High velocity flow in a fractal reservoir

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Abstract

We propose a generalized reservoir-to-well inflow equation for inhomogeneous reservoirs. The equation is based on the assumption that reservoir properties may be described by power law expressions. The objective of the present study is to provide an improved inflow performance relationship for reservoirs with sparse fracture networks. This study is focused on one phase flow of low pressure gas under steady state conditions. The fractal flow model leads to rock properties of power law dependency. Other conceptual models may lead to the same result.

Due to low viscosity, high velocity gas flow is likely to occur in the neighborhood of a well. High velocity flow leads to an additional pressure drop when compared against Darcy flow. The additional pressure loss may be taken into account by an additional component of the skin factor. This component depends on the production rate. The effect of damage and rate dependent skin is to reduce the production rates. We study the influence of the interaction of reservoir and well properties on production rates. The objective is to enable estimation of the improvement in production rate by infill drilling and/or stimulation. Application of the method may improve production forecasting and aid design of wells in sparsely fractured reservoirs of power law description, fractal or not.

Keywords: Fractals, High velocity flow, Inflow performance

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1. INTRODUCTION

1.1. Introductory remarks

Fluid production from sub surface reservoirs depends on the interaction of the reservoir and the production string. The production rate depends on reservoir and well characteristics. All reservoirs cannot be described by conventional models. A fractal description may provide a valid alternative. A fractal reservoir may be thought of as having a spatial dimension that falls in between the Eucledian space dimensions, 1, 2 and 3. An additional requirement for fractal behavior is self-similarity. We investigate the flow of gas in sparsely fractured reservoirs. A sparse fracture network may not fill the Euclidean space. Such networks could exist in conventional reservoir rocks, sandstone or carbonates, or gas shale. A fractured water and hot water reservoir may exist in any kind of rock. We believe the condition of self-similarity is not required to benefit from application of the methodology.

1.1. Previous work

Some studies investigate use of the concept "fractal network" in pressure transient testing. Despite considerable efforts, the methodology remains esoteric. This is disappointing since some pressure signatures may be consistent with a fractal model. The lack of interest may be due to less intuitive concepts than for conventional models. Also the economic significance of results obtained by use of the fractal concept may be harder to assess. For example, the economic consequence of a fractal dimension, D=1.8, may not be obvious. This study emphasizes practical rather than theoretical aspects of the methodology.

Hirata (1989) studied fault systems in Japan by use of the box-counting method. He observed fractal dimensions between 1.6 down to 0.72. This methodology provides a fractal dimension based on direct observations of the fault or fracture network. The technique has the advantage of intellectual as well as intuitive appeal. Fractal dimensions depend on fracture architecture.

Subsurface reservoirs are not open for visual inspection. Chang and Yortsos (1990) proposed a one-phase model of fluid flow in fractal networks. They found that permeability and porosity may be described by simple power law expressions. Their solution is for single well tests. They derived a limiting equation, of power law type, for pressure change at large values of time. The equation shows up as a straight line in a log-log coordinate system. The slope is related to the fractal dimension and the conductivity index.

A well test response is not unique. Different models may lead to the same limiting behavior. Jelmert (2003) found that the interaction between pressure transients and external boundaries may lead to pressure changes of power law appearance.

Beier (1990) applied the Chang and Yortsos fractal model to a non-fractured but disordered inhomogeneous reservoir. Aprillan et al. (1993) studied the pressure behavior of interference tests. Their methodology was based on a similarity solution and did not accommodate well bore storage, skin or possible faults. Jelmert (2006) studied the effect of no-flow boundaries on interference and single well testing. The effect of well bore storage and skin was included

Doe (1991) studied constant pressure well tests. He found that flow areas of power law dependency on the spatial coordinate could lead to well test pressure responses of fractal appearance in otherwise homogeneous reservoirs.

The rock properties, permeability and porosity, are obtained as average values associated with an elementary volume, Bear (1972). The permeability- and porosity (functions) depend on the length scale of the elementary volume. Jelmert (2009) found that power law description could be a result of the length scale associated with the elementary volume

2. Theory and discussion

2.1. Governing equations

Suppose the properties of a fractal reservoir may be described by a single spatial variable, r, then the diffusivity equation becomes a one-dimensional partial differential equation.

$$\frac{\partial}{\partial r}\left(\rho_g(r)q_g(r)\right)=\varphi(r)2\pi hr\frac{\partial\rho}{\partial t} \qquad \text{eq.(1)}$$

Integration of the above equation under the assumption of steady state flow yields:

$$\left(p_e^2-p_w^2\right)=\frac{z\mu Tp_{sc}q_{gsc}}{\pi k_w hT_{sc}}\left(\frac{1}{(1-\beta)}\left(r_{eD}^{1-\beta}-1\right)+S+F_Q q_{gsc}\right), \qquad \beta\neq 1 \qquad \text{eq.(2)}$$

and

$$\left(p_e^2-p_w^2\right)=\frac{z\mu Tp_{sc}q_{gsc}}{\pi k_w hT_{sc}}\left(\ln r_{eD}+S+F_Q q_{gsc}\right), \qquad \beta=1 \qquad \text{eq.(3)}$$

The high velocity flow factor is given by:

$$F_Q=\frac{\beta_Q k_w}{\mu}\frac{p_{sc}M}{2\pi hRT_{sc}}\frac{1}{r_w} \qquad \text{eq.(4)}$$

The dimensionless distance, r_{eD}, is defined as r_e/r_w. See the appendix for details.

Before entering the wellbore, the gas has to flow through the formation and the skin zone (near wellbore region). There is pressure loss associated with each region. The pressure loss associated with formation flow is:

$$\left(p_e^2-(p_w')^2\right)=\frac{z\mu Tp_{sc}q_{gsc}}{\pi k_w hT_{sc}}\ln r_{eD} \qquad \text{eq.(5)}$$

Superscript prime denote well pressure on the reservoir side of the skin.

The pressure drop across the skin is given by:

$$\Delta p^2{}_{skin} = (p_w')^2 - p_{wf}^2 = \frac{z\mu T p_{sc} q_{gsc}}{\pi k_w h T_{sc}} S' = \frac{z\mu T p_{sc} q_{gsc}}{\pi k_w h T_{sc}} (S + F_Q q_{gsc})$$

eq.(6)

Note that, eq.(6), is the same as for a conventional reservoir. This similarity reflects the fact that both share the same conceptual model. In both cases, the skin zone may be thought of as a thin coating around a cylindrical well.

When the apparent skin, S', is positive, the skin zone represents an impedance to flow. Hence one may consider removing the skin. Use of the above equations may be used as an aid to quantify the benefit of reducing the skin.

The flow rate of a well is determined by the interaction of the properties of the reservoir, the near wellbore region and the flow line. In the present study, we limit the flow line to a vertical tubing, see Katz and Lee, (1990, ch. 9). The flow through a horizontal flow line and the choke may be included by two additional equations.

Tubing flow equation – Dry gas

$$q_{gsc} = \left(\frac{\pi}{4}\right)\left(\frac{R}{M_{air}}\right)^{0.5}\left(\frac{T_{sc}}{P_{sc}}\right)\left[\frac{D_{iT}^5}{\gamma_g f_M z_{av} T_{av} L}\right]^{0.5}\left(\frac{se^S}{e^S - 1}\right)^{0.5}\left(\frac{p_1^2}{e^S} - p_2^2\right)^{0.5} \qquad \text{eq.(7)}$$

$$\frac{s}{2} = \frac{M_g g}{z_{av} R T_{av}} H = \frac{(28.97)\gamma_g g}{z_{av} R T_{av}} H \qquad \text{eq.(8)}$$

The vertical flow equation may be reduced to:

$$q_{gsc} = C_T \left(\frac{p_1^2}{e^s} - p_2^2\right)^{0.5} \qquad \text{eq.(9)}$$

Were index 1 and 2 refer to intake and outflow end and the tubing constant, C_T , is the product of all the constant factors of eq.(7).

$$C_T = \left(\frac{\pi}{4}\right)\left(\frac{R}{M_{air}}\right)^{0.5}\left(\frac{T_{sc}}{p_{sc}}\right)\left[\frac{D_{iT}^5}{\gamma_g f_M z_{av} T_{av} L}\right]^{0.5}\left(\frac{se^S}{e^S - 1}\right)^{0.5} \qquad \text{eq.(10)}$$

For a production well, the intake is the bottom hole-pressure and the outflow is the tubing head pressure. Hence eq.(9) will simplify to:

$$p_{wf} = p_1 = e^{s/2}\left(p_{wh}^2 + \frac{q_{gsc}^2}{C_T^2} \right)^{0.5} \qquad \text{eq.(11)}$$

We refer to eq.(2) and eq.(3) as the reservoir flow equations and eq.(11) as the vertical flow equation. The flow rate of the well may be computed from the condition that the bottom-hole pressure, obtained by both equations is the same. This can be obtained by the simultaneous solution of the reservoir- and vertical flow equations.

2.2 Discussion

A bottom-hole pressure vs. production rate, p_{wf} vs. q_{gsc}, plot is an important tool in well design. The traditional p_{wf} vs. q_{gsc} plot implies the assumptions of radial flow in a homogeneous reservoir which lead to logarithmic dependency of the spatial variable, eq.(3).

For some important cases the traditional assumptions may be too simplistic. For example, radial flow in homogeneous gas shale is unrealistic. Then an inhomogeneous model based on rock properties of power law description could be a valid alternative.

The present model can handle more complex cases. It will reduce to the conventional one for $\beta=1$. Both models, logarithmic and power law, share the property that the production rate may be computed by simple analytical expressions.

The challenge is to determine the best β-value. This problem may be alleviated with additional empirical and theoretical experience. Outcrop- and simulation studies may be used as aids to estimatw reasonable values. A finite element program has the capability to simulate flow in discrete and continuous fracture networks. Pictures of fracture systems of outcrops may shed light on fracture architecture.

The economy of a well depends critically on the production rate. Key terms that can be manipulated by engineering projects are the sum term terms of eq.(2). They are listed below. The first term is associated with flow through the formation, the two next with the near wellbore region.

$$\frac{1}{(1-\beta)}\left(r_{eD}^{1-\beta} - 1\right) + S + F_Q q_{gsc} \qquad \text{eq.(12)}$$

Inspection of eq.(2) shows that a decrease in the above sum leads to an improved production rate.

Since β shows up with a negative sign in the exponent of the spatial variable, eq.(12), it is clear that a low β-value leads to low production rate. Hence the effect of the spatial term may be reduced by infill drilling, which corresponds to a reduction of the external radius of the drainage area, r_e. The intuitive explanation is that the average pressure gradient is increased since since the drawdown is applied to a shorter distance.

Another strategy is to capture the gas deeper into the reservoir by increasing the wellbore radius r_w. This is problematic for deep wells. The alternative is to create a hydraulic fracture, which leads to an increase of the equivalent wellbore radius, r_{we}.
The concept of equivalent radius, r_{we}, for a fractal reservoir was discussed by Jelmert (2009).

We may compute the production rate as a function of each variable and the corresponding derivative. The derivative gives the sensitivity of the production rate to a small change. For example

$$\Delta q_{gsc} \approx -\frac{dq_{gsc}}{dF_Q}\Delta F_Q \qquad \text{eq.(13)}$$

It is important to locate variables with steep gradients if acceleration projects are up for evaluation. The production rate may be improved by several methods: infill drilling, hydraulic fracturing, acid treatment etc.

High velocity flow is a near wellbore phenomenon. In case of restricted entry or partial penetration the height of the inflow zone is essentially the perforated height, h_p. One may also capture the fluid out in the reservoir by hydraulic fracturing. A valid modification of the equation for F_Q as follows:

$$F_Q = \frac{\beta_Q k_w}{\mu}\frac{p_{sc}M}{2\pi h_p RT_{sc}}\frac{1}{r_{we}} \qquad \text{eq.(14)}$$

As pointed out by Beier (1990) the fractal model may also be applied to disordered reservoirs without important fracture networks.

2.3. Case studies

Fig. 1 shows the variation of the production rate with β-values and the dimensionless distance to the external boundary. As expected, increasing β-values and reduced dimensionless distance, $r_{eD,}$ leads to higher production rates.

In a deep well, the wellbore radus is in the order of 10 cm. A dimensionless distance of r_{eD} = 170 corresponds to 17 m. In a shallow well, the wellbore radius may be larger. Also hydraulic fracture mays increase the effective wellbore radius to even much larger values.

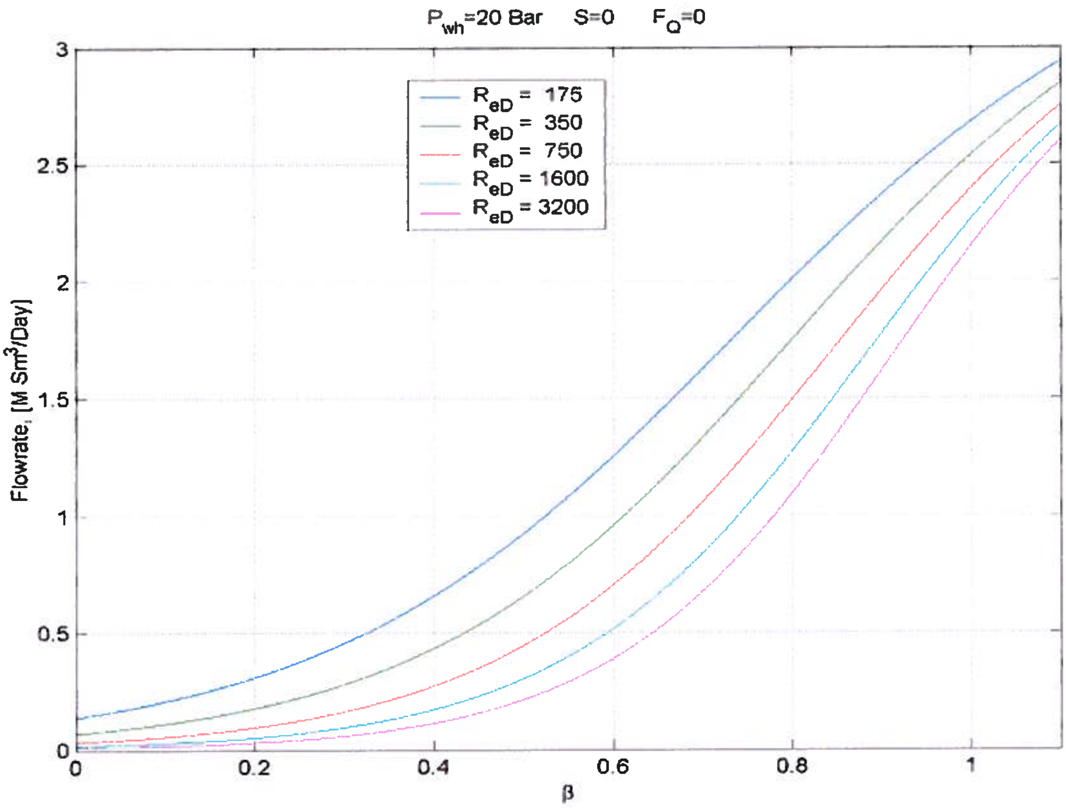

Fig.1: Effect of infill drilling on production rate.

The production rate is obtained by the simultaneous solution of the reservoir and vertical flow equations. The solution may be obtained by a graphical technique.The wellbore pressure is plotted vs. the gas flow rate. The key equations are: the reservoir flow equations, eq.(2) and eq.(3), and the vertical flow equation, eq.(11). We specify the reservoir pressure, p_{we}, and the wellhead pressure, p_{wh}, and calculate the corresponding bottom hole pressure, p_{wf}. The production rate may be obtained as the intercept of the curves associated with the reservoir- and vertical flow equations. Fig. 2 and 3 illustrates the technique.

Both Fig. 2 and Fig. 3 shows 9 possible flow rates depending on the reservoir flow curves (for 3 different β –values, unbroken curves) and the vertical flow curves (for 3 different values of wellhead pressures, broken lines). In each case the pressure at the external boundary is 100 bar. The skin factor is S=0.55. The high velocity flow factor, F_Q , is 0 and = 0.04 (S M m^3/day)$^{-1}$ respectively. There is not much difference between the two cases except for the high β-value. The interpretation is that the rate dependent skin is negligible in comparison to the reservoir flow term, r_{eD}.

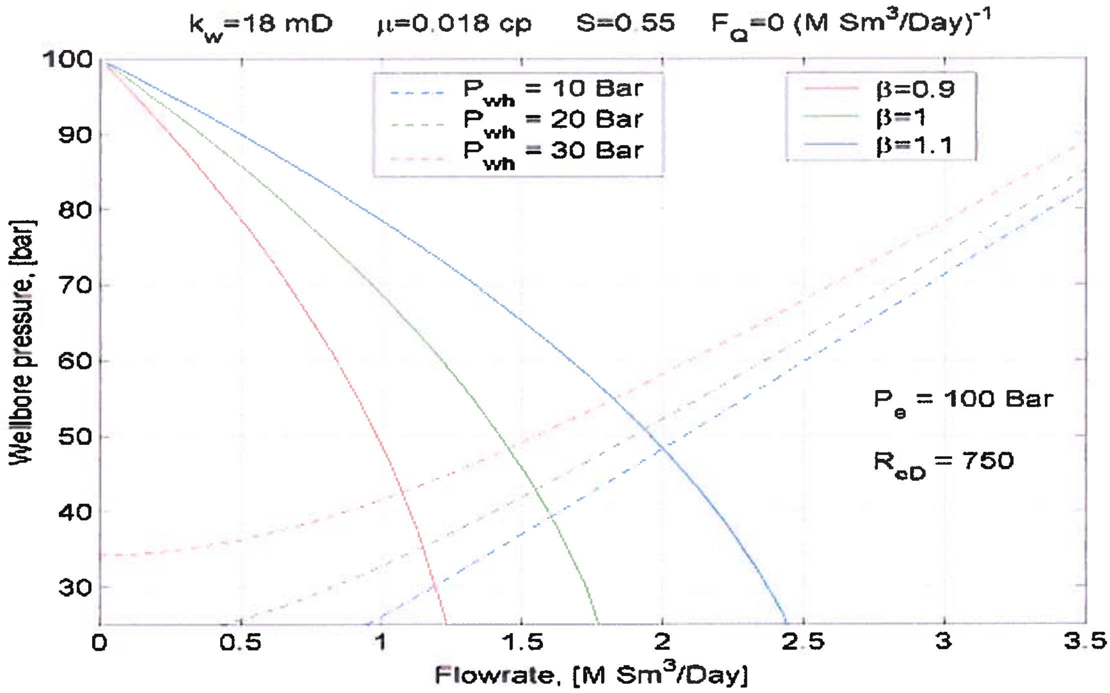

Fig.2: Variation of production rate with wellhead pressure and β-value. Rate define by intercept of curves

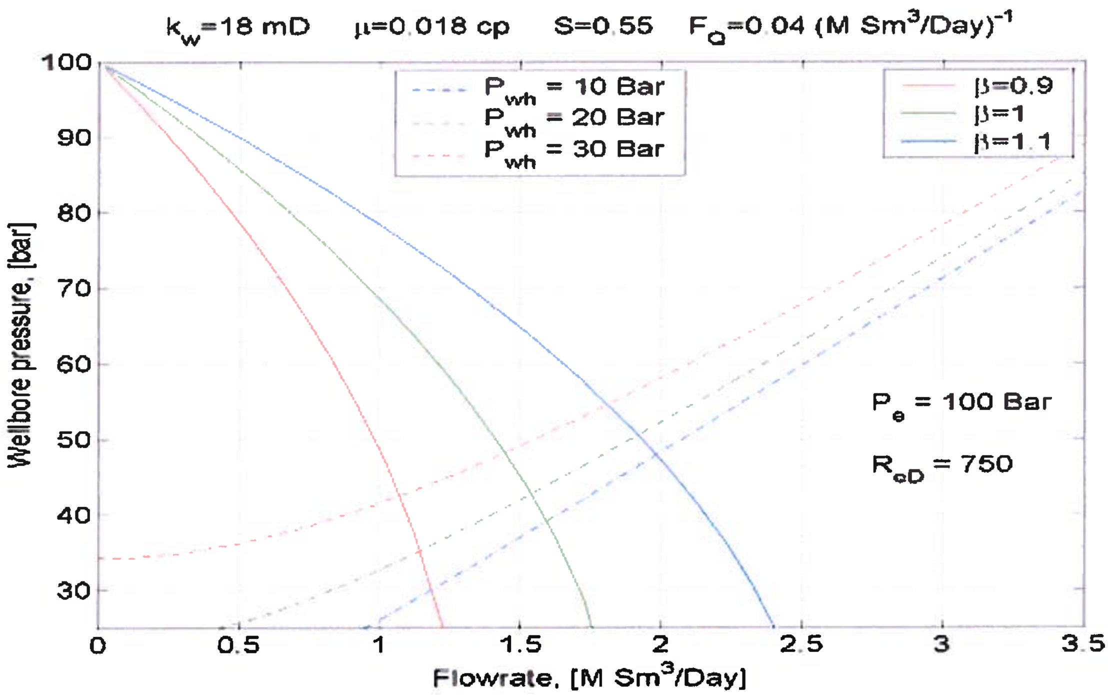

Fig.3: Variation of production rate with wellhead pressure and β-value.

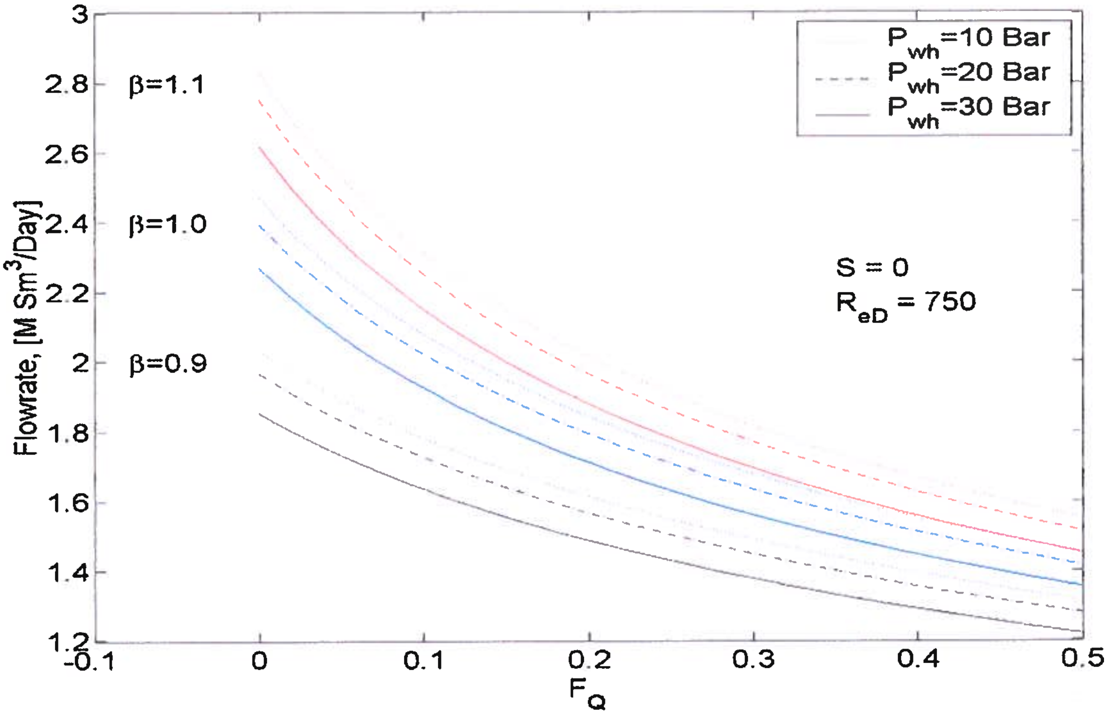

Fig.4: Production rate as a function of high velocity factor and wellhead pressure.

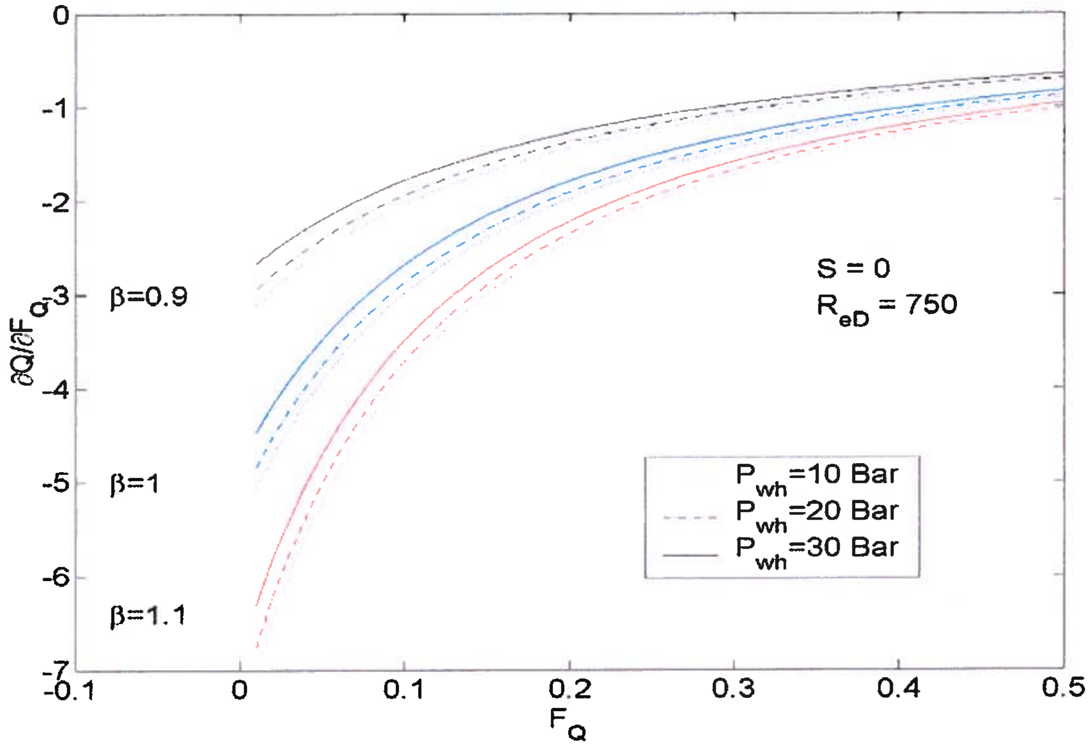

Fig.5: Sensitivity of production rate to high velocity factor.

Fig. 4 shows the influence of selected β-values and wellhead pressures on the production rate as a function of the high velocity flow factor, see eq.(2) and eq.(11). Fig. 5 shows the sensitivity (derivative) of the production rate to the same parameters.

Fig. 6 shows the variation of the production rate with the high velocity factor and the dimensionless distance to the external boundary

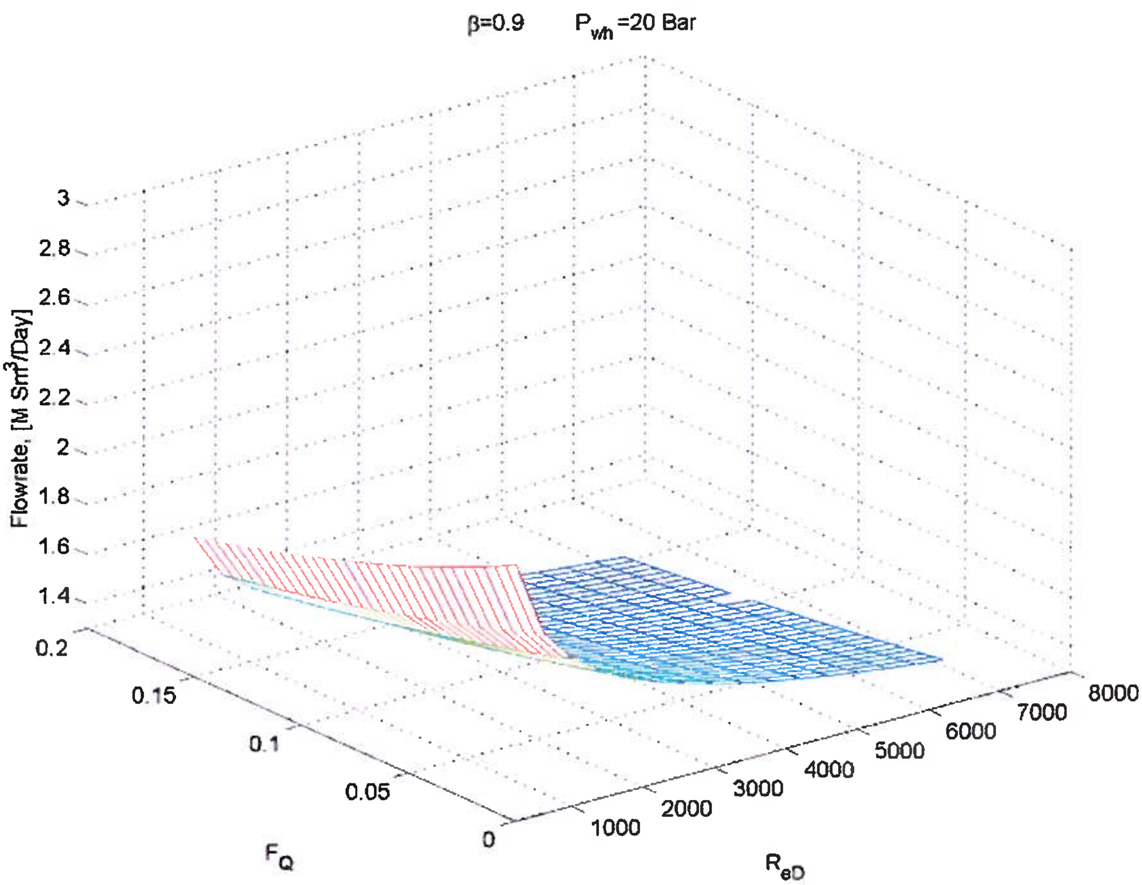

Fig.6: Production rate as a function of high velocity factor and dimensionless distance to external boundary.

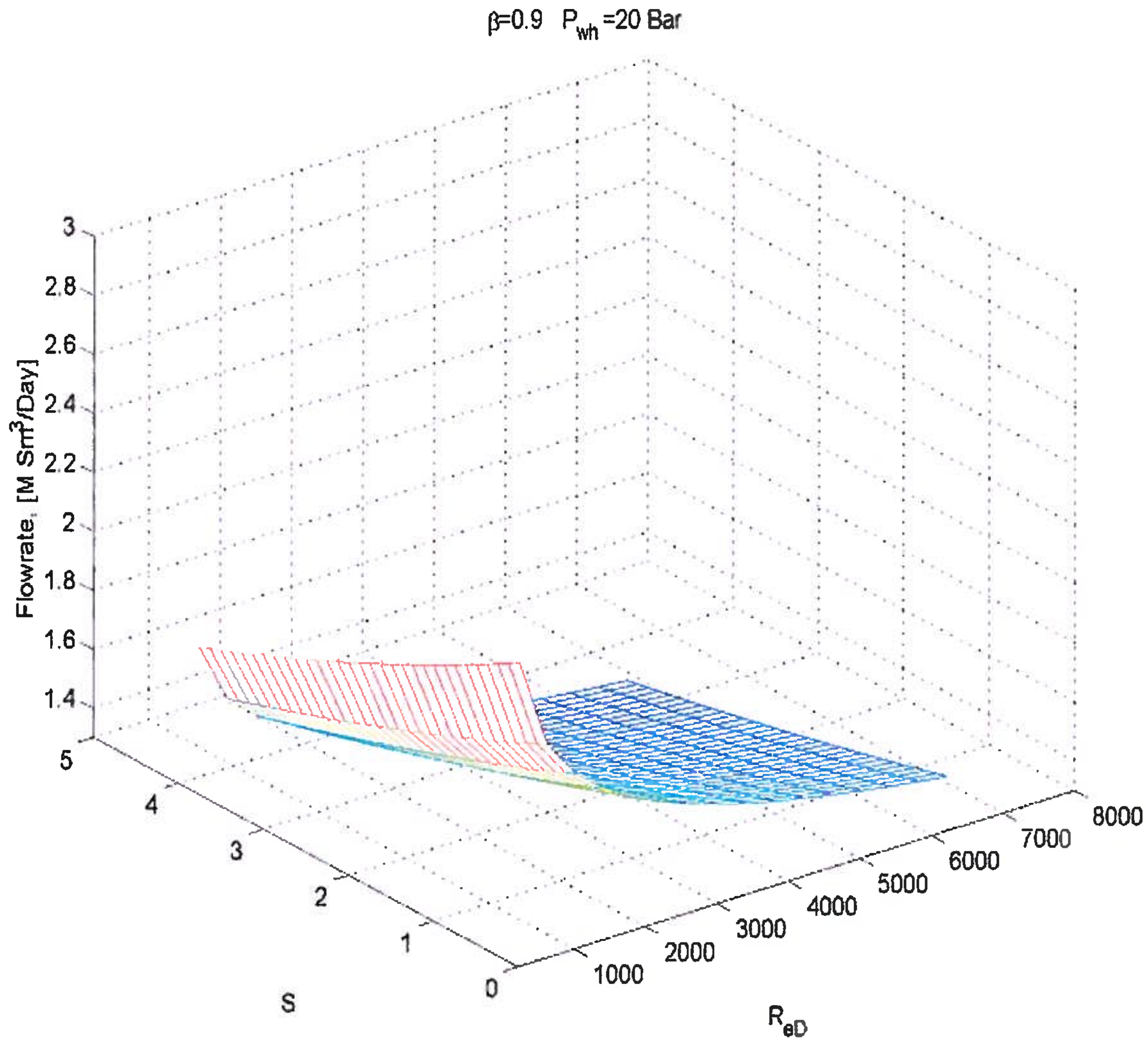

Fig.7: Variation of production rate with dimensionless distance to external boundary and skin factor

Fig. 7 shows a similar plot, but with the skin, S, and the dimensional distance on the horizontal axes.

A fractal reservoir will almost always have β-values less than 1. In Fig. 8 we plot the difference in production rate between homogeneous, β=1, and a fractal reservoir with β=0.75. The homogeneous reservoir will lead to higher production rates.

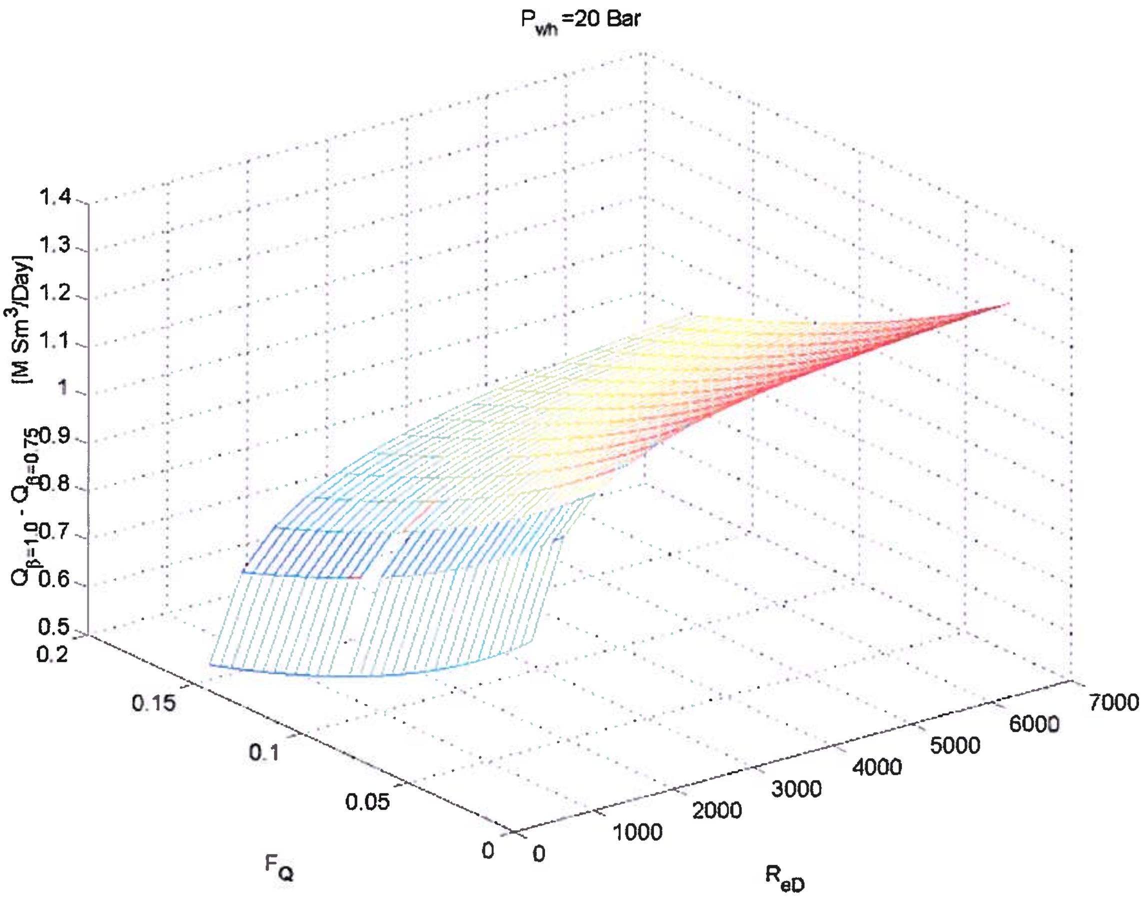

Fig.8: Production rate difference plot. β=1 and β=0.75

2.4 Conclusions

A generalized reservoir inflow equation has been proposed. It is valid for, but not limited to, fractal reservoirs, fractured or not. The traditional radial flow equation is included as a special case.

The proposed inflow equation is easy to program on a spreadsheet. Due to simplicity and flexibility we believe the method may be of use as an aid in production forecasting, well monitoring and design.

The power law model has an additional parameter to calibrate the reservoir flow equation to production data.

The method may need support of simulation and outcrop studies to define a reasonable β-value.

The production rate of a well has been computed by the simultaneous solution of the fractal reservoir flow equation and the vertical flow equation.

The sensitivity of the production rate to the reservoir- and near wellbore region properties has been investigated.

In many cases of interest the production rate may be improved by infill drilling and stimulation.

3. REFERENCES AND NOMENCLATURE

3.1 References

Apprillan, S. Abdassah, D. and Sumantri, R. 1993: Application of fractal reservoir model for interference test analysis in Kamojang géothermal Field, (Indonesia), 68th Annual Technical Conference and Exhibition, Dallas, TX, Oct 3-6

Bear, J., 1972. Dynamics of fluids in porous media, Elsevier, New York

Beier, R. A., 1990. Pressure transient field data showing fractal reservoir structure. Paper SPE 21553, CIM/SPE International technical meeting, Calgary, Alberta June 10-15.

Chang, J. & Yortsos, Y.C., 1990. Pressure transient behavior of fractal reservoirs. *SPE Formation Evaluation, (March)*: 31-38.

Doe, T. W. 1991: Fractal dimension analysis of constant- pressure well test, Paper SPE 22702, 66th Annual Technical Conference and Exhibition, Dallas, TX, Oct 6-9

Hirata, T. 1989, Fractal dimension of fault systems in Japan: Fractal structure in Rock Fracture at various scales. PAGEOPH, Vol. 131, Nos. ½

Jahn, F., Cook, M. and Graham, M., 1998 : Hydrocarbon Exploration and Production, Elsvier, Amsterdam, Ch. 9, p. 213

Jelmert, T.A.,2003. Bi-linear pressure signatures of horizontal wells. Buisness Briefing and Eploration : The Oil and Gas Review, Vol 2. (CD version for mathematical details)

Jelmert, T.A., 2009: Productivity of fractal reservoirs, Paper SPE12860, SPE Saudi Arabia Section Technical Symposium and Exhibition, AlKhobar, Saudi Arabia, 09–11 May 2009.

Katz, D.L. and Lee, R.L., 1990: Handbook of natural gas engineering. Production and storage. New York, McGraw-Hill, Ch. 6, p 231

Katz, D.L. and Lee, R.L., 1990: Handbook of natural gas engineering. Production and storage. New York, McGraw-Hill, Ch. 8, p. 327

3.2 Nomenclature

A = Area, L^2
D = fractal dimension, dimensionless
D_{iT} = Inner diameter of tubing
d = Eucledian dimension, dimensionless
C_T= Tubing constant, eq.(10)
F_Q = High velocity flow factor
f_M = Mody friction factor of tubing
H = True vertical height of vertical flow line (tubing)
h = Height
k = Permeability
L= Length of vertical flow line
M= Molecular weight of gas
$\dot{m}$ = Mass rate of flow
p= Pressure
p' = Pressure on the reservoir side of the skin
q = Production rate
R = Universal gas constant
r = distance
r_D = Dimensionless distance, r/r_e
S = skin
S' = Apparant skin, mechanical and rate dependent, Dimensionless
s = Vertical flow parameter, see eq.(8)
u = volumetric flux
V = Volume
z= Gas deviation factor

Greek Letters
α = Angle
β = Dimensionless exponent. For fractal reservoirs: $\beta = D - \theta - 1$
β_Q = High velocity coefficient
γ = Specific gravity of the gas relative to air, Dimensionless
Δ = Change
ρ = Density
φ = Porosity
θ = connectivity index, dimensionless
μ = Viscosity

Subscripts:
e = External
ew= Equivalent wellbore radius
D = Dimensionless
g = Gas
p = perforated
Q= Related to high velocity flow
sc = Standard conditions
ref = Arbitrary reference condition
wf = Well flowing

Conversion Factors

1 bara = 10^5 Pa
1 m^3/day = 86400 m^3/s
1 cp = 10^{-3} Pa s
1 mD = 10^{-15} m^2

4. APPENDIX

Conservation of mass leads to the following equation:

$$\frac{\partial}{\partial r}\left(\rho_g(r)q_g(r)\right)=\varphi(r)2\pi hr\frac{\partial \rho}{\partial t}$$ eq.(A.1)

Under steady state conditions, the right hand side is zero. Hence:

$$\frac{\partial}{\partial r}\left(\rho_g(r)q_g(r)\right)=0$$ eq.(A.2)

Integration leads to the conclusion that the mass rate of flow is independent of position but the volumetric flow rate and density depends on position.

$$\dot{m}=\rho_g(r)q_g(r)=\rho_g q_g=\rho_{gsc}q_{gsc}=Const$$ eq.(A.3))

The equation of state for a gas is given by:

$$\rho=\frac{pM}{zRT}$$ eq.(A.4)

The relationship between pressure gradients and volumetric flux is given by the Forscheimer equation.

$$\frac{dp}{dr}=\frac{\mu\cdot u}{k}+\beta_Q\rho_g\cdot u^2$$ eq.(A.5)

Where the volumetric flux, u, is flow rate pr. unit area.

$$u = \frac{q_g}{A} \qquad \text{eq.(A.6)}$$

$$\frac{dp}{dr} = \frac{\mu q_g}{kA} + \beta_Q \rho_g \left(\frac{q_g}{A}\right)^2 \qquad \text{eq.(A.7)}$$

Substitution of eq.(A.4) into eq.(A.7) yields:

$$\frac{dp}{dr} = \frac{zRT}{pM}\left(\frac{\mu \rho q_g}{kA} + \beta_Q \rho^2 \left(\frac{q_g}{A}\right)^2\right) \qquad \text{eq.(A.8)}$$

Substitution of eq.(A.3) and eq.(A.4) into the above equation leads to:

$$pdp = \frac{zRT}{M}\left(\frac{\mu \dot{m}}{2\pi khr} + \beta_Q \frac{\dot{m}^2}{4\pi^2 h^2 r^2}\right)dr \qquad \text{eq.(A.9)}$$

The permeability of the fractal flow model is given by a power law expression.

$$k(r) = k_w \left(\frac{r}{r_w}\right)^{D-d-\theta} \qquad \text{eq.(A.10)}$$

We restrict our analysis to the special case where a sparse fracture network of fractal dimension, D, is embedded in a cylindrical space of Euclidian dimension, d=2.

$$A(r) = 2\pi h r_w \frac{r}{r_w} \qquad \text{eq.(A.11)}$$

$$k(r) \cdot A(r) \cdot = 2\pi h r_w \frac{r}{r_w} \cdot k_w \left(\frac{r}{r_w}\right)^{D-2-\theta} = 2\pi h r_w \cdot k_w \left(\frac{r}{r_w}\right)^{\beta} \qquad \text{eq.(A.12)}$$

Where:

$$\beta = D - \theta - 1 \qquad \text{eq.(A.13)}$$

Since the mass rate of flow is constant it has no influence on the integration operation.

$$\int_{p_w}^{p} pdp = \frac{zRT\dot{m}}{M}\left(\frac{\mu r_w^{\beta}}{k_w 2\pi h r_w} \int_{r_w}^{r} \frac{1}{r^{\beta}} dr + \beta_Q \frac{\dot{m}}{4\pi^2 h^2} \int_{r_w}^{r} \frac{1}{r^2} dr\right) \qquad \text{eq.(A.14)}$$

The result is:

$$\frac{1}{2}\left(p^2 - p_w^2\right) = \frac{zRT\dot{m}}{M}\left(\frac{\mu}{2\pi k_w h} \frac{r_w^{\beta-1}}{(1-\beta)}\left(r^{1-\beta} - r_w^{1-\beta}\right) - \beta_Q \frac{\dot{m}}{4\pi^2 h^2}\left(\frac{1}{r_e} - \frac{1}{r_w}\right)\right) \qquad \text{eq.(A.15)}$$

Since r_e usually is large in comparison to r_w:

$$\left(p^2 - p_w^2\right) = \frac{z\mu T p_{sc} q_{gst}}{\pi k_w h T_{sc}}\left(\frac{1}{(1-\beta)}\left(\frac{r^{1-\beta}}{r_w^{1-\beta}} - 1\right) + \frac{\beta_Q k_w}{\mu} \frac{p_{sc} M q_{gst}}{2\pi h R T_{sc}} \frac{1}{r_w}\right) \qquad \text{eq.(A.16)}$$

The above equation has a singularity for β=1. Eq.(A.14) has to be integrated separately for this case. The result is the standard equation for the flow of a low pressure gas.

$$\left(p^2 - p_w^2\right) = \frac{z\mu T p_{sc} q_{gst}}{\pi k_w h T_{sc}} \left(\ln\left(\frac{r}{r_w}\right) + \frac{\beta_Q k_w}{\mu} \frac{p_{sc} M q_{gst}}{2\pi h R T_{sc}} \frac{1}{r_w} \right) \qquad \text{eq.(A.17)}$$

Contact resistance between the sand face and the wellbore may be taken into account by a skin factor. The total pressure change squared may be obtained as:

$$\left(p_e^2 - p_w^2\right) = \frac{z\mu T p_{sc} q_{gst}}{\pi k_w h T_{sc}} \left(\frac{1}{(1-\beta)} \left(\frac{r_e^{1-\beta}}{r_w^{1-\beta}} - 1 \right) + S + \frac{\beta_Q k_w}{\mu} \frac{p_{sc} M q_{gst}}{2\pi h R T_{sc}} \frac{1}{r_w} \right) \qquad \text{eq.(A.18)}$$

$$\left(p_e^2 - p_w^2\right) = \frac{z\mu T p_{sc} q_{gst}}{\pi k_w h T_{sc}} \left(\ln\left(\frac{r_e}{r_w}\right) + S + \frac{\beta_Q k_w}{\mu} \frac{p_{sc} M q_{gst}}{2\pi h R T_{sc}} \frac{1}{r_w} \right) \qquad \text{eq.(A19)}$$

$$\Delta p^2{}_{skin} = \frac{z\mu T p_{sc} q_{gst}}{\pi k_w h T_{sc}} S' = \frac{z\mu T p_{sc} q_{gst}}{\pi k_w h T_{sc}} \left(S + F_Q q_{gsc}\right) \qquad \text{eq.(A20)}$$

where

$$F_Q = \frac{\beta_Q k_w}{\mu} \frac{p_{sc} M}{2\pi h R T_{sc}} \frac{1}{r_w} \qquad \text{eq.(A.21)}$$