Rejection of Sinusoidal Disturbance Approach Based on High-Gain Principle

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Abstract— **This paper deals with the output stabilization of linear systems with unknown parameters and sinusoidal disturbance. The approach is based on a hybrid algorithm of frequency estimation, that is used for compensation of the harmonic disturbance, and the high-gain feedback principle for robust stabilization. Efficiency of the approach is demonstrated through numerical simulations.**

I. INTRODUCTION

Rejection of unknown disturbances is not new [14], but still actual problem in control [5], [7]–[13], [17], [21]–[24], [27]–[31]. This is an essential problem for many practical control applications, such as development of advanced research tools for nanotechnology [1] and increased density hard disks [16], where precise positioning is a critical demand for control system, dynamic positioning systems for vessels in the presence of waves and wind [33] etc. At the same time, in number of tasks harmonic nature of disturbances is reasonable assumption. Using this representation we can obtain fruitful results for certain implementations.

Anyway, this challenging problem remains unsolved for a number of special assumptions on disturbance as well as restrictions on plant dynamics. Cases, when amplitudes, frequencies, and phases are unknown constant parameters, are common for current publications in the field. Other variations in different works are connected with plant, which could linear or nonlinear, with known parameters or parametric uncertainty, fully measurable state or not etc.

On the other hand, output adaptive control methods development is nontrivial problem itself. It's highly motivated by practical applications, when state measurement is hard or even impossible to realize. A lot of different original results were obtained in this field for the last years [18]–[20], [25], [26], [32]. This paper is focused on the recent advantages in the development of adaptive output control approach using high-gain principle named by the authors as "consecutive compensator", that was considered in number of previous works [4]–[6], [8], [29], [30].

This work was supported by the Federal Target Program "R&D in Priority Areas of Russia's Scientific and Technological Development in 2007–2013" (project 11.519.11.4007).

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Let us briefly review works closely related to proposed approach and analyze main difference between them. In [21] control algorithm for linear stable plant with known parameters and relative degree one under conditions of biased harmonic disturbance was proposed. In contrast to [21] in [7] the same task for non-minimum phase plant with known parameters, but arbitrary relative degree was solved. In [10], [12], [28] improved approach was extended for time-delay systems with known parameters. Works [8], [11] are devoted to output control with sinusoidal disturbance rejection under conditions of plant parametric uncertainty. In [8] only linear plant was considered, while [11] is dealing with nonlinear systems, but for both works relative degree of the systems is assumed to be one.

In the current work authors discuss novel output control algorithm for linear parametrically uncertain plant with relative degree higher than one rejecting sinusoidal disturbance $\delta(t) = A\sin(\omega\, t + \varphi)$, where amplitude A, frequency ω, and phase shift φ are also a priori unknown. Merging two different results, presented at CDC last year (robust output controller [30] and frequency estimator [31]), we can provide better disturbance rejection compare to stand apart "consecutive compensator".

II. PROBLEM FORMULATION

Consider the linear plant

$$a(p)y(t) = b(p)[u(t) + \delta(t)], \tag{1}$$

where $p = \frac{d}{dt}$ is the differentiation operator, polynomials $a(p) = p^n + a_{n-1}p^{n-1} + ... + a_0$ and $b(p) = b_m p^m + b_{m-1}p^{m-1} + ... + b_0$ are unknown, and

$$\delta(t) = A\sin(\omega t + \varphi) \tag{2}$$

is the disturbance with the unknown amplitude A, frequency ω, and phase φ.

The purpose of control is to provide the asymptotic stability of nonlinear system (1).

$$\lim \lim_{t\to\infty} y(t) = 0. \tag{3}$$

Let us consider the following assumptions.

Assumption 1: Polynomial $b(p)$ is Hurwitz and the parameter $b_0 > 0$.

Assumption 2: The relative degree $r = n - m$ is known, while degrees of polynomials $a(p)$ and $b(p)$ are unknown.

Assumption 3: The lower bound $\omega_{\min}$ of frequency ω is known.

III. MAIN RESULT

A. Nominal Controller Design

In this subsection we consider the preliminary result assuming that the frequency ω of the disturbance is known. Rewrite (1) as

$$Y(s) = \frac{b(s)}{a(s)}U(s) + \frac{b(s)}{a(s)}\Psi(s) + \frac{D(s)}{a(s)}, \tag{4}$$

where s is the complex variable, $Y(s) = L\{y(t)\}$, $U(s) = L\{u(t)\}$ and $\Psi(s) = L\{\delta(t)\} = \frac{A_{\delta 1}s + A_{\delta 2}}{(s^2+\omega^2)}$ is the Laplace images of corresponding signals, $A_{\delta 1} = A\sin\varphi$ and $A_{\delta 2} = A\omega\cos\varphi$ are constants, polynomial $D(s)$ denotes the initial conditions.

To derive the main result temporary assume that first $r-1$ derivatives of the variable $y(t)$ are measurable and the frequency ω of the disturbance $\delta(t) = A\sin(\omega t + \varphi)$ is known. Choose the control law $u(t)$ as follows

$$u(t) = -k\frac{\alpha(p)(p+1)^2}{p^2+\omega^2}y(t), \tag{5}$$

where Hurwitz polynomial $\alpha(p)$ with degree $r-1$ and constant $k > 0$ are chosen such that all eigenvalues of a polynomial $\gamma(s) = a(s)(s^2+\omega^2) + kb(s)\alpha(s)(s+1)^2$ has a negative real part (more detailed choice of $\alpha(p)$ and $k > 0$ making the polynomial $\gamma(s)$ Hurwitz can be found, for example in [5], [6], [9], [15]).

Making the Laplace transformation in (5) and substituting obtained expression into (4), we have

$$\begin{aligned} Y(s) &= -k\frac{b(s)\alpha(s)(s+1)^2}{a(s)(s^2+\omega^2)}Y(s) \\ &\quad + \frac{b(s)}{a(s)}\frac{A_{\delta 1}s + A_{\delta 2}}{(s^2+\omega^2)} + \frac{D(s)}{a(s)} \\ Y(s) &= (A_{\delta 1}s + A_{\delta 2})\frac{b(s)}{\gamma(s)} + \frac{D(s)(s^2+\omega^2)}{\gamma(s)}. \end{aligned} \tag{6}$$

Since the polynomial $\gamma(s)$ is Hurwitz the inverse Laplace transformation in (6) yields

$$\lim_{t\to\infty} y(t) = 0.$$

It is easy to see that the controller (5) is realizable if the frequency ω is known and $r-1$ derivatives of $y(t)$ are measurable. However the considered problem is formulated so that the disturbance is unknown and only the output variable is measurable. Therefore we need to modify the controller (5) to exclude the unknown functions and parameters. The next subsection deals with this task.

B. Realizable Control Law

In this subsection the realizable adaptive and robust controller is presented that has iterative structure. Only the output $y(t)$ is accessible, parameters of polynomials $a(p)$, $b(p)$, and frequency ω are unknown. Following the results [5], [5], [6] choose the control law as follows

$$u(t) = -k\frac{\alpha(p)(p+1)^2}{p^2+\hat{\omega}^2}\xi_1(t), \tag{7}$$

$$\begin{cases} \dot{\xi}_1 = \sigma\xi_2, \\ \dot{\xi}_2 = \sigma\xi_3, \\ \dots \\ \dot{\xi}_{\rho-1} = \sigma\left(-k_1\xi_1 - \ldots - k_{\rho-1}\xi_{\rho-1} + k_1 y\right), \end{cases} \tag{8}$$

where k and $\alpha(p)$ chosen like in (5), the number $\sigma > k$, and parameters k_i are calculated for the system (8) to be asymptotically stable in the absence of $y(t)$, the constant parameter $\hat{\omega}$ is the estimate of the disturbance frequency.

Remark 1: If the relative degree equals one $r = 1$ then the controller is formed simpler. In (7) we have $\alpha(p) = 1$, $\xi_1(t) = y(t)$, and the system (8) is excluded.

Substitution (7) into (1), gives

$$\begin{aligned} y(t) &= \frac{kb(p)\alpha(p)(p+1)^2}{a(p)(p^2+\hat{\omega}^2) + kb(p)\alpha(p)(p+1)^2}\varepsilon(t) \\ &\quad + \frac{b(p)(p^2+\hat{\omega}^2)}{a(p)(p^2+\hat{\omega}^2) + kb(p)\alpha(p)(p+1)^2}\delta(t), \end{aligned} \tag{9}$$

where $\varepsilon(t) = y(t) - \xi_1(t)$.

Rewrite (9) as

$$y(t) = \frac{kb(p)\alpha(p)(p+1)^2}{a(p)(p^2+\hat{\omega}^2) + kb(p)\alpha(p)(p+1)^2}[\varepsilon(t) + w(t)], \tag{10}$$

where a signal $w(t) = \frac{(p^2+\hat{\omega}^2)}{k\alpha(p)(p+1)^2}\delta(t)$.

The similar model like (10) can be found in [6], [9]. Following the results of [6], [9] we write the input-state-output model

$$\dot{x} = Ax + kb(\varepsilon + w), \tag{11}$$

$$y = c^T x, \tag{12}$$

where $x \in R^n$ is the state vector of the model (11); A, b, and c are the corresponding matrices. In accordance to the well-known KYP lemma (e.g., [15]) one can take the positive symmetric matrix P, satisfying two following matrix equality

$$A^TP + PA = -Q_1, \qquad Pb = c, \tag{13}$$

where $Q_1 = Q_1^T$ is some positive definite matrix.

Let us rewrite model (7), (8) in the form

$$\dot{\xi}(t) = \sigma(\Gamma\xi(t) + dy(t)), \tag{14}$$

$$\hat{y}(t) = h^T\xi(t), \tag{15}$$

where $\Gamma = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_1 & -k_2 & -k_3 & \dots & -k_{\rho-1} \end{bmatrix}$, $d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ k_1 \end{bmatrix}$, and $h^T = \begin{bmatrix} 1 & 0 & 0 & \dots 0 \end{bmatrix}$.

Consider vector

$$\eta(t) = hy(t) - \xi(t), \tag{16}$$

then by force of vector h structure the error $\varepsilon(t)$ will become

$$\begin{aligned}\varepsilon(t) &= y(t) - \hat{y}(t) = h^T hy(t) - h^T \xi(t) \\ &= h^T (hy(t) - \xi(t)) = h^T \eta(t).\end{aligned} \tag{17}$$

For derivative of $\eta(t)$ we obtain

$$\begin{aligned}\dot{\eta}(t) &= h\dot{y}(t) - \sigma(\Gamma(hy(t) - \eta(t)) + dy(t)) \\ &= h\dot{y}(t) + \sigma\Gamma\eta(t) - \sigma(d + \Gamma h)y(t).\end{aligned} \tag{18}$$

Since $d = -\Gamma h$ (can be checked by substitution), then

$$\dot{x}(t) = Ax(t) + kb(\varepsilon(t) + w(t)), \quad y(t) = c^T x(t), \tag{19}$$

$$\dot{\eta}(t) = h\dot{y}(t) + \sigma\Gamma\eta(t), \quad \varepsilon(t) = h^T \eta(t), \tag{20}$$

where matrix Γ is Hurwitz by force of calculated parameters k_i of system (8) and

$$\Gamma^T N + N\Gamma = -Q_2, \tag{21}$$

where $N = N^T > 0$, $Q_2 = Q_2^T > 0$.

Proposition 3.1: Consider the system (1) with the control law (7), (8). Then the output variable $y(t)$ converges to a small area ε_0 for finite time t_1.

Proof: Following the ideas of [5], [6], [9] choose the Lyapunov function

$$V = x^T Px + \eta^T N\eta. \tag{22}$$

Differentiation (22) yields

$$\begin{aligned}\dot{V} &= x^T(A^T P + PA)x + 2kx^T Pbh^T\eta + 2kx^T Pbw \\ &+ \eta^T\sigma(\Gamma^T N + N\Gamma)\eta + 2\eta^T Nhc^T Ax \\ &+ 2k\eta^T Nhc^T bw + 2k\eta^T Nhc^T bh^T\eta.\end{aligned} \tag{23}$$

Using [5], [6], [9] consider inequalities

$$\begin{aligned}2kx^T Pbh^T\eta &\le k^{-1}\, x^T Pbb^T Px + k^3\eta^T hh^T\eta, \\ 2kx^T Pbw &\le k^{-1}\, x^T Pbb^T Px + k^3\, w^2, \\ 2k\eta^T Nhc^T bh^T\eta &\le k\eta^T Nhc^T bb^T ch^T N\eta + k\eta^T hh^T\eta, \\ 2\eta^T Nhc^T Ax &\le k\eta^T Nhc^T AA^T ch^T N\eta + k^{-1}\, x^T x, \\ 2k\eta^T Nhc^T bw &\le k\eta^T Nhc^T bb^T ch^T N\eta + k\, w^2.\end{aligned} \tag{24}$$

Thus

$$\begin{aligned}\dot{V} \le &-x^T Q_1 x - \sigma\eta^T Q_2\eta + k^{-1}\, x^T Pbb^T Px + k^3\eta^T hh^T\eta \\ &+ k^{-1}\, x^T Pbb^T Px + k^3\, w^2 + k\eta^T Nhc^T bb^T ch^T N\eta \\ &+ k\eta^T hh^T\eta + k\eta^T Nhc^T AA^T ch^T N\eta \\ &+ k^{-1}\, x^T x + k\eta^T Nhc^T bb^T ch^T N\eta + k\, w^2.\end{aligned} \tag{25}$$

Let numbers $k > 0$ and $\sigma > 0$ be such that

$$\begin{aligned}&-Q_1 + k^{-1}Pbb^T P + k^{-1}Pbb^T P + k^{-1}I \le -Q' < 0, \\ &-\sigma Q_2 + (k + k^3)hh^T + kNhc^T bb^T ch^T N \\ &\quad +kNhc^T AA^T ch^T N + kNhc^T bb^T ch^T N \le -Q'' < 0,\end{aligned}$$

then for the derivative of (22) we have

$$\dot{V} \le -x^T Q' x - \eta^T Q''\eta + (k^3 + k)w^2. \tag{26}$$

Hence using [5], [6], [9], [29], [30], it is easy to get the inequality

$$\dot{V} \le -\lambda V + (k^3 + k)w^2, \tag{27}$$

where $\lambda > 0$.

From (27) follows that the amplitude of the function (22) is decreased as soon as the estimate $\hat{\omega}$ converges to the true value ω. So in the case $\hat{\omega} = \omega$ the signal $w(t)$ exponentially converges to zero that means the Lyapunov function (22) goes to zero, and objective (3) âûïîëíÿåòñÿ. ■

Then it is necessary to design the identification scheme for unknown frequency ω of the disturbance $\delta(t)$ and substitute it in the controller (7). It should be noted that in control law we can substitute only constant values but not the time-varying functions. The next subsection devoted to the iterative procedure of frequency estimation.

C. Iterative Frequency Estimation

Identification of the unknown parameter ω can be made in a few steps. Firstly, we substitute in (7) the minimum value $\hat{\omega} = \omega_{\min}$ and fixed this parameter. Since the considered closed loop system is linear and state matrix A is Hurwitz the output variable $y(t)$ has a sinusoidal behavior with the frequency ω after transient time, i.e.

$$y(t) = A_1 \sin(\omega t + \varphi_1)$$

(see, for example, [27], [28]). Thus, identification scheme of the parameter ω can be based on [2], [3], [8]–[10], [12], [27], [28]. Following the ideas [13], [27], [28] we introduce the second order filter

$$\varsigma(s) = \frac{\gamma_0^2}{(s + \gamma_0)^2}, \tag{28}$$

where $\gamma_0 > 0$.

To identify the disturbance frequency we use the following algorithm

$$\hat{\omega}(t) = \sqrt{\left|\hat{\theta}(t)\right|}, \tag{29}$$

$$\hat{\theta} = \chi + k\dot{\varsigma}\ddot{\varsigma}, \tag{30}$$

$$\dot{\chi} = -k\dot{\varsigma}^2\hat{\theta} - k\ddot{\varsigma}^2, \tag{31}$$

When the estimate of the frequency ω is found we substitute it into the control law (7) instead of $\omega_{\min}$. However, one can ask what time should be chosen for substitution the value $\hat{\omega}$ from the algorithm (29) – (31) into (7), because the estimate $\hat{\omega}$ converges to the true value in the infinite time $\lim_{t\to\infty}(\omega - \hat{\omega}) = 0$. Obviously, that moment of time $t = \infty$ is not applicable. To solve this problem we propose to use the iterative procedure of identification.

The main idea of the iterative procedure is to substitute the frequency estimate $\hat{\omega}$ in (7) in discrete time periods. At the first step we use the value $\hat{\omega}_0 = \omega_{\min}$ that corresponds the minimum value of the frequency. The system works so for a some period of time t_1. In the moment t_1 the renewed value $\hat{\omega}_1 = \hat{\omega}(t_1)$ is taken from adaptive algorithm (29)–(31) and then it is substituted into the controller (7).

Until the moment of time t_2 the controller uses the estimate $\hat{\omega}_1$. In the moment t_2 the new estimate of the frequency $\hat{\omega}_2 = \hat{\omega}(t_2)$ is taken from (29) – (31) and applied to the controller (7) etc. Thus, the iterative update law can be written as

$$\bar{\omega}(t) = \begin{cases} \omega_{\min},\ t \leq t_1, \\ \hat{\omega}(t_i),\ t \in [t_i,\, t_{i+1})\,,\ i = \overline{2, N}, \end{cases} \tag{32}$$

where $\hat{\omega}(t_i)$ denotes the frequency estimates gotten from (30) – (29) in the moment t_i while $\bar{\omega}(t)$ is the constant value substituted to the controller (7) – (8).

In the general case moments of renewing can be not regular. For example, the first moment for updating should be chosen large in comparison with the following moments of time. This interval means the initialization period that is necessary that the estimate $\hat{\omega}$ would be as closer to true value ω as possible. Moreover, it will happen faster if undesired switchings in the system are eliminated.

It should be noted that theoretical analysis of rules of choosing switching moments t_i for iterative update law in the controller (7) – (8) is the nontrivial problem. Authors are planning to consider the detailed analysis of this problem as the extension of already obtained result. In this work we present the approach the effectiveness of which confirmed only empirically.

IV. Illustrative Example

To illustrate effectiveness and analyze properties of obtained control algorithm consider following simulation results.

For example, we need to stabilize linear parametrically uncertain unstable plant with simultaneous unknown sinusoid disturbance rejection having only system output measurements. The plant is described in input-output form as

$$y(t) = \frac{p+1}{p(p-1)(p+2)(p+3)}[u(t) + \delta(t)],$$

where relative degree of transfer function is $r = 3$.

Let us choose only the time of the first frequency update t_1 independently to have enough time for more accurate initial frequency estimation, while set subsequent iteration intervals τ fixed and equal.

Figs. 1–2 display transients in the closed-loop system for different parameters of external disturbance and iterative algorithm itself.

Obtained results show that proposed iterative control algorithm provides system stabilization with simultaneous disturbance rejection. Achieved stabilization accuracy and dynamic performance indexes are comparable with other known results. At the same time, one can see that overall system performance strictly depends on the parameters of iterative updates of estimations of the unknown disturbance frequency such as t_1 and τ which are strongly correlated with other controller parameters, therefore further research efforts will target this part to develop more sophisticated scheme and theoretically prove its workability.

V. Conclusions

Novel robust output control algorithm with unknown sinusoidal disturbance rejection for linear parametrically uncertain plant was derived. This method is based on high-gain principle and utilizing iterative substitution of the disturbance frequency estimations. In contrast to known results, proposed approach provides disturbance rejection for the systems with relative degree higher than one.

Obtained algorithm enclose stabilizing controller, described by (7) and (8), and unknown disturbance' frequency estimator (29) – (31). Simplified iterative scheme with fixed and even equal switching intervals have been validated for closed-loop system stability and performance analysis. More sophisticated strategies of real-time frequency estimation $\hat{\omega}_i$ substitution to output control algorithm (7) should be derived as the next steps.

Moreover, authors believe, that obtained results can be extended to nonlinear case.

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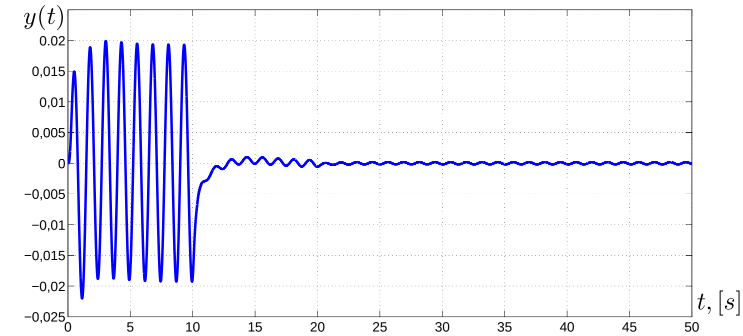
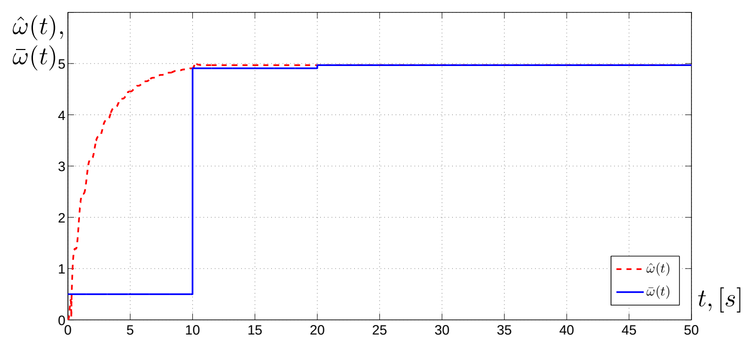
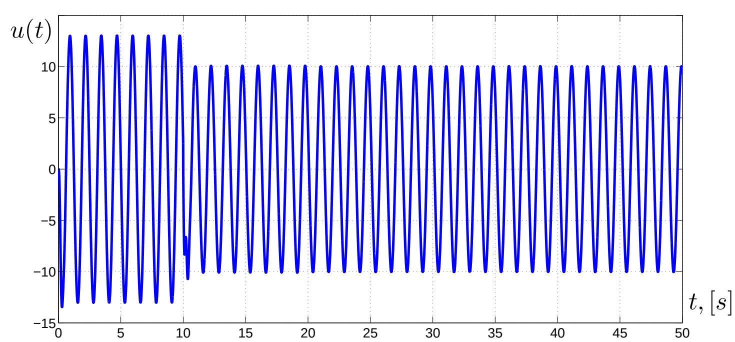


(a) System output $y(t)$.



(b) Disturbance frequency estimates $\hat{\omega}(t)$ and $\bar{\omega}(t)$.



(c) Control signal $u(t)$.

Fig. 1: Transients in the closed-loop system with $\delta(t) = 10\sin(5t+0.1)$, $t_1 = 10$, $\tau = 10$, $\omega_{\min} = 0.5$, $\alpha(p) = (p+4)(p+5)$, $k_1 = k_2 = 1$, $k = 14$, $\sigma = 30$, $k_a = 20$, $\gamma_0 = 20$.

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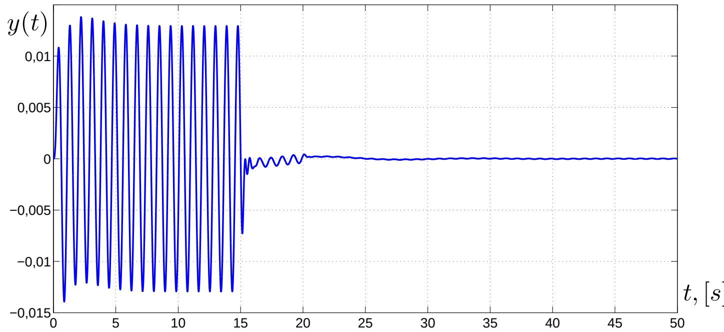
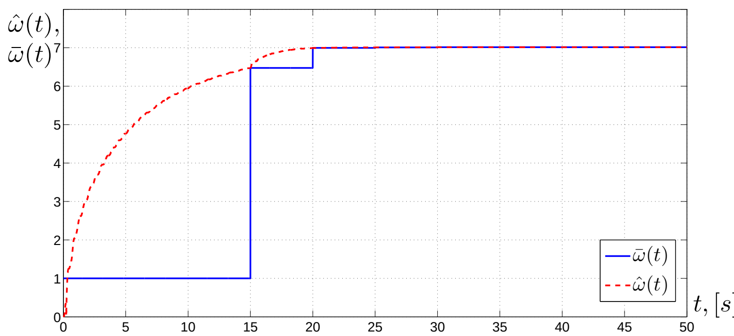
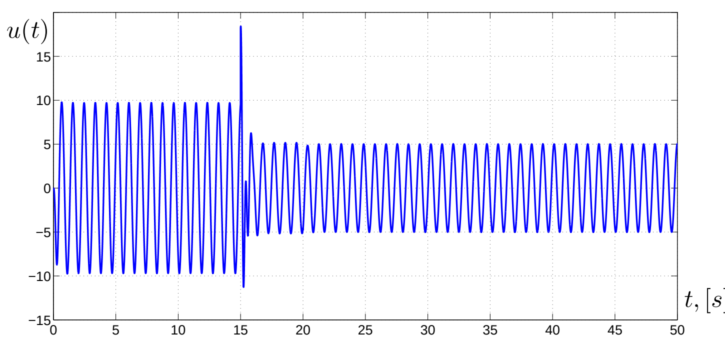


(a) System output $y(t)$.



(b) Disturbance frequency estimates $\hat{\omega}(t)$ and $\bar{\omega}(t)$.



(c) Control signal $u(t)$.

Fig. 2: Transients in the closed-loop system with $\delta(t) = 5\sin(7t + 0.3)$, $t_1 = 15$, $\tau = 5$, $\omega_{\min} = 1$, $\alpha(p) = (p + 4)(p + 5)$, $k_1 = k_2 = 1$, $k = 10$, $\sigma = 25$, $k_a = 15$, $\gamma_0 = 15$.

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