NTNU – Trondheim
Norwegian University of
Science and Technology

Experimental Cuttings Transport in Horizontal Wellbore

The Determination of Cuttings bed Height

Francis Effiong

Petroleum Engineering
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Supervisor: Pål Skalle, IPT

Norwegian University of Science and Technology
Department of Petroleum Engineering and Applied Geophysics

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ABSTRACT

Poor hole cleaning problems during drilling operations has been the reason for several unwanted issues encountered during drilling. Hence, the numerous researches work on cuttings transportation. This project work presents an experimental and theoretical approach to the determination of stationery bed height during drilling operations. An experimental flow loop was used to simulate the oil field drilling process in which flow velocity was used to obtain a stable bed height. In the experimental work, drilling fluids of varying viscosity was used in order to ascertain the effect of fluid viscosity on cuttings deposition and how it affects other parameters. Also, theoretical models from past studies were used to compare with the laboratory results. In order to ensure that the model was used independent of experimental work, the author developed a simple geometry transformation from bed perimeter to height based on circle geometry. The result obtained showed that the theoretical results for bed height and perimeter were approximately the same as that of the practical result. The minimum flow velocity obtained from the adopted model was higher than the recommended flow rate from pump due to restrictions, as a result of laboratory and safety conditions at the time experiment was carried out.

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1. Introduction

The force of gravity has influence on the movement of drill cuttings during deviated and horizontal well drilling. It causes the build-up or deposit of drill cuttings along the bottom side of the wellbore. Such deposits are commonly called cuttings beds. Cuttings bed problems occur because, in inclined wells, drilling fluid velocity has a reduced vertical component compared to vertical wells (Ramadan 2001). Therefore, particles slip through the mud to the bottom side of the wellbore, where chances of re-entrainment are low. As drilling progresses with time, these particles continue to accumulate and form cuttings bed. Building up of cuttings bed can lead to occurrence of drilling problem. Usually insufficient hole cleaning results in a increase of cuttings concentration in the annulus, consequently when running the drill string axially it gets stuck, lost circulation could occur and it also hinders the process of running of casing. Obviously, cuttings beds impede drill pipe movement and often the drill pipe gets stuck resulting in greater rig time and higher cost of operation. Therefore, design of a drilling operation that ensures the removal of cuttings bed with minimum cost of operation is the main objective of the study of hole cleaning and cuttings transportation.

In the midst of the numerous problems and dangers which surround the drilling processes of horizontal and deviated wellbores, are the financial benefits. This explains why oil and gas industry will continue to embark on horizontal and deviated drilling, regardless of the associated risk, some of which are; the risk of excessive over-pull on trips, high drag and torque, stuck pipe, hole pack-off, wellbore steering problems, excessive equivalent circulating density, formation break down, premature bit wear, slow ROP, and difficulty in running casing. High drilling fluid flow-rate is one way to overcome the influence of force of gravity. As a result, several models have been developed to determine the critical and sub-critical velocities to ensure effective management of rate of cuttings deposition. Although successes have been recorded in solving the problem, there are variances in methods, procedures and sometimes result presented by different authors. More research on hole cleaning and additional input into existing theories is necessary to get more accurate solution. Especially now that drilling depths are on the increase as a result of dwindling reserve and economic fluctuations which has made cost of drilling to increase. There is still need for additional knowledge on determination of stationery cuttings beds height. This would help in further control of rate of particle build-up and prevention of the imminent danger which may occur as a result of cuttings bed. Additional knowledge on stationery bed height therefore, will help improve the hole cleaning process as it would help ensure effective application of existing models for predicting critical flow rate of drilling fluid. Hence, in this research work the author would be focusing on theoretical and experimental investigation of cutting build up, to obtain stationery bed height in horizontal wellbore, the theoretical and experimental study will be carried out in parallel.

The theoretical study will address:

1. The acquisition of knowledge from available sources on cuttings build-up, evaluate all aspects of the problem and discuss the challenges involved. The stationary height of cuttings build-up would be the priority of this theoretical research.
2. Application of existing mathematical models to estimate stable bed height on basis of controllable parameters.
3. The theoretical results would be tested against experimental data.

Experimental study will address

1. The design and build of the flow loop for the experiment.
2. Design a test matrix that would be used to perform the experiment to ensure achievement of the goal.
3. Perform tests to obtain stationery bed height during circulation in straight wellbores.
4. Develop a simplified empirical model for cuttings bed height.
5. Compare empirical model with theoretical ones.

Through these activities, the author hopes to reveal some of the down hole secrets during drilling operations.

2. Published Knowledge on Cuttings Bed Height

Previous investigations carried out by different researchers has proposed several solutions to the problem of wellbore cleaning and also paved the way for further research works. Therefore, this chapter is intended to review some of those works. Hopkins and Leickesenring (1995) mentioned that one of every three stuck pipe problem is caused by insufficient hole cleaning. It was reported by Adari et. al. (2000) that to correct the problem of stuck pipe could cost as much as one week of non-productive time (NPT) depending on the situation. Today the net time spent on every stuck pipe problem have declined.

2.1 Critical Velocities for Cuttings Removal

An attempt to control the amount of cuttings accumulated in the wellbore, operations such as wiper trip are often performed, but reaming operations require time and can significantly add to the costs of drilling horizontal and deviated wells (Hyun 2001). Two investigations were carried out by Hopkins (1995) and Bizanti and Alkafeef (2003) who presented a simplified procedure to predict critical mud flow rate required in cleaning deviated and horizontal sections. The summary of their work is presented in the table 2.1:

Table 2.1: Summary of Formula from the Work of Hopkins (1995) and Bizanti & Alkafeef (2003)

Authors	Hopkins	Bizanti and Akafeef
Step 1	$FMW = 2.117 - 0.1648 \times \rho_m + 0.003681 \times \rho_m^2$	Get RF from tables based on PV, YP and Hole size
Step 2	$v_s = FMW \times v_{sv}$	Obtain AF based on hole deviation angle
Step 3	$v_{min} = (v_s \times \cos\theta) + (v_2 \times \sin\theta)$	$TI = SG \times RF \times AF$
Step 4	$v_2 = C * \left[\left[\left(\frac{\rho_s - \rho_m}{\rho_m}\right) * g^3 * \left(\left(\frac{d_h - d_p}{12}\right)\right)^3\right]\right]^{\frac{1}{6}}$	Obtain minimum mud flow rate from tables corresponding to the hole size.
Step 5	$Q_{crit} = 0.04079\,(d_h^2 - d_p^2) \times v_{min}$	

- FMW in step 1 of Hopkins (1995) method, is the mud weight correction factor, it is dimensionless. This step is used to obtain the effect of mud weight on the slip velocity. While in the Work of Bizanti and Akafeef (2003), RF is the Rheology Factor, it is obtained from tables, based on the Plastic Viscosity, Yield Point and the Hole Size.

- Step 2 on Hopkins (1995), entails a two in one procedure, first v_{sv} (slip velocity in ft/min for the vertical condition) is obtained, by the use of figure 1, in the Appendix. By imputing the Yield point and assuming the average cutting size. After which the adjusted vertical slip velocity considering effect of mud weight and yield point is obtained by the equation presented in the table. While in the Work of Bizanti and Akafeef (2003), involves the use of the hole deviation angle to obtain the Angle Factor (AF).
- In Hopkins (1995), the step 3. Involves calculation of the formula presented above to calculate the minimum velocity (v_{min}) required to transport the cuttings. in the Work of Bizanti and Akafeef (2003), the step 3 involves the calculation of the Transport index (IT)
- In Hopkins (1995), the step 4 entails the calculation of v_2, which is the minimum mud velocity in the non-vertical section is obtained by inculcating C an empirical constant which is based on a laboratory data. In the Work of Bizanti and Akafeef (2003) this is the last step, it entails the determination of the minimum mud flow rate from tables corresponding to the hole size.
- In Hopkins (1995), the step 5 which is the last entails the calculation of minimum flow rate in gal/min by engaging the formula presented in the table 2.1 above.

Definition of the parameters and calculations involving the equations is placed in nomenclature section of this project work. Also, Bizanti and Alkafeef (2003) mentioned that the procedure can be used by the drilling personnel to predict the minimum flow rate required for hole cleaning. They developed a table and compared it with Hopkin's method against actual field data; it showed fairly close answers for hole greater than 45 degrees. The minimum flow rates obtained was lower than Hopkin's value and yielded closer answers to the actual field data. The difference in results obtained from application of both methods presented above and other related ones, might be responsible for the assertion put forward by Zamora and Hanson (1990) that, many contemporary hole-cleaning theories are based on research conducted by different investigators over the last two decades. They said research efforts by the industry have been extensive, but by no means complete or definitive. Despite subsequent refinements in the field, problems persist. They added that this can be attributed, at least in part, to misapplication of certain "universal" guidelines to specific field situations.

Norton (2002) in the second edition of his work, presented methods through which the critical annular method and critical flow rate necessary for cuttings transport can be obtained.

In order to prevent the build-up of cuttings bed, the methods are based on the Power law Model. Brief summaries of the steps involved are presented in table below;

Table 2.2: Summary of Formula from the Work of Norton (2002)

Author	Norton (2002)	
Step 1	$n = 3.32 \log \frac{\theta 600}{\theta 300}$	$n = flow\ behaviour\ index\ (dimensionless)$ $\theta 600 = 600\ viscometer\ dial\ reading$ $\theta 300 = 300\ viscometer\ dial\ reading$
Step 2	$K = \frac{\theta 600}{1022^n}$	$K = Power\ Law\ consistency\ index\ (dimensionless)$
Step 3	$x = \frac{81600\ (K) n^{0.387}}{(Dh - Dp)^n MW}$	$MW = mud\ weight\ ppg$ $Dh = Hole\ Diameer, in$ $Dp = Pipe\ Diameter, in$
Step 4	$AVc\ =\ (x)^{1+2-n}$	$AVc = Critical\ Annular\ Velocity, ft/min$
Step 5	$\mathrm{GPMc} = \frac{\mathrm{AVc}\ (\mathrm{Dh}^2 - \mathrm{Dp}^2)}{24.5}$	$\mathrm{GPMc} = \mathrm{Critical\ flowrate, gpm}$

This method is based on Power Law Model; $\tau = K\gamma^n$ and the steps required to perform the calculation is as explained in the chapter three of this work.

Kelessidis and Bandelis (2004) presented an article on coiled tubing drilling (CTD), in which they provided a review of the state-of -the -art modelling for efficient cuttings transport during CTD, they examined the critical parameters involved and established range of values for the parameters based on reference to practical findings. Also they proposed a different approach for predicting the minimum suspension velocity during CTD.

Rubiandini (1999) said that hole-cleaning problems could be mastered by defining the minimum mud rate that had a capability to clean the drilling wellbore. He expressed the minimum mud rate as a sum of the slip velocity and velocity of the fallen cuttings. The cuttings velocity was dependent on the wellbore geometry and magnitude of ROP. He further explained that mud weight, inclination angle, and RPM were major factors affecting cuttings transport mechanisms. Therefore, corrections factor of these parameters played a main role in the model he proposed. In his equation, the angle correction factor was obtained by using Cartesian dimensionless plotting between slip velocity (u_{slip}) and inclination.

Ranjbar (2010) compared the work of Rubiandini (2010) to that of Larsen et al. (1993). The analysis of the two empirical models showed that both models show the same trend for required cuttings transport flow velocity and flow rate when drilling parameters, such as mud weight, ROP, mud rheology and drill-pipe diameter varied. For the horizontal case, He observed that Larsen predicts flow rate that are not far from the flow rates typical seen in operations, however it slightly over predicts required cuttings transport velocity. Rubiandini‟s model seems to predict high flow rate required for cuttings transport. However, for the vertical case, the predicted rate seems to coincide with flow rates typical in operations. He further mentioned that the main advantage of Rubiandini‟s model is that in his work, he considered RPM as a variable that could affect the cuttings transport. The results also indicated that Larsen‟s model and Rubiandini‟s model show the opposite effect on required cuttings transport velocity when the cuttings size is a variable parameter. In the Larsen‟s model, smaller cuttings required higher flow velocity to be transported, while in the Rubiandini‟s model, the opposite is observed; larger cuttings need a higher flow velocity for transport in the wellbore. In the conclusion, he mentioned several recommendations on how to achieve better cuttings transport and hole cleaning. In the work of Larsen et al (1997), they developed a model for determining the minimum velocity for circulation of cuttings, but the model is only valid for 55^{o}- 90^{o} inclination. Also, according to Rubiadini (1999), the work of Peden's et al (1990) in which they presented an analytical equation for determining minimum flow rate for transportation of cuttings is defect of two parameters, which are namely inclination and Revolution per Minute (RPM).

2.2 Bed Height

Li et al. (2007) developed a one-dimensional transient mechanistic model of cuttings transport with conventional (incompressible) drilling fluids in horizontal wells. The model was solved numerically to predict cuttings bed height as a function of drilling fluid flow rate and rheological characteristics (n, K), drilling rates, wellbore geometry and drill pipe eccentricity. The results of the sensitivity analysis showing the effects of various drilling operational parameters on the efficiency of solids transport were presented. They said the model developed in the study can be used to develop computer programs for practical design purposes to determine optimum drilling fluid rheology (n, K) and flow rates required for drilling horizontal wells. The following assumptions were also made for the development of the cuttings transport model in horizontal wells:

- Drilling fluid rheology is represented by a power-law model.
- The cuttings are assumed to be spherical with uniform sizes, shape and velocity at a given cross-sectional area of the well.

- Slippage exists between the drilling fluid and cuttings.

Finally, their proposed method of determining bed height was summarized as follows;

- Initially, it is assumed that are no cuttings bed formed and the solids are fully dispersed within the drilling fluid. Velocity and solids concentration distributions are then calculated along the horizontal well.
- The criterion for the determination of ‘if there is a bed formation or not’ is as follows:
  Case I: Building of cuttings bed height
  If $u_f < u_c$, then bed height will increase, and a guess is made for Δh_b.
  Case II: Stabilization of cuttings bed height
  If $u_f = u_c$, then the height of cuttings bed is stabilized and the solution is found.
- The formation of a solids bed imposes effects on the flow behaviour in the upper layer. It narrows the upper flow channel and thus reduces the hydraulic diameter of the flow channel. Therefore, the flow area and the hydraulic diameter have to be re-evaluated.
- The fluid velocity and solids concentration distributions are re-evaluated based on the changed terms in Step 3.
- Repeat Step 2 to step 4 until a convergent bed height is obtained.

A research work on the Erosion of velocity of a cuttings Bed During the circulation of Horizontal and Highly Inclined Wells, carried out by Martins et al. (1997). In the work, they presented a test result which relates to the erosion of a solid bed, formed in full scale flow loop, by different polymeric suspensions in annular flow, at several values of flow rates, wellbore inclinations and drill pipe rotational speeds. They employed adequate instrumentation in order to allow for recording of the evolution of solids concentration and bed height as function of time. They said that the analysis of experimental data enables the prediction of the rate and time of erosion of cuttings bed and can be used as guidelines for the optimization of circulation of horizontal and highly inclined wellbores. They proposed an exponential type of equation to characterize the decay of concentration of solids in the annulus due to the interruption of solids injection as;

$$\mathrm{C}_{(\mathrm{t})} = \mathrm{C}_{\mathrm{R}} + A\mathrm{e}^{-\mathrm{t}/\tau} \quad (11)$$

C_R, A and τ are the regression parameters. C_R Represents the residual concentration of solids at infinite time and will be zero if hydraulic conditions can assure the completer removal of solids greater than zero if there is not sufficient energy to completely remove solids bed. The parameter A represents the amount of solids removed after long time of circulation. The parameter τ, known as the time constant, is a measure of how long it takes for concentration to fall to 1/e = 37% of A, or for removing 63% of the solids that the present hydraulic conditions are able to remove. Consequently, in their test, $C_R + A$ represents the amount of solids in the annulus while drilling.

Ozbayoglu et al. (2002) stated that knowing the amount of cuttings that accumulate inside horizontal and highly-inclined wellbores is part of the information that is essential for controlling bottom-hole pressure, preventing stuck pipe and minimizing the circulation time for cleaning the wellbore. In their work, they conducted a dimensional analysis using basic drilling information such as pump rate, fluid densities and viscosity, drilling rate and well bore geometry, to develop three dimensional groups for estimating the height of stationery cuttings beds deposited in horizontal and highly inclined wellbores for a wide range of drilling fluids, including foams and compressible drilling fluids for underbalanced drilling. They used the result of the experiment conducted to develop two different models. The first model is a traditional least-square fit of the dimensionless groups to constants. The equation calculates bed heights that are within 15% of measurements for all fluids. The second model they developed is an Artificial Neural Network (ANN) program that uses the same dimensionless groups but has been "trained" by using the test data. They mentioned that the ANN model predicts bed height with an error of less than 10% over the entire range of measured data. They included in their work, that the disadvantage of the first model is that different correlations are needed for different flow regimes i.e. Turbulent flow requires a correlation that is different from one for laminar flow. It was also pointed out that one of the advantages of the ANN model is that it can accommodate data from all flow regimes and provide equally good results. They said it could also be extended to include other effects such as drill string rotation.

Wang et al. (2011) developed a dynamic three-layer model based on solid–liquid flow mechanics. They considered the interaction between layers, the slip between solid and liquid phase, and the effect of drill pipe rotation. By orthogonal experiment regression, a formula for calculating the thickness of the cutting bed was developed that is suitable for field application. It was stated that the dynamic thickness of the cutting bed can be predicted by measured equivalent circulating density (ECD). Also they mentioned in their work that research has shown that the field equipment cannot attain the capability to keep the borehole clean, and when the thickness of the cutting bed exceeds 10%, a flushing method must be used to keep the borehole clean

Martins et al. (1996) carried out an experimental work developed on a ‘large scale flow loop which was aimed at the quantification of empirical parameters such as the shear stresses at the interface between fluids and cuttings bed as well as the maximum cuttings-wall friction factor which avoids bed movement. The tests consisted on the visualization of sandstone bed erosion by different polymeric solutions flowing through an annular section. Pressure losses and steady state bed heights were recorded for several input parameters, such as fluid flow rate, rheology and density, annular geometry, eccentricity and particle size. The experiment was conducted to guarantee a steady state fluid flow above a stationery cuttings bed, the momentum balance equations were used to characterize the problem.

Gavignet and Sobey (1989) presented a two layer model for cuttings transport. Their assumptions were that cuttings were falling to the lower side of the wellbore due to inclination of well and gravity. Therefore, the heterogeneous layer of the flow would be at the top with cuttings bed on the bottom as represented in the diagram below;

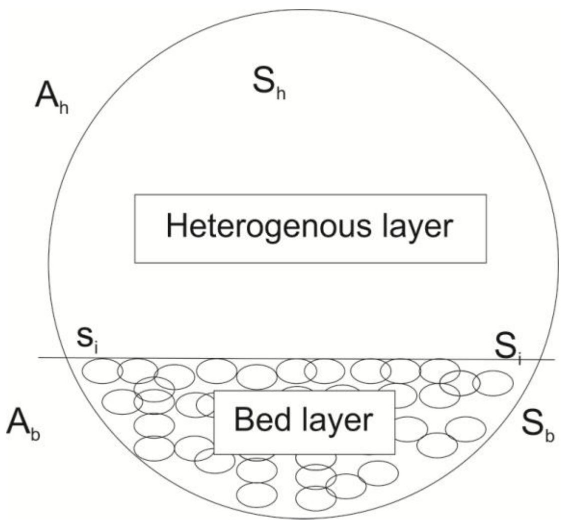

Figure 2.1: Schematic diagram for the two layer model representation of Heterogeneous and bed layer of cuttings in a wellbore during Circulation. (S = Perimeter, subscript-h = heterogeneous layer, subscript b = bed layer, subscript i= interfacial layer, A = Area)

In the work of Gavignet and Sobey (1989), it was also assumed that equal hydraulic pressure exists in both heterogeneous and the bed layer due to the closely packed nature of cuttings bed. Based on this assumption the hydrostatic pressure was neglected. The momentum balance for heterogeneous layer is;

$$A_h \left(\frac{\partial p}{\partial z}\right) = -\tau_h \times S_h - \tau_i - S_i \quad \text{(12)}$$

Momentum balance for the bed is expressed similarly by:

$$A_b \left(\frac{\partial p}{\partial z}\right) = -\tau_b \times S_b - \tau_i - S_i \quad \text{(13)}$$

The equation for total annulus area is given by:

$$A = \pi \left(r_o^{\,2} - r_i^{\,2}\right) \quad \text{(14)}$$

The diagram below describes the geometry of the cuttings bed and parameters required for its calculation;

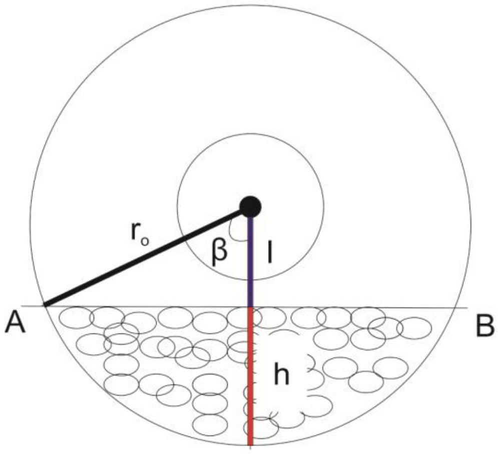

Figure 2.2: Geometry of cuttings bed and parameters required for calculation of bed height (h), used to develop equation (12) and (13). r_o = radius of pipe, l = length from centre of pipe to top of bed layer,

From Figure 2.2 it is seen that;

$$\cos\beta = \frac{l}{r_o} \quad \text{(15)}$$

Then bed height (h) is given by;

$$h = r_o - l = r_o(1 - \cos\beta) \quad \text{(16)}$$

The above equation would be used in the theoretical studies in next section to determine cuttings bed height theoretically.

Doron and Barnea (1993) stated that the two layer model developed by Doron et al (1987) does not have the ability to predict accurately, the existence of a stationery bed at low flow rates. They further explained that there are cases when stationery bed is observed, yet the model results only indicate flow with moving bed, as a result this leads to reduced reliability of pressure drop results for low flow rates (where a stationery bed can be expected). In order to solve the shortcomings, they introduced a three layer model.

3. THEORITICAL INVESTIGATION

This experimental work will be verified theoretically, hence, the theoretical aspect of this work, will entail the use of previous models presented in chapter two of this work to determine cuttings bed height, u_{min}, Q_{crit}, AVc and GPMc (eqn (3), (5), (9) and (10) respectively) .However, it is important that we should be able to picture a typical situation that occurs down-hole during drilling operation, before we can attempt a theoretical solution. Therefore, let us use Stokes law to describe a typical case.

Application of Stokes Law to describe cuttings transportation in Horizontal wells

In order to determine the stationery bed height in a wellbore during drilling operations, let us consider a horizontal pipe with solid-liquid mixture flowing through it. If the pump flow rate is high enough and the flow is turbulent then the velocity will be high enough to erode the bed and to suspend all the solid particles. However, if the flow rate is less, the flow velocity is accordingly less and the solid particles, which are denser than the carrier fluid, will eventually settle and agglomerate at the bottom of the pipe, in accordance with stokes;

$$F_{drag} = C_d A_c \rho_w \frac{{v_s}^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

The particles will then form a layer of bed, above which flows a heterogeneous mixture. Also, as the bed layer increases, the velocity increases at constant flow rate due to reduction in flow area, this causes the particles at the upper layer of the bed to be eroded. As the flow rate is reduced further, more solid particle will continue to settle and the height of the bed further increases. The bed becomes stationery when the sum of the driving forces acting on the bed is lower than the sum of the forces opposing the bed motion. If this situation continues for a long time, then the well gradually becomes filled up with layer of cuttings bed, if the perimeter of the bed can be measured or estimated and the diameter of the well can be measured, then the bed height can be calculated by using the approach described in Gavignet and Sobey (1989). Also, the minimum velocity required to keep cuttings suspended can be determined using the method described by Hopkins (1995)

3.1 Calculation of Minimum Critical Flow rate by Hopkins method

In this section, the Hopkins method presented in the literature review above was used to determine the critical velocities for cuttings removal and the result were compared to the velocities provided by the circulation pump. The results are presented in subsequent section and also conclusions as to why cuttings beds were formed in the wellbore. This method involves the following;

1. Calculation of the slip velocity using Hopkins slip velocity chart. However, in this work the slip velocity was calculated using analytical procedure because the drilling fluid in this case was water.

$$v_s = \frac{(\rho_c - \rho_m)^{0.667} \times 175 \times d_c}{\rho_m{}^{0.333} \times \mu^{0.333}} \quad \ldots\ldots\ldots\ldots \text{(17)}$$

2. Effect of mud weight on slip velocity is obtained from;

$$\mathrm{FMW} = 2.117 - 0.1648 \times \rho_m + 0.003681 \times \rho_m^2 \quad \ldots\ldots\ldots\ldots \text{(18)}$$

Then

$$v_s = \mathrm{Fmw} \times v_{sv} \quad \ldots\ldots\ldots\ldots \text{(19)}$$

3. Use the corrected slip velocity above to determine the minimum mud velocity as follows;

$$v_{\min} = (v_s \times \cos\theta) + (v_2 \times \sin\theta) \quad \ldots\ldots\ldots\ldots \text{(20)}$$

4. To calculate v_2 in eq. (17) above the following formula is used;

$$v_2 = \mathrm{C} * \left[\left[\left(\frac{\rho_s - \rho_m}{\rho_m}\right) * g^3 * \left(\left(\frac{d_h - d_p}{12}\right)\right)^3\right]\right]^{\frac{1}{6}} \quad \ldots\ldots\ldots\ldots \text{(21)}$$

5. Finally, the minimum flow required in gal/min is obtained using the formula below;

$$Q_{crit} = 0.04079\,(d_h^2 - d_p^2) \times v_{\min} \quad \ldots\ldots\ldots\ldots \text{(22)}$$

From the steps outlined above, it could be seen that this method is simple to understand and can be applied for field and laboratory purposes. This is the reason why the author chooses the Hopkins method, for comparative analysis with experimental work.

3.2 Calculation of Minimum Critical Flow rate by Norton (2002)

The important formulas necessary to use this method are presented in table 2.1. Hence, this section will be dedicated to giving a detailed explanation on how the formulas are applied. Then this method will be used to theoretically calculate the critical flow rate and compared with that obtained from experimental procedure. The required parameters are as follows;

- Mud Weight
- 600 viscometer dial reading
- 300 viscometer dial reading
- Hole diameter
- Pipe diameter or collar Outer Diameter

This method is based on Power Law Model; $\tau = K\gamma^n$ and the steps required to perform the calculation is as explained below;

1st Step: Determine n from $n = 3.32 \log \frac{\theta 600}{\theta 300}$

2nd Step: Calculate K from $K = \frac{\theta 600}{1022^n}$

3rd Step: Obtain x from $\boldsymbol{x} = \frac{\mathbf{81{,}600}\ (\boldsymbol{K})(\boldsymbol{n})^{\mathbf{0.387}}}{(\boldsymbol{Dh}-\boldsymbol{Dp})^{\boldsymbol{n}}\boldsymbol{MW}}$

4th Step: Calculate the critical annular Velocity (AVc) from $AVc = (x)^{1+2-n}$

5th Step: Determine the critical flow rate $(GPMc)$ from $GPMc = \frac{AVc\ (Dh^2 - Dp^2)}{24.5}$

The result obtained from application of the two theoretical methods of determining the critical flow rates presented above are presented in subsequent chapter of this work, while the calculation procedures are presented in the appendix of this research work.

3.3 Calculation of Bed Height

To determine the bed height through Gavignet and Sobey's method, Eqs. (12) and (13) would be used. These equations are based on the fundamental principle of trigonometric ratio. Consider the circular cross section of a pipe, as presented in Fig. 2. If the radius of the pipe is known, then a right angled triangle could be marked out as represented. With β as the angle of the marked out section, r_o is the radius of the pipe, *l* is the adjacent side of the angle which represents the distance from the centre of the pipe to the top of the bed layer and h is the height of the bed layer. Therefore, recalling Eqs. (12) and (13) the angle can be calculated as follows;

$$\cos\beta = \frac{l}{r_o} \quad \text{.......} \quad (23)$$

Then the angle can be used to determine bed height as in below;

$$h = r_o - l = r_o(1 - \cos\beta) \quad \text{.......} \quad (24)$$

The field and laboratory feasibility of this model, as well as the ease of its implementation for the determination of bed height has made it the preferred choice for the author of this work, in order to check for consistency of the laboratory results.

In order to be able to apply the model independent of the experimental investigation, the writer developed a simple geometrical transformation, using the knowledge of circle geometry. In this expression, the line AB is the same as the bed level which is presented in Fig. 2.2 above. Also some assumptions are made and listed as follows;

Assumptions

1. The wellbore is entirely circular.
2. The analysis made at a point can be generalized for the entire section under consideration.
3. The top of the bed layer is levelled, such that the length at a point can be determined by considering it as the length of a chord of circle (e.g. line AB in Fig. 3.2 below)

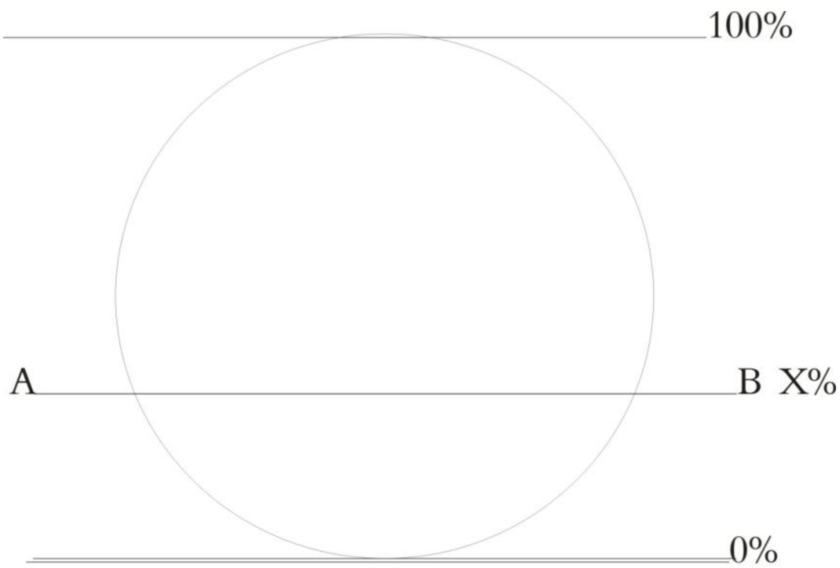

Figure 3.2: Diagram for geometry transformation (line AB, represents the assumed level of bed height which is equal to x % of the entire circular section at that point)

Considering a point in the circular wellbore with a known diameter, and selecting some positions along the vertical axis of the horizontal wellbore and representing it as in the diagram above (Position AB = x %). Taking the selected position (x %) and assuming the stationery bed layer in the wellbore is filled to that mark, then we can write a general formula for the expression for determining perimeter for a bed layer in the wellbore as follows;

$$Bed\ Perimeter\ (S)\ at\ x\ \%\ mark = \left(2\sqrt{r^2 - p^2}\right) + \left(\frac{x}{100} * 2\pi r\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

X is a position on the vertical axis of the point under consideration in the wellbore. It is to be determined by the Drilling engineer carrying out the analysis. The calculations done with the above equations are presented in the appendix.

Determination of Bed Perimeter by Gavignet and Sobey (1989)

Since the circulation of cuttings was done without drill string in the flow loop, the author uses the Case three, in order to compare results obtained to ensure that an effective conclusion is reached.

Under this method, the following steps are involved;

1st step: Cross sectional area in the heterogeneous layer

(A_h) is $A_h = A(\pi - \beta) - \sin(\pi - \beta)\ \cos(\pi - \beta) * r_0^2$ ……………………….(26)

2nd step: The cross sectional area of bed layer can be obtained from

$A_b = A - A_h$ ……..……………………………………………………………(27)

3rd step: The total wetted perimeter can be calculated as; $S = 2\pi(r_l + r_o)$

4th step: wetted Perimeter of heterogeneous layer is calculated as follows

$S_h = 2r_o \times \sin(\pi - \beta)$

5th step: wetted perimeter of bed layer; $S_b = S - S_h$

The result obtained from the above outlined 5 steps involved in calculating the perimeter of cuttings bed is presented in chapters four (4) of this work.

4. EXPERIMENTAL INVESTIGATION

In other to achieve this, the writer used different fluids with varying viscosity and flow velocity to carry out the experiment. Cuttings bed height was measured for each step of the processes involved. The results and observations would be used to make recommendations on effective determination of cuttings bed height during drilling of horizontal wellbore. When contributing factors controlling bed height in smooth wellbores are understood, the geometry complexity will be increased. Therefore an experimental setup (flow loop) was developed and used for the investigation of Cuttings Transport in Horizontal Wellbores. Image of the flow loop is shown in figure 1, as shown in the picture, the test section is supported on an horizontal bench which is constructed such that the test section is slightly tilted at an angle, in order to vary the angle of inclination of the test section.

4.1 TEST SET-UP

The entire test section is 6 m long with outer diameter (OD) 0.06 m and internal diameter (ID) 0.0545 m. A schematic representation of the flow loop is presented in Figure 4.1. In this experiment, the expansion section represented in the schematic diagram was not used; hence all pipe section had uniform size. As shown, the loop consists of a channel, which is used as the test section. The channel is made of a transparent PVC pipe that is connected at both ends to detachable steel joints of 1.73 m and 3.24 m respectively; the PVC itself is 0.94 m long.

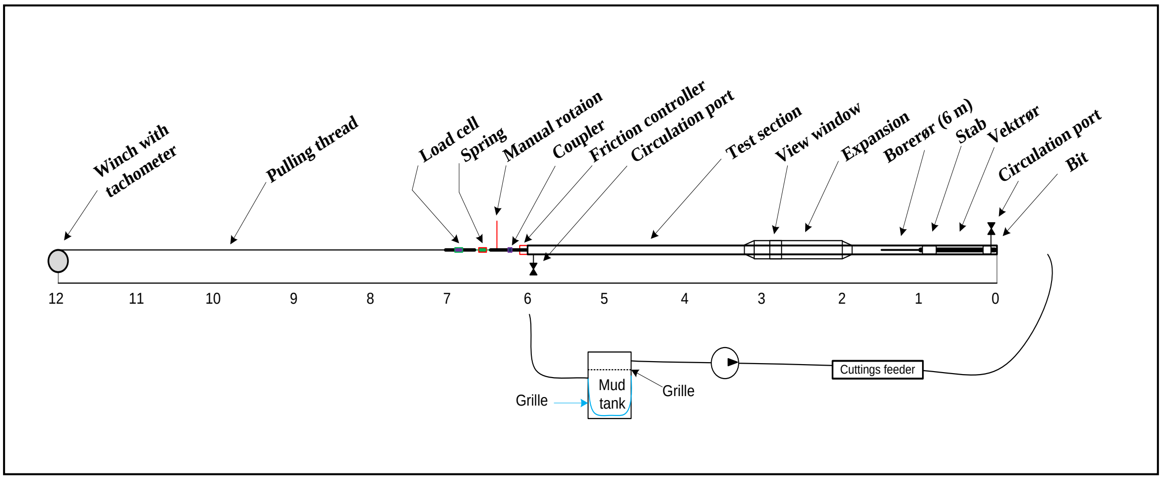

Figure 4.1: Diagram of the flow loop showing the major parts.(Skalle and Uduak 2012).

The loop is equipped with the necessary measuring equipments like the flow meter, connected to a personal computer for online displaying and recording. The main objective of the experiment was to observe bed height, hence more of observation was done in order to get the stable bed height and recordings were made. In order to avoid sand that has been transported from flowing into the channel. Therefore, a screen was placed inside the pit. The pit also served as a pumping tank for re-circulating the fluid. The temperature was maintained at room temperature. A manual controlled button on the flow meter was used to control the flow rate. The main objective was to determine the time it takes to form a stable bed height and then measure the bed height.

4.2 Test Matrix

A solid-liquid mixture was pumped through a horizontal pipe section and different physical parameters were monitored: flow frequency, stationary bed layer height (m), and Time of circulation before bed height was recorded. Pure water was used as the drilling fluid, after which varying concentration of HEC in water was used, the experimented flow frequency were 7, 10, 14, 19.5, 20, 25 and 30Hz. The inclination was 80^0, average cuttings specific gravity of 2.4 and average size of 2mm was used. Experiments were carried out at various flow frequencies and cuttings bed thicknesses were recorded. Amongst the equipments used during the process were; viscometer and mixer. The results are presented in subsequent chapter of this work.

Table 4.1 the Test Matrix

	Variables	**Variations**					**Total**
1	Flow rate	q_1	q_2	q_3	q_4	q_5	5
2	Viscosity	μ_1	μ_2	μ_3	μ_4		4
3	Cutting size	S_1					1

Where Concentrations are;

$$C_1 = 0.0\text{gHec/L}\, H_20$$

$$C_2 = 0.5\text{gHec/L}\, H_20$$

$$C_3 = 1\text{gHec/L}\, H_20$$

$$C_4 = 2\text{gHec/L}\, H_20$$

4.3 TEST FACILITY

4.3.1 Viscometer

The model 800 eight speed electronic viscometer by OFI Testing Equipment Inc. was used to obtain precise measurement of rheological properties of the fluids. Eight precisely regulated test speeds are provided by the OFI pulsed-power electronic speed regulator. The eight speeds are 3 (gel), 6, 30, 60, 100, 200, 300 and 600 RPM. A higher stirring speed is provided and speeds may be changed without stopping the rotor with a control knob selection switch.

Figure 4.2 Fann VG Viscometer

4.3.1.1 Procedures for Operation

1. Place a fresh sample of drilling fluid in the cup, filling it up to the scribed line inside the cup.
2. Immerse the rotor sleeve exactly to the scribed line by raising the platform and firmly tightening the lock nut on the platform as shown in fig 4.1 above.
3. Rotate the speed selector knob to the stir setting and mix the sample for a few seconds.
4. Rotate the knob to the 600 RPM setting, wait for the dial to reach a steady reading and record the 600RPM reading.
5. Rotate the speed selector Knob to the 300 RPM setting, wait for the dial to reach a steady reading and record the 300 RPM reading.

6. Rotate the speed selector knob back to the stir setting and re-stir the sample for a few seconds.
7. Rotate the speed selector Knob to the 200 RPM setting, wait for the dial to reach a steady reading and record the 200 RPM reading.
8. Rotate the speed selector Knob to the 100 RPM setting, wait for the dial to reach a steady reading and record the 100 RPM reading.
9. Rotate the speed selector Knob to the 6 RPM setting, wait for the dial to reach a steady reading and record the 6 RPM reading.
10. Rotate the speed selector Knob to the 3 RPM setting, wait for the dial to reach a steady reading and record the 3 RPM reading.

4.3.1.2 Calculations

$$Plastic\ Viscosity\ (PV), cP = 600\ RPM\ reading - 300\ RPM\ reading$$

$$Yield\ Point\ (YP), {}^{lb}/_{100\ ft^2} = 300\ RPM\ reading - Plastic\ Viscosity\ (PV)$$

$$Apparent\ Viscosity\ (AV), cP\ = 600\ RPM\ reading\ \div 2$$

The actual calculation procedures are presented in the appendix, while the result obtained are presented in the next chapter of this research work.

4.3.2 Mixer

In order to obtain an evenly mixed mixture of water and HEC, a mixer was used stir to vigorously, until the desired result was obtained. Below are images of the mixer and mixing process of the drilling fluid.

4.3.2.1 Procedure

1. Ensure that the fan of the mixer is firmly held in place, by using the screw knob available with the mixer for this purpose.
2. Obtain a wide enough container, which will enable the blade on the fan to rotate freely without hitting sides of the container.
3. Pour desired quantity of liquid and substance to be mixed into the container.
4. Ensure that the mixer is properly placed such that the blade is centralized in the container as in the figure below.
5. Connect the mixer to a power source and turn on the mixer
6. Rotate the handle at the rear of the mixer to obtain your desired mixing speed.

7. Allow it to mix, until the desired mixture is obtained.
8. Turn off the mixer and remove from the mixture.

Figure 4.3:The mixer, While not in use

Figure 4.4: The Mixer while in Use

4.3.3 Pump

This section outlines the steps involved in the processes of involved in the controlling of the pump and circulation of cuttings out of the flow-loop.

4.3.3.1 Procedures:

a. For controlling pump

1. Turn pump button to ON. Notice the green light as the white line on the button is in upward position.
2. Open the control panel of the pump
3. Regulate the pump flow rate
4. Press run: Mud is pumped through the system
5. Press stop: to stop the circulation
6. Close control panel
7. Turn the pump button off

b. For circulation during cuttings transport experiment

1. The mud tank was completely filled with water, since water was used as the drilling fluid.
2. The Prefill test section was filled with 2000 millilitres of cuttings from the rear, while pipe is approximately at an angle of repose.
3. The test section was connected to the circulation system
4. The fluid was pumped (according to the procedure for controlling pump above) until the cuttings formed a stationery bed for the selected pump rate.
5. The time it took to form stationery bed was measured and the bed height and perimeter was also measured.
6. The above procedure was repeated for 4000 millilitres of cuttings.
7. The result obtained was recorded, analyzed and compared with result obtained from theoretical studies.

Figure 4.5: The pump, while in use

Once the pump is started and circulation is in progress, the cuttings gradually moves from the wellbore to the mud pit. The pictures of different cross sections of the pipe during the circulation process are presented in Figure 4.6 to 4.8 below.

4.3.3.2 Pump Calibration

The pump is configured to work in Hertz. Therefore, in order to ensure ease of analysis, it is pertinent that the author determines the flow rate in litre Per Seconds (l/sec). there author, therefore carried out a manual calibration for each of the drilling fluid according to the process outlined below:

4.3.3.3 Manual Calibration Procedures

1. The drilling fluid was mixed to the desired specification (0.0g Hec/1Litre H_2O)
2. The fluid was poured into the mud tank
3. Then the pipe was disconnected from the flow loop and held over a 10 litre container.
4. A stop watch was held in position,
5. The pump speed was set to 5Hz
6. The stop watch and pump was started simultaneously.
7. The time to collect 10 litre of fluid was measured and recorded.
8. The process was repeated for four times and average value was determined and recorded.
9. The procedure 1 to 8 above was repeated for pump speed 10Hz, 15Hz, 20Hz, 25Hz and 30Hz.
10. Then the mud is discarded from the system
11. The author also repeated Procedure 1 to 10 above for drilling fluid specifications 0.5g Hec/l, 1g Hec/l and 2g Hec/l respectively.
12. The results were collated and used to determine the flowrate of the pump, the result are tabulated as in below.

4.4 Pump Calibration Result

The result obtained from the calibration procedure explained above, is tabulated below;

Table 4.2: Pump Calibration Result

	Quantity of Hec present in 1 Litre of H_2O			
Frequency (Hz)	**for 0.0gHec/l**	**for 0.5g HEC/l**	**for 1g HEC/l**	**for 2g HEC/l**
	Flowrate (Litre/min)	Flowrate (Litre/min)	Flowrate (Litre/min)	Flowrate (Litre/min)
5	28.6	26	23.5	13
10	33.3	29.6	26.3	15
15	38.9	34.5	35.9	20.1
20	44.8	41.6	37.5	35.2
25	49.1	42.9	38.7	35.3
30	51.3	50	41.7	37.7

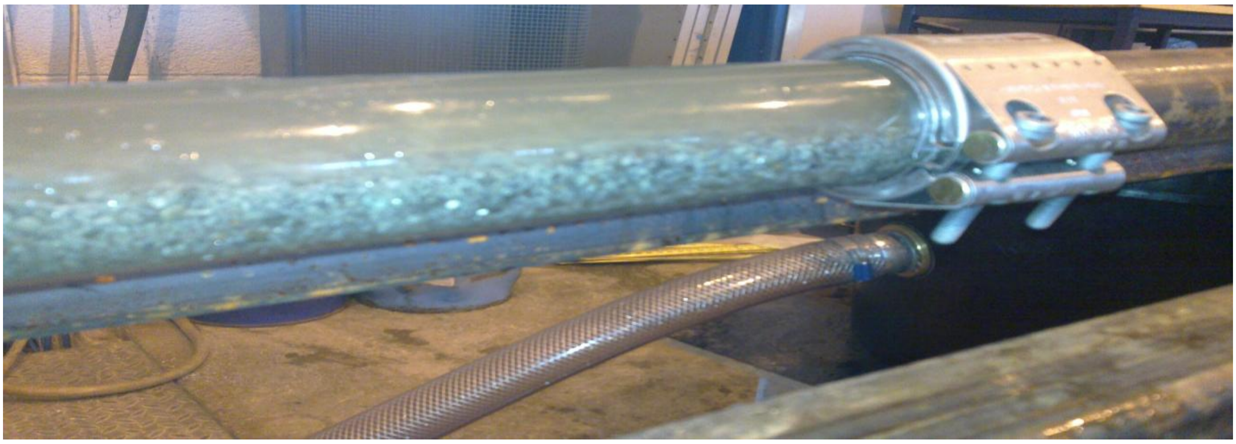

Figure 4.6: the circulation process of the cuttings

Fig 4.7: Another section of the wellbore, showing the circulation process cuttings.

Fig 4.8: Closer look at a cross section of the wellbore, as circulation progresses.

4.5 Experimental Results

Having carried out the processes outlined in the experimental procedures above, the author went ahead to obtain the desired readings and the results are tabulated below:

Table 4.3 Measured Laboratory Mud Rheology

Mud	**600 rpm**	**300 rpm**	**200 rpm**	**100 rpm**	**6 rpm**	**3 rpm**
0.5gHec/L H20	4	2.5	2	1	0.5	0
1.0gHec/L H20	4.5	3.5	2.5	2.0	0.5	0
2.0gHec/L H20	7	5	4	3	1	0.5

Table 4.4 Experimental Readings for 0.0gHec/L H_20 AS Drilling Mud

Flow	**Bed Height**			**Average**	**Bed Perimeter**			**Average**
Frequency	1^{st} Run	2^{nd} Run	3^{rd} Run		1^{st} Run	2^{nd} Run	3^{rd} Run	
10	0.04	0.043	0.042	**0.029**	0.14	0.15	0.16	**0.150**
15	0.033	0.035	0.035	**0.026**	0.14	0.14	0.14	**0.140**
20	0.032	0.032	0.31	**0.025**	0.13	0.135	0.135	**0.135**
25	0.030	0.029	0.031	**0.023**	0.128	0.132	0.130	**0.130**
30	0.027	0.028	0.029	**0.020**	0.121	0.120	0.19	**0.121**

Table 4.5 Experimental Readings for 0.5g HEC/ 1Litre H_2O Drilling Mud

Flow	**Bed Height**			**Average**	**Bed Perimeter**			**Average**
Frequency	1^{st} Run	2^{nd} Run	3^{rd} Run		1^{st} Run	2^{nd} Run	3^{rd} Run	
10	0.029	0.029	0.028	**0.029**	0.151	0.149	0.152	**0.151**
15	0.019	0.020	0.018	**0.019**	0.120	0.120	0.120	**0.120**
20	0.017	0.017	0.017	**0.017**	0.110	0.110	0.110	**0.110**
25	0.014	0.015	0.013	**0.014**	0.065	0.070	0.06	**0.065**
30	0.012	0.012	0.012	**0.012**	0.06	0.06	0.06	**0.060**

Table 4.6 EXPERIMENTAL READINGS FOR 1g HEC/ L H_2O DRILLLING MUD

Flow Frequency	Bed Height			Average	Bed Perimeter			Average
	1st Run	2nd Run	3rd Run		1st Run	2nd Run	3rd Run	
10	0.029	0.028	0.027	**0.028**	0.145	0.147	0.146	**0.146**
15	0.019	0.018	0.017	**0.018**	0.072	0.073	0.074	**0.073**
20	0.017	0.015	0.015	**0.015**	0.070	0.070	0.070	**0.070**
25	0.016	0.014	0.014	**0.014**	0.07	0.068	0.069	**0.069**
30	0.015	0.013	0.013	**0.013**	0.067	0.067	0.067	**0.067**

Table 4.7 Experimental Readings for 2g HEC/ L H_2O Drilling Mud

Flow Frequency	Bed Height			Average	Bed Perimeter			Average
	1st Run	2nd Run	3rd Run		1st Run	2nd Run	3rd Run	
10	0.022	0.020	0.020	**0.021**	0.120	0.120	0.120	**0.120**
15	0.019	0.016	0.017	**0.017**	0.112	0.112	0.112	**0.112**
20	0.015	0.014	0.014	**0.014**	0.065	0.070	0.060	**0.067**
25	0.012	0.011	0.012	**0.011**	0.05	0.051	0.05	**0.050**
30	0.01	0.01	0.010	**0.01**	0.02	0.02	0.02	**0.020**

Table 4.8 Results Obtained from Theoretical Calculations

	0.0g HEC/ 1LH_2O		**0.5g HEC/ 1LH_2O**		**1.0g HEC/ 1LH_2O**		**2.0g HEC/ 1LH_2O**	
	B.H	B.P	B.H	B.P	B.H	B.P	B.H	B.P
10	0.03	0.154	0.0249	0.149	0.029	0.146	0.0255	0.125
15	0.028	0.143	0.0245	0.1187	0.024	0.115	0.0235	0.112
20	0.0275	0.137	0.0235	0.112	0.0225	0.1053	0.0219	0.102
25	0.0265	0.13	0.0219	0.102	0.0219	0.1017	0.0205	0.0915
30	0.0249	0.128	0.021	0.0947	0.0215	0.0983	0.02	0.0879

B.H: Bed Height

B.P: Bed Perimeter

5 COMPARISON OF THEORITICAL AND EXPERIMENTAL RESULT

This chapter is committed to analyze and compare the results obtained from the research processes and procedures presented in previous chapters. From the experimental work, two outputs (bed height, bed perimeter) and fluid flow rate were obtained; these results were compared to that of the theoretical studies and the analyses are presented herein.

Table 4.3 through 4.7 displays the results obtained from both the experimental and theoretical work. Comparing the bed height and perimeter from, from the result obtained in both instances shows agreement. This therefore means that inferences can be drawn from further analysis of the results. In order to further point out the consistency in result of the experimental and theoretical work, a chart of bed height from the experimental studies and theoretical studies were plotted and the results shows that there is consistency in both works, as represented in figure 5.1 to 5.3.

Effect of Mud Rheology on Bed height can be observed from analysing the values presented in Table 4.3; from the table, the 2gHec/L H_2O obviously has the highest viscosity, while ordinary water is least viscous of the fluids. This property of the fluid impacts on the deposition of bed height. Generally, a more viscous fluid will lift and suspend more cuttings than a less viscous fluid, however when it comes to drilling, the effectiveness is not judged from lifting ability of the fluid alone. Therefore, looking at the Table 4.4 through 4.7 and considering the various level of bed heights obtained, it is evident that with ordinary water, the highest level of bed height was obtained at the beginning and at the end of the experiment. Also, a further look at the result obtained from varying the concentration of Hec per litre of water shows that as the quantity of Hec increased, there were reduction in bed height until a particular point where the change became minimal. Examining Table 4.4 through 4.7 for instance, and observing from the 10Hz to 20 Hz rows on each of the table, it is observed that for all level of concentration, there were remarkable changes in the level of bed height obtained with the increase of HEC concentration (i.e. increase in viscosity). After that point, the bed height seemed to remain the same, especially in Table 4.5 and 4.6. This therefore means that, with increase in viscosity of drilling fluid, there will be a corresponding decrease in the height of bed deposition. However, it takes more than just the viscosity to control bed deposition.

Effect of Pump rate on cutting deposition can be observed by examining and comparing the predicted and experimental flow rate. The summary of both flow rates are presented in Table 5.1 below;

Table 5.1: Theoretical Minimum flow rate and experimental minimum flow rate in litre/min

Concentration of HEC	Experimental	Method 1	Method 2
0.5g HEC/ 1LH_2O	50	83.27	94.63
1.0g HEC/ 1LH_2O	41.7	80.62	132.48
2.0g HEC/ 1LH_2O	37.7	79.49	162.76

From this table, it is observed that with increase in viscosity, there is a corresponding drop in the efficiency of the pump. Since, it is advisable to maximize flow rate during drilling and to enhance turbulence in the annulus if the ECD allows it, then it is pertinent that fluid pump flow rate be given optimum attention if bed height is to be controlled. Therefore, the pump rate calls for a re-examination of the effect of viscosity on height, it brings to book that even though increase in viscosity and gel strength, helps to maintain suspension of cuttings and will ultimately reduce bed height, it also impacts negatively on the pump by reducing the flow rate obtained at a point in time. This is because, with a more viscous fluid, flow is restricted as particles are closely bonded compared to when it is loosely packed. The flow rate shows that the experimental flow rate are less than the predicted flow rate from the models, which is presented in the appendix. This goes on to explain why the cuttings were not completely transported out of the wellbore. This shows that flow circulation rate, is a very important parameter during cuttings transport.

Figure 5.1 to 5.4: shows further, the comparison and change in the experimental and theoretical bed height with flow rate. As the flow rate increased, the bed height decreases, as the flow rate is further increased the height of the bed drops accordingly. Therefore, it could be further observed that the drilling fluid flow rate is a major factor controlling the formation of cuttings bed height. However, as stated earlier, other variables have a role to play in the deposition and accumulation of cuttings in a horizontal wellbore.

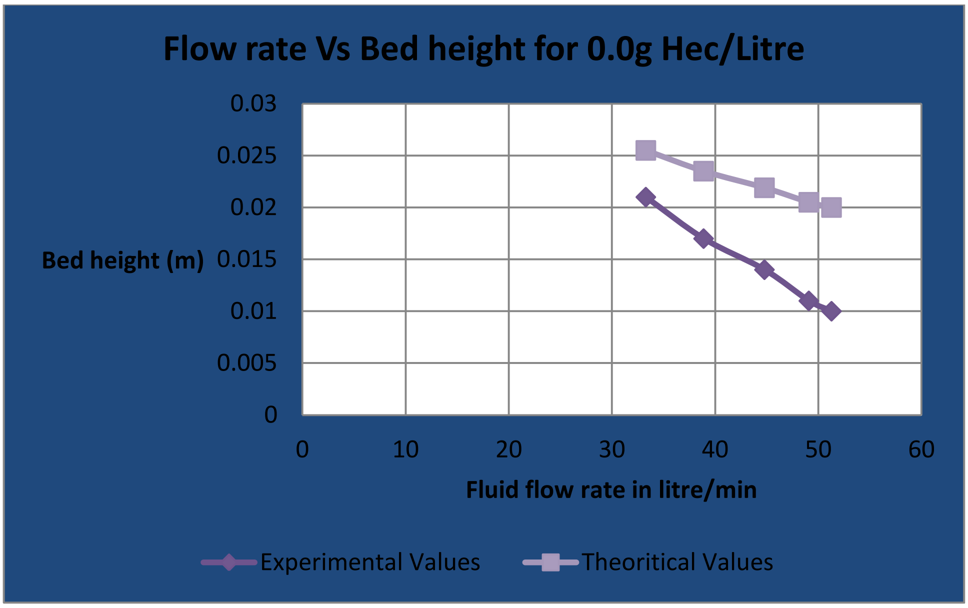

Figure 5.1: Theoretical and experimental bed height versus flow rate for 0.0g Hec/l H_20.

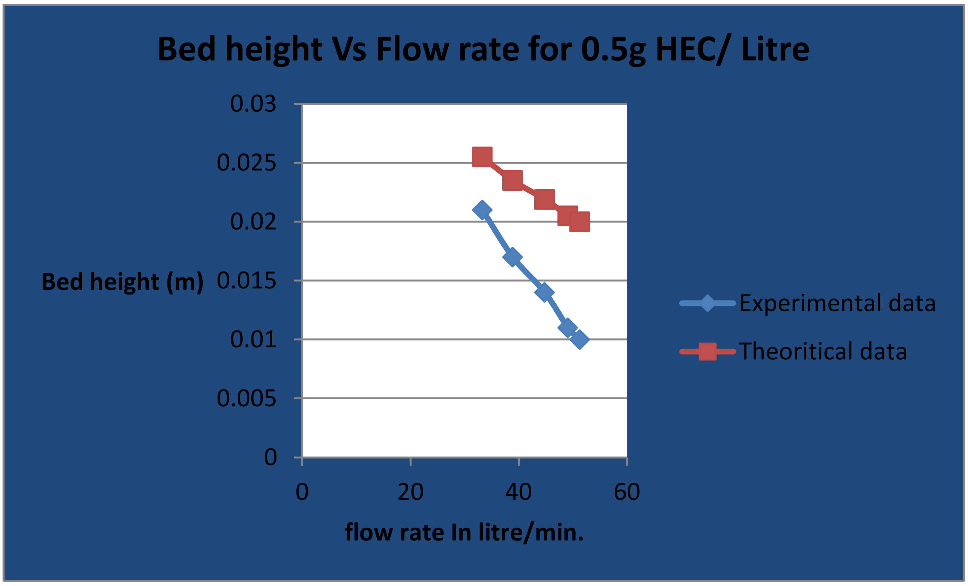

Figure 5.2: Theoretical and Experimental bed height versus Flow rate for 0.5g Hec/l H_20.

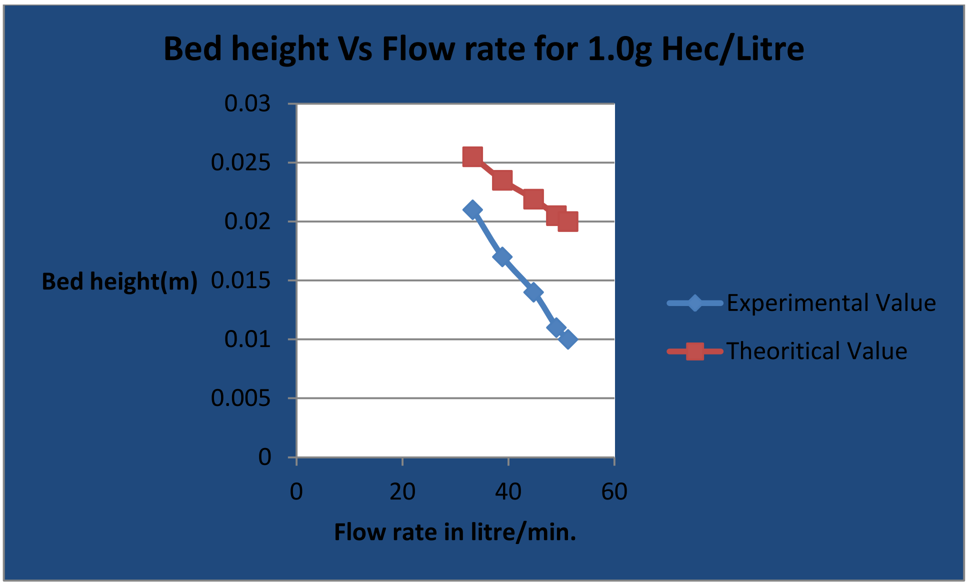

Figure 5.3: Theoretical and experimental bed height versus flow rate for 1.0g Hec/l H_20.

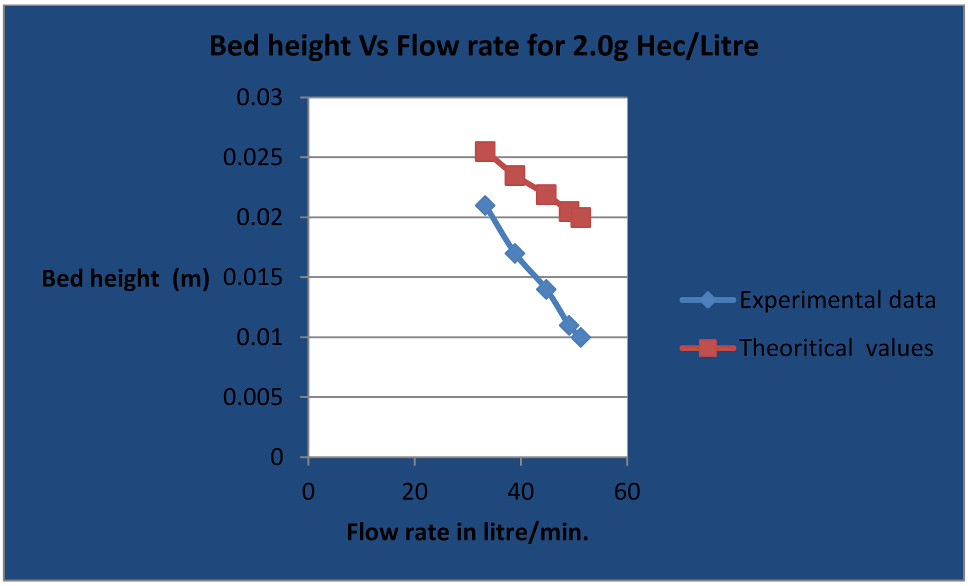

Figure 5.4: Theoretical and experimental bed height versus flow rate for 2.0g Hec/l H_20.

5.1 Discussion of Result

Basic parameters for field practice of horizontal well drilling were applied in carrying out theoretical investigation. For instance, the two layer method of Gavignet and Sobey (1989) for determination of bed height and bed perimeter was quite consistent because the differences between the experimental and theoretical readings, even with varying rheological properties of the fluids were observed to be less than 0.002 m. This can be easily seen from figure 5.5, 5.6 and 5.7 below, the chart shows that the experimental and theoretical bed heights are approximately equal. This helps to establish the validity of experimental results.

In order to further verify experimental result and ascertain the effectiveness of theoretical prediction, the Norton (2002) method of predicting minimum velocity and flow rate was used together with Hopkins method of obtaining critical flow rate and velocity. These helped to shed more light on the factors which influenced the result obtained in the experimental investigation. For example, it answered the question on why cuttings were left in the wellbore and established the factor responsible for cuttings bed layer formation. This answer is evident from Table 4.4 through 4.7, in that the experimentally supplied flow rate was measured to be less than the predictions of Hopkins (1995) and Norton (2002) method predicted minimum requirement for effective circulation of cuttings out of wellbore. Therefore, this helps drive home the point that flow rate of pump is a critical parameter during drilling operation for effective circulation of cuttings out of wellbore in order to avoid some problems encountered during drilling operation. Hence, based on the objective of this work, the bed height during circulation has been successfully measured and calculated experimentally and theoretically. More than one model has been used in order to ascertain the quality of theoretical results, viscosity of drilling fluid which is one of its rheological properties has been varied, in order to obtain more information concerning the prediction of bed height.

Nevertheless, more research needs to be carried out in this area of study, because in this work, only water and HEC was used as a drilling fluid, this is not usually the case during field practices, hence more investigation needs to be done in which more additive should be experimented so as to get more information in order to aid the process of suggesting conclusive approach for reducing problems encountered during oil well drilling operations. As a result, the author outlines below, some shortcomings which were experienced during the course of this work and how it could be improved upon.

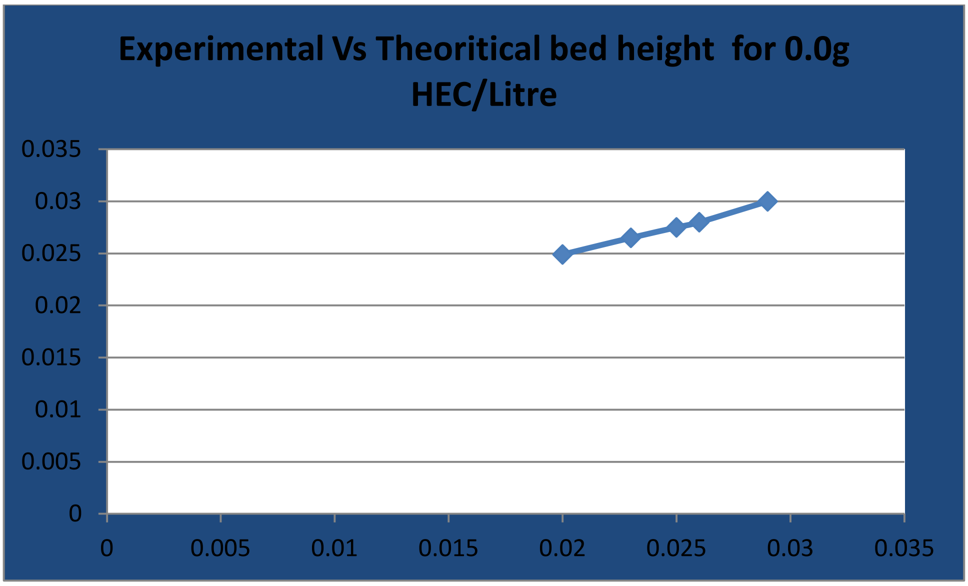

Figure 5.5: Experimental Vs Theoretical bed height for 0.0g Hec/l H_20

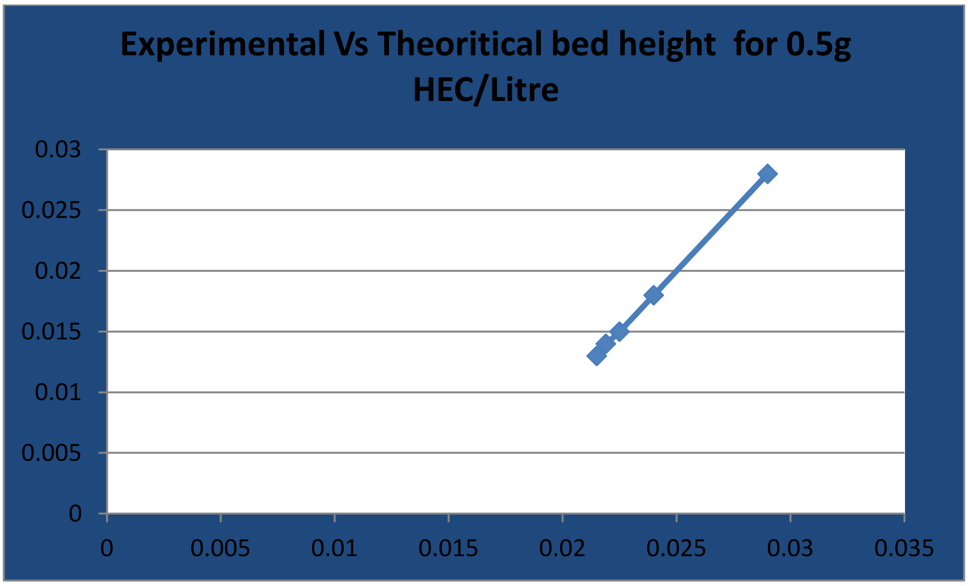

Figure 5.6: Experimental Vs Theoretical bed height for 0.5g Hec/l H_20

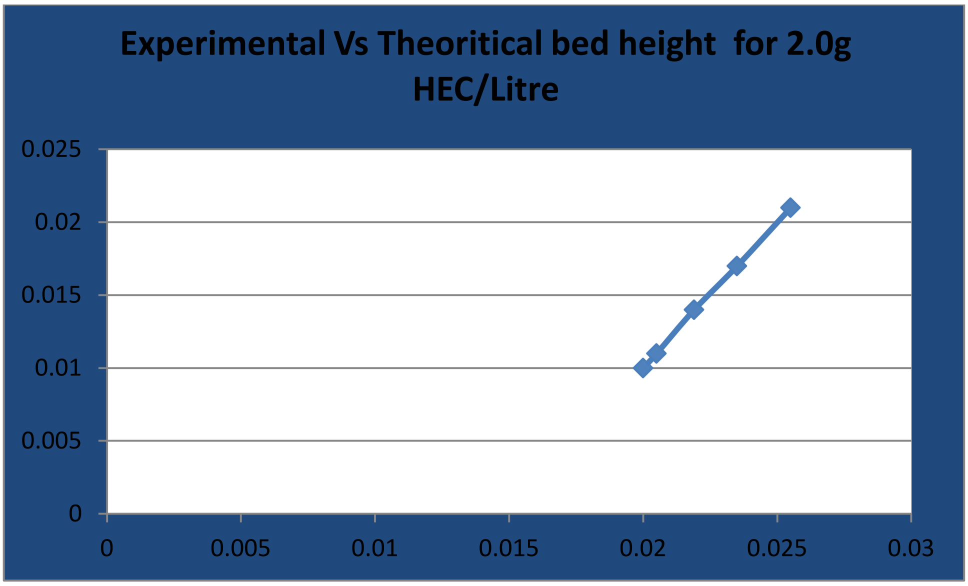

Figure 5.5: Experimental Vs Theoretical bed height for 2.0g Hec/l H_20

5.2 Shortcomings of theory and experiments

a. In the test set up, only HEC was the additives considered, this is not usually the case. This could as well affect the result, in that fluids of different density other than water are usually applied in the practical sense of drilling operations.

b. During the experiment, few data points were collected due to the restriction in allowable flow frequency of the pump.

c. In the set-up, the drill string was not used, therefore, further research work should be worked out to include the drill string in order to aid a better simulate a typical well scenario

d. Also, the calculations presented herein was done manually, this is impractical in some field work when a lot of calculations has to be done. Hence a computer programme is required to get effective analytical results.

5.3 Suggested Procedures to overcome Shortcomings;

In order to overcome the above mentioned shortcomings, the following have been suggested:

a. Fluids with different densities should be used to carry out the experiment, in order to better simulate a typical drilling field operation. So as to enhance the quality of result obtained.

b. It should be ensured that the restrictions on allowable flow frequency of the pump be reduced so as to allow running of the pump on higher flow frequency. This would help to give more data point to enable effective observation and conclusions obtained from the experiments.

c. The drill string should be included as part of the experiments.

6. Conclusion

Experimental and theoretical studies on cuttings transport in horizontal wellbore has been carried out, through the use of flow loop and engaging existing models to aid and compare with experimental data, in order to ensure validity of obtained result.

- The experiment showed that water can remove cuttings effectively from the wellbore, however, with a carefully increased viscosity of water by addition of viscofiers can enhance the result by and reducing the formation of cuttings bed height in the wellbore.
- The research has further proofed that pump flow rate could be affected by viscosity of the fluid if too high and tend to reduce the minimum critical velocity of flow. Also, it is primarily responsible for movement of cuttings from bottom hole to the surface.
- The geometry transformation included in this work, where utilized with the models, gave accurate estimation of bed height. This is evident from comparison and analysis of results in chapter five (5).
- This work could be a good start up point for any student who may want to research further into other variables responsible for cuttings bed deposition which were not considered in this research work.

The knowledge of the amount of drill cuttings inside horizontal wellbore is necessary for controlling wellbore pressure, preventing stuck pipe, and for minimizing the circulation time for cleaning of the wellbore. Hence, the importance of knowing the quantity of cuttings accumulated down-hole cannot be over-estimated. Therefore it is recommended that more research work be carried out in line with this work, in order to further ascertain effect of other variables not considered in this work as it affects the deposition of bed height. This will aid better prediction of cuttings bed and help in the process of decision making during drilling operation and minimize amount of problems encountered.

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Nomenclature

Latin

A_s = Sectional Area of the upper layer
A_B = Sectional Area of the bottom layer
$\frac{\Delta P}{\Delta Z}$ = Pressure gradient
S_s = Fluid to wall Perimeter
S_B = The bed to wall Perimeter
S_i = Fluid to bed interfacial perimeters
F = Bed to wall contact force
d_e = External diameter, m
k = diameters ratio
h = bed height, m
C = empirical constant based on laboratory data
p = the distance of top layer of bed height to the centre of the wellbore, m
C_d = Newton's drag coefficient
A_c = Cross sectional area of particle perpendicular to direction of motion
v_s = settling velocity of particle

Abbreviations

RF = rheology factor
TI = transportation Index
SG = mud specific gravity
AF = angle factor
FMW = mud weight correction factor, dimensionless
u_S = Slip velocity
u_2 = minimum velocity
Q_{crit} = minimum flow rate
C_t = concentration of solids in annulus
C_R = residual concentration of solids
A = amount of solids removed after long time of circulation

Greek

τ = Time constant

τ_s = Fluid to wall shear stress
τ_B = Bed to wall shear stress
τ_i = Fluid to bed interfacial shear stress
θ = Wellbore inclination, degrees
β = Auxiliary variable for geometric calculations, rad

Appendix

Hopkins Method

Theoretical calculations of Critical Velocities, Bed Perimeter and Bed Heights

Bed Height Calculation

Bed Height Calculation when circulating with 0.0g HEC/ L H_20

1st Case: this is at the assumed point where cuttings fill up to 50% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (S_B) = \left(\mathbf{2\sqrt{0.03^2 - 0}}\right) + \left(\mathbf{\tfrac{50}{100} \times 2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\mathbf{0.06}) + \mathbf{0.0942 = 0.1542}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$`

-

Position of top of 50% from centre of point under consideration = **0**m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0}}{\mathbf{0.03}} = \mathbf{0}$$

$$\boldsymbol{h = 0.03(1 - 0) = 0.03\ m}$$

2nd Case: this is at the assumed point where cuttings fill up to 44% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (S_B) = \left(\mathbf{2\sqrt{0.03^2 - 0.0018^2}}\right) + \left(\mathbf{\tfrac{44}{100} \times 2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\mathbf{2 \times 0.0299}) + \mathbf{0.0829 = 0.143}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 44% from centre of point under consideration = $\mathbf{0.0018}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0018, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0018}}{\mathbf{0.03}} = \mathbf{0.06}$$

$$\boldsymbol{h} = \mathbf{0.03(1 - 0.06) = 0.0282}\boldsymbol{m}$$

3rd Case: this is at the assumed point where cuttings fill up to 41.67% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B)} = \left(\mathbf{2}\sqrt{\boldsymbol{r^2 - p^2}}\right) + (\mathbf{2}\boldsymbol{\pi r})$$

$$\therefore (\boldsymbol{S_B}) = \left(\mathbf{2\sqrt{0.03^2 - 0.0025^2}}\right) + \left(\frac{\mathbf{41.67}}{\mathbf{100}} \times \mathbf{2} \times \mathbf{3.14} \times \mathbf{0.03}\right)$$

$$(\boldsymbol{S_B}) = (\mathbf{2 \times 0.029}) + \mathbf{0.0785 = 0.137}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 44% from centre of point under consideration = $\mathbf{0.0025}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0018, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0025}}{\mathbf{0.03}} = \mathbf{0.0833}$$

$$\boldsymbol{h} = \mathbf{0.03(1 - 0.0833) = 0.0275}\boldsymbol{m}$$

4th Case: this is at the assumed point where cuttings fill up to 38.33% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\boldsymbol{2\sqrt{0.03^2 - 0.0035^2}}\right) + \left(\boldsymbol{\frac{38.33}{100} \times 2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\boldsymbol{2 \times 0.029}) + \boldsymbol{0.0722 = 0.13m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 38.33% from centre of point under consideration = $\boldsymbol{0.0025}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0035, r_o = 0.03$$

$$\cos\beta = \frac{\boldsymbol{0.0035}}{\boldsymbol{0.03}} = \boldsymbol{0.1167}$$

$$\boldsymbol{h = 0.03(1 - 0.1167) = 0.0265m}$$

5th Case: this is at the assumed point where cuttings fill up to 33.33% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\boldsymbol{2\sqrt{0.03^2 - 0.005^2}}\right) + \left(\boldsymbol{\frac{33.33}{100} \times 2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\boldsymbol{2 \times 0.029}) + \boldsymbol{0.0628 = 0.1208m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 38.33% from centre of point under consideration = $\boldsymbol{0.0025}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0035, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.005}}{\mathbf{0.03}} = \mathbf{0.1667}$$

$$\boldsymbol{h} = \mathbf{0.03(1 - 0.1667) = 0.0249}\boldsymbol{m}$$

Bed Height Calculation when circulating with 0.5g HEC/ L H_20

1st Case: this is at the assumed point where cuttings fill up to 48.33% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\mathbf{2\sqrt{0.03^2 - 0.0005^2}}\right) + \left(\frac{\mathbf{48.33}}{\mathbf{100}} \times \mathbf{2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\mathbf{2 \times 0.029}) + \mathbf{0.09 = 0.149}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 48.33% from centre of point under consideration = $\mathbf{0.0005}$ m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0005, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0005}}{\mathbf{0.03}} = \mathbf{0.1667}$$

$$\boldsymbol{h} = \mathbf{0.03(1 - 0.1667) = 0.0249}\ \boldsymbol{m}$$

2nd Case: this is at the assumed point where cuttings fill up to 31.67% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\mathbf{2}\sqrt{\mathbf{0.03^2 - 0.0055^2}}\right) + \left(\frac{\mathbf{31.67}}{\mathbf{100}} \times \mathbf{2} \times \mathbf{3.14} \times \mathbf{0.03}\right)$$

$$(\boldsymbol{S_B}) = (\mathbf{2} \times \mathbf{0.0295}) + \mathbf{0.0597} = \mathbf{0.1187}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 31.67% from centre of point under consideration = $\mathbf{0.0055}$ m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0055, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0055}}{\mathbf{0.03}} = \mathbf{0.1833}$$

$$\boldsymbol{h} = \mathbf{0.03(1 - 0.1833) = 0.0245}\ \boldsymbol{m}$$

3rd Case: this is at the assumed point where cuttings fill up to 28.3% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter}\ (\boldsymbol{S_B}) = \left(\mathbf{2}\sqrt{\boldsymbol{r^2 - p^2}}\right) + (\mathbf{2}\boldsymbol{\pi r})$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\mathbf{2}\sqrt{\mathbf{0.03^2 - 0.0065^2}}\right) + \left(\frac{\mathbf{28.3}}{\mathbf{100}} \times \mathbf{2} \times \mathbf{3.14} \times \mathbf{0.03}\right)$$

$$(\boldsymbol{S_B}) = (\mathbf{2} \times \mathbf{0.029}) + \mathbf{0.0533} = \mathbf{0.112}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 28.33% from centre of point under consideration = $\mathbf{0.0065}$ m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0065, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0065}}{\mathbf{0.03}} = \mathbf{0.2167}\boldsymbol{m}$$

$$\boldsymbol{h} = \mathbf{0.03(1 - 0.2167) = 0.0235}\ \boldsymbol{m}$$

4th Case: this is at the assumed point where cuttings fill up to 23.33% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\boldsymbol{2\sqrt{0.03^2 - 0.008^2}}\right) + \left(\boldsymbol{\frac{23.33}{100} \times 2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\boldsymbol{2 \times 0.0289}) + \boldsymbol{0.044 = 0.102m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 23.33% from centre of point under consideration = $\boldsymbol{0.008}$ m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.008, r_o = 0.03$$

$$\cos\beta = \frac{\boldsymbol{0.008}}{\boldsymbol{0.03}} = \boldsymbol{0.2667m}$$

$$\boldsymbol{h = 0.03(1 - 0.2667) = 0.0219\ m}$$

5th Case: this is at the assumed point where cuttings fill up to 20.0% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\boldsymbol{2\sqrt{0.03^2 - 0.009^2}}\right) + \left(\boldsymbol{\frac{20}{100} \times 2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\boldsymbol{2 \times 0.0286}) + \boldsymbol{0.0377 = 0.0947m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 20% from centre of point under consideration = $\boldsymbol{0.009}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.009, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.009}}{\mathbf{0.03}} = \mathbf{0.3}\boldsymbol{m}$$

$$\boldsymbol{h} = \mathbf{0.03(1-0.3) = 0.021}\ \boldsymbol{m}$$

Bed Height Calculation when circulating with 1g HEC/ L $H_2$0

1st Case: this is at the assumed point where cuttings fill up to 46.66% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B)} = \left(\mathbf{2}\sqrt{\boldsymbol{r^2 - p^2}}\right) + \ (\mathbf{2}\boldsymbol{\pi r})$$

$$\therefore\ (\boldsymbol{S_B}) = \ \left(\mathbf{2}\sqrt{\mathbf{0.03^2 - 0.001^2}}\right) + \ (\frac{\mathbf{46.66}}{\mathbf{100}} \times \mathbf{2} \times \mathbf{3.14} \times \mathbf{0.03})$$

$$(\boldsymbol{S_B}) = (\mathbf{2} \times \mathbf{0.029}) + \ \mathbf{0.0879 = 0.146}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 46.66% from centre of point under consideration = $\mathbf{0.001}$ m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $$\cos\beta = \frac{l}{r_o} \text{ And } h = r_o - l = \ r_o(1-\cos\beta)$$

$$from\ above\ calculation, l = 0.001, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.001}}{\mathbf{0.03}} = \mathbf{0.0333}$$

$$\boldsymbol{h} = \mathbf{0.03(1-0.0333) = 0.029}\ \boldsymbol{m}$$

2nd Case: this is at the assumed point where cuttings fill up to 30% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B)} = \left(\mathbf{2}\sqrt{\boldsymbol{r^2 - p^2}}\right) + \ (\mathbf{2}\boldsymbol{\pi r})$$

$$\therefore\ (\boldsymbol{S_B}) = \ \left(\mathbf{2}\sqrt{\mathbf{0.03^2 - 0.006^2}}\right) + \ (\frac{\mathbf{30}}{\mathbf{100}} \times \mathbf{2} \times \mathbf{3.14} \times \mathbf{0.03})$$

$$(\boldsymbol{S_B}) = (\mathbf{2} \times \mathbf{0.0293}) + \ \mathbf{0.05652 = 0.115}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 30% from centre of point under consideration = $\mathbf{0.006}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1-\cos\beta)$

$$from\ above\ calculation, l = 0.006, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.006}}{\mathbf{0.03}} = \mathbf{0.2}$$

$$\boldsymbol{h} = \mathbf{0.03(1-0.2) = 0.024}\ \boldsymbol{m}$$

3rd Case: this is at the assumed point where cuttings fill up to 25% of the annulus

Step 1: Determination of bed perimeter as follows;

$\boldsymbol{Bed\ Perimeter\ (S_B)} = \left(\mathbf{2}\sqrt{\boldsymbol{r^2 - p^2}}\right) + (\mathbf{2}\boldsymbol{\pi r})$

$\therefore\ (\boldsymbol{S_B}) = \left(\mathbf{2}\sqrt{\mathbf{0.03^2 - 0.0075^2}}\right) + (\frac{\mathbf{25}}{\mathbf{100}} \times \mathbf{2} \times \mathbf{3.14} \times \mathbf{0.03})$

$(\boldsymbol{S_B}) = (\mathbf{2} \times \mathbf{0.0291}) + \mathbf{0.0471} = \mathbf{0.1053}\boldsymbol{m}$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 25% from centre of point under consideration = $\mathbf{0.0075}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1-\cos\beta)$

$$from\ above\ calculation, l = 0.0075, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0075}}{\mathbf{0.03}} = \mathbf{0.25}$$

$$\boldsymbol{h} = \mathbf{0.03(1-0.25) = 0.0225\ m}$$

4th Case: this is at the assumed point where cuttings fill up to 23.3% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ \boldsymbol{(S_B) = \left(2\sqrt{0.03^2 - 0.008^2}\right) + \left(\frac{23.3}{100} \times 2 \times 3.14 \times 0.03\right)}$$

$$\boldsymbol{(S_B) = (2 \times 0.02891) + 0.0439 = 0.1017m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 23.3% from centre of point under consideration = $\mathbf{0.008}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.008, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.008}}{\mathbf{0.03}} = \mathbf{0.2667}$$

$$\boldsymbol{h = 0.03(1 - 0.2667) = 0.0219}$$

5th Case: this is at the assumed point where cuttings fill up to 21.67% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ \boldsymbol{(S_B) = \left(2\sqrt{0.03^2 - 0.0085^2}\right) + \left(\frac{21.67}{100} \times 2 \times 3.14 \times 0.03\right)}$$

$$\boldsymbol{(S_B) = (2 \times 0.02877) + 0.0408 = 0.0983m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 21.67% from centre of point under consideration = $\mathbf{0.0085}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1-\cos\beta)$

$$from\ above\ calculation, l = 0.0085, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0085}}{\mathbf{0.03}} = \mathbf{0.2833}$$

$$\boldsymbol{h} = \mathbf{0.03(1-0.2833) = 0.0215}\boldsymbol{m}$$

Bed Height Calculation when circulating with 2g HEC/ L H_20

1st Case: this is at the assumed point where cuttings fill up to 35% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B)} = \left(\mathbf{2}\sqrt{\boldsymbol{r^2 - p^2}}\right) + (\mathbf{2}\boldsymbol{\pi r})$$

$$\therefore (\boldsymbol{S_B}) = \left(\mathbf{2\sqrt{0.03^2 - 0.0045^2}}\right) + \left(\frac{\mathbf{35}}{\mathbf{100}} \times \mathbf{2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\mathbf{2 \times 0.02966}) + \mathbf{0.06594 = 0.125}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 35% from centre of point under consideration = $\mathbf{0.0045}$ m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1-\cos\beta)$

$$from\ above\ calculation, l = 0.0045, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0045}}{\mathbf{0.03}} = \mathbf{0.15}$$

$$\boldsymbol{h} = \mathbf{0.03(1-0.15) = 0.0255}\ \boldsymbol{m}$$

2nd Case: this is at the assumed point where cuttings fill up to 28.3% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B)} = \left(\mathbf{2}\sqrt{\boldsymbol{r^2 - p^2}}\right) + (\mathbf{2}\boldsymbol{\pi r})$$

$$\therefore (\boldsymbol{S_B}) = \left(\mathbf{2\sqrt{0.03^2 - 0.0065^2}}\right) + \left(\frac{\mathbf{28.3}}{\mathbf{100}} \times \mathbf{2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\mathbf{2 \times 0.02928}) + \mathbf{0.0533 = 0.112}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 28.3% from centre of point under consideration = $\mathbf{0.0065}$ m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0065, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0065}}{\mathbf{0.03}} = \mathbf{0.2167}$$

$$\boldsymbol{h = 0.03(1 - 0.2167) = 0.0235\ m}$$

3rd Case: this is at the assumed point where cuttings fill up to 23.33% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\mathbf{2\sqrt{0.03^2 - 0.008^2}}\right) + \left(\frac{\mathbf{23.33}}{\mathbf{100}} \times \mathbf{2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\mathbf{2 \times 0.02891}) + \mathbf{0.0439 = 0.102}\boldsymbol{m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 23.33% from centre of point under consideration = $\mathbf{0.008}$ m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.008, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.008}}{\mathbf{0.03}} = \mathbf{0.2667}$$

$$\boldsymbol{h = 0.03(1 - 0.2667) = 0.0219\ m}$$

4^{th} **Case:** this is at the assumed point where cuttings fill up to 18.33% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\boldsymbol{2\sqrt{0.03^2 - 0.0095^2}}\right) + \left(\boldsymbol{\frac{18.33}{100} \times 2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\boldsymbol{2 \times 0.0285}) + \boldsymbol{0.0345 = 0.0915m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 18.33% from centre of point under consideration = $\mathbf{0.0095}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0095, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.0095}}{\mathbf{0.03}} = \mathbf{0.3167}$$

$$\boldsymbol{h = 0.03(1 - 0.3167) = 0.0205\ m}$$

5^{th} **Case:** this is at the assumed point where cuttings fill up to 16.66% of the annulus

Step 1: Determination of bed perimeter as follows;

$$\boldsymbol{Bed\ Perimeter\ (S_B) = \left(2\sqrt{r^2 - p^2}\right) + (2\pi r)}$$

$$\therefore\ (\boldsymbol{S_B}) = \left(\boldsymbol{2\sqrt{0.03^2 - 0.01^2}}\right) + \left(\boldsymbol{\frac{16.66}{100} \times 2 \times 3.14 \times 0.03}\right)$$

$$(\boldsymbol{S_B}) = (\boldsymbol{2 \times 0.02828}) + \boldsymbol{0.0314 = 0.0879m}$$

Diameter of well = 6mm = 0.06m $\therefore r = 0.03$

Position of top of 16.66% from centre of point under consideration = $\mathbf{0.01}$m

Now we have the perimeter, then using Eqs. 12 & 13

Step 2: $\cos\beta = \frac{l}{r_o}$ And $h = r_o - l = r_o(1 - \cos\beta)$

$$from\ above\ calculation, l = 0.0095, r_o = 0.03$$

$$\cos\beta = \frac{\mathbf{0.01}}{\mathbf{0.03}} = \mathbf{0.3333}$$

$$\boldsymbol{h = 0.03(1 - 0.3333) = 0.02m}$$

Critical velocities and flow rate for cuttings removal in 0.5 g HEC/l H_2O

Step 1: Since the rheology of the drilling fluid is changed, then we can calculate the Yield point as in below;

$$\boldsymbol{\mu_{pl} = \theta_{600} - \theta_{300} = 4 - 2.5 = 1.5}$$

$$\boldsymbol{YP = \theta_{300} - PV = 2.5 - 1.5 = 1\ lb/100ft^2}$$

Using the Hopkins chart, presented

$$\boldsymbol{V_{SV} = 60\ ft/min}$$

Step 2: $\boldsymbol{FMW = 2.11 - \ 0.1648 \times 8.33\ + 0.003681 \times 8.33\ ^2 = 0.9994}$

$$\boldsymbol{V_S = V_{SV} \times\ FMW = 60\ \times 0.99 = 59.4 \frac{ft}{min}}$$

Step 3: $\boldsymbol{V_2 = 40\ \times \left[\left(\frac{22.12-8.33}{8.33}\right) \times 32.3^3 \times \left(\left(\frac{2.15-0}{12}\right)^3\right)^{1/6}\right] = 104.8 \frac{ft}{min}}$

Step 4: $\boldsymbol{\therefore\ V_{min} = \ (57.42 Cos\ 80) + (104.8\ Sin\ 80) = 114 \frac{ft}{min}}$

Step 5: Therefore, the critical flow rate, can be calculated as in below;

$$\boldsymbol{Q_{crit} = 0.04079\ (2.15^2 - 0^2) \times 114 = 22 gal/min}$$

<u>Second Method</u>

Step 1: Determination of the constants n and K as in below;

$$\boldsymbol{n = 3.32\ log \frac{\theta_{600}}{\theta_{300}} = \ 3.32\ log \frac{4}{2.5} = 0.68}$$

$$\boldsymbol{k = \frac{\theta_{600}}{1022^n} = \frac{40}{1022^{0.68}} = 0.3595}$$

Step 2: determination of “x”

$$\boldsymbol{x = \frac{81600 \times (k) \times n^{0.387}}{(D_h - D_p)^2 \times 8.33} = \frac{81600 \times 0.3595 \times 0.68^{0.387}}{2.15^2 \times 8.33} = 656}$$

Step 3: Determination of "AV_c"

$$\boldsymbol{AV_c = \ (x)^{1 \div 2 - 0.68}}$$

$$\boldsymbol{AV_c = \ (656)^{1 \div (2 - 0.68)} = 136/min}$$

Step 4: Determine Critical flow rate “GPM_c”

$$GPM_c = \frac{136 \times 2.15^2}{24.5} = 25\ gal/min$$

Critical velocities and flow rate for cuttings removal in 1 g HEC/l H_2O

Step 1: Since the rheology of the drilling fluid is changed, then we can calculate the Yield point as in below;

$$\mu_{pl} = \theta_{600} - \theta_{300} = 4.5 - 3.5 = 1$$

$$YP = \theta_{300} - PV = 3.5 - 1 = 2.5\ lb/100ft^2$$

Using the Hopkins chart, presented

$$V_{SV} = 58\ ft/min$$

Step 2: $$FMW = 2.11 - 0.1648 \times 8.33 + 0.003681 \times 8.33^{\ 2} = 0.9994$$

$$V_S = V_{SV} \times FMW = 58 \times 0.99 = 57.42 \frac{ft}{min}$$

Step 3: $$V_2 = 40 \times \left[\left(\frac{22.12 - 8.33}{8.33} \right) \times 32.3^3 \times \left(\left(\frac{2.15 - 0}{12} \right)^3 \right)^{1/6} \right] = 104.8 \frac{ft}{min}$$

Step 4: $$\therefore V_{min} = (57.42 Cos\ 80) + (104.8 Sin\ 80) = 113 \frac{ft}{min}$$

Step 5: Therefore, the critical flow rate, can be calculated as in below;

$$Q_{crit} = 0.04079\ (2.15^2 - 0^2) \times 113 = 21.3\ gal/min$$

<u>**Second Method**</u>

Step 1: Determination of the constants n and K as in below;

$$n = 3.32\ log \frac{\theta_{600}}{\theta_{300}} = \frac{4.5}{3.5} = 0.36$$

$$k = \frac{\theta_{600}}{1022^n} = \frac{45}{1022^{0.36}} = 3.71$$

Step 2: determination of "x"

$$x = \frac{81600 \times (k) \times n^{0.387}}{(D_h - D_p)^2 \times 8.33} = \frac{81600 \times 3.71 \times 0.36^{0.387}}{2.15^2 \times 8.33} = 5294$$

Step 3: Determination of "AV_c"

$$AV_c = (x)^{1 \div 2 - n}$$

$$AV_c = (5294)^{1 \div (2 - 0.36)} = 186 ft/min$$

Step 4: Determine Critical flow rate "GPM_c"

$$GPM_c = \frac{186 \times 2.15^2}{24.5} = 35\ gal/min$$

Critical velocities and flow rate for cuttings removal in 2 g HEC/l H_2O

Step 1: Since the rheology of the drilling fluid is changed, then we can calculate the Yield point as in below;

$$\mu_{pl} = \theta_{600} - \theta_{300} = 7 - 5 = 2$$

$$YP = \theta_{300} - PV = 5 - 2 = 3\ lb/100ft^2$$

Using the Hopkins chart, presented

$$V_{SV} = 55\ ft/min$$

Step 2: $FMW = 2.11 - 0.1648 \times 8.33 + 0.003681 \times 8.33^{2} = 0.9994$

$$V_S = V_{SV} \times FMW = 55 \times 0.99 = 54.45 \frac{ft}{min}$$

Step 3: $V_2 = 40 \times \left[\left(\frac{22.12 - 8.33}{8.33} \right) \times 32.3^3 \times \left(\left(\frac{2.15 - 0}{12} \right)^3 \right)^{1/6} \right] = 104.8 \frac{ft}{min}$

Step 4: $\therefore V_{min} = (54.45 Cos\ 80) + (104.8 Sin\ 80) = 112 \frac{ft}{min}$

Step 5: Therefore, the critical flow rate, can be calculated as in below;

$$Q_{crit} = 0.04079\ (2.15^2 - 0^2) \times 112 = 21\ gal/min$$

<u>Second Method</u>

Step 1: Determination of the constants n and K as in below;

$$n = 3.32\ log \frac{\theta_{600}}{\theta_{300}} = 3.32\ log\ \frac{7}{5} = 0.4851$$

$$k = \frac{\theta_{600}}{1022^n} = \frac{70}{1022^{0.49}} = 2.3468$$

Step 2: determination of "x"

$$x = \frac{81600 \times (k) \times n^{0.387}}{(D_h - D_p)^2 \times 8.33} = \frac{81600 \times 2.3468 \times 0.49^{0.387}}{2.15^2 \times 8.33} = 3773$$

Step 3: Determination of "AV_c"

$$AV_c = (x)^{1 \div 2 - n}$$

$$AV_c = (3773)^{1 \div (2 - 0.49)} = 233 ft/min$$

Step 4: Determine Critical flow rate "GPM_c"

$$GPM_c = \frac{186\times2.15^2}{24.5} = 43\ gal/min$$

Critical velocities and flow rate for cuttings removal in 0.5 g HEC/l H_2O

Step 1: Since the rheology of the drilling fluid is changed, then we can calculate the Yield point as in below;

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$$V_{SV} = 60\ ft/min$$

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Step 3: $$V_2 = 40\ \times\left[\left(\frac{22.12-8.33}{8.33}\right)\times32.3^3\times\left(\left(\frac{2.15-0}{12}\right)^3\right)^{1/6}\right] = 104.8\frac{ft}{min}$$

Step 4: $$\therefore\ V_{min} = (57.42Cos\ 80) + (104.8\ Sin\ 80) = 114\frac{ft}{min}$$

Step 5: Therefore, the critical flow rate, can be calculated as in below;

$$Q_{crit} = 0.04079\ (2.15^2 - 0^2)\times114 = 22gal/min$$

<u>Second Method</u>

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$$GPM_c = \frac{136 \times 2.15^2}{24.5} = 25\ gal/min$$

Critical velocities and flow rate for cuttings removal in 1 g HEC/l H_2O

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Critical velocities and flow rate for cuttings removal in 2 g HEC/l H_2O

Step 1: Since the rheology of the drilling fluid is changed, then we can calculate the Yield point as in below;

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