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Airborne Wind Turbines for Ship Propulsion

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Marine Technology

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Abstract:

As the focus on the environment and how to produce “clean” and renewable energy takes a lot of focus these days, the research on this field is constantly increasing. Especially in the shipping industry there are frequently new rules and limitations being set into force. To keep up with these restrictions one needs to think outside the box and develop new ways to make it possible to keep the fuel consumption down, thus becoming more environmentally friendly.

AWTs (Airborne Wind Turbines) could be one of the options. An AWT operates on the same aerodynamic principles as a conventional wind turbine. It is connected with a tether to the ground, utilizing stronger wind high above ground, inducing large enough lift forces on the wing to keep it flying. The wing will then go into a crosswind motion and it will be able to achieve a fairly high incoming velocity. By mounting turbines on the wing, it will then be possible to produce power. The power equation is proportional to the incoming wind speed cubed, and due to the high incoming velocity the potential power output of the AWT is therefore very high.

In this master thesis it will be explored if it could be possible to mount an AWT on a ship to assist the propulsion. Also the forces on the wing have been calculated in order to check how large wind speed is needed in order for the wing to stay aloft.

A big concern to this idea is the forces in the tether. The idea will be considered feasible if the thrust to the propeller given by the produced power of the wing is higher than the resistance on the ship due to the horizontal forces in the tether.

The calculations show that a minimum wind speed of 4.5 [m/s] is needed for the wing to stay aloft. The propulsion given to the propeller from the power production of the wing is for all ship speeds too low to consider the idea feasible.

Keyword:

Airborne wind turbine propulsion

Fuel saving

Supervisor / Advisor:

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Sammendrag:

Ettersom fokuset på miljø og hvordan man kan produsere ren og fornybar energi tar mye fokus i disse dager, er dette et felt med stadig økende forskning. Spesielt i shippingbransjen er det stadig nye regler og begrensninger som blir satt i kraft. For å holde tritt med disse restriksjonene trenger man å tenke utenfor "boksen" og finne nye måter som gjør det mulig å holde drivstofforbruket nede og på den måten bli mer miljøvennlig.

AWTs (Airborne Wind Turbines) kan være en av mulighetene. En AWT fungerer etter samme aerodynamiske prinsippene som en konvensjonell vindturbin. Festet i en slags wire til bakken utnytter den de sterkere vindhastighetene høyt oppe som induserer høye nok løftkrefter til å holde den flygende. Vingen vil gå inn i en sirkulær bane, noe som vil gjøre det mulig å oppnå temmelig høy hastighet. Ved å montere turbiner på vingen kan den produsere energi. Ligningen for energiutvinningen er proporsjonal med innkommende vindhastighet i tredje, og på grunn av nettopp denne høye innkommende hastigheten er energiutvinningspotensialet veldig stort.

I denne masteroppgaven vil muligheten for å montere en slik AWT på et skip for å bistå fremdriften bli utforsket. Også kreftene som virker på vingen er blitt utregnet for å sjekke hvor høye vindhastigheter som trengs for å holde vingen flygende.

En stor bekymring til denne ideen er kreftene i wiren. Ideen vil bli regnet som gjennomførbar hvis kraften til propellen gitt av energien produsert av vingen er større en motstanden til skipet på grunn av den horisontale kraften i wiren. Utregningene viser at det kreves en minimum vindhastighet på 4.5 [m/s] for at vingen skal holde seg flygende. Propulsjonskraften til propellen fra energien produsert av vingen er for alle skipshastigheter for lav for å regne ideen som gjennomførbar.

Nøkkelord:

Airborne wind turbine propulsion

Drivstoffbesparelse

Veileder / Rådgiver:

Prof. Sverre Steen / Eirik Bøckmann



MASTER THESIS IN MARINE TECHNOLOGY

SPRING 2013

FOR

Kristian Malde Gilje

Airborne Wind Turbines for Ship Propulsion

Both due to environmental concerns and increasing fuel prices there is an increasing demand for energy saving in the international shipping fleet. In addition to the conventional energy saving devices, it is discussed to use wind and waves for auxiliary propulsion. It has been shown that a ship can be propelled directly against the wind by using one or more wind turbines mounted on the ship. For land-based electricity generation, flying wind turbines has been proposed. They have the benefit of working at higher altitude, where there is stronger winds, and to sweep a larger area than conventional wind turbines. The idea for this master thesis is to investigate the feasibility of using such flying wind turbines for ship propulsion. Thus, the objective of the thesis is to investigate the feasibility and potential of flying wind turbines for ship propulsion. An important factor, which is of very different significance for land and ship application, is the pull in the line holding the flying turbine. This pull must be explored, and its significance for the ship propulsion must be made clear. On this basis, the following activities are recommended:

- Give a brief overview of alternative concepts for wind propulsion of ships, with their pro's and con's.
- Discuss in more detail wind turbine propulsion.
- Describe airborne wind turbines, and provide simplified calculations of the power that can be extracted and the direction and pull in the mooring line – for different relative wind directions and wind speeds.
- Perform calculations of potential power savings for a case vessel.
- Give recommendations regarding the feasibility and attractiveness of this concept applied on ships.

In the thesis the candidate shall present his personal contribution to the resolution of problem within the scope of the thesis work.

Theories and conclusions should be based on mathematical derivations and/or logic reasoning identifying the various steps in the deduction.

The candidate should utilize the existing possibilities for obtaining relevant literature.

The thesis should be organized in a rational manner to give a clear exposition of results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Telegraphic language should be avoided.

The thesis shall contain the following elements: A text defining the scope, preface, list of contents, summary, main body of thesis, conclusions with recommendations for further work, list of symbols and acronyms, reference and (optional) appendices. All figures, tables and equations shall be numerated.



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The supervisor may require that the candidate, in an early stage of the work, present a written plan for the completion of the work. The plan should include a budget for the use of computer and laboratory resources that will be charged to the department. Overruns shall be reported to the supervisor.

The original contribution of the candidate and material taken from other sources shall be clearly defined. Work from other sources shall be properly referenced using an acknowledged referencing system.

The thesis shall be submitted electronically in DAIM:

- Signed by the candidate
- The text defining the scope, signed by the supervisor, included
- Electronic appendages (video, computer code etc.) can be uploaded in a zip-file.

Supervisor : Professor Sverre Steen
Advisor : Eirik Bøckmann
Start : 14.01.2013
Deadline : 30.06.2013

Trondheim, 13.06.2013

Sverre Steen
Supervisor

Preface

I would like to thank both my supervisor Prof. Sverre Steen and PhD. candidate Eirik Bøckmann at the Department of Marine Technology, Norwegian University of Science and Technology, for giving me guidance and the necessary literature in this master thesis.

Trondheim 13.06.2013

A handwritten signature in blue ink that reads "Kristian M. Gilje". The script is cursive and fluid, with the first name "Kristian" and last name "Gilje" clearly legible, and the middle initial "M." in between.

Kristian Malde Gilje

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Nomenclature

α	Angle of attack
ω	Rotational speed of wing in radians/second
ρ_a	Mass density of air
ρ_w	Mass density of water
ζ	Propulsive efficiency
a	Axial induction factor
A_{turb}	Area of turbines
Asp	Aspect area of the wing
AWT	Airborne Wind Turbine
$C_{D,i}$	Dimensionless induced drag force coefficient
$C_{D,\text{turb}}$	Dimensionless drag force coefficient of turbines
$C_{D,v}$	Dimensionless viscous drag force coefficient
C_D	Dimensionless drag force coefficient
C_L	Dimensionless lift force coefficient
C_P	Dimensionless power coefficient
D	Total drag force
D_{turb}	Drag force of turbines
D_T	Drag force projected in tether direction
D_{unit}	Drag unit vector
DOF	Degrees of freedom
F	Propeller thrust
J	Propeller advance number
L	Lift force

L'	Lift force normal to V_A
L_T	Lift force projected in tether direction
L_{unit}	Lift unit vector
n	Propeller revolution speed
N_{turb}	Number of turbines mounted on the wing
NED	North-East-Down
P	Power coefficient
P_D	Power produced by wing
R	Circling radius
S	Wing planform area
s	Wing span
T	Period of one orbit
TSR	Tip-speed ratio
u	Ship speed
V_A	Incoming velocity
V_W	Wind speed
W	Weight of the wing
W_T	Weight force projected in tether direction

1 Introduction

Today there is a big focus on the environment and how to produce "clean" energy. One of the resources that stands out is wind power, which is an enormous resource. An idea by a company called Makani [10] is to use airborne wind turbines instead of the conventional wind turbines that we know of today. Since 2006 research has been done on AWTs by this company and they have also tested their prototype wing with promising results, see fig. 1.1.



Figure 1.1: Step-by-step overview of the Makani prototype in one orbit [10]

An AWT has the advantage of going in big circles meaning the wind that the turbines will feel is very large at all times. It stays aloft due to the wind speed that induces lift forces. With turbines mounted, it produces power. Since the power produced by a windmill is proportional to the wind speed cubed, the potential using AWT's are very promising.

In this master thesis it will be explored the possibility of having an airborne wind turbine mounted to a ship to create additional propulsion making the ship possibly more environmentally friendly.

The thesis includes a part with some background, including other ways to assist the propulsion system of ships, theory of AWTs explaining how the forces are calculated and at last some results and conclusion.

2 Background

2.1 Utilize wind for ship propulsion

Finding new ways that will lower emissions such as NO_x , SO_x and CO_2 is an ongoing process, and important for our environment. Also in the shipping industry, with demands that continuously are getting stricter, it is sought to lower these emissions as much as possible. Previous solutions, when looking solely at wind power, are such as kites [14], Flettner rotors [20] and windmills [16], which in the following section will be briefly discussed.

2.1.1 SkySails

SkySails, founded in 2001, is a company that has found a solution using kites as an auxiliary propulsion system on vessels. The fuel costs are thereby lowered and thus the emission levels are significantly reduced. Figure 2.1 shows how this kite works on a modern cargo ship.



Figure 2.1: Cargo ship using the patented SkySails for auxiliary propulsion [14]

The system consists of three main components: a towing kite with rope, a launch and recovery system, and a control system for automated operation, seen in figure 2.2. The concept seems relevant to mention in regards to this master thesis as it is similar in many

ways. The difference is that instead of producing electricity as the AWT does, this kite works as a pulling system.

SkySails states that the fuel consumption can be lowered with an average of 10-15% pr. year.

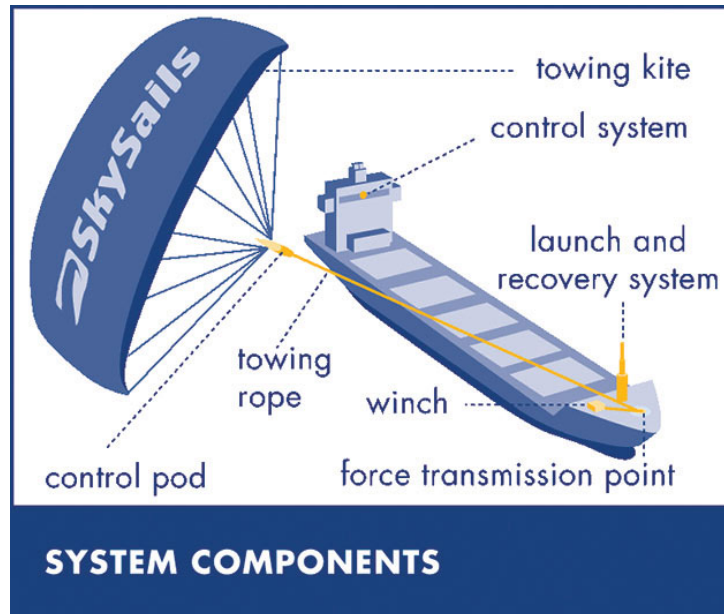


Figure 2.2: SkySails system [14]

2.1.2 Flettner rotors

Another concept is the so-called Flettner rotors. A Flettner ship is designed to use the Magnus effect for propulsion. To use this effect, vertical, spinning cylinders are needed as the Magnus effect is a force acting perpendicularly to the airstream around a spinning body. This is visualized in fig. 2.3 on the facing page.

Anton Flettner ¹ was the first to build a ship which attempted to use this effect, and in 1924 the *Buckau*, later *Baden Baden*, seen in fig. 2.4 on the next page, was finished. This idea worked, but it was seen that the propulsion force generated was less than the motor would have generated if it had been connected to a standard marine propeller [20]. Also the rotor system could not compete economically with the diesel engines at that time.

¹Anton Flettner (1885-1961) was a German engineer and inventor [21]

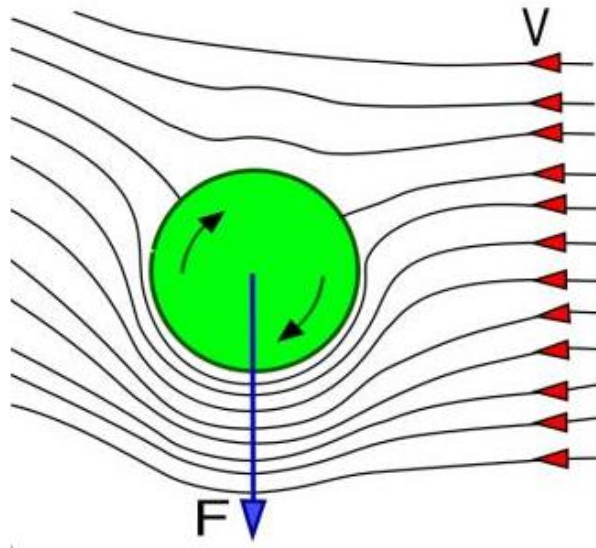


Figure 2.3: The Magnus effect and the direction of this force [17]

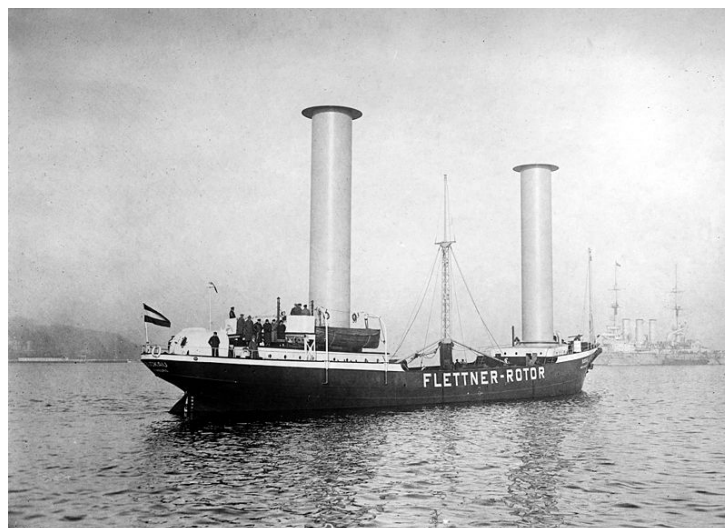


Figure 2.4: The Buckau Flettner Rotor Ship [20]

Renewed interest in the concept came in the later 20th century and in 2010 came *E-Ship1*, seen in fig. 2.5 on the following page. Energy efficiency has been the focus of this ship's design, and together with the 4 mounted Flettner-Rotors the 123-meter-long cargo ship is said to reduce the fuel consumption by up to 30-40%.



Figure 2.5: The E-Ship 1 [3]

2.1.3 Windmills

The idea of using a windmill mounted on a ship for propulsion has been done on the 36 foot catamaran *Revelation II*, seen in fig. 2.6 on the next page. The catamaran uses a water propeller which is mechanically coupled to a wind turbine to propel the boat. A system like this might lower the fuel consumption drastically if the wind turbine is custom designed for a specific displacement hull. The draw-back with this system is that if it is used on a large ship, a very large wind turbine would be needed, and the size of wind turbines is limited. A very large wind turbine could be a problem if the ship has to cross below bridges.

This is also the idea described in an article by Bøckmann and Steen, [16], and the theory described has been much applicable in the problem concerning this master thesis.



Figure 2.6: Catamaran *Revelation II* with a mounted wind turbine [6]

2.2 AWT for ship propulsion

In this master thesis the concept by the Makani group [10] will be looked more closely into. The Makani group is pursuing a way to extract high altitude wind energy by using a flying wing with turbines, so-called "Airborne Wind Turbine", mounted to the ground in a tether. The research has been ongoing since 2006, and they have so far come up with promising results in their field testing. In the following section some of the benefits with using such a wing for producing electricity will be discussed.

2.2.1 Why use the Makani wing?

The Makani wing operates on the same aerodynamic principles as a conventional wind turbine. Due to the air moving across the turbine blades, the turbine blades mounted on the Makani wing will be forced to rotate in the same way as the turbine blades on a wing turbine. But where the windmills have their limitation in size, the Makani solution is to only build the tip of the blade, as seen in fig. 2.7 on the following page.

This is an efficient way of extracting the most of the wind energy, as it is stated by

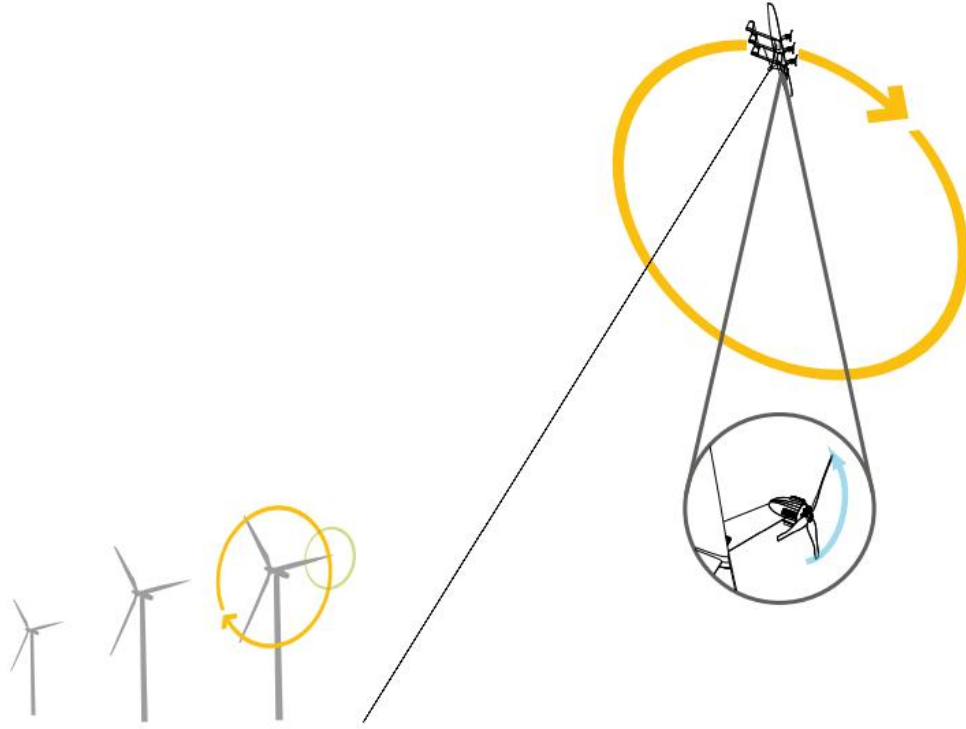


Figure 2.7: Windmills with varying size compared to the Makani wing [5]

Makani [10] that the last 25% of the blade on a windmill produces more than 50% of the total energy. It also shows in fig. 2.7 that the Makani wing is going in even bigger circles than the biggest windmill. This means that the incoming wind velocity on the turbines can be very high, and as the power produced is proportional to incoming velocity cubed, the potential power output is therefore very high. Another important aspect with this wing is that it works in a higher altitude than what a conventional wind turbine does. The empirical relation for calculating wind speed in different altitudes, is given in eq. 2.1.

$$V(z) = V_0 \left(\frac{z}{z_0} \right)^{0.143} \quad (2.1)$$

If the wind speed at 90 meters is then 10 [m/s], and the Makani wing operates at, say, 450 meters, the velocity is then 2.6 [m/s] higher. One might ask what is the big deal with increasing the speed with just another 2.6 [m/s]. The answer to this again leads to the power equation (eq. 2.2), which is proportional to incoming velocity cubed.

$$P = \frac{1}{2} \rho_a V^3 A \cdot C_P \quad (2.2)$$

The wing is also very light, and the need for materials is therefore lower than for a

conventional windmill.

What if the wind dies out?

A question that has been addressed in many forums discussing the feasibility of the Makani wing is what happens when the wind dies out. Makani states that a minimum of 3.5 [m/s] is needed in order for the wing to stay aloft. If the wind is even lower than this the Makani solution is that it is possible to put power into the system, meaning the turbines could for a while work as propellers. If this is just for a short period of time, power will be put into the system to keep the wing in its course, if, however, the wind seems to be lower over a longer period, the wing will go out of its crosswind motion, start hovering, almost similar as a helicopter, and safely move back to its perch on the ground, called "reeling in".

2.2.2 Prototype testing of the MAKANI M30

So far the Makani group has come up with promising results in the testing of their prototype model. This is an 8 meters wide wing with 4 turbines mounted, seen in fig. 2.8 and fig. 2.9 on the following page.

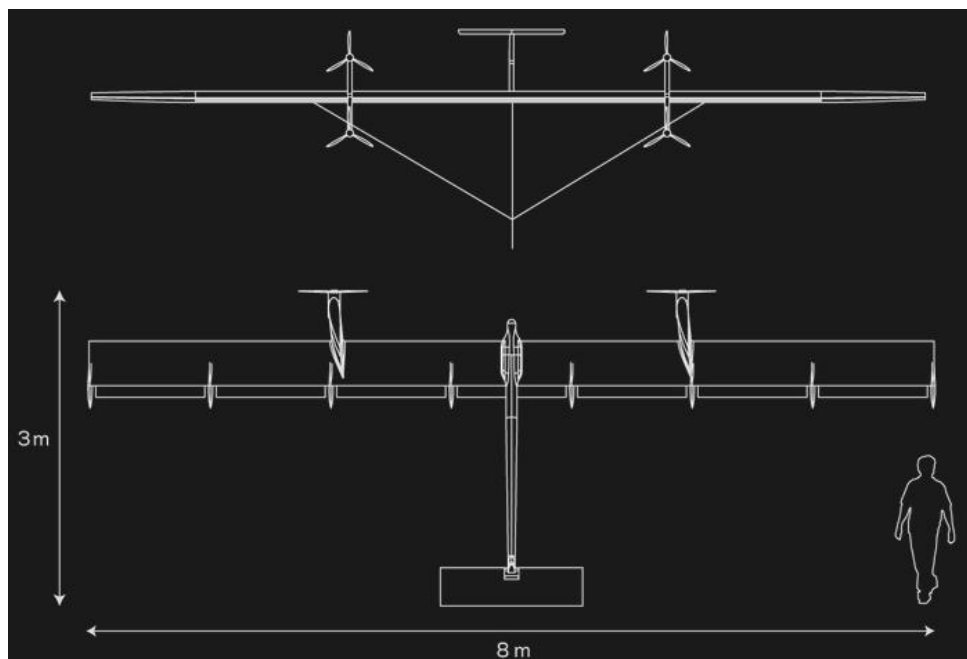


Figure 2.8: Makani M30 prototype wing [10]

Demonstrated in field testing they have done a fully autonomous run with successful results when it comes to:

- Take-off
- Hover
- Transition into crosswind
- Crosswind power generation
- Transition out of crosswind
- Reeling in and perching

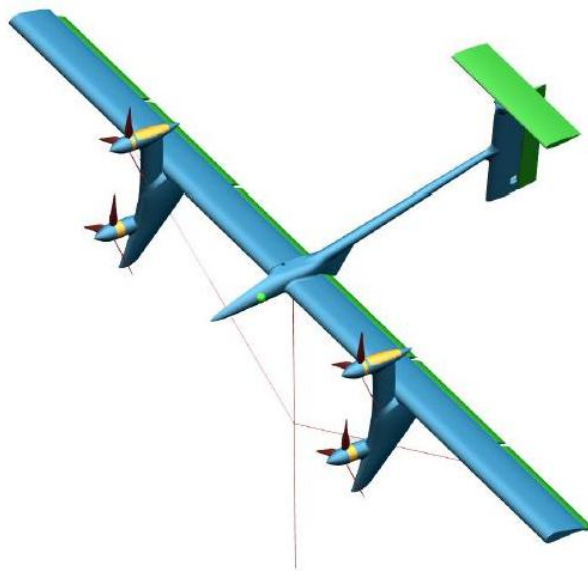


Figure 2.9: Makani M30 with 4 mounted turbines and the connecting tether [4]

As far as the prototype testing goes, Makani has been limited to the M30 wing. It is though interesting to look at Makani's future plans for development, seen in fig. 2.10 on the next page. The M5 wing is the one that has been used in the calculations in this master thesis.

2.2.3 Specifications of the Makani M5

The specifications of the Makani M5 wing is listed in tab. 2.1 on the facing page. Some of the values have been given by Makani, and some had to be estimated using figures from Makani's webpage [10].

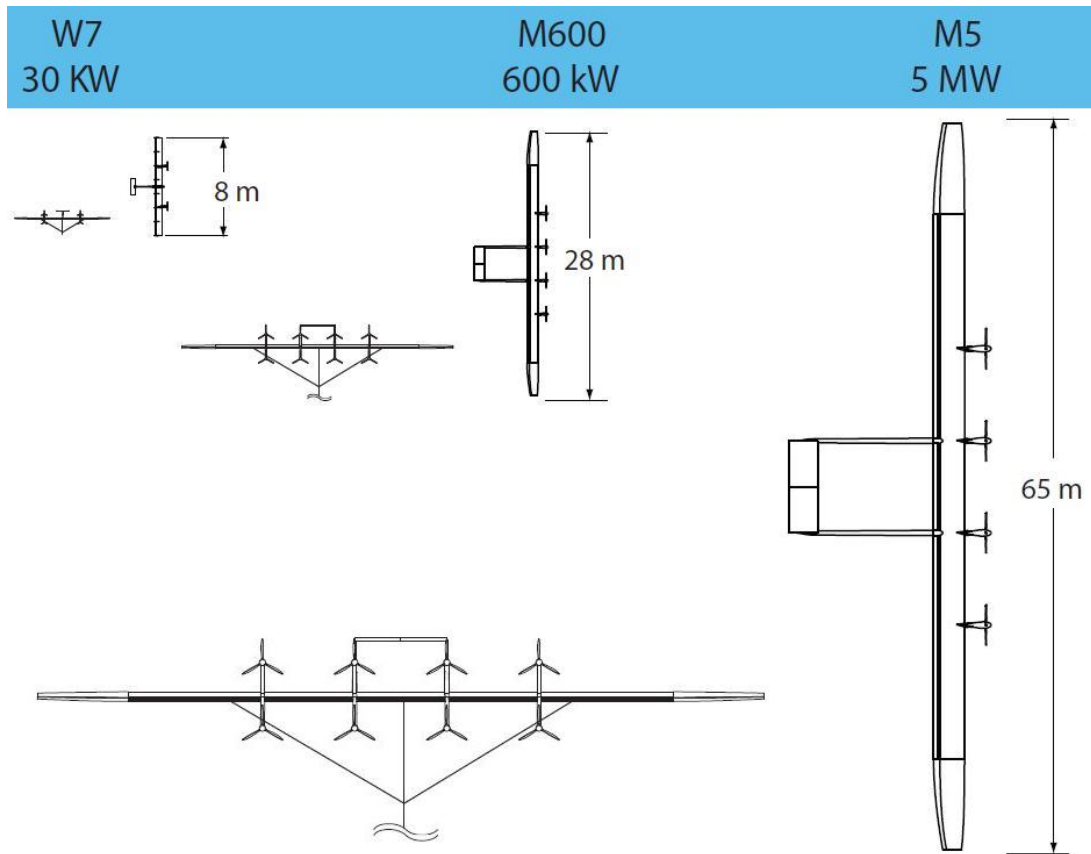


Figure 2.10: 3 different Makani wings [4]

Properties of the M5 wing	Value
Weight	9900 [kg]
Length of tether	1060 [m]
Circling radius	265 [m]
Top operational range	650 [m]
Wing span	65 [m]
Wing planform area	150 [m ²]
Aspect area of the wing	28.17 [m ²]
Area of turbines	12.57 [m ²]
Number of turbines	8 [-]

Table 2.1: Specifications of the Makani M5 [10] [12]

3 Theory

In the theory chapter the progress done in Matlab will be explained, which concerns the wing path, calculation of forces on the wing and the power production.

3.1 Project thesis

Previous to this master thesis the method described by Loyd [9] was used in a smaller project thesis. This method has been seen mentioned in most of the articles concerning AWTs as what started the interest of exploring this technology. This approach gave fair enough results in the project thesis, but too many uncertainties were introduced. Also the technology has developed a lot since when this article [9] was published. When it comes to for instance drag and weight of tether, there is a big difference between which numbers the Makani group are using and what is given in the article by Loyd.

In this master thesis the same idea as in the project thesis has been further explored, but with a different approach. Instead of using just the Loyd theory, it was sought to find a solution merely by looking at simple foil theory. To do this it was necessary to use a coordinate transformation between a local system on the wing, and a global system.

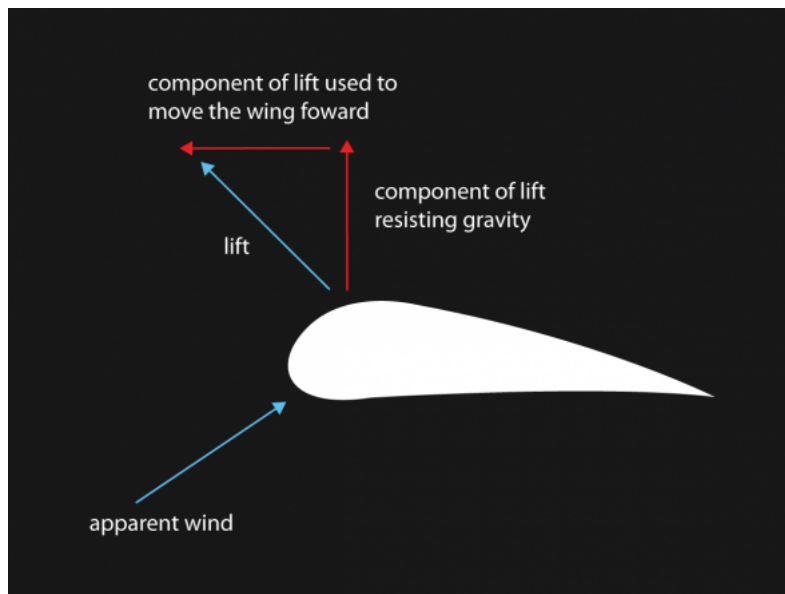


Figure 3.1: Lift force due to incoming wind on the wing [10]

3.2 Coordinate system

3.2.1 Wingpath

The first necessary step to do was to plot a realistic wingpath. Having studied a video of the Makani M30 prototype wing [11] made it possible to do this reasonable as long as the tether length, circling radius and top operational range was known (see tab. 2.1 on page 11).

In the project thesis the theory from Loyd, [9], was used fairly detailed as the wingpath could be replicated identical since some of the angles in the wing's path were given as constants (see fig. 3.2).

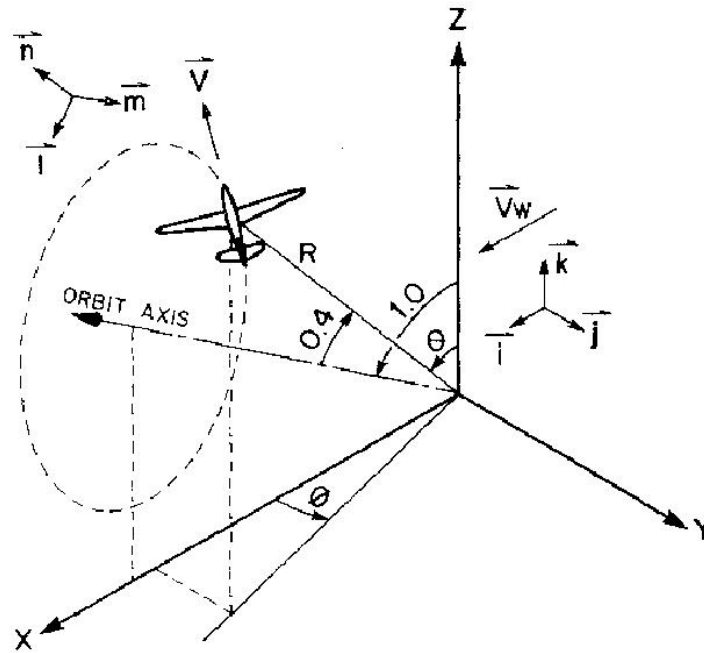


Figure 3.2: Wing coordinate system from Loyd, [9]

The article by Loyd has by many people been considered the "rebirth" of the research on AWTs. His calculations shows interesting results, and the article has also been referred to by the Makani group. His way of defining the axis system has been used in this master thesis, but with some modifications. He has for instance used fixed numbers for the angle between the orbit axis and the wing's position at any arbitrary point along its path and also between the z-axis and the orbit axis. These angles have been calculated by using given tether length, circling radius and top operational range.

A modification of a figure, found in [15], visualizes the wing seen from the side in 2D, 3.3. From this it can easily be explained how the fixed angles in the coordinate system by Loyd (see fig. 3.2 on the preceding page) can be calculated for this particular wing.

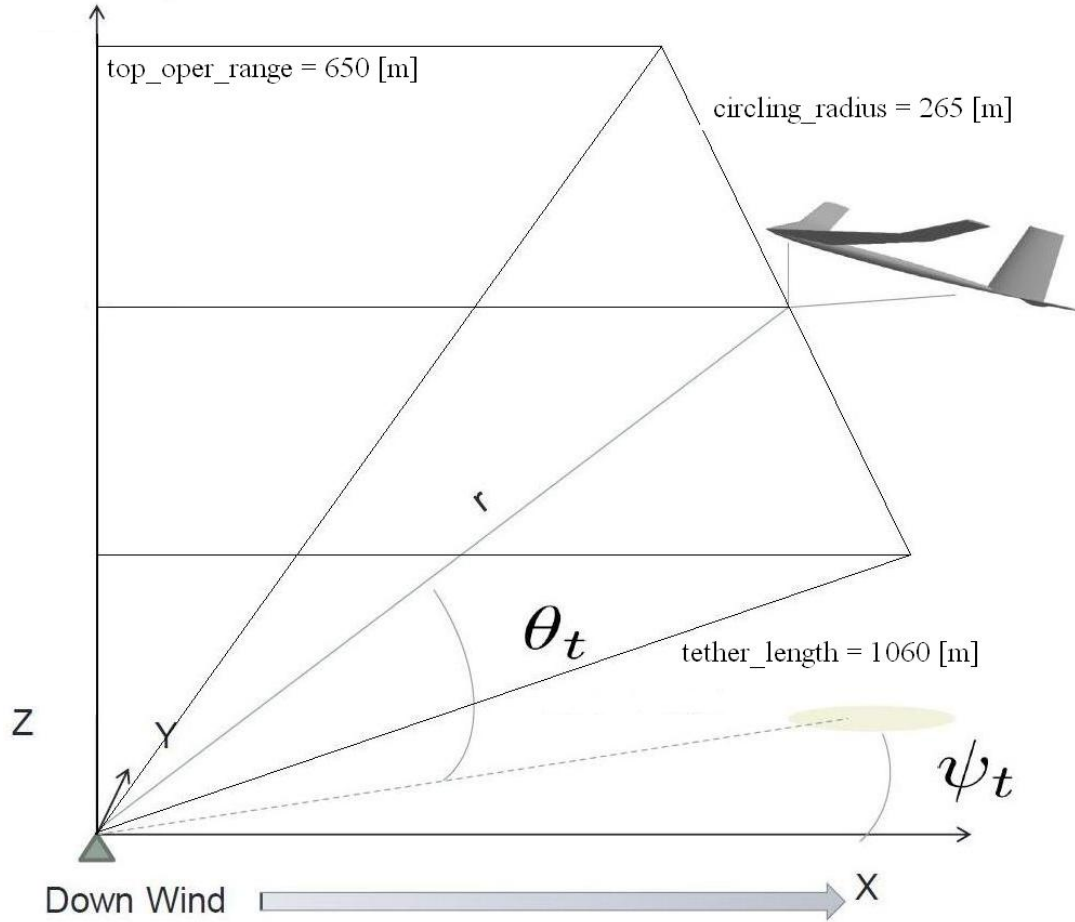


Figure 3.3: Modified wing seen in 2D [15]

The angle between center of orbit and the wingpath is calculated in eq. 3.1

$$rad_between_wing_and_orbit = \sin^{-1} \left(\frac{circling_radius}{length_of_tether} \right) \quad (3.1)$$

The angle between the z-axis and the orbit is calculated in eq. 3.2.

$$rad_between_z_axis_and_orbit = \cos^{-1} \left(\frac{top_oper_range}{length_of_tether} \right) + \quad (3.2)$$

$$rad_between_wing_and_orbit$$

Using these angles made it possible to get an idea of how the wing path would look like.

3.2.2 Motion Variables

When the wingpath has been plotted one may continue by looking closer into how the wing itself will move along this wing path. In Fossen [7] a lot of theory regarding ships and their motion variables are described. As much of the characteristics for a flying wing and a ship are similar, this theory can to a large extent also be applied for a wing. Therefore much of the theory described in this section has been found in Fossen. To see the similarity between the axis system of a ship and a wing (in this case represented as an airplane) it would be helpful to visualize this with the following figures, 3.4 and 3.5

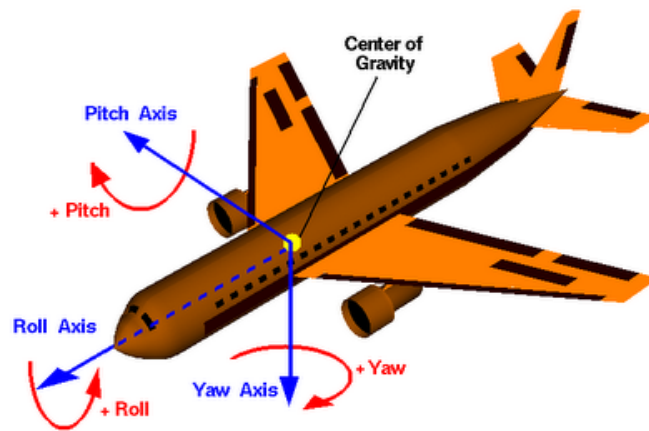


Figure 3.4: Coordinate system of an airplane [13]

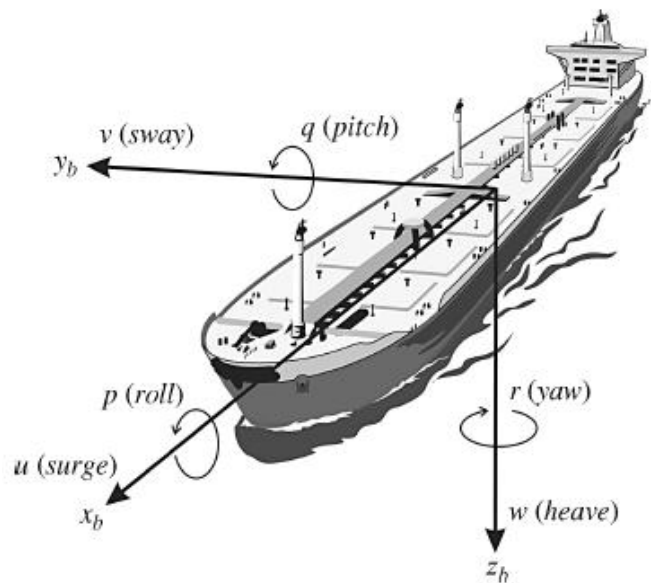


Figure 3.5: Coordinate system of a ship found in Fossen, [7]

For both the airplane and the ship one can see that there are six degrees of freedom (DOF). It is necessary to find these DOF at every point along the wing's path. To do this, the theory of Fossen [7] has been used for the transformation between two different coordinate systems.

DOF	Symbol	Explanation
Surge	η_1	Surging motion of wing (roll axis in fig. 3.4)
Sway	η_2	Sway motion of wing (pitch axis in fig. 3.4)
Heave	η_3	Heave motion of wing (yaw axis in fig. 3.4)
Roll	η_4	Movement of the wing tips of the wing up and down
Pitch	η_5	Movement of the nose of the wing up or down
Yaw	η_6	Movement of the nose of the wing from side to side

Table 3.1: Degrees of freedom of the wing

3.2.3 Transformation between body and NED

The two coordinate systems used are BODY and NED (see fig. 3.6).

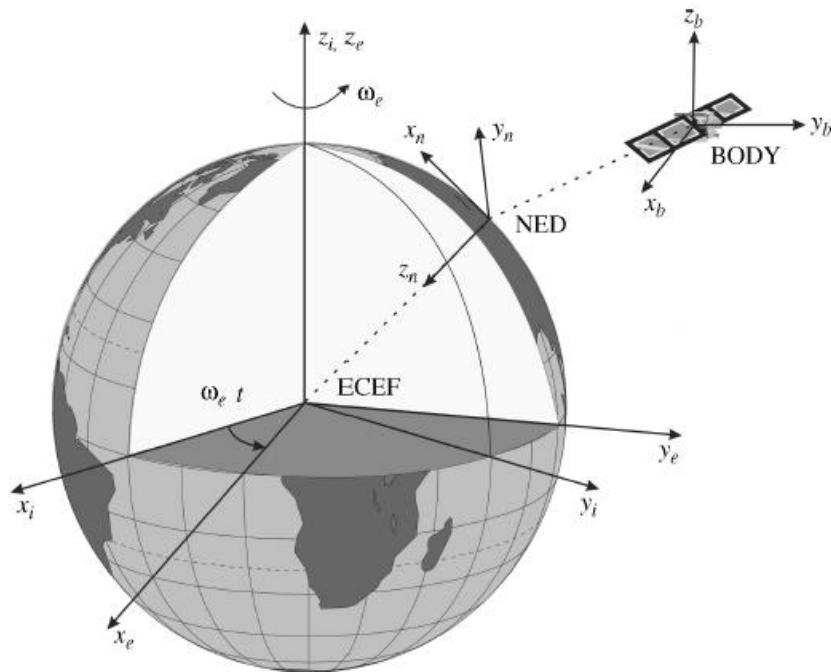


Figure 3.6: Connection between NED and BODY, [7]

BODY: A body reference coordinate system is simply enough a coordinate system fixed to the item of interest. This means that for the flying wing, the BODY coordinate system will change accordingly to the wing's movement in terms of roll, η_4 , pitch, η_5 , and yaw, η_6 (see fig. 3.4 on page 16).

NED: According to Fossen the *North-East-Down* (NED) coordinate system $\{n\} = (x_n, y_n, z_n)$ with origin o_n is defined relative to the Earth's reference ellipsoid (World Geodetic System, 1984). In easier terms this is our everyday life coordinate system. *North-East-Down* logically means that the axes are pointing in these directions. The x axis points towards true *North*, the y axis points towards *East* and the z axis points downwards normal to the Earth's surface.

Rotation matrices

Having already achieved a wingpath that is assumed to be representative for how the wing moves in the air and having defined the two coordinate systems, it was further explored how the wing itself would move in terms of roll, pitch and yaw by using rotation matrices. This theory is repeated in short and can be found more detailed in Fossen.

The rotation theorem was stated by Euler ¹ in 1776, and is now known as **Euler's rotation theorem**. In Fossen the theorem is phrased as follows:

Every change in the relative orientation of two rigid bodies or reference frames $\{A\}$ and $\{B\}$ can be produced by means of a simple rotation of $\{B\}$ in $\{A\}$

The rotation matrices are in Fossen, used to derive the kinematic equations of motion for a marine craft. But these matrices will work just as well for a flying wing with 6 DOF as for a marine craft with 6 DOF

A rotation matrix \mathbf{R} between two frames a and b is denoted as \mathbf{R}_b^a .

To explain how the transformation between the two different reference frames is connected it would be helpful to introduce vectors. If $v_{b/n}^b$ is a vector fixed in BODY and $v_{b/n}^n$ is a vector fixed in NED, one may by applying a rotation matrix and remembering Euler's

¹Leonhard Euler (1707-183) was a Swiss mathematician and physicist [22]

theorem state that it should be possible to express one of the vectors in terms of the other, as in eq. 3.3.

$$v_{b/n}^n = \mathbf{R}_b^n \mathbf{v}_{b/n}^b, \mathbf{R}_b^n := \mathbf{R}_{\lambda, \beta} \quad (3.3)$$

Where $\mathbf{R}_{\lambda, \beta}$ is the rotation matrix corresponding to a rotation β about the λ axis:

$$\mathbf{R}_{\lambda, \beta} = \mathbf{I}_{3 \times 3} + \sin(\beta)S(\lambda) + [1 - \cos(\beta)]S^2(\lambda) \quad (3.4)$$

$$\mathbf{S}(\lambda) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \quad (3.5)$$

and λ is the unit vector $\lambda = [\lambda_1, \lambda_2, \lambda_3]^\top$ with length equal to one.

If eq. 3.4 is expanded the following matrix elements are achieved:

$$\begin{aligned} R_{11} &= [1 - \cos(\beta)]\lambda_1^2 + \cos(\beta) \\ R_{22} &= [1 - \cos(\beta)]\lambda_2^2 + \cos(\beta) \\ R_{33} &= [1 - \cos(\beta)]\lambda_3^2 + \cos(\beta) \\ R_{12} &= [1 - \cos(\beta)]\lambda_1\lambda_2 - \lambda_3 \cos(\beta) \\ R_{21} &= [1 - \cos(\beta)]\lambda_2\lambda_1 - \lambda_3 \cos(\beta) \\ R_{23} &= [1 - \cos(\beta)]\lambda_2\lambda_3 - \lambda_1 \cos(\beta) \\ R_{32} &= [1 - \cos(\beta)]\lambda_3\lambda_2 - \lambda_1 \cos(\beta) \\ R_{31} &= [1 - \cos(\beta)]\lambda_3\lambda_1 - \lambda_2 \cos(\beta) \\ R_{13} &= [1 - \cos(\beta)]\lambda_1\lambda_3 - \lambda_2 \cos(\beta) \end{aligned} \quad (3.6)$$

The angles for roll, ϕ , pitch, θ and yaw, ψ can now be used to decompose the body-fixed vector $\mathbf{v}_{b/n}^b$ in the NED reference frame.

Let $\mathbf{R}_b^n(\theta_{nb})$ denote the Euler angle rotation matrix with argument $\theta_{nb} = [\phi, \theta, \psi]^\top$. This gives

$$v_{b/n}^n = \mathbf{R}_b^n(\boldsymbol{\theta}_{nb}) v_{b/n}^b \quad (3.7)$$

Finally the rotation matrices, can be obtained by setting $\lambda = [1, 0, 0]^\top$, $\lambda = [0, 1, 0]^\top$ and $\lambda = [0, 0, 1]^\top$ corresponding to the x, y and z axes, and $\beta = \phi$ and $\beta = \psi$, respectively, in the formula for $\mathbf{R}_{\lambda,\beta}$. This gives:

$$\mathbf{R}_{x,\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}, \mathbf{R}_{y,\theta} = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}, \mathbf{R}_{z,\psi} = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.8)$$

Combining these rotation matrices gives the total rotation matrix, eq. 3.9, that has been used in the calculations:

$$\mathbf{R}_b^n(\boldsymbol{\theta}_{nb}) = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi & c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + s\psi s\theta c\phi & \\ -s\theta & c\theta s\phi & c\theta c\phi & \end{bmatrix} \quad (3.9)$$

Using eq. 3.9 gives the local coordinate system of the wing with input values for roll, η_4 , pitch, η_5 and yaw, η_6 .

Finding an orientation of the wing that would represent a similiar movement as in the video uploaded by Makani [11] was found to be a challenge. The solution would be to set the input values as reasonable as possible. In the end the unit vectors of the local body system on the wing could be plotted, and one would be able to see if the orientation of the wing along its path would be realistic.

By setting ϕ as a constant $= \pi$, $\theta = -\frac{70}{180} \cdot \pi$ and letting ψ vary from 0 to 2π gave a plot that seemed reasonable, as seen in fig. 3.7 on the next page

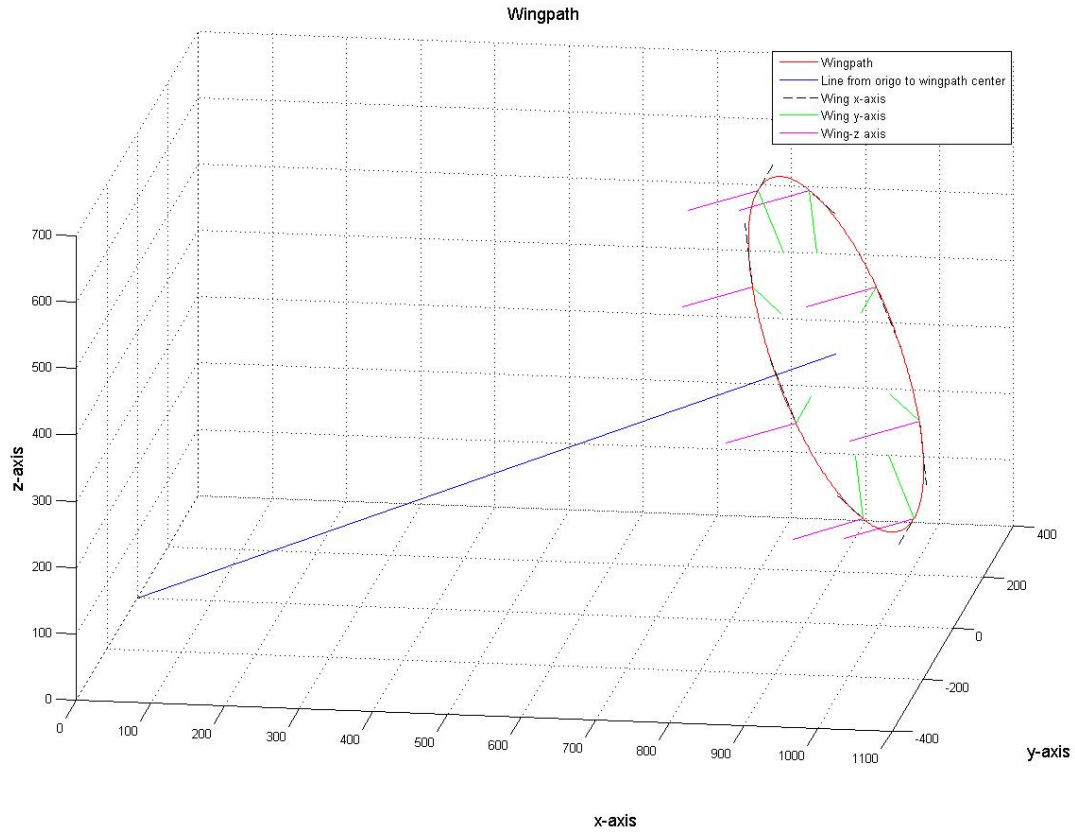


Figure 3.7: Plot of the wing orientation from Matlab

3.2.4 Incoming velocity V_A

The incoming velocity V_A is needed to calculate the lift and drag forces, and is also essential for the power production.

To find the incoming velocity V_A one has to consider the path of the wing, or the surge motion of the wing, combined with the wind speed. To do this a vector summation between the two needs to be done. If one first excludes the wind speed itself, the incoming wind will then only consist of the wind felt by the wing due to the crosswind motion along its orbit. The direction of this wind will then be just the opposite direction of the wing's surge motion. The surge motion will be the x-component on the local coordinate system of the wing, i.e. the transformed coordinate system. Naturally a flying wing like this will not be able to stay aloft if there is no wind. The researchers in the Makani-group [10] states that a minimum wind speed of 3.5 [m/s] is needed in order for the wing to keep its course in a crosswind motion.

At first it was only assumed a reasonable period of orbit of 15 [s]. Since the path is fixed, the orbit speed will then also be a constant. This is not a reasonable approach, since the speed of the wing along its orbit will naturally change with the magnitude of the wind speed. To do this an approach using tip speed ratio was done. The TSR of the wing using the period of 15 [s] in 11.5 [m/s] wind speed was calculated using eq. 3.10 from [23]. It is, however, important to notice that the choice of period of 15 [s] is based on guessing since there is no research available for this kind of wing, so far that is. A comparison with the circling speed of the Makani prototype and the circling speed achieved by setting the period to 15 [s] on the Makani M5 wing was what made this seem like a possible value to use.

$$TSR = \frac{Tip - speed\ of\ blade}{Wind\ Speed} = \frac{\omega R}{Wind\ speed} = \frac{\frac{2\pi}{T}R}{V_W} \quad (3.10)$$

Inserting $T = 15$ [s], $R_{M5} = 265$ [m] and $V_W = 11.5$ [m/s] gives **TSR_{M5} = 9.65 [-]**.

This value was further used to calculate the orbit speed of the wing, making it dependent on the different V_W used as input, using eq. 3.11.

$$Orbit - speed = V_W \cdot TSR_{M5} \quad (3.11)$$

When the orbit speed was calculated, one simply just add the wind, and the total incoming wind velocity V_A , is then known. In fig. 3.8 on the facing page the direction of these vectors, including lift, are shown.

3.2.5 Lift force

When the incoming wind velocity V_A is known, the magnitude and direction of the lift force may be calculated. When the homogenous wind speed in x-direction was added to the wind due to the orbit of the wing, the direction of the lift force will naturally also change, since this should be 90 [deg] on the incoming velocity V_A . How to solve this was shown to be not as simple as calculating V_A by just adding V and V_W . The solution to this was found by using a cross product between to vectors.

$$\vec{L}' = \vec{V}_A \times (\vec{L} \times \vec{V}_A) \quad (3.12)$$

The lift vector is found according to eq. 3.12. A check was done in Matlab with both dot product between two vectors and a plot of the vectors to confirm that this approach was correct.

When doing these cross products, L' , seen in fig. 3.8, will be plotted correctly 90 degrees on the incoming velocity V_A .

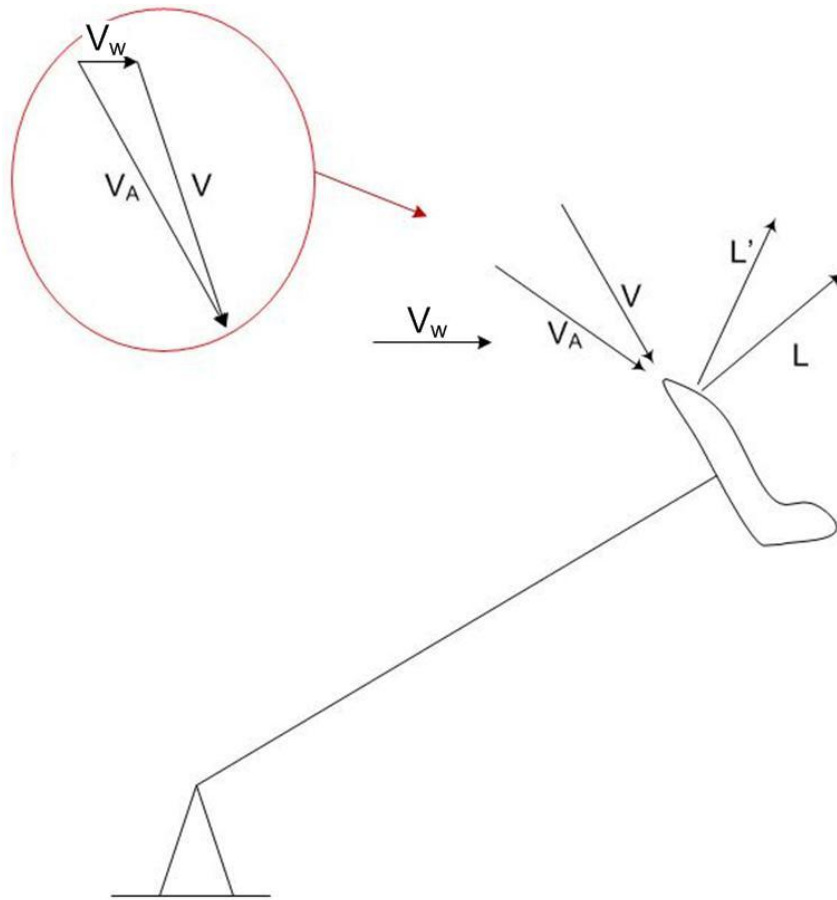


Figure 3.8: Incoming wind direction and lift force

As the magnitude of the lift vector is quite large after having done the cross product, it was found useful to divide this vector on the length of the vector to get the unit lift vector. The lift vector is given as $\vec{L}_i = [L_{x,i}, L_{y,i}, L_{z,i}]$ where i indicates the step along the wing path. The length or magnitude of this vector for each step is then found as $|\vec{L}|_i = \sqrt{L_{x,i}^2 + L_{y,i}^2 + L_{z,i}^2}$. Dividing the lift vector on the length for each step will then give a unit lift vector as seen in eq. 3.13 on the next page

$$\vec{L}_{unit,i} = \frac{\vec{L}_i}{|\vec{L}|_i} \quad (3.13)$$

The correct magnitude of the lift vector will then be calculated by using known foil theory and multiply with the unit lift vector, as seen in eq. 3.14.

$$L = \frac{1}{2} C_L \cdot \rho_a \cdot |\vec{V}_A|^2 \cdot S \cdot L_{unit} \quad (3.14)$$

and the lift coefficient, C_L , is calculated as [17]:

$$C_L = \frac{2\pi\alpha}{1 + \frac{2}{A_{sp}}} \quad (3.15)$$

The angle of attack, α , has been calculated as the the angle between V and V_A , seen in fig. 3.8 on the previous page.

$$\alpha = \cos^{-1} \left(\frac{\vec{V}_A \cdot \vec{V}}{|\vec{V}_A| \cdot |\vec{V}|} \right) \quad (3.16)$$

Makani states that the lift coefficient, C_L , is around actually around 1.5 [4]. The calculation of C_L using the calculated angle of attack as input from eq. 3.16 is not in correlation to this. It seems logical to assume that the wing is somewhat pitched. No sources are found that says what pitching angle the wing might have, but by assuming that C_L should be around 1.5 a reasonable added wing pitch of 9 [deg] in the angle of attack gives a value close to this.

By calculating the magnitude of V_A and using this as input value in eq. 3.14 together with inserting the area of the wing, the air density and the modified lift coefficient multiplied with the lift unit vector, the magnitude of the lift vector will then be known.

3.2.6 Drag

Drag is calculated using eq. 3.17 on the facing page and consists of three contributions, the induced drag D_i , the viscous drag D_v and the drag due to the mounted turbines on the wing, D_{turb} . A unit drag vector was calculated similarly as the unit lift vector.

$$D = \frac{1}{2} \underbrace{C_D}_{C_{D,i} + C_{D,v} + C_{D,turb}} \cdot \rho_a \cdot |\vec{V}_A|^2 \cdot S \cdot D_{unit}^{\rightarrow} \quad (3.17)$$

$C_{D,i}$ can be found from $\frac{C_L^2}{\pi \cdot A_{sp}}$ [17] and $C_{D,v}$ has been estimated from Abbott [1].

Drag of turbines

To find the drag coefficient of the turbines, $C_{D,turb}$, eq. 3.18 was used.

$$C_{D,turb} = \frac{D_{turb}}{0.5 \rho_a V_A^2 S} \quad (3.18)$$

The drag force on the turbines, D_{turb} , is given in eq.(1) in [16], repeated here in eq. 3.19.

$$D_{turb} = 2 \rho_a V_A^2 a (1 - a) A_{turb} \quad (3.19)$$

where ρ_a is the mass density of air, A_{turb} is the turbine rotor disk area, and a is the axial induction factor.

This means that the axial induction factor, a , must be known. This value can be found using eq.(7) in [16], repeated here in eq. 3.20, by using the embedded solver function in Matlab.

$$C_P = \frac{P}{\frac{1}{2} \cdot \rho_a \cdot V_A^3 \cdot A_{turb}} = 4a(1 - a)^2 \quad (3.20)$$

C_P is the coefficient of power. To calculate C_P the only unknown is the power, P . It was chosen to use a power curve given by Makani [10], which shows the power production for different wind speeds, seen in fig. 3.9 on the following page.

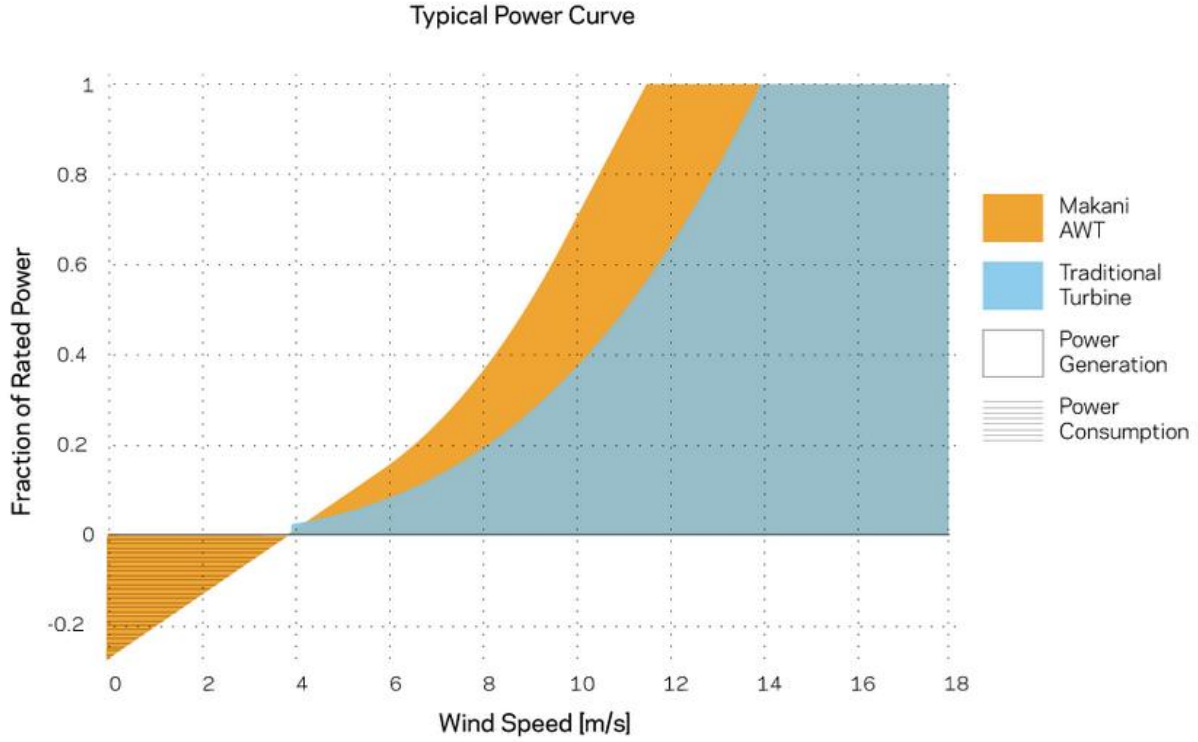


Figure 3.9: Power curve by Makani [4]

As can be seen in fig. 3.9, much of the power is dependent of the wind speed. The power production for the Makani wing is the orange graph, and the y-axis shows the fraction of rated power. Since it is stated by Makani that the full rated power at its maximum, is 5 [MW], the fraction of rated power can be multiplied with 5 [MW] to get the total power production for the different wind speeds. The values was then found by reading of the graph, and C_p was then acquired using eq. 3.20 on the preceding page. It is also important to notice that the axial induction factor, a , is then depending on the wind speed.

Finally the expression for the drag coefficient of the turbines were found by combining eq.3.18 and eq. 3.19 to get eq. 3.21.

$$C_{D,turb} = \frac{4a(1-a)A_{turb}}{S} \quad (3.21)$$

Now the terms needed in order to calculate the drag of the AWT are found.

3.3 Power production

How a conventional wind turbine works is that it extracts energy by slowing down the wind, or in other words, air goes into the disk, a slower stream tube of air is achieved and it goes into a bigger area (A_2) to conserve mass, as seen in fig. 3.10.

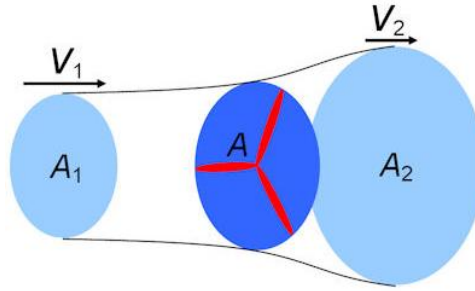


Figure 3.10: Flow across a disk [19]

The power it produces is dependent on the power coefficient C_P , which is equal to the electricity actually produced by the wind turbine divided by the total energy available in the wind. For a wind turbine to be 100% efficient it would need to stop 100% of the wind. But if this was the case, the rotor, that has to be a solid disk, would not be able to turn, and thus no kinetic energy would be converted [8]. This is where the so-called "Betz-Limit" is introduced. It defines the maximum amount of power that one is able to actually get out from a wind turbine. The "Betz-Limit", defined in eq. 3.22, was calculated by Albert Betz² who calculated that no wind turbine could convert more than 59.3% of the kinetic energy of the wind into mechanical energy turning a rotor.

$$C_{P,max} = \frac{\text{Electricity produced by wind turbine}}{\text{Total energy available in the wind}} = \frac{16}{27} = 0.593 \quad (3.22)$$

Wind turbines are fairly close to the optimum and there is not much room for improving the wind turbines in terms of achieving the Betz limit [5]. The C_P for an AWT is even lower than for a conventional wind turbine, actually very low, somewhere around 0.05, meaning a lower power production. But then again what stands out for an AWT is the high circling speed, meaning a high incoming velocity on the turbines and the power

²Albert Betz (1885-1968) was a German physicist and a pioneer of wind turbine technology [24]

produced by the wing also depends of number of turbines mounted on the wing, so that the power produced by the wing is then given as

$$P = \frac{1}{2} \rho_a V^3 A_{turb} \cdot C_P \cdot N_{turb} \quad (3.23)$$

4 Auxiliary propulsion and resistance

4.1 Resistance

The total resistance includes the resistance of the ship and the resistance of the tether. The resistance of the ship has been calculated using the empirical method by Hollenbach. The resistance of the tether affecting the ship is the horizontal component of the tether force.

4.1.1 Resistance of ship

Since the whole idea with the wing is to work as a an auxiliary propulsion for a ship, it is also necessary to include the resistance of the ship.

In experimental hydrodynamics [18] a script for calculating resistance using Hollenbach's empirical method was given, and has for this purpose been modified and used. Also the specifications for a car-carrier (seen in tab. 4.1) was given in this course, and have been used in the calculations.

	Symbol	Unit	Ship
Length in the waterline	L_{WL}	[m]	178.3
Beam	B	[m]	29
Mean draught	T	[m]	9.45
Wetted surface	S	[m ²]	6227.8
Block coef.	C_B	[-]	0.656
Propeller diameter	D_p	[m]	5

Table 4.1: Specifications of car-carrier [18]

Hollenbach's empirical formula for calculating resistance is given in eq. 4.1 on the next page.

$$C_{R,Hollenbach} = C_{R,Standard} \cdot C_{R,Fnkrit} \cdot k_L \cdot \left(\frac{T}{B}\right)^{a1} \cdot \left(\frac{B}{L}\right)^{a2} \cdot \left(\frac{L_{os}}{L_{wl}}\right)^{a3} \cdot \left(\frac{L_{wl}}{L}\right)^{a4} \cdot \left(1 + \frac{T_A - T_F}{L}\right)^{a5} \cdot \left(\frac{D_P}{T_A}\right)^{a6} \cdot (1 + N_{rud})^{a7} \cdot (1 + N_{brac})^{a8} \cdot (1 + N_{boss})^{a9} \cdot (1 + N_{Thr})^{a10} \quad (4.1)$$

More detailed progress on this method can be seen in Matlab-script in A.10 on page A-13.

4.1.2 Tether force

The forces in the tether is depending on the lift, drag and weight forces on the wing. To calculate this force, the lift, drag and weight of the wing was projected onto the tether direction, using eq. 4.2.

$$\begin{aligned} \vec{L}_T &= \frac{\vec{L} \cdot \vec{T}}{|\vec{T}|^2} \vec{T} \\ \vec{D}_T &= \frac{\vec{D} \cdot \vec{T}}{|\vec{T}|^2} \vec{T} \\ \vec{W}_T &= \frac{\vec{W} \cdot \vec{T}}{|\vec{T}|^2} \vec{T} \end{aligned} \quad (4.2)$$

where L_T , D_T and W_T denotes lift, drag and weight force in tether direction, respectively. The ' \cdot ' denotes that a dot product is used, and \vec{T} is the tether vector from origin and to the position of the wing. Summation of these forces gives the total tether force. However, it is in this case the x-component, or the horizontal force of the tether, which is of interest since it is this force that gives the added resistance on the ship.

4.2 Propulsion

When the power produced by the wing was obtained, the next step was to find the propulsion given to the water propeller. Eq (3) in [16] was used, repeated here in eq. 4.3 on the facing page.

$$F = \frac{P\zeta}{u} \quad (4.3)$$

where ζ is the overall propulsive efficiency, P is the power produced by the wing and u is the ship speed.

With increasing ship speed, u , the relative wind speed will be larger, and thereby a higher power generation may be achieved by the wing, given that the maximum power generation has not yet been reached. Still it can also be seen from the eq. 4.3 that as u increases, F will decrease.

4.2.1 Propulsive efficiency, ζ

To calculate the overall propulsive efficiency ζ , this was done according to the progress described in the master thesis by Bøckmann [2]. The same open water diagram as in [2] was used here. The program *GetData Graph Digitizer 2.25* was used to import the graphs into Matlab. Then the sub function Cftool was used to obtain smooth curves and to acquire the equation for the graphs. The plot in fig. 4.1 was obtained.

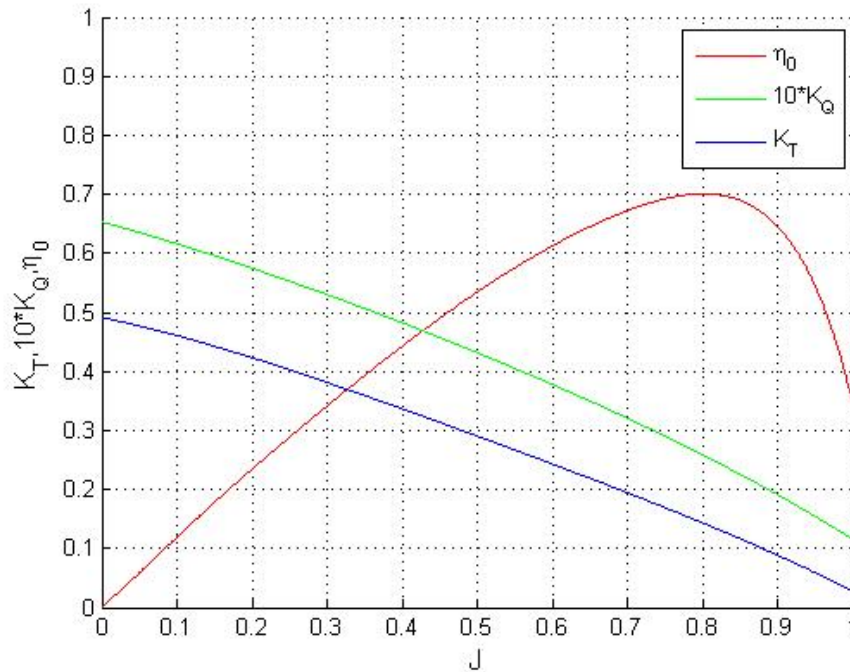


Figure 4.1: Open water diagram [2]

J on the x-axis in fig. 4.1 on the previous page is the advance number. This is given as:

$$J = \frac{u}{nD_p} \quad (4.4)$$

In fig 4.1 η_0 is given for each advance number, J . It will be more convenient to find this value for each ship speed. To eliminate the revolution speed, n , $\frac{K_Q}{J^3}$ vs. J was plotted as seen in fig. 4.2.

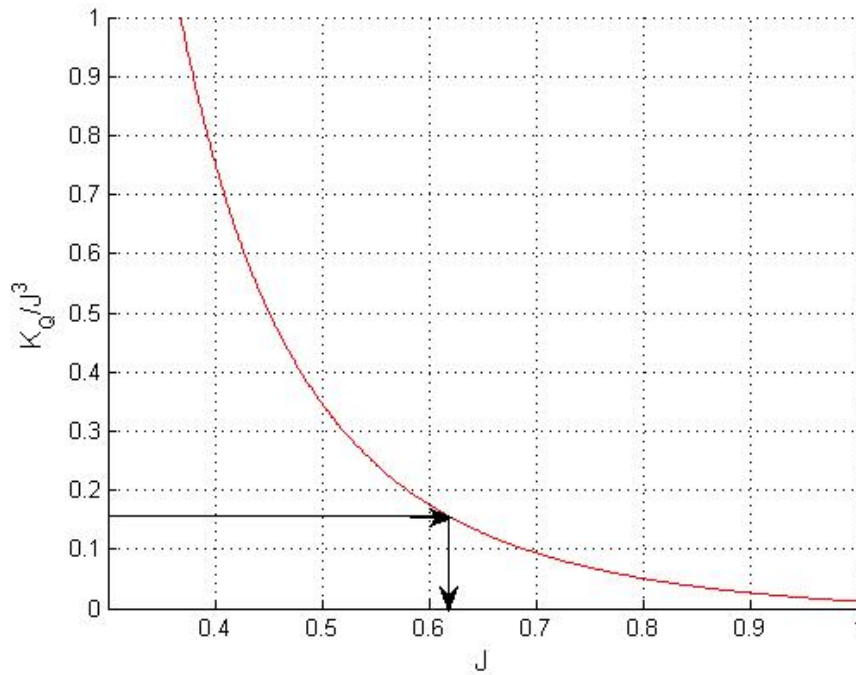


Figure 4.2: K_Q/J^3 vs. J

K_Q/J^3 can also be calculated as

$$\frac{K_Q}{J^3} = \frac{Q}{\rho_w n^2 D_p^5} \frac{n^3 D_p^3}{u^3} = \frac{P_D}{\rho_w 2\pi u^3 D_p^2} \quad (4.5)$$

This value is found for different ship speeds by entering the power produced by the wing, P_D , propeller diameter, D_p , and the density of water. When having acquired K_Q/J^3 for a number of ship speeds, it is then possible to use the plot in fig 4.2, and read of the corresponding J values as illustrated.

This makes it possible to finally find the propulsive efficiency for different ship speeds, η_p

(same as ζ in eq. 4.3 on page 31), by entering the J-values in the equation for η_0 and this gives the plot seen in fig. 4.3.

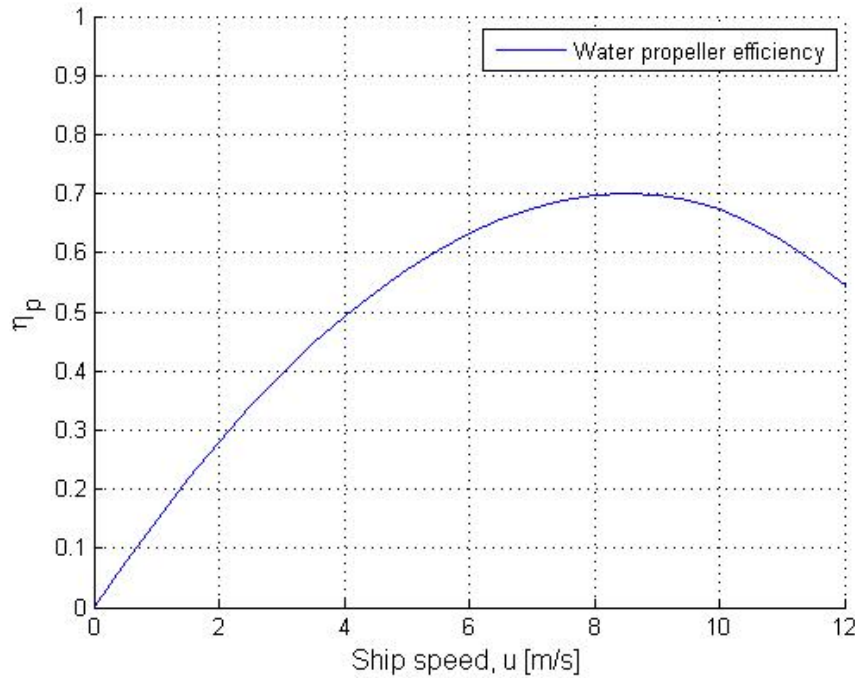


Figure 4.3: Propulsive efficiency

This plot, showing the propulsive efficiency for different ship speeds, will vary slightly with the chosen wind speed input. This is in correlation to the power produced by the wing, which is used as input for calculating $\frac{K_Q}{J^3}$ in eq. 4.5 on the preceding page.

5 Results and discussion

5.1 Wing results

One of the problems that was mentioned in the project thesis was the assumption made that the wing would at all times stay aloft. The focus was merely on the power production. This of course not realistic. If the wind speed is low, it is unphysical for the wing to be able to stay aloft. In this master thesis the analysis of the wing has been taken a step further, and this problem has been one of the challenges looked upon.

Since the weight is known and the lift and drag has been calculated for each point along the wing path, a reasonable way to see if the wing stays aloft would be to look at the z-components of these forces. This was done by calculating the mean z-component along one orbit and sum up the forces. The results are given in tab. 5.1.

V_W [m/s]	Summation of L, D and W in z-dir
1	-92373 [N]
2	-78135 [N]
3	-54406 [N]
4	-21185 [N]
5	21240 [N]
6	73320 [N]
7	134858 [N]

Table 5.1: Summation of lift, drag and weight in z-dir for increasing wind speed

For higher wind speed this will result in an even higher positive netforce. It can be seen that a positive force is obtained somewhere between 4 and 5 [m/s]. This is not that far from what Makani states on their webpage, that a minimum of 3.5 [m/s] is needed for the wing to stay aloft.

5.2 Power output

The power output was calculated using eq. 3.23 on page 28, which depends on the power coefficient C_P . This C_P was, however, found by using the power curve by Makani, and therefore it was expected to obtain similar power output as what was read off the power curve. Still it was of course interesting to see if this was actually true, and the results can be seen for different wind speeds in tab. 5.2, starting at 3.5 [m/s] as this is the first wind speed where the wing will be able to stay aloft (according to Makani) and thereby producing electricity.

V_w [m/s]	Power output
3.5	0.201 [MW]
4	0.402 [MW]
5	0.905 [MW]
6	1.559 [MW]
7	2.514 [MW]
8	3.871[MW]
9	5.028 [MW]
10	5.028 [MW]

Table 5.2: Power output for Makani M5

5.3 Propulsion vs. resistance

When the power output of the wing and the efficiency curve for converting the power produced into forward force to the propeller, together with the horizontal tether force and ship resistance, was acquired, the following plots were made:

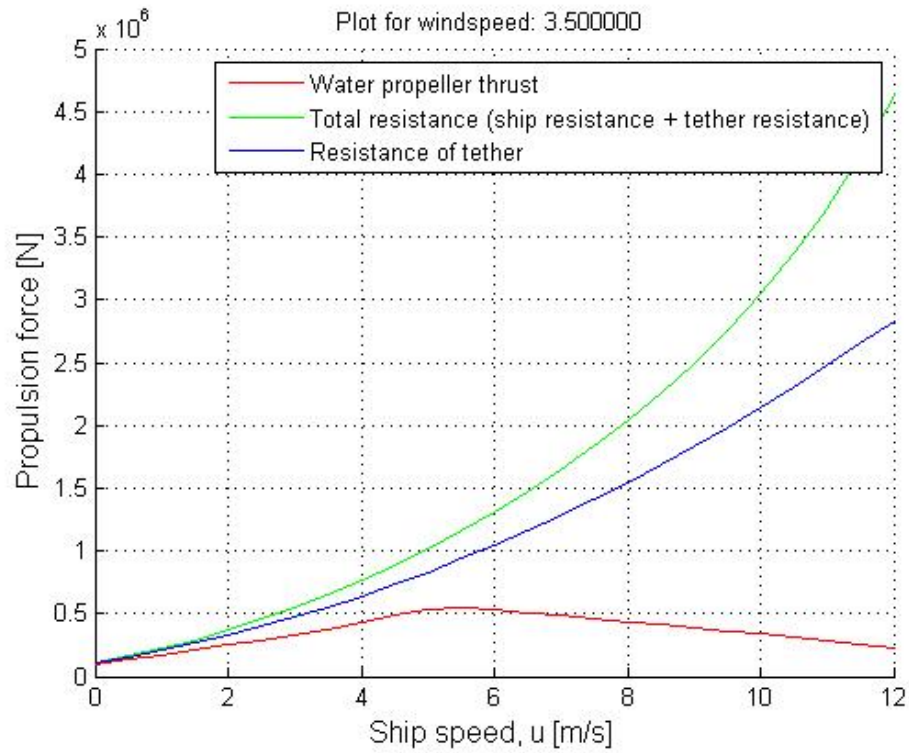


Figure 5.1: Propulsion vs. resistance for $V_W = 3.5$ [m/s]

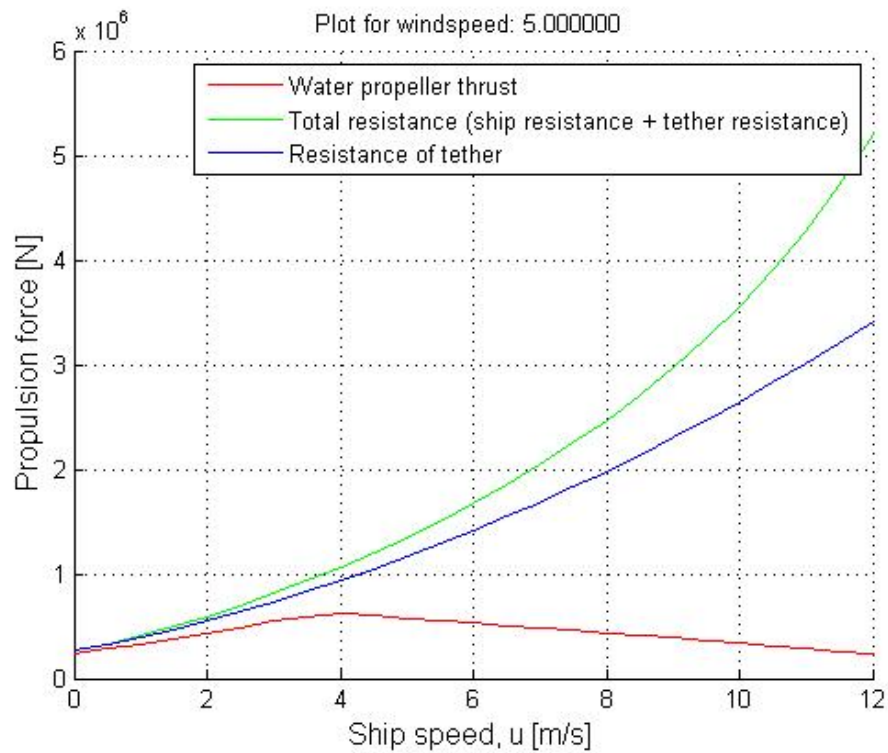


Figure 5.2: Propulsion vs. resistance for $V_W = 5$ [m/s]

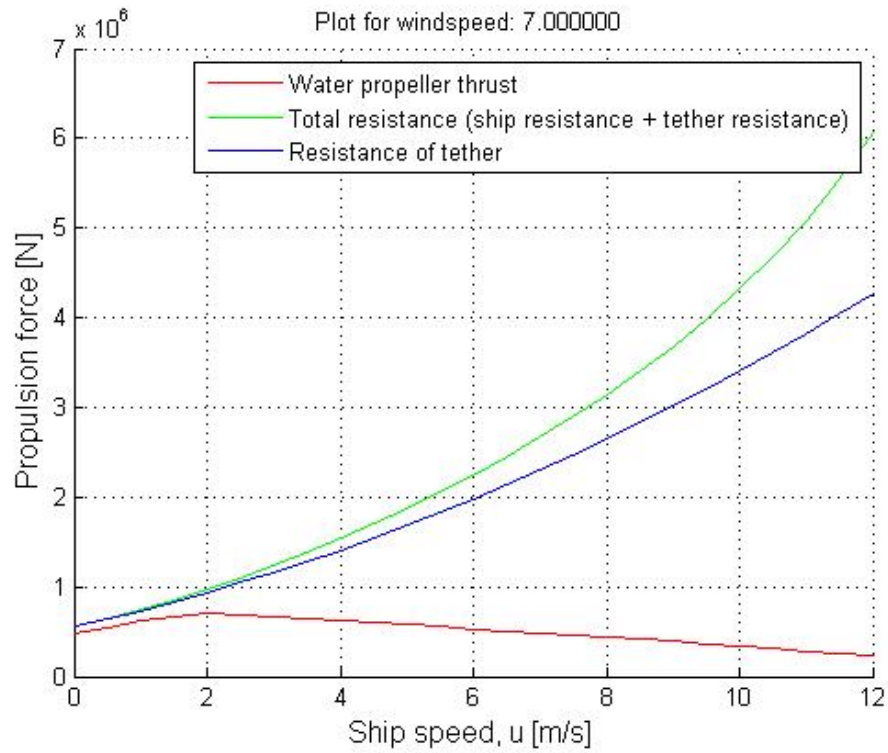


Figure 5.3: Propulsion vs. resistance for $V_W = 7$ [m/s]

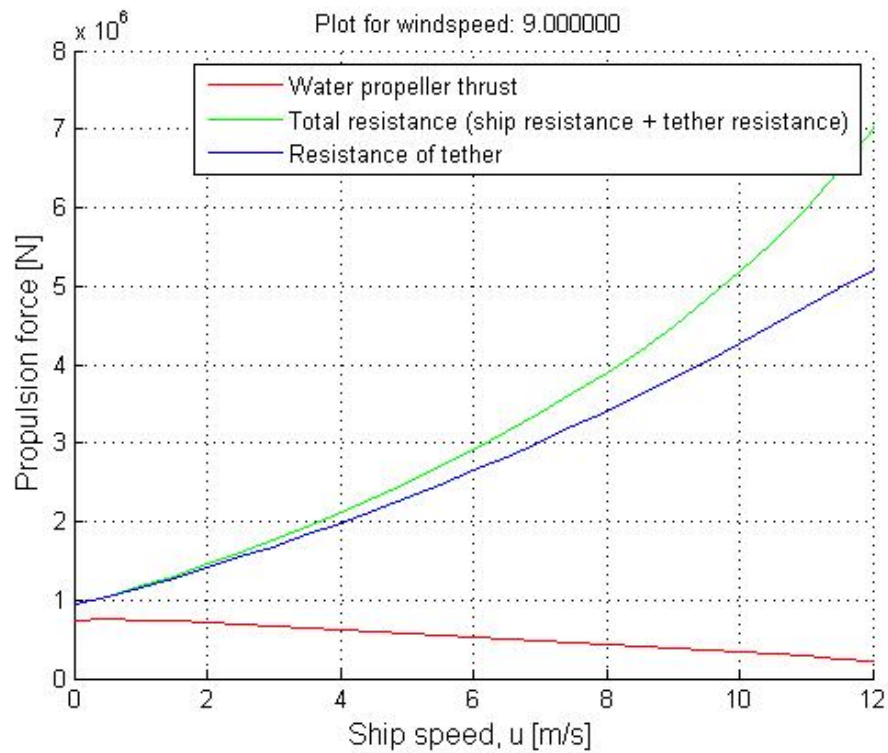


Figure 5.4: Propulsion vs. resistance for $V_W = 9$ [m/s]

Plot for other wind speeds may be acquired in Matlab, but it was found sufficient to use only the selected wind speeds in the plots above, since the first plot (fig. 5.1) is the lowest windspeed for power production (according to Makani) and the last plot (fig. 5.4) is the first wind speed where maximum power production is achieved.

At first it was only plotted the water propeller thrust vs. total resistance including ship resistance and horizontal tether force. It was seen that the total resistance was higher than the propeller thrust for every ship speed. Then the horizontal resistance of tether was included in the plot to see if the propeller thrust would exceed this resistance for any ship speed. The thought here was that if the propeller thrust was higher than the resistance of the tether, the system could be used on a ship for auxiliary propulsion without necessary giving the thrust needed to run the entire ship. As seen in the plots the horizontal tether resistance is too high for this to work.

6 Conclusion

The analysis of the Makani M5 wing was successfully done in Matlab. When doing the calculation of forces to see what wind speed that was needed in order for the wing to stay aloft this was calculated to a minimum of $V_w = 4.5$ [m/s] approximately. This result was satisfying since it was not that far from the reference value given by Makani ($V_w = 3.5$ [m/s]).

The results that has been calculated in the propulsion part was not as hoped. In the project thesis a plot using maximum power production 5[MW] for all ship speeds, u , was done. This showed that for ship speeds up to 6 [m/s] the calculated propulsion force was in fact higher than the total resistance. Since the power curve by Makani was used in these calculations it was expected to see that the system could at least be used as an auxiliary propulsion system (propulsion force $>$ horizontal resistance in tether), when achieving maximum power production by the wing. However, this was not achieved, and one can only conclude that the system is therefore not feasible.

Yet it must be mentioned that even more advanced calculations are needed in order to know for sure that the idea is not feasible. One big uncertainty is the orientation of the wing along the wing path. This has been approximated to look similar to the Makani prototype in a video uploaded by Makani [11]. The wing orientation is, however, more advanced than this.

When calculating the power production the power coefficient C_P had to be known. One solution considered was to just guess a value of C_P , since it was found during the research on the Makani wing that this value would be fairly low. Exactly which value to set, was though unknown. A more precise way would be to calculate this value using the Makani power curve. This meant that the power production achieved would therefore be connected to this C_P and therefore giving the same power output as what was read off the power curve.

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A Matlab script

A.1 run.m

```
%-----%  
%           MAKANI-WING M5           %  
%-----%  
  
clear all  
close all  
clc  
  
constants  
  
wingpath  
  
trans_matrices  
  
vectors  
  
powerproduction  
  
lift_and_drag  
  
tetherforce  
  
vectorplot  
  
hollenbach  
  
propulsion  
  
printvalues
```

A.2 constants.m

```
fprintf('—————Analysis of the Makani M5 wing—————\n\n')

%Constants

rho_a = 1.3;
rho_w = 1025;
gacc = 9.81;

%Defining windspeed
V_w_inp = input('Enter wind speed          ---->:' );
sprintf('Enter wind speed : %d\n\n', V_w_inp);
V_w = [V_w_inp 0 0];
fprintf('Windspeed is set to          ---->: %f [m/s]\n',V_w(1,1));

origin = [0 0 0]';

%Specs of Makani M5. Some estimated using figures in
%"technical specifications" from "www.makanipower.com"

w = 9900;                %Weight in kg found in FFA-Makani
w_in_N = [0 0 -w*gacc]; %Weight in Newton
s = 65;                  %Wing span in m
S = 150;                 %Wing planform area in m^2
Asp = s^2/S;            %Wing aspect ratio
length_of_tether = 1060; %Length of tether
circling_radius = 265;  %Circling radius
top_oper_range = 650;   %Operational range set in FAA to be 350–650m
A_turb = 12.57;         %Area of turbines
N_turb = 8;             %Number of turbines
e = 1;                 %Planform efficiency factor
period = 15;           %Assumed period for one orbit of the wing
C_Dv = 0.02;           %Viscous drag
etta_p0 = 0.45;        %Propulsion efficiency for zero-speed of ship
```


A.3 wingpath.m

```
%DEFINING A CIRCULAR PATH FOR THE WING IN 3D

%Various angles and lengths needed in order to follow the progress
%described in article by Loyd:
rad_between_wing_and_orbit = asin(circling_radius/length_of_tether);
rad_between_z_axis_and_orbit = acos(top_oper_range/length_of_tether)...
    +rad_between_wing_and_orbit;

length_to_center_of_orbit = cos(rad_between_wing_and_orbit)*length_of_tether;

x_length_to_center_of_orbit = sin(rad_between_z_axis_and_orbit)*...
    length_to_center_of_orbit;
z_length_to_center_of_orbit = cos(rad_between_z_axis_and_orbit)*...
    length_to_center_of_orbit;

%Check of geometry:
radius = sin(rad_between_wing_and_orbit)*length_of_tether;
%If radius equals 265 it should be ok. Per 08.06.2013 this corresponds.

normal = [x_length_to_center_of_orbit 0 z_length_to_center_of_orbit];
center = [x_length_to_center_of_orbit 0 z_length_to_center_of_orbit];

%TSR Makani M5 with period = 15[s] and windspeed 11.5 [m/s]
lambda_m5 = (2*pi*circling_radius)/(period*11.5);

t = 0:1/(10*pi):period;

gamma = (2*pi*t)/period;
%gamma_diff = (2*pi)/period;

v=null(normal);
points= repmat(center',1,size(gamma,2))+radius*(v(:,1)*cos(gamma)+v(:,2)*...
    sin(gamma));
plot3(points(1,:),points(2,:),points(3,:), 'r-');

figure(1)
title('Wingpath', 'fontsize',12.5)
grid on
hold on
xlabel('x-axis', 'fontsize',12.5)
ylabel('y-axis', 'fontsize',12.5)
zlabel('z-axis', 'fontsize',12.5)
xlim([0 1100])
ylim([-400 400])
zlim([0 700])
```

A.4 trans_matrices.m

```
%TRANSFORMATION MATRICES

%Comment:
%When the points along the path is known, it is important to find a
%possible movement of the wing in terms of roll, pitch and yaw (phi, theta
%and psi).

trans_matrix_phi = zeros(3,3,length(points));
trans_matrix_phi(1,1,:) = 1;
trans_matrix_theta = zeros(3,3,length(points));
trans_matrix_theta(2,2,:) = 1;
trans_matrix_psi = zeros(3,3,length(points));
trans_matrix_psi(3,3,:) = 1;

phi = zeros(1,length(points));
theta = zeros(1,length(points));
psi = zeros(1,length(points));

%Assumed movement of wing in each point along the path before using
%transformation matrix:
for i = 1:length(points)
    phi(i) = pi;
    theta(i) = -70/180*pi;
    psi(i) = -gamma(i);
end

%Transformation matrix from Fossen
for i = 1:length(points)
    trans_matrix_phi(2,2,i) = cos(phi(i));
    trans_matrix_phi(2,3,i) = -sin(phi(i));
    trans_matrix_phi(3,2,i) = sin(phi(i));
    trans_matrix_phi(3,3,i) = cos(phi(i));
    trans_matrix_theta(1,1,i) = cos(theta(i));
    trans_matrix_theta(1,3,i) = sin(theta(i));
    trans_matrix_theta(3,1,i) = -sin(theta(i));
    trans_matrix_theta(3,3,i) = cos(theta(i));
    trans_matrix_psi(1,1,i) = cos(psi(i));
    trans_matrix_psi(1,2,i) = -sin(psi(i));
    trans_matrix_psi(2,1,i) = sin(psi(i));
    trans_matrix_psi(2,2,i) = cos(psi(i));

    trans_matrix_tot(:, :, i) = trans_matrix_phi(:, :, i) * ...
        trans_matrix_theta(:, :, i) * trans_matrix_psi(:, :, i);
    %Unit vectors in x,y and z-direction
    n_vec(:, i) = [1 0 0]';
    e_vec(:, i) = [0 1 0]';
    d_vec(:, i) = [0 0 1]';

    %Transformed system
    n_in_NED(:, :, i) = trans_matrix_tot(:, :, i) * n_vec(:, i);
    e_in_NED(:, :, i) = trans_matrix_tot(:, :, i) * e_vec(:, i);
    d_in_NED(:, :, i) = trans_matrix_tot(:, :, i) * d_vec(:, i);
end
```

A.5 vectors.m

```
%PLOTING OF WING PATH AND THE WING'S ORIENTATION

%Seperate vectors for x,y and z coordinates
points_x = points(1,:);
points_y = points(2,:);
points_z = points(3,:);

%Draw a line from origin to the center of the circle
line_origin_to_center_x = [origin(1,1) center(1,1)];
line_origin_to_center_y = [origin(2,1) center(1,2)];
line_origin_to_center_z = [origin(3,1) center(1,3)];
line(line_origin_to_center_x, line_origin_to_center_y, ...
     line_origin_to_center_z);

%Vectors from origin to position along the wing path
for i = 1:length(points)
    x(i,:) = [origin(1) points_x(i)];
    y(i,:) = [origin(2) points_y(i)];
    z(i,:) = [origin(3) points_z(i)];
    %line(x(i,:),y(i,:),z(i,:));
    %This makes a line from origo to every single point on the path

    %Draw lines from point on the path to z-value zero (projected down on
    %the x-y-plane
    x_normal(i,:) = [points_x(i) points_x(i)];
    y_normal(i,:) = [points_y(i) points_y(i)];
    z_normal(i,:) = [points_z(i) origin(3)];
    %line(x_normal(i,:),y_normal(i,:),z_normal(i,:));
    %This makes a line that is projected down on the x-y-plane for every
    %point on the path
end

%Plotting a few points on the graph with the coord-system of the wing
for j = 1:59:length(points)
    %-----WING X DIRECTION-----%
    plot3([points_x(j) points_x(j)+100*n_in_NED(1,:,j)],...
          [points_y(j) points_y(j)+100*n_in_NED(2,:,j)],...
          [points_z(j) points_z(j)+100*n_in_NED(3,:,j)], 'k—')
    %-----%
    %-----WING Y DIRECTION-----%
    plot3([points_x(j) points_x(j)+100*e_in_NED(1,:,j)],...
          [points_y(j) points_y(j)+100*e_in_NED(2,:,j)],...
          [points_z(j) points_z(j)+100*e_in_NED(3,:,j)], 'g')
    %-----%
    %-----WING Z DIRECTION-----%
    plot3([points_x(j) points_x(j)+100*d_in_NED(1,:,j)],...
          [points_y(j) points_y(j)+100*d_in_NED(2,:,j)],...
          [points_z(j) points_z(j)+100*d_in_NED(3,:,j)], 'm')
    %-----%
end

legend('Wingpath', 'Line from origo to wingpath center','Wing x-axis',...
       'Wing y-axis','Wing-z axis')
```

A.6 powerproduction.m

```
%POWER PRODUCTION OF MAKANI-M5 WING AND CORRESPONDING POWER COEFFICIENT

wind_abs = 0:0.5:25; %Define a vector including wind speeds from 0-25.

%Power output. Read from graph by Makani
P_makani = 5*10^6*[0 0 0 0 0 0 0 0.04 0.08 0.12 0.18 0.24 0.31 ...
                  0.40 0.50 0.62 0.77 0.90 1 1 1 1 1 1 1 1 1 1 1 1 1 ...
                  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1];

figure(2)
grid on
plot(wind_abs,P_makani)

for j = 1:length(wind_abs)
    for i = 1:length(points)
        %Circling speed of wing using calculated TSR:
        circling_speed(j) = wind_abs(j)*lambda_m5;
        %Add windspeed to get incoming wind direction V_a:
        V_A = -n_in_NED*circling_speed(j);
        V_A(1,1,i) = V_A(1,1,i)+wind_abs(j);
        %Absolute value of incoming wind speed:
        V_A_abs(i) = sqrt(V_A(1,1,i)^2+V_A(2,1,i)^2+V_A(3,1,i)^2);
        %Using poweroutput gives corresponding powercoefficient Cp:
        Cp(i) = P_makani(j)/(0.5*rho_a*V_A_abs(i)^3*A_turb*N_turb);
    end
    C_P_avg(j) = mean(Cp); %Mean power coeff. Cp for each wind speed
    C_P_avg(1) = 0; %Set to zero to avoid NaN output
    for i = 1:length(points)
        %Calculating power output using calculated Cp as a check
        P_output(i) = N_turb*C_P_avg(j)*0.5*rho_a*V_A_abs(i)^3*A_turb;
    end
    %Calculating poweraverage for one orbit for each wind speed
    P_avg(j) = mean(P_output);
end

%Calculating "a" from eq. C_P = 4*a*(1-a)^2 using embedded solver function
tmp = zeros(length(C_P_avg)-10,3);
for i = 10:length(C_P_avg)
    syms a
    expr = 4.*a.*(1-a)^2 - C_P_avg(i);
    tmp(i,:) = solve(expr,a);
end

aval_tmp = double(tmp);
aval = real(aval_tmp); %Gives three values of a.
%Lowest one seems to be the only reasonable considering Betz limit

%"aval" gives a-values for each wind speed. To use correct a-value use the
%"find"-function and return the number in "wind_abs"-vector where
%windspeedinput = wind_abs:
val_of_Vw = find(wind_abs==V_w(1,1));
aval_lowest = min(aval(val_of_Vw,:)); %"a" value corresponding to chosen V_w
```

A.7 lift_and_drag.m

```
%LIFT AND DRAG

%Circling speed for selected wind speed:
v_a_negl_wind_value_abs = V_w(1,1)*lambda_m5;

wind_due_to_wing_motion = -n_in_NED*v_a_negl_wind_value_abs;

v_a = -n_in_NED*v_a_negl_wind_value_abs;

%Incoming wind velocity is also affected by the windspeed
v_a(1,1,:)=v_a(1,1,:)+V_w(1,1);

for i=1:length(points)
    v_a_abs(i) = sqrt(v_a(1,1,i)^2+v_a(2,1,i)^2+v_a(3,1,i)^2);
    n_in_NED_abs(i) = sqrt(n_in_NED(1,1,i)^2+n_in_NED(2,1,i)^2+ ...
        n_in_NED(3,1,i)^2);
end

%Lift force is normal to incoming wind. In this case it will be the opposite
%direction of the "Down-direction" of the wing. Therefore it is -d_in_NED!
L_normal_to_wing_z_dir = -d_in_NED;

%CROSSPRODUCT to find lift direction
L_cross_v_a = cross(L_normal_to_wing_z_dir,v_a);
dir_L = cross(v_a,L_cross_v_a);

%Define angle of attack and lift coefficient:
for i=1:length(points)
    %-----Temporary stored vectors for finding angle of attack-----%
    tmp1(i) = dot(v_a(:, :, i), wind_due_to_wing_motion(:, :, i));
    tmp2(i) = sqrt((v_a(1, :, i).^2)+(v_a(2, :, i).^2)+(v_a(3, :, i).^2));
    tmp3(i) = sqrt((wind_due_to_wing_motion(1, :, i).^2)+...
        (wind_due_to_wing_motion(2, :, i).^2)+...
        (wind_due_to_wing_motion(3, :, i).^2));
    tmp4(i) = tmp1(i) ./ (tmp2(i)*tmp3(i));
    %-----%
    wingpitch = 9*pi/180; %Adding an approximate wing pitch

    %Angle between wing x-direction and incoming wind + added wingpitch
    angle_of_attack(i) = acos(tmp4(i)) + wingpitch;

    %Lift coefficient
    C_L(i) = 2*pi*angle_of_attack(i)/(1+(2/Asp));
end

%Avg. lift coefficient for one orbit for given wind speed
C_L_mean = mean(C_L);

%DRAG

%Drag coefficient of turbines.
C_D_turb_tot = N_turb*4*aval_lowest*(1-aval_lowest)*A_turb/S;
```

```

for i = 1:length(points)
C_Di(i) = C_L(i)^2/(pi*Asp*e);           %Induced drag coefficient
C_Dtot(i) = C_Di(i)+C_Dv+C_D_turb_tot;   %Drag coefficient
end

%Avg. induced drag coefficient for one orbit for given wind speed
C_Di_avg = mean(C_Di);
%Avg. total drag coefficient for one orbit for given wind speed
C_Dtot_avg = mean(C_Dtot);

for i=1:length(points)
    %LENGTH OF THE "DIR_L"-vector:

    dir_L_length(i) = sqrt(dir_L(1,1,i)^2+dir_L(2,1,i)^2+dir_L(3,1,i)^2);
    dir_D_length(i) = sqrt((v_a(1,1,i)^2)+(v_a(2,1,i)^2)+(v_a(3,1,i)^2));

    %Lift and drag unit vectors:
    L_unit_vec(:, :, i) = dir_L(:, :, i)/dir_L_length(i);
    D_unit_vec(:, :, i) = v_a(:, :, i)/dir_D_length(i);

    %Check of lift and drag unit vectors (length should be equal to 1!)
    L_unit_vec_check(i) = L_unit_vec(1, :, i)^2+L_unit_vec(2, :, i)^2 ...
                        +L_unit_vec(3, :, i)^2;
    D_unit_vec_check(i) = D_unit_vec(1, :, i)^2+D_unit_vec(2, :, i)^2 ...
                        +D_unit_vec(3, :, i)^2;

    %Total lift vector
    L_vec(:, :, i) = 0.5*L_unit_vec(:, :, i)*C_L(i)*rho_a*v_a_abs(i)^2*S;
    %Total drag vector
    D_vec(:, :, i) = 0.5*D_unit_vec(:, :, i)*C_Dtot(i)*rho_a*v_a_abs(i)^2*S;

    %Summation of lift, drag and weight in z-direction
    L_D_and_W_in_z_dir(i) = L_vec(3, :, i)+D_vec(3, :, i)+w_in_N(1,3);
end

L_D_and_W_in_z_dir_avg = mean(L_D_and_W_in_z_dir);
%CHECK IF LIFT IS 90 deg on V_A (should be equal to zero)
lift_check = dot(v_a,L_unit_vec);

%LIFT, DRAG AND WEIGHT FORCES PROJECTED IN WINGPATH:
for i=1:length(points)

    %LIFT PROJECTED IN WINGPATH
    proj_L_in_wing_dir(:, i) = dot(n_in_NED(:, i), L_vec(:, i))* ...
                                n_in_NED(:, i)/n_in_NED_abs(i)^2;

    %MAGNITUDE PROJECTED LIFT IN WINGPATH
    mag_proj_L(i) = sqrt(proj_L_in_wing_dir(1,i)^2+ ...
                        proj_L_in_wing_dir(2,i)^2+proj_L_in_wing_dir(3,i)^2);

    %DRAG PROJECTED IN WINGPATH
    proj_D_in_wing_dir(:, i) = dot(n_in_NED(:, i), D_vec(:, i))* ...
                                n_in_NED(:, i)/n_in_NED_abs(i)^2;

    %MAGNITUDE PROJECTED DRAG IN WINGPATH

```

```
mag_proj_D(i) = sqrt(proj_D_in_wing_dir(1,i)^2+ ...  
                    proj_D_in_wing_dir(2,i)^2+proj_D_in_wing_dir(3,i)^2);  
  
%WEIGHT PROJECTED IN WINGPATH  
proj_W_in_wing_dir(:,i) = dot(n_in_NED(:,i),w_in_N)*n_in_NED(:,i)/ ...  
                            n_in_NED_abs(i)^2;  
  
%MAGNITUDE PROJECTED WEIGHT IN WINGPATH  
mag_proj_W(i) = sqrt(proj_W_in_wing_dir(1,i)^2+proj_W_in_wing_dir(2,i)^2 ...  
                    +proj_W_in_wing_dir(3,i)^2);  
  
end  
  
L_in_wingpath_mean = mean(mag_proj_L);  
D_in_wingpath_mean = mean(mag_proj_D);  
W_in_wingpath_mean = mean(mag_proj_W);  
L_D_W_in_wingpath_mean = L_in_wingpath_mean-D_in_wingpath_mean+W_in_wingpath_mean;
```

A.8 tetherforce.m

```
%FORCES PROJECTED IN TETHER DIRECTION:

for i = 1:length(points)

    %LENGTH OF POINTS-VECTOR
    points_L(i) = sqrt(points(1,i)^2+points(2,i)^2+points(3,i)^2);

    %LIFT PROJECTED IN TETHER DIRECTION:
    proj_L_in_T_dir(:,i) = dot(points(:,i),L_vec(:,i))*points(:,i)/ ...
        points_L(i)^2;

    %DRAG PROJECTED IN TETHER DIRECTION
    proj_D_in_T_dir(:,i) = dot(points(:,i),D_vec(:,i))*points(:,i)/ ...
        points_L(i)^2;

    %WEIGHT PROJECTED IN TETHER DIRECTION
    proj_W_in_T_dir(:,i) = dot(points(:,i),w_in_N)*points(:,i)/ ...
        points_L(i)^2;

    %SUMMATION OF DRAG, LIFT AND WEIGHT FORCES IN TETHER DIRECTION
    L_D_W_T_dir(:,i) = proj_L_in_T_dir(:,i)+proj_D_in_T_dir(:,i) ...
        +proj_W_in_T_dir(:,i);

    %MAGNITUDE OF FORCE IN TETHER
    mag_tetherforce(i) = sqrt(L_D_W_T_dir(1,i)^2+L_D_W_T_dir(2,i)^2 ...
        +L_D_W_T_dir(3,i)^2);

    %TETHER FORCE IN X-DIRECTION
    T_x_dir(i) = L_D_W_T_dir(1,i);
end

mean_tether = mean(mag_tetherforce);
mean_T_x_dir = mean(T_x_dir);
```


A.9 vectorplot.m

```
figure(3)
v=null(normal);
points= repmat(center',1,size(gamma,2))+radius*(v(:,1)*cos(gamma)+v(:,2)*...
            sin(gamma));
plot3(points(1,:),points(2,:),points(3,:), 'r-');
line_origin_to_center_x = [origin(1,1) center(1,1)];
line_origin_to_center_y = [origin(2,1) center(1,2)];
line_origin_to_center_z = [origin(3,1) center(1,3)];
line(line_origin_to_center_x, line_origin_to_center_y, ...
     line_origin_to_center_z);
title('Wing-path', 'fontsize',12)
grid on
hold on
xlabel('x-axis', 'fontsize',12)
ylabel('y-axis', 'fontsize',12)
zlabel('z-axis', 'fontsize',12)
xlim([0 1800])
ylim([-400 400])
zlim([0 700])

%DIFFERENT PLOTS USEFUL FOR CHECKING ORIENTATION OF WING AND DIRECTION OF
%FORCES ON THE WING. REMOVE "%" TO ENABLE PLOTS.
for j=1:59:length(points)
    %-----Plotting incoming wind, V_A-----%
    %plot3([points_x(j) points_x(j)-v_a(1,:,j)], ...
           %[points_y(j) points_y(j)-v_a(2,:,j)], ...
           %[points_z(j) points_z(j)-v_a(3,:,j)], 'k-')
    %-----%

    %-----%
    %Plotting incoming wind, when only taking into account the wing motion
    %and neglecting the windspeed
    %plot3([points_x(j) points_x(j)-wind_due_to_wing_motion(1,:,j)], ...
           %[points_y(j) points_y(j)-wind_due_to_wing_motion(2,:,j)], ...
           %[points_z(j) points_z(j)-wind_due_to_wing_motion(3,:,j)], 'b-')
    %-----%

    %-----%

    %plot3([points_x(j) points_x(j)+dir_L(1,j)], ...
           %[points_y(j) points_y(j)+dir_L(2,j)], ...
           %[points_z(j) points_z(j)+dir_L(3,j)], 'k')

    %plot3([points_x(j) points_x(j)], ...
           %[points_y(j) points_y(j)], ...
           %[points_z(j) points_z(j)+dir_L(3,j)], 'k')

    %-----PLOTING LIFT VECTOR (SCALED)-----%
    %plot3([points_x(j) points_x(j)+(L_vec(1,:,j)*10^-3)], ...
           %[points_y(j) points_y(j)+(L_vec(2,:,j)*10^-3)], ...
           %[points_z(j) points_z(j)+(L_vec(3,:,j)*10^-3)], 'm')
```

```

%-----PLOTTING LIFT IN Z-DIRECTION-----%
%plot3([points_x(j) points_x(j)], [points_y(j) points_y(j)],
%      [points_z(j) points_z(j)+(L_vec(3, :, j)*0.0005)], 'g')
%-----PLOTTING DRAG IN Z-DIRECTION (SCALED)-----%
%plot3([points_x(j) points_x(j)], [points_y(j) points_y(j)], ...
%      [points_z(j) points_z(j)+(D_vec(3, :, j)/5)], 'b')
%-----PLOTTING PROJECTED LIFT IN TETHER DIRECTION-----%
%plot3([points_x(j) points_x(j)+(proj_L_in_T_dir(1, j)*10^-4)], ...
%      [points_y(j) points_y(j)+(proj_L_in_T_dir(2, j)*10^-4)], ...
%      [points_z(j) points_z(j)+(proj_L_in_T_dir(3, j)*10^-4)], 'k')
%-----PLOTTING PROJECTED DRAG IN TETHER DIRECTION-----%
%plot3([points_x(j) points_x(j)+(proj_D_in_T_dir(1, j)*10^-2)], ...
%      [points_y(j) points_y(j)+(proj_D_in_T_dir(2, j)*10^-2)], ...
%      [points_z(j) points_z(j)+(proj_D_in_T_dir(3, j)*10^-2)], 'c')
%-----PLOTTING PROJECTED LIFT IN WINGPATH DIRECTION-----%
plot3([points_x(j) points_x(j)+(proj_L_in_wing_dir(1, j)*10^-3)], ...
      [points_y(j) points_y(j)+(proj_L_in_wing_dir(2, j)*10^-3)], ...
      [points_z(j) points_z(j)+(proj_L_in_wing_dir(3, j)*10^-3)], '—gs')
%-----PLOTTING PROJECTED DRAG IN WINGPATH DIRECTION-----%
plot3([points_x(j) points_x(j)+(proj_D_in_wing_dir(1, j)*10^-3)], ...
      [points_y(j) points_y(j)+(proj_D_in_wing_dir(2, j)*10^-3)], ...
      [points_z(j) points_z(j)+(proj_D_in_wing_dir(3, j)*10^-3)], 'b—')
%-----PLOTTING PROJECTED WEIGHT IN WINGPATH DIRECTION-----%
plot3([points_x(j) points_x(j)+(proj_W_in_wing_dir(1, j)*10^-3)], ...
      [points_y(j) points_y(j)+(proj_W_in_wing_dir(2, j)*10^-3)], ...
      [points_z(j) points_z(j)+(proj_W_in_wing_dir(3, j)*10^-3)], 'kd')
end

```

A.10 hollenbach.m

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               Hollenbach.m
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Ship velocities in m/s
Vsvec = wind_abs;
L = 178.3;
Lwl = 178.3;
Los = 178.3;
T = 9.45;
B = 29;
S_ship = 6227.8;
CB = 0.656;
TA = T;
TF = T;
Dp = 5;
NRud = 0;
NBrac = 0;
NThr = 0;
NBoss = 0;
k = 0.1075;      %Form factor to the ship

gravk = 9.81;    %Gravity
nu = 1.1395E-6; %Viscosity

%Calculation of 'Froude length', Lfn:
if Los/L < 1
    Lfn = Los;
elseif (Los/L ≥ 1) && (Los/L < 1.1)
    Lfn = L+2/3*(Los-L);
elseif Los/L ≥ 1.1
    Lfn = 1.0667*L;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Constants from Hollenbachs paper:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 'Mean' resistance coefficients
a = [-0.3382 0.8086 -6.0258 -3.5632 9.4405 0.0146 0 0 0 0];
%a1 means a(1) and so on
b = [-0.57424    13.3893    90.5960;    %b12 means b(1,2)
     4.6614     -39.721   -351.483;
     - 1.14215    -12.3296    459.254];
d = [0.854 -1.228 0.497];
e = [2.1701 -0.1602];
f = [0.17 0.20 0.60];
g = [0.642 -0.635 0.150];

% 'Minimum' resistance coefficients
a_min = [-0.3382 0.8086 -6.0258 -3.5632 0 0 0 0 0 0];
b_min = [-0.91424 13.3893 90.5960;...

```

```

        4.6614 -39.721 -351.483;...
        -1.14215 -12.3296 459.254];
d_min = [0 0 0];
e_min = [1 0];
f_min = [0.17 0.2 0.6];
g_min = [0.614 -0.717 0.261];

cc = 0;
% Loop over velocities
for Vs = Vsvec

    cc = cc + 1;

    % Froude's number
    Fn = Vs/sqrt(gravk*Lfn);
    Fnkrit = d*[1 CB CB^2]';

    c1 = Fn/Fnkrit;
    c1_min = Fn/Fnkrit;

    Rns = Vs*L/nu; % Reynold's number for ship
    CFs = 0.075/(log10(Rns)-2)^2; % ITTC friction line for ship

    % Calculation of C_R for given ship
    % Mean value

    CRFnkrit = max(1.0, (Fn/Fnkrit)^c1);

    kL = e(1)*L^(e(2));

    %There is an error in the hollenbach paper and in Minsaas' 2003
    %textbook, which is corrected in this formula by dividing by 10
    CRstandard = [1 CB CB^2]*(b*[1 Fn Fn^2]')/10;

    CR_hollenbach = CRstandard*CRFnkrit*kL*prod([T/B B/L Los/Lwl Lwl/L ...
        (1+(TA-TF)/L) Dp/TA (1+NRud) (1+NBrac) (1+NBoss) (1+NThr)].^a);

    CR = CR_hollenbach*B*T/S_ship; %Res. coef. scaled for wetted surface
    C_Ts = CFs + CR; %Total resistance coeff. ship
    R_T_mean = C_Ts*rho_w/2*Vs^2*S_ship;%Total resistance to the ship

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % When accounting for k, roughness and corr. coef., as given by Minsaas
    Rnm = 6*sqrt(6/L)*10^6/1.1395*Vs; % Reynold's number for model
    CFm = 0.075/(log10(Rnm)-2)^2; % ITTC friction line for model
    % Increase in friction due to roughness
    dCF = (110.31*(150*Vs/0.514)^0.21 - 403.33)*CFs^2;
    CA = -0.228*10^(-3); % Correlation coefficient
    CR_2 = CR_hollenbach*B*T/S_ship - k*CFm;% Resistance coefficient
    C_Ts_2 = (1+k)*(CFs + dCF) + CR_2 + CA; % Total resistance coeff. ship
    R_T_mean_2 = C_Ts_2*rho_w/2*Vs^2*S_ship;% Total resistance to the ship

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Minimum values

    %There is an error in the hollenbach paper and in Minsaas'2003 textbook,
    %which is corrected in this formula by dividing by 10

```

```

CRstandard_min = [1 CB CB^2]*(b_min*[1 Fn Fn^2]')/10;

CR_hollenbach_min = CRstandard_min*prod([T/B B/L Los/Lwl Lwl/L ...
      (1+(TA-TF)/L) Dp/TA (1+NRud) (1+NBrac) (1+NBoss) (1+NThr)].^a_min);

CR_min = CR_hollenbach_min*B*T/S_ship;

% Total resistance coefficient of the ship
C_Ts_min = CFs + CR_min;
% Total resistance
R_T_min = C_Ts_min*rho_w/2*Vs^2*S_ship;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% When accounting for k, roughness and corr. coef., as given by Minsaas
CR_min_2 = CR_hollenbach_min*B*T/S_ship - k*CFm; % Resistance coeff.
C_Ts_min_2 = (1+k)*(CFs + dCF) + CR_min_2 + CA; % Tot res coeff. ship
R_T_min_2 = C_Ts_min_2*rho_w/2*Vs^2*S_ship; % Tot res to the ship

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Store results
CFsvec(cc) = CFs;
CRvec(cc) = CR;
C_Tsvec(cc) = C_Ts;
C_Ts_2vec(cc) = C_Ts_2;
R_T_meanvec(cc) = R_T_mean;
R_T_mean_2vec(cc) = R_T_mean_2;
CR_minvec(cc) = CR_min;
C_Ts_minvec(cc) = C_Ts_min;
C_Ts_min_2vec(cc) = C_Ts_min_2;
R_T_minvec(cc) = R_T_min;
R_T_min_2vec(cc) = R_T_min_2;
end

```

A.11 propulsion.m

```
%Data for open water diagram is achieved from master thesis by Bockmann
load('open_water_diagram_10_K_Q.txt');
load('open_water_diagram_eta.txt');
load('open_water_diagram_K_T.txt');

etta_x = open_water_diagram_eta(:,1);
etta_y = open_water_diagram_eta(:,2);
K_Q_10_x = open_water_diagram_10_K_Q(:,1);
K_Q_10_y = open_water_diagram_10_K_Q(:,2);
K_T_x = open_water_diagram_K_T(:,1);
K_T_y = open_water_diagram_K_T(:,2);

%Smoother curve with same steps on x val for all lines are obt. from Cftool
%Coefficients obtained for etta
p1_etta = -23.23;
p2_etta = 65.71;
p3_etta = -74.42;
p4_etta = 42.23;
p5_etta = -12.64;
p6_etta = 1.58;
p7_etta = 1.12;
p8_etta = 0.0;
%Coefficients obtained for 10*K_Q
p1_KQ = -0.1467;
p2_KQ = 0.2282;
p3_KQ = -0.2743;
p4_KQ = -0.3439;
p5_KQ = 0.6532;
%Coefficients obtained for KT
p1_KT = -0.2797;
p2_KT = 0.5772;
p3_KT = -0.5004;
p4_KT = -0.2602;
p5_KT = 0.4913;

%Equations for etta, 10*KQ and KT
x = 0:0.001:1;
etta_line = p1_etta*x.^7 + p2_etta*x.^6 + p3_etta*x.^5 + p4_etta*x.^4 + ...
            p5_etta*x.^3 + p6_etta*x.^2 + p7_etta*x + p8_etta;
KQ_line = p1_KQ*x.^4 + p2_KQ*x.^3 + p3_KQ*x.^2 + p4_KQ*x + p5_KQ;
KT_line = p1_KT*x.^4 + p2_KT*x.^3 + p3_KT*x.^2 + p4_KT*x + p5_KT;

figure
hold on
grid on
ylabel('K_T,10*K_Q,\eta_0','fontsize',12)
xlabel('J','fontsize',12)
plot(x,etta_line,'r')
plot(x,KQ_line,'g')
plot(x,KT_line)
legend('\eta_0','10*K_Q','K_T')
ylim([0 1])
```

```

%Now the value  $K_Q/J^3$  is found and plotted on y-axis and J on the x-axis:
KQ_div_Jpow3_line = zeros(1,length(x));
for i = 1:length(x)
    KQ_div_Jpow3_line(i) = (KQ_line(i)/10)/x(i).^3;
end

%Calculating the values for:  $K_Q/J^3 = P/(\rho_w \cdot 2 \cdot \pi \cdot u^3 \cdot D^2)$ 

powerinp = P_avg(val_of_Vw:end);
V_ship_inp = wind_abs(1:length(wind_abs)-val_of_Vw+1);

for i = 1:length(powerinp)
    K_Q_div_J3(i) = powerinp(i) / (rho_w*2*pi*V_ship_inp(i).^3*Dp.^2);
    K_Q_div_J3(1) = 0;
end

figure
hold on
grid on
xlabel('J','fontsize',12)
ylabel('K_Q/J^3','fontsize',12)
plot(x,KQ_div_Jpow3_line,'r')
%plot(x,K_Q_div_J3(15))
ylim([0 1])

%If using Matlab 2013 this Jvalue should be used
for i = 1:length(K_Q_div_J3)
    for j=1:length(x)
        if KQ_div_Jpow3_line(j) ≤ K_Q_div_J3(i)
            break
        end
    end
    Jvalue(i) = x(j);
    Jvalue(1) = 0;
end

%Matlab version 2011b uses this "interp1" function perfect
J_value = interp1(KQ_div_Jpow3_line,x,K_Q_div_J3);
J_value(1) = 0;

etta_val = p1_etta*J_value.^7 + p2_etta*J_value.^6 + ...
    p3_etta*J_value.^5 + p4_etta*J_value.^4 + p5_etta*J_value.^3 + ...
    p6_etta*J_value.^2 + p7_etta*J_value + p8_etta;

figure
hold on
grid on
ylim([0 1])
xlim([0 12])
ylabel('\eta_p','fontsize',14)
xlabel('Ship speed, u [m/s]','fontsize',12)
plot(V_ship_inp,etta_val)
legend('Water propeller efficiency')

for i = 1:length(V_ship_inp)
    prop_force(i) = (powerinp(i)*etta_val(i))/V_ship_inp(i);
end

```

```

prop_force(1) = ((rho_w*pi*etta_p0.^2)^(1/3))*...
               (Dp.^2*powerinp(1).^2)^(1/3));
end

%Store values for horizontal tether forces for different wind speeds:
hor_tether_stored = [0 0 0 0 0 0 0 113653 158248 209943 266838 329603 ...
                    398329 473084 553784 640490 733246 831769 936040 1045764 1161467 ...
                    1283142 1410786 1544394 1683964 1829493 1980981 2138425 2301825 ...
                    2471179 2646486 2827746 3014958 3208122 3407236 3612300 3823315 ...
                    4040279 4263193 4492056 4726868 4967629 5214338 5466995 5725601 ...
                    5990154 6260655 6537104 6819501 7107845 7402137];

hor_tetherinp = hor_tether_stored(val_of_Vw:end);
shipres_and_tether = hor_tetherinp + ...
                    R_T_mean_2vec(1:length(R_T_mean_2vec)-val_of_Vw+1);

figure
str = sprintf('Plot for windspeed: %f ',V_w_inp);
title(str)
hold on
grid on
xlim([0 12])
plot(V_ship_inp,prop_force,'r')
plot(V_ship_inp,shipres_and_tether,'g')
plot(V_ship_inp,hor_tetherinp,'b')
ylabel('Propulsion force [N]','fontsize',12)
xlabel('Ship speed, u [m/s]','fontsize',12)
legend('Water propeller thrust', ...
       'Total resistance (ship resistance + tether resistance)', ...
       'Resistance of tether')

```


A.12 printvalues.m

```
fprintf('Corresponding circling speed          --->: %f [m]\n\n'...
      ,v_a_negl_wind_value_abs);

fprintf('-----Makani M5 wing specs-----\n\n')
fprintf('Weight of the Makani M5 wing          --->: %f [N]\n',w_in_N(1,3));

fprintf('Length of the tether                    --->: %f [m]\n'...
      ,length_of_tether);
fprintf('Circling radius                        --->: %f [m]\n'...
      ,circling_radius);
fprintf('Area of the turbines                    --->: %f [m^2]\n',A_turb);
fprintf('Number of turbines                    --->: %f [-]\n',N_turb);
fprintf('Wing span                                --->: %f [m]\n',s);
fprintf('Wing planform area                      --->: %f [m^2]\n',S);
fprintf('Aspect area of the wing                  --->: %f [m^2]\n\n',Asp);

fprintf('-----Results for selected windspeed-----\n\n')
fprintf('Max angle of attack                      --->: %f [deg]\n'...
      ,max(angle_of_attack)*(180/pi));
fprintf('Min angle of attack                      --->: %f [deg]\n'...
      ,min(angle_of_attack)*(180/pi));
fprintf('Mean C_L coeff.                          --->: %f [-]\n',C_L_mean);
fprintf('C_P coeff. average                       --->: %f [-]\n'...
      ,C_P_avg(val_of_Vw));
fprintf('Power output average                     --->: %f [W]\n'...
      ,P_avg(val_of_Vw));
fprintf('Axial induction factor "a"               --->: %f [-]\n',aval_lowest);
fprintf('Viscous drag coeff. (profile drag)       --->: %f [-]\n',C_Dv);
fprintf('Induced drag coeff.                     --->: %f [-]\n',C_Di_avg);
fprintf('Turbine drag coeff. (8 turbines)         --->: %f [-]\n',C_D_turb_tot);
fprintf('Tot drag coeff. (C_Di+C_Dv+C_D_turb)    --->: %f [-]\n',C_Dtot_avg);
fprintf('Mean tether force                        --->: %f [N]\n',mean_tether);
fprintf('Mean horizontal tether force             --->: %f [N]\n',mean_T_x_dir);
fprintf('Summation of L, D and W in z-dir.       --->: %f [N]\n'...
      ,L_D_and_W_in_z_dir_avg);
```


B Variables in MATLAB

Table B.1: Variables in MATLAB

Variable name	Explanation
s	Wing span
S	Wing planform area
Asp	Wing aspect ratio
length_of_tether	Length of tether connected to the wing
circling_radius	Circling radius of the wing
top_oper_range	Top operational range of the wing
A_turb	Area of the turbines on the wing
N_turb	Number of turbines on the wing
e	Planform efficiency factor
period	Period for one orbit of the wing
C_Dv	Viscous drag
etta_p0	Propulsion efficiency for zero-speed of ship
rad_between_wing_and_orbit	Radians between the wing position and the line to center of orbit
rad_betweeen_z_axis_and_orbit	Radians between the z-axis and the line to center of orbit
length_to_center_of_orbit	Length to the line from origin to center of orbit
x_length_to_center_of_orbit	Projected length to center of orbit on x-axis
z_length_to_center_of_orbit	Projected length to center of orbit on z-axis
normal, center	Vectors defined to set orbit path of the wing
lambda_m5	Reference TSR
t	Defining number of points used in for one orbit of the wing
gamma	Angles in one orbit to set wing path
points	X, Y and Z points for every position of the wing in orbit
points_x	X points for every position of the wing in orbit
Continued on next page	

Table B.1 – continued from previous page

Variable name	Explanation
points_y	Y points for every position of the wing in orbit
points_z	Z points for every position of the wing in orbit
line_origin_to_center_x	X-length from origin to center of orbit
line_origin_to_center_y	Y-length from origin to center of orbit
line_origin_to_center_z	Z-length from origin to center of orbit
line	Line from origin to center of orbit
phi	Wing's roll angle before trans. to wing's coord. system
theta	Wing's pitch angle before trans. to wing's coord. system
psi	Wing's yaw angle before trans. to wing's coord. system
trans_matrix_phi	Transformation matrix for wing's roll angle phi
trans_matrix_theta	Transformation matrix for wing's pitch angle theta
trans_matrix_psi	Transformation matrix for wing's yaw angle psi
trans_matrix_tot	Total transformation matrix for wing's rotation angles
n_vec, e_vec, d_vec	Unit-vectors [1,0,0], [0,1,0], [0,0,1]
n_in_NED	n_vec transformed to NED coord-system
n_in_NED_abs	Absolute value of n_in_NED
e_in_NED	e_vec transformed to NED coord-system
d_in_NED	d_vec transformed to NED coord-system
V_w_inp	Input value setting chosen wind speed
V_w	Wind speed vector
val_of_Vw	Finding position in "wind_abs" for selected V_w_inp
wind_abs	Defining a vector including wind speeds from 0-25
P_makani	Power output read off Makani power curve
P_output, poweraverage	Power calculated
Continued on next page	

Table B.1 – continued from previous page

Variable name	Explanation
circling_speed	Circling speed for wing
V_A, v_a	Incoming wind on wing
V_A_abs, v_a_abs	Absolute value of incoming wind on wing
v_a_negl_wind_value_abs	Constant circling speed for selected wind speed (neglecting wind)
wind_due_to_wing_motion	Circling speed vector for selected wind speed (neglecting wind)
Cp	Power coefficient
C_P_avg	Mean power coeff. for one orbit
C_P_avg_output	Mean power coeff. for one orbit for selected wind speed
expr	Used for calculating axial induction factor
aval	Axial induction factor calculated
aval_lowest	Lowest value for aval is the one being used
origin	Just defining the origin as [0,0,0]
w	Weight of wing
w_in_N	Weight of wing in [N]
Asp	Aspect ratio used in hydrodynamics of ref.ship
CA	Correlation coefficient used in hollenbach
CB	Block coefficient used in hollenbach
CFm	ITTC friction line for model. From hollenbach script
CFs, CFsvec	ITTC friction line for ship. From hollenbach script
CR, CRFnkrit, CR_2,	Coefficients used in hollenbach
CR_hollenbach, CR_hollenbach_min,	
CR_min, CR_min_2, CR_minvec,	
CRstandard. CRstandard_min, CRvec	
C_Ts_.....	
T, TA, TF	Depth of ship used in hollenbach
Continued on next page	

Table B.1 – continued from previous page

Variable name	Explanation
Dp	Prop. diameter
Fn, Fnkrit	Froude's number used in hollenbach
Loa, Lpp,Lwl,..	Length of ship
NBoss, NBrac, NRud, NThr	Used in hollenbach
R_T...	Resistance calculated in hollenbach
Rnm, Rns	Reynolds number calc. in hollenbach
S_ship	Wetted surface of ships
Vs, Vsvec	Ship speed used in hollenbach
L_normal_to_wing_z_dir	L in positive z-dir of wing
L_cross_v_a	Cross product to find direction of lift
dir_L	Gives correct direction of lift (normal to v_a)
wingpitch	Added wing pitch
angle_of_attack	Angle of attack for incoming wind on wing
C_L	Lift coefficient
C_L_mean	Average lift coefficient for one orbit
C_D_turb_tot	Drag coefficient of turbines
C_Di	Induced drag coefficient
C_Dtot	Total drag coefficient
C_Di_avg	Average induced drag coefficient for one orbit
C_Dtot_avg	Average total drag coefficient for on orbit
dir_L_length	Length of lift vector
dir_D_length	length of drag vector
L_unit_vec	Lift unit vector
D_unit_vec	Drag unit vector
L_unit_vec_check	Check if L_unit_vec is correct
D_unit_vec_check	Check if D_unit_vec is correct
L_vec	Total lift vector
D_vec	Total drag vector
L_D_and_W_in_z_dir	Summation of lift, drag and weight in z-direction
Continued on next page	

Table B.1 – continued from previous page

Variable name	Explanation
L_D_and_W_in_z_dir_avg	Avg summation of L, D and w in z-dir for one orbit
lift_check	Check if lift vector direction is correct
proj_L_in_wing_dir	Lift projected in wing direction
mag_proj_L	Magnitude of lift projected in wing direction
proj_D_in_wing_dir	Drag projected in wing direction
mag_proj_D	Magnitude of drag projected in wing direction
proj_W_in_wing_dir	Weight projected in wing direction
mag_proj_W	Magnitude of weight projected in wing direction
points_L	Length of points-vector
proj_L_in_T_dir	Lift projected in tether direction
proj_D_in_T_dir	Drag projected in tether direction
proj_W_in_T_dir	Weight projected in tether direction
L_D_W_T_dir	Summation of drag, lift and weight in tether direction
mag_tetherforce	Magnitude of force in tether
T_x_dir	Tether force in horizontal x-direction
mean_tether	Average tether force in one orbit for given wind speed
mean_T_x_dir	Average horizontal tether force in one orbit for given wind speed
etta_x	η x-values from open water diagram
etta_y	η y-values from open water diagram
K_Q_10_x	K_Q x-values from open water diagram
K_Q_10_y	K_Q y-values from open water diagram
K_T_x	K_T x-values from open water diagram
K_T_y	K_T y-values from open water diagram
p1_etta, p2_etta, ... , p8_etta	η values obtained in Cftool for smoother curve
p1_KQ, p2_KQ, ... , p5_KQ	K_Q values obtained in Cftool for smoother curve
p1_KT, p2_KT, ... , p5_KT	K_T values obtained in Cftool for smoother curve
Continued on next page	

Table B.1 – continued from previous page

Variable name	Explanation
x	Vector from 0 to 1 with steps 0.001 used in "propulsion"
etta_line	The smoother η line obtained using p1_etta, p2_etta, etc...
KQ_line	The smoother K_Q line obtained using p1_KQ, p2_KQ, etc...
KT_line	The smoother K_K line obtained using p1_KT, p2_KT, etc ...
KQ_div_Jpow3_line	K_Q / J^3 line
powerinp	Power output starting from given wind speed input and increasing with ship speed
V_ship_inp	Ship speed
K_Q_div_J3	Calculated K_Q / J^3 from formula
Jvalue, J_value	J-value read off K_Q / J^3 line using calculated value
etta_val	Propulsion efficiency graph calculated
prop_force	Calculated propulsion force to the propeller
hor_tether_stored	Vector with horizontal tether forces calculated for different wind speeds
hor_tetherinp	Horizontal tether forces starting from given wind speed input and increasing with ship speed
shipres_and_tether	Total resistance (ship resistance + horizontal tether force)
a, a_min,b, b_min,c, c_min,cc,DCF,d_min,e,e_min eta_...,f,f_min,g,g_min, kL gacc, gravk i, j k	Constants used in hollenbach Gravitational constant Counters in for-loops Form factor for ship
Continued on next page	

Table B.1 – continued from previous page

Variable name	Explanation
nu	Viscosity of water. Used in hollenbach
rho_a	Density of air
rho_w	Density of water
tmp, tmp1, ... , tmp4, aval_tmp	Temporary variables used
End of Table B.1	

C Additional files

Along with the thesis a zip-file is delivered with the following content:

- The Matlab files used in the calculation
- Poster from master thesis poster exhibition

