ISSN 0005-1179, Automation and Remote Control, 2012, Vol. 73, No. 8, pp. 1327–1336. © Pleiades Publishing, Ltd., 2012.
Original Russian Text © A.A. Bobtsov, S.A. Kolyubin, A.S. Kremlev, A.A. Pyrkin, 2012, published in Avtomatika i Telemekhanika, 2012, No. 8, pp. 64–75.

An Iterative Algorithm of Adaptive Output Control with Complete Compensation for Unknown Sinusoidal Disturbance

A. A. Bobtsov,[*,**] **S. A. Kolyubin,**[*] **A. S. Kremlev,**[*] **and A. A. Pyrkin**[*]

[*] *State Institute of Information Technologies, Mechanics and Optics, St. Petersburg, Russia*
[**] *Institute of Problems of Mechanical Engineering, Russian Academy of Sciences, St. Petersburg, Russia*

Received May 21, 2011

Abstract—The problem is considered for the output control of a linear, parametrically indeterminate object subjected to the effect of an external unknown, sinusoidal disturbing influence. The solution of this problem is found in the class of iterative adaptive algorithms involving the channels of stabilization and identification of the frequency of a disturbing influence.

DOI: 10.1134/S0005117912080061

1. INTRODUCTION

The problem of compensation for a sinusoidal or a multisinusoidal disturbing influence is not knew (see, for example, [1]), but so far it attracts interest (see, for example, [2–5]) because it is not solved for a number of mathematical tolerances both on the disturbance itself and on the object to which it is applied. At present most of the investigations, which are devoted to the development of the methods of compensation for harmonic disturbances, consider the case when amplitudes, phases, and frequencies are unknown constant parameters (see, for example, [1–11]). Further, variations begin to appear, which are associated with tolerances relative to the object itself, namely, the linearity or nonlinearity of dynamics, the parametric definiteness or its absence, the availability of measurements of all variables of the state or only their portion, etc. We will give a few examples of this kind of investigations.

The algorithm is suggested in [2] for the control of a linear stable object with known parameters and with the unitary relative degree defined for a linear system as a difference of the higher degrees of polynomials in the denominator and the numerator of the transfer function, the object being subjected to the effect of a biased harmonic disturbance. In contrast to [2], in [5] the algorithm of compensation for a disturbing influence is considered for the case of a nonminimum-phase linear object with known parameters, but of any relative degree. In [9, 10] this problem was extended to linear objects with known parameters, but with a delay in the control channel. The articles [6, 7, 11] are devoted to the counteraction of a sinusoidal disturbance under the conditions of the complete parametric indefiniteness of a controllable object. The adaptive controller was built up, which was based only on measurements of the output variable. The articles [6, 7] dealt with a linear object, and the article [11] with a nonlinear object; in this case it was assumed in [6, 7] and [11] that the relative degree was equal to unity.

This article is concerned with a new algorithm of the output control of a parametrically indefinite linear object subjected to the effect of the harmonic disturbance $\delta(t) = A\sin(\omega t + \phi)$ with the unknown amplitude, frequency, and phase. In the authors' opinion, the problem solved in this work significantly develops and supplements the approaches published in [1–11] because in contrast to the analogs [6, 7] that are the closest ones as to the problem statement, here it is assumed that the relative degree of an object can be higher than unity.

2. STATEMENT OF THE PROBLEM

We will consider a controllable object of the form

$$a(p)y(t) = b(p)[u(t) + \delta(t)], \tag{1}$$

where $y(t)$ is the object output, $u(t)$ is the control signal, $p = \frac{d}{dt}$ is the differentiation operator, the parameters of polynomials $a(p) = p^n + a_{n-1}p^{n-1} + a_{n-2}p^{n-2} + \ldots + a_0$ and $b(p) = b_m p^m + b_{m-1}p^{m-1} + b_{m-2}p^{m-2} + \ldots + b_0$ are unknown numbers, and $\delta(t) = A\sin(\omega t + \phi)$ is the disturbing influence with the unknown amplitude A, frequency ω, and phase ϕ.

The aim of control: it is necessary to find such a signal $u(t)$ that the following objective condition be fulfilled:

$$\lim_{t\to\infty} y(t) = 0. \tag{2}$$

We will solve this problem under the following assumptions.

Assumption 1. *The polynomial $b(p)$ is the Hurwitz one and the coefficient $b_0 > 0$.*

Assumption 2. *The relative degree $r = n - m$ is known, but the dimensions of the polynomials $a(p)$ and $b(p)$ are not known.*

Assumption 3. *The lower bound $\omega_{\min}$ of the frequency ω is known.*

3. BASIC RESULT

3.1. The Simplified Case of Controller Design

We will consider an auxiliary result, assuming that the frequency ω of the disturbing influence is known. We will present the Eq. (1) in the form

$$Y(s) = \frac{b(s)}{a(s)}U(s) + \frac{b(s)}{a(s)}\Psi(s) + \frac{D(s)}{a(s)}, \tag{3}$$

where s is a complex variable, $Y(s) = L\{y(t)\}$, $U(s) = L\{u(t)\}$ and $\Psi(s) = L\{\delta(t)\} = \frac{A_{\delta 1}s + A_{\delta 2}}{s^2+\omega^2}$ is the Laplace transform of appropriate signals, $A_{\delta 1} = A\sin\phi$ and $A_{\delta 2} = A\omega\cos\phi$ are constant numbers, the polynomial $D(s)$ denotes the sum of all terms containing nonzero initial conditions.

Let us temporarily assume that $r - 1$ derivatives of the variable $y(t)$ are known and also the frequency ω is known of the disturbing influence $\delta(t) = A\sin(\omega\, t + \phi)$. We will select the control law $u(t)$ in the form

$$u(t) = -k\frac{\alpha(p)(p+1)^2}{p^2+\omega^2}y(t), \tag{4}$$

where the Hurwitz polynomial $\alpha(p)$ of the degree $r-1$ and the constant coefficient $k > 0$ are chosen in such a way that all eigenvalues of the polynomial $\gamma(s) = a(s)(s^2+\omega^2) + kb(s)\alpha(s)(s+1)^2$ lie in the left half-plane (a more detailed description of the existence of the polynomial $\alpha(p)$ and the coefficient $k > 0$, which afford the Hurwitz property $\gamma(s)$, can be found, for example, in [12, 13]).

Then, performing the direct Laplace transformation with respect to (4) and substituting the obtained expression into the Eq. (3), we have

$$Y(s) = -k\frac{b(s)\alpha(s)(s+1)^2}{a(s)(s^2+\omega^2)}Y(s) + \frac{b(s)}{a(s)}\frac{A_{\delta 1}s + A_{\delta 2}}{s^2+\omega^2} + \frac{D(s)}{a(s)}$$

and

$$Y(s) = (A_{\delta 1}s + A_{\delta 2})\frac{b(s)}{\gamma(s)} + \frac{D(s)(s^2+\omega^2)}{\gamma(s)}.$$

Because the polynomial $\gamma(s) = a(s)(s^2+\omega^2)+kb(s)\alpha(s)(s+1)^2$ is the Hurwitz one, then carrying out the inverse Laplace transformation, we find

$$\lim_{t\to\infty} y(t) = 0.$$

It is evident that at the known value of the frequency ω and the measurability of $r-1$ derivatives of the variable $y(t)$, it is possible to use the control algorithm of the form (4). However, the problem considered in the article does not provide for similar assumptions, and, hence, $r-1$ derivatives of $y(t)$ and the frequency ω must be restored in a certain way. Namely, the restoration procedure of unknown functions and the parameter ω will be dealt with in the next subsection.

3.2. The Iterative Algorithm of Adaptive Control

Let us assume that the derivatives of the output signal $y(t)$ are not measured, while the coefficients of the polynomials $a(p)$, $b(p)$, and the parameter ω are unknown. In this case we will use the results published in [13–16] and select such a control law

$$u(t) = -k\frac{\alpha(p)(p+1)^2}{p^2+\hat{\omega}^2}\xi_1(t), \tag{5}$$

$$\begin{cases} \dot{\xi}_1 = \sigma\xi_2 \\ \dot{\xi}_2 = \sigma\xi_3 \\ \dots \\ \dot{\xi}_{r-1} = \sigma(-k_1\xi_1 - k_2\xi_2 - \ldots - k_{r-1}\xi_{r-1} + k_1 y), \end{cases} \tag{6}$$

where the number $k > 0$ and the polynomial $\alpha(p)$ are chosen in much the same way as (4), the number $\sigma > k$, and the coefficients k_i are estimated on the basis of requirements of the asymptotic stability of the system (6) at the zero input of $y(t)$, $\hat{\omega}$ is the estimate of the frequency of the disturbing influence $\delta(t)$.

Remark. At the relative degree $r = 1$, the control law is shaped up in the simplified form when in the expression (5) the polynomial $\alpha(p) = 1$, $\xi_1(t) = y(t)$, and the use of the system (6) is excluded.

Substituting (5) into (1) we obtain

$$y(t) = \frac{kb(p)\alpha(p)(p+1)^2}{a(p)(p^2+\hat{\omega}^2)+kb(p)\alpha(p)(p+1)^2}\varepsilon(t) + \frac{b(p)(p^2+\hat{\omega}^2)}{a(p)(p^2+\hat{\omega}^2)+kb(p)\alpha(p)(p+1)^2}\delta(t), \tag{7}$$

where $\varepsilon(t) = y(t) - \xi_1(t)$.

We will write (7) in the form

$$y(t) = \frac{kb(p)\alpha(p)(p+1)^2}{a(p)(p^2+\hat{\omega}^2)+kb(p)\alpha(p)(p+1)^2}[\varepsilon(t) + w(t)], \tag{8}$$

where the signal $w(t) = \frac{(p^2+\hat{\omega}^2)}{k\alpha(p)(p+1)^2}\delta(t)$.

The model close to (8) was considered in [14–16], therefore we will use the results from [14–16] and pass on to the input-state-output form:

$$\dot{x} = Ax + kb(\varepsilon + w), \tag{9}$$

$$y = c^{\mathrm{T}}x, \tag{10}$$

where $x \in R^n$ is the vector of variables of the state of the model (9); A, b and c are the matrices of transition from the input-output model to the input-state-output model; in this case in view of the known Yakubovich–Kalman lemma (see, for example, [12]), we can indicate the symmetric positive definite matrix P satisfying the two matrix equations

$$A^{\mathrm{T}}P + PA = -Q_1, \quad Pb = c, \tag{11}$$

where $Q_1 = Q_1^{\mathrm{T}}$ is a certain positive definite matrix.

We will rewrite (6) in the vector-matrix form

$$\dot{\xi} = \sigma(\Gamma\xi + dk_1 y), \tag{12}$$

$$\xi_1 = h^T\xi, \tag{13}$$

where $\xi \in R^{r-1}$ us the vector of variables of the state of the model (12), the matrix $\Gamma = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_1 & -k_2 & -k_3 & \dots & -k_{r-1} \end{bmatrix}$ is the Hurwitz one in view of the calculation of the coefficients k_i of the model (6), $d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$, $h = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$.

We will introduce for consideration the deviation vector

$$\eta = hy - \xi. \tag{14}$$

Differentiating (14), we obtain

$$\dot{\eta} = h\dot{y} - \sigma(\Gamma(hy - \eta) + dk_1 y) = h\dot{y} + \sigma\Gamma\eta - \sigma(dk_1 + \Gamma h)y = h\dot{y} + \sigma\Gamma\eta, \tag{15}$$

$$\varepsilon = y - \xi_1 = h^{\mathrm{T}}\eta, \tag{16}$$

where $dk_1 = -\Gamma h$.

Thus we have the system of differential equations

$$\dot{x} = Ax + kb(\varepsilon + w), \quad y = c^{\mathrm{T}}x, \tag{17}$$

$$\dot{\eta} = h\dot{y} + \sigma\Gamma\eta, \quad \varepsilon = h^{\mathrm{T}}\eta. \tag{18}$$

In view of the Hurwitz property Γ there exists a matrix $N = N^{\mathrm{T}}$ satisfying the Lyapunov equation

$$\Gamma^{\mathrm{T}}N + N\Gamma = -Q_2, \tag{19}$$

where $Q_2 = Q_2^{\mathrm{T}}$ is the positive definite matrix.

Assertion. *Let us consider the system of the form* (1) *closed with the use of the controller* (5) *and* (6). *Then the output variable* $y(t)$ *will be brought to a certain domain* ε_0 *and will not go out of it in the time interval* t_1.

Proof. As in [13–16], we will set up the Lyapunov function of the form

$$V = x^{\mathrm{T}}Px + \eta^{\mathrm{T}}N\eta. \tag{20}$$

Differentiating (20), we obtain

$$\dot{V} = x^{\mathrm{T}}(A^{\mathrm{T}}P + PA)x + 2kx^{\mathrm{T}}Pbh^{\mathrm{T}}\eta + 2kx^{T}Pbw + \eta^{\mathrm{T}}\sigma(\Gamma^{\mathrm{T}}N + N\Gamma)\eta$$
$$+ 2\eta^{\mathrm{T}}Nhc^{\mathrm{T}}Ax + 2k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bw + 2k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bh^{\mathrm{T}}\eta.$$

As in [13–16], we will consider the inequalities:

$$2kx^{\mathrm{T}}Pbh^{\mathrm{T}}\eta \le k^{-1}\,x^{\mathrm{T}}Pbb^{\mathrm{T}}Px + k^{3}\eta^{\mathrm{T}}hh^{\mathrm{T}}\eta,$$
$$2kx^{\mathrm{T}}Pbw \le k^{-1}\,x^{\mathrm{T}}Pbb^{\mathrm{T}}Px + k^{3}\,w^{2},$$
$$2k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bh^{\mathrm{T}}\eta \le k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bb^{\mathrm{T}}ch^{\mathrm{T}}N\eta + k\eta^{\mathrm{T}}hh^{\mathrm{T}}\eta,$$
$$2\eta^{\mathrm{T}}Nhc^{\mathrm{T}}Ax \le k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}AA^{\mathrm{T}}ch^{\mathrm{T}}N\eta + k^{-1}\,x^{\mathrm{T}}x,$$
$$2k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bw \le k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bb^{\mathrm{T}}ch^{\mathrm{T}}N\eta + k\,w^{2},$$

then

$$\dot{V} \le -x^{\mathrm{T}}Q_{1}x - \sigma\eta^{\mathrm{T}}Q_{2}\eta + k^{-1}\,x^{\mathrm{T}}Pbb^{\mathrm{T}}Px + k^{3}\eta^{\mathrm{T}}hh^{\mathrm{T}}\eta$$
$$+ k^{-1}\,x^{\mathrm{T}}Pbb^{\mathrm{T}}Px + k^{3}\,w^{2} + k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bb^{\mathrm{T}}ch^{\mathrm{T}}N\eta + k\eta^{\mathrm{T}}hh^{\mathrm{T}}\eta$$
$$+ k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}AA^{\mathrm{T}}ch^{\mathrm{T}}N\eta + k^{-1}\,x^{\mathrm{T}}x + k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bb^{\mathrm{T}}ch^{\mathrm{T}}N\eta + k\,w^{2}.$$

Let the numbers $k > 0$ and $\sigma > 0$ be such that

$$-x^{\mathrm{T}}Q_{1}x + k^{-1}\,x^{\mathrm{T}}Pbb^{\mathrm{T}}Px + k^{-1}\,x^{\mathrm{T}}Pbb^{\mathrm{T}}Px + k^{-1}\,x^{\mathrm{T}}x \le -x^{\mathrm{T}}Q'x < 0,$$
$$-\sigma\eta^{\mathrm{T}}Q_{2}\eta + k^{3}\eta^{\mathrm{T}}hh^{\mathrm{T}}\eta + k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bb^{\mathrm{T}}ch^{\mathrm{T}}N\eta + k\eta^{\mathrm{T}}hh^{\mathrm{T}}\eta$$
$$+ k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}AA^{\mathrm{T}}ch^{\mathrm{T}}N\eta + k\eta^{\mathrm{T}}Nhc^{\mathrm{T}}bb^{\mathrm{T}}ch^{\mathrm{T}}N\eta \le -\eta^{\mathrm{T}}Q''\eta < 0,$$

then the derivative of the function (20) will take the form $\dot{V} \le -x^{\mathrm{T}}Q'x - \eta^{\mathrm{T}}Q''\eta + (k^{3} + k)w^{2}$. Whence, using the Rayleigh relation (see, for example, [13–16]) for the quadratic forms $-x^{\mathrm{T}}Q'x$ and $-\eta^{\mathrm{T}}Q''\eta$, it is easy to obtain the inequality

$$\dot{V} \le -\lambda V + (k^{3} + k)w^{2}, \tag{21}$$

where the number $\lambda > 0$.

It follows from (21) that the closer the estimate $\hat{\omega}$ converges to the true value of the frequency ω, the less in amplitude is the function (20), while at $\hat{\omega} = \omega$ the signal $w(t)$ exponentially converges to zero, and, hence, the function (20) tends to zero, and the objective condition (2) is fulfilled.

Thus it is necessary to set up the identification scheme of the unknown parameter ω of the disturbing influence $\delta(t)$ and to substitute the obtained value into the control algorithm of the form (5). We will carry out the identification of the unknown parameter ω in a few stages. First, we will substitute $\hat{\omega} = \omega_{\min}$ into (5) and fix it. Since the system under consideration is linear and the matrix A is the Hurwitz one, the output variable $y(t)$ will be the sinusoidal function with the frequency ω, i.e., $y(t) = A_1 \sin(\omega t + \phi_1)$ (see, for example, [17]). Hence for the identification of the parameter ω, recourse can be made to various identification schemes, for example, algorithms published in [18–26]. As a basis, we will resort to the approach used in [26].

We will introduce a linear second-order filter of the form

$$\varsigma(s) = \frac{\gamma_0^2}{(s + \gamma_0)^2},$$

where $\gamma_0 > 0$.

To identify the disturbing influence frequency, we will use the following algorithm:

$$\hat{\theta}(t) = \chi(t) + k_a\gamma_0^2\dot{\varsigma}(t)y(t), \tag{22}$$

$$\dot{\chi}(t) = -k_a\dot{\varsigma}(t)(2\gamma_0\ddot{\varsigma}(t) + \gamma_0^2\dot{\varsigma}(t) + \dot{\varsigma}(t)\hat{\theta}(t)) - k_a\gamma_0^2\ddot{\varsigma}(t)y(t), \tag{23}$$

$$\hat{\omega}(t) = \sqrt{\left|\hat{\theta}(t)\right|}. \tag{24}$$

On finding the true value of the parameter ω, we substitute it into the control law (5) for $\omega_{\min}$. However, the question arises as to the choice of the instant of time for the substitution of a value $\hat{\omega}$ from the identification algorithm (22)–(24) into the Eq. (5) because $\lim_{t\to\infty}(\omega - \hat{\omega}) = 0$. It is obvious that the instant of time $t = \infty$ for actual systems is unacceptable. As a solution of this problem, the iterative identification procedure is suggested.

The essence of this procedure reduces to the following. At the first step the value $\hat{\omega}_0 = \omega_{\min}$, which corresponds to the lower bound for a possible disturbance frequency, is substituted into the Eq. (5). The system operates with the given value for a certain time interval up to a definite instant of time t_1. Further, at the instant t_1 the renewed value $\hat{\omega}_1 = \hat{\omega}(t_1)$ is taken from the identification algorithm (22)–(24) and is substituted into the Eq. (5). Now in the control law, up to the next instant of time t_2, use is made of the estimate of the frequency $\hat{\omega}_1$. At the instant of time t_2 the renewed value $\hat{\omega}_2 = \hat{\omega}(t_2)$ is again taken from the identification algorithm (22)–(24) and is substituted into the Eq. (5). Thus the analytically iterative renewal can be described by the system

$$\bar{\omega}(t) = \begin{cases} \omega_{\min}, & t \le t_1 \\ \hat{\omega}(t_i), & t \in [t_i,\ t_{i+1}), \quad i = \overline{2, N}, \end{cases}$$

where $\hat{\omega}(t_i)$ denotes the frequency value obtained at the instant of renewal directly from the identification algorithm (22)–(24), while $\bar{\omega}(t)$ represents a value used at each fixed interval of time in the control law (5) and (6).

In the general case, the renewal moments may limit unequal time intervals. In particular, it is expedient to select the time period, up to the first renewal, of a larger dimension in comparison with successive intervals. This time interval can be taken to be the initialization time necessary that the residual in the estimate of the disturbance frequency at the identifier output should reach an acceptable value, which will occur faster if undesirable switchings in the system are avoided.

It is necessary to note that the theoretical verification of the rules for selecting the time intervals t_i of the iterative renewal of frequency estimates in the control law (5) and (6) represents an independent nontrivial problem that the authors intend to solve with the aim of a progression of the obtained results. In this work a simplified approach is presented, the effectiveness of which is confirmed empirically.

4. ILLUSTRATING EXAMPLES

For the illustration of the effectiveness and special features of the developed iterative control algorithm, the simulation modeling of a closed system was carried out. As an example, it is possible to consider the stabilization problem of a linear unstable object with a relative degree $r = 3$, which is described by the equation

$$y(t) = \frac{p+1}{p(p-1)(p+2)(p+3)}[u(t) + \delta(t)]$$

with the simultaneous counteraction of the sinusoidal disturbance of the unknown amplitude, frequency, and initial phase, and under the assumption that the object parameters are also unknown.

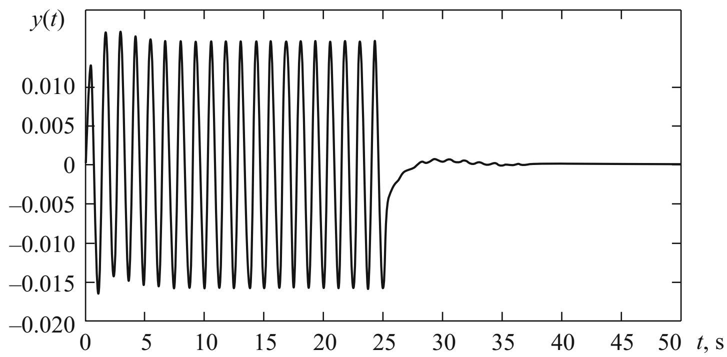

Fig. 1. Transient processes in the closed system for the signal $y(t)$ at $\delta(t) = 7\sin(5t + 0.1)$, $t_1 = 25$, $\tau = 10$, $\omega_{\min} = 1$, $\alpha(p) = (p + 4)(p + 5)$, $k_1 = k_2 = 1$, $k = 12$, $\sigma = 30$, $k_a = 15$, $\gamma_0 = 20$.

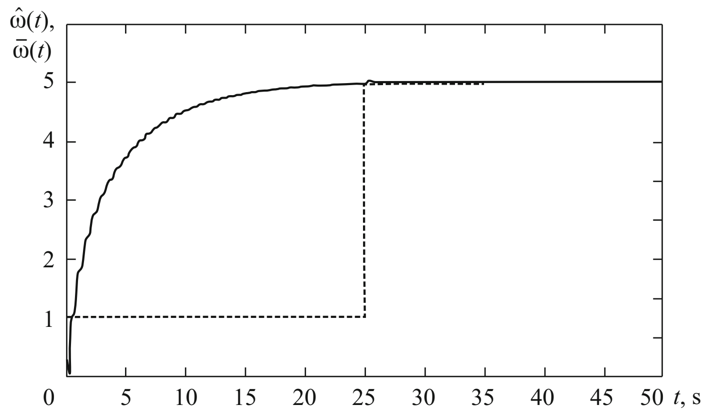

Fig. 2. Transient processes in the closed system for the signals $\hat{\omega}(t)$ (solid line) and $\bar{\omega}(t)$ (dashed line at $\delta(t) = 7\sin(5t + 0.1)$, $t_1 = 25$, $\tau = 10$, $\omega_{\min} = 1$, $\alpha(p) = (p + 4)(p + 5)$, $k_1 = k_2 = 1$, $k = 12$, $\sigma = 30$, $k_a = 15$, $\gamma_0 = 20$.

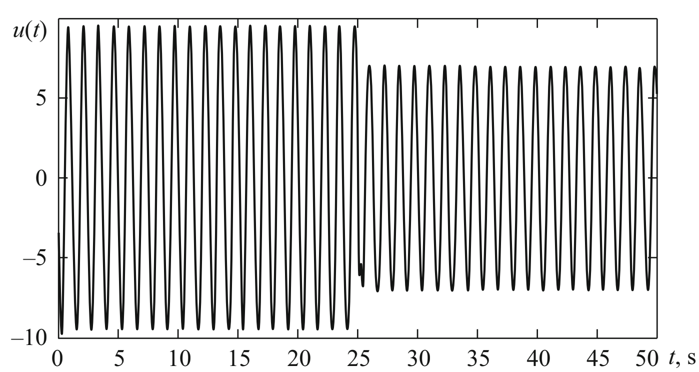

Fig. 3. Transient processes in the closed system for the signal $u(t)$ at $\delta(t) = 7\sin(5t + 0.1)$, $t_1 = 25$, $\tau = 10$, $\omega_{\min} = 1$, $\alpha(p) = (p + 4)(p + 5)$, $k_1 = k_2 = 1$, $k = 12$, $\sigma = 30$, $k_a = 15$, $\gamma_0 = 20$.

Figures 1–6 present the graphs of transient processes in the modeling of a closed system for two various sinusoidal disturbances and in the variation of parameters of the iterative procedure.

In the course of the modeling the time t_1 of the first renewal was chosen in the independent way, and the further intervals between the iterative substitutions of estimates of the disturbance frequency into the controller were set up for an equal length of τ.

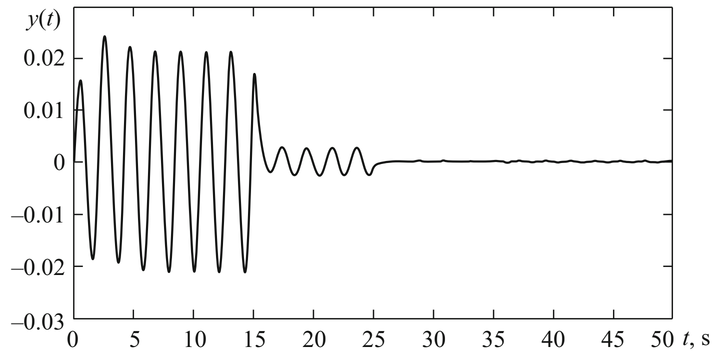

Fig. 4. Transient processes in the closed system for the signal $y(t)$ at $\delta(t) = 10\sin(3t + 0.1)$, $t_1 = 15$, $\tau = 10$, $\omega_{\min} = 1$, $\alpha(p) = (p + 4)(p + 5)$, $k_1 = k_2 = 1$, $k = 14$, $\sigma = 30$, $k_a = 20$, $\gamma_0 = 10$.

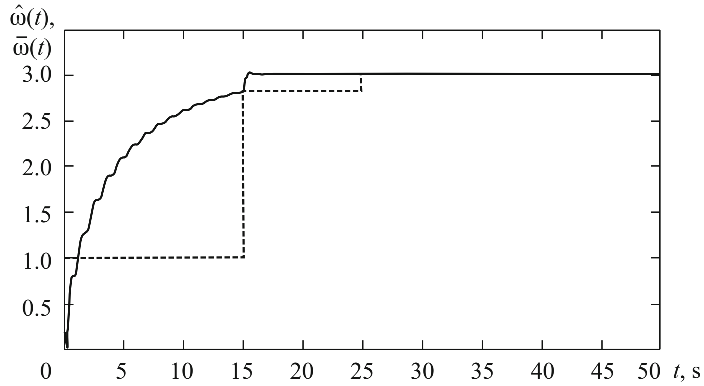

Fig. 5. Transient processes in the closed system for the signals $\hat{\omega}(t)$ (solid line) and $\bar{\omega}(t)$ (dashed line) at $\delta(t) = 10\sin(3t + 0.1)$, $t_1 = 15$, $\tau = 10$, $\omega_{\min} = 1$, $\alpha(p) = (p + 4)(p + 5)$, $k_1 = k_2 = 1$, $k = 14$, $\sigma = 30$, $k_a = 20$, $\gamma_0 = 10$.

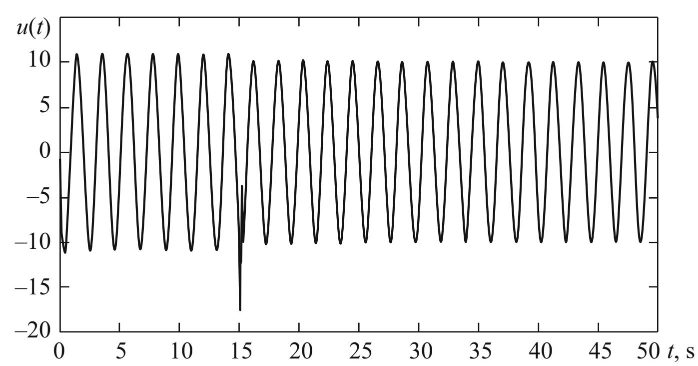

Fig. 6. Transient processes in the closed system for the signal $u(t)$ at $\delta(t) = 10\sin(3t + 0.1)$, $t_1 = 15$, $\tau = 10$, $\omega_{\min} = 1$, $\alpha(p) = (p + 4)(p + 5)$, $k_1 = k_2 = 1$, $k = 14$, $\sigma = 30$, $k_a = 20$, $\gamma_0 = 10$.

The modeling results show that the suggested adaptive algorithm of control with the iterative renewal of frequency estimates of the unknown sinusoidal disturbance does not give way in the accuracy and stabilization speed to the algorithms in [6, 7] and involves expenditures for control comparable to those in [6, 7]; at the same time it can be used for objects with an arbitrary relative degree. It is also evident that the transient processes in the system essentially depend not only on the parameters of the frequency indicator and the controller but also on the choice of intervals of the iterative renewal.

5. CONCLUSIONS

This work suggests a new iterative algorithm of adaptive output control that counteracts the sinusoidal disturbing influence acting on a linear parametrically indefinite object. The control algorithm envisages two channels: the stabilization channel represented by the expressions (5), (6) and the identification channel (Eqs. (22)–(24)). The suggestion is made of the iterative procedure involving the fact that at every instant of time t_i the value $\hat{\omega}_i$ obtained by the identification algorithm was substituted into the control algorithm (5). In contrast to the analogs [1–11], the suggested approach makes it possible to counteract the sinusoidal disturbing influence with the unknown amplitude, phase, and frequency, which can be performed under the conditions of the complete parametric indefiniteness of a controllable object and at any relative degree.

The authors reason that relying on this article and also on the works [13–16], the obtained result can be extended to nonlinear systems with parametric and functional uncertainties.

ACKNOWLEDGMENTS

This work was supported by the Federal Goal-oriented Program "Investigations and Developments in Priority Directions of the Progression of the Russian Scientific-Technological Complex" for 2007–2013 (state contract no. 11.519.11.4007).

REFERENCES

1. Bodson, M. and Douglas, S.C., Adaptive Algorithms for the Rejection of Periodic Disturbances with Unknown Frequencies, *Automatica*, 1997, vol. 33, pp. 2213–2221.
2. Marino, R., Santosuosso, G.L., and Tomei, P., Robust Adaptive Compensation for Biased Sinusoidal Disturbances with Unknown Frequency, *Automatica*, 2003, vol. 9, pp. 1755–1761.
3. Marino, R. and Tomei, R., Output Regulation for Linear Minimum Phase Systems with Unknown Order Exosystem, *IEEE Trans. Automat. Control*, 2007, vol. 52, pp. 2000–2005.
4. Bobtsov, A.A. and Kremlev, A.S., Synthesis of an Observer in the Problem of Compensation for Finite-Dimensional Quasiharmonic Disturbance, *Izv. Ross. Akad. Nauk, Teor. Sist. Upravlen.*, 2005, no. 3, pp. 5–11.
5. Bobtsov, A.A. and Kremlev, A.S., The Algorithm of Compensation for Unknown Sinusoidal Disturbance for a Linear Nonminimum-Phase Object, *Mekhatronika, Avtomatiz., Upravl.*, 2008, no. 10, pp. 14–17.
6. Bobtsov, A.A., Output Control Algorithm with the Compensation for Biased Harmonic Disturbances, *Autom. Remote Control*, 2008, vol. 69, no. 8, pp. 1289–1296.
7. Bobtsov, A.A., Adaptive Output Control with Compensation of Biased Harmonic Disturbance, *J. Comput. Syst. Sci. Int.*, 2009, vol. 48, no. 1, pp. 41–44.
8. Bobtsov, A.A. and Pyrkin, A.A., Compensation of Unknown Sinusoidal Disturbances in Linear Plants of Arbitrary Relative Degree, *Autom. Remote Control*, 2009, vol. 70, no. 3, pp. 449–456.
9. Bobtsov, A.A., Kolyubin, S.A., and Pyrkin, A.A., Compensation of Unknown Multi-harmonic Disturbances in Nonlinear Plants with Delay Control, *Autom. Remote Control*, 2010, vol. 71, no. 11, pp. 2383–2394.
10. Pyrkin, A., Smyshlyaev, A., Bekiaris-Liberis, N., and Krstic, M., Rejection of Sinusoidal Disturbance of Unknown Frequency for Linear Systems with Input Delay, *Am. Control Conf.*, Baltimore, 2010.

11. Bobtsov, A.A., Kremlev, A.S., and Pyrkin, A.A., Compensation of Harmonic Disturbances in Nonlinear Plants with Parametric and Functional Uncertainty, *Autom. Remote Control*, 2011, vol. 72, no. 1, pp. 111–118.

12. Miroshnik, I.V., Nikiforov, V.O., and Fradkov, A.L., *Nelineinoe i adaptivnoe upravlenie slozhnymi dinamicheskimi sistemami* (Nonlinear and Adaptive Control of Complex Dynamic Systems), St. Petersburg: Nauka, 2000.

13. Bobtsov, A.A. and Nikolaev, N.A., Fradkov Theorem-Based Design of the Control of Nonlinear Systems with Functional and Parametric Uncertainties, *Autom. Remote Control*, 2005, vol. 66, no. 1, pp. 108–118.

14. Bobtsov, A.A., The Algorithm of Compensation for Uncontrollable Disturbance in the Problem of Output Variable Stabilization of a Linear Object with Unknown Parameters, *Izv. Vyssh. Uchebn. Zaved., Priborostr.*, 2003, no. 1, pp. 22–27.

15. Bobtsov, A.A. and Nikolaev, N.A., Control Law Design for Stabilization of a Nonlinear System in Output Measurements with Compensation for Unknown Disturbance, *Izv. Ross. Akad. Nauk, Teor. Sist. Upravlen.*, 2005, no. 5, pp. 5–11.

16. Bobtsov, A.A., A Robust Control Algorithm for Tracking the Command Signal with Compensation for the Parasitic Effect of External Unbounded Disturbances, *Autom. Remote Control*, 2005, vol. 66, no. 8, pp. 1287–1295.

17. Babakov, N.A., Voronov, A.A., Voronova, A.A., et al., *Teoriya avtomaticheskogo upravleniya: uchebnik dlya vuzov po spetsial'nosti "Avtomatika i telemekhanika." I. Teoriya lineinykh sistem avtomaticheskogo upravleniya* (Theory of Automatic Control: Manual for Institution of Higher Educat. in specialty "Automation and Remote Control." Part 1. Theory of Linear Automatic Control Systems), Voronov, A.A., Ed., Moscow: Vysshaya Shkola, 1986.

18. Xia, X., Global Frequency Estimation Using Adaptive Identifiers, *IEEE Trans. Automat. Control*, 2002, vol. 47, pp. 1188–1193.

19. Hsu, L., Ortega, R., and Damm, G., A Globally Convergent Frequency Estimator, *IEEE Trans. Automat. Control*, 1999, vol. 46, pp. 967–972.

20. Marino, R. and Tomei, R., Global Estimation of Unknown Frequencies, *IEEE Trans. Automat. Control*, 2002, vol. 47, pp. 1324–1328.

21. Mojiri, M. and Bakhshai, A.R., An Adaptive Notch Filter for Frequency Estimation of a Periodic Signal, *IEEE Trans. Automat. Control*, 2004, vol. 49, pp. 314–318.

22. Aranovskii, S.V., Bobtsov, A.A., Kremlev, A.S., and Luk'yanova, G.V., A Robust Algorithm for Identification of the Frequency of a Sinusoidal Signal, *J. Comput. Syst. Sci. Int.*, 2007, vol. 46, no. 3, pp. 371–376.

23. Aranovskiy, S., Bobtsov, A., Kremlev, A., Nikolaev, N., et al., Identification of Frequency of Biased Harmonic Signal, *Eur. J. Control*, 2010, no. 2, pp. 129–139.

24. Bobtsov, A., New Approach to the Problem of Globally Convergent Frequency Estimator, *Int. J. Adapt. Control Signal Proc.*, 2008, no. 3, pp. 306–317.

25. Aranovskii, S.V., Bobtsov, A.A., and Pyrkin, A.A., Adaptive Observer of an Unknown Sinusoidal Output Disturbance for Linear Objects, *Autom. Remote Control*, 2009, vol. 70, no. 11, pp. 1862–1870.

26. Pyrkin, A.A., Adaptive Algorithm to Compensate Parametrically Uncertain Biased Disturbance of a Linear Plant with Delay in the Control Channel, *Autom. Remote Control*, 2010, vol. 71, no. 8, pp. 1562–1577.

This paper was recommended for publication by L.B. Rapoport, a member of the Editorial Board