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Three-Dimensional Seismic Diffraction Modeling

Thesis for the degree of Philosophiae Doctor

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Norwegian University of Science and Technology
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and Applied Geophysics

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The “where” is encoded in the phase, and
the “what” is encoded both in the phase and amplitude.

Tadeusz J. Ulrych,
University of British Columbia

Summary

Synthetic modeling of seismic wavefields scattered in the subsurface is of growing interest today due to its applicability in various forward and inverse problems of geophysics. It has been extensively used for general evaluation of the subsurface structure, in survey design and illumination studies, and also as the basis for imaging and inversion algorithms. A particular challenge is presented by models with complex geological structure containing strong-contrast or irregular reflectors and shadow zones, where conventional algorithms fail to simulate realistic wavefields. Therefore, the demand for advanced modeling techniques has dramatically increased.

This thesis is based on a new analytic approach to the description and modeling of three-dimensional reflected wavefields. The method combines surface singular integrals describing the wave propagation inside smoothly heterogeneous layers and effective reflection and transmission coefficients (ERC and ETC) at reflectors. The propagators are implemented using a seismic-frequency approximation of the Helmholtz-Kirchhoff integral. The approximation is based on the assumption that a small (compared to the predominant wavelength) part of the reflector acts as a secondary source, which, in accordance with Huygens' principle, emits a wave beam. The beam comprises not only the main reflection or transmission, but also the edge-diffracted and tip-diffracted waves. Because the tip-diffracted waves contribute most to the beam, the method is called the "tip-wave superposition method" (TWSM).

ERC and ETC generalize classical plane-wave reflection and transmission coefficients (PWRC and PWTC) for point sources and curved reflectors. Their definition accounts for the local interface curvature, sphericity of the wavefront, and finite frequency content of the incident wavelet. Therefore, they produce correct reflected and transmitted amplitudes at near-critical and post-critical incidence angles. Numerical experiments indicate that ERC and ETC also accurately reproduce amplitudes of the head waves. For a plane interface, ERC or ETC represents the exact reflection or transmission response at the reflector. For a curved interface, they are approximate and based on an “apparent“ source location, which depends on the incidence angle and the mean reflector curvature. Because ERC and ETC account for all amplitude-related effects, they are a useful tool for evaluation of reflected and transmitted wavefields at the reflector.

The new approach comprising the TWSM with ERC and ETC gives the possibility of reproducing complex wave phenomena such as caustics, diffractions, and head waves. The modeled full wavefield is represented as a set of separate events, each corresponding to a particular wavecode. This makes the approach event-oriented: an event of interest can be modeled separately. Numerical experiments demonstrate that the method simulates scattered 3D synthetic wavefields in layered media with accurate traveltimes and amplitudes.

Chapter 1 presents the basic principles of wave propagation in 3D media and an overview of existing modeling techniques. Chapter 2 contains a paper about 3D diffraction modeling of singly scattered acoustic wavefields. The paper introduces ERC and ETC for acoustic waves and gives a detailed overview of the acoustic version of TWSM. Chapter 3 contains a paper about 3D diffraction modeling of acoustic scattering in layered media. The paper deals with the extension of the acoustic version of TWSM for layered media. It provides a thorough analysis of the modeling results for 3D models with smoothly varying reflectors and for models containing diffracting edges. Chapter 4 contains a paper about ERC for curved interfaces in TI media. The paper analyzes the dependence of ERC on anisotropy parameters and reflector shape and demonstrates their advantages over PWRC in 3D diffraction modeling of PP and PS reflection data.

Preface

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Contents

Chapter 1

Thesis introduction

1.1 Wave propagation in 3D media

Synthetic seismic modeling of reflection data is a powerful tool with wide applications in various areas of oil and gas exploration (Sheriff and Geldart, 1995; Aki and Richards, 2002). It has been extensively used for general understanding of the subsurface structure, in survey design, for data interpretation, and velocity model verification. Also, modeling-based techniques are often exploited as the basis for imaging and inversion schemes (Claerbout, 1971; Gazdag and Sguazzero, 1984; Gray, 2001; Treitel and Lines, 2001; Ursin, 2004).

With the growing computing power, when usage of parallel machines becomes a day-to-day practice, and, with the modern acquisition trends, when three-dimensional (3D) and wide-azimuth (WAZ) datasets may be acquired, the need for advanced modeling and processing techniques pushes computational limits (Ramsden and Bennett, 2005). Seismic modeling evolves towards rather efficient and accurate methods, which allow better description of complex wave phenomena. With the development of quantitative seismic analysis and amplitude-versus-offset (AVO) studies, not only the phase, but also the amplitude information, is considered to be of major importance (Roden and Forrest, 2005; Ostrander, 1984; Rutherford and Williams, 1989; Hilterman, 2001).

Synthetic seismic modeling is a way of simulating the real reflection data given the elastic parameters of the subsurface (Carcione et al., 2002). A successful modeling algorithm is capable of producing a good match between modeled and real data.

Therefore, it is important to realize how seismic waves propagate and what effects may have significant influence on their amplitudes and phases.

Propagation of seismic waves inside layers with smoothly varying properties is described by the wave equation. The scalar wave equation describes acoustic pressure changes in a fluid medium (Aki and Richards, 2002):

$$\frac{1}{\rho v^2}\frac{\partial^2 p}{\partial t^2} - \nabla \cdot \left(\frac{1}{\rho}\nabla p\right) = f,$$

where $p = p(\mathbf{x}, t)$ is the acoustic pressure as function of spatial coordinate $\mathbf{x}$ and time t, $v = v(\mathbf{x})$ is the wave propagation velocity, $\rho = \rho(\mathbf{x})$ is the density, and $f(\mathbf{x}, t)$ is the body force. The vector wave equation for elastic waves deals with the particle displacement in a solid medium (Aki and Richards, 2002):

$$\rho\frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla \cdot \mathbf{T} = \mathbf{f},$$

where $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ is the displacement vector, $\mathbf{T} = \mathbf{T}(\mathbf{x}, t)$ is the stress tensor, and $\mathbf{f} = \mathbf{f}(\mathbf{x}, t)$ is the body force. The linear dependence between stress and displacement is described by the generalized Hooke's law:

$$\mathbf{T} = \mathbf{C} : \mathbf{E}.$$

Here $\mathbf{C} = \mathbf{C}(\mathbf{x})$ is the fourth-order stiffness tensor, and $\mathbf{E} = \mathbf{E}(\mathbf{x})$ is the strain tensor defined as

$$\mathbf{E} = \frac{1}{2}\left(\nabla\mathbf{u} + \mathbf{u}\nabla\right).$$

Additionally, full wavefields satisfy boundary conditions at strong reflectors. The boundary conditions for acoustic waves state the continuity of the pressure wavefields and their weighted normal derivatives at the reflectors:

$$p^{(1)} = p^{(2)},$$
$$\frac{1}{\rho^{(1)}}\frac{\partial p^{(1)}}{\partial n} = \frac{1}{\rho^{(2)}}\frac{\partial p^{(2)}}{\partial n},$$

where the superscripts (1) and (2) denote the medium just above and just below the reflector, correspondingly, and $\mathbf{n} = \mathbf{n}(\mathbf{x})$ is the normal to the reflector directed into the upper medium. The boundary conditions for elastic waves, in turn, state the continuity of the displacement wavefields and corresponding traction vectors at the reflectors:

$$\mathbf{u}^{(1)} = \mathbf{u}^{(2)},$$
$$\mathbf{n} \cdot \mathbf{T}^{(1)} = \mathbf{n} \cdot \mathbf{T}^{(2)}.$$

Also, the reflected wavefields obey a radiation condition, whose explicit form is of no particular importance here. The radiation condition states that there is no radiation from the infinity; i.e., the outgoing waves vanish at infinity and do not return. The closed system comprising the wave equation, boundary conditions, and radiation condition is often referred to as the transmission problem (Pao and Varatharajulu, 1976).

1.2 Seismic modeling methods

Whereas the transmission problem is well-studied in simpler subsurface models (such as, for example, horizontally layered models with homogeneous layers), solving it in arbitrarily inhomogeneous media becomes analytically and numerically nontrivial. Various approaches have been designed for synthetic seismic modeling in complex models. Based on the works of Carcione et al. (2002) and Ursin (1983), existing methods may be split into five main groups:

- Numerical or full-wave equation methods:
  - finite-difference modeling (FDM);
  - finite-element modeling (FEM);
  - spectral-element modeling (SEM).
- Asymptotic methods:
  - geometrical seismics or asymptotic ray theory (ART);
  - geometric theory of diffraction (GTD);
  - theory of edge and tip waves.
- Methods based on plane-wave decomposition:

  - reflectivity modeling in horizontally layered media.

- Surface and volume integral methods:
  - integral-equation approaches;
  - explicit surface or volume integral representations.

- Hybrid methods:
  - combination of volume integrals and spatial Fourier transforms;
  - combination of path surface integrals and local phase spatial decompositions;
  - combination of surface integral propagators and generalized spectral decompositions.

All modeling methods listed above possess strong sides, as well as some weaknesses. They have different areas of applicability, and are suited for particular types of geological models. A more detailed description of the methods is given below.

1.2.1 Numerical or full-wave equation methods

Numerical (or full-wave equation) modeling is widely used because of its ability to simulate full wavefields with highly accurate traveltimes and amplitudes in stratigraphically complex areas (Carcione et al., 2002). Numerical modeling is flexible and is generally not limited to particular types of models. Any numerical approach implies that the model is discretized in temporal and spatial coordinates. The quality of the obtained full wavefield is higher for finer grids. However, this type of modeling, in particular its application for 3D structures, requires heavy computations. This property limits the applicability of numerical approaches in models with large spatial extent.

Another property of a numerical algorithm is that it actually produces full wavefields. This may be considered both an advantage and a drawback. On the one hand, numerical modeling simulates the full wavefield, which includes all possible wave phenomena. Whenever this is the intention, numerical modeling is a good option for obtaining results with a high precision. On the other hand, modeling of full wavefields sometimes causes interpretational difficulties. For example, visual separation of wave events reflected at thin-layered structures or tuning reflectors may become challenging. In these cases, the ability of modeling full wavefields becomes a weak point of the approach.

An important point when addressing numerical methods is computational stability (Emerman et al., 1982). This has consequences for the choice of the modeling approach and for the spatial and temporal grid generation. FDM in particular experiences problems in areas with steeply dipping or curved reflectors. The grid is usually not suited for such types of reflectors, and the stair-case representation of reflectors generates unwanted grid diffractions. FEM and SEM are, on the contrary, free of this drawback because the mesh generation allows for grid fitting for the reflector shape (Seriani and Oliveira, 2007). In the vicinity of faults and curved reflectors, the mesh becomes essentially denser and is suited for the model geometry. A weak point of a denser grid is, of course, high computational costs. The choice of the grid size is always the trade-off of three factors: numerical stability, computational time, and quality of the modeled wavefields.

1.2.2 Asymptotic methods

The basis of asymptotic methods is the representation of the wavefield as a series over the reciprocal powers of the frequency ω (Červený, 2001). In high-frequency approximation, the power series is often reduced to the leading term only. This approximation gives rise to geometrical seismics, which is also often referred to as asymptotic ray theory (ART). Geometrical seismics thus becomes an analogue of geometrical optics, which is described in terms of optical rays obeying Fermat's principle.

Asymptotic modeling techniques are computationally much faster than numerical methods. They have wide applications in kinematic and dynamic ray tracing, which has been routinely used for its computational efficiency and flexibility. The approach allows modeling of reflected wavefields corresponding to the specified wavecodes. In order to obtain the full wavefield, estimation and computation of separate events corresponding to all possible wavecodes is necessary. This property makes ray-tracing modeling valuable not only for quantitative seismic modeling, but also for feasibility studies, for example, for survey design and analysis of illuminated zones in subsurface models (Gjøystdal et al., 2007a,b). However, because ray tracing is based on the geometrical-seismics approximation, it has its limitations. The methodology experiences difficulties in shadow zones, where the optical rays do not penetrate and the modeled amplitudes are zero. Another limitation of ray tracing reveals itself in the vicinity of caustic triplications. The amplitudes of the waves are defined through the reciprocal of the geometrical-spreading factor. Because the geometrical-spreading factor is zero at caustic cusps, special attention has to be paid to the amplitudes, which tend to become infinitely large.

Tsvankin and Chesnokov (1990) pointed out that the leading term of the power series may not be enough for the accurate description of the reflected waves. Often, the next term is needed as a correction to the leading term. For example, geometrical seismics assumes that there is no P-to-S conversion at normal incidence. However, observations show that for lower frequencies and shorter distances between the point source and reflector, a significant amplitude of the PS-wave may be recorded. This suggests that geometrical seismics can, in many cases, be corrected within the asymptotic approach. Another limitation of geometrical seismics is that it does not reproduce diffraction phenomena and head waves, which become important when the full-waveform solution is necessary.

Hanyga and Helle (1995) introduced generalized ray tracing (GenRT), which extends conventional ray tracing to areas containing caustics and caustic shadow regions. They use complex ray tracing as an analytic continuation of optical rays in the caustic shadow zones. They also exploit uniformly asymptotic expansions (Maslov integrals) to represent the contribution of a ray congruence to the wave amplitude. Inside the boundary layer adjacent to a caustic, they reduce uniformly asymptotic expansion to Airy functions. However, as the authors indicate, Airy functions are "physically irrelevant" for the description of the wavefield in the caustic shadow zones.

Typical velocity models contain shadow zones, where the optical rays are screened by parts of reflectors and therefore absent. According to geometrical seismics, the wavefield experiences a discontinuity at the border between the shadow and illuminated regions (often referred to as the shadow boundary). Keller (1962) introduced the geometric theory of diffraction (GTD), which smoothes the wavefield discontinuities at the shadow boundary. In addition to usual optical rays, he introduced diffracted rays produced by incident rays, which hit edges, corners, or vertices of reflectors. The points of discontinuities of reflectors act as secondary sources and generate diffracted waves in accordance with Huygens' principle. Diffracted waves thus obey all the principles of geometrical optics. As is done within the geometrical-seismics approach, diffracted waves are represented as an infinite series over the reciprocal and fractional orders of the frequency ω.

Klem-Musatov (1994) showed that GTD breaks down outside the regions of applicability of the transport equation, for example, in the vicinity of the shadow boundary. A better description of the diffraction phenomenon in such neighborhoods (often referred to as boundary layers) is needed. According to GTD, the diffracted wave amplitude in the boundary layer is $\sqrt{\omega}$ times lower than that of the reflected or transmitted wave. Therefore, for high frequencies, diffracted waves may be neglected without loss of the quality of the modeling results. However, within the boundary layer, the amplitudes of the reflected or transmitted and diffracted waves are comparable even for high frequencies. Klem-Musatov (1994) introduced the theory of edge and tip diffraction. For the description of the diffracted-wave amplitude in the boundary layer, they proposed a new function $W(w)$, which is closely related to the classical Fresnel integral. The parameter w is a derivative of the Fresnel-zone size and indicates the distance from the shadow boundary. Klem-Musatov (1994) also introduced the concept of vertex (tip) diffraction as a correction of the discontinuities of the edge waves at the secondary shadow boundaries caused by the vertices. The amplitude of the vertex-diffracted wave is proportional to $W(w_1)W(w_2)$, where the parameters w_1 and w_2 denote the distances from the secondary shadow boundaries of the two edge-diffracted waves. The two types of diffracted waves (edge-diffracted and tip-diffracted) complete the list of all possible waves generated at a wedge. The theory of edge and tip diffraction has certain limitations. For example, it breaks down at caustic cusps, just like geometrical seismics. Therefore, it should be carefully used in the areas of caustics and multipathing.

1.2.3 Methods based on the plane-wave decomposition

Schemes based on plane-wave decomposition are widely used for modeling of reflected wavefields in media with the velocity field varying with depth only, $v = v(z)$. Horizontally layered structures with no lateral velocity variations are examples where such modeling is largely applicable (Ursin, 1983; Tsvankin, 1995). The methodology is fundamental to a number of processing techniques, such as prediction and elimination of seabed and internal multiples, deterministic wavelet estimation, and decomposition of the full wavefield into upgoing and downgoing waves (Ikelle and Amundsen, 2005). Plane-wave decomposition is a powerful and computationally efficient tool. It serves as a basis for such approaches as phase-shift extrapolation, screen-propagator method, and Radon ($\tau - p$) transform (Gazdag, 1978; Stolt, 1978; Wu, 1994; de Hoop and Bleistein, 1997).

The main idea behind the method is the assumption that the wavefield generated by a point source can be represented as the superposition of plane waves traveling in all possible directions (Kennett, 1983; Tygel and Hubral, 1987). The reason for such representation is the simple and well-studied propagation and scattering of plane waves . For instance, reflection and transmission of plane waves is described by known plane-wave reflection coefficients given by Fresnel equations for acoustic media and Zoeppritz equations for elastic media (Aki and Richards, 2002). The superposition of plane waves is expressed by the Fourier-Bessel integrals, which can be evaluated numerically. The methodology allows researchers to represent and compute wavefields reflected in a general class of models: horizontally layered structures with vertical velocity variations.

Another useful application of plane-wave decomposition is the separation of upgoing and downgoing waves using the knowledge of pressure p and vertical particle velocity V_z. Because the vector comprising pressure and particle velocity is connected to the vector comprising amplitudes of upgoing and downgoing waves through a linear transformation in the frequency-wavenumber domain, the transition from one type of data to another can be realized in a straightforward manner. Also, pressure and particle velocity data are interconnected in the frequency-wavenumber domain (Ikelle and Amundsen, 2005). This leaves us with only one measurement, namely acoustic pressure, the one conventionally recorded in practice.

Even though the decomposition into upgoing and downgoing waves is derived for a stack of homogeneous layers, the methodology can be extended to inhomogeneous layers. The only restriction in this case is that the layer where the recoding system is placed has to be laterally homogeneous.

When the velocity field is a slowly varying function of the spatial coordinates, another practically useful implementation of the plane-wave decomposition of the upgoing and downgoing wavefields can be found. In this case, the two wavefields may be regarded as spatially decoupled. Thus the system of two differential equations splits into two independent one-way wave equations that can be resolved separately. This provides a relatively simple representation of the upgoing and downgoing wavefields through an exponential function containing the integrated vertical wavenumber component. Such representations are known as WKBJ-solutions (Ursin, 1983). The WBKJ-approximation is an integral formula that can be implemented in a straightforward manner; it is therefore valuable for modeling of seismic wavefields.

Much effort has been devoted to prediction and elimination of multiple reverberations within the water column in marine seismic data. One of the widely-used techniques for suppression of free-surface multiples is based on plane-wave decomposition (Verschuur, 2006). The multiple-free wavefield can be obtained by deconvolving the pressure data with the downgoing wavefield. The method preserves the amplitudes of the reflected wavefield and does not require knowledge of the subsurface below the receiver level. Velocity and density at the level of the receiver array are required information.

1.2.4 Surface and volume integral methods

Modeling based on surface and volume integrals requires more computational efforts than asymptotic methods and approaches based on plane-wave decomposition. However, this type of modeling is capable of reproducing quite complex wave events with rather accurate amplitudes. The approaches from this group fall into two main categories. The first group consists of methods based on integral equations, one of them being the Lippmann-Schwinger equation. The main idea is to extract a reference medium with a simple velocity field and a known wavefield. Then the difference between the actual and reference media acts as secondary sources and generates reflected wavefield as a correction to the wavefield in the reference medium (Ikelle and Amundsen, 2005).

Integral equations are usually solved iteratively, and the solution is written as a Neumann (or Born) series (Ursin and Tygel, 1997). In the Born approximation, only the first or first two terms of the series are calculated. The weak point of this approach is its inability to prove the convergence of the series. This makes it difficult to claim that just a few iterations will form the main part of the full wavefield. Also, Born-type integrals imply volume integration, which makes them computationally expensive. The approach is advantageous for modeling of wavefields in media with many small-scale scatterers, such as cracks or inclusions.

The second group of integral methods implies direct computation of integral representations (Pao and Varatharajulu, 1976). This group is, in a sense, a particular case of the first group, assuming that the actual and reference media are the same. Therefore, computation of the full wavefield reduces to direct evaluation of the surface or volume integral. This group comprises, for instance, Helmholtz-Kirchhoff surface integral approaches (Ursin and Tygel, 1997; Schleicher et al., 2001). Kirchhoff modeling, which is based on the Helmholtz-Kirchhoff integral, is a widely-used technique. It is mostly applicable for media with rough reflectors. This modeling approach is based on Huygens' principle stating that a point at the reflector is the secondary source of a spherical wave diverging from this point. The interference of the spherical waves diverging from all points of the reflector form the reflected wavefield at the receiver.

Modeling based on surface integrals is faster than Born-type modeling, in particular because it implies implementation of surface integrals over 2D reflectors. However, it requires knowledge of the Green's function and the boundary data for the actual model in the Helmholtz-Kirchhoff integral. Even relatively simple velocity models may generate quite complex Green's functions and boundary data, thus causing serious bottlenecks in conventional Kirchhoff modeling.

1.2.5 Hybrid methods

Classical plane-wave decomposition forms the basis for many other methods. Models with deviations from strictly horizontal layering may be fitted into the methodology in an approximate way. The generalized screen-propagator (GSP) method is one application of the plane-wave decomposition that is not limited to horizontally layered media (Wu, 1994). Within the GSP-approach, the wavefield is written as a Born-type volume integral, which follows from the perturbation theory. The Green's function and the wavefield in the reference medium are decomposed into plane waves. When interchanging the order of volume integration and the two decompositions, a wavenumber-domain formulation can be obtained. The scattered wavefield is thus written as a Fourier-type integral. The formula can be implemented in an efficient manner. It provides accurate modeling results in media with the properties varying in a single preferred direction. Also, the initial formulation of the GSP-approach is beneficial in modeling of small-angle scattering. However, for media with strong lateral velocity variations, the quality of amplitudes and traveltimes obtained with this approach substantially decreases.

Sen and Frazer (1991) proposed a hybrid approach based on the Helmholtz-Kirchhoff integral. The authors represent the Green's function in the integrand as an expansion into a set of plane waves. The coupling of the plane waves at the reflectors is determined by means of the Helmholtz-Kirchhoff integral using generalized reflection and transmission coefficients. The resulting integral (called the multifold phase space path integral) consists of a series of integrals over ray parameters and interfaces passed by the generalized ray. The method represents a generalization of conventional Kirchhoff modeling. It handles diffractions and head waves at curved interfaces in an approximate, but accurate, manner.

Another hybrid approach to the description of wavefields reflected and transmitted at curved interfaces was introduced by Klem-Musatov et al. (2004). Later, Klem-Musatov et al. (2005) extended the methodology for layered media with several curved reflectors. They showed that the propagation inside smooth layers can be described by the conventional Kirchhoff integral, whereas reflection and transmission is represented by reflection and transmission operators. If the reflector is plane, the reflection and transmission operators are expressed by the usual plane-wave decomposition. For curved reflectors, which are often found in typical geological settings, this decomposition breaks down. In this case, Klem-Musatov et al. (2005) suggest a generalized plane-wave decomposition. They decompose the reflected or transmitted wavefield into locally plane waves in the vicinity of the reflector. The decomposition is local and can not be used outside a thin layer surrounding the interface.

The new approach of Klem-Musatov et al. (2005) is computationally heavier than the conventional Kirchhoff modeling, which puts the method at a disadvantage. However, it proved its capability to accurately model complex wave phenomena, in particular caustic cusps, diffracted waves, and head waves. Although numerical simulation requires computational effort, the approach is advantageous for modeling in stratigraphically complex geological structures. The next section is devoted to a detailed explanation of the algorithmic implementation of the Kirchhoff integral and the reflection and transmission operators.

1.3 Tip-wave superposition method with effective coefficients

The Green's function for conventional Kirchhoff modeling is traditionally described using geometrical seismics (Schleicher et al., 2001). This approximation limits modeling to the areas of validity of the ray-tracing solutions (Frazer and Sen, 1985). Klem-Musatov and Aizenberg (1985) and later Klem-Musatov et al. (1993) proposed a high-frequency approximation of the Helmholtz-Kirchhoff integral based on diffraction theory. They showed that small rhombic parts of the reflector, in accordance with Huygens' principle, act as secondary sources and emit wave beams toward receivers. Beams are formed by the main reflection, four edge-diffracted, and eight tip-diffracted waves. Because the tip-diffracted waves are the greatest contributors, the approach is referred to as a "tip-wave superposition method," or TWSM (Aizenberg, 1992, 1993a,b). The method is computationally efficient and accurate for modeling caustic triplications and diffractions.

Within conventional Kirchhoff modeling, the boundary data are routinely generated by the multiplication of the incident wavefield and the plane-wave reflection coefficient (PWRC) (Ursin and Tygel, 1997; Ursin, 2004). PWRC is inadequate in describing the reflected wavefields at near- and post-critical incidence angles and in the presence of significant reflector curvature. Also, PWRC produces diffraction artifacts formed by the points of discontinuous slope of PWRC at the critical incidence angles (Frazer and Sen, 1985).

To handle curved reflectors in heterogeneous media, a rigorous theory of reflection and transmission for interfaces of arbitrary shape can be used Klem-Musatov et al. (2004). The authors showed that the boundary data in the acoustic Kirchhoff integral can be represented by a generalized plane-wave decomposition. Ayzenberg et al. (2007b) proved that the exact action of the reflection operator upon the incident wavefield may be approximately described by multiplication of the incident wavefield and the corresponding effective reflection coefficient (ERC) for each point at the interface. This formalism incorporates the local interface curvature into the reflection response and is not limited to small incidence angles and weak parameter contrasts across the reflector. Additionally, ERC correctly describes near- and post-critical reflection and includes head waves.

Ayzenberg et al. (2007b) provided the results of 3D diffraction modeling using

TWSM and ERC in order to illustrate the performance of the new approach for modeling complex wave phenomena, in particular multipathing, diffraction, and head waves. The tests demonstrated that the acoustic version of TWSM with ERC is capable of modeling accurate traveltimes and amplitudes. The paper also provided an overall comparison of TWSM modeling and conventional Kirchhoff modeling (in this case, represented by TWSM with PWRC) regarding the tradeoff between quality and computational speed.

Based on the work of Klem-Musatov et al. (2005) on reflection and transmission in layered media, Ayzenberg et al. (2008b) extended the TWSM and ERC formalism for 3D wave propagation in the overburden with several reflecting horizons. They showed that the total reflected wavefield may be represented as the sum of separate wave events coming from different interfaces and reflected in accordance with their own wavecodes. Also, the multilayered version of the TWSM accurately modeled amplitudes of multiple reflected and transmitted wavefields.

Ayzenberg et al. (2008a) generalized the method for reflection from curved reflectors between isotropic and bending transversely-isotropic (TI) layers. They studied the dependence of ERC and modeled reflected wavefields on the predominant frequency, local reflector curvature, and Thomsen anisotropy parameters. Modeling revealed strong dependence of the reflected amplitude on Thomsen parameters, in particular ε, for near- and post-critical incidence angles.

As for computational speed, TWSM with ERC is slower than conventional Kirchhoff modeling. Implementation of ERC implies computation of the Fourier-Bessel integrals, which are given by explicit algebraic formulas. Additionally, while PWRC are frequency-independent and need to be computed only once, ERC have to be computed for each frequency entering the bandwidth of the initial wavelet. In the case of almost plane reflectors, weak parameter contrasts across the reflectors and near offsets, ERC may be substituted with PWRC in order to decrease computational costs. However, in many situations, ERC produces considerably better modeling results.

1.4 Thesis content

This thesis consists of three papers published in or submitted to *Geophysics*.

Chapter 2 contains a paper about 3D diffraction modeling of singly scattered acoustic wavefields. The paper introduces ERC for acoustic waves and gives a detailed overview of the acoustic version of TWSM. It studies the properties of ERC and illustrates their dependence on the frequency content of the incident wave and reflectorgeometry. Also, the paper analyzes applicability, computational costs, and accuracy of 3D acoustic diffraction modeling.

Chapter 3 contains a paper about 3D diffraction modeling of acoustic scattering in layered media. The paper deals with the extension of the acoustic version of TWSM for layered media. It provides a thorough analysis of modeling results for 3D models with smoothly varying reflectors and for models containing diffracting edges. The paper also demonstrates that the modeling results are in agreement with FDM and diffraction theory.

Chapter 4 contains a paper about ERC for curved interfaces in TI media. Numerical tests in the paper demonstrate that the difference between ERC and PWRC for typical TI models can be significant, especially at low frequencies and in the post-critical domain. ERC are sensitive to the anisotropy parameters, local reflector curvature, and source-receiver geometry. The paper analyzes the dependence of ERC on reflector shape and demonstrates their advantages over PWRC in 3D diffraction modeling of PP and PS reflection data.

Chapter 2

3D diffraction modeling of singly scattered acoustic wavefields based on the combination of surface integral propagators and transmission operators

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Chapter 3

3D acoustic modeling in layered media using multiple tip-wave superposition method with effective coefficients

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Chapter 4

Effective reflection coefficients for curved interfaces in TI media

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Closing remarks

Conventional Kirchhoff modeling is based on the geometrical-seismics approximation. The two main assumptions are that the Green's function in the Helmholtz-Kirchhoff integral can be obtained from geometrical seismics and that the boundary data are the product of the amplitude of the incident wave and the corresponding plane-wave reflection coefficient (PWRC). The limitations of the geometrical-seismics approximation pose serious problems for dynamic ray tracing and, as a consequence, Kirchhoff integral modeling techniques. In particular, this approach produces artificial diffractions on synthetic data due to the discontinuous slope of the PWRC at the critical angle and does not correctly reproduce caustics, shadow zones, and head waves.

3D diffraction modeling is designed to overcome, to some degree, these limitations of the conventional Kirchhoff approach. Within the new method, the Kirchhoff-type surface integral propagator is realized in the form of a "tip-wave superposition method" (TWSM), which represents a seismic-frequency approximation of the Helmholtz-Kirchhoff integral. The implementation of TWSM involves splitting the reflector into small elements. According to Huygens' principle, each element acts as a secondary source emitting a tip-wave beam. The superposition of the tip-wave beams automatically produces correct reflection traveltimes at the observation point.

The boundary data are computed using "effective reflection and transmission coefficients" (ERC and ETC). ERC and ETC generalize PWRC and PWTC for wavefields from point sources at curved interfaces and are not limited to small incidence angles and weak parameter contrasts across the reflector. The difference between effective and plane-wave coefficients can be significant, especially at low frequencies and for the near-critical and post-critical incidence angles. Numerical results demonstrate that effective coefficients are formed by the interference of the reflected and head waves at the reflector. Effective coefficients are also sensitive to the model parameters, frequency range of the incident wavelet, and local reflector curvature. Numerous tests provided in the three papers show that Kirchhoff-type modeling with ERC and ETC removes the artificial diffractions generated at the critical angles and correctly simulates the amplitudes of the reflected and head waves.

The new approach is numerically stable and capable of simulating complex wave phenomena, such as caustic triplications, diffraction, and head waves at curved reflectors. For simpler models with weak parameter contrasts across reflectors, TWSM may be successfully used with PWRC and PWTC. In this case, 3D modeling becomes extremely, computationally efficient. However, amplitudes for near-critical and post-critical incidence angles are somewhat inaccurate. To evaluate the amplitudes with a higher precision, TWSM should be used with ERC and ETC. Because effective coefficients depend on frequency and, moreover, are represented by Fourier-Bessel integrals, 3D diffraction modeling becomes computationally difficult. (However, depending on the model, it can still be considerably cheaper than conventional FDM.) The disk-space requirements become more demanding because the tip-wave beams have to be stored separately for each frequency. Although data storage may present a logistical problem, minor changes of the model can be incorporated without recalculating all tip-wave beams. This advantage of TWSM becomes particularly valuable for modeling in layered models, as well as in survey design.

As a stand-alone technique, ERC may find useful applications in AVO-related studies. For plane interfaces, ERC represent the exact reflection response from point sources. For curved interfaces, ERC represent a sensible approximation of the exact reflection response. Because ERC incorporate the near-critical and post-critical effects at curved interfaces in an accurate manner, they exhibit a good match with the AVO response extracted from the data (Skopintseva et al., 2008). In particular, the inversion results in a simple model show that by accounting for the near-critical and post-critical incidence angles, the S-wave velocity and density can be more accurately estimated.

The author of this thesis hopes that the TWSM approach finds its place among other, more traditional, modeling techniques. Although the methodology shows promising results, there are still many unexplored areas. An important theoretical gap is finding a proper description for the Green's function in the presence of shadow zones. This may have serious consequences for the general understanding and future of Kirchhoff modeling. Also, there are numerical bottlenecks to be resolved. In particular, the computational cost for effective coefficients is prohibitively expensive and should be reduced in order to make it comparable with plane-wave coefficients. Also, the distortions brought in by the approximations in TWSM and effective coefficients must be estimated. TWSM and FEM must be compared regarding the quality of the modeled amplitudes and computational time.

Bibliography

Aizenberg, A., M. Ayzenberg, H. Helle, K. Klem-Musatov, and J. Pajchel, 2002, Single Reflection at Curvilinear Interface. Tip Wave Superposition Method with effective reflection coefficient: Open report, E&P Research Centre, Norsk Hydro a.s.

Aizenberg, A., M. Ayzenberg, H. Helle, and J. Pajchel, 2005, Reflection and transmission of acoustic wavefields at a curved interface of two inhomogeneous media: Continuum Dynamics, Acoustics of Inhomogeneous media, **123**, 73–79.

Aizenberg, A., H. Helle, K. Klem-Musatov, and J. Pajchel, 1996, The tip wave superposition method based on the refraction transform: 58th Annual Meeting of EAGE, Expanded Abstracts.

Aizenberg, A., K. Klem-Musatov, M. Ayzenberg, H. Helle, and J. Pajchel, 2006a, Integral reflection-transmission operators instead of reflection-transmission coefficient - A possibility to increase the resolution of seismic exploration: Russian Geology and Geophysics, **47**, 537–546.

Aizenberg, A. M., 1982, Scattering of seismic waves by broken edge of a flat boundary: Soviet Geology and Geophysics, **23**, 74–82.

———, 1992, A self-similar conformal analog of wave equation in 3D nonhomogeneous space: Russian Geology and Geophysics, **33**, 116–121.

———, 1993a, Special function of eddy diffusion equation in 3D inhomogeneous space: Russian Geology and Geophysics, **34**, 107–114.

———, 1993b, A system of irregular fundamental solutions to wave equation in a three-dimensional inhomogeneous medium: Russian Geology and Geophysics, **34**, 105–113.

Aizenberg, A. M. and A. A. Ayzenberg, 2008a, Feasible fundamental solution to the acoustic wave equation in a heterogeneous halfspace with a regular boundary: Proceedings of the International Conference on Mathematical Methods in Geophysics "MMG-2008", 1–6.

———, 2008b, Fundamental solution of the acoustic wave equation which satisfies the integral absorption condition at a regular boundary of a half-space: Proceedings of the International Conference "Differential Equations. Function Spaces. Approximation Theory" dedicated to the 100th anniversary of Sergei Sobolev, 89.

Aizenberg, A. M. and M. A. Ayzenberg, 2002, Simulation of interference wave field trace reflected/transmitted by plane interface of two acoustic media: Dynamics of solid media: Proceedings of Russian Academy of Sciences, **121**, 50–55.

Aizenberg, A. M., M. A. Ayzenberg, H. B. Helle, K. D. Klem-Musatov, and J. Pajchel, 2004, Modeling of single reflection by tip wave superposition method using effective coefficient: 66th Annual Meeting of EAGE, Expanded Abstracts.

Aizenberg, A. M., M. A. Ayzenberg, H. B. Helle, K. D. Klem-Musatov, J. Pajchel, and B. Ursin, 2006b, 3-D seismic diffraction modeling in multilayered media in terms of surface integrals: 68th Annual Meeting of EAGE, Expanded Abstracts.

Aki, K. and P. Richards, 2002, Quantitative Seismology (2nd edition): University Science Books.

Ayzenberg, M., A. Aizenberg, and J. Pajchel, 2007a, Representation of the interface conditions of the acoustic transmission problem at a curved contact of heterogeneous media in terms of transmission and reflection operators: Proceedings in Applied Mathematics and Mechanics.

Ayzenberg, M., I. Tsvankin, A. Aizenberg, and B. Ursin, 2008a, Effective reflection coefficients for curved interfaces in TI media: Submitted to Geophysics.

Ayzenberg, M. A., A. M. Aizenberg, H. B. Helle, K. D. Klem-Musatov, J. Pajchel, and B. Ursin, 2007b, 3D diffraction modeling of singly scattered acoustic wavefields based on the combination of surface integral propagators and transmission operators: Geophysics, **72**, SM19–SM34.

———, 2008b, 3D acoustic modeling in layered media using multiple tip wave superposition method with effective coefficients: Submitted to Geophysics.

Baker, B. B. and E. T. Copson, 1953, The Mathematical Theory of Huygens' Principle: Oxford: Clarendon Press.

Ben-Menahem, A. and S. J. Singh, 1998, Seismic Waves and Sources: Dover Publications.

Berkhout, A. J., 1987, Applied Seismic Wave Theory: Elsevier.

Bleistein, N. and R. A. Handelsman, 1975, Asymptotic Expansions of Integrals: Dover Publications.

Brekhovskikh, L. M., 1980, Waves in Layered Media: Academic Press.

Carcione, J., G. C. Herman, and A. P. E. ten Kroode, 2002, Seismic modeling: Geophysics, **67**, 1304–1325.

Červený, V., 2001, Seismic Ray Theory: Cambridge University Press.

Červený, V. and F. Hron, 1961, Reflection coefficients of spherical waves: Studia geoph. et geod., **4**, 20–41.

Chapman, C. H., 1994, Reflection/transmission coefficients reciprocities in anisotropic media: Geophysical Journal International, **161**, 498–501.

Claerbout, J., 1971, Toward a unified theory of reflector mapping: Geophysics, **36**, 467–481.

Clemmow, P. C. and T. B. A. Senior, 1971, A note on a generalized fresnel integral: Mathematical Proceedings of the Cambridge Philosophical Society, **9**, 570–572.

Costabel, M. and M. Dauge, 1997, On representation formulas and radiation conditions: Mathematical Methods in the Applied Sciences, **20**, 133–150.

Costabel, M. and E. P. Stephan, 1985, A direct boundary integral method for transmission problems: Journal of Mathematical Analysis and Applications, **106**, 367–413.

———, 1990, Integral equations for transmission problems in linear elasticity: Journal of Integral Equations and Applications, **2**, 211–223.

de Hoop, M. V. and N. Bleistein, 1997, Generalized Radon transform inversions for reflectivity in anisotropic elastic media: Inverse Problems, **13**, 669–690.

do Carmo, M. P., 1976, Differential Geometry of Curves and Surfaces: Prentice Hall.

Downton, J. E. and C. Ursenbach, 2006, Linearized amplitude variation with offset (AVO) inversion with supercritical angles: Geophysics, **71**, E49–E55.

Dwight, H. B., 1961, Tables of Integrals and Other Mathematical Data: New York, the MacMillan Company.

Emerman, S. H., W. Schmidt, and R. A. Stephen, 1982, An implicit finite-difference formulation of the elastic wave equation: Geophysics, **47**, 1521–1526.

Felsen, L. B. and N. Marcuvitz, 1973, Radiation and Scattering of Waves: Prentice Hall.

Fokkema, J. T. and P. M. van den Berg, 1993, Seismic Applications of Acoustic Reciprocity: Elsevier.

Frazer, L. N. and M. K. Sen, 1985, Kirchhoff-Helmholtz reflection seismograms in a laterally inhomogeneous multi-layered elastic medium - I. Theory: Geophysical

Journal of the Royal Astronomical Society, **80**, 121–147.

Friedlander, F., 1958, Sound Pulses: Cambridge University Press.

Gazdag, J., 1978, Wave equation migration with the phase-shift method: Geophysics, **43**, 1342–1351.

Gazdag, J. and P. Sguazzero, 1984, Migration of seismic data: Proceedings of the IEEE, **72**, 1302–1315.

Gjøystdal, H., Å. Drottning, I. Lecomte, and M. Branston, 2007a, Quantitative seismic modelling: First break, **25**, 95–102.

Gjøystdal, H., E. Iversen, I. Lecomte, T. Kaschwich, Å. Drottning, and J. Mispel, 2007b, Improved applicability of ray tracing in seismic acquisition, imaging and interpretation: Geophysics, **72**, SM261–SM271.

Goldin, S. V., 1992, Estimation of reflection coefficient under migration of converted and monotype waves: Russian Geology and Geophysics, **33**, 76–90.

Graebner, M., 1992, Plane-wave reflection and transmission coefficients for a transversely isotropic solid: Geophysics, **57**, 1512–1519.

Gray, S. H., 2001, Seismic imaging: Geophysics, **66**, 15–17.

Hanyga, A. and H. H. Helle, 1995, Synthetic seismograms from generalized ray tracing: Geophysical Prospecting, **43**, 51–75.

Hilterman, F., 2001, Seismic Amplitude Interpretation (Vol. Distinguished Instrustor Series No. 4): SEG.

Hönl, H., A. W. Maue, and K. Westpfahl, 1961, Theorie der Beugung: Springer-Verlag.

Ikelle, L. and L. Amundsen, 2005, Introduction to Petroleum Seismology (Investigations in Geophysics, No. 12): SEG.

Kampfmann, W., 1988, A study of diffraction-like events on DECORP 2-S by Kirchhoff theory: Journal of Geophysics, **62**, 163–174.

Keller, J. B., 1962, Geometrical theory of diffraction: Journal of the Optical Society of America, **52**, 116–130.

Kennett, B. L. N., 1983, Waves in Stratified Media.

———, 1984, Reflection operator methods for elastic waves I - Irregular interfaces and regions: Wave Motion, **6**, 407–418.

———, 1994, Representations of the seismic wavefield: Geophysical Journal International, **118**, 344–357.

Kleinman, R. E. and P. A. Martin, 1988, On single integral equations for the transmission problem of acoustics: Geophysical Journal International, **48**, 307–325.

Klem-Musatov, K., A. Aizenberg, J. Pajchel, and H. B. Helle, 2008, Edge and Tip Diffractions: Theory and Applications in Seismic Prospecting: SEG.

Klem-Musatov, K. D., 1994, Theory of Seismic Diffractions: Open File Publications #1, F. Hron and L. Lines (eds.): SEG.

Klem-Musatov, K. D. and A. M. Aizenberg, 1985, Seismic modelling by methods of the theory of edge waves: Journal of Geophysics, **57**, 90–105.

Klem-Musatov, K. D., A. M. Aizenberg, H. B. Helle, and J. Pajchel, 1993, Seismic simulation by the tip wave superposition method in complex 3D geological models: 55th Anual Meeting of EAGE, Expanded Abstracts.

———, 2004, Reflection and transmission at curvilinear interface in terms of surface integrals: Wave Motion, **39**, 77–92.

———, 2005, Reflection and transmission in multilayered media in terms of surface integrals: Wave Motion, **41**, 293–305.

Klem-Musatov, K. D., A. M. Aizenberg, J. Pajchel, and H. B. Helle, 1994, Edge and Tip Diffractions - Theory and Applications in Seismic Prospecting: Norsk Hydro.

Klimeš, L., 2003, Weak contrast reflection-transmission coefficients in a generally anisotropic background: Geophysics, **69**, 2063–2072.

Kottler, F., 1965, Diffraction at a black screen. Part I: Kirchhoff's theory: Progress in Optics, **IV**, 281–314.

Miranda, C., 1970, Partial Differential Equations of Elliptic Type: Springer-Verlag.

Ogilvie, J. S. and G. W. Purnell, 1996, Effects of salt-related mode conversions on subsalt prospecting: Geophysics, **61**, 331–348.

Ostrander, W. J., 1984, Plane-wave reflection coefficients for gas sands at nonnormal angles of incidence: Geophysics, **49**, 1637–1648.

Pao, Y.-H. and V. Varatharajulu, 1976, Huygens' principle, radiation conditions, and integral formulas for the scattering of elastic waves: Journal of the Acoustical Society of America, **59**, 1361–1371.

Ramsden, C. and G. Bennett, 2005, High-resolution 3D seismic imaging in practice: The Leading Edge, **24**, 423–428.

Roden, R. and M. Forrest, 2005, The impact of seismic amplitudes on prospect risk analysis: The Leading Edge, **24**, 706–711.

Rüger, A., 1997, P-wave reflection coefficients for transversely isotropic models with vertical and horizontal axis of symmetry: Geophysics, **62**, 713–722.

———, 2002, Reflection Coefficients and Azimuthal AVO Analysis in Anisotropic Media: SEG.

Rutherford, S. R. and R. H. Williams, 1989, Amplitude-versus-offset variations in gas sands: Geophysics, **54**, 680–688.

Schleicher, J., M. Tygel, B. Ursin, and N. Bleistein, 2001, The Kirchhoff-Helmholtz integral for anisotropic elastic media: Wave Motion, **34**, 353–364.

Sen, M. K. and L. N. Frazer, 1991, Multifold phase space path integral synthetic seismograms: Geophysical Journal International, **104**, 479–487.

Seriani, G. and S. P. Oliveira, 2007, Optimal blended spectral-element operators for acoustic wave modeling: Geophysics, **72**, SM95–SM106.

Sheriff, R. E. and L. P. Geldart, 1995, Exploration Seismology (2nd edition): Cambridge University Press.

Shuey, R. T., 1985, A simplification of the Zoeppritz equations: Geophysics, **50**, 609–614.

Skopintseva, L. V., M. A. Ayzenberg, M. Landrø, T. V. Nefedkina, and A. M. Aizenberg, 2008, AVO inversion of long-offset synthetic PP data based on effective reflection coefficients: 70th EAGE Conference & Exhibition, Extended Abstracts, 609–614.

Skudrzyk, E., 1971, Foundation of Acoustics: Springer-Verlag.

Stolt, R. H., 1978, Migration by Fourier transform: Geophysics, **43**, 23–48.

Stovas, A. and B. Ursin, 2003, Reflection and transmission responses of layered transversely isotropic viscoelastic media: Geophysical Prospecting, **51**, 447–477.

Thomsen, L., 1993, Weak anisotropic reflections: Offset Dependent Reflectivity (Castagna, J., and Backus, M., Eds.), 103–114.

Treitel, S. and L. Lines, 2001, Past, present, and future of geophysical inversion - A new millennium analysis.

Tsvankin, I., 1995, Seismic Wavefields in Layered Isotropic Media: Samizdat Press.

———, 2005, Seismic Signatures and Analysis of Reflection Data in Anisotropic Media: Elsevier.

Tsvankin, I. D. and E. M. Chesnokov, 1990, Synthesis of body wave seismograms from point sources in anisotropic media: Journal of Geophysical Research, **95**, 11,317–11,331.

Tygel, M. and P. Hubral, 1987, Transient Waves in Layered Media: Elsevier.

Ursin, B., 1983, Review of elastic and electromagnetic wave propagation in horizontally layered media: Geophysics, **48**, 1063–1081.

———, 2004, Tutorial: Parameter inversion and angle migration in anisotropic elastic media: Geophysics, **69**, 1125–1142.

Ursin, B. and M. Tygel, 1997, Reciprocal volume and surface scattering integrals for anisotropic elastic media: Wave Motion, **26**, 31–42.

van der Baan, M. and D. Smith, 2006, Amplitude analysis of isotropic P-wave reflections: Geophysics, **72**, C93–C103.

van Kampen, N. G., 1949, An asymptotic treatment of diffraction problems - I: Physica, **14**, 575–589.

———, 1950, An asymptotic treatment of diffraction problems - II: Physica, **16**, 817–821.

———, 1958, The method of stationary phase and the method of Fresnel zones: Physica, **24**, 437–444.

Verschuur, D. J., 2006, Seismic Multiple Removal Techniques: Past, Present and Future: SEG.

Weatherborn, C. E., 1930, Differential Geometry of Three Dimensions. Vol. II: Cambridge University Press.

Wenzel, F., K.-J. Stenzel, and U. Zimmermann, 1990, Wave propagation in laterally heterogeneous layered media: Geophysical Journal International, **103**, 675–684.

Wu, R.-S., 1994, Wide-angle elastic wave one-way propagation in heterogeneous media and an elastic wave complex-screen method: Journal of Geophysical Research, **99**, 751–766.