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Fluid Dynamics Research 28 (2001) 295–310

FLUID DYNAMICS RESEARCH

Pressure drop correlations for flow through regular helical coil tubes

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Received 27 September 1999; received in revised form 17 April 2000; accepted 11 October 2000

Abstract

An attempt has been made to arrive at a better characterizing dimensionless group for steady isothermal flow of Newtonian fluids in helically coiled circular tubes. Pressure drop versus flow rate data has been experimentally obtained for helical coils made from thick-walled polyethylene tubing. Generalized pressure drop correlations have been developed in terms of Euler number, Eu, Reynolds number, Re, and the obtained geometrical group, $G_{\mathrm{rhc}} = (d^{0.85} D_{\mathrm{eq}}^{0.15}/L_{\mathrm{c}})$, where d is the inside diameter of the tube, D_{eq} is the equivalent diameter which accounts for the torsion effect, and L_{c} is the length of the coil. The Fanning friction factor for helical coil tube is found to depend on Reynolds number and a geometrical number, d/D_{eq}, separately both, and not by a dimensionless number obtained by any combination of Reynolds number and some geometrical number as is the case of Dean number, $\mathrm{De} = \mathrm{Re}\sqrt{(d/D)}$, where D is the diameter of the coil. There exist four regimes of flow for the flow through helical-coiled circular tubes. The obtained pressure drop correlations are simple to use. © 2001 Published by The Japan Society of Fluid Mechanics and Elsevier Science B.V. All rights reserved.

1. Introduction

Due to the advantages of accommodating large heat transfer area within a small space, high heat transfer coefficient and small residence time distribution, tube coils are extensively used in industries as heat exchangers and reactors.

Increased pressure drop for flow in curved channels, as compared to the pressure drop for the same rate of flow in the corresponding straight channel of the same length, was first reported by Grindley and Gibson (1908). Their experiments concerned with the measurement of pressure drop

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PII: S0169-5983(00)00034-4

Nomenclature

A	curvature ratio $(=d/D)$
ai	constant power of d appearing in Eq. (2)
bi	constant power of D appearing in Eq. (2)
C	coiling effect factor $(=\Delta P_c/\Delta P_s)$
ci	constant power of p appearing in Eq. (2)
d	inside diameter of tube, cm
di	constant power of L_c appearing in Eq. (2)
D	diameter of coil, cm
De	Dean number $(=\mathrm{Re}\sqrt{(d/D)})$
D_{eq}	equivalent diameter of coil $\{=\sqrt{(p^2+(\pi D)^2)}/\pi\}$, cm
Eu	Euler number $(=\Delta P/(2\rho V^2))$
f_s	Fanning friction factor for straight tube $(=\{\Delta P_s/(2\rho V^2)\}\{d/L\})$
f_c	Fanning friction factor for coil $(=\{\Delta P_c/(2\rho V^2)\}\{d/L_c\})$
G	geometrical number
G_{rhc}	geometrical number for regular-helical coil
He	Helical number $(=\mathrm{Re}[(d/D)/\{1+(p/\pi D)^2\}^{1/2}]$
K	dynamical similarity parameter as defined by Dean $\{=2\,\mathrm{Re}^2(d/D)\}$
K_i	constant coefficient appearing in Eq. (2)
L	length of tube, cm
L_c	length of the coiled portion of coil, cm
n	number of turns in a coil
p	pitch of coil, cm
ΔP_c	pressure drop in coil, N/m^2
ΔP_s	pressure drop in straight tube, N/m^2
Re	Reynolds number $(=dV\rho/\mu)$
Recrit	critical Reynolds number
Recritll	critical Reynolds number for transition from low laminar to laminar flow
Recritl	critical Reynolds number for transition from laminar to mixed flow
Recritm	critical Reynolds number for transition from mixed to turbulent flow
V	average velocity, cm/s
Q_c	flow rate in coil, cm^3/s
Q_s	flow rate in straight tube, cm^3/s
Tn	torsion number $(=2\tau\mathrm{Re})$

Greek letters

α	constant coefficient
β	constant power
μ	viscosity of experimental fluid, cp
ρ	density of experimental fluid, g/cm^3
λ	ratio of torsion τ to curvature κ
κ	curvature of the centerline of pipe $\{=R/(R^2+p^2)\}$
τ	torsion $\{=p/(R^2+p^2)\}$

for the flow of air through helically coiled pipe for the determination of viscosity. Eustice (1910) was next to report this increased pressure drop for flow of water through flexible coiled tubing of various radii of curvature. Dean (1927), through his theoretical analysis for the flow of incompressible Newtonian fluids in torus, confirmed the observation of Eustice. Non-dimensionalizing his simplified Navier–Stokes equations for torii of small curvature ratio, $A = d/D$, he arrived at a dynamical similarity parameter, $K = 2\mathrm{Re}^2(d/D)$, which, he claimed, characterizes the flow in torus at small flow rates. Square root of half of K was later given the name Dean number, De. In his subsequent work, Dean (1928) succeeded in showing reduction in rate of flow with curvature and its functional dependence on K. Since these works, various experimental as well as theoretical attempts have been made to obtain correlations for pressure drop in one turn circular tubes and in regular helical-coiled tubes. Table 1 summarizes most of the available correlations. Following can be noted:

1. Most of the available correlations for laminar flow in these types of curved tubes are for the ratio of the Fanning friction factor for the curved tube to the Fanning friction factor for the flow of the same fluid in a straight length of the same diameter tube, f_c/f_s. For turbulent flow, the available correlations are for the Fanning friction factor, f_c, or for some product of f_c to the curvature ratio, d/D. Reason for seeking correlations in different characterizing groups for the two regimes of flow is, however, not obvious.
2. The friction factor ratio, f_c/f_s, is the ratio of the pressure gradient in the coiled tube to that in the corresponding straight tube at the same flow rate, $(\Delta P_c/L_c)/(\Delta P_s/L_s)$. Hence, it is called as coiling effect factor, C. It is also equal to the ratio of the square of flow rate in the corresponding straight tube to that in the coiled tube under the same pressure gradient, Q_s^2/Q_c^2. White's (1932) plot, $\log C$ versus $\log \mathrm{De}$, came out to be a single curve before deviating to separate curves at Reynolds numbers different for different curvature ratio coils; the point of break was inferred to correspond to laminar to turbulent flow transition point. Various investigators since then have tried to obtain correlations in C for the laminar flow in such curved tubes. However, the use of the value of f_s in the ratio f_c/f_s is confusing. For laminar flow, the value of f_s is 16/Re, and for turbulent flow, it may be taken to be $0.0791/\mathrm{Re}^{1/4}$. If f_c/f_s is visualized as a comparison of the pressure drop in coil to that in the corresponding straight tube, the former value should be used only up to the value of Reynolds number equal to the critical Reynolds number of flow in straight tubes, i.e. 2100. Beyond this value, the flow in the straight tube ceases to be laminar, whereas in coiled tubes it persists to be laminar up to much higher Reynolds number. However, the use of the former value of f_s up to $\mathrm{Re} = 2100$ and then the latter value up to the critical Reynolds number for the coil flow will result in an additional break in the $\log C$ versus $\log \mathrm{Re}$ curve at $\mathrm{Re} = 2100$. This makes the nature of the C versus Re curves more complex than f_c versus Re curve, which is continuous. Thus, it may be concluded that a correlation in f_c/f_s can be developed, at the most, up to $\mathrm{Re} = 2100$, whereas a correlation in f_c can be developed for the entire range of coil flow.
3. The available correlations are in terms of either the characterizing group Dean number or in terms of a separate or joint combination of Reynolds number and the curvature ratio. It needs to be noted that the dimensionless group Dean number obtained from the simplified Navier–Stokes equations for developed flow in torus is approximation only for slight curvature. In fact, both the Dean number and the curvature ratio appear separately in the full equations of motion. Ali's (1974) attempt to solve the complete Navier–Stokes equations for the flow by a method of weighted

Table 1
Available pressure drop correlations

Author	Correlation	Conditions	Characterizing groups
Dean (1928)	$f_c/f_s = 1.03058(\mathrm{De}^2/288)^2 + 0.01195(\mathrm{De}^2/288)^4$	Torus, analytical, laminar, small d/D, De < 20	f_c/f_s, De^2
White (1929)	$f_s/f_c = 1-[1-(11.6/\mathrm{De})^{0.45}]^{1/0.45}$ $=1$ for De < 11.6	Circular tube, empirical, laminar, $D/d = 15.15, 50, 2050$; $11.6 < \mathrm{De} < 2000$	f_c/f_s, De
White (1932)	$f_c = 0.08\,\mathrm{Re}^{-1/4} + 0.012\sqrt{(d/D)}$	Helical, empirical, turbulent, $15000 < \mathrm{Re} < 100,000$	f_c, Re, $\sqrt{(d/D)}$
Adler (1934)	$f_c/f_s = 0.1064\sqrt{\mathrm{De}}$	Experimental + theoretical, laminar, large De	f_c/f_s, $\sqrt{\mathrm{De}}$
Prandtl (1949)	$f_c/f_s = 0.37\,(0.5\mathrm{De})^{0.36}$	Empirical, laminar, $40 < \mathrm{De} < 2000$	f_c/f_s, De
Hasson (1955)	$f_c/f_s = 0.556 + 0.0969\sqrt{\mathrm{De}}$	Helical, empirical, laminar	f_c/f_s, $\sqrt{\mathrm{De}}$
Ito (1959)	$f_c/f_s = 21.5\mathrm{De}/[1.56 + \log_{10}\mathrm{De}]^{5.73}$	Empirical, laminar, $13.5 < \mathrm{De} < 2000$	f_c/f_s, De
Ito (1959)	$f_c\sqrt{(D/d)} = 0.0791[\mathrm{Re}(d/D)^2]^{-0.2}$	Circular tube, empirical, turbulent, $\mathrm{Re}(d/D)^2 > 6$	$f_c\sqrt{(D/d)}$, $\mathrm{Re}(d/D)^2$
Ito (1959)	$4f_c\sqrt{(D/d)} = 0.029 + 0.304 \times \{\mathrm{Re}(d/D)^2\}^{-1/4}$	Theoretical, turbulent, $0.034 < \mathrm{Re}(d/D)^2 < 300$	$f_c\sqrt{(D/d)}$, $\mathrm{Re}(d/D)^2$
Ito (1959)	$f_c\sqrt{(D/d)} = 0.0081 + 0.4 \times (Y^2\sqrt{(d/D)})^{-1.27}$, where $Y^3 e^Y = \mathrm{Re}\sqrt{(D/d)}$	Theoretical, turbulent, $Y^2\sqrt{(d/D)} < 12$	$f_c\sqrt{(D/d)}$, $\mathrm{Re}\sqrt{(D/d)}$, $\sqrt{(d/D)}$
Ito (1959)	$f_c = 0.2965/Y^2$	Theoretical, turbulent, $Y^2\sqrt{(d/D)} > 5.3$	f_c, $\mathrm{Re}\sqrt{(D/d)}$, $\sqrt{(d/D)}$
Kubair and Varrier (1961/1962)	$f_c = 0.7716\exp(3.553d/D)\,\mathrm{Re}^{-0.5}$	Helical, empirical, non-isothermal, $2000 < \mathrm{Re} < 9000$ $0.037 < d/D < 0.097$	f_c, d/D, Re
	$f_c = 0.003538\,\mathrm{Re}^{0.09}\exp(1.887d/D)$	Helical, empirical, turbulent, $10 < D/d < 27$; $9000 < \mathrm{Re} < 25,000$	f_c, Re, d/D
Barua (1963)	$f_c/f_s = 0.509 + 0.0918\sqrt{\mathrm{De}}$	Torus, theoretical, laminar, large De	f_c/f_s, $\sqrt{\mathrm{De}}$
Mori and Nakayama (1965)	$f_c/f_s = 0.1080\sqrt{\mathrm{De}}/[1 - 3.253/\sqrt{\mathrm{De}}]$	Circular tube, theoretical, experimentally verified, laminar, $13.5 < \mathrm{De} < 2000$	f_c/f_s, $\sqrt{\mathrm{De}}$
Mori and Nakayama (1967)	$f_c\sqrt{(D/d)} = 0.075[\mathrm{Re}(d/D)^2] - 0.2\{1 + 0.112[\mathrm{Re}(d/D)^2\}^{-0.2}]$	Circular tube, theoretical + experimental, turbulent	$f_c\sqrt{(D/d)}$, $\mathrm{Re}(d/D)^2$
Schmidt (1967)	$f_c/f_s = 1 + 0.14\,\mathrm{Re}^x$, where $x = 1 - 0.0644/(D/d)^{0.312}]/(D/d)^{0.97}$	Empirical	f_c, Re, D/d
Srinivasan et al. (1968)	$f_c = 32/\mathrm{Re}$	Helical, empirical $0.0097 < d/D < 0.135$, $\mathrm{Re}\sqrt{(d/D)} < 30$	f_c, Re, $\mathrm{Re}\sqrt{(d/D)}$
	$f_c = 5.22\,(\mathrm{Re}\sqrt{(D/d)})^{-0.6}$	$30 < \mathrm{Re}\sqrt{(d/D)} < 300$	f_c, $\mathrm{Re}\sqrt{(D/d)}$, $\mathrm{Re}\sqrt{(d/D)}$
	$f_c = 1.8\,(\mathrm{Re}\sqrt{(D/d)})^{-0.5}$	$300 < \mathrm{Re}\sqrt{(d/D)} < \mathrm{Recrit}\sqrt{(d/D)}$	f_c, $\mathrm{Re}\sqrt{(D/d)}$, $\mathrm{Re}\sqrt{(d/D)}$
	$f_c = 1.084\,(\mathrm{Re}\sqrt{(D/d)})^{-0.2}$	Re > Recrit	f_c, $\mathrm{Re}\sqrt{(D/d)}$
Ito (1969)	$f_c/f_s = 0.1033\sqrt{\mathrm{De}}[(1 + 1.729/\mathrm{De})^{0.5} - (1.729/\mathrm{De})^{0.5}]^{-3}$	Theoretical	f_c/f_s, De
Tarbell and Samuels (1973)	$f_c/f_s = 1 + [0.0008279 + 0.007964d/D]\mathrm{Re} - 2.096 \times 10^{-7}\,\mathrm{Re}^2$	Torus, numerical, $20 < \mathrm{De} < 500$, $3 < D/d < 30$	f_c/f_s, Re, d/D

Table 1. (Continued.)

Author	Correlation	Conditions	Characterizing groups
Ramana Rao and Sadasivudu (1974)	$f_c = 1.55\exp(14.12d/D)\,\mathrm{Re}^{-1}$	Helical, empirical, $0.0159 < d/D < 0.0556$, $\mathrm{Re} < 1200$,	f_c, Re, d/D
	$f_c = 1.55\exp(14.12d/D)\,\mathrm{Re}^{-0.64}$	„ , $1200 < \mathrm{Re} < \mathrm{Recrit}$	„
	$f_c = 0.0382\exp(11.17d/D)\,\mathrm{Re}^{-0.2}$	„ , $\mathrm{Recrit} < \mathrm{Re} < 27{,}000$	„
	$f_c = 0.01065(d^{0.94}/D^{0.1})\,\mathrm{Re}^{-0.2}$	„ , turbulent	Dimensional
Collins and Dennis (1975)	$f_c/f_s = 0.38 + 0.1028\sqrt{\mathrm{De}}$	Torus, numerical, laminar, large De	f_c/f_s, $\sqrt{\mathrm{De}}$
Van Dyke (1978)	$f_c/f_s = 0.47136\,\mathrm{De}^{1/4}$	Torus, theoretical, laminar, large $\mathrm{De} > 30$	f_c/f_s, De
Mishra and Gupta (1979)	$f_c/f_s = 1 + 0.033[\log_{10}\mathrm{He}]^4$	Helical, empirical, laminar, $1 < \mathrm{He} < 3000$	f_c/f_s, He
	$f_c = 0.0791\,\mathrm{Re}^{-1/4} + 0.0075\sqrt{(d/D)}$ where, $\mathrm{He} = \mathrm{Re}[(d/D)/\{1 + (p/\pi D)^2\}]^{1/2}$	Helical, empirical, turbulent, $4500 < \mathrm{Re} < 10^5$, $6.7 < D/d < 346$, $0 < p/D < 25.4$	f_c, Re, $\sqrt{d/D}$
Dennis (1980)	$f_c/f_s = 0.388 + 0.1015\sqrt{\mathrm{De}}$	Torus, numerical, laminar, large De	f_c/f_s, $\sqrt{\mathrm{De}}$
Manlapaz and Churchill (1980)	$f_c/f_s = [(1 - 0.18/\{1 + (35/\mathrm{He})^2\}^{0.5})^m + (1 + d/\{3D\})^2(\mathrm{He}/88.33)]^{0.5}$, where $m = 2$ for $\mathrm{De} < 20$, $=1$ for $20 < \mathrm{De} < 40$, $=0$ for $\mathrm{De} > 40$	Helical, numerical	f_c/f_s, He, d/D
Yanase et al. (1989)	$f_c/f_s = 0.557 + 0.0938\sqrt{\mathrm{De}}$	Toroidal tube, theoretical, laminar	f_c/f_s, $\sqrt{\mathrm{De}}$
Liu and Masliyah (1993)	$f_c\,\mathrm{Re} = [16 + (0.378\,\mathrm{De}\,\lambda^{1/4} + 12.1)\mathrm{De}^{1/2}\lambda^{1/2}\gamma^2]\times [1 + \{(0.0908 + 0.0233\lambda^{1/2})\mathrm{De}^{1/2} - 0.132\lambda^{1/2} + 0.37\lambda - 0.2\}/(1 + 49/\mathrm{De})]$, where $\lambda = (D/2)/[(D/2)^2 + (p/2\pi)^2]$, $\gamma = \eta/(\lambda\mathrm{De})^{1/2}$, $\eta = (p/2\pi)/[(D/2)^2 + (p/2\pi)^2]$	Helical, numerical, developing laminar	

residuals resulted in a converging series solution for flow rates corresponding to Reynolds number less than 50. Even at these small flow rates, his result shows separate dependence of the friction factor on the Reynolds number and the curvature ratio. Germano (1989) showed that the Dean equations when extended to a helical tube flow contain not only the Dean number but also the parameter λ/Re, where λ is the ratio of torsion τ to curvature κ of the centerline of the coiled tube. Thus, it may be concluded that an a priori assumption that De alone characterizes the helical flow is not advisable.

4. Most of the available correlations do not contain the geometrical parameter pitch p or torsion τ to account for the fact that helical coils do not bend in a plane. Srinivasan et al. (1968) noticed no significant effect of pitch on friction factor. Manlapaz and Churchill (1980) have concluded that the pitch effect is insignificant only for coils for which the increase in elevation per revolution of coils is less than the radius of the coil. Chen and Fan (1986) and Kao (1987) have concluded that the torsion effect on the flow rate can be ignored. However, Germano (1989) has shown

that the flow in a helical-circular pipe depends not only on the Dean number but also on the parameter λ/Re. Chen and Jan (1992) have shown that the flow in a helical pipe is controlled by three parameters: Reynolds number, Dean number and torsion number $\mathrm{Tn} = 2\tau\,\mathrm{Re}$. Mishra and Gupta (1979), Manlapaz and Churchill (1980), and Liu and Masliyah (1993) have accounted for p by obtaining their correlations in terms of a parameter He, named as helical number. For the laminar flow in a helical circular tube with fairly large pitches, Yamamoto et al. (1994,1999) have shown, by numerical computations, large effect of torsion on the friction factor. This has been further confirmed by the experimental study of Yamamoto et al. (1995).

5. Most of the investigators who have given correlations both for laminar as well as turbulent flow have mostly used different characterizing groups for the two regimes of flow. However, these groups, if they characterize the coil flow, are likely to be same.
6. It is generally believed that correlations for the one-turn circular tubes also hold good for the closely packed regular helical tubes. This seems not to be true as a large portion of such tubes contains undeveloped flow, whereas regular helical coil with more number of turns contains mostly fully developed flow. Also, to measure pressure drop, some of the investigators have placed pressure taps within the coiled length itself. The presence of secondary flow is likely to effect the measurement. Correlations developed from such data are likely to have unknown errors and can hardly correspond to fully developed flow in helical coils.
7. Lastly, because any correlation is only an approximation of the actual relationship valid only for a limited range of parameters, it needs to be simple. Some of the correlations listed above are quite complicated. This might have arisen from the failure of arriving at accurate characterizing groups.

This paper is a report of the attempt to arrive at better characterizing parameters and to obtain suitable correlations in terms of these parameters.

2. Experimental

Required pressure drop versus flow rate data, used here, for flow through helical tube coils was collected long back in 1977 by Ali and Zaidi who had failed to develop satisfactory correlation at that time. Regular-helical tube coils, used for testing, were made by winding thick-walled polyethylene tubings on wooden cylinders of different required diameters. All the coils thus formed were tangentially extended at their two ends. These straight tube extensions in sufficient length are required to subside fluid disturbances at the entry to the coil and to subside the secondary flow exiting from the coil. Pressure taps were attached to the two disturbance-free part of the straight lengths. Pressure-drop data for an equal straight length of the same tubing was obtained and subtracted from the pressure drop data of the coil-straight length combination so as to give the pressure drop data for the coiled portion only. A carbon tetrachloride manometer was used for smaller pressure drops and a mercury manometer was used for larger pressure drops. Water at ambient temperature was used as the test fluid. The inside diameter of the tubing was accurately measured by means of a travelling microscope by cutting the tubing at different lengths and measuring at two mutually perpendicular diameters. These observed values of diameters were further checked by noting the weight of water filled in a known length of the tubing. Visual inspection revealed no deformation of the tubing during winding and experimentation.

Table 2
Dimensions of coils and symbols used for data plotting

Coil no.	Symbol	d (cm)	D (cm)	p (cm)	L_c (cm)	n
1	O	0.603	11.6225	5	223.8	6
2	$\times$	0.603	11.6225	1	223.8	6
3	Δ	0.603	11.6225	5	112.3	3
4	$+$	0.603	11.6225	1	111.5	3
5	∇	0.464	11.6225	5	226.3	6
6	$\oplus$	0.464	11.6225	5	113.2	3
7	$*$	0.603	22.448	1	426	6
8	$\square$	0.603	22.448	1	213	3

The flow rate versus pressure drop data was obtained for a set of eight coils. These coils varied in geometry from each other in such a way that each of the geometrical parameters: d, the inside diameter of the tube, D, the coil diameter, L_c, the length of the coil, and p, the pitch of the coil, changed once with respect to a particular coil. These four geometrical parameters, d, D, L_c and p, are sufficient to uniquely specify the geometry of a regular-helical coil. Table 2 gives the dimensions of the coils used for experimentation and symbols used for them for the data plotting in the subsequent figures. The dimensions are such that d/D varies from 0.027 to 0.052, and p/D varies from 0.0445 to 0.43. All the correlations mentioned above are valid in these ranges.

3. Analysis of the experimental data

For flow in regular-helical tube coils, pressure drop, ΔP, as a dependent variable, depends on the independent variables of the fluid properties (ρ and μ), the flow rate variable (V), and the parameters of the coil geometry (d, D, p and L). Thus,

$$\Delta P = \Delta P(\rho, \mu, V, d, D, p, L). \tag{1}$$

Performing dimensional analysis on these variables, we get

$$\Delta P/(2\rho V^2) = \sum_i K_i d^{ai} D^{bi} p^{ci} L_c^{di} (\rho V/\mu)^{ai+bi+ci+di}. \tag{2}$$

The coefficient K_i and powers ai, bi, ci and di are to be obtained by fitting the experimental data. For the sake of simple expression, only one term on the right-hand side will be retained for obtaining pressure drop correlations.

Log–log plots of Euler number, $\mathrm{Eu} = \Delta P/(2\rho V^2)$, versus Reynolds number, $\mathrm{Re} = dV\rho/\mu$, are obtained from the pressure drop-flow rate data. Fig. 1 gives this plot for tube diameter changing coils. It appears that data points for coils 1 and 5 will fall on a single curve if we give an appropriate d-dependent shift to the Y coordinate. Similarly, data points for coils 3 and 6 will fall on a single curve by giving an appropriate Y-shift.

Fig. 2 is plot for coil diameter, D, changing coils 2 and 8. It appears that there is requirement for an appropriate D-dependent Y-shift for merging. Fig. 3 is a plot for coil length, L_c, changing coils. Data points for coils 1 and 3 and those for coils 5 and 6 need appropriate L_c-dependent Y-shift for

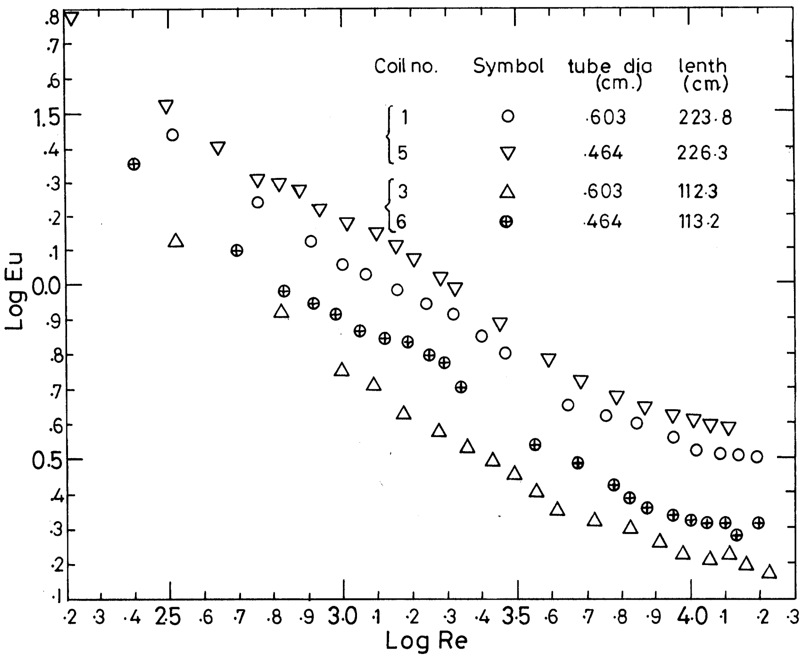

Fig. 1. Experimental data for tube diameter-changing coils.

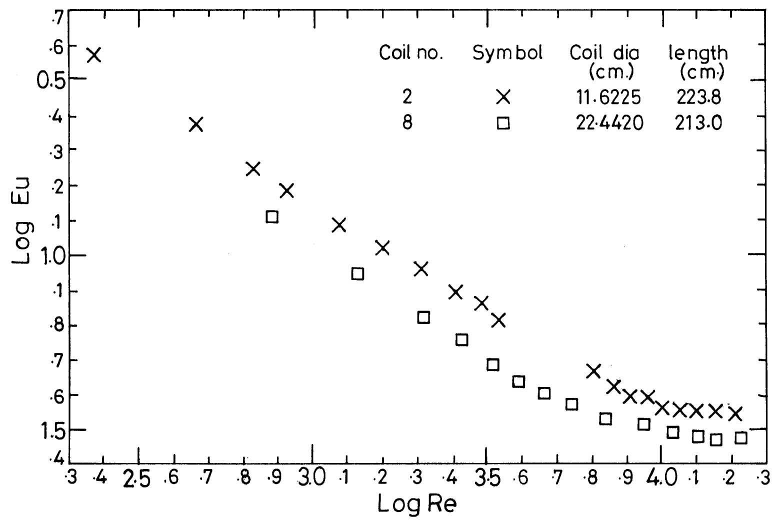

Fig. 2. Experimental data for coil diameter-changing coils.

merging. Similarly, Fig. 4 is a plot for pitch-changing coils 1–4. It appears that only a very small Y-shift is required.

Plots 1–4 indicate a very important conclusion that merging of data points for all the coils tested on a single curve does not require any X-shift, rather only Y-shifts are required. This shows that

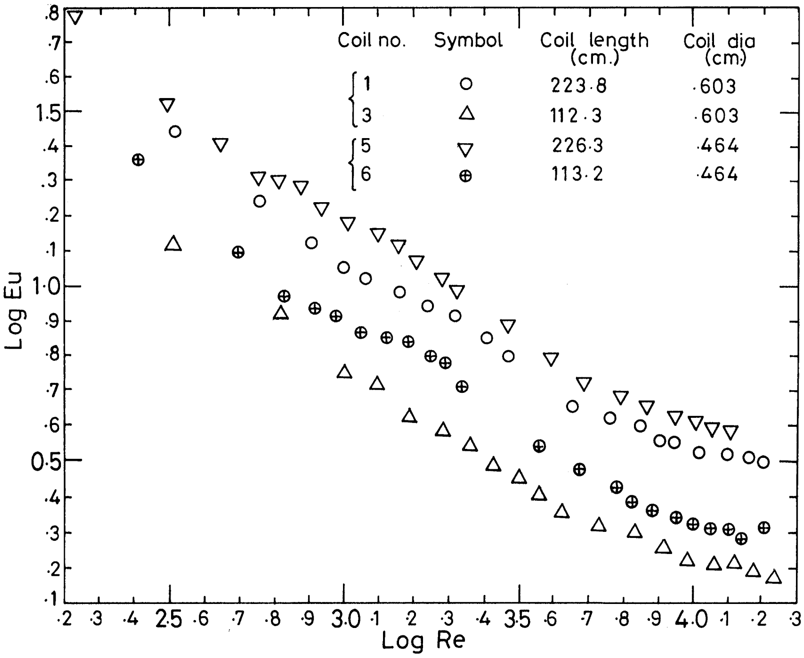

Fig. 3. Experimental data for coil length-changing coils.

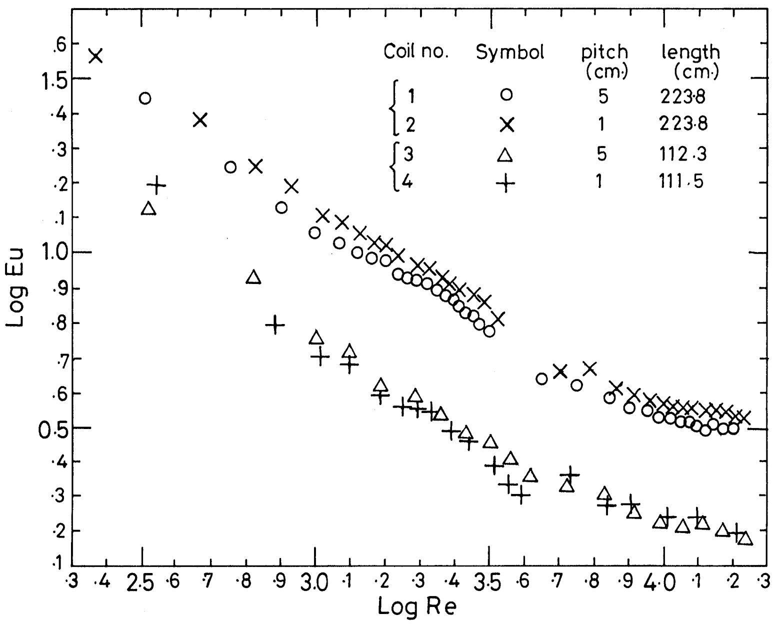

Fig. 4. Experimental data for pitch-changing coils.

Table 3
Requirement of shifts in coordinates

Coil	Symbol	Changing parameter	Laminar		Turbulent	
			X-shift	Y-shift	X-shift	Y-shift
1. *d changing coils*:						
5	∇	$d = 0.464$ cm	0.0	+0.1	0.0	0.08
1	0	$d = 0.603$ cm				
6	$\oplus$	$d = 0.603$ cm	0.0	+0.15	0.0	0.10
3	Δ	$d = 0.603$ cm				
2. *D changing coils*:						
2	$\times$	$D = 11.6225$ cm	0.0	+0.011	0.0	0.7
8	$\square$	$D = 22.448$ cm				
3. *L_c changing coils*:						
3	Δ	$L_c = 112.3$ cm	0.0	−0.3	0.0	−0.3
1	0	$L_c = 223.8$ cm				
6	$\oplus$	$L_c = 113.2$ cm	0.0	−0.3	0.0	−0.3
4	+	$L_c = 226.3$ cm				
4. *p changing coils*:						
2	$\times$	$p = 1$ cm	0.0	+0.03	0.0	+0.03
1	0	$p = 5$ cm				
4	+	$p = 1$ cm	0.0	0.0	0.0	0.0
2	$\times$	$p = 5$ cm				

Reynolds number and not a product of Reynolds number and some power of curvature ratio is required to characterize the regular helical coil flow.

Further, since the pitch, p, does not have any distinctive effect, it can be combined with D, the diameter of the coil, in an equivalent diameter, D_{eq}, as

$$D_{eq} = \sqrt{[p^2 + (\pi D)^2]}/\pi = L_c/n\pi. \tag{3}$$

Thus, a universal correlation for the helical flow is expected to be in the form

$$\mathrm{Eu}(d^a D_{eq}^b L_c^c) = \alpha \mathrm{Re}^{-\beta}. \tag{4}$$

A rough estimate of values of a, b and c can be obtained by finding the requirement of Y-shift for data points for d, D and L_c varying coils in Figs. 1–4. Table 3 gives these requirements.

Various attempts were made to obtain the best values of a, b and c to give a single Eu $(d^a D_{eq}^b L_c^c)$ versus Re curve. Eu $(d^{0.85} D_{eq}^{0.15}/L_c)$ versus Re curve, Fig. 5, resulted in the best merging indicating that

$$G_{rhc} = (d^{0.85} D_{eq}^{0.15}/L_c) \tag{5}$$

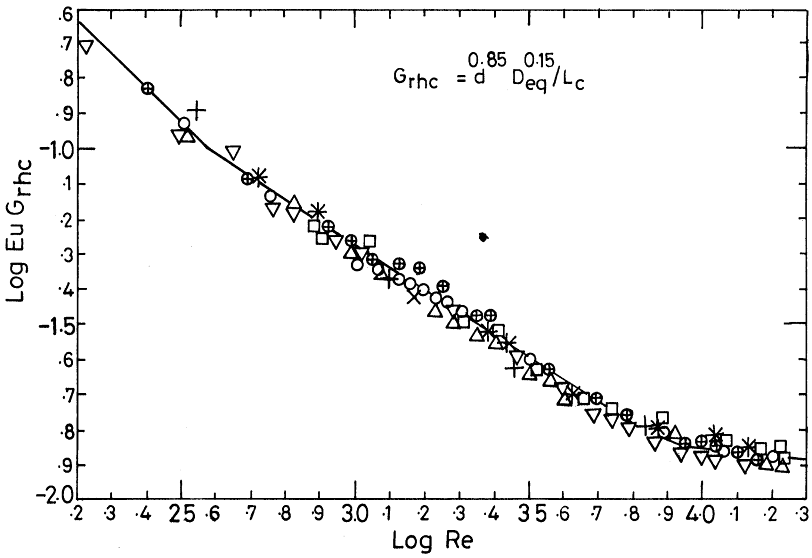

Fig. 5. $\log \mathrm{Eu}G_{\mathrm{rhc}}$ versus $\log \mathrm{Re}$ plot.

is the geometrical group which characterizes the regular-helical coil flow. Thus, the required universal correlation can be of the form

$$\mathrm{Eu}\,(d^{0.85}D_{\mathrm{eq}}^{0.15}/L_{\mathrm{c}}) = \alpha \mathrm{Re}^{-\beta} \tag{6}$$

which is equivalent to:

$$\mathrm{Eu}\,d/L_{\mathrm{c}} = \alpha(d/D_{\mathrm{eq}})^{0.15}\,\mathrm{Re}^{-\beta}. \tag{7}$$

A close look at Fig. 5 indicates that with the arrived new group G_{rhc}, data points fall on straight lines having three break points, indicating that helical flow changes nature at three critical values of Reynolds number. Hence, the flow may be thought to have four regimes of flow. The first regime is up to the first critical Reynolds number, say Recritll, and may be termed as low laminar flow regime. The second regime lies between the first critical Reynolds number and the second critical Reynolds number, say Recritl, and may be termed as laminar flow regime. The third regime lies between the second critical Reynolds number and the third critical Reynolds number, say Recritm, and may be termed as mixed flow regime. The last regime is that of turbulent flow and lies beyond the third critical Reynolds number. Straight-line fits are obtained in the low laminar, laminar, mixed, and turbulent flow regimes as

$$\mathrm{Eu}G_{\mathrm{rhc}} = 38\mathrm{Re}^{-1}, \quad \mathrm{Re} < \mathrm{Recritll}, \tag{8}$$

$$\mathrm{Eu}G_{\mathrm{rhc}} = 5.25\mathrm{Re}^{-2/3}, \quad \mathrm{Recritll}, < \mathrm{Re} < \mathrm{Recritl}, \tag{9}$$

$$\mathrm{Eu}G_{\mathrm{rhc}} = 0.31\mathrm{Re}^{-1/3}, \quad \mathrm{Recritl} < \mathrm{Re} < \mathrm{Recritm}, \tag{10}$$

$$\mathrm{Eu}G_{\mathrm{rhc}} = 0.045\mathrm{Re}^{-1/8}, \quad \mathrm{Re} > \mathrm{Recritm}, \tag{11}$$

where Recritll, Recritl, and Recritm are obtained as

$$\mathrm{Recritll} = 500, \tag{12}$$

$$\mathrm{Recritl} = 6300, \tag{13}$$

$$\mathrm{Recritm} = 10{,}000. \tag{14}$$

Average deviations of the experimental data points from the above correlations are within -16.8 to $+6.66\%$ in the low laminar flow regime, -11.6 to $+5.4\%$ in the laminar flow regime, -6.7 to $+7.2\%$ in the mixed flow regime, and -7.15 to $+7.16\%$ in the turbulent flow regime.

It needs to be emphasized that the almost complete merging of data points in Fig. 5 indicates success of obtaining a characterizing geometrical number, G_{rhc}. A slightly different set of correlations with different powers of Re, can also be fitted as

$$\mathrm{Eu}G_{\mathrm{rhc}} = 21.88\mathrm{Re}^{-0.9}, \quad \mathrm{Re} < 500 \tag{15}$$

with average deviation from -13.9% to $+8\%$,

$$\mathrm{Eu}G_{\mathrm{rhc}} = 5.25\mathrm{Re}^{-2/3}, \quad 500 < \mathrm{Re} < 6300 \tag{16}$$

with average deviation from -11.6% to $+5.4\%$,

$$\mathrm{Eu}G_{\mathrm{rhc}} = 0.56\mathrm{Re}^{-2/5}, \quad 6300 < \mathrm{Re} < 10{,}000 \tag{17}$$

with average deviation from -7.8% to $+6.1\%$, and

$$\mathrm{Eu}G_{\mathrm{rhc}} = 0.09\mathrm{Re}^{-1/5}, \quad \mathrm{Re} > 10{,}000 \tag{18}$$

with average deviation from -8.96 to $+6.14\%$.

The exact powers of Re, however, can only be deduced from the mathematical analysis of the flow, or can be predicted with very accurate experimental data.

4. Discussion of results

4.1. The form of the correlations

The obtained correlations (8)–(11) and (15)–(18) are of the form $\mathrm{Eu}G = \alpha\mathrm{Re}^{-\beta}$, which is the same form as that of Hagen–Poiseuille law $\mathrm{Eu}(d/L) = 16/\mathrm{Re}$, and Blasius resistance law $\mathrm{Eu}(d/L) = 0.0791\,\mathrm{Re}^{-0.25}$ for straight tube flow. G is some geometrical dimensionless group, which depends on the geometry of the tube. The correlations obtained by Ali and Seshadri (1971) for Archimedean spiral tube coils and those by Ali and Zaidi (1979) for ascending equiangular spiral tube coils are also in the same form. It is interesting to note that the correlations given by Adler (1934), Prandtl (1949), Kubair and Varrier (1961/1962), Srinivasan et al. (1968), Ramana Rao and Sadasivudu (1974) and Van Dyke (1978) for helical coil flow are also of the same form. A suitable adjustment in the coefficients in these correlations may result in better correlations even with the characterizing groups used by them.

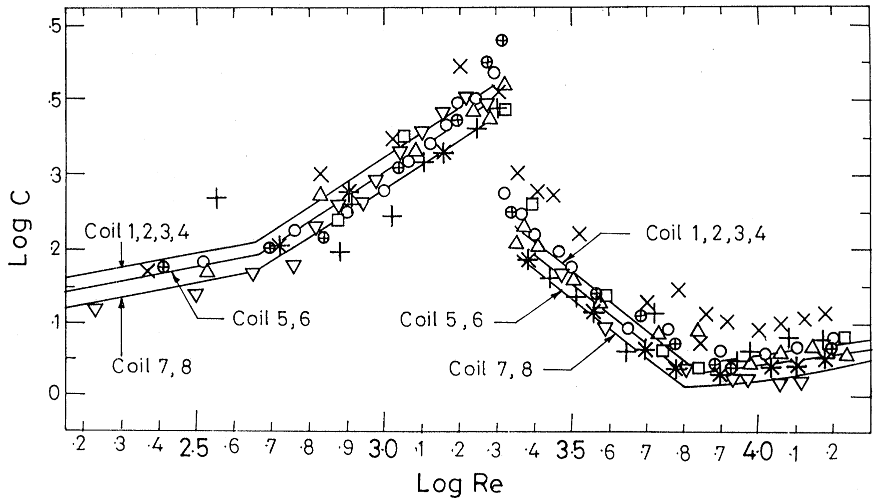

Fig. 6. Coiling effect factor versus Reynolds number plot.

4.2. Critical Reynolds numbers

From Fig. 5, the three critical Reynolds numbers have been guessed roughly as given by (12)–(14). The method of White (1929), originally suggested for curved pipes for the location of critical Reynolds numbers, was also tried. The method consists of observing breaks on the log–log plot of the ratio $\Delta P_c/\Delta P_s$ versus Re. Fig. 6 is this plot. Locations of the three critical Reynolds numbers are not distinct in this plot. A closer look at the trend of data points shows that variation of $\log C$ with $\log \mathrm{Re}$ is rather more gradual than linear, implying that the transitions from low laminar to laminar, laminar to mixed, and mixed to turbulent flow are gradual. However, the existence of the four regimes of flow, namely low laminar, laminar, mixed and turbulent flow is rather clear.

4.3. Comparison with available correlations

Since different workers have used different characterizing groups and their correlations differ in form, a comparison of the available correlations with the developed correlations (8)–(11) and (15)–(18) is somewhat difficult. However, for the ease of comparison, these correlations have been converted to the common form (7) and plotted in Fig. 7 for the coil 1. Following observations are made:

1. Most of the available correlations, including the present one, have good agreement with each other, and can be used to approximately predict pressure drop for regular-helical coil flow.
2. Since f_c and Re do not characterize the coil flow completely, we will get different curves for different coils which may differ from that shown in Fig. 7.
3. Since all the correlations result in almost straight lines in Fig. 7, it appears that they may be transformed into a common form.
4. Almost all the correlations show a break at the critical Reynolds number Recritl.

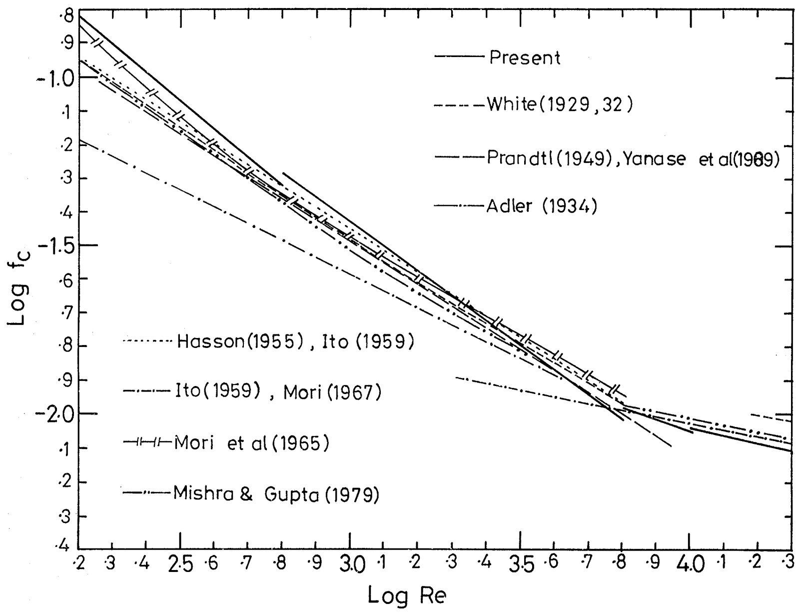

Fig. 7. Comparison with available correlations.

5. Conclusions and recommendation

The analysis of the experimental data in this work reveals a much better characterizing geometrical group, and the correlations developed are much simpler than most of the available correlations. It appears that there exist four regimes of flow. The low laminar flow regime extends approximately up to the first critical Reynolds number of 500. The secondary flow effect on pressure drop seems to be comparatively less in this regime and a comparison of correlation (8) with that of straight tube shows that pressure drop in regular helical coil in this regime is 2.375 $(d/D_{eq})^{0.15}$ times that of straight tube pressure drop. Although, there is continuous increase in the intensity of secondary flow with Reynolds number, in this regime of laminar flow, the effect of main axial flow on the flow resistance seems to dominate over the effect of secondary flow. The second regime of laminar flow extends up to the second critical Reynolds number of approximately 6300. In this regime, up to the Reynolds number of 2100, there exists laminar flow in both straight and coiled tubes; the secondary flow seems to increase further and the continuous increase of secondary flow in coiled tubes causes continuous increase in the coiling effect factor. Thereafter, flow in straight tube becomes turbulent while that in coil continues to be laminar. The effect of turbulence on the pressure drop tends to dominate over the secondary flow effect, and the coiling effect factor continuously decreases till it reaches a minimum. Hereafter, in the third regime of mixed flow, turbulence also sets in the coiled tube. The coiling effect factor tends to increase but slightly up to the third critical Reynolds number of approximately 10,000, till the intensity of turbulence in coil flow becomes equal to that in the straight tube. Thereafter, the presence of secondary flow over and above turbulence in the coiled tube tends to increase the coiling effect factor once again.

Apart from obtaining more accurate pressure drop correlations, importance of the present work lies in arriving at the better characterizing geometrical group G_{rhc} in (5). Incidentally, this group happens to have the same form as that in Adler's correlation and in Prandtl's correlation.

For more accurate prediction of power of Re in (6) and for the determination of more accurate values of the three critical Reynolds numbers, experimental data are required to be more precise.

Acknowledgements

Data analysis and correlation development were done with the help of a general computer program, **ftcoils.f**, developed by the author for performing computer interactive experiments on various types of helical and spiral coil flows. The program was developed under an education technology project funded by the HRD ministry, government of India.

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