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Optimal Scheduling of Battery Storage in the Future Power System

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Problem description

In 2009, the EU directive 2009/28/EC introduced ambitious targets to cut CO2 emissions by producing 20 % of the gross final energy consumption from Renewable Energy Sources (RES). The willingness of the European Union to cut greenhouse gas emissions was underpinned by the introduction of the "Energy Roadmap 2050" (EU directive 2009/29/EC) aiming to reduce CO2 emissions by 50 % below their 1990 levels. These targets will have a profound impact on transmission planning and system operation. In this context, an increased production flexibility of conventional power plants will be needed for a save and cost efficient integration of RES into the power system.

Electric Storage (ES) facilities might be able to effectively increase the production flexibility in the power system. While offering the possibility to store excessive power production from RES during times with high production, stored energy can be feed back into the grid and support the power system during periods with low RES production.

Distributed batteries among ES facilities are considered as responsive loads that can reduce the need for peak-load power plants, emitting a relatively high amount of CO2 when compared to base-load generators.

Objectives:
The purpose of this research work is to study the impact of optimal scheduling of battery storage in a power system.

The scope of the research is to:

1. Perform a literature survey and provide an overview over battery storage possibilities.
2. Establish an optimisation strategy for operation of batteries.
3. Perform optimal scheduling of distributed battery storage, and study the impact on the Norwegian power system using the established strategy, given the exogenous electricity prices from the electricity market.

Time-period: January - June 2016
Supervisor: Hossein Farahmand

Abstract

This research work has resulted in an optimisation strategy for operation of batteries in a power system. The model is formulated as a mixed integer linear program constrained by power system limitations on transmission capacities, production limits and PTDFs, and battery limitations on power/energy capacities and efficiency. In the work with the development of the optimisation strategy, two separate models is created. The first model is for optimisation of a power system determining the power production and power flow, based on the FBMC technology. The second model is for optimisation of battery dispatch with respect to the time for ant amount of charged and discharged power. The two models are merged and the result is the final optimisation model for operation of batteries in a power system. The formulation of the MILP is implemented in the modelling system GAMS, which is the program used to perform the optimisations.

The purpose of this research work is to study the impact of optimal scheduling of battery storage in a power system. An aggregated model of the Norwegian power system comprising of 22 buses and 33 lines is used. Discussion of the accuracy of the aggregated Norwegian power system relative to the real power flow situation is included. The formulation of the battery model is performed through the analyses on three different scenarios, to ensure properly behaviour. Verifications of the validity of the three scenarios are performed.

Scheduling of optimal battery dispatch using the established optimisation model and an aggregated model of the Norwegian power system is performed through seven case studies. The cases are constructed to investigate the behaviour of batteries in situations including large variations in day-ahead prices, congestion situations and planned generation outages. Moreover, analyses on two cases addressing a fictive future solar power scenario are performed.

The objective of the optimisation model is to minimise the total system operating costs, meaning costs for hydro production and battery operation. In the case studies, the exogenous electricity prices from the electricity market are used as marginal costs for hydro power production, and the charging costs and discharging earnings are set to the average electricity price. The batteries do not participate in the electricity market and are considered as price-takers. For the scope of the case studies, the batteries do not generate any additional income, but are assumed to be included in the grid to be able to provide services.

The results from the case studies show that the total system operating costs are decreased in all situations when including a battery to the power system. The performed cost-benefit analysis states that a battery investment of 50MW/200MWh will not be profitable for these optimisation scenarios. The total system operating cost reduction increases with increased volatility in the day-ahead price over a day. The charging pattern is strictly dependent on the day-ahead price, and will not deviate from the optimal pattern unless

limitations are given, or the network reaches its limit and the battery has to change the dispatch to help the system operate. It is also shown that batteries can help the system through transmission congestion and generation outages. The cases with solar polar power reduces the total system operating costs due to the decreased hydro power production requirement. In addition, revenue to the solar panel owners from selling energy to the grid and from production shifting by use of the solar panel owner's batteries give decreased total system operating costs.

It is assumed that the established model can be used by hydro power producers to schedule their production and by battery owners to schedule their desired charging pattern.

Sammendrag

Dette forskningsarbeidet har resultert i en optimaliseringsstrategi for drift av batterier i et kraftsystem. Modellen er beskrevet som et MILP-problem, begrenset av overføringskapasiteter, produksjonsbegrensninger og PTDF-matriser for kraftsystemet samt begrensninger for batterier for effekt/energi-kapasitet, og effektivitet.

Under arbeidet med utviklingen av optimaliseringsstrategien er to separate modeller utarbeidet. En modell er laget for optimalisering av et kraftsystem med variable for kraftproduksjon og kraftflyt, basert på FBMC-metoden. Den andre modellen er laget for optimalisering av batterier med der målet var å finne optimalt lademønster. De to modellene er slått sammen og resultatet er den endelige optimaliseringsmodellen for drift av batterier i et kraftsystem. Formuleringen av MILP-problemet er implementert i modelleringssystemet GAMS, som er programmet brukt til å utføre optimaliseringene.

Formålet med dette forskningsarbeidet er å studere virkningen av optimal drift av batterier i et kraftsystem. En aggregert modell av det norske kraftsystemet bestående av 22 noder og 33 linjer er brukt. Diskusjoner rundt nøyaktigheten av modellen av det norske kraftsystemet i forhold til den virkelige kraftflyten er også inkludert. Beskrivelsen av batterimodellen er utført gjennom analyser på tre forskjellige scenarier, for å sikre korrekt virkemåte. Verifisering av gyldigheten til de tre scenariene er gjennomført.

Planlegging av batteridrift ved bruk av den utviklede optimaliseringsmodellen er utført for syv caser. Situasjonene er konstruert for å undersøke et batteri sin oppførsel i ulike situasjoner med store variasjoner i elektrisitetspriser, overføringsbegrensninger og planlagte driftsstanser. Analyser av to tilfeller med en fiktiv situasjon for solenergi i fremtiden er også utført.

Målet med optimaliseringsmodellen er å redusere de totale driftskostnadene, altså kostnadene for vannkraftproduksjon og batteridrift. I casene er elektrisitetsprisene fra kraftmarkedet brukt som marginalkostnader for vannkraftproduksjon, og lade-prisen er satt til den gjennomsnittlige elektrisitetsprisen. Batteriene deltar ikke i kraftmarkedet, og er ansett som pristakere. I casene genererer ikke batteriene noen ekstra inntekt, men er plassert i nettet for å være i stand til å yte tjenester.

Resultatene fra casene viser at de totale driftskostnadene er redusert i alle situasjoner når et batteri er knyttet til kraftsystemet. En kostnadsanalyse er utført, og denne konkluderer med at en batteriinvestering i et 50MW/200MWh-batteri ikke vil være lønnsomt basert på casene. Den totale driftskostnadsreduksjoner øker med økt volatilitet i elektrisitetsprisen gjennom dagen. Lademønsteret er strengt avhengig av elektrisitetsprisen, og avviker ikke fra det optimale mønsteret med mindre begrensninger er gitt, eller at nettverket når sin grense for tillatt operasjon, og batteriet må endre lagringsmønster for å hjelpe systemet til å fungere. Det er også vist at batteriene kan hjelpe systemet gjennom overføringsbe-

gresninger og driftsstans i produksjon. I casene der solenergi er inkludert er de totale driftskostnadene redusert fordi vannkraftproduksjonen også kan reduseres i tillegg til at batteriene fører til reduserte driftskostnader. I tillegg genererer kombinasjonen av solceller og batterier inntekter til solcellepanelenes eiere.

Det antas at den utviklede optimeringsmodellen kan brukes av vannkraftprodusentene til å planlegge sin produksjon og av batterieiere til å planlegge sine ønskede lademønster.

Preface

This thesis is the resulting work of the 5 year study program Master in Energy and Environmental Engineering at the Norwegian University of Science and Technology (NTNU). The Master's thesis is carried out at the Department of Electric Power Engineering in the spring of 2016.

The research work resulting in this thesis is focused on optimisation of battery dispatch. Both operational research and power markets are topics of my interest, and when I in January 2016 came across the opportunity to work with optimisation of batteries in the power system, supervised by Hossein Farahmand, I did not hesitate.

I would like to thank my supervisor Associate Professor Hossein Farahmand for valuable guidance, availability and motivational inspiration throughout the semester. I also wish to thank Steve Völler at SINTEF for giving valuable feedback and for his dedication to my work.

Oda Karoline Halvorseth Sunde
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Abbreviations

ATC	=	Available Transfer Capacity
CAES	=	Compressed Air Energy Storage
CNE	=	Critical Network Elements
CNTC	=	Coordinated Net Transmission Capacity
DC OPF	=	DC Optimal Power Flow
DSO	=	Distribution System Operator
ES	=	Energy Storage
EV	=	Electric Vehicle
FBMC	=	Flow Based Market Coupling
MILP	=	Mixed Integer Linear Programming
NC CACM	=	Network Code on Capacity Allocation and Congestion Management
NP	=	Net Position
NTC	=	Net Transfer Capacity
PTDF	=	Power Transfer Distribution Factor
PV	=	Photo Voltaic
RES	=	Renewable Energy Source
SMES	=	Superconducting Magnetic Energy Storage
SPO	=	Solar Panel Owner
TSO	=	Transmission System Operator

1 | Introduction

The energy sector account for a large proportion of the global greenhouse gas emissions. In 2009, the EU directive 2009/28/EC introduced ambitious targets to cut emissions by producing 20 % of the gross final energy consumption from Renewable Energy Sources (RES). A reduction of the emissions require a change of the power system based on emission free energy sources. The restructuring of the power system has encouraged and is still encouraging the development of technology within production, transmission and energy storage [1]. Energy storage facilities might be able to effectively increase the production flexibility in the power system. While offering the possibility to store excessive energy from RES during times with high production, stored energy can be feed back into the grid and support the power system during periods with low RES production.

Energy storage will most likely be an important component of a reliable, low emission and cost-effective future power system [2]. Battery storage facilities will enable a larger share of renewable energy sources in addition to provide other services to the grid, utilities and consumers. Including batteries in the grid requires a strategy on how to utilise them. Such a strategy can be focused on technical aspects or economical aspects, or both. The purpose of this research work is to study optimal scheduling of battery operation with respect to economical aspects.

1.1 Objectives

The objective of this work is to establish an operation strategy for optimised battery dispatch and study the impact of optimal scheduling of a battery in a power system model. The strategy is aimed to give an optimised solution that reflects realistic power flows and gives reasonable and cost-saving battery dispatch. The established model may contribute to improve currently used power system models as the amount of batteries in the grid is assumed to increase. In addition, the model may help a battery owner to decide when to charge and discharge, to maximise profit. It may also support battery owners of larger battery facilities to participate in the electricity market. The model can also encourage to battery storage investment decisions, as batteries are expected to decrease the total system operating costs.

1.2 Scope of work

The scope of the work:

- Perform a literature survey and provide an overview over battery storage possibilities.
- Establish an optimisation strategy for operation of batteries in a power system.
- Perform optimal scheduling of distributed battery storage, and study the impact on the Norwegian power system using the established strategy, given the exogenous electricity prices from the electricity market.

The strategy will be formulated as a mixed integer linear program (MILP) minimizing the total hourly power system operating costs, constrained by grid limitations and battery restrictions, and will be implemented in the modelling system GAMS. The strategy will be based on a flow-based power flow modelling algorithm, giving information on the physical power flows described by PTDFs. The model will deal with a period of 24 hours.

1.3 Literature review

The work resulting in this report is partly based on a literature study performed in the beginning of this research work. The theory and background material for this report is mainly based on information gathered from internet sources. IEEE Xplore Digital Library and Elsevier are search facilities used frequently during this work.

Typical key words used for search are:

- Energy storage
- Battery energy storage
- Flow based market coupling
- Optimal scheduling of batteries
- Optimal battery dispatch
- Ancillary services battery storage

In the search facilities, papers and articles can be sorted by the number of times they have been cited. This has been used to find reliable and readable references during the literature study.

The literature study is focused on three topics. One part of the literature study addresses the advantages regarding battery energy storage in the grid and the current use of battery energy storage. This part will be presented in chapter 2. The overview over the ancillary services provided by the batteries included in chapter 2 is based on article: "The economy of battery energy storage" from the Rocky Mountain Institute, [3]. The second topic is about the Nordic power system and electricity market, and is presented in chapter 3. This part is mainly based on information from the electricity market Nord Pool's home page. The third topic addresses power system modelling with flow based methodology and is presented in section 3.4. The theory in this part is obtained from the collaborative report written by the TSOs in Norway, Sweeden, Finland and Denmark: "Methodology and concepts for the Nordic Flow-Based Market Coupling Approach" [4]. The formulation of the part of the optimisation model dealing with battery storage constraints is inspired by the Elsevier paper: "Optimal scheduling of distributed battery storage for enhancing the security and the economics of electric power systems with emission constraints", [5]. The Norwegian grid model used in this work is based on data from a Ph.D. thesis written by B. H. Bakken, NTNU: "Technical and economic aspects of operation of thermal and hydro power system" [6]. The literature review also involves the references in the text recognized with square brackets.

1.4 Structure of report

This report is composed of 7 chapters. The main content of each chapter is briefly described below:

Chapter 2: Background provides an introduction to the topic of battery energy storage systems with a focus on the benefits of battery energy storage and current storage facilities in the grid.

Chapter 3: The Nordic power system comprises a description of the Nordic power system, the electricity market and an overview over power system modelling using PTDFs. It also contains a description of an aggregated model of the Norwegian power system, and a discussion of its accuracy.

Chapter 4: Mathematical modelling of optimal scheduling of distributed battery dispatch describes the development of the formulation of the optimisation model, and verifications of the scenarios.

Chapter 5: Case studies gives an overview over the cases analysed regarding the impact of optimal battery dispatch on the power system. The results and some discussion aspects are also included here. In the end of the chapter, a cost-benefit analysis was performed.

Chapter 6: Discussion provides a discussion on improvement suggestions to the model. It also includes an overall discussion of the results from chapter 5, and some additional discussion aspects.

Chapter 7: Conclusion gives a short summary of the most important results. Information of further work is also given in this chapter.

2 | Background

Grid energy storage is a collection of technologies that are used to store electrical energy within a grid. An overview of some existing storage technologies and their primary applications is given in table 2.1 [7]. The scope of this work is limited to assess battery energy storage, and the focus of this report will be on battery systems.

Batteries have traditionally not been widely used for large scale energy storage, due to their cost. In addition they have required high maintenance and have a limited lifespan. It has been a lot of research and technology improvement on batteries the past years, and batteries are now used for energy and power applications to some extent [8]. Battery energy storage systems already connected to the grid are mainly used to provide ancillary services or for supporting solar and wind integration by providing grid stabilization, frequency regulation and wind and solar energy smoothing [8].

2.1 Benefits of battery energy storage

With an increase in the use of electricity as an energy source, the operation, regulation and capacity of the grid becomes even more important than before. Increased power production from unregulated generation assets such as wind and solar has introduced major reliability challenges for power grid operators [5]. Generation from wind resources are in general negatively correlated with the consumption, as the wind resources often are larger during night times when the demand is low [5]. Batteries may ease some of the challenges dealing with these issues by storing excess energy from unregulated renewable energy sources during periods of low demand, and discharge in periods with higher demand. This will result in utilisation of a larger portion of the produced renewable energy [9].

However, storage is not only used for charging during periods with excess renewable energy. Battery storage can also actively be used for peak shaving, which involves charging when there is excess energy, either from renewable power plants or base power plants, during low demand periods and inject the stored energy back into the system during peak load periods.

Table 2.1: Overview over battery storage technologies.

Technology	**Primary applications**
Compressed air energy storage (CAES)	Backup power Renewable integration
Pumped hydro	Backup power Regulation
Fly wheels	Load levelling Frequency regulation Peak shaving and off peak storage Power quality: Transient stability
Advanced Lead-Acid	Load levelling Regulation Power quality: Grid stabilization
NaS	Power quality Congestion relief Renewable integration
Li-Ion	Power quality Frequency regulation
Flow Batteries	Peak shaving Load levelling Frequency regulation Power quality
Superconducting magnetic energy storage (SMES)	Power quality Frequency regulation
Electrochemical Capacitors	Power quality Frequency regulation
Thermochemical Energy Storage	Load levelling Regulation Power quality: Grid stabilization

2.1.1 Services that batteries can provide

Energy storage can provide benefits both technically and economically. To get an overview over some of the most important services batteries can provide to the grid, they are listed in table 2.2. The overview is based on information from report [3]. Services that batteries are able to provide can benefit several stakeholders and the main stakeholder groups are the distribution system operators (DSOs), transmission system operators (TSOs), utilities and customers. The services can be applied on different levels in the power system, and the system level for each service is also given in table 2.2.

Table 2.2: Services that batteries can provide to the grid. T = Transmission level, D = Distribution level and C = Consumer level

Service	Main stakeholder	System level
1. Load levelling	DSO/TSO	T, D and C
2. Load following/regulation	DSO/TSO	T, D and C
3. Frequency regulation	DSO/TSO	T, D and C
4. Spinning reserve	DSO/TSO	T, D and C
5. Power quality	DSO/TSO	T, D and C
6. Black start	DSO/TSO	T, D and C
7. Resource adequacy	Utility	T, D and C
8. Distribution/transmission deferral	Utility	D/T and C
9. Transmission congestion relief	Utililty	T, D and C
10. Time-of-use bill management	Customer	C
11. Increased PV self-consumption	Customer	C
12. Demand charge reduction	Customer	C
13. Backup power	Customer	C

1. Load levelling
Charging energy during low demand hours and discharge the energy during peak hours to ensure a uniform load for generation, transmission and distribution systems [10]. This is also referred to as energy arbitrage because it is possible to make profit on load levelling, meaning purchase of electricity when the price is low and sale of electricity when the price is high. This is highly relevant for unregulated energy sources.

2. Load following/regulation
Management of the differences between day-ahead scheduled generation, actual generation and actual consumption. Regulation responds to rapid load fluctuations in the order of one minute, and load following responds to slower changes in the order of five to thirty minutes.

3. Frequency regulation
Immediate and automatic response of power to a change in local system frequency. Regulation is needed to make sure that the generation matches load to avoid frequency spikes or dips creating instability.

4. Spinning reserve
Spinning reserve is unused generation capacity that is online and able to serve load immediately in response to an unexpected contingency event.

5. Power quality
Involving mitigation of disturbances in the grid, such as flicker effects and voltage dips. Covering transient stability, voltage stability and power oscillation damping [10]. Ensures reliable and continuous electricity flow across the grid.

6. Black start
Enabling the power system to be restarted after a black out [10].

7. Resource adequacy
Grid operators and utilities may pay for energy storage to incrementally defer or reduce the need for new generation capacity and minimize the risk of over investment in the area.

8. Distribution/transmission deferral
Delay, reduce the size of, or entirely avoid utility investments in distribution/transmission systems necessary to meet expected load growth in specific regions.

9. Transmission congestion relief
Energy storage can be placed downstream of congested transmission lines to discharge during congested periods and minimize congestion in the system. This may also lead to transmission system upgrade deferral.

10. Time-of-use bill management
Minimizing electricity purchased during peak-hour when time-of-use rates are highest and shifting the purchases to periods of lower demand, and rates to reduce the bill.

11. Increased PV self-consumption
Benefit from a larger share of behind-the-meter solar power production, by storing produced energy and use it in no-production periods.

12. Demand charge reduction
Reducing the peak demand which consumers sometimes has to pay for, as an extra charge in addition to the charge for the total consumption. In Norway this is relevant in areas with AMR systems.

13. Backup power
In the event of grid failure, energy storage paired with a local generator can provide backup power at multiple scales.

In this report only some of the services are investigated. These are:
1. Load levelling
8. Distribution/transmission deferral
9. Transmission congestion relief
11. Increased PV self-consumption
13. Backup power

In addition to the above mentioned specific services batteries can also benefit otherwise. Batteries may lead to loss reduction by decreasing network usage in the higher load periods and increasing it in the lower load periods. Storage can increase the efficiency of the network and reduce energy transmission costs [10]. Compared with conventional generators, battery storage has a faster response and better performance [2]. Batteries can match total generation to total load precisely on a second by second basis, while conventional power plants may take several minutes or even hours to come online [8]. With batteries installed, traditional fuel-based power plants will to a greater extent be allowed to operate at constant production levels, providing higher efficiency and less contamination/pollution. The batteries can relieve the strain on the production units during peak load, and expensive fast generation can be avoided. [9] However, the life of a battery is not eternal, and frequent charging/discharging might decrease the battery life, especially when providing fast regulation service [2].

Batteries have the ability to change from energy supplier to power producer. Providing power means delivering electricity for short periods, and providing energy means delivering electricity for longer periods. Different applications require only seconds or minutes of power, such as frequency control, and others may require hours, such as load levelling [10].

One of the main reasons for installing batteries in the grid is to support the integration of more renewable power, to reach the emission reduction targets. However, batteries have an environmental footprint. They do not have tailpipe emissions, but the production of batteries leads to environmental burdens [11] that needs to be considered when estimating the value of batteries. Large investments on storage are not hugely attractive from an economical point of view. This is due to the investment costs and the insufficient remuneration of ancillary services [10].

2.2 Battery storage in the world today

Battery storage possibilities are receiving an increasing amount of attention within industry, politics and academia in the recent years, due to their beneficial technological characteristics, and their contribution to reduced costs. Batteries have been connected to electricity grids since the 1870s, but large-scale energy storage has mainly been achieved using pumped-storage hydroelectricity schemes [12]. However, increasing activities in the grid battery energy storage sector are observes. The development of advanced batteries that are efficient enough to allow for storage in the grid in addition to be safe and low-cost is important to encourage to investments in this area.

Navigant Research expects growth in advanced batteries for utility-scale energy storage applications to be robust over the next 10 years [13]. Factors expected to influence the amount and type of energy storage installed in a given region are: level of unregulated generation penetration, government support like subsidies, grid structure, market structure, demographics and grid stability [13]. The market structure of an electricity market is an important consideration when examining the potential for energy storage [13]. It is expected that new models for electricity markets will be developed when the amount of energy storage facilities increases. This is further discussed in chapter 3.

Report [14] has studied existing battery storage projects and a database of projects across the world was developed. [14] shows that Lithium ion batteries make up for more than half of the battery energy storage projects in the world today, and this may be because of their ability to provide both energy and power applications to a certain extent [14]. Lithium-ion batteries have a high efficiency and reliability, a high energy density and a slow self-discharge rate. The the efficiency can reach up to 85-90 % [10].

[14] also informs that the largest share of the projects studied is dedicated to arbitrage applications (36 %). This mainly involves electric bill management at consumer level. Other

applications involve wholesale arbitrage at the grid or generator level. Power quality applications also accounts for a large share of applications (33 %), such as frequency regulation (16 %) and voltage support (3 %), or by renewable intermittent energy output smoothing and ramping (14 %). 69 % of the projects studied also had secondary applications listed.

A single grid-connected battery system can be as small as a few kWh or as large as hundreds of MWh [12]. According to [15], September 21st 2015, Europe's largest battery energy storage project has recently opened in Feldheim, Germany. This is a project conducted by the companies Enercon and Energiequelle, and comprises a 10MW/10MWh energy storage plant. The primary reason for building this energy storage plant was to stabilise the frequency in the transmission system, but the plant also serves other applications. According to [16], the worlds largest battery storage facility started operating in Buzen, Japan on the 3rd of March, 2016. This is a NaS battery energy storage system with a total output of 50 MW and a storage capacity of 300 MWh. The battery system was installed to improve the electricity supply/demand balance, control the grid voltage and smooth the delivery of renewable energy to the grid.

Another real life example of a battery storage facility is the Tehachapi Energy Storage Project in California, which is an experiment in storing energy from wind power production in lithium-ion batteries. The size of the battery is 8MW/32MWh [17], with a price tag of $ 50 millions. The project efforts to demonstrate how energy storage can improve the power grid and reduce emissions. [18]

Electrification of the transport sector facilitates the integration of energy between electrical vehicles (EV) and the grid. Norway has several policies to speed up the quantity of EVs, and the amount of EVs in Norway is quite high. EVs are in general functioning as loads and the increasing amount of electrical vehicles leads to higher load peaks in the power system than before. Electrical vehicles can function as distributed storage, and vehicle to grid (V2G) technology demonstrates the bilateral role of EVs as both supplier and customer of energy [5]. By introducing bilateral storage, and using the EVs as a provider of energy as well, the EVs may help reducing the load peaks. The electrification of the transport sector also enables the opportunities for potential second-life applications for EV batteries for stationary use [7].

2.3 Optimal scheduling of distributed battery storage

Optimal scheduling of battery storage has been investigated and elaborated in a number of reports and papers before. There are several ways to optimise a battery dispatch, depending on what it is supposed to be optimised regarded to. Following the price, minimizing ageing of the battery, providing the best services to the grid and minimizing production costs are subjects to base an optimisation strategy on. Different solution algorithms are also used, some takes a grid into account, and others do not.

Paper [2] claims to provide an optimal bidding strategy in electricity markets considering performance-based regulation and battery cycle life. Providing fast regulation service largely affects battery life, so the bidding strategy aims to maximise the profit but also take the battery life into account. [2]. considers the batteries as price-takers, in opposite to [19], which considers the batteries as prices-makers. [19] investigates the charge and discharge schedules to maximize the total profit, when the batteries are investor-owned. The objective function is to maximize the total profit of the storage units. An optimal supply and demand bidding, scheduling, and deployment framework is proposed for battery systems in [20]. The model takes into account the day-ahead prices and the case studies are based on real market data with batteries as price-takers. The objective function is to maximize the total expected value of the battery system. [21] presents and analyses two MILP models for hourly scheduling of energy storage systems in day-ahead markets. The objective function is to minimize the total production cost. As mentioned, EVs can function as distributed storage. In paper [5], a model of scheduling of electric power systems as a MILP problem is proposed, using EVs as distributed storage. The objective function in this model is to minimize the power system operating cost.

A different optimisation strategy is presented in [22]. The model presented in this paper is using second order conic programming, and the objective is to minimize the total operating cost of the system. [23] considers installing energy storage systems to reduce network investment costs. The model is formulated as a MILP problem with an objective function to minimize the investment cost on transmission network elements while satisfying nodal energy balance, line capacity and other constraints. [23] and [24] are papers focusing on the use of batteries to improve efficiency, reliability and investment decision making, while [2], [11], [19] and [19] are focused on profitability.

This is just a a selection of reports dealing with optimal battery dispatch. In this research work, the aim of the optimisation strategy will be to minimize the total system operating costs.

There are several issues with modelling of batteries. Batteries are not linear, but are often assumed to be linear in the models. In addition, optimisation models will aim to empty the battery if there are no restrictions, and this might be a problem. These issues should be considered, but they are not assumed to limit this research work to a large extent.

3 | The Nordic power system

The purpose of this research work is to study optimal scheduling of battery operation in a power system. The power system to be investigated will be the Norwegian power system. As the Norwegian power system is closely related to the Nordic power system, an overview over the Nordic power system and electricity market is given in this chapter.

3.1 Description of the Nordic power system

The Nordic power system is connected over country borders and between price areas and an overview over the power system is visualised in figure 3.1. Transmission lines between countries enables export and import of electric power over borders and can be utilised both in a physical/technical and economical way. The Norwegian power system is connected to Sweden, Finland and Russia with AC lines, and to Denmark and the Netherlands via DC cables. The transmission capacities are listed in table 3.1. Further, cables to Germany and Great Britain are planned. The connection between borders increases stability, flexibility and security of supply for each country in addition to facilitate increased for renewable energy sources. In general, trading energy between countries contribute to a more effective utilisation of existing and future energy production in Europe. If all available energy resources are distributed, the installed capacity in each country can remain on a lower level and still maintain the security of supply. [1]

Table 3.1: Transmission capacities between Norway and neighbouring countries [1].

Countries	**Transmission capacity**
Norway - Denmark (Skagerrak 1-4)	1700 MW
Norway - Netherlands (NorNed)	700 MW
Norway - Sweeden	3600 MW
Norway - Finland	100 MW
Norway - Russia	50 MW (Import)

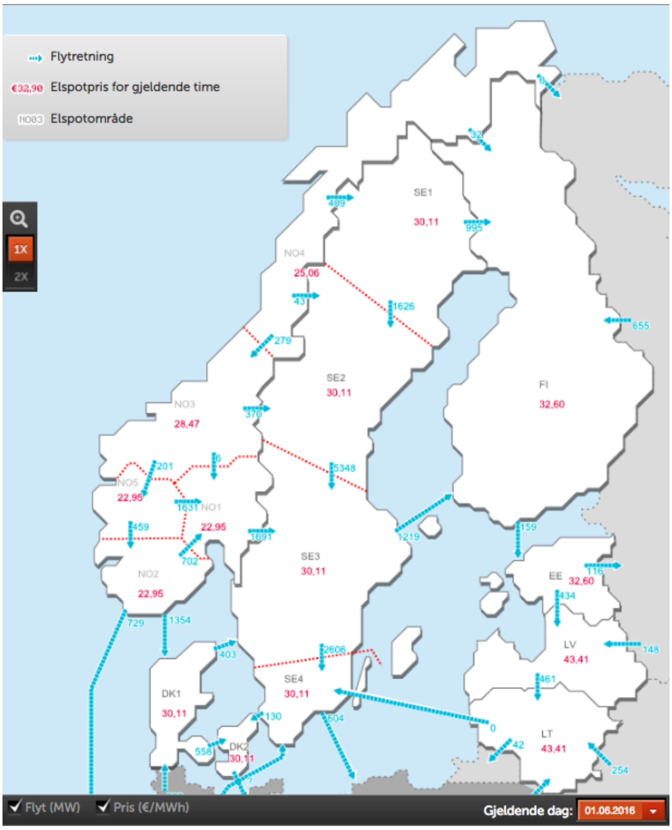

Figure 3.1: The Nordic power system [25].

The total exchange capacity between Norway and the neighbouring countries is currently 5500 MW, of which 2400 MW are sea cables. The cables from Norway to Germany and Great Britain are planned to have a capacity of 1400 MW each. [1]

Electricity produced in Norway is primarily based on renewable, regulative hydro power production. The situation for Europe as a whole is different and the share of renewable energy production is not nearly as high as in Norway. The Norwegian and the European production portfolios can be seen in respectively figures 3.2 and 3.3. Although the share of renewable energy production is lower in Europe, the share of variable generation assets are higher. This is due to the installed solar and wind production. The planned expansion of transmission capacity between Norway and Europe can support in using the regulated hydro power in Norway as a green battery towards unregulated power production in Europe in the future.

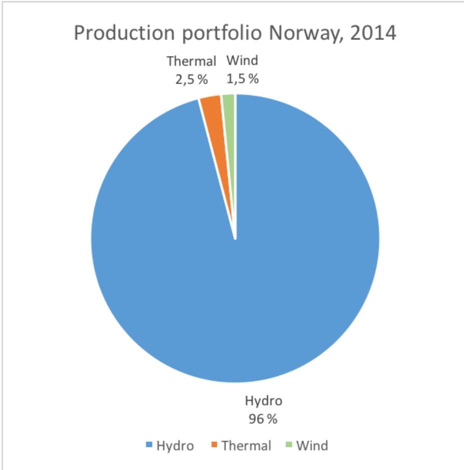

Figure 3.2: Production portfolio for Norway, 2014 [26].

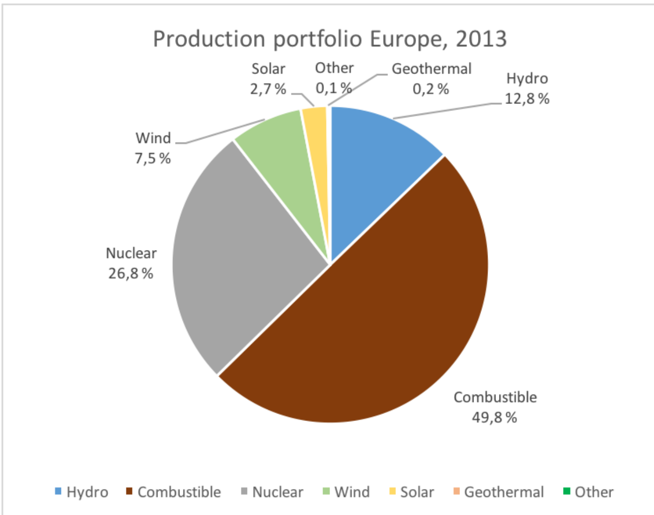

Figure 3.3: Production portfolio for Europe, 2013 [27].

ENTSO-E, the European Network of Transmission System Operators is an association representing TSOs from 34 countries in Europe. ENSTSO-E is given legal mandates by the EU, and are supposed to work towards a liberalised electricity markets. A closer cooperation across Europe's TSOs is one of the issues ENTSO-E is working with. The suggested cooperation and liberalisation of the electricity markets will support the implementation of EU's energy policy. This will further help to achieve Europe's energy and climate policy objectives, which are changing the nature of the power system. ENSTSO-E is working with developing the worlds largest electricity market. [28]

3.2 The Nordic Electricity Market

The Nordic countries (Denmark, Finland, Norway and Sweden) deregulated their electricity markets in the 1990s, and brought their individual markets together into a common Nordic market called Nord Pool. Estonia, Latvia and Lithuania joined the Nord Pool market in 2010-2013. Nord Pool is Europe's leading electricity market, and offers trading, clearing, settlement and associated services in both day-ahead and intraday markets across nine European countries, including Norway. Nord Pool AS is licensed by the Norwegian Water Resources and Energy Diractorate (NVE) to organise and operate a market for trading energy, and by the Norwegian Ministry of Petroleum and Energy to facilitate the electricity market with other countries. [29], [30]

The day-ahead market Elspot is an auction where members can place their orders hour by hour for the next day. The equilibrium between the aggregated supply and demand curves is established for all bidding zones, and the system and area prices are calculated. The system price is calculated based on the sale and purchase orders disregarding the available transmission capacity between the bidding zones. The day-ahead market is divided into bidding zones, and the area prices are established taking the transmission capacity between the areas into consideration[31]. Bottlenecks can occur where if large volumes has to be transmitted to meet demand. The price areas are introduced to relieve the congestion caused by the bottleneck. When the transmission capacity is constrained, the prices is increased to reduce the demand in the affected areas[32]. Note that bidding zones may differ from price zones as one price area can contain more than one bidding zones [4].

In a power system, situations where production and consumption deviates will occur. This can be caused by an unplanned generation outage, intermittent renewable energy producers not able to predict their production, or simply the fact that the consumption changes every second. When the production and load in the system does not correspond, the voltage and frequency will deviate from the desired system quantities and the system stability will be decreased. To maintain the power balance and system stability, the production or demand needs to be adjusted. This is taken care of by Nord Pool Intraday market, acting as a balancing market, or regulating market [33]. In the balancing market, participants places price bids to change their production or consumption. The TSOs are responsible for the organisation of regulating participants.

Battery energy storage devices can provide regulating capacity, it could be useful for the battery owners to somehow take part in the electricity market. As they are able to both charge and discharge, they will act like a producer or a load depending on the situation. Batteries can in theory place bids in both day-ahead markets and regulation markets [2].

An important aspect of balancing is ancillary services, referring to a set of services which TSOs contract so that they can guarantee system security, described further in chapter 2. This is especially relevant for battery applications. However, the Nordic electricity market is not designed to accommodate the participation of batteries. Due to insufficient remuneration of ancillary services [10], changes in the electricity market is required to accommodate batteries in the grid. Several countries aims to facilitate for increased participation for small consumers in the power and balance market [34]. ENTSO-E Working

Ground Ancillary Services is working towards creating a harmonised approach to ancillary services provision [35]. This will benefit battery owners participating in the electricity market in the future.

The European electricity market is already facing a comprehensive restructuring process, supported by ENTSO-E. The reason for the changeover is political goals regarding energy and climate, the economical development in Europe and international electricity markets. A challenge will be to integrate renewable power production in a way that will retain the security of supply and does not give large costs for the consumers. This will require a sufficient market design, support mechanisms and integration of the European markets. With today's market design, the prices are decided based on the marginal costs for production. By integrating more renewable production, the prices will be decreased, due to their low marginal production costs. But the increase in unregulated renewable power production will increase the volatility in the market. As a result negative electricity prices has been observed. [34]

Batteries on the distribution level requires fast response times. For secondary services, the batteries have to provide energy/power from between 30 seconds up to 15 minutes. This can be a challenge, especially regarding smaller batteries, as they may be totally discharged during these 15 minutes, and may not be able to provide the service they are supposed to. Due to the batteries fast response time, it will be beneficial to utilise the batteries for the first couple of minutes when a service is needed, and then connect conventional generators with a larger response time. This will also exclude the problem regarding too fast discharge of the batteries. This type of operation is not supported in the Nordic electricity market, and underpins the need for a change in the market design.

The batteries analysed in this research work is assumed to be on the transmission level, and the amount of energy is assumed be large enough to avoid the previously mentioned fast discharging issue. In this research work, the batteries are not participating in the market, thus assumed to be price-takers.

3.3 Day-ahead market clearing

The current practice to manage bottlenecks in the Nordic system is by use of area pricing, as mentioned in the previous section. In order for Nord Pool to be able to calculate the area prices for the day-ahead market, the available transmission capacity has to be taken into account. This is not straightforward because electricity does not flow directly from generator to consumer, but spreads out over parallel paths in the network. Thus, commercial flows differ from physical flows. Commercial flows describe the flow directly between the generator and consumer. This means that the transmission capacity between two market zones can not be fully allocated to commercial trade between these market zones, because some of the capacity is used by transit flows resulting from trade between other market zones. Different capacity allocation methods exists, amongst others the currently used Available Transfer Capacity (ATC) method also called Net Transfer Capacity (NTC) or Coordinated Net Transfer Capacity (CNTC), and the new Flow Based Market Coupling

(FBMC) method. [36]

The day-ahead market clearing can be expressed as a constrained optimisation problem where the objective is to maximise the social welfare, constrained to keep the supply and demand in balance, and by the transmission capacities [4]. The development of the optimisation model considering the Norwegian power system in this research work will use the flow based power flow modelling technique for capacity allocation. A further description of this method will be presented in the next sections.

3.3.1 Flow Based Market Coupling

FBMC is distinguishing from CNTC by the introduction of a simplified grid model to give information on the physical flows in the power system. This enables the market to prioritise flows that are most economically efficient in managing congestions. With the current methodology of CNTC only commercial exchanges between bidding zones are considered by the market algorithm. The real physical flows are managed by the TSO, and this requires the TSOs to make decisions on capacity allocations in advance of the market clearing, based on assumptions of the market outcome and physical flows. The flow based algorithm is expected to increase the social welfare compared to CNTC based calculations as they represent the physics of power flow more correctly. [4], [36]

This results in better utilisation of the physical infrastructure which leads to an increased solution domain for FBMC when compared to CNTC based auctions, because no transmission capacity has to be withheld from the market by prioritising capacity on certain borders in advance. This is visualized in figure 3.4. All CNTC market solutions are available to the FBMC, but the FBMC gives additional solutions not available with the CNTC method. [4]

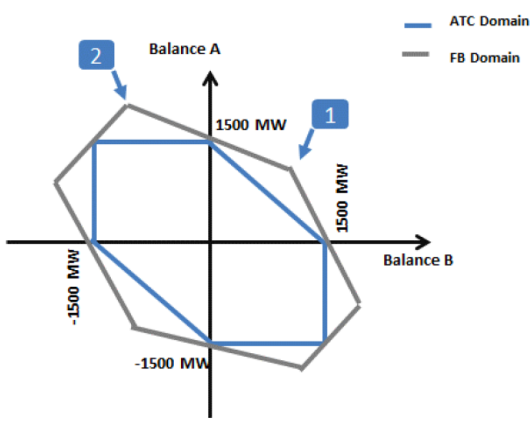

Figure 3.4: Solution spaces for FBMC and CNTC methodology [4].

The Flow Based Market Clearing is based on PTDF-factors, describing the incremental distribution factors associated with power transfers between two regions. The physical characteristics of the grid are known, and based on these characteristics, one can calculate the amount of power flowing in each line depending on the node of the power injection. These calculations result in the matrix of sensitivity factors commonly referred to as Power Transfer Distribution Factors (PTDF), and is based on DC OPF (DC Optimal Power Flow). Instead of modelling the whole grid, only interconnections and so-called Critical Network Elements (CNE) are considered. This gives a linear approximation of the physical reality. The PTDF-factors translates the change in net position into physical line flows, and expresses the DC power flow of the grid. If there are no congestions, the market price will be equal for all areas. [4]

The FBMC methodology is the preferred design in the Network Code on Capacity Calculation and Congestion Management (NC CACM), and the EU guidelines require a shift from the current CNTC to the FBMC methodology to increase the overall electricity market efficiency. The Flow Based Market Coupling has been in operation in the Central Western European day-ahead market since May 20, 2015 (in Belgium, the Netherlands, France, Germany and Austria). [4], [36]

3.4 Mathematical power system modelling

To be able to use the flow based methodology to calculate the physical power flows in a power system, the PTDF matrix needs to be obtained. The PTDFs are in reality provided to the market by the TSOs. A stepwise derivation on how to calculate them, obtained from [4], is provided in the next section.

3.4.1 Calculation of PTDF

The power transfer distribution factors (PTDFs) describe the impact of a certain node on a given branch, by translating the change in net position into physical line flows [4]. The calculation of the PTDFs is based on the AC power flow equations, and will be further described in this section.

The active power flow in a line can be calculated with equation 3.1 obtained through simplifications on the AC power flow equations [4]:

$$P_{ik} = B_{i,k}(\delta_i - \delta_k) \tag{3.1}$$

$B_{i,k}$ represents the susceptance between node i and k with negative sign.

A 3 node network is constructed to demonstrate how the PTDF matrix is calculated, and can be seen in figure 3.5.

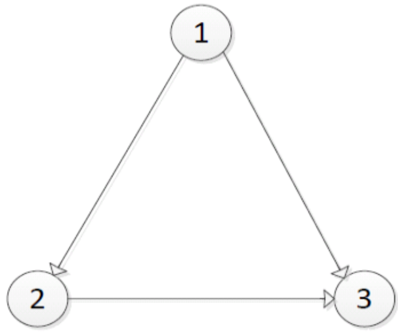

Figure 3.5: 3 node system.

For a 3 node network, equation 3.1 can be written as follows:

$$\mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} B_{1,2} + B_{1,3} & -B_{1,2} & -B_{1,3} \\ -B_{2,1} & B_{2,1} + B_{2,3} & -B_{2,3} \\ -B_{3,1} & -B_{3,2} & B_{3,1} + B_{3,2} \end{bmatrix} \cdot \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \mathbf{B} \cdot \boldsymbol{\delta} = \boldsymbol{Y_{bus}} \cdot \boldsymbol{\delta} \tag{3.2}$$

Where the **B**-matrix is the node admittance matrix, called $\boldsymbol{Y_{bus}}$. The admittance matrix represents the dependence of the branch currents towards the nodes on the node electric potentials relative to a reference node.

In order for equation 3.2 to have a unique solution, at least one of the diagonal elements must contain an additional value, creating a reference node also called slack bus. The slack bus can be defined by adding a "+1" to one of the diagonal elements of the bus impedance matrix, $\boldsymbol{Z_{bus}}$, where the $\boldsymbol{Z_{bus}}$ is the inverted $\boldsymbol{Y_{bus}}$. The voltage angles are given as:

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} 1 + B_{1,2} + B_{1,3} & -B_{1,2} & -B_{1,3} \\ -B_{2,1} & B_{2,1} + B_{2,3} & -B_{2,3} \\ -B_{3,1} & -B_{3,2} & B_{3,1} + B_{3,2} \end{bmatrix}^{-1} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \boldsymbol{Z_{bus}} \cdot \mathbf{P} \tag{3.3}$$

To derive the PTDF, an example assuming injection of additional power, $\varDelta P_1$, in node 1 is considered. This gives:

$$\varDelta\delta_1 = \varDelta P_1(1 + B_{1,2} + B_{1,3}) = \varDelta P_1 Z_{bus,11} \tag{3.4}$$

$$\Delta\delta_2 = \Delta P_1(-B_{2,1}) = \Delta P_1 Z_{bus,21} \tag{3.5}$$

The injected power in node 1, ΔP_1, induces a power change in line 1-2 that can be calculated by combining equation 3.1, 3.4 and 3.5:

$$\Delta P_{1-2} = B_{1,2}(\Delta\delta_1 - \Delta\delta_2) = B_{1,2}(Z_{bus,11} - Z_{bus,21})\Delta P_1 \tag{3.6}$$

The change in power flow in line 1-2 can be expressed per unit net power injected in node 1, by setting ΔP_1 to unity. This results in the PTDF value for affection of power injection in node 1 on line 1-2.

$$PTDF_{12,1} = B_{1,2}(Z_{bus,11} - Z_{bus,21}) \tag{3.7}$$

Or more generic:

$$PTDF_{ik,n} = B_{i,k}(Z_{bus,in} - Z_{bus,kn}) \tag{3.8}$$

The PTDF-matrix for the 3 node example is given under. The PTDF-matrix includes each node's PTDF for each line in the network. $PTDF_{12,1}$ describes the contribution of node 1 on line 1-2.

$$\mathbf{PTDF} = \begin{array}{c} \\ \text{Line 1-2} \\ \text{Line 1-3} \\ \text{Line 2-3} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Node1} & \text{Node 2} & \text{Node 3} \end{array} \\ \begin{bmatrix} PTDF_{12,1} & PTDF_{12,2} & PTDF_{12,3} \\ PTDF_{13,1} & PTDF_{13,2} & PTDF_{13,3} \\ PTDF_{23,1} & PTDF_{23,2} & PTDF_{23,3} \end{bmatrix} \end{array}$$

3.4.2 Physical power flow calculated using PTDF

A PTDF-matrix can be directly used to calculate the physical active power flows in a network. The physical power flow is dependent on the net power injection in a node, called net position (NP). A positive net position indicates that the node is a net exporter, and a negative net position indicates a net importer. As the PTDF describes the change in power flow in a line per unit net power injected in a node, the power flow in a line would be the sum of the PTDFs for the line multiplied by the net position in the node.

$$P_{ik} = \sum_n PTDF_{ik,n} \cdot NP_n \tag{3.9}$$

3.5 Optimisation model for a power system

The day-ahead market clearing can as mentioned be expressed as a constrained optimisation problem. The optimisation problem provides a solution on capacity allocation. The same method will be used in this research work to obtain optimized power flows and production levels.

The optimisation will aim to minimize the operating cost for the power production. The optimisation problem is described as a Mixed Integer Linear Programming (MILP) problem. A MILP solution is the optimisation of a linear function with some of the variables accepting only integer values [5].

The information required as input to the model are the parameters given in the model formulation below.

Sets

t	Hour index
n	Node
l	Line

Parameters

$D_{n,t}$	Production cost for node n at time t
$K_{n,l}$	Connection matrix telling which node is connected to which line
$P_n^{gen,min}$	Minimum generation level for generator at node n
$P_n^{gen,max}$	Maximum generation level for generator at node n
$P_{n,t}^{dem}$	Load demand for node n at time t
P_l^{cap}	Transmission capacity for line l
$P_{n,t}^{exc}$	Power exchange with neighbouring countries from node n at time t
$PTDF_{l,n}$	PTDF-factors for line l by node n

Variables

$P_{n,t}^{gen}$	Generation level at node n at time t
$P_{l,t}^{flow}$	Power flow in transmission line l at time t

Objective function

$$\min \sum_t \sum_n D_{n,t} P_{n,t}^{gen} \tag{3.10}$$

Constraints

$$P^{gen,min} \leqslant P^{gen}_{n,t} \leqslant P^{gen,max} \tag{3.11}$$

$$P^{flow}_{l,t} = \sum_{n} PTDF_{l,n} \cdot NP_{n,t} = \sum_{n} PTDF_{l,n}(P^{gen}_{n,t} - P^{dem}_{n,t}) \tag{3.12}$$

$$|P^{flow}_{l,t}| \leqslant P^{cap}_{l} \tag{3.13}$$

$$P^{gen}_{n,t} - P^{dem}_{n,t} - P^{exc}_{n,t} - \sum_{l} P^{flow}_{l,t} \cdot K_{n,l} = 0 \tag{3.14}$$

Equation 3.11 indicates that the generation in one node must be between its minimum and maximum generating capacity. The power flow in each line at a given time is given by equation 3.12. The power flow has to be lower than the transmission capacity for the specific line, indicated by equation 3.13. Finally, the energy balance in each node needs to be fulfilled. This means that the sum of the production, load, power exchange between countries and power flow to/from the node has to be equal to zero for every hour, according to equation 3.14.

A network topology can be described by an incidence matrix/connection matrix, K, that indicates the connection of lines to nodes. The connection matrix for a 3 node system as in figure3.5 will look like:

$$K_{n,l} = \begin{array}{c} \\ \text{Node 1} \\ \text{Node 2} \\ \text{Node 3} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Line 1-2} & \text{Line 1-3} & \text{Line 2-3} \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \end{array}$$

3.6 Model of the Norwegian power system

This research work will result in a battery/grid optimisation model to be used to study the impact of batteries on the Norwegian power system. The Norwegian power system is fairly complex, thus modelling a copy of the physical grid will be time consuming. An aggregated model of the Norwegian power system described by 22 nodes and 33 lines will therefore be used. The original development of this model is described in B. H. Bakkens PhD Thesis[6], and further developed in [37]. This is a simplified model, but it is assumed to describe the main flows in the Norwegian power system quite well.

The parameters for the aggregated Norwegian power system model are obtained from [6]. Figures 3.6 and 3.7 pictures nodes and lines in the power system model. The maps are obtained from Nord Pools internet pages [38], and the nodes and lines are added. Norway is split into 5 price zones, NO1, NO2, NO3, NO4 and NO5, defined by the electricity market Nord Pool.

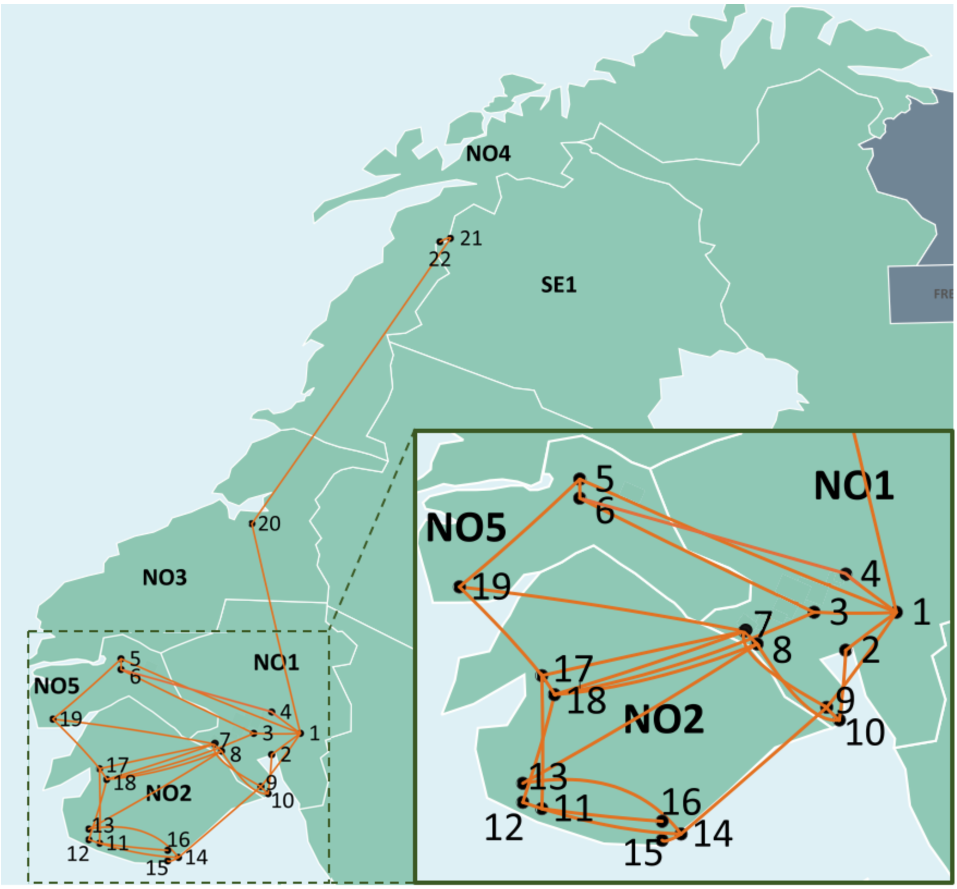

Figure 3.6: Nodes and lines in the Norwegian power system model.

The connection between the lines are described by a connection matrix given in appendix A. The PTDF-matrix is given in the same appendix. The admittance matrix is obtained from [6], and the PTDF-values are calculated by using MatPower, a package of MatLab M-files for solving power flow and optimal power flow problems.

Production units and loads in the Norwegian power system model are distributed over the area of Norway, and placed in 10 nodes. These 10 nodes are 1, 5, 7, 9, 11, 14, 17, 19, 20 and are marked with red dots in the map in figure 3.7. Each node has a minimum and maximum production capacity. All actual production in an area is added together, and assumed to be fed into the nodes in the network. The minimum and maximum production limits for each node are given in table 3.2. As most of the electricity produced in Norway is from hydro power plants, the production units are assumed to be hydro power plants, unless otherwise is specified. Hydro power plant usually have flexibility in order to meet the load fluctuations, and the output power can rapidly be changed to meet the demand. Hence, start-up and shut-down times are neglected, equally to the ramp-up and ramp-down times.

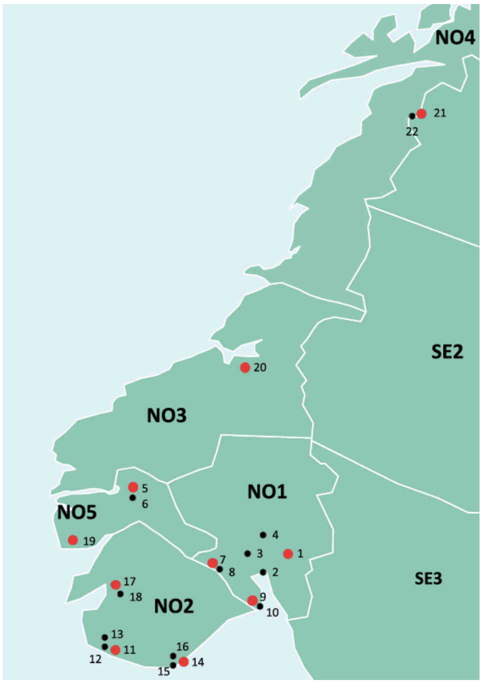

Figure 3.7: Nodes with production capacity in the Norwegian power system model.

The other 12 nodes in the system are present to connect transmission lines and secure the power flow to be more realistic. Table A.4 in appendix A shows an overview over which nodes are located in which area. The transmission capacity for each line is listed in table 3.3.

Table 3.2: Generation limits for the nodes in the Norwegian power system model.

Node	Min. production, $P_n^{gen,min}$ [MW]	Max. production, $P_n^{gen,max}$[MW]
1	0	2636
5	0	4028
7	0	2783
9	0	1465
11	0	4174
14	0	147
17	0	2050
19	0	4247
20	0	2783
21	0	4980

Table 3.3: Transmission capacity for each line in the Norwegian power system model.

Line	Nodes	Transmission capacity, P_l^{cap} [MW]
1	14-15	INF
2	11-16	INF
3	2-1	INF
4	3-1	2000
5	4-1	INF
6	1-5	INF
7	1-9	INF
8	1-20	INF
9	2-10	1732
10	3-6	1000
11	3-18	2065
12	4-6	1436
13	5-6	INF
14	5-19	774
15	7-8	INF
16	7-9	INF
17	7-17	900
18	7-19	850
19	8-10	2065
20	8-13	2957
21	8-18	2065
22	7-18	225
23	9-10	INF
24	9-14	1472
25	11-12	1000
26	11-14	1210
27	11-17	INF
28	12-18	2515
29	14-13	1000
30	17-18	INF
31	17-19	INF
32	20-21	INF
33	21-22	1000

In addition to power flow within Norway, exchange with neighbouring countries are considered. These are modelled as additional load (for export) or production (for import) in the nodes where the different countries are connected to Norway. The exchange with Finland and Russia is neglected, as these are small (and often equal to zero) compared to the rest of the system. The exchange nodes are:

Table 3.4: Exchange connections with neighbouring countries in the Norwegian power system model.

From node	From area	To area
2	NO1	SE3
15	NO2	DK1
16	NO2	NL
20	NO3	SE2
22	NO4	SE1
21	NO4	SE2

To be able to perform optimisation, data for day-ahead prices, consumption and exchange with neighbouring countries needed. As no forecasts are present, historical data from NordPool [39] is used. These data are organised in excel files. The excel files from Nord Pool give the day-ahead prices for different cities in Norway. These correspond to the different price areas as seen in table 3.5. The names of the excel files are given in table 3.6.

Table 3.5: Price areas described by cities in Nord Pools data for day-ahead prices.

Price area	Corresponding city
NO1	Oslo
NO2	Kristiansand
NO3	Trondheim/Molde
NO4	Tromsø
NO5	Bergen

Table 3.6: File names for the files obtained from Nord Pool [39].

Parameter	Nord Pool excel file name
Day-ahead price	Elspot Prices_2016_Hourly_EUR
Consumption	Consumption NO areas_2016_Hourly
Exchange with neighbouring countries	Exchange NO connections_2016_Hourly
Power flow between areas	Elspot flow NO_2016_Hourly
Transmission capacities	Elspot capacities NO_2016_Hourly

The consumption data from Nord Pool is given as total consumption for each area. The consumption is distributed over the nodes in each area according to IEE-EU Tradewind Project [40]. Table 3.7 reflects the share of consumption where the shares for each area adds up to 1.

Table 3.7: Consumption share for the different nodes in the Norwegian power system model.

Area	Node	Consumption share
NO1	1	0.761
NO1	7	0.017
NO1	9	0.222
SUM		**1**
NO2	11	0.871
NO2	14	0.128
NO2	17	0.001
SUM		**1**
NO5	5	0.062
NO5	19	0.938
SUM		**1**
NO3	20	**1**
NO4	21	**1**

3.7 Verification of the model of the Norwegian power system

A verification of the validity of the Norwegian power system will be conducted. The marginal cost of production for hydro is assumed to be correlated to the day-ahead price. The reason for the decision is elaborated in section 4.1.6. The optimisation time horizon is 24 hours with hourly resolution.

It is desired to test the power system model of Norway for extreme cases to see how it handles these situations. The model should handle situations pushing the limits of the transmission and production, such as situations with large consumption. The situation on Thursday 21st of January 2016 was characterized by large consumption, and can be used to analyse the behaviour of the power system model during large consumption. Large consumption can cause challenges regarding production limits and power transmission capacity, and it is desired to examine whether the model handles the real situation of the 21st of January 2016. A simulation is done using data for day-ahead prices, consumption and exchange between countries for the given date obtained from Nord Pool [39].

By using the original limits for the transmission lines and production units, the simulation did not give a feasible solution. To cope with this problem, transmission limits and production capacities were adjusted according to IEE-EU Tradewind Project [40]. After this adjustment, a feasible solution was obtained. The changes are already accounted for in the table of the transmission capacities, table 3.3.

The feasible solution obtained means that the power system model handles the situation of 21st of January 2016. Hence, it is concluded that the production and transmission levels used in the aggregated power system model are large enough.

It is assumed that the model of the Norwegian power system, outlined in section 3.6, describes the main flows in the Norwegian grid quite well. To verify this assumption, and also see how close to reality the power flows are, an analysis on the power flow between the areas is done. Adding the simulated power flows in all transmission lines crossing the border between to areas gives the total power flow between two areas. The inter-area lines are listed in table 3.8. Comparing the simulated power flow to the actual power flow between the areas obtained from Nord Pool [39] gives an indication on how accurate the model is. The power flows between the areas from the simulation are given in table 3.9. The real power flows between the areas are given in table 3.10.

Table 3.8: Lines crossing the border between two areas in the Norwegian power system model.

From area - To area	Lines crossing the border between areas
NO1-NO2	11, 17, 20, 21, 22, 24
NO3-NO4	32
NO1-NO3	8
NO1-NO5	6, 10, 12, 18
NO2-NO5	31

By comparing tables 3.9 and 3.10, it is easy to see that the simulated flows over border NO3-NO4 and NO1-NO3 is quite out of range compared to the actual flows. It is concluded that these results are not sufficient for the scope of this work, and improvements of the model needs to be done. The overview over the actual transmission capacities between the areas obtained from Nord Pool, [39], reflects some interesting aspects. The transmission capacity for NO3-NO4 is given by Nord Pool to be -900 MW for a long time period on 21st of January 2016. This is why the actual power flow never exceeds -900 MW, as seen in table 3.10. In the simulated case, the power flow is larger than -3000 MW for some hours. Table 3.8 informs that 32 (from node 20 to 21) is the only line crossing border NO3-NO4. This means that power flow in line 32 can directly be compared with the flow between areas NO3 and NO4. As the capacity of line 32 is given to be infinitely large in the model of the Norwegian power system, seen in table 3.3, there is no wonder why the simulated power flow can exceed the limitation of -900 MW. Based on these findings, the transmission capacity in line 32 (between NO3 and NO4) is decreased from infinity to 900 MW in the Norwegian power system model.

Line 8 (node 1 to 8) is the only line crossing the border between NO1 and NO3 and the power flow and transmission capacity for line 8 and flow from NO1-NO3 can be directly compared. The same yields line 31 (from node 17 to 19) and border NO2-NO5. The transmission capacity between NO1 and NO3 (line 8) is given by Nord Pool to vary between 0 and 100 MW over the day of 21st of January 2016, and it is limited to flow in the direction from NO1 to NO3. The power flow from NO1-NO3 listed in table 3.9 equals the transmission capacity, meaning that the power flow between these nodes are on the limit during the whole day. Studying the overview over the capacities from Nord Pool, it can be seen that the capacity from NO1-NO3 varies from 0 to 400 MW over time. Assuming that the capacity is 400 MW when no lines are out supports the decision on decreasing the capacity in line 8 (between NO1 and NO3) on the Norwegian power system model from infinity to 400 MW.

Table 3.9: Simulated power flow between areas, 21.01.2016. Verification of the Norwegian power system model.

Time	**Simulated power flow [MW]**				
	NO1-NO2	NO3-NO4	NO1-NO3	NO1-NO5	NO2-NO5
1	-2337	-482	2347	-2436	-284
2	-2519	-353	2382	-2520	-314
3	-2406	-121	2586	-2524	-324
4	-940	-3230	-3316	1337	839
5	-713	-3230	-3309	1407	837
6	-2256	-2595	231	-2840	-431
7	-1982	-2871	-1235	-2799	-423
8	-1327	-2647	-2074	-2739	-428
9	-847	-2366	-1647	-2730	-454
10	-1011	-2336	-747	-2738	-448
11	-1193	-2580	-617	-2740	-437
12	-1292	-2543	-230	-2736	-426
13	-1633	-2613	-2153	-2764	-423
14	-1565	715	3926	-2526	-280
15	-1700	981	4229	-2527	-274
16	-1753	741	4044	-2532	-278
17	-1624	-2617	-1969	-2734	-398
18	-1725	-2786	-2170	-2733	-385
19	-1728	-2749	185	-2736	-394
20	-1975	-2739	555	-2738	-373
21	-2066	-2803	466	-2746	-372
22	-2194	-2966	-1023	-2748	-356
23	-2175	-3026	-581	-2762	-365
24	-2322	-3120	-105	-2783	-370

The simulated power flow from NO2-NO5 (line 31) does not deviate as much from the actual power flow as some of the other lines, but the transmission capacity in this line is nevertheless also decided to be changed according to Nord Pool. The new capacity in this line 31 is set to 300 MW as this is the maximum capacity for this border listed by Nord Pool.

According to the data from Nord Pool [39], the transmission capacity is not always equal in both directions for all lines and times. However, in the Norwegian power system model the transmission capacity is equal for both directions for all lines, and it is assumed that this does not change the essence and main characteristics of the power flows.

Table 3.10: Actual power flow between areas obtained from Nord Pool, 21.01.2016 [39]. Verification of the Norwegian power system model.

Time	Actual power flow [MW]				
	NO1-NO2	NO3-NO4	NO1-NO3	NO1-NO5	NO2-NO5
1	-2278	-691	100	-3900	-8
2	-2114	-720	100	-3900	-26
3	-2300	-714	100	-3900	-40
4	-2364	-725	100	-3900	-37
5	-2409	-719	100	-3900	-25
6	-2535	-639	100	-3866	0
7	-2500	-874	50	-3900	-149
8	-2500	-900	0	-3900	-202
9	-2500	-900	0	-3900	-176
10	-2500	-900	0	-3900	-135
11	-2500	-900	0	-3900	-117
12	-2500	-900	0	-3900	-109
13	-2500	-900	0	-3900	-138
14	-2472	-900	0	-3900	-151
15	-2427	-900	0	-3900	-160
16	-2438	-900	0	-3900	-157
17	-2500	-900	0	-3900	-151
18	-2500	-900	0	-3900	-172
19	-2500	-900	0	-3900	-224
20	-2500	-900	0	-3900	-202
21	-2500	-900	0	-3900	-216
22	-2500	-900	0	-3900	-122
23	-2500	-900	50	-3900	-11
24	-2541	-708	100	-3859	0

A simulation with the changed capacities for line 8, 31 and 32 are done, and the results are shown in 3.11. The simulated power flows given are now closer to to the actual power flows given in table 3.10. The power flows deviating the most are the flows from area NO1 to NO3, where the direction of the flows are opposite in some time periods. This is because the transmission capacity in reality was restricted to be between 0 and 100 MW for the given time period (21st of January 2016), and limited to flow in the direction from NO1 to NO3. Hence the simulated power flow is assumed to be representative when the capacity is not restricted to 0-100 MW in only one direction.

Table 3.11: Simulated power flow between areas after correction of transmission capacities, 21.01.2016. Verification of the Norwegian power system model.

Time	Simulated power flow [MW]				
	NO1-NO2	NO3-NO4	NO1-NO3	NO1-NO5	NO2-NO5
1	-1753	-900	-400	93	300
2	-1905	-900	-400	56	300
3	-1782	-900	-400	100	300
4	-1626	-900	-400	158	300
5	-1397	-900	-400	223	300
6	-2125	-900	400	-2670	-300
7	-1859	-900	-54	-2645	-300
8	-1199	-900	-327	-2578	-300
9	-693	-900	-181	-2535	-300
10	-863	-900	-277	-2551	-300
11	-1056	-900	-307	-2568	-300
12	-1166	-900	-318	-2578	-300
13	-1510	-900	-400	-2610	-300
14	-1504	-900	400	-1704	-219
15	-1632	-900	400	-1551	-206
16	-1855	-900	400	-1663	-206
17	-1526	-900	-252	-2613	-300
18	-1640	-900	-284	-2631	-300
19	-1634	-900	-340	-2620	-300
20	-1902	-900	-389	-2655	-300
21	-1994	-900	-400	-2666	-300
22	-2138	-900	46	-2692	-300
23	-2110	-900	400	-2693	-300
24	-2252	-900	400	-2698	-300

For a situation with high consumption it will be realistic to expect that the power will flow from areas with a large amount of production to areas with larger cities and consumption. In Norway this will in general be from the west to the east and south. This seems to be the trend for the simulation, as the power flows in the direction of NO1 for the entire day.

Based on these results, the model seems to be accurate enough for the scope of this work. To verify that the model is sufficient, another day is also analysed. The 1st of February 2016 is randomly chosen and the results can be found in table A.5 and A.6 in appendix B.

The simulated power flows are not expected to be equal to the actual power flows, as the simulations are done with an aggregated model. In addition the decision about production is made solely on the results from the optimisation problem, not considering weather conditions, transmission losses, outages of production units, start-up and shut-down times, market bids and costs of different types of production units. The production planning and performance in the real world is much more complex. This is mentioned to emphasize the fact that the simulated power flows between the areas are not expected to be equal to the real values, only reflecting the magnitude and direction to some extent. The results from the simulation after correcting the capacities are assumed to be adequate to give a sufficient indication on how the Norwegian power system behaves. The transmission capacities after correction are listed in table 3.12.

Table 3.12: Transmission capacity for each line after correction of capacities from the Trade Wind project [40] and Nord Pool capacities [39].

Line	Between nodes	Transmission capacity, P_l^{cap} [MW]
1	14-15	INF
2	11-16	INF
3	2-1	INF
4	3-1	2000
5	4-1	INF
6	1-5	INF
7	1-9	INF
8	1-20	400
9	2-10	1732
10	3-6	1000
11	3-18	2065
12	4-6	1436
13	5-6	INF
14	5-19	774
15	7-8	INF
16	7-9	INF
17	7-17	900
18	7-19	850
19	8-10	2065
20	8-13	2957
21	8-18	2065
22	7-18	225
23	9-10	INF
24	9-14	1472
25	11-12	1000
26	11-14	1210
27	11-17	INF
28	12-18	2515
29	14-13	1000
30	17-18	INF
31	17-19	300
32	20-21	900
33	21-22	1000

4 | Mathematical modelling of optimised scheduling of battery dispatch

An model for optimised production and power flow is elaborated in section 3.6. To optimise the battery dispatch in a power system, an optimisation model for battery operation needs to be developed. The development is conducted in several steps, and begins with an optimisation of a simple model of batteries before the power system is considered.

4.1 Battery optimisation model

This section describes a MILP problem that aims to optimise the battery dispatch. To be able to utilise a battery, the time aspect is necessary. The optimisation problem is developed through studies and analyses on three scenarios. After each scenario, the model is tested for several cases to enable verification of the validity of the model. Selected cases and results are presented in this report.

- **Scenario 1:** The first scenario is without any kind of production or load, only considering batteries and day-ahead prices.
- **Scenario 2:** The second scenario includes production and load, but no physical grid structure.
- **Scenario 3:** In the third scenario, a model of a physical grid structure is added.

4.1.1 Batteries

Battery storage systems can be characterized by its power and energy capacity, round tip efficiency and self-discharge. Energy capacity is the maximum amount of energy the battery can store, measured in watt hours. Power capacity is the maximum rate of charging and discharging, measured in watts. Round trip efficiency is the ratio of output-to-input energy for a battery, and self-discharge is the loss of energy due to parasitic losses in an energy storage system. These parasitic losses are a result of mechanical friction and chemical reactions. [41] A simple model of batteries are used in the model, considering the power and energy capacities and an efficiency factor.

The battery used in this report is said to be a lithium-ion battery, and is assumed to be linear, meaning that the maximum rate of charge is constant for every energy level. The primary applications of lithium-ion batteries are listed in table 2.1, but the services are not limited by these applications.

4.1.2 Battery optimisation model: Scenario 1

The aim for scenario 1 is to optimise the battery dispatch with respect to exogenous prices from the electricity market. The objective of this optimisation problem is to optimise the battery dispatch versus the price variation in day-ahead price. The owner of the batteries will sell and buy energy to the day-ahead price. It is assumed that the batteries are price takers, and does not affect the day-ahead prices. The optimisation formulation is given under.

The model is based on hourly resolution, hence power and energy magnitudes can be directly compared because the duration of a power injection is one hour.

Indicies

c	Charging mode
dc	Discharging mode
NT	Number of periods under study

Sets

t	Hour index
v	Battery stations

Parameters

λ_t	Day-ahead price for electricity
η_v	Efficiency of battery station v
$P_v^{c,min}$	Minimum charging capacity of battery station v
$P_v^{c,max}$	Maximum charging capacity of battery station v
$P_v^{dc,min}$	Minimum discharging capacity of battery station v
$P_v^{dc,max}$	Maximum discharging capacity of battery station v
E_v^{min}	Minimum energy stored in battery station v
E_v^{max}	Maximum energy stored in battery station v
$M_{v,t}$	Status of grid connection of battery station v at time t

Variables

$P^{c}_{v,t}$	Charged power of battery station v at time t
$P^{d}c_{v,t}$	Discharged power of battery station v at time t
$I^{c}_{v,t}$	Indicator of charging mode for battery station v at time t
$I^{dc}_{v,t}$	Indicator of discharging mode for battery station v at time t
$E_{v,t}$	Available energy in battery station v at time t
$E^{net}_{v,t}$	Net charged energy in battery station v at time t

Objective function

$$\min \sum_t \sum_v \lambda_t (P^{c}_{v,t} - P^{dc}_{v,t}) \tag{4.1}$$

Constraints

$$E^{net}_{v,t} = \eta_v P^{c}_{v,t} - P^{dc}_{v,t} \tag{4.2}$$
$$E_{v,t} = E_{v,t-1} + E^{net}_{v,t} \tag{4.3}$$
$$E^{min}_{v} \leqslant E_{v,t} \leqslant E^{max}_{v} \tag{4.4}$$
$$I^{c}_{v,t} P^{c,min}_{v} \leqslant P^{c}_{v,t} \leqslant I^{c}_{v,t} P^{c,max}_{v} \tag{4.5}$$
$$I^{dc}_{v,t} P^{dc,min}_{v} \leqslant P^{dc}_{v,t} \leqslant I^{dc}_{v,t} P^{dc,max}_{v} \tag{4.6}$$
$$I^{c}_{v,t} + I^{dc}_{v,t} \leqslant M_{v,t} \tag{4.7}$$
$$P^{dc}_{v,t} \leqslant E_{v,t-1} \tag{4.8}$$
$$E_{v,0} = 0 \tag{4.9}$$
$$E_{v,0} = E_{v,NT} \tag{4.10}$$

In this scenario, the day-head prices are used as charging costs and discharging earnings for the battery owners. In theory the battery will charge (buy from electricity market) when the electricity price is low and discharge (sell stored energy to electricity market) when the electricity price is high. It is here assumed that the operation of batteries has a direct correlation with the variation of the prices in the market. As the objective function, equation 4.1, to be minimised is the difference of the charging cost and the discharge earnings, the worst possible solution would be when the objective function is zero, and the batteries do not provide any economical benefits at all.

The net hourly injected energy to the battery is given by the constraint described by equation 4.2, which illustrates that the difference between the energy injected to a battery, v, and the energy injected back to the grid is given by the charging cycle efficiency, η, of the battery. Equation 4.3 gives the available amount of energy in a battery, given by the sum of the net injected energy to the battery, and the energy already stored in the battery. The minimum and maximum storage limits of a battery are given by 4.4. Equations 4.5 and 4.6 make sure that the rate of charge and discharge do not exceed the limits. $I^{c}_{v,t}$ and $I^{dc}_{v,t}$ are binary variables. As $M_{v,t}$ is equal to 0 or 1, constraint 4.7 restricts the battery to not be in charging mode and discharging mode at the same time. The discharged energy in a given time step can not exceed the amount of stored energy in the battery for the previous time step, and is taken care of by 4.8. The available amount of energy in the start and in the end of each period are equal, and set to be zero by equations 4.9 and 4.10. Another option would be to force the energy to be e.g. 50 % of the maximum capacity. If longer periods where the battery is not in use occurs, a lot of stored energy might be spilled. This is why the battery in this case is forced to be empty in the evening, to make sure that most of the energy stored will be utilized.

4.1.3 Verification of battery optimisation model: Scenario 1

To verify the battery model in the first scenario, an optimisation with two battery stations, A and B is performed with the day-ahead prices for the 21th of January 2016 [39]. The time period lasts for 24 hours, and $NT = 24$. The values for the parameters are chosen and listed in table 4.1.

Table 4.1: Battery parameters, scenario 1.

Parameter	Battery station A	Battery station B
η_v [%]	60	90
$P_v^{c,min}$ [MW]	0	0
$P_v^{c,max}$ [MW]	0,9	0,5
$P_v^{dc,min}$ [MW]	0	0
$P_v^{dc,max}$ [MW]	0,9	0,5
E_v^{min} [MWh]	0	0
E_v^{max} [MWh]	7	2

The objective value of the optimisation problem is -1231 EUR. This means that the battery owner will earn 1231 EUR this day from charging and discharging the batteries. From the results visualised in figure 4.1 it can be seen that the batteries are charging during periods with low day-ahead price, and discharging when the prices are higher. The price difference is quite large, and both charging and discharging takes place in two periods during the day.

The batteries A and B stops charging in respectively hour 7 and 6. Figure 4.1 shows that battery station B reaches its maximum energy capacity of 2 MWh at hour 6. The maximum limit is also the reason why the slope of the curve is slightly reduced between hour 5 and 6. The power rate is larger for battery station A, and this is visualised by the steeper slope of the available energy, both during charging and discharging. Battery station B has a larger efficiency than station A. This means that battery station B can charge for higher prices than battery station A and still make profit. Because the price varies so abruptly in this case, this situation does not occur here. Other simulations not included in this report have shown it this will occur for days with more uniform pricing pattern.

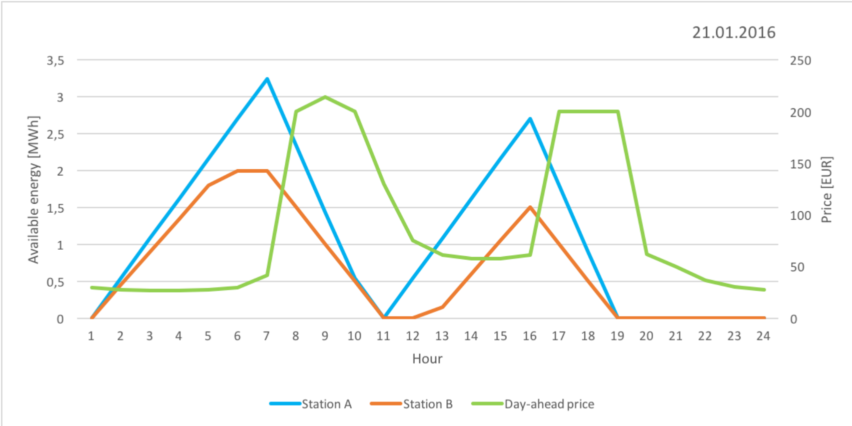

Figure 4.1: Available energy, $E_{v,t}$, for station A and B compared with day-ahead prices, scenario 1

4.1.4 Battery optimisation model: Scenario 2

Production and load is added to the model developed in scenario 1. The production level and load are assumed to be known, and will function as parameters in the model. The only difference between scenario 1 and scenario 2 is that production and load is present in scenario 2, thus a new constraint given by equation 4.11 needs to be added. The added constraint describes the energy balance and forces the sum of the generation, demand, charged energy and discharged energy to be zero for each time step.

Added parameters

P_t^{gen}
P_t^{dem}

Added constraint

$$P_t^{gen} - P_t^{dem} - P_{v,t}^{c} + P_{v,t}^{dc} = 0 \tag{4.11}$$

4.1.5 Verification of battery optimisation model: Scenario 2

To verify the second scenario of the battery model, a simple system including one battery (A), one wind production unit and a load is analysed. The load and production from the wind production unit are assumed to be perfectly forecasted in advance. The sum of generation and the sum of the consumption over time are equal to each other. The aim of this analysis is to investigate whether the battery can store excess wind energy produced in the start of the time period and serve as a production unit to cover the consumption in the last part of the time period. For the solution to be feasible when the total production equals the total demand, the efficiency of the battery is set to be 100 %. The parameters in this scenario are dummy variables and not linked to a specific day or network. System parameters and battery parameters are given in tables 4.2 and 4.3 and the results are given in table 4.4. The energy balance is visualised in figure 4.2.

Table 4.2: System parameters, scenario 2.

Parameter	**Value for hour nr.**					
	1	**2**	**3**	**4**	**5**	**6**
λ_t [EUR]	25	25	25	25	25	25
${P_t}^{dem}$ [MW]	20	10	50	90	30	30
${P_t}^{gen}$ [MW]	40	80	60	20	10	20

Table 4.3: Battery parameters, scenario 2.

Parameter	**Value**
η_A [%]	100
${P_A}^{c,min}$ [MW]	0
${P_A}^{c,max}$ [MW]	100
${P_A}^{dc,min}$ [MW]	0
${P_A}^{dc,max}$ [MW]	100
${E_A}^{min}$ [MWh]	0
${E_A}^{max}$ [MWh]	120

Table 4.4: Results, scenario 2.

Parameter	**Value for hour nr.**					
	1	**2**	**3**	**4**	**5**	**6**
P_t^c [MW]	20	70	10	0	0	0
P_t^{dc} [MW]	0	0	0	70	20	10
E_t [MWh]	20	90	100	30	10	0

As seen in figure 4.2, the energy balance is fulfilled, as the energy over and under the x-axis are equal. In the first three hours the wind production is larger than the load, and the battery is able to store excessive energy represented by the red bars. As the total load is equal to the total wind production, but the hourly production and load is not the same, the battery has to charge in hours with excess energy production in order for the system to be able to operate. In the last three hours, the load is larger than the production, and the battery discharges the stored energy to compensate the generation scarcity of wind generation. The discharged energy is represented by the purple bars. In this case, the day-ahead price is indifferent because there is only one solution to the problem.

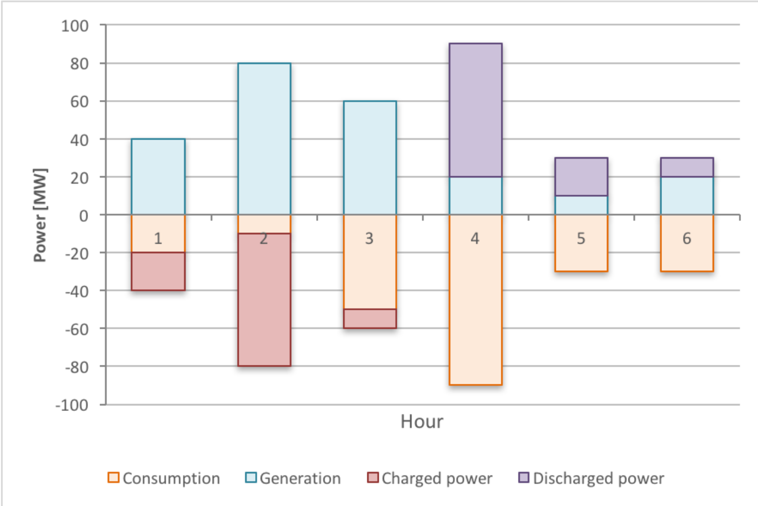

Figure 4.2: Energy balance, scenario 2.

4.1.6 Battery and power system optimisation model: Scenario 3

The model developed in scenario 2 uses batteries, prices, production and load to estimate a battery dispatch. To make the model more applicable for the scope of this research work, a model that is able to take a network structure as input is developed. Transmission constraints, production constraints, demand, prices, battery characteristics and energy exchange with neighbouring countries will influence the system, and will be implemented in the model. This is done by merging the model developed in scenario 2 in section 4.1.4, and the optimisation model for a power system elaborated in section 3.5. The production level is no longer assumed to be known, and is now functioning as a variable instead of a parameter. The consumption is still assumed to be known.

Hydro power operating costs

Hydro power production is assumed to have no variable costs dependent on the output, as operation and maintenance costs are assumed to be dimensioned to the power plant capacities [42]. Fixed costs and investment costs for hydro power plants are neglected in the analysis as the optimisation problem only considers already installed hydro power plants. As hydro generation has no variable costs, and fixed costs (such as maintenance and operational costs) that are sunk is not of interest in a management problem, another way to describe the production costs for hydro generation is needed. To maximise the future expected income, the producers can use the concept of water values. There are limited amounts of water stored in the reservoirs for the hydro power plants, and the concept of water values can be used as the decision variable to decide whether producing now or at a later point of time. The water value reflects the marginal value of passing more water to a different time period. The water values can be used as marginal production costs for hydro power generation [43]. To maximise the value of the water in the reservoir, it is profitable to produce when the water value is low. The aim of the optimisation model will be to minimize these so-called production costs.

Hydro power producer will produce if the electricity price is larger than the water value, and withhold production in the opposite case [44]. This is because they expect a greater income if the production is shifted a later point of time. Thus the hydro power production will be dependent on the relationship between the day-ahead price and the water value [44]. The calculation of water values is complex, and based on expectations on future market- and hydrology conditions. The water reservoirs in Norway have different locations and characteristics, and the water values will therefore not be equal for all hydro power plants. Due to the complexity regarding the water values, they are disregarded in this report. Instead, the day-ahead prices are used as a short term marginal cost of hydro power. This is possible because a decrease in the day-ahead price implies a decreased water value. They are not 100 % correlated, but they are assumed to follow each other to a large extent. Based on these assumptions, the objective function regarding the power production will be to minimize the production costs described as the day-ahead price multiplied with the production level.

Reservoir hydro power producers do not in theory profit from storing energy in batteries in a short-term perceptive, because they have their own storage possibility in the reservoirs. Actually, energy storage in batteries are not economically beneficial for hydro power producers because they will lose energy in the batteries that they would have retained by keeping it in the reservoir. Because batteries are used not only for economical reasons, but for the sake of providing services to the grid, the optimal time for charging will be when the water value is low. The main reasons for installing batteries in the grid are their ability to provide the different services listed in section 2.1.1. Increasing penetration of intermittent renewable energy sources and exchange capacity between the Nordic system to the European system pinpoint the importance of primary frequency control and fast response to frequency deviations. Applying frequent primary frequency control on hydro power plants brings a problem of increasing wear and tear of turbines. In this respect, batteries have much faster response times than hydro power plants, and can be a good alternative to provide these services to future power systems. In a long-term perspective batteries may be beneficial for hydro power producers after all, and in combination with a new approach to ancillary service provision in the electricity markets, they may also generate additional income.

Charging costs

The charging cost and discharging earnings are set to equal the average day-ahead price in this scenario, unlike the case in scenario 1 and 2. This is done because the battery is not participating in the electricity market, either when charging or discharging. Either is it inserted in the power system to generate additional income by utilising the day-ahead price as in scenario 1, but is supposed to be an alternative for the production units. Batteries are assumed to even out the day-ahead price variations, thus using the day-ahead price as charging cost and discharging earnings could be deceptive. As the hydro production has an operating cost assumed to be equal to the day-ahead price, they will aim to produce when the price is low, and withhold production when the price is high. If the battery capacity allows, the production units will produce more than the demand if the price is low, and then store the excess energy in the battery until the price is high and withhold of power production is desirable. A battery owner will use the same battery dispatch to maximize profit if the charging costs and discharging earning also followed the day-ahead prices. Meaning that the optimal dispatch for hydro power operating cost reduction obtained with the given cost function will be the same as the optimal dispatch if the aim was to maximise the battery income according to day-ahead prices as in scenario 1. However, the total system operating cost will be different.

The approach in this scenario will aim to minimise the operating costs for the hydro power producers, and the operating costs for the batteries. In this scenario, the batteries are not included in the grid to generate income, but to minimise the operating costs for hydro and to be able to provide different types of services.

The battery and power system optimisation model

To make the model more legible, the battery variable, v, from scenario 1 and 2 is removed. This is because the analyses planned throughout this work does not need several different battery types in one simulation. The battery type can easily be changed between the simulations. It will still be possible to add two batteries of the same type to one node or in several different nodes, and this is flexibility enough for this research work.

The optimisation model is made to be able to predict the optimal dispatch of batteries and the power flow and production in the power system. In the work with developing and formulating the optimisation problem, historical data from Nord Pool is used. Historical data are also used in the simulations and analyses later in the report. If the model is to be used as a scheduling tool for future time periods, forecasts and predicted values for day-ahead prices, consumption and export with neighbouring countries are needed. Day-ahead prices might be known depending on when the scheduling is performed, as the day-ahead market is cleared 12 hours in advance. The results from the optimisation model are day-ahead clearing.

It is assumed that the energy can be stored in a battery for up to 24 hours, but the batteries are forced to be emptied in the end of each day. This is because the time scope of the model is 24 hours, and enabling the battery to hold energy between two days would require the model to be able to handle a time scope of several days. This is a possibility, and simulations on such a case would require historical data input for several days. But for the use of the model to make a battery dispatch for an upcoming period, the results may not be trustworthy because the ability to predict the consumption and day-ahead prices for several days in advance is challenging.

It is assumed that once a battery is placed in a node, it will stay there for the rest of the simulation period. The batteries are stationary. Therefore, the status of grid connection described by M is only dependent on n and not on t in this scenario, unlike scenario 1 and 2.

Indicies

c	Charging mode
dc	Discharging mode
NT	Number of periods under study

Sets

t	Hour index
n	Node
l	Line

Parameters

$\lambda_{n,t}$	Day-ahead price for node n at time t
η	Efficiency for battery in node n
$P^{c,min}$	Minimum charging power for battery in node n
$P^{c,max}$	Maximum charging power for battery in node n
$P^{dc,min}$	Minimum discharging power for battery in node n
$P^{dc,max}$	Maximum discharging power for battery in node n
E^{min}	Minimum energy capacity in battery in node n
E^{max}	Maximum energy capacity in battery in node n
M_n	Status of grid connection of battery in node n
$K_{n,l}$	Connection matrix telling which node is connected to which line
$C_{n,t}$	Charging cost and discharging earnings for battery at node n at time t
$P_n^{gen,min}$	Minimum generation level for generator at node n
$P_n^{gen,max}$	Maximum generation level for generator at node n
$P_{n,t}^{dem}$	Load demand for node n at time t
P_l^{cap}	Transmission capacity for line l
$P_{n,t}^{exc}$	Power exchange with neighbouring countries from node n at time t
$PTDF_{l,n}$	PTDF-factors for line l by node n

Variables

$P_{n,t}^{c}$	Charge power of battery in node n at time t
$P_{n,t}^{dc}$	Discharge power of battery in node n at time t
$I_{n,t}^{c}$	Indicator of charging mode for battery in node n at time t
$I_{n,t}^{dc}$	Indicator of discharging mode for battery in node n at time t
$E_{n,t}$	Available energy in battery in node n time t
$E_{n,t}^{net}$	Net charged energy in battery in node n at time t
$P_{n,t}^{gen}$	Generation level in node n at time t
$P_{l,t}^{flow}$	Power flow in transmission line l at time t

Objective function

$$\min \sum_{t} \sum_{n} C_{n,t}(P^{c}_{n,t} - P^{dc}_{n,t}) + \sum_{n} \sum_{t} \lambda_{n,t} P^{gen}_{n,t} \tag{4.12}$$

Constraints

$$E^{net}_{n,t} = \eta P^{c}_{n,t} - P^{dc}_{n,t} \tag{4.13}$$
$$E_{n,t} = E_{n,t-1} + E^{net}_{n,t} \tag{4.14}$$
$$E^{min} \leqslant E_{n,t} \leqslant E^{max} \tag{4.15}$$
$$I^{c}_{n,t} P^{c,min} \leqslant P^{c}_{n,t} \leqslant I^{c}_{n,t} P^{c,max} \tag{4.16}$$
$$I^{c}_{n,t} P^{dc,min} \leqslant P^{dc}_{n,t} \leqslant I^{dc_{n,t}} P^{dc,max} \tag{4.17}$$
$$I^{dc}_{n,t} + I^{dc}_{n,t} \leqslant N_{n,t} \tag{4.18}$$
$$P^{dc}_{n,t} \leqslant E_{n,t-1} \tag{4.19}$$
$$E_{n,0} = 0 \tag{4.20}$$
$$E_{n,0} = E_{n,NT} \tag{4.21}$$
$$P^{gen,min} \leqslant P^{gen}_{n,t} \leqslant P^{gen,max} \tag{4.22}$$
$$P^{flow}_{l,t} = \sum_{n} PTDF_{l,n} \cdot NP_{n,t} = \sum_{n} PTDF_{l,n}(P^{gen}_{n,t} - P^{dem}_{n,t} + P^{dc}_{n,t} \cdot M_n - P^{c}_{n,t} \cdot M_n) \tag{4.23}$$
$$|P^{flow}_{l,t}| \leqslant P^{cap}_{l} \tag{4.24}$$
$$P^{gen}_{n,t} - P^{dem}_{n,t} - P^{c}_{n,t} \cdot M_n + P^{dc}_{n,t} \cdot M_n - P^{exc}_{n,t} - \sum_{l} P^{flow}_{l,t} \cdot K_{n,l} = 0 \tag{4.25}$$

The objective function to be minimised is the sum of the production costs and the total charging costs. The constraints described by equations 4.13 to 4.21 are equal to the constraints from scenario 2, and are explained in section 4.1.2.

Equation 4.22 indicates that the generation in one node must be between its minimum and maximum generating capacity. The power flow in each line at a given time is given by equation 4.23. It is important to include both generation, demand, charged energy and discharged energy in the net position when including batteries. The power flow has to be lower than the transmission capacity for the specific line, indicated by equation 4.24. Finally, the power balance in each node needs to be fulfilled. This means that the sum of the production, load, charging, discharging, exchange between countries and power flow to/from the node has to be equal to zero, according to equation 4.25.

The model developed in this section is implemented in the modelling system GAMS. This is further described in appendix G.

4.1.7 Verification of battery and power system optimisation model: Scenario 3

In the end, the developed optimisation model is supposed to be applicable for all physical grid structures described by PTDF-matrices. To get an overview over the optimisation model, and to easier understand and analyse the results, optimisation for a 3 node grid structure, as seen in figure 4.3 is preformed. The PTDF-matrix is given in equation 4.26, and is obtained from Lars Åmellems project work at NTNU [45]. Exchange with neighbouring countries is not considered. Node 1 and 2 consists of production units, and node 3 consists a load. A battery is connected to node 3. Two cases are studied, one with and one without congestion. The amount of time for this analysis is limited to 6 hours, and $NT = 6$. The parameters for the battery are listed in table 4.5.

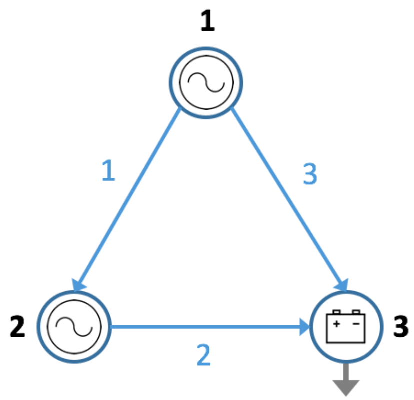

Figure 4.3: 3 node system.

$$PTDF_{l,n} = \begin{bmatrix} 0,4127 & -0,3175 & 0 \\ 0,5873 & 0,3175 & 0 \\ 0,4127 & 0,6825 & 0 \end{bmatrix} \tag{4.26}$$

Table 4.5: Battery parameters, scenario 3.

Parameter	Value
η [%]	80
$P^{c,min}$ [MW]	0
$P^{c,max}$ [MW]	100
$P^{dc,min}$ [MW]	0
$P^{dc,max}$ [MW]	100
E^{min} [MWh]	0
E^{max} [MWh]	100

Case 1: Without congestion

The input parameters for this case is given below.

Table 4.6: Generation limitations: case 1, scenario 3.

Parameter	**Node 1**	**Node 2**	**Node 3**
$P_n^{gen,min}$ [MW]	0	0	0
$P_n^{gen,max}$ [MW]	230	200	0

Table 4.7: Transmission capacities: case 1, scenario 3.

Parameter	**Line 1 (node 1-2)**	**Line 2 (node 2-3)**	**Line 3 (node 1-3)**
P_l^{cap} [MW]	300	400	400

Table 4.8: Day-ahead prices: case 1, scenario 3.

$\lambda_{n,t}$	**Value for hour nr.**					
	1	**2**	**3**	**4**	**5**	**6**
$\lambda_{1,t}$ [EUR/MWh]	10	15	40	50	30	10
$\lambda_{2,t}$ [EUR/MWh]	15	20	45	55	60	35
$\lambda_{3,t}$ [EUR/MWh]	0	0	0	0	0	0

Table 4.9: Charging prices: case 1, scenario 3.

$C_{n,t}$	**Value for hour nr.**					
	1	**2**	**3**	**4**	**5**	**6**
$C_{1,t}$ [EUR/MWh]	0	0	0	0	0	0
$C_{2,t}$ [EUR/MWh]	0	0	0	0	0	0
$C_{3,t}$ [EUR/MWh]	100	100	100	100	100	100

Table 4.10: Demand: case 1, scenario 3.

$P_{n,t}^{dem}$	**Value for hour nr.**					
	1	**2**	**3**	**4**	**5**	**6**
$P_{1,t}^{dem}$ [MW]	0	0	0	0	0	0
$P_{2,t}^{dem}$ [MW]	0	0	0	0	0	0
$P_{3,t}^{dem}$ [MW]	400	410	200	300	240	430

The energy balance is given in figure 4.4 and the power flows are given in table 4.11 and visualised in figure 4.5.

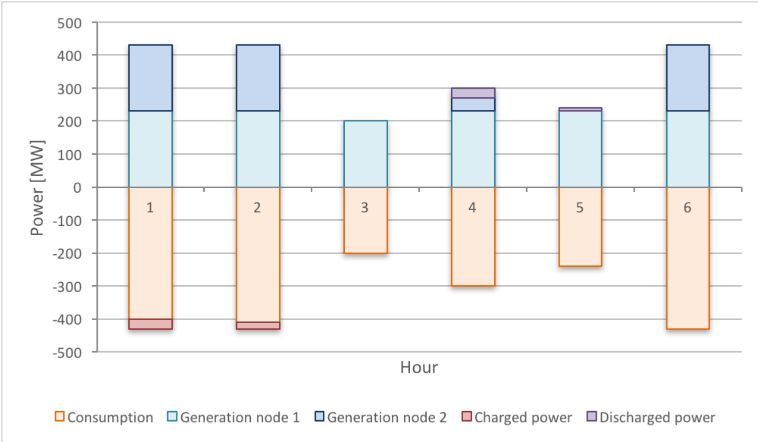

Figure 4.4: Energy balance: case 1, scenario 3.

Table 4.11: Power flow: case 1, scenario 3.

$P_{l,t}^{flow}$	**Value for hour nr.**					
	1	**2**	**3**	**4**	**5**	**6**
$P_{1,t}^{flow}$ [MW]	31.4	31.4	82.5	82.2	94.9	31.4
$P_{2,t}^{flow}$ [MW]	198.6	198.6	117.5	147.8	135.0	198.6
$P_{3,t}^{flow}$ [MW]	231.4	231.4	82.5	122.2	94.9	231.4

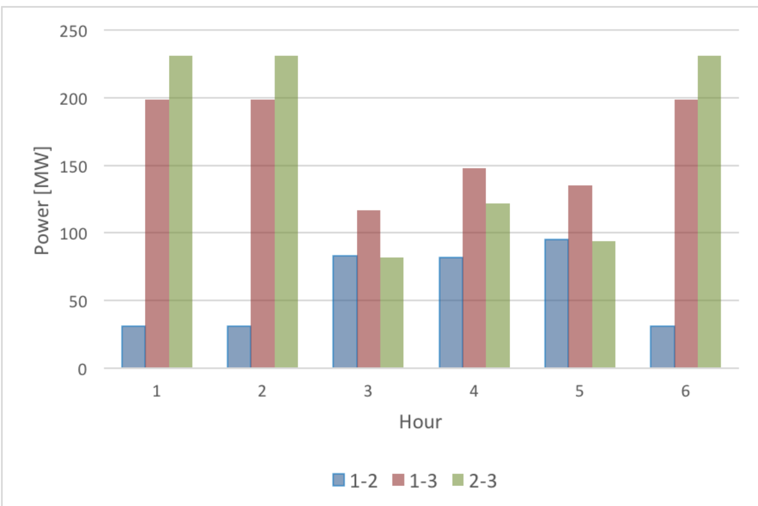

Figure 4.5: Power flow in the three lines: case 1, scenario 3.

The production level in node 1 is at its maximum level for every hour, except for hour 3, where the load is lower than the maximum production level for node 1. This is because the production cost in node 1 is lower than the cost for node 2.

As seen in figure 4.4, the amount of energy in each time step sums up to zero, hence the energy balance is fulfilled. The battery will charge when the charging cost is so low that it is profitable, meaning that the sum of the total charging/discharging costs (production cost + charging cost - discharging earnings) needs to be lower than the largest production price for the cheapest available production unit over the time period of 6 hours.

Due to losses in the charging/discharging cycle in the battery, the amount of energy charged is larger than the energy able to be delivered back to the system. The efficiency in this case is assumed to be 80 %, meaning that the charged energy needs to be $\frac{1}{0{,}8} = 125\%$ greater than the energy to be discharged later. Since the price for charging and the earnings for discharging are equal, the cost for one unit charged energy will be $1,25 \cdot C_{n,t} - C_{n,t} = 1,25 \cdot 100 - 100 = 25$ EUR/MWh. For this particular case, it will be profitable to charge when the sum of the total costs for charging/discharging and the production cost is lower than 60 EUR/MWh (maximum cost for generation unit node 2) for all time steps except for time step 3 where production at node 1 is available, and the total costs for charging/discharging should be less than 50 EUR/MWh (maximum cost for generation unit node 1). It is also important to remember that the amount of energy produced for charging also needs to be multiplied with 1,25, due to the losses in the battery.

The only time steps that it will be profitable to charge is in hour 1 and 2. Discharging takes place when the production cost is largest, in this case hour 4 and 5. There is a reason why the discharging does not only take place where the price is highest, in step 5. First, for time step 5, the load is only 10 MWh larger than the maximum production limit of production unit 1. The total cost for charging/discharging will be larger than the cost of production at time step 5 for node 1, meaning that the profitable amount of energy discharged in time step 5 will be 10 MWh. The second largest production cost is present in time step 4. The rest of the charged energy will be discharged in this time step.

It is profitable to charge in time step 1 and 2 because the total charging/discharging cost is lower than the largest production cost for both production units. In these time steps, all excess energy is charged and stored in the battery. In time step 3, it is not profitable to charge, and the size of the load is small enough to be covered by the cheapest production unit. In time step 4, the load exceeds the maximum production of node 1, and needs node 2 to produce as well. It is, as previously discussed, profitable to discharge some of the energy from the battery, and the production in node 2 can be decreased. In node 5, the rest of the stored energy is discharged.

Case 2: Congestion

To make sure that the model handles congestion, and that the batteries can help overcome congestion problems, the transmission capacity in line 1-3 is decreased. The results from case 1 show that the maximum transmission in line 1-3 is 198,6 MW. In this case, the transmission capacity in line 1-3 is set to be 185 MW. The other parameters are equal to the parameters in case 1.

Table 4.12: Transmission capacities: case 2, scenario 3.

Parameter	Line 1 (node 1-2)	Line 2 (node 2-3)	Line 3 (node 1-3)
P_l^{cap} [MW]	300	185	400

The results are presented in figure 4.6, 4.7 and table 4.13.

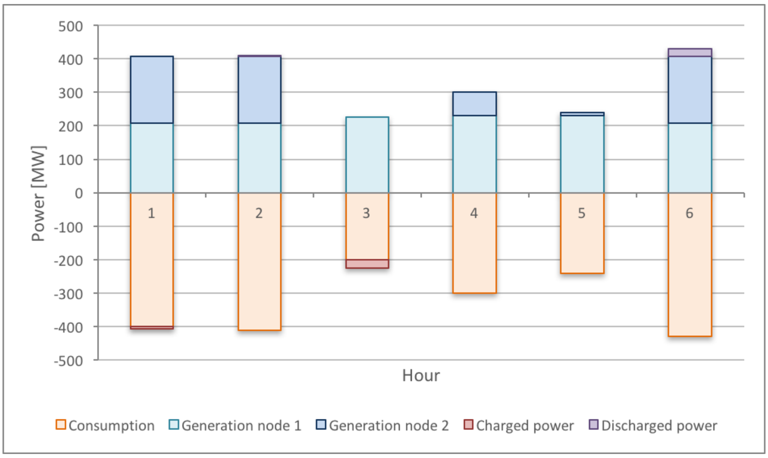

Figure 4.6: Energy balance: case 2, scenario 3.

Table 4.13: Power flow: case 2, scenario 3.

$P_{l,t}^{flow}$	Value for hour nr.					
	1	**2**	**3**	**4**	**5**	**6**
$P_{1,t}^{flow}$ [MW]	21.9	21.9	93.2	72.7	91.7	21.9
$P_{2,t}^{flow}$ [MW]	185.0	185.0	132.7	157.3	138.3	185.0
$P_{3,t}^{flow}$ [MW]	221.9	221.9	93.2	142.7	101.7	221.9

Comparing figure 4.4 and 4.6 shows that the charging and discharging pattern changes during congestion. First of all, the charging in time step 1 is forced to decrease, because power flow is limited by capacity scarcity in line 1-3. The power flow to the load is prioritised. In time step 2 energy is discharged to overcome the congestion problem. It may be hard to see the discharged energy in figure 4.6, as the amount of discharged energy is small compared to the total energy. Charging takes place in time step 3, unlike the situation in case 1. Even though it is not economically profitable to charge in time step 3, the system is forced to do it due to the congestion. This is the desired results, as it shows that the model handles congestion situations. Discharge takes place in node 6.

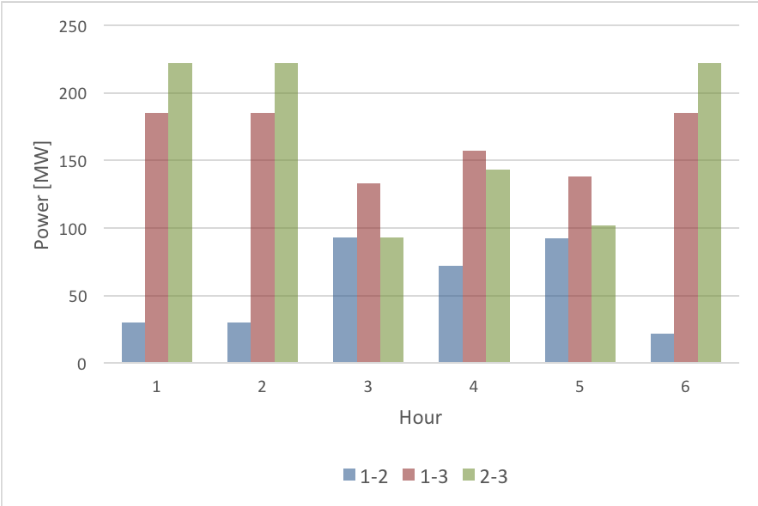

Figure 4.7: Power flow , $P_{l,t}^{flow}$, in the three lines: case 2, scenario 2.

The power flows are quite similar with and without congestion when comparing figures 4.5 and 4.7. This is because the loads are the same, and the transmission capacity is only slightly decreased. However, it can be seen that the flow from 1-3 is decreased from the level of almost 200 MW down to 185 MW in the start. Of course, this affects the flow in the other two lines. The production in node 1 is increased to the maximum point where the limit of the line is reached. This is done to charge the battery in the earlier time steps, to be able to cover the load in time step 6.

5 | Case studies

An optimisation model has been developed, formulated and then verified, and it is now possible to use the model to perform optimisation on a system including both a network structure and a battery model. This section will provide in-depth analysis of optimal battery dispatch in the Norwegian transmission system. The optimisation model to be used is developed and elaborated in scenario 3 in chapter 4, and the model of the Norwegian power system is described in section 3.6. It is assumed that the size of the batteries are large enough to support the transmission system. The cases will be conducted by using historical data for different days. As this model is supposed to be used to find the optimal battery dispatch for a future day, forecasts could have been used. However, for these case studies, it is more practical to use historical data, as they reflect real situations. If nothing else is specified, the generation units are hydro power plants.

The production costs for hydro power production are described by the day-ahead prices in the price area where the hydro power plant is located. The charging costs and discharging earnings are equal to the average day-ahead price for the price area where the battery is placed. The use of these costs are justified in section 4.1.6.

The cases are constructed to investigate the behaviour of batteries in different situations. Such situations are e.g. situations with large variations in day-ahead prices, congestion situations and planned generation outages. Moreover, analyses on some cases with a future solar power scenario are conducted. The case studies are based on time periods of 24 hours with hourly resolution, representing a whole day. This is reasonable as the data from Nord Pool is based on hourly values for consumption, exchange and day-ahead prices. Hour 1 represents the hour between 00:00 and 01:00. Hour 2 represents the hour between 01:00 and 02:00, and so on. Historical real data for the dates to be simulated are gathered from Nord Pools home pages [39].

As mentioned in chapter 2, the largest battery storage facility in the world today is a 50MW/300MWh battery in Buzen, Japan. It is assumed that a battery of the size of 50MW/200MWh realistically can be installed in the Norwegian power system. Most of the upcoming optimisation cases will implement a battery of this size, specified in table 5.1. If nothing is specified, this is the battery referred to throughout the report.

Table 5.1: Battery parameters for the case studies.

Parameter	**Value**
Battery efficiency, η [%]	90
Minimum charging power, $P^{c,min}$ [MW]	0
Maximum charging power, $P^{c,max}$ [MW]	50
Minimum discharging power, $P^{dc,min}$ [MW]	0
Maximum discharging power, $P^{dc,max}$ [MW]	50
Minimum stored energy, E^{min} [MWh]	0
Maximum stored energy, E^{max} [MWh]	200

The map of the areas, nodes and lines from chapter 3 is reposted with line numbers included in figure and 5.1, as the areas, nodes and lines are frequently referred to in this chapter.

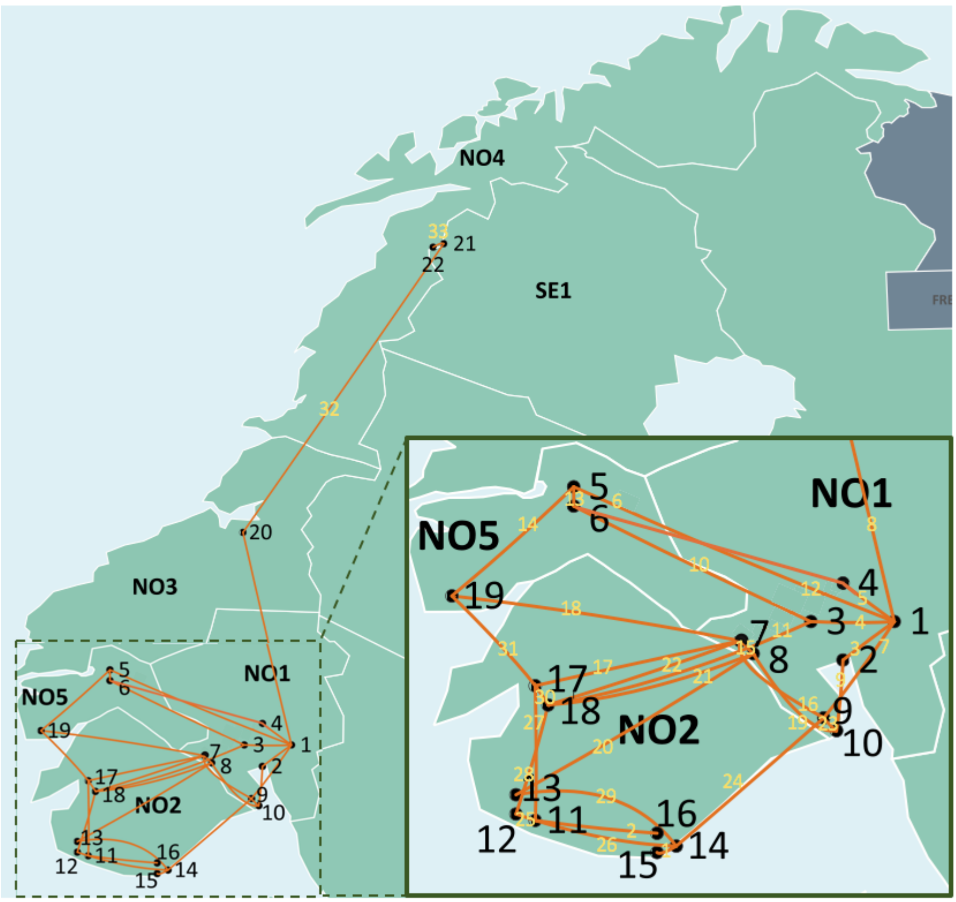

Figure 5.1: Lines with numbering labels in the Norwegian power system model.

5.1 Case 1: Battery located in node 1

The first case addresses a situation with a battery specified by table 5.1 connected to node 1. Node 1 is located in area NO1, and represents Oslo and the areas around. In general, the consumption in node 1 is a lot larger than the maximum generation capacity in node 1. This means that large energy import to node 1 from other nodes is expected, especially on cold winter days with large consumption.

5.1.1 Thursday 21st of January 2016

The date, 21.01.2016, studied in the verification of the optimisation model in section 3.7, is also used for simulation. This day is an interesting day both because of the large consumption and due to the large variation in the day-ahead prices over the day and between the price areas. The large price variations throughout the day can be utilised by power producers by using batteries to decrease their operational costs. The day-ahead prices for 21.01.2016 can be seen in figure 5.2.

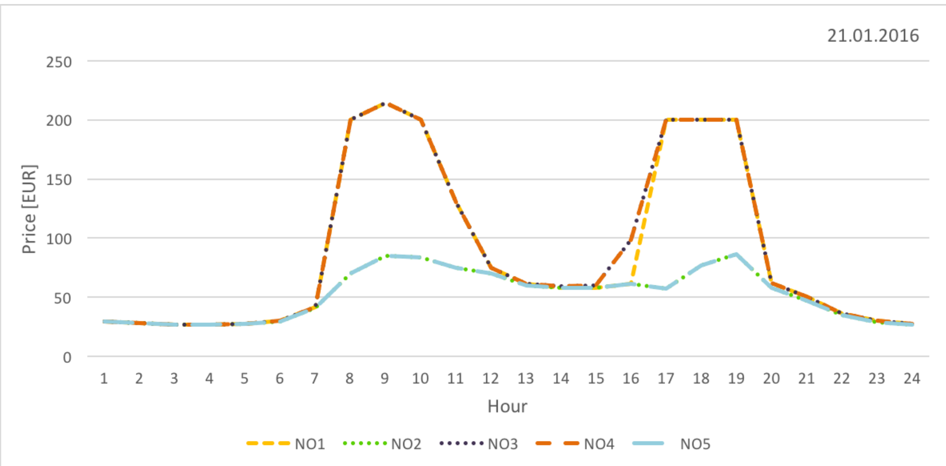

Figure 5.2: Day-ahead prices, 21.01.2016.

Figure 5.3 pictures the energy balance in node 1. The generation is lower than the consumption during the whole day. This means that import from other nodes is needed. The import of energy to node 1 is represented by the green bars. Export would have been represented by green bars on the negative side of the x-axis. The energy generation in node 1, represented by the blue bars, varies over the day. The small red and purple bars represent respectively charging and discharging.

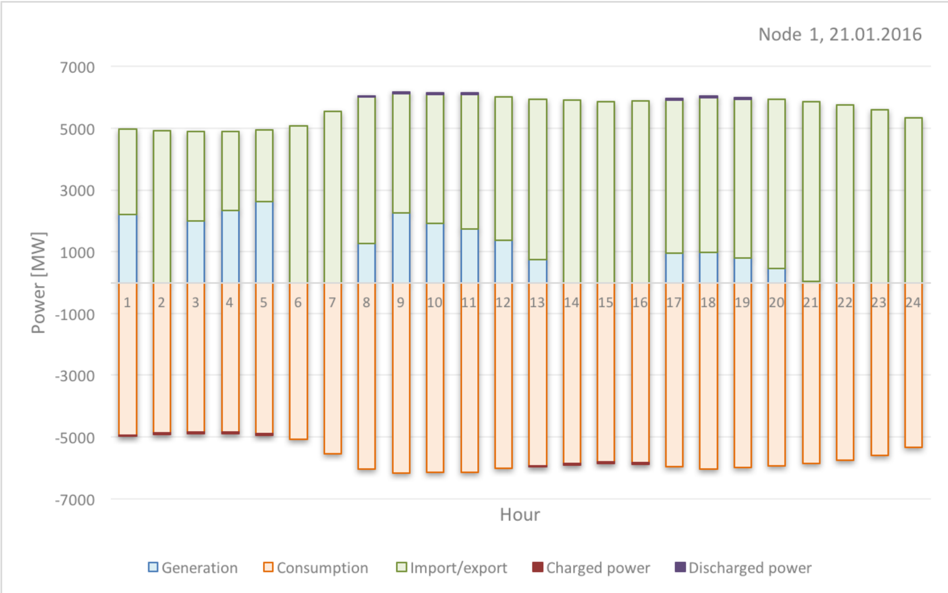

Figure 5.3: Energy balance in node 1 when battery located in node 1, 21.01.2016.

The generation level in each node is dependent on several factors such as day-ahead prices in the other areas and the network structure affecting the power flows. The hydro power plant model used in this research work is simple, and is only limited by its maximum production level. The model does not have any hard restrictions like start-up time and losses in the lines. This is why the generation in node 1 is allowed to be completely shut of in hour 2 and be over 2000 MW in hour 1 and 3. For the optimisation model it does not matter which unit produces as long as it gives a solution. Since the optimisation model uses the first optimal solution, the results for production might look strange. This might not be quite realistic, and should be kept in mind when reading the results.

The day-ahead price for all areas are equal from hour 1-5, as seen in figure 5.2. Hence, the generation pattern is only dependent on the network structure for this time period. The day-ahead prices in area NO1, NO3, and NO4 are significantly larger than the prices for NO2 and NO5 in hours 8-11 and 17 to 19. Production in node 1 is still present even if the day-ahead price is high for this time period. The reason might be limited production and transmission capacities in other parts of the network.

The three generation units in area NO2 (node 11, 14 and 17) are on their maximum capacity limit for every hour throughout the day. This is most likely because the day-ahead price for this area is lower than for NO1, NO3 and NO4. This means that the hydro generation units minimise their overall production costs by producing in the areas where the cost is lowest. The situation in area NO5 is not the same as the situation in NO2. The generation units in area NO5 (node 5 and 19) are not on the limit of the capacity for the whole day. This is a result of limited power transmission capacities, and power flow patterns specified by the network structure. The line between nodes 6 and 3 (line 10) is congested (reached maximum capacity limit) between hour 6 to 13, and between hour 17 to 24. Specified by the PTDSs a large share of the energy produced in node 5 will be transported via line 13, 10 and 4 (from node 5 to node 6 to node 3 to node 1). Due to congestion in line 10, the production in node 5 is limited. The same type of situation occurs for the production in node 19, as line 31 (node 17 to 19) is congested for the whole day, except from hour 14-16. The production units in node 1 do not have to produce for these hours, and can be see in in figure 5.3. This is also the reason why there are production in node 1 also for the time periods when the prices in NO1 are large.

The results describing the power flows for all lines and generation levels for all nodes for this case are given in figures A.8 and A.10 in appendix D. As the situation on this day, 21.01.2016, was out of the ordinary it is not surprising that the power system is congested for some time periods.

As discussed, the location of the hydro generation is not totally reflected by the day-ahead prices, due to production capacity, transmission capacity and power flow constraints defined by the PTDFs, but they definitely have an impact.

The battery connected to node 1 charges when the prices are low and discharges when the prices are higher. This is visualised in figure 5.4. As the battery is places in node 1 (NO1), the charging pattern will be dependent on the day-ahead prices for area NO1. The day-ahead price is pictured in figure 5.4.

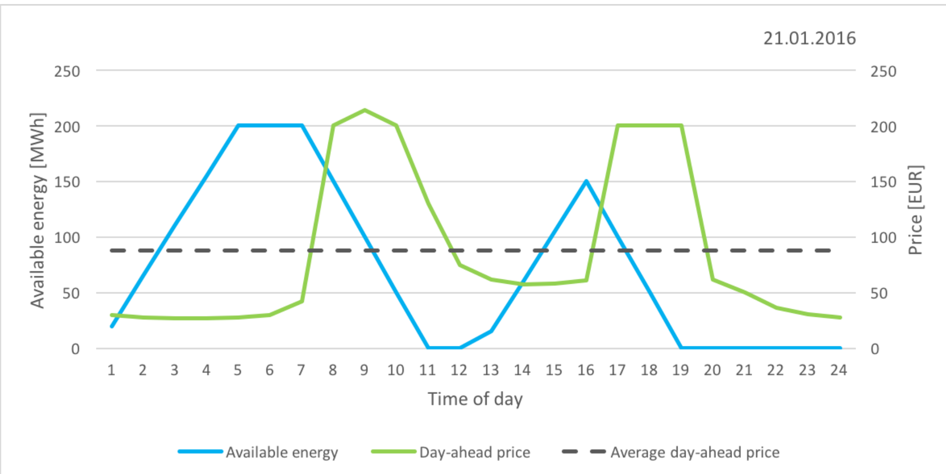

Figure 5.4: Available energy compared with day-ahead price when battery located in node 1, 21.01.2016.

The charging dispatch leads to decreased total system operating cost, as the production during low-price periods is increased, and during high-price periods is decreased. The lowest value of the day-ahead price takes place in the first hours of the day until hour 6, and in a short period right before midnight. The battery is expected to charge in these periods because the price is low. It starts to charge right after midnight, and charges to the maximum energy capacity limit of 200 MWh as fast as possible, limited by the charging rate of 50 MW. At the point where the day-ahead price starts to increase, from hour 7, the battery immediately starts to discharge. When the price decreases below the average day-ahead price, the battery starts to charge again.

The reason why not all the available energy is discharged exactly when the price is at its highest, in hour 9, is due to the battery charge/discharge rate limit of 50 MW, meaning that the battery can only discharge 50 MWh in one hour.

From figure 5.3 it can be seen that the energy absorbed and discharged by the battery is quite small compared to the total amount of energy in node 1. Node 1 is the node where the load is generally highest, and hence also the node with the largest amount of energy in the system. The battery connected is a quite large one with a capacity of 200 MWh, but compared to the amount of energy in node 1 of about 6000 MWh, the percentage is quite small.

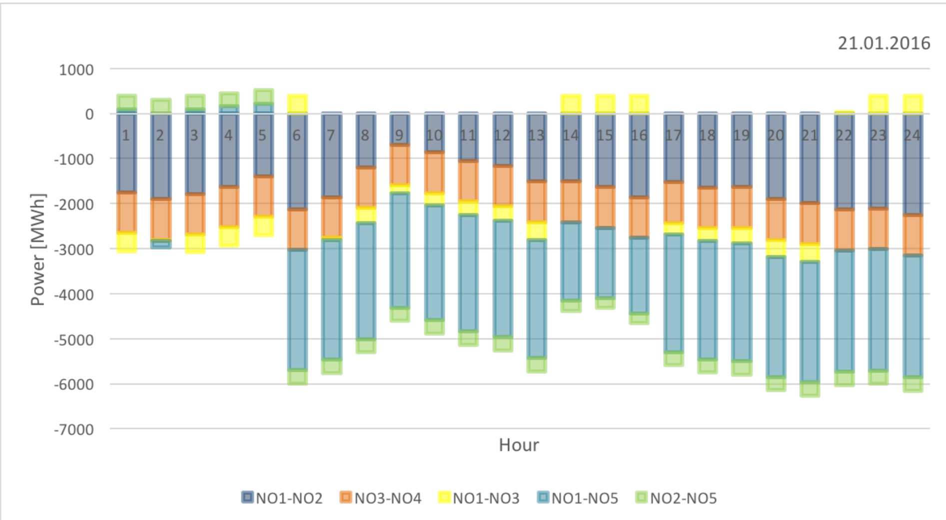

Figure 5.5: Power flow between areas when battery located in node 1, 21.01.2016.

As previously stated, the power flow to node 1 is constantly positive on 21.01.2016. This is not surprising, as the production in node 1 is lower than the consumption. Figure 5.5 shows that a large amount of energy is transported from different areas to area NO1, where node 1 is located. The day-ahead prices are equal for all areas between hour 1 and 5, and for the hours between 6 and 13 the day-ahead price for area NO1, NO3 and NO4 are larger than for NO2 and NO5, leading to the fact that there is transmission bottleneck between these areas. From figure 5.5 it is obvious that there is a change in the situation around hour 5-6. At this point, the power flow from area NO5 to NO1 increases significantly. The power flow from NO2 to NO1 is also increased. This is probably due to the larger prices in NO1 than in NO2 and NO5. Another significant change in the flow pattern occurs in hour

13-14. Here, the prices in NO1, NO2 and NO5 are marginally lower than the prices in NO3 and NO4. The power flow from NO1 to NO3 increases from hour 13, in line with the price variations. As mentioned earlier, the power flow can not solely be described based on the day-ahead prices, due to the network structure and limitations on this.

A battery's ability to overcome transmission congestion situations is also studied. Congestion occurs when the scheduled flow over a line is constrained below desired levels. This normally occurs during high-consumption situations, such as on 21.01.2016. On this particular day, the transmission capacity in several lines were on the maximum limit. This yields line 4, line 10, line 22, line 31 and line 32. To investigate this case, the power flow is further investigated. For hour 2 it turns out that the power flow in two of the lines is decreased from the maximum transmission capacity limit when the battery is connected. Hence, the battery can help the system operator to overcome operating challenges brought by transmission congestion to power system. However, there are still congestion situations in other periods of the day, and the battery could only overcome the congestion for one hour. A larger battery, or several batteries might be a better solution to this issue. The power flow results for both the case with battery connected and without battery connected can be found in appendix D. Information on the power flows in hour 2 in line 8 and 22 can be found in table 5.2.

Table 5.2: Case1: Transmission congestion relief in hour 2, 21.01.2016.

Battery status	**Line**	**Transm. capacity, $P^{cap}_{l,t}$ [MW]**	**Power flow, $P^{flow}_{l,t}$ [MW]**
Without battery	8	400	400
	22	225	225
With battery	8	400	28
	22	225	133

Line 8 is the line between node 1 and node 20, and the only line connection between area NO1 and NO3. The power flow in this line is from node 20 to node 1 for hour 2 in both the case with and without a battery connected. Connecting a battery in node 1 lead to battery charging from hour 1 to 5. This means that the total load in node 1 increases in this time period, and a larger power flow to node 1 is required. By studying the power flow and production in the nodes, it can be seen that the power flow from node 7 to 9 and from 9 to 1 increases drastically in hour 2, in combination with an increased energy production in node 7. It can also be observed that the production in node 1 is zero in hour 2. The production in node 1 is shifted to node 7, and this is probably to effect the power flow described by the PTDFs, so the power flow in the line from node 7 to 9 and from 9 to 1 could be increased, and the power flow in line 8 (node 20-1) could be decreased.

Table 5.3: Case 1: Total system operating cost, 21.01.2016.

Battery situation	**Total system operating cost [€]**
Without battery	32 829 240
Battery connected to node 1	32 781 353

Connecting a battery to node 1 decreases the total system operating cost by 47 887 €. This is 0,16 % of the total cost. Even if it is a small percentage, it is still a considerable amount of money. The size of the battery is small compared to the amount of energy in the system, but the battery is still economically utilised.

5.1.2 Monday 1st of February 2016

The simulation in section 5.1.1 is repeated for the date of 01.02.2016. This date has different day-ahead prices, consumption and exchange with neighbouring countries. The day-ahead prices are presented in figure 5.6.

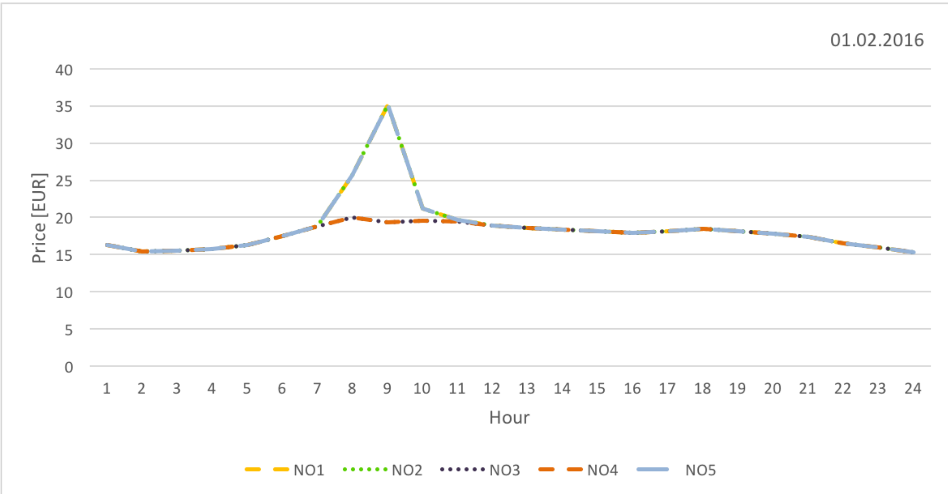

Figure 5.6: Day-ahead prices, 01.02.2016.

The energy balance for node 1 on 01.02.2016 with a battery connected to node 1 can be seen in figure 5.7.

The day-ahead prices are equal for all areas between hour 1-7. Between hour 8 and 11, the prices for area NO1, NO2 and NO5 are equal but larger than the prices in areas NO3 and NO4, as seen in figure 5.6. The production in node 1 in this time period is low, and is a result of the larger prices in area NO1. Production is shifted to area NO3 or NO4. The line between NO3 and NO4 is maximum loaded for every hour during the day. Hence, the production in node 21 (NO4) is already maximised to minimise the production costs.

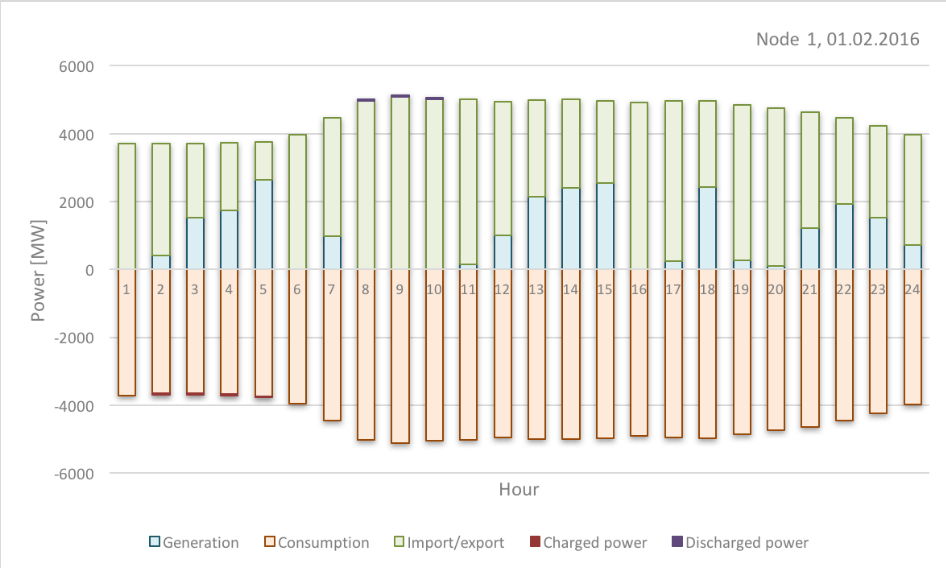

Figure 5.7: Energy balance in node 1 when battery located node 1, 01.02.2016.

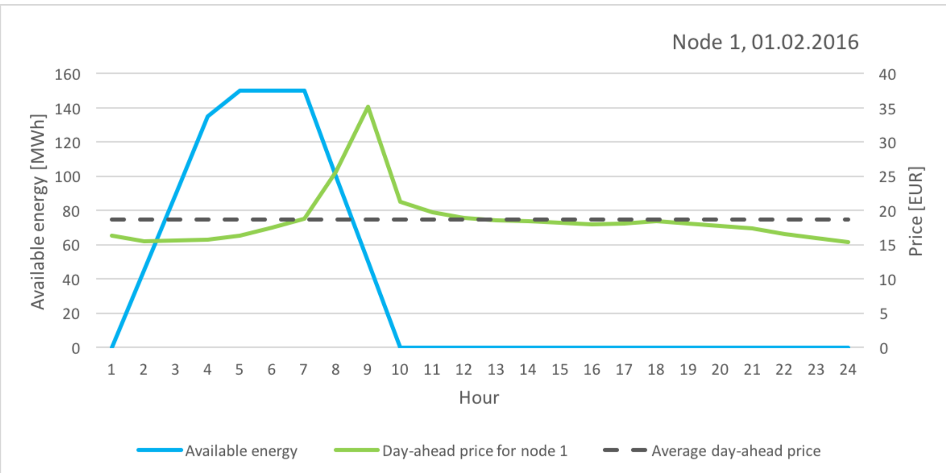

Figure 5.8: Available energy compared with day-ahead price with battery connected to node 1, 01.02.2016.

The charging pattern for the 01.02.2016 follows the same logic as expected, by charging in low-price periods, and discharging in high-price periods. The pattern is quite different from the dispatch on 21.01.2016. This is due to the different day-ahead price variation through the day. For the 01.02.2016, the day-ahead price only has one peak, and after the peak it decreases slowly through the day, and discharging takes place only once. As the battery needs to be emptied before midnight, the low price period before midnight can not be utilised to increase the profitability. The difference in charging pattern reflects the importance of optimisation of battery dispatch with respect to the status of the power system.

Table 5.4: Case 1: Total system operating cost, 01.02.2016

Battery situation	Total system operating cost [€]
Without battery	8 477 436 €
Battery connected to node 1	8 476 256 €

The total system operating cost is decreased with 1180 € when inserting a battery in node 1 on 01.02.2016. This represents 0,014 % of the original operating cost without a battery installed. The reason why the saved costs are not larger is the short time period where the day-ahead price is high. The total system operating cost is also a lot smaller than for the case of 21.01.2016, as the day-ahead prices generally are lower and less volatile.

5.2 Case 2: Transmission line outage

Transmission line outage will reduce the available transmission capacity between two nodes. In this case, transmission line outage forces the system to exchange energy up to the maximum available transmission capacity, and result in congestion.

5.2.1 Thursday 21st of January 2016

To determine whether a battery in node 1 can be utilised during congestion, the transmission capacity for line 4, between node 1 and 3, is decreased. First, simulations without a battery connected are performed. The capacity is decreased to the limit of what the system can handle. At a capacity of 726 MW, a solution to the optimisation problem does not exist, meaning that the demand can not be met, and a blackout might occur. If the capacity is increased with one megawatt, to 727 MW, a solution to the problem does exist. This scenario may be unrealistic, and the situation might also affect the day-ahead prices, but the assumption is made for the sake of illustration on how a battery can support the system with reduced transmission capacity.

Simulations with reduced transmission capacity from 2000 MW to 727 MW are performed both with and without a battery connected to node 1. The total system operating costs are listed in table 5.5. Implementing the transmission capacity reduction to 727 MW without a battery connected, the total system operating cost increases with 1 415 236 €, which is 4 %. This indicates that transmission congestion is highly undesirable.

After connecting the battery, the total system operating cost is decreased by 107496 €, which is a decrease of 0,31 %. The total cost will still be higher compared to the case without outage, because the production pattern is forced to be changed from the original optimal pattern.

To see how much the transmission capacity can be reduced when the battery is connected, simulations were done by decreasing the transmission capacity until there were no feasible solution. Infeasible solution was obtained with a transmission capacity of 711 MW, meaning that a transmission capacity of 712 MW is feasible when a battery is connected. This is a further decrease of 15 MW. This means that the operating limits of the network can be shifted by inserting a battery. Thus the network can handle more extreme situations if a battery is included.

Table 5.5: Case 2: Total system operating cost, 21.01.2016

Battery situation	**Capacity line 4 [MW]**	**Total system op. cost [€]**
Without battery	2000	32 829 240
Without battery	727	34 244 476
Battery connected to node 1	727	34 136 980
Battery connected to node 1	712	34 217 444

The charging pattern for both the situation with 727 MW and the situation with 712 MW are equal to the charging pattern in case 1, with battery connected to node 1 and with a transmission capacity of 2000 MW, seen in figure 5.4. This is because the optimal charging pattern from case 1 is the charging pattern that minimises the total system operating cost, and it also helps the system back to a feasible solution when the transmission capacity is reduced.

A battery storage system may be useful during outages of transmission lines, both in the manner of helping the system operate when it is out of its original limits, and in an economical perspective. This means that batteries can help the system operator to avoid large investments in upgrading the transmission system, at least for marginal values, i.e. 15 MW as this case has shown. Though, it is important to keep in mind that the outage can lead to a large increase in total system operating costs.

5.2.2 Monday 1st of February 2016

The charging pattern on 21.01.2016 was equal during normal operation and during the transmission outage case presented in this section. To see if the charging pattern can be forced to be changed, the same congestion case is performed on data from 01.02.2016.

Decreasing the transmission capacity in line 4 (node 1 to 3) from 2000 MW to 799 MW gives infeasible solution when there is no connected battery. A transmission capacity of 800 MW gives a feasible solution. With a battery connected, the system can handle a further decrease of the transmission capacity down to 785 MW. The total system operating costs are listed in table 5.6.

Table 5.6: Case 2: Total system operating cost, 01.02.2016

Battery situation	**Capacity line 4 [MW]**	**Total system op. cost [€]**
Without battery	2000	8 477 436
Without battery	800	8 477 436
Battery connected to node 1	800	8 476 256
Battery connected to node 1	785	8 476 426

As seen in table 5.6, the total system operating cost was not changed when the transmission capacity was decreased from 2000 MW to 800 MW, before the battery was inserted. One might think that this is because the generation pattern stays the same in the two cases, but figures 5.9 and 5.10 show that this is not the case.

The congestion forces a larger share of the production to take place in node 1, because the power flow to node 1 is limited due to the reduced transmission capacity. The optimisation model was able to find a solution with the same total operating costs during congestion. This is probably because of the day-ahead prices are quite similar and stable over the day, except for the time period between 7 and 11, where the prices in area NO1, NO2 and NO5 are higher. In this time period, the energy production in node 1 is lower for both situations. The day-ahead prices for all areas can be seen in figure 5.6, and the day-ahead price for area NO1 is given in figure 5.12.

Inserting a battery when the transmission capacity is set to 800 MW gives reduced cost, but with the same charging pattern as the situation with original transmission capacity of 2000 MW, seen in figure 5.8. This is not surprising. As the situation is still within the network limits, the charging pattern will follow the day-ahead prices to give the most profitable charging pattern. With a battery connected, the system gives a solution when the transmission capacity is further decreased to 785 MW. The energy balance for the situation with decreased capacity to 785 MW is shown in figure 5.11. If the transmission capacity is lower than 785 MW for this specific line, the situation is not feasible any more.

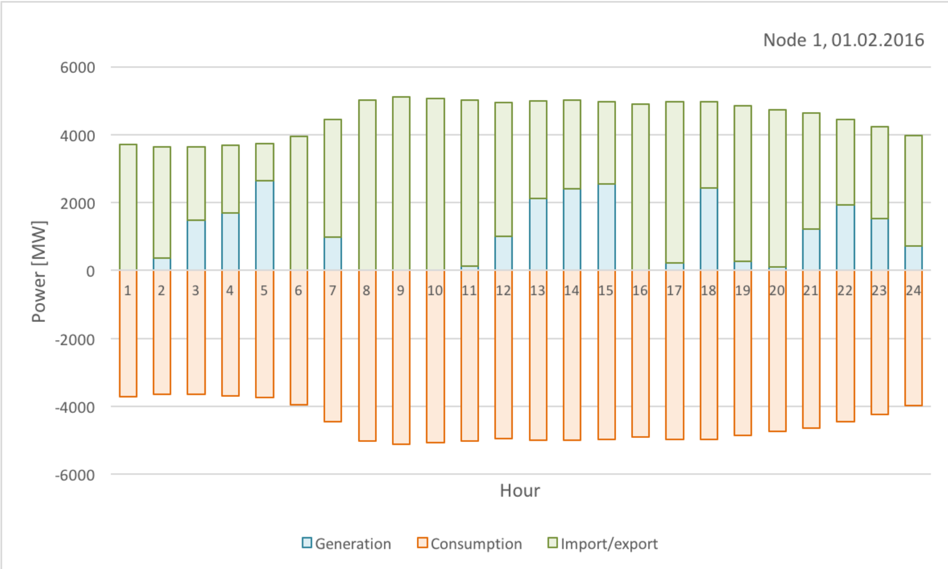

Figure 5.9: Energy balance in node 1 in original situation without battery connected, 01.02.2016.

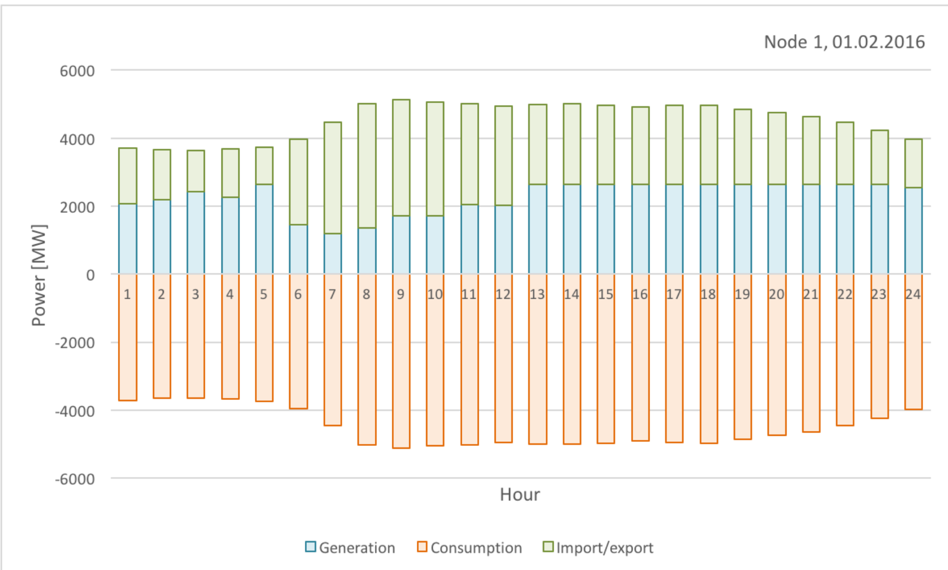

Figure 5.10: Energy balance in node 1 when transmission capacity in line 4 is reduced to 800 MW, without battery connected, 01.02.2016.

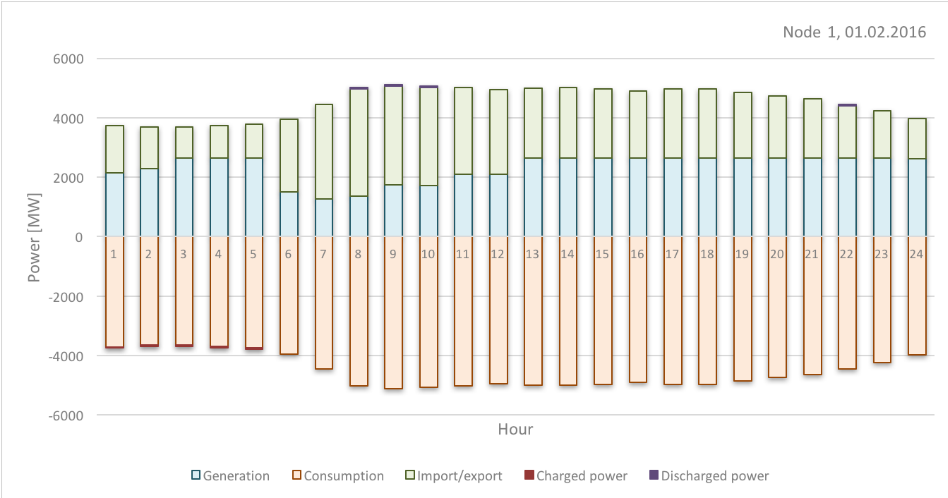

Figure 5.11: Energy balance in node 1 when the capacity in line 4 is reduced to 785 MW, with battery connected to node 1, 01.02.2016.

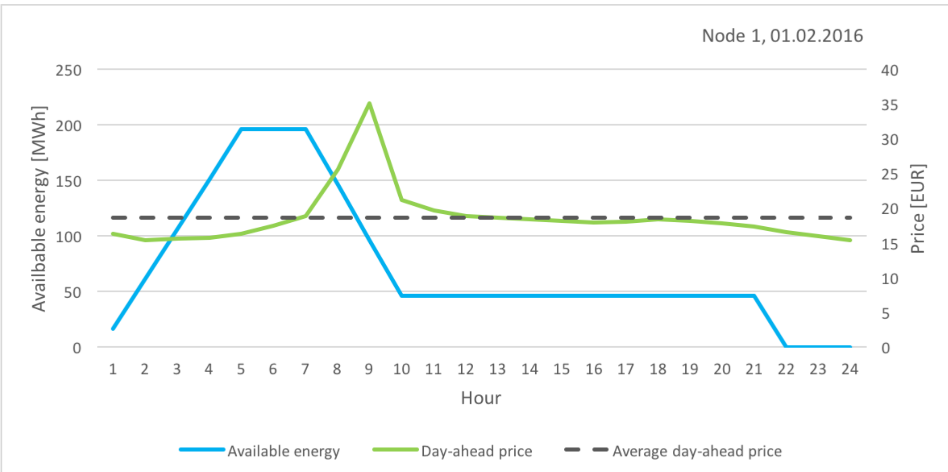

Figure 5.12: Available energy compared with day-ahead price, during reduced capacity in line 4 to 785 MW, with battery connected to node 1, 01.02.2016.

The charging pattern during the reduction of transmission capacity to 785 MW looks somewhat similar to the charging pattern without congestion. The main difference is the discharge in hour 21. This discharge is not conducted for economical reasons, but to make the system within the limits of operation and to give a solution to the optimisation problem. Hence, hour 21 is an hour with a critical bottleneck fixed by the battery. The battery charges up to 200 MWh (instead of up to 150 MWh in the case without congestion) to be able to discharge energy in hour 21, and get the earnings for discharging in hours 7-10. The reason why the transmission capacity can not be reduced even more without resulting in an infeasible solution might be that the amount of energy in the beginning and end of a period is forced to be zero, and the fact that the charging/discharging rate is only 50 MW. These factors limit the operation of the batteries quite much.

5.3 Case 3: Battery located in node 5

In this case the battery is moved from node 1 to node 5. The demand and generation situation for node 5 is quite different from the situation in node 1. Node 5 has in general a much larger generation capacity than consumption, and is able to produce energy to distribute to other nodes in energy deficit. Node 5 is located in area NO5.

5.3.1 Thursday 21st of January 2016

The prices in area NO1, NO3 and NO4 are significantly larger than in NO2 and NO5 for the time periods form hour 8-11 and 16-19 on 21.01.2016. The generation in node 5 is expected to be significant in these time periods. It is, and this can be seen in figure 5.13.

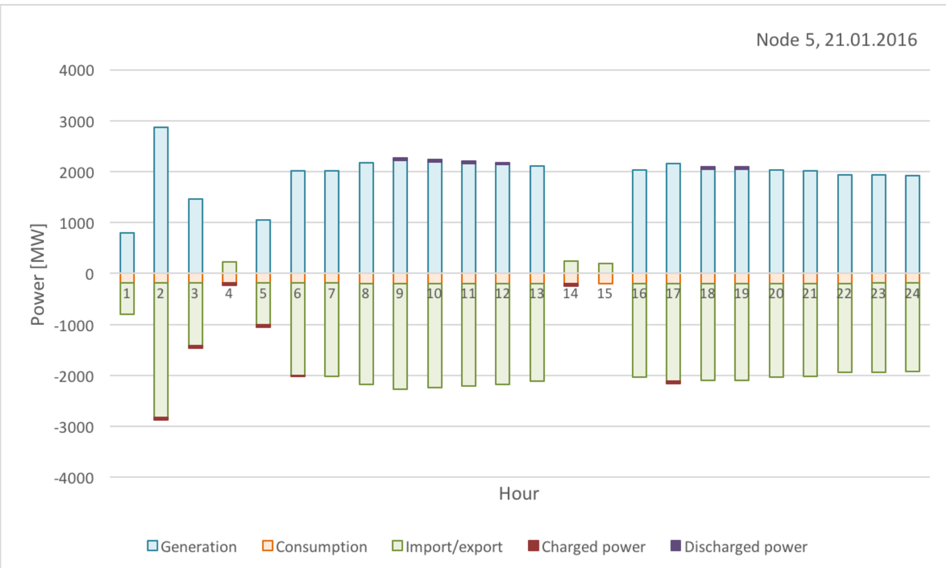

Figure 5.13: Energy balance in node 5 when battery located in node 5, 21.01.2016.

From figure 5.13 it can also be seen that the generation level is relatively high and stable throughout the day. It is though not near the production capacity limit of 4028 MW. The limiting factor for generation level will be the transmission capacity. According to the

PTDF-matrix, a large share of the energy produced in node 5 flows through line 10, the line between node 3 and 6. Line 10 is congested for every hour where the generation in node 5 is above 1900 MW. This is probably the reason why generation in node 5 is not even larger.

The green bars representing the import and export are almost constantly negative, and means that the node exports energy to the rest of the system. Node 5 imports in hour 4, 14 and 15, as there is no production in node 5 in these hours. In the same hours, the day-ahead price in area NO5 is equal to the price in NO1 and NO2, and the production is shifted to these areas.

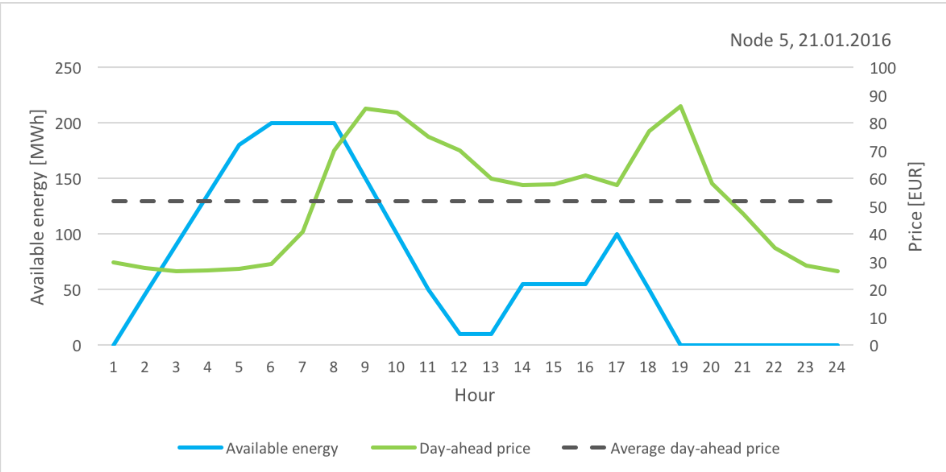

Figure 5.14: Available energy compared with day-ahead price, with battery connected to node 5, 21.01.2016.

The charging pattern when the battery is placed in node 5, pictured in figure 5.14, is different from the charging pattern with the battery connected in node 1 from case 1, pictured in figure 5.4. This is because the day-ahead price in node 5 (area NO5) is different from the day-ahead price in node 1 (area NO1).

One of the interesting aspects with this case is that the battery starts charging again in hour 13, even if the day-ahead price is above average. This is because the day-ahead price between hour 18 and 19 is so high that it is profitable to charge even if the price is above average.

Table 5.7: Case 3: Total system operating cost, 21.01.2016

Battery situation	**Total system operating cost [€]**
Battery connected to node 1	32 781 353
Battery connected to node 5	32 819 621

According to table 5.7 it is more profitable to locate the battery in node 1. This because the day-ahead price pattern in node 1 serves a more profitable charging strategy, as the prices are more volatile.

The south-eastern part of Norway is the most densely populated area, and the largest consumption is there. A large part of the production is located in the western part of Norway, as the production in node 5. During situations with large consumption, as the situation on 21.01.2016, congestion between east and west will occur. This leads to differences in area prices. As the area prices in areas with a large consumption will be larger and most likely more volatile, it will be more profitable to locate batteries in load centres. This yields especially in extreme cases.

5.3.2 Monday 1st of February 2016

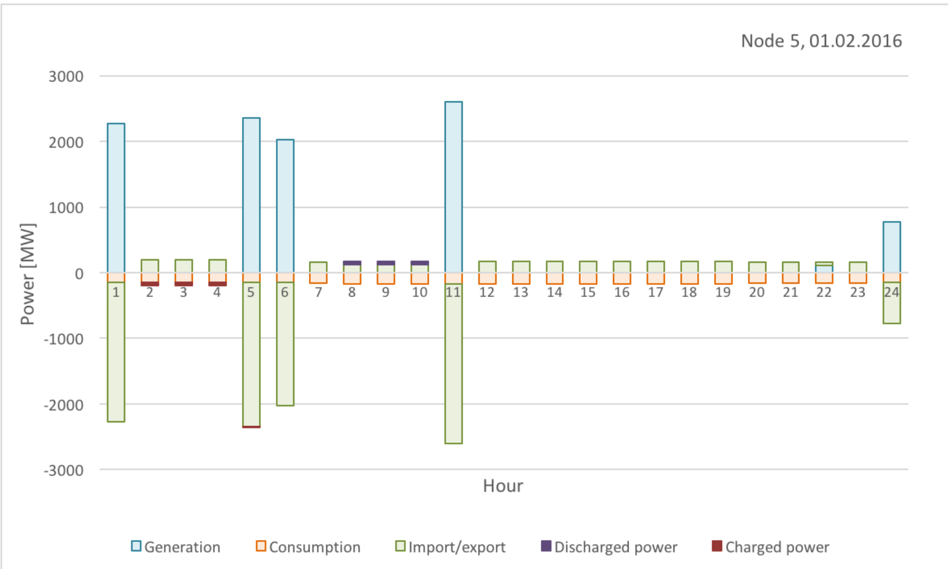

Figure 5.15: Energy balance in node 5 when battery located in node 5, 01.02.2016.

The energy balance in node 5 can be seen in figure 5.15. The production in hour 1, 5, 6, 11 and 24 is quite high. The demand is low, so most of the energy produced in these hours is exported to cover other loads in the system.

The prices are equal for all areas, expect for the period between hour 8 and 10, where the price in area NO1, NO2 and NO5 are larger. The generation in node 5 (NO5) is zero for this time period, and this makes sense as it will be cheaper to produce in another area.

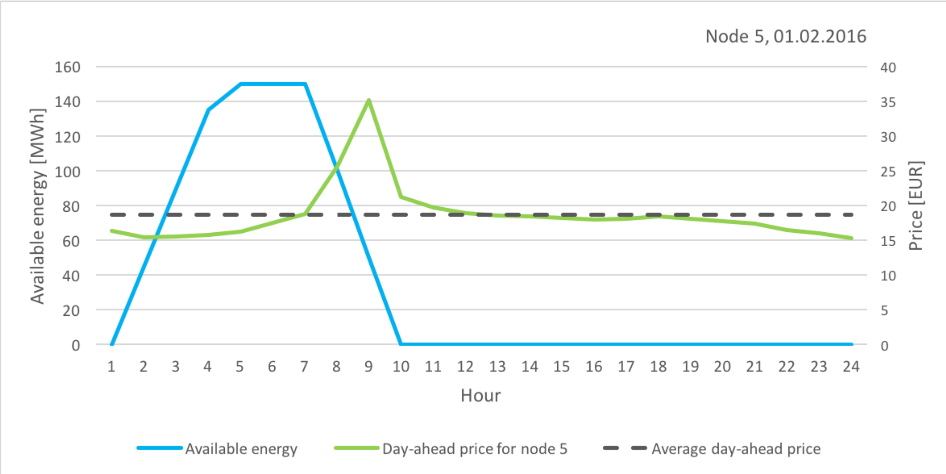

Figure 5.16: Available energy compared with day-ahead price, with battery connected to node 5, 01.02.2016.

Comparing figure 5.8 and 5.16 shows that the charging patterns for the battery are equal for the situations where the battery is placed in node 1 and node 5. This is because the price in node 1 and node 5 for this day is similar, as there is no congestion between area NO1 and NO5. As seen in table 5.8, the total system operating costs are also equal. This underpins that the location of the batteries is not equally important in a non-congested situation as in extreme cases.

Table 5.8: Case 3: Total system operating cost, 01.02.2016

Battery situation	**Total system operating cost [€]**
Battery connected to node 1	8 476 256
Battery connected to node 5	8 476 256

5.4 Case 4: Battery located in node 21

In this case, the battery is moved to the northern part of Norway. In this part of the country, the transmission grid is weaker than in the rest of the country. They are more dependent on their own production, as import from other areas are restricted. The impact of battery on operating challenges in this area is to be studied.

5.4.1 Thursday 1st of February 2016

The battery is moved to node 21 (NO4), and the charging pattern might change, as the day-ahead price is different for this area. In general, the prices are equal over the day, but between hour 8 and 10, the prices in area NO1, NO2 and NO5 are larger.

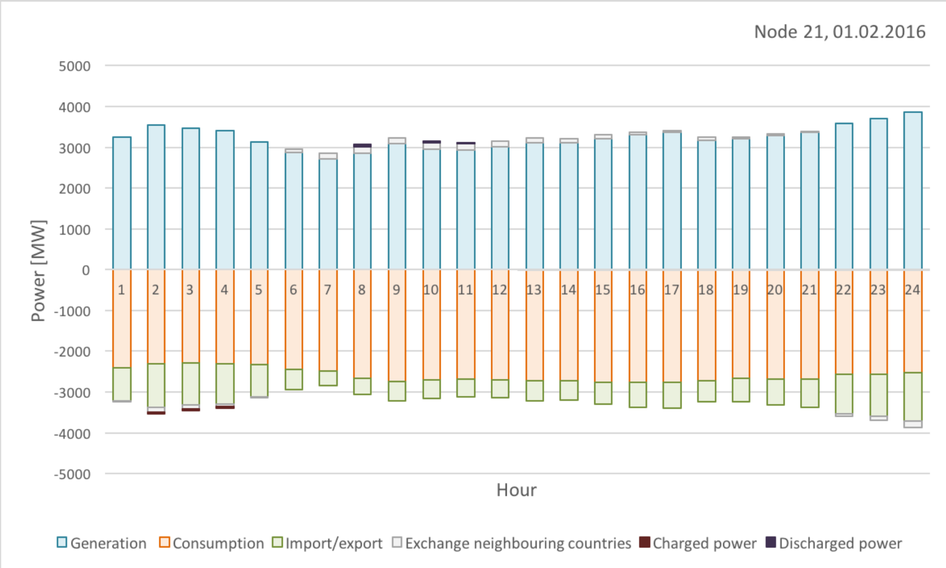

Figure 5.17: Energy balance in node 21 when battery located in node 21, 01.02.2016.

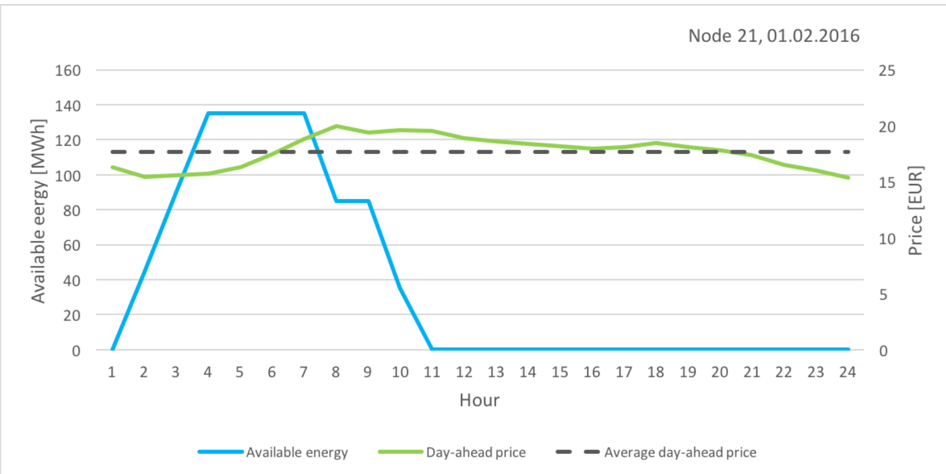

Figure 5.18: Available energy compared with day-ahead price when battery located in node 21, 01.02.2016.

Figure 5.17 shows that the generation level is stable and high throughout the whole day. The power flow in line 32 (node 21 to 20) is congested the whole day. This can be seen in figure A.14 in appendix E. This line is the only line connecting the northern part of

Norway to the southern part. The production limit for node 21 of 4980 MW is not reached, but since the transmission line from node 21 to node 20 is maximum loaded for the whole day, producing more in node 21 will not be possible in this case.

The charging dispatch differs for the situation when the battery is placed in node 1 and 5, because the day-ahead price is different. This can be seen by comparing figure 5.8 and figure 5.18. Hence, the total system operating cost will also be different. The reason why the total system operating cost is lower when connecting the battery to node 21, is the smaller day-ahead price variation over the day for node 21.

Table 5.9: Case 4: Total system operating cost, 01.02.2016

Battery situation	**Total system operating cost [€]**
Battery connected to node 1	8 476 256
Battery connected to node 5	8 476 256
Battery connected to node 21	8 477 377

5.5 Case 5: Generation outage in node 21

To investigate how the system handles outage of generation, the production level in one of the nodes is decreased. Because the production level will be given as an input to the system, this can be considered as a planned outage of generation, for instance in relation to maintenance of a production unit.

5.5.1 Thursday 1st of February 2016

The production capacity in node 21 is decreased from to 4980 to 3550 MW, due to generation outage. The optimisation problem does not have a solution for this case without a battery connected. But when connecting a battery, the model finds a solution by utilising the battery. The energy balance is given in figure 5.19.

The charging pattern during the generation outage in node 21 is visualised in figure 5.20. By comparing figure 5.18 and figure 5.20 it is possible to reveal where the generation outage causes trouble, namely hour 24. Preferably, the battery would discharge all the available energy before hour 11, but this is not possible during this outage. By withholding some of the energy in the battery for a longer time, and then discharge it in hour 24, the system is able to operate even with this kind of outage.

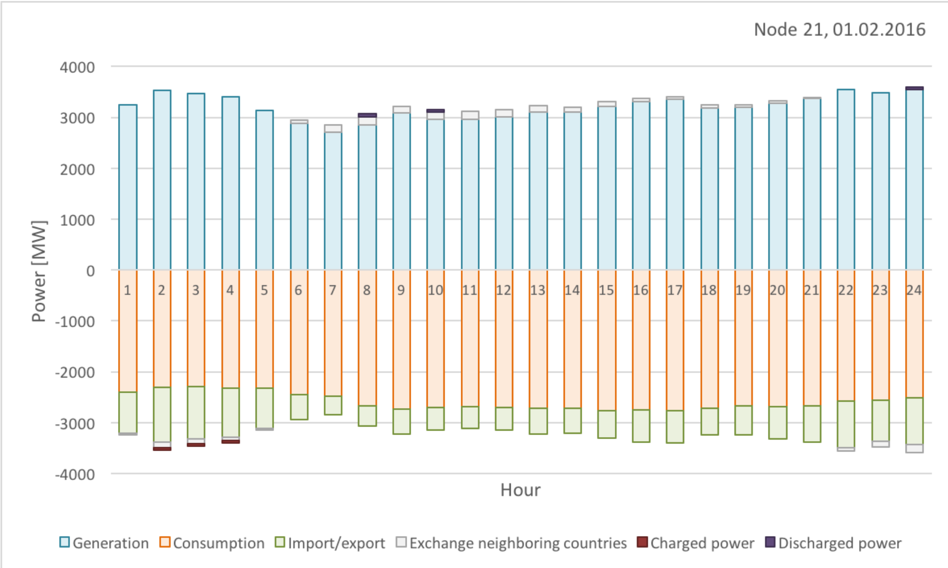

Figure 5.19: Energy balance in node 21, when generation outage and battery located in node 21, 01.02.2016.

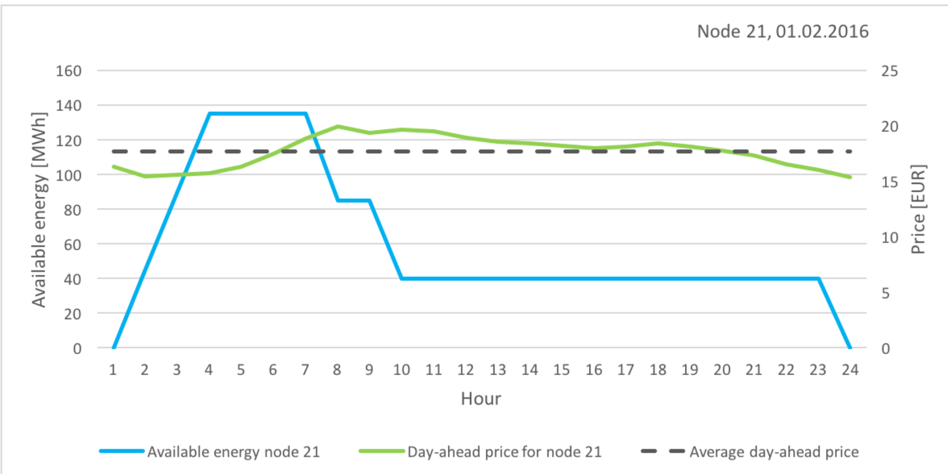

Figure 5.20: Available energy compared with day-ahead price when generation outage and battery located in node 21, 01.02.2016.

Batteries can support the system to restore from generation outage. This is especially important in the northern part of Norway, as they are more reliable on their own production due to a weaker transmission grid. The system can not withstand this incident because the connection of this area to the entire system is not strong enough. A situation with an equivalent outage in the southern part of Norway might not need a battery to keep operating.

Table 5.10: Case 5: Total system operating cost, 01.02.2016

Battery situation	Total system operating cost [€]
Without battery and before generation outage	8 477 436
Battery connected to node 21	8 477 546

It is interesting to see that the total system operating costs for the original case without batteries, are almost equal to the case with generation outage and battery connected. Connecting a battery will restore the system in addition to keep the costs down.

5.6 Case 6: Oslo and Akershus solar power future scenario

Solar power production is an upcoming and increasingly popular technology. An rise in the number of roof-top solar panels is already seen, and this growth is expected to continue in the near future. The main reason for this development is the decrease in price for both solar panels and battery storage options. A fictive scenario based on the trend towards a larger share of solar power production is created, and the impact on the network is to be studied. The scenario is based on a share of 80 % of the detached houses in Oslo and Akershus has installed roof-top solar panels. The houses with solar panels have also installed a battery energy storage facility. The data for the battery storage facilities is based on the Tesla Power Wall battery[46].

According to [47], the average installed solar power in the German private market is 7 kW. Based on this, it is assumed that the average size of solar panels in Norway will be in the same range, and the average power per roof-top is set to be 7 kW. Information on the solar power scenario is given in 5.11.

Table 5.11: Information on Oslo and Akershus solar power future scenario.

Number of detached houses in Oslo and Akershus, [48]	150 016
Share of detached houses in Oslo and Akershus with solar power	80 %
Number of detached houses in Oslo and Akershuswith solar panels	120 012
Average installed solar power per house	7 kW
Total installed solar power in Oslo and Akershus	**840 090 kW**
Average installed battery energy capacity	6,4 kWh
Total installed battery energy capacity in Oslo and Akershus	**768 077 kWh**
Average installed battery power capacity	3,3 kW
Total installed battery power capacity in Oslo and Akershus	**396 040 kW**

Norway´s leading solar panel supplier is Solcellespesialisten [49], and they provide roof-top solar panel facilities from 2,1 kW to 7,3 kW [47]. The size of the 7,3 kW installation is 42,6 m^2. Based on this, the size of the 7 kW installation is calculated to be $\frac{7kW}{7{,}3kW} \cdot 42,6m^2 = 40,8m^2$.

Data for solar radiation given inn W/m^2 is found in [50]. The location for the solar radiation measurements is on the roof top of "Statens strålevern" in Oslo, and the measurements are assumed to be performed on a horizontal surface. The measurements are assumed to be representative for the whole area of Oslo and Akershus, although local differences will occur. The solar radiation on a horizontal surface in Oslo is 951 kWh/m^2 pr year. On a 30 degree surface directed to the south, the solar radiation in Oslo is 1149 kWh/m^2 pr year. This means that an angled roof-top directed to the south will have a larger solar radiation than the measurement at Statens Strålevern. Nevertheless, the values obtained from Statens Strålevern is used for this work, because a lot of houses in does not have a surface to the south, and the values are assumed to be representative though maybe somewhat lower than the actual values.

Ideally, the solar power production should be predicted using forecasts, and not looking back in time, because the model is to be used for future days. As for day-ahead prices, consumption and exchange with neighbouring countries, real historical data for solar radiation is used in this case. The radiation within the scope of an hour is assumed to be constant, as the model developed in this work is based on hourly resolution. To calculate the expected solar energy production based on the solar radiation data, equation 5.1 is used.

$$\begin{aligned}\text{Expected solar power } &= \text{Solar radiation}[\frac{W}{m^2}] * \text{Area}[m^2] * \\ &\quad \text{Efficiency} * \text{Number of houses}\end{aligned} \tag{5.1}$$

Typical efficiency for a solar cell is between 12-20 % [51]. Improvement is expected in solar cell efficiency in the future, hence the efficiency for this case is set to be in the upper range, to 20 %.

For the simulations, the operational costs for solar power production is set to zero. By doing this, parts of the the energy imported from other areas is substituted with energy from the solar panels without any costs, which in turns will decrease the total system operating cost. An alternative would be to consider the maintenance cost of solar power plants as an operating cost. For the scope of this study, the solar power plant maintenance cost is neglected.

To be able to model the solar power production in the model in GAMS, an extra node is added to the system. This node is called node 23, and is connected only to node 1 via a new line 34 with infinite transmission capacity. The demand in this node is set to zero. From this point, node 1 and 23 will be referred to as node 1, as they in practice will function as the same node.

The owners of the houses with roof-top solar panels will most likely want to use their own free production of energy when they need it. A typical load pattern can be useful to estimate approximately how much energy each household will need for themselves. A publication from SSB, [52], states that there are limited information about the variation between the different customer groups regarding the short-term variations in the electricity consumption. Based on measured data from 3930 household customers of Skagerak Nett in 2006, an average load profile is made. The average load profile for June can be seen in figure 5.21.

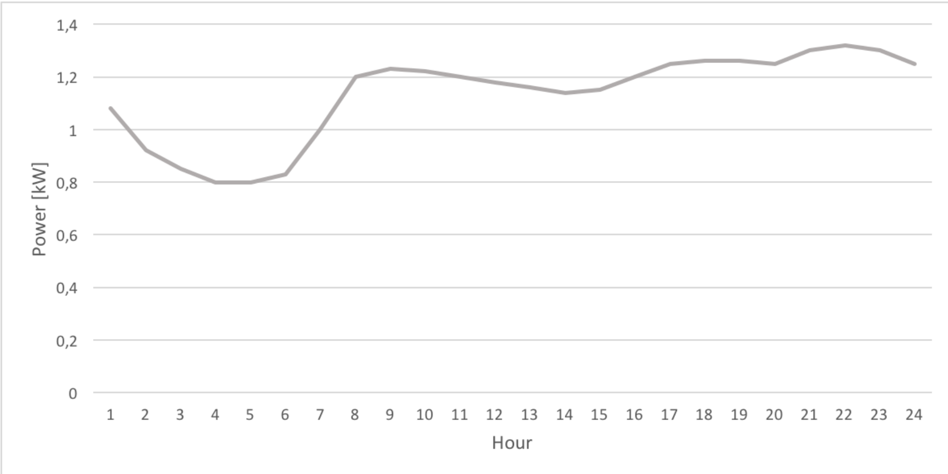

Figure 5.21: Average load on a weekday for household customers [52].

Skagerak Nett has customers in a large part of southern Norway (Grenland and Vestfold), and the weather conditions in these areas are comparable with the weather conditions in Oslo and Akershus. The growth in electricity consumption in Norway lasted until 1999, and for the past years the electricity consumption has stabilised [34]. There are 120 012 houses in this study, and the average load profile seen in figure 5.21 is assumed fit very well when aggregating the load in every household. The amount of power will be multiplied with the number of houses, and the scaled load profile is assumed to be a good consumption prediction for the solar panel roof-top houses in Oslo and Akershus.

Limitations on battery charging

It is assumed that the solar panel owners (SPOs) will use their own produced energy from their solar panels when they need it. It is also assumed that they will store excess energy from their own production, if they assume that they will need the energy in the afternoon or evening. If not, it is sold to the grid. Based on the load pattern in figure 5.21, the assumed consumption for the evening is given.

The situation is handled by forcing discharging in the evening hours, when the solar power production is not sufficient to cover the consumption of the SPOs. In this way, the battery will make sure that there is enough energy for the evening consumption. This approach is only possible if the excess energy from the solar panels during the day is sufficient to cover the evening load. If not, they will have to buy energy from the grid, as regular customers.

The charging cost for charging energy from the solar panels and discharging earnings to cover evening load are set to be zero, because the charging and discharging for the SPOs for their own use is free.

If the total capacity of the batteries is not used by the solar panel owners themselves, the rest of the capacity can be utilised to minimise the hydro power operation costs further.

5.6.1 Thursday 25th of June 2015

As the sun shines a lot more during the summer months than in the winter, an analysis on a summer day will be performed to see a large effect of the solar power and battery installations. The weather on the 25th of June 2015 was partly clouded [53], but the sun was shining in some hours. The solar radiation in Oslo on 25.06.2015 is quite variable, as seen in figure 5.22. The variation is probably caused by clouds passing by.

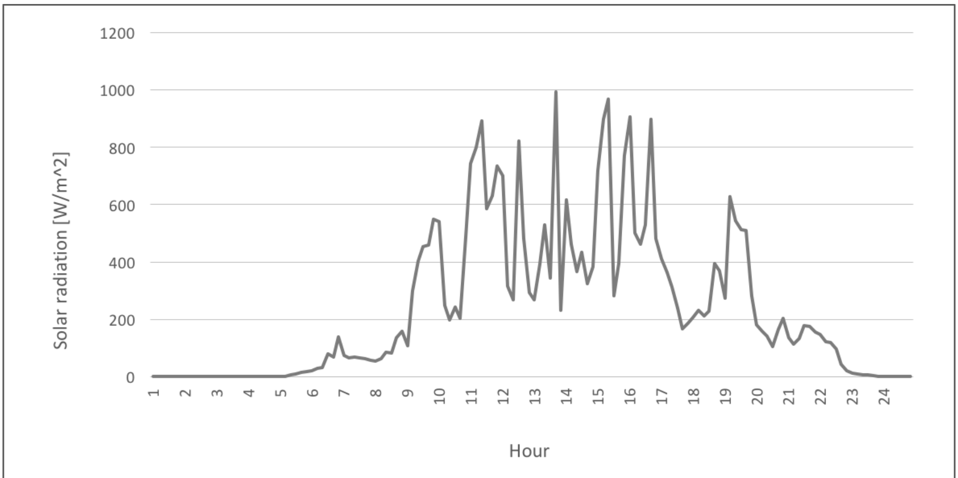

Figure 5.22: Solar radiation, Oslo 25.06.2015.

Based on this solar radiation curve, the expected power production is calculated with equation 5.1. The results are presented in table 5.12.

Table 5.12: Solar radiation and expected solar power production in Oslo and Akershus, 25th of June 2015.

Hour	**Solar radiation** [W/m^2]	**Expected production,** $P^{gen}_{23,t}$ **[MW]**
1	0	0
2	0	0
3	0	0
4	0	0
5	21	20.6
6	74	72.5
7	54	52.9
8	107	104.8
9	541	529.8
10	744	728.6
11	701	686.5
12	269	263.4
13	616	603.2
14	717	702.2
15	906	887.2
16	412	403.5
17	207	202.7
18	274	268.3
19	180	176.3
20	135	132.2
21	146	143.0
22	13	12.7
23	0	0
24	0	0

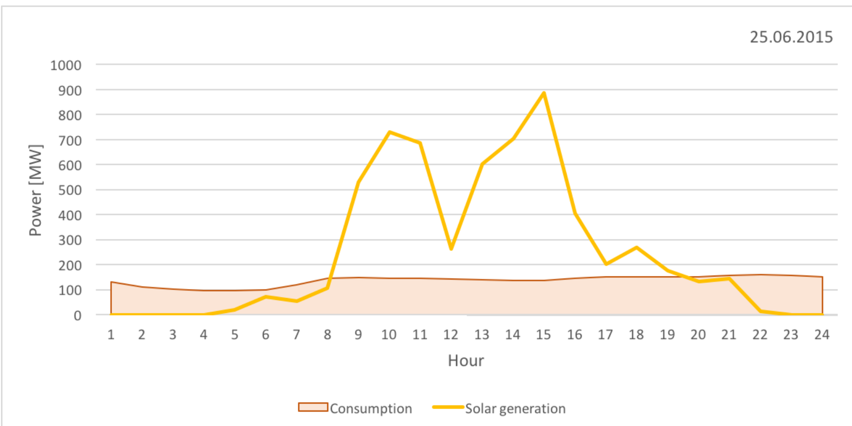

Figure 5.23: Expected solar power production compared to load profile for 25.06.2015.

The total consumption on the 25th of June 2015 for the households with installed rooftop solar panels is plotted in figure 5.23. The calculated production is also included in the figure, and it indicates that the expected solar power production is larger than the consumption in the time period from 08.00 to 19.00. It is assumed that the owner of the solar panels would want to store the excess energy in their installed batteries. But because energy can be stored in batteries for only a limited amount of time, the batteries needs to be emptied at midnight.

Figure 5.24 pictures the energy balance in node 1 including the solar panel owners and their batteries. The solar production is represented by the yellow bars, and follows the pattern in figure 5.23. Figure 5.25 pictures the energy balance only considering the solar panel owners and their batteries.

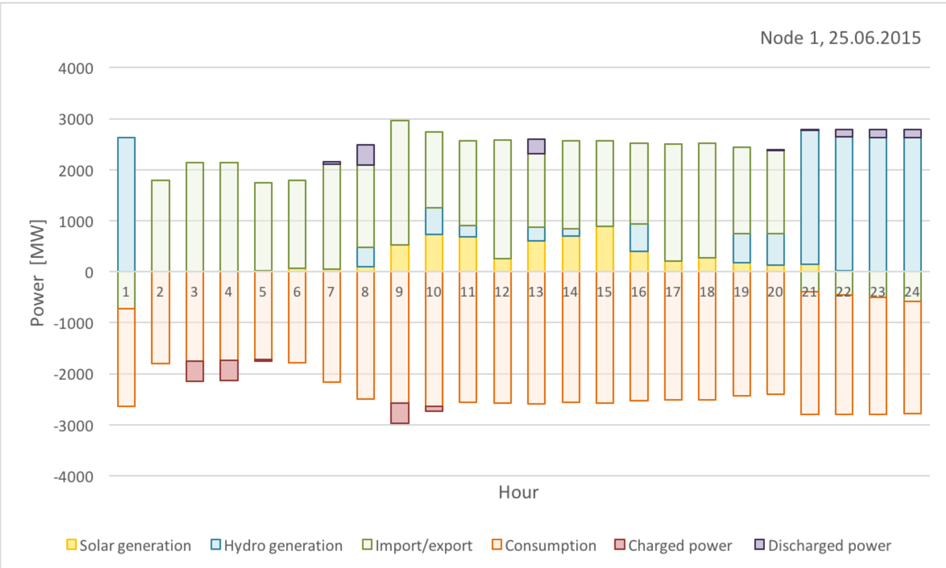

Figure 5.24: Case 6: Energy balance in node 1, 25.06.2015.

First of all, the energy needed in the evening is charged in hour 9 and 10, directly after the solar power production starts. This is visualized by the red bars in figure 5.24 and 5.25. The charged energy for evening consumption is charged in this time period because this is the time period where the day-ahead price is lowest during the period with excess solar energy, as seen in figure 5.26. In hour 9, no energy is sold from the SPOs to the grid, as all the energy from the solar panels is charged to the batteries. This energy is stored until the solar generation becomes lower than the consumption, in hour 20. Then the stored energy is discharged according to their evening load, and the amount of energy in the batteries at midnight will be zero, see figure 5.26.

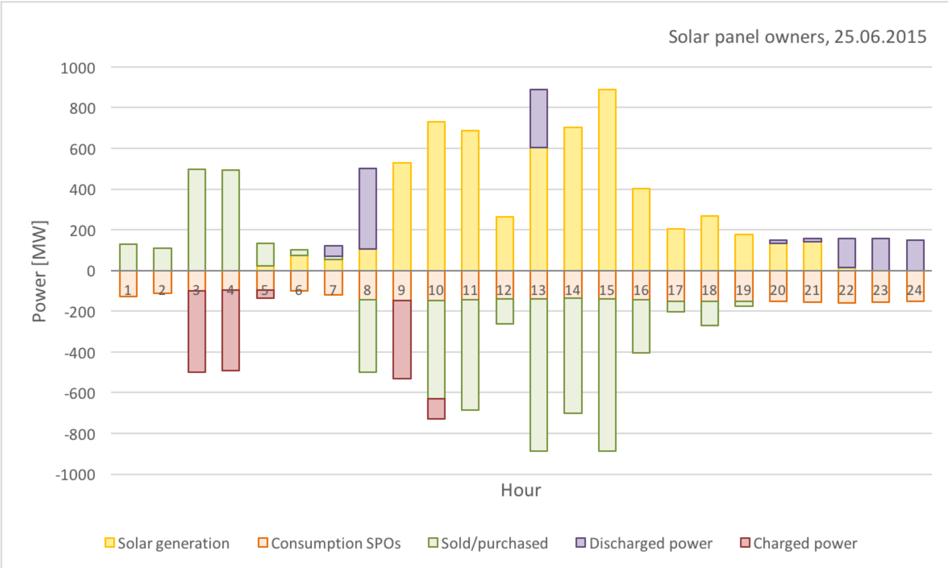

Figure 5.25: Case 6: Energy balance for solar panel owners, 25.06.2015.

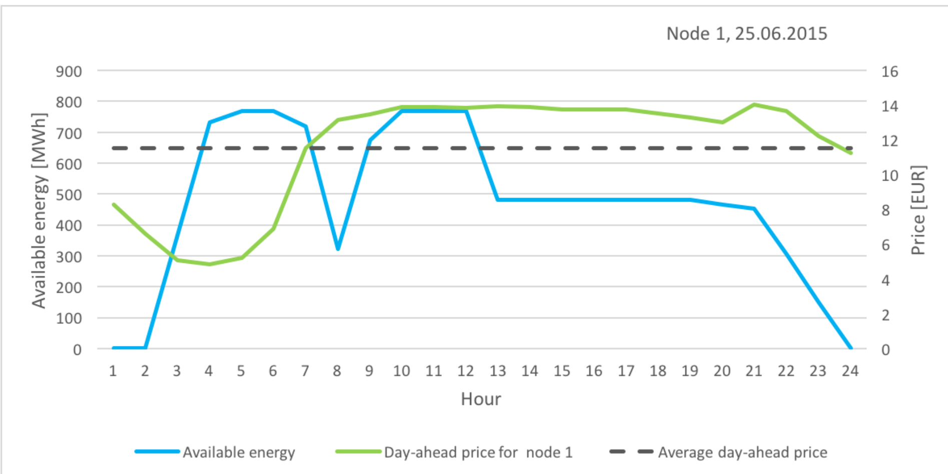

Figure 5.26: Case 6: Available energy compared with day-ahead prices, 25.06.2015.

As the SPOs evening consumption is covered, the batteries are free to be used to minimise the hydro power operating cots. The batteries charge early in the morning, and then discharge right before the solar energy production exceeds the demand. This is to make room for the excess energy from the solar energy production to be stored and used for evening consumption. A precondition for this case study was that the stored energy for the evening consumption for the SPOS was supposed to be excess energy from the solar energy production. In this case, it would be more profitable to not discharge the energy in hour 7 to make room for the excess energy from the solar power generation, but keep the energy in the battery and sell the excess energy directly to the grid.

The revenue for the solar panel owner for charging (buying) and discharging (selling) energy for the day-ahead price is not considered in this case. This is not a realistic case as the SPOs would not bother to charge/discharge only to decrease the hydro power operating costs without provision. For this case to be realistic, close communication between solar panel owners and the hydro power producers are required. However, it illustrates an opportunity for the solar panel owners to utilise the day-ahead prices to gain profit, and for the hydro power producers to minimise their production costs by utilising the batteries when the solar panel owners do not use them.

Table 5.13: Case 6: Total system operating cost, 25.06.2015

Battery situation	**Total system operating cost [€]**
Without solar or battery	2 597 023
With solar production and household batteries	2 491 301

The total system operating cost in this case is reduced by 105 722 €, and equals a reduction of 4,1 %. The reduction is due to the reduced hydro power operating cost as a consequence of battery charging and solar power free of charge. If the solar panel owners were given the benefit from buying and selling to day-ahead prices, and these costs and earnings was included in the objective function, the total system operating costs would have decreased even more. Batteries can support solar power to be integrated, as they have the ability to store free energy to be used on a later time period.

The power flow between the areas was changed when the solar power was introduced to the system. This is because the production in node 1 is larger than for the other cases, meaning that area NO1 is more self-sufficient than before. Though, a lot of energy is still transported to area NO1. The power flow between areas can be seen in figure 5.27.

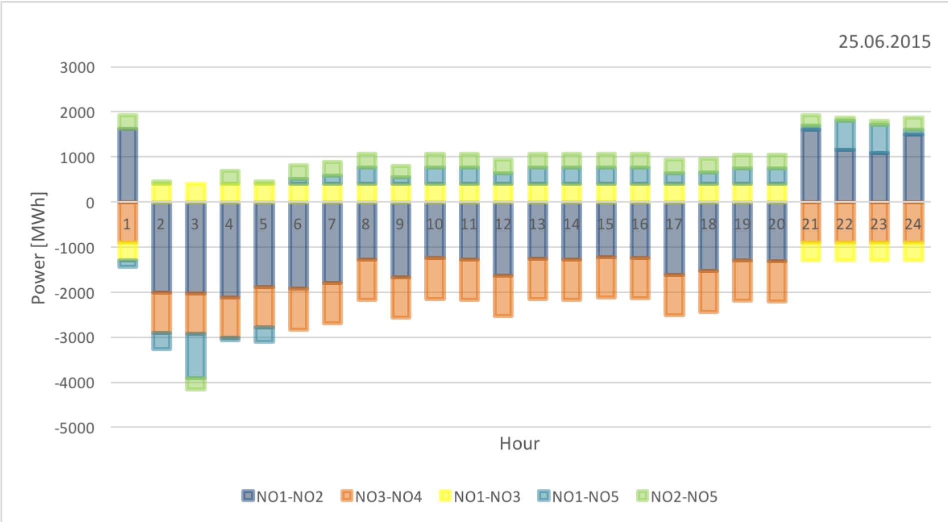

Figure 5.27: Case 6: Energy flow between areas, 25.06.2015.

5.7 Case 7: Oslo and Akershus solar power future scenario limited to solar power charging

In this case the charging of the household batteries is limited to only charge energy generated from the solar panels. This means that the batteries can not be used to be charged with energy from hydro power production bought from the grid. The charging price is set to zero. The discharging price is set to zero when discharging to cover the solar panel owners own consumption, but is set to the day-ahead price when selling to the grid. The energy sold directly to the grid is also considered, and the earnings follows the day-ahead price. The objective function for this case will be to minimise the operational costs for hydro power generation, and to maximise the income for the solar panel owners.

The model is manipulated to force the batteries to not charge before excess energy generated from the solar panels is present. This means that the batteries are first of all used to cover the consumption for the solar panel owners themselves. It is made sure that the SPOs have stored enough energy to cover their own assumed consumption for the afternoon and evening. The model is left to find the best charging time period for the energy for the evening consumption.

5.7.1 Thursday 25th of June 2015

The energy balance for node 1 is pictured in figure 5.28, and the specific energy balance situation for the solar panel owners is shown in figure 5.29. The solar power production and consumption is equal to the situation in case 6, pictured in figure 5.25, but the charging pattern in this case is forced to be changed.

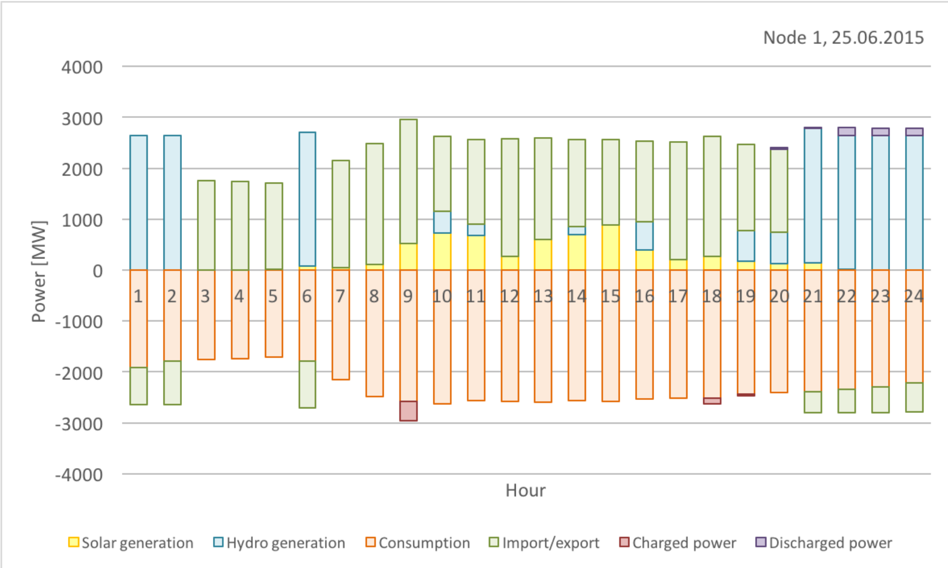

Figure 5.28: Case 7: Energy balance in node 1, 25.06.2015

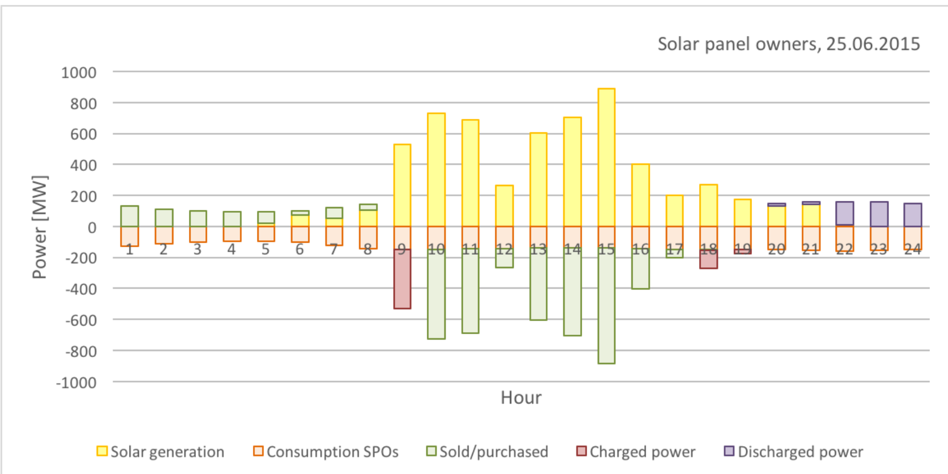

Figure 5.29: Case 7: Energy balance for solar panel owners, 25.06.2015

Figure 5.29 shows that the batteries are charged in hour 9, and in hour 18 and 19. The energy discharged is consumed by the solar panel owners themselves. This means that the

SPOs do not charge any of the energy to sell it to the grid later, but is sold right away. This is due to the day-ahead price pattern, shown in figure 5.30.

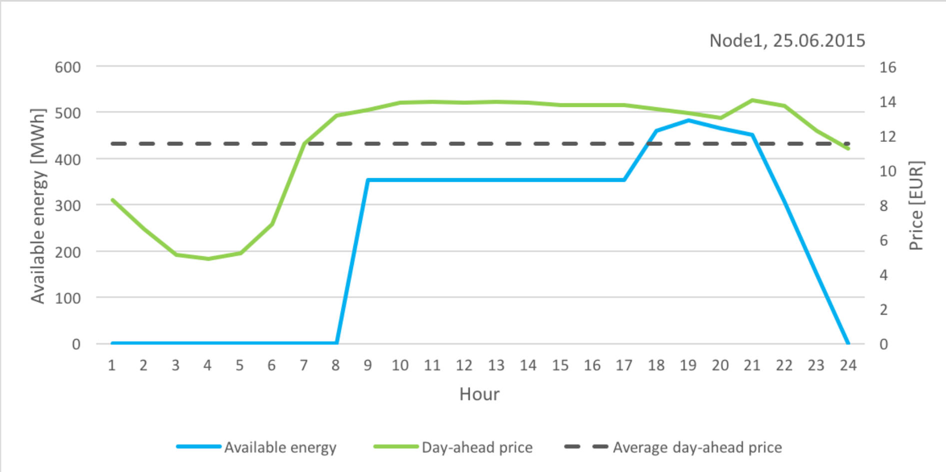

Figure 5.30: Case 7: Available energy compared with day-ahead prices, 25.06.2015

The day-ahead price is low in the start of the of the day, but is almost increased to the maximum at the point where the energy produced from the solar panels exceeds the SPOs consumption, and charging is allowed. In hour 9, the battery charges all the excess energy, and stores it. This is not enough to cover the evening load for the solar panel owners, so the battery needs to be filled with more energy. This is done in hour 18 and 19, as these are the hours with lowest day-ahead price, except from hour 9, in the time period with excess solar energy, and hence minimises the total hydro production cost. The discharge takes place in the evening to cover the load for the SPOs. The reason why there is no charging in the small price dip in hour 20 is because there is no excess energy from solar power production to be charged at this point of time.

Table 5.14: Case 7: Total system operating cost, 25.06.2015

Battery situation	**Total system operating cost [€]**
Without solar or battery	2 597 023
With solar production and household batteries	2 451 026

The total system operating cost is decreased, and reduced total system operating costs are not due to charging/discharging, because the batteries are only used to cover the solar panel owners own consumption. The reduction in costs are due to the solar power production leaving the hydro power plants to decrease the production level for parts of the day. The revenue of the solar panel owners for selling energy to the grid is included in this cost estimation. The energy is sold to the day-ahead price.

The total system operating costs is decreased with 45 996 €, a percentage of 1,8 %. The total revenue for the solar panel owners from the energy sold to the grid this day is 46 276 €. The number of solar panel owners is set to be 120 012. This gives a revenue of 0,4 € for this day. The reason why the batteries do not store more energy, is that it is more profitable to sell the energy to the grid directly. The restriction on charging energy only from the solar panels is not beneficial for the hydro power plants as production shifting is limited.

5.7.2 Thursday 11th of June 2015

On the 25.06.2015, the hydro power producers did not profit on the advantage of the household batteries owned by the SPOs, due to the pattern of the day-ahead price. To analyse a situation where the hydro power producers has an advantage of the batteries, the situation on 11.06.2015 is simulated.

The energy balance for node 1 can be seen in figure 5.31. The energy balance for the solar panel owners is pictured in figure 5.32.

In addition to discharge in the evening hours, discharge also takes place in hour 9 and 16. This is to cover the consumption of the solar panel owners, but it is discharged and sold to the grid. This is done during high-price hours, as this is the most profitable for the solar panel owners and the hydro power producers. Energy is charged in hour 8, 12, 13 and 14 with excess energy from the solar panels and lower day-ahead prices as seen in figure 5.33.

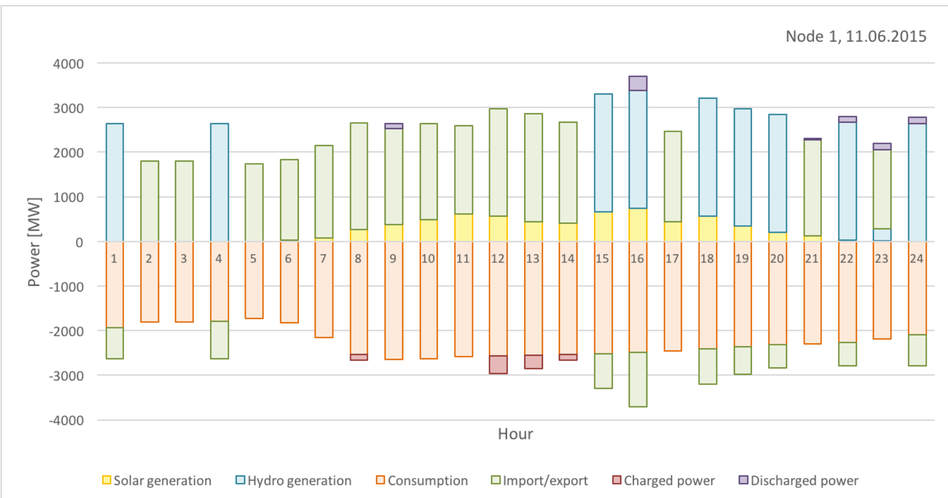

Figure 5.31: Case 7: Energy balance in node 1, 11.06.2015

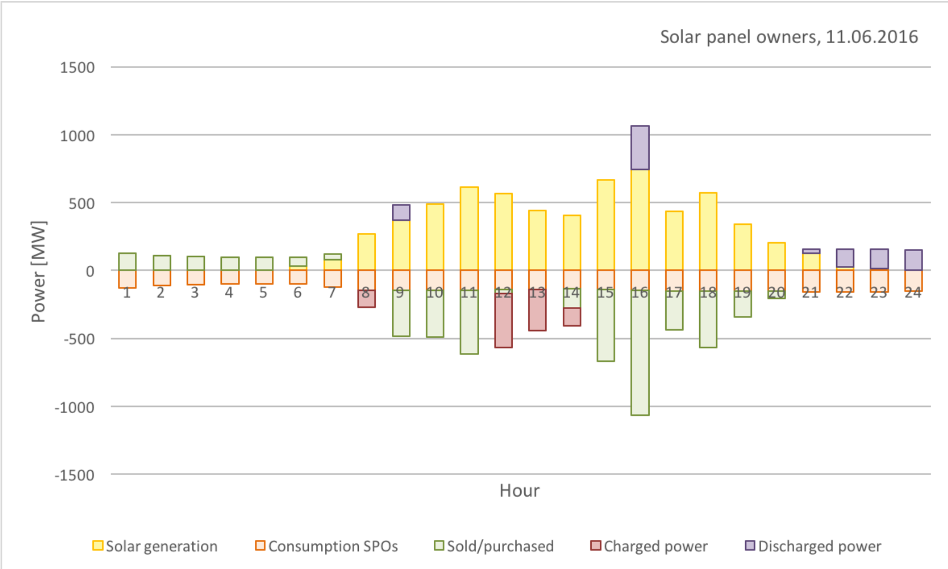

Figure 5.32: Case 7: Energy balance for solar panel owners, 11.06.2015

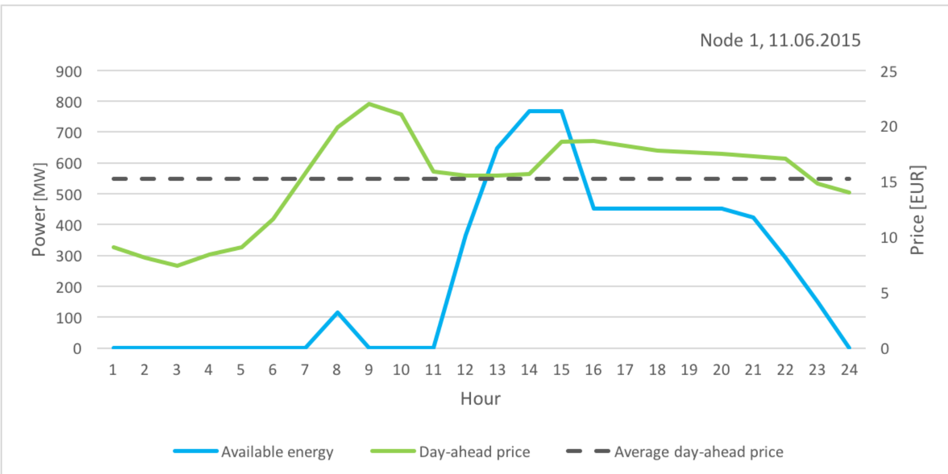

Figure 5.33: Case 7: Available energy compared with day-ahead prices, 11.06.2015

The total system operating costs can be seen in table 5.15.

Table 5.15: Case 7: Total system operating cost, 11.06.2015

Battery situation	Total system operating cost [€]
Without solar or battery	3 273 554
With solar production and household batteries	3 098 686

The total system operating costs is decreased with 174 864 €, a percentage of 4,3 %. The total revenue for the solar panel owners from the energy sold to the grid this day is 68 455 €. The number of solar panel owners is set to be 120 012. This gives a revenue of 0,6 € for this day. The reduction of total system operating cost for the 11.06.2015 is quite much larger than the reduction for the 25.06.2015. This underpins that solar power in combination with batteries can generate reduced hydro power operating costs, but the magnitude of the reduction is largely dependent on the day-ahead price. As the solar radiation generally is largest when the prices are highest and solar radiation in the morning is low, the batteries will not generate the same advantage for hydro power producers as if the batteries were not restricted to charge energy from solar panels.

5.8 Cost-benefit analysis

A cost-benefit analysis will be conducted based on the battery of 50MW/200MWh used in cases 1-5. The capital cost of battery storage varies a lot across reports. Initial capital cost for battery energy storage in a transmission system is according to [54] in the range of \$399 - \$1051/kWh. As the battery installed in most of the previous cases is a large battery of 50MW/200MWh, it is assumed that the price per kWh is in the lower range. The capital cost of the battery is assumed to be \$600/kWh = 540 €/kWh. Expected useful life for Lithium-ion batteries is 5-15 years [54].

$$\text{Investment cost} = 540\text{€/kWh} \cdot 200\text{MWh} = 108\text{mill €}. \tag{5.2}$$

The reduced total system operating costs for the different dates analysed in this report are listed in table 5.16.

Table 5.16: Reduced total system operating costs for the analysed dates.

Date	Cost reduction by inserting battery [€]
11.06.2015	1888
25.06.2015	1291
21.01.2016	47887
01.02.2016	1180
Average	12636.5

By using the average cost reduction as the expected reduced cost per day, and an expected life time of 15 years, the present value of the future cash flow will be:

$$PV = C\frac{(1+r)^n - 1}{r(1+r)^n} = 47.9\text{mill €} \tag{5.3}$$

Where
C = 12636.5 € *365 days = 4612322.5 €
r = 5 %
n = 15 years

The present value of the future cash flow is lower than the investment capital cost for the battery. From a financial point of view, the investment should not be performed, as the net present value will be negative, and the investor will lose money. The break down value for the investment is:

$$\frac{47.9\text{mill €}}{200MWh} = 239.5\text{€/kWh} \tag{5.4}$$

The reduced cost for 21.06.2015 might not be representative, because the prices variations on this day was particularly large. The net present value when disregarding this date will be even more negative. Calculating the present value with the average reduced cost without the 21.06.2016 gives the following:

$$PV = C\frac{(1+r)^n - 1}{r(1+r)^n} = 5.5\text{mill €} \tag{5.5}$$

Where
C = 1453 € *365 days = 530 345 €
r = 5 %
n = 15 years

In calculation of the present values, the value of the other services the battery can provide is not considered. The value of these services are hard to predict, but according to [3], energy storage service values can be up to $ 900/kW-year. Lithium-ion technology is expected to dominate the commercial market for energy storage in the next years because Lithium-ion system costs are expected to fall by 30 % to 50 % in the next 5 years [55]. The possibility of buying used EV batteries and use them as energy grid storage will also decrease the investment costs. Decreased battery prices in combination with a more sufficient market design for provision of ancillary services may make this battery investment profitable in the future.

The case studies throughout this report only analyse a few different dates. Hence, this cost-benefit analysis might not be trustworthy. In this analysis, maintenance costs are neglected.

6 | Discussion

This research work has resulted in an optimisation strategy for battery dispatch. Throughout this report it has been shown that the model gives reasonable results regarding power production, power flow and battery dispatch. The model is assumed to be sufficient for the scope of this work. However, the model has its deficiencies and a potential for improvement. Some of the items to improve are briefly discussed in this chapter.

The production level in the nodes varies abruptly in several of the case studies analysed in chapter 5. The optimisation model does not have restrictions for start-up, ramp-up and ramp-down times for the generation units. Nor does it deal with losses in the transmission lines. The lack of these hard restrictions is one of the reasons why the production level in a node is allowed to change fast. The optimisation program GAMS uses the first feasible optimisation solution. The hard restrictions mentioned should be included to implement a more realistic solution for the production and power flows. Hydro power plants have a relatively quick ramp possibility, and are able to start-up and shut-down in only a few minutes [56]. Even if a hydro power plant may have the ability to operate as the results from the case studies propose, it might not be a long-term solution. Traditionally, Norwegian hydro power plants have been built based on a stable production levels with a limited number of start-ups and shut-downs. By an increased number of start-ups and shut-downs, the wear on the technical components in the power plant will increase. [57]

The simple hydro power model used in the optimisation strategy is only restricted by its maximum limit of production. This means that the reservoirs are assumed to be full at all times. This is not a realistic assumption, and should be taken into consideration. Some hydro power plants have requirements for the minimum production level due to environmental issues for e.g. rivers and should also be considered.

The transmission capacities given by Nord Pool [39] are dependent on the direction of the power flow. This is not catered for in the optimisation model where all transmission lines provide the same capacity in both directions. To give more realistic power flows the directionality should be implemented. Exchange with Russia and Finland was neglected during the development of the model. There are no explicit reason for excluding them, and they should be included. It is however not a large operation to include them in the model.

It would be interesting to investigate the operation with several different batteries in the grid. The model is not developed to include two different batteries by now, due to limited time. It is possible to expand the model to be able to deal with different battery types at the same time.

One of the main reasons for inserting batteries in the grid is to be able to provide services and applications described in chapter 2. For a battery to be able to provide these services, it needs to contain energy. In the optimisation model developed throughout this research work, there are no requirements for the battery regarding the amount of energy stored in the battery at each time, except for the constraint forcing the battery to be empty at midnight. Another approach would be to set the available amount of energy at midnight to 50 % of the maximum, or require a minimum amount of energy to always be available. This would increase the opportunities of the battery to provide services when it is needed. Though, restrictions on the battery level limit the operation of the battery quite much. To further improve the battery's ability to provide services, a maximum available energy limit could be set lower than its actual maximum limit to be able to charge when there excess energy in the system.

The optimisation model suggests an optimal dispatch for batteries, but also for production units and transmission flows. This is to investigate the impact of the battery on the power system, and see how a battery can be a support in the power system. The model can be used for day-ahead clearing.

For the analyses on the cases studies performed with the developed optimisation model, day-ahead prices are used as the marginal cost of hydro power as they are assumed to reflect the water values. As they are not equal, the total system operating costs calculated in the cases will not directly reflect the real costs, but the percentage-wise increase/decrease will be representative. The charging costs/earnings for the analyses in chapter 5 are set to equal the average day-ahead price. Another approach for optimising the battery dispatch would be to study the revenue of the batteries by buying and selling energy to the grid at day-ahead price. The model is applicable for all production costs and charging/discharging costs, and can be used to study different scenarios. From a a short-term economical perspective, a reservior hydro power producer would not want to store energy in a battery when it is possible store it in the reservoir. In the future electricity market, larger payments for providing services as regulation are assumed to be present. Hence, producing energy in low-price hours to store in a battery for ancillary services may be beneficial in a short-term manner after all.

The cases analysed in chapter 5 use historical data as input for day-ahead prices, exchange and consumption. As the model is aimed to be used for time periods in the near future, forecasts are needed. As it is difficult to predict exchange and consumption in the future, the model may give an optimal dispatch that is not actually optimal in real time. This is not a limitation of the model, but a limitation of forecast methods, and a result of that the future is not perfectly predictable. If the model is used less than 12 hours prior to the period start, the day-ahead prices will already be known, an does not need to be predicted. This limits the battery owner to be a price-taker. The optimisation model can be used as a bidding strategy in the electricity market, but then predictions of day-ahead prices are needed as well. The future operation of batteries will probably be based on forecasts and participation by bidding in the electricity market.

In all cases analysed in chapter 5, inserting a battery in the grid gave reduced total system operating costs. This is because the production was shifted from periods with high day-ahead prices to periods with lower day-ahead prices, and reduces the hydro power production costs. This can be described as the opposite of load levelling. It is also concluded through analyses on case 2 that the a battery of a certain size can help the power system to survive congestion situations. This can also lead to transmission upgrade deferral. It is also concluded through the analysis of case 5 that in planned outage situations, a battery can function as backup power. It is however also known that batteries can serve this function during unplanned outages as well. As the optimisation model does not handle unplanned events, this can not be shown by the model.

Comparing the results from case 1 and case 3 confirms that the location of batteries is not indifferent. Batteries located in load centres will in general provide a larger economical benefit for the power system, as the area prices normally are highest where the consumption is largest.

Cases 6 and 7 shows that installed solar power in combination with house hold batteries may generate a large benefit by reducing the total system operating cost. Solar power in combination with batteries opens for solar power self-consumption and revenue by selling energy to the grid. The total system operating cost reduction is a result of a lower hydro power production level, revenue to the solar panel owners from selling energy to the grid and hydro production shifting by use of the solar panel owners batteries. Though, including solar power will in theory lead to decreased water values. This decrease was not considered in the case studies, and means that the revenues from the batteries calculated in case 7 were probably to high.

The direction of the power flow between the areas is more or less similar in all cases. This can be seen by studying figures A.1- A.8 in appendix F. However, changes can be seen in case 6 and 7, when solar power is installed in node 1. This is not surprising, as the production level in area NO1 is increased, and the share of consumption in the different parts of the country are more or less constant.

Increased load shifting will have a smoothing effect on the electricity price. This will reduce the economical benefit of the batteries [34], as batteries are concerned with the price difference across their charge and discharge cycles. This effect was not analysed in the case studies, as the case studies used historical data. Though, the charging cost and discharging earnings for the batteries are equal, and the revenue to the battery owners are not important in this case. Less volatile prices would though affect the total system operating costs when batteries are included.

The economical benefits form batteries in the grid will be reduced when the price volatility reduces. This result is also shown in the case studies. This can both be an advantage and a disadvantage, depending on the stakeholder. For a consumer, more stable electricity prices is desirable, but for a battery owner or in this case a hydro power plant owner, the situation might be opposite. Though, with a larger share of unregulated renewable energy sources, the variations in day-ahead prices are expected to increase. Hence, to predict the exact development of the electricity prices is impossible.

A cost-benefit analysis was conducted in the last part of chapter 5. The analysis showed that with today's battery prices it is not economically profitable to install a battery of 50MW/200MWh, when the optimisation objective is reduced total system operating costs based on day-ahead prices. However, the prices are predicted to decrease by 30-50 % the next few years. By including batteries in the electricity market, the profitability of the battery owners will probably increase. These factors may make battery storage profitable in the future. It is also worth mentioning that by calculating the revenue of a battery that will buy and sell electricity to the day-ahead prices, a different reduced price estimate would have been the result and may have given a different cost-benefit result. Expected increased payments for ancillary services in the future will increase the benefit from the batteries as well, and might also make battery investments profitable.

7 | Conclusion

The developed optimisation model for battery dispatch gives reasonable results regarding power production, power flows, and battery dispatch. However, improvements are necessary to make the model more precise.

In the results from the case studies using the developed optimisation strategy for optimal battery dispatch, it is be seen that the total system operating cost is decreased for all cases when including a battery. The performed cost-benefit analysis states that a battery investment of 50MW/200MWh is not profitable for the optimisation scenarios in this report. However, this may change when including battery charging income and with the expected reduction of battery prices in addition to increased provision for provided services.

The total system operating cost reduction increases with increased volatility in the day-ahead price over a day. The charging pattern is strictly dependent on the day-ahead price, and does not deviate from the optimal pattern unless limitations are given, or the network reaches its limit and the battery has to change the dispatch to help the system operate. It is also shown that batteries can help the system through transmission congestion and generation outages.

The combination of solar power and batteries is beneficial, as batteries increases the value of solar power. In addition, solar power reduces the total system operating costs due to the decreased hydro power production requirement. Revenue to the solar panel owners from selling energy to the grid and from production shifting by use of the solar panel owner's batteries gives decreased total system operating costs.

It is assumed that the established model can be used by hydro power producers to schedule their production and by battery owners to schedule their desired charging pattern both when participating in the electricity market as a price-taker, or without participating in the electricity market.

7.1 Suggestions for further work

For the optimisation model:

- Include hard restrictions in the hydro power production units such as ramp-up and -down times and losses on the transmission lines. Also include other production restrictions than the maximum production capacity.
- Implement directionality of the transmission capacity.
- Implement the ability to use different types of batteries simultaneously.
- Include line losses and directionality of transmission capacity.
- Investigate different levels of stored energy at midnight.

For the case studies:

- Perform case studies on a wind power production.
- Perform case studies on several batteries in the grid and different battery sizes.
- Perform case studies on micro grids including wind, solar and conventional production in addition to batteries.
- Perform case studies with different approaches regarding the price and cost functions for production and charging/discharging.

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Appendix

A The Norwegian power system model

Table A.1: PTDF matrix, $PTDF_{n,l}$, for the Norwegian power system model, node 1-11.

	1	2	3	4	5	6	7	8	9	10	11
1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
3	0,000	0,854	0,036	0,018	0,085	0,060	0,398	0,408	0,482	0,545	0,391
4	0,000	0,036	0,857	0,159	0,439	0,528	0,244	0,246	0,121	0,113	0,261
5	0,000	0,006	0,049	0,765	0,179	0,223	0,039	0,036	0,019	0,017	0,038
6	0,000	-0,006	-0,033	-0,045	-0,240	-0,148	-0,046	-0,042	-0,022	-0,020	-0,043
7	0,000	-0,098	-0,025	-0,012	-0,058	-0,041	-0,274	-0,268	-0,356	-0,305	-0,266
8	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
9	0,000	0,146	-0,036	-0,018	-0,085	-0,060	-0,398	-0,408	-0,482	-0,545	-0,391
10	0,000	-0,007	0,093	-0,162	-0,424	-0,536	-0,050	-0,041	-0,023	-0,021	-0,042
11	0,000	-0,030	0,050	0,002	-0,014	0,008	-0,194	-0,205	-0,098	-0,092	-0,219
12	0,000	-0,006	-0,049	0,235	-0,179	-0,223	-0,039	-0,036	-0,019	-0,017	-0,038
13	0,000	0,012	-0,045	-0,073	0,603	-0,241	0,089	0,077	0,042	0,038	0,080
14	0,000	-0,019	0,011	0,028	0,157	0,093	-0,134	-0,118	-0,063	-0,058	-0,123
15	0,000	-0,003	-0,001	0,002	0,013	0,007	0,147	-0,220	0,007	-0,008	-0,052
16	0,000	-0,023	0,034	0,018	0,086	0,060	0,443	0,291	-0,111	-0,072	0,289
17	0,000	0,007	-0,011	-0,003	-0,010	-0,009	0,137	-0,050	0,029	0,020	-0,133
18	0,000	0,012	-0,009	-0,016	-0,088	-0,053	0,104	0,053	0,041	0,036	0,037
19	0,000	-0,021	0,020	0,009	0,043	0,031	0,174	0,302	-0,027	-0,065	0,206
20	0,000	-0,001	0,002	0,001	0,005	0,004	0,026	0,080	-0,010	-0,003	-0,130
21	0,000	0,019	-0,023	-0,008	-0,035	-0,027	-0,053	0,398	0,044	0,060	-0,128
22	0,000	0,007	-0,014	-0,002	-0,001	-0,005	0,168	-0,074	0,034	0,023	-0,141
23	0,000	-0,126	0,016	0,009	0,042	0,029	0,224	0,105	0,509	-0,390	0,186
24	0,000	0,004	-0,007	-0,003	-0,015	-0,011	-0,054	-0,083	0,023	0,014	-0,163
25	0,000	0,002	-0,003	-0,001	-0,005	-0,004	-0,015	-0,005	0,009	0,007	0,443
26	0,000	-0,003	0,005	0,002	0,010	0,007	0,029	0,003	-0,014	-0,011	0,293
27	0,000	0,001	-0,002	-0,001	-0,005	-0,003	-0,013	0,002	0,005	0,004	0,265
28	0,000	0,002	-0,003	-0,001	-0,005	-0,004	-0,015	-0,005	0,009	0,007	0,443
29	0,000	0,001	-0,002	-0,001	-0,005	-0,004	-0,026	-0,080	0,010	0,003	0,130
30	0,000	0,001	-0,010	0,008	0,055	0,028	0,094	-0,113	0,011	0,002	0,045
31	0,000	0,007	-0,003	-0,012	-0,069	-0,041	0,030	0,065	0,022	0,022	0,087
32	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
33	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000

Table A.2: PTDF matrix, $PTDF_{n,l}$, for the Norwegian power system model, node 12-22.

	12	13	14	15	16	17	18	19	20	21	22
1	0,000	0,000	0,000	-1,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,000	0,000	0,000	0,000	-1,000	0,000	0,000	0,000	0,000	0,000	0,000
3	0,391	0,416	0,417	0,417	0,391	0,377	0,376	0,321	0,000	0,000	0,000
4	0,262	0,228	0,226	0,226	0,261	0,279	0,284	0,299	0,000	0,000	0,000
5	0,038	0,033	0,033	0,033	0,038	0,041	0,040	0,071	0,000	0,000	0,000
6	-0,044	-0,039	-0,038	-0,038	-0,043	-0,048	-0,046	-0,089	0,000	0,000	0,000
7	-0,266	-0,284	-0,286	-0,286	-0,266	-0,255	-0,254	-0,219	0,000	0,000	0,000
8	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-1,000	-1,000	-1,000
9	-0,391	-0,416	-0,417	-0,417	-0,391	-0,377	-0,376	-0,321	0,000	0,000	0,000
10	-0,042	-0,038	-0,038	-0,038	-0,042	-0,048	-0,043	-0,132	0,000	0,000	0,000
11	-0,220	-0,190	-0,188	-0,188	-0,219	-0,231	-0,241	-0,167	0,000	0,000	0,000
12	-0,038	-0,033	-0,033	-0,033	-0,038	-0,041	-0,040	-0,071	0,000	0,000	0,000
13	0,080	0,071	0,071	0,071	0,080	0,090	0,083	0,203	0,000	0,000	0,000
14	-0,124	-0,110	-0,109	-0,109	-0,123	-0,137	-0,129	-0,292	0,000	0,000	0,000
15	-0,051	-0,112	-0,099	-0,099	-0,052	-0,008	-0,034	0,063	0,000	0,000	0,000
16	0,291	0,214	0,205	0,205	0,289	0,346	0,335	0,330	0,000	0,000	0,000
17	-0,134	-0,067	-0,069	-0,069	-0,133	-0,222	-0,141	-0,023	0,000	0,000	0,000
18	0,037	0,045	0,044	0,044	0,037	0,024	0,038	-0,391	0,000	0,000	0,000
19	0,206	0,203	0,190	0,190	0,206	0,209	0,218	0,158	0,000	0,000	0,000
20	-0,124	-0,443	-0,384	-0,384	-0,130	0,019	0,024	0,019	0,000	0,000	0,000
21	-0,133	0,128	0,095	0,095	-0,128	-0,236	-0,275	-0,114	0,000	0,000	0,000
22	-0,143	-0,079	-0,080	-0,080	-0,141	-0,141	-0,198	0,021	0,000	0,000	0,000
23	0,185	0,213	0,226	0,226	0,186	0,167	0,158	0,163	0,000	0,000	0,000
24	-0,160	-0,283	-0,308	-0,308	-0,163	-0,076	-0,077	-0,053	0,000	0,000	0,000
25	-0,538	0,170	0,192	0,192	0,443	-0,019	-0,040	-0,014	0,000	0,000	0,000
26	0,284	-0,274	-0,308	-0,308	0,293	0,058	0,053	0,035	0,000	0,000	0,000
27	0,255	0,104	0,116	0,116	0,265	-0,039	-0,013	-0,020	0,000	0,000	0,000
28	0,462	0,170	0,192	0,192	0,443	-0,019	-0,040	-0,014	0,000	0,000	0,000
29	0,124	-0,557	0,384	0,384	0,130	-0,019	-0,024	-0,019	0,000	0,000	0,000
30	0,034	-0,029	-0,018	-0,018	0,045	0,627	-0,245	0,274	0,000	0,000	0,000
31	0,087	0,065	0,065	0,065	0,087	0,113	0,091	-0,317	0,000	0,000	0,000
32	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-1,000	-1,000
33	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-1,000

Table A.3: Connection matrix, $K_{l,n}$ for the Norwegian power system model.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
1	0	0	-1	-1	-1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	-1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
11	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0
14	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	-1	0	0	1	0	0	0	0
15	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	-1	0	0	1	1	0	0
18	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	-1	0	-1	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
20	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1

Table A.4: Correlation between node numbers and price areas for the Norwegian power system model.

Node	Price area
1	NO1
2	NO1
3	NO1
4	NO1
5	NO5
6	NO5
7	NO1
8	NO1
9	NO1
10	NO1
11	NO2
12	NO2
13	NO2
14	NO2
15	NO2
16	NO2
17	NO2
18	NO2
19	NO5
20	NO3
21	NO4
22	NO4

B Verification of Norwegian grid model

Table A.5: Simulated power flow between areas, 01.02.2016. Verification of the Norwegian power system model.

Time	Simulated power flow [MW]				
	NO1-NO2	NO3-NO4	NO1-NO3	NO1-NO5	NO2-NO5
1	-1351	-900	-233	-1815	300
2	-569	-900	-146	-1922	151
3	-1445	-900	-178	-2644	-300
4	-1319	-900	-229	-2624	-300
5	-1008	-900	287	55	300
6	-1711	-900	400	-1593	300
7	-2574	-900	-400	-181	300
8	-2727	-900	-400	-279	300
9	-2649	-900	-303	-289	300
10	-2732	-900	-400	-302	300
11	-2786	-900	-400	-326	300
12	-2031	-900	-400	-164	300
13	-2030	-900	-193	-1166	-300
14	-2030	-900	-59	-1170	-300
15	-1986	-900	-33	-1164	-300
16	-1834	-900	169	-2247	-300
17	-1616	-900	42	-2082	-300
18	-1823	-900	13	-1156	-300
19	-1833	-900	-41	-2693	-300
20	-1665	-900	-44	-2138	-300
21	-1227	-900	27	-1757	-300
22	-980	-900	140	-1555	-300
23	-1046	-900	180	-1654	-300
24	-1281	-900	129	-1903	-300

Table A.6: Actual power flow between areas obtained from Nord Pool [39], 01.022016. Verification of the Norwegian power system model.

Time	**Simulated power flow [MW]**				
	NO1-NO2	NO3-NO4	NO1-NO3	NO1-NO5	NO2-NO5
1	-1165	-546	100	-523	0
2	-1907	-156	100	-354	0
3	-1578	-308	100	-387	0
4	-1756	-338	100	-424	0
5	-1649	-396	100	-555	0
6	-1643	-660	50	-1397	0
7	-2060	-900	0	-2359	0
8	-1853	-900	-100	-3746	0
9	-1683	-900	-100	-3756	0
10	-1652	-900	-100	-3631	0
11	-1742	-900	-100	-2897	0
12	-1931	-900	-100	-2520	0
13	-2242	-900	-100	-2156	0
14	-2219	-900	-100	-2026	0
15	-2058	-900	-100	-1965	0
16	-1878	-900	-100	-1845	0
17	-2037	-900	-100	-1948	0
18	-2264	-900	-100	-2124	0
19	-2025	-900	-100	-1863	0
20	-2109	-900	-100	-1547	0
21	-2247	-900	-100	-1119	0
22	-2138	-845	0	-658	0
23	-1972	-675	50	-620	0
24	-1700	-446	100	-536	0

C GAMS code

```
Sets
         n node          / 1*22 /
         l line          / 1*33 /
         t hour index    / 1*24 /
         a area          / NO1, NO2, NO3, NO4, NO5 / ;

Alias(l,ll);

*=== Import from Excel: Network parameters =========================
Parameter Pmin(n) Minimum production capacity in node n [MW];
$call GDXXRW parameters.xlsx trace=3 par=Pmin rng=pmin!A1:B22 rdim=1 cdim=0
$GDXIN parameters.gdx
$LOAD Pmin
$GDXIN

Parameter Pmax(n) Maximum production capacity in node n [MW];
$call GDXXRW parameters.xlsx trace=3 par=Pmax rng=pmax!A1:B22 rdim=1 cdim=0
$GDXIN parameters.gdx
$LOAD Pmax
$GDXIN

Parameter Pcap(l) Transmission capacity in line l [MW];
$call GDXXRW parameters.xlsx trace=3 par=Pcap rng=transmission!A1:B33
rdim=1 cdim=0
$GDXIN parameters.gdx
$LOAD Pcap
$GDXIN

Parameter Nc(n) Indicator of battery connection (=0 if no battery in node n
and =1 if 1 batery in node n);
$call GDXXRW parameters.xlsx trace=3 par=Nc rng=batteryconnection!A1:B22
rdim=1 cdim=0
$GDXIN parameters.gdx
$LOAD Nc
$GDXIN
*==================================================================

*=== Import from Excel: Battery parameters =========================
Parameter eta Charging efficiency for battery;
$call GDXXRW battery.xlsx trace=3 par=eta rng=battery!B1:B1 rdim=0 cdim=0
$GDXIN battery.gdx
$LOAD eta
$GDXIN

Parameter Pcmin Minimum charging power for battery [MW];
$call GDXXRW battery.xlsx trace=3 par=Pcmin rng=battery!B2:B2 rdim=0 cdim=0
$GDXIN battery.gdx
$LOAD Pcmin
$GDXIN

Parameter Pcmax Maximum charging power for battery [MW];
$call GDXXRW battery.xlsx trace=3 par=Pcmax rng=battery!B3:B3 rdim=0 cdim=0
$GDXIN battery.gdx
$LOAD Pcmax
$GDXIN

Parameter Pdcmin Minimum discharging power for battery [MW];
$call GDXXRW battery.xlsx trace=3 par=Pdcmin rng=battery!B4:B4 rdim=0
cdim=0
$GDXIN battery.gdx
$LOAD Pdcmin
```

```
$GDXIN

Parameter Pdcmax Maximum discharging power for battery [MW];
$call GDXXRW battery.xlsx trace=3 par=Pdcmax rng=battery!B5:B5 rdim=0
cdim=0
$GDXIN battery.gdx
$LOAD Pdcmax
$GDXIN

Parameter Emin Minimum available energy for battery [MWh];
$call GDXXRW battery.xlsx trace=3 par=Emin rng=battery!B6:B6 rdim=0 cdim=0
$GDXIN battery.gdx
$LOAD Emin
$GDXIN

Parameter Emax Maximum available energy for battery [MWh];
$call GDXXRW battery.xlsx trace=3 par=Emax rng=battery!B7:B7 rdim=0 cdim=0
$GDXIN battery.gdx
$LOAD Emax
$GDXIN
*====================================================================

*=== Import from Excel: Day-ahead prices from NordPool =============
$onecho > nordpool.txt
I="%system.fp%nordpool.xlsx"
R=dayahead!A1:F25
O=dayahead.inc
$offecho
$call =xls2gms @nordpool.txt
table price(t,a)
$include dayahead.inc
;

Parameter pricet(a,t)  Day-ahead price transposed [EUR];
pricet(a,t) =  price(t,a);

Parameter lambda(n,t) Day-ahead price [EUR];
lambda('1',t) = pricet('NO1',t);
lambda('2',t) = pricet('NO1',t);
lambda('3',t) = pricet('NO1',t);
lambda('4',t) = pricet('NO1',t);
lambda('5',t) = pricet('NO5',t);
lambda('6',t) = pricet('NO5',t);
lambda('7',t) = pricet('NO1',t);
lambda('8',t) = pricet('NO1',t);
lambda('9',t) = pricet('NO1',t);
lambda('10',t) = pricet('NO1',t);
lambda('11',t) = pricet('NO2',t);
lambda('12',t) = pricet('NO2',t);
lambda('13',t) = pricet('NO2',t);
lambda('14',t) = pricet('NO2',t);
lambda('15',t) = pricet('NO2',t);
lambda('16',t) = pricet('NO2',t);
lambda('17',t) = pricet('NO2',t);
lambda('18',t) = pricet('NO2',t);
lambda('19',t) = pricet('NO5',t);
lambda('20',t) = pricet('NO3',t);
lambda('21',t) = pricet('NO4',t);
lambda('22',t) = pricet('NO4',t);

*=== Import from Excel: Average day-ahead prices from NordPool =====
```

```
Parameter CNO1 Charging cost & discharging earning for NO1 [EUR per MWh];
$call GDXXRW.EXE nordpool.xlsx trace=3 par=CNO1 rng=dayahead!B26:B26
rdim=0 cdim=0
$GDXIN nordpool.gdx
$LOAD CNO1
$GDXIN

Parameter CNO2 Charging cost & discharging earning for NO2 [EUR per MWh];
$call GDXXRW.EXE nordpool.xlsx trace=3 par=CNO2 rng=dayahead!C26:C26
rdim=0 cdim=0
$GDXIN nordpool.gdx
$LOAD CNO2
$GDXIN

Parameter CNO3 Charging cost & discharging earning for NO3  [EUR per MWh];
$call GDXXRW.EXE nordpool.xlsx trace=3 par=CNO3 rng=dayahead!E26:E26
rdim=0 cdim=0
$GDXIN nordpool.gdx
$LOAD CNO3
$GDXIN

Parameter CNO4 Charging cost & discharging earning for NO4 [EUR per MWh];
$call GDXXRW.EXE nordpool.xlsx trace=3 par=CNO4 rng=dayahead!F26:F26
rdim=0 cdim=0
$GDXIN nordpool.gdx
$LOAD CNO4
$GDXIN

Parameter CNO5 Charging cost & discharging earning for NO5 [EUR per MWh];
$call GDXXRW.EXE nordpool.xlsx trace=3 par=CNO5 rng=dayahead!D26:D26
rdim=0 cdim=0
$GDXIN nordpool.gdx
$LOAD CNO5
$GDXIN

Parameter C(n) Charging cost & discharging earning [EUR per MWh];
C('1') = CNO1;
C('2') = CNO1;
C('3') = CNO1;
C('4') = CNO1;
C('5') = CNO5;
C('6') = CNO5;
C('7') = CNO1;
C('8') = CNO1;
C('9') = CNO1;
C('10') = CNO1;
C('11') = CNO2;
C('12') = CNO2;
C('13') = CNO2;
C('14') = CNO2;
C('15') = CNO2;
C('16') = CNO2;
C('17') = CNO2;
C('18') = CNO2;
C('19') = CNO5;
C('20') = CNO3;
C('21') = CNO4;
C('22') = CNO4;
*=====================================================================

*=== Import from Excel: Consumption data from NordPool =============
```

```
$onecho > nordpool.txt
I="%system.fp%nordpool.xlsx"
R=demand!A1:F25
O=demand.inc
$offecho
$call =xls2gms @nordpool.txt
table demand(t,a)
$include demand.inc
;

Parameter demandt(a,t) Demand transposed [MW];
demandt(a,t) = demand(t,a);

Parameter  Pdem(n,t) Demand [MW];
Pdem('1',t) = demandt('NO1',t)*0.7608;
Pdem('2',t) = 0;
Pdem('3',t) = 0;
Pdem('4',t) = 0;
Pdem('5',t) = demandt('NO5',t)*0.0620;
Pdem('6',t) = 0;
Pdem('7',t) = demandt('NO1',t)*0.0171;
Pdem('8',t) = 0;
Pdem('9',t) = demandt('NO1',t)*0.2221;
Pdem('10',t) = 0;
Pdem('11',t) = demandt('NO2',t)*0.8706;
Pdem('12',t) = 0;
Pdem('13',t) = 0;
Pdem('14',t) = demandt('NO2',t)*0.1283;
Pdem('15',t) = 0;
Pdem('16',t) = 0;
Pdem('17',t) = demandt('NO2',t)*0.0011;
Pdem('18',t) = 0;
Pdem('19',t) = demandt('NO5',t)*0.9380;
Pdem('20',t) = demandt('NO3',t);
Pdem('21',t) = demandt('NO4',t);
Pdem('22',t) = 0;
*======================================================================

*=== Import from Excel: Exchange data between Norway and neighbouring
countries, from NordPool
$onecho > nordpool.txt
I="%system.fp%nordpool.xlsx"
R=exchange!A2:G26
O=exchange.inc
$offecho
$call =xls2gms @nordpool.txt
table exchange(t,n)
$include exchange.inc
;

Parameter  exchanget(n,t) Exchange transposed [MW] ;
exchanget(n,t) =  exchange(t,n);

Parameter Pexc(n,t) Exchange [MW];
Pexc('1',t) = 0;
Pexc('2',t) = exchanget('2',t);
Pexc('3',t) = 0;
Pexc('4',t) = 0;
Pexc('5',t) = 0;
Pexc('6',t) = 0;
Pexc('7',t) = 0;
```

```
Pexc('8',t) = 0;
Pexc('9',t) = 0;
Pexc('10',t) = 0;
Pexc('11',t) = 0;
Pexc('12',t) = 0;
Pexc('13',t) = 0;
Pexc('14',t) = 0;
Pexc('15',t) = exchanget('15',t);
Pexc('16',t) = exchanget('16',t);
Pexc('17',t) = 0;
Pexc('18',t) = 0;
Pexc('19',t) = 0;
Pexc('20',t) = exchanget('20',t);
Pexc('21',t) = exchanget('21',t);
Pexc('22',t) = exchanget('22',t);
*======================================================================


*=== Import from Excel: PTDF-matrix ===================================
$CALL GDXXRW.EXE Norway.xlsx par=PTDF rng=PTDF!A1:W34
Parameter PTDF(l,n);
$GDXIN Norway.gdx
$LOAD PTDF
$GDXIN
*======================================================================

*=== Import from Excel: Connection-matrix ============================
$CALL GDXXRW.EXE Norway.xlsx par=K rng=connection!A3:AH25
Parameter K(n,l);
$GDXIN Norway.gdx
$LOAD K
$GDXIN
*======================================================================

Variables
         z               Total costs
         cb              Total charging costs
         Pc(n,t)         Charging power
         Pdc(n,t)        Discharging power
         Ic(n,t)         Indicator of charging mode
         Idc(n,t)        Indicator of discharging mode
         E(n,t)          Available energy in battery
         Enet(n,t)       Net charged energy in battery
         Pgen(n,t)       Generation
         Pflow(l,t)      Flow in transmission line;

Positive variable P;

Binary variables  Ic, Idc;

*=== Fixing variable E (available energy) to 0 in the end of a period.
E.fx('1','24') = 0;
E.fx('2','24') = 0;
E.fx('3','24') = 0;
E.fx('4','24') = 0;
E.fx('5','24') = 0;
E.fx('6','24') = 0;
E.fx('7','24') = 0;
E.fx('8','24') = 0;
E.fx('9','24') = 0;
E.fx('10','24') = 0;
```

```
E.fx('11','24') = 0;
E.fx('12','24') = 0;
E.fx('13','24') = 0;
E.fx('14','24') = 0;
E.fx('15','24') = 0;
E.fx('16','24') = 0;
E.fx('17','24') = 0;
E.fx('18','24') = 0;
E.fx('19','24') = 0;
E.fx('20','24') = 0;
E.fx('21','24') = 0;
E.fx('22','24') = 0;

*=== Equation definitions ========================================
Equations
cost                     Total cost
energynet(n,t)           Net hourly absorbed or injected energy
in battery
chargpowlow(n,t)         Charging power lower boundary
chargpowup(n,t)          Charging power upper boundary
dischargpowlow(n,t)      Discharging power lower boundary
dischargpowup(n,t)       Discharging power upper boundary
energybal(n,t)           Energy balance
energylimlow(n,t)        Energy limitations of batteries lower  boundary
energylimup(n,t)         Energy limitations of batteries upper boundary
gridcon(n,t)             Charge or discharge mode
energyatm(n,t)           Ensures there to be enough energy to discharge a
battery

generatingpowlow(n,t)    Generated energy lower boundary
generatingpowup(n,t)     Generated energy upper boundary
flowtrans(l,t)           Power flow in transmission lines
plineconstrf(l,t)        Transmission constraint 1
plineconstrt(l,t)        Transmission constraint 2
powerbalance(n,t)        Power balance in node n;

*=== Objective function ==========================================
cost..                        z =e= sum((n,t),
lambda(n,t)*Pgen(n,t))+sum((n,t),C(n)*(Pc(n,t)-Pdc(n,t)));

*=== Battery contraints ==========================================
energynet(n,t)..          Enet(n,t) =e= eta*Pc(n,t) - Pdc(n,t);
chargpowlow(n,t) ..       Ic(n,t)*Pcmin =l= Pc(n,t);
chargpowup(n,t) ..        Pc(n,t) =l= Ic(n,t)*Pcmax;
dischargpowlow(n,t) ..    Idc(n,t)*Pdcmin =l= Pdc(n,t);
dischargpowup(n,t) ..     Pdc(n,t) =l= Idc(n,t)*Pdcmax;
energybal(n,t) ..         E(n,t) =e= E(n,t--1) + Enet(n,t);
energylimlow(n,t) ..      Emin =l= E(n,t);
energylimup(n,t) ..       E(n,t) =l= Emax;
gridcon(n,t) ..           Idc(n,t) + Ic(n,t) =l= Nc(n);
energyatm(n,t) ..         Pdc(n,t) =l= E(n,t--1);

*=== Network constraints =========================================
generatingpowlow(n,t) .. Pmin(n) =l= Pgen(n,t);
generatingpowup(n,t) ..  Pgen(n,t) =l= Pmax(n);
flowtrans(l,t)..         Pflow(l,t) =e= sum(n, PTDF(l,n)*(Pgen(n,t)-
Pdem(n,t)-Pexc(n,t)+ (Pdc(n,t) - Pc(n,t))*Nc(n)));
plineconstrf(l,t) ..     Pflow(l,t) =l= Pcap(l);
plineconstrt(l,t) ..     -Pflow(l,t) =l= Pcap(l);
powerbalance(n,t) ..     Pgen(n,t) - Pdem(n,t)-Pexc(n,t) - sum(ll,
K(n,ll)*Pflow(ll,t))+ (Pdc(n,t) - Pc(n,t))*Nc(n) =e= 0;
```

```
Model gen /all/ ;

Solve gen using mip minimizing z ;

Display  Pgen.l, Pflow.l ,Pmax, Pmin, ll,PTDF, Pdem, Pexc, lambda, E.l, C,
eta, Emax;

*=== Export to Excel: Results ============================================
execute_unload 'res.gdx', Pgen, Pflow, E, z, Pc, Pdc, Ic, Idc, Enet, Pdem,
Pexc;
execute 'gdxxrw.exe res.gdx Squeeze=N var=Pgen.l     rng=powergen!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N var=Pflow.l   rng=powerflow!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N var=E.l      rng=availableenergy!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N var=Enet.l   rng=netchargedenergy!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N var=z.l      rng=optimalsolution!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N var=Pc.l     rng=chargedpower!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N var=Pdc.l    rng=dischargedpower!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N var=Ic.l     rng=chargeindication!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N var=Idc.l
rng=dischargeindication!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N par=Pdem     rng=demand!A1'
execute 'gdxxrw.exe res.gdx Squeeze=N par=Pexc     rng=exchange!A1'
```

D Results Case 1

Table A.7: Power flow with battery in node 1 (hour 1-12), 21.01.2016

	1	2	3	4	5	6	7	8	9	10	11	12
1	-845	-1047	-1091	-1142	-1099	-1088	-1173	-925	-508	-600	-748	-826
2	-227	-102	79	265	388	108	91	1	0	0	0	0
3	1599	2564	1719	1617	1542	2169	2063	1495	1128	1214	1256	1431
4	431	1060	418	325	247	1581	1558	1368	1241	1284	1328	1363
5	38	144	37	21	8	471	469	452	440	444	448	451
6	-37	-163	-35	-17	-1	-615	-613	-595	-584	-588	-592	-595
7	-261	-950	-275	-186	-109	-651	-788	-494	-300	-360	-418	-481
8	-400	-28	-400	-400	-400	400	-54	-327	-181	-277	-307	-318
9	29	-985	52	194	321	-516	-797	-430	-184	-272	-374	-430
10	4	-142	5	26	44	-1000	-1000	-1000	-1000	-1000	-1000	-1000
11	-435	-918	-423	-351	-292	-581	-558	-368	-241	-284	-328	-363
12	-38	-144	-37	-21	-8	-471	-469	-452	-440	-444	-448	-451
13	35	286	32	-5	-36	1471	1469	1452	1440	1444	1448	1451
14	-254	-625	-243	-187	-142	-267	-246	-72	43	4	-36	-67
15	17	330	-10	-28	-19	56	54	90	128	118	103	96
16	953	2008	894	762	679	1203	975	771	630	682	739	768
17	-388	-105	-384	-378	-350	-451	-408	-344	-304	-320	-337	-345
18	165	307	167	170	184	-584	-563	-531	-511	-519	-528	-532
19	480	920	463	395	338	626	529	378	266	305	350	373
20	-19	-33	-91	-87	-46	-29	-16	147	320	273	210	175
21	-445	-557	-382	-336	-311	-540	-459	-435	-458	-461	-456	-452
22	-225	133	-225	-225	-198	-225	-182	-122	-82	-98	-115	-122
23	-509	65	-514	-589	-659	-110	268	52	-81	-33	24	57
24	-242	-426	-277	-248	-200	-299	-236	-78	71	27	-29	-59
25	-124	-146	-174	-252	-339	-297	-416	-629	-744	-695	-649	-621
26	-40	-53	-190	-271	-309	-197	-312	-321	-215	-225	-259	-276
27	-128	-152	-157	-202	-254	-250	-320	-446	-514	-485	-458	-441
28	-124	-146	-174	-252	-339	-297	-416	-629	-744	-695	-649	-621
29	19	33	91	87	46	29	16	-147	-320	-273	-210	-175
30	1229	1488	1203	1164	1140	1643	1615	1553	1525	1538	1548	1557
31	300	300	300	300	300	-300	-300	-300	-300	-300	-300	-300
32	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900
33	-713	-743	-736	-694	-707	-637	-530	-466	-335	-344	-419	-414

Table A.8: Power flow with battery in node 1 (hour 13-24), 21.01.2016

	13	14	15	16	17	18	19	20	21	22	23	24
1	-1031	-1032	-1177	-1377	-987	-877	-1123	-1053	-1050	-704	-576	-595
2	0	0	0	0	-7	-294	-13	-317	-276	-621	-537	-483
3	1673	2665	2742	2610	1585	1554	1617	1749	1972	2148	2320	2123
4	1451	1581	1515	1587	1452	1482	1474	1551	1584	1674	1677	1698
5	459	356	322	344	460	462	462	469	472	480	480	482
6	-603	-448	-401	-430	-603	-606	-605	-612	-615	-623	-623	-625
7	-611	-1253	-1280	-1309	-602	-619	-632	-723	-782	-886	-890	-799
8	-400	400	400	400	-252	-284	-340	-389	-400	46	400	400
9	-598	-1504	-1527	-1639	-615	-666	-664	-799	-828	-971	-910	-830
10	-1000	-646	-561	-607	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000
11	-451	-935	-954	-980	-452	-482	-474	-551	-584	-674	-677	-698
12	-459	-356	-322	-344	-460	-462	-462	-469	-472	-480	-480	-482
13	1459	1002	883	951	1460	1462	1462	1469	1472	1480	1480	1482
14	-148	-724	-774	-774	-149	-177	-169	-239	-270	-354	-357	-376
15	73	401	377	349	74	76	64	58	54	90	97	116
16	872	1871	1862	1899	884	942	911	1018	1047	1206	1211	1405
17	-377	-69	-84	-113	-382	-407	-389	-430	-440	-459	-460	-464
18	-548	-301	-289	-305	-550	-563	-554	-574	-579	-589	-589	-591
19	451	846	856	899	458	490	481	549	568	640	634	718
20	74	119	61	-20	82	98	35	18	3	73	103	85
21	-452	-564	-540	-530	-465	-513	-453	-510	-517	-622	-640	-686
22	-153	225	206	174	-159	-183	-166	-205	-214	-225	-225	-225
23	147	657	671	740	157	176	183	250	260	331	276	112
24	-151	-283	-321	-387	-149	-153	-187	-225	-242	-230	-211	-264
25	-525	-527	-499	-452	-509	-430	-494	-360	-323	-180	-167	-123
26	-307	-230	-279	-335	-280	-174	-326	-206	-188	58	112	142
27	-384	-395	-378	-349	-375	-328	-366	-287	-265	-182	-174	-148
28	-525	-527	-499	-452	-509	-430	-494	-360	-323	-180	-167	-123
29	-74	-119	-61	20	-82	-98	-35	-18	-3	-73	-103	-85
30	1582	1801	1788	1788	1586	1608	1588	1626	1638	1702	1709	1732
31	-300	-222	-206	-207	-300	-300	-300	-300	-300	-300	-300	-300
32	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900
33	-449	-447	-462	-412	-431	-499	-507	-470	-508	-607	-597	-658

Table A.9: Power generation with battery in node 1 (hour 1-12), 21.01.2016

	1	2	3	4	5	6	7	8	9	10	11	12
1	2200	0	1996	2324	2636	0	0	1273	2255	1926	1740	1367
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	1997	2024	2176	2271	2238	2206	2180
6	0	0	0	0	0	0	0	0	0	0	0	0
7	633	2783	551	409	406	113	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	523	1465	1465	1465	1465	1465	1465
10	0	0	0	0	0	0	0	0	0	0	0	0
11	4174	4174	4174	4174	4174	4174	4174	4174	4174	4174	4174	4174
12	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0
14	147	147	147	147	147	147	147	147	147	147	147	147
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	2050	2050	2050	2050	2050	2050	2050	2050	2050	2050	2050	2050
18	0	0	0	0	0	0	0	0	0	0	0	0
19	2542	2680	2432	2366	2331	3843	3958	3943	3856	3876	3912	3949
20	2329	1863	2207	2197	2204	1526	2183	2783	2783	2783	2783	2783
21	2715	2593	2581	2650	2650	2783	3009	3233	3514	3544	3300	3337
22	0	0	0	0	0	0	0	0	0	0	0	0

Table A.10: Power generation with battery in node 1 (hour 13-24), 21.01.2016

	13	14	15	16	17	18	19	20	21	22	23	24
1	744	0	0	0	953	988	807	451	38	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	2110	925	707	803	2114	2092	2098	2039	2012	1942	1937	1916
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	2259	2202	2135	0	0	0	0	0	152	159	361
8	0	0	0	0	0	0	0	0	0	0	0	0
9	1465	1465	1465	1465	1465	1465	1465	1465	1465	1465	1377	798
10	0	0	0	0	0	0	0	0	0	0	0	0
11	4174	4174	4174	4174	4174	4174	4174	4174	4174	4174	4174	4174
12	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0
14	147	147	147	147	147	147	147	147	147	147	147	147
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	2050	2050	2050	2050	2050	2050	2050	2050	2050	2050	2050	2050
18	0	0	0	0	0	0	0	0	0	0	0	0
19	3959	4247	4247	4247	4044	4085	4056	4108	4105	4151	4115	4061
20	2743	1911	1948	2003	2783	2783	2783	2783	2769	2257	1812	1718
21	3267	3296	3334	3316	3263	3094	3131	3141	3077	2914	2854	2760
22	0	0	0	0	0	0	0	0	0	0	0	0

Table A.11: Power flow without battery connected (hour 1-12), 21.01.2016

	1	2	3	4	5	6	7	8	9	10	11	12
1	-845	-1047	-1091	-1142	-1099	-1088	-1173	-925	-508	-600	-748	-826
2	-227	-102	79	265	388	108	91	1	0	0	0	0
3	1599	1662	1719	1617	1523	2169	2063	1495	1128	1214	1256	1431
4	431	487	418	325	235	1581	1558	1368	1241	1284	1328	1363
5	38	48	37	21	6	471	469	452	440	444	448	451
6	-37	-49	-35	-17	1	-615	-613	-595	-584	-588	-592	-595
7	-261	-329	-275	-186	-96	-651	-788	-494	-300	-360	-418	-481
8	-400	-400	-400	-400	-400	400	-54	-327	-181	-277	-307	-318
9	29	-83	52	194	340	-516	-797	-430	-184	-272	-374	-430
10	4	-11	5	26	47	-1000	-1000	-1000	-1000	-1000	-1000	-1000
11	-435	-476	-423	-351	-282	-581	-558	-368	-241	-284	-328	-363
12	-38	-48	-37	-21	-6	-471	-469	-452	-440	-444	-448	-451
13	35	59	32	-5	-41	1471	1469	1452	1440	1444	1448	1451
14	-254	-284	-243	-187	-135	-267	-246	-72	43	4	-36	-67
15	17	8	-10	-28	-26	56	54	90	128	118	103	96
16	953	1008	894	762	658	1203	975	771	630	682	739	768
17	-388	-389	-384	-378	-356	-451	-408	-344	-304	-320	-337	-345
18	165	165	167	170	181	-584	-563	-531	-511	-519	-528	-532
19	480	522	463	395	330	626	529	378	266	305	350	373
20	-19	-91	-91	-87	-47	-29	-16	147	320	273	210	175
21	-445	-423	-382	-336	-308	-540	-459	-435	-458	-461	-456	-452
22	-225	-225	-225	-225	-206	-225	-182	-122	-82	-98	-115	-122
23	-509	-439	-514	-589	-670	-110	268	52	-81	-33	24	57
24	-242	-301	-277	-248	-197	-299	-236	-78	71	27	-29	-59
25	-124	-111	-174	-252	-339	-297	-416	-629	-744	-695	-649	-621
26	-40	-121	-190	-271	-311	-197	-312	-321	-215	-225	-259	-276
27	-128	-120	-157	-202	-253	-250	-320	-446	-514	-485	-458	-441
28	-124	-111	-174	-252	-339	-297	-416	-629	-744	-695	-649	-621
29	19	91	91	87	47	29	16	-147	-320	-273	-210	-175
30	1229	1235	1203	1164	1135	1643	1615	1553	1525	1538	1548	1557
31	300	300	300	300	300	-300	-300	-300	-300	-300	-300	-300
32	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900
33	-713	-743	-736	-694	-707	-637	-530	-466	-335	-344	-419	-414

Table A.12: Power flow without battery connected (hour 13-24), 21.01.2016

	13	14	15	16	17	18	19	20	21	22	23	24
1	-1031	-1032	-1177	-1377	-987	-877	-1123	-1053	-1050	-704	-576	-595
2	0	0	0	0	-7	-294	-13	-317	-276	-621	-537	-483
3	1673	2660	2730	2598	1585	1554	1617	1749	1972	2148	2320	2123
4	1451	1559	1498	1571	1452	1482	1474	1551	1584	1674	1677	1698
5	459	347	317	339	460	462	462	469	472	480	480	482
6	-603	-436	-395	-423	-603	-606	-605	-612	-615	-623	-623	-625
7	-611	-1250	-1272	-1300	-602	-619	-632	-723	-782	-886	-890	-799
8	-400	400	400	400	-252	-284	-340	-389	-400	46	400	400
9	-598	-1499	-1515	-1627	-615	-666	-664	-799	-828	-971	-910	-830
10	-1000	-625	-549	-596	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000
11	-451	-934	-949	-974	-452	-482	-474	-551	-584	-674	-677	-698
12	-459	-347	-317	-339	-460	-462	-462	-469	-472	-480	-480	-482
13	1459	972	866	935	1460	1462	1462	1469	1472	1480	1480	1482
14	-148	-731	-774	-774	-149	-177	-169	-239	-270	-354	-357	-376
15	73	400	373	345	74	76	64	58	54	90	97	116
16	872	1867	1848	1885	884	942	911	1018	1047	1206	1211	1405
17	-377	-69	-88	-117	-382	-407	-389	-430	-440	-459	-460	-464
18	-548	-296	-290	-305	-550	-563	-554	-574	-579	-589	-589	-591
19	451	844	851	894	458	490	481	549	568	640	634	718
20	74	118	61	-21	82	98	35	18	3	73	103	85
21	-452	-562	-538	-528	-465	-513	-453	-510	-517	-622	-640	-686
22	-153	225	202	169	-159	-183	-166	-205	-214	-225	-225	-225
23	147	655	664	733	157	176	183	250	260	331	276	112
24	-151	-282	-319	-385	-149	-153	-187	-225	-242	-230	-211	-264
25	-525	-527	-499	-451	-509	-430	-494	-360	-323	-180	-167	-123
26	-307	-231	-280	-336	-280	-174	-326	-206	-188	58	112	142
27	-384	-395	-377	-349	-375	-328	-366	-287	-265	-182	-174	-148
28	-525	-527	-499	-451	-509	-430	-494	-360	-323	-180	-167	-123
29	-74	-118	-61	21	-82	-98	-35	-18	-3	-73	-103	-85
30	1582	1798	1784	1784	1586	1608	1588	1626	1638	1702	1709	1732
31	-300	-219	-206	-206	-300	-300	-300	-300	-300	-300	-300	-300
32	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900
33	-449	-447	-462	-412	-431	-499	-507	-470	-508	-607	-597	-658

E Results Case 4

Table A.13: Power flow with battery in node 21 (hour 1-12), 01.02.2016

	1	2	3	4	5	6	7	8	9	10	11	12
1	332	803	797	537	-21	-314	-923	-1556	-1619	-1586	-1389	-760
2	23	553	126	469	666	105	-491	-723	-728	-728	-730	-730
3	281	18	-627	-668	-63	864	1404	2148	2158	2007	1740	1494
4	1151	1583	1408	1326	748	1692	1686	1146	1230	1230	1276	1680
5	181	472	455	448	95	442	469	158	172	172	180	386
6	-213	-615	-599	-592	-106	-567	-607	-180	-196	-196	-206	-487
7	-798	-459	-148	-62	-499	-794	-693	-981	-1064	-1052	-1075	-921
8	-245	-146	-178	-229	287	400	400	-400	-303	-400	-400	19
9	-1591	-999	-634	-473	-1113	-1346	-931	-1222	-1390	-1426	-1582	-1370
10	-231	-1000	-1000	-1000	-81	-878	-964	-163	-181	-181	-192	-711
11	-921	-583	-408	-326	-667	-815	-721	-983	-1049	-1049	-1084	-969
12	-181	-472	-455	-448	-95	-442	-469	-158	-172	-172	-180	-386
13	412	1472	1455	1448	176	1320	1434	321	353	353	372	1097
14	-774	16	-114	-38	-429	-159	-26	-674	-726	-725	-752	-774
15	482	509	204	166	419	377	91	203	275	259	283	288
16	2260	1836	1291	1126	1792	2047	1619	2047	2238	2229	2331	2213
17	-135	-34	-460	-451	-44	-131	-412	-273	-199	-224	-216	-352
18	-53	164	-589	-585	337	294	153	222	260	247	251	-535
19	1008	732	596	521	768	939	850	1028	1098	1105	1155	1077
20	375	602	506	452	312	156	-136	-236	-225	-233	-226	38
21	-900	-825	-898	-807	-661	-718	-624	-588	-599	-613	-646	-828
22	146	225	-225	-225	195	107	-225	-54	36	7	21	-74
23	583	267	38	-48	345	407	81	195	292	321	427	293
24	-203	45	40	38	-143	-310	-457	-592	-613	-620	-635	-446
25	-92	-381	-175	-290	-315	-77	185	137	83	125	155	-24
26	574	559	655	458	230	279	152	-25	-60	-26	18	210
27	-131	-297	-179	-246	-252	-111	52	18	-16	10	26	-96
28	-92	-381	-175	-290	-315	-77	185	137	83	125	155	-24
29	-375	-602	-506	-452	-312	-156	136	236	225	233	226	-38
30	1767	1564	1706	1648	1449	1503	1385	1489	1529	1529	1554	1895
31	12	151	-300	-300	300	300	300	300	300	300	300	-300
32	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900
33	-85	178	141	75	-116	-406	-535	-497	-421	-452	-471	-463

Table A.14: Power flow with battery in node 21 (hour 13-24), 01.02.2016

	13	14	15	16	17	18	19	20	21	22	23	24
1	-67	18	90	348	609	407	471	731	1166	1524	1584	1569
2	-729	-727	-729	-729	-728	-727	-728	-728	-728	-727	-728	-728
3	880	743	699	1287	1248	777	1010	358	497	-638	114	218
4	977	948	1057	1843	1725	947	1651	949	1488	1407	1411	1586
5	174	166	199	436	401	166	478	166	330	455	307	359
6	-210	-199	-242	-553	-506	-199	-621	-200	-413	-599	-383	-452
7	-509	-480	-531	-959	-888	-470	-781	-443	-719	-138	-661	-763
8	400	-59	400	169	42	13	-41	-44	27	140	180	129
9	-778	-774	-896	-1531	-1403	-741	-1208	-852	-1358	-622	-1391	-1555
10	-259	-240	-311	-818	-743	-240	-1000	-242	-591	-1000	-542	-655
11	-718	-708	-746	-1024	-982	-707	-651	-707	-897	-407	-869	-931
12	-174	-166	-199	-436	-401	-166	-478	-166	-330	-455	-307	-359
13	433	406	509	1254	1143	406	1478	409	921	1455	849	1014
14	-774	-774	-774	-774	-774	-774	-336	-774	-774	-95	-774	-774
15	242	239	284	544	564	290	182	342	608	530	638	629
16	1731	1719	1837	2571	2512	1757	1270	1821	2424	1666	2396	2481
17	-399	-411	-376	-161	-147	-383	-470	-357	-127	15	-125	-154
18	-559	-565	-547	-440	-433	-551	-594	-537	-423	-352	-422	-436
19	812	807	851	1129	1078	796	638	808	1005	606	976	1036
20	268	286	316	442	534	410	389	512	719	799	841	800
21	-838	-854	-884	-1027	-1049	-915	-845	-978	-1116	-875	-1179	-1207
22	-141	-154	-110	159	175	-119	-225	-85	196	225	201	173
23	-34	-33	45	402	324	-55	570	44	353	16	415	518
24	-201	-188	-187	-223	-147	-110	-31	-50	-3	211	85	37
25	-106	-87	-94	-127	-159	-89	-48	-86	-196	-188	-178	-86
26	416	460	497	651	739	623	620	764	945	995	1123	1168
27	-141	-130	-135	-164	-183	-132	-105	-132	-205	-167	-195	-140
28	-106	-87	-94	-127	-159	-89	-48	-86	-196	-188	-178	-86
29	-268	-286	-316	-442	-534	-410	-389	-512	-719	-799	-841	-800
30	1803	1803	1833	2019	2015	1829	1769	1856	2012	1245	2025	2051
31	-300	-300	-300	-300	-300	-300	-300	-300	-300	-300	-300	-300
32	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900	-900
33	-394	-416	-357	-281	-263	-377	-326	-271	-193	63	126	287

F Flow between areas

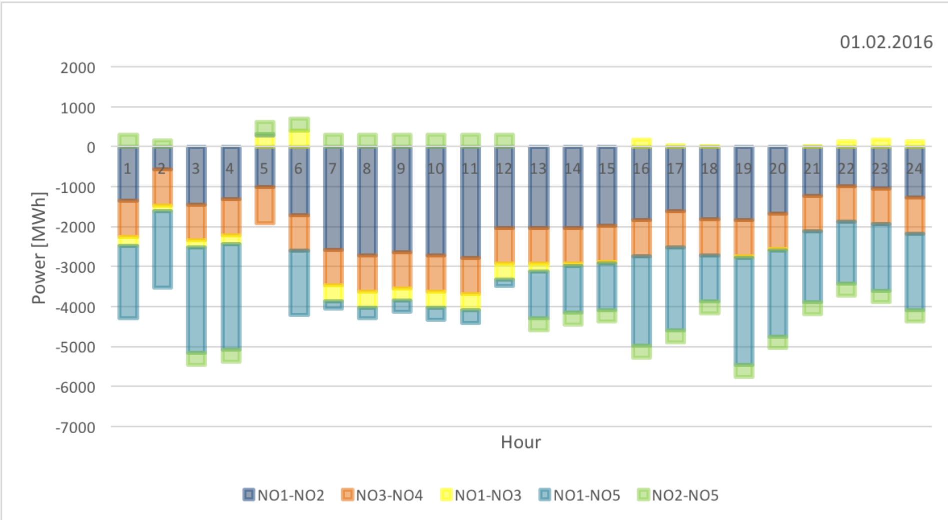


Figure A.1: Case 1: Power flow between areas, with battery connected to node 1, 01.02.2016.

Figure A.2: Case 2: Power flow between areas, congestion line 4 to 785 MW, with battery in node 1, 01.02.2016.

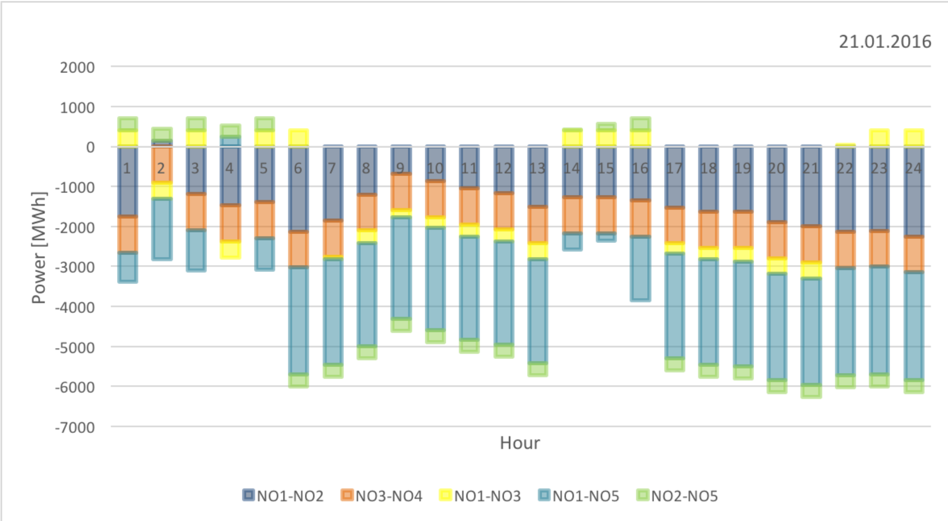


Figure A.3: Case 3: Power flow between areas, with battery in node 5, 21.01.2016.

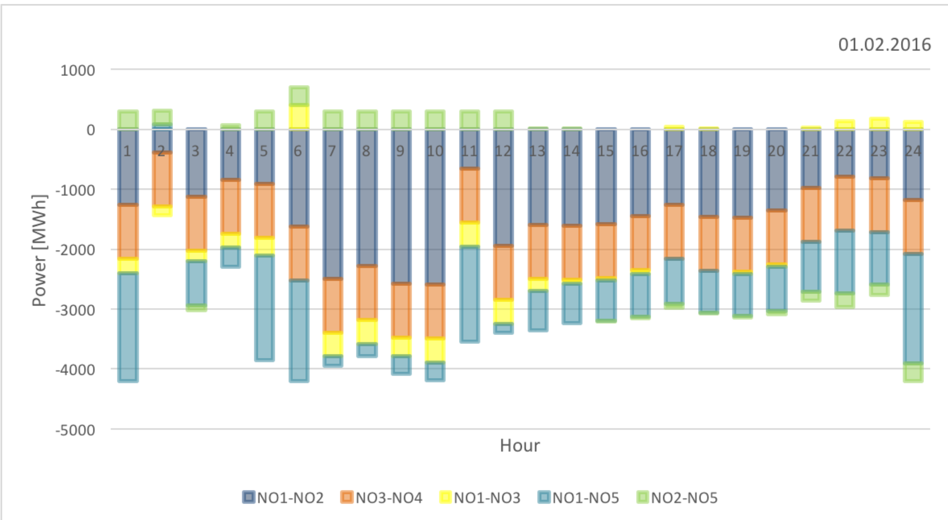


Figure A.4: Case 3: Power flow between areas, with battery connected to node 5, 01.02.2016.

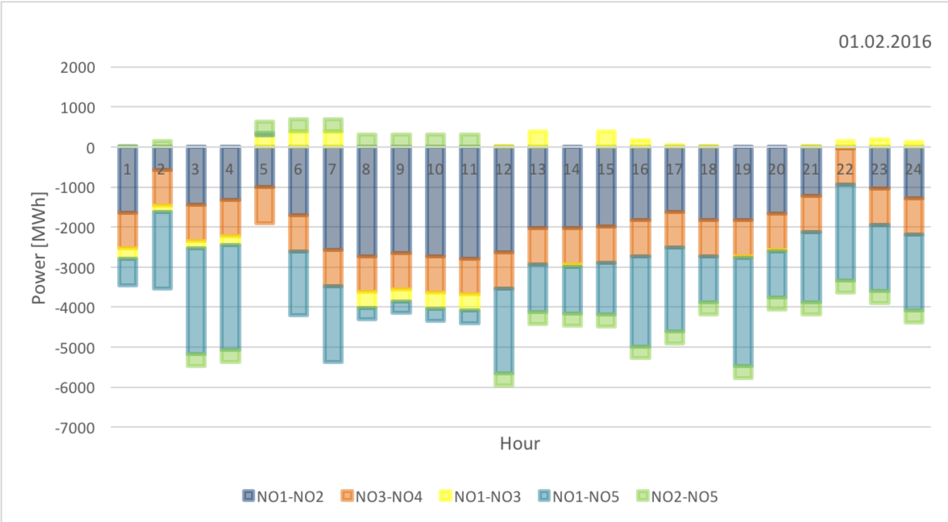


Figure A.5: Case 4: Power flow between areas, with battery connected to node 21, 01.02.2016.

Figure A.6: Case 5: Power flow between areas, with battery connected to node 21, during generation outage, 01.02.2016.

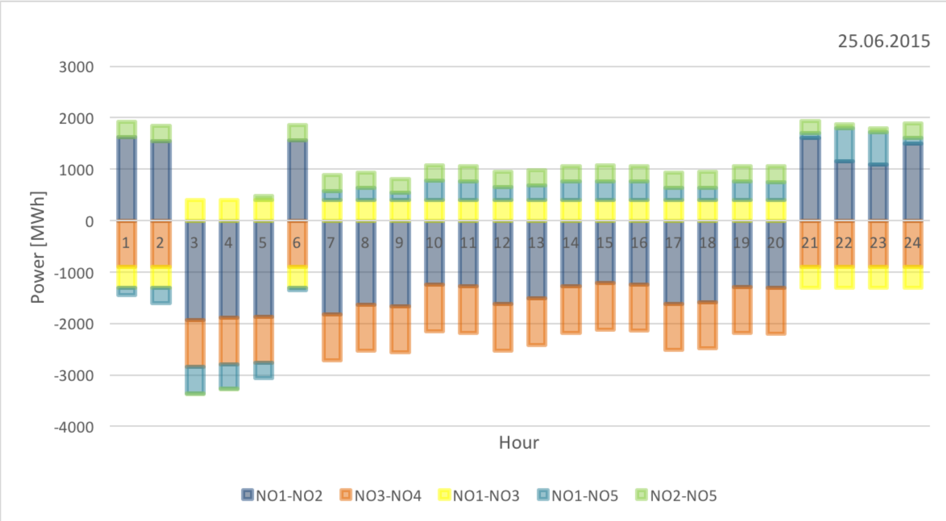


Figure A.7: Case 7: Power flow between areas, solar power scenario with solar charge, 25.06.2015.

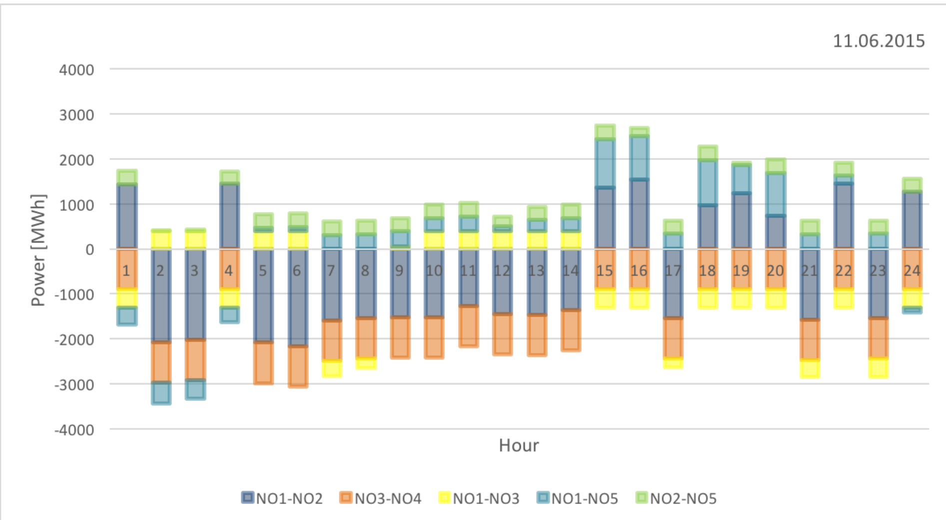


Figure A.8: Case 7: Power flow between areas, solar power scenario, 11.06.2015.

G Implementation in GAMS

The optimisation problem formulated in section 4.1 is implemented in the modelling system GAMS, General Algebric Modeling System. GAMS is a high-level modelling system for mathematical programming and optimisation, and is the program used to perform the optimisations.

The writer of this report was not familiar with GAMS before the start of this work. Time was spent to read about GAMS' features and learn how to use the program. GAMS was used actively during the work with the formulation of the optimisation problem. The formulation of the optimisation problem has been developed by testing and analysing results from GAMS, and to adjust the formulation when needed. In the beginning, a large part of the work dealt with debugging in GAMS. Later when the model was up running, time was spent to tweak the model to fit the scope of the work. Verification of the model was done to make sure that it was suitable for this work. This is further discussed in section 4.1.

The input data is organised in excel files that are read into GAMS. The four excel files needed to run the optimisation problem in GAMS are listed in table A.15.

Table A.15: Excel input files to GAMS

File name	**Sheet**	**Parameter**	**Description**
Norway.xlsx	PTDF	$PTDF_{n,l}$	PTDF-matrix
	connection	$K_{n,l}$	Connection matrix
parameters.xlsx	pmin	$P_n^{gen,min}$	Minimum production
	pmax	$P_n^{gen,max}$	Maximum production
	transmission	P_l^{cap}	Transmission capacity
	batteryconnection	M_n	Battery connection
nordpool.xlsx	dayahead	n, t	Day-ahead price
	demand	$P_{n,t}^{dem}$	Demand
	exchange	$P_{n,t}^{exc}$	Exchange with neighbouring countries
battery.xlsx	battery	eta	eta
		$P^{c,min}$	Minimum charging power
		$P^{c,max}$	Maximum charging power
		$P^{dc,min}$	Minimum discharging power
		$P^{dc,max}$	Maximum discharging power
		E^{max}	Minimum energy capacity
		E^{max}	Maximum energy capacity

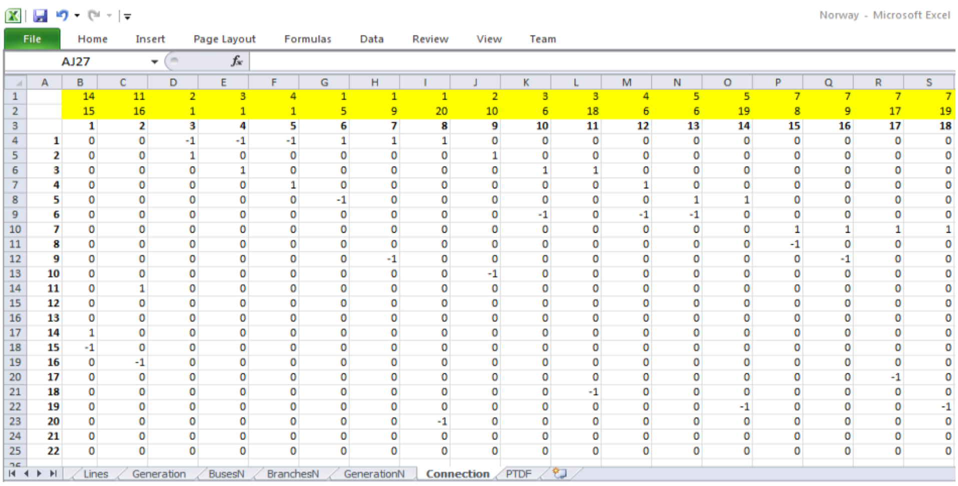
For GAMS to be able to read the excel files, they have to be organised in a certain way. The organization of the excel sheets can be seen appendix G. The excel files and the GAMS project needs to be located in the same folder for GAMS to be able to find the files.

The results from the simulations in GAMS are written to an excel file containing the variables in the model, listed in table A.16.

Table A.16: Excel output file from GAMS.

File name	Sheet	Parameter	Description
res.xlsx	powergen	$P_{n,t}^{gen}$	Generation level
	powerflow	$P_{l,t}^{flow}$	Power flow
	availableenergy	$E_{n,t}$	Available energy
	netchargedenergy	$E_{n,t}^{net}$	Net charged energy
	optimalsolution	Objective function	Total system operating cost
	chargedpower	$P_{n,t}^{c}$	Charged power
	dischargedpower	$P_{n,t}^{dc}$	Discharged power
	chargeindication	$I_{n,t}^{c}$	Indicator of charging mode
	dischargeindication	$I_{n,t}^{dc}$	Indicator of discharging mode

The excel file Norway.xlsx contains information about the connection matrix and the PTDF-matrix. The connection matrix describing the connection between the nodes and lines is organized as shown in figure A.9 and A.10. The rows describe the nodes, while the columns describe the lines. The yellow cells contains information about which nodes a given line is connected to.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1		14	11	2	3	4	1	1	1	2	3	3	4	5	5	7	7	7	7
2		15	16	1	1	1	5	9	20	10	6	18	6	6	19	8	9	17	19
3		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
4	1	0	0	-1	-1	-1	1	1	1	0	0	0	0	0	0	0	0	0	0
5	2	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
6	3	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0
7	4	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
8	5	0	0	0	0	0	-1	0	0	0	0	0	0	1	1	0	0	0	0
9	6	0	0	0	0	0	0	0	0	0	-1	0	-1	-1	0	0	0	0	0
10	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
11	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0
12	9	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	-1	0	0
13	10	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
14	11	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	14	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	15	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	16	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0
21	18	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0
22	19	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-1
23	20	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
24	21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure A.9: Norway.xlsx: Connection matrix line 1-18, $K_{n,l}$.

Formula Bar	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH
8	8	8	7	9	9	11	11	11	12	14	17	17	20	21
10	13	18	18	10	14	12	14	17	18	13	18	19	21	22
19	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
-1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0
0	0	0	0	0	-1	0	-1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-1	0	0	1	1	0	0
0	0	-1	-1	0	0	0	0	0	-1	0	-1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1

Figure A.10: Norway.xlsx: Connection matrix line 19-33, $K_{n,l}$.

The PTDF-matrix is organized with the lines in the rows and nodes in the columns, and can be seen in figures A.11 and A.12.

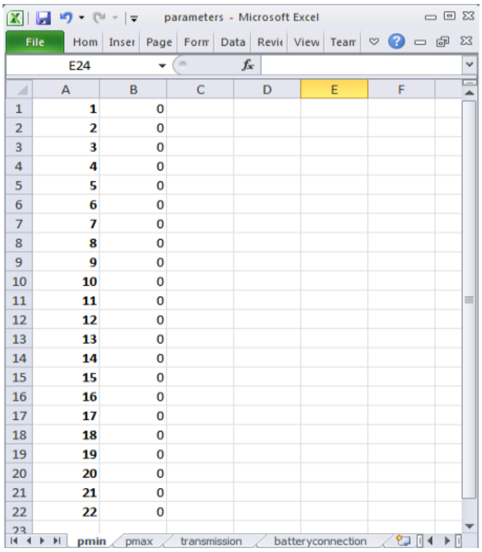
	1	2	3	4	5	6	7	8	9	10	11	12
1	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
2	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
3	0,0000	0,8535	0,0364	0,0182	0,0850	0,0604	0,3980	0,4079	0,4822	0,5447	0,3914	0,3908
4	0,0000	0,0364	0,8570	0,1593	0,4385	0,5276	0,2436	0,2460	0,1211	0,1131	0,2612	0,2620
5	0,0000	0,0056	0,0486	0,7654	0,1786	0,2231	0,0387	0,0360	0,0186	0,0173	0,0378	0,0379
6	0,0000	-0,0065	-0,0334	-0,0447	-0,2398	-0,1479	-0,0455	-0,0415	-0,0218	-0,0202	-0,0435	-0,0436
7	0,0000	-0,0980	-0,0246	-0,0124	-0,0580	-0,0411	-0,2742	-0,2685	-0,3563	-0,3047	-0,2661	-0,2657
8	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
9	0,0000	0,1465	-0,0364	-0,0182	-0,0850	-0,0604	-0,3980	-0,4079	-0,4822	-0,5447	-0,3914	-0,3908
10	0,0000	-0,0068	0,0931	-0,1617	-0,4243	-0,5356	-0,0499	-0,0410	-0,0229	-0,0211	-0,0422	-0,0422
11	0,0000	-0,0296	0,0499	0,0024	-0,0142	0,0080	-0,1937	-0,2051	-0,0982	-0,0921	-0,2191	-0,2199
12	0,0000	-0,0056	-0,0486	0,2346	-0,1786	-0,2231	-0,0387	-0,0360	-0,0186	-0,0173	-0,0378	-0,0379
13	0,0000	0,0123	-0,0445	-0,0729	0,6029	-0,2414	0,0886	0,0770	0,0415	0,0383	0,0799	0,0801
14	0,0000	-0,0188	0,0111	0,0282	0,1573	0,0935	-0,1341	-0,1185	-0,0633	-0,0585	-0,1234	-0,1236
15	0,0000	-0,0025	-0,0006	0,0022	0,0132	0,0074	0,1471	-0,2201	0,0071	-0,0078	-0,0518	-0,0512
16	0,0000	-0,0231	0,0336	0,0181	0,0857	0,0598	0,4435	0,2911	-0,1110	-0,0719	0,2888	0,2905
17	0,0000	0,0065	-0,0108	-0,0027	-0,0096	-0,0090	0,1371	-0,0499	0,0289	0,0203	-0,1333	-0,1336
18	0,0000	0,0117	-0,0085	-0,0160	-0,0879	-0,0530	0,1043	0,0532	0,0411	0,0365	0,0369	0,0369
19	0,0000	-0,0209	0,0203	0,0094	0,0428	0,0310	0,1743	0,3024	-0,0272	-0,0648	0,2057	0,2062
20	0,0000	-0,0010	0,0023	0,0011	0,0050	0,0036	0,0257	0,0799	-0,0098	-0,0030	-0,1296	-0,1240
21	0,0000	0,0193	-0,0231	-0,0082	-0,0346	-0,0272	-0,0529	0,3976	0,0441	0,0601	-0,1279	-0,1333
22	0,0000	0,0074	-0,0137	-0,0016	-0,0015	-0,0053	0,1681	-0,0744	0,0340	0,0229	-0,1406	-0,1427
23	0,0000	-0,1256	0,0161	0,0089	0,0423	0,0294	0,2237	0,1055	0,5093	-0,3904	0,1857	0,1847
24	0,0000	0,0044	-0,0071	-0,0032	-0,0146	-0,0106	-0,0545	-0,0829	0,0234	0,0138	-0,1630	-0,1598
25	0,0000	0,0022	-0,0033	-0,0011	-0,0045	-0,0036	-0,0154	-0,0049	0,0088	0,0068	0,4427	-0,5384
26	0,0000	-0,0035	0,0049	0,0021	0,0096	0,0071	0,0287	0,0030	-0,0136	-0,0108	0,2926	0,2839
27	0,0000	0,0013	-0,0016	-0,0010	-0,0051	-0,0034	-0,0133	0,0019	0,0048	0,0040	0,2647	0,2545
28	0,0000	0,0022	-0,0033	-0,0011	-0,0045	-0,0036	-0,0154	-0,0049	0,0088	0,0068	0,4427	0,4616
29	0,0000	0,0010	-0,0023	-0,0011	-0,0050	-0,0036	-0,0257	-0,0799	0,0098	0,0030	0,1296	0,1240
30	0,0000	0,0007	-0,0098	0,0085	0,0547	0,0281	0,0939	-0,1132	0,0114	0,0023	0,0449	0,0342
31	0,0000	0,0071	-0,0026	-0,0122	-0,0694	-0,0405	0,0298	0,0653	0,0222	0,0220	0,0865	0,0867
32	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
33	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Figure A.11: Norway.xlsx: PTDF-matrix node 1-12, $PTDF_{n,l}$.

N	O	P	Q	R	S	T	U	V	W
13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
0,0000	0,0000	-1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
0,0000	0,0000	0,0000	-1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
0,4156	0,4166	0,4166	0,3914	0,3766	0,3762	0,3210	0,0000	0,0000	0,0000
0,2282	0,2260	0,2260	0,2612	0,2791	0,2840	0,2994	0,0000	0,0000	0,0000
0,0334	0,0331	0,0331	0,0378	0,0414	0,0401	0,0707	0,0000	0,0000	0,0000
-0,0385	-0,0382	-0,0382	-0,0435	-0,0479	-0,0459	-0,0895	0,0000	0,0000	0,0000
-0,2842	-0,2862	-0,2862	-0,2661	-0,2550	-0,2538	-0,2194	0,0000	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,0000	-1,0000	-1,0000
-0,4156	-0,4166	-0,4166	-0,3914	-0,3766	-0,3762	-0,3210	0,0000	0,0000	0,0000
-0,0380	-0,0376	-0,0376	-0,0422	-0,0482	-0,0428	-0,1323	0,0000	0,0000	0,0000
-0,1902	-0,1884	-0,1884	-0,2191	-0,2310	-0,2411	-0,1671	0,0000	0,0000	0,0000
-0,0334	-0,0331	-0,0331	-0,0378	-0,0414	-0,0401	-0,0707	0,0000	0,0000	0,0000
0,0714	0,0707	0,0707	0,0799	0,0895	0,0830	0,2029	0,0000	0,0000	0,0000
-0,1099	-0,1089	-0,1089	-0,1234	-0,1375	-0,1288	-0,2924	0,0000	0,0000	0,0000
-0,1122	-0,0989	-0,0989	-0,0518	-0,0077	-0,0335	0,0626	0,0000	0,0000	0,0000
0,2140	0,2046	0,2046	0,2888	0,3460	0,3348	0,3296	0,0000	0,0000	0,0000
-0,0671	-0,0692	-0,0692	-0,1333	-0,2216	-0,1412	-0,0225	0,0000	0,0000	0,0000
0,0446	0,0436	0,0436	0,0369	0,0245	0,0381	-0,3906	0,0000	0,0000	0,0000
0,2027	0,1904	0,1904	0,2057	0,2093	0,2181	0,1575	0,0000	0,0000	0,0000
-0,4428	-0,3843	-0,3843	-0,1296	0,0186	0,0239	0,0186	0,0000	0,0000	0,0000
0,1280	0,0949	0,0949	-0,1279	-0,2356	-0,2755	-0,1135	0,0000	0,0000	0,0000
-0,0794	-0,0800	-0,0800	-0,1406	-0,1411	-0,1981	0,0209	0,0000	0,0000	0,0000
0,2129	0,2261	0,2261	0,1857	0,1673	0,1582	0,1635	0,0000	0,0000	0,0000
-0,2831	-0,3077	-0,3077	-0,1630	-0,0763	-0,0772	-0,0533	0,0000	0,0000	0,0000
0,1703	0,1918	0,1918	0,4427	-0,0191	-0,0399	-0,0143	0,0000	0,0000	0,0000
-0,2741	-0,3081	-0,3081	0,2926	0,0577	0,0534	0,0347	0,0000	0,0000	0,0000
0,1038	0,1163	0,1163	0,2647	-0,0386	-0,0135	-0,0204	0,0000	0,0000	0,0000
0,1703	0,1918	0,1918	0,4427	-0,0191	-0,0399	-0,0143	0,0000	0,0000	0,0000
-0,5572	0,3843	0,3843	0,1296	-0,0186	-0,0239	-0,0186	0,0000	0,0000	0,0000
-0,0286	-0,0183	-0,0183	0,0449	0,6268	-0,2454	0,2740	0,0000	0,0000	0,0000
0,0653	0,0653	0,0653	0,0865	0,1130	0,0908	-0,3169	0,0000	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,0000	-1,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,0000

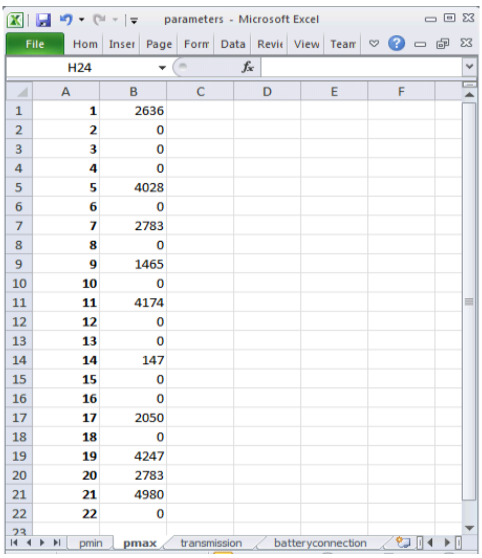
Figure A.12: Norway.xlsx: PTDF-matrix node 13-22, $PTDF_{n,l}$.

The parameters for minimum and maximum generation capacity, transmission capacity and status of battery connection is given in the excel file parameters.xlsx.



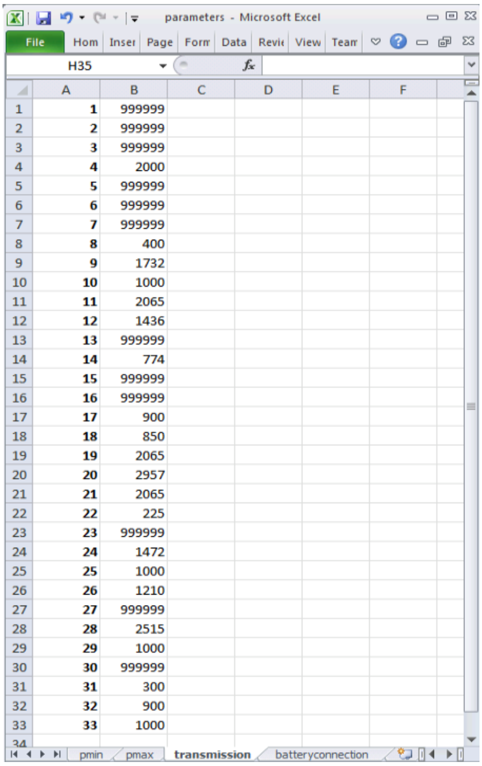
	A	B
1	1	0
2	2	0
3	3	0
4	4	0
5	5	0
6	6	0
7	7	0
8	8	0
9	9	0
10	10	0
11	11	0
12	12	0
13	13	0
14	14	0
15	15	0
16	16	0
17	17	0
18	18	0
19	19	0
20	20	0
21	21	0
22	22	0

Figure A.13: parameters.xlsx: Minimum production, $P_n^{gen,min}$.



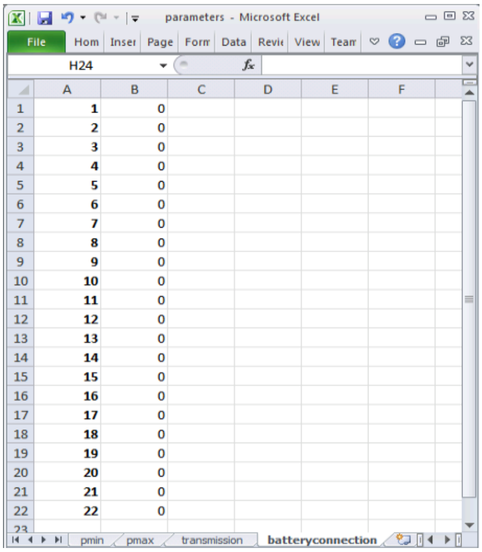
	A	B
1	1	2636
2	2	0
3	3	0
4	4	0
5	5	4028
6	6	0
7	7	2783
8	8	0
9	9	1465
10	10	0
11	11	4174
12	12	0
13	13	0
14	14	147
15	15	0
16	16	0
17	17	2050
18	18	0
19	19	4247
20	20	2783
21	21	4980
22	22	0

Figure A.14: parameters.xlsx: Maximum production, $P_n^{gen,max}$.



	A	B
1	1	999999
2	2	999999
3	3	999999
4	4	2000
5	5	999999
6	6	999999
7	7	999999
8	8	400
9	9	1732
10	10	1000
11	11	2065
12	12	1436
13	13	999999
14	14	774
15	15	999999
16	16	999999
17	17	900
18	18	850
19	19	2065
20	20	2957
21	21	2065
22	22	225
23	23	999999
24	24	1472
25	25	1000
26	26	1210
27	27	999999
28	28	2515
29	29	1000
30	30	999999
31	31	300
32	32	900
33	33	1000

Figure A.15: parameters.xlsx: Transmission capacity, P_l^{cap}.



	A	B
1	1	0
2	2	0
3	3	0
4	4	0
5	5	0
6	6	0
7	7	0
8	8	0
9	9	0
10	10	0
11	11	0
12	12	0
13	13	0
14	14	0
15	15	0
16	16	0
17	17	0
18	18	0
19	19	0
20	20	0
21	21	0
22	22	0

Figure A.16: parameters.xlsx: Battery connection, M_n.

Information from Nord Pool is gathered in the excel file nordpool.xlsx. The data includes day-ahead prices, demand and exchange with neighbouring countries. Be aware that the order of the areas (NO1-NO5) is not necessarily equal for the different information types in Nord Pool.

	NO1	NO2	NO3	NO4	NO5
1	29,65	29,65	29,65	29,65	29,65
2	27,82	27,82	27,82	27,82	27,82
3	26,65	26,65	26,65	26,65	26,65
4	26,96	26,96	26,96	26,96	26,96
5	27,47	27,47	27,47	27,47	27,47
6	29,81	29,09	29,81	29,81	29,09
7	41,78	40,78	41,78	41,78	40,78
8	199,96	70,01	199,96	199,96	70,01
9	214,25	85,05	214,25	214,25	85,05
10	200	83,54	200	200	83,54
11	130,06	74,94	130,06	130,06	74,94
12	74,97	70,12	74,97	74,97	70,12
13	61,39	59,91	61,39	61,39	59,91
14	57,69	57,69	59,11	59,11	57,69
15	57,99	57,99	60,01	60,01	57,99
16	61,05	61,05	98,6	98,6	61,05
17	199,97	57,43	199,97	199,97	57,43
18	200,09	77,13	200,09	200,09	77,13
19	199,87	86,01	199,87	199,87	86,01
20	61,61	58,21	61,61	61,61	58,21
21	50,09	47,04	50,09	50,09	47,04
22	36,34	35,04	36,34	36,34	35,04
23	30,36	28,6	30,36	30,36	28,6
24	27,64	26,56	27,64	27,64	26,56
AVG	86,39458	51,86417	88,1025	88,1025	51,86417

Figure A.17: nordpool.xlsx: Day-ahead prices, $\lambda_{n,t}$.

	NO1	NO2	NO3	NO4	NO5
1	6499	5390	3061	2694	2935
2	6387	5315	2979	2620	2838
3	6350	5301	2954	2608	2832
4	6363	5322	2943	2627	2824
5	6434	5385	2937	2636	2849
6	6685	5526	3051	2694	2869
7	7289	5894	3218	2806	3037
8	7957	6396	3454	2949	3241
9	8123	6486	3513	3071	3292
10	8074	6408	3451	3112	3263
11	8068	6363	3455	2966	3250
12	7895	6331	3434	2987	3252
13	7787	6192	3399	2970	3159
14	7693	6118	3378	2992	3199
15	7638	6122	3397	3025	3174
16	7663	6099	3437	2965	3157
17	7829	6139	3511	2941	3245
18	7946	6202	3503	2856	3247
19	7869	6173	3457	2904	3234
20	7812	6139	3429	2866	3192
21	7707	6003	3401	2848	3151
22	7578	5858	3359	2789	3101
23	7349	5674	3285	2725	3059
24	7002	5497	3156	2684	2979

Figure A.18: nordpool.xlsx: Demand, $P^{dem}_{n,t}$.

	A	B	C	D	E	F	G
1		NO2-DK	NO2-NL	NO1 - SE3	NO3 - SE2	NO4 - SE1	NO4 - SE2
2		15	16	2	20	22	21
3	1	-845	-227	-1628	-232	-713	-166
4	2	-1047	-102	-1579	-244	-743	-184
5	3	-1091	79	-1771	-247	-736	-191
6	4	-1142	265	-1811	-246	-694	-183
7	5	-1099	388	-1863	-233	-707	-179
8	6	-1088	108	-1653	-225	-637	-174
9	7	-1173	91	-1266	-189	-530	-167
10	8	-925	1	-1065	-98	-466	-150
11	9	-508	0	-944	-11	-335	-122
12	10	-600	0	-942	-45	-344	-124
13	11	-748	0	-882	-79	-419	-147
14	12	-826	0	-1001	-69	-414	-136
15	13	-1031	0	-1075	-156	-449	-154
16	14	-1032	0	-1161	-167	-447	-149
17	15	-1177	0	-1215	-149	-462	-129
18	16	-1377	0	-971	-134	-412	-137
19	17	-987	-7	-970	-80	-431	-147
20	18	-877	-294	-888	-104	-499	-163
21	19	-1123	-13	-953	-114	-507	-166
22	20	-1053	-317	-950	-135	-470	-155
23	21	-1050	-276	-1144	-132	-508	-163
24	22	-704	-621	-1177	-156	-607	-168
25	23	-576	-537	-1410	-173	-597	-174
26	24	-595	-483	-1293	-138	-658	-166

Figure A.19: nordpool.xlsx: Exchange with neighbouring countries, $P_{n,t}^{exc}$.

The excel file battery.xlsx contains information about the battery size and efficiency, and can be seen in figure A.20.

	A	B
1	eta	0,9
2	Pcmin	0
3	Pcmax	50
4	Pdcmin	0
5	Pdcmax	50
6	Emin	0
7	Emax	200

Figure A.20: battery.xlsx: Battery parameters, eta: $P^{c,min}$, $P^{c,max}$, $P^{dc,min}$, $P^{dc,max}$, E^{min}, E^{max}.