NTNU – Trondheim
Norwegian University of
Science and Technology

Hull Dimensions of a Semi-Submersible Rig

A Parametric Optimization Approach

Joakim Rise Gallala

Marine Technology
Submission date: June 2013
Supervisor: Bjørn Egil Asbjørnslett, IMT

Norwegian University of Science and Technology
Department of Marine Technology

Abstract

Semi-submersible rigs are utilized to perform drilling, production and intervention operations in the oil and gas industry. The design of the rigs is a complex task which today is highly dependent on manual iteration and experience. In this thesis, the prospect of using optimization methods to establish the main dimensions of the hull was investigated. A nonlinear optimization model which minimizes the hull weight was developed taking the most important properties of the semi-submersible rig such as stability, motion characteristics and air gap into account. The model was solved in Microsoft Excel to enhance the availability and make it easier to implement for engineers with solid experience using the software. The add-in algorithms solved the model in matter of minutes and the output was compared to other rigs operating in the North Sea. The results were verified by experienced engineers in Aker Solutions and were found feasible and interesting. Based on the results from the computational study and the sensitivity analysis performed, it is concluded that the model can be used as a decision support tool to establish the main dimensions of the hull structure.

Sammendrag

Halvt nedsenkbare plattformer benyttes til boring, produksjon og intervensjonsoperasjoner i olje og gass industrien. Utviklingen av plattformdesignet er en kompleks oppgave som i stor grad er basert på manuell iterasjon og erfaring. I denne oppgaven undersøkes mulighetene for å benytte optimeringsmetoder til å bestemme hoveddimensjonene på skrogstrukturen. En ikke lineær optimeringsmodell som minimerer skrogvekten ble utviklet ved å ta hensyn til de viktigste egenskapene til halvt nedsenkbare plattformer som stabilitet, bevegelseskarakteristikk og air gap. Microsoft Excel ble valgt som løsningsverktøy for å gjøre modellen mer tilgjengelig og lettere å implementere for ingeniører som har solid erfaring med programvaren. De innebygde algoritmene løste modellen i løpet av få minutter og resultatet ble sammenliknet med rigger som opererer i Nordsjøen. Resultatene ble undersøkt av erfarne ingeniører fra Aker Solutions som konkluderte med at de var realistiske og interessante. Basert på resultater fra testkjøring av modellen og følsomhetsanalysen, konkluderes det med at modellen kan brukes som et beslutningsstøtte verktøy for å fastsette hoveddimensjoner til skrogstrukturen.

Scope of Work

MASTER THESIS IN MARINE TECHNOLOGY

SPRING 2013

For stud.techn.

JOAKIM RISE GALLALA

Hull Dimensions of a Semi-Submersible Rig- A Parametric Optimization Approach

Background

The latest development in the oil and gas industry has showed an increased demand for semi-submersible rigs. The design of the rigs is a complex process with several objectives and constraints acting as design drivers. The main goal is to design a platform which fulfills the relevant functional requirements at a satisfying construction cost. Typical objectives may be identified as variable deckload capacity, deck area and motion characteristics. The engineers will try to manually optimize these and similar objectives, while the relevant requirements such as stability, structural strength, air gap and motion characteristics are satisfied. In this thesis the preliminary design process of the semi-submersible hull will be investigated.

Objective

Develop an optimization model which can be utilized as a decision support tool in the establishment of the main dimensions of the hull structure of a semi-submersible rig.

Tasks

a) Identify the main objectives and constraints in the design process

b) Develop an optimization model which take the main objectives and constrains into account

c) Perform a computational study employing commercial optimization software

d) Benchmarking of the results with rigs operating in the North Sea

General

In the thesis the candidate shall present his personal contribution to the resolution of a problem within the scope of the thesis work.

Theories and conclusions should be based on a relevant methodological foundation that through mathematical derivations and/or logical reasoning identify the various steps in the deduction.

The candidate should utilize the existing possibilities for obtaining relevant literature.

The thesis should be organized in a rational manner to give a clear statement of assumptions, data, results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Telegraphic language should be avoided.

The thesis shall contain the following elements: A text defining the scope, preface, list of contents, summary, main body of thesis, discussion of results and conclusions with recommendations for further work, list of symbols and acronyms, reference and (optional) appendices. All figures, tables and equations shall be numerated.

The supervisor may require that the candidate, in an early stage of the work, present a written plan for the completion of the work.

The original contribution of the candidate and material taken from other sources shall be clearly defined. Work from other sources shall be properly referenced using an acknowledged referencing system.

Deliverable

- The thesis shall be submitted in two (2) copies:
- Signed by the candidate
- The text defining the scope included
- In bound volume(s)
- Drawings and/or computer prints that cannot be bound should be organized in a separate folder.
- The bound volume shall be accompanied by a CD or DVD containing the written thesis in Word or PDF format. In case computer programs have been made as part of the thesis work, the source code shall be included. In case of experimental work, the experimental results shall be included in a suitable electronic format.

Supervision:

Main supervisor: Professor Bjørn Egil Asbjørnslett

Co-supervisor: Professor Kjetil Fagerholt

Company supervisor: Anders Martin Moe

Executive Summary

This master thesis investigates the utilization of optimization methods to establish the main dimensions of the hull structure on a semi-submersible rig. During the preliminary design phase, engineers decide the hull dimensions based on an early sizing of the topside. The process can be time consuming and is to a large extent based on manual iteration and experience, and do not guarantee a robust and cost effective design. The use of an optimization model may provide the engineers with a valuable decision support tool and reduce the amount of human resources needed to complete the hull design.

The model is developed by combining theory from marine engineering relevant for the semi-submersible platform with mathematical modeling from optimization theory. The result is a nonlinear optimization model which holds for four legged semi submersibles with rectangular cross sectional pontoons and columns. Further on, low construction cost, favorable motion characteristics, large deck area and high variable deckload are identified as the four main objectives. The model has constraints related to stability, Eigen periods and air gap. After discussions with Aker Solutions it was agreed that the model should seek to minimize the weight of the hull structure, which is closely related to the construction cost. The three remaining main objectives are handled through various constraints and input parameters. The model is implemented and solved in Microsoft Excel. Most engineers are familiar with the software, which hopefully will make the model more available and easier to implement in a company like Aker Solutions.

To test and evaluate the model a computational study is performed using input parameters developed in collaboration with Aker Solutions. The model was solved using the algorithms implemented in the Excel solver add-in. A sensitivity and robustness analysis is conducted to highlight the most valuable aspects of the model. For instance, the designers get information on how each parameter is affecting the overall solution. The design obtained from the model is an 8100 mt hull structure which can carry a topside weight of 7000 mt together with a variable deckload of 4100 mt. Finally, the proposed hull designs are compared with rigs with similar amount of deckload capacity and are found to be competitive in terms of hull weight. The model was reviewed with experienced engineers in Aker Solution and their feedback was that the results were feasible and promising.

It is concluded that the model can be implemented and used as a decision support tool to establish the main dimensions of the hull structure. The main features of the model are that it suggests designs swiftly and input parameters like variable deckload can easily be changed. Additionally, the sensitivity analysis provides the decision makers with valuable information about the interaction between different parameters and the final design. The objective function might need to be revised to give a more realistic picture between main dimensions and overall construction costs. However, the objective function can be changed and the model can be extended to be valid for rigs with six or eight legs. The utilization of the model will hopefully reduce the amount of human resources needed in the early design process and provide the decision makers with the necessary tools to make right decisions.

Preface

This report is submitted to fulfill the requirement to the degree of Master of Science in Marine Technology at the Norwegian University of Science and Technology (NTNU). The scope of work was developed in collaboration with Aker Solutions and NTNU. The work with this report was conducted during the spring 2013 and is written entirely by Joakim Rise Gallala

I would like to thank my two supervisors Professor Bjørn Egil Asbjørnslett and Professor Kjetil Fagerholt for valuable counseling and guidance during the work with this thesis. I would also like to thank my company contact Anders Martin Moe and his colleagues at Aker Solutions which has given me valuable knowledge of the design process of the semi-submersible rig.

Trondheim, 10.06.2013

Joakim Rise Gallala

Contents

Abbreviations

CAPEX	Capital Expenditures
COB	Center of Buoyancy
COG	Center of Gravity
DNV	Det Norske Veritas
GM	Metacentric Height
GRG	General Reduced Gradient
MC	Metacenter
NTNU	Norwegian University of Science and Technology
RAO	Response Amplitude Operator
VDL	Variable Deckload

1. Introduction

Background

From the beginning of offshore oil production, engineers have relied on different constructions in order to drill wells and extract oil and gas reserves. The first rigs were simple fixed constructions in shallow waters, but when the worlds demand for oil kept growing, the oil companies moved to deeper waters to discover new resources. The need of floating drilling rigs became obvious and the semi-submersible rig concept was introduced in the Gulf of Mexico in 1961. The floating rig Blue Water Rig No.1 was overloaded and did not have sufficient buoyancy to carry the topside at the designed draft configuration. In order to get the rig to shore it was towed between two locations at submerged draft. During transit the engineers from Shell and Blue Water Company noticed that the platform had favorable motion characteristics at this new submerged draft. The result was that the idea and principle of the semi-submersible as a drilling rig was born. The main features of the semi-submersible rig are favorable motion characteristics and ability to carry heavy topside along with a large deck area.

Several attempts have been made to describe the design of a marine vessel by a systematical approach. One suggestion is the design spiral introduced by Evans (Amdahl, et al., 2005). The design spiral clearly emphasizes the iterative process, where the first step is to identify the functional requirements of the vessel. The rig is typically a drilling, intervention or a production unit. The engineers will focus on the functional requirements and try to estimate a necessary deck area. After the required deck area has been estimated the designers will move on to the sizing of the topside based on the equipment needed to perform the tasks defined in the functional requirements. The next step is to design a hull structure which gives sufficient buoyancy to carry the topside weight and at the same time provides the rig with satisfying features. When a first proposal has been established, engineers will typically perform hydro dynamical analysis to ensure that the rig has favorable motion characteristics and sufficient stability. This part of the design is an iterative process, where the engineers will alter hull dimensions, topside weights and other parameters repeatedly in order to give the rig the desired properties. Once a satisfying result has been obtained the more detailed design can commence, which involves more accurate weight estimates, mooring, power and structural analysis. Early in the design process, the use of spreadsheets is widespread as an iteration tool in the sizing of the hull and the topside. The manual optimization procedure is time consuming and the process is often disturbed by alterations in functional requirements and topside layout. The design of the hull structure and topside are often shared between two different departments, which also creates communicational challenges throughout the preliminary design phase. During the design process the estimated topside weight will change, which will impact on the hull design. Hence, the hull engineers may have to alter the design several times, as the topside input changes. This can be a time consuming task which requires much experience and knowledge on how the different parameters affects the main features of the rig. The final solution will be a result of different objectives and constraints, and in the end it is hard to get knowledge of how good the final design actually is compared to other feasible designs. The result is that the company responsible for the hull design utilizes resources to create a design which may not be the best in order to satisfy the functional requirements.

After the introduction of modern computers, it is unthinkable to design a rig without the extensive use of state of the art software and technology. However, this software is often used in the detail design phase, once the main parameters already have been established. If the computer software indicates some challenges with the design, like poor stability or bad motion characteristics, the proposed design needs to be altered. The result is that the use of computational power is utilized too late and not sufficiently early in the design process where much of the iterations are done manually. The consequence is that resources are exploited to achieve a result which may be quite far from the optimal solution. The use of computational power at an early stage may save the company from expensive design changes later on in the design process.

An approach to enhance the effectiveness during the preliminary design phase and to obtain more robust and cost effective designs is to create an optimization model. The model could act as a decision support tool to establish the main dimensions of the hull structure, taking the most important functional requirements into account. The development of modern optimization software has made it possible to solve complex models with the use of effective algorithms. It is possible to model the most important constraints related to the design of the hull and maximize or minimize a certain objective function. One of the advantages with an optimization model is that the engineers easily can change input parameters like desired variable deckload and the model will suggest new hull designs which can be further investigated. The model will calculate new optimal solutions automatically and the manual iteration is not necessary. Another important feature with an optimization model is that the user gets comprehensive knowledge about the overall design problem through running sensitivity analyses. The engineers can extract valuable information on how each parameter and constraint is affecting the overall design.

State of the Art

During the development of optimization theory the applications areas have grown wider. As a result of the modern computer, optimization can be utilized in large and complex problems. Today, optimization theory is applied in various areas like manning problems, transportation problems, product mix planning, finance and production planning. In the aviation industry optimization is used as a design tool in order to develop an optimal design on foils of commercial airplanes. In marine technology optimization are becoming more common in the area of shipping and routing problems. From a design perspective, some research has been focused on establishment of the main dimensions of the semi-submersible hull structure. Birk and Claus describe how hydro dynamical relationships can be implemented and modeled in optimization problems related to design of marine structures (Birk & Clauss, Automated Hull Optimization of Offshore Structures Based on Rational Seakeeping Criteria, 2001). Applications areas, such as design of semi submersibles and fixed structures are suggested. Further on, the lack of use of optimization software early in the design process is questioned. In a later article the same authors employ relationships defined in earlier publications to optimize the hull dimensions of a semi-submersible hull structure (Birk & Clauss, Parametric Hull Design and Automated Optimization of Offshore Structures, 2002) .The main focus is to optimize the motion characteristics through changes in pontoons and columns geometrical shape for a four legged semi-submersible. The final results revealed considerable improvements regarding downtime and heave response. The article concludes that further research is needed to convince the industry of the great potential of using automated optimization models in offshore engineering. Birk and Clauss further states that optimal results only can be achieved through utilize computational power early in the design process. In another article published on the 23rd International Conference on Offshore Mechanics and Arctic Engineering the parametric optimization of offshore floaters are further investigated. The authors emphasize the design spiral developed by Evans which describes the iterative approach based on manual iteration (Birk, Clauss, & Lee, Practical Applications of Global Optimization to the Design of Offshore Structures, 2004). The authors introduce three alternative algorithms to solve the optimization problem, which seek to minimize the amount of down time of a semi-submersible rig. The results show that the use of these algorithms improves the initial design based on experience considerably with a reduction of downtime around 50%. Further on, a minimization model of the stress in the tethers of a TLP is developed. The results revealed a considerable growth in the estimated lifetime of the platform compared with the initial design.

In an article published on the International Conference on Offshore Mechanics and Arctic engineering a parametric optimization of a semi-submersible platform with heave plates are performed (Aubalt, Cermelli, & Roddier, 2007). The authors build a general optimization model with the objective of minimizing capital expenditures (CAPEX) by changing the hull dimensions. The CAPEX are divided into the four segments; hull and deck fabrication, mooring manufacturing, riser fabrication and installation. The model is formulated by using general constraints related to stability, Eigen periods and mooring systems. Finally, the model is solved using a genetic optimization algorithm. The authors emphasize that the optimization of hull dimensions often creates complex nonlinear models which are hard to solve. The challenge is to implement all necessary constraints, without creating a model which is too complicated to solve. For instance, constraints related to structural strength and risers are not included in the model. The results showed that the genetic algorithm employed slowed down its convergence rate once it closed in on a local optimum. The authors conclude that further research should focus on improving algorithms so that more complex model with a higher number of constraints can be solved.

It is important to know the limitations of the optimization programming. Complex problems with several nonlinearities can be hard to solve. However, many search algorithms have been developed in order to solve complicated models. A general reduced gradient method was developed in 1975 (Lasdon, Waren, Arvind, & Ratner, 1975). Another powerful method is the particle swarm algorithm introduced by Kennedy and Eberhart in 1995 (Kennedy & Eberhart, 1995). The method deploys an evolutionary approach to investigate the solution space. The development of these and other algorithms have together with the modern computational power made it possible to utilize optimization theory to solve complex design problems.

Objective

Even though some research has been conducted in the utilization of optimization models in naval architecture, the industry is yet to accept it as an efficient tool to create robust and cost effective designs. The models are often difficult to formulate and are solved using somewhat complicated algorithms that need to be formulated in a programming language. The lack of knowledge about optimization modeling applied in naval architecture along with the complicated solution algorithms may suggest why optimization not is used to a wider extent in design of offshore floaters. The main objective of this thesis is to develop an optimization model which can be used as a decision support tool for engineers designing hulls for semi-submersible platforms. Finally the model will be solved in Excel which is widely used in the iterative design process. The use of Excel will hopefully make the model easier available and understandable for the designers. The software is heavily relied on throughout the industry and with only a small amount of training it is possible to make use of the add-in solver.

Structure

The first part of the thesis introduces important aspects related to the marine technology of the semi-submersible rig. Further a brief introduction to optimization theory is given, afore the optimization model is formulated. The model is implemented and solved in Excel, and a computational study is performed. Finally the results are discussed and compared with rigs operating in the North Sea.

Notation and numbering

It should be noted that equations used to develop the model are numbered by roman numbers, while equations implemented in the model are numbered with Hindu-Arabic numbers. This is to create a clear segregation between the optimization model and formulas used in the development of the model. All formulas are numbered on the left hand side.

2. The Semi-Submersible Rig

After the discovery of the semi-submersible concept in 1961, it was quickly accepted by the oil and gas industry. Today, the semi-submersible rig is a widely used floating structure which performs drilling, production and intervention operations. A modern semi-submersible is illustrated in figure 1.

Figure 1 The modern semi-submersible rig

Figure 1 illustrates that the semi-submersible rig consists of several components. The area above the columns is the topside, where operation equipment, accommodations, drilling derrick and drilling deck is located. The columns are supporting the topside and provide the rig with sufficient air gap between the water surface and the deck. The columns are also used for ballasting and storage of various bulk loads, such as mud and fuel. The number of columns varies from four to eight columns, dependent on the stability requirements and variable deckload (VDL) capacity required. In the lower part of the hull structure the pontoons are connected to the columns. The pontoons main function is to provide the rig with sufficient buoyancy and act as catamaran hulls during transit. This part of the hull is also used to store mud, fuel and the majority of the water ballast. The rigs are typically designed with two pontoons, or a ring pontoon, connecting all the columns. The hull is usually equipped with some kind of bracing between the pontoons and columns in order to enhance the structural integrity of the rig. The bracing can be arranged in various configurations, dependent on the environmental loads governing in the operating area. The rig is typically designed for three different draft configurations which are the operational, survival and transit draft. In the operating condition the draft is at the maximum magnitude. This gives low pressure variation on the pontoons which ensures favorable motions that is required during the operations, because severe response may damage valuable equipment. If extreme weather approaches, the rig will halt operations and de-ballast to increase the air gap from the water surface to the rig. The increase in air gap will prevent slamming of waves into the deck structure. Slamming can damage the deck and destroy equipment and should not occur. In the transit condition the pontoons act as catamaran hulls and they are not totally submerged. The large waterplane area will give the rig the necessary stability for the transit.

In the following section the stability and the motion characteristics of the semi-submersible rig will be further discussed.

2.1. Stability

The stability of a marine vessel is strongly dependent on the outer geometry and weight distribution. When a rig is subjected to forces from wave, wind and currents the forces will create a heeling moment which will affect the heeling angle of the rig. The stability can be interpreted as the ability to withstand heeling moments and return to the upright position after the external forces subdue. Figure 2 illustrates the most important stability features of a marine vessel.

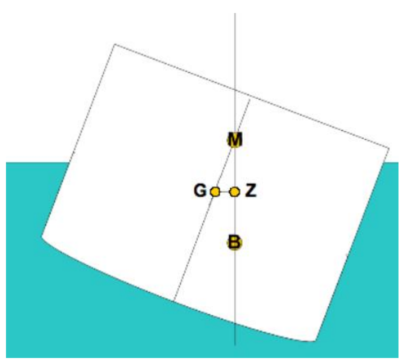

Figure 2 Important stability properties

Where:

G Center of Gravity

M Metacenter

B Center of Buoyancy

The metacenter (MC) can be interpreted as the intersection between the center line and the vertical line through the center of buoyancy (COB). For heeling angles smaller than 10 degrees it is assumed that the sum of the buoyancy forces will act through the MC (Sillerud, 2003). For larger heeling angles the MC will tend to move due to considerably changes in the geometry of the waterplane area. When a rig floats in the upright position the COB is directly below the center of gravity (COG). If the rig is subjected to heeling moments it will start to heel and the center of buoyancy will shift towards the direction of the heeling. The sum of the buoyancy forces acting along the line between the COB and the MC and the forces of gravity acts along different axes. This creates a righting moment which increases as the heeling angle grows. When the righting moment is equal or larger to the heeling moment, the vessel will stop the heeling motion and subsequently return to the upright position once the environmental loads diminish. The righting moment is calculated using equation (I) (Sillerud, 2003).

(I) $$RM = \mathrm{GZ} \times \Delta$$

Where:

RM Righting moment

GZ Length of arm of the righting moment

Δ Weight displacement

The arm length of the righting moment can be calculated through equation (II) once the location of the MC and the COG is known.

(II) $$\mathrm{GZ} = \mathrm{GM} \times \sin\theta$$

Where:

GZ Length of arm of the righting moment

GM The vertical distance from the COG to the MC

θ The angle of heeling

For all marine vessels, the requirement for floating without capsizing is to have a metacentric height (GM value) greater than zero. A negative GM value implies that the MC is located below the COG which will give a negative righting moment. This implies that the righting moment will act in the same direction as the heeling moment. The GM value is given by equation (III).

(III) $$GM = KB + BM - KG$$

Where:

GM The vertical distance from the COG to the MC

KB Vertical distance from keel to the COB

BM Vertical distance from the COB to the MC

KG Vertical distance from keel to the COG

The GM values have a serious impact on the overall performance of the rig. Too high GM values will give large righting moments, resulting in increased accelerations in pitch and roll. This will increase the response in pitch and roll, and create short rolling periods which are uncomfortable for the crew. On the other hand, too low GM values will make the vessel vulnerable to large heeling moments. To calculate the GM value it is necessary to define the variables given in equation (III). The distance between the COB and the MC is given by equation (IV) (Sillerud, 2003).

(IV) $$BM = \frac{I}{\nabla}$$

Where:

BM Vertical distance from COB to MC

I Second moment of area for the waterplane area

∇ Volume displacement

The vertical distance between the keel and the COB is given by equation (V) employing theory of moments.

(V) $$KB = \frac{\sum_{i=1}^{N} kb_i \times \nabla_i}{\sum_{i=1}^{N} \nabla_i}$$

Where:

KB The vertical distance from keel to the COB

kb_i The vertical distance from keel to the COB for submerged part *i*

∇_i The volume displacement of submerged part *i*

The vertical distance from the keel to the COG is given by equation (VI).

(VI) $$KG = \frac{\sum_{i=1}^{N} kg_i \times m_i}{\sum_{i=1}^{N} m_i}$$

Where:

KG The vertical distance from keel to the COG

kg_i The vertical distance from keel to the COG for mass *i*

m_i The weight of mass *i*

Furthermore, it is important to account for the free surface effects when stability calculations are performed. Most marine vessels use water ballast to control the draft and the trim. Rigs usually carry other liquids such as fuels, chemicals and drilling mud. When a vessel with liquid cargoes or ballast starts to heel, the liquids in the tanks will translate in the direction of the heeling. This causes a translation of the COG towards the heeling side. The result is that the arm of the righting moment will be reduced and the overall stability of the vessel is deteriorated. The free surface effects are often accounted for by raising the COG to a new imaginary COG which will give a more realistic picture of the vessels stability. The mathematical expression for the raising of COG is given by equation (VII) (Sillerud, 2003).

(VII) $$\delta G = \frac{\sum_{n=1}^{N} \rho_n^L i_n}{\rho \nabla}$$

Where:

δG Vertical elevation of the COG

ρ_n^L Density of liquid in tank n

i_n Second moment of area of the liquid area for tank n

ρ Density of seawater

∇ The volume displacement of submerged part i

It is important to notice that totally empty and full tanks not will affect the stability of the vessel. If a compartment is full or empty, no movements of liquids are allowed.

Due to the geometry, semi-submersible rigs usually have robust stability. Engineers can adjust the GM values by altering the geometry of the platform early in the design phase. Equation (IV) illustrates that the location of the MC is dependent on the second moment of area for the waterplane area. If engineers want to raise the transversal GM values they can increase the distance between the pontoons. Similarly, an increase in the distance between the columns will increase the longitudinal GM values and enhance the longitudinal stability. Larger waterplane area will also strengthen the stability of the rig. It is also possible to alter the center of gravity to some extent using water ballast.

The engineers will design the hull structure so the rig can carry VDL stated in the functional requirements. Loading of cargo onto the deck will raise the COG towards the deck, thereby reducing the GM value. This explains why stability requirements often are limiting the VDL capacity of a rig. To compensate for the raising of the COG the engineers will usually concentrate the ballast water in the pontoons, limiting the raising of the COG.

In this chapter the most important stability aspects of the semi-submersible rig has been discussed. The challenge is to design a stable platform which can carry the required amount of deckload without being too stiff. To achieve this, it is necessary to design a hull structure with a geometry which gives the rig GM values which lie inside a certain interval.

2.2. Motion Characteristics

The favorable motion characteristic is one of the main features of the semi-submersible rig. In this chapter the most important parameters affecting the motion characteristic will be discussed.

As figure 3 illustrates, floating structures has six degrees of freedom. The three translations degrees of freedom is heave, sway and surge while the rotational movements are yaw, roll and pitch.

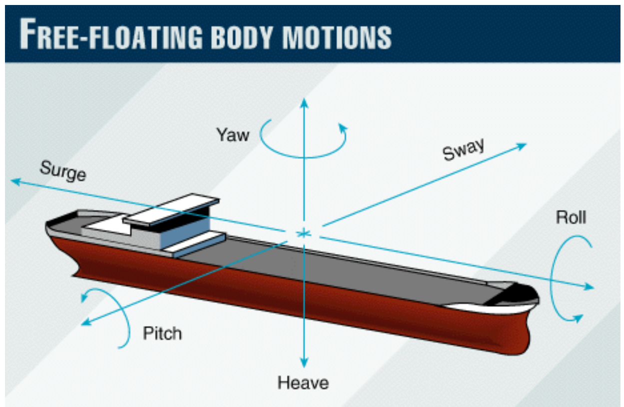

Figure 3 The six degrees of freedom for a marine vessel

For a semi-submersible rig, the motions characteristics are dominated by heave, pitch and roll. The other motions are kept low because of mooring systems and/or dynamic positioning, and will not be further discussed in this report (Aker Solutions, 2012). When performing a hydro dynamical analysis of a vessel, one of the most important output parameters is the response amplitude operator (RAO). For instance, the RAO in heave gives the response of the vessel in heave per meter wave amplitude. Similarly the RAO in roll will give information of how many degrees the vessel will rotate per meter wave amplitude. Figure 4 illustrates a typical heave RAO for a semi-submersible rig (Matos, 2011).

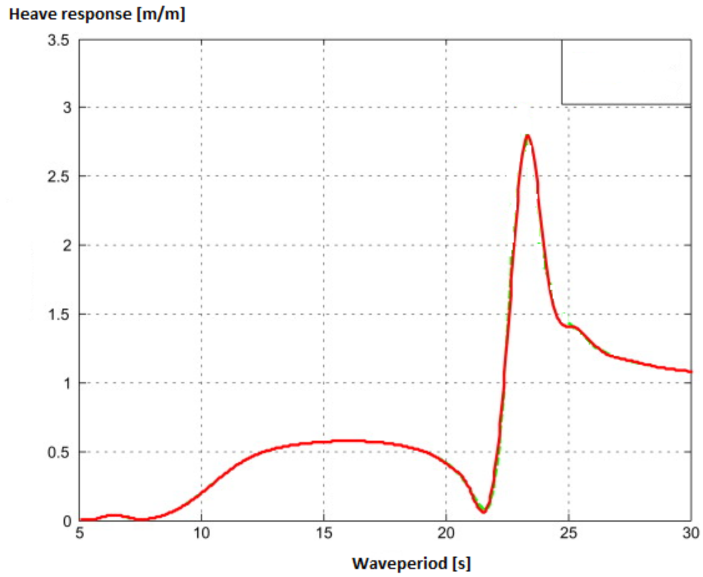

Figure 4 Heave RAO for a semi-submersible rig

Figure 4 illustrates that the semi-submersible rig will have a peak around the Eigen Period. The magnitude of the peak is dependent on viscous damping (Faltinsen, 1990). A rig will always have viscous damping due to wave making and vortex shedding around the pontoons. The damping will slow down the motions and will enhance the motion behavior of the vessel. It should also be noted that the semi-submersible have a neutral period around 21 seconds when the pressure on the top and bottom of the pontoons are equal. When the wave periods are above 25 seconds the wavelength are high, so the semi-submersible will tend to float with the waves. This explains convergence towards 1 for the RAO in heave in figure 4. The RAO for pitch and roll are comparable to the RAO for heave, with a peak around the Eigen period. The response of a rig can be found by multiplying the RAO and the wave spectrum governing the operational area. A wave spectrum describes the distribution of the wave energy as a function of wave periods. The wave spectrum applied in the North Sea is the JONSWAP spectrum, illustrated in figure 5. It should be noted that the given spectrum is for extreme conditions and usually the wave periods are below 20 seconds for more than 99% of the time (DNV, 2010). The wave spectrum illustrated in figure 5 was obtained from the DNV software POSTRESP. The designer of a marine vessel will always try to obtain Eigen Periods which lie above or below the wave periods dominating the operating area so the peak in the RAO falls outside the energy peak in the wave spectrum. For Eigen periods above 21 seconds the peak in the RAO falls outside the energy peak in the wave spectrum and will seriously reduce the probability of resonance behavior.

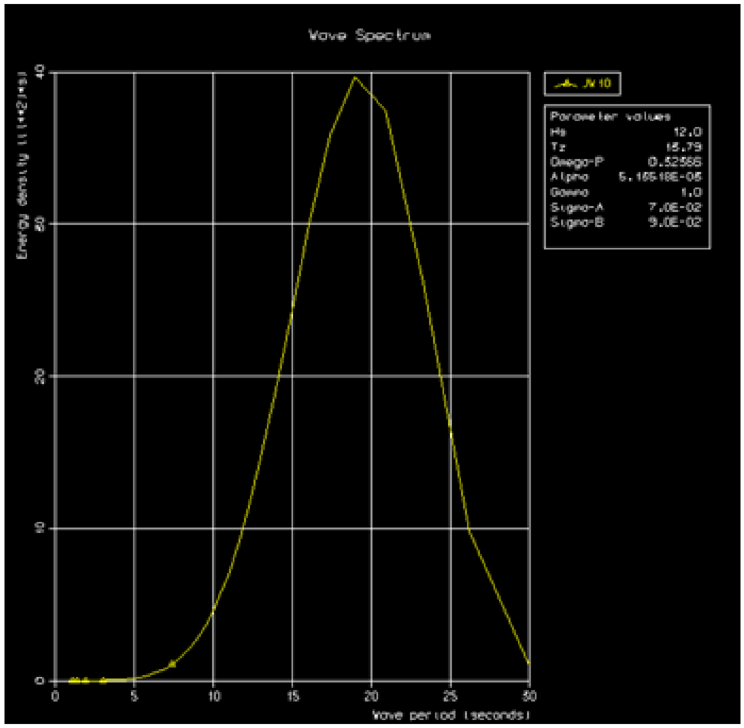

Figure 5 JONSWAP spectrum for a 100 year storm in the North Sea

During operations the rigs are connected to drillings risers, blowout preventers and other valuable equipment. Even though the rigs are equipped with heave compensators, large heave oscillations will put the equipment employed under serious stress. If the heave motions are large enough the operations will therefore have to terminate. In the North Sea the day rate of a rig is currently around 450 000 $, thus the revenues of the rig suffers a serious impact when operations are stopped. Engineers will try to minimize the down time of the rig through improving the motion characteristics, preferably through high Eigen periods.

When a marine vessel is accelerating through water, it will tend to accelerate water particles which are located near the hull. The particles that are accelerated will again cause further movements for the neighboring particles. The result is that a body moving through water will accelerate a certain amount of water particles. Thus, a semi-submersible rig will accelerate water particles once it translates or rotates. The result is that the rig will behave as it has a mass which is larger than the structural mass alone. In naval architecture this additional mass is usually referred to as the added mass of the vessel. This is an important property which must be taken into account to when the hydro dynamic properties of the vessel are analyzed. From the literature it is possible to determine the added mass of standard geometrical shapes like spheres, rectangles, circular cylinders using results obtained through experiments (Pettersen, 2004).

A well-known method applied to calculate the added mass for long slender vessels is strip theory. Similar to carving bread, the vessel is divided into small slices. This makes it easier to mimic the geometry of ships using standard shapes like rectangles and circles. Strip theory is therefore well suited for calculating the added mass in heave for the pontoons. A slice of the pontoon is illustrated in figure 6, assuming equal cross section throughout the pontoon.

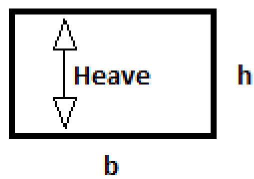

Figure 6 Strip theory applied on the pontoons

Using equation (VIII) the added mass for a rectangular slice of one meter length, oscillating in heave can be calculated (Pettersen, 2004).

(VIII) $$A_{HEAVE}^{2D} = C^{AM}\rho\pi\frac{b^2}{4}$$

Where:

A_{HEAVE}^{2D}	Two dimensional added mass for strip of one meter length
C^{AM}	Added mass coefficient in heave
ρ	Density of seawater
b	Pontoon breadth

The added mass coefficient is dependent on the breadth height ratio of the pontoon, and is usually decided from empirical studies. The total added mass can be formulated mathematically by summing all the strips for both pontoons. The total added mass in heave given by equation (IX)

(IX) $$A_{HEAVE}^{3D} = C^{AM} \rho \pi \frac{b^2 l}{2}$$

Where:

A_{HEAVE}^{3D}	Total added mass in heave for the rig
C^{AM}	Added mass coefficient in heave
ρ	Density of seawater
b	Pontoon breadth
l	Pontoon length

It should be noted that the rig also will have added moment of inertia in the rotational degrees of freedom. The added mass for a ship or a floating structure can be as high as one half of the structural mass. This explains why it is an important parameter when predicting the hydro dynamical behavior of the rig. If damping is neglected the Eigen Period in heave is given by equation (X) (Pettersen, 2004).

(X) $$T_{heave} = 2\pi \sqrt{\frac{\Delta + A_{heave}}{\rho g A_w}}$$

Where:

T_{heave}	Eigen period in heave
Δ	The weight displacement of the rig
A_{heave}	Added mass in heave for the rig
ρ	Density of seawater
g	Gravitational constant
A_w	The waterplane area of the rig

The large displacement and relatively small waterplane area provides the semi-submersible rig with high Eigen Periods in heave, which are one of the main reasons behind the favorable motion characteristics. If damping effects are neglected the Eigen periods in roll and pitch is given by equation (XI) and (XII) (Pettersen, 2004).

(XI) $$T_{roll} = 2\pi\sqrt{\frac{\Delta r_{roll}^2 + A_{roll}}{\rho g \nabla GM_t}}$$

Where:

T_{roll}	Eigen period in roll
Δ	The weight displacement of the rig
r_{roll}	The radius of gyration in roll
A_{roll}	Added moment of inertia in roll for the rig
ρ	Density of seawater
g	Gravitational constant
∇	The volume displacement of the rig
GM_t	The transversal GM value

(XII) $$T_{pitch} = 2\pi\sqrt{\frac{\Delta r_{pitch}^2 + A_{pitch}}{\rho g \nabla GM_l}}$$

Where:

T_{pitch}	Eigen Period in pitch
Δ	The weight displacement of the rig
r_{pitch}	The radius of gyration in pitch
A_{pitch}	Added moment of inertia in pitch for the rig
ρ	Density of seawater
g	Gravitational constant
∇	The volume displacement of the rig
GM_l	The longitudinal GM value

Equation (XI) and (XII) illustrates why high GM values should be avoided. Too high GM values will give the rig low Eigen periods in pitch and roll, which will lie inside the area where most of the wave energy is focused. This will result in poorer motion characteristics and the possibility for resonance behavior increases. The Eigen periods in pitch and roll are typically high for semi-submersible rigs as a result of a large moment of inertia along with low GM values.

Table 1 presents typical Eigen Periods for heave for different deep water floaters (DNV, 2010).

Table 1 Eigen Periods for deep water floaters

Eigen Periods [s]			
Floater	FPSO	Tension Leg Platform	Semi- submersible
Surge	>100	>100	>100
Sway	>100	>100	>100
Heave	5-12	<5	20-50
Roll	5-30	<5	30-60
Pitch	5-12	<5	30-60
Yaw	>100	>100	>100

Table 1 illustrates that heave is the most critical degree of freedom with Eigen periods down to 20 seconds. The Eigen periods in pitch and roll are usually controlled by keeping low GM values within defined boundaries, in the preliminary design phase (Aker Solutions, 2012). During the design process the engineers will alter the geometry of the rig to obtain satisfying Eigen periods.

The part of the heave RAO to the left of the Eigen period peak is strongly dependent on the draft of the rig. From the hydro dynamics the dynamical pressure under a wave is given by equation (XIII) (Pettersen, 2004).

(XIII) $$p_{dyn} = \rho g \, \zeta_a e^{-kz} \cos \omega t$$

Where:

p_{dyn} Dynamical pressure

ζ_a Wave amplitude

ρ Density of seawater

g Gravitational constant

z Water depth

k Wave number

ω Wave frequency

t Time

The wave number is given by equation (XIV).

(XIV) $$k = \frac{2\pi}{L}$$

Where:

k Wave number

L Wave length

As formula (XIII) illustrates the pressure decreases exponential with the water depth. This leads to small variation in dynamical pressure on the pontoons for high drafts, which give favorable motion characteristics. The operational draft for a rig operating in the North Sea will typically lie in the interval 19-25 meters, while in the more benign waters of the Gulf of Mexico, shallower drafts can be accepted.

The two aspects discussed in this chapter explain the favorable motion characteristics of the semi-submersible rig. The geometry of rig gives Eigen Periods which lie outside the energy density peak in the wave spectrum. For only a small amount of time, the rig will be subjected to waves of periods equal to the Eigen periods. The low pressure variation on the deeply submerged pontoons further enhances the motion characteristics of the rig.

3. Optimization Theory

In optimization theory a problem is described using mathematical modeling in order to obtain the best solution out of various alternatives. The theory is often used to describe and give decision support in both technical and economic systems to identify the best possible decision based on the information available.

The optimization problem usually contains an objective function which is maximized or minimized by altering the decision variables. The problem is typically limited by certain constraints which need to be satisfied.

The application areas are vast, and optimization is used in areas like production planning, logistics, telecommunication, structural design and manning problems. In the maritime industry optimization is typically applied in ship scheduling and routing problems.

There is usually necessary to make some basic assumptions and simplifications of a real life problem in order to create an optimization model which can be solved. Real life problems are often unlimited with a large degree of uncertainty involved. Once the problem is simplified sufficiently an optimization model can be created and solved by appropriate solution methods.

In the following section model formulation, linear and nonlinear optimization problems will be discussed.

3.1. Model Formulation

The optimization models are often divided into

- Indices and sets

Indices and sets are utilized in order to write the model more compact. As an example, if the problem contains five different factories, it is possible to denote the factories using the index i. i=1 indicates factory one and so on. All the factories define the set of factories I. Indices are usually denoted by lower case while sets are denoted by capital letters.

- Parameters

The parameters provide all the relevant data for the problem. Typical parameters are production cost, maximum production capacity and so on. Parameters are usually denoted by capital letters.

- Variables

The variables are the part of the problem the decision makers can affect. The decision makers want to find the optimal combination of these variables which maximize or minimize the objective function. The variables are usually denoted by lower case letters.

- The objective function

The objective function can be interpreted as the overall goal with the optimization problem, and is typically used to minimize or maximize certain values, which are dependent on the variables. Example on an objective function may be to minimize costs or to maximize profit.

- Constraints

The optimization problem is usually bounded by certain constraints. The constraints define the solution space and are typically related to limitations regarding time, capacity and resources.

A general optimization problem is illustrated in equation (XV) and (XVI).

(XV) $$\min\ z$$

$Subject\ to$

(XVI) $$g_i(x) \geq b_i$$

Where z is the objective function and $g_i(x)$ and b_i describes the constraints.

3.2. Linear Optimization Problems

In a linear problem, the objective function and the constraints only contain linear terms. An example on a linear model is given below.

(XVII) $$\max z = 2x_1 + 3x_2$$

$Subject\ to$

(XVIII) $$x_1 + x_2 \leq 6$$

(XIX) $$x_1, x_2 \geq 0$$

When an optimization model is formulated mathematically the solution space is defined by feasible and non-feasible regions separated by the constraints. A graphical presentation of the linear model is given in figure 7.

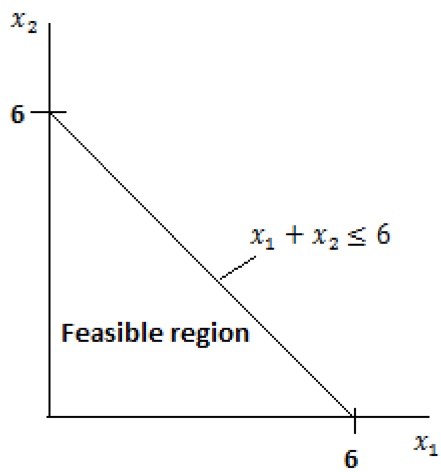

Figure 7 Graphical representation of a linear optimization problem

Since George B. Dantzig introduced the simplex method in 1947, it has been the standard procedure for solving linear problems. The algorithm moves around the feasible corner point solutions, improving the objective value, until the optimum is found. The simplex method is widely recognized because it is very robust. It solves any linear problem; it detects redundant constraints in the problem formulation; it identifies unbounded problems and can solve problems with more than one optimal solution (MIT Web Education). Another important aspect is that the algorithm is self-initiating. The simplex method can be used to find a feasible start solution and from that point it will find the optimum after a number of iterations. The output of the algorithm will not only give the optimal solution, but will also give valuable information related to the sensitivity of the problem.

3.3. Nonlinear Optimization

A nonlinear optimization problem contains mathematical terms that are nonlinear. Compared to figure 7 the solution space will be divided by nonlinear functions. Nonlinear problems are significantly harder to solve because the solution space may contain several local optimums which often are hard to separate from the global optimum. If only parts of the problem are nonlinear it is often possible to convert the problem to a linear one, by using linear approximation and simplifying the model through assumptions. In order to discuss solution methods it is necessary to separate the convex and non-convex problems.

A mathematical function is convex if the line segment between any two points on the graph of the function lies above the graph in a vector space of at least two dimensions. A function is concave if the negative of the function is convex. The relationship between convex and concave functions is illustrated in figure 8.

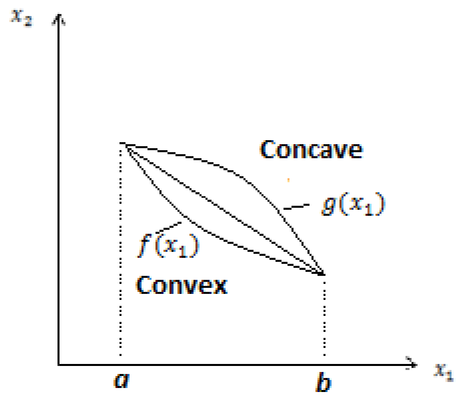

Figure 8 Convex and concave functions

As figure 8 illustrates $g(x_1)$ is concave on the interval between a and b because all points on the function lies above the line segment in the interval. Similarly $f(x_1)$ is convex on the interval because all points on the function lies below the line segment between a and b. Further on it is necessary to define convex and non-convex set which describes the solution space. A set in a vector space is called a convex set if the line segment joining any pair of points in the region S lies entirely in the region S (Wolfram, 2013). If a feasible region is an intersection between several convex sets, the feasible region is also a convex set. Convex and non-convex sets are illustrated in figure 9.

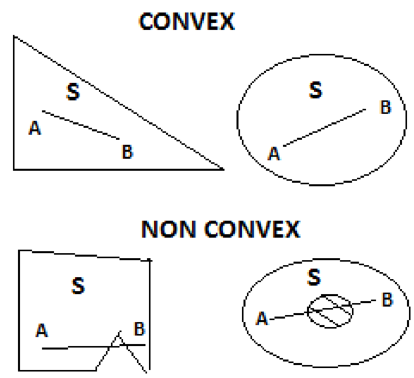

Figure 9 Convex and non-convex sets

The definition of convex functions and sets are important in order to decide whether a problem is convex or non-convex. A minimization problem is defined as a convex problem if the objective function is convex and the feasible region defined by the constraints $g_i(x) \geq b_i$ is a convex set. The problem is also convex if it is defined as a maximization problem, the objective function is concave and the feasible region is a convex set (Lundgren, 2010). A feasible point is a global optimum if no other feasible points got better objective value. For a convex problem, each local minimum or maximum is the global optimum (Lundgren, 2010). So a convex problem is significantly easier to solve than a non-convex problem, because once a local optimum is found it is indeed the global optimum. For a non-convex problem it is often hard to decide whether the local optimum obtained is the true global optimum. The convexity of the problem often decides which solution method that is suitable and what solution quality to expect when applying this method (Lundgren, 2010).

For nonlinear problems it is not possible to find a single algorithm that applies to all problems like the simplex algorithm does for linear programming. The solution method to apply is dependent on the structure of the problem. The goal is to find the optimal solution or a solution which lays close to the optimum by some convergence criteria. Some algorithms such as the Frank-Wolfe are used primarily to solve convex nonlinear problems with linear constraints (Lundgren, 2010). This method performs a Taylor series expansion of the objective function to make it linear around the initial point. Other examples of algorithms are evolutionary algorithms like the particle swarm algorithm. Evolutionary algorithms mimic biological processes with the birth and death of candidate solutions. The particle swarm algorithm can be described by visualizing a large football field as the solution space and a piece of food as the optimal solution. A swarm of birds are released and the birds got different mass, direction and velocity. The birds settle out on the football field and communicate about the solution found. Some birds can see the piece of food because they are in a promising area of the solution space. During the next iteration the new birds are reborn in the most promising area while birds far from the solution die. The birds are released once again and the swarm will investigate the most promising areas. In the end, all the birds have gathered around the optimal solution and the algorithm will stop. Another example of algorithms is the generalized reduced gradient method which is an extension of the Frank Wolfe algorithm that is employed in Microsoft Excel solver add-in.

There are several algorithms to solve nonlinear problems, but the problem is that many of these algorithms easily get trapped in a local optimums. The solution space may span many dimensions and be extremely complicated. As discussed, non-convex problems may have several local optimum points. It can be complicated and even impossible to tell which one is the global optimum. To decide whether a problem is convex can be fairly complicated as well. Especially if the problem involves several variables and the functions in the problem are of high orders. Linear problems are always convex and this explains why they are so much easier to solve than nonlinear problems. In the solution of a complex nonlinear problem the main challenge is to find an appropriate solution tool and to interpret the results correctly.

4. Problem Description

To formulate the model mathematically it is important to have a compact problem description which gives a written formulation of the mathematical model.

The overall objective is to minimize the hull structure weight of a semi-submersible rig by establishing the main dimensions of the hull. The design objectives will be further discussed in chapter 5.4

The rig should have GM values within an interval that ensures that the vessel have sufficient stability in all conditions without being too stiff. To ensure favorable motion characteristics the rig ought to have Eigen periods in heave which lies above a lower boundary for the survival and operational conditions. The motion characteristics of the vessel are to be further controlled by designing the rig for draft configurations which are comparable with other rigs operating in the North Sea. The steel hull should be able to carry the topside with a VDL of a magnitude so that drilling or intervention can be performed and the rig can compete with other platforms in the same segment. The rig should also have a sufficient deck area to arrange all necessary equipment. To avoid wave slamming issues on the deck structure, the rig must have sufficient air gap for all the draft configurations. The geometry of the rig should be similar to the typical semi-submersible design with columns which is supported by the pontoons, and topside which is supported by the columns. The breadth-height ratio of the pontoons should lie inside a specific interval to ensure sufficient structural stiffness. All hull dimensions should be defined with a lower and upper boundary based on state of the art rigs.

5. The Mathematical Model

When establishing a decision support tool it is important to be able to communicate the problem. In this chapter, the model is developed together with explanations of all constraints. A more compact presentation of the model is given in appendix A.

5.1. Assumptions

As discussed in chapter 3 it is necessary to make some general assumptions to formulate the problem mathematically.

5.1.1. Columns and Pontoons

All semi-submersible rigs are required to carry some VDL to perform the tasks specified in the functional requirements. The necessary capacity is dependent on the tasks and the operating depth of the rig. Rigs operating in deep waters usually require more drill pipe and equipment to perform operations. A large VDL requires high GM values, which often are obtained by increasing the number of columns. To choose the optimal number of columns, results from a screening study performed by Aker Solutions is utilized. The heave RAO`s from the study are presented in figure 10.

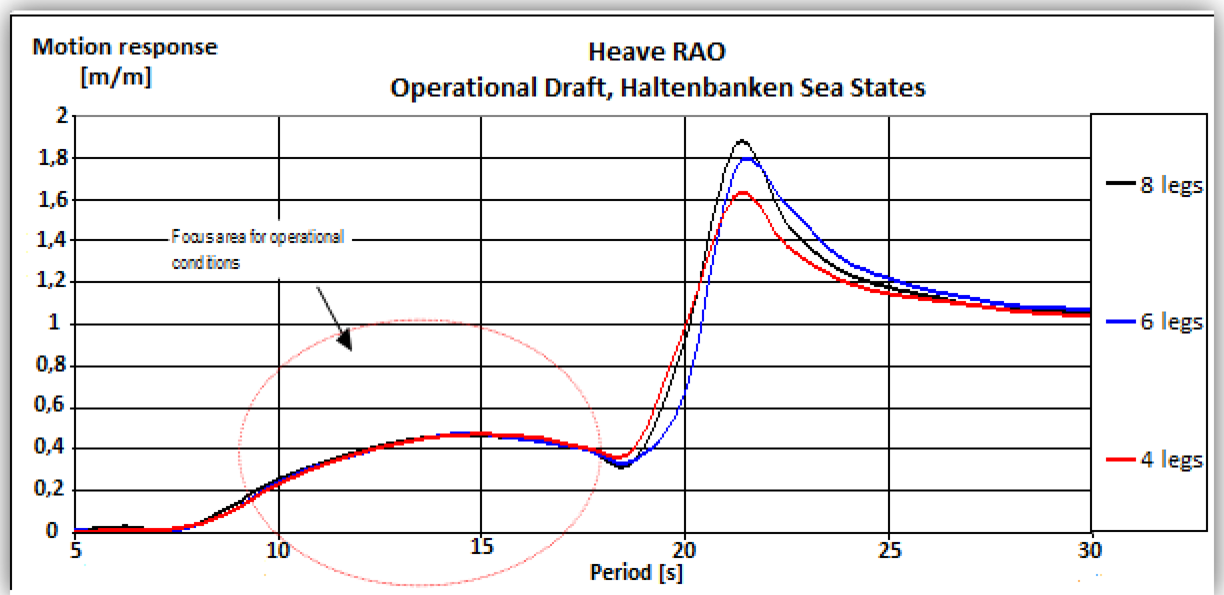

Figure 10 Number of columns impact on the heave RAO

Figure 10 illustrates that the number of legs have similar heave RAO`s in the operating area. All configurations peak around the same Eigen period, but the four legged configuration has a smaller response than the others. This study is primarily focused on rigs operating in the shallow waters on the Norwegian continental shelf where the depth ranges down to 500 meters. Hence, it is assumed that four columns will provide the rig with sufficient stability to carry the necessary VDL to operate in these shallow waters. Experience from previous rig studies has showed that a rectangular cross section in the pontoons and columns will increase the vortex shedding and hence increase the viscous damping of the vessel (Faltinsen, 1990). Thus, the rig will be designed with cross sectional pontoons and columns.

5.1.2. Bracing

Bracing come in various configurations and are designed in order to enhance the structural strength of the rig. As a simplification for the model, Aker Solutions agreed that the weight of the braces is given as a fraction of the total hull weight. The mass and volume of the braces are assumed to be distributed evenly from the start of the bracing to the top of the deck.

5.2. Notation

As mentioned in chapter 3 the model is usually formulated using different indexes and sets. In the mathematical model one possible approach is to separate the different draft configurations. In the model, these states will be denoted by the subscript t, where t is defined by the set T which consists of the survival, operational and transit condition. Figure 11 illustrates the coordinate system to be used in the mathematical formulation.

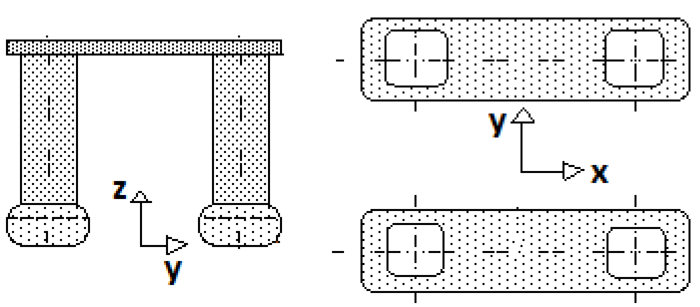

Figure 11 Coordinate system applied in the mathematical formulation

The input parameters are denoted by capital letters, with the exception of the water density and the gravitational acceleration. All variables are denoted by lower case letters. Subscripts refer to an index, while superscripts are used to explain parameters and variables. For instance, the superscript P is used if the variable or parameter is related to the pontoons. C is used for columns while B is used for the braces. All superscripts are in capital letters, while subscripts are denoted by lower case letters.

To develop a compact model which is comprehensible it is necessary to introduce certain auxiliary variables. The auxiliary variables are typically introduced to calculate certain sizes, such as GM values or Eigen periods. These variables will be explained during the development of the model.

The model will be numbered starting at (1) for the objective function and (2) for the first constraint and so on. This is to clearly differentiate the functions contained in the model from the equations that are used to formulate the model which are numbered with roman numbers.

5.3. Decision Variables

The main dimensions of the hull structure are defined as decision variables and are illustrated in figure 12.

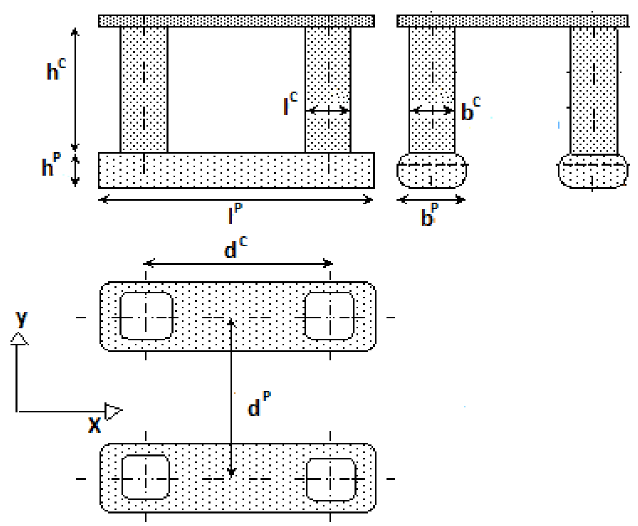

Figure 12 The eight decision variables

Where:

l^P	Pontoon length
h^P	Pontoon height
b^P	Pontoon breadth
l^C	Column length
h^C	Column height
b^C	Column breadth
d^P	Distance between pontoons
d^C	Distance between columns

5.4. Objectives

Four design objectives were identified during the development of the objective function. Of course, it is important that the rig have low construction costs. Good motion characteristics are also important to reduce the down time. Further, the rig is required to have good capacity in terms of deck area and VDL to perform operations. Initially, a multi objective function was considered, but after discussions with Aker Solutions it was agreed that the objective function should minimize hull weight which gives an indication of the overall construction cost. The three other objectives will be controlled through input parameters and constraints. Alternative objective functions are further discussed in chapter 7.1.2.

The objective function is formulated in equation (1).

(1) $$\min z = w^P + w^C + w^B$$

Where:

z	Hull weight
w^P	Weight of pontoons
w^C	Weight of columns
w^B	Weight of braces

The objective function was implemented by defining three auxiliary variables w^P ,w^C and w^B which are to be calculated by constraints defined in chapter 5.5.3.

It is important to notice that the objective function can be replaced by functions that target other objectives such as VDL, deck area or CAPEX.

5.5. Constraints

From the theory discussed in chapter 2 it is possible to define the most important constraints to ensure that all requirements are satisfied. The constraints can be divided into six groups.

- Stability constraints
- Motion characteristics related constraints
- Weight constraints
- Air gap constraints
- Geometrical constraints
- Deck area constraints

The stability constraints are formulated to ensure that the rig has sufficient stability without being too stiff. Constraints related to motion characteristics is defined to ensure satisfying motion behavior. The weight constraints will create equilibrium between weight and buoyancy of the rig. The weight constraints are also necessary to calculate the weight of the hull. Further on, the air gap constraints will provide the rig with sufficient air gap in all conditions. It is also necessary to define some geometrical constraints so that the desired geometry is obtained when the model is used. Finally, deck area constraints will ensure that the rig have the necessary deck area.

5.5.1. Stability Constraints

As discussed in chapter 2.1 all marine vessels need to have positive GM values in order to have sufficient stability. However, the engineers want to avoid high GM values which will create short uncomfortable rolling periods along with increased response and accelerations. Constraints that ensure that the stability lies in the desired area is therefore formulated. Constraint (2) and (3) are implemented to ensure that the GM values lies above a lower boundary for all conditions.

(2) $$g_t^{TRANSVERSAL} \geq S_t^{TRANSVERSAL,MIN} \qquad t\epsilon T$$

(3) $$g_t^{LONGITUDINAL} \geq S_t^{LONGITUDINAL,MIN} \qquad t\epsilon T$$

Where:

$g_t^{TRANSVERSAL}$	Transversal GM value in condition t
$S_t^{TRANSVERSAL,MIN}$	Minimum required transversal GM value for condition t
$g_t^{LONGITUDINAL}$	Longitudinal GM value in condition t
$S_t^{LONGITUDINAL,MIN}$	Minimum required longitudinal GM value for condition t

The constraints were formulated introducing two new auxiliary variables which will be calculated through constraints defined later on in this chapter.

It is also necessary to establish a maximum boundary for the GM values. The constraint is formulated in equation (4) and (5). The GM values in transit condition will be very high due to the large waterplane area. The transit for semi-submersible are usually performed in quiet weather and the GM value in this condition is of little concern to the designers. Thus the transit condition is not included in constraint (4) and (5).

(4) $$g_t^{TRANSVERSAK} \leq S_t^{TRANSVERSAL,MAX} \qquad t = Su, Op$$

(5) $$g_t^{LONGITUDINAL} \leq S_t^{LONGITUDINAL,MAX} \qquad t = Su, Op$$

Where:

$g_t^{TRANSVERSAL}$	Transversal GM value in condition t
$g_t^{LONGITUDINAL}$	Longitudinal GM value in condition t
$S_t^{TRANSVERSAL,MAX}$	Maximum allowed transversal GM value in condition t
$S_t^{LONGITUDINAL,MAX}$	Maximum allowed longitudinal GM value in condition t

Further, it is necessary to establish some relations who enable the model to calculate the GM values which are expressed using equation (III) given in chapter 2.1 and introducing four auxiliary variables related to the COG, COB and MC. After discussions with Aker Solutions, it was agreed to treat the free surface effects through an input parameter which reduces the GM values. The constraints which enable the model to calculate the GM values are given by equation (6) and (7).

(6) $$g_t^{TRANSVERSAL} = d_t^{KEEL-COB} + d_t^{COB-META,TRANSVERSAL} - d_t^{KEEL-COG} - O_t \qquad t\epsilon T$$

(7) $$g_t^{LONGITUDINAL} = d_t^{KEEL-COB} + d_t^{COB-META,LONGITUDINAL} - d_t^{KEEL-COG} - O_t \qquad t\epsilon T$$

Where:

$g_t^{TRANSVERSAL}$	Transversal GM value in condition t
$g_t^{LONGITUDINAL}$	Longitudinal GM value in condition t
$d_t^{KEEL-COB}$	Vertical distance from the keel to COB in condition t
$d_t^{COB-META,TRANSVERSAL}$	Vertical distance from COB to transversal MC in condition t
$d_t^{COB-META\ ,LONGITUDINAL}$	Vertical distance from COB to longitudinal MC in condition t
$d_t^{KEEL-COG}$	Vertical distance from keel to COG in condition t
O_t	Reduction in GM values due to free surface effects in condition t

As discussed in chapter 2.1 the distance from the keel to the COB for a submerged structure is given by equation (V). Using the model notation, the equality constraint which ensures that the distance is calculated correctly is formulated in equation (8). Auxiliary variables related to the displacement and COB of the pontoons, columns and bracing are introduced. For all parts of the submerged structure, the volume is multiplied with the vertical distance from the respective volume center to the keel, and divided by the total submerged volume.

(8) $$d_t^{KEEL-COB} = \frac{d_t^{P,COB}\nabla_t^P + d_t^{C,COB}\nabla_t^C + d_t^{B,COB}\nabla_t^B}{\nabla_t^P + \nabla_t^C + \nabla_t^B} \qquad t\epsilon T$$

$d_t^{KEEL-COB}$	Vertical distance from keel to COB of pontoons in condition t
$d_t^{P,COB}$	Vertical distance from keel to COB of pontoons in condition t
∇_t^P	The volume displacement of the pontoons in condition t
$d_t^{C,COB}$	Vertical distance from keel to COB for columns in condition t
∇_t^C	The volume displacement of the columns in condition *t*
$d_t^{B,COB}$	Vertical distance from keel to COB of braces in condition t
∇_t^B	The volume displacement of the braces is condition t

The COG for a floating structure can be determined employing the theory discussed in chapter 2.1 and equation (VI). Each weight is multiplied with the distance to the keel to create a vertical moment. The vertical moments for the entire rig is summed up and divided by the total mass of the rig. The vertical distance from the keel to the COG is calculated by equation (9) which contains auxiliary variables for the mass and the vertical moments of the different components.

(9) $$d_t^{KEEL-COG} = \frac{m^P + m^C + m^B + m^{TS} + m_t^{VDL} + m_t^{BALLAST}}{w^P + w^C + W^{TS} + w^B + w_t^{BALLAST} + W_t^{VDL}} \qquad t\epsilon T$$

Where:

$d_t^{KEEL-COG}$	Vertical distance from keel to COG in condition t
m^P	Vertical moment of the pontoons
m^C	Vertical moment of the columns
m^B	Vertical moment of the braces
m^{TS}	Vertical moment of the topside
m_t^{VDL}	Vertical moment of the VDL in condition t
$m_t^{BALLAST}$	Vertical moment of the ballast in condition t
w^P	Weight of the pontoons
w^C	Weight of the columns
w^B	Weight of the braces
W^{TS}	Topside Weight
$w_t^{BALLAST}$	Weight of the ballast water in condition t
W_t^{VDL}	VDL capacity in condition t

It should be noticed that the weight of the VDL and the topside weight is given as input parameters which can be adjusted. The other auxiliary variables will be calculated through constraints defined in chapter 5.5.3.

As discussed in chapter 2.1 the vertical distance from the COB to the MC is calculated by equation (IV). The transversal and longitudinal values are calculated by introducing three new auxiliary variables in constraint (10) and (11).

(10) $$d_t^{COB-MC,TRANSVERSAL} = \frac{i_t^{XX}}{\nabla_t} \qquad t\epsilon T$$

(11) $$d_t^{COB-MC,LONGITUDINAL} = \frac{i_t^{YY}}{\nabla_t} \qquad t\epsilon T$$

Where:

$d_t^{COB-MC,TRANSVERSAL}$	Vertical distance from COB to transversal MC in condition t
$d_t^{COB-MC,LONGITUDINAL}$	Vertical distance from COB to the longitudinal MC in condition t
i_t^{XX}	Second moment of area around the x axis for the waterplane area in condition t
i_t^{YY}	Second moment of area around the y axis for the waterplane area in condition t
∇_t	Volume displacement of the rig in condition t

Figure 13 illustrates a rectangular cross section with height h and breadth b.

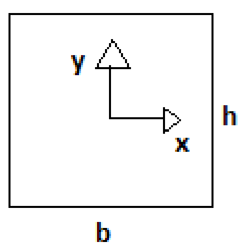

Figure 13 Second moment of area for a rectangle

The second moment area can be calculated using equation (XX) and (XXI) from mechanics.

(XX) $$i^{XX} = \frac{1}{12}bh^3$$

(XXI) $$i^{YY} = \frac{1}{12}hb^3$$

Where:

i^{XX}	Second moment of area around the x-axis
i^{YY}	Second moment of area around the y-axis
h	Height
b	Breadth

It is also necessary to employ Steiner's theorem to calculate the second moment of area. The theorem states that the second moment of area for a figure around any axis is equal to the sum of the second moment of area around the parallel axis and the product of the area of the figure and the distance between the two parallel axes squared. This is formulated in equation (XXII)

(XXII) $$i^{ZZ} = i^{XX} + Ar^2$$

Where:

i^{ZZ}	Second moment of area of *R* around the parallel axis
i^{XX}	Second moment of area of *R* around the centroid of *R*
A	Area of the region *R*
r^2	The distance from the new axis *z* to the centroid of the plane region *R*

The water plane area for the operational and survival condition is given by the rectangular cross section of the four columns. For the transit draft the waterplane area of the rig is defined by the waterplane area of the pontoons. The second moments of area for the waterplane area of the rig can now be formulated using equation (XX), (XXI) and (XXII). The result is four constraints which ensure that the model calculates the correct second moment of area in all conditions.

(12) $$i_t^{XX} = \frac{1}{3} \times l^C \times (b^C)^3 + b^C \times l^C \times (d^P)^2 \qquad t = Su, Op$$

(13) $$i_t^{XX} = i_t^{XX} = \frac{1}{6} l^P (b^P)^3 + \frac{1}{2} l^P b^P (d^P)^2 \qquad t = Tr$$

(14) $$i_t^{YY} = \frac{1}{3} \times b^C \times (l^C)^3 + b^C \times l^C \times (d^C)^2 \qquad t = Su, Op$$

(15) $$i_t^{YY} = \frac{1}{6} b^P (l^P)^3 \qquad t = Tr$$

Where:

i_t^{XX}	Second moment of area of the waterplane area around the x axis in condition t
i_t^{YY}	Second moment of area of the waterplane area around the y axis in condition t
l^C	Column length
b^C	Column breadth
l^P	Pontoon length
b^P	Pontoon breadth
d^C	Distance between columns
d^P	Distance between pontoons

All the constraints formulated in this chapter ensure that the GM values of the rig lies inside an interval defined by input parameters. Constraint (1) and (2) ensures that the GM values lie above a lower boundary, while (3) and (4) will keep the GM value below an upper boundary. All other constraints are equality constraints formulated so that the GM values can be calculated properly. The constraints formulated will ensure that the rig have sufficient stability within reasonable limits.

5.5.2. Constraints related to Motion Characteristics

In order to perform drilling and intervention services in the harsh conditions of the North Sea it is necessary to formulate some constraints related to the motion characteristics of the vessel. As discussed in chapter 2.2 the most critical degree of translation is the heave oscillations, which magnitude is dependent on the Eigen period in heave. Constraint (16) is implemented to ensure that the Eigen period in heave lies above a lower boundary.

(16) $$p_t^{HEAVE} \geq T_t^{HEAVE,MIN} \qquad t \epsilon T$$

Where:

p_t^{HEAVE}	Eigen period in heave in condition t
$T_t^{HEAVE,MIN}$	Lower boundary for Eigen period in heave in condition t

Further on it is necessary to formulate some constraints which enable the model to calculate the Eigen periods in heave. From formula (X) given in chapter 2.2 the Eigen Period in heave without damping is given by equation (17), introducing four new auxiliary variables.

$$p_t^{HEAVE} = \sqrt{\frac{\Delta_t + w^{AM,HEAVE}}{\rho g a_t^{WATERLINE}}} \qquad t \epsilon T \tag{17}$$

Where:

p_t^{HEAVE}	Eigen period in heave in condition t
Δ_t	Weight displacement in condition t
$w^{AM,HEAVE}$	Total added mass in heave
ρ	Density of seawater
g	Gravitational acceleration
$a_t^{WATERLINE}$	Waterplane area in condition t

Constraint (17) enables the model to calculate the Eigen period in heave for all conditions once the auxiliary variables for weight displacement, added mass and waterline area are defined. Using formula (IX) from chapter 2.2 the auxiliary variable for the added mass in heave is given by equation (18).

$$w^{AM,HEAVE} = C^{AM,HEAVE} \rho \pi \frac{l^P (b^p)^2}{2} \tag{18}$$

Where:

$w^{AM,HEAVE}$	Total added mass in heave
$C^{AM,HEAVE}$	Added mass coefficient in heave for the pontoons
ρ	Density of seawater
l^P	Pontoon length
b^p	Pontoon breadth

The added mass coefficient is dependent on the height-breadth ratio of the pontoons and will be given as an input parameter. Further on it is necessary to create a constraint which defines the waterplane area which is used in the calculation of the Eigen periods in heave. The waterplane area of the rig is given as the cross sectional area of the columns in the survival and operational condition, and by the waterplane area of the pontoons during transit. The waterplane area is given by equation (19) and (20).

(19) $$a_t^{WATERLINE} = 4l^C b^C \qquad t = Su, Op$$

(20) $$a_t^{WATERLINE} = 2l^P b^P \qquad t = Tr$$

Where:

$a_t^{WATERLINE}$	Waterplane area in condition t
l^C	Column length
b^C	Column breadth
l^P	Pontoon length
b^P	Pontoon breadth

The constraints formulated in this section enable the model to control that the Eigen period in heave is above a lower boundary. Two other important properties are the Eigen periods in pitch and roll. One approach would have been to formulate the constraints for the pitch and roll Eigen periods similar to constraint (16). However, discussions with Aker Solutions showed that these periods usually are satisfying, given a low GM value. The GM values are already bounded by constraint (2) - (5). Together with Aker Solutions it was agreed that no further constraints to control the Eigen Periods in pitch and roll was required.

Another important property which affects the motion characteristics is the draft configurations. As discussed in chapter 2.2 the dynamical pressure decreases exponentially with the water depth. It is the pressure variation on the large volume pontoons which creates the majority of motion for the rig. Pontoons which are deeply submerged below the water surface will be exposed to smaller variation in dynamical pressure which implies more favorable motion characteristics. Discussion with Aker Solutions led to the final approach; the drafts for the operation and survival condition are given by the input parameter F_t. This enables the engineers to control the drafts and try out different configurations. The output from an analysis will also show how a change in the draft configurations will affect the solution. The draft in the transit condition is often decided by rules and regulations and the amount of freeboard from the pontoon top down to the water surface should always be larger than a minimum boundary. The draft of the transit condition can then be expressed by constraint (21).

(21) $$f = h^P - K$$

Where:

f	Draft in transit condition
h^P	Pontoon height
K	Freeboard from top of pontoon to the water surface in transit condition

In this chapter the constraints related to motions characteristics of the rig have been discussed. Constraints which ensure that the Eigen periods in heave are above a lower boundary were implemented. Further on, several auxiliary variables and equality constraints were established in order to enable the model to calculate the Eigen periods in heave. It was further decided to keep the operational and survival drafts as input parameters to avoid too many variables and allow the engineers to try out different draft configurations. The impact of changing the target drafts can be further investigated in the sensitivity analysis of the model.

5.5.3. Weight and Buoyancy Constraints

The design of a semi-submersible hull structure is driven by the weight of the topside because the hull must provide the rig with sufficient buoyancy and stability. It is necessary to establish constraints which ensure equilibrium between the buoyancy of the rig and the weight. The weight can be divided into the following components:

- Topside weight
- Hull weight
- Weight of variable deckload
- Weight of ballast

The hull weight can further be divided into weight of pontoons, columns and braces. In collaboration with Aker Solutions it was agreed that no dry ballast should be used because wet ballast can be relocated and gives the designers more flexibility. Thus, only water ballast is considered in the model.

Pontoons

The pontoons will provide the craft with buoyancy and ballasting capacity. The displacement of the pontoons is dependent on the draft. For the operational and the survival condition the pontoons are totally submerged, while the pontoons only are partially submerged in the transit condition. The displacement of the pontoons in condition t is thus formulated in equation (22) and (23).

(22) $$\nabla_t^P = 2l^P b^P h^P \qquad t = Su, Op$$

(23) $$\nabla_t^P = 2l^P b^P f \qquad t = Tr$$

Where:

∇_t^P	The volume displacement of the pontoons in condition t
l^P	Pontoon length
b^P	Pontoon breadth
h^P	Pontoon height
f	Draft in transit condition

Early in the design process the engineers can estimate the weight of the pontoons by assuming a linear relationship between volume and weight (Aker Solutions, 2012). This linear relationship is based on experience, and the density factor of the pontoons is given as an input parameter. The weight of the pontoons is formulated as an equality constraint in equation (24).

(24) $$w^P = 2l^P b^P h^P V^P$$

Where:

w^P	Weight of pontoons
V^P	Weight density of the pontoons
l^P	Pontoon length
b^P	Pontoon breadth
h^P	Pontoon height

Further on it is necessary to define some of the auxiliary variables utilized in the stability constraints in chapter 5.5.1. From the geometrical shape of the pontoons the COB of the pontoons is expressed as two equality constraints in equation (25) and (26).

(25) $$d_t^{P,COB} = \frac{1}{2} h^P \qquad t = Su, Op$$

(26) $$d_t^{P,COB} = \frac{1}{2} f \qquad t = Tr$$

Where:

$d_t^{P,COB}$	Vertical distance from keel to COB of pontoons in condition t
h^P	Pontoon height
f	Draft in the transit condition

The pontoons are equipped with some pumping equipment and are split into compartments to hold water ballast. The equipment located in the pontoons is distributed fairly symmetrical. Thus it is assumed that the COG is located in the center of the pontoon. The relationship is implemented through equality constraint (27).

(27) $$d^{P,COG} = \frac{1}{2} h^P$$

Where:

$d^{P,COG}$	Vertical distance from keel to COG of pontoons
h^P	Pontoon height

The vertical moment is formulated through constraint (28).

(28) $$m^P = w^P d^{P,COG}$$

Where:

m^P	Vertical moment of the pontoons
w^P	Weight of the pontoons
$d^{P,COG}$	Vertical distance from keel to COG of the pontoons

Columns

The four columns are attached to the top of the pontoons and will therefore not be submerged during transit. During survival and operational condition all columns will be partially submerged. The displacement of the columns is formulated through constraint (29) and (30).

(29) $$\nabla_t^C = 4l^C b^C (F_t - h^P) \qquad t = Su, Op$$

(30) $$\nabla_t^C = 0 \qquad t = Tr$$

Where:

∇_t^C	The volume displacement of the columns in condition *t*
l^C	Column length
b^C	Column breadth
F_t	Draft in survival and operational condition
h^P	Pontoon height

The weight is estimated by assuming a linear relationship between the volume and the weight of the columns using a density factor based on experience. The weight of the columns is calculated through constraint (31).

(31) $$w^C = 4l^C b^C h^C V^C$$

Where:

w^C	Weight of the columns
V^C	Weight density of the columns
l^C	Column length
b^C	Column breadth
h^C	Column height

Further on, the distance from the keel to the COB of the columns is given by constraint (32) and (33). The COB for the transit condition is set equal to zero because none of the columns are submerged.

(32) $$d_t^{C,COB} = h^P + \frac{F_t - h^P}{2} \qquad t = Su, Op$$

(33) $$d_t^{C,COB} = 0 \qquad t = Tr$$

Where:

$d_t^{C,COB}$	Vertical distance from keel to COB of the columns in condition *t*
F_t	Draft in survival and operational condition
h^P	Pontoon height

The columns are constructed symmetrically and after discussions with Aker Solutions it was assumed that the COG is located at the volume center of the columns. Thus the vertical distance between the keel and the COG of the columns is formulated through constraint (34).

(34) $$d^{C,COG} = \frac{1}{2} h^C + h^P$$

Where:

$d^{C,COG}$	Vertical distance from columns to COG of columns
h^C	Column height
h^P	Pontoon height

The vertical moment is given by constraint (35).

(35) $$m^{C} = w^{C} d^{C,COG}$$

Where:

m^{C}	Vertical moment of the columns
w^{C}	Weight of columns
$d^{C,COG}$	Vertical distance from keel to COG of columns

Braces

The bracing comes in various configurations depending on the structural loads the floater must withstand. To limit the complexity of the model, it was assumed that weight of the braces is decided by a fraction of the total weight, which is given as an input parameter. The weight of the braces is then given as a function of the pontoon and column weights and is given by constraint (36).

(36) $$w^{B} = \frac{J^{B}}{1 - J^{B}}(w^{P} + w^{C})$$

Where:

w^{B}	Weight of braces
w^{P}	Weight of pontoons
w^{C}	Weight of the columns
J^{B}	Bracing weight fraction of total hull weight

The volume displacement for the braces can be expressed by assuming a volume and weight distribution which is evenly distributed from the start of the bracing to the top of the deck. It is necessary to express the displacement as a function of the draft. By introducing a density factor for the braces the total volume can be found by dividing the weight of the braces by the density. It is now possible to divide the total volume by the height of the braces to obtain an expression which gives the displacement of the braces as a function of the draft. Bracing usually start right above the top of pontoons and ranges up to the deck structure, hence it is assumed that no bracing are submerged during transit. The volume displacement of the bracing is expressed through constraints (37) and (38).

(37)	$\nabla_t^B = \frac{w^B F_t}{V^B(h^c - Q^B)}$	$t = Su, Op$
(38)	$\nabla_t^B = 0$	$t = Tr$

Where:

∇_t^B	The volume displacement of the braces is condition t
w^B	Weight of the braces
F_t	Draft in survival and operational condition
V^B	Weight density of braces
h^c	Column height
Q^B	Distance from top of pontoons to the start of the bracing

Based on the assumption of evenly distributed volume, the COB for the braces can be expressed through constraint (39) and (40).

(39)	$d_t^{B,COB} = h^P + (F_t - \frac{F_t - h^P}{2})$	$t = Su, Op$
(40)	$d_t^{B,COB} = 0$	$t = Tr$

Where:

$d_t^{B,COB}$	Vertical distance from keel to COB of the braces in condition t
h^P	Pontoon height
F_t	Draft in survival and operational condition

Using the same assumptions, the center of gravity of the braces is expressed by constraint (41).

(41) $$d^{B,COG} = h^P + Q^B + \frac{1}{2}(h^c - Q^B)$$

Where:

$d^{B,COG}$	Vertical distance from keel to COG of the braces
h^P	Pontoon height
h^c	Column height
Q^B	Distance from top of pontoons to the start of the bracing

The vertical moment of the braces is expressed through constraint (42).

(42) $$m^B = w^B d^{B,COG}$$

Where:

m^B	Vertical moment of the braces
w^B	Weight of the braces
$d^{B,COG}$	Vertical distance from keel to COG of the braces in condition t

Buoyancy and Weight Equilibrium

Archimedes principle states that a body submerged in fluid will experience an upward buoyant force equal to the weight of the displaced volume of the fluid. It is necessary to formulate some constraints which ensure this equilibrium between weight of the rig and amount of displaced water. The total volume displacement of the floater is given by the sum of pontoons, columns and bracing displacement. This is formulated through constraint (43).

(43) $$\nabla_t = \nabla_t^P + \nabla_t^C + \nabla_t^B \qquad t \epsilon T$$

Where:

∇_t	Volume displacement in condition t
∇_t^P	The volume displacement of the pontoons in condition t
∇_t^C	The volume displacement of the columns in condition t
∇_t^B	The volume displacement of the braces is condition t

The weight displacement is given by multiplying the volume displacement with the density of seawater and is formulated in constraint (44).

(44) $$\Delta_t = \rho \nabla_t \qquad t \epsilon T$$

Where:

Δ_t	Weight displacement in condition t
ρ	Density of seawater
∇_t	Volume displacement in condition t

The total weight of the rig can be expressed as the sum of the weight of the pontoons, columns, braces, ballast, topside and the variable deckload. To ensure equilibrium between the buoyancy and weight the weight displacement are set equal to the total weight of the rig. Because the displacement is a function of the draft, the equilibrium equation can be reformulated to an equality constraint demanding that the amount of ballast will create equilibrium between buoyancy and weight at the targeted draft. The relationship is expressed in constraint (45).

(45) $$w_t^{BALLAST} = \Delta_t - w^P - w^C - w^B - W^{TS} - W_t^{VDL} \qquad t \epsilon T$$

Where:

$w_t^{BALLAST}$	Weight of the ballast water in condition t
Δ_t	Weight displacement in condition t
w^P	Weight of the pontoons
w^C	Weight of the columns
w^B	Weight of the braces
W^{TS}	Topside weight
W_t^{VDL}	VDL capacity in condition t

This is an important constraint which decides the necessary amount of ballast to reach a targeted draft. It is also necessary to formulate a constraint which ensures that the ballast must have a positive mass. This is implemented through constrain (46).

(46) $$w_t^{BALLAST} \geq 0 \qquad t \epsilon T$$

Where:

$w_t^{BALLAST}$ Weight of the ballast water in condition t

Further on it is necessary to ensure that the amounts of ballast water in the pontoons are controlled by a upper boundary which is dependent on the ballast capacity of the pontoons. This relationship is implemented through constraint (47).

(47) $$w_t^{BALLAST} \leq 2l^P b^P h^P Z \qquad t\epsilon T$$

Where:

$w_t^{BALLAST}$	Weight of the ballast water in condition t
l^P	Pontoon length
b^P	Pontoon breadth
h^P	Pontoon height
Z	Factor describing the ballast capacity of the pontoons

If the maximum ratio of water ballast in the pontoon is equal to one implies that the total volume of the pontoons can be filled with ballast water. The value of the factor describing the filling capacity of the pontoons should not exceed one.

In order to maintain a low center of gravity to maximize the amount of VDL the rig is usually ballasted by using the pontoons. When the rig is ballasted the engineers will fill the pontoons, tank by tank, in order to avoid large free surface effects. After discussions with Aker Solutions it was agreed that the COG of the ballast is assumed to be located in the center of the pontoons. This is formulated through constraint (48).

(48) $$d^{BALLAST,COG} = \frac{1}{2}h^P$$

Where:

$d^{BALLAST,COG}$	Vertical distance from keel to COG of the ballast water
h^P	Pontoon height

The vertical moment of the ballast used in the stability calculations is formulated through constraint (49).

(49) $$m_t^{BALLAST} = d^{BALLAST,COG} w_t^{BALLAST} \qquad t\epsilon T$$

Where:

$m_t^{BALLAST}$	Vertical moment of the ballast in condition t
$d^{BALLAST,COG}$	Vertical distance from keel to COG of the ballast water
$w_t^{BALLAST}$	Weight of the ballast water in condition t

It is also necessary to formulate constraints which enable the model to calculate the distance from the keel to the COG for the VDL and the topside. In the early design phase engineers estimates the COG relative to the deck structure. The distance from keel to COG is then formulated through constraint (50) and (51).

(50) $$d^{TS,COG} = G^{TS} + h^P + h^C$$

(51) $$d^{VDL,COG} = G^{VDL} + h^P + h^C$$

Where:

$d^{TS,COG}$	Distance from keel to COG of topside
$d^{VDL,COG}$	Distance from keel to COG of VDL
G^{TS}	Vertical distance from deck to COG of the topside
G^{VDL}	Vertical distance from deck to COG of the VDL
h^P	Pontoon height
h^C	Column height

The vertical moment of the topside and VDL used in stability calculations are formulated in constraint (52) and (53).

(52) $$m^{TS} = d^{TS,COG} W^{TS}$$

(53) $$m_t^{VDL} = d^{VDL,COG} W_t^{VDL} \qquad t\epsilon T$$

Where:

m^{TS}	Vertical moment of the topside
m_t^{VDL}	Vertical moment of the VDL in condition t
$d^{TS,COG}$	Vertical distance from keel to COG of the topside
$d^{VDL,COG}$	Vertical distance from keel to COG of the VDL
W^{TS}	Topside weight
W_t^{VDL}	VDL capacity in condition t

The constraints defined in this chapter enable the model to calculate the weight, displacement, COB and COG for the pontoons, columns and braces. A constraint which ensures that the amount of ballast water at the targeted draft creates equilibrium between the rigs buoyancy and weight were also formulated. Auxiliary variables used in stability calculations where defined through constraints related to weight, COG and COB for the different parts of the rig.

5.5.4. Air gap Constraints

As discussed in chapter 2 the air gap is an important parameter for all semi submersibles due to the risk of wave slamming. Slamming of waves into the deck structure results in very high structural loads and may damage valuable equipment on deck. It is necessary to implement a constraint which ensures that the floater has sufficient air gap in the survival and operational condition. The air gap requirements are implemented through constraint (54), where the left hand side gives the air gap in the two conditions.

(54) $$h^P + h^C - F_t \geq A_t \qquad t = Su, Op$$

Where:

h^P	Pontoon height
h^C	Column height
F_t	Draft in survival and operational condition
A_t	Minimum air gap for the survival and operation condition

It is not necessary to formulate a constraint for the transit condition, because the air gap will always be sufficient in this condition. Later in the design process it is necessary to perform hydro dynamical analyses and model testing to ensure that the air gap is sufficient.

5.5.5. Geometrical Constraints

The semi-submersible rig should have the classical semi-submersible rig design where the columns are supported by the pontoons and the deck structure is supported by the columns. It is necessary to implement some geometrical constraints to ensure that the output design from the model is feasible. For instance, it is not possible to have columns which have larger breadth than the pontoon it is supported by. The following constraints are defined to ensure that the model will give feasible solutions that actually are possible to construct.

Pontoons

For the pontoons it is required that they stay inside a certain interval based on an upper and lower boundary. The interval should be based on rigs operation in the operation areas which are relevant for the rig. If an interval is defined the model will be easier to solve because many unrealistic solutions is removed from the solution space. Constraint (55)-(60) ensures that the pontoon dimensions lie inside a given interval.

(55) $l^P \leq L^{P,MAX}$

(56) $l^P \geq L^{P,MIN}$

(57) $h^P \leq H^{P,MAX}$

(58) $h^P \geq H^{P,MIN}$

(59) $b^P \leq B^{P,MAX}$

(60) $b^P \geq B^{P,MIN}$

Where:

l^P Pontoon length

h^P Pontoon height

b^P Pontoon breadth

$L^{P,MAX}$ Upper boundary for pontoon length

$L^{P,MIN}$ Lower boundary for pontoon length

$H^{P,MAX}$ Upper boundary for pontoon height

$H^{P,MIN}$ Lower boundary for pontoon height

$B^{P,MAX}$ Upper boundary for pontoon breadth

$B^{P,MIN}$ Lower boundary for pontoon breadth

It is also necessary to formulate a constraint which ensures that the pontoon is broader than the columns so that the column can be supported from below. This is formulated through constraint (61) which ensures that the breadth of the columns is smaller than the breadth of the pontoon multiplied by an input factor which not should exceed one.

(61) $$b^C \leq Ub^P$$

Where:

b^C Column breadth

b^P Pontoon breadth

U Factor restricting max column breadth as a function of pontoon breadth

It is also necessary to implement a constraint which ensures some structural robustness of the pontoons. If the breadth-height ratio gets too high, the structural stiffness of the pontoons may be insufficient. The breadth height ratio is controlled by constraint (62) and (63).

(62) $$\frac{b^P}{h^P} \leq R^{MAX}$$

(63) $$\frac{b^P}{h^P} \geq R^{MIN}$$

Where:

b^P	Pontoon breadth
h^P	Pontoon height
R^{MAX}	Maximum allowed breadth-height ratio for pontoon
R^{MIN}	Minimum required breadth-height ratio for pontoon

Columns

To reduce the solution space for the columns a feasible region ought to be defined. The interval should be based on other rigs operating rigs, but it should be wide enough to allow the model to investigate new designs. The constraints restricting the feasible region for columns dimensions are given in equation (64)-(69).

(64) $$l^C \leq L^{C,MAX}$$

(65) $$l^C \geq L^{C,MIN}$$

(66) $$h^C \leq H^{C,MAX}$$

(67) $$h^C \geq H^{C,MIN}$$

(68) $$b^C \leq B^{C,MAX}$$

(69) $$b^C \geq B^{C,MIN}$$

Where:

l^C	Column length
h^C	Column height
b^C	Column breadth
$L^{C,MAX}$	Upper boundary for column length
$L^{C,MIN}$	Lower boundary for column length
$H^{C,MAX}$	Upper boundary for column height

$H^{C,MIN}$ Lower boundary for column height

$B^{C,MAX}$ Upper boundary for column breadth

$B^{C,MIN}$ Lower boundary for column breadth

Distance between Pontoons and Columns

The allowable interval for the distance between the pontoons and columns should also be restricted. To large or small intervals may cause challenges related to structural strength and constructability. The interval allowed is formulated to through constraints (70)-(73).

(70) $d^P \leq D^{P,MAX}$

(71) $d^P \geq D^{P,MIN}$

(72) $d^C \leq D^{C,MAX}$

(73) $d^C \geq D^{C,MIN}$

Where:

d^P Distance between pontoons

d^C Distance between columns

$D^{P,MAX}$ Upper boundary for distance between the pontoons

$D^{P,MIN}$ Lower boundary for distance between the pontoons

$D^{C,MAX}$ Upper boundary for distance between the columns

$D^{C,MIN}$ Lower boundary for distance between the columns

It is also necessary to introduce constraint (74) which ensures that the distance between the columns don’t exceed the length of the pontoons.

(74) $d^C \leq Y(l^P - l^C)$

Where:

d^C	Distance between columns
l^P	Pontoon length
l^C	Column length
Y	Factor that restricts the distance between columns as a function of pontoon length

The factor $Y$is an input parameter which controls the allowable distance between the columns. If Y is equal to one, the four columns can be located in each corner of the pontoons. If Y is reduced the allowed interval shrinks and the distance between the columns will be reduced. The factor should never exceed one.

5.5.6. Deck Area Constraints

The sizing of a semi-submersible rig is strongly dependent on the required deck area. In the development of the objective function, the deck area identified as one of the four main objectives. In this model, the deck area is controlled through constraints. Constraint (75) is created to estimate the deck area based on the distance between columns and pontoons. The estimation formula was developed in cooperation with Aker Solutions and should give a fair estimate on the deck area.

(75) $$a^{DECK} = (d^C + l^C)(d^P + b^C)$$

Where:

a^{DECK}	Estimated deck area
d^C	Distance between columns
l^C	Column length
d^P	Distance between pontoons
b^C	Column breadth

Further on constraint (76) ensures that the deck area is larger than a lower boundary.

(76) $$a^{DECK} \geq A^{DECK,MIN}$$

Where:

a^{DECK} Estimated deck area

$A^{DECK,MIN}$ Minimum required deck area

Constraint (75) and (76) ensures that the deck area is sufficient based on the input parameters, and the engineers can alter the required deck area and obtain various design suggestions.

5.6. The Convexity of the Model

The mathematical optimization model defined in chapter 5 is a complex problem with numerous constraints. As the objective function and many of the constraints shows, the problem is nonlinear. As discussed in chapter 3.3 it is sometimes possible to linearize nonlinear problems using various methods. For instance, in an article on optimization of stowage plans for a RoRo ship a stability constraint is formulated as an upper boundary of torque moment for the ship (Øvstebø, Hvattum, & Fagerholt, 2011). This is an example of how constraints can be reformulated and simplified to avoid nonlinearities. Several challenges arise if the model in this thesis is linearized. For instance, the stability constraints are hard to reformulate. In the article regarding the RoRo stowage optimization, the ships stability is known together with the highest allowable center of gravity. In the model regarding the semi-submersible rig, the geometry is yet to be decided and the stability must be calculated. Further on, it is hard to find a reasonable linear estimation of the Eigen period in heave. Many variables are multiplied in the calculation of waterplane area and added mass. Together with the expressions for the weight and buoyancy of the vessel, some of the constraints related to geometry are impossible to formulate linearly. Hence it is assumed that the model cannot be converted to a linear model. As discussed in chapter 3.3, a nonlinear model will be more complex to solve because algorithms may be trapped in local optimums.

To prove mathematically whether the model is convex or non-convex is extremely complicated. The complexity of the problems grows with the number of variables involved. In this problem, eight variables are involved along with 75 constraints. This creates an eight-dimensional solution space which is bounded by several nonlinear constraints along a nonlinear objective function. To get an indication whether the problem is convex, it is possible to utilize multi start algorithms and diagnosing tools. A multi start algorithm will generate several start solutions. If the algorithms converge towards a single solution for all start solutions, it indicates that the problem is convex. If different start solutions yields different end solutions, the problem are probably non-convex. The convexity of the model will be further discussed in chapter 6.

5.7. Summary of the Model

The model was developed with 1 objective function and 75 constraints. The number of decision variables was 8 while the auxiliary variables counted 37. This illustrates that the model is comprehensive. Therefore, a more compact summary of the model is included in Appendix A. Pictures of the model in Excel are given in appendix B

6. Computational Study

To test and evaluate the model a computational study was performed. As discussed in chapter 3.3, different computer software can be used to solve optimization problems. As Excel is used extensively in the design process, it was chosen as the tool to solve the problem. Further, Excel is relatively in-expensive, readily available and used by most companies. This will hopefully make the model easier to use and implement in a company. One of the add-ins is the solver which can solve optimization models. The solver utilizes the simplex method to solve linear problems while nonlinear problems can be solved with two different algorithms. The first alternative is the general reduced gradient (GRG) algorithm developed by Leon S Lasdon of the University of Texas at Austin and Allan Warren of Cleveland State University (Microsoft, 2011). The algorithm is a typical reduced gradient method which is based on unconstrained methods (Biegler, 2011). First the problem is initialized and the objective function is divided into three partitions consisting of the basic, non-basic and superbasic parts. The basic part consists of basic variables, while the non-basic part contains non-basic variables which are fixed at a bound. The super basic variables are the ones which not are fixed at their bound and can be changed. The idea is to calculate the reduced gradient which is done by differentiate the objective function with respect to the super basic variables to find the most promising search direction. Because the non-basic variables are locked to their bounds, algorithms for non-constrained optimization like the Quasi-Newton method can be applied to find the gradient projecting search direction (Biegler, 2011). When the search direction is obtained a line search is performed and the optimal step size is determined and the algorithm moves to the next point. New iterations are performed until the algorithm is stopped by a convergence criterion. Another possible solution strategy in Excel is to utilize the evolutionary algorithm. As the name suggests this is a typical genetic algorithm which employs different populations and evolutionary principles to find the optimal solution.

The input parameters developed in collaboration with Aker solutions is illustrated in table 2.

Table 2 Input parameters

		Condition		
Input Parameter		**Transit**	**Survival**	**Operational**
$S_t^{TRANSVERSAL,MIN}$	[m]	1.5	1.5	1.5
$S_t^{LONGITUDINAL,MIN}$	[m]	1.5	1.5	1.5
$S_t^{TRANSVERSAL,MAX}$	[m]	Not defined	4	4
$S_t^{LONGITUDINAL,MAX}$	[m]	Not defined	4	4
O_t	[m]	0.5	0.5	0.5
W_t^{VDL}	[mt]	1500	4000	4000
W^{TS}	[mt]	7000	7000	7000
ρ	[mt/m^3]	1.025	1.025	1.025
g	[m/s^2]	9.81	9.81	9.81
$T_t^{HEAVE,MIN}$	[s]	0	19	20
$C^{AM,HEAVE}$	[-]	1.1	1.1	1.1
V^P	[mt/m^3]	0.270	0.270	0.270
V^C	[mt/m^3]	0.270	0.270	0.270
F_t	[m]	Not defined	17	22
J^B	[-]	0.1	0.1	0.1
Q^B	[m]	1	1	1
V^B	[mt/m^3]	0.270	0.270	0.270
G^{TS}	[m]	10	10	10
G^{VDL}	[m]	6	6	6
Z	[-]	0.5	0.5	0.5
A_t	[m]	19	19	14
K	[m]	0.3	Not defined	Not defined
$A^{DECK,MIN}$	[m^2]	4000	4000	4000
$L^{P,MAX}$	[m]	115	115	115
$L^{P,MIN}$	[m]	87	87	87
$H^{P,MAX}$	[m]	13	13	13

$\boldsymbol{H^{P,MIN}}$	[m]	8	8	8
$\boldsymbol{B^{P,MAX}}$	[m]	6	6	6
$\boldsymbol{B^{P,MIN}}$	[m]	16	16	16
$\boldsymbol{U}$	[-]	0.9	0.9	0.9
$\boldsymbol{R^{MAX}}$	[-]	2.5	2.5	2.5
$\boldsymbol{R^{MIN}}$	[-]	1.5	1.5	1.5
$\boldsymbol{L^{C,MAX}}$	[m]	20	20	20
$\boldsymbol{L^{C,MIN}}$	[m]	7	7	7
$\boldsymbol{H^{C,MAX}}$	[m]	30	30	30
$\boldsymbol{H^{C,MIN}}$	[m]	10	10	10
$\boldsymbol{B^{C,MAX}}$	[m]	20	20	20
$\boldsymbol{B^{C,MIN}}$	[m]	7	7	7
$\boldsymbol{D^{P,MAX}}$	[m]	85	85	85
$\boldsymbol{D^{P,MIN}}$	[m]	40	40	40
$\boldsymbol{D^{C,MAX}}$	[m]	74	74	74
$\boldsymbol{D^{C,MIN}}$	[m]	40	40	40
Y	[-]	0.95	0.95	0.95

It should be noted that only the parameters which are indexed are able to have different values in the transit, survival and operational condition. All other parameters are equal for all conditions. The minimum GM values were set to 1.5 meter in both transversal and longitudinal directions, which will ensure sufficient stability for the rig. In order to maintain high periods in roll and pitch the maximum GM values were decided to be 4 meters for the survival and the operational conditions. The effect of free water surface was set to 0.5 meter based on previous rig studies performed by Aker Solutions. To compete with other operating rigs on the Norwegian shelf the required VDL capacity was set to 1500 mt in the transit condition and 4000 mt in the survival and operational condition. The estimated topside weight was estimated to 7000 mt, which holds for all conditions. The COG of the topside and VDL usually is, based on discussions with Aker Solutions, located 8-12 and 4-8 meters above the deck respectively. Hence the COG of the topside and the variable deckload were set to 10 and 6 meters above the top of columns. The uncertainty in these estimates will be further discussed in chapter 6.4.3. The lower boundary for Eigen periods in heave where set 19 and 20 seconds for the survival and operational conditions respectively. This will ensure that the peak in the RAO`s fall outside the critical areas in the wave spectrums dominating in the North Sea. No requirement where formulated for the transit condition. This is because the large water plane area will give very low Eigen periods which will fall outside the critical peak in the wave spectrum. However, the constraint was not removed from the problem in case the user wish to formulate a lower boundary. The added mass was set to 1.1 based on empirical data for added mass for rectangular cross sections (Pettersen, 2004). The added mass factor is dependent on the breadth/ height ratio for the pontoon which is controlled by restriction (62) and (63). The weight densities of the pontoons, columns and braces are set to 270 kg/m^3. This number is based on previous rig studies performed by Aker Solutions, and is a conservative estimate. Earlier rig studies have showed density factors around 250 kg/m^3. Discussions with Aker Solutions showed that the bracing typically counts for 5-15% of the total weight. In this model, the bracing is assumed to count for 10% of the total hull weight. It is further assumed that the bracing starts one meter above the pontoons and reaches up to the top of the columns. Based on other rigs operating in the North Sea, the drafts were set to 17 and 22 meters for the survival and operational condition respectively. The high draft will enhance the motion characteristics in both conditions. The factor controlling the ballasting capacity of the pontoons where set to 0.5. The required air gap A_t where set to 19 and 14 meters for the survival and operational condition. The vertical distance from the top of the pontoon to the water surface in the transit condition was set to 0.3 meters, a safety margin which is controlled by rules and regulations. The minimum deck area $A^{DECK,MIN}$ was set to 4000 m^2 based on deck areas for various platforms operating in the North Sea. The breadth-height ratio interval was set to 1.5-2.5 to ensure that the pontoons have the necessary structural stiffness. The geometrical parameters where developed in collaboration with Aker Solutions. The values are based on similar rigs and the allowed intervals are wide to give the model a certain degree of freedom. Change in these boundaries will be further discussed in chapter 6.4.7.

All input parameters were discussed with Aker Solutions and should correspond well to the values used in state of the art rig designs. The parameters can be changed in the Excel model and the impact on the optimal solution can then be further investigated through a sensitivity analysis.

6.1. The Results

The solution was obtained by employing the two algorithms applied to nonlinear problems in the Solver add-in. The diagnosing tool in excel suggested that the model was nonlinear and non-convex as feared. However after an amount of computation time using 1000 start solutions the solver gave the message that the solution found was probably the global optimum. The solver will give this message if a Bayesian test suggests that all local optimums have been discovered. Once this criteria is fulfilled and the solver cannot improve the objective value, the solver will stop and suggest that a global optimum have been discovered. However, there is no guarantee that this indeed is the global optimum. Even though the model was run with a single start solution, the optimal solution converged rapidly towards the same solution which gives an indication of a convex problem. Regardless of whether or not the solution represents the global optimum it will at least provide the engineers with a starting point which satisfy all requirements in a limited amount of time. It is up to the engineers to interpret the results from the model which is to be used for decision support. The results and the computational time using the two algorithms are illustrated in table 3.

Table 3 Results and computational time using the add-in solver in Excel

	GRG	**Genetic algorithm**
Start solutions [-]	2000	2000
Computational time [s]	103	349
Objective [mt]	8093	8093

The optimum derived from running the two different algorithms where analogous. The results are illustrated in table 4.

Table 4 Optimal solution obtained from Excel

	Decision variables	Value	
l^P	Pontoon length	87.00	[m]
h^P	Pontoon height	8.04	[m]
b^P	Pontoon breadth	12.06	[m]
l^C	Column length	9.11	[m]
h^C	Column height	27.96	[m]
b^C	Column breadth	9.92	[m]
d^P	Distance between pontoons	73.97	[m]
d^C	Distance between columns	74.00	[m]
z	Hull weight	8093	[mt]

Table 4 gives the optimal hull dimensions obtained from the model when all constraints are satisfied. The results showed that the GM values in the survival condition was acting as binding constraints and had a value equal to 1.5 meters. Further on the Eigen Periods of the vessel where above the lower limit with values of 22.11 and 22.70 seconds for the survival and operational conditions. Both of the air gap constraints were binding with air gaps equal to the minimum values. The geometry constraints which acted as binding were identified as the lower boundary of the pontoon length, and the upper boundary for the distance between columns. For the pontoons, the breadth height ratio was also binding with a breadth height ratio equal to 1.5. The rig had a survival and operational displacement equal to 22 500 and 25 000 mt respectively. The rig had an estimated deck area equal to 6970 m^2 which are large compared to similar rigs. The large deck area is a result of the large distance between the pontoons and the columns, which can be explained by analyzing the stability constraints. There are several ways to improve the stability of the rig. One approach is to increase the waterplane area, which will increase the overall weight. Another and more effective approach is to increase the distance between the pontoons and columns, in other words increase the two variables d^P and d^c. This gives no additional weight in the model and this explains the large values of d^P and d^c. The model is trying to fulfill the stability requirements by increasing the two variables towards the allowed boundary until the stability requirements are satisfied. Another important aspect that was identified during the running of the model was that the distance between the pontoons varied in optimal solutions found by the model. For instance, the model could find objective values equal to 8093 mt with different values for the distance between pontoons. The model will increase the distance between the pontoons until the transversal GM constraint related to the survival condition is satisfied, then the value of d^P stops somewhere in the allowed interval. This was identified as weakness in the model because larger distance between pontoons will usually increase the amount of bracing needed to carry the structural loads. Together with Aker Solutions it was agreed that alterations of the model was needed to better mirror the actual design process. These alterations will be further discussed in chapter 6.2.

6.2. Changing the Objective Function

Two alternative approaches was considered to stop the model from treating the distance between pontoons as a free variable as long as stability requirements where fulfilled. The first approach is to tighten the allowed interval. The disadvantage with this approach is that the input parameter for the upper boundary must be altered continuously based on other input parameters. The other alternative which gives a more effective and realistic approach is to implement a penalty term in the objective function which will increase the objective weight once the distance between the pontoons is increased. A new objective function is suggested in equation (1).

(1) $$min z = w^P + w^C + w^B + d^P X$$

Where:

z	Hull weight
w^P	Weight of pontoons
w^C	Weight of columns
w^B	Weight of braces
d^P	Distance between pontoons
X	Penalty constant for distance between the pontoons

It should be noticed that the penalty constant $X$was multiplied with the distance between pontoons to better represent the added weight resulting from high distance between the pontoons. This implies that the penalty constant will have a unit of mt/m. As an initial value, the penalty constant was given the value 0.1 mt/m. The low value was chosen to enforce a penalty for increasing the distance between the pontoons without changing the objective value too much, because bracing weight already counts for 10% of the total weight. Scaling of this penalty function is to be further discussed under the sensitivity analysis in chapter 6.4.9.

6.3. Results after altering the Objective Function

The model was solved with the new objective using the same algorithms in Excel. The results illustrated that the penalty function had the desired effect on the model. Instead of treating the distance between the pontoons as a free variable after the stability requirements is satisfied, the model increased the variable until the transversal GM value requirements were satisfied. The optimal solution was obtained and showed that the penalty function had increased the objective value to 8101 mt. All other dimensions remained unchanged, with the exception of the distance between the pontoons which stopped at 73.96 meters from all start solutions. As expected, the stability requirements in the survival condition where both binding. Once the model acted in a satisfactory way the sensitivity and robustness analysis could be performed.

6.4. Sensitivity and Robustness Analysis

The optimal solution obtained in the computational study is only optimal if the input parameters of the problem remain unchanged. However, input parameters are frequently changed during the preliminary design phase. Furthermore it is often hard to estimate the right value for an input parameter, such as COG and weight of the topside. The value of the optimal solution can be considerably reduced if the input parameters deviate much from reality. Small changes in the input parameters may cause the optimal solution to change considerably. In the following section a sensitivity analysis is performed to investigate the impact of changing certain input variables. The analysis was performed using a bracing penalty factor equal to 0.1 mt/m in the objective function.

6.4.1. Stability Requirements

As discussed in chapter 2.1 the expressions for the GM values are strongly dependent on the geometry of the hull. The solution of the optimization model showed that two of the binding constraints are the stability constraint related to the longitudinal and transversal GM values in the survival condition. It is of interest to investigate how a change in the stability requirements will affect the solution. There are two possible scenarios for how a change in an input parameter in a constraint will affect the overall solution. If the GM requirements are reduced it may give a new objective value with the same binding constraints. Another possibility is that the optimal solution shifts, and new constraints are binding. This can lead to larger changes in the objective value and decision variables. Initially, the input requirements for the GM values where set to 1.5 meters after correction of free surface effects. The sensitivity of the stability was investigated by altering the GM requirements stepwise. In cooperation with Aker Solutions it was agreed that the lower boundaries for the GM values are equal for all conditions. The results are presented in figure 14.

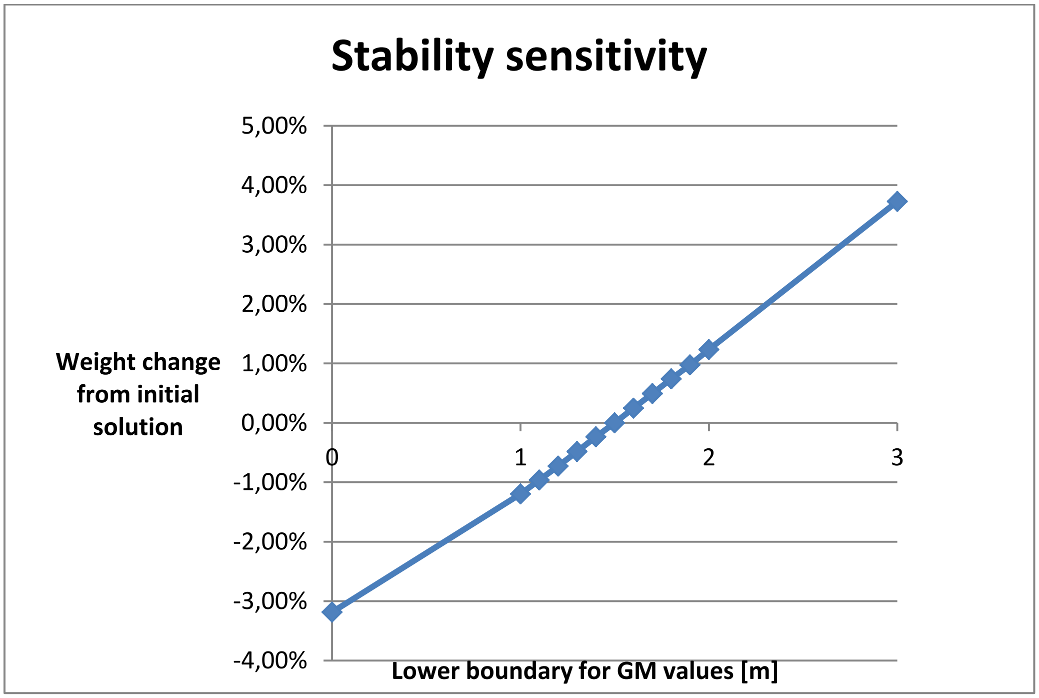

Figure 14 Change in hull weight when GM requirements are altered

The new solution showed small changes in the overall dimensions. The breadth of the columns was reduced by 0.24 meters when a 0.5 meter slack in GM values was introduced. Similar, but opposite results were obtained if the GM requirements were increased. All other parameters remained more or less unchanged. Further investigations showed that the GM values in the survival condition remained binding for all new values of the GM requirement. The total weight reduction was 3.2% when the lower GM boundary was reduced to 0 meters, which is a small reduction for an unreasonable reduction in GM requirements. The input parameter for free surface effect is set to 0.5 meters. This conservative, but it is unlikely that the effect will be much smaller than this without the hull being split up into very small compartments. A change in free surface effects will have the same impact a as a right hand change in the GM constraints which are described above. The results illustrated that small changes in the overall objective function was achieved when the GM requirements where altered.

The sensitivity analysis illustrated that the GM constraints is bounding in a large interval and changes in the requirements will only lead to small changes in objective function. The total savings is below 200 mt per meter change in GM values. Compared to the overall weight this is a small change related to a large change in the GM value requirement. It is concluded that the model is not very sensitive to changes in the stability requirements.

6.4.2. Eigen Period Requirements

As discussed in chapter 2.2 the Eigen period in heave affects the motion characteristics of the rig. It the current solution, none of the Eigen period constraints is binding. Thus, a lowering of required Eigen period will not affect the optimal solution. On the other hand, the engineers might want to increase the requirements to enhance the motion characteristics of the rig. It is of interest to see how this affects the overall design and the objective value. Initially, requirements where set to 19 and 20 seconds for the survival and operational condition respectively. The initial solution showed Eigen periods on 22.11 and 22.70 seconds for the survival and operational condition. Due to higher displacement and equality in waterplane area, the Eigen periods in the operational condition will always be higher than for the survival condition. A sensitivity analysis was performed in order to see how the optimal solution changes once the Eigen periods become a binding constraint. To make one of the constraints binding, the requirement to survival condition was increased. The results from the sensitivity analysis are illustrated in table 5.

Table 5 Increasing the lower boundary for heave Eigen period in survival condition

Minimum Eigen period in survival condition [s]	Actual Eigen period in survival [s]	Actual Eigen period in operational [s]	Objective weight [mt]	Change in hull weight [%]
19.00	22.11	22.70	8101	Initial
22.40	22.40	22.98	8188	1.07
22.70	22.70	23.28	8294	2.38
23.00	23.00	23.57	8401	3.70
23.30	23.30	23.86	8508	5.02
23.60	23.60	24.16	8615	6.34
23.90	23.90	24.45	8772	8.28

Another approach to enhance the motion characteristics is to demand a higher Eigen period in heave in the operational condition. After all, the rig spends the majority of its lifetime in this condition. The results of increasing the Eigen period in heave for the operational conditions are illustrated in table 6.

Table 6 Increasing the lower boundary of the heave Eigen Period in the operational condition

Minimum Eigen period in operational condition [s]	Actual Eigen period in survival [s]	Actual Eigen period in survival [s]	Objective weight [mt]	Change in hull weight [%]
20.00	22.11	22.70	8101	Initial
23.00	22.42	23.00	8194	1.15
23.30	22.73	23.30	8304	2.51
23.60	23.03	23.60	8412	3.84
23.90	23.34	23.90	8521	5.18
24.20	23.64	24.20	8630	6.53
24.50	23.95	24.50	8740	7.89

Both tables illustrate that higher Eigen periods can be achieved by increasing the hull weight. In order to increase the Eigen periods, the model will increase the breadth of the pontoons. This is because the added mass will considerably higher, resulting in increased Eigen periods. The total increase of the added mass was around 25 % when the requirement for the survival Eigen period where increased to 23.9 seconds. The breadth of the pontoons increased 11% in the same interval. Apart from this, all decision variables stayed basically unchanged. The results showed that it is possible to increase the Eigen periods without having a large impact on the hull weight. The sensitivity analysis illustrates some of the major advantages with the model. If the engineers find that the Eigen period constraint is binding, it is easy to investigate the cost of changing the periods. The model also gave the valuable information that the cheapest way to rise the Eigen periods is by raising the added mass, not by reducing the waterplane area.

The sensitivity regarding the added mass coefficient where investigated. The results are given in table 7.

Table 7 Sensitivity related to changes in the added mass coefficient

Added mass coefficient [-]	**Eigen Period in Heave Survival [s]**	**Eigen Period in Heave Operational [s]**	**Change in hull weight [%]**
0.9	21.09	21.70	No
1.0	21.61	22.21	No
1.1	22.11	22.70	Initial
1.2	22.61	23.18	No
1.3	23.09	23.66	No

The changes in the added mass coefficient had no impact on the optimal solution apart from small changes in Eigen periods. An increase or reduction of the added mass coefficient of 18% gave changes in Eigen Periods of approximately 4.5%. For the semi-submersible rig with two rectangular pontoons, the added mass will tend to lie between 0.9 and 1.3 for all allowed breadth height ratios which are controlled by constraint (61) and (62) (Pettersen, 2004).

6.4.3. Topside Weight and COG

Because the hull must provide stability and buoyancy for the entire unit, the design is driven by the topside weight and area requirements. It is therefore important to investigate how a change in topside parameters will affect the overall solution. The sensitivity of the topside was investigated by changing the topside weight stepwise with 250 mt intervals. The results are presented in table 8.

Table 8 Sensitivity related to the topside weight

Topside weight [mt]	Hull weight [mt]	Hull fraction of total lightship weight [%]	Change in hull weight [%]
5500	7654	58.19	-5.52
5750	7720	57.31	-4.70
6000	7786	56.48	-3.89
6250	7852	55.68	-3.07
6500	7918	54.92	-2.26
6750	7984	54.19	-1.44
7000	8101	53.65	Initial
7250	8288	53.34	2.31
7500	8475	53.05	4.62
7750	8661	52.78	6.91
8000	8846	52.51	9.20
8250	9031	52.26	11.48
8500	9215	52.02	13.75

The variables primarily affected by a change in topside weight were the variables related to the pontoons. In order to carry a heavier topside, the pontoon dimensions were increased once the topside weight was increased. The results also illustrates that the hull will count for a smaller part of the overall lightship weight for larger topsides. The initially binding constraints continued to bound the solution for all topside weights.

It is often hard to determine the exact location of the COG and the weight of the topside in an early phase. The uncertainty related to these parameters will affect the quality of the optimal solution because the GM constraints related to the survival condition are binding. The sensitivity analysis was performed changing the COG for the topside with intervals of 0.5 meters. The results obtained are presented in table 9.

Table 9 Sensitivity related to the topside COG

Vertical distance from deck to COG of topside [m]	Hull weight [mt]	Change in hull weight [%]
8.0	7978	-1.52
8,5	8009	-1.14
9.0	8040	-0.75
9.5	8070	-0.38
10.0	8101	Initial
10.5	8131	0.37
11.0	8162	0.75
11.5	8192	1.12
12.0	8223	1.51

As the results in table 9 suggests, the hull weight is reduced by 1.52% when the COG of the topside is decreased from 10 to 8 meters. If the COG of the topside is increased the current solution is non-feasible. The result of increasing the topside COG is an increase in the hull weight of 1.41%. This is analogous with the results obtained in the sensitivity analysis of the stability constraints. A change in the topside weight will simply change the stability of the rig which implies that the terms in constraints (2)-(5) are changed.

Discussions with Aker Solutions indicated that the location of the COG is located 8-12 meters above the deck. It is of interest to investigate the consequence of choosing a solution based on biased input parameters. In the following case it is assumed that the model has been solved and an optimal solution is obtained. The final results from the topside department show that the COG of the topside are changed considerably from the preliminary phase estimate. The results are presented in table 10.

Table 10 Robustness of the solution if COG of topside deviates from estimate

Vertical distance from deck to COG of topside [m]	Transversal GM values in survival condition [m]	Longitudinal GM values in survival condition [m]
8.0	2.12	2.12
8.5	1.97	1.97
9.0	1.81	1.81
9.5	1.66	1.66
10.0	1.50	1.50 (initial)
10.5	1.34	1.34
11.0	1.19	1.19
11.5	1.03	1.03
12.0	0.88	0.88

As the result illustrated in table 10 shows, the GM values are strongly dependent on the COG of the topside. Once the COG value is larger than the input value, the rig will violate the stability constraints. This can be handled by using weight margins on weights and COG. This will give more allowance to uncertainties in estimates. Further on it is interesting to investigate the robustness related to the weight of the topside. Similarly to the location of the COG the weight of the topside is an estimate, and subject to change early in the design process. The robustness of the solution was analyzed varying the topside weight with intervals of 250 mt. The results are presented in table 11. The results illustrates that the rig will get a negative GM value if the weight of the topside is underestimated by 11%. Discussions with Aker solutions showed that the usual procedure is to define a “not to exceed” vertical moment from the topside. If the vertical moment is too high, the hull design department must re-design the hull. This emphasizes the value of the model. When the topside department concludes that the vertical moment of the topside is too large the model can be solved with new input parameters.

Table 11 Robustness of solution when Topside weight deviates from estimate

Topside weight [mt]	Transversal GM values in survival condition [m]	Longitudinal GM values in survival condition [m]
6500	2.43	2.43
6750	1.97	1.97
7000	1.50	1.50
7250	1.03	1.03
7500	0.57	0.57
7750	0.10	0.10
8000	-0.36	-0.36

6.4.4. VDL Capacity and COG

How the input VDL capacity affects the solution is of great interest for the designers. A rig with a high VDL is capable of drilling and operating in larger depths. Larger VDL implies that the rig is more flexible and can accept a big aspect of different contracts. Performing a sensitivity analysis on the VDL impact on the solution will tell the engineers if the VDL capacity can be raised and at what cost. The sensitivity was investigated by changing the topside weight at 250 mt intervals. The results are illustrated in table 12.

Table 12 Sensitivity related to the VDL weight

VDL [mt]	Hull weight [mt]	VDL-hull weight ratio [-]
3000	7811	0.38
3250	7871	0.41
3500	7934	0.44
3750	8071	0.47
4000	8101	0.49
4250	8183	0.52
4500	8265	0.54
4750	8348	0.57
5000	8430	0.59

The results illustrated that the VDL-hull ratio decreases once the VDL increases. The results will give the engineers a estimation of the cost of increasing the VDL capacity of the rig.

In the previous chapter the robustness of the solution was discussed regarding a change in the vertical moment of the topside. Similar conclusion can be drawn related to the vertical moment of the VDL. However, the vertical moment of the VDL will be easier to control because the cargo capacity can be changed and monitored during operations. A change in the weight of the VDL or COG will have smaller impact on the stability because the vertical moment of the topside is significantly larger than that of the VDL.

6.4.5. Air gap Requirements

The air gap requirements were identified as one of the binding constraints in the optimal solution. It is of interest to see which impact a change in air gap requirements will have on the optimal solution. Table 13 gives the results when the required air gap in the survival condition was altered, while the operational air gap required was held constant.

Table 13 Sensitivity related to the survival air gap requirement

Survival Air gap required[m]	Operational Air gap required[m]	Hull weight [mt]	Change in hull weight [%]
17.5	14	8101	0
18.0	14	8101	0
18.5	14	8101	0
19.0	14	8101	0
19.5	14	8243	1.75
20.0	14	8388	3.54
20.5	14	8536	5.37

As table 13 illustrates the reduction of the survival air gap gave no improvement in the objective value. The design will not change because the solution is controlled by the draft requirements and the operational air gap requirement. The result is that the air gap remains at 14 and 19 meters in the survival condition, but the restriction is not binding anymore. Further reduction of requirements will not change the optimal design but the slack in the air gap restriction will increase. If the air gap requirement is raised the solution must change because the initial optimum are not feasible anymore. When the air gap required in the survival condition is increased to 19.5

meters the air gap in the operational condition will increase by 0.5 meters. There are small increases in pontoon dimensions, but the column height is increased by 0.46 meters. The result is a weight increase of 1.75% to meet the new requirements. Further increases of the air gap requirements for the survival condition confirmed the trend with increasing hull weight. The solutions obtained by altering the requirements to the operational air gap are given in table 14.

Table 14 Sensitivity related to the operational air gap requirement

Operational air gap required[m]	Survival Air gap required [m]	Hull weight [mt]	Change in hull weight [%]
12.5	19.0	8101	0
13.0	19.0	8101	0
13.5	19.0	8101	0
14.0	19.0	8101	0
14.5	19.0	8243	1.75
15.0	19.0	8388	3.54
15.5	19.0	8636	6.60

The results given in table 14 illustrates that the solution remained unchanged when the requirements to the operational air gap is reduced. Similar to the sensitivity related to the survival air gap, the solution is controlled by the draft parameters and the constraint related to the air gap in the survival condition. The design was altered slightly once the requirement to air gap was increased, because the optimal solution is not feasible anymore. The column height increased by around 0.9 meters per meter increase in air gap requirements.

Furthermore, it is of interest to analyze the impact of changing both air gap requirements. For instance if the rig have very favorable motion behavior, the engineers may want to introduce some slack in the constraints. The solutions obtained when changing input parameters for both the survival and operational condition are illustrated in table 15.

Table 15 Sensitivity related to survival and operational air gap requirements

Operational Air gap required[m]	Survival Air gap required[m]	Hull weight [mt]	Change in hull weight [%]
12.5	17.5	7756	-4.26
13.0	18.0	7852	-3.07
13.5	18.5	7962	-1.72
14.0	19.0	8101	Initial
14.5	19.5	8243	1.75
15.0	20.0	8388	3.54
15.5	20.5	8536	5.37

The results given in table 15 revealed an important opportunity. If both requirements are lowered, the model is able to find new solutions. When the input parameters for the air gap are reduced with 1 meter the new optimum will have a reduced the column height by 0.96 meter and the objective value improves 3.07%. So the model will simply reduce the column height once a slack in the constraints is given. The air gap in the transit condition was not analyzed because this will never be a constraint that affects the solution.

The sensitivity of the air gap input parameters illustrated that the reduction of one parameter at a time not will affect the current solution. The solution is simply controlled by draft constraints and the air gap requirement related to the other condition. Increasing one of the air gap requirements illustrated that the initial optimal design becomes unfeasible and the model will find a new solution which increases the hull weight. A change in both input parameters at the same time revealed an impact on the objective function. A reduction of 1.5 meter in the required air gap showed a possible decrease of 4.26% of the initial hull weight.

6.4.6. Draft Configurations

As explained in chapter 2 the rig will de ballast to reach sufficient air gap once extreme conditions are expected. So the input drafts should be given with the same difference as the air gap requirements, which in this condition is five meters. An alteration of this difference will just cause an imbalance between the draft regulations and the air gap requirements. If both of the input drafts are adjusted and the difference corresponds to that of the air gap the model will give a more realistic picture of the actual de ballasting process. The result of further de-ballasting after sufficient air gap is reached will be to deteriorate the motion characteristics. The sensitivity results obtained by changing both drafts simultaneously are presented in table 16.

Table 16 Sensitivity analysis of different draft configurations

Draft [m]	Draft survival [m]	Hull weight [mt]	Change in hull weight [%]
20.0	15.0	7722	-4.68
20.5	15.5	7804	-3.67
21.0	16.0	7886	-2.65
21.5	16.5	7986	-1.42
22.0	17.0	8101	Initial
22.5	17.5	8216	1.42
23.0	18.0	8332	2.85
23.5	18.5	8449	4.30
24.0	19.0	8566	5.74

Table 16 illustrates that it is possible to increase or decrease the hull weight by altering the input drafts. A reduction of the input drafts will make the air gap constraints easier to fulfill. The model will then reduce the column height by approximately one meter per meter reduction of draft. The same trend continues when the drafts are further reduced until the objective weight is reduced by 4.68% for drafts of 15 and 20 meters for the survival and operational condition. If the draft inputs are increased the column height grows accordingly. The final results revealed a weight increase of 5.76% when the input drafts were increased by 2 meters. This aspect gives the engineers information of the cost of increasing the draft to enhance the motion characteristics of the rig.

6.4.7. Changes in Geometrical Constraints

The variables bounded by the geometrical constraints where identified as the following:

- Minimum length of pontoon
- Maximum distance between columns
- Breadth-height ratio of the pontoons

The lower boundary for the length of the pontoons is based on statistical data from drilling rigs operating in the North Sea. However, it is of interest to see if slack in this constraint will suggest a new optimal solution. The sensitivity of the pontoon length where altered with 1 meter intervals. The results are illustrated in table 17.

Table 17 Sensitivity analysis of allowed interval for pontoon length

Lower boundary of pontoon length [m]	Optimal pontoon length [m]	Hull weight [mt]	Change in hull weight [%]
85.00	86.18	8096	-0.06
86.00	86.18	8096	-0.06
87.00	87.00	8101	Initial solution
88.00	88.00	8108	0.09
89.00	89.00	8164	0.78
90.00	90.00	8219	1.46
91.00	91.00	8276	2.16

If the lower boundary for the length of the pontoon where lowered to 86 meters the new optimum gave a pontoon length of 86.18 meters. This reveals that the constraint related to the lower boundary for the pontoon length are not acting as a binding constraint anymore and further reduction of the lower boundary will not affect the solution. The results further shows that the solution improved by only 0.06% when the minimum requirement was reduced. An increase in the lower boundary will make the current solution infeasible and model will alter the optimal solution. When the lower boundary was increased the model found new optimums with the pontoon length still acting as a binding constraint. The results revealed an increase in the hull weight of 1.46% when the input parameter where changed from the initial 87 meters to 90 meters.

An increase of the upper boundary of the pontoon length will not affect the current solution because the constraint is not binding.

Further on the constraint related to the breadth-height ratio for the pontoon was investigated. The ratio is acting as a binding constraint where the ratio is reaching its lower limit at 1.5. The effect of decreasing this lower boundary is illustrated in table 18.

Table 18 Sensitivity of breadth-height ratio for the pontoons

Lower Breadth height ratio [-]	h^p**[m]**	b^p**[m]**	**Hull weight [mt]**	**Change in hull weight [%]**
1.0	9.64	9.64	7713	-4.79
1.1	9.25	10.18	7806	-3.64
1.2	8.91	10.69	7889	-2.62
1.3	8.59	11.17	7976	-1.54
1.4	8.30	11.62	8036	-0.80
1.5	8.04	12.06	8101	Initial solution
1.6	8.00	12.80	8400	3.69
1.7	8.00	13.60	8749	8.00
1.8	8.00	14.40	9098	12.31
1.9	8.00	15.20	9446	16.60
2.0	8.00	16.00	9795	20.91

Table 18 illustrates the breadth-height ratio impact on the optimal solution. A reduction from 1.5 to 1.0 for the lower boundary of the ratio showed an improvement of 4.79%. When slack is introduced the model tends to increase the height of the pontoons, while the breadth is reduced. This is because the increased height in pontoons allows the model to reduce the column height, and still satisfy the air gap requirements. The results also revealed large impacts in the objective value when raising the lower boundary of the ratio. This is because the columns must be increased in order to satisfy the air gap requirements. It should be noted that the distance between the columns not will be investigated as an increase in the allowed interval will allow the columns to be located outside the pontoons, which will cause both structural and constructability challenges.

6.4.8. Change in the Weight Density of the Hull

As discussed in chapter 6.1 the input parameters for the weight densities were set to 0.270 mt/m^3 initially. Discussions with Aker Solutions suggested that similar densities could be assumed for each part of the hull structure. The impact of altering the input density to 0.250 mt/m^3 and 0.290 mt/m^3 are illustrated in table 19 and 20 respectively.

Table 19 The optimal dimensions with 0.250 [mt/m^3] as density factor

Decision variable		**Value**		**Change in hull weight [%]**
l^P	Pontoon length	86.17	[m]	-0.95
h^P	Pontoon height	8.00	[m]	-0.53
b^P	Pontoon breadth	12.00	[m]	-0.53
l^C	Column length	8.28	[m]	-9.09
h^C	Column height	28.00	[m]	0.15
b^C	Column breadth	10.80	[m]	8.92
d^P	Distance between pontoons	73.89	[m]	-0.10
d^C	Distance between columns	74.00	[m]	0
z	Hull weight	7384	[mt]	-8.85

Table 20 The optimal dimensions with 0.290 [mt/m^3] as density factor

Decision variable		**Value**		**Change in hull weight [%]**
l^P	Pontoon length	86.05	[m]	-1.09
h^P	Pontoon height	8.30	[m]	3.16
b^P	Pontoon breadth	12.44	[m]	3.16
l^C	Column length	8.16	[m]	-10.40
h^C	Column height	27.70	[m]	-0.91
b^C	Column breadth	11.20	[m]	12.95
d^P	Distance between pontoons	73.87	[m]	-0.13
d^C	Distance between columns	74.00	[m]	0
z	Hull weight	8995	[mt]	11.04

The results presented in table 19 and 20 illustrated that the model were able to suggest new optimal solutions once the coefficients in the objective function were altered. In future work the engineers can investigate the consequence of operating with different densities for the pontoons, columns and bracing. To limit the scope of the computational study, the sensitivity of the different objective coefficients was not further investigated.

6.4.9. Scaling of the Penalty Function

The penalty term was introduced in objective function to force the model to choose the solution with the shortest distance between the pontoons which satisfy the transversal stability requirements. The scaling of the penalty should be an expression of the extra bracing needed when the distance is increased. A sensitivity study was conducted by increasing the penalty input parameter stepwise.

Table 21 Sensitivity related to the bracing penalty factor

Penalty constant ***X*** [mt/m]	Total bracing weight [mt]	Bracing penalty weight [mt]	Bracing weight fraction of total hull weight [%]
0.1	816	7	10.08
0.5	845	37	10.41
1.0	882	74	10.81
2.5	993	185	12.00
5.0	1178	370	13.93
7.5	1363	555	15.77
10.0	1548	740	17.53

The results shows how different penalty factors impact on the bracing penalty. For Aker Solutions, which can access sensitive data regarding bracing, it should be possible to scale the penalty function properly.

7. Discussion

In the following chapters, the most important aspects of the model and results are discussed.

7.1. The Model

In this chapter various aspects of the model are discussed and evaluated.

7.1.1. Assumptions

During this thesis an optimization model for design hull structures of semi-submersible rigs have been developed. To be able to make use of optimization modeling it was necessary to make some basic assumptions and simplifications of the problem. As discussed in chapter 5.1 the number of legs on semi-submersible rigs varies from four to eight. The number of legs required is dependent on the maximum defined VDL capacity. A high VDL will give challenges related to the stability because of the high COG. Large VDL rigs are therefore more likely to have more columns in order to increase the waterplane area, which will enhance the stability. The design VDL in the computational study was 4000 mt which is a typical value for drilling rigs operating on the Norwegian Shelf. From the deckload capacity targeted it was assumed that a four legged platform would provide sufficient stability. This decision was also based on previous studies from Aker Solutions, which indicated that four legged platforms has got more favorable motion characteristics than rigs of the same displacement size with six or eight legs. A possible extension of the model will be to include the possibility of choosing platforms with six or eight legs. An extension can be solved using two different approaches. One possibility is to use binary variables and force the model to choose between four, five, six or eight legs. The use of binary variables and implementation of several nonlinear constraints will make the model more complex and more difficult to solve. Another approach is to simply develop new models for six and eight legged platforms. The initial model can be used as a basis and many of the constraints are equal. There will be some changes in the calculations, but the problem is pretty much described by the same model. The advantage with this approach is that the engineers will get designs with four, six and eight legs. This gives the designers more alternatives and more flexibility in terms choosing the right hull structure.

The columns and pontoons where assumed to have a rectangular cross sectional area, while some rigs have a circular cross section. The majority of rigs are equipped with rectangular pontoons with some curvature at the fore and aft part. However, there are larger variations in column configuration. The column configurations vary between circular and rectangular cross sections, or a mix between the two. The rectangular cross sections were studied mainly because of the low construction complexity and the high viscous damping factor. Circular columns and pontoons can be implemented in the model by changing constraints which are affected by the column and pontoon shapes. The model does not take the end curvature of the columns and pontoons which is created to reduce resistance in transit condition, into account. Together with Aker Solutions it was agreed that the end curvature will have little impact on the overall dimensions. The optimal solution from the model may be used as a basis, introducing the curvature at a later stage, with a negligible impact on overall properties.

7.1.2. The Objective Function

After the basic assumptions where made it was possible to formulate the optimization model. Several objective functions where considered. As discussed in chapter 1, some research has been focused on minimizing motion behavior or CAPEX. Four main objectives were identified as low construction cost, large VDL capacity, favorable motion characteristics and large deck area. Initially, a multi objective model was considered. The advantage with a multi objective model is that all of the defined objectives will have an impact on the objective value. This will give a more realistic description of the economy of the problem. For instance, good motion behavior will lead to less down time for the rig. This will affect the revenue, so it might be acceptable to increase the costs to improve the motion characteristics. The normal procedure is to construct the multi objective function by using different weights which enable the model to summarize the objective function terms. Engineers can alter the weighting in the objective function. For instance, if the motion behavior is more important than the deck area, it is possible to increase the weighting of the motion behavior term, while the weight for the deck area is decreased. The multi objective function would have allowed the engineers to change the weights and get various design configurations. However, some of the objectives are difficult to formulate mathematically. Large draft and high Eigen periods, generally enhances the motion behavior. But it is difficult to quantify how much a change in one of these parameters will alter the motion characteristics. The conclusion is that it is challenging to formulate reasonable terms in the objective function for VDL, deck area and motion characteristics. Another important aspect is that it would be difficult to interpret the results, and they are dependent on the weighting of the objectives. In the end the engineers would have ended up with unlimited possible designs. The model would be hard to communicate and it would be challenging to agree on the correct weighting of the objectives.

During the previous semester, a study of the design process of the Cat-B rig was conducted as a part of the basis for this master thesis. Aker Oilfield Services have won a contract to design and operate the rig which will perform intervention services for Statoil. In the early design phase, Statoil have already defined a list of functional requirements. The most important features for the rig stated, such as the Eigen period in heave, the required VDL capacity and the required deck equipment. Based on these input parameters, Aker Solutions will design and build the rig at the least cost which fulfill all requirements. Thus, the established model gives a good picture of the actual design process, where most input parameters are defined and the company will try to minimize the construction cost. In this thesis, the weight was used as an expression for the construction cost. In further development of the model other, more accurate cost functions should be considered. One of the main reasons for minimizing the weight was that it is difficult to establish a reasonable cost function. Additionally, cost data is very sensitive. However, the model is formulated so that a change in the objective function easily can be implemented.

7.1.3. Decision Variables

The eight main dimensions of the hull were identified is decision variables. The draft was treated as an input parameter, because it provides engineers with the opportunity to try out different configurations. Another approach would have been to treat the different draft configuration as variables. The allowed interval for the draft could have been constructed by constraints which gave an upper or lower boundary. However, three additional variables would have made the model more complex and harder to solve. It is assumed that the possibility to try out different draft configurations along with the sensitivity analysis will give the engineers sufficient information to decide on the appropriate drafts for the rig.

The VDL could also been treated as a variable, but then it would have been necessary to include it in the objective function so it could be maximized. Because the weight of the hull was chosen as the objective function it was agreed with Aker Solutions that the VDL should be treated as an input variable, again allowing engineers to try out different configurations.

7.1.4. The Constraints

The constraints related to the Eigen periods in pitch and roll where neglected because they are usually satisfactory due to the GM constraints. When more accurate hydro dynamical analysis is performed, the Eigen periods should be investigated more closely.

A total of 75 constraints where developed. However, most constraints are equality constraints which enable the model to calculate the help variables applied in the non-equality constraints. It should be noticed that constraints easily can be added or removed from the model.

7.2. The Results

In the following section, the results will be compared with various rig designs operating in the North Sea.

7.2.1. Comparison with other Rigs

To evaluate the results it is necessary to benchmark the results from the model with rigs operating in the North Sea. The four legged GVA 4000 (GVA, 2013) is designed by the Swedish company GVA. The rig has a VDL capacity of 4200 mt. The operation draft is 20.5 and 16.2 meters in the operational and survival condition. The main dimensions are given in table 22.

Table 22 Optimal solution compared with GVA 4000

	Results from model [m]	**GVA 4000 [m]**
Pontoon length	87.0	80.6
Pontoon height	8.04	7.5
Pontoon breadth	10.8	Unknown
Column length	9.11	14.2 (diameter)
Column height	27.96	29.0
Column breadth	9.92	14.2 (diameter)
Distance between pontoons	73.97	73.40
Distance between columns	74.00	Unknown

Table 22 illustrates that the rig has circular columns with a diameter of 14.2 meters which gives the rig a total waterplane of 633 m^2 compared to 351 m^2 from the optimal solution derived from the model. This shows that much of the stability is obtained by the large waterplane area. The displacement in the operational condition is 29 700 mt at 20.5 meter draft for the GVA 4000 compared to 25 000 mt at 22 meter draft for the model design. This may indicate that the hull of the GVA 4000 have a larger volume and weight than the design obtained from the model. The larger draft of the model design should also give better motion characteristics. It should be noted that the GVA 4000 can carry 200 mt more than the model design. To further compare the two rigs the model was tested with an input VDL of 4200 mt. The final results revealed a hull weight of 8167 mt and a displacement of 25 143 mt at 22 meters operational draft. Again the model seems to develop lighter hull structures than the comparison rig.

Further on the four legged GVA 3800 (GVA, 2013) design was investigated. The rig has a deckload capacity of 5000 mt and the survival and operational drafts are set to 16 and 20 meters. The rig has an operating displacement of 3000 mt. The main dimensions of the rig are given in table 23.

Table 23 Optimal solution compared with GVA 3800

	Results from model [m]	**GVA 3800 [m]**
Pontoon length	87.0	81.6
Pontoon height	8.04	8.4
Pontoon breadth	10.8	Unknown
Column length	9.11	12.0
Column height	27.96	27.1
Column breadth	9.92	12.5
Distance between pontoons	73.97	70.7
Distance between columns	74.00	Unknown

Table 23 illustrates the GVA rigs tend to have larger columns while the pontoons are shorter. The rigs operate in smaller drafts with a higher displacement compared to the model design, which may indicate a larger hull weight. The model was run with a VDL of 5000 mt and suggested a design with 8430 mt steel hull and a displacement of 25 800 mt, which is 14% smaller than the GVA 3800 displacement. The input operation draft was set to 22 meters.

Deepsea Bergen is a drilling rig of the Aker H-3.2 design with eight legs operating on the Norwegian shelf with a deckload capacity of 4100 mt and an operational draft of 22 meters.

Table 24 Optimal solution compared with Deepsea Bergen

	Results from model [m]	Deepsea Bergen [m]
Pontoon length	87.0	92.5
Pontoon height	8.04	7.2
Pontoon breadth	10.8	17.2
Column length	9.11	Unknown
Column height	27.96	27.3
Column breadth	9.92	Unknown
Distance between pontoons	73.97	67.2
Distance between columns	74.00	Unknown

The rig is operating with the same draft as the rig design from the model and the displacement is 28 000 mt compared to 25 000 mt. This again suggests that the design from the model have a lighter hull structure. The Deepsea Bergen can carry 2.5% more cargo, but the displacement is 12% higher than the rig suggested by the model. The pontoons of the Deepsea Bergen have a larger breadth-height ratio which will reduce the overall air gap of the rig. When the model was run with input VDL of 4100 mt, a design with an operational displacement of 25 060 mt was suggested.

From the comparison with other rigs it was assumed that the design suggested by the model is feasible due to the similarities. The GVA rigs showed a smaller length of pontoons and larger waterplane area. When the sensitivity of the pontoon length where studied, the model found small advantages by reducing the length of the pontoons more than 86 meters. The Deepsea Bergen rig had eight legs and very broad pontoons. The large breadth-height ratio will give the rig a smaller air gap than the model design. The results indicated the model suggested lighter hull structures compared to the other rigs. Further on, the designs obtained from the model was reviewed by experienced engineers in Aker Solutions and found to be feasible and promising.

8. Conclusion

The main objective with this master thesis was to develop an optimization model which could be used as a decision support tool in the establishment of the main dimensions of the hull structure on a semi-submersible rig. The nonlinear model was formulated for four legged semi-submersibles with rectangular cross sections in both pontoons and columns.

During the development of the model it was noticed that today, the use of optimization theory in the design of semi submersibles rigs is somewhat absent. This thesis explains the development of the model thoroughly and demonstrates a computational study. As discussed in chapter 1, much of the research conducted on the relevant area requires a solid knowledge basis in optimization and marine technology to comprehend. The researchers often solve their models using programming and complex algorithms, while the developed model were solved using Microsoft Excel. This will hopefully make the model easier available for engineers which often rely on Excel and have experience using the software.

The designs obtained from the model were compared with three rigs currently operating in the North Sea. When using similar variable deck loads as input parameters, the model designs operated in deeper drafts with smaller displacements. This suggests that the hull structures obtained from the model are lighter than the structures of the comparison rigs. The results were discussed with Aker Solutions, and they concluded that the model designs appeared feasible and cost efficient.

Initially, the model was developed for a four legged semi-submersible rig but can be converted to hold for six and eight legs as well. This will give the decision makers more alternative designs to investigate further. The objective function was formulated to minimize the weight of the hull. In further applications it might be considered to change the objective to better model the overall cost which is dependent on several factors.

It is concluded that the model can be a convenient tool, supporting Aker Solutions during early design stages, potentially saving time, money and human resources.

Bibliography

Aker Solutions. (2012). *Well Intervention Platform Study*. Oslo: Aker Solutions.

Amdahl, Endal, Minsaas, Rasmussen, Sillerud, Sortland, et al. (2005). *Marin Teknikk 1*. Trondheim: Marin Teknisk Senter.

Aubalt, A., Cermelli, B., & Roddier, C. (2007). Parametric Optimization of a Semi- Submersible Platform with Heave Plates. *OMAE2007* (pp. 1-8). San Diego: OMAE.

Aubault, A., Cermelli, C., & Roddier, D. (2007). Parametric Optimization of a Semi-Submersible Platform With Heave Plates. *Offshore Mechanics and Arctic Engineering* (pp. 1-8). San Diego: OMAE.

Biegler, L. (2011). *Nonlinear Programming: Consepts and Algorithms for Process Optimization*. Retrieved April 25, 2013, from Carnegie Mellon University: http://cepac.cheme.cmu.edu/pasi2011/library/biegler/PASI2011_LB_nlp.pdf

Birk, Claus, G., & Lee. (2004). Practical Applications of Global Optimization to the Design of Offshore Structure. (pp. 1-13). Vancouver: American Society of Mechanical Engineers.

Birk, Clauss, & Lee. (2004). Practical Applications of Global Optimization to the Design of Offshore Structures. *Americal Society of Mechanical Engineers* (pp. 1-10). Vancouver: Americal Society of Mechanical Engineers.

Birk, L., & Clauss. (2002). Parametric Hull Design and Automated Optimization of Offshore Structures. *International Maritime Association of the Mediterranean* (pp. 1-8). Crete: International Maritime Association of the Mediterranean.

Birk, L., & Clauss, G. (2001). Automated Hull Optimization of Offshore Structures Based on Rational Seakeeping Criteria. *International Offshore and Polar Engineering Conference* (pp. 382-389). Stavanger: The international Society of Offshore and Polar Engineers.

Birk, L., & Clauss, G. F. (2002). Parametric Hull Design and Automated Optimization of Offshore Structures. *International Maritime Association of the Mediterranean* (pp. 1-8). Crete: International Maritime Association of the Mediterranean.

DNV. (2010). *DNV RP C-205 ENVIRONMENTAL CONDITIONS AND ENVIRONMENTAL LOAD.* Oslo: DNV.

DNV. (2010). *DNV-RP-F205.* Oslo: DNV.

Faltinsen, O. M. (1990). *Sea Loads On Ships And Offshore Structures.* Cambridge: Cambridge University Press.

GVA. (2013, January 1). *GVA rigs* . Retrieved April 5, 2013, from GVA website: http://www.gvac.se/gva-400/

Hillier, L. (2010). *Introductions to Operations Research.* New York: McGraw-Hill.

Kennedy, H., & Eberhart, R. (1995). Particle Swarm Optimization. *Institute of Electrical and Electronics Engineers* (pp. 1942-1948). Washington DC: Institute of Electrical and Electronics Engineers.

Lasdon, L. S., Waren, A. D., Arvind, J., & Ratner, M. W. (1975). *Design and Testing of a Generalized Gradient Code for Nonlinear Optimization.* Cleveland: National Technical Information Service.

Lundgren, R. V. (2010). *Optimization.* Stockholm: Studentlitteratur.

Matos, V. (2011). Second-order resonant heave, roll and pitch motions of a deep-draft semi-submersible: Theoretical and experimental results. *Ocean Engineering*, 2227–2243.

Microsoft. (2011, September 19). *Support:Microsoft*. Retrieved April 3, 2013, from Microsoft Support Web Site: http://support.microsoft.com/kb/82890

MIT Web Education. (n.d.). *www.mit.com*. Retrieved April 4, 2013, from web.mit.edu: http://web.mit.edu/15.053/www/AMP-Chapter-02.pdf

Øvstebø, B., Hvattum, L., & Fagerholt, K. (2011). Optimizatiobn of Stowage plans for RORO ships. *Computers and Operations Research*, 1425-1434.

Pettersen, B. (2004). *Hydro Dynamics.* Trondheim: Marine Technology Centre.

Pike, R. W. (1985). *Optimization for Engineering Systems* . New York: Van Nostrand.

Sillerud, B. O. (2003). *Marin Teknikk 1.* Trondheim: Marin Teknisk Senter.

Wolfram. (2013). *www.wolfram.com*. Retrieved Aprli 4, 2013, from http://mathworld.wolfram.com: http://mathworld.wolfram.com/ConvexSet.html

9. Appendix A

9.1. Summary of the model

9.1.1. Sets and Indexes

t:	Condition t
T:	Set of all conditions, {Transit, Survival, Operation}

9.1.2. Parameters

$S_t^{TRANSVERSAL,MIN}$	Minimum required transversal GM value for condition t
$S_t^{LONGITUDINAL,MIN}$	Minimum required longitudinal GM value for condition t
$S_t^{TRANSVERSAL,MAX}$	Maximum allowed transversal GM value for survival and operational condition
$S_t^{LONGITUDINAL,MAX}$	Maximum allowed longitudinal GM value for survival and operational condition
O_t	Reduction in GM values due to free surface effects in condition t
W_t^{VDL}	VDL capacity in condition t
W^{TS}	Topside Weight
ρ	Density of seawater
g	Gravitational acceleration
$T_t^{HEAVE,MIN}$	Lower boundary for Eigen period in heave in condition t
$C^{AM,HEAVE}$	Added mass coefficient for the pontoons
V^P	Weight density of the pontoons
V^C	Weight density of the columns
V^B	Weight density of the braces
F_t	Draft in survival and operational condition
J^B	Bracing weight fraction of total hull weight
Q^B	Distance from top of pontoons to the start of the bracing
G^{TS}	Vertical distance from deck to COG of the topside

G^{VDL}	Vertical distance from deck to COG of the VDL
A_t	Minimum air gap for the survival and operation conditions
K	Freeboard from top of pontoon to the water surface in transit condition
Z	Factor describing the ballast capacity of the pontoons
$A^{DECK,MIN}$	Minimum required deck area
$L^{P,MAX}$	Upper boundary for pontoon length
$L^{P,MIN}$	Lower boundary for pontoon length
$H^{P,MAX}$	Upper boundary for pontoon height
$H^{P,MIN}$	Lower boundary for pontoon height
$B^{P,MAX}$	Upper boundary for pontoon breadth
$B^{P,MIN}$	Lower boundary for pontoon breadth
U	Factor restricting max column breadth as a function of pontoon breadth
R^{MAX}	Maximum allowed breadth/height ratio for pontoon
R^{MIN}	Minimum required breadth/height ratio for pontoon
$L^{C,MAX}$	Upper boundary for column length
$L^{C,MIN}$	Lower boundary for column length
$H^{C,MAX}$	Upper boundary for column height
$H^{C,MIN}$	Lower boundary for column height
$B^{C,MAX}$	Upper boundary for column breadth
$B^{C,MIN}$	Lower boundary for column breadth
$D^{P,MAX}$	Upper boundary for distance between the pontoons
$D^{P,MIN}$	Lower boundary for distance between the pontoons
$D^{C,MAX}$	Upper boundary for distance between the columns
$D^{C,MIN}$	Lower boundary for distance between the columns
Y	Factor that restricts the distance between columns as a function of pontoon length

9.1.3. Variables

Decision Variables

l^P:	Pontoon length
h^P:	Pontoon height
b^P:	Pontoon breadth
l^C:	Column length
h^C:	Column height
b^C:	Column breadth
d^P:	Distance between pontoons
d^C	Distance between columns

Objective function

(1) $$\min z = w^P + w^C + w^B$$

Auxiliary variables

$g_t^{TRANSVERSAL}$	Transversal GM value in condition t
$g_t^{LONGITUDINAL}$	Longitudinal GM value in condition t
$d_t^{KEEL-COB}$	Vertical Distance from the keel to COB in condition t
$d_t^{COB-META,TRANSVERSAL}$	Vertical distance from COB to transversal metacenter in condition t
$d_t^{COB-META,LONGITUDINAL}$	Vertical distance from COB to longitudinal metacenter in condition t
$d_t^{KEEL-COG}$	Vertical distance from keel to COG in condition t
i_t^{XX}	Second moment of area of the waterplane area around the x axis in condition t
i_t^{YY}	Second moment of area of the waterplane area around the y axis in condition t
p_t^{HEAVE}	Eigen period in heave in condition t
$w^{AM,HEAVE}$	Total added mass in heave
$a_t^{WATERLINE}$	Waterplane area in condition t

∇_t^P	The volume displacement of the pontoons in condition t
w^P	Weight of the pontoons
$d_t^{P,COB}$	Vertical distance from keel to COB of pontoons in condition t
$d^{P,COG}$	Vertical distance from keel to COG of pontoons
m^P	Vertical moment of the pontoons
∇_t^C	The volume displacement of the columns in condition t
w^C	Weight of the columns
$d_t^{C,COB}$	Vertical distance from keel to COB of the columns in condition t
$d^{C,COG}$	Vertical distance from keel to COG of columns
m^C	Vertical moment of the columns
w^B	Weight of the braces
∇_t^B	The volume displacement of the braces is condition t
$d_t^{B,COB}$	Vertical distance from keel to COB of the braces in condition t
$d^{B,COG}$	Vertical distance from keel to COG of the braces
m^B	Vertical moment of the braces
∇_t	Volume displacement in condition t
Δ_t	Weight displacement in condition t
$w_t^{BALLAST}$	Weight of the ballast water in condition t
$d^{BALLAST,COG}$	Vertical distance from keel to COG of the ballast water
$m_t^{BALLAST}$	Vertical moment of the ballast in condition t
$d^{TS,COG}$	Vertical distance from keel to COG of the topside
m^{TS}	Vertical moment of the topside
$d^{VDL,COG}$	Vertical distance from keel to COG of the VDL
m_t^{VDL}	Vertical moment of the VDL in condition t
a^{DECK}	Estimated deck area
f	Draft in transit condition

9.1.4. Constraints

Stability

(2) $$g_t^{TRANSVERSAL} \geq S_t^{TRANSVERSAL,MIN} \qquad t\epsilon T$$

(3) $$g_t^{LONGITUDINAL} \geq S_t^{LONGITUDINAL,MIN} \qquad t\epsilon T$$

(4) $$g_t^{TRANSVERSAL} \leq S_t^{TRANSVERSAL,MAX} \qquad t = Su, Op$$

(5) $$g_t^{LONGITUDINAL} \leq S_t^{LONGITUDINAL,MAX} \qquad t = Su, Op$$

(6) $$g_t^{TRANSVERSAL} = d_t^{KEEL-COB} + d_t^{COB-META,TRANSVERSAL} - d_t^{KEEL-COG} - O_t \qquad t\epsilon T$$

(7) $$g_t^{LONGITUDINAL} = d_t^{KEEL-COB} + d_t^{COB-META\,,LONGITUDINAL} - d_t^{KEEL-COG} - O_t \qquad t\epsilon T$$

(8) $$d_t^{KEEL-COB} = \frac{d_t^{P,COB}\nabla_t^P + d_t^{C,COB}\nabla_t^C + d_t^{B,COB}\nabla_t^B}{\nabla_t^P + \nabla_t^C + \nabla_t^B} \qquad t\epsilon T$$

(9) $$d_t^{KEEL-COG} = \frac{m^P + m^C + m^B + m^{TS} + m_t^{VDL} + m_t^{BALLAST}}{w^P + w^C + W^{TS} + w^B + w_t^{BALLAST} + W_t^{VDL}} \qquad t\epsilon T$$

(10) $$d_t^{COB-MC,TRANSVERSAL} = \frac{i_t^{XX}}{\nabla_t} \qquad t\epsilon T$$

(11) $$d_t^{COB-MC,LONGITUDINAL} = \frac{i_t^{YY}}{\nabla_t} \qquad t\epsilon T$$

(12) $$i_t^{XX} = \frac{1}{3}l^C(b^C)^3 + b^C l^C (d^P)^2 \qquad t = Su, Ops$$

(13) $$i_t^{XX} = \frac{1}{6}l^P(b^P)^3 + \frac{1}{2}l^P b^P (d^P)^2 \qquad t = Tr$$

(14) $$i_t^{YY} = \frac{1}{3}b^C(l^C)^3 + b^C l^C (d^C)^2 \qquad t = Su, Op$$

(15) $$i_t^{YY} = \frac{1}{6}b^P(l^P)^3 \qquad t = Tr$$

Motion characteristics

(16)	$p_t^{HEAVE} \geq T_t^{HEAVE,MIN}$	$t \epsilon T$
(17)	$p_t^{HEAVE} = \sqrt{\frac{\Delta_t + w_t^{AM,HEAVE}}{\rho g a_t^{WATERLINE}}}$	$t \epsilon T$
(18)	$w^{AM,HEAVE} = C^{AM,HEAVE} \rho \pi \frac{(b^p)^2}{2} l^P$	
(19)	$a_t^{WATERLINE} = 4 l^C b^C$	$t = Su, Op$
(20)	$a_t^{WATERLINE} = 2 l^P b^P$	$t = Tr$
(21)	$f = h^P - K$	

Weight and buoyancy

(22)	$\nabla_t^P = 2 l^P b^P h^P$	$t = Su, Op$
(23)	$\nabla_t^P = 2 l^P b^P f$	$t = Tr$
(24)	$w^P = 2 l^P b^P h^P V^P$	
(25)	$d_t^{P,COB} = \frac{1}{2} h^P$	$t = Su, Op$
(26)	$d_t^{P,COB} = \frac{1}{2} f$	$t = Tr$
(27)	$d^{P,COG} = \frac{1}{2} h^P$	
(28)	$m^P = w^P d^{P,COG}$	
(29)	$\nabla_t^C = 4 l^C b^C (F_t - h^P)$	$t = Su, Op$

(30) $\nabla_t^C = 0$ $t = Tr$

(31) $w^C = 4l^C b^C h^C V^C$

(32) $d_t^{C,COB} = h^P + \frac{F_t - h^P}{2}$ $t = Su, Op$

(33) $d_t^{C,COB} = 0$ $t = Tr$

(34) $d^{C,COG} = \frac{1}{2} h^C + h^P$

(35) $m^C = w^P d^{P,COG}$

(36) $w^B = \frac{J^B}{1 - J^B}(w^P + w^C)$

(37) $\nabla_t^B = \frac{w^B F_t}{V^B (h^c - Q^B)}$ $t = Su, Op$

(38) $\nabla_t^B = 0$ $t = Tr$

(39) $d_t^{B,COB} = h^P + H^B + \frac{1}{2}(F_t - h^P - Q^B)$ $t = Su, Op$

(40) $d_t^{B,COB} = 0$ $t = Tr$

(41) $d^{B,COG} = h^P + Q^B + \frac{1}{2}(h^c - Q^B)$

(42) $m^B = w^B d^{B,COG}$

(43) $\nabla_t = \nabla_t^P + \nabla_t^C + \nabla_t^B$ $t \epsilon T$

(44) $\Delta_t = \rho \nabla_t$ $t \epsilon T$

(45) $w_t^{BALLAST} = \Delta_t - w^P - w^C - W^{TS} - W_t^{VDL} - w^B$ $t \epsilon T$

(46) $w_t^{BALLAST} \geq 0$ $t \epsilon T$

(47) $w_t^{BALLAST} \leq 2 l^P b^P h^P Z$ $t \epsilon T$

(48) $d^{BALLAST,COG} = \frac{1}{2} h^P$

(49) $m_t^{BALLAST} = w_t^{BALLAST} d^{BALLAST,COG}$ $t \epsilon T$

(50) $d^{TS,COG} = G^{TS} + h^P + h^C$

(51) $d^{VDL,COG} = G^{VDL} + h^P + h^C$

(52) $m^{TS} = W^{TS} d^{TS,COG}$

(53) $m_t^{VDL} = W_t^{VDL} d^{VDL}$ $t \epsilon T$

Air gap constraints

(54) $h^P + h^C - F_t \geq A_t$ $t = Su, Op$

Geometrical constrains

(55) $l^P \leq L^{P,MAX}$

(56) $l^P \geq L^{P,MIN}$

(57) $h^P \leq H^{P,MAX}$

(58) $h^P \geq H^{P,MIN}$

(59) $$b^P \leq B^{P,MAX}$$

(60) $$b^P \geq B^{P,MIN}$$

(61) $$b^C \leq Ub^P$$

(62) $$\frac{b^P}{h^P} \leq R^{MAX}$$

(63) $$\frac{b^P}{h^P} \geq R^{MIN}$$

(64) $$l^C \leq L^{C,MAX}$$

(65) $$l^C \geq L^{C,MIN}$$

(66) $$h^C \leq H^{C,MAX}$$

(67) $$h^C \geq H^{C,MIN}$$

(68) $$b^C \leq B^{C,MAX}$$

(69) $$b^C \geq B^{C,MIN}$$

(70) $$d^P \leq D^{P,MAX}$$

(71) $$d^P \geq D^{P,MIN}$$

(72) $$d^C \leq D^{C,MAX}$$

(73) $$d^C \geq D^{C,MIN}$$

(74) $$d^C \leq Y(l^P - l^C)$$

Other constraints

(75) $$a^{DECK} = (d^C + l^C)(d^P + b^C)$$

(76) $$a^{DECK} \geq A^{DECK,MIN}$$

Constraint	Description
(2)	Ensures that the transversal GM values are above a lower boundary in condition t
(3)	Ensures that the longitudinal GM values are above a lower boundary in condition t
(4)	Ensures that the transversal GM values are lower than a upper boundary in the survival and operational condition
(5)	Ensures that the longitudinal GM values are lower than a upper boundary in the survival and operational condition
(6)	Enables the model to calculate the transversal GM values in condition t
(7)	Enables the model to calculate the longitudinal GM values in condition t
(8)	Enables the model to calculate vertical distance from keel to COB in condition t
(9)	Enables the model to calculate vertical distance from keel to COG in condition t
(10)	Enables the model to calculate the vertical distance between the COB and the transversal metacenter in condition t
(11)	Enables the model to calculate the vertical distance between the COB and the longitudinal metacenter in condition t
(12)	Enables the model to calculate the second moment of area for waterplane area around the x axis for the survival and operational condition
(13)	Enables the model to calculate the second moment of area for waterplane area around the x axis for the transit condition
(14)	Enables the model to calculate the second moment of area for waterplane area around the y axis for the survival and operational condition
(15)	Enables the model to calculate the second moment of area for waterplane area around the y axis for the transit condition
(16)	Ensures that the Eigen Period in heave are above a lower boundary in condition t
(17)	Enables the model to calculate the Eigen Period in heave for condition t

(18) Enables the model to calculate the total added mass in heave

(19) Enable the model to calculate the waterplane area for the survival and operation conditions

(20) Enable the model to calculate the waterplane area for the transit condition

(21) Determines the draft in the transit condition

(22) Gives the volume displacement of the pontoons in survival and operational condition

(23) Gives the volume displacement of the pontoons for the transit condition

(24) Gives the linear relationship between volume and weight for the pontoons

(25) Gives the vertical distance from the keel to COB of the pontoons in survival and operational condition

(26) Gives the vertical distance from the keel to COB of the pontoons in the transit condition

(27) Gives the vertical distance from the keel to COG of the pontoons

(28) Gives the vertical moment of the pontoons

(29) Gives the volume displacement of the columns for the survival and operational condition

(30) Gives the volume displacement of the columns for the transit condition

(31) Gives the linear relationship between volume and weight for the columns

(32) Gives the vertical distance from the keel to COB of the columns in survival and operational condition

(33) Gives the vertical distance from the keel to COB of the columns in the transit condition

(34) Gives the vertical distance from the keel to COG of the columns

(35) Gives the vertical moment of the columns

(36) Enables the model to estimate the bracing weight based on a input parameter which gives the bracing weight as a fraction of the total weight

(37) Gives the volume displacement of the braces as a function of drafts for the survival and operational condition

(38) Gives the volume displacement of the braces for the transit condition

(39) Gives the vertical distance from the keel to COB of the braces in survival and operational condition

(40) Gives the vertical distance from the keel to COB of the braces in the transit condition

(41) Gives the vertical distance from the keel to COG of the braces

(42) Gives the vertical moment of the bracing

(43) Gives the total volume displacement for condition t

(44) Gives the relationship between the volume and weight displacement

(45) Gives necessary amount of ballast water to achieve equilibrium between weight and buoyancy in condition t

(46) Ensures that the amount of ballast not can be negative in any of the conditions

(47) Ensures that the ballast water in the pontoons not exceed the ballast capacity

(48) Gives the vertical distance from the keel to COG of the ballast water

(49) Gives the vertical moment of the ballast water in condition t

(50) Gives the vertical distance from the keel to COG of the topside

(51) Gives the vertical distance from the keel to COG of the VDL

(52) Gives the vertical moment of the topside

(53) Gives the vertical moment of the VDL in condition t

(54) Ensures sufficient air gap in survival and operational condition

(55) Ensures that the pontoon length is smaller than a upper bound

(56) Ensures that the pontoon length is larger than a lower bound

(57) Ensures that the pontoon height is smaller than a upper bound

(58) Ensures that the pontoon height is larger than a lower bound

(59) Ensures that the pontoon breadth is lower than a upper bound

(60) Ensures that the breadth of the pontoons are bigger or equal to the breadth of the columns

(61) Ensures that the breadth of the columns are smaller than the breadth of pontoons by a constant which should be smaller than 1

(62) Ensures that the breadth height ratio of the pontoon are smaller than a upper

boundary

(63) Ensures that the breadth height ratio of the pontoon are larger than a lower boundary

(64) Ensures that the column length is lower than a upper bound

(65) Ensures that the column length is larger than a lower bound

(66) Ensures that the column height is lower than a upper bound

(67) Ensures that the column height is larger than a lower bound

(68) Ensures that the column breadth is lower than a upper bound

(69) Ensures that the column breadth is larger than a lower bound

(70) Ensures that the distance between the pontoons are lower than a upper bound

(71) Ensures that the distance between the pontoons are larger than a lower bound

(72) Ensures that the distance between the columns are lower than a upper bound

(73) Ensures that the distance between the columns are larger than a lower bound

(74) Ensures that distance between the columns are restricted by the length of the pontoons multiplied by a constant which should be smaller than 1

(75) Enables the model to estimate the deck area

(76) Ensures that the estimated deck area is larger than a lower boundary

10. Appendix B

The following section gives some pictures of the model in Excel.

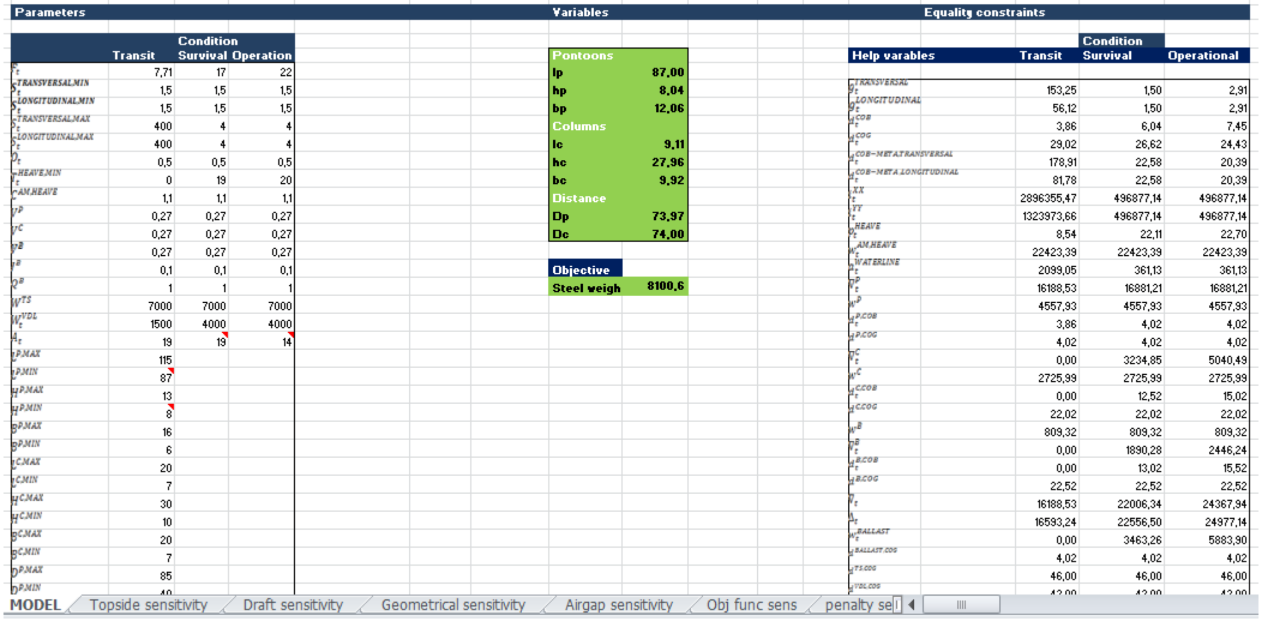

Figure 15 The optimization model in Microsoft Excel

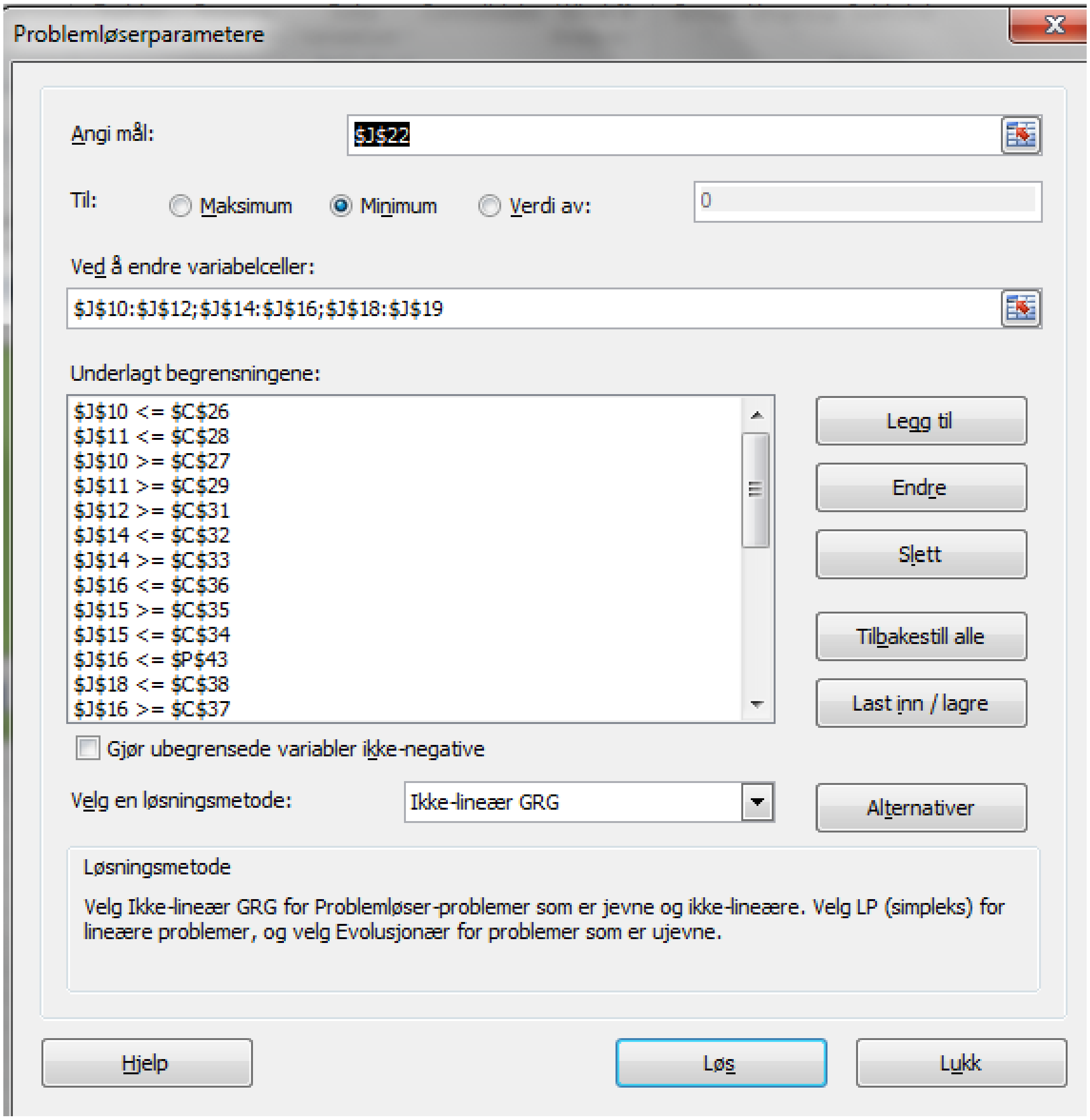

Figure 16 Setting up the add-in solver

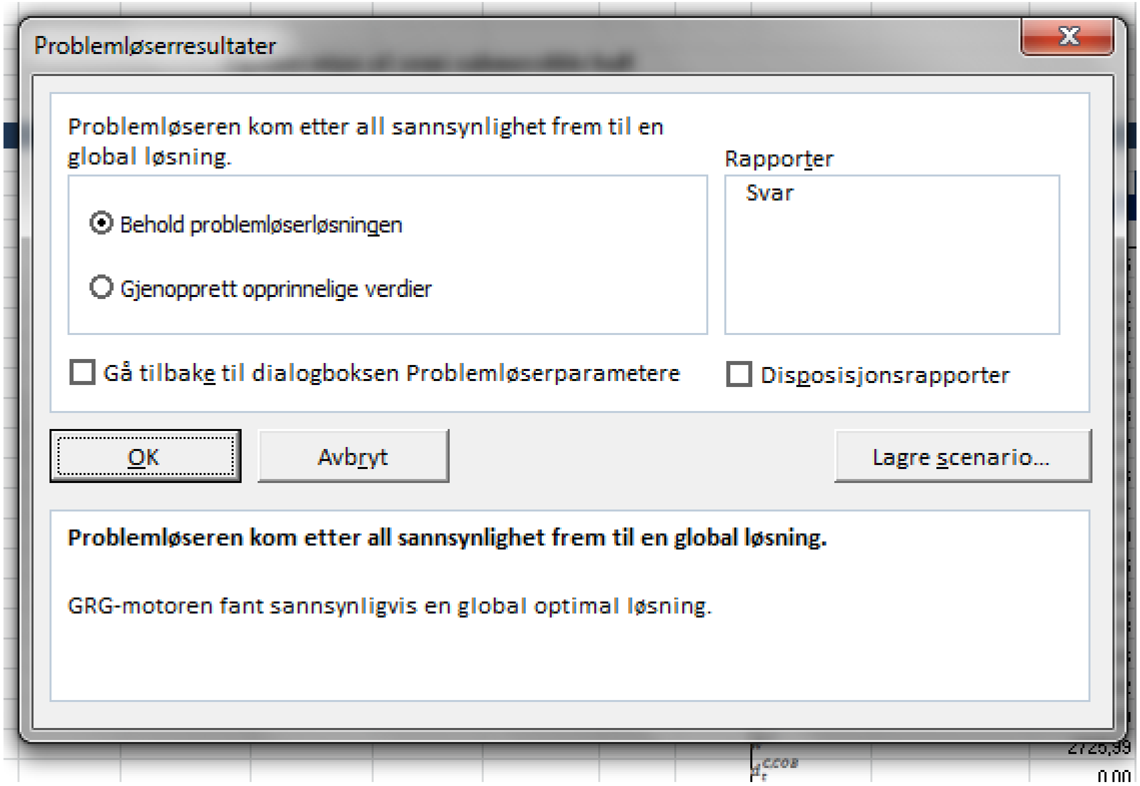

Figure 17 Solution message from the solver