert estimates shown in Table B.1. The maximum likelihood estimates of α and β have been used as deterministic (best) estimate. They were calculated by using the approach described in Paper 5.

The two distributions have approximately the same location, however, the deterministic approach yields a distribution that is somewhat broader than the distribution calculated by the probabilistic approach. One could conclude that the best estimate approach yields a distribution that is more conservative with respect to the sojourn time T. However, this conclusion can not be generalized. In other cases, $f(t; \hat{\boldsymbol{\theta}})$ is narrower than $\tilde{f}(t)$ or the location is somewhat displaced to each other.

Further analyses are required to judge the influence of the deterministic approach on the costs and on the results of the maintenance model presented here. The following section presents therefore an attempt to propagate the sojourn time distribution parameter uncertainty through the whole maintenance model.

Expert judgement		
Estimate: 10th percentile	Mean	
used as: 25th percentile	Median	
Expert 1	10	15
Expert 2	15	25
Data (uncensored)		
18.3	64.4	37.9
51.7	50.0	46.4
31.3	32.0	40.1
29.0		

Table B.1. Expert judgement and data used for the Bayesian analysis.

B.2 Example with cost figures

Consider the following example: Two expert judgements for the four sojourn time distributions have been requested (see Table B.2). The expert judgements are requested as 10th percentile and mean of the sojourn time T_i and are used as 10th percentile and median in the following analysis. The resulting best estimates (mean values of the state-of-knowledge distributions) have been calculated by using the approach described in Paper 5. The estimates are shown in Table B.3. A maintenance strategy is considered where preventive maintenance is carried out when the component is classified in state 3 or state 4. Failure is self-announcing and corrective maintenance is carried out immediately. The objective of the analysis is to find the inspection interval
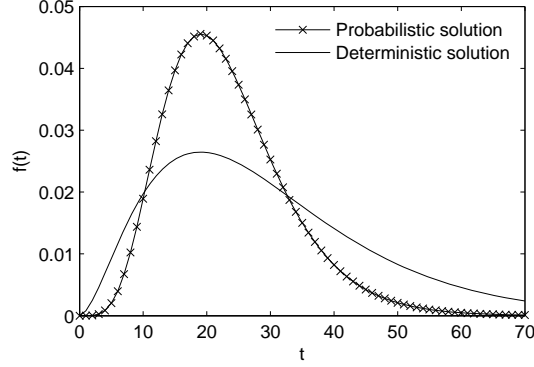
Figure B.1. Sojourn time distribution: Probabilistic solution vs. deterministic.

lengths in state 1 and state 2, τ_1 and τ_2, respectively, that minimize the annual long-term total cost. The costs for a single inspection, preventive maintenance action and corrective maintenance action are 5, 50 and 1500, respectively.

Assume that the parameter uncertainty of the sojourn time distribution of the second state is propagated through the maintenance model, whereas all other parameters are kept constant (best estimates, ignoring parameter uncertainty; see Table B.3). The following approach is used to calculate an approximative probabilistic solution for the optimal inspection strategy:

1. The state-of-knowledge distribution $\pi(\boldsymbol{\theta}_2)$ is approximated by a discrete joint probability distribution $\pi(\boldsymbol{\theta}_{2,k})$, $k = 1, ..., 9$.
2. For each $\boldsymbol{\theta}_{2,k}$, the cost surface $C_k(\tau_1, \tau_2 | \boldsymbol{\theta}_{2,k})$ is calculated.

| State j | —- Expert 1 —- | | —- Expert 2 —- | |
	10th percentile [years]	Mean [years]	10th percentile [years]	Mean [years]
1	7	15	8	25
2	1	5	3	8
3	1	2	0.5	2
4	0.2	0.5	0.25	1

Table B.2. Expert judgement results. Requested: 10th percentile and mean. Used as: 10th percentile and median.

State j	α_j	β_j
1	2.2	9.9
2	1.7	4.2
3	2.2	1.1
4	1.5	0.7

Table B.3. Best estimates for the sojourn time distribution parameters.

3. The probability that this cost surface represents the reality is $p_k = \pi(\boldsymbol{\theta}_{2,k})$.
4. The cost surface of the probabilistic solution is calculated as
   $C(\tau_1, \tau_2) = \sum_k [p_k \cdot C_k(\tau_1, \tau_2 | \boldsymbol{\theta}_{2,k})]$.
5. The probabilistic solution is given by τ_1 and τ_2 that minimize $C(\tau_1, \tau_2)$.

The resulting cost surface is shown as contour plot in Figure B.2(c). The results are compared with the cost surface that is obtained by a deterministic model, that is, when T_2 is modelled as gamma distribution where the model parameters are constants (best estimates). Figure B.2(a) shows a deterministic solution where the mean of $\pi(\boldsymbol{\theta})$ is used as best estimate for the sojourn time distribution parameters, and Figure B.2(b) shows a deterministic solution where the maximum likelihood estimates (MLE) are used as best estimate for the sojourn time distribution parameters. The cost surfaces are very similar and the optimal solution is approximately the same ($\tau_1 \approx 44$ months and $\tau_2 \approx 12$ months).

The approach can be extended to the case where the parameter uncertainty in both T_1 and T_2 is driven through the maintenance model:

1. The state-of-knowledge distribution $\pi(\boldsymbol{\theta}_1)$ is approximated by a discrete joint probability distribution $\pi(\boldsymbol{\theta}_{1,l})$, $l = 1, ..., 9$.
2. The state-of-knowledge distribution $\pi(\boldsymbol{\theta}_2)$ is approximated by a discrete joint probability distribution $\pi(\boldsymbol{\theta}_{2,k})$, $k = 1, ..., 9$.
3. For each possible combination of $\boldsymbol{\theta}_{1,l}$ and $\boldsymbol{\theta}_{2,k}$, the cost surface
   $C_{l,k}(\tau_1, \tau_2 | \boldsymbol{\theta}_{1,l}, \boldsymbol{\theta}_{2,k})$ is calculated.
4. Assuming independence[1] between $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$, the probability that this cost surface represents the reality is $p_{l,k} = p_l \cdot p_k = \pi(\boldsymbol{\theta}_{1,l}) \cdot \pi(\boldsymbol{\theta}_{2,k})$.
5. The cost surface of the probabilistic solution is calculated as
   $C(\tau_1, \tau_2) = \sum_l \sum_k [p_{l,k} \cdot C_k(\tau_1, \tau_2 | \boldsymbol{\theta}_{1,l}, \boldsymbol{\theta}_{2,k})]$.
6. The probabilistic solution is given by τ_1 and τ_2 that minimize $C(\tau_1, \tau_2)$.

A contour plot of the cost surface is shown in Figure B.2(d). In this case, the optimal solution of the inspection intervals is $\tau_1 \approx 40$ months and $\tau_2 \approx 12$ months.

Although the analysis in the previous section has shown that the sojourn time distributions can differ considerably if either the deterministic approach or the probabilistic approach is used (Figure B.1), the results calculated with the maintenance model does not differ much, which is a surprising finding. From the results it can be concluded that the treatment of the parameter uncertainty in the maintenance model does not influence the optimal solution much. The calculated minimum costs (C_{min}) does not differ much. They vary between 4.21 (Figure B.2(b)) and 4.43 (Figure B.2(d)). The increased modelling effort by using the probabilistic approach seems not to be justified by a considerable improvement of the results. Thus, it is suggested to use

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[1] The assumption of independence is not very realistic, but it allows calculating $p_{l,k}$ in a simple way, since defining dependence between $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ seems practically impossible.
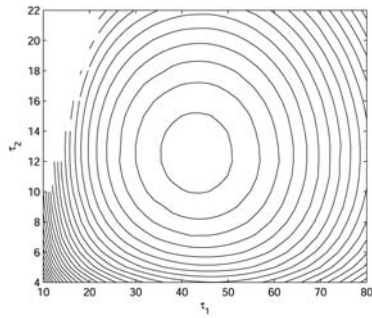
the deterministic approach in practice. Nevertheless, it would be desirable to analyse the problem in more detail in further work to confirm the results.
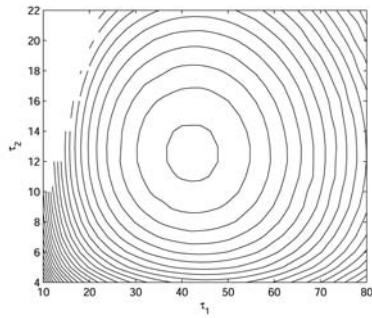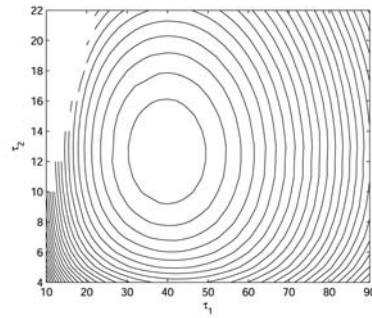

(a) Deterministic model; mean for T_2.
$C_{\min} = 4.25$ at $\tau_1 \approx 42$ and $\tau_2 \approx 13$
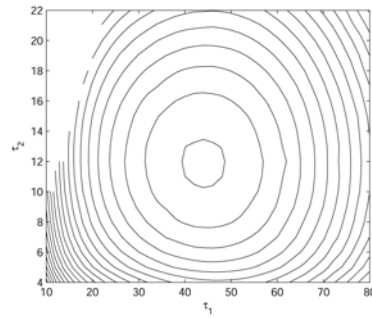
(b) Deterministic model; MLE for T_2.
$C_{\min} = 4.21$ at $\tau_1 \approx 44$ and $\tau_2 \approx 12$

(c) Probabilistic model for T_2.
$C_{\min} = 4.30$ at $\tau_1 \approx 44$ and $\tau_2 \approx 12$

(d) Probabilistic model for T_1 and T_2.
$C_{\min} = 4.43$ at $\tau_1 \approx 40$ and $\tau_2 \approx 12$

Figure B.2. Contour plot of cost surface for deterministic solution and probabilistic solution; τ_1 and τ_2 er given in months.

Part II

Papers

Paper 1

Markov state model for optimization of maintenance and renewal
of hydro power components

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KTH, Stockholm, Sweden – 11-15 June, 2006

1

Markov State Model for Optimization of Maintenance and Renewal of Hydro Power Components

Thomas M. Welte, Jørn Vatn and Jørn Heggset

Abstract—In this paper a reliability model is presented which can be used for scheduling and optimization of maintenance and renewal. The deterioration process of technical equipment is modeled by a Markov chain. A framework is proposed how the parameters in the Markov process can be estimated based on a description of the technical condition of components and systems in hydro power plants according to the Norwegian Electricity Industry Association. A time dependent solution of the Markov model is presented. Imperfect periodic inspection can be modeled by the proposed approach. The length of the inspection interval depends on the system condition revealed by the previous inspection. The model can be used to compute performance measures and operational costs over a finite time horizon. Finally, simulation results for a dataset for a Norwegian hydro power plant are presented.

Index Terms—deterioration model, imperfect inspection, maintenance optimization, Markov model

I. NOMENCLATURE

b inspection interval ratio factor
C costs
d discount rate
f_{red} reduction factor
i state in the final Markov chain
k main state (gamma distributed)
l sub state (exponential distributed)
L_k number of sub states in main state k
m maintenance limit in the final Markov chain
n number of events
$P_i(t)$ probability that the system is in state i at time t
$q_{j|k}$ probability that the system is classified to be in main state j when the real state is k
r failure state in the final Markov chain
t time variable
$t_{0.1,k}$ 10th percentile of T_k
T_k duration of main state k
$T_{k,l}$ duration of sub state l in main state k
u proportion of uncertainty
y year
z time of inspection
α_k shape parameter
β_k scale parameter
λ_i transition rate from state i to state $i + 1$
τ_k inspection interval for a system regarded to be in main state k

II. INTRODUCTION

SCHEDULING and optimization of maintenance and renewal require reliability models describing the deterioration process of technical equipment. Suitable models should be capable of evaluating the residual service life, the failure probability and the change of operational costs as a function of maintenance and renewal. A common objective of many of these models is to find the maintenance and renewal strategy where the total costs of repairs, inspections, production losses and other consequences are minimal.

Various models were proposed over the past decades. Anders, Endrenyi and Leite da Silva presented in [1] and [2] maintenance optimization models based on a Markov chain and computer software for maintenance management. An often applied method to model deterioration is the use of the gamma process. A short overview about work on this topic is given by Kallen and Noortwijk in [3]. Extensive analyses have been carried out on handling different inspection strategies and special situations in the models. Here, the work done by Grall, Dieulle, Bérenguer and Roussignol [4] could be mentioned. However, by using the gamma process the deterioration has, in average, a linear trend. Whether this applies to the case of interest or not has to be considered. Another frequently surveyed and suggested method is the delay time concept [5]. Other maintenance models and an overview about their application is given e.g. in [6], [7] and [8].

In spite of the great number of methods, mathematical models for maintenance optimization are hardly used in practice [9]. One reason can be that there are often difficulties in providing the proper amount of data. To overcome this problem, the model described in this paper is built on an already existing state definition of the technical condition of hydro power components [10]. This definition is used by many of the Norwegian power companies. It can be expected

that a model based on an already existing approach is far easier to implement in existing maintenance procedures than models based on more abstract statistical concepts.

The state definition is presented in Section III. In Section IV a probabilistic model based on this state definition is described including an approach that transfers this model into a Markov chain. In the next Section we define a maintenance model. For this model a time-dependent solution for the Markov chain is presented (Section VI). The model can be used to calculate performance measures. In Section VIII, the model is applied to a real example in order to show its potential.

This work is part of ongoing R&D activities within hydro power operation and maintenance in Norway. An overview over the activities and recent results were presented by Wiborg, Solvang, Heggset and Daleng in [11].

III. DESCRIPTION OF TECHNICAL CONDITION

The technical condition of a system in hydro power plants is characterized on a scale from 1 to 4 according to the Norwegian Electricity Industry Association (EBL) [10]. Thus, the continuous degradation of a component is simplified by dividing it into four states. The state description is given in Table I and in the following, these four states will be denoted *main states k*. A component as-good-as-new is in state $k = 1$. When the condition is characterized as critical, the state is $k = 4$ and normally maintenance actions must be taken immediately.

TABLE I
TECHNICAL CONDITION STATES (MAIN STATES)

State	Description
1	No indication of degradation.
2	Some indication of degradation. The condition is noticeably worse than "as good as new".
3	Serious degradation. The condition is considerably worse than "as good as new".
4	The condition is critical.

In addition to this general state specification, more detailed descriptions are given in the handbooks [10] for different failure modes of all main components in a hydro power plant. Thus, the maintenance personnel have a guideline for the interpretation of different inspections and measurement results in order to define the condition of the system according to the four-state-scale.

Failure is always assumed to occur when there is a transition out of state 4 in a state 5 as indicated in Fig. 1. The length T_k of each main state k may vary from several years (state 1) to only a few years or months (state 4). If the length of T_k is known, the concept of life curves [2] can be applied and in principle the deterioration process can be sketched as shown in Fig. 1. However, the length of the four main states has an element of uncertainty, which can be represented by a probability distribution. The Gamma distribution was used in this paper in order to model T_k.
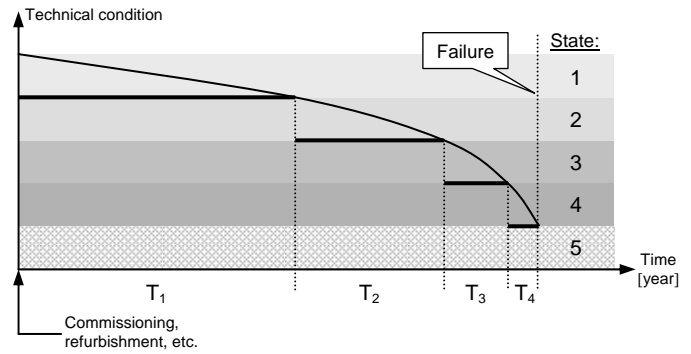

Fig. 1. Technical condition levels (main states) and life curve

IV. MAIN STATE MODELING

Preferably, the estimation of suitable probability distributions that describe the length of the four main states is based on analyses of reliability data and real observations. However, reliability data is often scarce. Hence, T_k has to be modeled by expert judgment. Currently, EBL is establishing a national database for reliability data for power plant components. The data will be collected in a standardized way. Thus, the amount and quality of the collected data will be largely improved in the future. At the moment, however, the opinion of experts and maintenance personal is playing a decisive role in the analysis process.

There exist many proposals how expert judgment can be carried out. A good overview about expert judgment techniques and a practical guideline is given in [12]. In the examples presented in this paper the experts express their opinion about the main state length by assessing the expectation $E(T_k)$ and the 10th percentile $t_{0.1,k}$ of T_k. A gamma distribution is fitted to these values. Due to the fact that the definition of the technical condition states is based on the handbook descriptions and that the described state definition is already in use in the Norwegian hydro power industry for many years, the plant experts are familiar with the systematic and have a lot of experience with the described state definition. Thus, feasible estimates can be expected from experienced experts. Nevertheless, expert judgment should only be regarded as a preliminary source until the estimates can be improved by reliability data from plant operation.

In the next step, the gamma distributed main states are transferred into a Markov chain. A known theoretical result states that a sum of n identical and independent distributed exponential variables is gamma distributed with shape parameter n. In the following, an approach is described for the approximation of each main state k by L_k exponential distributed states. These exponential distributed states are denoted *sub states*. This means that during a stay in the main state k the condition of the system runs through the sub states $l = 1, 2, ..., L_k$. The number of sub states L_k must be at least as big as the shape parameter α_k.

$$L_k \geq \alpha_k \qquad L_k \in \{1,2,3,...\} \qquad (1)$$

The deterioration process often accelerates towards the end of life. This yields a life curve similar to the one shown in Fig. 1. If one wants to reflect the accelerating deterioration behavior in the Markov model, the expectation of the exponential distributed states in the Markov chain has to become stepwise smaller. Thus, we pragmatically set

$$E(T_{k,l}) = f_{\text{red},k} \cdot E(T_{k,l-1}) \tag{2}$$

where $f_{\text{red},k}$ is a reduction factor ($f_{\text{red},k} < 1$). $E(T_{k,l})$ and $E(T_{k,l-1})$ denote the expectation of the length of sub state l and sub state l-1, respectively, in main state k.

The probability density function of the gamma distribution, which is used to model the four main states k, can be written in the following form:

$$f(t) = \frac{1}{\beta^{\alpha_k} \Gamma(\alpha_k)} \cdot t^{\alpha_k - 1} \cdot e^{-\frac{t}{\beta_k}} dt \tag{3}$$

where α_k is a shape parameter and β_k a scale parameter. The shape parameter α_k can be expressed by the expectation and the variance as

$$\alpha_k = \frac{E(T_k)^2}{Var(T_k)} \tag{4}$$

The variance of the exponential distributed sub states is

$$Var(T_{k,l}) = E(T_{k,l})^2 \tag{5}$$

If we assume independence between the Markov states, the expectation and variance of the chain of exponential distributed sub states in one main state k is given by

$$E(T_k) = \sum_{l=1}^{L_k} E(T_{k,l}) \tag{6}$$

and

$$Var(T_k) = \sum_{l=1}^{L_k} Var(T_{k,l}) \tag{7}$$

respectively. The expectation of the first sub state is $E(T_{k,1})$. According to (2), the expectation of an arbitrary sub state l can be expressed as

$$E(T_{k,l}) = E(T_{k,1}) \cdot (f_{\text{red},k})^{(l-1)} \tag{8}$$

Equation (6) and (8) yield

$$E(T_k) = E(T_{k,1}) \cdot \sum_{l=1}^{L_k} f_{\text{red},k}^{(l-1)} \tag{9}$$

and with (5) and (8) the variance (7) becomes

$$Var(T_k) = E(T_{k,1})^2 \cdot \sum_{l=1}^{L_k} \left[f_{\text{red},k}^{(l-1)} \right]^2 \tag{10}$$

The expectation and variance of the chain of exponential distributed sub states should be equal to the expectation and variance in the primary gamma distribution. Thus, the shape parameter α_k can be expressed with (9) and (10) as

$$\alpha_k = \frac{\left[\sum_{l=1}^{L_k} f_{\text{red},k}^{(l-1)} \right]^2}{\sum_{l=1}^{L_k} \left[f_{\text{red},k}^{(l-1)} \right]^2} \tag{11}$$

$E(T_{k,1})$ and $f_{\text{red},k}$ have to be chosen such that both (9) and (11) are satisfied.

The described procedure yields the final Markov chain which is used for further analyses in Section VI. The states in the final Markov process are denoted i and the number of states is

$$r = \sum_{k=1}^{4} L_k + 1 \tag{12}$$

where r is the failure state. The first exponential distributed state i_k that represents the beginning of a main state k is given by

$$i_1 = 1, \qquad i_2 = L_1 + 1, \qquad i_3 = L_1 + L_2 + 1$$
$$i_4 = L_1 + L_2 + L_3 + 1, \qquad i_5 = r \tag{13}$$

V. MODEL SPECIFICATIONS

In the following sections, a system is analyzed that is maintained according to the following specifications:

- The system is subjected to a deterioration process. The deterioration is modeled as a Markov chain with r states.
- The system is periodically inspected. However, the inspection interval is not constant. The time of next inspection depends on the system state revealed by the previous inspection.
- The inspections are considered to be imperfect, i.e. there is a probability that the inspection results in a wrong assessment of the technical condition.
- If the system excesses an intervention threshold level, preventive maintenance (PM) will be carried out.
- If the system fails, corrective maintenance (CM) will be carried out.
- PM and CM are modeled as perfect and the system is replaced or repaired to an "as good as new" state.

VI. TIME DEPENDENT MARKOV CHAIN SOLUTION

Now, we want to consider the final Markov process as specified in the end of Section IV. The transition rate from state i to state $i+1$ is denoted λ_i. As described in the previous section, a PM action will be taken if an inspection reveals that the technical condition of the system has exceeded a maintenance limit. CM is carried out directly after failure. The maintenance limit and the failure state are denoted m and r, respectively. PM is triggered by $i \geq m$. For the analyses in this paper it was assumed that PM is carried out as soon as it is observed that the system is in main state 4, i.e. $m = i_4$.

However, the maintenance limit could also be set to any other state i. The time of the next inspection depends on the result of the previous inspection, i.e. if the inspection reveals, that the system is in main state 1, 2 or 3, the next inspection is after a time period τ_1, τ_2 and τ_3, respectively. This definition implies that the answer on the question "maintain or don't maintain" is given by the system state. If the system appears to be in state 1, 2 or 3, the system is not maintained but the inspection interval is adjusted to the assumed system state. If the system appears to be in state 4 at an inspection point a PM activity is carried out. System failure could occur between inspections leading to an immediate CM action. The situation is shown in Fig. 2.
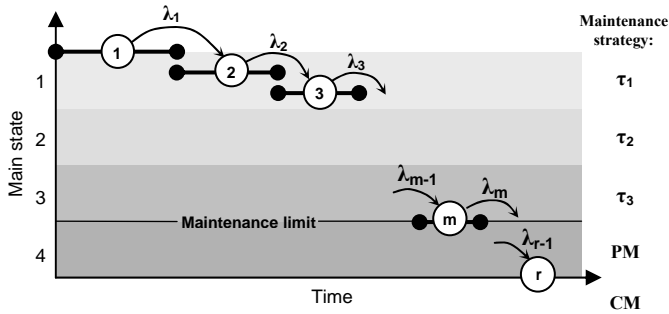

Fig. 2. Maintenance strategy in the Markov model

We will now find the probability that the system is in the various states as a function of the time t. We let $P_i(t)$ denote the probability that the system is in the state i at time t. By standard Markov considerations [13], we obtain the Markov differential equation:

$$P_i(t + \Delta t) = P_i(t) \cdot (1 - \lambda_i \cdot \Delta t) + P_{i-1}(t) \cdot \lambda_{i-1} \cdot \Delta t \qquad (14)$$

where Δt is a small time interval. The transition rate of the ith state is given as the reciprocal of the expectation of state i.

$$\lambda_i = \frac{1}{E(T_i)} \qquad (15)$$

For a constant inspection interval the integration of (14) is straight forward and we can easily handle the situation when the system is inspected at time τ, 2τ, etc. However, a challenge is to model different inspection and maintenance strategies. The inspection interval is not assumed to be constant but the time of the next inspection depends on the inspection result. In addition, the time of next inspection will be rejected and rescheduled if a failure occurs unexpectedly early.

These problems are handled in the following practical way: The state probabilities $P_i(t)$ are written in a vector with r elements.

$$\vec{P}_z = \left[P_{z,1}(t), P_{z,2}(t), \ldots P_{z,r}(t) \right] \qquad (16)$$

where z is the time of the next inspection and $P_{z,1}(t)$, $P_{z,2}(t)$, … $P_{z,r}(t)$ are the probabilities to be in Markov state 1, 2, … r at time t. For a new component for example, at $t = 0$ all information is placed in one vector where z is the time

of the first inspection after a period of τ_1 and $P_{z,1}(t) = 1$.

Now, the Markov differential equation can be written as

$$P_{z,i}(t + \Delta t) = P_{z,i}(t) \cdot (1 - \lambda_i \cdot \Delta t) + P_{z,i-1}(t) \cdot \lambda_{i-1} \cdot \Delta t \qquad (17)$$

By applying (17) the deterioration of the system can be modeled. Additional considerations are necessary in order to simulate inspections, maintenance and failure. Let $q_{j|k}$ be the probability that the system is classified to be in main state j when the real state is k. An imperfect inspection and preventive maintenance can be simulated in the following way:

1) Integration according to (17), $t = t + \Delta t$
2a) Inspection and PM ➜ equations (18)-(22) for all vectors where $z \leq t$:

$$P_{t+\tau_1,i}(t) = P_{t+\tau_1,i}(t) + P_{z,i}(t) \cdot q_{1|k(i)} \qquad (18)$$

$$P_{t+\tau_2,i}(t) = P_{t+\tau_2,i}(t) + P_{z,i}(t) \cdot q_{2|k(i)} \qquad (19)$$

$$P_{t+\tau_3,i}(t) = P_{t+\tau_3,i}(t) + P_{z,i}(t) \cdot q_{3|k(i)} \qquad (20)$$

$$P_{t+\tau_1,1}(t) = P_{t+\tau_1,1}(t) + P_{z,i}(t) \cdot q_{4|k(i)} \qquad (21)$$

for $1 \leq i < r$

and $k(i) = \begin{cases} 1 & \text{if} & 1 \leq i < i_2 \\ 2 & \text{if} & i_2 \leq i < i_3 \\ 3 & \text{if} & i_3 \leq i < i_4 \\ 4 & \text{if} & i_4 \leq i < r \end{cases}$

Set afterwards

$$P_{z,i}(t) = 0 \qquad\qquad i < r \qquad (22)$$

For the simulation of corrective maintenance the following procedure can be used:

2b) CM ➜ equations (23) and (24) for all vectors z:

$$P_{t+\tau_1,1}(t) = P_{t+\tau_1,1}(t) + P_{z,r}(t) \qquad (23)$$

and

$$P_{z,r}(t) = 0 \qquad (24)$$

Equations (18)-(20) represent the situation when the system is inspected and when it is assumed that the system is in state $k = 1$, $k = 2$ and $k = 3$, respectively. Thus, the next inspection will be carried out at $t + \tau_k$. Equation (21) represents the situation that the system is classified in main state 4 and PM is carried out. During PM the system is renewed, i.e. the system state is set to $i = 1$ and the time of the next inspection is chosen as $t + \tau_1$. Equation (23) is used for the modeling of failures. Also in this case the new system state is $i = 1$ and the next inspection will be carried out after a period τ_1.

The equations outlined in this section can be used for the calculation of performance measures like e.g. the expected number of inspections and maintenance actions per year. By choosing Δt sufficiently small, the proposed numerical

procedure provides good results. The presented equations represent the basic principle. During the simulation procedure the number of vectors increases. Nevertheless, the probability to be in state i at time t is the sum of $P_{z,i}$ over all vectors z.

$$P_i(t) = \sum_z P_{z,i}(t) \qquad (25)$$

In addition, it must hold for an arbitrary chosen t that

$$\sum_{i=1}^{r}\left[\sum_z P_{z,i}(t)\right] = 1 \qquad (26)$$

At first view, the computational effort seems to be large. However, by smart programming the computing time can be reduced considerably. Thus, the method provides a quite effective way to calculate the time dependent solution of the Markov chain.

VII. COSTS

The overall objective is to find the inspection or renewal strategy which gives the lowest costs. The costs are expressed as their current value, i.e. the present value (PV) is calculated for all future costs. For the calculation of the present value of a future amount C in year y, the following equation is used:

$$C_{\mathrm{PV}} = C \cdot (1+d)^{-y} \qquad (27)$$

where d is the discount rate. In this paper we consider the following expenses:

C_{CM} Costs per corrective maintenance (= costs for failure consequences, incl. production loss due to unscheduled plant down time and costs for replacement)

C_{PM} Costs per preventive maintenance (= costs for replacement incl. production loss due to scheduled plant down time)

C_{I} Costs for one inspection

If $E[n_{\mathrm{CM}}(y)]$, $E[n_{\mathrm{PM}}(y)]$ and $E[n_{\mathrm{I}}(y)]$ denote the expected number of CM, PM and inspections, respectively, in the yth year, the total costs expressed as PV in the beginning of year one is:

$$C_{\mathrm{tot}} = \sum_y \left\{(1+d)^{-y}\cdot\left(C_{\mathrm{CM}}E[n_{\mathrm{CM}}(y)] + C_{\mathrm{PM}}E[n_{\mathrm{PM}}(y)] + C_{\mathrm{I}}E[n_{\mathrm{I}}(y)]\right)\right\} \qquad (28)$$

In the presented examples in Section VIII, all costs are calculated for a time horizon of 30 years. The costs are assumed to be constant over the analysis period. Typically, the discount rate d is set to 0.08 for this type of analysis in the Norwegian power production sector.

VIII. EXAMPLE AND SIMULATION RESULTS

In this section we want to analyze and discuss an example for a Norwegian hydro power plant. Data and results are given for the degradation of a special type of stator winding connection. The degradation process can result in overheating and short circuit. Two expert groups were asked to express their opinion about the length of the main states k. The expert opinion given as expectation and percentiles resulted in a gamma distribution with expectation and standard deviation as shown in Table II. The average of the two expert opinions (bold numbers in Table II) were used for the further modeling. The life curves for the main states and for all exponential distributed states after having transferred the gamma distributions into a Markov chain as described in Section IV are shown in Fig. 3.

TABLE II
MAIN STATES MODELING

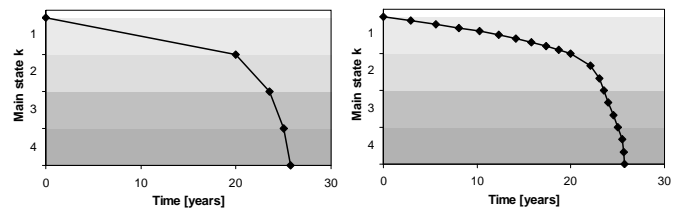| Main state k	----- Expert 1 -----		----- Expert 2 -----		---- Combined ----	
	$E(T_k)$ [years]	$SD(T_k)$ [years]	$E(T_k)$ [years]	$SD(T_k)$ [years]	$E(T_k)$ [years]	$SD(T_k)$ [years]
1	15	4.14	25	8.40	**20**	**6.54**
2	2	0.87	5	4.04	**3.5**	**2.34**
3	1	0.43	2	1.46	**1.5**	**0.87**
4	0.5	0.22	1	1.01	**0.75**	**0.53**

Fig. 3. Life curve with main states (left) and all sub states (right)

Now, after having defined the final Markov process, the equations presented in Section VI can be used to calculate performance measures. For the given data the expected number of CM, PM and inspections have been calculated for a new system. In addition Monte Carlo simulations have been carried out in order to verify the results. The chosen lengths of the inspection intervals τ_1, τ_2 and τ_3 are assumed to be 36 months, 6 months and 1 month, respectively. $E[n_{CM}(y)]$, $E[n_{PM}(y)]$ and $E[n_{I}(y)]$ will reach a steady state after a while. As an example, results for the expected number of PM are shown in Fig. 4. The steady state situation is reached after approximately three replacements, i.e. after $3 \cdot E(T)$. In the analyzed case this is after around 80 years. However, as shown in Fig. 5, expenses incurred in the steady state situation are not of interest considered from a present point of view because the present value (PV) of these costs converges to zero after few decades. Thus, the PV of the costs for CM, PM and inspections are only summed up in the total costs figure C_{tot} for the first decades. In the presented example the analysis period is 30 years. The expenses were estimated for the failure, repair and inspection costs of a medium size generator (100 MW) as $C_{CM} = 2\,500\,000$ EUR, $C_{PM} = 800\,000$ EUR and $C_I = 7\,000$ EUR, respectively.
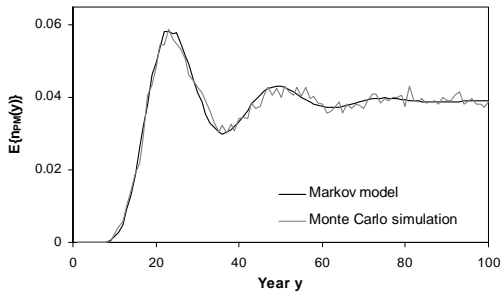
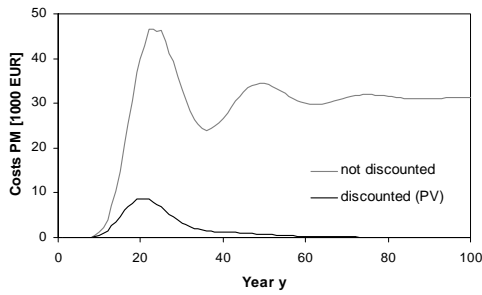Fig. 4. Development of the expected number of PM per year

Fig. 5. Expected annual costs for PM

In order to find a good inspection strategy, the length of the inspection interval τ_1 and τ_3, respectively, are defined as follows:

$$\tau_1 = b \cdot \tau_2 \tag{29}$$

$$\tau_3 = \frac{1}{b} \cdot \tau_2 \tag{30}$$

This means that dependent on the choice of the length of one inspection interval, τ_1 is always b times τ_2 and τ_3 is b times smaller than τ_2. The total costs (present value calculated over a time horizon of 30 years) are plotted in Fig. 6 versus τ_2 for various values of the factor b. The plot shows clearly that the optimal inspection strategy is given by $b \approx 4.5$ and $\tau_2 \approx 0.7$ years, i.e. a plant operator should choose τ_1, τ_2 and τ_3 around 38 months, 8 months and 2 months, respectively. The different cost drivers and the total costs are shown versus τ_2 for the optimal value of b in Fig. 7.
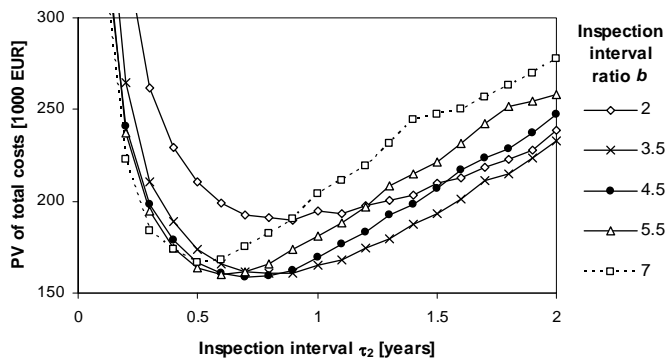

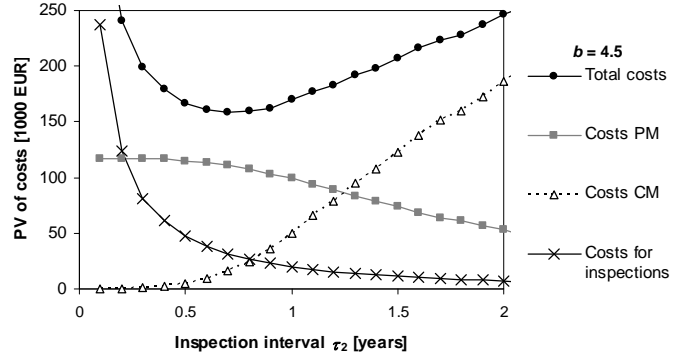Fig. 6. Total costs vs. τ_2 for different values of factor b

Fig. 7. Present value of total costs and costs for CM, PM and inspections
vs. τ_2; $b = 4.5$

Until now, the inspections were assumed to be perfect. In the next step we want to analyze the influence of imperfect inspections. By introducing the proportion of uncertainty u, $q_{j|k}$ can be defined as shown in Table III. For $u = 0$ the inspection is perfect. A value $0 < u < 0.5$ means that the system is wrong classified with probability u in one of the adjacent states. Fig. 8 shows the influence of imperfect inspections on the number of expected failures per year for different values of u. The influence of imperfect inspections on the costs is shown in Fig. 9 for $u = 0.1$. As expected, the total costs increases and the inspection intervals have to be chosen somewhat shorter in order to compensate the uncertainty of the inspection method.

TABLE III
DEFINITION OF $q_{j|k}$ BY USING THE UNCERTAINTY PROPORTION u

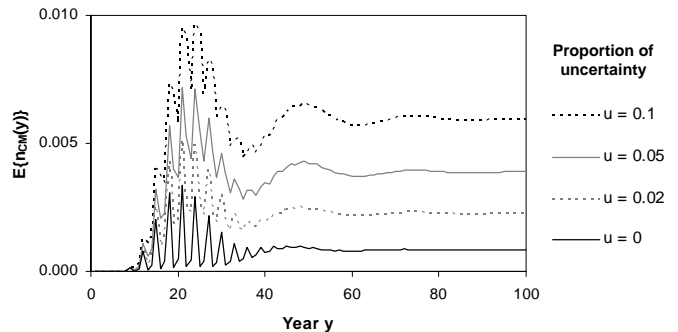|			real main state k		
		1	**2**	**3**	**4**
classified in main state j	**1**	$1 - u$	u	0	0
	2	u	$1 - 2u$	u	0
	3	0	u	$1 - 2u$	u
	4	0	0	u	$1 - u$

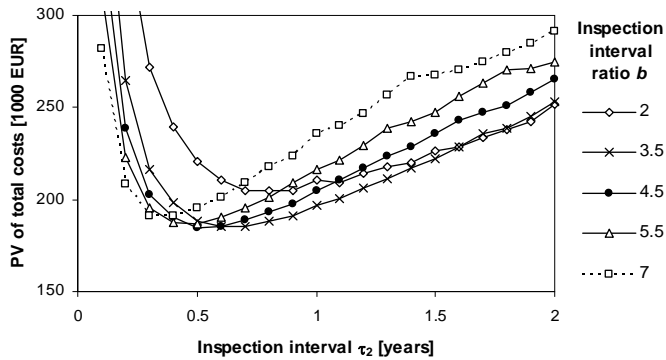Fig. 8. Expected number of failures per year for different values of u

Fig. 9. Total costs vs. τ_2 for different values of factor b and $u = 0.1$

IX. DISCUSSION AND CONCLUSIONS

In this paper a reliability model is presented which is based on a state definition of the technical condition according to the Norwegian Electricity Industry Association. It was shown how expert opinion can be used to model these states and an approach was proposed to transfer this model into a Markov chain. A time dependent solution for the Markov chain was provided for a maintenance strategy as specified in Section V. This solution was used in a maintenance optimization task. The results were verified by Monte Carlo simulations.

An advantage of the proposed framework is that the probabilistic model is based on an already existing state definition. Instead of introducing a new concept for the description of the technical condition of a system we take advantage of the already existing way of thinking in order to simplify the implementation of new maintenance models.

The proposed Markov chain solution is an alternative to Monte Carlo simulations. For rare events, which require many repetitions in a Monte Carlo simulation (like e.g. failures of high reliable systems), the numerical integration of the Markov model provides stable results. The simulation results are not subjected to random fluctuations such as Monte Carlo simulation results show even though hundred thousands or millions of repetitions are carried out.

The presented model has a number of limitations. Neither the option to repair from an arbitrary state to any other state nor the possibility to model imperfect repair is currently included in the model. In the following, we discuss some issues related to improving our model. If the maintenance action does not result in a system "as good as new" one could take this into account by setting the condition of the system to state 2 or 3 instead to state 1; or possibly to any sub state corresponding to these main states. This corresponds to a non-perfect repair, but the system performance from that state is identical to a new system. Another way to look at this would be that for a maintained system the deterioration goes faster than for a new system. This could be realized in the model by increasing the transition rates by some amount. However, several problems might arise if the model becomes too complex. Firstly, the model requires additional parameters that can be difficult to estimate, and secondly, the determination of cost figures might become complex.

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XI. BIOGRAPHIES

Thomas M. Welte was born on August 10, 1976 in Böblingen, Germany. He received his MSc degree in mechanical engineering from University Stuttgart, Germany. Currently, he is PhD student at the Norwegian University of Science and Technology (NTNU), Department of Production and Quality Engineering, Trondheim, Norway. He is working with degradation models for hydro power plants, maintenance planning and optimization.

Jørn Vatn was born on January 24, 1961 in Inderøy, Norway. He received his MSc degree in mathematical statistics from the University of Science and Technology (NTNU), Trondheim, Norway in 1986, and his PhD from the same university in 1996 with a thesis on maintenance optimization. He has almost 20 years of experience as a researcher at SINTEF Safety and Reliability, and current he holds a position as professor at NTNU, Department of Production and Quality Engineering.

Jørn Heggset was born on May 11, 1962 in Molde, Norway. He received his MSc degree in electrical engineering from the Norwegian University of Science and Technology (NTNU), Trondheim, Norway in 1987. He is a research scientist since 1993 at SINTEF Energy Research, Department of Energy Systems, working mainly with reliability assessment, application of fault and interruption data and maintenance optimization.

Paper 2

A theoretical study of the impact of different distribution classes
in a Markov model

A theoretical study of the impact of different distribution classes in a Markov model

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Norwegian University of Science and Technology, Trondheim, Norway

ABSTRACT: In this paper it is shown that a semi-Markov process with sojourn times given as a general life-time distribution can be approximated by a conventional Markov process with exponentially distributed so-journ times. This means that the general lifetime distribution is replaced by a sum of exponentially distributed times. A replacement of the gamma distribution by a chain of exponential distributions gives good results in a maintenance model. Different approximation approaches are presented and discussed. If the general lifetime distribution is a Weibull distribution, however, an approximation becomes more difficult. One way to estimate the general lifetime distribution in the semi-Markov model is the use of expert opinion. It is recommended asking the experts to assess the mean sojourn time and a percentile in the same order of magnitude as the inspection interval of the system. It is shown that such a strategy will give rather small modeling errors, even when the 'correct' lifetime distribution is unknown.

1 INTRODUCTION

Components and systems in hydro power plants require high reliability and availability. The downtime of a plant due to failures or due to unexpected maintenance tasks causes usually high costs. Apart from the expenses for repair or replacement, the failure of a component can also cause hazards for humans, environment and other components. For a production unit like a hydro power plant an additional cost factor is the loss of production due to the unscheduled down-time. Thus, the operation of a power plant calls for models for scheduling of maintenance and renewal. Objective of the models is to describe the relationship between maintenance decisions (e.g. inspection or replacement frequency) and the operational costs in order to find the best strategy that gives the lowest overall costs.

In an ongoing research program, a Markov model with five main states was established to be used as a general maintenance model of hydro power plants (Welte et al. 2006). Each main state in the model represents a certain, well-defined technical condition of a system (component, item, …) in the plant. Four states in the Markov model stand for different stages of deterioration and the fifth state is the fault state. The state definition is based on a classification system used in the condition monitoring handbooks (EBL 2006) from EBL Kompetanse, a subsidiary of the Norwegian Electricity Industry Association (EBL). A verbal definition of the main states is given in Table 1. If no maintenance is carried out, the system's technical condition will run through all main states until failure occurs. The progression of the system condition in a situation where no maintenance is carried out is sketched in Figure 1.

Table 1. Technical condition states (EBL 2006).

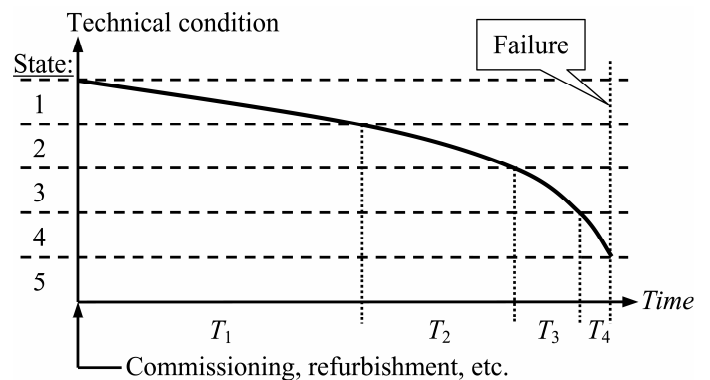| Main state	Description
1	No indication of deterioration.
2	Some indication of deterioration. The condition is noticeably worse than 'as good as new'.
3	Serious deterioration. The condition is considerably worse than 'as good as new'.
4	The condition is critical.
5	Failure

Figure 1. Condition propagation in a situation without maintenance.

The sojourn times in each main state k, denoted T_k, are believed to have an increasing failure rate. The sojourn times are modeled by a 'general' (i.e. non-exponential) probability distribution like e.g. the Weibull distribution or the gamma distribution. This means that deterioration and failures are modeled as a semi-Markov process. In addition, it is possible to include different maintenance situations in the model like e.g. preventive maintenance and various inspection strategies. The model may be solved by means of Monte Carlo simulations. In order to get sufficiently accurate results, however, Monte Carlo simulations are often a time-consuming task. Therefore, a numerical solution was proposed in Welte et al. (2006). However, to apply the suggested algorithm, the sojourn times must be split into a number of virtual, exponentially distributed sojourn times. This means that the semi-Markov process used for modeling deterioration is approximated by a conventional Markov process. This approximation simplifies the solution of the model.

A challenge in maintenance modeling is the estimation of the model parameters like e.g. the parameters of the probability distribution describing the sojourn time in the Markov model. If reliability data is scarce, expert judgment can be used to provide missing or additional information. Expert judgment techniques have been the object of much research activity in the past decades. An overview about the topic and practical guidelines are given e.g. in Cooke (1991), Meyer & Booker (1991) and Øien & Hokstad (1998). A general problem in estimating probability distributions by means of expert opinion is, however, that it is often unknown which probability distribution is the 'correct' one to model the problem at hand. Thus, an approach is suggested asking the experts to provide estimates for the expectation and the pth percentile of the distribution where p is chosen such that the pth percentile is in the order of magnitude of the inspection/replacement interval of the chosen maintenance strategy. In Section 3 we show, that this will result in relatively small modeling errors although if the 'correct' probability distribution is unknown. The representation of different distribution classes by a sum of exponential distributions is discussed in Section 2. In Section 4 the results are summarized and conclusions are drawn.
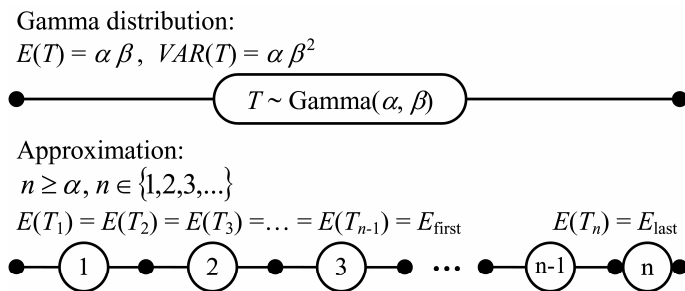
Gamma distribution:
$E(T) = \alpha\beta$, $VAR(T) = \alpha\beta^2$

Approximation:
$n \geq \alpha$, $n \in \{1,2,3,...\}$
$E(T_1) = E(T_2) = E(T_3) = \ldots = E(T_{n-1}) = E_{first}$ $E(T_n) = E_{last}$

Figure 2. Basic principle of Approach B.

2 APPROXIMATING DIFFERENT DISTRIBUTION CLASSES

In this section several approaches are discussed how to approximate a gamma or a Weibull distribution by a sum of exponentially distributed lifetimes.

2.1 *Gamma distribution*

A known theoretical result states that a sum of n identically and independently exponentially distributed variables is gamma distributed with shape parameter n. Thus, a gamma distribution with a shape parameter n that is an integer (also known as Erlang distribution) can be modeled by a sum of n exponential distributions. However, this result can only be applied to gamma distributions where n is an integer. If n is not an integer, other approximations have to be found.

The probability density function of the gamma distribution is

$$f(t) = \frac{1}{\beta^\alpha \Gamma(\alpha)} t^{\alpha-1} e^{-t/\beta} \qquad (1)$$

where α is a shape parameter and β a scale parameter. Expectation and variance of the gamma distribution are given by $E(T) = \alpha\beta$ and $VAR(T) = \alpha\beta^2$, respectively.

2.1.1 *Approach A*
One approach to approximate a gamma distribution by a chain of exponential distributions was presented by Welte et al. (2006). The sojourn times in the exponentially distributed states become stepwise smaller by a reduction factor f_{red}. This corresponds to an accelerated deterioration process. Furthermore, it is assumed that expectation and variance of the chain of exponential sojourn times is equal expectation and variance of the original gamma distribution. These assumptions result in a set of equations that gives a definite solution for the length of the exponentially distributed states (Welte et al. 2006). However, this approach does not give the best approximation of the gamma distribution. Therefore, another approach will be discussed in the following paragraph.

2.1.2 *Approach B*
The principle of this approach is sketched in Figure 2. As in Approach A, the number n of exponentially distributed sojourn times is equal or bigger as the shape parameter α of the gamma distribution. As a premise we assume that expectation and variance of the sum of exponential distributions is equal expectation and variance of the correct gamma distribution. Finally, the first $(n-1)$ sojourn times have the same length E_{first}, i.e.

$$E(T_1) = E(T_2) = ... = E(T_{n-1}) = E_{\text{first}} \qquad (2)$$

and the last sojourn time has the length E_{last}, i.e.

$$E(T_n) = E_{\text{last}} \qquad (3)$$

If we assume independence between the exponential sojourn times this yields:

$$E(T) = \sum_{i=1}^{n} E(T_i) = (n-1) \cdot E_{\text{first}} + E_{\text{last}} \qquad (4)$$

$$VAR(T) = \sum_{i=1}^{n} VAR(T_i) = (n-1) \cdot E_{\text{first}}^2 + E_{\text{last}}^2 \qquad (5)$$

The system of the two equations (4) and (5) can easily be solved and results in a quadratic equation with two solutions. If the relative distribution function (confer Section 2.1.4) for both solutions is plotted one can easily see that the best solution is given by the largest E_{first}, i.e.

$$E_{\text{first}} = \frac{E(T) \cdot (n-1) + \sqrt{W}}{n^2 - n} \qquad (6)$$

where

$$W = \left[E(T) \cdot (n-1) \right]^2 - (n^2 - n) \cdot \left[E(T)^2 - VAR(T) \right] \qquad (7)$$

If we consider a gamma distribution with shape parameter α, where $(n-1) \le \alpha \le n$, then, the proposed solution converges both to $(n-1)$ equal, exponentially distributed sojourn times if the shape parameter α approaches to $(n-1)$ and to n equal, exponentially distributed times if the shape parameter α approaches to n.

2.1.3 *Examples*

The influence of the presented approaches on the calculation results in a maintenance model is analyzed in this section. In Welte et al. (2006) the maintenance model described in Section 1 was used for maintenance optimization (example 1). In a second application the model was used for the assessment of the current maintenance strategy of a Norwegian power distribution company (example 2). The former application and a slightly modified version of the latter application were used as examples to analyze the error caused by the approximation of the gamma distribution by a sum of exponential distributions. The expected number of corrective maintenance actions per year (expected failure rate in year y) was calculated by means of Monte Carlo simulations, and by means of the numerical solution presented in Welte et al. (2006). Both Approach A and Approach B were used to transfer the semi-Makov process into a conventional Markov process. The parameters for the gamma distribution in the semi-Markov process are given in Table 2 and the inspection strategy is given by the length of the inspection intervals τ_1-τ_3 in Table 3. The general maintenance

strategy is to reduce the length of the next inspection interval if the previous inspection reveals that the system has deteriorated from state 1 to state 2 or 3. The system is replaced or renewed if an inspection has revealed that the system is in main state 4 or after failure (confer also Figure 1). The expected failure rates for a system that is as good as new at $t = 0$ are shown in Figure 3 and Figure 4, respectively.

As it can be seen from the diagrams, both approximations are rather good compared with results calculated by Monte Carlo simulations using the correct gamma distributions. In both examples, costs were assigned to a failure event and the present values were calculated. In the first example (Figure 3)

Table 2. Gamma distribution parameters for example 1 & 2.

State k	Example 1		Example 2	
	Shape α_k	Scale β_k	Shape α_k	Scale β_k
1	9.37	2.13	5.32	7.52
2	2.23	1.57	1.53	3.26
3	2.99	0.50	2.62	1.15
4	2.02	0.37	5.32	0.19

Table 3. Inspection interval length for example 1 & 2.

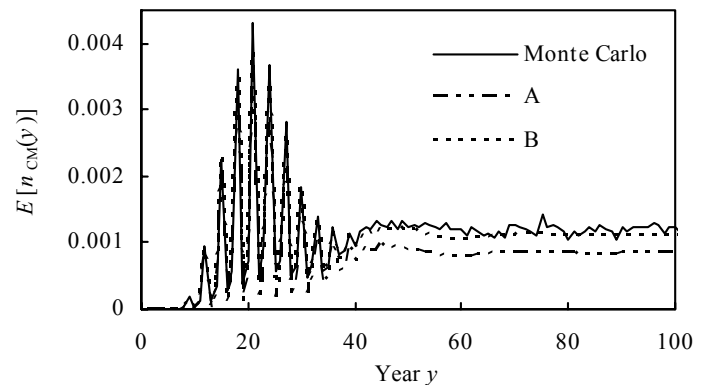| State k	Inspection interval τ_k	
	Example 1	Example 2
	months	months
1	36	48
2	6	12
3	1	3

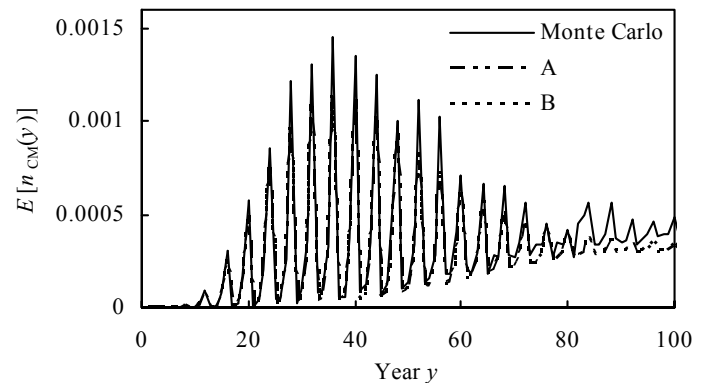Figure 3. Expected yearly failure rate, $E[n_{\text{CM}}(y)]$, example 1.

Figure 4. Expected yearly failure rate, $E[n_{\text{CM}}(y)]$, example 2.

Approach B gives rather good results, whereas the failure rate calculated with Approach A is noticeably too low in a steady state situation. However, the steady state situation has no influence on the discounted costs because the present value of incurred expenses for $y > 60$ years is approximately zero; confer Welte et al. (2006).

2.1.4 Relative distribution function

Instead of considering the results in the Markov model we analyze the relative distribution function $r(t)$ defined by

$$r(t) = \frac{F_{\text{Approach } j}(t)}{F_{\text{Gamma distribution}}(t)} \qquad j = A, B \qquad (8)$$

where we want to set the distribution function of the approaches that are presented in the sections before, $F_{\text{Approach } j}(t)$, in relation to the distribution function of the gamma distribution, $F_{\text{Gamma distribution}}(t)$. A good distribution fit is given by $r(t) \approx 1$.

Figure 5 shows several plots of the relative distribution function for a gamma distribution with different shape parameters and an approximation by Approach A and B. One can see that there is a large deviation in the left tail of the distribution. This means that the approximations are not suitable to model the lower left tail of the distribution. However, in a practical application, as for example in the presented examples, this seems to have only minor influence on the results. The risk is given by the probability to get a serious deterioration within one inspection interval. In those models, however, the length of the inspection interval is not situated in the extreme left tail of the distribution.

2.2 Partial fit of the distribution function

The weakness revealed in the previous chapter leads to the solution to fit the distribution function of the sum of exponential d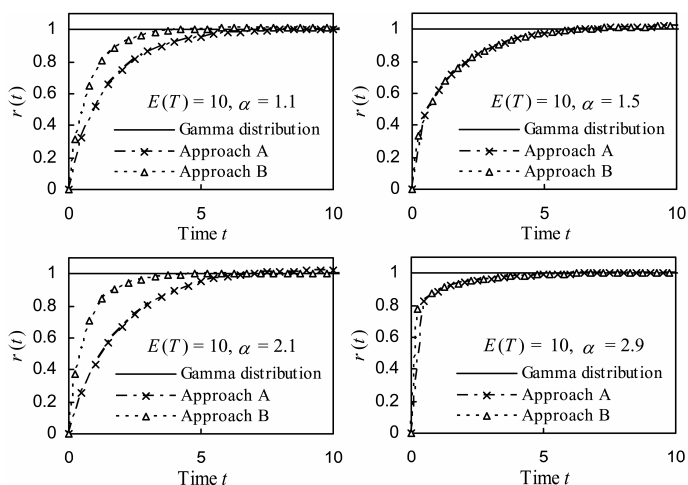istributions *partially* to the distribution function of the original gamma distribution. In the order of magnitude of t where accurate results are required, denoted t_f, the exponential distribution parameters can be adjusted such that $F(t_f)$ of the chain of exponential distributions is approximately equal $F(t_f)$ of the gamma distribution.

The probability density function of the sum of two independent random variables is given by the convolution rule (Papoulis 1991):

$$f_z(z) = \int_0^z f_x(z - y) \cdot f_y(y) \, dy \qquad (9)$$

Multiple application of the convolution rule leads to the probability density function of the sum of several random variables. A last integration step yields the distribution function $F(t)$. For very simple combinations, like e.g. two exponential distributions, the integrals can easily be solved analytically. For longer and more complicate sums of random variables, however, the analytic solution becomes cumbersome or impossible. Numerical methods can be used alternatively. However, the numerical solution of multiple integrals can become a tricky task. Thus, the evaluation of the exponential parameters by partial distribution fit can only be recommended if the other approaches give unsatisfying results.

Figure 6 shows two examples for a gamma distribution where the distribution function of the sum of exponential distributions is fitted to the distribution function of the original gamma distribution at around $t_f = 2$. Distribution fit 'i' fulfils the requirement that $r(t) \approx 1$ at t_f. Distribution fit 'ii' gives rather good results for $F(t)$ for $t = 1 \ldots 5$. However, the requirement that the expectation of the sum of exponential distributions is equal the expectation of the original gamma distribution was relaxed. Moreover, the solution presented in Section 2.1.2 is better for $t > 5$.
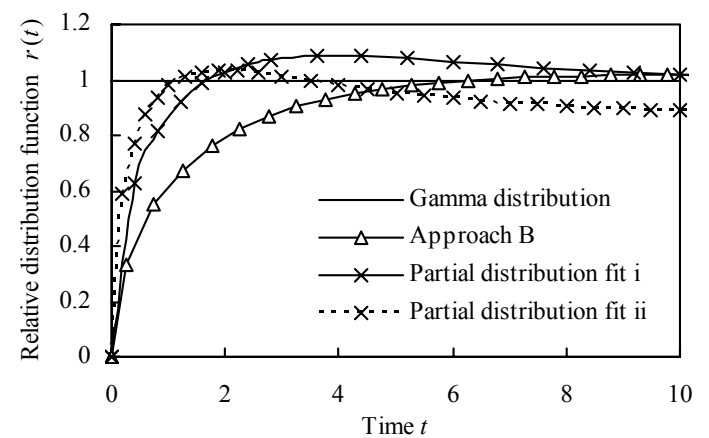

Figure 6. Relative distribution function, $r(t)$, for two examples of partial distribution fit.
Gamma distribution: $\alpha = 1.5$, $\beta = 6.67$ → $E(T) = 10$
Approach B: $E(T_1) = 7.89$, $E(T_2) = 2.11$
Partial distribution fit i: $E(T_1) = 8.8$, $E(T_2) = 1.2$
Partial distribution fit ii: $E(T_1) = 11.94$, $E(T_2) = 0.66$

Figure 5. Relative distribution function $r(t)$ for different shape parameters α.
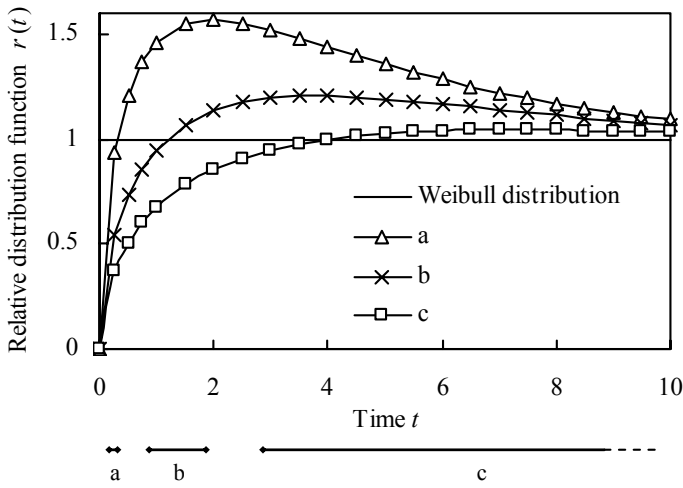
Figure 7. Three examples for partial distribution fit of the Weibull distribution. The section of t where the distribution is fitted is indicated by the bars below the graph.
Weibull distribution: $E(T) = 10$, $VAR(T) = 46.1$
a: $E(T_1) = 4$, $E(T_2) = 6$
b: $E(T_1) = 8$, $E(T_2) = 2$
c: $E(T_1) = 1$, $E(T_2) = 9$

2.3 *Weibull distribution*

There is no useful relationship between the Weibull distribution and the exponential distribution as it is given for the Erlang distribution by the sum of exponential distributions. Thus, a simple approximation rule giving acceptable results could not be found for the Weibull distribution. However, a partial distribution fit can be carried out. Figure 7 shows three partial distribution fits for a Weibull distribution. The time ranges where the distribution function of the Markov chain is fitted to the distribution function of the Weibull distribution is indicated in Figure 7. Distribution fit 'c' gives good results for $t \geq 3$. For smaller t, however, the approximation becomes rather poor.

3 EXPERT JUDGMENT IN A MAINTENANCE MODEL

A common problem in reliability analysis is the lack of data. A common way to overcome this problem is the use of expert knowledge. A typical expert judgment task is the estimation of expectation, median or percentiles of a stochastic variable (e.g. failure time, or sojourn time in a Markov model). Thereafter, a suitable probability distribution can be fitted to these estimates.

In this paper an expert judgment approach is considered where the expert assesses the expectation and the pth percentile. We want to assume that the pth percentile is smaller than the expectation, i.e. from a reliability engineer point of view, the expert has to assess an unexpected event like e.g. an early failure. The expert's task is to assess the probability q that a component fails after a (short) time interval t_q, or

vice versa, the time t_q when $(q \cdot 100)\%$ of the components has failed, i.e. $F(t_q) = q$. The time t_q is denoted the $(q \cdot 100)^{th}$ percentile.

Two groups of persons that are involved in an expert judgment process are the analyst(s) and the expert(s). From an analyst point of view, the challenge is to choose the 'correct' distribution class for the analysis and to choose q and t_q, respectively. Qualitative understanding about the failure mechanisms might be utilized in this process, e.g., crack initiation is often modeled by a Weibull distribution because a crack will start at the largest of many small material defects, whereas a gamma distribution is relevant when the failure process is driven by a series of random shocks. Analysts may tend to choose distribution types that can easily be handled mathematically, or they may use a certain distribution because the used model requires this type of probability distribution (e.g. the exponential distribution in Markov models). Consequentially, the expert should recommend a suitable distribution if he has good knowledge about the failure mechanisms.

Typical measures in maintenance modeling are e.g. the expected number of system failures or the mean time between failures as a function of the maintenance interval and the intervention strategy. The probability of a system failure is given by the probability that the system fails before it is replaced or before an inspection reveals that the system condition is in a failed or unacceptable state. The inspection interval is normally shorter than the expectation, median and mode of the distribution. Thus, the main interest will be in the left tail of the probability distributions, i.e. the risk of failure is due to this left part of the distribution. Even if the Weibull and the gamma distribution have very similar properties, there can be significant differences in the left tail of these distributions.

We recommend asking the expert to assess the expectation, and a percentile being in the same order of magnitude as the replacement/inspection interval of the system. Such a strategy will give rather small modeling errors, even when there is no knowledge about the failure mechanisms, i.e. even when the 'correct' distribution class is unknown. This is shown in this section by means of two examples. The first example is an age replacement strategy in a binary system and the second example is a multistate Markov model.

To ask experts questions that are related to the length of maintenance intervals has also practical reasons (van Noortwijk et al. 1992). Experts are usually familiar with thinking in terms of maintenance intervals and their knowledge about the system's behavior is closely related to events and degrees of degradation occurring within one interval length.

3.1 Age replacement

This first and very simple example considers an age replacement policy as described in Rausand & Høyland (2004). Under an age replacement policy a system is replaced upon failure or at a specified operational age t_0, whichever comes first. It is assumed that the lifetime T of the system can be described by a probability distribution and that the system is binary, i.e. the system has only two states: Functioning or failed.

According to Rausand & Høyland (2004), the mean time between failures, *MTBF*, can be calculated by

$$MTBF = \frac{1}{F(t_0)} \int_0^{t_0} \left(1 - F(t)\right) dt \qquad (10)$$

where t_0 = replacement age; and $F(t)$ = cumulative distribution function of lifetime T.

Now, we want to assume that the 'correct' lifetime distribution is unknown, i.e. it is unknown which type of probability distribution should be used for the modeling. Thus, we want to consider both the gamma and the Weibull distribution as potentially 'correct' lifetime distribution.

In this example, it is assumed that the failure time of the real system is gamma distributed. Shape and scale parameter are given by $\alpha = 2.29$ and $\beta = 17.47$, respectively. However, neither the expert estimating the lifetime properties nor the analyst eliciting the expert opinion knows this fact. If no failure occurs, the system is regularly replaced after a period t_0. This is, however, well known for both the expert and the analyst.

Two cases (I and II) are analyzed in this example. In the former case the replacement age is 12 years, whereas in the latter case the replacement age is 4 years, i.e. $t_{0,\text{I}} = 12$ years and $t_{0,\text{II}} = 4$ years. If the lifetime of the system is gamma distributed with $\alpha = 2.29$ and $\beta = 17.47$, the probability getting a failure before replacement is 10% (case I) and 1.1% (case II), respectively. This means that $t_{0,\text{I}}$ is the 10th percentile of the lifetime distribution and $t_{0,\text{II}}$ is approximately the 1st percentile of the distribution.

As described above, the elicitation process is carried out by asking the expert to assess expectation and a pth percentile of the distribution. The remaining degrees of freedom in this expert judgment task are given by the distribution class and the choice of p, i.e. either the expert or the analyst has to decide what type of distribution is used in the following analysis steps and which percentile is used.

Now, we assume that the analyst (or maybe the expert) chooses p and the distribution class and the expert gives an estimate for expectation and pth percentile of T. The chosen distribution (either the gamma or the Weibull distribution) is fitted to the expert estimate and afterwards *MTBF* is calculated by means of Equation 10.

Figure 8 and Figure 9 show the mean time between failures for different combinations of p and distribution class for case I and case II, respectively. The chart curve for the gamma distribution is a straight line, i.e. *MTBF* is constant for all p. As mentioned before, the real system behavior is a gamma distribution. At this point, we want to make use of the term 'model of the world' (Chhibber et al. 1992). In the current example the 'world' is gamma distributed, whereas the 'model of the world' can be a gamma or a Weibull distribution. The expert's experience is based on observations, this means it is based on observations of the real world. Thus, a perfect (unbiased) expert gives estimates that correspond to the percentiles in a gamma distribution. If now the calculations are carried out with the correct 'model of the world' (gamma distribution), *MTBF* is a constant value, regardless at which percentile the expert gives his (unbiased) estimate. However, if the wrong 'model of the world' (Weibull distribution) is used, the calculated *MTBF* will vary depending on the percentile used for the expert judgment.

Both plots have a common feature: The graphs for the gamma and the Weibull distribution have an intersection at one point. This means that there is one p where the choice of the distribution has no influence on *MTBF*. This specific point is given by p where the pth percentile is approximately equal to the length of the replacement interval t_0. In other words,
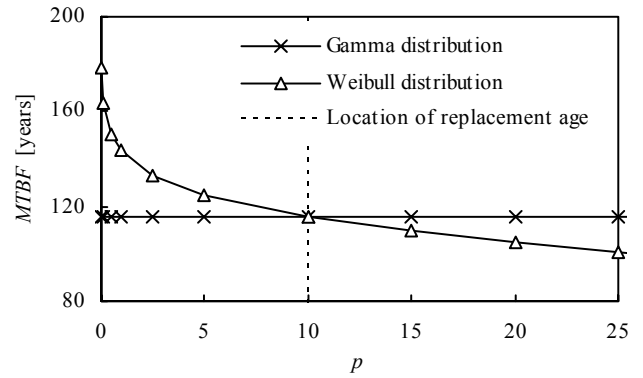

Figure 8. *MTBF* vs. p in the age replacement model. Case I: Replacement age t_0 is the 10th percentile of the lifetime T.
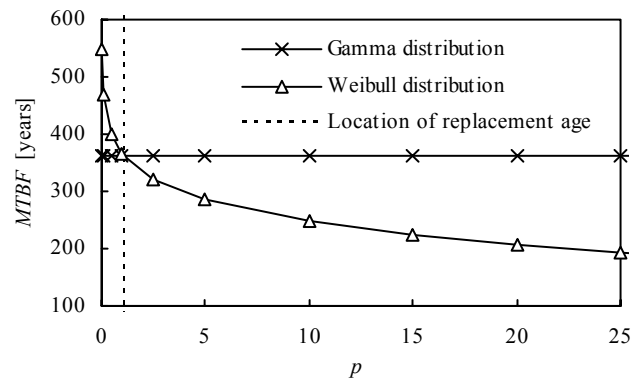

Figure 9. *MTBF* vs. p in the age replacement model. Case II: Replacement age t_0 is the 1.1th percentile of the lifetime T.
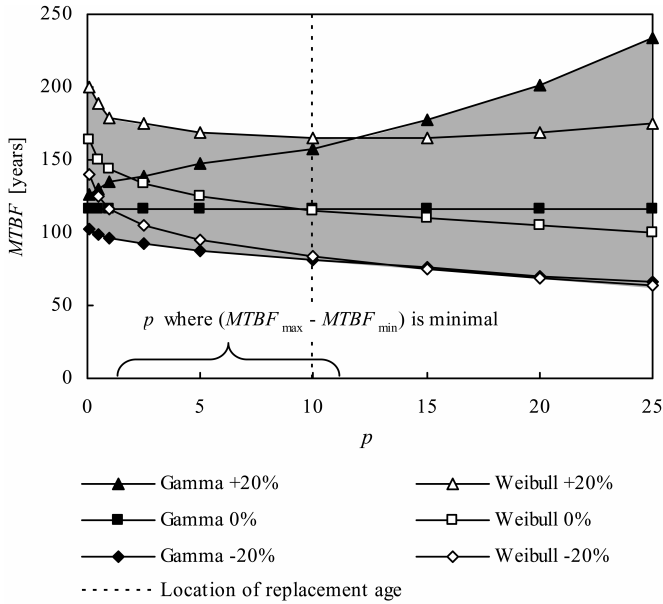
Figure 10. *MTBF* vs. *p* in the age replacement model with uncertain estimate for the pth percentile.

by choosing *p* such that $t_p \approx t_0$, the distribution class has no influence on the calculation of *MTBF*.

If we look on Equation 10, this result is not surprising. We can expect that the survivor function, $(1-F(t))$, is very close to one in the interval $(0, t_0)$ because we usually require high reliability of technical systems. This holds regardless of the distribution type used for the modeling. Thus, the integral expression in Equation 10 has a result varying not much for different distributions. $F(t)$ is equal for the gamma distribution and the Weibull distribution at the point where the expert estimate is given, i.e. at $t = t_p$. Thus, *MTBF* hardly depends on the choice of the distribution type if $t_p = t_0$, i.e. if the pth percentile corresponds to the replacement age t_0.

The assumption that the expert is perfect and unbiased is unrealistic. We generally expect good estimates from experienced experts. However, expert errors are possible without any doubt. Thus, it remains the question if the suggested rules also are valid if the expert gives biased estimates.

If we e.g. assume that the expert systematically over-/underestimates the pth percentile and if the minimum or the maximum error is +/-20% of t_p, then, the sample space for *MTBF* for different combinations of *p*, distribution class and error is given by the grey area in the diagram in Figure 10. If *p* is chosen approximately in the order of magnitude of the replacement interval, the possible spread for *MTBF* ($= MTBF_{max} - MTBF_{min}$) becomes small and is part of the area where the calculation error attain its minimum value. The diagram shows the curves both for the gamma and the Weibull distribution for the minimal possible (-20%) and the maximal possible (+20%) error. In addition the curves for perfect estimates (0% error) are shown. Again, each pair of curves (Gamma/Weibull) at a given error has one intersection at *p* such that $t_p \approx t_0$.

3.2 *Markov model*

The Markov model described in Section 1 is considered in the second example. The sojourn times, T_k, are estimated by experts. As suggested before, this can be done by asking the experts for expectation and p^{th} percentile of T_k. Similarly to the example presented in Section 3.1, it is analyzed how the choice of *p* and distribution class influences the calculated results. The effective failure rate is calculated for different combinations of *p* and distribution class. Once again, the real system behavior is

Table 4. Duration of sojourn times.

State *k*	Shape *α*	Scale *β*	Expectation years	Standard deviation years
1	2.29	17.47	40	26.44
2	2.29	8.73	20	13.21
3	2.29	4.37	10	6.61
4	2.29	2.18	5	3.30

Table 5. Inspection interval lengths for both case I and II.

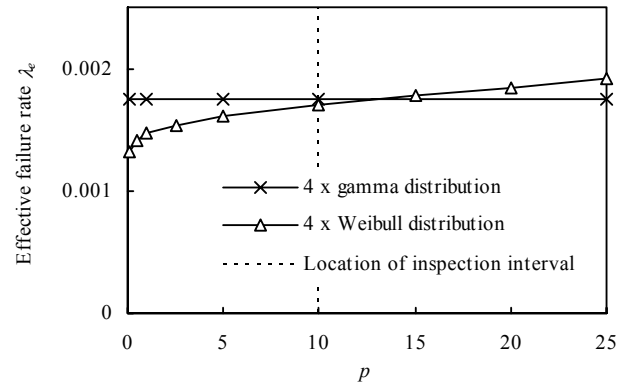| State *k*	Inspection interval τ_k	
	Case I years	Case II years
1	12	4
2	6	2
3	3	1

Figure 11. Effective failure rate vs. *p* in the Markov model. Case I: Inspection interval length τ_k is 10th percentile of sojourn time T_k.
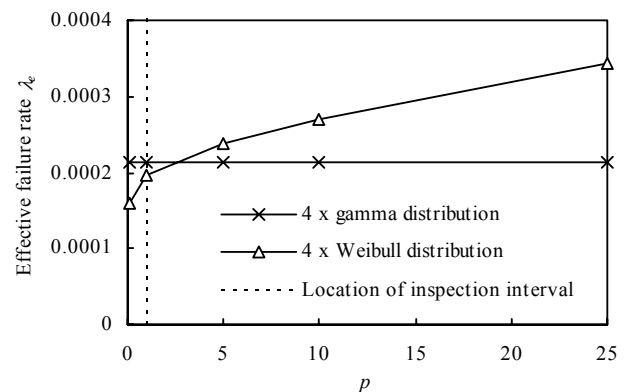

Figure 12. Effective failure rate vs. *p* in the Markov model. Case II: Inspection interval length τ_k is 1.1^{th} percentile of sojourn time T_k.

assumed to be gamma distributed, i.e. $T_k \sim \text{gamma}(\alpha_k, \beta_k)$. Shape and scale parameters for T_1 - T_4, as well as expectation and standard deviation, are given in Table 4. Two cases are analyzed with inspection interval lengths τ_k ($k = 1, 2, 3$) as given in Table 5. The two cases correspond to the situations where the length of τ_k is equal to the length of the 10th percentile of T_k and the 1.1^{th} percentile of T_k, respectively.

The effective failure rate for different combinations of p and distribution class is shown in Figure 11 for case I and in Figure 12 for case II. Once again, there is an intersection between the curve for the gamma distribution and the curve for the Weibull distribution. This intersection is close to the point where the percentile assessed by the expert is in the same order of magnitude as the inspection interval of the system. This means, if p is chosen such that $t_p \approx \tau$, the choice of the distribution class has only minor influence on the effective failure rate.

4 SUMMARY AND CONCLUSIONS

In this paper was shown that a general lifetime distribution like the gamma distribution or the Weibull distribution can be approximated by a sum of exponential distributions in a maintenance model. Such an approximation can be especially useful in semi-Markov models with sojourn times following a general lifetime distribution. Thus, the semi-Markov process is transferred into a conventional Markov-process. This simplifies the solution of the model.

Two simple approaches for an approximation of the gamma distribution for the use in maintenance modeling were presented in this paper. The approaches are based on simple rules. They can easily be applied and they give quite good results in many applications. However, the two approaches have weaknesses in the lower left tail of the distribution. Hence, a partial distribution fit can be carried out in applications where the suggested approaches give unsatisfying results and where a good distribution fit for the lower left tail is necessary. This will yield a good approximation at least in a smaller range of t. A partial distribution fit must also be applied for the approximation of the Weibull distribution. This can give good results for the central part and the right tail of the distribution. However, this method does not provide a good approximation of the left tail of the Weibull distribution.

In the second part of this paper was shown that if the expert judgment is carried out by assessing expectation and a lower percentile of the distribution, a recommended approach is to ask the expert to assess a percentile being in the same order of magnitude as the inspection/renewal interval of the system. By means of several examples it was shown that this approach will give rather small modeling errors, even when there is no knowledge about the correct lifetime distribution.

This finding can be utilized in the presented Markov model. Even if it is difficult to represent the Weibull distribution by exponential distributions, a case with Weibull distributed sojourn times can be modeled if the parameter estimation is based on expert judgment and the experts assess percentiles according to the recommendations above.

The recommendations are useful if the maintenance modeling is carried out for a situation with a constant replacement/inspection interval. If, however, the maintenance model is used to find an optimal maintenance strategy, it is not possible asking the expert to assess a percentile corresponding to the replacement/inspection interval because this interval is an unknown variable. In this case, one could try to collect the expert data at time t_p that approximately corresponds to the expected optimal inspection interval. If, after having calculated the optimal solution, the optimal interval diverges much from the expected solution, the experts could update their estimates and the calculations could be repeated.

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Paper 3

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Paper 4

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Paper 5

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Paper 6

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Paper 7

Influence of different operating patterns
on the life of Francis turbine runners

Proceedings of the HydroVision conference

Portland, Oregon, 31 Juli - 4 August 2006