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Bestemme GM ved hjelp av rulleperiodetest

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Determining GM from a Roll Decay Test
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Preface

This is a master thesis conducted at the Norwegian University of Science and Technology (NTNU) during the autumn of 2015. The outline and guidance of the thesis was conducted by Professor Håvard Holm.

The purpose of this thesis is to study alternative methods of determining the metacentric height and how it is affected by the roll damping effects. Different model tests have been conducted, such as roll decay tests and measurement of rolling motions in irregular waves.

I would like to thank my supervisor Professor Håvard Holm for his help and guidance throughout the thesis. Especially during the model tests at the MC Lab his help was of significance. I also would like to thank Torgeir Wahl for his support during the model tests.

Vegard Nesset Hjelvik
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Abstract

The intact stability is an essential parameter in the design phase of a vessel, and has significant affect on the main dimension and hull form. For the safety of crew and vessel, governments have specific stability requirements, which often refer to the International Maritime Organization's Code on Intact Stability. A vessels stability is defined by the metacentric height (GM), which is usually found by conducting an inclining test. Although, in certain circumstances it might not be possible due to time or cost. In such cases, one may estimate the metacentric height based on a roll decay test or the response spectre of the vessel in irregular waves.

Rolling motions of a vessel is complex due to the viscous effects and dependence on forward speed. Viscous damping is also an important factor with regards to reducing the rolling motion to avoid damages on the cargo or even capsizing. One way of reducing the rolling motion is use of bilge keels, which increases the viscous damping. The roll damping effect have been studied in this thesis.

Model tests have been conducted in order to study the possibility of determining the metacentric height by roll decay tests or based on the response spectre, as well as the affect of viscous damping on the roll period and amplitude. Two different ship models were tested in various loading conditions, with and without bilge keels. Initially an inclining test was carried out, in order to determine the metacentric height, before roll decay tests were conducted.

Based on existing hull lines for one of the vessels, the geometry was imported into a computer program in order to calculate the hydrostatic data. These data have then been used to predict the damping coefficients by Kawahara's simple method, which is based on Ikeda's well know prediction method. The process of digitalizing the hull lines proved to be challenging.

Sammendrag

Intaktstabilitet er en viktig parameter i designfasen for et fartøy, og har stor innvirkning på hoveddimensjoner og skrogform. For at fartøyer og mannskap skal være trygge, settes det derfor krav til stabiliteten fra myndighetene, som ofte refererer til International Maritime Organizations intaktkode. Et fartøys stabilitet defineres av metasenterhøyden (GM), som normalt blir funnet ved å utføre en krengeprøve. I noen tilfeller er det derimot ikke mulig eller ønskelig å gjennomføre en krengeprøve på grunn av tid eller kostnad. Dermed er det noen ganger ønskelig å beregne metasenterhøyden ved hjelp av andre metoder, som rulledempingstest eller basert på responsspekteret til et fartøy i bølger.

Rullebevegelsen til et fartøy er den mest komplekse bevegelsen, på grunn av viskøse effekter og kobling til fart. Viskøs demping er også en vesentlig faktor med tanke på reduksjon av rullebevegelser, for å unngå skader på last eller i verste fall kantring. En av metodene som ofte blir brukt for å redusere rullebevegelsen er slingrekjøler, hvor reduserte bevegelser følger av økt viskøs demping.

Modellforsøk har blitt gjennomført for å undersøke om det er mulig å bestemme metasenterhøyden ved hjelp av rulledempingstester eller basert på responsspekteret, og påvirkningen av rulledempingens effekt på periode og amplitude. To ulike modeller har blitt testet, hvor den ene modellen har blitt testet med to ulike slingrekjøler. Innledningsvis ble en krengeprøve er utført for å bestemme metasenterhøyden, før rulleperiodetester ble gjennomført.

Videre har geometrien til den ene modellen blitt modellert ved hjelp av et dataprogram som beregner hydrostatiske data på bakgrunn av skrogform. Disse dataene har så blitt brukt som input i Kawaharas metode, som er en metode for å beregne dempingskoeffisienten. Prosessen med å modellere den eksisterende geometrien og bruke de hydrostatiske dataene for å predikere dempingskoeffisienten viste seg å være utfordrende.

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Nomenclature

Accronyms & Abbreviations:

2008 IS Code	International code on intact stability 2008
IMO	International Maritime Organization
NTNU	Norwegian University of Science and Technology
RAO	Response Amplitude Operator
SOLAS	Safety of Life at Sea
DOF	Degrees of freedom

Greek Symbols:

$\ddot{\Phi}$	Roll angular acceleration
Δ	Displaced weight
$\dot{\Phi}$	Roll angular velocity
η_{4a}	Roll amplitude
ν	Fluid kinematic viscosity
Φ	Angle of heel
Φ	Roll angle
ρ	Density
ρ	Fluid density
σ	Coefficient of the ship hull section
ζ	Damping ratio

Symbols:

$(I_{44+A_{44}})$	Virtual moment of inertia
A_{44}	Roll added mass
A_{44}	Roll added moment
B_{44}^{V1}	Linear damping coefficient
B_{44}^{V2}	Non-linear damping coefficient
B_e	Roll damping coefficient
C_B	Block coefficient
I_{44}	Ship longitudinal moment of inertia
r_{44}	Roll radii of gyration
r_s	Average radius of roll

T_N	Natural roll period
D	Draft
g	gravity
GM	Metacentric height
GZ	Righting lever
GZ curve	Righting lever curve
L	Length
OG	Vertical distance from origin to center of gravity
RM	Righting moment
S	Wetted surface
U	Velocity

1 Introduction

Intact stability is one of the main criteria when designing a vessel and is important when determining the main dimensions, hull form and arrangement. During years of accidents at sea, the rules and regulations stated by International Maritime Organization (IMO) have been revised and changed in order to build safer vessels. The International Convention for the Safety of Life at Sea (SOLAS) is generally regarded as the most important of all international treaties concerning the safety of merchant ships, and the Code on Intact Stability are generally viewed as requirements for most vessels. Mathematical simulation, experiments and investigation of empirical data have been conducted to create stability criteria which satisfies the operational requirements of the vessel and need for safe environment to the seafarers. In an ever more demanding market, ship safety is an increasingly important factor and the intact stability requirements will continually be revised, taking both experience and further development into consideration [13].

The vessels stability, which is measured by the metacentric height, is dependent on the weight and centre of gravity. The weight and centre of gravity can normally be found by conducting an inclining test, but at certain circumstances such exercise is not possible or desirable. In such cases, one will need other methods for determining the metacentric height, such as a free roll decay test or estimation based on the ships response spectra.

Rolling motions are probably the most important phenomenon for vessels, as small changes have huge impact on the ship response. They are hard to overcome and difficult to estimate, due to their complexity. Viscous effects gives a clearly non-linear behaviour, which makes estimating even more difficult. The effects have been studied comprehensively by an empirical and experimental approach, and later by a technique of dividing the roll damping into different components. Most of the commercial computer programs are based on the latter approach, hence that is the main focus in this thesis.

Based on ship motion theory, including the viscous roll damping presented in chapter 2.5 based on the review of Himeno [8] and the traditional strip theory developed by Salvesen et al. [18], computer programs such as MARINTEK's VERES and University of Michigan's SHIPMO have been made for calculating vessel responses [5]. From the vessel data, one are able to calculate the motion transfer function for six degrees of freedom. These motion transfer functions, also knows as response amplitude operators (RAO), can be combined with wave spectres to obtain response spectra. This way, designers, owners and operators can calculate the accelerations at certain points of the vessel for dimensioning equipment or planning operations.

1.1 Objective

This thesis will evaluate how the rolling motion is affected by the damping effect. The objective is to investigate if a roll decay test or response spectra will give sufficient accuracy when calculating the metacentric height, for vessels which do not have the opportunity to conduct a complete inclining test. Model tests will be conducted for two different ship models with various loading conditions, load distributions, sizes of bilge keels, in still water and waves with different headings. The damping coefficient is calculated based on the results from model tests and compared to estimated values from a numerical prediction method. Four main objectives are to be studied:

- Review of alternative methods for estimating the metacentric height
- Model tests to assess if the metacentric height can be estimated by roll decay tests
- Model tests to assess if the metacentric height can be estimated by the ships response spectra
- Numerical calculations to predict the damping coefficients

1.2 Structure of the Thesis

Chapter 2: A brief review of previous research on the topic is presented. Then theory of stability and the general stability criteria are introduced, before a more thorough review of the rolling motion equation and roll viscous damping are presented.

Chapter 3: The models and setups, test procedures, results and analysis of the model tests are presented, before a short discussion with conclusions.

Chapter 4: Kawahara's prediction method is presented, and the predicted damping coefficients are compared with experimental data from the model tests.

Chapter 5: General discussions with conclusions and proposals for further work.

2 Theory

An introduction to previous research on roll damping and the theory of stability and stability requirements are briefly presented, before a more thorough review is done for the theory of rolling motions and the viscous damping effects.

2.1 Previous research

The strip method was established in 1970 by Salvesen, Tuck and Faltinsen to predict heave, pitch, sway, roll and yaw motions [18] and was based on potential flow theories (Ursell-Tasai method, source distribution method, etc). As the roll damping is more complex than the other motions due to its non-linear behaviour and dependency on forward speed, it has been subject to loads of studies. Two different approaches have been made to the prediction of roll damping; empirical models based on data from regression analysis of model experiments and later the analytical treatment, where the damping component is divided into different parts. The latter were investigated in Japan, where Himeno, Tanaka and Ikeda at the Osaka Prefecture University published several papers. Ikeda created a prediction method, known as Ikeda's method based on existing formulas such as Kato's formula for the friction coefficient [14], his own work for eddy making [10] and bilge keel damping, and the strip method for wave damping. Late 1978, the method was summarized in a computer program to determine the damping coefficient [19]. A complete review of the method was done in 1981 by Himeno [8] with references both to the Japanese and non-Japanese litterature on the topic. As Ikeda's method is too complicated to use in the design phase, Kahawara, Maekawa and Ikeda created a simple roll damping prediction method by using regression analysis in 2008 [15]. As that method could not be used for modern ships with high position of centre of gravity, such as cargo ships. Kawahara et al. investigated the method's limits and proposed a new prediction method in 2009 [16]. As some of the key references are not available in English and some contains typographical errors, Falzarano et al. presented 'An overview of the prediction methods for roll damping of ships' in 2015 [5] to provide a comprehensive summary of the state of the art method in one place.

2.2 Stability

2.2.1 Definition of stability

If a floated body returns to its initial position when a force or moment causes a small change in its position, it is considered as stable. The second alternative is that the vessel is unstable, the condition when the body continues change. The last option is when the vessel is in neutral equilibrium [1].

2.2.2 Metacentric height

The metacentre is the intersection between two vertical lines, one through the centre of buoyancy of a hull in equilibrium, the other through the centre of buoyancy when the hull is inclined slightly to one side or toward one end.

The metacentric height, $\overline{GM}$, is the distance from the centre of gravity, G, to the metacentre M:

$$\overline{\mathrm{GM}} = \overline{KB} + \overline{BM} - \overline{KG} \tag{1}$$

A ship is initially stable if its initial metacentre is above the centre of gravity and unstable if it is below. The condition of initial stability is expressed as:

$$\overline{GM} > 0 \tag{2}$$

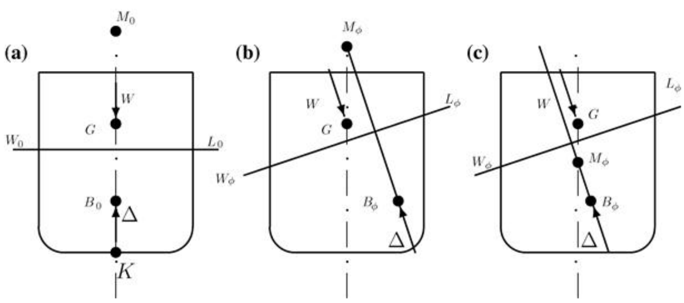

Figure 1: Initial stability, [1]

(a) Vessel is in upright position and stable. In (b) the vessel is stable, as the metacentre is above the vertical centre of gravity. (c) The vessel is unstable as the metacentre is below the centre of gravity.

The metacentric height of a vessel varies due to the operational tasks. E.g. a passenger ship will have totally different GM than an offshore vessel with need for stability in heavy crane or cable laying operations. As the metacentric height depends on the centre of gravity, it will also vary significantly in different loading conditions.

2.2.3 The righting lever

The righting lever (GZ) and righting lever curve (GZ curve) are important when determining if a vessel have sufficient intact stability. A ship whose waterline in upright position is shown in figure 2, where the forces of weight and buoyancy produce a righting moment whose values is [1]

$$M_R = \Delta GZ \tag{3}$$

The righting lever can also be calculated by

$$GZ = GM \cdot \sin(\theta) \tag{4}$$

for small angles of heel, θ, up to approximate 10 deg.

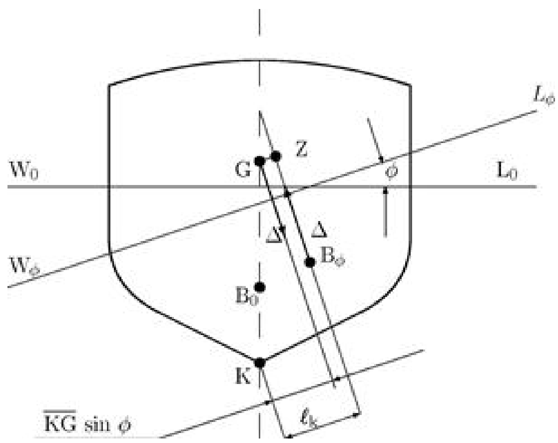

Figure 2: Definition of righting arm, [1]

2.2.4 The righting lever curve

The righting lever curve is the righting arm, GZ, plotted against the heel angle, often called the GZ curve.

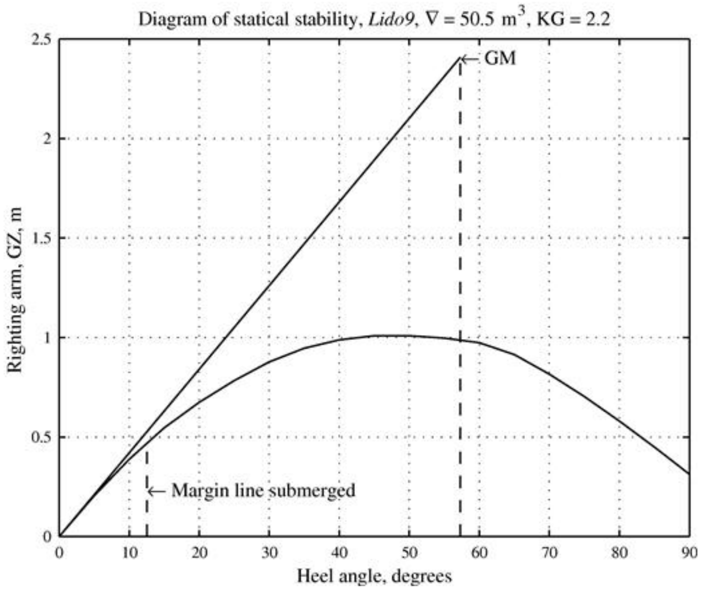

Figure 3: Righting lever curve, [13]

The properties of the GZ curve are important with regards to the intact stability requirements in the Code on Intact Stability.

2.3 Stability criteria

The International Code on Intact Stability 2008 (2008 IS Code) is the latest revision of the IMO regulations. It contains the general stability criteria based on righting arm characteristics which was adopted in 1968. Later, in 1985, the Weather Criterion was included [7]. Since the requirements first was adopted, it have been reviewed and revised continuously. Specific criteria are available for different types of vessels, such as cargo ships, passenger ships, fishing vessels, offshore supply vessels (OSV) or mobile offshore drilling units (MODU). In those cases where specific codes are given, the other codes should be taken as the prevailing instrument [13].

2.3.1 Criteria regarding righting lever curve properties

The Code on Intact Stability have specific requirements regarding the lever curve properties, such as area under the GZ curve, position of max GZ and minimum GZ at certain angles of heel. The requirement varies for different types of vessels, but the general criteria are shown below.

2.2.1 The area under the righting lever curve (GZ curve) shall not be less than 0.055 metre-radians up to $\varphi = 30°$ angle of heel and not less than 0.09 metre-radians up to $\varphi = 40°$ or the angle of down-flooding φ_f[5] if this angle is less than 40°. Additionally, the area under the righting lever curve (GZ curve) between the angles of heel of 30° and 40° or between 30° and φ_f, if this angle is less than 40°, shall not be less than 0.03 metre-radians.

[5] φ_f is an angle of heel at which openings in the hull, superstructures or deckhouses which cannot be closed weathertight immerse. In applying this criterion, small openings through which progressive flooding cannot take place need not be considered as open.

2.2.2 The righting lever GZ shall be at least 0.2 m at an angle of heel equal to or greater than 30°.

2.2.3 The maximum righting lever shall occur at an angle of heel not less than 25°. If this is not practicable, alternative criteria, based on an equivalent level of safety[6], may be applied subject to the approval of the Administration.

2.2.4 The initial metacentric height GM_0 shall not be less than 0.15 m.

2.3.2 Severe wind and rolling criterion (weather criterion)

The weather criterion was adopted in 1985, after IMO recommended that external forces affecting ships in seaway that may lead to capsizing or unacceptable angle of heel, were taken into the regulations [7]. An excerpt from the criterion is shown below

2.3 Severe wind and rolling criterion (weather criterion)

2.3.1 The ability of a ship to withstand the combined effects of beam wind and rolling shall be demonstrated, with reference to the figure 2.3.1 as follows:

.1 the ship is subjected to a steady wind pressure acting perpendicular to the ship's centreline which results in a steady wind heeling lever (l_{w1});

.2 from the resultant angle of equilibrium (φ_0), the ship is assumed to roll owing to wave action to an angle of roll (φ_1) to windward. The angle of heel under action of steady wind (φ_0) should not exceed 16° or 80% of the angle of deck edge immersion, whichever is less;

.3 the ship is then subjected to a gust wind pressure which results in a gust wind heeling lever (l_{w2}); and

.4 under these circumstances, area *b* shall be equal to or greater than area *a*, as indicated in figure 2.3.1 below:

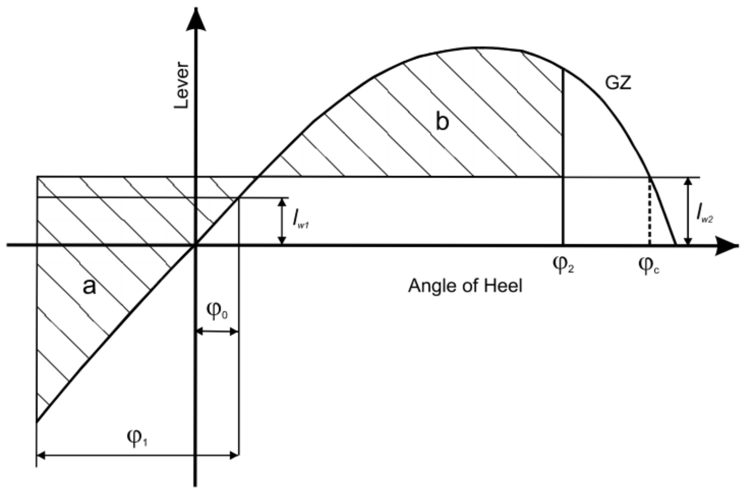

Figure 4: Severe wind and rolling criterion, [13]

2.4 Equation of roll motion

Rolling motion is dependent on the other ship responses, such as pitch, heave, sway and yaw. In order to reduce the complexity, the equation of roll motion is most often presented in a simple single-degree-of-freedom form and can be written as:

$$(I_{44} + A_{44})\ddot{\Phi} + B_{44}(\dot{\Phi})\dot{\Phi} + C_{44}\Phi = M_{44}(\omega t) \quad (5)$$

where $\ddot{\Phi}$ is the roll angular acceleration, $\dot{\Phi}$ is the roll angular velocity and Φ is the roll angle [21]. Even if the coupling effects are being neglected, the accuracy have been proven to be very good.

The ship longitudinal mass moment of inertia I_{44} is controlled by the mass distribution of the vessel and can be calculated

$$I_{44} = M \cdot r_{44}^2 \quad (6)$$

where M is mass and r_{44} is the radii of gyration which can be calculated by

$$r_{44} = \sqrt{\frac{\sum(y^2 + z^2) \cdot \Delta M}{M}} \quad (7)$$

where the coordinates y and z are given relative to the center of gravity. ΔM is the weight of the item at (x,y) and M is the total weight of the vessel [6].

Typical values for r_{44} are between 0.3 · B - 0.45 · B, where B is the vessel breadth [6]. According to Faltinsen 0.35 · B is a typical value [4]. Loaded vessels have typically radius of gyration in the lower part of the range, while vessels with large superstructures or deck cargo have radius in the upper part of the range.

Roll added inertia A_{44} is also called added mass, which determines the necessary work done to change the kinetic energy associated with the motion of the fluid [2]. It is determined by the hull shape and the appendices such as keel, bilge keel and stabilizer fins. A rounded hull form with no keel will have low added mass, when a v-shaped hull form with keel and bilge keels will have a significantly higher added mass. Added mass can be calculated quite accurate by special computer software, and is typically 20-40%, depending og hull form and appendices.

The virtual moment of inertia, $(I_{44} + A_{44})$ is often used in calculations, as it can be determined directly from experiments.

B_{44} represents roll damping moment, which are to be predicted by formulas in the next part. The damping moment can be composed into linear and non-linear terms of the roll velocity, which are time dependent. The damping coefficients have been presented in different ways, depending on whether the roll damping is expressed as linear or non-linear form [3], where

$$B(\dot{\Phi}) = B_1\dot{\Phi} + B_2|\dot{\Phi}|\dot{\Phi} + B_3\Phi^3 \quad (8)$$

is a presentation of a non-linear model. The total damping is often approximated by an equivalent linear term

$$B(\dot{\Phi}) = B_{eq}\dot{\Phi} \quad (9)$$

where B_{eq} can be expressed as

$$B_{eq} = B_1 + \frac{8}{3\pi} B_2(\omega R_0) + \frac{3}{4} B_3(\omega R_0) \tag{10}$$

and R_0 is the roll amplitude.

C_{44} is the restoring force term, and can be calculated by $g \cdot \Delta \cdot \overline{GZ}$. To achieve better accuracy, the righting arm can be approximated as polynomial of third or fifth order.

$$\overline{GZ}(\Phi) = C_1\Phi + C_3\Phi_3 + C_5\Phi^5 \tag{11}$$

where $C_1 = \overline{GM}$ and $\overline{GM}$ is the metacentric height.

The restoring moment can be estimated linearly for roll angles up to approximate 7 deg, which may be extended for wall-sided ships [21]

$$C_{44} = g \cdot \Delta \cdot \overline{GM} \tag{12}$$

M_{44} is the excitation moment due to waves or external forces and is non-linear due to its dependency on time and the radian frequency, ω.

Then the linear form of roll equation of motion can be expressed as

$$(I_{44} + A_{44})\ddot{\Phi} + B_{44}\dot{\Phi} + g \cdot \Delta \cdot \overline{GM}\Phi = 0 \tag{13}$$

2.5 Viscous roll damping

By using the potential strip method, where the fluid is assumed to be homogeneous, non-viscous and incompressible, one can calculate all of the terms in the equation of ship motions by practical accuracy, except from the roll damping. The roll damping has proven difficult to estimate due to the viscous effect and dependence on forward speed [8]. Roll damping is mainly due to wave generation, but for certain bilge forms or hulls with bilge keels and at certain frequencies, the viscous effects becomes important, which makes the response non-linear and much more complex to calculate.

The estimation of roll damping has been widely studied, as accurate estimates gives a good base for predicting the rolling motion, which is considered the most critical ship motion with regards to cargo shifting or even capsizing. The main purpose of determining the roll damping is to analyse the effect on roll amplitudes [17]. The damping effects and added mass are usually neglected by the stability regulations as it is conservative when determining the metacentric height. But some regulations like IMO, takes it indirectly into account by using different parameters for ships fitted with sharp bilges og bilge keels [1]. For accurate prediction of the rolling motion, for a vessel with known geometry, one has to determine the damping coefficients and the added mass, either by experiments or predicting methods. Often special computer software are used, based on empirical or theoretical models and data.

For a long time the Watanabe-Inoue formula was the simplest method for estimating the roll damping moment. The formula is empirical and experimental, based on analysis of model tests on actual ships and some theoretical considerations on the pressure distribution on the hull caused by the ship roll motion [8]. As the first proposed formula only covered a few ship forms, its was slightly modified in 1963 [20] to cover for a larger range of hull forms. Another simple method based on the same approach is the Tasai-Takaki's table.

In the 1970s professor Ikeda et al. studied the different components of the roll damping and divided the equivalent linear damping B_{eq} into seven separate components [8].

$$B_{eq} = B_F + B_E + B_L + B_W + B_{BKN} + B_{BKH} + B_{BKW} \tag{14}$$

where B_F is the frictional component, B_E is the eddy making component, B_L is the lift component, B_W is the wave component, B_{BKN} is the bilge keel component for natural force, B_{BKH} is the bilge keel component for hull pressure and B_{BKW} is the bilge keel component for waves.

The bilge keel terms are often summed up:

$$B_{BK} = B_{BKN} + B_{BKH} + B_{BKW} \tag{15}$$

which gives the equivalent linear damping B_{eq} with five components

$$B_{eq} = B_F + B_E + B_L + B_W + B_{BK} \tag{16}$$

where the values varies with the roll amplitude Φ_A and the frequency ω.

Lift damping B_L is linear, independent of ω and proportional to ship speed, hence there is no damping due to lift when the vessel has zero forward speed [8]. Thus it will not be studied further in this thesis.

2.5.1 Friction damping

Friction damping B_F is the damping caused by the viscous skin friction stress acting on the hull surface and was studied by Kato in 1958 [14] where he, in the absence of forward speed, applied Blasius formula for laminar flow

$$B_{F0} = \frac{4}{3\pi}\rho S r_s^3 \phi_A \omega C_f \tag{17}$$

Then he applied Hughes formula to adjust for turbulent flow [8]. The formula can be expressed in terms of an equivalent linear damping coefficient as follows

$$B_{F0} = 0.787\rho S r_s^2 \sqrt{\omega\nu}(1 + 0.00814(\frac{r_s^2\phi_A^2\omega}{\nu})) \tag{18}$$

where B_{F0} is the friction damping coefficient at zero speed, ρ is the density and ν is the kinematic viscosity of the fluid.

The wetted surface S and the average roll radius can be expressed approximately by the following formulas

$$S = L(1.7D + C_B \cdot B) \tag{19}$$

$$r_s = \frac{1}{\pi}((0.887 + 0.145C_B)\frac{S}{L} - 2OG) \tag{20}$$

L, d, B and C_B are length, draft, breadth and block coefficient of the ship respectively. OG denotes the distance between the water surface and the centre of gravity, where positive values are defined as below the water surface.

Ikeda et al. confirmed the validity of Kato's formula in practical use through the measurements of the velocity profile in the boundary layer on two-dimensional cylinders of ship like sections [9].

2.5.2 Eddy damping

Eddy damping at zero speed B_{E0} is caused by vortices generated by flow separation at the bilge of cross section. The pressure drop in the separation increases the damping.

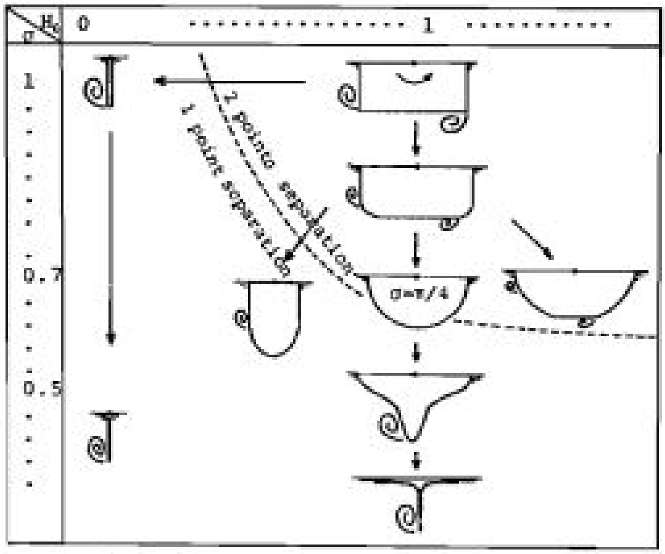

Figure 5: Eddies near hull, [10]

Ikeda et.al. studied this effect and proposed the following equation for the two dimensional cross section coefficient [10]

$$B'_{E0} = \frac{4}{3\pi}\rho d^4 \omega \phi_A (-\frac{r_{max}}{d})^2 \cdot F(\frac{R}{d}, H_0, \sigma, \frac{OG}{d}) \cdot C_P \tag{21}$$

where ω is natural frequency, ϕ_A is the roll angle, H_0 is half the beam-draft ratio and C_P is the pressure coefficient which can be found from

$$C_P = 0.5[0.87exp(-\gamma) - 4exp(-0.187\gamma) + 3] \tag{22}$$

where γ is the velocity-increment ratio which are expressed in Appendix A.1.

The eddy damping value can be found by integrating the sectional value over the ship length [8]. For ship rolling, Ikeda et al. confirmed that the eddy damping coefficient safely can be considered constant.

2.5.3 Bilge keel damping

Bilge keel damping B_{BK} is damping due to normal forces on bilge keels, and includes the interaction effects among the bilge keels, waves and hull [8]. B_{BK} is usually the largest damping component and creates 50-80% of the total roll damping [16]. It is normally divided into two components, the normal force component B_{BKN} and the hull pressure component B_{BKH}, as the wave damping component B_{BKW} can be neglected for bilge keels of normal breadth.

The normal force damping of bilge keels can be presented as

$$B'_{BKN0} = \frac{8}{3\pi}\rho r^2 b_{BK}^2 \omega f^2 (\frac{22.5}{\pi f} + 2.4\frac{r\phi_A}{b_{BK}}) \tag{23}$$

$$f = 1 + 0.3exp(-160(1-\sigma)) \tag{24}$$

where r is the mean distance from centre of gravity to the bilge keel and b_{BK} is the breadth of the bilge keel and f is an empirical coefficient of velocity increment at the bilge circle [8].

Hull pressure damping due to bilge keels can be determined

$$B_{BKH0} = \frac{4}{3\pi}\rho r^2 d^2 \omega \phi_A f^2 I \tag{25}$$

$$I = \frac{1}{d^2}\int C_P l_0 ds \tag{26}$$

where the integration of I must be done around the whole girth, and multiplicated by the moment lever l_0 around the rotation axis [8].

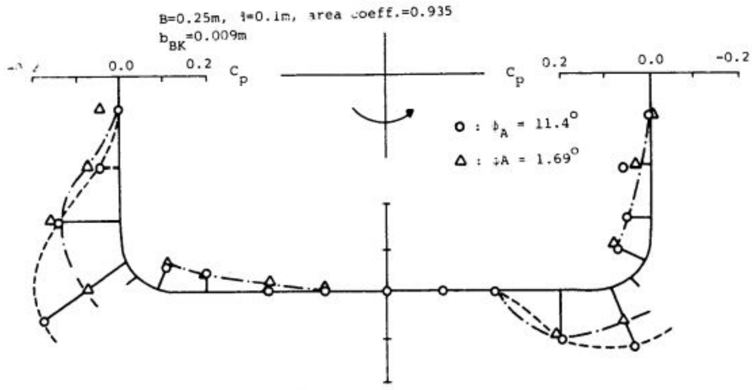

Figure 6: Pressure distribution on hull due to bilge keels, [8]

2.5.4 Wave damping

The wave damping coefficient for a ship section B'_{W0} can be calculated with practical accuracy for ship forms at zero speed by strip theory or any other potential theory [16]. The wave damping for a ship section is calculated from the solution of a two-dimensional wave problem

$$B'_{W0} = \rho N_S (l_w - OG)^2 \tag{27}$$

where N_S is the sway damping coefficient and l_w is the moment lever measured from the point O due to the sway damping force [8].

2.5.5 Prediction of total damping

Below is a schematic overview of the damping components and their contribution at different Froude number.

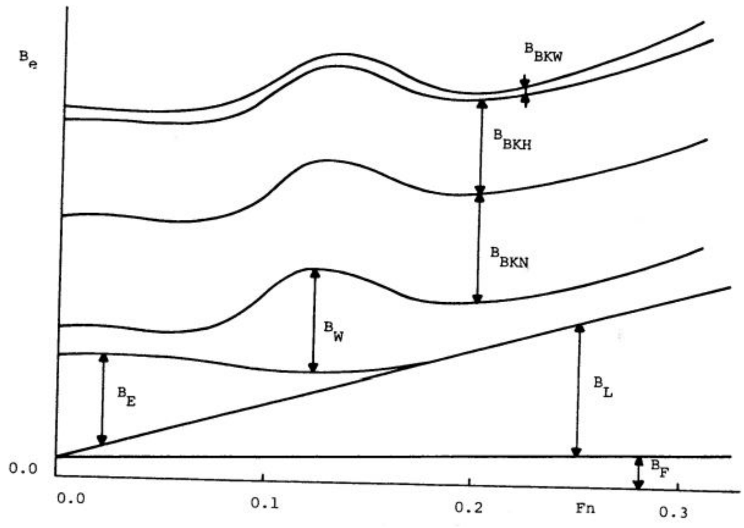

Figure 7: Roll damping coefficients with advance speed, [8]

As we can see from figure 7, B_{BKW}, B_F are small compared to the other components. The friction scale can normally be omitted for full scale, but at small scale model tests it can take about 5%-10% of the total roll damping [11]. B_L is zero when Froude's number F_n is zero, thus eddy damping, bilge keel damping and wave damping are the main components of roll damping when the vessel have no forward speed.

3 Model testing

The metacentric height is a significant value for most stability criteria, hence accurate estimation will be of significant importance. An inclining test is the common and accurate way to determine the metacentric height. In some cases it might not be possible to conduct a complete inclining test, and then a roll decay test can be an alternative.

With regards to the Code on Intact Stability, the stability documentation for a vessel requires an inclining test report or a summary of other methods used to determine the stability data. The stability data are included in a loading computer for the captain to be able to check if the vessel have sufficient stability in the given loading condition or for a certain operation.

As masters of small ships requested alternative methods for determining the initial stability, attention was given to roll decay tests in the late 1980s. Experiments showed that the roll decay test could be an alternative to inclining tests, when the latter was not practical and simple formulas were deduced. [13].

3.1 Models and setups

Several model tests have been conducted in Lilletanken and in the Marine Cybernetics Lab at NTNU to examine the influence of viscous damping in a roll decay test when determining the metacentric height. Inclining tests and roll decay tests were conducted for two ship models with different geometry. Both models were tested with different setups, such as various drafts, weight distributions and bilge keels.

Model	Breadth (m)	Length (m)	Depth (m)	Mass (kg)
1	0.300	0.890	0.260	11.6
2	0.250	1.070	0.200	13.6

Model 1 was tested with three different loading conditions; lightship, 4 kg spread in the two cargo rooms and 7 kg spread in the two cargo rooms.

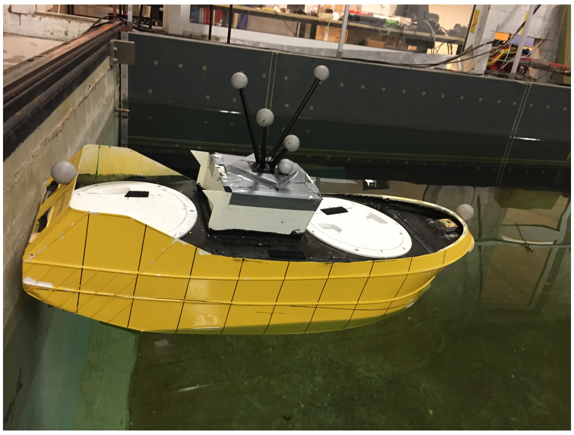

Figure 8: Model 1

Model 2 was tested with two different loading conditions; lightship and weights of 4 kg on deck.

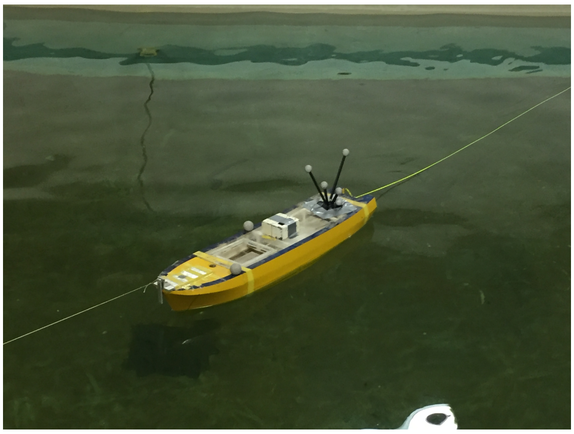

Figure 9: Model 2

3.2 Inclining test

According to IMO, any vessel of 24 m and upwards is required to be inclined upon its completion. It also requires any vessel to be re-inclined whenever, in comparison with the approved stability information, a deviation from the lightship displacement exceeding 2 % or a deviation of the longitudinal centre of gravity exceeding 1 % is found, or anticipated.

The vessel should be as close to completion as possible at the time of the inclining test, and should also be as clean as possible, as any weights extra weight which are not accounted for will affect the accuracy of the results. Preferably all tanks should be empty or full during the test due to the free surface effect. Any liquids in tanks necessary for the test are to be measured and included in the final calculations. The complete inclining test procedure is found in the Code on Intact Stability annex 1 [13].

The inclining tests were carried out at 'Lilletanken' at MARINTEK. Two ship models were tested by moving small weights from each side of the vessel to create a known moment. The weights are divided in four weight groups to get several heeling angles during the test. In total eight weight shifts are carried out, starting with equal weights on both sides, to determine the metacentric height, GM.

Initially the vessel is weighed to find the displacement. For full scale vessels, the displacement is found by draught readings. The test weights are also measured in order to ensure accurate results.

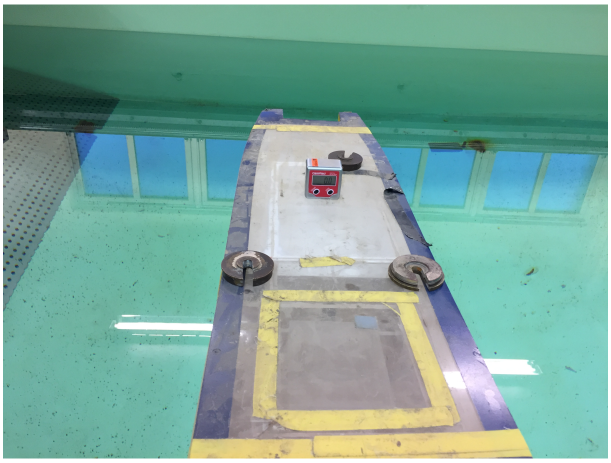

Figure 10: Inclining step 0 - zero heel

Then the vessel is heeled by moving the inclining weights. In a standard test, 8 weight shifts are carried out, where the last shift is done to confirm the initial heeling angle.

Figure 11: Inclining step 2 - max heel

The inclining angles are measured by a digital angle gauge for each shift and the GM value for each weight shift is calculated by equation (28), which can be found from equation (3 - 4). The average of single GM for each weight shift gives GM for the specific load case.

$$GM = \frac{RM}{\Delta \sin(\Phi)} \tag{28}$$

where RM is the righting moment, Δ is the displaced weight and Φ is the measured heel angle. The righting moment can be calculated by multiplying the inclining weight with the shifted distance. To ensure the results are accurate, one can plot the moment against heel angle and the results will be in a straight line if correct. For full scale inclining tests, there might be inaccuracy due to wind, waves, free surface effect in tanks, tight mooring, etc. If the results are too inaccurate, they will not be accepted by class societies, and a new inclining test must be carried out.

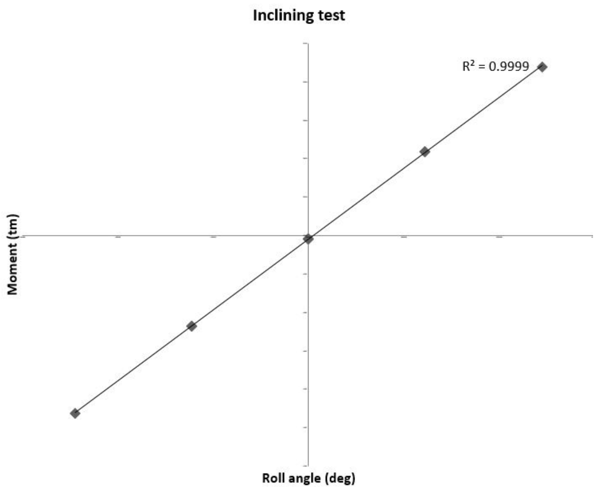

Figure 12: Moment-heel angle plot

3.3 Roll decay test

A procedure for the roll decay test is presented in the Code on Intact Stability which was adopted in 1993, Resolution A.749(18), for vessels below 70 meters. It was later removed and is not included in the latest revision of the intact code.

3.3.1 Calculation of metacentric height and roll period

In Resolution A.749(18), equation (29) is given in the test procedure for roll decay tests:

$$GM_0 = \left(\frac{fB}{T_f}\right)^2 \tag{29}$$

where f is the factor for the rolling period, B is breadth and T_f is roll period. Factor f is of greatest importance for the accuracy of the results and varies for general ship types, as shown in figure 13, which are based on numerous inclining tests and roll decay tests. The values are given as mean values of the experiments, where observed values generally are within ± 0.05 [12]. Thus one need good knowledge of the ship which is to be tested in order to get accurate results.

	f values
Empty ship or ship carrying ballast	$f \approx 0.88$
Ship fully loaded and with liquids in tanks comprising the following percentage of the total load on board (i.e. cargo, liquids, stores, etc.):	
20% of total load	$f \approx 78$
10% of total load	$f \approx 0.75$
5% of total load	$f \approx 0.73$
Double-boom shrimp fishing boats	$f \approx 0.95$
Deep sea fishing boats	$f \approx 0.80$
Boats with a live fish well	$f \approx 0.60$

Figure 13: Factor f for different ship types, [12]

Experiments have shown that the results of the rolling test method get increasingly less reliable the nearer they approach GM values of 0.20 m and below.

As the factor f in equation (29) is based on experiments conducted many years ago and given for different ship types, modern ships may deviate significantly. Therefore one need a more accurate formula which takes the specific ship into account. A formula for natural roll period T_N is given in Ship Motions and Sea Loads by Faltinsen [4]

$$T_N = 2\pi\left(\frac{Mr_{44}^2 + A_{44}}{\rho g \Delta GM}\right)^{\frac{1}{2}} \tag{30}$$

where Mr_{44}^2 equals mass moment of inertia (I_{44}), which is given by the weight distribution and added mass (A_{44}), which is given by the hull form and appendices of the vessel. The formula can be rewritten to calculate the metacentric height:

$$GM = 4\pi^2\left(\frac{Mr_{44}^2 + A_{44}}{\rho g \Delta T_N^2}\right) \tag{31}$$

3.3.2 Calculation of coefficients

Calculation of coefficients for the linear equation of roll motion are presented by Zeraatgar and Asghari in 'A study of the Roll Motion by Means of a Free Decay Test' [21].

$$(I_{44} + A_{44})\ddot{\Phi} + B_{44}\dot{\Phi} + \Delta g\overline{GM}\Phi = 0 \tag{32}$$

When estimating the motion from a roll decay test, with the initial conditions $\dot{\Phi(0)}$=0 and $\Phi(0) = \Phi_0$, the analytical solution is

$$\Phi(t) = \frac{\Phi_0}{\sqrt{1-\zeta^2}exp(-\zeta\omega_n t)sin(\omega_d t + \alpha)} \tag{33}$$

where

$$\alpha = tan^{-1}(\sqrt{\frac{1-\zeta^2}{\zeta}}) \tag{34}$$

ζ is the damping ratio, which increases as the damping increases. ω_n and ω_d are the natural frequency and the damped frequency.

$$\zeta = \frac{B_{44}}{2\sqrt{I_{44} + A_{44}\Delta g\overline{GM}}} \tag{35}$$

$$\omega_n^2 = \frac{\Delta g\overline{GM}}{I_{44} + A_{44}} \tag{36}$$

When the roll period T_N is found from the roll decay test, the damped roll frequency can be obtained from equation (37 - 38)

$$\omega_n = \frac{2\pi}{T_N} \tag{37}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{38}$$

The damping factor can be found from the decrement of two successive positive or negative peaks

$$\zeta = \frac{ln(\frac{\Phi_{n+2}}{\Phi_n})^2}{4\pi + ln(\frac{\Phi_{n+2}}{\Phi_n})^2} \tag{39}$$

with Φ_n as the roll angle at oscillation n, and Φ_{n+2} as the roll angle at oscillation n + 2

When the damping factor is found, the virtual moment of inertia $I_{44} + A_{44}$ can be calculated by

$$(I_{44} + A_{44}) = \frac{\Delta g\overline{GM}(1-\zeta^2)}{\omega_d^2} \tag{40}$$

the linear damping coefficient B_{44} is expressed as

$$B_{44} = \zeta \cdot 2\sqrt{(I_{44} + A_{44})\Delta g\overline{GM}} \tag{41}$$

and finally the restoring coefficient C_{44}

$$C_{44} = g \cdot \Delta \cdot GM \tag{42}$$

where g is gravity, Δ is the displaced mass and $\overline{GM}$ is the metacentric height.

3.3.3 Test cases

The roll decay tests were carried out at the Marine Cybernetic Laboratory at NTNU. Four different test cases were chosen to determine the damping coefficient B_{44}, the mass moment of inertia I_{44}, the impact of different bilge keels on the roll damping and finally to assess the possibility to determine GM from the response spectra:

- Roll decay test in still water for both ship models
- Roll decay test for model 1 with two different bilge keels
- Roll decay test for model 2 with different weight distribution
- Model 2 in waves with 45 ° and 90 ° headings

Before the free roll decay test starts, the GM is determined by an inclining test. Further the vessel is rolled to a given angle and released. Then the subsequent motions are measured by an optical system, and the roll amplitude Φ_A and natural roll frequency ω_n can be determined. In the software all 6 degrees-of-freedom are registered, together with the position, time and wave height. The roll motion was wrongly calibrated as pitch in the tests, hence results in the pitch column were used.

File comment:										
Time DEVI	FrameCou	ErrorCode(	Xpos	Ypos	Zpos	Roll	Pitch	Yaw	Residual	WP1
s	-	-	m	m	m	deg	deg	deg	-	m
0	160243	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.02	160243	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.04	160243	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.06	160245	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.08	160245	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.1	160245	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.12	160246	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.14	160246	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.16	160247	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.18	160247	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.2	160248	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793
0.22	160248	0	5.7827	-2.6326	0.1293	-173.659	0.728	-0.404	-1	-0.66793

Figure 14: Roll decay test output

The rolling motion can be plotted against time to analyse and determine the roll period T_N and the amplitude ϕ_A. Based on those factors, one can calculate the damping factor ζ and then the damping coefficient B_{44} and the virtual moment of inertia $(I_{44} + A_{44})$ from the decay test results and equations (39) - (41).

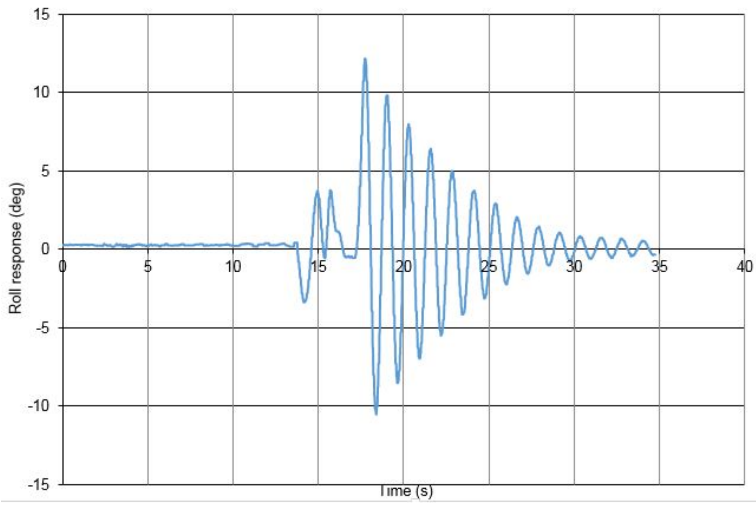

Figure 15: Roll decay test result

The roll period is determined from the graph in figure 15, where the ships natural period is averaged over at least 5 peaks and the amplitude is given for each oscillation. As the ship model was calibrated only once for each ship model, the heel angle deviates slightly for each loading condition and the mean of the initial angle is used for correction of the curve.

3.4 Results and analysis

3.4.1 Inclining test - results

The metacentric heights for the two different ship models are calculated by equation (28) from the inclining tests results and presented below

Model	Load	Δ	GM
1	0	11.975	0.0253
1	4	15.975	0.0331
1	7	18.975	0.0284
2	0	13.975	0.0550
2	4	17.975	0.0224

Table 1: Inclining test result

As the weights on model 1 is placed low in the cargo rooms, the metacentric heights are increased. The metacentric height is significantly reduced for model 2 when the weights of 4 kg are placed on deck as the centre of gravity is shifted upwards, closer to the metacentre.

Shift	Moment (tm)	tan (heel)
0	0	-0.0017
1	0.000025	0.0437
2	0.000049	0.0875
3	0.000025	0.0437
4	0	-0.0017
5	-0.000025	-0.0437
6	-0.000049	-0.0928
7	-0.000025	-0.0472
8	0	-0.0017

Table 2: Moments and heel angles

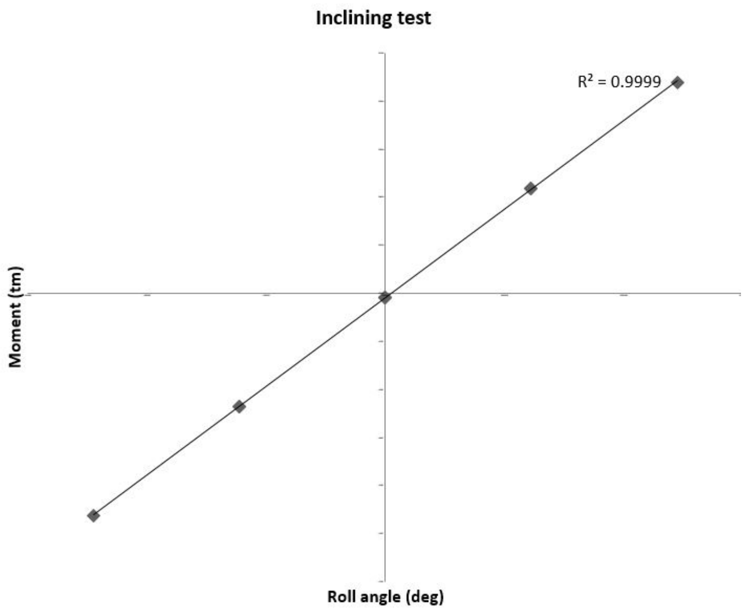

Figure 16: Moment-heel angle plot

As seen from figure 16 which shows the moment - heel angle plot for model 1 with 4 kg load, all weight shifts are perfectly in line, which shows that the accuracy of the inclining test is very good and that the metacentric height can be used as reference for the roll decay tests.

3.4.2 Roll decay test - calculation of damping coefficients

Model 2 was tested in two different loading conditions to calculate the damping coefficient. As the hull lines of this ship model are available, one can compare the results of this test with the predicted coefficients in ch. 4.2. The results from the roll decay test of model 2 with no deck load is shown below:

N	$\Phi(deg)$	Time (s)	$\zeta_{N:N+2}$
0	17.65	0	0.040
1	-15.30	0.44	0.047
2	13.70	0.90	0.043
3	-11.35	1.34	0.040
4	10.43	1.74	0.038
5	-8.8	2.20	0.041
6	8.2	2.64	0.041
7	-6.78	3.10	0.038
8	6.34	3.54	0.029
9	-5.34	3.94	0.031
10	5.28	4.40	0.032
11	-4.4	4.84	0.039
12	4.33	5.30	0.032

Table 3: Roll decay test results - model 1

The damping ratio for small heel angles is assumed to be linear. The average damping ratio shown in figure 17, is approximately 0.037.

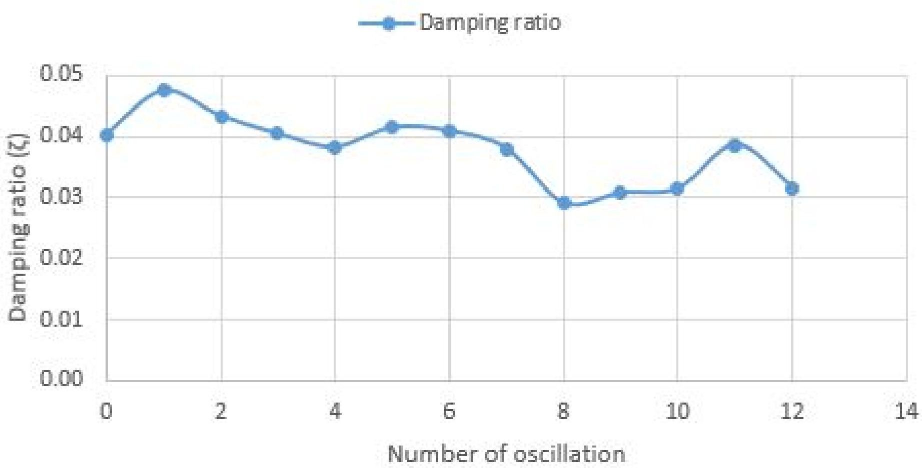

Figure 17: Damping ratio as function of oscillation

From equations (40) - (42) one can find the virtual moment of inertia, $(I_{44} + A_{44})$, the damping coefficient (B_{44}) and the restoring coefficient (C_{44}) based on the results from the roll decay test:

Model	Load	$\Delta(kg)$	GM (m)	$T_N(s)$	$(I_{44} + A_{44})$	(B_{44})	(C_{44})
2	0	13.975	0.0550	0.883	0.151812	0.080999	7.545147
2	4	17.975	0.0224	1.457	0.212111	0.117622	3.944608

Table 4: Model 2 - roll decay test

When the coefficients of the roll motion equation are calculated, the second order differential equation can be solved numerically by a Runge-Kutta solver in MAPLE with $\dot{\Phi(0)}=0$ and $\Phi(0) = \Phi_0$ as the initial conditions. Then the rolling motion based on the calculated coefficients can be compared with the actual measured motions.

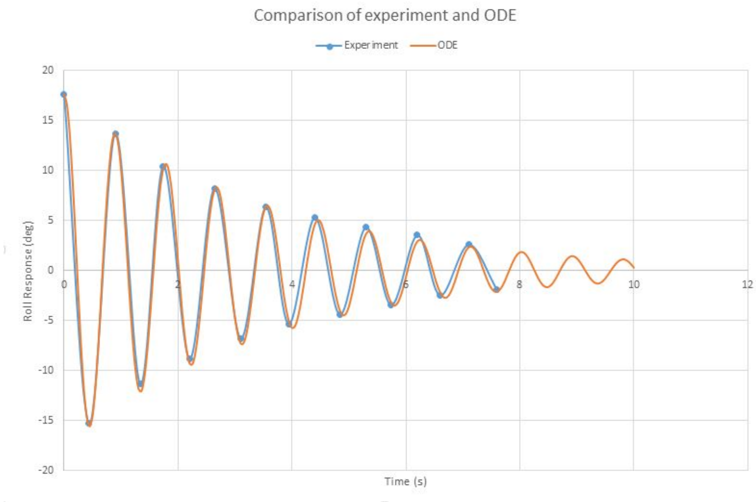

Figure 18: Comparison of experiment and solution of ODE

There are slight discrepancies on measured rolling motions in the experiment and the numerical solution of the roll equation, which can be due to the assumptions made, such as linear damping and restoring moment. This can be calculated more accurately by using a quadratic or cubic model for the damping and restoring terms. Another source of error can be inaccuracy in the measurements from the model tests.

3.4.3 Roll decay test - various bilge keels

Model 1 was tested with two different bilge keels installed, with same length of 250 mm and breadth of 15 mm and 30 mm respectively, in order to determine the shift of natural period due to the damping effect.

Figure 19: Fitting of bilge keels

In table 5 are the results from the roll decay test of model 1 with 4 kg load, with the two different bilge keels presented. The virtual moment of inertia ($I_{44} + A_{44}$), damping coefficient (B_{44}) and restoring coefficient (C_{44}) are calculated by equations (40) - (42).

Model	Load	Bilge keel	T_N	$(I_{44} + A_{44})$	(B_{44})	(C_{44})
1	4	None	1.400	0.221347	0.025166	4.458378
1	4	15 mm	1.407	0.223996	0.060812	4.458378
1	4	30 mm	1.450	0.235995	0.092514	4.458378

Table 5: Roll period - various bilge keels

In order to compare the rolling motions for three different setups with various initial heel angle, the numerical solutions of the roll equation with calculated coefficient were plotted.

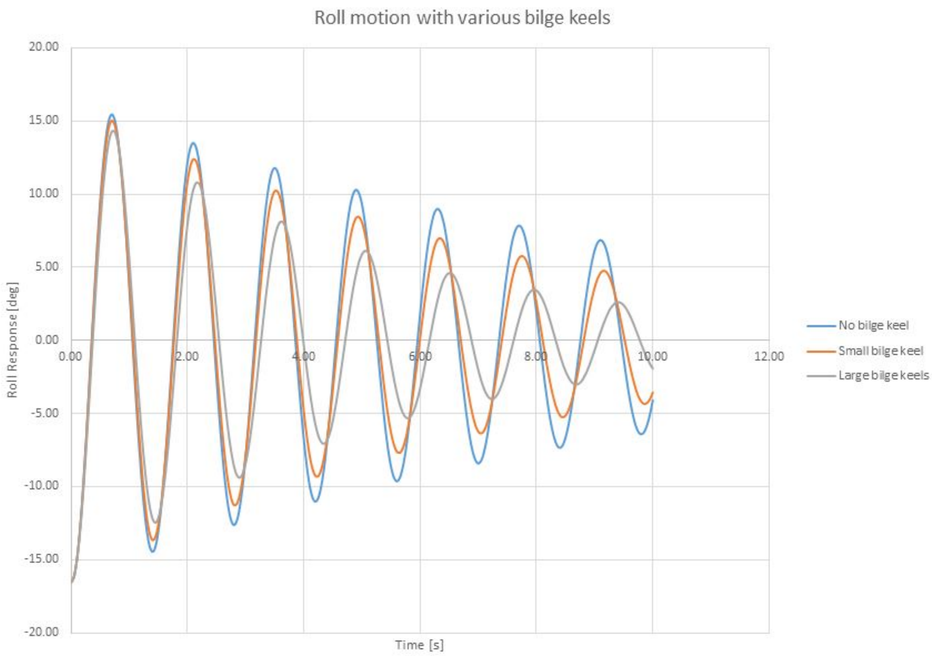

Figure 20: Roll motion with various bilge keels

As seen in figure 20, the roll period is slightly increased when the bilge keels are added, mainly due to the increased added mass A_{44} shown in table 5, as the moment of inertia remains the same. The increased weight from bilge keels is negligible.

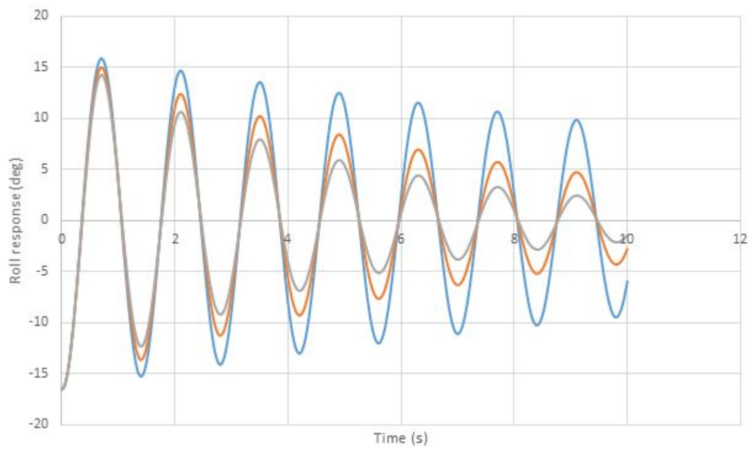

Figure 21: Change in roll period due to roll damping

When plotting the numerical solutions of the roll equation with various damping factors, one can see that the damping effect on the roll period is negligible as shown in figure 21, while it has significant impact on the amplitudes.

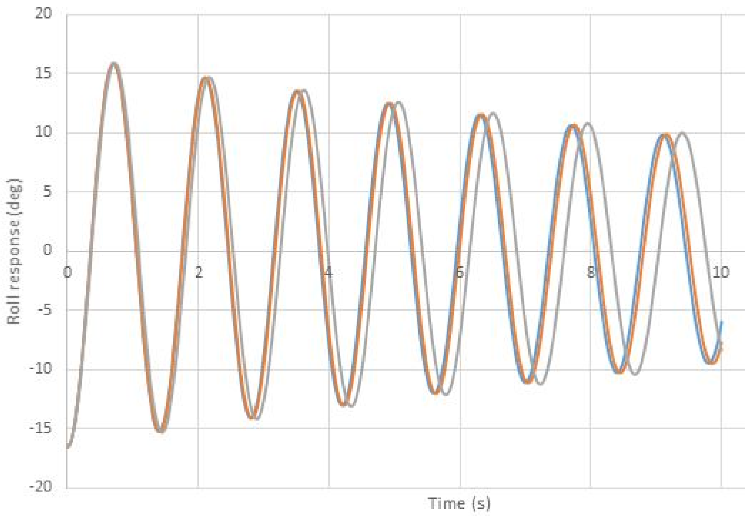

Figure 22: Change in roll period due to added mass

When plotting the numerical solutions of the roll equation with various virtual moment of inertia, due to change in added mass, one can see that the added mass shifts the roll period, as shown in figure 22, while the amplitudes remains the same.

3.4.4 Determining GM from response spectre

Model 2 was tested in irregular waves with headings, 45 ° and 90 °. A JONSWAP wave spectrum was applied in the wave maker, with significant wave height H_s of 0.015 m. The ship model was restrained to stay in position by two light ropes and the rolling motions were measured.

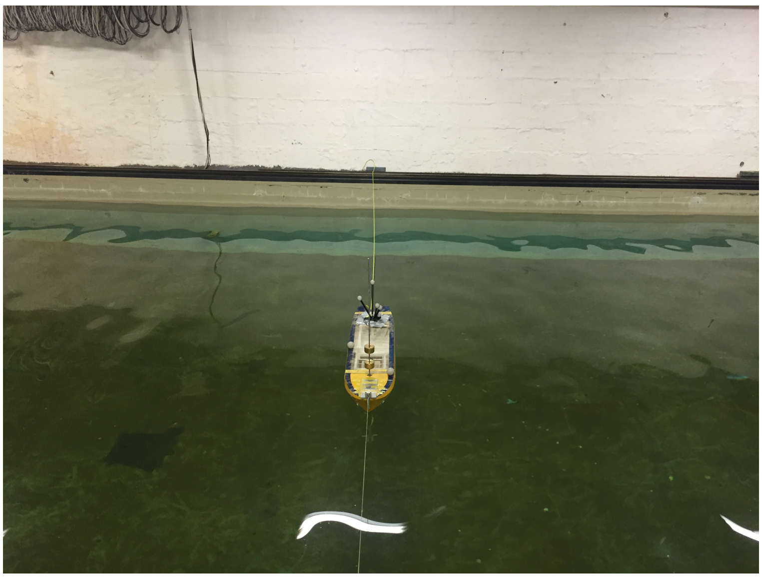

Figure 23: Roll period test in 90 deg waves

Then the natural frequency of the vessel could be calculated by averaging the time between positive oscillations from the response spectre.

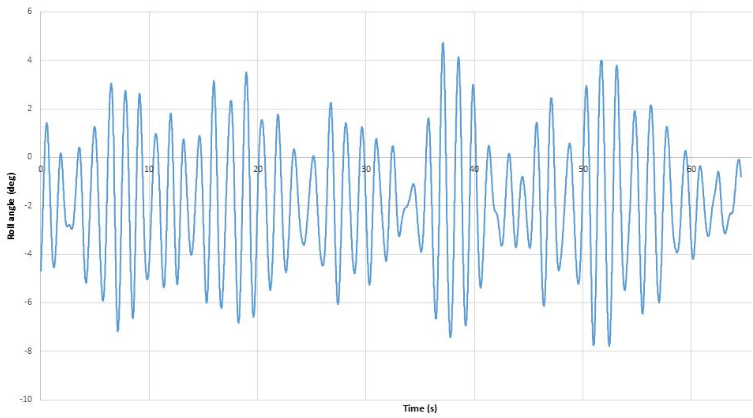

Figure 24: Roll period test - JONSWAP wave spectre

The measured roll periods of the four different tests in waves are shown in table 6.

Model	Load	Heading (deg)	$T_N(s)$
2	4	45	1.473
2	4	45	1.357
2	4	90	1.392
2	4	90	1.385

Table 6: Roll period - waves

From table 1 we find that model 2, with 4 kg deck load has a metacentric height of 0.0224 m. Applying equation (30), with the metacentric heigh from the inclining test and estimated virtual moment of inertia from equation (6) with $r_{44} = 0.35 \cdot B$ and A_{44} as 40 % of I_{44}, the natural period T_N is 1.3886 seconds. This corresponds well with the average of the measured roll period, which is 1.402. The predicted virtual moment of inertia 0.1927 is a little lower than the virtual moment of inertia from the roll decay test, which is 0.2121, hence the roll period is shorter.

r_{44}	$A_{44}(\% of I_{44})$	$(I_{44} + A_{44})$	$T_N(s)$
0.3	20	0.1213	1.1020
0.35	20	0.1651	1.2856
0.45	20	0.2730	1.6529
0.3	40	0.1416	1.1902
0.35	40	0.1927	1.3886
0.45	40	0.3185	1.7854

Table 7: Comparison of virtual moment of inertia

As we see from table 7, the natural period T_N is highly dependent of the virtual moment of inertia $(I_{44} + A_{44})$, hence accurate calculations of the mass moment of inertia I_{44} and added mass A_{44}, are crucial in order to get an accurate estimation based on the response spectra.

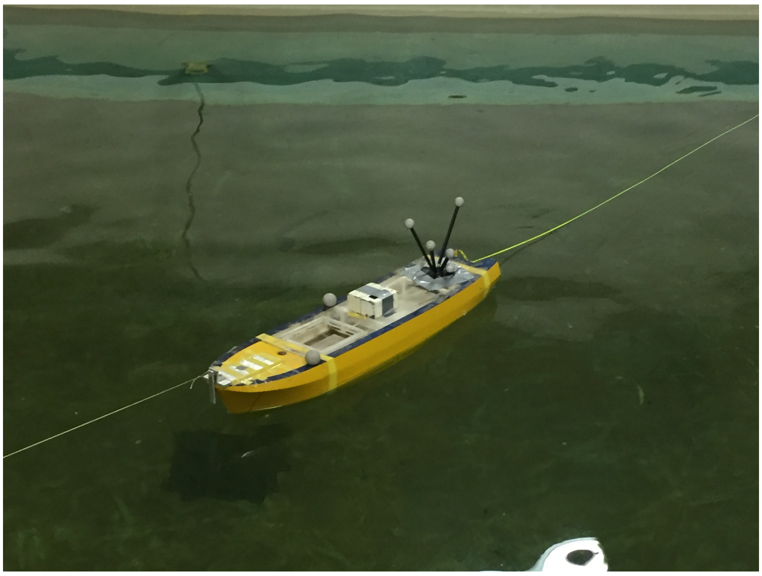

Figure 25: Rolling motions in waves

4 Prediction

When no inclining test have been conducted, there is a need for another approach to predict roll damping coefficients. The damping coefficients can be calculated by Ikeda's or Kawahara's prediction methods. The roll damping is divided into five damping components, and the interaction between the them is ignored [19].

4.1 Hull geometry

One of the challenges with regards to the damping prediction and comparison with the existing model, was to obtain the hydrostatic data such as block coefficient, mid ship coefficient, etc. The hull lines were not available in digital form, hence the model had to be obtained from the hull lines on paper.

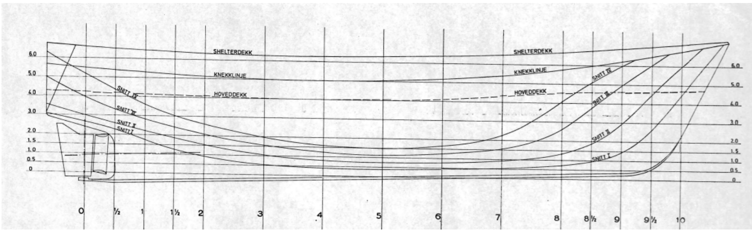

Figure 26: Profile

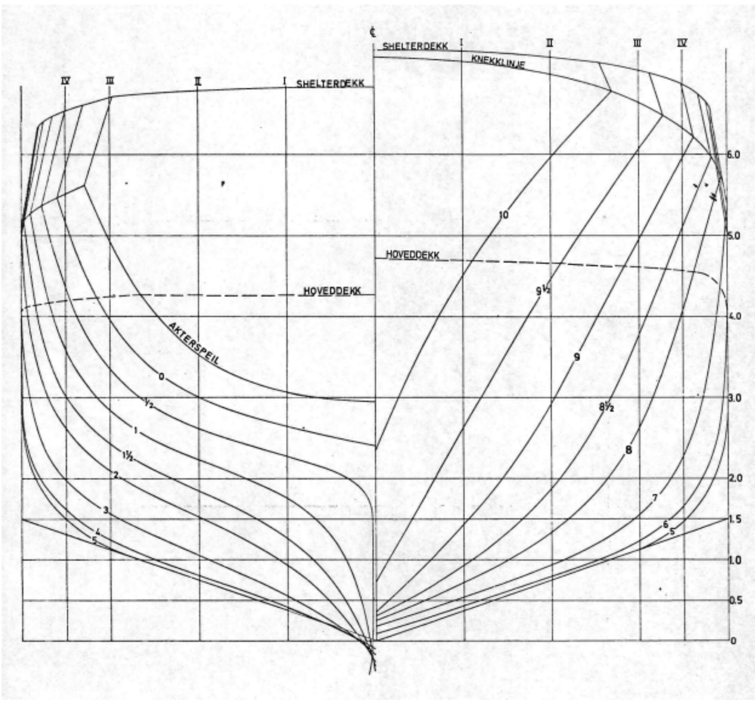

Figure 27: Body lines

The computer software DelftShip is a free computer software which was used to establish the hull model from existing hull lines, in order to extract the hydrostatics. The body plan and the profile was imported, and the model was defined to correspond with the imported lines drawing.

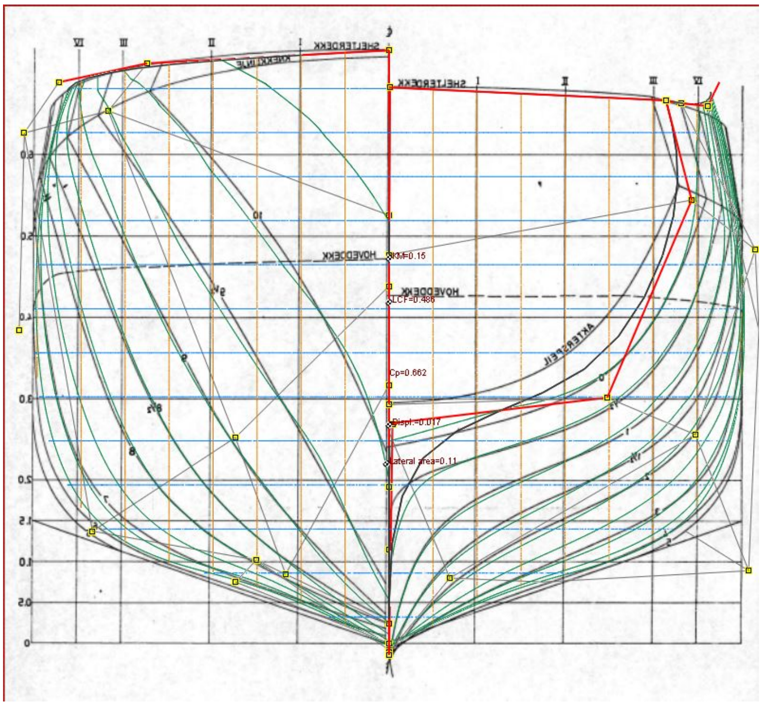

Figure 28: Defining body lines in FreeShip

Then the hydrostatic data such as the block coefficient C_B and the midship coefficient C_M were calculated by the software, which could be used in the prediction of the damping coefficients.

Design hydrostatics report

Designer			
Created by			
Comment			
Filename		New model.fbm	
Design length	1.056 (m)	**Midship location**	0.528 (m)
Length over all	0.969 (m)	**Relative water density**	1.0250
Design beam	0.284 (m)	**Mean shell thickness**	0.0010 (m)
Maximum beam	0.266 (m)	**Appendage coefficient**	1.0000
Design draft	0.132 (m)		

Volume properties		**Waterplane properties**	
Moulded volume	0.016 (m^3)	Length on waterline	0.922 (m)
Total displaced volume	0.016 (m^3)	Beam on waterline	0.266 (m)
Displacement	0.017 (tonnes)	Entrance angle	34.216 (Degr.)
Block coefficient	0.5014	Waterplane area	0.207 (m^2)
Prismatic coefficient	0.6617	Waterplane coefficient	0.8425
Vert. prismatic coefficient	0.5832	Waterplane center of floatation	0.486 (m)
Wetted surface area	0.326 (m^2)	Transverse moment of inertia	0.001 (m^4)
Longitudinal center of buoyancy	0.508 (m)	Longitudinal moment of inertia	0.011 (m^4)
Longitudinal center of buoyancy	-2.197 %		
Vertical center of buoyancy	0.086 (m)		
Total length of submerged body	0.922 (m)		
Total beam of submerged body	0.266 (m)		

Midship properties		**Initial stability**	
Midship section area	0.027 (m^2)	Transverse metacentric height	0.149 (m)
Midship coefficient	0.7577	Longitudinal metacentric height	0.801 (m)

Lateral plane	
Lateral area	0.105 (m^2)
Longitudinal center of effort	0.529 (m)
Vertical center of effort	0.072 (m)

Figure 29: Hydrostatics

4.2 Kawahara's prediction method

As presented in chapter 2.5, Ikeda divided the damping coefficient into five different parts. In 1978 he created a small computer program for calculation of the different damping coefficients in Fortran where the total damping is estimated by adding up the predicted values of the five components [19]. In 2008 Kawahara et al. presented a modification of the prediction method, deduced by using regression analysis, as Ikeda's prediction method is too complicated to use in the design phase [15]. The following year they revised the method to improve accuracy for ships with a high centre of gravity [16].

4.2.1 Friction damping

As the frictional component in Ikeda's method is calculated for the whole ship, the same simple formula is used in Kawahara's method.

$$B_{F0} = \frac{4}{3\pi}\rho S r_s^3 \phi_A \omega C_f \tag{43}$$

where B_F is the frictional component of the roll damping, S_f is the

$$C_f = 1.328[\frac{3.22 r_s^2 \phi_A^2 \omega}{2\phi \ nu}] \tag{44}$$

and S_f and r_s are given by Eq. (15) and (16).

The frictional component is normally negligible for full scale ships, but approximately 5-10% of the total roll damping in model scale.

4.2.2 Wave damping

The wave damping component can normally be calculated by any potential theory with practical accuracy. Although Kawahara deduced simpler formulas for the wave damping component B_W which are presented in appendix A.2.

4.2.3 Eddy damping

In Ikeda's method the eddy damping pressure on the hull surface by the flow separation is assumed as a simple shape for each section. The total eddy damping is then calculated by integrating the eddy damping of a sechting B'_E over the ships length. The formulas are presented in chapter 2.5.

Kawahara presented the following formulas, based on the eddy components predicted by Ikeda's method

$$\hat{B_E} = \frac{4L_{pp}d^4\hat{\omega}\phi_A}{3\pi\Delta B^2}C_R = \frac{\hat{\omega}\phi_A}{3\pi C_b(B/d)^3} \tag{45}$$

$x_1 = B/d, x_2 = C_b, x_3 = C_m, x_4 = OG/d$

$$C_R = A_E \cdot exp(B_{E1} + B_{E2} \cdot C_m^{B_{E3}}) \tag{46}$$

$$A_E = (-0,0182x^2 + 0,0155) \cdot (x_1 - 1.8)^3 - 79.414x_2^4 + 215.695x_2^3 - 215.883x_2^2 + 93.894x_2 - 14.848 \tag{47}$$

$$B_{E1} = (-0.2x_1 + 1.6)(3.98x_2 - 5.1525) \cdot x_4 \cdot ((0.9717x_2^2 - 1.55x_2 + 0.723) \cdot x_4 + (0.04567x_2 + 0.9408) \tag{48}$$

$$B_{E2} = (0.25x_4 + 0.95) \cdot x_4 - 219.2x_2^3 + 443.7x_2^2 - 283.3x_2 + 59.6) \tag{49}$$

$$B_{E3} = (46.5 - 15x_1)x_2 + 11.2x_1 - 28.6 \tag{50}$$

4.2.4 Bilge keel damping

In Ikeda's method, the pressure on both sides of the bilge keel is calculated by formulas found in chapter 2.5. Kawahara et al. deduced a simple prediction formula by fitting the bilge keel components

$$\hat{B_{BK}} = A_{BK} \cdot exp(B_{BK1} + B_{BK2} \cdot C_b(B/d)^3) \cdot \hat{\omega} \tag{51}$$

$$A_{BK} = f_1(B/d) \cdot f_2(C_b) \cdot f_3(\phi_A) \cdot f_4(b_{BK}/B) \cdot f_5(l_{bk}/L_{pp}, C_b) \tag{52}$$

$$f_1 = 0.6(B/d)^2 - 3.5(B/d) + 10.6 \tag{53}$$

$$f_2 = -1.07C_b + 1.26 \tag{54}$$

$$f_3 = 0.23\phi_A + 1 \tag{55}$$

$$f_4 = 2.7(b_{BK}/B)^2 + 0.023(b_{BK}/B) + 0.0021 \tag{56}$$

$$f_5 = (-1.6275C_b^2 + 2.5725C_b - 1.1725) \cdot (l_{BK}/L_{pp})^2 + 0.33(l_{BK}/L_{pp}) - 0.01 \tag{57}$$

$$B_{BK1} = (0.52(B/d) - 0.017\phi_A + 5.13(b_{BK}/B) - 0.09(l_{BK}/L_{pp}) - 2.7) \cdot (OG/d) \tag{58}$$

$$B_{BK2} = -15(b_{BK}/B) + 0.05(OG/d) + 1.2C_b + 1.25 \tag{59}$$

$$B_{BK3} = -0.625\phi_A + 26.5 \tag{60}$$

4.3 Limitations and accuracy

Kawahara et al. studied the accuracy of the original simple prediction method [15] and found an increased error margin for vessels with high centre of gravity compared to Ikeda's method.

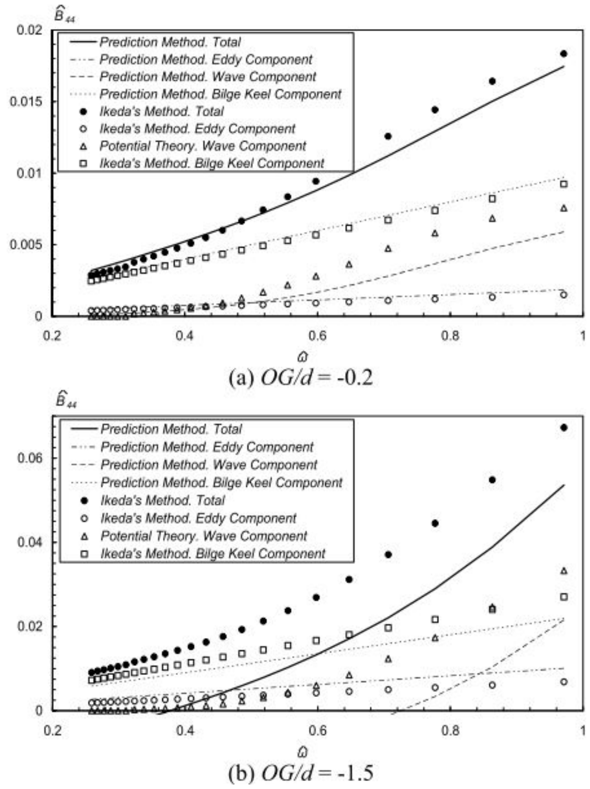

Figure 30: Error margins with increasing centre of gravity, [16]

When the new formula for wave damping was introduced the revised method gave significantly better results compared to Ikeda's method, as shown in figure 31.

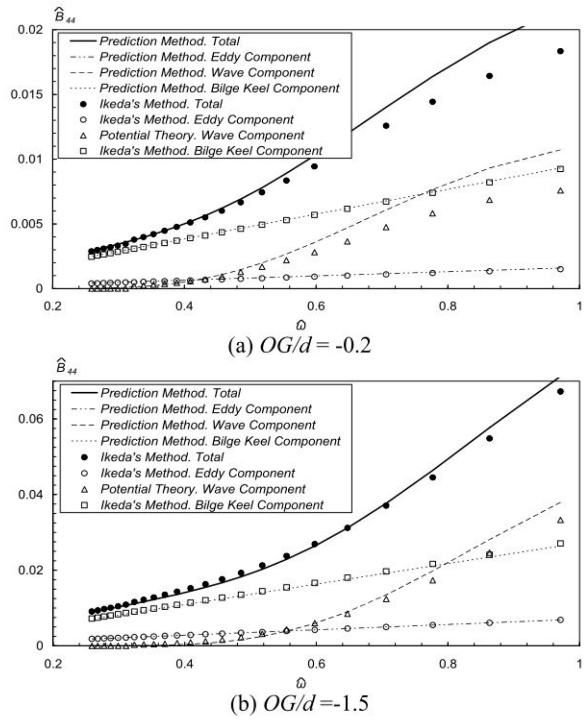

Figure 31: Error margins with increasing centre of gravity - revised method, [16]

Further Kawahara et al. tested the method on three different vessels to validate the accuracy, a large passenger ship, a pure car carrier and a wide breadth and shallow draft car carrier.

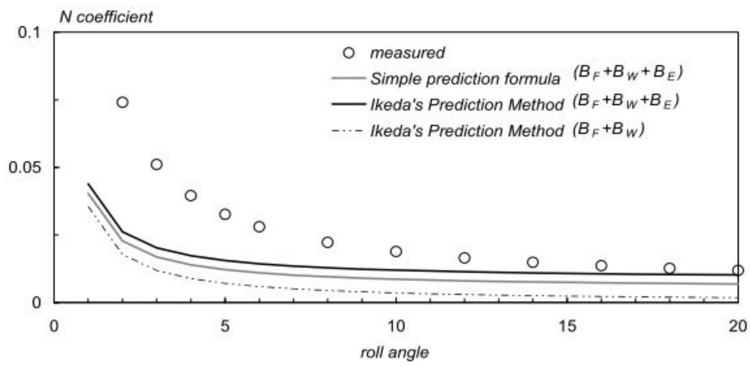

Figure 32: Large passenger ship, [16]

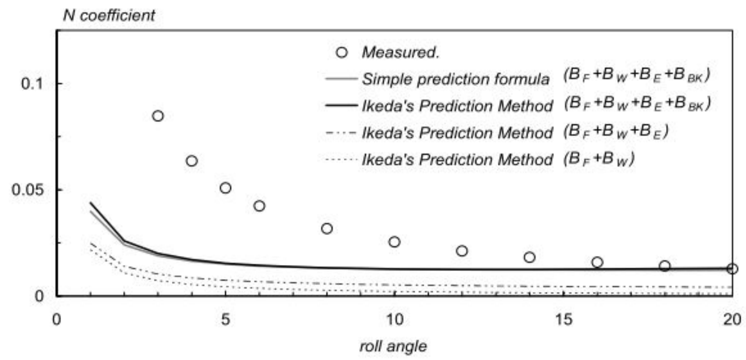

Figure 33: Pure car carrier, [16]

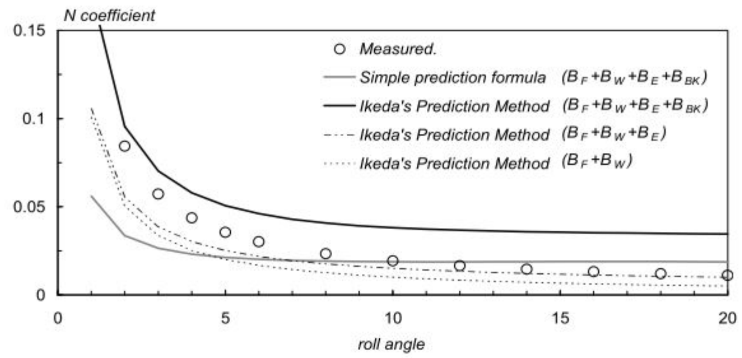

Figure 34: Wide breadth and shallow draft car carrier, [16]

As we can see from figures (32-34), the damping prediction is lower than the measured values for both Ikeda's and Kawahara's method (Simple prediction formula) on the two first ships. For the latter, Ikeda's prediction method including the bilge keel component is higher then the measured results, although the rest of the methods predicts lower values at low roll angles.

4.4 Implementing the method

At Osaka Prefecture University's web page [11], the computer program created by Kawahara et al. is available for download together with the code in fortran. The input to the program is limited to certain ranges for hydrostatic data such as breadth-draft ratio, block coefficient and midship coefficient. The ranges are:

$0.5 \leq C_b \leq 0.85$

$2.5 \leq B/d \leq 4.5$

$\hat{\omega} \leq 1.0$

$-1.5 \leq OG/d \leq 0.2$

$0.9 \leq C_m \leq 0.99$

As the B/d ratio for the ship model is outside the range, the computer program could not be used to predict the damping coefficient. Thus the formulas were implemented in excel, with no restrictions on the range, but lesser accuracy. To verify the formulas in the spreadsheet, the same input was used in the computer program and in the spreadsheet and compared.

Damping component	Spreadsheet
L_{pp}	1
L_{pp}/B	4
B/d	2.5
C_b	0.5
C_m	0.9
OG/d	-0.02
Φ_A	10
T_w	1
l_{BK}/L_{pp}	0.35
b_{BK}/B	0.05

Table 8: Input

Damping component	Spreadsheet	*Rolldamping09.exe*
B_F	0.000966	0.000966
B_W	0.004432	0.004432
B_E	0.003896	0.003896
B_{BK}	0.015233	0.015233
B_{44}	0.024527	0.024527

Table 9: Results

As the spreadsheet gives the same results as the computer program made by Kawahara et al. with the same input, it is assumed that all formulas are correctly implemented.

4.5 Results and analysis

Damping component	Spreadsheet	Experiment
B_F	0.001156	
B_W	0.025158	
B_E	0.009856	
B_{BK}	0	
B_{44}	0.036170	0.080999

Table 10: Comparison of results

The predicted method gives lower roll damping than measured in the experiment, as expected for low roll angles.

From table 4 the virtual moment of inertia, damping coefficient and restoring coefficient is found for model 2, with load of 4 kg. When the experimental damping coefficient is substituted with the predicted damping coefficient, the amplitudes are significantly affected as shown in figure 35.

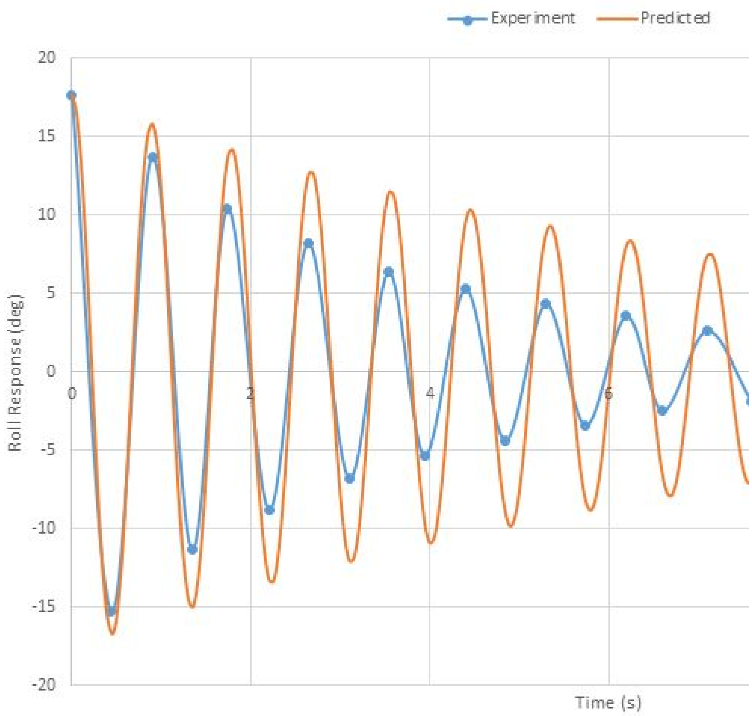

Figure 35: Comparison of rolling motion from experiment and with predicted roll damping

As the simple prediction method gives lower values for the damping coefficients, the amplitudes will not be sufficiently reduced when rolling motion is based on predicted coefficients. The results from this test is also likely to be affected by the inaccurate determination of the centre of gravity and the draft.

5 Conclusion and further work

5.1 Conclusion

Model tests were conducted to assess the possibility of estimating the metacentric height of a vessel based on roll decay tests and the response spectra. Initially the inclining test were conducted and the results seems very accurate, based on the moment-heel plot. The accuracy of the estimation from roll decay test also proved to be very good, although some of the measurements were biased or faulty and discarded. Comparing the roll motion equation with the experiments shows that one can get quite accurate results with the basic linear model.

One model was tested with bilge keels and the damping coefficients were estimated based on the results. The roll damping increases as larger bilge keels were installed, as expected. Results from the model tests conducted in this thesis indicates that the roll damping does not affect the roll period, which is in line with the literature. The virtual moment of inertia was also increased, and the results clearly shows that the shift in roll period origins from the increased added mass. Comparisons were made between numerical calculations of the roll equation, which showed no shift in roll period when the damping coefficient was altered.

When determining the metacentric height from the wave spectra, one should average the natural frequency over larger samples in order to get accurate results. Roll periods from the tests in waves were slightly lower than what was found for same model in still water, hence further work should be done in order to conclude if it is due to short samples or other factors.

Prediction of the damping coefficients by numerical calculations proved to be challenging. The numerical calculations by the simple prediction method of Kawahara et al. are based on the hydrostatic data from the hull geometry, which were not available in a digitalized format. There is a lot of uncertainty when calculating the hydrostatic data from old hull lines, which can have huge impact on the calculated hydrostatic data. The draft and centre of gravity of the ship model were not measured, and had to be estimated based on pictures and weight of the model, hence there is a lot of uncertainty in that regard. As the method is very sensitive for different values of the OG/d ratio and draft, the margin errors between experimental and predicted damping coefficients can to a certain extent be due to inaccurate input. Still, as both Ikeda's method and the simple prediction method gives lower values than experiments, some of the margin error is due to the inaccuracy of the method.

- Based on the model test results, determining the metacentric height from a roll decay test should be possible with sufficient accuracy.
- The shift in roll period is due to added mass when different bilge keels are installed.
- Results indicates that it is possible to determine the metacentric height from response spectra with sufficient accuracy for stability calculations. Although more studies must be conducted in order to determine the margin errors between still water results and response spectra from tests in various wave headings.

- Numerical predication by Kawahara's method did not prove to be sufficient accurate in order to determine rolling motions, hence more advanced commercial software is still needed. In the early design phase, it might give some indications on the roll damping, in order to determine the main dimensions, hull form and appendices.

5.2 Further work

The analysis shows that most of the results comply with the expected results, but there are still some factors which are not investigated in this master thesis.

- Results should be verified for full scale vessels
- Studies of how various trim will affect the results should be conducted
- Comparison of results from simple response spectra analysis with commercial software

As there are uncertainties with regards to some of the hydrostatic data, new model tests should be carried out in order to verify the results of the prediction method. One should also analyse the results based on the tests in waves with longer data samples.

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A Appendices

A.1 Formulas

Velocity-increment ratio

$$\gamma = \frac{\sqrt{\pi} f_3}{2[D - OG]\sqrt{H_0 \sigma}} [r_{max} + \frac{2M}{H_1} \sqrt{A_1^2 + B_1^2}] \quad (61)$$

with

$$M = \frac{B}{2(1 + a_1 + a_3)} \quad (62)$$

$$M = \frac{H_0}{1 - OG/d} \quad (63)$$

$$\sigma' = \frac{\sigma - OG/d}{1 - OG/d} \quad (64)$$

$$H = 1 + a_1^2 + 9a_3^2 + 2a_1(1 - 3a_3)cos(2\psi) - 6a_3 cos(4\psi) \quad (65)$$

$$A = -2a_3 cos(5\psi) + a_1(1 - a_3)cos(3\psi) + ((6 - 3a_1)a_3^2 + (a_1^2 - 3a_1 a_3 + a_1^2)cos\psi \quad (66)$$

$$B = -2a_3 sin(5\psi) + a_1(1 - a_3)sin(3\psi) + ((6 - 3a_1)a_3^2 + (a_1^2 - 3a_1 a_3 + a_1^2)sin\psi \quad (67)$$

$$r_{max} = M(((1 + a_1)sin\psi - a_3 sin(3\psi)^2 + ((1 - a_1)cos\psi + a_3 cps(3\psi))) \quad (68)$$

A.2 Code of Ikeda's method

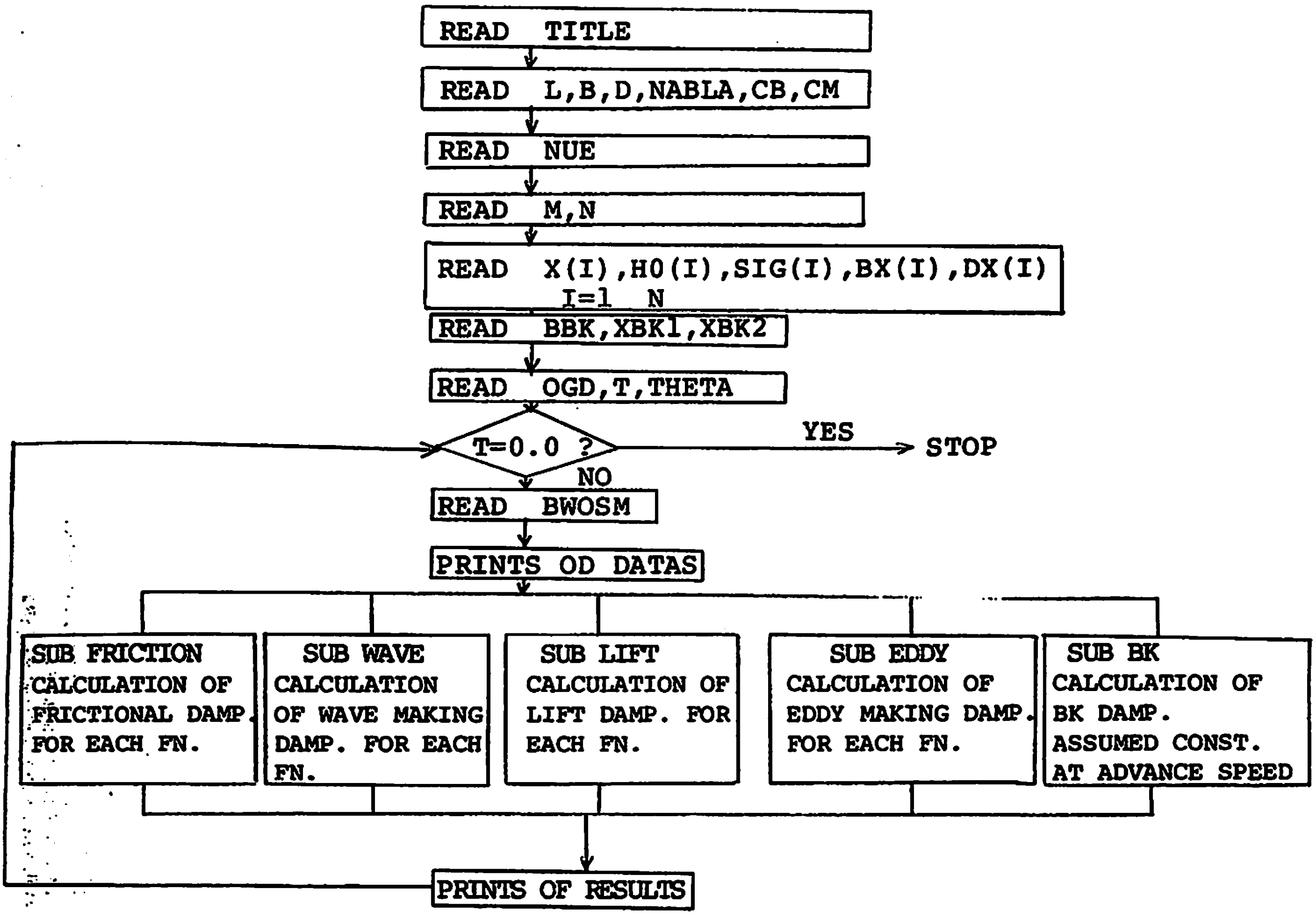
Appendix Computer program of roll damping prediction

(1) INPUT

1) Title (70 words) In put type 70A1
2) L,B,D,NABLA,CB,CM 6F10.0
   L ; length B ; breadth D ; draft (m)
   NABLA ; displacement volume (m^3)
   CB ; block coeff. CM ; midship coeff.
3) NUE F10.0
   NUE ; kinematic viscosity (m^3/s)
   Empirical formula
   $\nu=0.0178/(1+0.0336t+0.000221t^2)$ cm^2/s
   t ; tenperrature (°C)
4) M,N 2I5
   M ; the number of calculation (interval: $F_n=0.02$)
   N ; the number of input datas of sections
   (= the number of data cards of 5))
5) X(I),H0(I),SIG(I),BX(I),DX(I) 5F10.0
   X(I) ; the number of S.S. (0.0 at A.P.&10.0 at F.P.)
   H0(I); half breadth/draft (=BX(I)/DX(I))
   SIG(I); area coefficient (=cross section area/BX(I)DX(I))
   BX(I); breadth (m)
   DX(I); draft (m)
6) BBK,XBK1,XBK2 3F10.0
   BBK ; breadth of bilge keel (m)
   XBK1; SS number of aft end of bilge keel
   XBK2; SS number of for end of bilge keel
7) $\bar{O}$GD,T,THETA 3F10.0
   $\bar{O}$GD; $=\bar{O}G/d$
   T ; roll period (sec)
   THETA; roll amplitude (rad)
8) BW$\bar{O}$SM F10.0
   BW$\bar{O}$SM; $\hat{B}_W$ at $F_n=0.0$ caiculated by potential theory

(2) OUTPUT

1) prints of input datas
2) distribution of the coefficient C_R of the eddy making component,
   ($C_R=M_{RE}/(0.5\rho d^4\ \theta|\theta|$, M_{RE}; roll moment due to the eddy making component)
3) distributions of $\Delta\hat{B}_{BK}$ and $\Delta\hat{B}_N/\Delta\hat{B}_{BK}$.
   $\Delta\hat{B}_{BK}$; $\hat{B}_{BK}$ for unit length
   $\Delta\hat{B}_N$; $\hat{B}_N$ for unit length
4) each components $\hat{B}_F,\hat{B}_W,\hat{B}_E,\hat{B}_L,\hat{B}_{Bk}$ } for each F_n
   rates of each component } for each F_n
   total roll damping coefficient $\hat{B}_{44}$ } for each F_n



READ TITLE
READ L,B,D,NABLA,CB,CM
READ NUE
READ M,N
READ X(I),H0(I),SIG(I),BX(I),DX(I) I=1 N
READ BBK,XBK1,XBK2
READ OGD,T,THETA
T=0.0 ?
YES
STOP
NO
READ BWOSM
PRINTS OD DATAS
SUB FRICTION CALCULATION OF FRICTIONAL DAMP. FOR EACH FN.
SUB WAVE CALCULATION OF WAVE MAKING DAMP. FOR EACH FN.
SUB LIFT CALCULATION OF LIFT DAMP. FOR EACH FN.
SUB EDDY CALCULATION OF EDDY MAKING DAMP. FOR EACH FN.
SUB BK CALCULATION OF BK DAMP. ASSUMED CONST. AT ADVANCE SPEED
PRINTS OF RESULTS

(1)

```
C         ***** ESTIMATION OF ROLL DAMPING *****
C                 CODED BY Y.IKEDA.
      REAL  L,NUE,NABLA
      DIMENSION   X(25),HO(25),SIG(25),BX(25),DX(25),B44HAT(100),
     *                 BWHAT(100),BEHAT(100),BLHAT(100)
     *           ,FN(100),TITLE(70) ,BFHAT(100)
         READ (5,100)  (TITLE(I),I=1,70)
  100      FORMAT (70A1)
C       *** PRINCIPAL DIMENSIONS ***  L=LENGTH,B=BREADTH,D=DRAFT,NABLA=VO      LUME
C                                   ***  CB=BLOCK COEFF.,CM=MIDSHIP COEFF..
        READ (5,101)  L,B,D,NABLA,CB,CM
  101     FORMAT (6F10.0)
        READ (5,103) NUE
  103     FORMAT (F10.0)
C      ***  NUMBERS OF FN AND DATAS ***
        READ (5,104)  M,N
  104     FORMAT (2I5)
C      ***  PARAMETERS OF EACH SECTION ***    HO=B/2D,SIG=S/BD,BX=BREADTH
C                                               DX=DRAFT
        DO 1   I=1,N
    1   READ (5,105)  X(I),HO(I),SIG(I),BX(I),DX(I)
  105     FORMAT (5F10.0)
C      *** BILGE KEELS DATA ***   BBK=BREADTH OF B.K.,XBK1=X OF B.K. END      (AFT)
C                                 XBK2=X OF B.K. END(FOR)
        READ (5,106)  BBK,XBK1,XBK2
  106     FORMAT (3F10.0)
    5   CONTINUE
C      *** CONDITION ***  OGD=OG/D,T=PERIOD,THETA=AMP. OF ROLL
        READ (5,102)  OGD,T,THETA
  102     FORMAT (3F10.0)
        IF (T.LT.0.000001)   STOP
C      *** WAVE MAKING COMPONENT AT FN=0.0  ***
        READ (5,103) BWOSM
        WRITE (6,200)  (TITLE(I),I=1,70)
  200   FORMAT (1H1,///,7X,5H*****,2X,70A1,2X,5H*****)
        WRITE (6,300) L,B,D,NABLA,CB,CM
  300   FORMAT (1H ,//,5X,8H**DATA**,2X,2HL=,F8.5,2X,2HB=,F8.5,2X,2HD=,
     *        F8.5,2X,6HNABLA=,F8.5,2X,3HCB=,F8.5,2X,3HCM=,F8.5)
        WRITE (6,301) OGD,T,THETA,NUE,BWOSM
  301   FORMAT (1H ,5X,5HOG/D=,F8.3,2X,2HT=,F8.3,2X,6HTHETA=,F8.3,3HRAD
     *        ,2X,4HNUE=,F12.8,2X,6HBWOSM=,F15.10)
        WRITE (6,302)  BBK,XBK1,XBK2
  302   FORMAT (1H ,5X,4HBBK=,F8.5,2X,5HXBK1=,F8.3,2X,5HXBK2=,F8.3)
        WRITE (6,304)
  304   FORMAT (1H ,//,4X,25H**DATAS OF EACH SECTION**)
        DO 15  I=1,N
   15   WRITE (6,303) X(I),HO(I),SIG(I),BX(I),DX(I)
  303   FORMAT(1H ,4X,3HSS=,F8.3,3X,3HHO=,F8.5,3X,6HSIGMA=,F8.5,3X,2HB=
     *        ,F8.5,3X,2HD=,F8.5)
C      ************************************************
        OMEGA=6.28318/T
        DO 2   I=1,M
    2   FN(I)=0.0+0.02*FLOAT(I-1)
        CALL FRICT     (L,B,D,CB,NABLA,OGD,NUE,OMEGA,FN,BFHAT,M)
        CALL WAVE      (L,D,OMEGA,BWOSM,FN,BWHAT,M)
        CALL LIFT      (L,B,D,CM,NABLA,OGD,FN,BLHAT,M)
```

(2)

```
      CALL EDDY    (X,HO,SIG,BX,DX,B,D,NABLA,OGD,OMEGA,THETA,FN,
     *              BEHAT,M,N,L)
      IF (BBK.LT.0.0000000001) GO TO 20
      CALL BK       (X,HO,SIG,BX,DX,B,D,NABLA,OGD,OMEGA,THETA,BBK,
     *              XBK1,XBK2,BBKHAT,M,N,L)
   20 IF(BBK.LT.0.0000000001) BBKHAT=0.0
      DO 3   I=1,M
    3 B44HAT(I)=BFHAT(I)+BWHAT(I)+BLHAT(I)+BEHAT(I)+BBKHAT
C    ***************************************************
      WRITE (6,201)
  201   FORMAT (1H ,///,4X,2HFN,6X,5HBFHAT,5X,5HBWHAT,5X,5HBEHAT,5X,
     *          5HBLHAT,5X,6HBBKHAT,5X,2H**,4X,6HBF/B44,4X,6HBW/B44,
     *          4X,6HBE/B44,4X,6HBL/B44,4X,7HBBK/B44,4X,2H**,2X,
     *          6HB44HAT,2X,2H**)
      DO 4  I=1,M
       BFF=BFHAT(I)/B44HAT(I)
       BWW=BWHAT(I)/B44HAT(I)
       BEE=BEHAT(I)/B44HAT(I)
       BLL=BLHAT(I)/B44HAT(I)
       BBKK=BBKHAT    /B44HAT(I)
    4 WRITE(6,202)FN(I),BFHAT(I),BWHAT(I),BEHAT(I),BLHAT(I),BBKHAT,
     *           BFF,BWW,BEE,BLL,BBKK,B44HAT(I)
  202   FORMAT (1H ,2X,F6.3,2X,F8.5,2X,F8.5,2X,F8.5,2X,F8.5,2X,F8.5
     *         ,4X,2H**,3X,F8.5,2X,F8.5,2X,F8.5,2X,F8.5,2X,F8.5,4X,
     *         2H**,1X,F8.5,1X,2H**)
      GO TO 5
      END

C*************************************************************************    *******
      SUBROUTINE LIFT (L,B,D,CM,NABLA,OGD,FN,BLHAT,M)
C         LIFT COMPONENT
C         REF.   Y.IKEDA ET AL  (JZK.NO.143)
      REAL  L,NABLA,KAPA,KN,LO,LR
      DIMENSION  FN(100),BLHAT(100)
      IF (CM.LE.0.92)  KAPA=0.0
      IF (CM.LE.0.97.AND.CM.GT.0.92)  KAPA=0.1
      IF (CM.GT.0.97)  KAPA=0.3
      KN=6.28319*D/L+KAPA*(4.1*B/L-0.045)
      OG=OGD*D
      LO=0.3*D
      LR=0.5*D
      DO 1  I=1,M
    1 BLHAT(I)=L*D*KN*LO*LR*FN(I)*0.5/(NABLA*B**2)*SQRT(0.5*L*B)*(1.0-
     *        1.4*OG/LR+0.7*OG**2/(LO*LR) )
      RETURN
      END

C *************************************************************************    ********
      SUBROUTINE  WAVE  (L,D,OMEGA,BWOSM,FN,BWHAT,M)
C         WAVE MAKING COMPONENT
C         REF.  Y.IKEDA ET AL (JZK,NO,143)
      REAL   L,LOMEGA
      DIMENSION   FN(100),BWHAT(100)
      GUZAID=OMEGA**2*D/9.80665
      A1=1.0+GUZAID**(-1.2)*EXP(-2.0*GUZAID)
```

```
      A2=0.5+GUZAID**(-1.0)*EXP(-2.0*GUZAID)
      DO 1  I=1,M
      LOMEGA=OMEGA*FN(I)*SQRT(L/9.80665)
    1 BWHAT(I)=BWOSM*0.5*(((A2+1.0)+(A2-1.0)*TANH(20.0*(LOMEGA-0.3)))+
     *          (2.0*A1-A2-1.0)*EXP(-150.0*(LOMEGA-0.25)**2))                   (3)
      RETURN
      END

C ***********************************************************************      ********
      SUBROUTINE  FRICT    (L,B,D,CB,NABLA,OGD,NUE,OMEGA,FN,BFHAT,M)
C         FRICTIONAL COMPONENT
C         REF. H.KATO (JZK.NO.102) AND S.TAMIYA ET AL (JZK.NO.132)
      REAL  NABLA,NUE,L
      DIMENSION   FN(100),BFHAT(100)
      SF=L*(1.7*D+CB*B)
      RF=((0.887+0.145*CB)*(1.7*D+CB*B)-2.0*OGD*D)/3.1415
      DO 1  I=1,M
    1 BFHAT(I)=0.787*SF*RF**2*SQRT(OMEGA*NUE*B/19.6133)/(NABLA*B**2)
     *         *(1.0+4.1*FN(I)/OMEGA*SQRT(9.80665/L))
      RETURN
      END

C ***********************************************************************      ********
      SURROUTINE  EDDY   (X,HO,SIG,BX,DX,B,D,NABLA,OGD,OMEGA,THETA,FN,
     *                    BEHAT,M,N,L)
C         EDDY MAKING COMPONENT
C         REF.  Y.IKEDA ET AL  (JZK.NO.142,JZK.NO.143)
      REAL  NABLA,LTHETA,L
      DIMENSION  X(25),HO(25),SIG(25),BX(25),DX(25),FN(100),BEHAT(100),
     *           RMAX1(2),V(2),CR(25) ,CR1(25) ,X1(30)
      DO 1  J=1,N
      AHO=HO(J)/(1.0-OGD)
      SIGMA=(SIG(J)-OGD)/(1.0-OGD)
      E=(AHO-1.0)/(AHO+1.0)
      E2=E**2
      A=4.0*SIGMA*(1.0-E2)/3.1415+E2
      O=-A/(A+3.0)
      O2=SQRT(O**2-(A-1.0)/(A+3.0))
      A3=O+O2
      A1=E*(1.0+A3)
      AM=BX(J)/(1.0+A1+A3)*0.5
      AA1=A1*(1.0+A3)/A3*0.25
      IF(AA1.GT.1.0)  AA1=1.0
      IF(AA1.LT.-1.0) AA1=1.0
      DO 2   I=1,2
      LTHETA=0.5*ARCOS(AA1)
      IF (I.EQ.1)  LTHETA=0.0
      AH=1.0+A1**2+9.0*A3**2+2.0*A1*(1.0-3.0*A3)*COS(2.0*LTHETA)-6.0*A3
     *     *COS(4.0*LTHETA)
      AA=-2.0*A3*COS(5.0*LTHETA)+A1*(1.0-A3)*COS(3.0*LTHETA)+((6.0-3.0*
     *    A1)*A3**2+(A1**2-3.0*A1)*A3+A1**2)*COS(LTHETA)
      BB=-2.0*A3*SIN(5.0*LTHETA)+A1*(1.0-A3)*SIN(3.0*LTHETA)+((6.0+3.0*
     *    A1)*A3**2+(3.0*A1+A1**2)*A3+A1**2)*SIN(LTHETA)
      V(I)=2.0*AM*SQRT(AA**2+BB**2)/AH
    2 RMAX1(I)=AM*SQRT(((1.0+A1)*SIN(LTHETA)-A3*SIN(3.0*LTHETA))**2
     *          +((1.0-A1)*COS(LTHETA)+A3*COS(3.0*LTHETA))**2)
```

(4)

```
      RMAX=RMAX1(1)
      VMAX=V(1)
      IF (RMAX1(1).LE.RMAX1(2)) GO TO 8
      GO TO 9
    8 RMAX=RMAX1(2)
      VMAX=V(2)
    9 CONTINUE
      RMEAN=2.0*DX(J)*(1.0-OGD)*SQRT(AHO*SIGMA/3.1415)
      P1=VMAX/RMEAN
      P2=RMAX/RMEAN
      PP3=P1+P2
      GAMMA=(1.0+4.0*EXP(-165000.0*(1.0-SIGMA)**2))*PP3
      CP=0.5*(0.87*EXP(-GAMMA)-4.0*EXP(-0.187*GAMMA)+3.0)
      F1=0.5*(1.0+TANH(20.0*(SIG(J)-0.7)))
      F2=0.5*(1.0-COS(3.1415*SIG(J)))-1.5*(1.0-EXP(-5.0*(1.0-SIG(J))))*
     *   SIN(3.1415*SIG(J))**2
      R=2.0*DX(J)*SQRT(HO(J)*(SIG(J)-1.0)/(-0.8584))
      RD=R/DX(J)
      IF (HO(J).LE.1.0.AND.RD.GE.AHO)  R=0.5*BX(J)
      IF (HO(J).GT.1.0.AND.RD.GE.1.0)     R=DX(J)
      RD=R/DX(J)
    1 CR1(J)=RMAX**2/DX(J)**2*CP*((1.0-F1*RD)*(1.0-OGD-F1*RD)+F2*
     *      (HO(J)-F1*RD)**2)
      WRITE (6,452)
  452 FORMAT (1H ,//,4X,4H****,31HLONGITUDINAL DISTRIBUTION OF CR,
     *      4H****)
      DO 10  J=1,N
   10 WRITE (6,453) X(J),CR1(J)
  453 FORMAT (1H ,4X,3HSS=,F8.5,4X,3HCR=,F8.5)
      DO 3   K=1,21
      X1(K)=0.0+0.5*FLOAT(K-1)
      MAX=N
    3 CALL HOKAN1 (X,CR1, 25,MAX,X1(K),CR(K),DAM,1,0)
      CR(1)=1.5*(1.0-OGD)
      CR(21)=1.5*(1.0-OGD)
      SAM=0.0
      DO 4  K=1,10
      K2=2*K
      K1=K2-1
      K3=K2+1
      SAM1=CR(K1)+4.0*CR(K2)+CR(K3)
    4 SAM=SAM+SAM1
        CRT=SAM/60.0
      BEHAT(1)=4.0*L*D**4/3.0/3.1415*OMEGA*SQRT(B/19.6)/NABLA/B**2*CRT
     *        *THETA
      DO 5   I=2,M
      AK=OMEGA/FN(I)*SQRT(L/9.8)
    5 BEHAT(I)=BEHAT(1)*(0.04*AK)**2/((0.04*AK)**2+1.0)
      RETURN
      END

C **********************************************************************      ********
      SUBROUTINE  BK (X,HO,SIG,BX,DX,B,D,NABLA,OGD,OMEGA,THETA,BBK,
     *                XBK1,XBK2,BKHAT,M,N,L)
C         DAMPING DUE TO BILGE KEELS.
C         REF.  Y.IKEDA ET AL.  (KZK.NO.161, KZK.NO.165)
      REAL  NABLA,       M1,M2,M3,M4,M5,M6,M7,M8,L
```

```
      DIMENSION X(25),HO(25),SIG(25),BX(25),DX(25),BBKHAT(25), XBK(25)
     *        ,RATIO(25)                                                (5)
      XBK(1)=XBK1
      XBK(11)=XBK2
      DO 1   I=2,10
    1 XBK(I)=XBK(I-1)+(XBK2-XBK1)*0.1
      MAX=N
      DO 2   I=1,11
       CALL HOKAN1 (X,HO, 25,MAX,XBK(I),HO1,DAM,1,0)
       CALL HOKAN1 (X,SIG, 25,MAX,XBK(I),SIG1,DAM,1,0)
       CALL HOKAN1 (X,DX, 25,MAX,XBK(I),DX1,DAM,1,0)
       CALL HOKAN1 (X,BX, 25,MAX,XBK(I),BX1,DAM,1,0)
      R=2.0*DX1*SQRT(HO1*(SIG1-1.0)/(-0.8585))
      RD=R/DX1
      IF (HO1.LE.1.0.AND.RD.GE.HO1)   R=0.5*BX1
      IF (HO1.GT.1.0.AND.RD.GE.1.0)   R=DX1
      RD=R/DX1
      F=1.0+0.3*EXP(-160.0*(1.0-SIG1))
      RBK=DX1*SQRT((HO1-0.2929*RD)**2+(1.0-OGD-0.2929*RD)**2)
      M1=RD
      M2=OGD
      M3=1.0-M1-M2
      M4=HO1-M1
      M5=(0.414*HO1+0.0651*M1**2-(0.382*HO1+0.0106)*M1)/((HO1-0.215
     *        *M1)*(1.0-0.215*M1))
      M6=(0.414*HO1+0.0651*M1**2-(0.382+0.0106*HO1)*M1)/((HO1-0.215
     *    *M1)*(1.0-0.215*M1))
      SO=0.3*(3.1415*F*RBK*THETA)+1.95*BBK
      M7=SO/DX1-0.25*3.1415*M1
      R1=0.25*3.1415*R
      IF (SO.LT.R1)  M7=0.0
      M8=M7+0.414*M1
      IF (SO.LT.R1)   M8=M7+1.414*(1.0-COS(SO/R))*M1
      A=(M3+M4)*M8-M7**2
      BB=M4**3/3./(HO1-0.215*M1)+(1.0-M1)**2*(2.0*M3-M2)/6.0/(1.0-0.215
     *   *M1)+M1*(M3*M5+M4*M6)
      CPPLAS=1.2
      CPMINS=-22.5*BBK/(3.1415*RBK*F*THETA)-1.2
      CD=CPPLAS-CPMINS
C     ***  BBKHAT FOR UNIT LENGTH  ***
      RATIO(I)=RBK*BBK*CD/(RBK*BBK*CD+0.5*DX1**2*(-A*CPMINS+BB*CPPLAS))
    2 BBKHAT(I)=8.0*RBK**2*OMEGA*SQRT(B/19.6)*THETA*F**2/(3.0*3.1415
     *        *NABLA*R**2)*(RBK*BBK*CD+0.5*DX1**2*(-A*CPMINS+BB*CPPLAS
     *        ))
      WRITE (6,100)
  100 FORMAT (1H ,//,4H****,35HLONGITUDINAL DISTRIBUTION OF BBKHAT,
     *        4H****)
      DO 7  I=1,11
    7 WRITE (6,101) XBK(I),BBKHAT(I),RATIO(I)
  101 FORMAT (1H ,4X,3HSS=,F8.5,3X,7HBBKHAT=,F13.8,3X,22HNORMAL FORCE/T
     *OTAL BK=,F13.8)
C     ****   BBKHAT FOR THREE DIMENSIONAL SHIP FORM ****
      SAM=0.0
      DO 3 I=1,5
      I2=2*I
      SAM1=BBKHAT(I2-1)+4.0*BBKHAT(I2)+BBKHAT(I2+1)
    3 SAM=SAM+SAM1
      BKHAT=SAM*(XBK2-XBK1)*0.1/3.0*L*0.1
      RETURN
      END
```

(G)

```
C ******************************************************************************          ********
      SUBROUTINE HOKAN1 (X1,Y1,MAX,N,X,Y,YX,M1,M2)
C     LAGRANGE 3 POINTS INTERPOLATION
      DIMENSION X1(MAX),Y1(MAX),WX(3),WY(3)
      N1=N-1
            DO 10  I=2,N1
            IF (X.LE.X1(I)) GO  TO  1
   10       CONTINUE
    1 I1=I-1
            IF(X.GT.X1(N1))  I1=N-2
      I2=I1+2
            DO 20  I=I1,I2
      II=I+1-I1
      WX(II)=X1(I)
   20 WY(II)=Y1(I)
            IF(M1.NE.1) GO  TO  2
            CALL LAG3(WX,WY,X,Y)
    2       CONTINUE
            IF(M2.NE.1)  RETURN
      YX=0.0
      RETURN
      END

C ******************************************************************************          ********
      SUBROUTINE LAG3 (WX,WY,X,Y)
      DIMENSION WX(3),WY(3)
      Y=0.0
            DO  11  I=1,3
      W=1.0
      Z=1.0
            DO  12  J=1,3
            IF(J.EQ.I)  GO  TO  12
      W=W*(X-WX(J))
      Z=Z*(WX(I)-WX(J))
   12       CONTINUE
      Y=Y+WY(I)*W/Z
   11          CONTINUE
      RETURN
      END
```

```
*****   SR108 CONTAINER                                                  *****

**DATA**  L= 1.75000  B= 0.25400  D= 0.09500  NABLA= 0.02400  CB= 0.57100  CM= 0.97000
 OG/D=   0.       T=   1.200  THETA=    0.175RAD  NUE= 0.00000114  BWOSM=     0.00095620
 BBK= 0.00450  XBK1=    3.750  XBK2=    6.250

**DATAS OF EACH SECTION**
 SS=    0.        HO= 2.10600   SIGMA= 0.72700   B= 0.03000   D= 0.00700
 SS=    0.500     HO= 0.47300   SIGMA= 0.27700   B= 0.09000   D= 0.09500
 SS=    1.000     HO= 0.74100   SIGMA= 0.46000   B= 0.14100   D= 0.09500
 SS=    1.500     HO= 0.95600   SIGMA= 0.56500   B= 0.18200   D= 0.09500
 SS=    2.000     HO= 1.12000   SIGMA= 0.65800   B= 0.21300   D= 0.09500
 SS=    2.500     HO= 1.23000   SIGMA= 0.74600   B= 0.23400   D= 0.09500
 SS=    3.000     HO= 1.29500   SIGMA= 0.82500   B= 0.24600   D= 0.09500
 SS=    3.500     HO= 1.32800   SIGMA= 0.89000   B= 0.25300   D= 0.09500
 SS=    4.000     HO= 1.33500   SIGMA= 0.94000   B= 0.25400   D= 0.09500
 SS=    4.500     HO= 1.33500   SIGMA= 0.96600   B= 0.25400   D= 0.09500
 SS=    5.000     HO= 1.33500   SIGMA= 0.97000   B= 0.25400   D= 0.09500
 SS=    5.500     HO= 1.33400   SIGMA= 0.95200   B= 0.25400   D= 0.09500
 SS=    6.000     HO= 1.30600   SIGMA= 0.91400   B= 0.24800   D= 0.09500
 SS=    6.500     HO= 1.22800   SIGMA= 0.86800   B= 0.23300   D= 0.09500
 SS=    7.000     HO= 1.10500   SIGMA= 0.81800   B= 0.21000   D= 0.09500
 SS=    7.500     HO= 0.93700   SIGMA= 0.77200   B= 0.17800   D= 0.09500
 SS=    8.000     HO= 0.73800   SIGMA= 0.73500   B= 0.14000   D= 0.09500
 SS=    8.500     HO= 0.52700   SIGMA= 0.72000   B= 0.10000   D= 0.09500
 SS=    9.000     HO= 0.33100   SIGMA= 0.75100   B= 0.07000   D= 0.09500
 SS=    9.500     HO= 0.16400   SIGMA= 0.92300   B= 0.03100   D= 0.09500
 SS=   10.000     HO= 0.07700   SIGMA= 1.00000   B= 0.00700   D= 0.09500

****LONGITUDINAL DISTRIBUTION OF CR****
 SS= 0.           CR= 0.01930
 SS= 0.50000      CR= 1.08838
 SS= 1.00000      CR= 0.53705
 SS= 1.50000      CR= 0.10031
 SS= 2.00000      CR= 0.02335
 SS= 2.50000      CR= 0.00043
 SS= 3.00000      CR= 0.00151
 SS= 3.50000      CR= 0.03689
 SS= 4.00000      CR= 0.17439
 SS= 4.50000      CR= 0.37376
 SS= 5.00000      CR= 0.42150
 SS= 5.50000      CR= 0.24781
 SS= 6.00000      CR= 0.07310
 SS= 6.50000      CR= 0.01108
 SS= 7.00000      CR= 0.00040
 SS= 7.50000      CR= 0.00034
 SS= 8.00000      CR= 0.02475
 SS= 8.50000      CR= 0.15268
 SS= 9.00000      CR= 0.42203
 SS= 9.50000      CR= 0.57702
 SS=10.00000      CR= 0.34357
```

(8)

****LONGITUDINAL DISTRIBUTION OF BBKHAT****

SS= 3.75000	BBKHAT=	0.00453689	NORMAL FORCE/TOTAL BK= 0.47981367
SS= 4.00000	BBKHAT=	0.00580597	NORMAL FORCE/TOTAL BK= 0.40740543
SS= 4.25000	BBKHAT=	0.00714898	NORMAL FORCE/TOTAL BK= 0.35315630
SS= 4.50000	BBKHAT=	0.00841900	NORMAL FORCE/TOTAL BK= 0.31508720
SS= 4.75000	BBKHAT=	0.00920128	NORMAL FORCE/TOTAL BK= 0.29615364
SS= 5.00000	BBKHAT=	0.00906588	NORMAL FORCE/TOTAL BK= 0.29918894
SS= 5.25000	BBKHAT=	0.00810482	NORMAL FORCE/TOTAL BK= 0.32434918
SS= 5.50000	BBKHAT=	0.00676648	NORMAL FORCE/TOTAL BK= 0.36655595
SS= 5.75000	BBKHAT=	0.00533023	NORMAL FORCE/TOTAL BK= 0.42870754
SS= 6.00000	BBKHAT=	0.00416363	NORMAL FORCE/TOTAL BK= 0.49768392
SS= 6.25000	BBKHAT=	0.00318729	NORMAL FORCE/TOTAL BK= 0.57803215

FN	BFHAT	BWHAT	BEHAT	BLHAT	BBKHAT	**	BF/B44	BW/B44	BE/B44	BL/B44	BBK/B44	**	B44HAT	**
0.	0.00068	0.00096	0.00113	0.	0.00298	**	0.11816	0.16648	0.19703	0.	0.51833	**	0.00574	**
0.020	0.00070	0.00096	0.00108	0.00027	0.00298	**	0.11754	0.16022	0.17979	0.04532	0.49714	**	0.00599	**
0.040	0.00073	0.00100	0.00094	0.00054	0.00298	**	0.11789	0.16093	0.15197	0.08777	0.48144	**	0.00618	**
0.060	0.00075	0.00120	0.00078	0.00081	0.00298	**	0.11558	0.18458	0.11882	0.12477	0.45625	**	0.00653	**
0.080	0.00078	0.00184	0.00062	0.00109	0.00298	**	0.10670	0.25180	0.08527	0.14862	0.40761	**	0.00731	**
0.100	0.00080	0.00274	0.00050	0.00136	0.00298	**	0.09610	0.32681	0.05938	0.16208	0.35563	**	0.00837	**
0.120	0.00083	0.00316	0.00040	0.00163	0.00298	**	0.09229	0.35110	0.04435	0.18112	0.33115	**	0.00899	**
0.140	0.00085	0.00307	0.00032	0.00190	0.00298	**	0.09369	0.33637	0.03542	0.20821	0.32631	**	0.00913	**
0.160	0.00088	0.00281	0.00027	0.00217	0.00298	**	0.09668	0.30857	0.02913	0.23852	0.32709	**	0.00910	**
0.180	0.00091	0.00264	0.00022	0.00244	0.00298	**	0.09860	0.28703	0.02400	0.26606	0.32431	**	0.00918	**
0.200	0.00093	0.00260	0.00019	0.00271	0.00298	**	0.09894	0.27613	0.01971	0.28861	0.31661	**	0.00940	**
0.220	0.00096	0.00259	0.00016	0.00299	0.00298	**	0.09882	0.26826	0.01630	0.30872	0.30789	**	0.00967	**
0.240	0.00098	0.00259	0.00014	0.00326	0.00298	**	0.09862	0.26089	0.01362	0.32749	0.29939	**	0.00995	**
0.260	0.00101	0.00259	0.00012	0.00353	0.00298	**	0.09839	0.25379	0.01150	0.34510	0.29123	**	0.01022	**
0.280	0.00103	0.00259	0.00010	0.00380	0.00298	**	0.09814	0.24698	0.00979	0.36168	0.28341	**	0.01051	**
0.300	0.00106	0.00259	0.00009	0.00407	0.00298	**	0.09789	0.24047	0.00840	0.37730	0.27594	**	0.01079	**
0.320	0.00108	0.00259	0.00008	0.00434	0.00298	**	0.09763	0.23425	0.00726	0.39205	0.26881	**	0.01108	**
0.340	0.00111	0.00259	0.00007	0.00461	0.00298	**	0.09737	0.22832	0.00632	0.40599	0.26200	**	0.01137	**
0.360	0.00113	0.00259	0.00006	0.00489	0.00298	**	0.09711	0.22265	0.00554	0.41921	0.25549	**	0.01165	**
0.380	0.00116	0.00259	0.00006	0.00516	0.00298	**	0.09686	0.21724	0.00488	0.43174	0.24928	**	0.01194	**
0.400	0.00118	0.00259	0.00005	0.00543	0.00298	**	0.09661	0.21207	0.00432	0.44365	0.24335	**	0.01224	**

A.3 Code of Kawahara's method

```
C    *********************************************************************
C    *  Simple Prediction Formula of Roll Damping                       *
C    *                   on the Basis of Ikeda's Method            *
C    *                                                       *
C    *  Roll_Damping.for          coded by Yoshiho IKEDA               *
C    *                            Yuki KAWAHARA                *
C    *                            Kazuya MAEKAWA               *
C    *                            Osaka Prefecture University    *
C    *                            Graduate School of Engineering *
C    *  last up date 2009.07.08                             *
C    *********************************************************************
     PROGRAM  MAIN
     implicit none
     double precision,parameter :: PI = 3.14159
     double precision,parameter :: RO = 102
     double precision,parameter :: KVC = 1.14e-6
C    KVC : Kinematic Viscosity Coefficient

     double precision :: LPP,LB,BD,CB,CMID,OGD,PHI,TW,LBKL,BBKB
     double precision :: OMEGA,BRTH,DRAFT,OMEGAHAT,X1,X2,X3,X4,X5
     character :: BKCOMP,OK
     double precision :: CF,RF,SF,BF,BFHAT
     double precision A111,A112,A113,A121,A122,A123,A124,A131,A132,
    & A133,A134,A11,A12,A13,AA111,AA112,AA113,AA121,AA122,AA123,AA11,
    & AA12,AA1,A1,A2,A31,A32,A33,A34,A35,A36,A37,AA311,AA31,AA32,XX4,
    & AA3,A3,BWHAT
     double precision :: FE1,FE2,AE,BE1,BE2,BE3,CR,BEHAT,B44HAT
     double precision :: FBK1,FBK2,FBK3,FBK5,ABK,BBK1,BBK2,BBK3,BBKHAT

C    *******************************************
C    ***    Input principal particulars     ***
C    *******************************************
90   write(*,*) '----- INPUT PRINCIPAL PARTICULARS -----'
     write(*,*) ' SHIP LENGTH - Lpp [m] : '
     read(*,*) LPP
C
     write(*,*) ' RATIO of SHIP LENGTH to BREADTH - Lpp/B : '
     read(*,*) LB
C
     write(*,*) ' RATIO of BREADTH to DRAFT - B/d [2.5?…B/d?…4.5] : '
100  read(*,*) BD
```

```
      if (BD.lt.2.5 .or. 4.5.lt.BD) then
       write(*,*) ' Please confirm the range of B/d [2.5?…B/d?…4.5]. '
       go to 100
      end if
C
200   write(*,*) ' BLOCK COEFFICIENT - Cb [0.5?…Cb?…0.85] : '
      read(*,*) CB
      if (CB.lt.0.5 .or. 0.85.lt.CB) then
       write(*,*) ' Please confirm the range of Cb [0.5?…Cb?…0.85]. '
       go to 200
      end if
C
300   write(*,*) ' MIDSHIP SECTION COEFFICIENT - Cm [0.9?…Cm?…0.99] : '
      read(*,*) CMID
      if (CMID.lt.0.9 .or. 0.99.lt.CMID) then
       write(*,*) ' Please confirm the range of Cm [0.9?…Cm?…0.99]. '
       go to 300
      end if
C
400   write(*,*) ' RATIO of CENTER of GRAVITY to DRAFT
     & - OG/d [-1.5?…OG/d?…0.2] : '
      write(*,*) ' Downward direction is positive. '
      read(*,*) OGD
      if (OGD.lt.-1.5 .or. 0.2.lt.OGD) then
       write(*,*) ' Please confirm the range of OG/d [-1.5?…OGd?…0.2]. '
       go to 400
      end if
C
      write(*,*) ' ROLL ANGLE - ƒÓ [deg.] : '
      read(*,*) PHI
      PHI=ABS(PHI)
C
      write(*,*) ' WAVE PERIOD - Tw [sec.] [ƒÖHAT?…1.0] : '
      read(*,*) TW
      OMEGA=2*PI/TW

      BRTH=LPP/LB ; DRAFT=BRTH/BD ; OMEGAHAT=OMEGA*SQRT(BRTH/2/9.81)
      if (OMEGAHAT.gt.1.0) then
       write(*,*) ' Please confirm the range of ƒÖHAT [ƒÖHAT?…1.0]. '
       go to 400
      end if
```

```
C    *** Input Bilge Keel ***
      write(*,*) '----- INPUT BILGE KEEL DATA -----'
      write(*,*)
     & ' Do you calculate roll damping of the Bilge Keel component?
     &   - (Y or N) '
      read(*,*) BKCOMP
      if (BKCOMP .eq. 'N' .or. BKCOMP .eq. 'n') then
       go to 700
      end if
C
500   write(*,*) ' RATIO of BILGE KEEL LENGTH to SHIP LENGTH - lBK/Lpp
     & [0.05?...lBK?...0.4] : '
      read(*,*) LBKL
      if (LBKL.lt.0.05 .or. 0.4.lt.LBKL) then
       write(*,*)
     & ' Please confirm the range of lBK/Lpp [0.05?...lBK/Lpp?...0.4]. '
       go to 500
      end if
C
600   write(*,*) ' RATIO of BILGE KEEL BREADTH to SHIP BREADTH - bBK/B
     & [0.01?...bBK?...0.06] : '
      read(*,*) BBKB
      if (BBKB.lt.0.01 .or. 0.06.lt.BBKB) then
       write(*,*)
     &  ' Please confirm the range of bBK/B [0.01?...bBK/B?...0.06]. '
       go to 600
      end if

C    *** Data Confirmation ***
700   write(*,*) '----- PRINCIPAL PARTICULARS -----'
      write(*,710) LPP
      write(*,720) LB
      write(*,730) BD
      write(*,740) CB
      write(*,750) CMID
      write(*,760) OGD
      write(*,770) PHI
      write(*,780) TW

      if (BKCOMP .eq. 'N' .or. BKCOMP .eq. 'n') then
```

```
      go to 900
     end if

     write(*,790) LBKL
     write(*,800) BBKB

710   format(7x,'Lpp',6x,'[m]',':',4x,f6.2)
720   format(7x,'L/B',9x,':',4x,f6.2)
730   format(7x,'B/d',9x,':',4x,f6.2)
740   format(7x,'Cb',10x,':',4x,f6.2)
750   format(7x,'Cm',10x,':',4x,f6.2)
760   format(7x,'OG/d',8x,':',4x,f6.2)
770   format(7x,'ƒÓ',4x,'[deg.]',':',4x,f6.2)
780   format(7x,'Tw',4x,'[sec.]',':',4x,f6.2)
790   format(7x,'lBK/Lpp',5x,':',4x,f6.2)
800   format(7x,'bBK/B',7x,':',4x,f6.2)

900   write(*,*) ' Is it OK (Y or N) ? '
     read(*,*) OK
     if (OK .eq. 'N' .or. OK .eq. 'n') then
      go to 90
     end if
C     ******************************************************************
C     *** Calculation of roll damping by the proposed predition method ***
C     ******************************************************************

C     *** Frictional Component ***
     RF=DRAFT*((0.887d0+0.145d0*CB)*(1.7d0+CB*BD)-2.0d0*OGD)/PI
     SF=LPP*(1.75d0*DRAFT+CB*BRTH)
     CF=1.328*((3.22*RF**2*(PHI*PI/180)**2)/(TW*KVC))**-0.5
     BF=4.0/3.0/PI*RO*SF*RF**3*(PHI*PI/180)*OMEGA*CF
     BFHAT=BF/(RO*LPP*BRTH**3*DRAFT*CB)*SQRT(BRTH/2.0/9.81)

C     *** Wave Component ***
     X1=BD ; X2=CB ; X3=CMID
     X5=OMEGAHAT
     X4=1-OGD
     A111=-0.002222d0*X1**3+0.040871d0*X1**2-0.286866d0*X1
    &    +0.599424d0
     A112=0.010185d0*X1**3-0.161176d0*X1**2+0.904989d0*X1
    &    -1.641389d0
```

```
 A113=-0.015422d0*X1**3+0.220371d0*X1**2-1.084987d0*X1
&    +1.834167d0
 A121=-0.0628667d0*X1**4+0.4989259d0*X1**3+0.52735d0*X1**2
&         -10.7918672d0*X1+16.616327d0
 A122=0.1140667d0*X1**4-0.8108963d0*X1**3-2.2186833d0*X1**2
&         +25.1269741d0*X1-37.7729778d0
 A123=-0.0589333d0*X1**4+0.2639704d0*X1**3+3.1949667d0*X1**2
&         -21.8126569d0*X1+31.4113508d0
 A124=0.0107667d0*X1**4+0.0018704d0*X1**3-1.2494083d0*X1**2
&         +6.9427931d0*X1-10.2018992d0
 A131=0.192207d0*X1**3-2.787462d0*X1**2+12.507855d0*X1
&    -14.764856d0
 A132=-0.350563d0*X1**3+5.222348d0*X1**2-23.974852d0*X1
&    +29.007851d0
 A133=0.237096d0*X1**3-3.535062d0*X1**2+16.368376d0*X1
&    -20.539908d0
 A134=-0.067119d0*X1**3+0.966362d0*X1**2-4.407535d0*X1
&    +5.894703d0

 A11=A111*X2**2+A112*X2+A113
 A12=A121*X2**3+A122*X2**2+A123*X2+A124
 A13=A131*X2**3+A132*X2**2+A133*X2+A134

 AA111=17.945d0*X1**3-166.294d0*X1**2+489.799d0*X1-493.142d0
 AA112=-25.507d0*X1**3+236.275d0*X1**2-698.683d0*X1+701.494d0
 AA113=9.077d0*X1**3-84.332d0*X1**2+249.983d0*X1-250.787d0
 AA121=-16.872d0*X1**3+156.399d0*X1**2-460.689d0*X1+463.848d0
 AA122=24.015d0*X1**3-222.507d0*X1**2+658.027d0*X1-660.665d0
 AA123=-8.56d0*X1**3+79.549d0*X1**2-235.827d0*X1+236.579d0

 AA11=AA111*X2**2+AA112*X2+AA113
 AA12=AA121*X2**2+AA122*X2+AA123

 AA1=(AA11*X3+AA12)*(1-X4)+1.0

 A1=(A11*X4**2+A12*X4+A13)*AA1
 A2=-1.402d0*X4**3+7.189d0*X4**2-10.993d0*X4+9.45d0

 A31=-7686.0287d0*X2**6+30131.5678d0*X2**5
&  -49048.9664d0*X2**4+42480.7709d0*X2**3-20665.147d0*X2**2
&  +5355.2035d0*X2-577.8827d0
```

```
      A32=61639.9103d0*X2**6-241201.0598d0*X2**5+392579.5937d0*X2**4
     &   -340629.4699d0*X2**3+166348.6917d0*X2**2-43358.7938d0*X2
     &   +4714.7918d0
      A33=-130677.4903d0*X2**6+507996.2604d0*X2**5
     &    -826728.7127d0*X2**4+722677.104d0*X2**3-358360.7392d0*X2**2
     &    +95501.4948d0*X2-10682.8619d0
      A34=-110034.6584d0*X2**6+446051.22d0*X2**5-724186.4643d0*X2**4
     &   +599411.9264d0*X2**3-264294.7189d0*X2**2+58039.7328d0*X2
     &   -4774.6414d0
      A35=709672.0656d0*X2**6-2803850.2395d0*X2**5+
     &    4553780.5017d0*X2**4-3888378.9905d0*X2**3+1839829.259d0*X2**2
     &    -457313.6939d0*X2+46600.823d0
      A36=-822735.9289d0*X2**6+3238899.7308d0*X2**5
     &   -5256636.5472d0*X2**4+4500543.147d0*X2**3-2143487.3508d0*X2**2
     &   +538548.1194d0*X2-55751.1528d0
      A37=299122.8727d0*X2**6-1175773.1606d0*X2**5
     &   +1907356.1357d0*X2**4-1634256.8172d0*X2**3+780020.9393d0*X2**2
     &   -196679.7143d0*X2+20467.0904d0

      AA311=(-17.102d0*X2**3+41.495d0*X2**2-33.234d0*X2+8.8007d0)*X4
     &    +36.566d0*X2**3-89.203d0*X2**2+71.8d0*X2-18.108d0

      AA31=(-0.3767d0*X1**3+3.39d0*X1**2-10.356d0*X1+11.588d0)*AA311
      AA32=-0.0727d0*X1**2+0.7d0*X1-1.2818d0

      XX4=X4-AA32

      AA3=AA31*(-1.05584d0*XX4**9+12.688d0*XX4**8-63.70534d0*XX4**7
     & +172.84571d0*XX4**6-274.05701d0*XX4**5+257.68705d0*XX4**4
     & -141.40915d0*XX4**3+44.13177d0*XX4**2-7.1654d0*XX4-0.0495d0*X1**2
     & +0.4518d0*X1-0.61655d0)

      A3=A31*X4**6+A32*X4**5+A33*X4**4+A34*X4**3+A35*X4**2
     &    +A36*X4+A37+AA3

      BWHAT=A1/X5*EXP(-A2*(LOG(X5)-A3)**2/1.44)

C    *** Eddy Component ***
      FE1=(-0.0182d0*CB+0.0155d0)*(BD-1.8d0)**3
      FE2=-79.414d0*CB**4+215.695d0*CB**3
     &          -215.883d0*CB**2+93.894d0*CB-14.848d0
```

```
      AE=FE1+FE2
      BE1=(3.98d0*CB-5.1525d0)*(-0.2d0*BD+1.6d0)*OGD*
     & ((0.9717d0*CB**2-1.55d0*CB+0.723d0)*OGD+0.04567d0*CB+0.9408d0)
      BE2=(0.25*OGD+0.95)*OGD
     &          -219.2d0*CB**3+443.7d0*CB**2-283.3d0*CB+59.6d0
      BE3=-15d0*CB*BD+46.5d0*CB+11.2d0*BD-28.6d0
      CR=AE*EXP(BE1+BE2*CMID**BE3)
      BEHAT=4.0*OMEGAHAT*PHI*PI/180/(3.0*PI*CB*BD**3.0)*CR

C    *** Bilge Keel Component ***
      if (BKCOMP .eq. 'N' .or. BKCOMP .eq. 'n') then
       BBKHAT=0.0
      else
       FBK1=(-0.3651d0*CB+0.3907d0)*(BD-2.83d0)**2-2.21d0*CB+2.632d0
       FBK2=0.00255d0*PHI**2+0.122d0*PHI+0.4794d0
       FBK3=(-0.8913d0*BBKB**2-0.0733d0*BBKB)*LBKL**2
     &        +(5.2857d0*BBKB**2-0.01185d0*BBKB+0.00189d0)*LBKL
       ABK=FBK1*FBK2*FBK3
       BBK1=(5.0d0*BBKB+0.3d0*BD-0.2d0*LBKL
     &          +0.00125d0*PHI**2-0.0425d0*PHI-1.86d0)*OGD
       BBK2=-15.0d0*BBKB+1.2d0*CB-0.1d0*BD
     &          -0.0657d0*OGD**2+0.0586d0*OGD+1.6164d0
       BBK3=2.5d0*OGD+15.75d0
       BBKHAT=ABK*EXP(BBK1+BBK2*CMID**BBK3)*OMEGAHAT
      endif
C    *** Total Roll Damping ***
      B44HAT=BFHAT+BWHAT+BEHAT+BBKHAT
C    *******************************
C    ***     Output to files     ***
C    *******************************
      open(10,FILE='output.csv')
      write(10,*) ' ----- INPUT PRINCIPAL PARTICULARS ----- '
      write(10,*) 'Lpp',LPP
      write(10,*) 'Lpp/B',LB
      write(10,*) 'B/d',BD
      write(10,*) 'Cb',CB
      write(10,*) 'Cm',CMID
      write(10,*) 'OG/d',OGD
      write(10,*) 'ƒÓ',PHI
      write(10,*) 'Tw',TW
```

```
      if (BKCOMP .eq. 'N' .or. BKCOMP .eq. 'n') then
       go to 1000
      end if

      write(10,*) 'lBK/Lpp',LBKL
      write(10,*) 'bBK/B',BBKB

1000  write(10,*) ' ----- Cal. by simplified prediction method ----- '
      write(10,*) ' BFHAT ',' BWHAT ',' BEHAT ',' BBKHAT ',' B44HAT '
      write(10,*) BFHAT,BWHAT,BEHAT,BBKHAT,B44HAT
      close(10)

      END  PROGRAM
```

A.4 Implementation of Kawahara's method in spreadsheet

	Experiment	Kawahara's method
		0.011132
B_F		0.001156
B_w		0.025158
B_E		0.009856
B_{BKN}		
B_{44}	0.0810	0.036170

Input

L	1.07
L/B	4.28
B/D	1.923076923
Cb	0.5
Cm	0.9
OG/d	-0.153846154
ϕA	10
T	1
lbk/L	0.23364486
bbk/B	0.06
B	0.25
d	0.13
ρ	1000
OG	-0.02
ν	1.14E-06
ω	6.283185307
ω_{hat}	0.709251677
bbk	0.015
rbk	0.15
rmax	0.15
lbk	0.25

bf

C_f	3.82E-02
r_f	0.11840714
S_f	0.377175

bw

x1	1.923
x2	0.500
x3	0.900
x4	1.154
x5	0.709
x6	1.359

a134	5.15260166030427000000000000E-01
a133	-4.49911731642164000000000000E-01
a132	-2.76811951291918000000000000E-01
a131	3.46719153781669000000000000E-01

a124	-1.3105994548057900000000000E+00
a123	2.3516021641037600000000000E+00
a122	-1.8648932710088000000000000E+00
a121	5.0190797946502000000000000E-01
a113	4.5298565708292600000000000E-01
a112	-4.2468582610260500000000000E-01
a111	1.8311783514732600000000000E-01
a13	2.6444120660907400000000000E-01
a12	-5.3828319307298200000000000E-01
a11	2.8642220281845500000000000E-01
aa123	1.6379067503480000000000000E+01
aa122	-4.7320681032994900000000000E+01
aa121	3.6316676767776000000000000E+01
aa113	-1.7375826180041000000000000E+01
aa112	5.0271492244231100000000000E+01
aa111	-3.8592925180685100000000000E+01
aa12	1.7978961789265500000000000E+00
aa11	-1.8883113530967300000000000E+00
a37	-8.9919765624945300000000000E+00
a36	3.9546073437471900000000000E+01
a35	-6.6495615625004600000000000E+01
a34	5.6191143750002100000000000E+01
a33	-2.5158492187492800000000000E+01
a32	5.6991734374978500000000000E+00
a31	-5.1444218750009400000000000E-01
aa311	5.4633379999999600000000000E-01
aa32	-2.0453943830000000000000000E-01
aa31	8.3623674007450200000000000E-01
aa3	-9.9188803789411100000000000E-02
aa1	9.8484394198451800000000000E-01
a3	1.7736080129798300000000000E-01
a2	4.1831903538720000000000000E+00
a1	2.4321146650545400000000000E-02
bw	2.5157598484361100000000000E-02

Be

Cr	6.671306160129380E-01
AE	1.267619319071450E-01
BE1	5.524583744538010E-01
BE2	1.334763313609460E+00
BE3	1.765384615384600E+00
x1	1.923076923076920E+00
x2	5.000000000000000E-01
x3	9.000000000000000E-01
x4	-1.538461538461540E-01

bk

x1	1.923076923076920E+00
x2	5.000000000000000E-01
x3	9.000000000000000E-01

x4	-1.538461538461540E-01
x5	7.092516767214440E-01
x6	1.000000000000000E+01
x7	6.000000000000000E-02
x8	2.336448598130840E-01
f1	1.698205345650890E+00
f2	1.954400000000000E+00
f3	4.306135120971260E-03
Bbk1	2.045855223137760E-01
Bbk2	1.113521893491120E+00
Bbk3	1.536538461538460E+01
abk	1.429194416637920E-02
Bbk	1.550779365419960E-02

A.5 Attachments

In addition to the written master thesis, a zip-file is submitted including the results of the model tests.