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Diffusion-Based Molecular Communications in Wireless Pacemakers

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Problem description

The goal of this master project is to suggest an optimal point-to-point diffusion-based molecular communication system for communication between wireless pacemakers. This will consist of an encoder, a channel and a detector. Parameters as Distortion, Intersymbol Interference, Data Rate and Capacity will be essential for this decision.

Abstract

Molecular communications is a novel communication technique that has many advantages over the traditional radio frequency technologies used in body area medical applications. It is based on the body`s own biological communication system. This makes this type of communication biocompatible, which is a huge advantage when the body is the environment. Heart diseases is an increasing problem in the society today. Pacemakers are one option to use for treatment of these types of diseases. The newest development is leadless pacemakers, which improve the performance significantly. Since the mortality of these diseases is high, the importance of having the best medical devices and communication technologies is vital. This report studies the combination of diffusion-based molecular communications and leadless pacemakers. We show that the definition of the concentration of the molecules in pheromone communications, can be transferred to the molecular communications used in pacemakers. A deterministic diffusion-based communication system with this concentration, is first proved a suitable model for the diffusion channel of the communication system. It is then shown that a stochastic Single-Input Single-Output (SISO) communication system with on-off keying as the encoder, a binary hypothesis testing channel and a simplified Neyman Pearson detector, gives good performance for the diffusion-based molecular communication between leadless pacemakers. The most important results leading to this conclusion, is a relatively high capacity of 0,958 bits, a low average error probability (BER) of 0,39 % and low levels of Intersymbol interference (ISI) with the optimal signaling interval of 150 seconds. A stochastic SIMO communication system with the same encoder, channel and decoder, but with Maximum Ratio Combining Diversity, is also evaluated as a suitable communication system for this problem. This approach gives a capacity of 0,23 bits, an average error probability of 0,74 % and low levels of ISI with the optimal signaling interval of 200 seconds. Therefore, the SIMO communication system achieves a lower capacity than the SISO communication system, but is optimal with higher signaling intervals. Last, it is shown that a MIMO communication system with the same encoder, channel and decoder, but with Spatial Multiplexing, is not an optimal alternative for the molecular communication between leadless pacemakers. The reason for this is the significantly lower capacity of $8{,}47*10^{-7}$ bits, a high level of distortion where BER is 24,41 % and a rate distortion that does not match the theory. Even though, the ISI levels are reasonable, it is not sufficient to declare this a suitable communication scheme for molecular communication between leadless pacemakers.

Nomenclature

AM Amplitude modulation

ASK Amplitude shift keying

BER Bit error rate

CLT Central Limit Theorem

FM Frequency modulation

ISI Intersymbol interference

ISM Industrial Scientific Medical

MICS Medical Implement Communication Services

MIMO Multiple input multiple output

MLE Maximum likelihood estimation

MRC Maximum-ratio Combining

OOK On-off keying

OPTA Optimum Performance Theoretically Attainable

PDF Probability Density Function

PM Phase modulation

RF Radio frequency

SIMO Single-Input Multiple-Output

SISO Single-Input Single-Output

SM Spatial Multiplexing

Content

1 Introduction

This report is the outcome of the 5th year specialization project within signal processing at the Norwegian University of Science and Technology (NTNU). The topic of this master thesis is molecular communications for wireless pacemakers.

1.1 Motivation

Today, we have an increasing number of people suffering from chronic non-communicable diseases, like congestive heart failure. This is critical because the mortality connected to these diseases, are high, which gives significant consequences. Hence, there is a huge need of technology that helps detecting the diseases early on, and prolonging and saving the lives of the people suffering from them. The condition congestive heart failure is when the pump function of the heart becomes impaired [1]. A pacemaker can be used to treat this disease among other types of chronic heart diseases. Several complications, however, have been connected to the traditional pacemaker, mostly because of the leads. This is why the latest development among pacemakers, namely the leadless pacemaker is such important.

The traditional communication techniques are not sufficient for use in intra body sensor communications. The classical radio frequency (RF) technology has several disadvantages. It has a high energy consumption, electromagnetic interference may occur and there is also security issues [2]. Nor is this technology biocompatible, which is a huge disadvantage when the environment is inside the body. In the latest years, however, a new health monitoring technique has arisen, namely the body area network (BAN) technologies. The development in mm- and nm-scale has opened up for a new communication technique in these networks, molecular communications. A lot of benefits are connected to this form of communications. It is for instant energy efficient, biocompatible because it is based on the biological environment of cells and molecules, and it gives communication ability in aqueous environment that the traditional communication techniques cannot [3].

1.2 Objective and limitations

The objective of this project is to suggest a complete point-to-point molecular communication system that is optimal for leadless pacemakers. This will include an encoder, a diffusion channel and a decoder. A Single-Input Single-output (SISO) system, a Single-Input Multiple-Output (SIMO) system and a Multiple-Input Multiple-Output (MIMO) system will be tested to investigate what will optimise the communication system. Why can molecular communication methods improve the efficiency of the pacemaker for the heart, and how do we implement it? What model can be used to evaluate the performance of this form of communication?

To simplify the complexity of the simulations, the communication system will mostly be displayed in one dimension, all though the environment in the heart is in three dimensions. All the decisions, however, will be based on what optimises a three dimensional problem.

1.3 Structure of the report

First, some background connected to molecular communications and pacemakers are being addressed in Chapter 2. The development in communication techniques towards this form of communication, and the development in pacemakers from needing leads to being leadless is important. Some relevant theory needed to understand the essence of this report is given in Chapter 3. In Chapter 4, a literature study is given. Most of the reviewed articles are dated from only a few years back. Hence, it is clearly a novel technology. The methodology is explained in Chapter 5, where a model of the studied communication system including the pacemaker system is being shown. The communication system is first deterministic, then stochastic with SISO, SIMO and MIMO communications. In Chapter 6, the results from all types of communications systems, is being displayed and discussed. Last, the conclusion will be presented in Chapter 7.

2 Background

2.1 The development of communication technologies towards molecular communications

Communication technologies used in pacemakers, together with other intra body medical applications, have developed in the last two decades. The traditional radio frequency (RF) communication technology dominated for a long time. At first, narrowband wireless technologies were used widely for medical applications [4]. The disadvantage of applying narrow bands is that they only support low transfer rates [5].

The use of medical implement communication services (MICS) frequency bands from 402 to 405 Hz, provides improved properties [6]. This is a frequency band intended just for use with medical devices like pacemakers. It is now possible to have low rates, and ultra-low power. The antennas can also be quite small. The size of the antennas can, however, not be reduced enough to be used in devices in mm-scale.

A higher frequency band that gives further improved capabilities, is the industrial scientific medical (ISM) band [7]. These bands are unlicensed frequency bands intended for use in industrial, scientific and medical applications. Higher frequencies are beneficial for many reasons. They support higher data rates, have lower power consumption, longer range and are less susceptible for interference [8]. Another benefit is that small, efficient antennas, can be applied. This opens up for wireless implantable sensors and actuators, in this case a pacemaker, to be even smaller [9].

Even though the radio frequency communication technology has developed, it still has many disadvantages when used in intra body medical applications. Nowadays, a novel communication scheme has become highly important in this area. Molecular communications has among other abilities, better energy efficiency than the radio frequency technology. With use of molecular communications, the transmitter and receiver devices can be very small. The reason is that this technology communicates well between micro- and nano-sized devices. The last type of devices is called nanomachines.

Molecular communications has still many challenges. To mention some of them, their communication channels are substantial different than the traditional channel in wireless communication systems, the messenger molecules used to transfer information are vulnerable to noise, and their propagation speed is slow, [10]. Therefore, further research on this topic is important.

2.2 The development in pacemakers from pacemakers with leads to leadless pacemakers

The traditional pacemaker consists of a pulse generator part, an actuator and one to three leads, which include electrodes [11]. The electrodes both sends the impulses from the actuator to the heart muscle and works as a sensor. There are, however, some consequences using a pacemaker with leads [12]. This is because the leads are the weakest part of the pacemaker system. The most critical problems are infections, damaging of vascular and heart structure like the tricuspid heart valve and chronic lead reliability problems. In fact, in 21 % of the cases where the patient has a pacemaker with leads, the patient will experience a lead based consequence within a 10 years period after the implementation. Therefore, it is preferable to use no leads in the communication system at all.

Leadless pacemakers are the newest type of pacemakers [13]. They can, in contrast to traditional pacemakers, be placed directly in the heart. They are also less invasive since the pacemaker does not need use of a surgical pocket or leads. This device is much smaller and is formed like a capsule. Inside the capsule, there is a pulse generator and a sensor part with a battery unit. It is placed in the right ventricle with a catheter. Since the size of this capsule is in mm-scale, it is defined as a nanomachine. This is a challenge, which may be solved with use of molecular communications as mentioned above.

3 Theory

3.1 The heart

The heart is a large muscle, which pumps blood through the body. It consist of four chambers [14]. The right and left atrium are the top champers and the right and left ventricles are the bottom champers. The sinus node is a group of muscle cells located in the right atrium, and serves as the body's natural pacemaker. Electrical signals are sent from the sinus node and move through the heart from the top chambers down to the bottom chambers. This makes the heart contract and pump blood. The contraction of first the top chambers and then the bottom chambers is called a heartbeat. A model of the heart is shown in Figure 1.

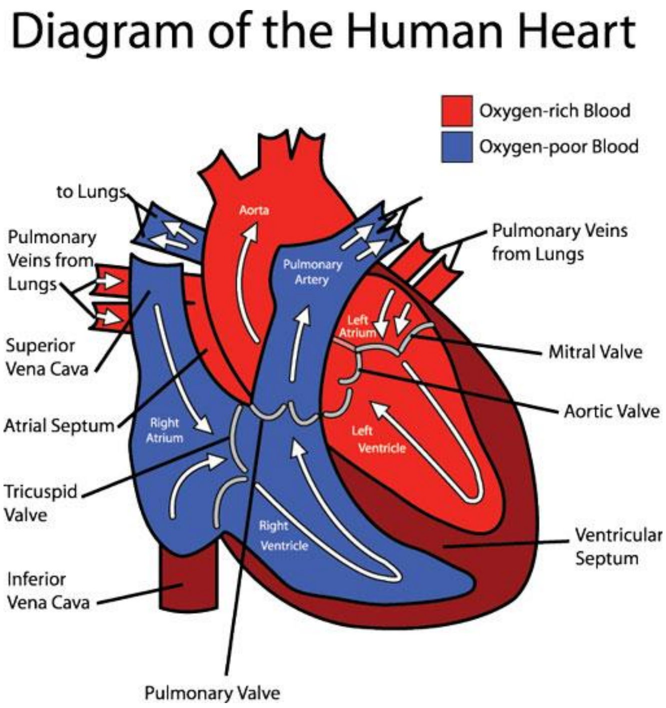

Figure 1: A model of the heart [15]

3.2 The pacemaker

If the electrical signals from the sinus node do not reach all the muscle cells, the rate of the heartbeat can get too low or irregular. This is called Arrhythmias [16]. A pacemaker can help getting normal rate of the heartbeat. It is a small electronic device placed in the right ventricle and works only when needed. Therefore, if the heart rate gets too low, the pacemaker sends out an electrical impulse, which makes the heart contract.

3.3 Molecular communications

Molecular communications is a relatively new communication form where molecules are used to transport information between a transmitter and a receiver [17]. These components are often of mm-scale, and are therefore called nanomachines. The movement of the molecules between the nanomachines, is described by the Brownian motion model explained in the next chapter. There are three types of molecular communications based on the molecular propagation; walkway based, flow-based and diffusion-based [18]. In this report, the last one is being used. Therefore, that is the only one that is going to be explained. In diffusion-based molecular communications, the environment is a fluid like water or blood. The molecules propagate through this medium with diffusion. This diffusion can be either free diffusion or turbulent diffusion. In this case we have the last type of diffusion.

3.4 Brownian motion model

The Brownian motion model is a model that describes the random movement of molecules in a gas or liquid. [19] Molecules released in an environment like that, will move in a zig zag motion as a result of crashing with the molecules from the environment. The botanist Robert Brown from Scotland first discovered it, when he experimented with pollen grains. Later, Albert Einstein developed the diffusion equation for Brownian particles, which describes the numerical movement of molecules.

3.5 Diffusion

Diffusion means "to spread out" in Latin, and is a process connected to the movement of molecules [20]. When molecules are emitted, they tend to move from the area where their concentration is high to an area where their concentration is low. This can be seen as waves spreading out from their original position. This happens because of their kinetic energy and the fact that the molecules move with an independent random speed. Equilibrium is reached when the concentration of the molecules is equally dispersed in the environment.

3.6 Turbulent diffusion

Turbulent diffusion is a form of diffusion. This is when irregular turbulent velocity fluctuations lead to spreading of a quantity [21]. The main difference between free diffusion and turbulent diffusion is that the turbulent diffusion is quicker, which makes it more efficient. If you have a quantity of molecules in a bottle of water and you shake it, this dispersion can describe the turbulent diffusion.

3.7 Eddy diffusivities

Eddy diffusivities are diffusion coefficients [21]. They are used for modelling the turbulence transport caused by fluctuations.

3.8 Modulation

Modulation is an important tool when transmitting information from a transmitter to a receiver [22]. A carrier signal is changed to be able to transfer information from one place to another. There are two main categories of modulation, analog and digital modulation. In analog modulation, a signal may take any value, while a digital modulated signal only takes two values. There are three main types of modulation within these two; Amplitude modulation (AM), Frequency modulation (FM) and Phase modulation (PM). What separates these types of modulation is which parameter being changed.

3.8.1 Amplitude modulation (AM)

In Amplitude modulation (AM) the amplitude or the intensity of the carrier signal is being modified [23]. The carrier signal f(t) is given in Eq. 3.1, where A is the unmodulated amplitude, m(t) is the message and ω is the carrier frequency.

$$f(t) = [A + m(t)]cos(\omega t) \tag{3.1}$$

The amplitude, $[A + m(t)]$, gets varied in relation to the message m(t) that is being transmitted.

3.8.2 Amplitude shift keying (ASK) modulation

Amplitude shift keying is the digital version of amplitude modulation [24]. The amplitude of the carrier sinusoid is here shifted between two fixed values.

3.8.3 On-off keying (OOK)

On-off keying (OOK) is the simplest form of binary ASK modulation [24]. In this form of modulation the message signal m(t) equals either 1 or 0. That gives either a presence or an absence of the bit data.

3.9 Binary hypothesis testing channel

A binary hypothesis testing channel is a channel where there is two available hypotheses, the null hypothesis, H_0, and the alternative hypothesis, H_1 [25]. They correspond to two different distributions, namely $p(Z_i; H_0)$ if H_0 is true and $p(Z_i; H_1)$ if H_1 is true, where Z_i is a random independent variable. The goal is to test which distribution that is in use. In this scenario, there is two main probabilities, the false alarm probability, P_F and the detection probability, P_D. The false alarm probability is the probability for transmitting a one, but receiving a zero, while the detection probability is transmitting a one, and receiving a one. This is shown in Eq. 3.2 [26]

$$P(Y_i = 1|X_i = 0) = P_F$$

$$P(Y_i = 1|X_i = 1) = P_D$$

$$P(Y_i = 0|X_i = 0) = 1 - P_F$$
$$P(Y_i = 0|X_i = 1) = 1 - P_D \tag{3.2}$$

where X_i is the input signal and Y_i is the output signal.

3.10 The Neyman-Pearson detector

The Neyman-Pearson detector uses the Maximum likelihood test to maximize the detection probability, P_D, while making sure that the false alarm probability, P_F, is equal to or below a specified value, α [27]. The maximum likelihood is defined as the ratio between the two distributions mentioned in chapter 3.9. The maximum likelihood test is defined as the comparison of the maximum likelihood and a threshold, λ in Eq. 3.3

$$\frac{p(Z_i;H_1)}{p(Z_i;H_0)} = \Lambda \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{3.3}$$

The threshold, λ, can be found from Eq. 3.4.

$$P(\Lambda > \lambda|H_0) = P_F \tag{3.4}$$

3.11 The normal (Gaussian) distribution

A distribution describes how the probability of every outcome of a random variable Z is distributed based on their probability density function (PDF) [28]. The Normal distribution, also called the Gaussian distribution is the most used distribution. The probability density function (PDF) of a normal distributed random variable Z is defined by

$$f_z(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3.5}$$

where μ is the mean value and σ^2 is the variance of z. The graph of the PDF of z, when $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$ is shown in figure 2.

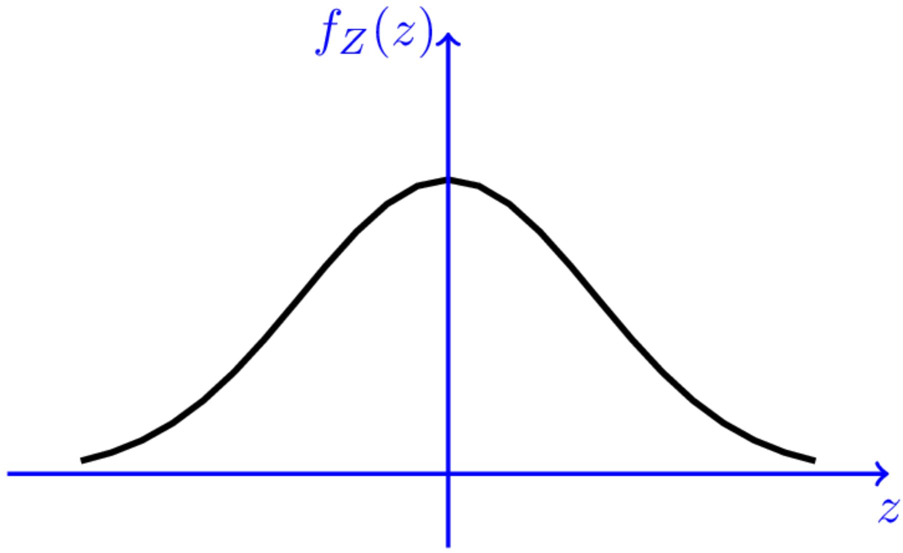

Figure 2: The PDF of z

3.11.1 The central limit theorem (CLT)

The central limit theorem is the reason why the normal distribution is so important [28]. It states that all large numbers N of random variables converge towards the normal distribution when N goes towards infinity.

3.11.2 Lindeberg-Feller Central Limit theorem

Lindeberg-Feller Central Limit theorem is a special case of the central Limit theorem, which holds if the Lindeberg's condition is fulfilled [29]. It states that the sum of random independent variables converges towards the normal distribution. The Lindeberg's condition is defined in Eq. 3.6.

$$for\ every\ \varepsilon > 0, \frac{1}{s_n^2}\sum_{i=1}^{n} E(Y_i^2 I\{|Y_i| \geq \varepsilon s_n\}) \rightarrow 0\ as\ n \rightarrow \infty \tag{3.6}$$

The variables Y_n, T_n and s_n is defined in Eq. 3.7

$$Y_n = X_n - \mu_n$$

$$T_n = \sum_{i=1}^{n} Y_i$$

$$s_n = var(T_n) = \sum_{i=1}^{n} \sigma_i^2 \tag{3.7}$$

where X_n is random independent variables with the mean value, μ_n, and the variance σ_n^2.

3.12 Bernoulli distribution

The Bernoulli distribution is based on two possible outcomes, either success or failure [30]. A random variable Z that is Bernoulli distributed, is assigned 1 for success with the probability p and 0 for failure with the probability 1-p. The PDF of this distribution is defined by Eq. 3.8.

$$f_z(z) = \begin{cases} 0 & if\ z < 0 \\ 1-p & if\ 0 \leq z \leq 1 \\ p & if\ z \geq 1 \end{cases} \tag{3.8}$$

The mean value is defined as

$$E\{z\} = p \tag{3.9}$$

The variance is defined as

$$Var\{z\} = p(p-1) = p^2 - p \tag{3.10}$$

3.13 Bit error rate (BER)

The bit error rate (BER) is the rate errors occur at when transmitting information [31]. In a digital communication system, there is an error if 0 is sent, but 1 is received, or 1 is sent, but 0 is received. It is defined by

$$BER = \frac{The\ number\ of\ bits\ with\ error}{The\ total\ number\ of\ bits} \tag{3.11}$$

The mathematical formula for calculating this parameter is given in Eq. 3.12

$$P_b = P\{n > A\} = \int_A^{\infty} f(x)dx \tag{3.12}$$

where A is the threshold that the test variable has to be above to have an error, and f(x) is the PDF [32].

When the distribution is normal distributed, the BER is defined as

$$P_b = P\{n > A\} = \int_A^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu_x)^2}{2\sigma^2}} \tag{3.13}$$

3.14 Intersymbol interference (ISI)

Intersymbol interference (ISI) is a distortion of the signal in a communication system [33]. Overlapping symbols can cause noise and make the signal less reliable. ISI is an undesirable effect that can occur because of multipath propagation among other reasons.

3.15 Optimum Performance Theoretically Attainable (OPTA)

Optimum Performance Theoretically Attainable (OPTA) is the performance limit of a communication system [34]. This limit is attained if the distortion rate function is equal to the capacity of the channel.

3.15.1 Rate distortion

Rate distortion is the relationship between the data rate and the distortion of the signal and is displayed in Figure 3. When the rate increases, the distortion decreases [35].

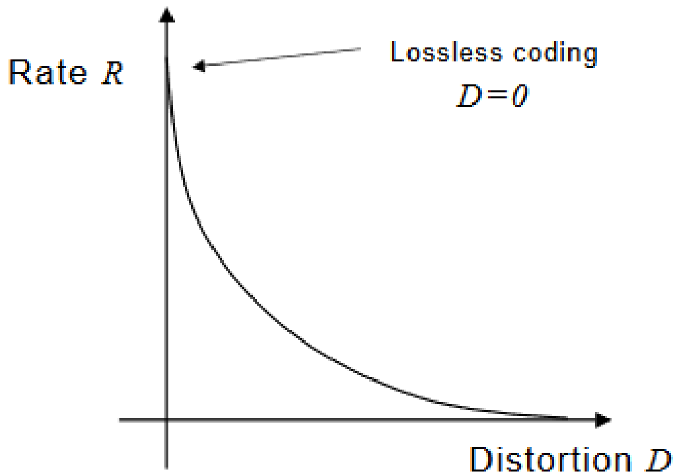

Figure 3: Rate distortion

The distortion rate function describes the properties of the source. It determines the minimum number of bits that needs to be transmitted to achieve the desired quality [34].

3.15.2 Capacity

The capacity describes the channel properties [34]. It is defined by the maximum mutual information that can be transmitted through the channel, where the mutual information is defined in Eq. 3.14.

$$I(X_i;Y_i) = \sum_{X_i=0}^{1} \sum_{Y_i=0}^{1} P(Y_i|X_i)P(X_i)\, log \frac{P(Y_i|X_i)}{P(Y_i)} \qquad (3.14)$$

3.18 Fading channels

A fading channel is a channel with variations [36]. There are different degrees of fading. Therefore, the fading channels is divided in slow fading channels and fast fading channels.

3.18.1 Fast fading

Unlike a slow fading channel, which is approximately constant, a fast fading channel is varying over the time dimension [36]. One of the most common definitions of a fast fading channel is that the coherence time T_c of the channel is much shorter than the delay requirements [37]. Fast fading channels makes it possible to transmit coded symbols over multiple fades of the channel.

3.19 Multiple-input multiple-output (MIMO) channels

Multiple-input multiple-output (MIMO) channels is a channel with multiple antennas both at the transmitter and at the receiver [38].

Variations of the MIMO channel is single-input multiple-output (SIMO) and multiple-input single-output (MISO) channels [39]. The first one has only one antenna at the transmitter, but multiple antennas at the receiver, while the second one has multiple antennas at the transmitter, but only one at the receiver. In Figure 4 this is displayed with two transmit antennas and/or two receive antennas.

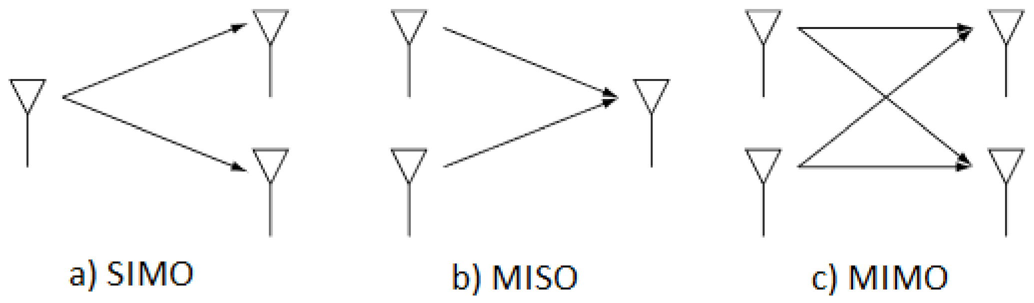

Figure 4: a) SIMO channel, b) MISO channel, c) MIMO channel

3.20 Diversity

Diversity is a technique that can improve the performance of the communication over fading channels [40]. The idea is to transmit signals with the exact same information over multiple channel paths with independent fading. In this way, the information can be received if at least one of the channel paths is functional. This makes the detection more reliable. There are different methods of achieving diversity. The techniques can be performed over time, space or frequency. Some of the most used is transmit diversity, receive diversity and time diversity.

3.20.1 Antenna diversity

Antenna diversity is a diversity technique performed over the space dimension [41]. This can be achieved by using multiple antennas at the transmitter and/or the receiver with SIMO, MISO or MIMO channels. The distance between the antennas is an important factor. They have to be sufficiently far apart to obtain independent traveling paths.

3.20.1.1 Receive diversity

Receive diversity is a form of Antenna diversity where there is multiple antennas at the receiver [42]. This is called a Single-Input Multiple-Output (SIMO) channel. If there is L antennas at the receiver and they are placed appropriately far apart, the diversity gain is L.

3.20.1.2 Maximum-ratio Combining (MRC) Diversity

Maximum-ratio Combining (MRC) Diversity is a communication technique where the idea is to amplify the strong signal components and attenuate the weak signal components to improve the performance of the communication system [43]. This is done by adding a weight to the signal.

3.21 Spatial multiplexing

Spatial multiplexing is a communication technique for MIMO channels [38]. Independently encoded signals are transmitted through different channel paths. With this technique, it is possible to exploit the degrees of freedom and increase the capacity of the communication system.

4 Literature study

In this chapter a summary from the literature study will be given. The most promising approach to similar problems will be explained and later used in simulations of the point to point molecular communication system.

4.1 Channel model approaches with deterministic signaling

The past few years, there have been a rise of published articles connected to diffusion-based communications. They conclude that diffusion-based molecular communications is one of the most promising approaches for communication between nano machines. Many of the articles that address the channel model, focus on the general use of this type of communications, not in a specific environment like the heart in this case.

One of the reviewed articles where the environment is specified, studies the establishment of digital molecular communications in blood vessels [44]. This article has as this report the environment situated in the cardiocirculatory system. The transfer of digital information is done by sending molecules from mobile transmitters. They then diffuse in the blood towards the receivers, as the molecules also will do in the heart. The receivers are fixed and attached by the vessel walls. To review their performance, simulations were applied. This technique is also going to be used in this report. The main difference between these two topics is that the molecules in the blood vessels move in one direction, while the molecules in the heart move in three dimensions.

Another of the reviewed articles has a three dimensional approach to diffusion-based molecular communications [45]. The article studies the use of a plant pheromone channel were pheromone diffusion is used to transfer molecules between the transmitter and the receiver. These pheromones are chemicals that induce a physiological reaction or behaviour on the receiver end. The information is being transferred from a plant, which is the transmitter to another plant, which is the receiver, by use of diffusion in the air. The pheromone communications is quite similar to the molecular communications that can be used in pacemakers. They transfer information in the same three-dimensional way. The main difference is that the environment in this report is the blood in the heart instead of the wind, and that the transmitter and receiver are pacemakers instead of plants.

4.2 Complete communication system approaches

As mentioned above, the interest for studying diffusion-based molecular communications has increased the last couple of years. There is some diversity in what their focus is, but when it comes to the communication system, there is significantly many common factors. Three main parts of the communication system with SISO communication, will be focused on; the encoder, the channel and the decoder.

The encoder of the communication system is mostly the same in the reviewed articles. They use the amplitude modulation, on-off keying (OOK). One paper uses stochastic signaling where a large random number of molecules are sent [26], while in another only one molecule is sent [46]. In this report, the encoder will be closest to the first approach, because sending one molecule only works as a theoretical basis of a communication system. There is also been done research with use of multilevel amplitude modulation, and even type-based modulation [47].

The physical channel in diffusion-based molecular communications is what makes the communication system differ the most from traditional communication systems. Because of the Brownian motion model, the molecules move randomly with different speed and arrives at the receiver at different times. Hence, a crossover effect can arise [47]. This means that molecules could arrive in another signaling interval than intended. The most used channel in the research material is a binary hypothesis testing channel, which has four different probabilities of the output given the input; the detection probability, the false alarm probability and their inverses [26]. Therefore, this is also going to be used in this report .

For the decoder, there is a bit of variety. Many articles, however, use an information theoretical approach to find the optimal decision threshold for 0 and 1. This threshold is derived for maximal mutual information. The detection is often done by two methods, one where the receiver knows the a priori information for the input, and one where it does not. When the a priori information is known, the decoder is a Neyman-Pearson detector. In one article, the authors conclude that knowing the a priori information is important to get an optimal decision threshold when the molecules are moving in three dimensions [26]. When they only use one or two dimensions, however, you get equally optimal results without knowing. In another of the articles, the detector is defined as a one-shot detector [46]. Since the blood moves in three dimensions, the a priori information is assumed known and the Neyman-Pearson detector is applied in this report.

4.3 Multi-Input Multi-Output (MIMO) approaches

There have also been published articles on how to improve the performance of the nanomachines in diffusion-based molecular communication systems by use of MIMO systems. The authors of one of these, claim that the diffusion channel have similar characteristics as a fast fading channel [48]. This is because the nanomachines can travel several micrometres per second, the range of the channel is usually in tens of micrometres and they have long signaling intervals. Multi-Input Multi-Output (MIMO) communication systems is a good method to improve the efficiency of these channels, instead of transmitting more molecules. Diversity techniques can decrease the error probability significantly. The authors concluded, however, that a dynamic switching technique between Diversity mode in a SIMO system and Spatial Multiplexing (SM) mode in a MIMO system gave the best results. These two techniques is therefore going to be tested in this report.

5 Method and Implementation

In this chapter, the methodology and implementation will be explained.

The simulations is mostly based on a one-dimensional communication scheme to simplify the complexity of the communication system. All decisions, however, is based on what will optimise a three-dimensional communication scheme, which communication between pacemakers in the heart is defined as. This makes it easy to convert to the actual problem.

5.1 The approach to the problem

The research methods used in this report were mostly literature searching and simulations. To study the transport of molecules through the fluent environment inside a human heart, a model of the molecular communication system had to be defined. The communication system consists of three parts, the transmitting of the molecules, the diffusion of the molecules in the channel towards the receiver, and the receiving of the molecules. This is shown in Figure 5.

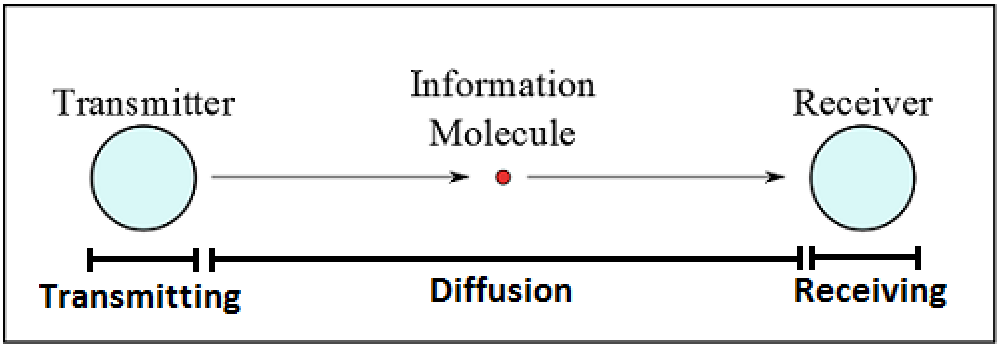

Figure 5: The molecular communication system [20]

First, only the diffusion part of the communication system was studied. Therefore, a model of the concentration was defined. In this study, the molecules were only transmitted once through the channel with deterministic signaling in a Single-Input Single-Output (SISO) communication system. To evaluate if the model were a suitable model for the diffusion channel, a performance analysis was conducted.

When the physical model of the channel was established, the communication system was expanded to include modulation and coding schemes optimal for the diffusion channel. Molecules were now transmitted several times over the channel with stochastic signaling. A performance analysis with parameters like distortion, Intersymbol interference and the maximum mutual information and rate, were conducted to evaluate the functionality of the complete communication system.

To try to improve the performance of the communication system, a Single-Input Multiple-Output (SIMO) and a Multiple-Input Multiple-Output (MIMO) system was applied instead of the SISO system. Diversity and Spatial Multiplexing techniques were tested on the communication system to see if they improved the functionality. The same performance analysis were applied to these two communication systems.

5.2 The model of the physical diffusion channel with deterministic signaling

5.2.1 The concentration as the model and definitions of the parameters

The channel model of the communication system was first based on molecular communications in blood vessels in the cardiovascular system instead of in the heart. This research is discussed in [44]. Since the blood flow velocity of the molecules in blood vessels moves only in one direction, while the blood flow velocity in the heart moves in three directions, this research only gave a simplified solution to the problem.

To get a more accurate model for the molecular communication system, it was based on molecular communications using pheromones between plants as presented in the literature study. Research on this topic is discussed in [45]. To study the performance of this communication, the concentration of the molecules were studied. The formula for that in pheromone communications is given as

$$c(\vec{x},t) = \frac{Q_T}{8(\pi r)^{3/2}} e^{(-(x-ut)^2-y^2)/4r}\left[e^{-(z-H)^2/4r} + e^{-(z+H)^2/4r}\right] \tag{5.1}$$

where $c(\vec{x},t)$ is the concentration of molecules at position $\vec{x} = (x,y,z)$ at time t. This is the 3D version of the impulse response. With the definition $r = \frac{Kx}{u}$, we get another formula for the concentration given as

$$c(\vec{x},t) = \frac{Q_T}{8(\pi \frac{Kx}{u})^{3/2}} e^{(-(x-ut)^2-y^2)/\frac{4Kx}{u}}\left[e^{-(z-H)^2/\frac{4Kx}{u}} + e^{-(z+H)^2/\frac{4Kx}{u}}\right] \tag{5.2}$$

Since both the pheromone communications and the molecular communications in this report use diffusion to transfer information from the transmitter to the receiver, this formula was also used for the concentration of the molecules in this case. The parameters, however, have different definitions in the molecular communications used in the heart. The definitions from pheromone communications were that u is the wind velocity, H is the height of the emitting leaf according to the ground, Q_T is a scalar related to the number of molecules released per second and K is the eddy diffusivities. The new definitions is that u is the blood flow velocity and the position $\vec{x} = (x,y,z)$ is now located in the heart instead of in the air. K is also here the eddy diffusivities since they exist in both the wind and the blood. Q_T is still the number of molecules released, but from the pacemaker instead of the plants. H is defined as the distance from the pacemaker to the closest wall of the heart. To use this parameter, some assumptions had to be made. The first assumption is that the molecules is not going to reach other walls than the closest one. Another important assumption is that the pacemaker system includes two pacemakers; one transmitter pacemaker, T_x and one receiver pacemaker, R_x. The final assumption is that these two pacemakers is placed in equal distance from their closest wall of the heart. The last one makes it possible to have one value for H, since $H_1 = H_2 = H$. In Table 1 the correspondence between these parameters are shown.

Table 1: Correspondence between the parameters in this report and for the pheromone communications

Parameters	**The meaning of the parameter in pheromone communications**	**The meaning of the parameter in molecular communications in the heart**
$c(\vec{x}, t)$	The concentration of the molecules in the wind	The concentration of the molecules in the blood
u	The wind velocity	The blood flow velocity
$\vec{x} = (x, y, z)$	The position of the molecules in the wind	The position of the molecules in the heart
K	The eddy diffusivities in the wind	The eddy diffusivities in the blood
Q_T	The number of molecules released from the plants per second	The number of molecules released from the transmitter pacemaker, T_x, per second
H	The height of the emitting leaf according to the ground	The distance from the pacemaker to the closest wall of the heart

5.2.2 The values of the parameters

The values of the parameters $\vec{x} = (x, y, z)$, u, K, H and Q_T had to be found to be able to do the performance analysis of the molecular communication system. First, a figure of the communication system in the correct environment was made, shown in Figure 6.

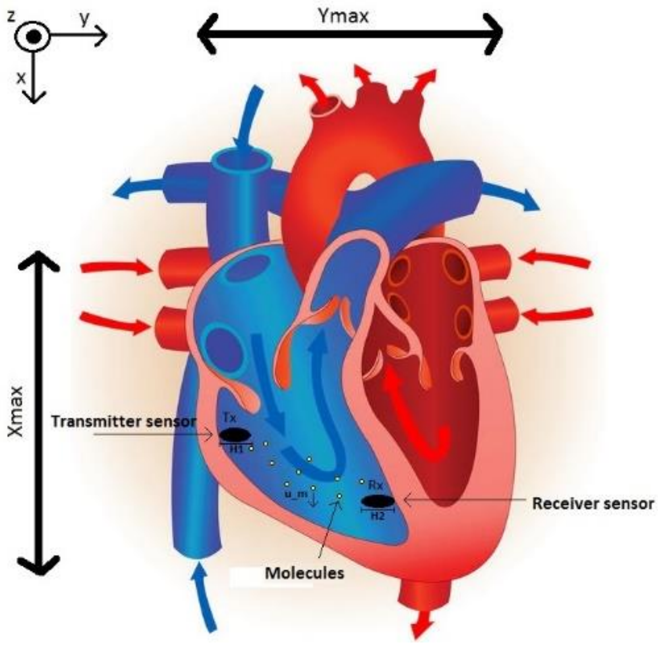

Figure 6: A model of the heart with the pacemaker system implemented [49]

In Figure 6, a model of the heart is shown with the pacemaker system implemented. The two pacemakers have the names transmitter sensor, Tx and receiver sensor, Rx because it is only the sensor part that is being used in this report. The actuator part is out of scope. In addition, the x, y and z axis is being displayed. X is defined as the height of the heart, y is the width and z is the depth. The yellow dots in the figure, is the messenger molecules emitted from Tx and received by Rx. They have each their individual random travelling path towards the receiver pacemaker. The parameters H1 and H2 from before, is also displayed in the figure.

To find the maximum values of the parameters x, y and z, the approximation that the heart is of the size of a closed fist was made. Measurements of my own hand gave the height, xmax ≈ 7,5 cm, the width, ymax ≈ 8 cm, and the depth zmax ≈ 5,5 cm. Since the parameters x, y and z give a position in the heart, their values are a number between zero and their maximum value. In reality, the molecules can only move in the right ventricle were the pacemaker is placed, so the xmax, ymax and zmax would have a smaller value.

The blood flow velocity u, were first set to simple values like [1 2 3 4 5] for a test drive. Since the pacemaker is placed in the right ventricle, the blood flow velocity there had to be used for a more accurate value. From figure 1 in the theory, we can see that the Inferior Vena Cava and Superior Vena Cava are the two large veins, which carries deoxygenated blood from the body to the right atria of the heart. Then the blood moves through the Triscuspid valve to the right ventricle. Therefore, the blood flow velocity in the Inferior Vena Cava and Superior Vena Cava can be used. This value is measured to be approximately 15 cm/s [50]. Here the assumption that the blood flow velocity is constant, also from the large veins and through the Triscuspid valve, is made.

The value of the eddy diffusivities K is defined as

$$K = RD \tag{5.3}$$

where R is the Reynolds number and D is the diffusion coefficient of molecules [51]. The value of D is, $D = e^{-6}\, m^2/s$ [51]. The Reynolds number is defined as

$$R = \frac{uL}{\nu} \tag{5.4}$$

where u is the blood flow velocity, L is the diameter of the heart and ν is the blood kinematic viscosity [52]. Eq. 5.5 defines the blood kinematic viscosity

$$\nu = \frac{\mu}{\rho} \tag{5.5}$$

where μ is the blood viscosity and ρ is the blood density which have the values, $\mu = 0{,}0488e^{-1}$ and $\rho = 1{,}06e^{3}$ [51] [53]. Matlab were used to calculate the final value for the eddy diffusivities.

The distance of the pacemaker to the closest wall, H were set to the value 7,5 mm. We can see from the formula that the concentration does not depend heavily on the magnitude of this parameter. It is, however, less than ymax, as the pacemaker cannot be placed further away from the heart wall than the width of the heart. Hence, H is given an arbitrary value. The number of emitted molecules, Q_T, is not important for the purpose of this report. Hence, it was given the arbitrary value 500.

5.3 The performance analysis

After the model of the concentration was defined, the performance analysis was carried out by use of simulations. For it to be a suitable model, the concentration had to be dispersed over time and distance by the concepts of diffusion. To determine if this was the case, the gain and the delay of the concentration were evaluated. A reference direction for the blood flow velocity was defined to simplify this evaluation. In Figure 6, this blood flow velocity parameter is presented, namely the blood flows main direction, u_m. This parameter is defined to move along the x-axis. By use of this main direction of the blood flow, a simplification of the model in Figure 6 could be made as a cylinder, where the transmitter pacemaker Tx, and the receiver pacemaker Rx is placed directly above each other. This is shown in Figure 7.

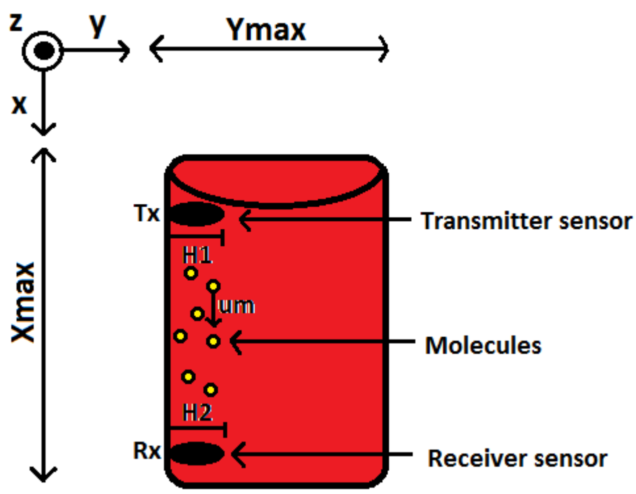

Figure 7: Simplification of the heart model

To understand better how the concentration should disperse over time and distance, for the model to be realistic, a figure of this is shown in Figure 8. The time, t = 0, is when the molecules are being emitted, and t = tmax, is when the molecules have reached the receiver.

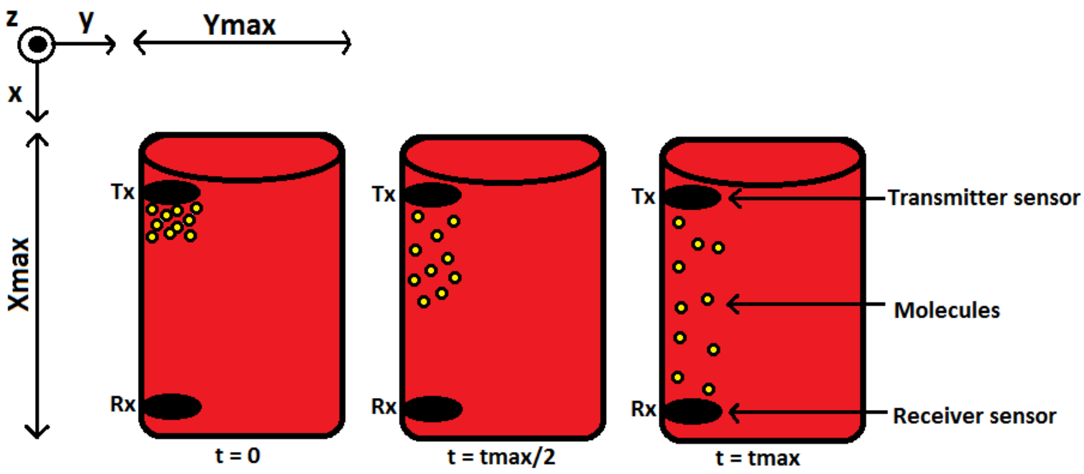

Figure 8: The diffusion process

5.4 The complete communication system with stochastic signaling

Until now, a number of molecules, Q_T, of 500 molecules are transmitted once randomly through the communication system with deterministic signaling. The concentration of the molecules with deterministic signaling and only one signaling interval, is defined in Eq. 5.2.

We now expend the communication system to involve the molecules being sent a number of times. A signaling interval, T_s, is defined as the time slope from the time the molecules are transmitted until the molecules are transmitted the next time. Hence, the molecules are transmitted at $t = iT_s$, where $i = 0,1, \dots N$, and N is the number of interfering signaling intervals. The concentration of the molecules can now be defined as the sum of all the intervals in Eq. 5.6.

$$c(\vec{x},t) = \sum_{j=i-N}^{i} X_j \frac{Q_j}{8(\pi\frac{Kx}{u})^{3/2}} e^{(-(x-u(t-jT_s)^2-y^2)/\frac{4Kx}{u}} \left[e^{-(z-H)^2/\frac{4Kx}{u}} + e^{-(z+H)^2/\frac{4Kx}{u}}\right] \quad (5.6)$$

The number of emitted molecules was previously denoted by a deterministic variable, Q_T, but is now exchanged with a stochastic variable, Q_j. To be able to do this an input variable, X_j, is added to the equation. The time variable, t, is replaced by $(t - jT_s)$ to include all the overlapping intervals. The mean value and the variance of Q_j is defined in Eq. 5.7

$$\mu_Q = 1\ mmol$$

$$\sigma_Q = 0{,}3\mu_Q \quad (5.7)$$

where 1 mol is defined as 6,022*10^{23} molecules. The diffusion coefficient, D for molecules is set to 0,5 m^2/s with the modified communication system.

An information theoretical approach for the communication system is, as mentioned, frequently used. When the molecules move in one or two dimensions, the optimization of the decision is not dependent on whether the a priori information is known or unknown [26]. When they move in three dimensions, however, the a priori information should be known to get the optimal decision. Since the molecules diffuses in three dimensions in the heart, the a priori information is assumed known in this report.

The communication system consists of three main parts, the encoder, the channel and the decoder.

5.5 The encoder

As mentioned in the literature study, the amplitude modulation On-Off Keying (OOK) is an appropriate encoder for the communication system [26]. With this form of modulation, molecules are sent if the input signal, X_i = 1, and no molecules are sent if the input signal is X_i = 0.

5.6 The physical channel

For the diffusion channel, a binary hypothesis testing channel is a good option for getting the optimal detection [26]. A model of this is shown in Figure 9.

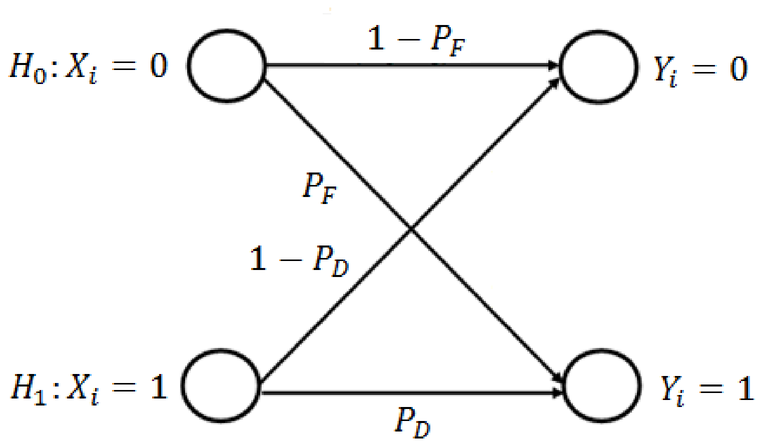

Figure 9: Binary hypothesis testing channel

This channel decides between two hypotheses, where the first one, H_1, is that molecules are sent, while the other one, H_0, is that no molecules are sent.

$$H_1: Molecules\ are\ sent\ at\ t = iT_s$$

$$H_0: No\ molecules\ are\ sent \tag{5.8}$$

Because of the random diffusion, the channel is not symmetric. Therefore, there are four different probabilities in this channel; the detection probability, P_D, which is that molecules are sent and received, the false alarm probability, P_F, which is that molecules are sent, but not detected, and the inverses of these two. From the definition of a binary channel, we have

$$P(Y_i = 1|X_i = 0) = P_F$$

$$P(Y_i = 1|X_i = 1) = P_D$$

$$P(Y_i = 0|X_i = 0) = 1 - P_F$$

$$P(Y_i = 0|X_i = 1) = 1 - P_D \tag{5.9}$$

5.7 The decoder

An information theoretical approach is a good method to define the detector [26]. The optimal decision threshold between the decision that molecules are sent or not sent can then be derived from the maximal mutual information. The definition of mutual information is given in Eq. 5.10.

$$I(X_i; Y_i) = \sum_{X_i=0}^{1} \sum_{Y_i=0}^{1} P(Y_i|X_i)P(X_i)\, log\, \frac{P(Y_i|X_i)}{P(Y_i)} \tag{5.10}$$

The probability, $P(Y_i|X_i)$, is defined in Eq. 5.9. Molecules are sent with the a priori probability, $P(X_i) = p_1$. From these two variables, the probability $P(Y_i)$ can be derived as in Eq. 5.11. It is therefore a combination of P_F, P_D and p_1.

$$P(Y_i) = \sum_{X_i=0}^{1} P(Y_i|X_i)P(X_i) \tag{5.11}$$

By use of these notations, the mutual information can be written as $I(P_F, P_D, p_1)$.

From [26], it is shown that the mutual information is a monotonically increasing function of P_D given P_F. Since the Neyman-Pearson detector maximizes P_D for a given P_F, it can be used to find the optimal decision, if P_F is fixed to a desired value. First, we define a random variable Z_i as the integral of the concentration at the receiver from the start of one signaling interval to the next one, as shown in Eq. 5.12.

$$Z_i = \int_{iT_s}^{(i+1)T_s} c(\vec{x}, t)\, dt \tag{5.12}$$

The Newman-Pearson detector uses the maximum likelihood test to find the optimal decision. Hence, the likelihood ratio is compared to a threshold defined as in Eq. 5.13.

$$\frac{p(Z_i; H_1)}{p(Z_i; H_0)} = \Lambda \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{5.13}$$

where $p(Z_i; H_1)$ is the probability density function of Z_i given hypothesis H_1 is true, and $p(Z_i; H_0)$ is the probability density function of Z_i given hypothesis H_0 is true. The threshold, λ, can be found from the solution of

$$P(\Lambda > \lambda | H_0) = P_F \tag{5.14}$$

To find the likelihood ratio, the distribution of the input bits has to be known. From the reviewed articles, the Bernoulli distribution with success probability p_1, seems to be a good choice. With the Bernoulli distribution, we get Z_i as a sum of independent variables, defined in Eq. 5.15

$$H_1: Z_i = a_0 Q_i + \sum_{j=i-N}^{i-1} a_{i-j} X_j Q_j$$

$$H_0: Z_i = \sum_{j=i-N}^{i-1} a_{i-j} X_j Q_j \tag{5.15}$$

where N is the memory of the channel and a_j is defined as

$$a_j = \int_0^{T_s} \frac{1}{8(\pi \frac{Kx}{u})^{3/2}} e^{(-(x-u(t-jT_s)^2-y^2)/\frac{4Kx}{u}} \left[e^{-(z-H)^2/\frac{4Kx}{u}} + e^{-(z+H)^2/\frac{4Kx}{u}} \right] dt \tag{5.16}$$

Because of the overlapping intervals, we get Intersymbol interference (ISI) in the communication system.

When Lindeberg´s condition is assumed fulfilled for the independent random variables in Eq. 5.15, the Lindeberg-Feller Central Limit theorem states that the sum of these variables converges towards the normal distribution when N increases towards infinity. This means that we have the hypothesis testing problem

$$H_1: z \sim \mathcal{N}(\mu_{Z_1}, \sigma_{Z_1}^2)$$

$$H_0: z \sim \mathcal{N}(\mu_{Z_0}, \sigma_{Z_0}^2) \tag{5.17}$$

where μ_{Z_i} is the mean value and $\sigma_{Z_i}^2$ is the variance of the random parameter Z_i. The probability density function for the normal distribution is defined as

$$p(Z_i; H_i) = \frac{1}{\sqrt{2\pi\sigma_{Z_i}^2}} e^{-\frac{(z-\mu_{Z_i})^2}{2\sigma_{Z_i}^2}} \tag{5.18}$$

To be able to find the likelihood ratio, we also need to find the mean value and the variance. This is found by the definition of these two values, stated in Eq.5.19.

$$\mu_{Z_i} = E\{Z_i\}$$

$$\sigma_{Z_i}^2 = E\{(Z_i - \mu_{Z_i})^2\} = E\{{Z_i}^2\} - 2E\{Z_i\}\mu_{Z_i} + \mu_{Z_i}^2 \tag{5.19}$$

Hence, we get the mean value of Z_0 and Z_1 as

$$\mu_{Z_0} = E\{Z_0\} = E\{X_j\}E\{Q_i\}\sum_{j=1}^{N} a_j = p_1\mu_Q \sum_{j=1}^{N} a_j$$

$$\mu_{Z_1} = E\{Z_0\} = a_0E\{Q_i\} + E\{X_j\}E\{Q_i\}\textstyle\sum_{j=1}^{N} a_j = a_0\mu_Q + \mu_{Z_0} \tag{5.20}$$

The variance of Z_0 and Z_1 is then given as

$$\sigma_{Z_0}^2 = (\sigma_X^2 + p_1^2)(\sigma_Q^2 + \mu_Q^2)\sum_{j=1}^{N} a_j^2 - 2p_1^2\mu_Q^2\sum_{j=1}^{N} a_j^2 + p_1^2\mu_Q^2\sum_{j=1}^{N} a_j^2$$

$$= \left((\sigma_X^2 + p_1^2)(\sigma_Q^2 + \mu_Q^2) - p_1^2\mu_Q^2\right)\sum_{j=1}^{N} a_j^2$$

$$\sigma_{Z_1}^2 = a_0^2\sigma_Q^2 + \sigma_{Z_0}^2 \tag{5.21}$$

From the definition of the variance of a Bernoulli distributed variable, we have that $\sigma_X^2 = (p_1 - p_1^2)$. By calculation of the parenthesis and use of this variance, the variance of Z_i is given as

$$\sigma_{Z_0}^2 = \left(p_1^2\sigma_Q^2 + \mu_Q^2(p_{1}-p_1^2) + \sigma_Q^2(p_{1}-p_1^2)\right)\sum_{j=1}^{N} a_j^2$$

$$\sigma_{Z_1}^2 = a_0^2\sigma_Q^2 + \sigma_{Z_0}^2 \tag{5.22}$$

All the parameters needed to find the likelihood ratio and by that define the detector, is now given. Hence, we get

$$\frac{p(Z_i;H_1)}{p(Z_i;H_0)} = \Lambda = \frac{\frac{1}{\sqrt{2\pi\sigma_{Z_1}^2}}e^{-\frac{(z-\mu_{Z_1})^2}{2\sigma_{Z_1}^2}}}{\frac{1}{\sqrt{2\pi\sigma_{Z_0}^2}}e^{-\frac{(z-\mu_{Z_0})^2}{2\sigma_{Z_0}^2}}} = \frac{\sigma_{Z_0}}{\sigma_{Z_1}}e^{\frac{(\sigma_{Z_1}^2-\sigma_{Z_0}^2)z^2-2(\mu_{Z_0}\sigma_{Z_1}^2-\mu_{Z_1}\sigma_{Z_0}^2)z+\mu_{Z_0}^2\sigma_{Z_1}^2-\mu_{Z_1}^2\sigma_{Z_0}^2}{2\sigma_{Z_0}^2\sigma_{Z_1}^2}} \tag{5.23}$$

By comparing the likelihood ratio with the threshold as in Eq. 5.13 and taking the natural logarithm of both sides, we get

$$\ln\frac{\sigma_{Z_0}}{\sigma_{Z_1}} + \frac{(\sigma_{Z_1}^2 - \sigma_{Z_0}^2)z^2 - 2(\mu_{Z_0}\sigma_{Z_1}^2 - \mu_{Z_1}\sigma_{Z_0}^2)z + \mu_{Z_0}^2\sigma_{Z_1}^2 - \mu_{Z_1}^2\sigma_{Z_0}^2}{2\sigma_{Z_0}^2\sigma_{Z_1}^2} \underset{H_0}{\overset{H_1}{\gtrless}} ln\,\lambda$$

$$(\sigma_{Z_1}^2 - \sigma_{Z_0}^2)z^2 - 2(\mu_{Z_0}\sigma_{Z_1}^2 - \mu_{Z_1}\sigma_{Z_0}^2)z + \mu_{Z_0}^2\sigma_{Z_1}^2 - \mu_{Z_1}^2\sigma_{Z_0}^2 \underset{H_0}{\overset{H_1}{\gtrless}} 2\sigma_{Z_0}^2\sigma_{Z_1}^2\left(ln\,\lambda - \ln\frac{\sigma_{Z_0}}{\sigma_{Z_1}}\right) \tag{5.24}$$

By solving the equation for z, we get a test statistic of z, stated in Eq. 5.25.

$$z \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\mu_{Z_0}\sigma_{Z_1}^2 - \mu_{Z_1}\sigma_{Z_0}^2 + \sqrt{\left(\mu_{Z_0}\sigma_{Z_1}^2 - \mu_{Z_1}\sigma_{Z_0}^2\right)^2 - \left(\sigma_{Z_1}^2 - \sigma_{Z_0}^2\right)\left(\mu_{Z_0}^2\sigma_{Z_1}^2 - \mu_{Z_1}^2\sigma_{Z_0}^2 - 2\sigma_{Z_0}^2\sigma_{Z_1}^2\left(ln\,\lambda - \ln\frac{\sigma_{Z_0}}{\sigma_{Z_1}}\right)\right)}}{\left(\sigma_{Z_1}^2 - \sigma_{Z_0}^2\right)} = \eta$$

(5.25)

where η is defined as the decision threshold. The decision whether the molecules are being sent or not is now based directly on the parameter z and is the hypothesis testing that is going to be used further on. This is a simplified version of the Neyman Pearson detector, which holds under the condition

$$\frac{\mu_{Z_0}^2}{2\sigma_{Z_0}^2} - \frac{\mu_{Z_1}^2}{2\sigma_{Z_1}^2} + \ln\frac{\sigma_{Z_0}}{\sigma_{Z_1}} < ln\,\lambda \tag{5.26}$$

This condition is derived from Eq. 5.24 when z=0. The false alarm probability, P_F, and the detection probability, P_D, can then be derived using the definition for bit error probability, BER, and is defined in Eq. 5.27.

$$P_F = \int_{\eta}^{\infty} p(Z_i; H_0)\,dz = Q\left(\frac{\eta - \mu_{Z_0}}{\sigma_{Z_0}}\right)$$

$$P_D = \int_{\eta}^{\infty} p(Z_i; H_1)\,dz = Q\left(\frac{\eta - \mu_{Z_1}}{\sigma_{Z_1}}\right) \tag{5.27}$$

The optimal value of η, η_{opt} can be found from the argument of the maximal value of the mutual information.

$$\eta_{opt}(p_1) = arg\max_{\eta} I\left(P_F(\eta_{opt}), P_D(\eta_{opt}), p_1\right) \tag{5.28}$$

Since it is possible for the receiver to control the a priori probability, the optimal value for p_1 is given as

$$p_1^*(\eta) = arg\max_{p_1} I\left(P_F(\eta_{opt}(p_1)), P_D(\eta_{opt}(p_1)), p_1\right) \tag{5.29}$$

A summary of the mean values and the variances of the parameters for the SISO system is given in Table 2.

Table 2: Mean value and variance for SISO communication system

Variables	**Definitions**
Mean value of Q_i, μ_Q	1 mmol
Variance of Q_i, σ_Q^2	$(0{,}3\mu_Q)^2$
Mean value of X_i, μ_X	p_1
Variance of X_i, σ_X^2	$p_1 - p_1^2$
Mean value of Z_0, μ_{Z_0}	$\mu_Q \sum_{j=1}^{N} a_j$
Mean value of Z_1, μ_{Z_1}	$a_0\mu_Q + \mu_{Z_0}$
Variance of Z_0, $\sigma_{Z_0}^2$	$\left(p_1^2\sigma_Q^2 + \mu_Q^2(p_1 - p_1^2) + \sigma_Q^2(p_1 - p_1^2)\right)\sum_{j=1}^{N} a_j^2$
Variance of Z_1, $\sigma_{Z_1}^2$	$a_0^2\sigma_Q^2 + \sigma_{Z_0}^2$

5.8 Performance analysis for the SISO communication system

To evaluate the performance of the complete SISO communication system including the encoder, channel and decoder, a rate distortion analysis was applied. Distortion is defined as the difference between the output signal and the input signal. The signaling rate is defined in Eq. 5.30.

$$Signaling\ rate = \frac{Mutual\ information}{Signaling\ interval} = \frac{I(P_D,P_F,p_1)}{T_S} per\ second \tag{5.30}$$

The difference between the output signal and input and the data rate were evaluated at different signaling intervals to see how the signaling interval effected the performance.

An analysis of the distributions of the random variables, Z_0, and Z_1, the Intersymbol interference (ISI) and the mutual information was also performed to study the performance of the communication system.

5.9 Single-input Multiple-output (SIMO) communication system with Receiver Diversity

According to the authors in [48] , the use of more than one sensor at the transmitter and/or receiver end would improve the performance of the communication system. To see if this is the case for this particular communication system, a combination between a SIMO system in Diversity mode and a MIMO system with Spatial Multiplexing mode is going to be implemented.

For the diversity mode, Receive diversity with Maximum-ratio combining (MRC) is going to be tested. Up until now, only one transmit antenna and one receive antenna has been used. The communication system will for the SIMO case, however, consist of one transmit antenna and two receiver antennas with the distances d_{11} and d_{12} between the transmit and receive antennas. This is shown in Figure 10.

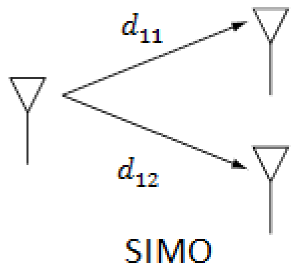

Figure 10: SIMO communication system with distances d_{11} and d_{12} between the transmit and receive antennas

With this new combination of antennas and the use of MRC, the random independent variable, Z_i, will get a new definition as

$$Z_i = \sum_{n=1}^{2} b_n Z_n \tag{5.31}$$

where Z_n is the previously used random independent variable from the SISO system, n indicates which channel path the molecules is transmitted through, and b_n is the weight multiplied with Z_n to strengthen or attenuate the signal components. b_n is defined in Eq. 5.32.

$$b_n = \frac{h_p(d_n)}{\sigma_{I_n}^2} \tag{5.32}$$

where $h_p(d_n)$ is a constant dependent on the distance from the transmitter to the receiver, as defined in Eq. 5.33.

$$h_p = \left(\frac{4\left(\frac{Kd_n}{u}\right)}{8\pi\left(\frac{Kd_n}{u}\right)^{3/2}}\right)\left(2e^{-\frac{H^2}{4\left(\frac{Kd_n}{u}\right)}}\right) \tag{5.33}$$

$\sigma_{I_n}^2$ is the variance of the interference, which equals the variance of Z_0 from the SISO system and is defined in Eq. 5.34.

$$\sigma_{I_n}^2 = \sigma_{Z_0,SISO}^2 \tag{5.34}$$

The complete communication system with use of MRC is shown in Figure 11.

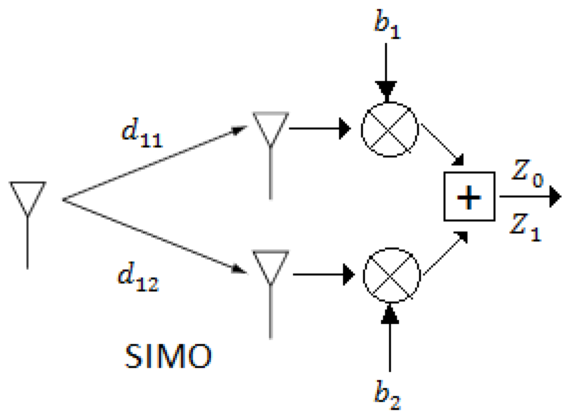

Figure 11: The SIMO communication system with use of the diversity technique, MRC

The new mean value and variance of Z_i is calculated with the same formulas as before, and defined as

$$\mu_{Z_0} = \sum_{n=1}^{2} b_n \mu_{I_n}$$

$$\mu_{Z_1} = \sum_{n=1}^{2} b_n (\mu_Q h_p(d_n) + \mu_{I_n})$$

$$\sigma_{Z_0}^2 = \sum_{n=1}^{2} b_n^2 \sigma_{I_n}^2$$

$$\sigma_{Z_1}^2 = a_0^2 \sigma_Q^2 + \sigma_{Z_0}^2 \qquad (5.35)$$

Where μ_{I_n} is the mean value of the interference, which equals the mean value of Z_0 from the SISO system and is defined in Eq. 5.36.

$$\mu_{I_n} = \mu_{Z_0,SISO} \qquad (5.36)$$

A summary of the mean values and the variances of the variables in the SIMO communication system is given in Table 3.

Table 3: Mean value and variance for SIMO communication system

Variables	**Definitions**
Mean value of Q_i, μ_Q	1 mmol
Variance of Q_i, σ_Q^2	$(0{,}3\mu_Q)^2$
Mean value of X_i, μ_X	p_1
Variance of X_i, σ_X^2	$p_1 - p_1^2$
Mean value of Z_0, μ_{Z_0}	$\sum_{n=1}^{2} b_n \mu_{I_n}$
Mean value of Z_1, μ_{Z_1}	$\sum_{n=1}^{2} b_n(\mu_Q a_j(d_n) + \mu_{I_n})$
Variance of Z_0, $\sigma_{Z_0}^2$	$\sum_{n=1}^{2} b_n^2 \sigma_{I_n}^2$
Variance of Z_1, $\sigma_{Z_1}^2$	$a_0^2\sigma_Q^2 + \sigma_{Z_0}^2$

5.10 MIMO communication system with Spatial Multiplexing (SM)

For the MIMO case, the communication system will consist of two transmit antennas and two receive antennas, and is shown in Figure 12 [48].

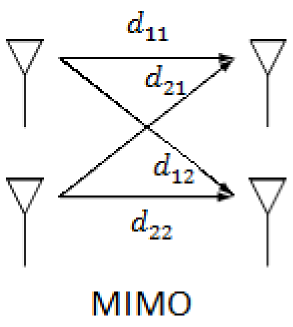

Figure 12: MIMO communication system with distances d_{11}, d_{12}, d_{21} and d_{22} between the transmit and receive antennas

In contrast to diversity techniques, where the same bit sequence is sent through both channel paths, different bit sequences will be sent through the channel paths with Spatial Multiplexing (SM). The MIMO system with SM will be equivalent to the sum of two SIMO systems, but with a different definition of the parameter Z_i and its mean value and variance. To simplify the calculations and easier be able to decide the optimal threshold, the SIMO communication system will be divided in two parts where only one of the transmitters will be considered at a time. However, the interference in the signal paths from the other transmitter will be taken into account. In Figure 13 and 14, the complete MIMO communication system is displayed where the black parts in the figure is in use, and the red part is temporarily unused.

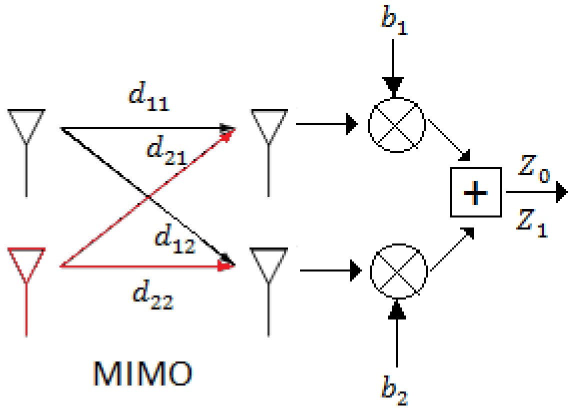

Figure 13: The MIMO system part 1 where transmitter 1 is considerated

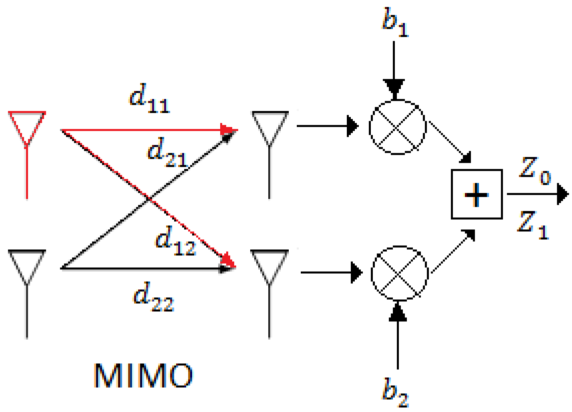

Figure 14: The MIMO system part 2 where transmitter 2 is considerated

The new definitions of the Z_i variable, the mean values, μ_{Z_0} and μ_{Z_1} and the variances, $\sigma^2_{Z_0}$ and $\sigma^2_{Z_1}$ is given in Eq. 5.37.

$$Z_i = X_n Q_n h_p(d_{nn}) + I_n + \sum_{m=1,m\neq n}^{2} X_m Q_m h_p(d_{mn})$$

$$\mu_{Z_0} = \mu_{I_n} + \sum_{m=1,m\neq n}^{2} p_m \mu_Q h_p(d_{mn})$$

$$\mu_{Z_1} = \mu_Q h_p(d_{nn}) + \mu_{Z_0}$$

$$\sigma^2_{Z_0} = \sigma^2_{I_n} + \sum_{m=1,m\neq n}^{2} (p_m - p_m^2){\mu_Q}^2 h_p(d_{mn})^2$$

$$\sigma^2_{Z_1} = a_0^2 \sigma_Q^2 + \sigma^2_{Z_0} \tag{5.37}$$

In the MIMO communication system the interference is defined as the signals through all the other paths than the one that the molecules are sent through in that instant. This is defined in Eq. 5.38, where I_{11} is the interference when the molecules are sent from transmit sensor 1 to receive sensor 1, I_{12} is the interference when the molecules are sent from transmit sensor 1 to receive sensor 2, I_{21} is the interference when the molecules are sent from transmit sensor 2 to receive sensor 1 and I_{22} is the interference when the molecules are sent from transmit sensor 2 to receive sensor 2.

$$I_{11} = Z_{12} + Z_{21} + Z_{22}$$

$$I_{12} = Z_{11} + Z_{21} + Z_{22}$$
$$I_{21} = Z_{11} + Z_{12} + Z_{22}$$
$$I_{22} = Z_{11} + Z_{12} + Z_{21} \quad (5.38)$$

That means that the filter coefficient, a_0 has to be included in the mean value and variance of the interference. The mean value and the standard deviation of the interference is defined as

$$\mu_{I_n} = 2 * 10^{16}$$
$$\sigma_{I_n} = 0{,}3\mu_{I_n} \quad (5.39)$$

A summary of the mean values and the variances of the variables in the MIMO communication system is given in Table 4.

Table 4: Mean value and variance for MIMO communication system

Variables	**Definitions**
Mean value of Q_i, μ_Q	1 mmol
Variance of Q_i, σ_Q^2	$(0{,}3\mu_Q)^{\mathbf{2}}$
Mean value of X_i, μ_X	p_1
Variance of X_i, σ_X^2	$p_1 - p_1^2$
Mean value of Z_0, μ_{Z_0}	$\mu_{I_n} + \sum_{m=1, m\neq n}^{2} p_m \mu_Q a_j(d_{mn})$
Mean value of Z_1, μ_{Z_1}	$\mu_Q a_j(d_{nn}) + \mu_{Z_0}$
Variance of Z_0, $\sigma_{Z_0}^2$	$\sigma_{I_n}^2 + \sum_{m=1, m\neq n}^{2} (p_m - p_m^2){\mu_Q}^2 a_j(d_{mn})^2$
Variance of Z_1, $\sigma_{Z_1}^2$	$a_0^2\sigma_Q^2 + \sigma_{Z_0}^2$

5.8 Performance analysis for the SIMO and the MIMO communication system

To be able to determine if the SIMO and MIMO communication system increased the performance of the communication, the same performance analysis was performed for these two as was done for the SISO system. The difference between the input signal and the output was investigated. The data rate and the distortion were compared. Additionally, the parameters maximum mutual information and ISI was displayed and discussed.

6 Results and discussion

In this chapter, the results will be displayed and discussed. The results from both the SISO communication scheme with deterministic signaling and only one signaling interval, and the SISO system with stochastic signaling and multiple signaling intervals, will be given. A plot of the concentration of the molecules through the diffusion channel will be shown first in both parts. In the simplest version, the gain and the delay will be displayed to be able to discuss how suitable the physical channel is for this communication system.

In the extended SISO communication system with stochastic signaling, histograms of the distributions of Z_0 and Z_1, will be presented to see if they are as expected as Gaussian distributed variables. The optimal value of the thresholds to decide between the two variables, Z_0 and Z_1, and the optimal value of the a priori probability, p_1 will be presented. With this established, the results from comparing the input and output signal will be given with different signaling intervals, T_S. Parameters as the bit error rate (BER), and the rate distortion will be displayed to discuss how different signaling intervals and bit rate effect the performance of the communication system. The ISI effect with different signaling intervals will also be evaluated.

The results from the SIMO and the MIMO communication will then be displayed. To be able to discuss if these two communication schemes with diversity and Spatial Multiplexing techniques did optimise the performance system compared to the SISO communication system, the same parameters will be presented and discussed.

6.1 The deterministic communication system

6.1.1 The concentration of the molecules c(x,y,z,t) with deterministic signaling

In the simulations, the concentration of the molecules, c(x,y,z,t) was plotted as contour plot. This is displayed in Figure 15.

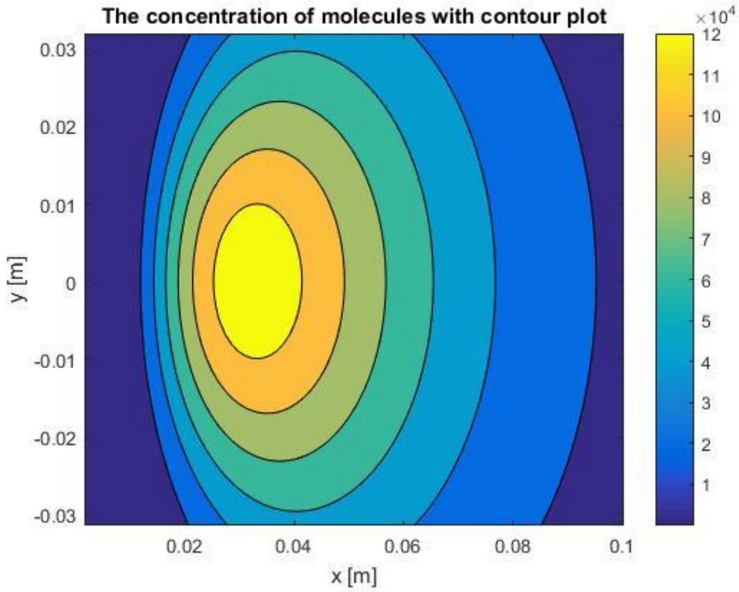

Figure 15: The concentration of the molecules in the deterministic SISO communication system dependent on the distance travelled by the molecules in the x- and y-dimension

Here we can see the diffusion part of a transmission scheme. The inner circle is where the molecule is emitted from and the diffusion is the waves out from this point. If you throw a stone in the water, you get circles spreading out from every spot the stone hits in the water. These waves moving out from the emitting start point could be compared with the emitting of molecules and the diffusion towards the receiver. This plot shows that the model used, is a reasonable model for the molecular communications in the pacemaker since the concentration diffused like anticipated. Next, we are discussing the performance analysis of some important parameters.

6.1.2 Performance analysis

In the next chapters a performance analysis of the deterministic SISO communication system will be presented. The concentration of the molecules depending on the distance the molecules travel in x-direction and the time it takes them is first displayed. The gain and the delay of the concentration is then displayed.

6.1.2.1 The concentration

In the performance analysis, two parameters were studied. The first one is the gain or the attenuation of the concentration. This will be defined as the ratio between the maximum concentration varied over time, c_t, and the absolute maximum value of the concentration at any time, c_0, in Eq. 6.1.

$$gain = \frac{c_t}{c_0} = \frac{\max\{c(x,y,z,t)\}}{\max[\max\{c(x,y,z,t)\}]} \tag{6.1}$$

The value, c_0, should be close to the transmitter in space and soon after the molecules have been emitted in time to fulfil the concept of diffusion.

The second parameter is the delay of the molecules, which means how long time it takes from the molecules are emitted at the transmitter until we have the maximum concentration. This will be defined as the argument of the maximum concentration in Eq. 6.2.

$$delay = argmax\{c(x, y, z, t)\} \tag{6.2}$$

To visualize these two expressions, a new plot of the concentration was made with regular plot. Here the concentration depends on the time. This is displayed in figure 16.

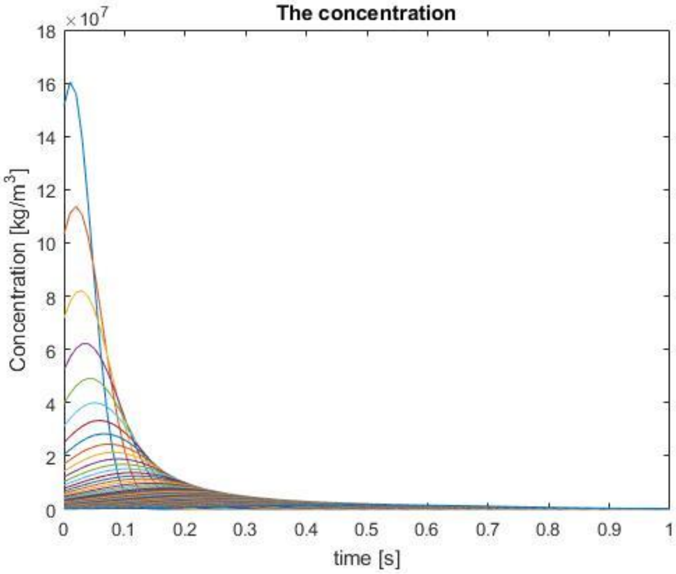

Figure 16: The concentration of the molecules in the deterministic SISO communication system dependent on the distance travelled by the molecules in the x-direction in meters and the time in seconds, where each curve represents different distances

From the plot we can see that the absolute maximum value of the concentration, c_0, is the peak of approximately $16 \cdot 10^7\ kg/m^3$. The delay is the time where we find this peak value, which is at approximately 0,01 s.

The different curves in Figure 16 represent the concentration of the molecules at different distances from the transmitter varied over time. Since the curve with the highest concentration represents the distance closest to the transmitter and the curve with the lowest concentration represents the distance furthest away from the transmitter, we see that c_0, is placed where and when it is expected to be. Hence, the model for the concentration achieves one of the effects that should be present when there is diffusion. After this maximum point, we can see that the concentration disperses over time and distance and therefore decreases when the molecules diffuses towards the receiver. In reality, there is a delay before the maximum concentration of the molecules is reached. This can also be seen in the plot.

We can see from the plot in Figure 16 that the curves from the different distances from the transmitter have their maximum concentration at different times. Thus, their delay is different. The delay of the maximum concentration increases with the distance because the molecules are dispersing away from where they have their maximum concentration. This makes the maximum concentration move from the transmitter towards the receiver. We can also see that this maximum concentration decreases gradually in magnitude while the molecules move. The reason for this is that diffusion reaches equilibrium when the dispersion is equally distributed at all the distances. Therefore, in theory, there should be the same amount of molecules at the transmitter than at the receiver at the time where the molecules have reached the receiver. In reality, some of the molecules will also disappear to the environment. Hence, the concentration of the molecules will be less at the receiver than the transmitter at the time they have reached equilibrium. Neither of this is the case for the plot. In the plot, at approximately 0,08 s after the molecules are emitted, the concentration further away from the transmitter is actually higher than near the transmitter. This makes a flaw of the suggested model of the concentration. After some time, however, the concentration at different distances eventually evens out, as it should.

6.1.2.2 The gain

To see clearer how the gain and delay changes with the distance from the transmitter, two plots of the gain and the delay separately depending on the distance the molecule is moving, were made. This is shown in Figure 17 and 18, respectively.

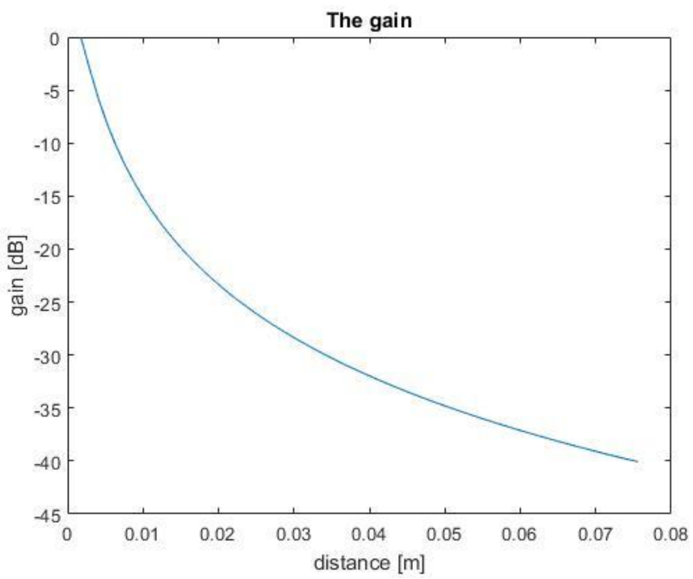

Figure 17: The gain of the concentration in the deterministic SISO communication system

The gain is now plotted in a logarithmic scale. This is because it is easier to see the small differences. From the plot in Figure 17, we can see that the gain is equal to 0 dB at the closest distance to the transmitter, which means that the ratio between the maximum concentration and the absolute maximum concentration is 1. Thus, these two values are the same. Then the gain, decreases depending on the distance. Hence, the maximum concentration decreases relatively to the absolute maximum concentration while the molecules move towards the receiver. This is the anticipated effect of the concentration, because the diffusion makes the molecules disperse. Therefore, the result from the gain supports the model of the communication system.

The distance in this plot is the moving distance for the molecules from the transmitter towards the receiver, in only x-direction. This approximation can be made because the x-direction is the same direction that the blood velocity's main direction, u_m.

The more accurate distance could be expressed by a distance d and an angle θ. The parameter d is defined as

$$d = \sqrt{(x_R - x_T)^2 + (y_R - y_T)^2} \tag{6.3}$$

where x_T is the start point for the molecules at the transmitter sensor in x-direction, x_R is the endpoint at the receiver sensor in x-direction, y_T is the start point for the molecules at the transmitter sensor in y-direction, y_R is the endpoint at the receiver sensor in y-direction. We can disregard the z-direction, because the movement in the depth is of less meaning to the gain and delay. The parameter θ is defined in Eq. 6.4.

$$\theta = tan^{-1}\left\{\frac{y_R - y_T}{x_R - x_T}\right\} \tag{6.4}$$

The angle θ is the angle the molecules are moving towards from the transmitter. These two parameters are going to be used in later work, but we are keeping the approximation for now.

6.1.2.3 The delay

The delay of the maximum concentration is plotted in Figure 18.

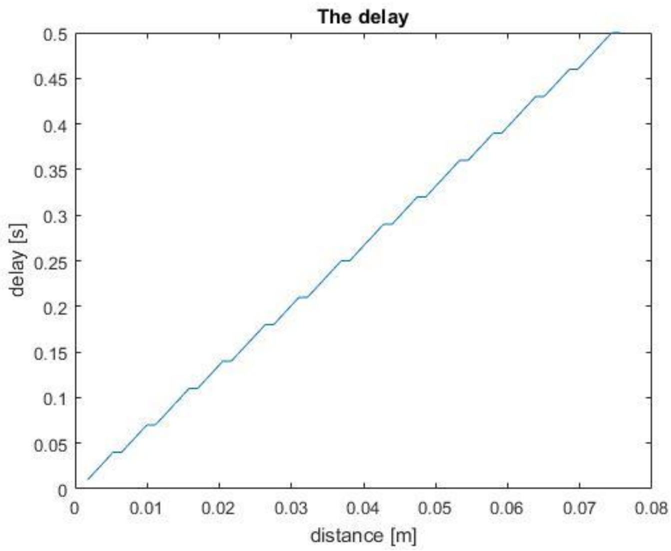

Figure 18: The delay of the concentration in the deterministic SISO communication system

In the plot of the delay, we see that it almost has a linear ascending curve dependent on the distance. The reason for this is the assumed constant blood velocity and the definition of the main blood flow velocity direction along the x-axis. Because of the diffusion, the maximum concentration moves towards the receiver with this blood velocity. Thus, the delay will increase slowly with the distance from the transmitter. This effect can be seen in the plot. When the distance from the transmitter increases, so will the delay. Therefore, there will not be several distances with the same delay. The parts in Figure 18, where the delay is constant for a short distance at a time, is most likely due to the time resolution in the simulations. With shorter time between every time step, we would not have these constant delays. Hence, the delay is also showing that the model of the concentration is a fit model for the communication system.

6.2 The stochastic SISO communication system

The results from the stochastic SISO communication system will in the following chapters be displayed and discussed. Most figures and tables will be based on 10000 transmitted bits to get accurate results. Some of the plots, however, will be based on 1000 transmitted bits to make it easier to see the details in the plots. The signaling interval, T_s used in most of the plots, is 150 seconds. The reason for this is that this signaling interval gives desired results with limited ISI as we will see in Chapter 6.2.8. The distance between the transmitter and the receiver is sat to 0,03 meters.

6.2.1 The concentration of the molecules for the SISO communication system with stochastic signaling and multiple signaling intervals, T_s

A plot of the concentration of the molecules through the diffusion channel with stochastic signaling, is displayed in Figure 19. This plot is based on 1000 transmitted bits and a signaling interval of T_s=150 s.

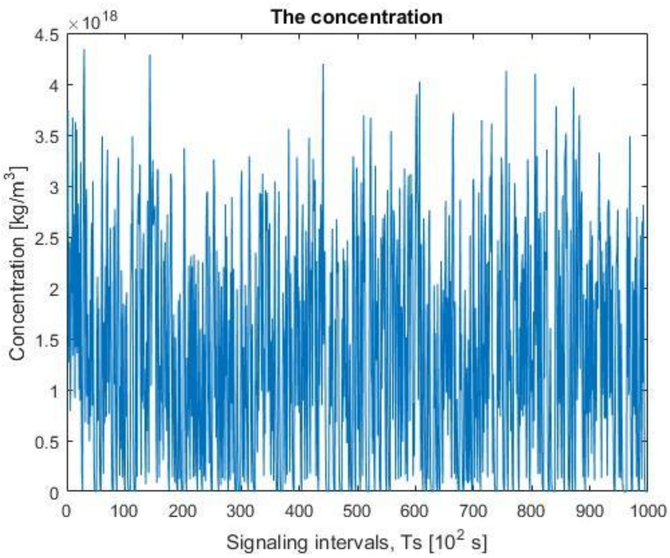

Figure 19: The concentration of the molecules in the stochastic SISO communication system dependent on the number of signaling intervals, T_s, where each signaling interval is 150 seconds

From the plot we can see a large peak in the concentration every time molecules are transmitted, which is when X_i equals one. These peaks should have individual amplitudes because of the stochastic input signal, $Q_{i,}$, which means that random amounts of molecules are sent per second for every signaling interval, $\mathrm{T_s}$. This can be seen in the plot. As expected, the amplitude drops for every signaling interval where no molecule is sent. However, the amplitudes is never zero even though no molecules are sent. The Brownian motion of the molecules and the memory of the channel is the cause of this. The stochastic version of the concentration is therefore also a good model for the communication system.

6.2.2 The distribution of Z_0 and Z_1

A histogram of the distribution of the random variable, Z_1, is displayed in Figure 20. It is based on 10000 transmitted bits and a signaling interval of 150 seconds.

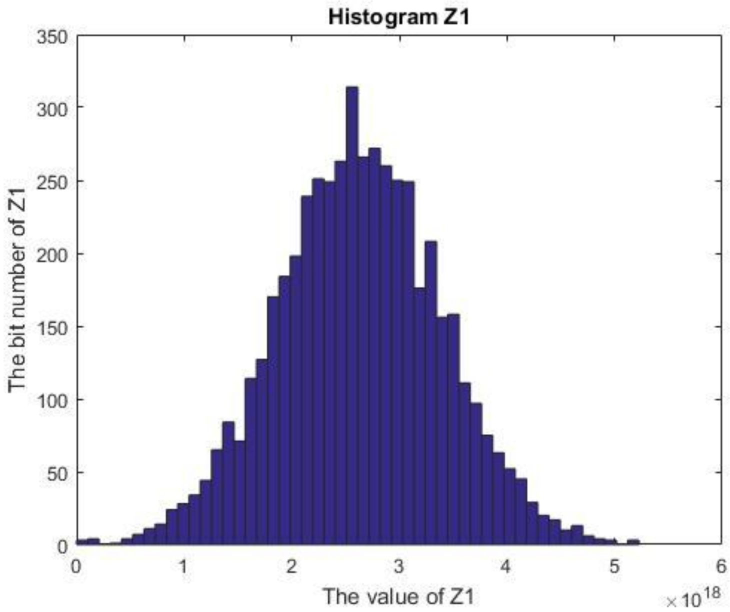

Figure 20: The distribution of Z_1 in the stochastic SISO communication system when T_s = 150 s

As expected, we can see from Figure 20, that the distribution of Z_1 has a Gaussian shape. The mean value is at approximately 2,6*10^{18}. This is about the same value as the mean value of Q, μ_Q, multiplied by the filter coefficient, a_0. As explained in the methodology, Q is the number of transmitted molecules per second, while a_0 is the current attenuation of the concentration. The mean value of Z_1 is therefore reasonable. The variance is about 5,3*10^{18}.

A histogram of the distribution of the random variable, Z_0, is displayed in Figure 21. This is based on the same amount of transmitted bits and signaling interval as in Figure 20.

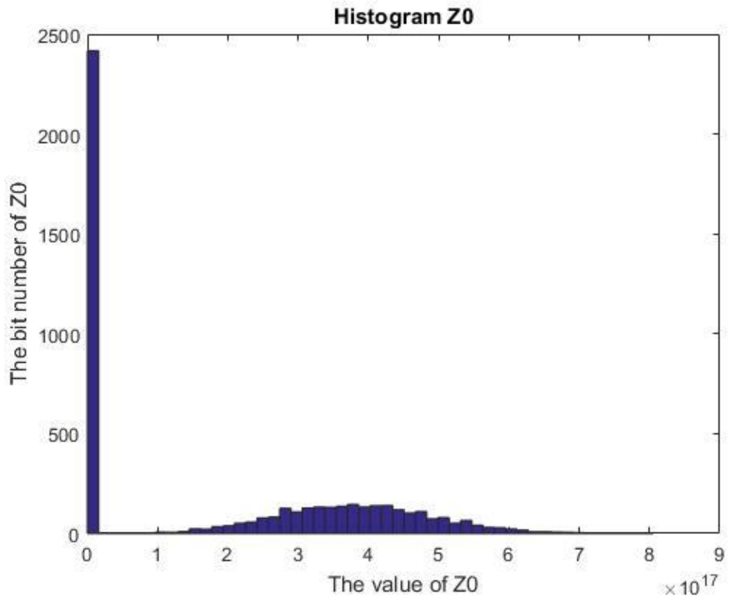

Figure 21: The distribution of Z_0 in the stochastic SISO communication system when T_s= 150 s

From the plot we can see that the distribution of Z_0 also has a Gaussian shape as expected. The mean value is at approximately 3,8*10^{17} and the variance is close to 6,7*10^{17}, which is closer to zero compared to the values of Z_1, by a factor of 10. In addition to this, the distribution has a significant peak at zero. Since the variable Z_0 is supposed to represent the case that no molecules are transmitted, it is reasonable that the values is this low compared to Z_1. When Z_0 equals zero, it means that two zeros has been transmitted consecutively. This will by statistics, happen with 25 % probability since the probability for transmitting a zero at one random bit number is 50 % and the probability for transmitting a zero given that the last bit was a zero is 50 % multiplied by 50 %, which equals 25 %. Since the peak is at approximately 2450 out of 10000 transmitted bits which equals 24,5 %, the distribution of Z_0 is as expected.

Figure 20 and 21 shows that the distributions of Z_1 and Z_0 have a small overlapping interval. This is the cause of ISI and distortion, which will be investigated closely in Chapter 6.2.6 and 6.2.8.

6.2.3 The optimal thresholds, η and λ, and the optimal a priori probability, p_1 with different signaling intervals, T_s

The maximum mutual information is used to find the optimal values of the threshold, η, the threshold, λ and the a priori probability, p_1. Since the simplified version of the Neyman Pearson detector is used, only the threshold, η, is used to decide between the two hypothesis that molecules are transmitted or not. The threshold, λ, is only stated as information. A plot of the mutual information dependent on the threshold, η, when the signaling interval, T_s=150 s based on 10000 transmitted bits, is shown in Figure 22.

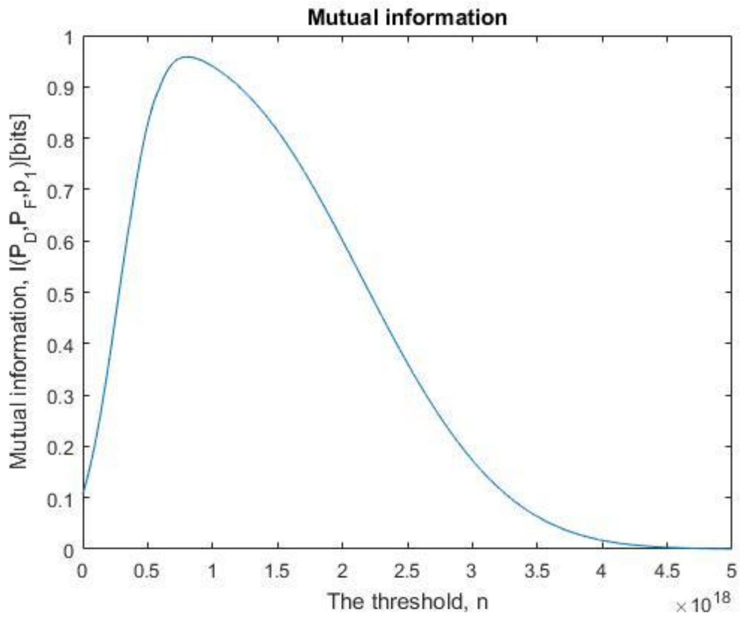

Figure 22: The mutual information in the stochastic SISO communication system dependent on the optimal threshold, η with T_s = 150 s

From the plot we can see that the maximum mutual information is approximately 0,96 bits and occur when the threshold, η is about 0,7*10^{18} with the signaling interval, T_S=150. In Table 5, the maximum mutual information, the optimal threshold, η, the optimal threshold, λ and the optimal a priori information, p_1 with different signaling intervals, T_s, is displayed.

Table 5: The maximum mutual information, the optimal thresholds, η and, λ and the optimal a priori probability p_1

T_s	1 s	10 s	50 s	60 s	100 s	150 s	500 s
η	6,4*10^{16}	43,2*10^{16}	118,9*10^{16}	133,7*10^{16}	128,4*10^{16}	81*10^{16}	7,8*10^{16}
λ	3,1036*10^{5}	0,1999	0,0033	0,0032	0,0160	1,1966	4527,5
p_1	0,1	0,1	0,3	0,4	0,5	0,5	0,5
I_{max}	20,76*10^{-3} bits	0,05 bits	0,32 bits	0,40 bits	0,74 bits	0,958 bits	0,997 bits

From the table, we can see that the maximum mutual information, I_{max} increases when the signaling interval, T_s increases. This is as expected. The a priori probability, p_1 is also increasing. When the capacity of the channel increases in form of the maximum mutual information, it is possible to transmit through the channel, it is reasonable that the optimal probability for sending molecules is rising. From the table, we also see that the optimal threshold, η is increasing until 60 s, and is decreasing with higher signaling intervals. The optimal threshold, λ, has the opposite effect. It is decreasing up until the signaling interval is 60 s, and after that it is increasing.

6.2.4 The data rate

The data rate, R, is defined as the mutual information divided by the signaling interval, T_s. The data rate, R based on 10000 transmitted bits with the signaling interval of 150 seconds, is shown in Figure 23.

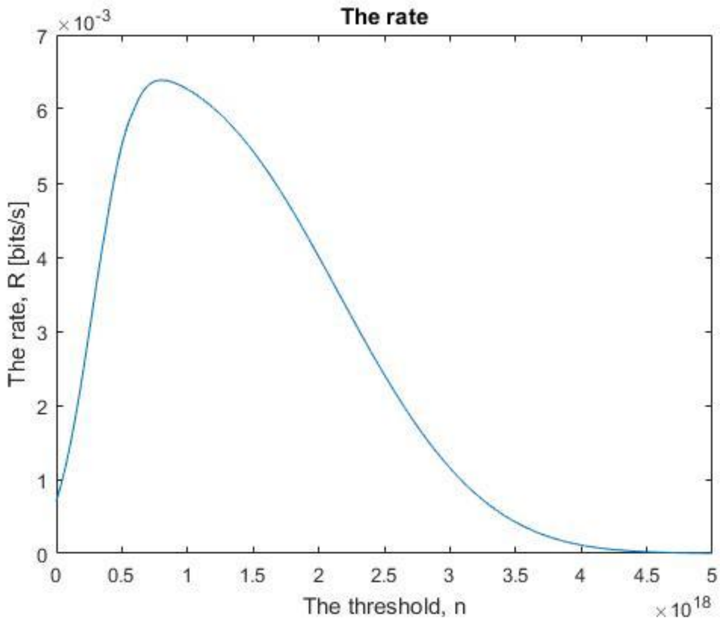

Figure 23: The rate, R with the stochastic SISO communication system when T_s = 150 s

From the plot we can see that the maximum data rate is about 6,5*10^{-3} bits/seconds and occur when the threshold, η is about 0,7*10^{18}. In Table 6, the maximum rate with different signaling intervals is displayed.

Table 6: The maximum rate, R_{max}

T_s	1 s	10 s	50 s	60 s	100 s
R_{max}	20,76*10^{-3} bits/s	4,99*10^{-3}bits /s	6,35*10^{-3}bits/s	6,68*10^{-3}bits/s	7,40*10^{-3}bits/s
T_s	110 s	130 s	150 s	200 s	500 s
R_{max}	7,37*10^{-3}bits/s	6,99*10^{-3}bits /s	6,39*10^{-3}bits/s	4,96*10^{-3}bits/s	1,99*10^{-3}bits/s

From the plot we can see that the maximum rate increases with higher signaling intervals up until the signaling interval equals 100 seconds. After that, the maximum rate decreases. The reason for this will be discussed in Chapter 6.2.9.

6.2.5 The output signal, Y_i vs the input signal, X_i

The output signal, and the input signal based on 1000 transmitted bits with signaling interval, T_s of 150 seconds, is plotted in Figure 24 and 25. The first plot shows the signals separately with the input signal in red and the output signal in blue, and the second plot shows the signals with overlapping plots.

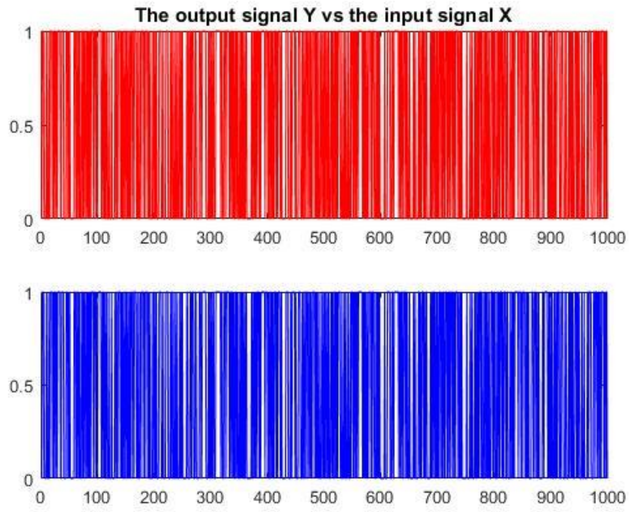

Figure 24: The output signal, Y_i (blue) vs the input signal, X_i (red) with the stochastic SISO communication system

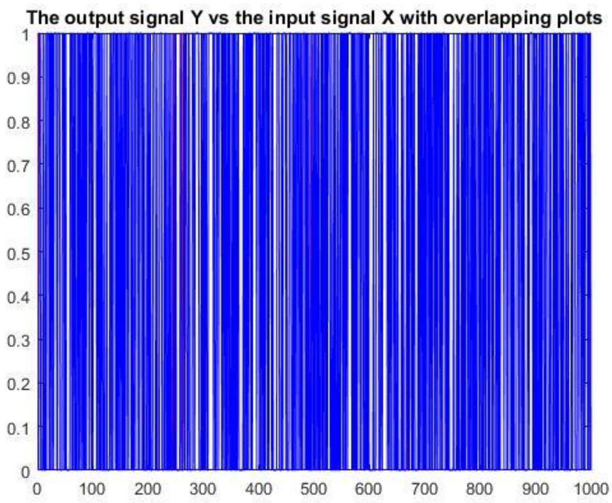

Figure 25: The output signal, Y_i (blue) vs the input signal, X_i (red) with the stochastic SISO communication system with overlapping plots

From Figure 24 and 25 we can see that when the signaling interval is 150 seconds, the input and output signals is almost identical. There is minimal ISI in the SISO communication system.

6.2.6 The distortion

The distortion, D, is defined as the difference between the output signal, Y_i and the input signal, X_i. In Figure 26 to 29, the bit number where the output signal and the input signal differ based on 10000 transmitted bits, is shown with different signaling intervals.

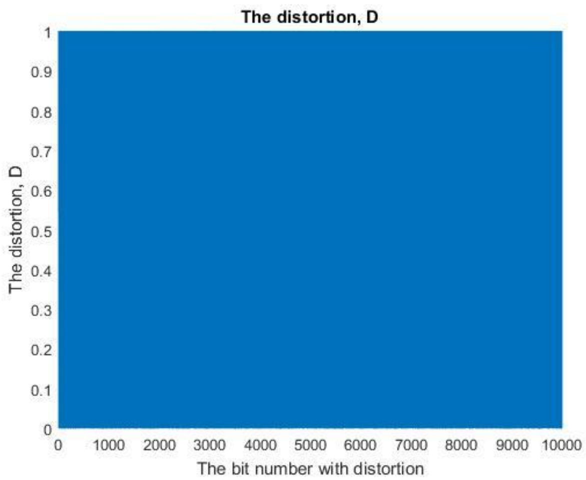

Figure 26: The distortion, D s with the stochastic SISO communication system and signaling interval T_s = 50 s

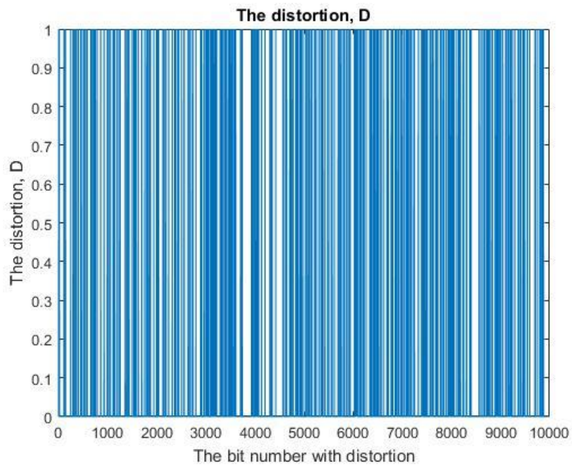

Figure 27: The distortion, D s with the stochastic SISO communication system and signaling interval T_s = 100 s

In Figure 26, the signaling interval, T_s is only 50 seconds. With 10000 transmitted bits, the number of bits that differ is so significant that the plot is completely blue, and you would have to zoom in to see where they are. The purpose of these plots is only to get the roughly impression of how the distortion changes with an increasing signaling interval. The accurate average error probability will, however be given in Chapter 6.8.1. With a signaling interval of 100 seconds, the distortion is clearly reduced. This can be seen in Figure 27.

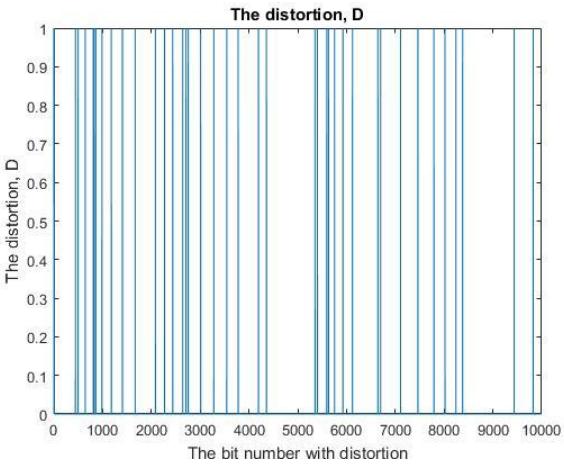

Figure 28: The distortion, D s with the stochastic SISO communication system and signaling interval T_s = 150 s

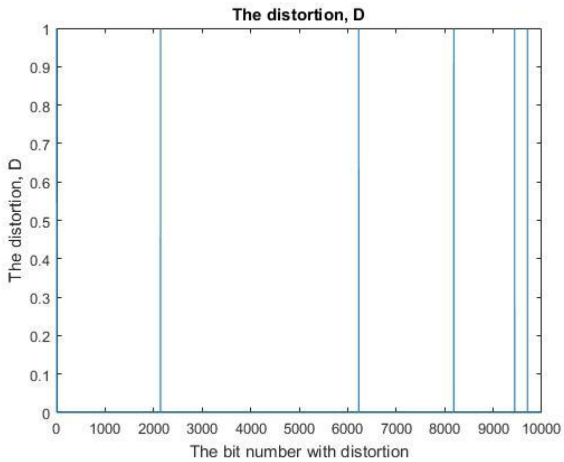

Figure 29: The distortion, D s with the stochastic SISO communication system and signaling interval T_s = 200 s

In Figure 28, the signaling interval is 150 seconds. The distortion is further reduced. This is a reasonable amount of distortion because a small amount of ISI is expected in a communication system like this.

The signaling interval is 200 seconds in Figure 29. In this case, the distortion is almost not present. Hence, the signaling interval has a significant influence on the amount of distortion we get, and therefore the quality of the communication system. This low amount of distortion is probably too low to be representing a realistic communication system. Therefore, the signaling interval of 150 seconds seems to be an optimal interval.

6.2.6.1 The average error probability, BER

The average error probability, BER, is defined as the number of bit errors divided by the number of transmitted bits. Based on 10000 transmitted bits, Table 7 shows the development in BER when T_s increases.

Table 7: The average error probability, BER

T_s	1 s	10 s	50 s	80 s	100 s	110 s	130 s	150 s	500 s
BER	0,5012	0,4973	0,2087	0,0955	0,0401	0,0270	0,0103	0,0039	0,001

As expected, the average error probability, BER, decreases, when the signaling interval increases.

6.2.7 The rate distortion

The rate distortion is defined as the relationship between the rate and the distortion. In Table 8, these two parameters is displayed with different signaling intervals, T_s.

Table 8: The rate distortion

T_s	1 s	10 s	50 s	80 s	100 s
R_{max}	20,76*10^{-3} bits/s	4,99*10^{-3}bits /s	6,35*10^{-3}bits/s	7,25*10^{-3}bits/s	7,40*10^{-3}bits/s
BER	0,5012	0,4973	0,2087	0,0955	0,0401
T_s	110 s	130 s	150 s	200 s	500 s
R_{max}	7,37*10^{-3}bits/s	6,99*10^{-3}bits /s	6,39*10^{-3}bits/s	4,96*10^{-3}bits/s	1,99*10^{-3}bits/s
BER	0,0270	0,0103	0,0039	0,0005	0,001

From the table, it is given that the maximum rate increases when the distortion decreases when the signaling is between 10 and 100 seconds. When the signaling interval is above 100 seconds, the rate decreases when the distortion decreases. The reason for this will be explained in Chapter 6.2.9.

6.2.8 Intersymbol interference (ISI)

The Intersymbol interference (ISI) in the SISO communication system, is the values of the random independent Z variable not equal to zero, that correspond to transmitting X_i=0. That means that the Z variable has a value other than zero, when it should have been equal to zero. Since the variable Z_0 represent this case of the Z variable, a histogram of Z_0 is plotted. The ISI effect is shown with different signaling intervals, T_s , in Figure 30 to 35. All the plots is based on 10000 transmitted bits.

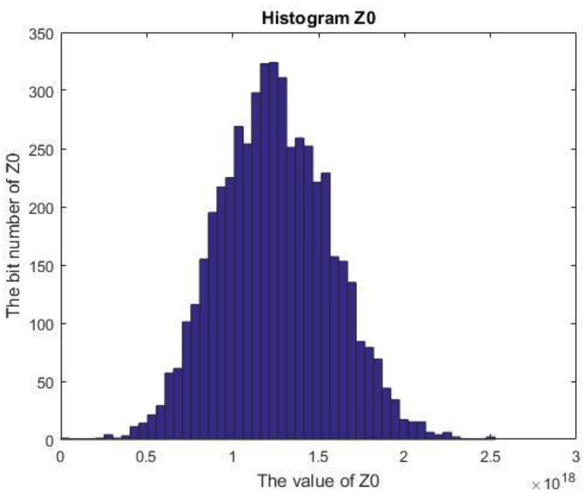

Figure 30: The ISI with the stochastic SISO communication system and signaling interval T_s = 10 s

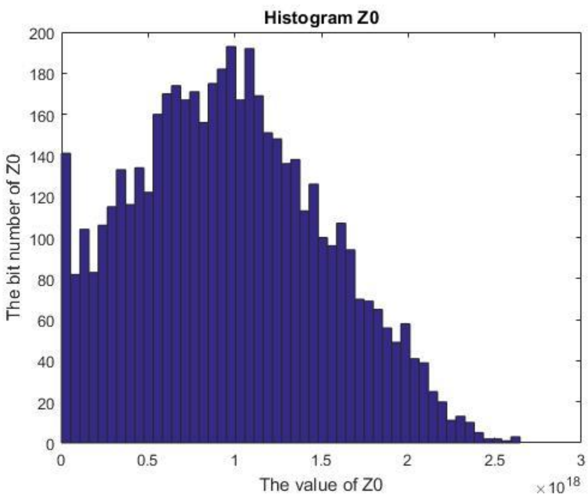

Figure 31: The ISI with the stochastic SISO communication system and signaling interval T_s = 40 s

When the signaling interval, T_s is 10 seconds, there is a lot of ISI in the communication system. There is, in fact, no value of Z that equals zero, and the mean value is about 1,25*10^{18}. There is however, a Gaussian shape of the ISI, which is as expected. This can be seen in Figure 30.

In Figure 31, the signaling interval, T_s is 40 seconds. There is about 140 Z-values equal to zero, which is an improvement from the plot in Figure 17. The mean value has also decreased to about 1*10^{18}. Therefore, there is less ISI when the signaling interval is 40 seconds, compared to when it is 10 seconds. However, there is still a significantly amount of ISI.

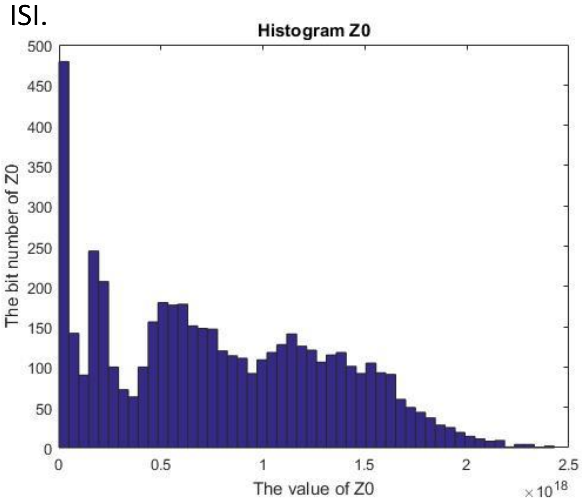

Figure 32: The ISI with the stochastic SISO communication system and signaling interval T_s = 60 s

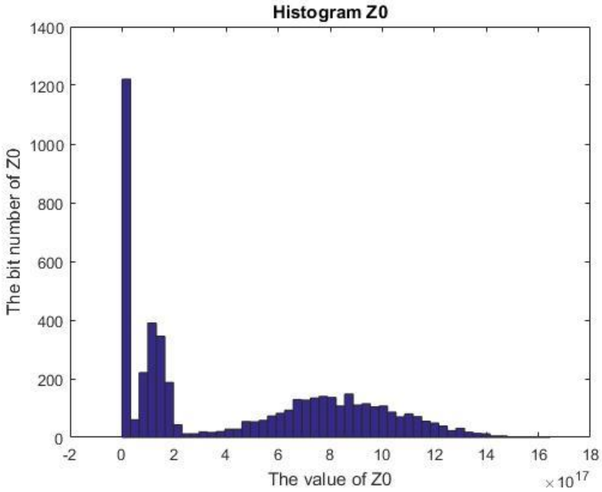

Figure 33: The ISI with the stochastic SISO communication system and signaling interval T_s = 100 s

When the signaling interval, T_s, is 60 seconds, the peak at zero has increased to about 470. This is shown in Figure 32. The mean value is harder to distinguish than before because the Gaussian shape is not as apparent as desired. Nevertheless, an increased number of the Z-values is closer to zero than with lower signaling intervals, so the ISI has decreased.

In Figure 33, the signaling interval, T_s, is 100 seconds. At this point, the ISI has decreased significantly. The Gaussian form starts to show, and the peak at zero is about 1220, which is a percentage of 12,2 of the transmitted bits. There is also a large peak close to zero with 400 bits. The mean value is now about 8,5*10^{17}, which is a huge decrease.

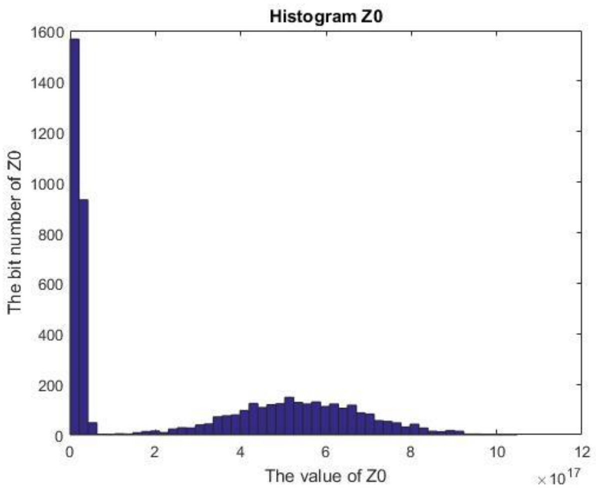

Figure 34: The ISI with the stochastic SISO communication system and signaling interval T_s = 130 s

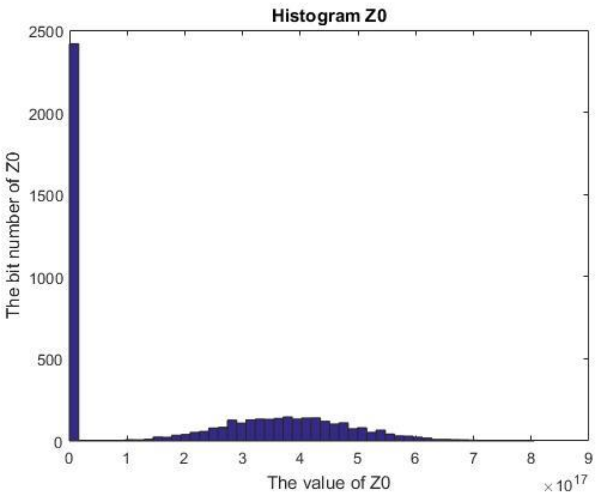

Figure 35: The ISI with the stochastic SISO communication system and signaling interval T_s = 150 s

In Figure 34, with signaling interval, T_s of 130 seconds, the ISI has again decreased significantly. The peak at zero is 1600 bits. There is also a peak adjacent to this peak of almost 1000 bits. The mean value has decreased to about 5,2*10^{17}.

When the signaling interval is 150 seconds, the ISI has decreased to a desired low value. This can be seen in Figure 35. The peak at zero is about 2450, which is as mentioned before a desired percentage of the 10000 bits. The mean value is here at approximately 3,8*10^{17}. At this point, the channel is almost ISI free, and the output signal will be nearly identical to the input signal, which we also can see from the BER values in Chapter 6.2.6.1.

6.2.9 Summary of different parameters for the SISO communication system

In Table 9, an overview over some important parameters discussed earlier is presented. These values are from the simulations and are based on 10000 transmitted bits from one random transmission.

Table 9: Summary of different parameters of the SISO communication system

T_s	1 s	10 s	30 s	50 s	60 s
η	6,4*10^{16}	43,2*10^{16}	109,2*10^{16}	118,9*10^{16}	133,7*10^{16}
λ	3,1036*10^{5}	0,1999	0,0302	0,0033	0,0032
p_1	0,1	0,1	0,3	0,3	0,4
BER	0,5012	0,4973	0,2968	0,2087	0,1738
I_{max}	20,76*10^{-3}bits	0,05 bits	0,17 bits	0,32 bits	0,40 bits
R_{max}	20,76*10^{-3} bits/s	4,99*10^{-3}bits/s	5,64*10^{-3}bits/s	6,35*10^{-3}bits/s	6,68*10^{-3}bits/s
a_0	36,89$10^{-6}$	0,36*10^{-3}	0,0011	0,0018	0,0021

T_s	70 s	80 s	90 s	100 s	110 s
η	133,0*10^{16}	130,9*10^{16}	127,1*10^{16}	128,4*10^{16}	122*10^{16}
λ	0,0040	0,0066	0,0140	0,0160	0,0375
p_1	0,4	0,4	0,4	0,5	0,5
BER	0,1370	0,0955	0,0599	0,0401	0,0270
I_{max}	0,49 bits	0,58 bits	0,66 bits	0,74 bits	0,81 bits
R_{max}	6,99*10^{-3}bits/s	7,25*10^{-3}bits/s	7,35*10^{-3}bits/s	7,40*10^{-3}bits/s	7,37*10^{-3}bits/s
a_0	0,0024	0,0027	0,0029	0,0032	0,0034

T_s	130 s	150 s	200 s	500 s	1000 s
η	$103*10^{16}$	$81*10^{16}$	$35*10^{16}$	$7,8*10^{16}$	10^{14}
λ	0,2218	1,1966	44,16	4527,5	NaN
p_1	0,5	0,5	0,5	0,5	0
BER	0,0103	0,0039	0,0005	0,0001	0,0001
I_{max}	0,91 bits	0,958 bits	0,992 bits	0,997 bits	0
R_{max}	$6,99*10^{-3}$bits/s	$6,39*10^{-3}$bits/s	$4,96*10^{-3}$bits /s	$1,99*10^{-3}$bits/s	0
a_0	0,0037	0,0040	0,0044	0,0047	0,0047

The mutual information is increasing towards one when the signaling interval is increasing, until it reaches its limit. From Table 9 it can be seen that when the signaling interval is 1000 seconds, the mutual information is zero. That is why we cannot use signaling intervals higher than 500 seconds, even though the BER is more optimised with higher T_s values. That is also why the rate only increases from T_s is 10 seconds, until T_s= 100 s, and decreases above this value. The rate is defined as the mutual information divided by the signaling interval. When the mutual information only increases a bit at the end, but T_s continue to increase, the rate will start to decrease. The reason why the rate also decreases from the signaling interval of one second to the signaling interval of 10 seconds is probably that there is too much ISI and distortion to get accurate values. So all the values are reasonable.

The a_0 values is the filter coefficients of the communication system at the current signaling interval. These values is between zero and one with all the different signaling intervals. Since the molecules are transported from the transmitter to the receiver with diffusion, which disperses the molecules in the environment, it is reasonable that the filter attenuate the signal. The a_0 values increases with the signaling interval. Hence, the higher signaling interval used, the more the signal is attenuated.

To summarize, it is observed that the distortion and ISI decreases with increasing signaling intervals. It is the opposite case for the mutual information and the a0 coefficient, which increases. Up until T_s equals 100 seconds, the rate increases, after that it decreases. The optimal threshold, η, first increases, then decreases with the signaling interval, while the optimal threshold, have the opposite order of change. The optimal a priori probability, p_1, increases with less distortion and higher signaling interval. As can be seen, the signaling interval has a significant role in the performance of the communication system, and has an optimal value of 150 seconds, for this particular SISO communication system.

6.3 The stochastic SIMO communication system

The results from the stochastic SIMO communication system will mostly be presented with a signaling interval, T_s of 200 seconds based on 10000 transmitted bits. The large amount of transmitted bits is chosen to get results that are more accurate. In some cases, 1000 bits are transmitted to be able to study the details in the plots. The signaling interval of 200 seconds is chosen because this is defined as the optimal signaling interval for the MIMO communication system, as we will see in the next chapters. The distance between the transmitter and the first receiver is 0,03 meters and the distance between the transmitter and the second receiver is 0,04 meters.

6.3.1 The distribution of Z_0 and Z_1 for the SIMO communication system

The distribution of Z_1 for the SIMO communication system based on 10000 transmitted bits with a signaling interval of 200 seconds, is displayed in a histogram in Figure 36.

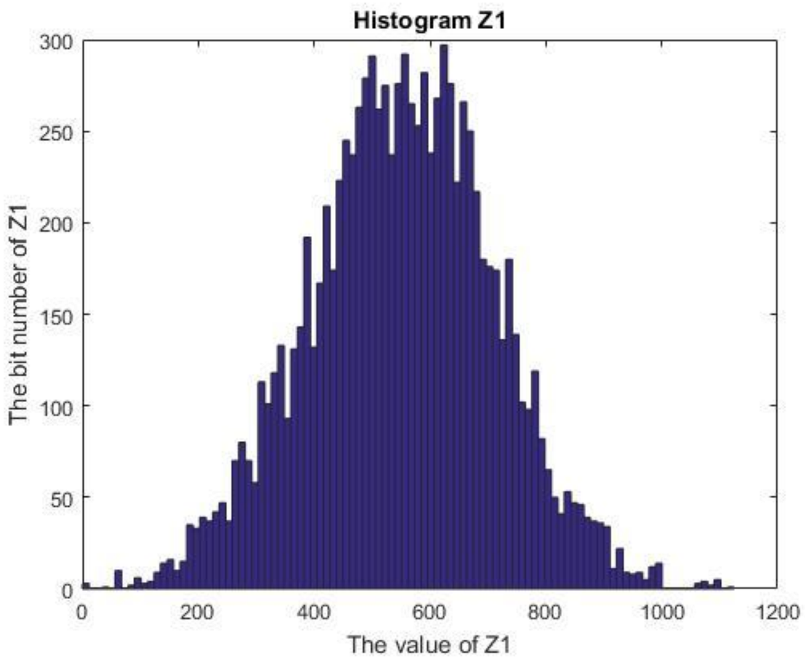

Figure 36: The distribution of Z_1 with SIMO communication and the signaling interval, T_s = 200 s

From the histogram, we can see that the distribution of Z_1 has a Gaussian shape as for the SISO communication system. The mean value, μ_{Z_1}, is about 580. When Maximum-Ratio combining is applied, the Z variable is multiplied by a weight, b_n. This is the reason why μ_{Z_1} is not approximately equal to the mean value of Q, μ_Q, multiplied by the mean value of the filter coefficients, $a_{0,1}$ and $a_{0,2}$ as for the SISO communication system. The variance, σ_{Z_1}, from the histogram is about 1100.

A histogram of the distribution of Z_0 for the SIMO communication system based on 10000 transmitted bits with a signaling interval of 200 seconds, is displayed in Figure 37.

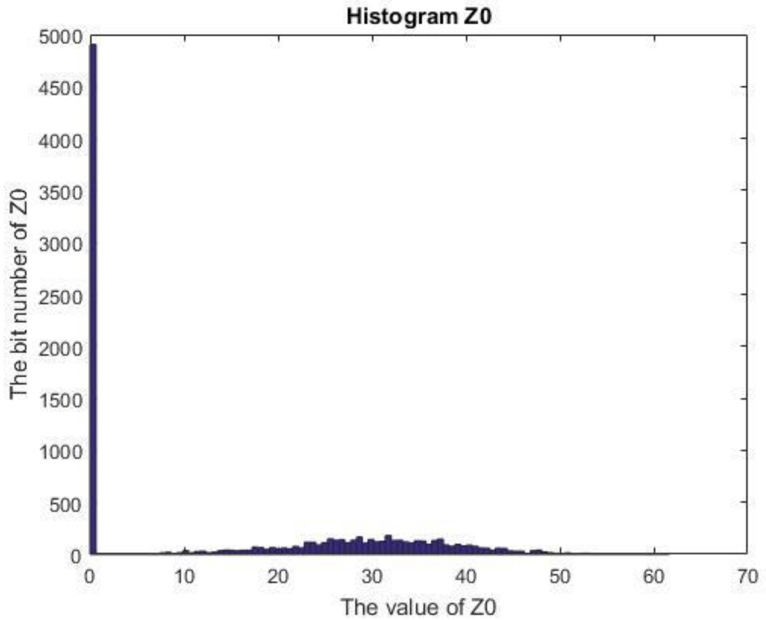

Figure 37: The distribution of Z_0 with SIMO communication and the signaling interval, T_s = 200 s

From the plot in Figure 37, we can see that the distribution of Z_0 is quite similar as for the SISO communication system. It has a Gaussian shape and a large peak at zero. The mean value, μ_{Z_0}, is significantly lower than the mean value for the Z_1 variable, as expected. The main difference from the SISO system is that the peak at zero is approximately 4900 instead of 2450. The probability for transmitting two consecutive zeros, hence, getting a zero value of Z_0, is doubled for the SIMO communication system because there is added another travelling path for the molecules. Therefore the distributions of Z_0 and Z_1 is reasonable for the SIMO communication system.

Also for the SIMO communication system the distributions of Z_0 and Z_1 have a small overlap, which means there will be some ISI and distortion in the system.

6.3.2 The optimal thresholds, η and λ, and the optimal a priori probability, p_1 with different signaling intervals, T_s

The mutual information of the SIMO communication system with a signaling interval of 200 seconds based on 10000 transmitted bits, is displayed in Figure 38.

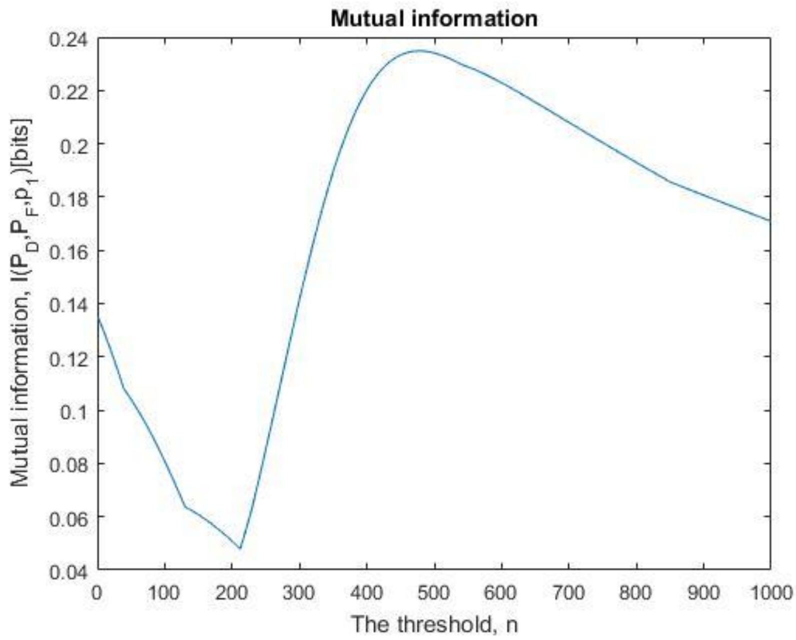

Figure 38: The mutual information in the stochastic SIMO communication system dependent on the optimal threshold, η with T_s = 200 s

The plot shows that the maximum mutual information is approximately 0,23 with the optimal signaling interval 200 seconds. This value for the mutual information is achieved when the threshold, η, is about 480. That makes 480 the optimal threshold for this signaling interval.

In Table 10, the numerical values of the maximum mutual information, the optimal thresholds, η, and λ, and the optimal a priori probability, p_1 is displayed.

Table 10: The maximum mutual information, I, the optimal thresholds, η and, λ and the optimal a priori probability p_1

T_s	1 s	10 s	50 s	100 s	150 s	200 s	300 s
η	1157	172	82	113	163	479	9106
λ	1,03*10^{15}	3519,2	1,6998	12,5048	39,5004	146,0118	427,7360
p_1	0,1	0,1	0,1	0,1	0,3	0,4	0,4
I_{max}	2,25*10^{-5} bits	0,11*10^{-3} bits	4,72*10^{-3} bits	35,60*10^{-3} bits	0,12 bits	0,23 bits	0,32 bits

From the table we can see that the maximum mutual information is increasing with the signaling interval as expected. Compared to the SISO communication system, these values are much smaller. With the optimal signaling interval of 150 seconds, the SISO system has a maximum mutual information of 0,958 bits. The maximum mutual information for the SIMO system with the optimal signaling interval of 200 seconds is 0,23 bits. The SISO communication system, therefore has an increased capacity compared to the SIMO system.

The optimal thresholds, η and λ is decreasing from the signaling interval is 1 seconds to 50 seconds, and then increasing. Compared to the SISO system, the a priori probability is lower. This is probably corresponding to that the maximum mutual information is lower.

With increasing signaling intervals and maximum mutual information, the a priori probability increases, but with the optimal signaling interval of 200 seconds, p_1 equals 0,4. The a priori probability for the SISO communication system with optimal signaling interval is 0,5.

6.3.3 Data rate, R

The data rate, R is displayed in Figure 39 with the signaling interval of 200 seconds. It is based on 10000 transmitted bits.

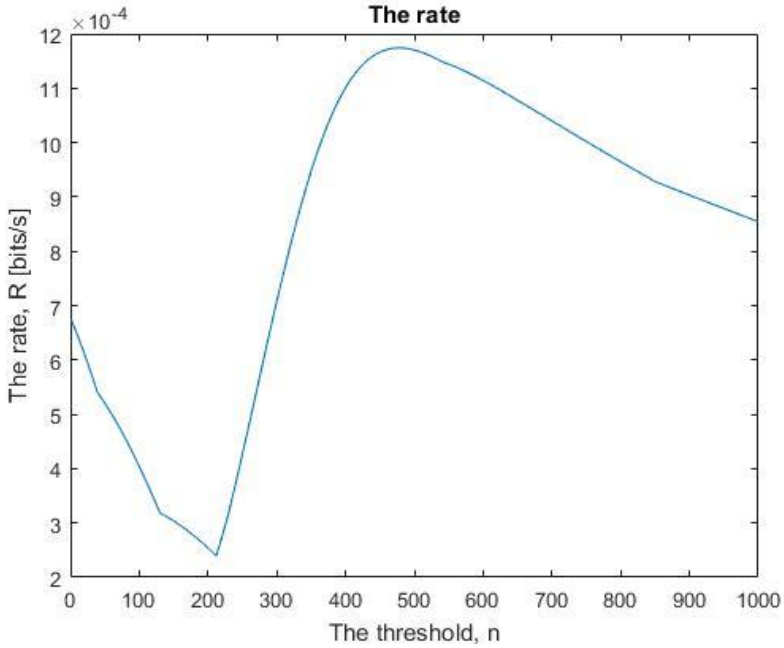

Figure 39: The rate, R with the stochastic SIMO communication system when T_s = 200 s

From the plot we can see that the maximum rate, R_{max} is about 1,18*10^{-3} bits/seconds when the threshold, η is about 480. The maximum rate with different signaling intervals is displayed in Table 11.

Table 11: The maximum rate, $\boldsymbol{R_{max}}$

$\boldsymbol{T_s}$	1 s	10 s	50 s	60 s	100 s
$\boldsymbol{R_{max}}$	2,25*10^{-5}bits/s	1,09*10^{-5}bits /s	9,44*10^{-5}bits /s	0,13*10^{-3}bits /s	0,36*10^{-3}bits /s
$\boldsymbol{T_s}$	110 s	130 s	150 s	200 s	300 s
$\boldsymbol{R_{max}}$	0,43*10^{-3}bits/s	0,62*10^{-3}bits /s	0,83*10^{-3}bits /s	1,17*10^{-3}bits /s	1,05*10^{-3}bits /s

From the table we can see that the maximum rate decreases from T_s is 1 seconds to 10 seconds. Then it increases from T_s is 10 seconds to 200 seconds, and decreases again from 200 seconds to 300 seconds.

6.3.4 The output signal, Y_i vs the input signal, X_i

The output signal of the SIMO communication system versus the input signal is displayed in Figure 40 and 41 with a signaling interval of 200 seconds based on 1000 transmitted bits. In Figure 40, they are plotted separately, while they have overlapping plots in Figure 41. In both figures, the input signal is in red and the output signal is in blue.

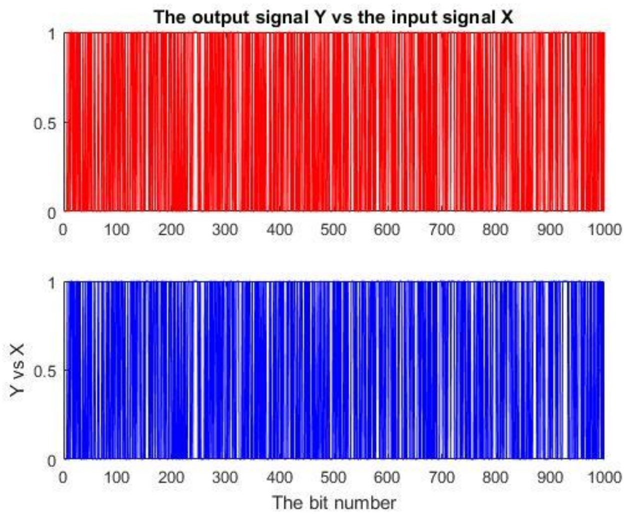

Figure 40: The output signal, Y_i (blue) vs the input signal, X_i (red) with the stochastic SIMO communication system

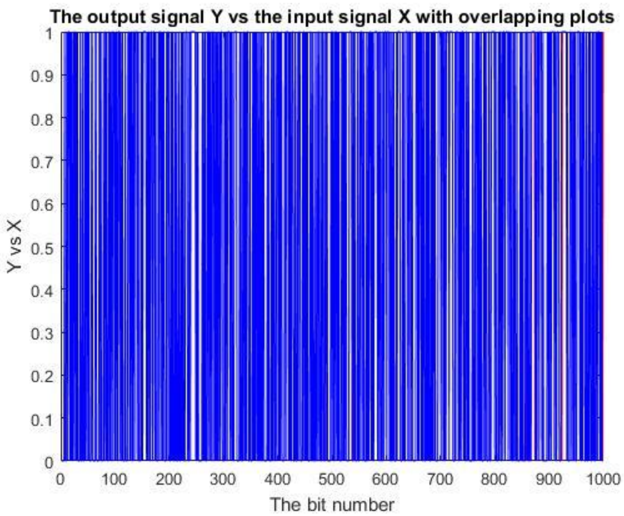

Figure 41: The output signal, Y_i (blue) vs the input signal, X_i (red) with the stochastic SIMO communication system with overlapping plots

From the plots, we can see that with the optimal signaling interval of 200 seconds, the input and output signal is nearly identical. A more accurate study of the deviation between the input and the output signal is given in the next chapter when the distortion is studied.

6.3.5 Distortion

The distortion, D, for the SIMO communication system with different signaling intervals, is displayed in Figure 42 to 46.

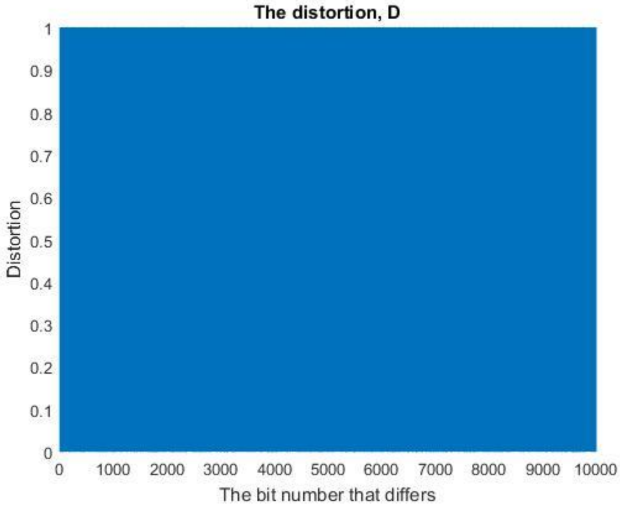

Figure 42: The distortion, D s with the stochastic SIMO communication system and signaling interval T_S = 50 s

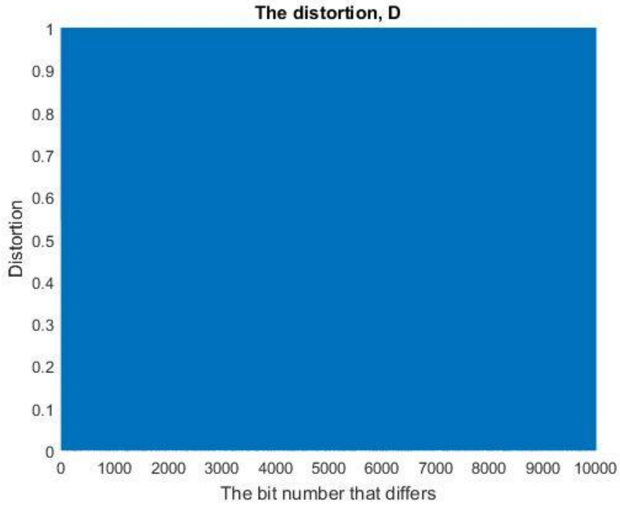

Figure 43: The distortion, D s with the stochastic SIMO communication system and signaling interval T_S = 100 s

In Figure 42, the signaling interval is 50 seconds. The number of bits with distortion is so high that they blend together in the plot. The result is a completely blue plot since the blue amplitudes represents the bit with distortion. As for the SISO system, this signaling interval is therefore, far from optimal for this communication system.

In the SISO system a signaling interval of 100 seconds gave an improved system with visible less distortion. This is not the case in the SIMO communication. In Figure 43, there seems to be just as much distortion as it was with the signaling interval of 50 seconds. Nevertheless, the distortion has been reduced as can be seen in Table 12 in the next chapter.

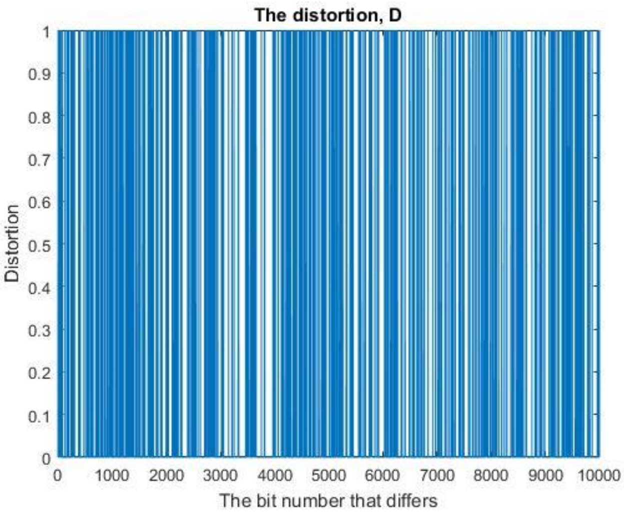

Figure 44: The distortion, D s with the stochastic SIMO communication system and signaling interval T_S = 150 s

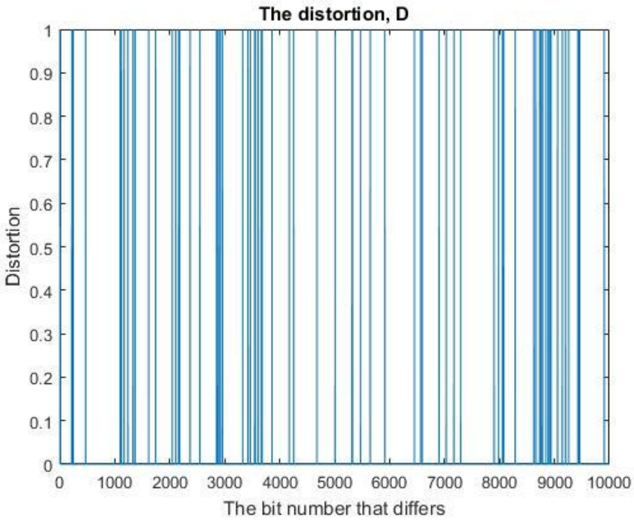

Figure 45: The distortion, D s with the stochastic SIMO communication system and signaling interval T_S = 200 s

When the signaling interval is 150 seconds, the distortion has decreased. This can be seen in Figure 44. In the SISO communication system, this was the optimal signaling interval with only a reasonable amount of distortion. For the SIMO communication system, there is still too much distortion. In fact, the plot is quite similar to the plot of the distortion for the SISO system with a signaling interval of 100 seconds.

In Figure 45, the signaling interval is 200 seconds. At this point the distortion is reduced to approximately the same amount as for the SISO communication system with a signaling interval of 150 seconds. Therefore, this is chosen as the optimal signaling interval.

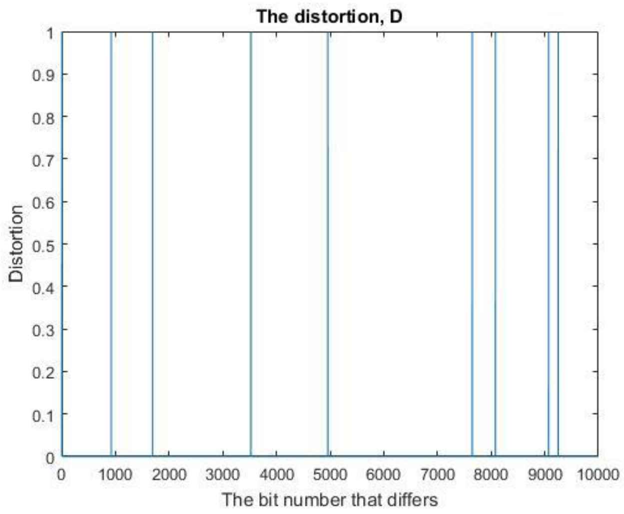

Figure 46: The distortion, D s with the stochastic SIMO communication system and signaling interval T_s = 250 s

In Figure 46, the distortion is shown with a signaling interval of 250 seconds. The presence of distortion is in this case too low to represent the distortion from a realistic communication system. That is why the signaling interval of 250 seconds is not chosen as the optimal signaling interval despite that the distortion is not desirable.

6.3.5.1 The number of errors and total average error probability, BER

In Table 12, the total average error probability, BER, is displayed with different signaling intervals, T_s.

Table 12: The average error probability, BER

T_s	1 s	10 s	50 s	80 s	100 s	110 s	130 s	150 s	200 s	300 s
BER	0,4959	0,4954	0,3271	0,2338	0,2186	0,2116	0,1430	0,0627	0,0074	0,0003

The average error probability decreases with increasing signaling intervals as expected. When the signaling interval is one and ten seconds, the BER is approximately the same for the SISO and SIMO system. With higher signaling intervals, however, the average error probability decreases slower for the SIMO communication system. For the SIMO system, higher signaling intervals gives an improved performance than for the SISO system.

6.3.6 Rate distortion

The rate distortion is studied in Table 13, which displays the parameters maximum rate and BER with different signaling intervals.

Table 13: The rate distortion

T_s	1 s	10 s	50 s	80 s	100 s
R_{max}	2,25*10^{-5}bits/s	1,09*10^{-5}bits /s	9,44*10^{-5}bits /s	9,44*10^{-5}bits /s	0,36*10^{-3}bits /s
BER	0,4959	0,4954	0,3271	0,2338	0,2281
T_s	110 s	130 s	150 s	200 s	300 s
R_{max}	0,43*10^{-3}bits/s	0,62*10^{-3}bits /s	0,83*10^{-3}bits /s	1,17*10^{-3}bits /s	1,05*10^{-3}bits /s
BER	0,2204	0,1430	0,0627	0,0074	0,0003

It can be seen from table 13, that the maximum rate increases when the distortion decreases from one second to 200 seconds. From the signaling interval is 200 seconds to 300 seconds, the rate decreases when the distortion decreases.

6.3.7 Intersymbol interference (ISI)

In Figures 47 to 53, the Intersymbol interference in the SIMO communication system with different signaling intervals, T_s, is displayed. The plots is based on 10000 transmitted bits.

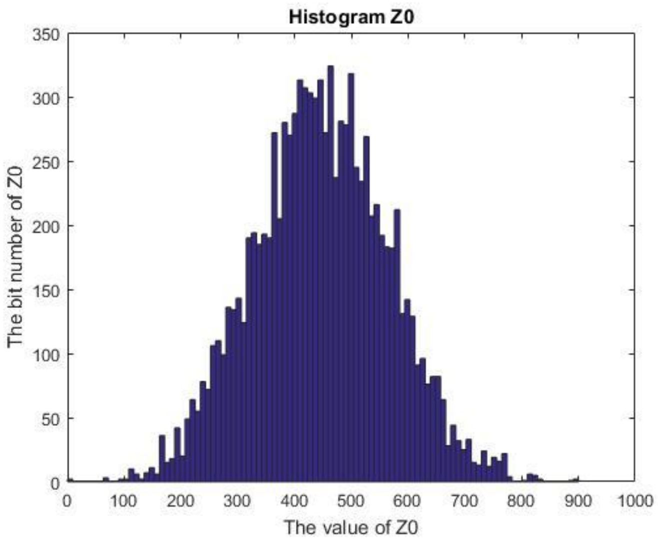

Figure 47: The ISI with the stochastic SIMO communication system and signaling interval T_s = 10 s

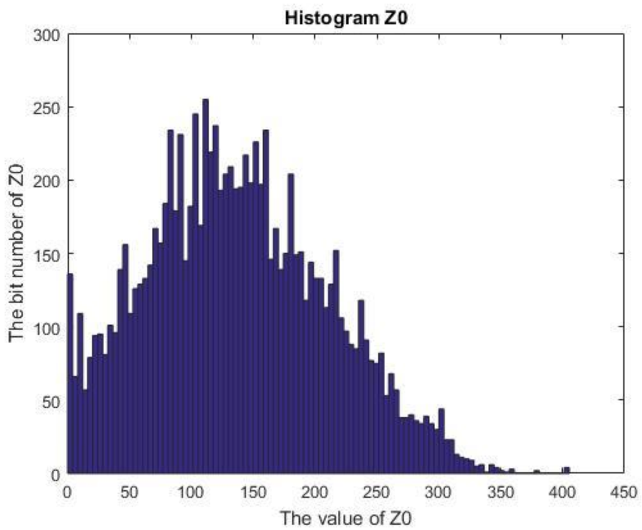

Figure 48: The ISI with the stochastic SIMO communication system and signaling interval T_s = 40 s

In Figure 47, the ISI effect is displayed with the signaling interval, T_s, of 10 seconds. In this case, there is almost as much interference as there is signal. The peak at zero is almost not present at all and the mean value of the interference is approximately 460. Therefore, this is not an optimal signaling interval for the communication system.

As for the SISO communication system, the interference in the SIMO communication system decreases when the signaling interval increases to 40 seconds. This can be seen in Figure 48. The mean value of the interference is about 130, which is closer to zero than with shorter signaling intervals. The peak at zero has also increased to almost 150, which is about the same as for the SISO system at this signaling interval. There is, however, still a great amount of ISI in the communication system.

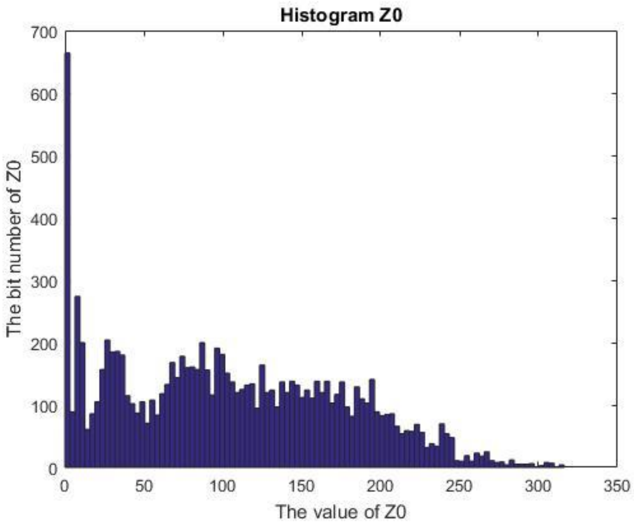

Figure 49: The ISI with the stochastic SIMO communication system and signaling interval T_s = 60 s

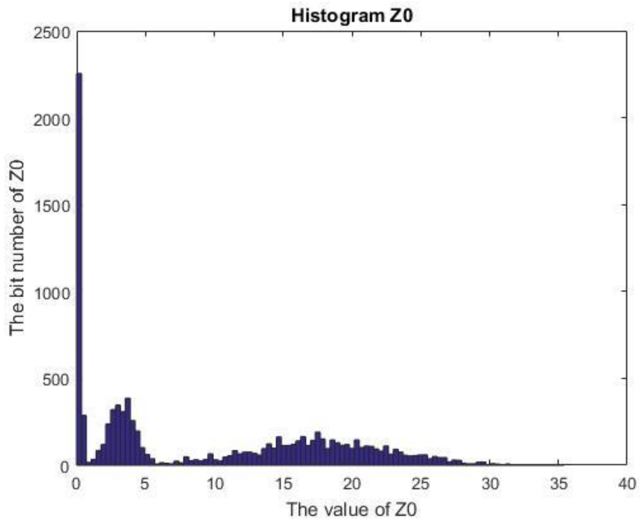

Figure 50: The ISI with the stochastic SIMO communication system and signaling interval T_s = 100 s

With a signaling interval of 60 seconds, the peak at zero has reached almost 700 bits, which is a decent improvement from when the signaling interval was 40 seconds. This can be seen in Figure 49. Compared to the SISO communication system, which had a peak at almost 500 bits at the same signaling interval, this is a higher amount. The mean value is also hard to tell at this signaling interval in the SIMO system since there is no evident peak.

In Figure 50, the signaling interval is 100 seconds. At this point, the mean value of the interference has decreased significantly to about 17. The peak at zero has increased to about 2300 which is a greater number than the peak with the signaling interval of 60 seconds, and is almost doubled from the peak at zero for the SISO system at about 1200.

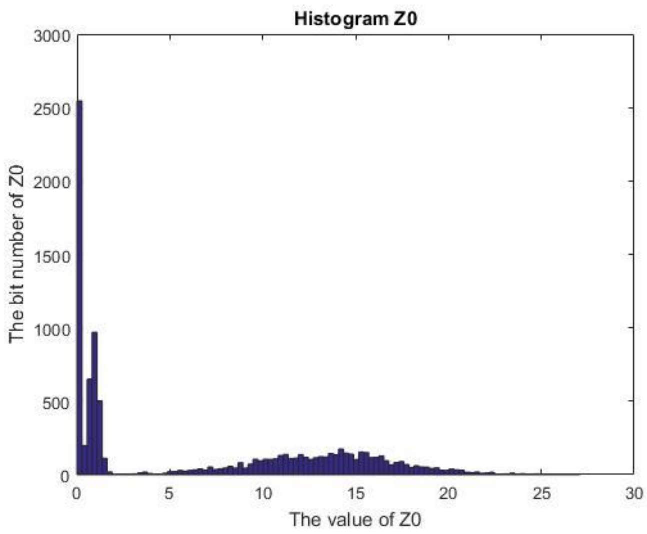

Figure 51: The ISI with the stochastic SIMO communication system and signaling interval T_s = 130 s

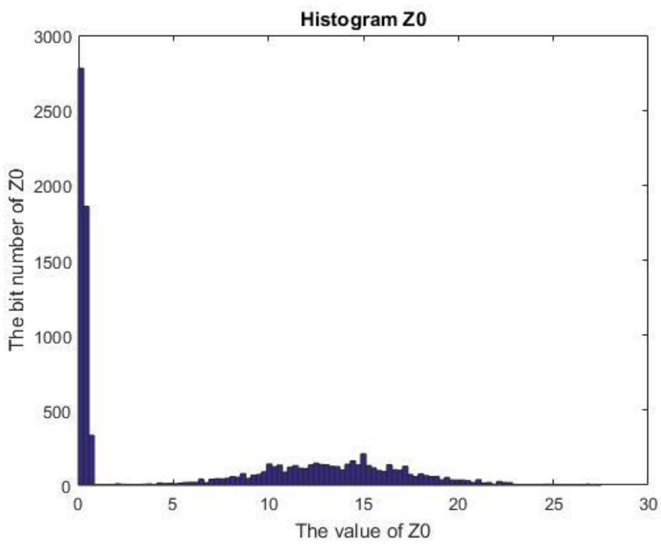

Figure 52: The ISI with the stochastic SIMO communication system and signaling interval T_s = 150 s

The signaling interval is 130 seconds in Figure 51. We can see that the performance of the communication system has further improved since the ISI has decreased. The mean value is now below 15 and the peek at zero has passed 2500 bits. There is also a peak close to zero at about 1000 bits. It is desired for this peak to merge with the peak at zero, hence there is still a too high amount of ISI in the system.

In Figure 52, the signaling interval is 150 seconds, which is the optimal signaling interval for the SISO communication system. There is, however, still a peak that is adjacent to the peak at zero for the SIMO communication system. The peak at zero is approximately 2800 bits, which should be about 5000 to be optimal as discussed in Chapter 6.3.1. Therefore, there is still space for improvement. The mean value is about the same value as it was with the signaling interval of 130 seconds.

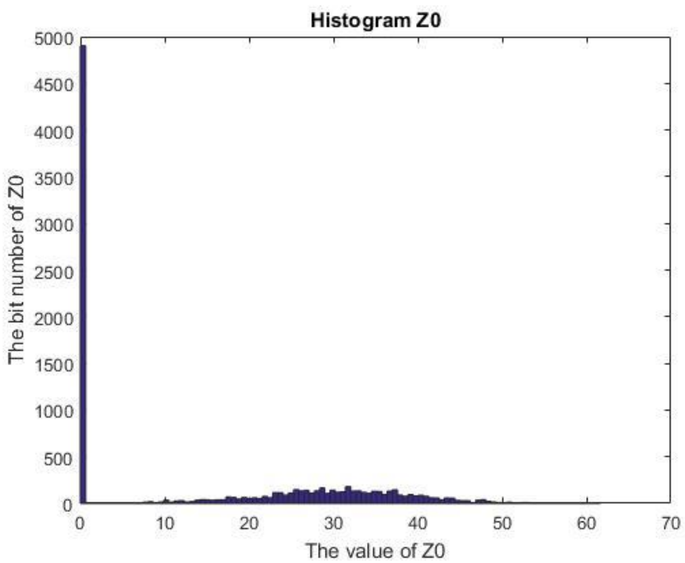

Figure 53: The ISI with the stochastic SIMO communication system and signaling interval T_s = 200 s

With a signaling interval of 200 seconds, the ISI is as desired. This can be seen in Figure 53. The peak at zero is almost 5000 bits and there is no additional peak next to it. The mean value, however, has increased a bit, to about 30. Since the increase is relatively small, the Gaussian shape is as expected and the peak at zero is as desired, this is the chosen optimal signaling interval for the SIMO communication system. That means the SIMO system works with a higher signaling interval than the SISO system.

6.3.8 Summary of different parameters for the SIMO communication system

In Table 14, an overview over the same important parameters as for the SISO communication system is presented. These values are from the simulations and are based on 1000 transmitted bits from one random transmission.

Table 14: Summary of different parameters for the SIMO communication system

T_s	1 s	10 s	30 s	50 s	60 s
η	1157	172	93	82	83
λ	1,03*10^{15}	3519,2	0,8533	1,6998	2,6496
p_1	0,1	0,1	0,1	0,1	0,1
BER	0,4959	0,4954	0,4184	0,3271	0,2943
I_{max}	2,25*10^{-5}bits	0,11*10^{-3}bits	1,28*10^{-3}bits	4,72*10^{-3}bits	7,83*10^{-3}bits
R_{max}	2,25*10^{-5}bits/s	1,09*10^{-5}bits/s	4,27*10^{-5}bits/s	9,44*10^{-5}bits/s	0,13*10^{-3}bits/s
$a_{0,11}$	3,69*10^{-5}	0,37*10^{-3}	0,0011	0,0018	0,0021
$a_{0,12}$	2,40*10^{-5}	0,24*10^{-3}	0,71*10^{-3}	0,0012	0,0014

T_s	80 s	100 s	110 s	130 s	150 s
η	93	113	129	128	163
λ	6,2542	12,5048	17,4220	20,9271	39,5004
p_1	0,1	0,1	0,1	0,2	0,3
BER	0,2338	0,2281	0,2204	0,1430	0,0627
I_{max}	0,18*10^{-3}bits	35,60*10^{-3}bits	47,05*10^{-3}bits	80,01*10^{-3}bits	0,12 bits
R_{max}	0,23*10^{-3}bits/s	0,36*10^{-3}bits/s	0,43*10^{-3}bits/s	0,62*10^{-3}bits/s	0,83*10^{-3}bits/s
$a_{0,11}$	0,0027	0,0032	0,0034	0,0037	0,0040
$a_{0,12}$	0,0011	0,0021	0,0023	0,0026	0,0028

T_s	175 s	200 s	250 s	300 s	500 s
η	277	479	188	9106	2
λ	66,2506	146,0118	279,5168	427,7360	1,20*10^{-6}
p_1	0,3	0,4	0,4	0,4	0,7
BER	0,0333	0,0074	0,0008	0,0006	0,2498
I_{max}	0,18 bits	0,23 bits	0,29 bits	0,29 bits	0,13 bits
R_{max}	1,05*10^{-3}bits/s	1,17*10^{-3}bits/s	1,18*10^{-3}bits/s	1,18*10^{-3}bits/s	0,26*10^{-3}bits/s
$a_{0,11}$	0,0043	0,0044	0,0046	0,0046	0,0047
$a_{0,12}$	0,0030	0,0032	0,0034	0,0035	0,0035

To summarize, the capacity is higher for the SISO communication system than the SIMO communication system, which improves the performance of the communication system. With the optimal signaling intervals for the two systems, the maximum mutual information is 0,23 for the SIMO communication system, while it is 0,958 bits for the other. The a priori probability is also higher for the SISO system, which has a value of 0,5 with the optimal signaling interval of 150 seconds. For the SIMO system, this value is 0,4 at the optimal signaling interval. That is most likely connected to the difference in capacity. However, the optimal signaling interval is higher for the SIMO communication system. Therefore, it is possible to use higher signaling intervals for the SIMO communication system then for the SISO. The optimal thresholds, η and λ, and the filter coefficients has about the same development for both communication systems. Both the communication systems get almost distortion-free with increasing signaling intervals. They reach this almost distortion-free state with different signaling intervals as mentioned earlier.

6.4 The stochastic MIMO communication system

The results from the stochastic MIMO communication system, will in the next chapters, be presented with a signaling interval, T_s of 300 seconds based on 10000 transmitted bits. This signaling interval is defined optimal for the MIMO communication system, as will be shown later. As for the SISO and SIMO communication system, some of the plots will be based on 1000 transmitted bits to see more details. The distance between the first transmitter and the first receiver, and the second transmitter and the second receiver is sat to 0,02 meters. The distance between the first transmitter and the second receiver, and the second transmitter and the first receiver is 0,03 meters.

6.4.1 The distribution of Z_0 and Z_1

The distribution of Z_1 from transmit sensor 1 and transmit sensor 2 is displayed in Figure 54 and 55, respectively, with a signaling interval of 300 seconds based on 10000 transmitted bits.

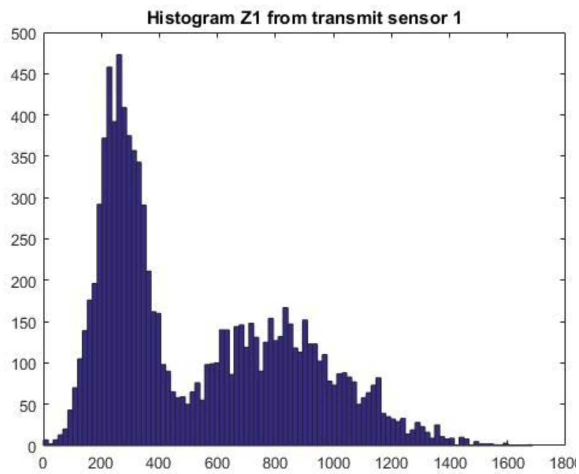

Figure 54: The distribution of Z_1 from transmit sensor 1 with MIMO communication and the signaling interval, T_s = 300 s

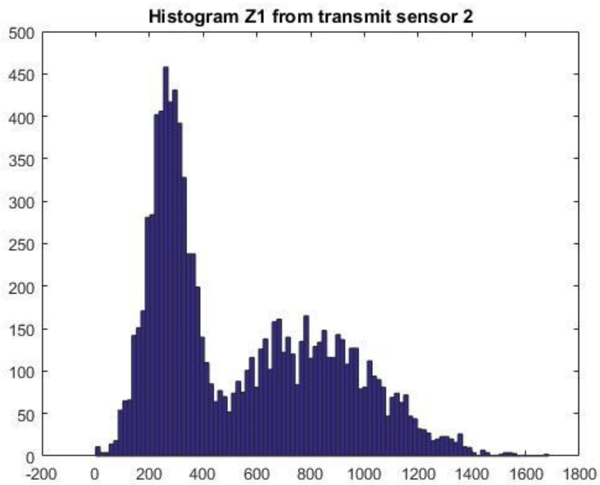

Figure 55: The distribution of Z_1 from transmit sensor 2 with MIMO communication and the signaling interval, T_s = 300 s

From the histograms, we can see that the distributions of Z_1 from the two transmit sensors is very similar. These signals does not have the optimal Gaussian form that the SISO and SIMO system did. It looks more like an overlap of two Gaussian distributions. It is hard to find the mean value from the plot, but from the simulations we have that the mean value of Z_1 from transmit sensor 1 is and the mean value of Z_1 from transmit sensor 2 is. Because of the deviation from the Gaussian shape, it will be hard to find the optimal threshold from the mutual information and will therefore cause more distortion in the communication system than it did in the SISO and SIMO system. This will be discussed further in Chapter 6.4.5

In Figure 56 and 57, the distribution of Z_0 from transmit sensor 1 and 2 with a signaling interval of 300 seconds based on 10000 transmitted bits is displayed.

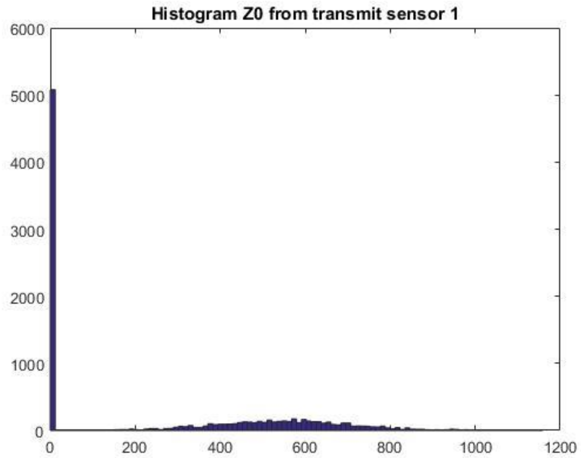

Figure 56: The distribution of Z_0 from transmit sensor 1 with MIMO communication and the signaling interval, T_s = 300 s

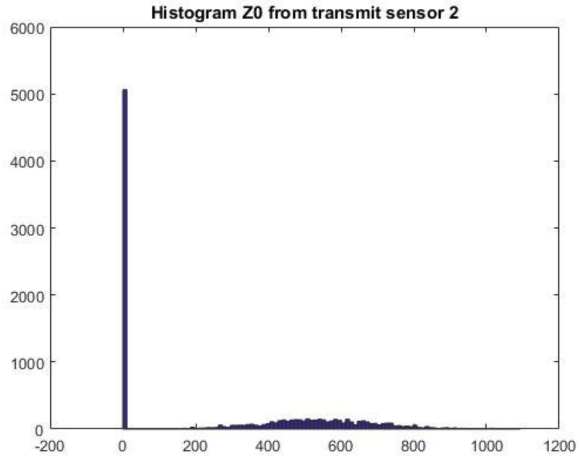

Figure 57: The distribution of Z_0 from transmit sensor 2 with MIMO communication and the signaling interval, T_s = 300 s

The distributions of Z_0 is also very alike. For that reason, only plots from transmit sensor 1 will be displayed in the next chapters when the performance of the communication system is discussed. Unlike the distributions for Z_1, the distributions for Z_0 is reasonable when the optimal signaling interval of 200 seconds is used. The peak at zero is approximately 5000 with each of the transmit sensors, which is 50 % of the transmitted bits in one channel. Since, the two transmit sensors is looked at separately, this is reasonable for the MIMO communication system. However, the mean values of Z_0 is too high compared to the mean value of Z_1. That gives a too large overlap, which will lead to more distortion than with the SISO and SIMO communication systems. This is a highly undesirable consequence which will be studied in the chapter about distortion.

6.4.2 The optimal thresholds, η and λ, and the optimal a priori probability, p_1 with different signaling intervals, T_s

The mutual information, I, depending on the threshold, η, is displayed in Figure 58 with the signaling interval of 300 seconds based on 10000 transmitted bits.

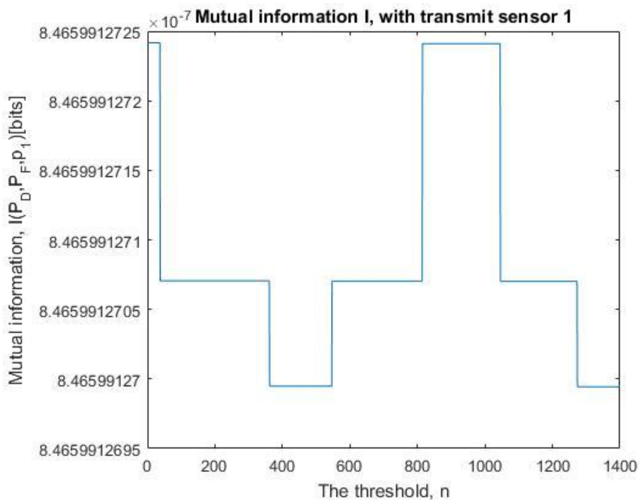

Figure 58: The mutual information in the stochastic MIMO communication system dependent on the optimal threshold, η with T_s = 300 s

From the figure we can see that the maximum mutual information, $\boldsymbol{I_{max,1}}$, is approximately 8,47*10^{-7} bits when the threshold, η_1 is 2. The early peak of the mutual information gives a really low optimal threshold. The reason why that gives relatively good results compared to with use of different signaling intervals with higher optimal thresholds, is that the distributions of Z_0, and Z_1 is pretty close as we saw in chapter 6.4.1. When the threshold equals two, almost only the Z variables that equals zero will lead to a zero for the output signal, Y_i, and the rest will lead to ones. Since the distributions is close, and the peak at zero for Z_0 is high, this is relatively reasonable.

In Table 15, the maximum mutual information, $I_{max,1}$ and $I_{max,2}$, the optimal thresholds, η_1 and η_2 and λ_1 and λ_2 and the optimal a priori probability p_1 is displayed.

Table 15: The maximum mutual information, $I_{max,1}$, and $I_{max,2}$, the optimal thresholds, η and, λ and the optimal a priori probability p_1

T_s	1 s	10 s	50 s	100 s	150 s	200 s	300 s	500 s
η_1	68035	363	363	816	816	816	2	308
η_2	68035	363	363	816	816	816	2	308
λ_1	∞	∞	∞	∞	∞	∞	∞	∞
λ_2	∞	∞	∞	∞	∞	∞	∞	∞
p_1	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$I_{max,1}$	8,47* 10^{-7} bits	8,47* 10^{-7} bits	8,47* 10^{-7} bits	8,47*10^{-7} bits	8,47* 10^{-7} bits	8,47* 10^{-7} bits	8,47* 10^{-7} bits	8,47* 10^{-7} bits
$I_{max,2}$	8,47* 10^{-7} bits	8,47* 10^{-7} bits	8,47* 10^{-7} bits	8,47*10^{-7} bits	8,47* 10^{-7} bits	8,47* 10^{-7} bits	8,47* 10^{-7} bits	8,47* 10^{-7} bits

From the table, we can see that the maximum mutual information is fixed unlike in the SISO and SIMO communication system. The fixed value is 8,47*10^{-7} bits, which is really low compared to the other two communication systems. This is not optimal because that gives a low capacity, which was the opposite effect than expected and decreases the performance of the communication system.

The thresholds, η_1, and η_2 have the same value which is varying between 2 and 68035. The thresholds, λ_1, and λ_2 is equal to ∞, hence it would not be possible to use the original Neyman Pearson detector for this specific communication system.

6.4.3 Data rate, R

The data rate, R is displayed in Figure 59 with signaling interval, T_s, of 300 seconds.

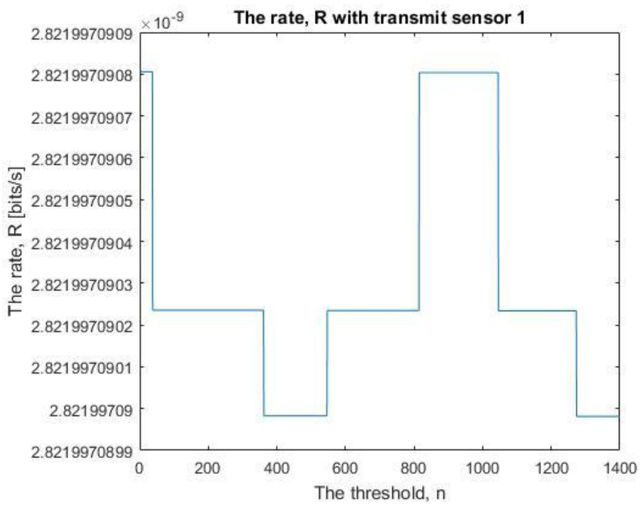

Figure 59: The rate, R with the stochastic MIMO communication system when T_s = 300 s

From the plot, we can see that the maximum value of the data rate, $R_{max,1}$ is equal to 2,82*10^{-9} bits/s. This is achieved when the threshold, η_1 is 2. In Table 16, maximum values of the rate with different signaling intervals is displayed for both transmit sensors.

Table 16: The maximum rate, R_{max}

T_s	1 s	10 s	50 s	100 s	150 s	200 s	300 s	500 s
$R_{max,1}$	8,47*10^{-7} bits/s	8,47*10^{-8} bits/s	1,69*10^{-8} bits/s	8,47*10^{-9} bits/s	5,64*10^{-9} bits/s	4,23*10^{-9} bits/s	2,2*10^{-9} bits/s	1,69*10^{-9} bits/s
$R_{max,2}$	8,47*10^{-7} bits/s	8,47*10^{-8} bits/s	1,69*10^{-8} bits/s	8,47*10^{-9} bits/s	5,64*10^{-9} bits/s	4,23*10^{-9} bits/s	2,82*10^{-9} bits/s	1,69*10^{-9} bits/s

From the table we can see that the rate is decreasing with increasing signaling interval.

6.4.4 The output signal, Y_i vs the input signal, X_i

The output signal in blue and the input signal in red is displayed with the signaling interval of 300 seconds in Figure 60 and 61.

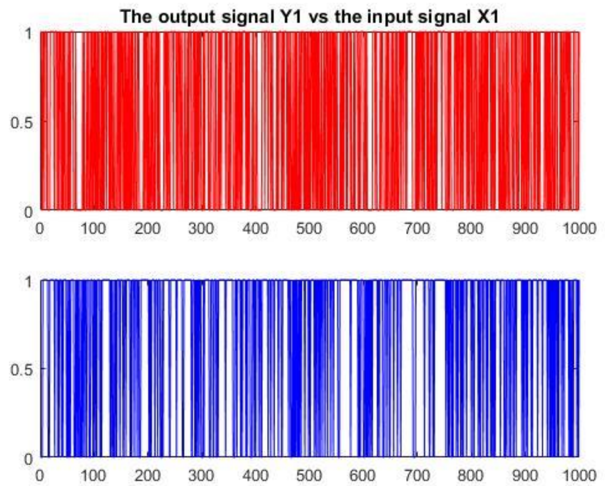

Figure 60: The output signal, Y_i (blue) vs the input signal, X_i (red) with the stochastic MIMO communication system with transmit sensor 1

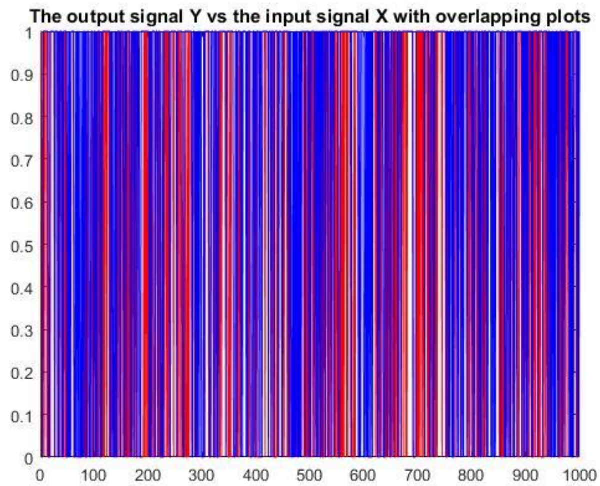

Figure 61: The output signal, Y_i (blue) vs the input signal, X_i (red) with the stochastic MIMO communication system with sensor 1 with overlapping plots

From Figure 60 and 61, we can see that the input and output signal differs significantly more than for the SISO and SIMO communication system with optimal signaling intervals. This is another proof that the two former communication systems had better performance. The distortion will be studied closer in the next chapter.

6.4.5 Distortion, D

In Figure 62 to 64, the distortion, D is displayed with different signaling intervals based on 1000 transmitted bits. The reason why 1000 transmitted bits is used instead of 10000, is that the distortion is relatively high with every signaling intervals, so it would be hard to tell the difference with a higher amount of transmitted bits.

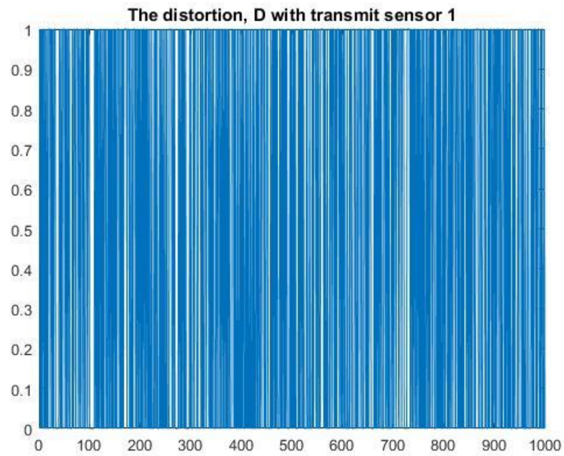

Figure 62: The distortion, D s with the stochastic MIMO communication system and signaling interval T_s = 50 s

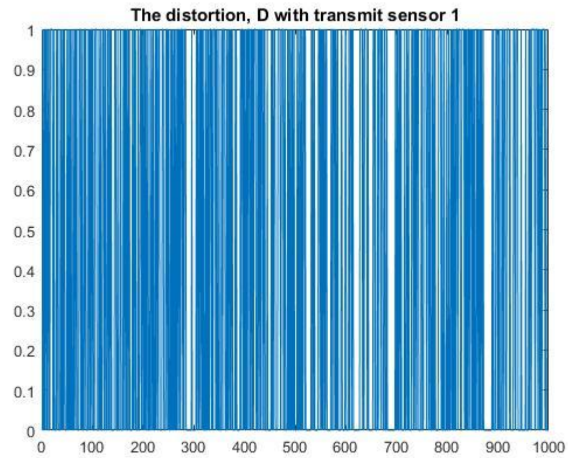

Figure 63: The distortion, D s with the stochastic MIMO communication system and signaling interval T_s = 100 s

terval is doubled to 100 seconds, there seems to be almost as much distortion. There is only a few more open areas without distortion in the plot. This is displayed in Figure 63.

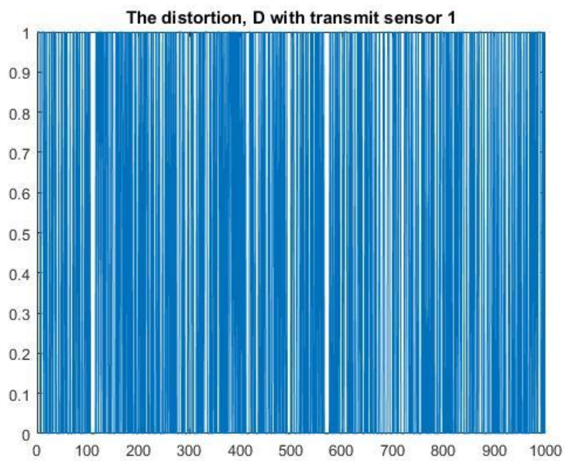

Figure 64: The distortion, D s with the stochastic MIMO communication system and signaling interval T_s = 200 s

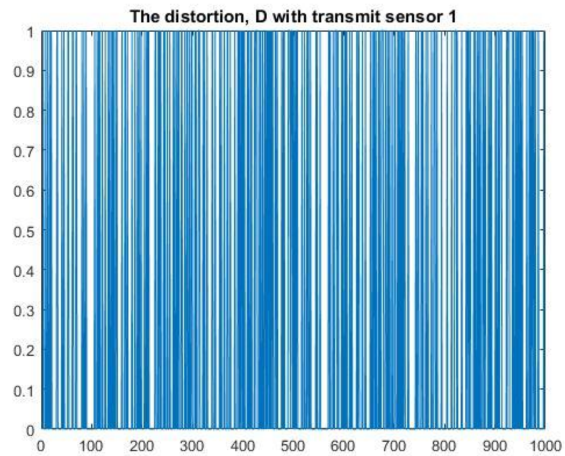

Figure 65: The distortion, D s with the stochastic MIMO communication system and signaling interval T_s = 300 s

With a signaling interval of 200 seconds, the distortion does not appear decreased. This can be seen in Figure 64. With the numerical values in the next chapter, it shows that the distortion has decreased, but just a bit.

In Figure 65, the optimal signaling interval for the MIMO communication system of 300 seconds is applied. It is possible to see a small improvement, in terms of a smaller amount of distortion. There is, however, still excessively much distortion, for it to be the optimal signaling interval. This is another indication that the MIMO system is not optimal for this particular communication system.

6.4.5.1 The total average error probability, BER

The total average error probability, BER with different signaling intervals, based on 10000 transmitted bits is displayed in Table 17.

Table 17: The average error probability, BER

T_s	1 s	10 s	50 s	100 s	150 s	200 s	300 s	500 s
BER_1	0,5028	0,4979	0,4615	0,4013	0,3745	0,3768	0,2441	0,4119
BER_2	0,5000	0,4892	0,4525	0,4071	0,3794	0,3803	0,2513	0,4121

From the table we can see that there is a small difference between the error probability when the transmit sensor 1 and when the transmit sensor 2 is used. They follow the same development when the signaling interval increases, as they both decreases. This effect is reasonable. However, it would be desirable for the BER to decrease a lot more with the signaling interval than it does. The lowest values of the average error probability is at the defined optimal signaling interval of 300 seconds, and is 24,41 % and 25,13 %.

6.4.6 Rate distortion

In Table 18, the rate distortion is displayed in form of a comparison between the maximum rate, R_{max} and the average error probability, BER.

Table 18: The rate distortion

T_s	1 s	10 s	50 s	100 s	150 s	200 s	300 s	500 s
$R_{max,1}$	8,47* 10^{-7} bits/s	8,47* 10^{-8} bits/s	1,69* 10^{-8} bits/s	8,47* 10^{-9} bits/s	5,64* 10^{-9} bits/s	4,23* 10^{-9} bits/s	2,2*10^{-9} bits/s	1,69* 10^{-9} bits/s
$R_{max,2}$	8,47* 10^{-7} bits/s	8,47* 10^{-8} bits/s	1,69* 10^{-8} bits/s	8,47* 10^{-9} bits/s	5,64* 10^{-9} bits/s	4,23* 10^{-9} bits/s	2,82* 10^{-9} bits/s	1,69* 10^{-9} bits/s
BER_1	0,5028	0,4979	0,4615	0,4013	0,3745	0,3768	0,2441	0,4119
BER_2	0,5000	0,4892	0,4525	0,4071	0,3794	0,3803	0,2513	0,4121

From the table we can see that the rate decreases when the distortion decreases. This is not reasonable since they should have opposite effects as explained in the theory.

6.4.7 Intersymbol interference (ISI)

The Intersymbol interference (ISI) for the stochastic SIMO communication system is displayed in Figure 66 to 71 with different signaling intervals, T_S. The signaling intervals displayed, is a bit different than for the SISO and SIMO communications because higher signaling intervals is optimal for the MIMO communication system. The plots with the nearest signaling intervals from the different communication systems will be compared.

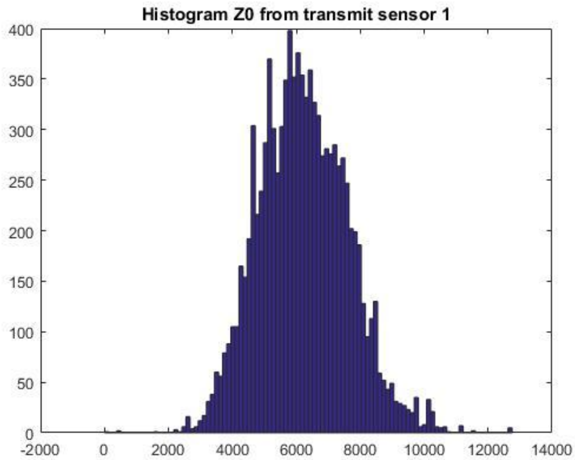

Figure 66: The ISI with the stochastic MIMO communication system and signaling interval T_S = 10 s

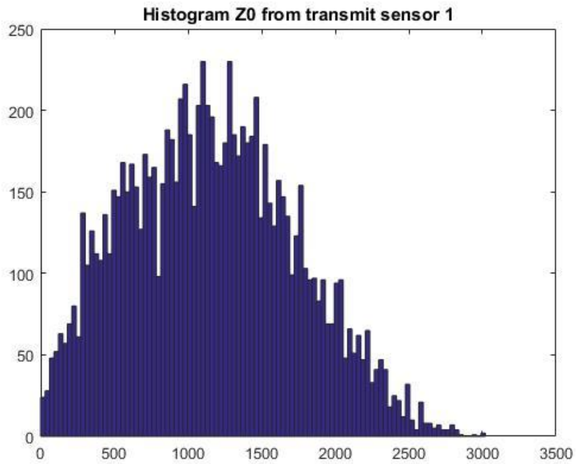

Figure 67: The ISI with the stochastic MIMO communication system and signaling interval T_S = 50 s

In Figure 66, the ISI is displayed with signaling interval of 10 seconds. As for the SISO and SIMO communication system, there is a huge amount of ISI at this signaling interval. There is no peak at zero, and the mean value is about 6000. Since we want minimum ISI in the communication system to get a reliable signal, this is clearly not an optimal signaling interval.

With a signaling interval of 50 seconds, the ISI is decreased a bit, as can be seen in Figure 67. The mean value has decreased to about 1200. The shape of the ISI is quite similar to the shape of the ISI for the SISO and SIMO system with a signaling interval of 40 seconds. There is now a small peak at zero of about 25 bits. This is a very low amount compared to the SISO system with 140 bits and the SIMO system with 150 bits with a signaling interval of 40 seconds. Hence, there is room for improvement.

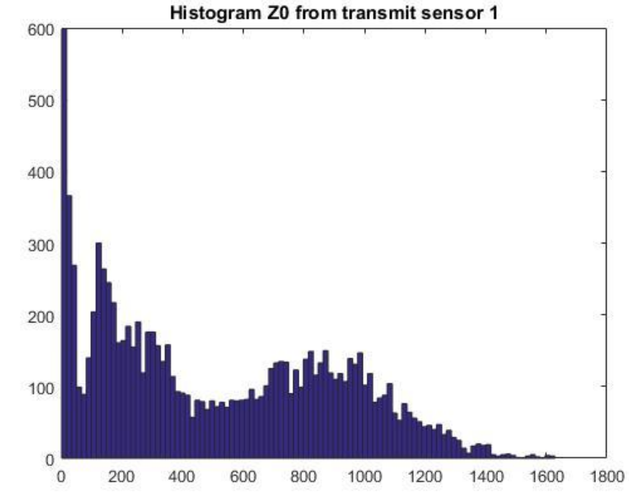

Figure 68: The ISI with the stochastic MIMO communication system and signaling interval T_S = 100 s

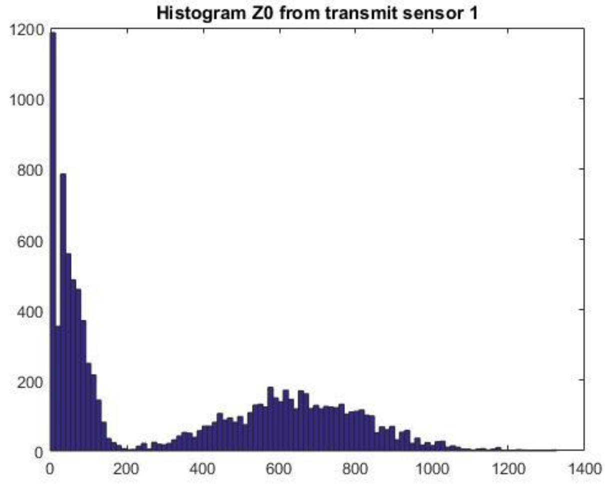

Figure 69: The ISI with the stochastic MIMO communication system and signaling interval T_S = 150 s

In Figure 68, the ISI is displayed with a signaling interval of 100 seconds. The peak at zero has increased significantly to 600 bits, hence there is less ISI in the communication system. However, it is still only half of the value for the SISO system and 25 % of the value for the SIMO system with the same signaling interval. The mean value is not easy to distinguish since the shape of the ISI is not quite Gaussian at this point.

With a signaling interval of 150 seconds, the ISI has decreased and the Gaussian shape starts to form again. This can be seen in Figure 69. The mean value is decreased to about 650, almost a halving from the interval of 50 seconds. The peak at zero has increased to almost 1200, which is a huge raise, but still quite low compared to the SISO and the SIMO system, which has a value of almost 2500 and 2800 respectively. There is also a peak close to zero of 800 bits. The signaling interval is therefore not optimal yet.

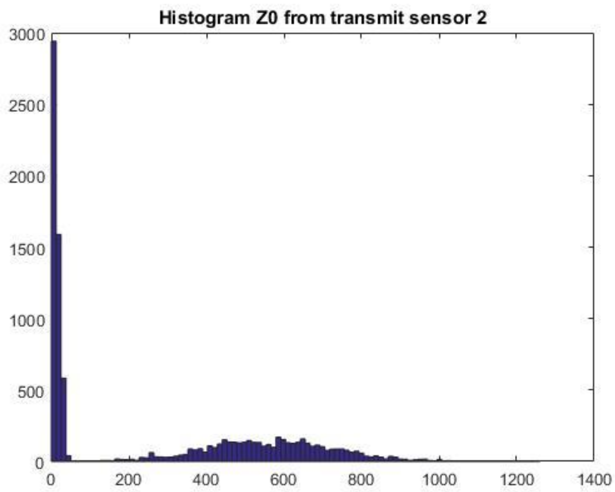

Figure 70: The ISI with the stochastic MIMO communication system and signaling interval T_S = 200 s

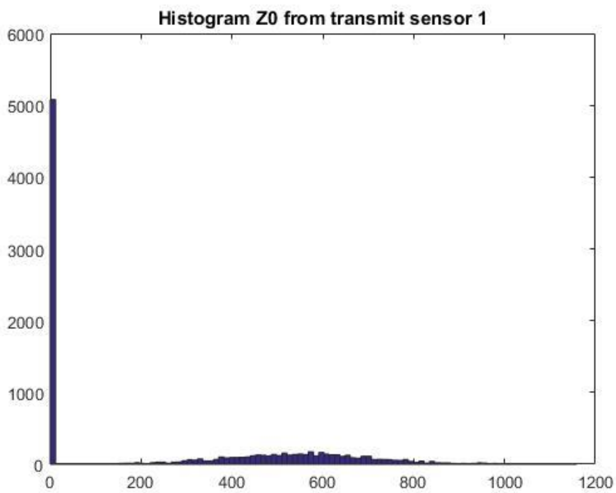

Figure 71: The ISI with the stochastic MIMO communication system and signaling interval T_S = 300 s

From the histogram in Figure 70, we can see that the ISI is significantly decreased with a signaling interval of 200 seconds. The mean value is about 550, which is a bit lower than with the signaling interval of 150 seconds. There has also been a huge increase in the peak at zero, which is almost 3000 bits. Compared to the SIMO system and the expected value of this peak with an optimal signaling interval, it is still too low. The peak next to zero has almost merged with the peak at zero. Hence, the ISI of the communication system is almost reduced enough at this signaling interval.

In Figure 71, the signaling interval is 300 seconds. At this point the ISI is reduced to an appropriate level. The mean value is about the same value as the mean value with the signaling interval of 200 seconds. The peak at zero has now reached the desired value of 5000 bits. For the MIMO communication system, the optimal signaling interval is therefore 300 seconds.

6.4.8 Summary of different parameters for the MIMO system

In Table 19, a summary of the parameters in the MIMO system is given. All values are based on 10000 transmitted bits.

Table 19: Summary of different parameters from the MIMO communication system

T_s	1 s	10 s	50 s	100 s	150 s
η_1	68035	363	363	816	816
η_2	68035	363	363	816	816
λ_1	∞	∞	∞	∞	∞
λ_2	∞	∞	∞	∞	∞
p_1	0,5	0,5	0,5	0,5	0,5
BER_1	0,5028	0,4979	0,4615	0,4013	0,3745
BER_2	0,5000	0,4892	0,4525	0,4071	0,3794
$I_{max,1}$	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits
$I_{max,2}$	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits
$R_{max,1}$	8,47*10^{-7} bits/s	8,47*10^{-8} bits/s	1,69*10^{-8} bits/s	8,47*10^{-9} bits/s	5,64*10^{-9} bits/s
$R_{max,2}$	8,47*10^{-7} bits/s	8,47*10^{-8} bits/s	1,69*10^{-8} bits/s	8,47*10^{-9} bits/s	5,64*10^{-9} bits/s
$a_{0,11}$	6,78*10^{-5}	6,76*10^{-4}	0,0032	0,0054	0,0065
$a_{0,12}$	3,69*10^{-5}	3,68*10^{-4}	0,0018	0,0032	0,0040
$a_{0,21}$	3,69*10^{-5}	3,68*10^{-4}	0,0018	0,0032	0,0040
$a_{0,22}$	6,78*10^{-5}	6,76*10^{-4}	0,0032	0,0054	0,0065

T_s	200 s	300 s	500 s	750 s	1000 s
η_1	816	2	308	308	308
η_2	816	2	308	308	308
λ_1	∞	∞	∞	∞	∞
λ_2	∞	∞	∞	∞	∞
p_1	0,5	0,5	0,5	0,5	0,5
BER_1	0,3768	0,2441	0,4119	0,4129	0,4120
BER_2	0,3803	0,2513	0,4121	0,4089	0,4109
$I_{max,1}$	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits
$I_{max,2}$	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits	8,47*10^{-7} bits
$R_{max,1}$	4,23*10^{-9} bits/s	2,2*10^{-9} bits/s	1,69*10^{-9} bits/s	1,13*10^{-9} bits/s	8,47*10^{-10} bits/s
$R_{max,2}$	4,23*10^{-9} bits/s	2,82*10^{-9} bits/s	1,69*10^{-9} bits/s	1,13*10^{-9} bits/s	8,47*10^{-10} bits/s
$a_{0,11}$	0,0069	0,0070	0,0070	0,0070	0,0070
$a_{0,12}$	0,0044	0,0046	0,0047	0,0047	0,0047
$a_{0,21}$	0,0044	0,0046	0,0047	0,0047	0,0047
$a_{0,22}$	0,0069	0,0070	0,0070	0,0070	0,0070

To summarize, the maximum mutual information has a fixed value at 8,47*10^{-7} bits for the MIMO communication system. This is a significantly lower value than for the SISO and SIMO communication systems, which means that the MIMO system has a lower capacity. This value is also supposed to increase with the signaling interval. Because of the fixed value of the maximum mutual information, the maximum rate is decreasing with increasing signaling interval. That causes the rate distortion analysis to be wrong since the rate should increase when the distortion decreases, and the distortion decreases when the signaling interval increases.

The distortion is a lot worse for the MIMO communication system than for the two others. The smallest average error probability is 24,41 %, while the SISO and SIMO communication systems is almost error free at best. This is an important reason for why the SISO and the SIMO communication system have a better performance than the MIMO system.

The threshold, λ for the original Neyman Pearson Decision is ∞ at all signaling intervals. That means that this detection does not work on this communication system. The threshold η, has very varying values. Since the BER has very high values, the simplified version of the Neyman Pearson detector does not work optimally either.

6.5 A comparison of the stochastic SISO, SIMO and MIMO communication systems

In this chapter, a comparison of the most important parameters from the previous chapters will be given for the stochastic SISO, SIMO and MIMO communication systems. For the SISO communication system, the optimal signaling interval was defined as 150 seconds. This was the value were all the parameters of the communication system were optimised. The distributions of Z_0 and Z_1 had the Gaussian shape as expected. A large peak at zero of almost 2500 bits out of 10000 was found in the distribution of Z_0. This is reasonable, because Z_0 is supposed to represent the case were no molecules is sent. Additionally, the mean value was much lower for the distribution of Z_0, than for Z_1, which is reasonable because Z_1 represent the signal. There were still a small overlap between the two distributions, which causes distortion and ISI. This is realistic in a communication system.

The maximum mutual information increased with an increasing signaling interval for the SISO system, as it should. At the optimal signaling interval, the maximum mutual information was almost 1 bit, which means that the capacity of the communication system were high.

To study the distortion of the communication system, the average error probability was measured. As expected, this value decreased with the increase of the signaling interval. At the optimal signaling interval, the BER had a value of 0,39 %. This shows that the performance of the SISO communication system was acceptable.

The maximum rate increased with the increase of the signaling interval and the decrease of the distortion when the signaling intervals were not too low. That meets the theory about rate distortion.

The ISI was substantially high with low signaling intervals. At worst, there were not even a small peak at zero, where there should be about 25 % of the bits as mentioned before. With higher signaling intervals, the ISI decreased as anticipated, and with the optimal signaling interval of 150 seconds, the communication system were nearly ISI free. All the parameters showed that the SISO communication system worked well.

To study the performance of the SIMO communication system and be able to compare it to the SISO communication system, the same parameters were retrieved from the simulations. The optimal signaling interval was now 200 seconds, hence, it was higher with this change. That means that it is possible to use higher signaling intervals with the SIMO communication system than with the SISO communication system.

The distributions of Z_0 and Z_1 were quite similar with SISO and SIMO communication. The Gaussian shape was still there and it was a relatively big separation between the mean values of the two distributions with a small overlap. However, the peak at zero for Z_0 were now almost 5000 instead of 2500 for the SISO system. This is reasonable because there is added another signaling path.

The maximum mutual information had the expected increase when the signaling interval was increasing. However, it had a much lower value than for the SISO communication system at the optimal signaling interval. The maximum mutual information for the SISO communication system at the optimal signaling interval of 150 seconds, were 0,958 bits, while it for the SIMO system with the optimal signaling interval of 200 seconds, were 0,23 bits. Therefore, the SISO communication system has a better capacity than the SIMO communication system.

Both the SISO and the SIMO communication system had low distortion with the optimal signaling interval, in fact below 1 %. Therefore, both systems works well. With lower signaling intervals, the distortion increased for the SIMO communication system as it did for the SISO communication system.

Also for the SIMO system, the maximum rate increased when the signaling interval increased and the distortion decreased. Another indication that the performance of the SIMO communication system was relatively good since the results matched the theory.

The ISI had a similar development that it had for the SISO communication. It decreased with increasing signaling intervals. The only difference was that the signaling interval where the ISI was decreased properly, was higher than for the SISO system. The parameters showed that the SIMO communication system worked well, but had a lower capacity than the SISO system, and worked with higher signaling intervals.

The same parameters were evaluated in the MIMO communication system. The optimal signaling interval was in this case even higher than for the SIMO system, with the value of 300 seconds. The results from the two transmit sensors were so similar that only the results from the first transmit sensor were plotted, but both were displayed in the tables. It was early clear that the MIMO communication system did not work as good as the SISO and SIMO communication systems. The distribution of Z_1, which represented the signal was no longer Gaussian at the optimal signaling interval. It looked like two Gaussian distributions overlapped. The two distributions of Z_1 and Z_0 had a too large overlap to get a good threshold between them. That caused a raise in the distortion.

The average error probability became never lower than 24,41 %, which is a significantly high value of distortion. The ISI, however, decreased as wanted, and had the Gaussian shape as expected with a peak at zero of about 5000 bits.

The maximum mutual information had with the MIMO system, a fixed value at every signaling interval. This value was also quite low compared to the other communication systems. With the optimal signaling interval, the maximum mutual information was 0,958 bits for the SISO system, 0,23 bits for the SIMO system and 8,47*10^{-7} bits for the MIMO system. Therefore, the capacity of the MIMO communication system was significantly low. Because of the fixed value for the maximum mutual information, the maximum rate decreased with higher signaling intervals and decreasing distortion. That is inconsistent with the rate distortion theory. Most of the parameters show that the MIMO communication system with Spatial Multiplexing combined with on-off keying, binary hypothesis testing and Neyman Pearson detection was not optimal for the diffusion-based molecular communication between leadless pacemakers.

In Table 20, the most important parameters from the previous chapters are compared between the stochastic SISO, SIMO and MIMO communication systems. The Capacity, BER and maximum rate is defined for the optimal signaling interval for the three communication systems.

Table 20: Comparison of the most important parameters for the SISO, SIMO and MIMO communication system:

	SISO	SIMO	MIMO
Optimal signaling interval, T_s	150 s	200 s	300 s
Distortion (BER)	0,39 %	0,74 %	24,41 %
Capacity, C (I_{max})	0,958 bits	0,23 bits	8,47*10^{-7} bits
R_{max}	6,39*10^{-3}bits/s	1,17*10^{-3}bits/s	2,2*10^{-9} bits/s

7 Conclusion

Diffusion-based molecular communications used in leadless pacemakers, has been studied in this report. A model of the communication system with deterministic signaling and only one transmission, was first defined. Since only the diffusion part of the communication system was investigated, the concentration of the molecules was used as the model. The formula for the concentration of the molecules mapped from pheromone communications, proved a relatively good model for the diffusion part of the communication system. The performance analysis of the gain and delay showed that the concentration behaved like anticipated with the diffusion from the transmitter pacemaker to the receiver. The concentration had a maximum peak near the transmitter just after the molecules were emitted. Then, the concentration decreased in time and distance towards the receiver.

There were also discovered some flaws with the model. One of the anticipated effects were that the concentration should be evenly dispersed between the transmitter pacemaker and the receiver at the time where the messenger molecules reached the receiver pacemaker. This was not the case for the model. Approximately 0,08 seconds after the molecules were emitted from the transmitter, the concentration of the molecules was larger closer to the receiver than the transmitter. In reality, this value should even be a bit smaller at the receiver than the transmitter at this time because some of the molecules are absorbed by the environment. Therefore, the model needs a bit improvement. Since the concentration eventually evened out, this is not a major problem. Therefore, the overall performance of the model was approved.

To study the molecular diffusion between leadless pacemakers in a more realistic setting, the signaling was made stochastic instead of deterministic and with a varying signaling interval, T_s. First, a SISO communication system with an information theoretical approach was tested. As the encoder, on-off keying (OOK) was implemented, where the molecules either were transmitted or not. A binary hypothesis-testing channel was defined as the diffusion channel. A simplified version of the Neyman Pearson detector, which is based on the maximum likelihood test, was used as the detector.

In an attempt to improve the communication system, a SIMO approach with the same specifications, but with the receive diversity, Maximum-Ratio Combining, was tested. The mean values and variances of the random, independent variable, Z_i were updated accordingly. For the same reason, a MIMO approach was implemented with Spatial Multiplexing and updated mean values and variances. The encoder, channel and detector were still the same as with the SISO communication system.

The results from the stochastic SISO communication system showed a promising model. With increasing signaling interval, the distortion and ISI decreased significantly. A reliable communication system with appropriate low levels of distortion and ISI were achieved with the signaling interval, T_s of 150 seconds. This was therefore, defined as the optimal signaling interval for the SISO communication system. The capacity of the communication system was relatively high as the maximum mutual information was 0,958 bits at the optimal signaling interval. The data rate increased with higher signaling intervals and a decrease in the distortion. This correspond to the known theory. Overall, the SISO communication system worked as desired.

The SIMO communication system, had quite desired results as well. As for the SISO approach, the distortion and ISI decreased to a reasonable level when the optimal signaling interval was reached. This was the case at 200 seconds. Hence, the SIMO communication system had a higher optimal signaling interval than the SISO system, which gave the opportunity to work with higher signaling intervals. The capacity, however, were not as optimal as for the SISO system. With the optimal signaling interval, the maximum mutual information was 0,23 bits, which is quite a bit lower. The rate behaved as anticipated with higher signaling interval and lower distortion for the SIMO communication system as well. With all the parameters taken into account, this approach gave desired results.

The implementation of the MIMO communication system was not equally successful. All though the distortion decreased with increasing signaling intervals, it did not decrease to an appropriate level. At the optimal signaling interval, the BER was 24,41 %, compared to 0,39 % and 0,74 % at the SISO and SIMO communication system, respectively. The capacity was also substantially lower than for the other approaches, with the maximum mutual information of $8{,}47 * 10^{-7}$ bits at the optimal signaling interval. Additionally, the rate behaved inverse than anticipated. With higher signaling intervals and decreasing distortion, the rate decreased. The reason for this was the fixed value of the maximum mutual information at every signaling interval. However, the ISI behaved like desired. It decreased with the increase of the signaling interval, and reached an appropriate level at the optimal signaling interval. This happened for the MIMO system, at 300 seconds, which was even higher than for the SIMO communication system. Although, the ISI was reasonable, the performance of the MIMO communication system was not sufficient. This approach is therefore not recommended for a communication system with the same specifications as in this report.

To summarize, both SISO communication and SIMO communication with the given combination of the communication components can be used for diffusion-based molecular communication between leadless pacemakers. If high capacity is wanted, it is favourable to use the SISO communication system. If you want to prioritize high signaling intervals, SIMO can be used, but both works fine. MIMO communication with the given combination of communication components is not recommended.

7.1 Further work

In this report, a couple of simplifications has been made. The simulations were mostly based on one-dimensional data, even though the decisions were made for optimizing a three-dimensional problem. In further work, the simulations should be done with three-dimensional data to get a more accurate result.

The blood velocity is in this report set to a fixed value. In a more realistic approach, the blood velocity should be time-varying. Additionally, the values of the distance between the pacemakers and the closest wall, and the distance between the receiver and the transmitter sensors should be more accurate.

The scope of this report does not include the physical model of the pacemakers and the actuator part. In further work, this should be studied as well. A device for collecting and displaying the data from the communication system is also an important study.

Overall, this novel study has a lot of potential in the following years.

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A Matlab code

8.1 The matlab code for the deterministic SISO communication system

8.1.1 The communication system

```
%% The deterministic SISO communication system

close all;
clc;
clear;

%% Molecule properties, diffusion coeffisient of molecules
D = 1e-6;

%% The position x =(x,y,z)
% Height
xmax = 0.075; %m
% Width
ymax = 0.08; %m
% Depth
zmax = 0.055; %m

%% The distance from the pacemaker to the closest wall of the heart
H = 0.0075; %m

%% The diameter of the heart
d = 0.035; %m

%% The number of molecules released per second
Qt = 500 ;

%% The blood flow velocity
u = 0.15; %m/s

%% Blood fluid properties
rho = 1.06*1e3; % Blood density
m = 0.0488*1e-1; % Blood viscosity
nu = m/rho; % Blood kinematic viscosity
R = u*d/nu; % Reynolds number
K = R*D; % Eddy diffusitives

%% pi
pi = 3.14;

%% The time
t0 = 0.01; %s

%% The concentration
N = 64;

c = @(x,y,z,t)  (Qt/(8*(pi*(K*abs(x)/u))^(3/2)))*exp((-(x-u*t)^2-y^2)...
    /(4*(K*x/u)))*(exp(-(z-H)^2/(4*(K*abs(x)/u)))...
    +exp(-(z+H)^2/(4*(K*abs(x)/u))));
```

```
%% Making the x-axis, y-axis, z-axis and time-axis
dr = xmax/N; %Step size
dt = 0.01; %Step size

xaxis = (dr):dr:(4*xmax/3);
yaxis = -(2*ymax/5):dr:(2*ymax/5);
zaxis = 0:dr:zmax;
taxis = 0:dt:1;
xaxis = xaxis+dr/2; %Take away nan numbers
yaxis = yaxis+dr/2;

%% Making a matrix for the concentration with varying values for x- and y,
% and fixed values for z and t
C = zeros(length(xaxis),length(yaxis));
for x = xaxis
    for y = yaxis
        C(x == xaxis,y == yaxis) = c(x,y,zmax,t0);
    end
end

%% Plotting the concentration depending on the distance from the
% transmitter in x- and y-direction with surf and contour plot
figure;
surf(yaxis,xaxis,C);
view([90 90]);
colorbar;
xlabel('y [m]');
ylabel('x [m]');
title('The concentration of molecules with surf plot');

figure;

contourf(xaxis,yaxis,C');
colorbar
ylabel('y [m]');
xlabel('x [m]');
title('The concentration of molecules with contour plot');
```

8.1.2 The performance analysis

```
%% The deterministic SISO communication system - performance analysis

close all;
clc;
clear;

%% Molecule properties, diffusion coeffisient of molecules
D = 1e-6; %m^2/s

%% The position x =(x,y,z)
% Height
xmax = 0.075; %m
% Width
ymax = 0.08; %m
% Depth
zmax = 0.055; %m
```

```
%% The distance from the pacemaker to the closest wall of the heart
H = 0.0075; %m

%% The diameter of the heart
d = 0.035; %m

%% The number of molecules released per second
Qt = 500 ;

%% The blood flow velocity
u = 0.15; %m/s

%% Blood fluid properties
rho = 1.06*1e3; % Blood density
m = 0.0488*1e-1; % Blood viscosity
nu = m/rho; % Blood kinematic viscosity
R = u*d/nu; % Reynolds number
K = R*D; % Eddy diffusitives

%% pi
pi = 3.14;

%% The time
t0 = 0.01; %s

%% The concentration
N = 64;

c = @(x,y,z,t) (Qt/(8*(pi*(K*abs(x)/u))^(3/2)))*exp((-(x-u*t)^2-y^2)...
    /(4*(K*x/u)))*(exp(-(z-H)^2/(4*(K*abs(x)/u)))...
    +exp(-(z+H)^2/(4*(K*abs(x)/u))));

%% Making the x-axis, y-axis, z-axis and time-axis
dr = xmax/N; %Step size
dt = 0.01; %Step size

xaxis = (dr):dr:(xmax);
yaxis = (dr):dr:(ymax/2);
zaxis = 0:dr:zmax;
taxis = 0:dt:1;
xaxis = xaxis+dr/2; %Take away nan numbers
yaxis = yaxis+dr/2;

%% Making a matrix for the concentration with varying values for x and t,
% and fixed values for y and z
h = zeros(length(taxis),length(xaxis));
for t = taxis
    for x = xaxis
        h(t == taxis,x == xaxis) = c(x,0,0,t);
    end
end

%% Defining the delay and the gain of the concentration
delay = zeros(1,length(xaxis));
maxc = zeros(1,length(xaxis));
gain = zeros(1,length(xaxis));

testmax = max(max(h));
```

```
for x = xaxis
    [m,i] = max(h(:,x == xaxis));
    maxc(x == xaxis) = m;
    gain(x == xaxis) = (max(h(:,x == xaxis))/(testmax)); % finds the max
    % value of all the max concentrations
    delay(x == xaxis) = taxis(i);
end

%% Plotting the concentration depending on the distance from the
% transmitter in x-direction and the time elapsing from the molecules are
% emitted
figure;
plot(taxis,h);
title('The concentration');
xlabel('time [s]');
ylabel('Concentration [kg/m^3]');

%% Plotting the gain in dB depending on the distance from the transmitter
%in x-direction
figure;
plot(xaxis,20*log10(gain))
title('The gain');
xlabel('distance [m]');
ylabel('gain [dB]');

%% Plotting the delay in dB depending on the distance from the transmitter
%in x-direction
figure;
plot(xaxis,delay);
title('The delay');
xlabel('distance [m]');
ylabel('delay [s]');
```

8.2 The matlab code for the stochastic SISO communication system

```
%% The stochastic SISO communication system

close all;
clc;
clear;

%% Molecule properties, diffusion coeffisient of molecules
D = 0.5; %m^2/s

%% The number of particles in a mol
mol = 6.022*10^23; % Particles

%% The position x =(x,y,z)
% Height
xmax = 0.075; %m
% Width
ymax = 0.08; %m
% Depth
zmax = 0.055; %m
```

```
%% The distance from the pacemaker to the closest wall of the heart
H = 0.0075; %m

%% The diameter of the heart
d = 0.035; %m

%% The distance between transmit sensor and receive sensor
d11 = 0.03; %m

%% The blood flow velocity
u = 0.15; %m/s

%% Blood fluid properties
rho = 1.06*1e3; % Blood density
m = 0.0488*1e-1; % Blood viscosity
nu = m/rho; % Blood kinematic viscosity
R = u*d/nu; % Reynolds number
K = R*D; % Eddy diffusitives

%% pi
pi = 3.14;

%% The signaling interval, Ts
Ts = 150; %s

%% Transmitted sequence with On-off keying (OOK)

M = 10000;                           %The number of bits that is transmitted
X = round(rand(1,M));             %The transmitted sequence X
N = 20;                             %The number of interfering intervals

b = sprintf('%d',X);
fprintf('X = %s\n',b);

%% The mean value and variance of the stocastic number of emitted molecules
% , Q
mean_Q = 0.001*mol;
var_Q = (0.3*mean_Q)^2;

%% Defining the random variable Z as a Bernoulli distributed variable
a0 = zeros();
Z = zeros();
fun = zeros();                   %The filter
con = zeros();
c = zeros();                     %The concentration of the molecules

% The number of molecules released per second with stocastic signaling
Q = abs(randn(1,M)*sqrt(var_Q)+mean_Q);

for i = 0:M-1
        if X(i+1) == 0
            fun = @(x,y,z,t,j) (1/(8*(pi*(K*abs(x)/u)).^(3/2)))...
                *exp((-(x-u*(t+j*Ts)).^2-y.^2)/(4*(K*x/u)))...
                *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
                +exp(-(z+H).^2/(4*(K*abs(x)/u))));
            con = @(x,y,z,t,j) ((X(j+1)*Q(j+1))...
                /(8*(pi*(K*abs(x)/u)).^(3/2)))...
                *exp((-(x-u*(t-j*Ts)).^2-y.^2)/(4*(K*x/u)))...
                *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
```

```
            +exp(-(z+H).^2/(4*(K*abs(x)/u))));
        dt = 0.1;
        taxis = i*Ts:dt:((i+1)*Ts)-dt;
        Ztemp = 0;
        for j = max(0,(i-N)):(i-1)
            a_ij = integral(@(t) fun(d11,0,0,t,(i-j)),0,Ts);
            a0 = integral(@(t) fun(d11,0,0,t,0),0,Ts);
            Ztemp = Ztemp + X(j+1)*a_ij*Q(j+1);
        end
        Z(i+1) = Ztemp;

        ctemp = 0;
        for j = max(0,(i-N)):i
            c_j = con(d11,0,0,taxis,j).*dt;
            ctemp = ctemp + sum(c_j);
        end
        c(i+1) = ctemp;
    elseif X(i+1) == 1
        fun = @(x,y,z,t,j) (1/(8*(pi*(K*abs(x)/u)).^(3/2)))...
            *exp((-(x-u*(t+j*Ts)).^2-y.^2)/(4*(K*x/u)))...
            *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
            +exp(-(z+H).^2/(4*(K*abs(x)/u))));
        con = @(x,y,z,t,j) ((X(j+1)*Q(j+1))...
            /(8*(pi*(K*abs(x)/u)).^(3/2)))...
            *exp((-(x-u*(t-j*Ts)).^2-y.^2)/(4*(K*x/u)))...
            *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
            +exp(-(z+H).^2/(4*(K*abs(x)/u))));
        dt = 0.1;
        taxis = i*Ts:dt:((i+1)*Ts)-dt;
        Ztemp = 0;
        for j = max(0,(i-N)):(i-1)
            a_ij = integral(@(t) fun(d11,0,0,t,(i-j)),0,Ts);
            Ztemp = Ztemp + X(j+1)*a_ij*Q(j+1);
        end
        Z(i+1) = a0*Q(i+1)+Ztemp;

        ctemp = 0;
        for j = max(0,(i-N)):i
            c_j = con(d11,0,0,taxis,j).*dt;
            ctemp = ctemp + sum(c_j);
        end
        c(i+1) = ctemp;
    end
end
%% The filter coeffiecients a
for j = 0:N
    a(j+1) = integral(@(t) fun(d11,0,0,t,j),0,Ts);
end

%% Making the time axis
dt = 0.01; %Step size
taxis = i*Ts:dt:Ts-dt;

%% The concentration with stocastic signaling and the signaling interval Ts
figure;
plot(c);
title('The concentration');
xlabel('Signaling intervals, Ts [10^2 s]');
ylabel('Concentration [kg/m^3]');
```

```
%% Histogram of the random z variabel that corresponds to X=0 and X=1
Z_one_temp = 0;
Z_zero_temp = 0;
Z_one = zeros();
Z_zero = zeros();

k = 1;
p = 1;
for l = 1:length(X)
    if X(l) == 0
        Z_zero(k) = Z(l);
        k = k+1;

    elseif X(l) == 1
        Z_one(p) = Z(l);
        p = p+1;
    end
end
figure;
hist(Z_zero,50);
title('Histogram Z0');
xlabel('The value of Z0')
ylabel('The bit number of Z0');

figure;
hist(Z_one,50);
title('Histogram Z1');
xlabel('The value of Z1')
ylabel('The bit number of Z1');

%% Finding the optimal threshold n and the optimal a priori probability p1
mean_Z0_opt = 0;
mean_Z1_opt = 0;
var_Z0_opt = 0;
var_Z1_opt = 0;

Itemp = 0;
I_temp = zeros();
PF_opt = 0;
PD_opt = 0;

mm = 1;
start = 1;
step = 10^14;
stop = 5*10^18;
for nn = start:step:stop
    b = 1;
    for p1temp = 0:0.1:1
        mean_Z0_opt = p1temp*mean_Q*sum(a(2:end));
        mean_Z1_opt = a0*mean_Q+mean_Z0_opt;
        var_Z0_opt = (p1temp.^2*var_Q+mean_Q.^2*(p1temp-p1temp.^2)...
            +var_Q*(p1temp-p1temp.^2))*sum(a(2:end).^2);
        var_Z1_opt = a0.^2*var_Q+var_Z0_opt;

        PF_opt(mm) = qfunc((nn-mean_Z0_opt)/sqrt(var_Z0_opt));
        PD_opt(mm) = qfunc((nn-mean_Z1_opt)/sqrt(var_Z1_opt));

%% Mutual information, I
        PY0 = (1-PF_opt(mm))*(1-p1temp)+(1-PD_opt(mm))*p1temp;
        PY1 = PF_opt(mm)*(1-p1temp)+PD_opt(mm)*p1temp;
```

```
        if PY0 == 0 || PY1 == 0
            Itemp = 0;
        else
        Itemp = ((PF_opt(mm)).*(1-p1temp).*log2((PF_opt(mm))/PY1)... %
Yi=1|Xi=0
        +((1-PF_opt(mm)).*(1-p1temp).*log2((1-PF_opt(mm))/PY0)) ... %
Yi=0|Xi=0
        +((PD_opt(mm)).*(p1temp).*log2((PD_opt(mm))/PY1)) ... % Yi=1|Xi=1
        +((1-PD_opt(mm)).*(p1temp).*log2((1-PD_opt(mm))/PY0))); % Yi=0|Xi=1
        end

        I_temp(b) = Itemp;

        b = b+1;
    end
    [Imaxtemp, p1_temp] = max(I_temp);
    I(mm) = Imaxtemp;
    p1opt(mm) = p1_temp;
    mm = mm+1;

end

%% Making the n-axis
naxis = start:step:stop;
figure;
plot(naxis, I);
title('Mutual information');
xlabel('The threshold, n');
ylabel('Mutual information, I(P_D,P_F,p_1)[bits]')

%% The rate
rate = zeros();
for i = 1:length(I)
rate(i) = I(i)/Ts;
end

figure;
plot(naxis,rate);
title('The rate');
xlabel('The threshold, n');
ylabel('The rate, R [bits/s]');

%% The maximum mutual information Imax and the optimal threshold n
[Imax, n] = max(I);
fprintf('Maximum value of the mutual information_p1 = %d\n',Imax);
fprintf('Optimal n, n_opt = %d\n',start+(n*step));

%% The maximum rate, R_max
R_max = Imax/Ts;
fprintf('Maximum value of the rate, R_max = %d\n',R_max);

%% The optimal a priori probability p1
switch p1opt(n)
    case {1}
        p1 = 0;
    case {2}
        p1 = 0.1;
    case {3}
        p1 = 0.2;
    case {4}
```

```
        p1 = 0.3;
    case {5}
        p1 = 0.4;
    case {6}
        p1 = 0.5;
    case {7}
        p1 = 0.6;
    case {8}
        p1 = 0.7;
    case {9}
        p1 = 0.8;
    case {10}
        p1 = 0.9;
    case {11}
        p1 = 1;
end
fprintf('Optimal p1, p1_opt = %1g\n',p1);

%% The mean value and the variance of the independent random variable Z

mean_Z0 = p1*mean_Q*sum(a(2:end));
mean_Z1 = a0*mean_Q+mean_Z0;
var_Z0 = (p1.^2*var_Q+mean_Q.^2*(p1-p1.^2)+var_Q*(p1-p1.^2))...
    *sum(a(2:end).^2);
var_Z1 = a0.^2*var_Q+var_Z0;


%% The distribution of Z given the two hypothesis H0 and H1
z = Z;
Atemp = 0;
pZi_H1_temp = 0;
pZi_H0_temp = 0;

for i = 1:length(z)
    pZi_H1_temp = (1/(sqrt(2*pi*var_Z1)))*exp(-(z(i)-
mean_Z1).^2/(2*var_Z1));
    pZi_H0_temp = (1/(sqrt(2*pi*var_Z0)))*exp(-(z(i)-
mean_Z0).^2/(2*var_Z0));
    if pZi_H0_temp == 0
        Atemp = 0;
    else
        Atemp = pZi_H1_temp/pZi_H0_temp;
    end
    pZi_H1(i) = pZi_H1_temp;
    pZi_H0(i) = pZi_H0_temp;

%% Likelihood ratio function, A
    A(i) = Atemp;

end
figure;
plot(pZi_H1);
title('PDF of Z1');

figure;
plot(pZi_H0);
title('PDF of Z0');
```

```
%% The threshold, n
n = start+(n*step);

%% The threshold, lambda
exponent = log(sqrt(var_Z0/var_Z1))+((((var_Z1-var_Z0)*n...
    -(mean_Z0*var_Z0)).^2-((mean_Z0*var_Z1)-(mean_Z1*var_Z0)).^2)...
    /(2*var_Z0*var_Z1*(var_Z1-var_Z0)))+(((mean_Z0.^2*var_Z1)...
    -(mean_Z1.^2*var_Z0))/(2*var_Z0*var_Z1));
thres = exp(exponent);

%% False alarm probability, PF and detection probability, PD
PF = qfunc((n-mean_Z0)/sqrt(var_Z0));
PD = qfunc((n-mean_Z1)/sqrt(var_Z1));

fprintf('The calculated false alarm probability PF = %1g\n',PF);
fprintf('The calculated detection probability PD = %1g\n',PD);
%% Binary hypothesis testing channel with the original threshold lamda
% for l = 1:length(A)
%     if A(l) > thres
%         Y(l) = 1;
%     elseif A(l) < thres
%         Y(l) = 0;
%     end
% end
% g = sprintf('%d',Y);
% fprintf('Y = %s\n',g);

%% Binary hypothesis testing channel simplified with the threshold, n
for l = 1:length(z)
    if z(l) > n
        Y(l) = 1;
    elseif z(l) < n
        Y(l) = 0;
    end
end
g = sprintf('%d',Y);
fprintf('Y = %s\n',g);

%% Plotting the output vector Y, versus the input vector X
figure;
plot(X,'r');
title('The output signal Y vs the input signal X with overlapping plots');
hold on
plot(Y,'b');


figure;
subplot(2,1,1);
plot(X,'r');
title('The output signal Y vs the input signal X');
subplot(2,1,2);
plot(Y,'b');


%% The distortion, D
figure;
plot(abs(X-Y));
title('The distortion, D');
xlabel('The bit number with distortion');
ylabel('The distortion, D');
```

```
%% The number of bit errors and average total bit error probability, BER
[biterror_number, ratio] = biterr(X,Y);
fprintf('The number of bit errors = %1g\n',biterror_number);
fprintf('Average total bit error probability, BER = %1g\n',ratio);
```

8.2 The matlab code for the stochastic SIMO communication system

```
%% The stochastic SIMO communication system with Maximum-Ratio Combining

close all;
clc;
clear;

%% Molecule properties, diffusion coeffisient of molecules
D = 0.5;

%% The number of particles in a mol
mol = 6.022*10^23;

%% The position x =(x,y,z)
% Height
xmax = 0.075; %m
% Width
ymax = 0.08; %m
% Depth
zmax = 0.055; %m

%% The distance from the pacemaker to the closest wall of the heart
H = 0.0075; %m

%% The diameter of the heart
d = 0.035; %m

%% The blood flow velocity
u = 0.15; %m/s

%% Blood fluid properties
rho = 1.06*1e3; % Blood density
m = 0.0488*1e-1; % Blood viscosity
nu = m/rho; % Blood kinematic viscosity
R = u*d/nu; % Reynolds number
K = R*D; % Eddy diffusitives

%% pi
pi = 3.14;

%% The signaling interval, Ts
Ts = 200; %

%% Transmitted sequence with On-off keying (OOK)

M = 10000;                    %The number of bits that is transmitted
X = round(rand(1,M));       %The transmitted sequence X
N = 20;                       %The number of interfering intervals
```

```
b = sprintf('%d',X);
fprintf('X = %s\n',b);

%% The mean value and variance of the stocastic number of emitted molecules
% , Q
mean_Q = 0.001*mol;
var_Q = (0.3*mean_Q)^2;

%% The distances between the transmit sensor and the two receiver sensors

d11 = 0.03;
d12 = 0.04;


%% Defining the random variable Z as a Bernoulli distributed variable
a11_ij = zeros();
a12_ij = zeros();
a11 = zeros();
a12 = zeros();
a_j = zeros();
a11_0 = zeros();
a12_0 = zeros();
Z11 = zeros();
Z12 = zeros();
fun = zeros();
con = zeros();

% The number of molecules released per second with stocastic signaling
Q = abs(randn(1,M)*sqrt(var_Q)+mean_Q);

for i = 0:M-1
        if X(i+1) == 0
            fun = @(x,y,z,t,j) (1/(8*(pi*(K*abs(x)/u)).^(3/2)))...
                *exp((-(x-u*(t+j*Ts)).^2-y.^2)/(4*(K*x/u)))...
                *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
                +exp(-(z+H).^2/(4*(K*abs(x)/u))));
            Z11temp = 0;
            Z12temp = 0;
            for j = max(0,(i-N)):(i-1)
                a11_ij = integral(@(t) fun(d11,0,0,t,(i-j)),0,Ts);
                a12_ij = integral(@(t) fun((d12),0,0,t,(i-j)),0,Ts);
                a11_0 = integral(@(t) fun(d11,0,0,t,0),0,Ts);
                a12_0 = integral(@(t) fun((d12),0,0,t,0),0,Ts);

                Z11temp = Z11temp + X(j+1)*a11_ij*Q(j+1);
                Z12temp = Z12temp + X(j+1)*a12_ij*Q(j+1);
            end
            Z11(i+1) = Z11temp;
            Z12(i+1) = Z12temp;
        elseif X(i+1) == 1
            fun = @(x,y,z,t,j) (1/(8*(pi*(K*abs(x)/u)).^(3/2)))...
                *exp((-(x-u*(t+j*Ts)).^2-y.^2)/(4*(K*x/u)))...
                *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
                +exp(-(z+H).^2/(4*(K*abs(x)/u))));
            Z11temp = 0;
            Z12temp = 0;
            for j = max(0,(i-N)):(i-1)
                a11_ij = integral(@(t) fun(d11,0,0,t,(i-j)),0,Ts);
                a12_ij = integral(@(t) fun((d12),0,0,t,(i-j)),0,Ts);
```

```
                Z11temp = Z11temp + X(j+1)*a11_ij*Q(j+1);
                Z12temp = Z12temp + X(j+1)*a12_ij*Q(j+1);
            end
            Z11(i+1) = a11_0*Q(i+1)+Z11temp;
            Z12(i+1) = a12_0*Q(i+1)+Z12temp;
        end
end
%% The coeffiecients a11 and a12
for j = 0:N
    a11(j+1) = integral(@(t) fun(d11,0,0,t,j),0,Ts);
    a12(j+1) = integral(@(t) fun((d12),0,0,t,j),0,Ts);
end


%% The weight in Maximum-ratio Combining

hp_1 = (1/(8*(pi*(K*d11/u)).^(3/2)))/(4*(K*d11/u))...
    *(exp(-H.^2/(4*(K*d11/u)))+exp(-(H).^2/(4*(K*d11/u))));
hp_2 = (1/(8*(pi*(K*d12/u)).^(3/2)))/(4*(K*d12/u))...
    *(exp(-H.^2/(4*(K*d12/u)))+exp(-(H).^2/(4*(K*d12/u))));

%% Finding the optimal threshold n and the optimal a priori probability p1
mean_Z0_opt = 0;
mean_Z1_opt = 0;
var_Z0_opt = 0;
var_Z1_opt = 0;

Itemp = 0;
I_temp = zeros();
PF_opt = 0;
PD_opt = 0;

mean_Z0_opt = 0;
mean_Z1_opt = 0;
var_Z0_opt = 0;
var_Z1_opt = 0;

mm = 1;
start = 1;
step = 1;
stop = 1000;

for nn = start:step:stop
    b = 1;
    for p1temp = 0.1:0.1:0.9
        xtemp = 0;

        %% The mean value and the variance of the independent random
        % variable Z
        mean11_Z0opt = p1temp*mean_Q*sum(a11(2:end));
        mean12_Z0opt = p1temp*mean_Q*sum(a12(2:end));
        mean11_Z1opt = a11_0*mean_Q+mean11_Z0opt;
        mean12_Z1opt = a12_0*mean_Q+mean12_Z0opt;
        var11_Z0opt = (p1temp.^2*var_Q+mean_Q.^2*(p1temp-p1temp.^2)...
            +var_Q*(p1temp-p1temp.^2))*sum(a11(2:end).^2);
        var12_Z0opt = (p1temp.^2*var_Q+mean_Q.^2*(p1temp-p1temp.^2)...
            +var_Q*(p1temp-p1temp.^2))*sum(a12(2:end).^2);
        var11_Z1opt = a11_0.^2*var_Q+var11_Z0opt;
```

```
        var12_Z1opt = a12_0.^2*var_Q+var12_Z0opt;

        %% The variable b (a_n from article)
        if (var11_Z0opt == 0)
            b_1 = 0;
        else
            b_1 = 10^26*hp_1/var11_Z0opt;
        end

        if (var12_Z0opt == 0)
            b_2 = 0;
        else
            b_2 = 10^26*hp_2/var12_Z0opt;
        end


        %% The optimal combined mean value and variance of the independent
        % variable Z from both channel paths

        mean_Z0_opt = b_1*mean11_Z0opt+b_2*mean12_Z0opt;
        mean_Z1_opt = b_1*(mean_Q*hp_1+mean11_Z0opt)...
            +b_2*(mean_Q*hp_2+mean12_Z0opt);

        var_Z0_opt = b_1^2*var11_Z0opt+b_2^2*var12_Z0opt;
        var_Z1_opt = b_1^2*a11_0^2*var_Q+b_2^2*a12_0^2*var_Q+var_Z0_opt;

        PF_opt(mm) = qfunc((nn-mean_Z0_opt)/sqrt(var_Z0_opt));
        PD_opt(mm) = qfunc((nn-mean_Z1_opt)/sqrt(var_Z1_opt));

%% Mutual information, I
        PY0 = (1-PF_opt(mm))*(1-p1temp)+(1-PD_opt(mm))*p1temp;
        PY1 = PF_opt(mm)*(1-p1temp)+PD_opt(mm)*p1temp;
        Itemp = ((PF_opt(mm)).*(1-p1temp).*log2((PF_opt(mm))/PY1)... %
Yi=1|Xi=0
        +((1-PF_opt(mm)).*(1-p1temp).*log2((1-PF_opt(mm))/PY0)) ... %
Yi=0|Xi=0
        +((PD_opt(mm)).*(p1temp).*log2((PD_opt(mm))/PY1)) ... % Yi=1|Xi=1
        +((1-PD_opt(mm)).*(p1temp).*log2((1-PD_opt(mm))/PY0))); % Yi=0|Xi=1
        I_temp(b)=Itemp;

        b = b+1;

     end
    [Imaxtemp, p1_temp] = max(I_temp);
    I(mm) = Imaxtemp;
    p1opt(mm) = p1_temp;
    mm = mm+1;
end
naxis = start:step:stop;
figure;
plot(naxis, I);
title('Mutual information');
xlabel('The threshold, n');
ylabel('Mutual information, I(P_D,P_F,p_1)[bits]')

%% The rate
rate = zeros();
for i = 1:length(I)
rate(i) = I(i)/Ts;
```

```
end

figure;
plot(rate);
title('The rate');
xlabel('The threshold, n');
ylabel('The rate, R [bits/s]');

%% The maximum mutual information Imax and the optimal threshold n
[Imax, n] = max(I);
fprintf('Maximum value of the mutual information_p1 = %d\n',Imax);
fprintf('Optimal n, n_opt = %d\n',start+(n*step));

%% The maximum rate, R_max
R_max = Imax/Ts;
fprintf('Maximum value of the rate, R_max = %d\n',R_max);

%% The optimal a priori probability p1
switch p1opt(n)
    case {1}
        p1 = 0.1;
    case {2}
        p1 = 0.2;
    case {3}
        p1 = 0.3;
    case {4}
        p1 = 0.4;
    case {5}
        p1 = 0.5;
    case {6}
        p1 = 0.6;
    case {7}
        p1 = 0.7;
    case {8}
        p1 = 0.8;
    case {9}
        p1 = 0.9;
end
fprintf('Optimal p1, p1_opt = %1g\n',p1);

%% The variable b (a_n from article)
Ztemp = 0;
Z = zeros();

mean_Z0 = 0;
mean_Z1 = 0;
var_Z0 = 0;
var_Z1 = 0;

%% The mean value and the variance of the independent random variable Z
mean11_Z0 = p1*mean_Q*sum(a11(2:end));
mean12_Z0 = p1*mean_Q*sum(a12(2:end));
mean11_Z1 = a11_0*mean_Q+mean11_Z0;
mean12_Z1 = a12_0*mean_Q+mean12_Z0;
var11_Z0 = (p1.^2*var_Q+mean_Q.^2*(p1-p1.^2)+var_Q*(p1-p1.^2))...
    *sum(a11(2:end).^2);
var12_Z0 = (p1.^2*var_Q+mean_Q.^2*(p1-p1.^2)+var_Q*(p1-p1.^2))...
    *sum(a12(2:end).^2);
var11_Z1 = a11_0.^2*var_Q+var11_Z0;
var12_Z1 = a12_0.^2*var_Q+var12_Z0;
```

```
if (var11_Z0 == 0)
    b_1 = 0;
else
    b_1 = 10^26*hp_1/var11_Z0;
end

if (var12_Z0 == 0)
    b_2 = 0;
else
    b_2 = 10^26*hp_2/var12_Z0;
end

for p = 1:M
    %% The combined random variable Z
    Ztemp = Ztemp + (b_1).*Z11(p)+(b_2).*Z12(p);
    Z(p) = Ztemp;
    Ztemp = 0;
end

%% The combined mean value and variance of the independent variable Z
% from both channel paths

mean_Z0 = mean_Z0 + b_1*mean11_Z0+b_2*mean12_Z0;

%% The combined mean value and variance of the independent variable Z
% from both channel paths

mean_Z0 = b_1*mean11_Z0+b_2*mean12_Z0;
mean_Z1 = b_1*(mean_Q*hp_1+mean11_Z0)+b_2*(mean_Q*hp_2+mean12_Z0);

var_Z0 = b_1^2*var11_Z0+b_2^2*var12_Z0;
var_Z1 = b_1^2*a11_0^2*var_Q+b_2^2*a12_0^2*var_Q+var_Z0;

Z(p) = Ztemp;
Ztemp = 0;

%% Histogram of the random z variabel that corresponds to X=0 and X=1
Z_one_temp = 0;
Z_zero_temp = 0;
Z_one = zeros();
Z_zero = zeros();

for l = 1:M
    if X(l) == 0
        Z_zero_temp = Z(l);
    elseif X(l) == 1
        Z_one_temp = Z(l);
    end
    Z_zero(l) = Z_zero_temp;
    Z_one(l) = Z_one_temp;
end
figure;
hist(Z_zero,100);
title('Histogram Z0');
xlabel('The value of Z0')
ylabel('The bit number of Z0');

figure;
```

```
hist(Z_one,100);
title('Histogram Z1');
xlabel('The value of Z1')
ylabel('The bit number of Z1');

%% The distribution of Z given the two hypothesis H0 and H1
z = Z;
Atemp = 0;
pZi_H1_temp = 0;
pZi_H0_temp = 0;

for i = 1:length(z)
    pZi_H1_temp = (1/(sqrt(2*pi*var_Z1)))*exp(-(z(i)-mean_Z1).^2/...
        (2*var_Z1));
    pZi_H0_temp = (1/(sqrt(2*pi*var_Z0)))*exp(-(z(i)-mean_Z0).^2/...
        (2*var_Z0));
    if pZi_H0_temp == 0
        Atemp = 0;
    else
        Atemp = pZi_H1_temp/pZi_H0_temp;
    end
    pZi_H1(i) = pZi_H1_temp;
    pZi_H0(i) = pZi_H0_temp;

%% Likelihood ratio function, A
    A(i) = Atemp;

end
figure;
plot(pZi_H1);
title('PDF of Z1');

figure;
plot(pZi_H0);
title('PDF of Z0');


%% The threshold, n
n = start+(n*step);

%% The threshold lambda
exponent = log(sqrt(var_Z0/var_Z1))+((((var_Z1-var_Z0)*n...
    -(mean_Z0*var_Z0)).^2-((mean_Z0*var_Z1)-(mean_Z1*var_Z0)).^2)...
    /(2*var_Z0*var_Z1*(var_Z1-var_Z0)))+(((mean_Z0.^2*var_Z1)...
    -(mean_Z1.^2*var_Z0))/(2*var_Z0*var_Z1));
thres = exp(exponent);

%% False alarm probability, PF and detection probability, PD
PF = qfunc((n-mean_Z0)/sqrt(var_Z0));
PD = qfunc((n-mean_Z1)/sqrt(var_Z1));

fprintf('The calculated false alarm probability PF = %1g\n',PF);
fprintf('The calculated detection probability PD = %1g\n',PD);
%% Binary hypothesis testing channel with the original threshold lamda
% for l = 1:length(A)
%     if A(l) > thres
%         Y(l) = 1;
%     elseif A(l) < thres
%         Y(l) = 0;
```

```
%     end
% end
% g=sprintf('%d',Y);
% fprintf('Y = %s\n',g);

%% Binary hypothesis testing channel simplified with the threshold, n
for l = 1:length(z)
    if z(l) > n
        Y(l) = 1;
    elseif z(l) < n
        Y(l) = 0;
    end
end
g = sprintf('%d',Y);
fprintf('Y = %s\n',g);

%% Plotting the output vector Y, versus the input vector X
figure;
plot(X,'r');
title('The output signal Y vs the input signal X with overlapping plots');
xlabel('The bit number');
ylabel('Y vs X');
hold on
plot(Y,'b');
xlabel('The bit number');
ylabel('Y vs X');


figure;
subplot(2,1,1);
plot(X,'r');
title('The output signal Y vs the input signal X');
subplot(2,1,2);
plot(Y,'b');
xlabel('The bit number');
ylabel('Y vs X');

%% The difference between the input bits and the utput bits
figure;
plot(abs(X-Y));
title('The distortion, D');
xlabel('The bit number that differs');
ylabel('Distortion');


%% The number of bit errors and average total bit error probability
[biterror_number, ratio] = biterr(X,Y);
fprintf('The number of bit errors = %1g\n',biterror_number);
fprintf('Average total bit error probability, BER = %1g\n',ratio);
```

8.3 The matlab code for the stochastic MIMO communication system

```
%%  The stochastic MIMO communication system with Spatial Multiplexing

close all;
clc;
clear;
```

```
%% Molecule properties, diffusion coeffisient of molecules
D = 0.5;

%% The number of particles in a mol
mol = 6.022*10^23;

%% The position x =(x,y,z)
% Height
xmax = 0.075; %m
% Width
ymax = 0.08; %m
% Depth
zmax = 0.055; %m

%% The distance from the pacemaker to the closest wall of the heart
H = 0.0075; %m

%% The diameter of the heart
d = 0.035; %m

%% The blood flow velocity
u = 0.15; %m/s

%% Blood fluid properties
rho = 1.06*1e3; % Blood density
m = 0.0488*1e-1; % Blood viscosity
nu = m/rho; % Blood kinematic viscosity
R = u*d/nu; % Reynolds number
K = R*D; % Eddy diffusitives

%% pi
pi = 3.14;

%% The signaling interval, Ts
Ts = 300; %

%% The distance from the two transmit sensors to the two receiver sensors
d11 = 0.02;
d12 = 0.03;
d21 = d12;
d22 = d11;

%% Transmitted sequence with On-off keying (OOK)from sensor 1
M = 1000;                        %The number of bits that is transmitted
N = 20;                          %The number of interfering intervals
X1 = round(rand(1,M));           %The transmitted sequence X1


b = sprintf('%d',X1);
fprintf('X1 = %s\n',b);

%% Transmitted sequence with On-off keying (OOK) from sensor 2
X2 = round(rand(1,M));           %The transmitted sequence X2

v = sprintf('%d',X2);
fprintf('X2 = %s\n',v);
```

```
%% The mean value and variance of the stocastic number of emitted molecules
% , Q
mean_Q = 0.001*mol;
var_Q = (0.3*mean_Q)^2;

%% The mean value and the variance of the interference
mean_I = 2*10^16;
var_I = (0.3*mean_I)^2;

%% A priori probability p1
p1 = 0.5;

%% Defining the random variable Z as a Bernoulli distributed variable
a11_ij = zeros();
a12_ij = zeros();
a21_ij = zeros();
a22_ij = zeros();

a11 = zeros();
a12 = zeros();
a21 = zeros();
a22 = zeros();

a11_0 = zeros();
a12_0 = zeros();
a21_0 = zeros();
a22_0 = zeros();

Z11 = zeros();
Z12 = zeros();
Z21 = zeros();
Z22 = zeros();

fun = zeros();
con = zeros();

% The number of molecules released per second with stocastic signaling, Q
Q = abs(randn(1,M)*sqrt(var_Q)+mean_Q);

%% Transmitting of molecules from sensor 1
for i = 0:M-1
        if X1(i+1) == 0
            fun = @(x,y,z,t,j) (1/(8*(pi*(K*abs(x)/u)).^(3/2)))...
                *exp((-(x-u*(t+j*Ts)).^2-y.^2)/(4*(K*x/u)))...
                *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
                +exp(-(z+H).^2/(4*(K*abs(x)/u))));
            Z11temp = 0;
            Z12temp = 0;
            for j = max(0,(i-N)):(i-1)
                a11_ij = integral(@(t) fun(d11,0,0,t,(i-j)),0,Ts);
                a12_ij = integral(@(t) fun(d12,0,0,t,(i-j)),0,Ts);
                a11_0 = integral(@(t) fun(d11,0,0,t,0),0,Ts);
                a12_0 = integral(@(t) fun(d12,0,0,t,0),0,Ts);

                Z11temp = Z11temp + X1(j+1)*a11_ij*Q(j+1);
                Z12temp = Z12temp + X1(j+1)*a12_ij*Q(j+1);
            end
            Z11(i+1) = Z11temp;
            Z12(i+1) = Z12temp;
```

```
        elseif X1(i+1) == 1
            fun = @(x,y,z,t,j) (1/(8*(pi*(K*abs(x)/u)).^(3/2)))...
                *exp((-(x-u*(t+j*Ts)).^2-y.^2)/(4*(K*x/u)))...
                *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
                +exp(-(z+H).^2/(4*(K*abs(x)/u))));
            Z11temp = 0;
            Z12temp = 0;
            for j=max(0,(i-N)):(i-1)
                a11_ij = integral(@(t) fun(d11,0,0,t,(i-j)),0,Ts);
                a12_ij = integral(@(t) fun(d12,0,0,t,(i-j)),0,Ts);
                Z11temp = Z11temp + X1(j+1)*a11_ij*Q(j+1);
                Z12temp = Z12temp + X1(j+1)*a12_ij*Q(j+1);
            end
            Z11(i+1) = a11_0*Q(i+1)+Z11temp;
            Z12(i+1) = a12_0*Q(i+1)+Z12temp;
        end
end
%% The coeffiecients a11 and a12
for j = 0:N
    a11(j+1) = integral(@(t) fun(d11,0,0,t,j),0,Ts);
    a12(j+1) = integral(@(t) fun(d12,0,0,t,j),0,Ts);
end

%% Transmitting of molecules from sensor 2
for i = 0:M-1
        if X2(i+1) == 0
            fun = @(x,y,z,t,j) (1/(8*(pi*(K*abs(x)/u)).^(3/2)))...
                *exp((-(x-u*(t+j*Ts)).^2-y.^2)/(4*(K*x/u)))...
                *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
                +exp(-(z+H).^2/(4*(K*abs(x)/u))));
            Z21temp = 0;
            Z22temp = 0;
            for j = max(0,(i-N)):(i-1)
                a21_ij = integral(@(t) fun(d21,0,0,t,(i-j)),0,Ts);
                a22_ij = integral(@(t) fun(d22,0,0,t,(i-j)),0,Ts);
                a21_0 = integral(@(t) fun(d21,0,0,t,0),0,Ts);
                a22_0 = integral(@(t) fun(d22,0,0,t,0),0,Ts);

                Z21temp = Z21temp + X2(j+1)*a21_ij*Q(j+1);
                Z22temp = Z22temp + X2(j+1)*a22_ij*Q(j+1);
            end
            Z21(i+1) = Z21temp;
            Z22(i+1) = Z22temp;
        elseif X2(i+1) == 1
            fun = @(x,y,z,t,j) (1/(8*(pi*(K*abs(x)/u)).^(3/2)))...
                *exp((-(x-u*(t+j*Ts)).^2-y.^2)/(4*(K*x/u)))...
                *(exp(-(z-H).^2/(4*(K*abs(x)/u)))...
                +exp(-(z+H).^2/(4*(K*abs(x)/u))));
            Z21temp = 0;
            Z22temp = 0;
            for j = max(0,(i-N)):(i-1)
                a21_ij = integral(@(t) fun(d21,0,0,t,(i-j)),0,Ts);
                a22_ij = integral(@(t) fun(d22,0,0,t,(i-j)),0,Ts);
                Z21temp = Z21temp + X2(j+1)*a21_ij*Q(j+1);
                Z22temp = Z22temp + X2(j+1)*a22_ij*Q(j+1);
            end
            Z21(i+1) = a21_0*Q(i+1)+Z21temp;
            Z22(i+1) = a22_0*Q(i+1)+Z22temp;
        end
end
%% The coeffiecients a11, a12, a21 and a22
```

```
for j=0:N
    a11(j+1) = integral(@(t) fun(d11,0,0,t,j),0,Ts);
    a12(j+1) = integral(@(t) fun(d12,0,0,t,j),0,Ts);
    a21(j+1) = integral(@(t) fun(d21,0,0,t,j),0,Ts);
    a22(j+1) = integral(@(t) fun(d22,0,0,t,j),0,Ts);
end

%% The weight in Maximum-ratio Combining

hp_11 = (1/(8*(pi*(K*d11/u)).^(3/2)))/(4*(K*d11/u))...
    *(exp(-H.^2/(4*(K*d11/u)))+exp(-(H).^2/(4*(K*d11/u))));
hp_12 = (1/(8*(pi*(K*d12/u)).^(3/2)))/(4*(K*d12/u))...
    *(exp(-H.^2/(4*(K*d12/u)))+exp(-(H).^2/(4*(K*d12/u))));
hp_21 = (1/(8*(pi*(K*d21/u)).^(3/2)))/(4*(K*d21/u))...
    *(exp(-H.^2/(4*(K*d21/u)))+exp(-(H).^2/(4*(K*d21/u))));
hp_22 = (1/(8*(pi*(K*d22/u)).^(3/2)))/(4*(K*d22/u))...
    *(exp(-H.^2/(4*(K*d22/u)))+exp(-(H).^2/(4*(K*d22/u))));

%% The mean value and the variance of the independent random variable Z
% from transmit antenna 1
mean11_Z0 = p1*mean_Q*sum(a11);
mean12_Z0 = p1*mean_Q*sum(a12);
mean11_Z1 = a11_0*mean_Q+mean11_Z0;
mean12_Z1 = a12_0*mean_Q+mean12_Z0;

var11_Z0 = (p1.^2*var_Q+mean_Q.^2*(p1-p1.^2)+var_Q*(p1-p1.^2))*sum(a11.^2);
var12_Z0 = (p1.^2*var_Q+mean_Q.^2*(p1-p1.^2)+var_Q*(p1-p1.^2))*sum(a12.^2);
var11_Z1 = a11_0.^2*var_Q+var11_Z0;
var12_Z1 = a12_0.^2*var_Q+var12_Z0;

%% The mean value of independent random variable Z from transmit antenna 2
mean21_Z0 = p1*mean_Q*sum(a21);
mean22_Z0 = p1*mean_Q*sum(a22);
mean21_Z1 = p1*mean_Q*sum(a21);
mean22_Z1 = p1*mean_Q*sum(a22);

%% The weight b in MRC from sensor 1 (a_n from article)
if (var11_Z0 == 0)
    b_11 = 0;
else
    b_11 = 10^27*hp_11/var11_Z0;
end

if (var12_Z0 == 0)
    b_12 = 0;
else
    b_12 = 10^27*hp_12/var12_Z0;
end

%% The combined mean value and variance of the independent variable Z from
% sensor 1
mean_Z0_1 = mean_I + p1*mean_Q*(hp_11+hp_12);
mean_Z1_1 = mean_Q*(hp_11+hp_12) + mean_Z0_1;

var_Z0_1 = var_I + (p1-p1.^2)*mean_Q.^2*hp_11.^2 + (p1-p1.^2)...
    *mean_Q.^2*hp_12.^2;
var_Z1_1 = var_Z0_1 + a11_0.^2*var_Q*hp_11.^2 + a12_0.^2*var_Q*hp_12.^2;
```

```
%% The mean value and the variance of the independent random variable Z
% from transmit sensor 2
mean21_Z0 = p1*mean_Q*sum(a21);
mean22_Z0 = p1*mean_Q*sum(a22);
mean21_Z1 = a21_0*mean_Q+mean21_Z0;
mean22_Z1 = a22_0*mean_Q+mean22_Z0;

var21_Z0 = (p1.^2*var_Q+mean_Q.^2*(p1-p1.^2)+var_Q*(p1-p1^2))*sum(a21.^2);
var22_Z0 = (p1.^2*var_Q+mean_Q.^2*(p1-p1.^2)+var_Q*(p1-p1.^2))*sum(a22.^2);
var21_Z1 = a21_0.^2*var_Q+var21_Z0;
var22_Z1 = a22_0.^2*var_Q+var22_Z0;

%% The weight b in MRC from sensor 2 (a_n from article)

if (var21_Z0 == 0)
    b_21 = 0;
else
    b_21 = 10^27*hp_21/var21_Z0;
end

if (var22_Z0 == 0)
    b_22 = 0;
else
    b_22 = 10^27*hp_22/var22_Z0;
end

%% The optimal combined mean value and variance of the independent variable
% Z from sensor 2

mean_Z0_2 = mean_I + p1*mean_Q*(hp_21+hp_22);
mean_Z1_2 = mean_Q*(hp_21+hp_22) + mean_Z0_2;

var_Z0_2 = var_I + (p1-p1.^2)*mean_Q.^2*hp_21.^2 + (p1-p1.^2)...
    *mean_Q.^2*hp_22.^2;
var_Z1_2 = var_Z0_2 + a21_0.^2*var_Q*hp_21.^2 + a22_0.^2*var_Q*hp_22.^2;

%% Finding the optimal threshold n and the optimal a priori probability p1

Itemp_1 = 0;
I_temp_1 = zeros();
PF_opt_1 = 0;
PD_opt_1 = 0;

Itemp_2 = 0;
I_temp_2 = zeros();
PF_opt_2 = 0;
PD_opt_2 = 0;

mm = 1;
start = 1;
step = 1;
stop = 1800;
for nn_1 = start:step:stop

    %% The optimal detection probability PD and false alarm probability PF
    PF_opt_1(mm) = qfunc((nn_1-mean_Z0_1)/sqrt(var_Z0_1));
    PD_opt_1(mm) = qfunc((nn_1-mean_Z1_1)/sqrt(var_Z1_1));

    %% Mutual information, I from transmit sensor 1
```

```
    PY0_1 = (1-PF_opt_1(mm))*(1-p1)+(1-PD_opt_1(mm))*p1;
    PY1_1 = PF_opt_1(mm)*(1-p1)+PD_opt_1(mm)*p1;

    Itemp_1 = ((PF_opt_1(mm)).*(1-p1).*log2((PF_opt_1(mm))/PY1_1)... %
Yi=1|Xi=0
    +((1-PF_opt_1(mm)).*(1-p1).*log2((1-PF_opt_1(mm))/PY0_1)) ... %
Yi=0|Xi=0
    +((PD_opt_1(mm)).*(p1).*log2((PD_opt_1(mm))/PY1_1)) ... % Yi=1|Xi=1
    +((1-PD_opt_1(mm)).*(p1).*log2((1-PD_opt_1(mm))/PY0_1))); % Yi=0|Xi=1
    I_1(mm) = Itemp_1;

    mm = mm+1;
end

naxis = start:step:stop;
figure;
plot(naxis, I_1);
title('Mutual information I, with transmit sensor 1');
xlabel('The threshold, n');
ylabel('Mutual information, I(P_D,P_F,p_1)[bits]')

%% The rate
rate_1 = zeros();
for i = 1:length(I_1)
    rate_1(i) = I_1(i)/Ts;
end

figure;
plot(naxis, rate_1);
title('The rate, R with transmit sensor 1');
xlabel('The threshold, n');
ylabel('The rate, R [bits/s]');

%% The maximum mutual information Imax and the optimal threshold n_1 for
% sensor 1
[Imax_1, n_1] = max(I_1);
fprintf('Maximum value of the mutual information with transmit sensor 1=
%d\n',Imax_1);
fprintf('Optimal n, n_opt for transmit sensor 1 =
%d\n',(start+(n_1*step)));

%% The maximum rate, R_max
R_max_1 = Imax_1/Ts;
fprintf('Maximum value of the rate, R_max for transmit sensor 1 =
%d\n',R_max_1);

%% The optimal threshold n for transmit sensor 2
mm = 1;

for nn_2 = start:step:stop

    PF_opt_2(mm) = qfunc((nn_2-mean_Z0_2)/sqrt(var_Z0_2));
    PD_opt_2(mm) = qfunc((nn_2-mean_Z1_2)/sqrt(var_Z1_2));

    %% Mutual information, I, with transmit sensor 2
    PY0_2 = (1-PF_opt_2(mm))*(1-p1)+(1-PD_opt_2(mm))*p1;
    PY1_2 = PF_opt_2(mm)*(1-p1)+PD_opt_2(mm)*p1;
```

```
    Itemp_2 = ((PF_opt_2(mm)).*(1-p1).*log2((PF_opt_2(mm))/PY1_2)... %
Yi=1|Xi=0
    +((1-PF_opt_2(mm)).*(1-p1).*log2((1-PF_opt_2(mm))/PY0_2)) ... %
Yi=0|Xi=0
    +((PD_opt_2(mm)).*(p1).*log2((PD_opt_2(mm))/PY1_2)) ... % Yi=1|Xi=1
    +((1-PD_opt_2(mm)).*(p1).*log2((1-PD_opt_2(mm))/PY0_2))); % Yi=0|Xi=1
    I_2(mm) = Itemp_2;

    mm = mm+1;

end

naxis = start:step:stop;
figure;
plot(naxis, I_2);
title('Mutual information, I with transmit sensor 2');
xlabel('The threshold, n');
ylabel('Mutual information, I(P_D,P_F,p_1)[bits]')

%% The rate
rate_2 = zeros();
for i = 1:length(I_2)
    rate_2(i) = I_2(i)/Ts;
end

figure;
plot(naxis, rate_2);
title('The rate, R with transmit sensor 2');
xlabel('The threshold, n');
ylabel('The rate, R [bits/s]');

%% The maximum mutual information Imax and the optimal threshold n
[Imax_2, n_2] = max(I_2);
fprintf('Maximum value of the mutual information, I with transmit sensor 2
= %d\n',Imax_2);
fprintf('Optimal n, n_opt with transmit sensor 2 = %d\n',start+(n_2*step));

%% The maximum rate, R_max
R_max_2 = Imax_2/Ts;
fprintf('Maximum value of the rate, R_max with transmit sensor 2 =
%d\n',R_max_2);

%% The combined random variable Z
Z1temp = 0;
Z2temp = 0;
for p = 1:M

    In11 = Z12(p) + Z21(p) + Z22(p);
    In12 = Z11(p) + Z21(p) + Z22(p);
    In21 = Z11(p) + Z12(p) + Z22(p);
    In22 = Z11(p) + Z12(p) + Z21(p);

    sumtemp11 = sum(X1(p)*Q(p)*hp_12+X2(p)*Q(p)*hp_21 + X2(p)*Q(p)*hp_22);
    sumtemp12 = sum(X1(p)*Q(p)*hp_11+X2(p)*Q(p)*hp_21 + X2(p)*Q(p)*hp_22);
    sumtemp21 = sum(X1(p)*Q(p)*hp_11+X1(p)*Q(p)*hp_12 + X2(p)*Q(p)*hp_22);
    sumtemp22 = sum(X1(p)*Q(p)*hp_11+X1(p)*Q(p)*hp_12 + X2(p)*Q(p)*hp_21);

    Z_11 = X1(p)*Q(p)*hp_11 + In11 + sumtemp11;
    Z_12 = X1(p)*Q(p)*hp_12 + In12 + sumtemp12;
```

```
    Z_21 = X2(p)*Q(p)*hp_21 + In21 + sumtemp21;
    Z_22 = X2(p)*Q(p)*hp_22 + In22 + sumtemp22;

    %% The combined random variable Z1 from antenna 1 to te to receivers
    % and from antenna 2 to the two receivers

    Z1temp = Z1temp + (b_11).*Z_11+(b_12).*Z_12;
    Z2temp = Z2temp + (b_21).*Z_21+(b_22).*Z_22;

    Z_1(p) = Z1temp;
    Z_2(p) = Z2temp;
    Z1temp = 0;
    Z2temp = 0;
end

%% The combined mean value and variance of the independent variable Z from
% sensor 1

mean_Z0_1 = mean_I + p1*mean_Q*(hp_11+hp_12);
mean_Z1_1 = mean_Q*(hp_11+hp_12) + mean_Z0_1;

var_Z0_1 = var_I+(p1-p1.^2)*mean_Q.^2*hp_11.^2 + (p1-p1.^2)...
    *mean_Q.^2*hp_12.^2;
var_Z1_1 = var_Z0_1 + a11_0.^2*var_Q*hp_11.^2 + a12_0.^2*var_Q*hp_12.^2;

%% The combined mean value and variance of the independent variable Z from
% sensor 2

mean_Z0_2 = mean_Z0_2 + mean_I + p1*mean_Q*(hp_21+hp_22);
mean_Z1_2 = mean_Z1_2 + mean_Q*(hp_21+hp_22) + mean_Z0_2;

var_Z0_2 = var_I + (p1-p1.^2)*mean_Q.^2*hp_21.^2 + (p1-p1.^2)...
    *mean_Q.^2*hp_22.^2;
var_Z1_2 = var_Z0_2 + a21_0.^2*var_Q*hp_21.^2 + a22_0.^2*var_Q*hp_22.^2;

%% Histogram of the random variabel z that corresponds to X1=0 and X1=1
Z_one_temp_1 = 0;
Z_zero_temp_1 = 0;
Z_one_1 = zeros();
Z_zero_1 = zeros();

for l = 1:M
    if X1(l) == 0
        Z_zero_temp_1 = Z_1(l);
    elseif X1(l) == 1
        Z_one_temp_1 =Z_1(l);
    end
    Z_zero_1(l) = Z_zero_temp_1;
    Z_one_1(l) = Z_one_temp_1;
end
figure;
hist(Z_zero_1,100);
title('Histogram Z0 from transmit sensor 1');

figure;
hist(Z_one_1,100);
title('Histogram Z1 from transmit sensor 1');

%% Histogram of the random z variabel that corresponds to X2=0 and X2=1
```

```
Z_one_temp_2 = 0;
Z_zero_temp_2 = 0;
Z_one_2 = zeros();
Z_zero_2 = zeros();
for l = 1:M
    if X2(l) == 0
        Z_zero_temp_2 = Z_2(l);
    elseif X2(l) == 1
        Z_one_temp_2 = Z_2(l);
    end
    Z_zero_2(l) = Z_zero_temp_2;
    Z_one_2(l) = Z_one_temp_2;
end
figure;
hist(Z_zero_2,100);
title('Histogram Z0 from transmit sensor 2');

figure;
hist(Z_one_2,100);
title('Histogram Z1 from transmit sensor 2');

%% The distribution of Z given the two hypothesis H0 and H1 from sensor 1
z_1 = Z_1;
Atemp_1 = 0;
pZi_H1_temp_1 = 0;
pZi_H0_temp_1 = 0;

for i = 1:length(z_1)
    pZi_H1_temp_1 = (1/(sqrt(2*pi*var_Z1_1)))...
        *exp(-(z_1(i)-mean_Z1_1).^2/(2*var_Z1_1));
    pZi_H0_temp_1 = (1/(sqrt(2*pi*var_Z0_1)))...
        *exp(-(z_1(i)-mean_Z0_1).^2/(2*var_Z0_1));
    if pZi_H0_temp_1 == 0
        Atemp_1 = 0;
    else
        Atemp_1 = pZi_H1_temp_1/pZi_H0_temp_1;
    end
    pZi_H1_1(i) = pZi_H1_temp_1;
    pZi_H0_1(i) = pZi_H0_temp_1;



%% Likelihood ratio function, A
    A_1(i) = Atemp_1;

end
figure;
plot(pZi_H1_1);
title('PDF of Z1 with transmit sensor 1');

figure;
plot(pZi_H0_1);
title('PDF of Z0 with transmit sensor 1');


%% The distribution of Z given the two hypothesis H0 and H1 from sensor 2
z_2 = Z_2;
Atemp_2 = 0;
pZi_H1_temp_2 = 0;
```

```
pZi_H0_temp_2 = 0;

for i = 1:length(z_2)
    pZi_H1_temp_2 = (1/(sqrt(2*pi*var_Z1_2)))...
        *exp(-(z_2(i)-mean_Z1_2).^2/(2*var_Z1_2));
    pZi_H0_temp_2 = (1/(sqrt(2*pi*var_Z0_2)))...
        *exp(-(z_2(i)-mean_Z0_2).^2/(2*var_Z0_2));
    if pZi_H0_temp_2 == 0
        Atemp_2 = 0;
    else
        Atemp_2 = pZi_H1_temp_2/pZi_H0_temp_2;
    end
    pZi_H1_2(i) = pZi_H1_temp_2;
    pZi_H0_2(i) = pZi_H0_temp_2;



%% Likelihood ratio function, A
    A_2(i) = Atemp_2;

end
figure;
plot(pZi_H1_2);
title('PDF of Z1 with transmit sensor 2');

figure;
plot(pZi_H0_2);
title('PDF of Z0 with transmit sensor 2');

%% The threshold, n
n_1 = (start+(n_1*step));
n_2 = (start+(n_2*step));

%% The threshold lambda with transmit antenna 1
exponent_1 = log(sqrt(var_Z0_1/var_Z1_1))+((((var_Z1_1-var_Z0_1)...
    *n_1-(mean_Z0_1*var_Z0_1)).^2-((mean_Z0_1*var_Z1_1)...
    -(mean_Z1_1*var_Z0_1)).^2)/(2*var_Z0_1*var_Z1_1...
    *(var_Z1_1-var_Z0_1)))+(((mean_Z0_1.^2*var_Z1_1)...
    -(mean_Z1_1.^2*var_Z0_1))/(2*var_Z0_1*var_Z1_1));
thres_1 = exp(exponent_1);

%% The threshold lambda with transmit antenna 2
exponent_2 = log(sqrt(var_Z0_2/var_Z1_2))+((((var_Z1_2-var_Z0_2)...
    *n_2-(mean_Z0_2*var_Z0_2)).^2-((mean_Z0_2*var_Z1_2)...
    -(mean_Z1_2*var_Z0_2)).^2)/(2*var_Z0_2*var_Z1_2...
    *(var_Z1_2-var_Z0_2)))+(((mean_Z0_2.^2*var_Z1_2)...
    -(mean_Z1_2.^2*var_Z0_2))/(2*var_Z0_2*var_Z1_2));
thres_2 = exp(exponent_2);

%% False alarm probability, PF and detection probability, PD for transmit
% antenna 1
PF_1 = qfunc((n_1-mean_Z0_1)/sqrt(var_Z0_1));
PD_1 = qfunc((n_1-mean_Z1_1)/sqrt(var_Z1_1));

fprintf('The calculated false alarm probability PF_1 = %1g\n',PF_1);
fprintf('The calculated detection probability PD_1 = %1g\n',PD_1);

%% False alarm probability, PF and detection probability, PD for transmit
% antenna 2
```

```
PF_2 = qfunc((n_2-mean_Z0_2)/sqrt(var_Z0_2));
PD_2 = qfunc((n_2-mean_Z1_2)/sqrt(var_Z1_2));


fprintf('The calculated false alarm probability PF_2 = %1g\n',PF_2);
fprintf('The calculated detection probability PD_2 = %1g\n',PD_2);

%% Binary hypothesis testing channel with the original Neyman Pearson
% detector and threshold lamda with transmit sensor 1
% for l = 1:length(A_1)
%     if A_1(l) > thres_1
%         Y_1(l) = 1;
%     elseif A_1(l) < thres_1
%         Y_1(l) = 0;
%     end
% end
% g=sprintf('%d',Y_1);
% fprintf('Y_1 = %s\n',g);

%% Binary hypothesis testing channel with the original Neyman Pearson
% detector and threshold lamda with transmit sensor 2
% for l = 1:length(A_2)
%     if A_2(l) > thres_2
%         Y_2(l) = 1;
%     elseif A_2(l) < thres_2
%         Y_2(l) = 0;
%     end
% end
% g=sprintf('%d',Y_1);
% fprintf('Y_2 = %s\n',g);

%% Binary hypothesis testing channel simplified with the threshold, n, and
% transmit sensor 2
for l = 1:length(z_2)
    if z_2(l) > n_2
        Y_2(l) = 1;
    elseif z_2(l) < n_2
        Y_2(l) = 0;
    end
end
g = sprintf('%d',Y_2);
fprintf('Y2 = %s\n',g);

%% Plotting the output vector Y, versus the input vector X with transmit
% sensor 1
figure;
plot(X1,'r');
hold on
plot(Y_1,'b');
title('The output signal Y1 vs the input signal X1 with overlapping plots
with transmit sensor 1');

%% Plotting the output vector Y, versus the input vector X with transmit
% sensor 2
figure;
plot(X2,'r');
title('The output signal Y1 vs the input signal X1 with overlapping plots
with transmit sensor 2');
hold on
plot(Y_2,'b');
```

```
figure;
subplot(2,1,1);
plot(X1,'r');
title('The output signal Y1 vs the input signal X1');
subplot(2,1,2);
plot(Y_1,'b');

figure;
subplot(2,1,1);
plot(X2,'r');
title('The output signal Y2 vs the input signal X2');
subplot(2,1,2);
plot(Y_2,'b');


%% The difference between the input bits and the utput bits with transmit
% sensor 1
figure;
plot(abs(X1-Y_1));
title('The distortion, D with transmit sensor 1');

%% The difference between the input bits and the utput bits with transmit
% sensor 2
figure;
plot(abs(X2-Y_2));
title('The distortion, D with transmit sensor 2');

%% The number of bit errors and average total bit error probability with
% transmit sensor 1
[biterror_number_1, ratio] = biterr(X1,Y_1);
fprintf('The number of bit errors = %1g\n',biterror_number_1);
fprintf('Average total bit error probability, BER for X1 = %1g\n',ratio);

%% The number of bit errors and average total bit error probability with
% transmit sensor 2
[biterror_number_2, ratio] = biterr(X2,Y_2);
fprintf('The number of bit errors = %1g\n',biterror_number_2);
fprintf('Average total bit error probability, BER for X2 = %1g\n',ratio);
```