NTNU
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M.Sc.THESIS

Title:	Delivered:
Parametric Roll Resonance of a Fishing Vessel as function of Forward Speed and Sea State	June 14, 2011
	Availability:
Student:	**Number of pages:**
Per Martin Martinussen	83 + appendix(78)

Abstract:

The subject of this thesis is sea keeping and stability of a fishing vessel in regular waves, with focus on parametric roll resonance. Parametric roll resonance is resonance in roll due to time variation of a parameter, in this case the metacentric height or the restoring term. This variation is caused by the ship moving in waves, and is hence a function of heave, roll, pitch roll and time. We have made a mathematical model based on strip theory that can calculate the linear ship motions. The restoring term in roll has been modified to be non-linear and to vary with time, and we have added a viscous damping term in roll due to bilge keels. We look at any wave heading, and the equations of motions are coupled in terms of heave-pitch and sway-roll-yaw. The two dimensional added mass and damping coefficients are calculated beforehand by a separate program and used as input to this model. The linear ship motions are presented in the frequency domain in terms of transfer functions and the simulations of parametric roll resonance are presented by polar diagrams as function of forward speed and wave heading in terms of safe and unsafe domains with respect to resonance.

We may get resonance both in head and bow sea as well as following and quartering sea. For head and bow sea, the dangerous forward speeds and headings are the ones giving a ratio of the period of encounter and the natural period in roll in the vicinity of $T_e/T_n \approx 0.5$. For the following and quartering sea case this ratio is in the vicinity of $T_e/T_n \approx 1$ for the sea states analyzed here. The speed range giving resonance decreases for head or bow sea when the wave period is increasing and increases for following and quartering sea. The roll amplitude is small close to the lower speed limits of the speed range for head sea, but increases rapidly when T_e/T_n approaches 0.51-0.52, and a general trend seems to be that the roll amplitude increases with increasing forward speed.

During parametric resonance, the vessel will oscillate in roll with its natural frequency, even though the wave excitation moment in roll oscillates with the frequency of encounter.

The vessel analyzed here, a 90 ft purse seiner, has natural periods in coupled heave-pitch that are close to or equal to half the natural period coupled sway-roll-yaw. The vessel will then have its maximum vertical motions and hence maximum change of metacentric height when there is danger of parametric resonance. This makes this kind of vessel particularly vulnerable to parametric resonance in roll. It is not sufficient that the vessel complies the intact stability rules in order to avoid parametric resonance.

If parametric resonance has occurred, an effective way to escape it is to slow down and increase the heading relative to the waves.

Keyword:

Parametric Roll Resonance
Stability
Sea Keeping

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MASTER THESIS IN MARINE TECHNOLOGY

SPRING 2011

FOR

Per Martin Martinussen

Parametric Roll Resonance of a Fishing Vessel as Function of Forward Speed and Sea State

(Parametrisk rull resonans av fiskefartøy som funksjon av hastighet og sjøtilstand)

Parametric roll resonance is a phenomenon caused by change of the metacentric height at a certain frequency. Ships that are vulnerable are ships with pronounced change of geometry around the mean water line, such as container vessels, cruise vessels and fishing vessels. In order to catch the variation of the stability, it is important to calculate the vertical ship motions properly. Parametric roll resonance may hence be looked upon as a combined sea keeping and stability problem. In the literature the main focus has traditionally been head or following seas, but we may also get instability or resonance at other wave headings. Parametric rolling may lead to damage on the ship, cargo and crew, and may ultimately lead to capsizing. Factors that affect occurrence of the phenomenon are forward speed, heading relative to the incident waves, damping and initial stability.

Objective
The objective of the project thesis is to calculate the vertical ship motions and investigate the occurrence of parametric roll resonance at different wave headings and forward speeds for a typical coastal fishing vessel.

On this background, it is recommended that the candidate shall do the following in the master thesis:

1. Give an overview of previous work, methods and assumptions used in the analysis of parametric roll resonance and ship motions in regular waves.
2. Develop a mathematical model based on the known strip theory to calculate the linear wave induced motions for an arbitrary wave heading, with or without forward speed.
3. Include viscous roll damping due to bilge keels into the model.
4. Include a non-linear restoring moment in roll into the model.
5. Expand the model so that it can take into account how heave and pitch motions and the wave elevation for an arbitrary wave heading affects the time variation of the non-linear restoring moment in roll.
6. Simulate parametric rolling for different forward speeds and wave headings. This should also be done for different wave periods and wave heights it time permits it.
7. Give conclusions and recommendations for further work.

The candidate should in his report give a personal contribution to the solution of the problem formulated in this text. All assumptions and conclusions must be supported by mathematical models and/or references to physical effects in a logical manner.

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The candidate should apply all available sources to find relevant literature and information on the actual problem.

In the thesis the candidate shall present his personal contribution to the resolution of problem within the scope of the thesis work.

Theories and conclusions should be based on mathematical derivations and/or logic reasoning identifying the various steps in the deduction.

The candidate should utilize the existing possibilities for obtaining relevant literature.

The thesis should be organized in a rational manner to give a clear exposition of results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Telegraphic language should be avoided.

The thesis shall contain the following elements: A text defining the scope, preface, list of contents, summary, main body of thesis, conclusions with recommendations for further work, list of symbols and acronyms, reference and (optional) appendices. All figures, tables and equations shall be numerated.

The supervisor may require that the candidate, in an early stage of the work, present a written plan for the completion of the work. The plan should include a budget for the use of computer and laboratory resources that will be charged to the department. Overruns shall be reported to the supervisor.

The original contribution of the candidate and material taken from other sources shall be clearly defined. Work from other sources shall be properly referenced using an acknowledged referencing system.

The thesis shall be submitted in two copies:

- Signed by the candidate
- The text defining the scope included
- In bound volume(s)
- Drawings and/or computer prints that cannot be bound should be organized in a separate folder.
- The bound volume shall be accompanied by a CD or DVD containing the written thesis in Word or PDF format. In case computer programs have been made as part of the thesis work, the source code shall be included. In case of experimental work, the experimental results shall be included in a suitable electronic format.

Supervisor : Professor Odd Magnus Faltinsen
Start : 17.01.2011
Deadline : 14.06.2011

Odd Magnus Faltinsen
Supervisor

Abstract

The subject of this thesis is sea keeping and stability of a fishing vessel in regular waves, with focus on parametric roll resonance. Parametric roll resonance is resonance in roll due to time variation of a parameter, in this case the metacentric height or the restoring term. This variation is caused by the ship moving in waves, and is hence a function of heave, roll, pitch roll and time. We have made a mathematical model based on strip theory that can calculate the linear ship motions. The restoring term in roll has been modified to be non-linear and to vary with time, and we have added a viscous damping term in roll due to bilge keels. We look at any wave heading, and the equations of motions are coupled in terms of heave-pitch and sway-roll-yaw. The two dimensional added mass and damping coefficients are calculated beforehand by a separate program and used as input to this model. The linear ship motions are presented in the frequency domain in terms of transfer functions and the simulations of parametric roll resonance are presented by polar diagrams as function of forward speed and wave heading in terms of safe and unsafe domains with respect to resonance.

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Preface

This report is the result of the work on my master thesis in Marine Hydrodynamics at the Department of Marine Technology, Norwegian University of Science and Technology, Trondheim. It is carried out during the spring of 2011. The subject in the thesis is stability and sea keeping of a fishing vessel with focus on parametric roll resonance, and the idea of this subject was initiated by SINTEF Fisheries and Aquaculture.

The Matlab code used is a further development from the code developed in the specialization project delivered December 20, 2010. The old code was only able to simulate head or following sea, and the expanded code has been generalized to simulate any heading. This further development has been demanding and time consuming.

The work has been carried out under supervision of Professor Odd M. Faltinsen at the Department of Marine Technology, Norwegian University of Science and Technology. His continuous guidance and support is most appreciated, specially when I had a hard time and nothing seemed to work out for me he continued to support and encourage me. Thank you! I would also like to thank Birger Enerhaug at SINTEF Fisheries and Aquaculture for providing me with the information I needed about the example vessel used in this project. Finally I would like to thank Dr. Rentao Skejic at MARINTEK for giving me and helping me with a program for calculation of added mass and damping coefficients for two dimensional ship sections. This contribution was essential in order to obtain a realistic picture of the vertical ship motions.

Trondheim, June 14, 2011

Per Martin Martinussen

Contents

Nomenclature

$\boldsymbol{A}$	Transformation matrix
$\boldsymbol{r}$	Position vector
$\boldsymbol{s}$	Motion of any point at the ship
$\boldsymbol{t}$	Surface tangent vector
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vector in x-, y- and z-direction respectively
$\mathbf{n}$	Normal vector of surface. Positive direction into the fluid
$\mathbf{U}$	Body velocity vector
$\mathbf{V}$	Velocity vector
$\overline{BM}$	Metacentric radius
$\overline{GM}$	Transverse metacentric height
$\overline{GM}_L$	Longitudinal metacentric height
$\overline{GZ}$	Righting arm about center of gravity
$\overline{KB}$	Distance from keel to the vertical center of buoyancy
$\overline{KG}$	Distance from keel to the vertical center of gravity
A_S	Sectional area
A_W	Water plane area
a_x, a_y, a_z	Fluid particle acceleration in x-, y- and z-direction respectively
A_{jk}	$3D$ added mass in mode j due to acceleration in mode k
a_{jk}	$2D$ added mass in mode j due to acceleration in mode k
B	Ship beam, center of buoyancy
B_S	Sectional beam
B_v	Viscous roll damping coefficient
B_{jk}	$3D$ damping in mode j due to velocity in mode k
b_{jk}	$2D$ damping in mode j due to velocity in mode k
$B_{v,eqv}$	Equivalent damping
C	Constant
C_S	Sectional area coefficient
C_x	Cross section at position x
C_{jk}	Linear restoring coefficient in mode j due to displacement in mode k
F_j	Total excitation force amplitude, for $j = 1..6$
F_j^D	Diffraction force amplitude, for $j = 1..6$
f_j^D	Sectional diffraction force amplitude, for $j = 1..6$
F_j^{FK}	Froude-Kriloff force amplitude, for $j = 1..6$
f_j^{FK}	Sectional Froude-Kriloff force amplitude, for $j = 1..6$
Fn	Froude number
G	Center of gravity
g	Acceleration of gravity
I	Second moment of area

i	Imaginary unit
I_{jk}	Moment of inertia
K	Keel
k	Wave number
L	Ship length, in this case L_{PP}
L_{OA}	Length over all
L_{PP}	Length between perpendiculars
M	Mass, metacenter
M_R	Non-linear restoring moment in roll
M_V	Volume moment
$M_{\phi F}$	False metacenter
P	Engine power
p	Pressure in general
p_a	Atmospheric pressure
p_{dyn}	Linear dynamic pressure
p_{stat}	Hydrostatic pressure
r_{jj}	Radius of gyration
S	Surface
s	Frame spacing
S_B	Mean wetted surface
T	Mean draught
t	Time
T_e	Period of encounter
T_n	Natural period
T_S	Sectional draught
T_0	Wave period of incident waves
T_{n246}	Natural period in coupled sway-roll-yaw
T_{n35}	Natural periods in coupled heave pitch
U	Ship forward speed
u, v, w	Fluid particle velocity in x-, y- and z-direction respectively
x', y', z'	Coordinates in a body-fixed coordinate system
x, y, z	Coordinates in the Cartesian coordinate system
x_T	x-coordinate of transom stern
Z	Vertical distance from the mean free surface
z_B	Vertical center of buoyancy
z_G	Vertical center of gravity

Greek symbols

$\bar{\eta}_j$	Complex motion amplitude, for $j = 1..6$
β	Angle between ship heading and wave propagation
$\boldsymbol{\omega}$	Vorticity vector, rotational rigid body motions
ϵ	Phase angle
η_j	Modes of rigid body motions. Here $j = 1..6$ corresponds to surge, sway, roll, heave, pitch and yaw respectively
η_{Ij}	Imaginary part of complex motion amplitude, for $j = 1..6$
η_{Rj}	Real part of complex motion amplitude, for $j = 1..6$
ω_0	Wave frequency
ω_e	Frequency of encounter
ω_n	Natural frequency
Φ	Velocity potential

ϕ	Static heel angle
ϕ_f	Flooding angle
ϕ_v	Angle of vanishing stability
ρ	Mass density of water
θ	Angle in general
φ	Velocity potential in general
φ_0	Incoming velocity potential
φ_7	Diffraction potential
φ_j	Velocity potential describing mode $j = 1..6$
ξ	Damping ratio
ζ	Wave elevation
ζ_a	Wave amplitude

Mathematical operators and special symbols

$\dot{x}$	Time derivative of a variable, x
∇	Differential operator, Volume displacement
$\Re$	Real part of a complex number

Abbreviations

$2D$	Two dimensional
$3D$	Three dimensional
CFD	Computational Fluid Dynamics
COG	Center of gravity
GT	Gross tonnage
IMO	International Maritime Organization
LCG	Longitudinal center of gravity
NMD	Norwegian Maritime Directorate

Chapter 1

Introduction

1.1 Parametric roll resonance

When a ship is moving in waves, the shape of its submerged volume will change. This will cause the stability of the ship to vary with time. In linear sea keeping calculations this fact is neglected, since taking it into account will lead to non-linearities. However, by neglecting this fact we may miss a very important phenomenon; Parametric Roll Resonance. This may happen when the stability changes at a frequency around twice the natural frequency in roll, and it may occur both in head or following waves as well as oblique waves. It is called parametric resonance because it is resonance caused by time variation of one or more parameters, in this case the stability or restoring term. What characterizes this resonance motion is that when initiated it rapidly builds up to a large roll amplitude, typical 30-40° (Shin et al., 2004), and then it continues in a more or less steady-state motion. This large roll motion may cause damage to the ship, its equipment cargo and crew and may ultimately lead to capsizing. A typical example of how the roll amplitude develops under parametric resonance is shown in figure 1.1. Ships

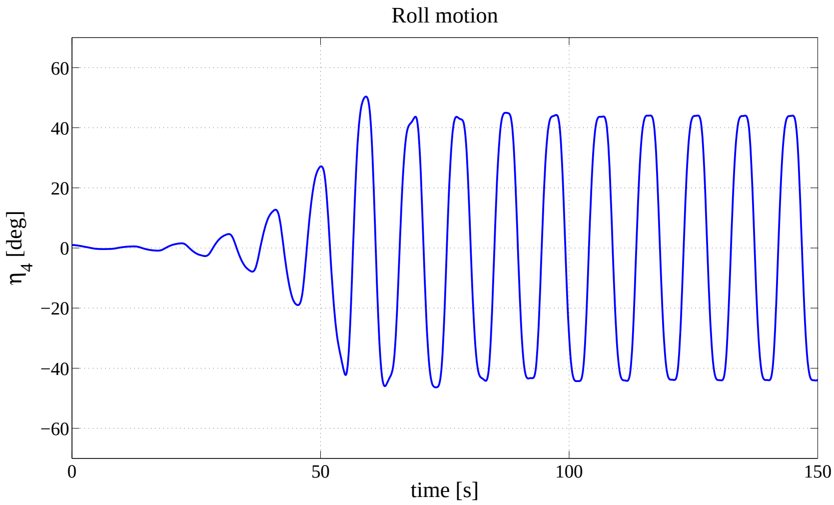


Figure 1.1: Development of roll amplitude during parametric roll resonance. The amplitude grows rapidly and reaches a steady state condition around $50°$.

that have a very pronounced change of geometry around the mean water line in the bow and stern regions are particularly vulnerable to parametric resonance (Shin et al., 2004). These ships may be container ships, cruise ships and fishing vessels.

We will in this report calculate the ship motions using linear theory. Further we will combine these motions with a time varying and non-linear restoring moment in roll, and use this to simulate parametric roll resonance for different forward speeds and wave headings in regular waves. All this will be done with a typical coastal fishing vessel as an example vessel. Parametric roll resonance will be further discussed in chapter 5.

1.2 Sea keeping

In order to find the variation of the stability it is essential to calculate the ship motions. We have several options when choosing which method to use in this context. These are simplified methods such as strip theory using potential theory, and more advanced methods such as computational fluid dynamics (CFD). In strip theory we divide the ship into vertical two dimensional ($2D$) strips or sections and calculate the forces and moments and hydrodynamic coefficients on each strip and sum the results. We may also apply a panel method where we divide the ship hull into small panels instead of vertical sections. When we solve the Navier-Stokes equations, as done in many CFD codes, it is possible to catch some of the viscous effects and also some non-linear effects such as green water on deck and bottom slamming. Nowadays CFD is more and more popular (Faltinsen & Timokha, 2009), but strip theory is still common in commercial software like VERES due to its speed and fairly good accuracy. CPU-time and computer costs is a disadvantage of CFD. We will in this text apply strip theory, since this is fairly accurate and possible to code by oneself.

1.2.1 Fundamental assumptions in potential theory

When assuming that the fluid is incompressible, inviscid and irrotational a velocity potential, Φ, can be found (White, 2005). This potential contains all the information about the fluid, such as pressure and velocity distribution, but the potential itself is pure mathematical and a scalar and has no physical meaning in itself. This velocity potential has to satisfy certain boundary conditions. These are the Laplace equation,

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0, \tag{1.1}$$

which basically is conservation of mass, and this follows because the fluid is incompressible. Another condition is that the fluid is irrotational meaning

$$\boldsymbol{\omega}=\nabla\times\mathbf{V}=0. \tag{1.2}$$

Here $\mathbf{V}$ is the velocity vector, given as

$$\mathbf{V}=\nabla\boldsymbol{\Phi}\equiv\mathbf{i}\frac{\partial\Phi}{\partial x}+\mathbf{j}\frac{\partial\Phi}{\partial y}+\mathbf{k}\frac{\partial\Phi}{\partial z}. \tag{1.3}$$

In equation (1.3) $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the unit vectors in x-, y- and z-directions respectively. In a mathematical sense equation (1.2) states that the curl is equal to zero everywhere in the fluid. This assumption is questionable. The velocity potential also have to satisfy the body boundary condition, meaning no fluid flow through a body in the fluid or through the seabed. For a moving body we express this condition by

$$\frac{\partial\varphi}{\partial n}=\mathbf{U}\cdot\mathbf{n} \tag{1.4}$$

on the body surface. Here $\mathbf{n}$ is the component in the n-direction of the body surface unit normal vector and $\mathbf{U}$ is the body velocity. In equation (1.4) we have rewritten the velocity potential to (Faltinsen, 2005)

$$\Phi = Ux + \varphi. \tag{1.5}$$

This is convenient if we have a ship with a forward speed U, and we observe the flow from a ship-fixed coordinate system. The term Ux is a uniform flow. The last conditions the velocity potential has to satisfy are the kinematic and dynamic free surface conditions. The kinematic free surface condition is in linear theory expressed as

$$\frac{\partial \zeta}{\partial t} = \frac{\partial \varphi}{\partial z}. \tag{1.6}$$

Here ζ is the surface elevation and the expression is to be evaluated in the mean free surface, $z = 0$, since this is linear theory. The physical explanation for the kinematic free surface condition is continuity in the layer between the water and the air, meaning that a fluid particle at the surface stays at the surface. The dynamic free surface condition in linear theory is given as

$$g\zeta + \frac{\partial \varphi}{\partial t} = 0, \tag{1.7}$$

where g is the acceleration of gravity. Also here we evaluate the expression at $z = 0$. The physics behind the dynamic free surface condition is that the pressure at the free surface is equal to pressure in the air at the free surface. We find the pressure in the fluid from Bernoulli's equation

$$p + \rho\frac{\partial \Phi}{\partial t} + \frac{\rho}{2}|\nabla\Phi|^2 + \rho g z = C, \tag{1.8}$$

where ρ is the mass density of the fluid. The constant C is found from the dynamic free surface condition in equation (1.7) and is associated with the atmospheric pressure, p_a. By substituting equation (1.5) into equation (1.8) and solving for the pressure, we obtain

$$p = -\rho\frac{\partial \varphi}{\partial t} - \frac{\rho}{2}\left[\left(U + \frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2\right] - \rho g z + p_a + \frac{\rho}{2}U^2. \tag{1.9}$$

Since we are applying linear theory we will disregard the second order terms in φ in Bernoulli's equation. The linear dynamic pressure is hence given as

$$p_{dyn} = -\rho\frac{\partial \varphi}{\partial t} - \rho U\frac{\partial \varphi}{\partial x}. \tag{1.10}$$

Here we are not interested in the hydrostatic pressure and the atmospheric pressure.

1.2.2 Regular waves

A very common way to describe regular waves is by assuming a sinusoidal shape. For this assumption to be valid the waves need to be small, meaning that the wave amplitude-to-length ratio, ζ_a/λ, is small. The velocity potential for incident sinusoidal waves at infinite water depth is given as (Fathi & Hoff, 2010)

$$\varphi_0 = \frac{g\zeta_a}{\omega_0}e^{kz}e^{-ik(x\cos\beta + y\sin\beta)}e^{i\omega_e t}. \tag{1.11}$$

Here i is the imaginary unit, β is the heading of the ship relative to the incident waves and k is the wave number, defined as

$$k = \frac{2\pi}{\lambda}, \tag{1.12}$$

where λ is the wave length. Further ω_e is the frequency of encounter, given as

$$\omega_e = \omega_0 + \frac{\omega_0^2}{g} U \cos\beta, \tag{1.13}$$

where ω_0 is the frequency of the incident waves. The frequency of encounter is the frequency of the response of the vessel. The relationship between the period of encounter and frequency of encounter is given as

$$T_e = \frac{2\pi}{\omega_e}. \tag{1.14}$$

We see from equation (1.3) that we find the fluid velocities by taking the derivative of the velocity potential in the actual directions. The vertical fluid particle velocity is hence given by

$$w = \frac{\partial \varphi_0}{\partial z} = \omega_0 \zeta_a e^{kz} e^{-ik(x\cos\beta + y\sin\beta)} e^{i\omega_e t}. \tag{1.15}$$

To find the accelerations we take the derivative of the velocities with respect to time. The vertical fluid particle acceleration then becomes

$$a_z = \frac{\partial^2 \varphi_0}{\partial z \partial t} = i\omega_e \omega_0 \zeta_a e^{kz} e^{-ik(x\cos\beta + y\sin\beta)} e^{i\omega_e t}. \tag{1.16}$$

We find the dynamic pressure from equation (1.10) to be

$$p_{dyn} = -i\rho g \zeta_a e^{kz} e^{-ik(x\cos\beta + y\sin\beta)} e^{i\omega_e t}. \tag{1.17}$$

Equation (1.17) is only valid up to $z = 0$ so we need to make an assumption about the pressure below the wave crest. We assume that the pressure here is hydrostatic, see figure 1.2. This way we fulfill the

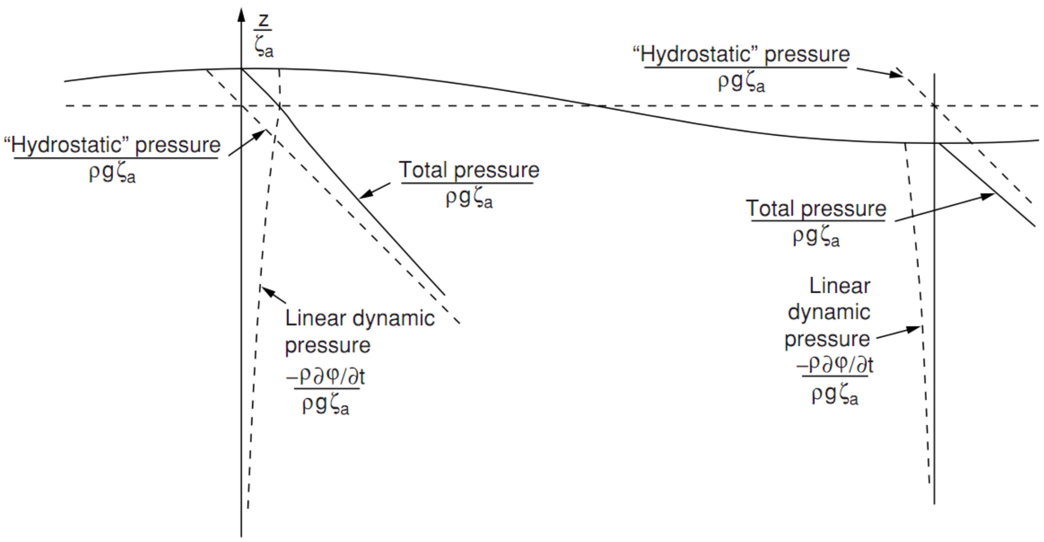


Figure 1.2: Total pressure under a wave crest and wave trough. We see that the dynamic free surface condition is exactly satisfied at the wave crest and that we have a higher order error in the wave trough. The pressure above $z = 0$ is hydrostatic. This figure is from Faltinsen (2005).

dynamic free surface condition. Below $z = 0$ the total pressure is equal to the sum of the hydrostatic and the dynamic pressure. Because of the term e^{kz} we get a small error in the pressure at the surface in the wave trough. The total pressure hence becomes

$$p = \begin{cases} \rho g (\zeta - z) & \text{if } z > 0 \\ -\rho g z + p_{dyn} & \text{if } z \leq 0 \end{cases} \tag{1.18}$$

where ζ is the wave elevation, expressed as

$$\zeta = -i\zeta_a e^{-ik(x\cos\beta + y\sin\beta)} e^{i\omega_e t}. \tag{1.19}$$

When operating with complex numbers it is always the real part that has physical meaning.

1.3 Coordinate system

We will use the same coordinate system as used in the sea keeping analysis by Faltinsen (1990) and Faltinsen (2005), see figure 1.3. This system is right-handed and has a positive x-direction towards the

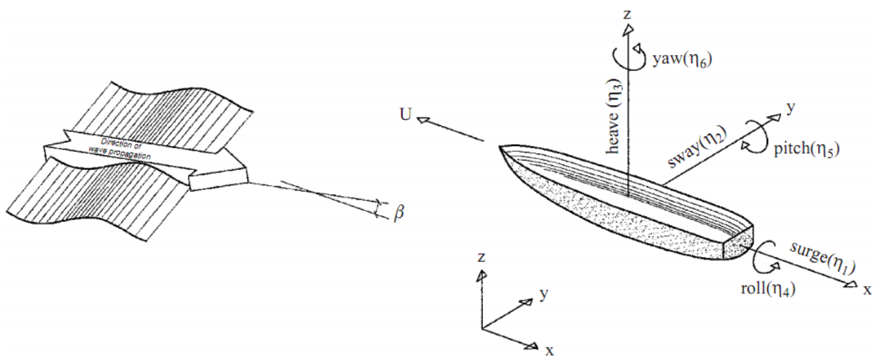


Figure 1.3: Definition of coordinate system. The xy-plane is in the mean water line and the z-axis goes through the longitudinal center of gravity. The coordinate system moves with the ship with the mean forward speed U. The figure is from Faltinsen (2005).

stern of the vessel. Further the positive z-axis is pointing upwards and goes through the longitudinal center of gravity (LCG) and the origin is in the mean water plane. The rigid body motions consist of three translations and three rotations. The translations along the x-, y- and z-axes respectively are called surge, sway and heave and are noted η_1, η_2 and η_3 respectively. The rotations around the same axes are called roll, pitch and yaw and are noted η_4, η_5 and η_6 respectively. We also need to define the different wave headings. This is shown in figure 1.4. The wave directions are called head, bow, beam, quartering and following and the angles are shown in figure 1.4.

1.4 Reference ship - Trønderhav

In this project we will use data of the 90 ft purse seiner *Trønderhav* as an example vessel, as shown in figure 1.5. The vessel was built in 2001 and was lengthened in 2010. We will use the data from the original vessel. The data about the vessel are provided by Birger Enerhaug at SINTEF Fisheries and Aquaculture. Its principal particulars are summarized in table 1.1 and the body plan is shown in figure 1.6. The breadth, volume displacement and water plane area are calculated by the Matlab code, and the length and mean draught are given as input.

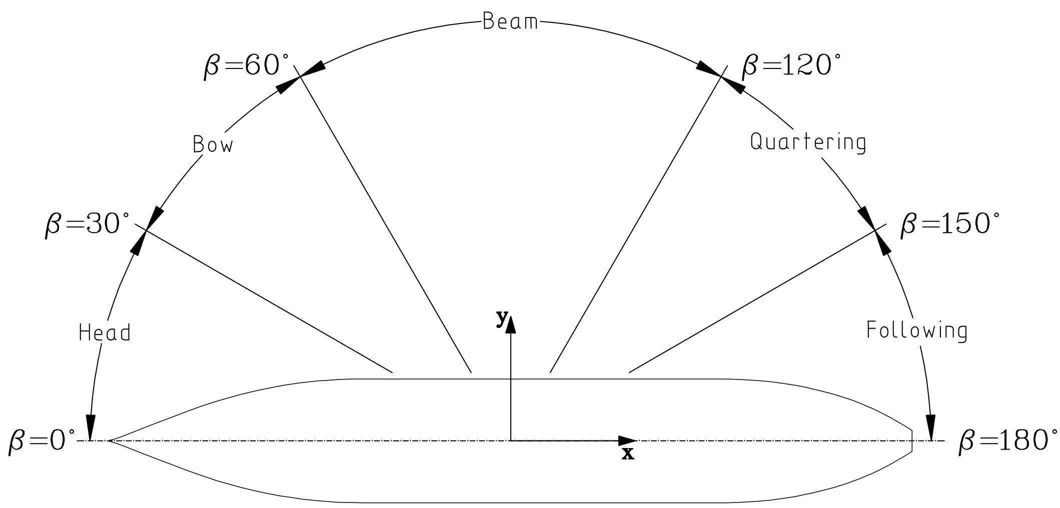


Figure 1.4: Definition of wave heading angles.

Figure 1.5: Trønderhav before it was rebuilt. We see that it has considerable forward trim in loaded condition. We also see the freeing ports in the net bin at port quarter.

Table 1.1: Principal particulars Trøndherhav.

Ship info Trøndherhav		
Length over all	L_{OA}	27.4 m
Length between perpendiculars	L_{PP}	24.0 m
Breadth	B	8.524 m
Mean draught	T	3.998 m
Volume displacement	∇	530.00 m^3
Water plane area	A_W	193.60 m^2
Main engine power	P	1125 kW

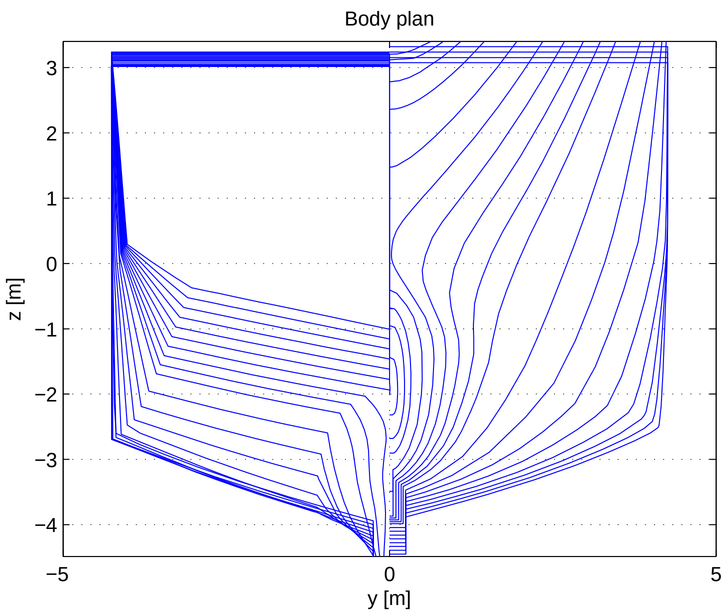


Figure 1.6: Body plan of Trøndherhav

The reason for choosing this vessel as example vessel, is that before it was rebuilt, its sea kindliness was a disaster (Enerhaug, 2010, personal comm.). It is suspected that parametric resonance might have been the reason, so this vessel should be well suited to use when analyzing this phenomenon numerically. Vessels of this kind are known to have bad sea keeping characteristics (Enerhaug, 2010, personal comm.). One reason for this may be the regulations controlling the size of these vessels. For the coastal fishing vessels there traditionally was a limitation in maximum length, but no limitation in gross tonnage (GT) (Aasjord et al., 2003). The limit was set to 90 ft, or 27.45 m, for vessels using active fishing gears like purse seine. For vessels using passive gears, like gillnets, this limitation was 28 m. This may lead to a vessel that is designed to carry as much catch as possible. As a result of this, we often end up with vessels that are extremely plump and have extreme length-to-beam ratios. This is believed to worsen the vessel's sea kindliness, and hence increase the chance of parametric resonance. However, this limitation was in 2008 repealed and replaced by a limitation in cargo hold capacity of 300 m^3 (Fiskeridirektoratet, 2008). Therefore it should now be possible to design coastal fishing vessels that are more optimized for sea keeping. Nevertheless, there still sail many vessels that were built according to the old regulations so the problem is still highly relevant.

1.5 Outline of report

This report is divided into the following chapters:

- **Chapter 1** gives an introduction to parametric roll resonance and the assumptions behind potential theory. We also define the coordinate system here, and give the principal particulars of ship used in the calculations.
- **Chapter 2** presents the hydrodynamic loads in terms of excitation forces and moments, added mass, damping and restoring forces and moments calculated by means of strip theory.
- **Chapter 3** describes the equations of motions and how to solve them in the frequency and time domain.
- **Chapter 4** introduces the basic concepts in ship stability and shows how the non-linear restoring moment in roll is calculated.
- **Chapter 5** combines the linear sea keeping theory with the non-linear restoring moment and presents simulations of parametric roll resonance at different wave headings and forward speeds.
- **Chapter 6** concludes and makes suggestions for further work.

Chapter 2

Hydrodynamic loads

As previously mentioned we will apply strip theory (Salvesen et al., 1970) in this text. Here we reduce the three dimensional ($3D$) problem to a two dimensional problem by dividing the hull into vertical two-dimensional sections along the ship length, see figure 2.1. Each strip has constant cross section. We also need to do some assumptions when applying strip theory. We need the flow to be almost two

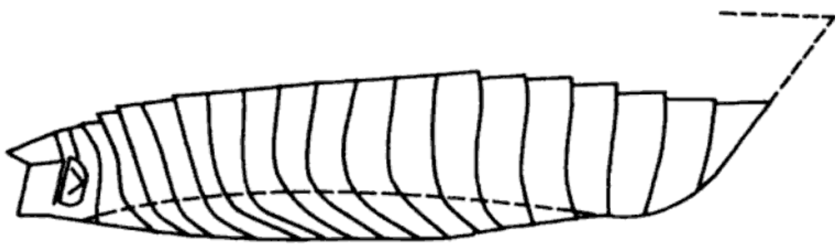


Figure 2.1: Strip model of a ship. Each strip has constant cross section. The figure is from Faltinsen (1990)

dimensional, meaning that the variation of flow is much larger across each strip than along the strip. This is a fairly good assumption in the midship area, but becomes questionable towards the ends of the ship. Also the ship needs to be slender, i.e. having a high length-to-beam ratio. This assumption is also questionable for a 90 ft fishing vessel. Maybe the most critical assumption used in strip theory is that the frequency needs to be high (Salvesen et al., 1970). However, for long waves hydrostatic effects dominate the heave and pitch motions so it is believed that this will have minor effects on the final results (Salvesen et al., 1970). The strip model of Trønderhav is shown in figure 2.2.

A common approach when calculating the hydrodynamic loads is to split the hydrodynamic problem into two sub-problems, sub-problems A and B (Faltinsen, 1990). In sub-problem A we restrain the body (ship) from moving when exposed to incoming waves. This will give us the wave excitation forces and moments, which again are split into Froude-Kriloff forces and moments and diffraction forces and moments. In sub-problem B we have no incoming waves, but we oscillate the ship in all rigid-body degrees of freedom with the frequency of encounter corresponding to the wave frequency in sub-problem A. This will give us the added mass, damping and restoring terms, A_{jk}, B_{jk} and C_{jk}. Since we are working in the linear world we simply superimpose the results from the two sub-problems in order to get the final hydrodynamic loads, in terms of the equations of motion. The uncoupled equation of motion in heave is given as

$$(M + \underbrace{A_{33})\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3}_{\text{Sub-problem B}} = \underbrace{F_3 e^{i\omega_e t}}_{\text{Sub-prb. A}} \tag{2.1}$$

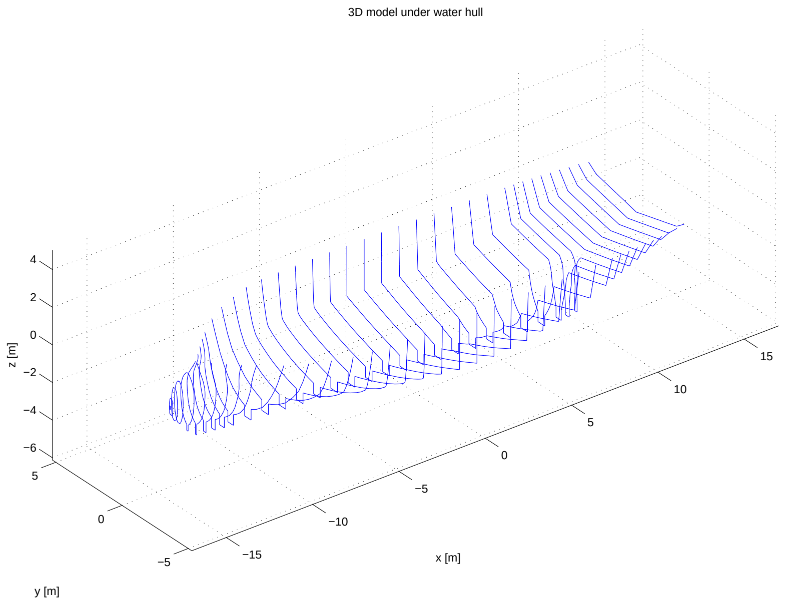


Figure 2.2: Strip model of Trønderhav, a 90 ft purse seiner.

Here dot means time derivative, and M is the ship mass. See also figure 2.3 for an illustration of the two sub-problems.

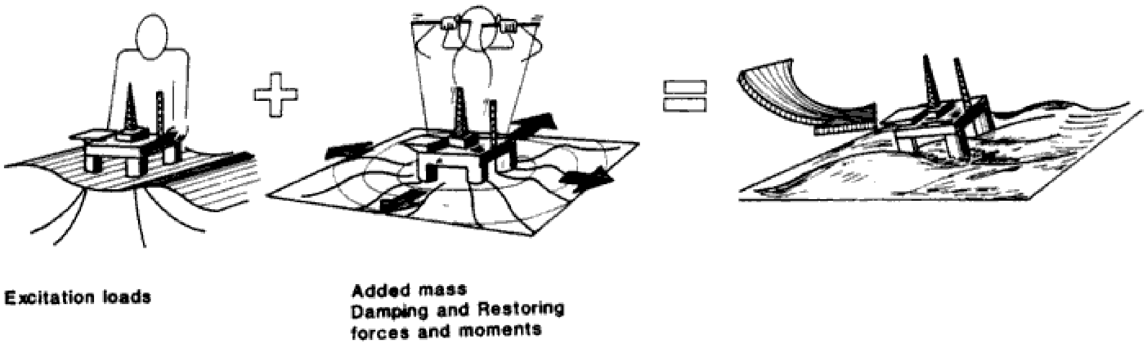


Figure 2.3: Sub-problems A and B. Sub-problem A gives the wave excitation loads and B gives the added mass, damping and restoring terms. The sub-problems are superimposed. The figure is from Faltinsen (1990).

2.1 Added mass, damping and restoring terms

In this section we will focus on sub-problem B, where the vessel is forced to oscillate harmonically in all degrees of freedom with the frequency of encounter. We have no incident waves in this case, only waves propagating away. The way we do this is to solve a boundary value problem for each section, see Faltinsen (1990) for details. The result of this is the added mass, damping and restoring forces and moments, i.e. the coefficients at the left-hand side except the mass in the equation of motion, as shown in equation (2.1). In this project the added mass and damping for each ship section is calculated by the Fortran program *HydroDyn2D*, developed by Dr. Renato Skejic at MARINTEK. The results from this program are used as input to the Matlab code.

2.1.1 Added mass

Added mass and damping forces and moments occur because of forced harmonic oscillations. Because of this oscillations the surrounding fluid will also oscillate, and this implies that we have pressure fields in the fluid which again creates forces. A common misunderstanding of the added mass is that added mass is an amount of water that oscillates with the ship. This is wrong. Added mass is to be understood as a hydrodynamic force (Faltinsen, 1990). If we use pure heave motion as an example, the added mass and damping force is by definition (Faltinsen, 1990)

$$F_3 \equiv -A_{33}\frac{\mathrm{d}^2\eta_3}{\mathrm{d}t^2} - B_{33}\frac{\mathrm{d}\eta_3}{\mathrm{d}t} \tag{2.2}$$

Here A_{33} and B_{33} are the three dimensional added mass and damping coefficients in heave respectively. The three dimensional added mass coefficients will depend on both the frequency of encounter and the forward speed. To find the total added mass we just sum the contribution from each strip. In addition we get a contribution due to the forward speed. In this section we will state the formulas used for the three dimensional added mass in each mode. All formulas below are taken from Fathi & Hoff (2010), and the interested reader is referred to Salvesen et al. (1970) for details.

$$A_{22} = \int_L a_{22}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} b_{22}(x_T), \tag{2.3}$$

$$A_{24} = \int_L a_{24}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} b_{24}(x_T), \tag{2.4}$$

$$A_{26} = \int_L x a_{22}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2}\int_L b_{22}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} x_T b_{22}(x_T) - \frac{U^2}{\omega_e^2} a_{22}(x_T), \tag{2.5}$$

$$A_{33} = \int_L a_{33}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} b_{33}(x_T), \tag{2.6}$$

$$A_{35} = -\int_L x a_{33}(x)\,\mathrm{d}x + \frac{U}{\omega_e^2}\int_L b_{33}(x)\,\mathrm{d}x + \frac{U}{\omega_e^2} x_T b_{33}(x_T) + \frac{U^2}{\omega_e^2} a_{33}(x_T), \tag{2.7}$$

$$A_{42} = \int_L a_{24}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} b_{24}(x_T), \tag{2.8}$$

$$A_{44} = \int_L a_{44}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} b_{44}(x_T), \tag{2.9}$$

$$A_{46} = \int_L x a_{24}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2}\int_L b_{24}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} x_T b_{24}(x_T) - \frac{U^2}{\omega_e^2} a_{24}(x_T), \tag{2.10}$$

$$A_{53} = -\int_L x a_{33}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2}\int_L b_{33}(x)\,\mathrm{d}x + \frac{U}{\omega_e^2} x_T b_{33}(x_T), \tag{2.11}$$

$$A_{55} = \int_L x^2 a_{33}(x)\,\mathrm{d}x + \frac{U^2}{\omega_e^2}\int_L a_{33}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} x_T^2 b_{33}(x_T) - \frac{U^2}{\omega_e^2} x_T a_{33}(x_T), \tag{2.12}$$

$$A_{62} = \int_L x a_{22}(x)\,\mathrm{d}x + \frac{U}{\omega_e^2}\int_L b_{22}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} x_T b_{22}(x_T), \tag{2.13}$$

$$A_{64} = \int_L x a_{24}(x)\,\mathrm{d}x + \frac{U}{\omega_e^2}\int_L b_{24}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} x_T b_{24}(x_T), \tag{2.14}$$

$$A_{66} = \int_L x^2 a_{22}(x)\,\mathrm{d}x + \frac{U^2}{\omega_e^2}\int_L a_{22}(x)\,\mathrm{d}x - \frac{U}{\omega_e^2} x_T^2 b_{22}(x_T) - \frac{U^2}{\omega_e^2} x_T a_{22}(x_T). \tag{2.15}$$

Here a_{jk} and b_{jk} is the two dimensional added mass and damping coefficient respectively for each ship section. These are calculated by *HydroDyn2D* as already mentioned. Further x_T is the x-coordinate of the transom stern, and the integration is performed along the ship length, L. The frame at the transom stern is shown in figure 2.4. The first subscript index in a_{jk}, b_{jk} and A_{jk} represents the direction of the

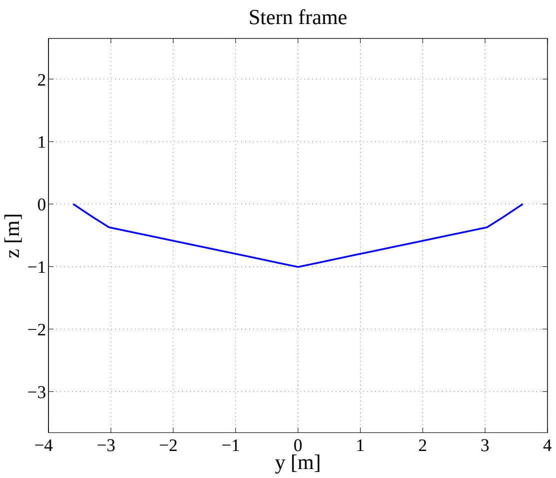
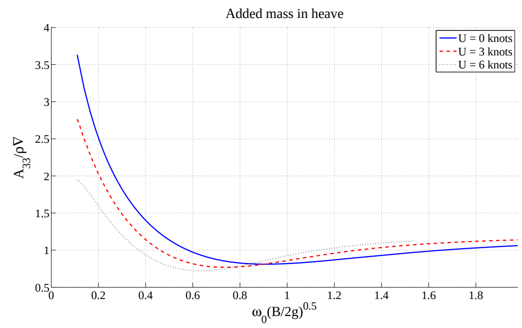
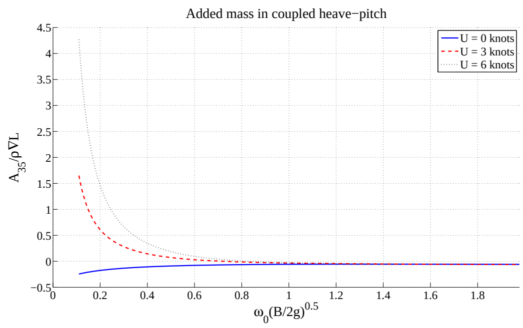


Figure 2.4: Submerged part of the frame at the transom stern.

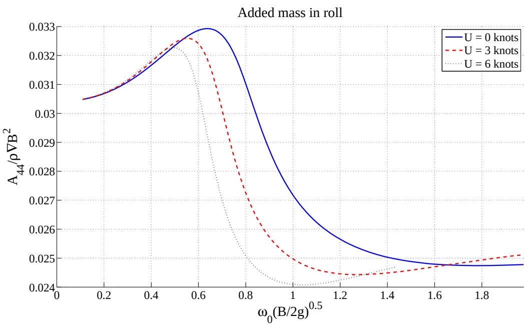
added mass, and the second subscript index represents the direction of the motion. This means that A_{33} is added mass in heave due to an acceleration in heave, A_{26} is added mass in sway due to an acceleration in yaw and so on. Coupled motions will be discussed in somewhat more detail in section 3.1. The non-dimensional $3D$ added mass coefficients for heave, coupled heave-pitch, roll and pitch is shown in figure 2.5 as a function of frequency and forward speed in head sea. The remaining added mass coefficients are shown in appendix D. Further the non-dimensional $2D$ added mass in heave and roll for the transom stern is shown in figure 2.6 as a function of frequency. We from figure 2.5 and 2.6 that the added mass



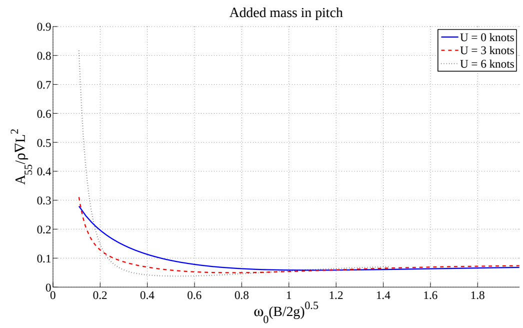
(a) A_{33}



(b) A_{35}

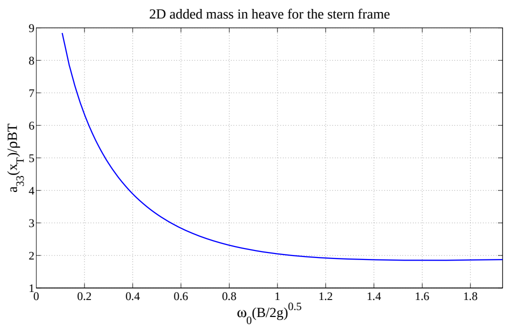


(c) A_{44}

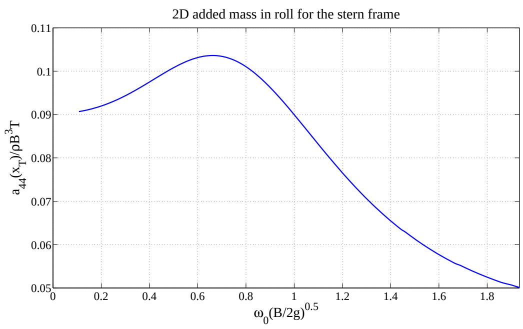


(d) A_{55}

Figure 2.5: Non-dimensional $3D$ added mass coefficients in heave, coupled heave-pitch, roll and pitch as function of frequency and forward speed.



(a) $a_{33}\,(x_T)$



(b) $a_{44}\,(x_T)$

Figure 2.6: Non-dimensional $2D$ added mass in heave and roll at the transom stern.

for the vertical motions goes to infinity when the frequency goes towards zero. We also see that these added masses approaches an asymptotic value irrespective of forward speed then the frequency goes to infinity.

2.1.2 Damping

Mechanical damping is usually related to friction, but since we use potential theory where the viscosity is neglected we cannot have shear stress at the hull surface. The damping in potential theory is therefore only related to waves propagating away from the body (Faltinsen, 1990). We will in this section also just state the formulas for the three dimensional damping, as we did in the previous section. All formulas below are taken from Fathi & Hoff (2010).

$$B_{22} = \int_L b_{22}(x)\,\mathrm{d}x + U a_{22}(x_T), \tag{2.16}$$

$$B_{24} = \int_L b_{24}(x)\,\mathrm{d}x + U a_{24}(x_T), \tag{2.17}$$

$$B_{26} = \int_L x b_{22}(x)\,\mathrm{d}x + U\int_L a_{22}(x)\,\mathrm{d}x + U x_T a_{22}(x_T) - \frac{U^2}{\omega_e^2} b_{22}(x_T), \tag{2.18}$$

$$B_{33} = \int_L b_{33}(x)\,\mathrm{d}x + U a_{33}(x_T), \tag{2.19}$$

$$B_{35} = -\int_L x b_{33}(x)\,\mathrm{d}x - U\int_L a_{33}(x)\,\mathrm{d}x - U x_T a_{33}(x_T) + \frac{U^2}{\omega_e^2} b_{33}(x_T), \tag{2.20}$$

$$B_{42} = \int_L b_{24}(x)\,\mathrm{d}x + U a_{24}(x_T), \tag{2.21}$$

$$B_{44} = \int_L b_{44}(x)\,\mathrm{d}x + U a_{44}(x_T), \tag{2.22}$$

$$B_{46} = \int_L x b_{24}(x)\,\mathrm{d}x + U\int_L a_{24}(x)\,\mathrm{d}x + U x_T a_{24}(x_T) - \frac{U^2}{\omega_e^2} b_{24}(x_T), \tag{2.23}$$

$$B_{53} = -\int_L x b_{33}(x)\,\mathrm{d}x + U\int_L a_{33}(x)\,\mathrm{d}x - U x_T a_{33}(x_T), \tag{2.24}$$

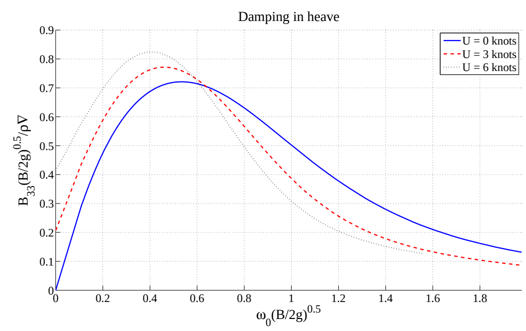
$$B_{55} = \int_L x^2 b_{33}(x)\,\mathrm{d}x + \frac{U^2}{\omega_e^2}\int_L b_{33}(x)\,\mathrm{d}x + U x_T^2 a_{33}(x_T) - \frac{U^2}{\omega_e^2} x_T b_{33}(x_T), \tag{2.25}$$

$$B_{62} = \int\limits_L x b_{22}(x)\,\mathrm{d}x - U \int\limits_L a_{22}(x)\,\mathrm{d}x + U x_T a_{22}(x_T)\,, \tag{2.26}$$

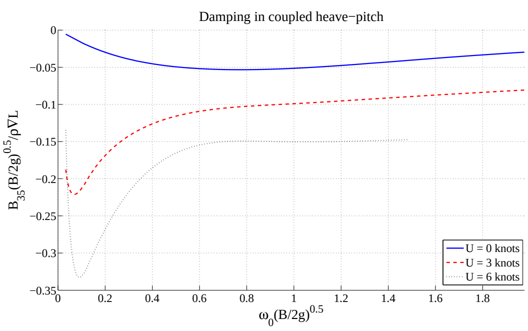
$$B_{64} = \int\limits_L x b_{24}(x)\,\mathrm{d}x - U \int\limits_L a_{24}(x)\,\mathrm{d}x + U x_T a_{24}(x_T)\,, \tag{2.27}$$

$$B_{66} = \int\limits_L x^2 b_{22}(x)\,\mathrm{d}x + \frac{U^2}{\omega_e^2} \int\limits_L b_{22}(x)\,\mathrm{d}x + U x_T^2 a_{22}(x_T) - \frac{U^2}{\omega_e^2} x_T b_{22}(x_T)\,, \tag{2.28}$$

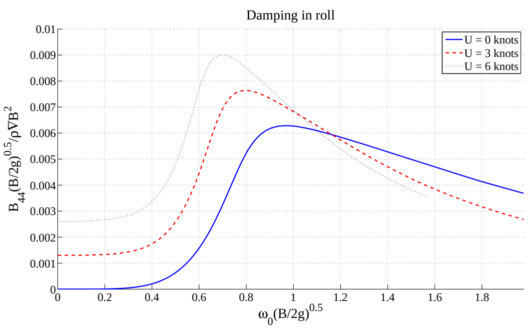
The non-dimensional $3D$ damping coefficients for heave, coupled heave-pitch, roll and pitch is shown in figure 2.7 as a function of frequency and forward speed in head sea. The remaining damping coefficients are shown in appendix E. Further the non-dimensional $2D$ damping in heave and roll for the transom



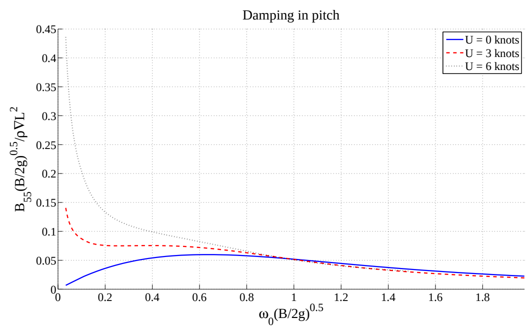
(a) B_{33}



(b) B_{35}



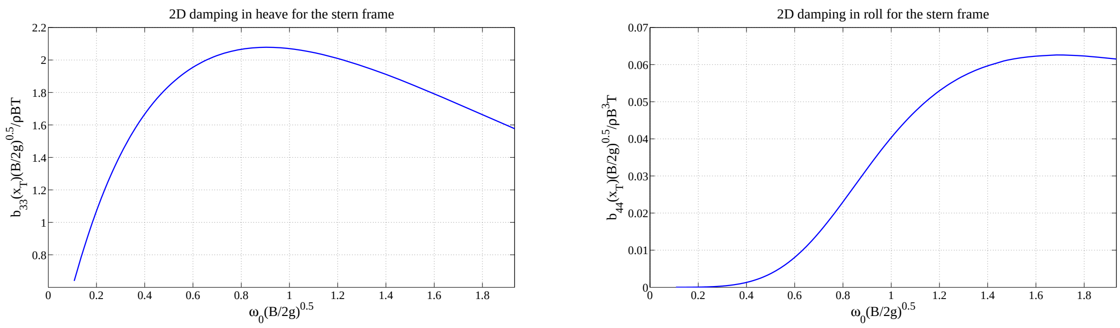
(c) B_{44}



(d) B_{55}

Figure 2.7: Non-dimensional $3D$ damping coefficients in heave, coupled heave-pitch, roll and pitch as function of frequency and forward speed.

stern is shown in figure 2.8 as a function of frequency. We see that the damping goes toward zero for very low and very high frequencies when the forward speed is zero. This is because we will not generate outgoing waves when $\omega \to 0$ or $\omega \to \infty$. When the forward speed is different from zero, we see that the lift effects give some contribution to the damping for low frequencies, but it still goes towards zero for high frequencies.



(a) $b_{33}\ (x_T)$ **(b)** $b_{44}\ (x_T)$

Figure 2.8: Non-dimensional $2D$ damping in heave and roll at the transom stern.

2.1.2.1 Roll damping

Strip theory shows very good agreement with experiments for the vertical ship motions, such as heave and pitch, and also sway and yaw (Salvesen et al., 1970). However, when analyzing the roll motion, the results scatter (Himeno, 1981). One reason for this may be the roll damping. As already mentioned, the potential damping is related to waves propagating away from the ship. The roll motion will not generate waves of any significance, so the potential damping in roll is small. In reality we will have a lot of viscous effects and this will matter (Faltinsen, 1990), especially when we have large roll amplitudes (Ibrahim & Grace, 2010), which is the case for parametric roll resonance. In order to catch these effects, it is convenient to split the total roll damping into components. The viscous terms, B_v, are proportional to the roll velocity squared. This means that we write the uncoupled equation of motion in roll as

$$\left(I_{44} + A_{44}\right)\ddot{\eta}_4 + B_{44}\dot{\eta}_4 + B_v|\dot{\eta}_4|\dot{\eta}_4 + C_{44}\eta_4 = F_4 e^{i\omega_e t}, \tag{2.29}$$

Here I_{44} is the mass moment of inertia in roll. This equation has to be solved in the time domain because of the term $B_v|\dot{\eta}_4|\dot{\eta}_4$. If we want to do a frequency domain analysis, we need to linearize the non-linear damping. One way of doing that is equivalent linearization, which can be done when we have harmonic oscillations. When linearizing we find an equivalent damping coefficient, that will give the same amount of damping work during one cycle as the non-linear damping. Linearizing of non-linear damping when the load has a sinusoidal cycle is extensively discussed by Langen & Sigbjörnsson (1979).

According to Himeno (1981), the roll damping may be divided into the following components:

<u>Friction damping</u>, which is caused by the skin friction. This will depend of both the viscosity of the fluid and of the roughness of the hull surface. This component is believed to be small, so it will be neglected in this text.

<u>Eddy damping of the naked hull</u>. This is damping due to vortex shedding from the bilge when the ship oscillates or rolls. It is non-linear of nature. Since the vessel analyzed in this text has bilge keels, this damping will be disregarded. However, the bilge keels are not covering the whole ship length, so we will probably get eddies and vortex shedding along some parts of the hull. In addition, this vessel has a pronounced skeg, as shown in figure 1.6, and this is probably a source of eddies. Disregarding the eddy damping is hence a simplification, and we probably underestimate the damping because of that. This may affect the results and contribute to make the results conservative, since damping is favorable in order to avoid large resonant motions.

<u>Lift damping</u>. When the vessel has forward speed, we will get a lift effect due to the roll motion. This

lift will cause a damping moment that is proportional to the forward speed. This damping is linear, and is included in equation (2.22) as the term $Ua_{44}(x_T)$.

Bilge keels. It is common that fishing vessels are equipped with bilge keels, due to its cheap and simple construction and due to its large effect on roll damping. Usually these keels have a length of $0.25 - 0.5$ times the ship length (Pettersen, 2007), and consists of a flat bar or similar welded to the bilge of the vessel. In figure 2.9 we see a typical bilge keel installed on a naval vessel.

Figure 2.9: A typical bilge keel. The picture is taken from (Australian Government, 2011).

In order to reduce the resistance from the bilge keels, it is common to shape them according to the streamlines of the flow around the bilge. These streamlines are usually determined by a paint test of a model of the ship.

When it comes to roll damping by the bilge keels, the contribution may be divided in two. The first contribution comes from normal forces on the bilge keels itself (Ikeda & Tanaka, 1976), and the second contribution comes from the pressure created by the bilge keels (Ikeda et al., 1977). The damping due to the normal force on the bilge keel is simply a drag force that occurs because the ship, and hence the bilge keel, is oscillating. How to calculate the damping from bilge keels, according to Ikeda & Tanaka (1976) and Ikeda et al. (1977) is shown in appendix A. It turns out that the damping due to the bilge keels is proportional to the roll amplitude, η_{4a} (Ikeda & Tanaka, 1976; Ikeda et al., 1977). The non-dimensional damping coefficient for Trønderhav due to bilge keels is shown in figure 2.10 for three different roll amplitudes as a function of frequency. In figure 2.10, the relationship between the viscous damping coefficient and the equivalent linear damping coefficient is given as, see Ikeda et al. (1977) for details,

$$B_{v,eqv} = \frac{3\pi}{8} B_v. \tag{2.30}$$

In figure 2.11 we have shown how the bilge keel damping influences the total roll damping, where the roll amplitude is assumed to be $10°$. We see that the higher frequency, the higher contribution, and that the bilge keels may contribute to as much as $50\ \%$ of the roll damping.

The breadth and position (y and z position) of the bilge keels on Trøndherhav are given in table 2.1

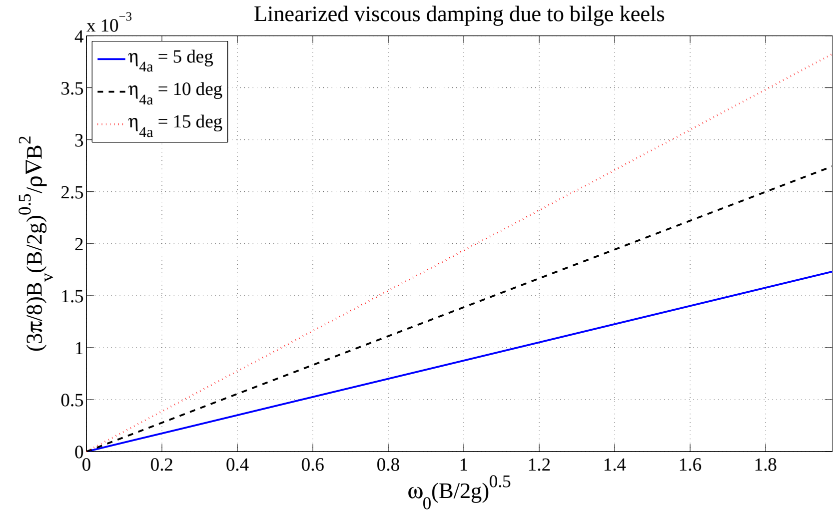


Figure 2.10: Linearized non-dimensional damping coefficient due to bilge keels.

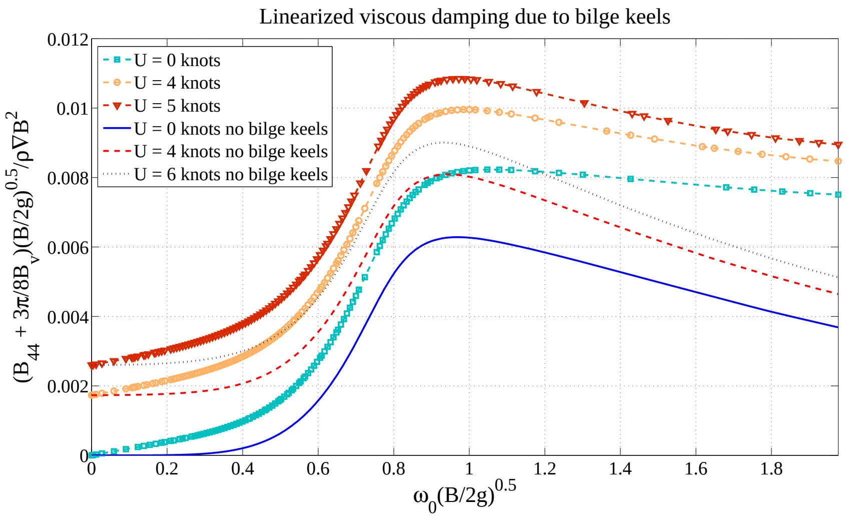


Figure 2.11: Linearized non-dimensional total damping coefficient in roll as function of frequency and forward speed in beam sea. We see that bilge keels give an important contribution to the roll damping.

Table 2.1: Dimensions and position of bilge keel at sections on Trønderhav

Bilge keel on Trøndherhav									
y [m]	3.80	3.94	3.99	4.00	3.98	3.81	3.66	3.52	3.36
z [m]	-2.74	-2.77	-2.78	-2.79	-2.79	-2.78	-2.76	-2.73	-2.69
b [m]	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30

2.1.3 Restoring forces and moments

The restoring forces are hydrostatic forces that make the vessel able to return to its static equilibrium position after a disturbance. These forces and moments are proportional to the displacements and rotations and exist for a free floating vessel only for motions in the vertical plane, i.e. heave, roll and pitch.

Let us consider a barge with dimensions length×beam×draught = $L \times B \times T$ in heave motion. When the barge is out of its equilibrium position, the restoring force will be the difference between its gravity and its buoyancy. Another way of saying that is that the restoring force is equal to the force from the changed volume of fluid. For a barge with vertical sides and a positive heave motion, see figure 2.12,

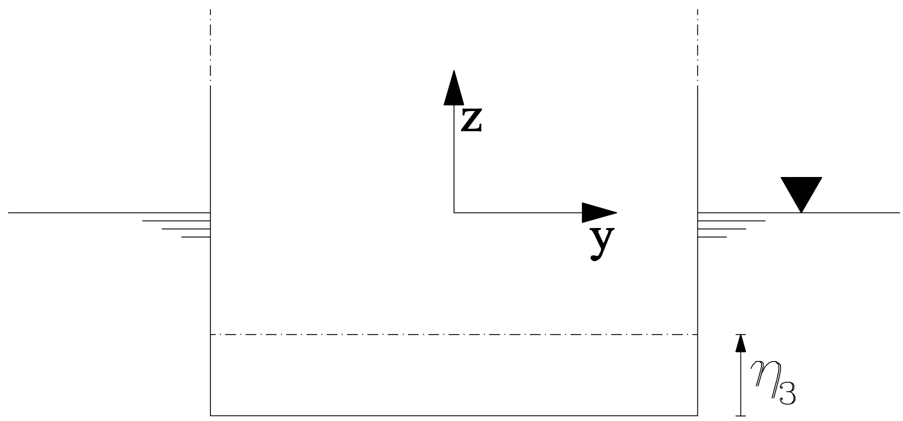


Figure 2.12: The restoring force in heave is equal to the difference between the gravity and buoyancy force when the vessel is out of its equilibrium position.

this is equal to

$$F_3 = -\rho g L B \eta_3. \tag{2.31}$$

The restoring force in heave is by definition given as

$$F_3 \equiv -C_{33}\eta_3, \tag{2.32}$$

which leads to the restoring coefficient in heave

$$C_{33} = \rho g A_W, \tag{2.33}$$

where A_W is the mean water plane area.

If the vessel is not symmetric about the midship section, the heave motion will also cause a pitch moment. The reduced vertical force on a strip on a vessel with a triangular water plane with a positive heave motion η_3, see fig 2.13, is

$$\mathrm{d}F_3 = -\rho g 2 y \eta_3 \mathrm{d}x. \tag{2.34}$$

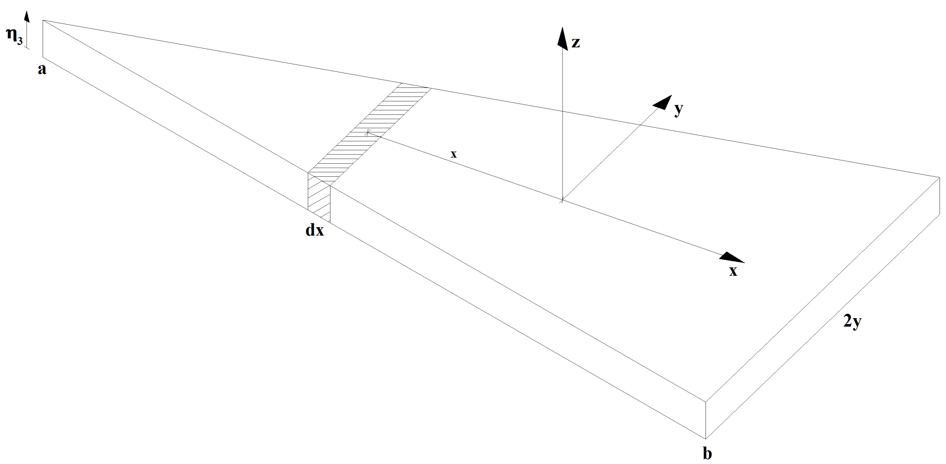


Figure 2.13: When the vessel is forced into heave, the added or reduced buoyancy creates a pitch moment due to dissymmetry.

The moment in general is given as

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ F_1 & F_2 & F_3 \end{vmatrix}$$
$$= (yF_3 - zF_2)\,\mathbf{i} - (xF_3 - zF_1)\,\mathbf{j} + (xF_2 - yF_1)\,\mathbf{k}. \tag{2.35}$$

A common assumption is that the surge force, F_1, can be neglected (Salvesen et al., 1970). Hence the pitch moment on the same strip can be written as

$$\mathrm{d}F_5 = -x\mathrm{d}F_3. \tag{2.36}$$

The total pitch moment due to a heave displacement is therefore given as

$$F_5 = \rho g \int_a^b 2yx\mathrm{d}x\eta_3 \equiv -C_{53}\eta_3. \tag{2.37}$$

From this we see that the coupled heave-pitch restoring coefficient for a ship with a triangular water plane is may be written as

$$C_{53} = -\rho g \int_a^b 2yx\mathrm{d}x. \tag{2.38}$$

This can be generalized to

$$C_{53} = -\rho g \iint_{A_W} x\mathrm{d}S, \tag{2.39}$$

for a vessel with an arbitrary shaped water plane.

Similarly, if the vessel has a pitch angle, it will create a heave force. Doing the same analysis as for the coupled heave-pitch restoring force, it follows that the coupled pitch-heave restoring force becomes the same, and hence

$$C_{35} = -\rho g \iint_{A_W} x \mathrm{d}S. \tag{2.40}$$

If we force the vessel to roll, change of volume distribution in the water plane will cause a restoring moment, see figure 2.14. The moment per unit ship length due to a small volume element is for small

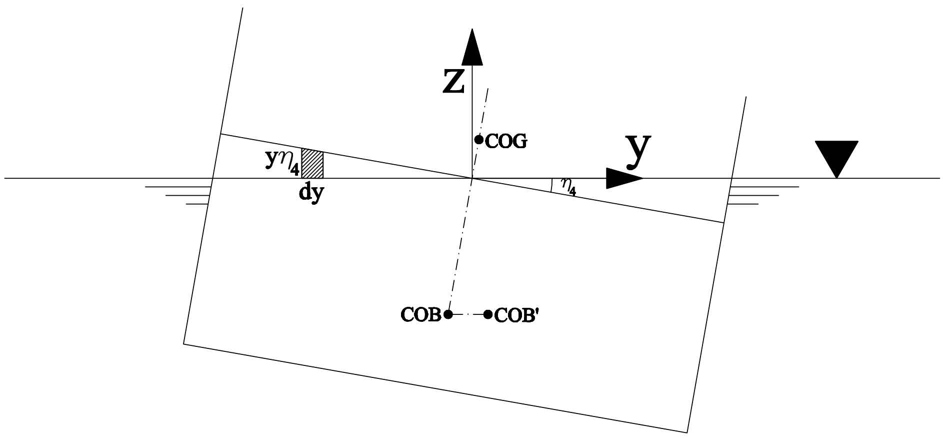


Figure 2.14: The linear restoring moment in roll has two contributions. One from the moment created by the water plane area and one created by the weight acting through the center of gravity.

heel angles equal to

$$\mathrm{d}M = -\rho g y \eta_4 \mathrm{d}y y. \tag{2.41}$$

For a ship with an arbitrary shaped water plane, the restoring moment in roll may for small heel angles be generalized to

$$\mathrm{d}M = -\rho g \iint_{A_W} y^2 \mathrm{d}S \eta_4. \tag{2.42}$$

However, since the origin of the coordinate system does not go through the center of gravity (COG), we get a contribution to the restoring moment from the weight and the buoyancy of the ship. When the vessel heels, the center of buoyancy shifts towards the side the vessel heels and hence creates an uprighting moment. For small angles this is equal to

$$M_{z_B} = -\rho g \nabla z_B \eta_4. \tag{2.43}$$

Here ∇ is the volume displacement of the ship and z_B is the vertical center of buoyancy. The weight acts through the center of gravity and will cause a destabilizing moment. In the same way as the righting moment from the center of buoyancy, this is for small heel angles given as

$$M_{z_G} = \rho g \nabla z_G \eta_4, \tag{2.44}$$

where z_G is the vertical center of gravity. Since the linear restoring moment in roll by definition is given as

$$F_4 = -C_{44}\eta_4, \tag{2.45}$$

the linear restoring coefficient in roll becomes

$$C_{44} = \rho g \nabla \left(z_B - z_G\right) + \rho g \iint\limits_{A_W} y^2 \mathrm{d}S. \tag{2.46}$$

Similarly the linear restoring coefficient in pitch is given as

$$C_{55} = \rho g \nabla \left(z_B - z_G\right) + \rho g \iint\limits_{A_W} x^2 \mathrm{d}S. \tag{2.47}$$

The linear restring coefficients for Trønderhav are summarized in table 2.2.

Table 2.2: Linear restoring coefficients of Trøndherhav

Restoring coefficients		
C_{33}	$1.9467 \cdot 10^6$	N/m
C_{35}	$-2.3142 \cdot 10^6$	N
C_{44}	$2.6105 \cdot 10^6$	Nm
C_{53}	$-2.3142 \cdot 10^6$	N
C_{55}	$8.2909 \cdot 10^7$	Nm

2.2 Excitation loads

We will now take a closer look into the wave excitation loads. These are split into Froude-Kriloff loads and diffraction loads, i.e. sub-problem A. Since we are applying strip theory we will calculate the forces and moments on each strip separately and summarize them in order to get the total load.

2.2.1 Froude-Kriloff forces and moments

A hydrodynamic force is in reality the pressure integrated over a surface, S, as given in equation (2.48).

$$\mathbf{F} = -\iint\limits_{S} p\mathbf{n}\mathrm{d}S. \tag{2.48}$$

We find the Froude-Kriloff forces and moments by integrating the linear dynamic pressure from the incoming velocity potential over the mean wetted surface of the hull. Hence we can express the Froude-Kriloff force in heave as

$$F_3^{FK} = -\iint\limits_{S_B} p_{dyn} n_3 \mathrm{d}S, \tag{2.49}$$

where S_B is the mean wetted surface and n_3 is the vertical component of the unit normal vector, $\mathbf{n}$, of the hull. This vector describes the hull geometry and has positive direction into the fluid. By substituting equation (1.10) into equation (2.49) we find the Froude-Kriloff force amplitude on each strip to be

$$f_3^{FK}(x) = i\rho g\zeta_a \int\limits_{C_x} n_3 e^{-ik(x\cos\beta + y\sin\beta)} e^{kz} \mathrm{d}l. \tag{2.50}$$

We integrate the pressure along the length of the cross section, C_x, at position x. We also see that the magnitude of the Froude-Kriloff force is independent of the forward speed. The absolute value of the total Froude-Kriloff force in heave for a wave amplitude of $\zeta_a = 1$ m in head sea is shown in figure 2.15.

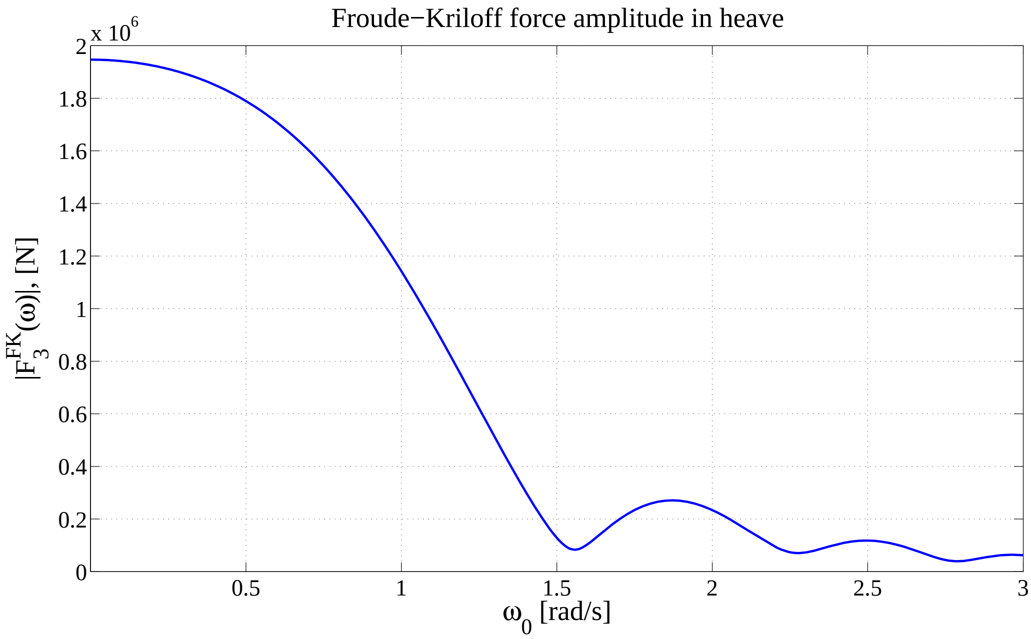


Figure 2.15: Absolute value of the Froude-Kriloff amplitude in heave as function of wave frequency in head sea.

Similarly the sectional Froude-Kriloff force in sway is given as

$$f_2^{FK}(x) = i\rho g\zeta_a \int\limits_{C_x} n_2 e^{-ik(x\cos\beta + y\sin\beta)} e^{kz} \mathrm{d}l. \tag{2.51}$$

Here n_2 is the horizontal component of the unit normal vector of the hull.

Similar to equation (2.36), the Froude-Kriloff moment in pitch can be written as

$$F_5^{FK} = \int\limits_L -x f_3^{FK}(x)\,\mathrm{d}x = \iint\limits_{S_B} x p_{dyn} n_3 \mathrm{d}S. \tag{2.52}$$

The absolute value of the total Froude-Kriloff moment in pitch for a wave amplitude of $\zeta_a = 1$ m in head sea is shown in figure 2.16.

If we substitute equation (2.48) into equation (2.35) we can write

$$\mathbf{M} = -\iint\limits_S p\,(\mathbf{r} \times \mathbf{n})\,\mathrm{d}S. \tag{2.53}$$

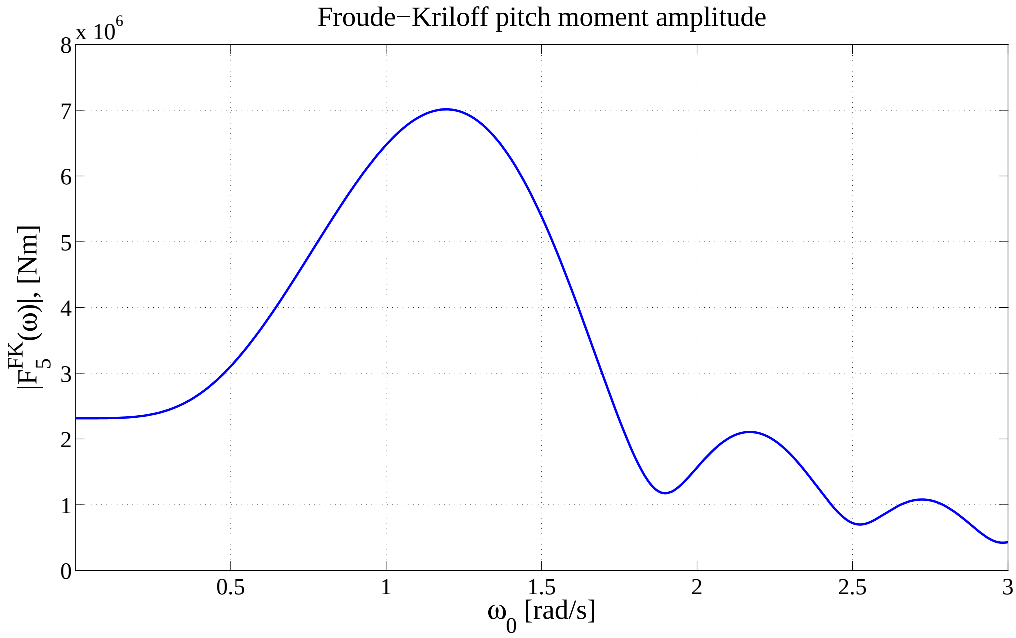


Figure 2.16: Absolute value of the Froude-Kriloff amplitude in pitch as function of wave frequency in head sea.

Now the Froude-Kriloff moment in roll is the **i**-component of

$$F_4^{FK} = -\iint_S p_{dyn} \left(\mathbf{r} \times \mathbf{n}\right) \mathrm{d}S. \tag{2.54}$$

This implies that we can write the sectional Froude-Kriloff moment in roll as

$$f_4^{FK}(x) = i\rho g \zeta_a \int_{C_x} n_4 e^{-ik(x\cos\beta + y\sin\beta)} e^{kz} \mathrm{d}l, \tag{2.55}$$

where

$$n_4 = y n_3 - z n_2. \tag{2.56}$$

The absolute value of the total Froude-Kriloff moment in roll for a wave amplitude of $\zeta_a = 1$ m and zero forward speed in beam sea is shown in figure 2.17.

The most tricky part here is to determine the normal vectors. The first we have to do is to discretize each section or strip. This is usually no problem because we have the offset points of each frame as input. We approximate the hull by straight lines between these points, see figure 2.18. If the offset points at each end of the red line shown in figure 2.18 have coordinates (y_0, z_0) and (y_1, z_1) the vector between them has components

$$\mathbf{t} = \left[\left(y_1 - y_0\right), \left(z_1 - z_0\right)\right] \tag{2.57}$$

and length

$$|\mathbf{t}| = \sqrt{\left(y_1 - y_0\right)^2 + \left(z_1 - z_0\right)^2}. \tag{2.58}$$

The vector has positive direction from left to right, i.e. the left point has coordinates (y_0, z_0). To find the unit normal vector we may rotate this vector 90° clockwise and divide by $|\mathbf{t}|$. We do this rotation by multiplying with the transformation matrix, **A**, see Edwards & Penney (1988) for details.

$$\mathbf{A} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \tag{2.59}$$

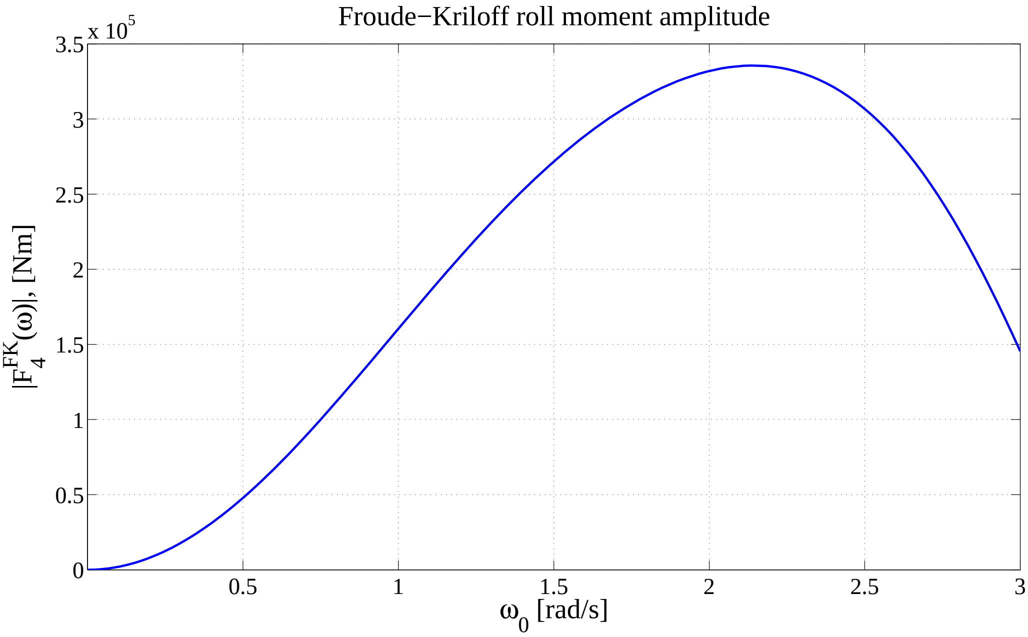


Figure 2.17: Absolute value of the Froude-Kriloff amplitude in roll as function of wave frequency at zero forward speed in beam sea.

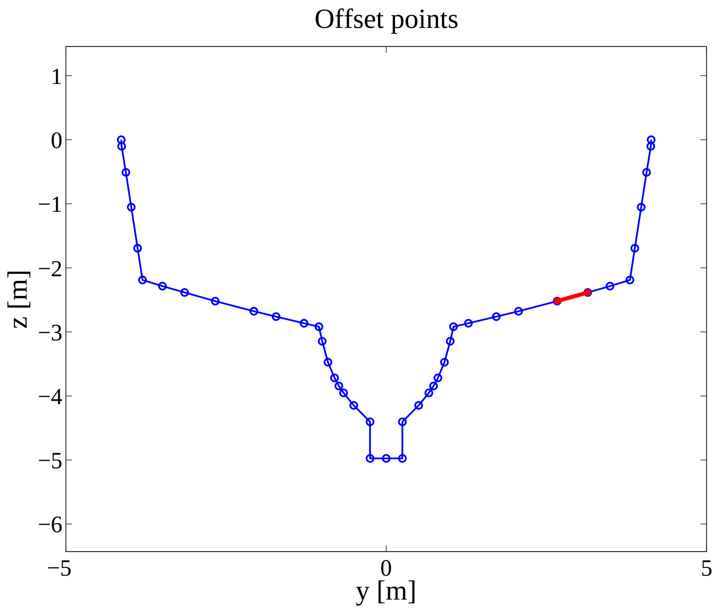


Figure 2.18: Offset points for a frame towards the stern of the vessel. The line marked in red shows an example of how the frame is discretized by straight lines (vectors). Positive direction of the vector is from left to right.

where θ is the rotation angle, in this case $90°$. Hence equation (2.59) becomes

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

The two dimensional unit normal vector can now be written as

$$\mathbf{n} = [n_2, n_3] = \frac{\mathbf{t}}{|\mathbf{t}|}\mathbf{A} \tag{2.60}$$

When integrating the pressure over the wetted surface, the surface area will be approximated by the sum off all elements in each strip. The area of each element is given as $|\mathbf{t}| \cdot s$, where s is the distance between the frames. The pressure will be evaluated at the midpoint of each element. The integral will in practice therefore be replaced by a finite sum of element areas multiplied by the pressure at each element and its vertical component of the normal vector.

2.2.2 Diffraction forces and moments

When assuming that the undisturbed pressure, i.e. the Froude-Kriloff pressure is acting on the hull surface we have implicitly said that we have flow through the hull. This is unphysical and, off course, totally unacceptable from a practical naval architect's point of view. Therefore we need to find a diffraction potential, φ_7, that makes sure that there is no flow through the surface. Another way of saying that is that this potential has to induce an opposite velocity in the direction of the hull surface. This can be expressed as

$$\frac{\partial \varphi_7}{\partial n} = -\frac{\partial \varphi_0}{\partial n}. \tag{2.61}$$

The reason for the calling this potential φ_7 is that $\varphi_1 - \varphi_6$ are the potentials related to the six rigid-body motions while φ_0 is the incoming potential.

If we now consider heave motion in head sea, equation (2.61) may be approximated as

$$\frac{\partial \varphi_7}{\partial n} \approx -n_3 \frac{\partial \varphi_0}{\partial z} = -n_3 w \tag{2.62}$$

The diffraction potential have to satisfy the same boundary conditions as the velocity potential for the heave motion. We can hence write, see Faltinsen (2005)

$$\varphi_7 = -\varphi_3 \frac{\partial \varphi_0}{\partial z}. \tag{2.63}$$

In order to solve this for the mean position of the vessel we need to average equation (2.63) in space. We then write

$$\varphi_7 \approx -\varphi_3 \overline{\frac{\partial \varphi_0}{\partial z}} = -\varphi_3 w. \tag{2.64}$$

We average this by evaluating the vertical fluid particle velocity in a "mean" z-position. According to Salvesen et al. (1970) a convenient value may be $-T_S C_S$, where T_S is the sectional draught and C_S is the sectional area coefficient. This is given as

$$C_S = \frac{A_S}{B_S T_S}, \tag{2.65}$$

where B_S is the sectional beam.

We can express the two dimensional diffraction force in heave as

$$f_3(x) = -\rho \int\limits_{C_x} n_3 \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \varphi_3 w \mathrm{d}S. \tag{2.66}$$

This is the same equation used in slender body theory by for instance Newman (1977), and it follows from Bernoulli's equation. Since both φ_3 and w may vary with time, we need to apply the product rule when differentiating with respect to time. Equation (2.66) then becomes

$$f_3(x) = -\rho \int\limits_{C_x} n_3 \varphi_3 \mathrm{d}S \frac{\partial w}{\partial t} - \rho \int\limits_{C_x} n_3 \frac{\partial \varphi_3}{\partial t} \mathrm{d}S w - U \frac{\partial}{\partial x} \left(\rho \int\limits_{C_x} n_3 \varphi_3 \mathrm{d}S w \right). \tag{2.67}$$

The forward speed independent diffraction force may now be written as

$$f_3^D(x) = a_{33}(x)\, a_z + b_{33}(x)\, w. \tag{2.68}$$

Here we have used that (see Faltinsen (2005) and Newman (1977) for details)

$$-\rho \int\limits_{C_x} \varphi_3 n_3 \mathrm{d}l = a_{33}(x) \tag{2.69}$$

and

$$-\rho \int\limits_{C_x} \frac{\partial \varphi_3}{\partial t} n_3 \mathrm{d}l = b_{33}(x). \tag{2.70}$$

In order to find the total diffraction force amplitude in heave we integrate the sectional forces along the ship length.

$$F_3^D = \int\limits_L \left(a_{33}(x)\, a_z + b_{33}(x)\, w \right) \mathrm{d}x + U a_{33}(x_T)\, w \tag{2.71}$$

Here a_{33} and b_{33} is the sectional added mass and damping respectively and a_z and w is the fluid particle acceleration and velocity respectively and they are to be evaluated at $z = -T_S C_S$ at time $t = 0$. We also see that we get a contribution from the forward speed and that the shape of the aft body of the ship is of importance when it comes to sea keeping. The absolute value of the total diffraction force in heave for a wave amplitude of $\zeta_a = 1$ m and zero forward speed in head sea is shown in figure 2.19.

Similarly, we have the sectional diffraction force in sway

$$f_2^D(x) = a_{22}(x)\, a_y + b_{22}(x)\, v. \tag{2.72}$$

We can now write the total excitation force amplitude in heave as

$$F_3 = \int\limits_L \left(f_3^{FK}(x) + f_3^D(x) \right) \mathrm{d}x + U a_{33}(x_T)\, w. \tag{2.73}$$

Similarly as for heave the excitation force in sway becomes

$$F_2 = \int\limits_L \left(f_2^{FK}(x) + f_2^D(x) \right) \mathrm{d}x + U a_{22}(x_T)\, v. \tag{2.74}$$

The absolute value of the total wave excitation force in heave for a wave amplitude of $\zeta_a = 1$ m in head sea is shown in figure 2.20. In order to consider if this force is calculated correctly, we may look at the

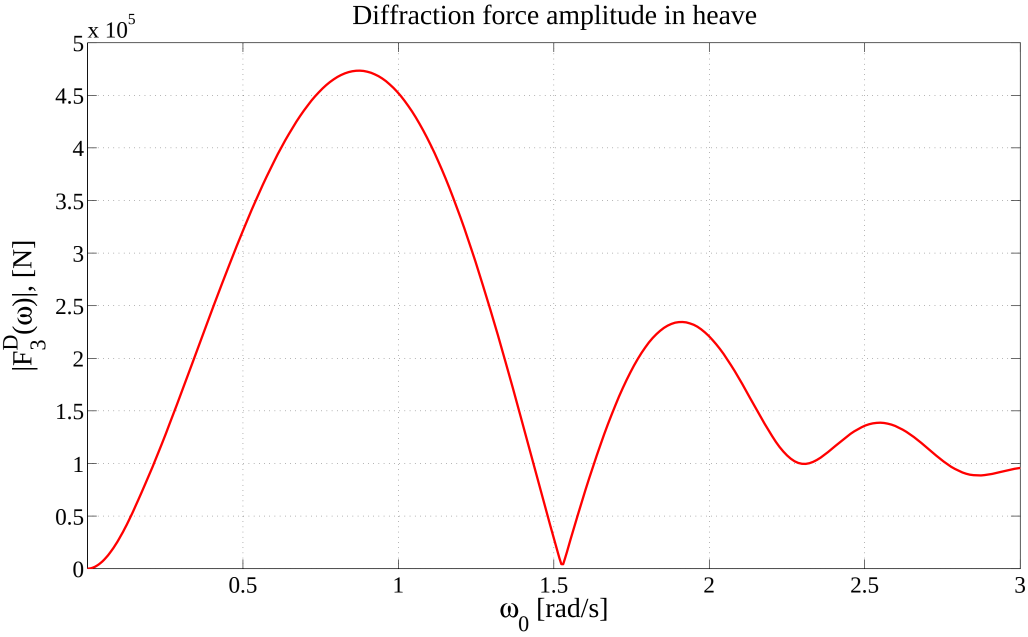


Figure 2.19: Absolute value of the diffraction amplitude in heave as function of wave frequency at zero forward speed in head sea.

asymptotic values, when $\omega \to \infty$ and $\omega \to 0$. From the dispersion relation (Faltinsen, 1990) for infinite depth, we have that

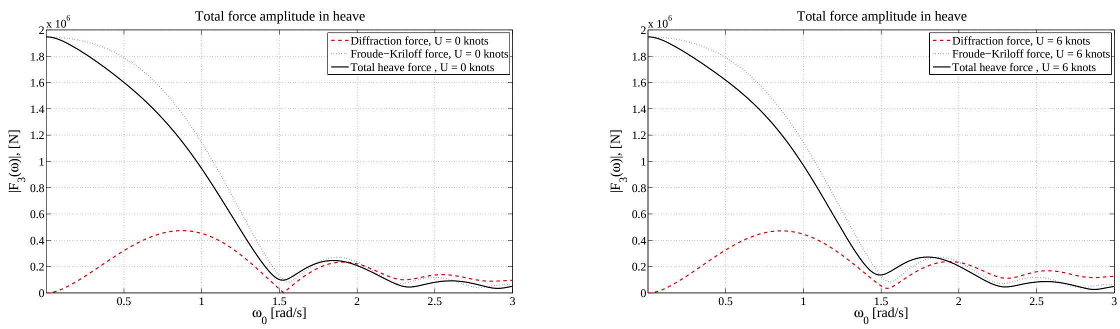
$$\omega^2 = kg. \tag{2.75}$$

This can be re-written as

$$\lambda = \frac{2\pi}{\omega^2 g}. \tag{2.76}$$

From equation (2.76) we see that the wave length decreases rapidly as the frequency increases. For short wave lengths high waves do not exist, so it makes sense that the excitation force approaches zeros as the frequency increases. For low frequencies, i.e. very long waves, the ship will follow the wave motion, so the forces are hydrostatic or quasi-steady. This means that the wave excitation force should balance the restoring terms in the heave equation of motion, such that

$$C_{33}\eta_3 + C_{35}\eta_5 \approx F_3, \tag{2.77}$$



(a) $U = 0$ knots **(b)** $U = 6$ knots

Figure 2.20: Absolute value of the wave excitation force in heave as function of wave frequency at two different forward speeds in head sea.

when $\omega \rightarrow 0$. When the ship is following the wave motion, the heave amplitude is equal to the wave amplitude and the pitch amplitude is equal to the wave slope. The wave slope is given as $\zeta_a k$ (Myrhaug, 2006). This means that we can write equation (2.77) as

$$C_{33}\zeta_a + C_{35}\zeta_a k \approx F_3. \tag{2.78}$$

If we now assume a wave amplitude of 1 m and a very long wave period, say 500 s, we see from table 2.2 and figure 2.20 that equation (2.77) is fulfilled. This is shown in table 2.3. We also wee from figure

Table 2.3: Comparison of restoring force amplitude and excitation force amplitude in heave for quasi-steady condition.

Quasi-steady heave force		
$C_{33}\eta_3 + C_{35}\eta_5$	=	$1.9467 \cdot 10^6$ N
F_3	=	$1.9462 \cdot 10^6$ N

2.20 that for low wave frequencies the Froude-Kriloff force is dominating and that the diffraction force goes to zero. This makes sense, since both the fluid particle velocity and acceleration are proportional to the wave frequency.

To find the sectional diffraction moment in pitch we multiply by the heave force by minus the horizontal distance from the lontitudinal center of gravity (LCG) to the section, see equation (2.35). We write this as

$$\begin{aligned} f_5(x) &= -x f_3(x) \\ &= -x\left(a_{33}a_z + b_{33}w\right) - x\frac{\partial}{\partial x}\left(Ua_{33}\right). \end{aligned} \tag{2.79}$$

When we integrate this over the ship length we need to apply integration by parts on the last term. When then end up with the following expression for the diffraction moment in pitch

$$F_5^D = -\int_L \left[x\left(a_{33}a_z + b_{33}w\right) - Ua_{33}w\right] \mathrm{d}x - Ux_T a_{33}\left(x_T\right) w. \tag{2.80}$$

The absolute value of the total diffraction moment in pitch for a wave amplitude of $\zeta_a = 1 \text{ m}$ and zero forward speed i head sea is shown in figure 2.21.

Now the total pitch moment becomes

$$F_5 = -\int_L \left[x\left(f_3^{FK}(x) + f_3^D(x)\right) - Ua_{33}\left(x_T\right)w\right] \mathrm{d}x - Ux_T a_{33}\left(x_T\right) w. \tag{2.81}$$

Here $f_3^{FK}(x)$ and $f_3^D(x)$ are given by equation (2.50) and (2.68) respectively.

The absolute value of the total wave excitation moment in pitch for a wave amplitude of $\zeta_a = 1 \text{ m}$ in head sea is shown in figure 2.22.

To find out if this makes sense we may argue as we did for the heave force. The excitation moment must balance the restoring terms in pitch for a quasi-steady condition. This is given as

$$C_{53}\eta_3 + C_{55}\eta_5 \approx F_5. \tag{2.82}$$

The comparison for the same wave condition as we did for the heave force is given in table 2.4. Remember that the pitch moment is plotted in terms of absolute value in figure 2.22.

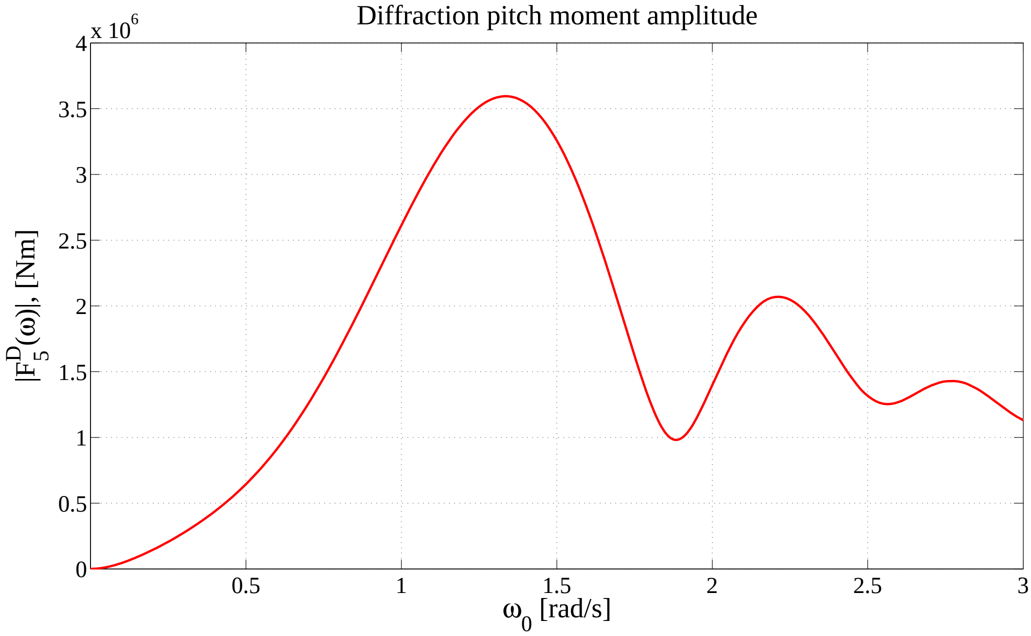
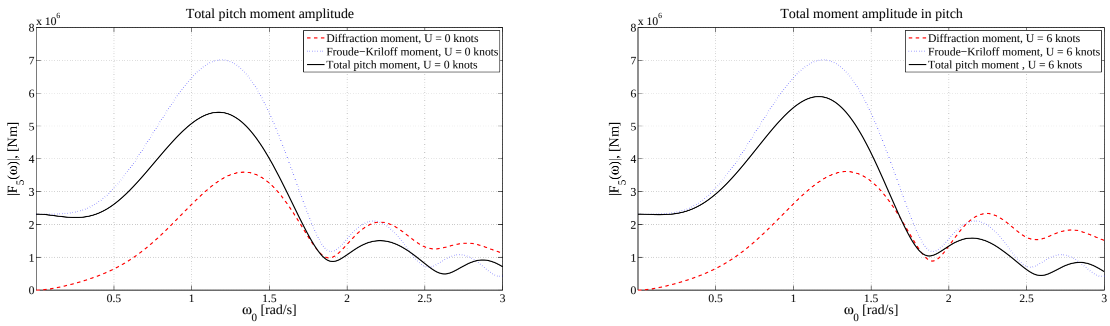


Figure 2.21: Absolute value of the diffraction moment amplitude in pitch as function of wave frequency at zero forward speed in head sea.



(a) $U = 0$ knots

(b) $U = 6$ knots

Figure 2.22: Absolute value of the wave excitation moment in pitch as function of wave frequency at two different forward speeds in head sea.

Table 2.4: Comparison of restoring moment amplitude and excitation moment amplitude in pitch for quasi-steady condition.

Quasi-steady pitch moment		
$C_{53}\eta_3 + C_{55}\eta_5$	=	$-2.3129 \cdot 10^6$ N
F_5	=	$-2.3134 \cdot 10^6$ N

The yaw moment is found by following the same approach as for the pitch moment, and the this is given as

$$F_6 = \int\limits_L \left[x\left(f_2^{FK}(x) + f_2^D(x)\right) - Ua_{22}(x_T)\,v\right]\mathrm{d}x + Ux_T a_{22}(x_T)\,v. \tag{2.83}$$

The roll motion may be considered as a combination of heave and sway. Equation (2.61) then becomes

$$\frac{\partial\varphi_7}{\partial n} \approx -n_2\overline{\frac{\partial\varphi_0}{\partial y}} - n_3\overline{\frac{\partial\varphi_0}{\partial z}} = -n_2 v - n_3 w, \tag{2.84}$$

hence we can write the diffraction potential in roll as

$$\varphi_7 = -\varphi_2 v - \varphi_3 w. \tag{2.85}$$

Then the sectional diffraction roll moment becomes

$$f_4 = -\rho\int\limits_{C_x} n_4\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\varphi_2 v\mathrm{d}S \tag{2.86}$$

$$\underbrace{-\rho\int\limits_{C_x} n_4\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\varphi_3 w\mathrm{d}S}_{=0}\,. \tag{2.87}$$

The last term is equal to zero due to symmetry. By following the same procedure as for heave and the same analogy as in equation (2.69) and (2.70) we find that the total diffraction moment in roll is expressed as

$$F_4^D = \int\limits_L (a_{42}a_y + b_{42}v)\,\mathrm{d}x + Ua_{42}(x_T)\,v. \tag{2.88}$$

The absolute value of the total Diffraction moment in roll for a wave amplitude of $\zeta_a = 1$ m and zero forward speed in beam sea is shown in figure 2.23.

The total excitation moment in roll may now be written as

$$F_4 = \int\limits_L \left(f_4^{FK}(x) + f_4^D(x)\right)\mathrm{d}x + Ua_{42}(x_T)\,v, \tag{2.89}$$

where

$$f_4^D(x) = a_{42}a_y + b_{42}v. \tag{2.90}$$

The absolute value of the total wave excitation moment in roll for a wave amplitude of $\zeta_a = 1$ m in beam sea is shown in figure 2.24.

When solving the equations of motions in the time domain it is $\Re\left(F_j e^{i\omega_e t}\right)$ that has physical meaning. Here $\Re$ means the real part.

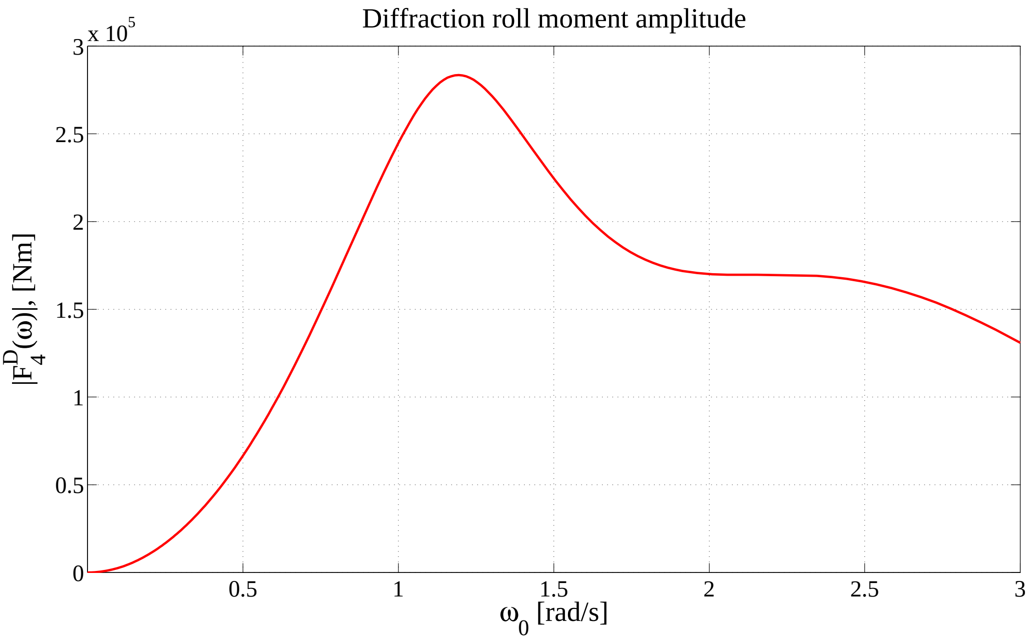
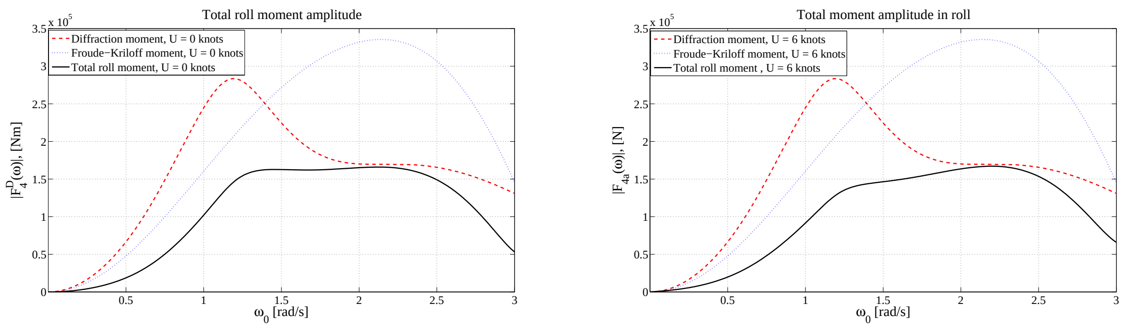


Figure 2.23: Absolute value of the Diffraction moment amplitude in roll as function of wave frequency at zero forward speed in beam sea.



(a) $U = 0$ knots **(b)** $U = 6$ knots

Figure 2.24: Absolute value of the wave excitation moment in roll as function of wave frequency at two different forward speeds in beam sea.

Chapter 3

Response in regular waves

Having obtained the excitation forces and the hydrodynamic coefficients we can find the motions of the ship. The most basic equation in mechanics is Newton's second law, stating that the mass of a body multiplied by its acceleration equals the forces acting on that body.

$$Ma = \sum_i F_i \tag{3.1}$$

The forces, F_i, are found from sub-problem A and B. This leads to the equations of motion, and we find the motions by solving these equations. The equations of motion will be discussed in next section.

3.1 Coupled motions

For most ship types the modes of motion will be coupled. This means that one mode of motion will induce another, for instance a coupling between heave and pitch. This means that a ship that has heave motion also will get a pitch motion because of that heave motion. In principle all six rigid-body motions are coupled, but for a ship with starboard-port symmetry, surge-heave-pitch will be decoupled from sway-roll-yaw (Salvesen et al., 1970). As previously mentioned, surge is neglected. We then end up with two sets of equations, one for heave-pitch and one for sway-roll-yaw. This means that we can treat heave and pitch independently from sway, roll and yaw, but we cannot treat heave and pitch independently from each other. Nor can we treat sway, roll and yaw independently from each other.

3.1.1 Heave and pitch motion

The equations that describe the heave and pitch motions are given as (Salvesen et al., 1970)

$$(M + A_{33})\,\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 + A_{35}\ddot{\eta}_5 + B_{35}\dot{\eta}_5 + C_{35}\eta_5 = F_3 e^{i\omega_e t} \tag{3.2}$$

$$A_{53}\ddot{\eta}_3 + B_{53}\dot{\eta}_3 + C_{53}\eta_3 + (I_{55} + A_{55})\,\ddot{\eta}_5 + B_{55}\dot{\eta}_5 + C_{55}\eta_5 = F_5 e^{i\omega_e t} \tag{3.3}$$

Here I_{55} is the moment of inertia in pitch. This is given similarly as for roll as

$$I_{55} = Mr_{55}^2, \tag{3.4}$$

where r_{55} is the radius of gyration in pitch. It is common to give this as a fraction of the ship length. For this kind of vessel this value may be around $r_{55}/L_{PP} = 0.33 - 0.35$ (Enerhaug, 2011, personal comm.). We choose a value of $r_{55}/L_{PP} = 0.34$ in this case.

3.1.2 Sway, roll and yaw motion

The linear coupled equations in sway, roll and yaw are expressed as (Salvesen et al., 1970)

$$(M + A_{22})\,\ddot{\eta}_2 + B_{22}\dot{\eta}_2 + (-Mz_G + A_{24})\,\ddot{\eta}_4 + B_{24} + A_{26}\ddot{\eta}_6 + B_{26}\dot{\eta}_6 = F_2 e^{i\omega_e t}, \tag{3.5}$$

$$\begin{aligned}(-Mz_G + A_{42})\,\ddot{\eta}_2 + B42\dot{\eta}_2 + (A_{44} + I_{44})\,\ddot{\eta}_4 + B44\dot{\eta}_4 \\ + C_{44}\eta_4 + (A_{46} - I_{46})\,\ddot{\eta}_6 + B_{46}\dot{\eta}_6 = F_4 e^{\omega_e t},\end{aligned} \tag{3.6}$$

$$A_{62}\ddot{\eta}_2 + B_{62}\dot{\eta}_2 + (A_{64} - I_{46})\,\ddot{\eta}_4 + B_{64}\dot{\eta}_4 + (A_{66} + I_{66})\,\ddot{\eta}_6 + B_{66}\dot{\eta}_6 = F_6 e^{i\omega_e t}. \tag{3.7}$$

Here I_{46} is the product of inertia in coupled roll-yaw. In practice this can be neglected (Faltinsen, 2005), and we have done so in this text. Further I_{66} is the moment of inertia in yaw, given as

$$I_{66} = Mr_{66}^2, \tag{3.8}$$

where r_{66} is the radius of gyration in yaw. In this case this is set equal to $r_{66}/L_{PP} = 0.25$. The reason for the term $-Mz_G$ is that the origin of the coordinate system does not go through the center of gravity, hence this creates a moment that needs to be accounted for. We also have the additional roll damping term due to viscous effects, as discussed in section 2.1.2.1. If we include this, equation (3.6) now becomes

$$\begin{aligned}(-Mz_G + A_{42})\,\ddot{\eta}_2 + B42\dot{\eta}_2 + (A_{44} + I_{44})\,\ddot{\eta}_4 + B_{44}\dot{\eta}_4 + B_v|\dot{\eta}_4|\dot{\eta}_4 \\ + C_{44}\eta_4 + (A_{46} - I_{46})\,\ddot{\eta}_6 + B_{46}\dot{\eta}_6 = F_4 e^{\omega_e t},\end{aligned} \tag{3.9}$$

3.2 Solving the equations of motion

When we are going to solve the equations of motions we have two opportunities, i.e. frequency domain and time domain. In the frequency domain we solve for the motion amplitude over a range of frequencies and in the time domain we get the actual time history of the motion for a given frequency. In this section we will discuss these two methods further.

3.2.1 Frequency domain

When assuming linear theory we can calculate a transfer function in order to find the response in the frequency domain. The transfer function gives the ratio between the amplitude of a given motion of the ship, e.g. heave displacement, and the wave amplitude as a function of frequency. The transfer function can also serve as an indicator of the quality of the result, e.g. when developing a computer program for wave induced motions, we can easily compare the transfer function for a given ship to the "true" transfer function for that ship, obtained from either another program or a model test. By linear theory we mean that the motions are proportional to the wave amplitude.

In order to find the transfer function, we need to determine the steady state amplitude for the actual motion. This we can find from the particular solution of the equation of motion. If we assume that the motions oscillating harmonically, we can write

$$\begin{aligned}\eta_3 &= \bar{\eta}_3 e^{i\omega_e t} \\ \dot{\eta}_3 &= i\omega_e \bar{\eta}_3 e^{i\omega_e t} \\ \ddot{\eta}_3 &= -\omega_e^2 \bar{\eta}_3 e^{i\omega_e t}\end{aligned} \tag{3.10}$$

for heave displacement, velocity and acceleration respectively and where $\bar{\eta}_3$ is the complex heave amplitude, given as (Faltinsen, 2005)

$$\bar{\eta}_3 = \eta_{R3} + i\eta_{I3}. \tag{3.11}$$

where η_{R3} and η_{I3} are the real and imaginary parts of $\bar{\eta}_3$ respectively.

It is convenient to express the equations of motion on matrix form. The modes of motion matrix is given as

$$\boldsymbol{\eta} = \begin{bmatrix} \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{bmatrix}. \tag{3.12}$$

We also need to write the mass, added mass coefficients, damping coefficients, restoring coefficients and excitation forces on matrix form. The mass matrix is given as (Faltinsen, 2005)

$$\mathbf{M} = \begin{bmatrix} M & 0 & -Mz_G & 0 & 0 \\ 0 & M & 0 & 0 & 0 \\ -Mz_G & 0 & I_{44} & 0 & -I_{46} \\ 0 & 0 & 0 & I_{55} & 0 \\ 0 & 0 & -I_{46} & 0 & I_{66} \end{bmatrix}. \tag{3.13}$$

Further the added mass and damping matrices respectively are given as (Salvesen et al., 1970)

$$\mathbf{A} = \begin{bmatrix} A_{22} & 0 & A_{24} & 0 & A_{26} \\ 0 & A_{33} & 0 & A_{35} & 0 \\ A_{42} & 0 & A_{44} & 0 & A_{46} \\ 0 & A_{53} & 0 & A_{55} & 0 \\ A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix}, \tag{3.14}$$

$$\mathbf{B} = \begin{bmatrix} B_{22} & 0 & B_{24} & 0 & B_{26} \\ 0 & B_{33} & 0 & B_{35} & 0 \\ B_{42} & 0 & B_{44} & 0 & B_{46} \\ 0 & B_{53} & 0 & B_{55} & 0 \\ B_{62} & 0 & B_{64} & 0 & B_{66} \end{bmatrix}. \tag{3.15}$$

The added mass and damping coefficients are found from equations (2.3) to (2.15) and (2.16) to (2.28) respectively. The matrix containing the linear restoring coefficients is given as

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & C_{53} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3.16}$$

where the coefficients are given by equations (2.33), (2.40), (2.46), (2.39) and (2.47). The numerical values are given in table 2.2. The last matrix is the excitation force matrix. This is given as

$$\mathbf{F} = \begin{bmatrix} F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}, \tag{3.17}$$

where F_2 to F_6 is given by equation (2.74), (2.73), (2.89), (2.81) and (2.83) respectively.

The equations of motion can now be written as

$$(\mathbf{M} + \mathbf{A})\,\ddot{\boldsymbol{\eta}} + \mathbf{B}\dot{\boldsymbol{\eta}} + \mathbf{C}\boldsymbol{\eta} = \mathbf{F}e^{i\omega_e t}. \tag{3.18}$$

The force amplitudes in equation (3.17) are written on complex form, meaning that they have the form $x + iy$, so the phases of each force are taken into account by the real and imaginary parts.

If we assume that all modes of motion are oscillating harmonically, we may do as we showed for the heave motion in equation (3.10). This means that we can write the response matrix as

$$\begin{aligned}
\boldsymbol{\eta} &= \bar{\boldsymbol{\eta}}e^{i\omega_e t} \\
\dot{\boldsymbol{\eta}} &= i\omega_e\bar{\boldsymbol{\eta}}e^{i\omega_e t} \\
\ddot{\boldsymbol{\eta}} &= -\omega_e^2\bar{\boldsymbol{\eta}}e^{i\omega_e t},
\end{aligned} \tag{3.19}$$

where $\bar{\boldsymbol{\eta}}$ are the complex motion amplitudes, defined for each mode the same way as for heave, shown in equation (3.11). We can now substitute equation (3.19) into the equations of motion, which means that equation (3.18) now becomes

$$-\,\omega_e^2\,(\mathbf{M} + \mathbf{A})\,\bar{\boldsymbol{\eta}}e^{i\omega_e t} + i\omega_e\mathbf{B}\bar{\boldsymbol{\eta}}e^{i\omega_e t} + \mathbf{C}\bar{\boldsymbol{\eta}}e^{i\omega_e t} = \mathbf{F}e^{i\omega_e t}. \tag{3.20}$$

This has to be true irrespective of time, so we divide equation (3.20) by $e^{i\omega_e t}$. We can now write the complex response matrix as a function of frequency as

$$\bar{\boldsymbol{\eta}}\,(\omega) = \mathbf{H}\,(\omega)\,\mathbf{F}\,(\omega)\,, \tag{3.21}$$

where $\mathbf{H}\,(\omega)$ is called the mechanical transfer function given as

$$\mathbf{H}\,(\omega) = \left[-\omega_e^2\,(\mathbf{M} + \mathbf{A}) + i\omega_e\mathbf{B} + \mathbf{C}\right]^{-1}. \tag{3.22}$$

The final response amplitude for each mode is given by the absolute value of the complex response amplitude, meaning that

$$\eta_j = \sqrt{\eta_{Rj}^2 + \eta_{Ij}^2}, \tag{3.23}$$

for $j = 2..6$. The phase angle, ϵ_j, between the load and response for each mode is found from the ratio between the imaginary and real part of the response, see fig 3.1

$$\epsilon_j = \arctan\left(\frac{\eta_{Ij}}{\eta_{Rj}}\right), \tag{3.24}$$

for $j = 2..6$.

This way of solving the equations of motion in the frequency domain is called the frequency-response method and is extensively discussed by for instance Langen & Sigbjörnsson (1979).

We see the transfer functions for heave and pitch in head sea for two different forward speeds in figure 3.2. We see that the value of the transfer function goes towards 1 for long waves and 0 for short waves. This indicates that the calculations are realistic, or at least physical, as discussed for the excitation forces in section 2.2.2. The transfer function in pitch is made dimensionless by dividing the pitch amplitude on the wave slope.

The transfer functions for sway, heave, roll and yaw in beam sea for two different forward speeds are shown in figure 3.3. We see that the asymptotic trends are the same as for heave and pitch in head sea, and that there is a strong coupling between sway, roll and yaw around the natural period in roll.

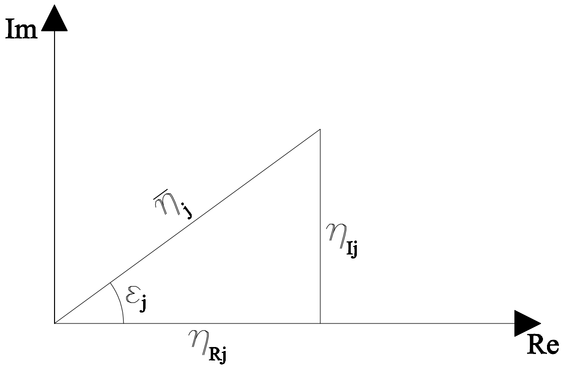
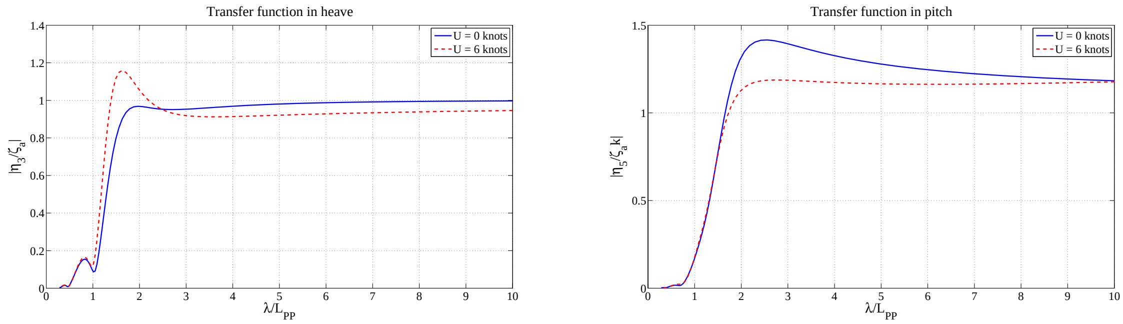


Figure 3.1: Illustration of the response in the complex plane.



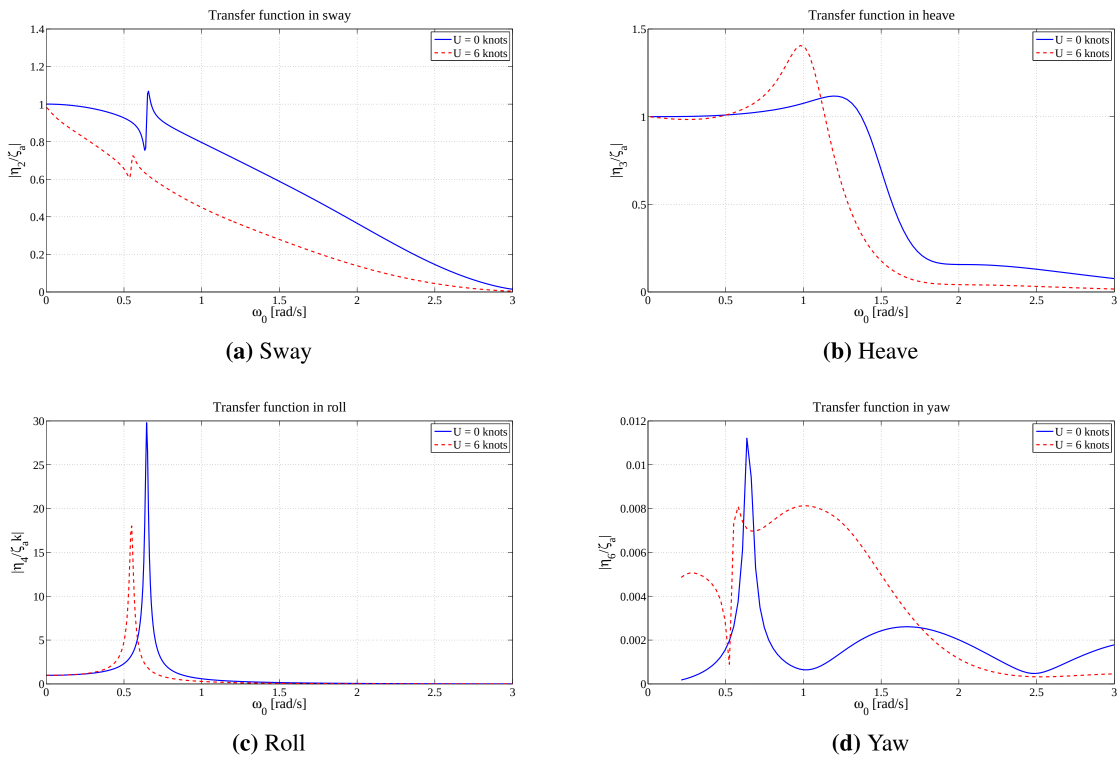
(a) Heave

(b) Pitch

Figure 3.2: Transfer functions in heave and pitch in head sea for two different forward speeds.

3.2.2 Time domain

When we solve the equations of motion in the time domain, we find the time history of the motions at a given frequency or frequencies. This is a useful way to solve the equations if we are interested in how the response develops in time, such as the occurrence of resonance for instance. We are also able to account for non-linearities, such as the viscous damping or the non-linear restoring moment, variables whose magnitude is dependent of the response itself. When we have non-linearities involved, we have to use a numerical time integration method. Here we have some options, but we will in this project direct time integration by use of the built-in function *ode45* in Matlab. This function is based on the Dormand-Prince (4,5) pair (Dormand & Prince, 1980), which is an improvement of an explicit Runge-Kutta (4,5) formula (Shampine & Reichelt, 2009). We will not go into further details about this, but an interested reader is referred to Shampine & Reichelt (2009) and Dormand & Prince (1980) for further reading.



(a) Sway

(b) Heave

(c) Roll

(d) Yaw

Figure 3.3: Transfer functions in sway, heave, roll and yaw in beam sea for two different forward speeds.

Chapter 4

Ship stability

Hydrostatic stability is maybe the most basic and fundamental subject within the field of naval architecture. A ship being stable and floating with the right side up is one of the three fundamental requirements to a ship (Amdahl et al., 2003). The other two requirements are that that ship should be seaworthy and it should be safe for its passengers and cargo (Amdahl et al., 2003). In this chapter we will take a closer look into the most basic terms in stability of an intact ship and show how we have calculated the non-linear restoring moment in this project.

4.1 Initial stability

Stability is a ship's ability to return to its equilibrium position after an external disturbance. In order to understand how this works, we need to take a look at the midship section in figure 4.1. As we saw in section 2.1.3, when a ship heels, the center of buoyancy shifts towards the side it heels. This is marked by B and B' in figure 4.1. For small heel angles the buoyancy acts through the metacenter, M, and creates an uprighting moment. If we consider the vertical center of gravity, in this case called G, as the moment axis, the arm of this restoring moment is $\overline{GZ}$. We see from figure 4.1 that if the metacenter was below the center of gravity, the $\overline{GZ}$-arm would cause a negative restoring moment, and the ship would continue to heel, i.e. be unstable. The key in this case is therefore the distance between the vertical center of gravity and the metacenter, $\overline{GM}$. This has to be greater than zero in order for the ship to be stable. This distance is called the initial metacentric height, and we see from figure 4.1 that we may express this as

$$\overline{GM} = \overline{KB} + \overline{BM} - \overline{KG}. \tag{4.1}$$

Here $\overline{KB}$ is the vertical distance from the keel to the center of buoyancy, $\overline{BM}$ is the vertical distance from the center of buoyancy to the metacenter, called metacentric radius, and $\overline{KG}$ is the distance from the keel to the vertical center of gravity. Equation (4.1) is exactly correct for zero heel angle, i.e. initial stability, and approximately correct for heel angles up to 10° (Amdahl et al., 2003). From figure 4.1 we see that for small angles this arm is equal to

$$\overline{GZ} = \overline{GM}\sin\phi \approx \overline{GM}\phi, \tag{4.2}$$

where ϕ is the heel angle. Note the difference between ϕ and η_4, where ϕ is static and η_4 is dynamic, even though booth quantities describe an inclination angle. For heel angles larger than 10° we can no longer assume that the metacenter will not move (Amdahl et al., 2003). We will discuss this more into detail in section 4.2.

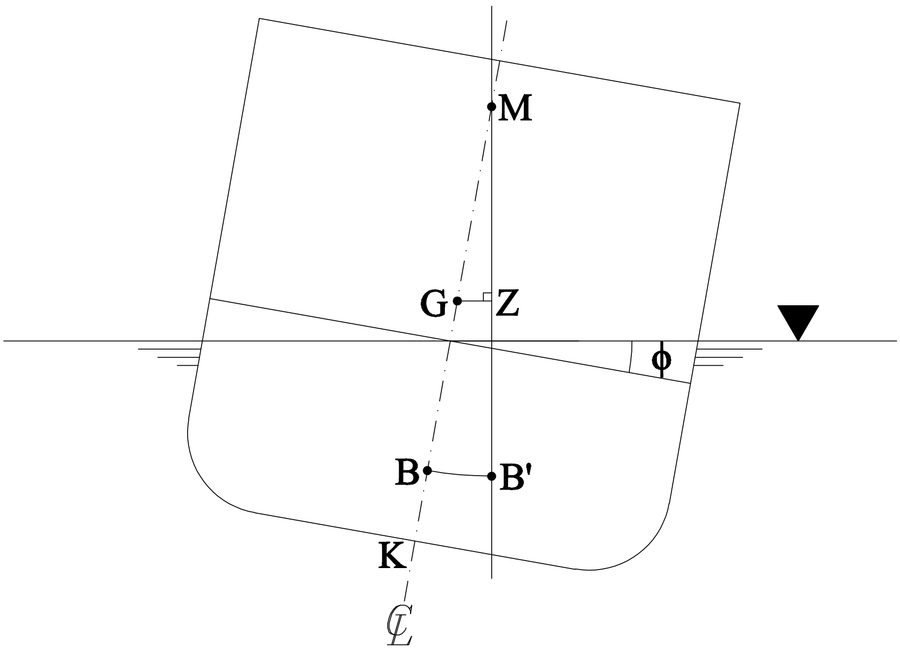


Figure 4.1: Midship section with a small static heel angle, ϕ. The center of buoyancy moves from B to B' and the buoyancy acts through the metacenter, M, and creates an uprighting moment around the center of gravity. The arm of this moment is $\overline{GZ}$.

The vertical center of buoyancy is found from the center of volume of the submerged part of the hull. We find this by calculating the vertical volume moment of each section and divide by the total volume. We find the vertical center of gravity by the same procedure, but here we calculate the vertical moment of every single mass and divide by the total mass. In this project the vertical center of gravity will be given as an input value.

4.1.1 Metacentric radius

Having found $\overline{KB}$ and $\overline{KG}$ we only miss $\overline{BM}$ in order to find the metacentric height and to determine of the ship is stable or not. Let us consider a rectangular barge with length L and breadth B with a small heel angle, see figure 4.2. The center of buoyancy shifts because of the change of volume from the left to the right triangle. The volume per unit ship length of each triangle is given as

$$V = \frac{1}{2} \cdot \frac{B}{2} \cdot \frac{B}{2} \cdot \tan\phi = \frac{B^2}{8} \tan\phi. \tag{4.3}$$

The shift of these volumes creates a volume moment, M_V, given as

$$M_V = V \cdot L \cdot \frac{2B}{3} = \frac{1}{12} B^3 L \tan\phi. \tag{4.4}$$

This volume moment may also be expressed as

$$M_V \approx \overline{BB'} \nabla, \tag{4.5}$$

where $\overline{BB'}$ is the distance the center of buoyancy has shifted, see figure 4.2. The reason for that this is approximately the same, is that the center of buoyancy will not shift along a straight line as shown

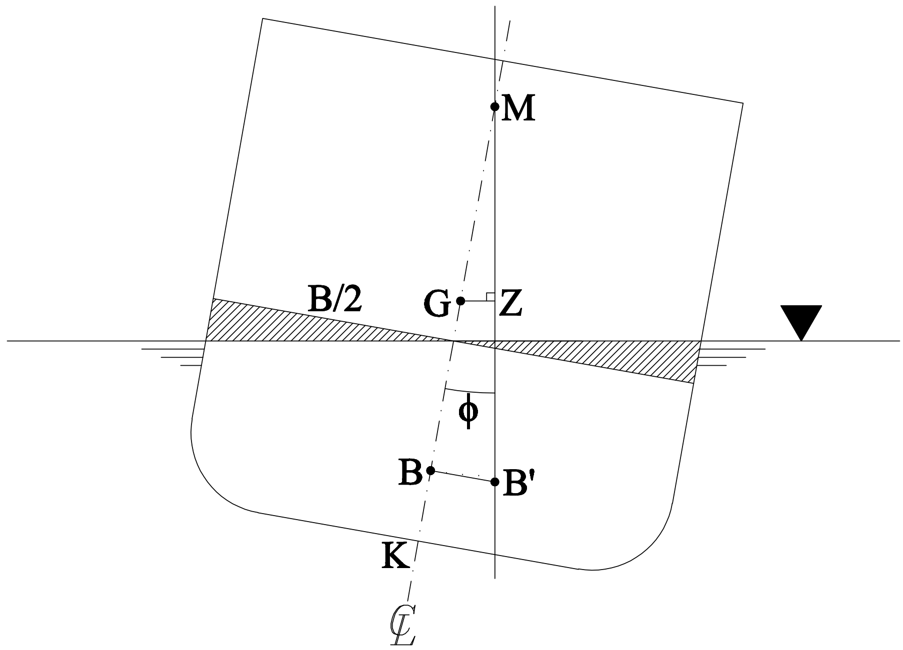


Figure 4.2: The shift of center of buoyancy for a barge with a rectangular water plane is caused by the change of volume distribution in the triangles.

in figure 4.2, but will have a more circular-like path (Biran, 2003). However, for small heel angles this assumption is ok (Amdahl et al., 2003), and hence we may write equation (4.5) as

$$M_V \approx \overline{BM}\nabla \tan \eta_4. \tag{4.6}$$

If we now equal equation (4.4) and (4.6), we find that the metacentric radius is given as

$$\overline{BM} = \frac{I}{\nabla}, \tag{4.7}$$

where I is the transverse second moment of area, which for a rectangular water plane is expressed as

$$I = \frac{1}{12}B^3L. \tag{4.8}$$

This may for an arbitrary water plane shape be written as

$$I = \iint_{A_W} y^2 \mathrm{d}S. \tag{4.9}$$

If we now take a closer look at equation (4.7) and (4.9) and compare this to the linear restoring coefficient in roll given by equation (2.46), we see that this may be written as

$$C_{44} = \rho g \nabla \overline{GM}. \tag{4.10}$$

The linear restoring coefficient in pitch may be written the same way, meaning

$$C_{55} = \rho g \nabla \overline{GM}_L, \tag{4.11}$$

where $\overline{GM}_L$ is the longitudinal metacentric height.

From equation (4.10) and (4.11) we see that the initial stability have a direct influence of the sea keeping characteristics of a ship.

4.2 Stability at larger heel angles

In the previous section we looked upon hydrostatic stability for small heel angles. By small heel angles we mean angles up to approximately $10°$. For angles larger than this, we cannot assume that the metacenter will not move. As the heel angle increases, the metacenter will begin to move upwards. To avoid misunderstandings we call the metacenter that has moved upwards for false metacenter, $M_{\phi F}$, since the metacenter is defined exactly for $\phi = 0°$. When the false metacenter moves upwards, we get an another contribution to the $\overline{GZ}$-arm. This contribution comes from the residual stability which is a pure geometric property of the ship. We can see the lever of the residual stability, $\overline{MS}$, in figure 4.3.

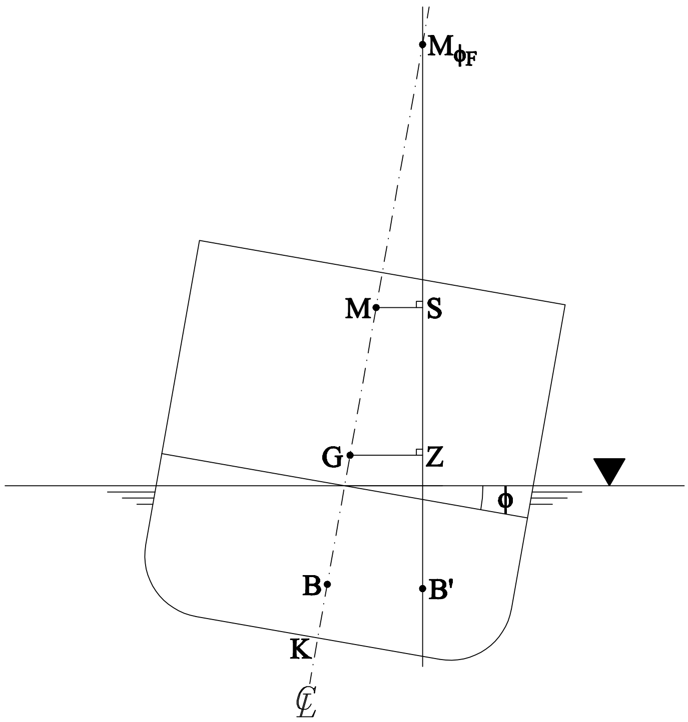


Figure 4.3: Midship section for larger heel angles. We get another contribution to the righting arm from the residual stability, $\overline{MS}$, and the false metacenter, $M_{\phi F}$, moves upwards.

This means that we can write the righting arm as

$$\overline{GZ} = \overline{GM}\sin\phi + \overline{MS}\,(\phi)\,, \tag{4.12}$$

and hence, the non-linear restoring moment in roll as

$$M_R = \rho g \nabla \overline{GZ}. \tag{4.13}$$

4.2.1 The $\overline{GZ}$-curve

A very common way to describe a ship's stability is to plot the $\overline{GZ}$-arm as a function of heel angle in a $\overline{GZ}$-curve. If we Taylor expand this curve, we find that the slope at zero heel angle is the metacentric height. This can also be seen from

$$\frac{\partial \overline{GZ}}{\partial \phi}|_{\phi=0} = \overline{GM}. \tag{4.14}$$

An example of a typical $\overline{GZ}$-curve is shown in figure 4.4. The maximum value of the $\overline{GZ}$-curve,

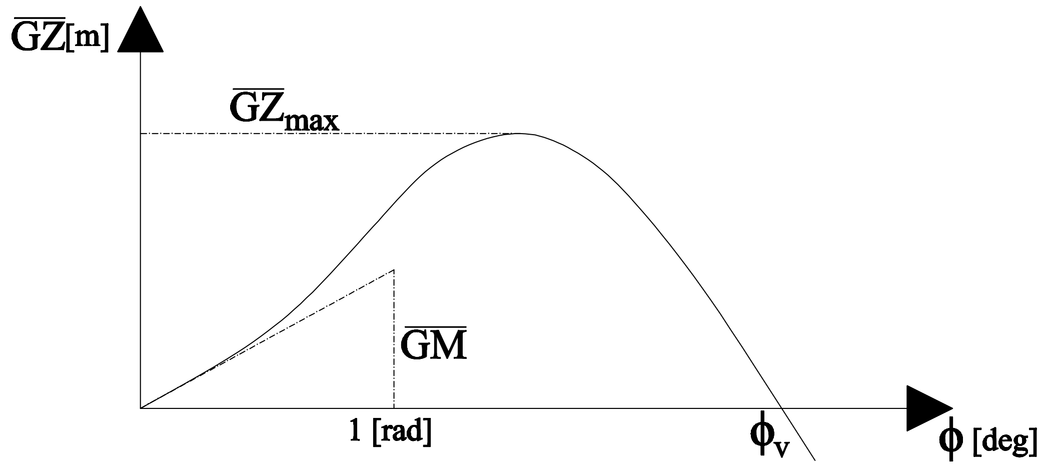


Figure 4.4: A typical $\overline{GZ}$-curve. $\overline{GM}$ is given by the slope at zero heel angle, $\overline{GZ}_{max}$ is where we have submergence of the deck corner and ϕ_v is called the angle of vanishing stability.

$\overline{GZ}_{max}$ corresponds approximately to where we have submergence of the deck corner. This is because the residual stability decreases. The angle where the curve becomes negative, ϕ_v, is called the angle of vanishing stability. This is where the ship will capsize.

The area under the $\overline{GZ}$-curve is an indication of how much energy that can be absorbed by the ship when it is rolling, i.e. dynamic stability (Amdahl et al., 2003). The rules and regulations regarding an intact ship are related to the $\overline{GZ}$-curve, i.e. the area, slope at zero heel, extension and maximum value. The general rules are decided by the International Maritime Organization (IMO) and can be found in IMO (1993). In addition, there are some rules that applies for Norwegian fishing vessels related to the metacentric height and angle of vanishing stability. These rules are set by the Norwegian Maritime Directorate (NMD) (Ellingsen & Endal, 2007) and are to a large extent based on the The Torremolinos International Convention for the Safety of Fishing Vessels from 1977 (Torremolinos, 1977). The area under the $\overline{GZ}$-curve should exceed 0.055 mrad for a heel angle of $30°$, and 0.09 mrad for a heel angle of $40°$ or the flooding angle, ϕ_f, if this is less. By flooding angle we mean the angle where water starts to enter through openings that are not water tight (IMO, 1993). In addition the area should be no less than 0.03 mrad between $30°$ and $40°$ or ϕ_f whichever less. Further the maximum $\overline{GZ}$-arm should occur at a heel angle larger than $25°$ and at $30°$ it should be at least 0.20 m. For Norwegian fishing vessels, the metacentric height should be no less than 0.35 m and it should have a positive $\overline{GZ}$-arm up to a heel angle of $80°$, i.e. the angle of vanishing stability should be larger than $80°$. These minimum requirements are summarized in tab 4.1.

Table 4.1: Minimum intact stability requirements for a Norwegian fishing vessel according to IMO (1993) and Torremolinos (1977).

Minimum intact stability requirements		
Area under $\overline{GZ}$-curve up to 30°	>	0.055 mrad
Area under $\overline{GZ}$-curve up to 40° or ϕ_f	>	0.09 mrad
Area under $\overline{GZ}$-curve between 30° - 40° or ϕ_f	>	0.03 mrad
Heel angle at maximum $\overline{GZ}$	>	25°
$\overline{GZ}$ at a heel angle of 30°	>	0.20 m
$\overline{GM}$, 100 % loaded	>	0.35 m
Angle of vanishing stability	>	80°

4.2.1.1 Calculating the $\overline{GZ}$-curve

In section 4.1.1 we considered the restoring moment by change of volume distribution when the vessel is heeling, which led to the important conclusion that the second moment of area of the water plane is important for stability. Another way of considering the restoring moment is by integrating the pressure over the submerged part of the hull when forcing the vessel to heel. The static restoring moment in roll for calm water is given similar as in equation (2.54) as the **i**-component of

$$M_R = -\iint_S p_{stat}\left(\mathbf{r}\times\mathbf{n}\right)\mathrm{d}S, \tag{4.15}$$

where p_{stat} is the hydrostatic pressure given as

$$p_{stat} = -\rho g Z. \tag{4.16}$$

Here Z is the vertical distance from the mean free surface. Since we are interested in the $\overline{GZ}$-arm, we need to take the restoring moment around the center of gravity. In equation (4.15) $\mathbf{r}$ is therefore the coordinates of the hull surface relative to the COG in a body-fixed coordinate system and $\mathbf{n}$ the unit normal vector in a body-fixed coordinate system, see figure 4.5. We write the $\mathbf{r}$-vector as

$$\mathbf{r} = \left[x', y', z' + z_G\right], \tag{4.17}$$

where x', y' and z' are coordinates in the body-fixed coordinate system. It is essential that we are able to transform the coordinates from the body-fixed coordinate system to the global coordinate system given in figure 1.3. From the figure we can derive the following relationships when the section rotates around the x-axis (Faltinsen & Timokha, 2009).

$$y = y'\cos\phi - z'\sin\phi, \tag{4.18}$$
$$z = y'\sin\phi + z'\cos\phi. \tag{4.19}$$

If we rewrite the **i**-component of equation (4.15) we obtain the following expression for the restoring moment in calm water

$$M_R = \rho g \iint_S \left(y' n_3 - \left(z' + z_G\right) n_2\right) Z\mathrm{d}S. \tag{4.20}$$

Here Z is given by the transformation in equation (4.19). Having obtained the restoring moment we divide by $\rho g \nabla$ in order to get $\overline{GZ}$, see equation (4.13).

The resulting $\overline{GZ}$-curve by using this procedure on Trønderhav is shown in figure 4.6. From the slope at zero heel angle we find that the $\overline{GM}$ value is 0.49 m.

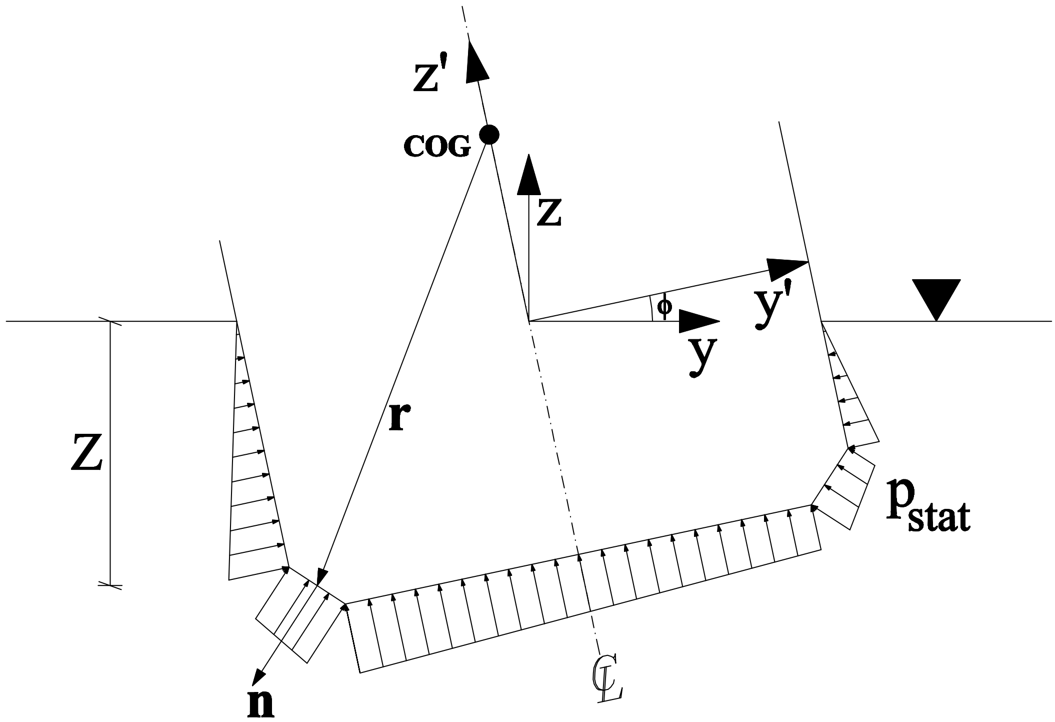


Figure 4.5: Calculation of restoring moment in calm water by integrating the pressure over the wetted surface. We measure the moment about the center of gravity. The distance from the center of gravity to the hull surface in a body-fixed coordinate system is denoted $\boldsymbol{r}$ and the unit normal vector of the surface is $\boldsymbol{n}$.

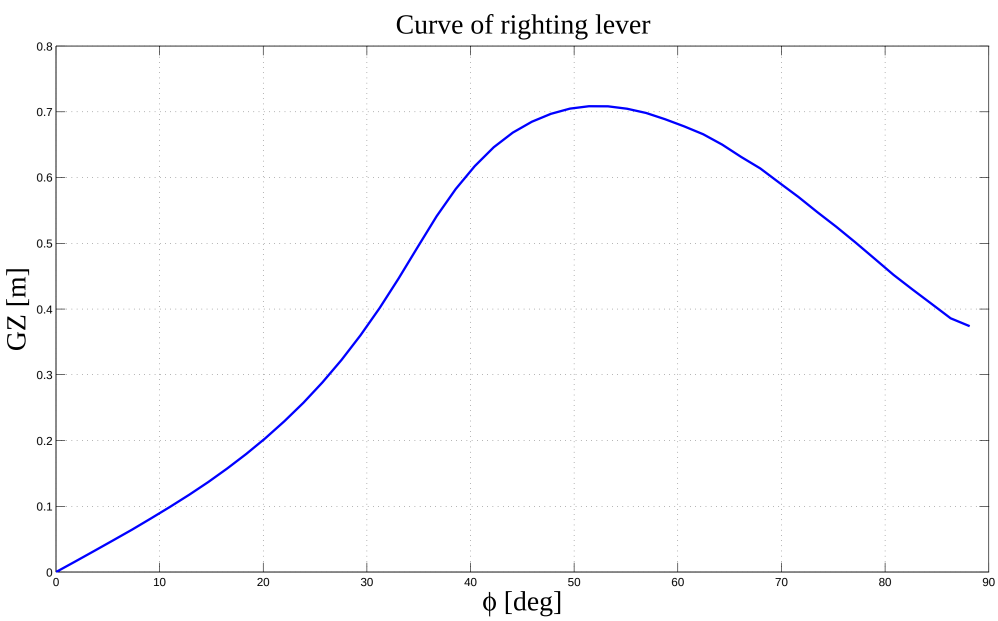


Figure 4.6: $\overline{GZ}$-curve for the calm water case.

However, the procedure described above is not the whole truth. Because, when the ship is heeling the longitudinal pressure distribution will change since the ship not has fore-aft symmetry. This will lead to an unbalanced moment in trim and an unbalanced force in heave. The vessel will therefore in reality change both its sinkage and trim when heeling. We have not taken this into account in this project. The vessel rotates around a fixed axis coaxial to the x-axis, and that is a simplification. If we should have taken this into account we would need an iteration procedure in two dimensions, and this would make the computer program slower. In commercial software for hydrostatic calculations it is common to take into account the imbalance in vertical forces, so the calculations are performed with constant volume displacement. The imbalance in trim moment is also usually taken into account, but not always (Sillerud, 2010, personal comm.). In the software called Hydromax, a part of Maxsurf, it is possible to choose whether one want to calculate the $\overline{GZ}$-curve with or without free trim, i.e. taking the trim moment into account or not. Fixed trim is a faster method than free trim, but this method tend to over predict $\overline{GZ}$ (Formsys, 2009). We will in section 5.4 describe how we can calculate the non-linear restoring moment in roll for the vessel moving in waves.

Having obtained the $\overline{GZ}$-curve we can analyze it and compare the results with the stability requirements given in table 4.1. This is summarized in table 4.2. We see that all the intact stability requirements are

Table 4.2: Comparison of the actual stability and the stability requirements.

Minimum intact stability requirements			Value	Status
Area under $\overline{GZ}$-curve up to 30°	>	0.055 mrad	0.081 mrad	OK
Area under $\overline{GZ}$-curve up to 40° or ϕ_f	>	0.09 mrad	0.17 mrad	OK
Area under $\overline{GZ}$-curve between 30° - 40° or ϕ_f	>	0.03 mrad	0.086 mrad	OK
Heel angle at maximum $\overline{GZ}$	>	25°	52°	OK
$\overline{GZ}$ at a heel angle of 30°	>	0.20 m	0.61 m	OK
$\overline{GM}$, 100 % loaded	>	0.35 m	0.49 m	OK
Angle of vanishing stability	>	80°	> 90°	OK

fulfilled with good margin. But it is not for sure that this is enough to avoid parametric roll resonance. We will come back to this in section 5.5.

Chapter 5

Parametric Roll Resonance

As we mentioned in section 1.1, the stability will vary when the ship is moving in waves. In reality this means that the metacentric height, and hence $\overline{GZ}$, is varying with time. This means that the restoring coefficient in roll in the equation of motion also will vary with time. In this chapter we will show how we can calculate the restoring moment in roll as a function of heave, roll, pitch and time. In addition we will simulate the ship motions for different forward speeds and wave headings and try to determine dangerous areas with respect to parametric roll resonance.

5.1 Variation of the metacentric height

A common assumption in the literature is that the metacentric height has a sinusoidal variation when the ship is moving in regular waves (Shin et al., 2004; Gunsing & Dallinga, 2010; Moideen & Falzarano, 2010). This enables us to model the resonance by the Mathieu equation, as will be further discussed in section 5.1.1. However, the variation in stability is caused by the ship and wave motions, i.e. the relative position between the ship and waves, and a ship is usually not symmetric around the mean water plane. Hence the assumption that the metacentric height varies harmonically is questionable (Moideen & Falzarano, 2010). A qualitative example of how the variation of the metacentric height could look like is shown in figure 5.1. We see that the variation is periodical, but it is not symmetric about the mean value. Both its shape and value is different at each side.

5.1.1 Mathieu type of instability

If we for simplicity assume that the variation of the metacentric height is harmonic, even though this is not correct, we may write the restoring coefficient in roll as

$$C_{44}(t) = \rho g \nabla \left(\overline{GM} + \delta \overline{GM} \sin(\omega_e t) \right), \tag{5.1}$$

where $\delta\overline{GM}$ is the amplitude of the variation. If we also assume that the roll motion is uncoupled from sway and yaw and insert equation (5.1) into the uncoupled equation of motion in roll for head or following waves, we get the Mathieu equation, see Faltinsen (2005).

$$(I_{44} + A_{44}) \ddot{\eta}_4 + B_{44}\dot{\eta}_4 + \rho g \nabla \left(\overline{GM} + \delta\overline{GM} \sin(\omega_e t) \right) \eta_4 = 0 \tag{5.2}$$

Equation (5.2) can also be written as (Faltinsen, 2005)

$$\ddot{\eta}_4 + 2\xi\omega_n\dot{\eta}_4 + \omega_n^2 \left(1 + \frac{\delta\overline{GM}}{\overline{GM}} \sin(\omega_e t) \right) \eta_4 = 0, \tag{5.3}$$

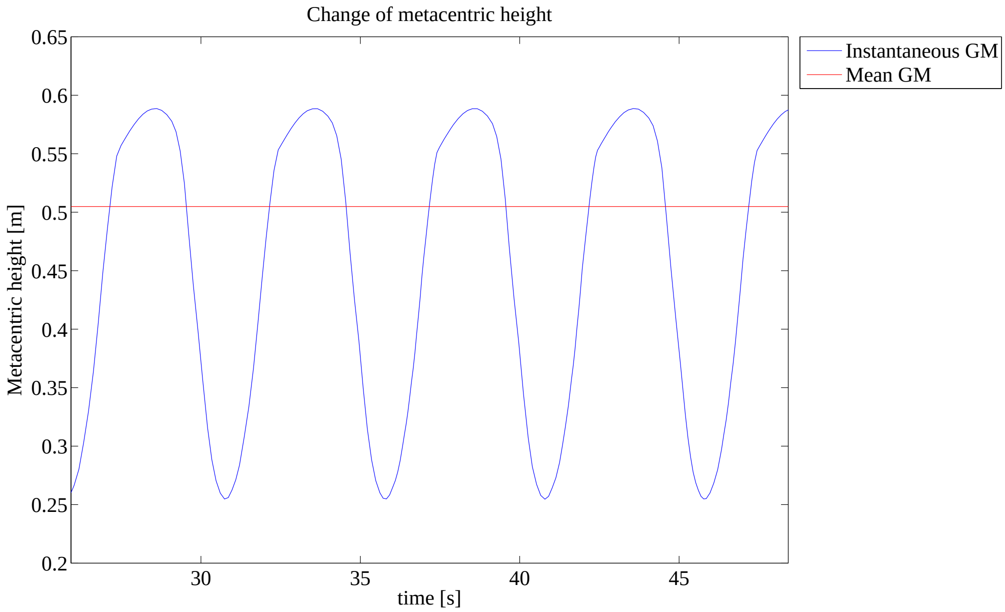


Figure 5.1: Change of $\overline{GM}$ as a function of time.

where ξ is the damping ratio

$$\xi = \frac{B_{44}}{2\sqrt{(I_{44} + A_{44})\,\rho g \nabla \overline{GM}}}, \tag{5.4}$$

and ω_n is the undamped natural frequency in roll

$$\omega_n = \sqrt{\frac{\rho g \nabla \overline{GM}}{I_{44} + A_{44}}}. \tag{5.5}$$

When solving equation (5.3), we can get an unstable solution, meaning that the resulting motion, in this case η_4, grows with increasing time. Whether we get instability or not depends on the values of ω_n/ω_e, $\delta\overline{GM}/\overline{GM}$ and ξ. It is possible to plot the stable and unstable domains as a function of these values. This diagram, called Ince-Strutt diagram (Moideen & Falzarano, 2010), is shown in figure 5.2.

We see from figure 5.2 that we may get an unstable solution, meaning parametric roll resonance, when the period of encounter approximately equals a half number of the natural period in roll, i.e. $T_e/T_n \approx 0.5, 1, 1.5$ and so. We also see that high damping and having a good initial stability, i.e. high $\overline{GM}$, is favorable in order to avoid this.

5.1.2 Physical explanation

In the following section we will try to give a simplified, but physical and qualitative explanation of why we may get resonance when $T_e/T_n \approx 0.5$. Imagine a ship moving at forward speed in head sea, with a wave encounter period corresponding to half its natural period in roll. When the ship is passing a wave crest, meaning wave crest at midships, the metacentric height has its minimum. If the ship now gets a small disturbance that causes a heel angle, it will heel one quarter of a roll period by the time it is passing the following wave trough. This is because half a wave encounter period corresponds to a quarter of the natural period in roll. Now the metacentric height has its maximum, and the heel angle, which also has its maximum, cause a restoring moment that uprights the ship. The large metacentric

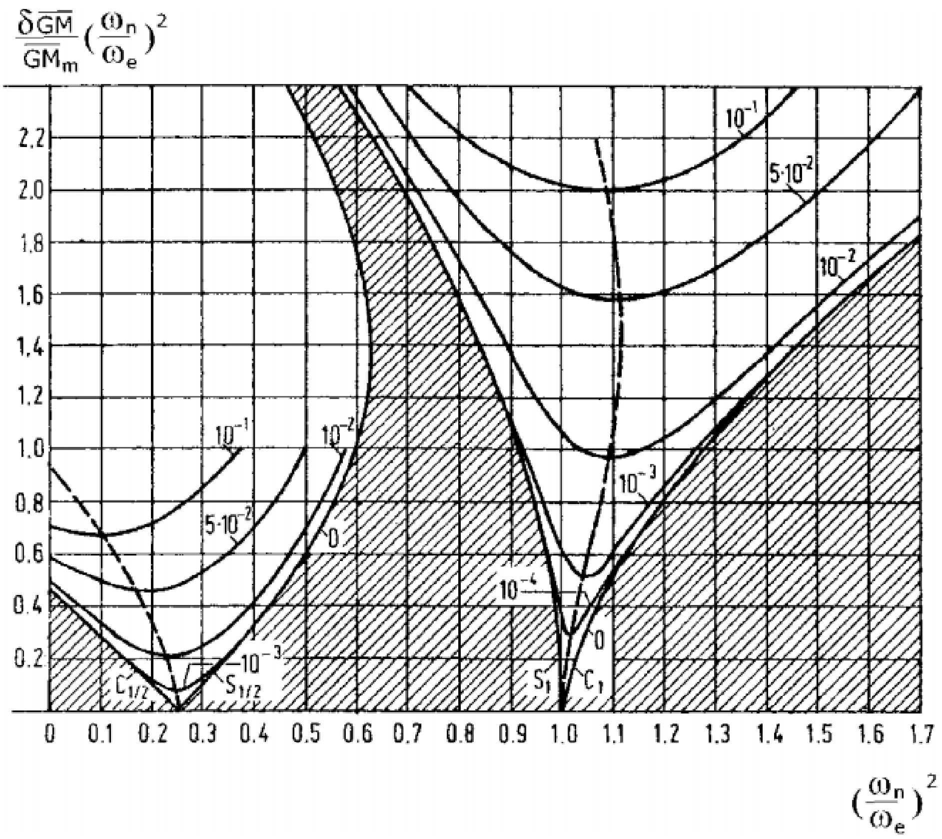


Figure 5.2: Stability diagram showing where the Mathieu equation gives stable or unstable solutions. The shaded areas correspond to the stable domain when $\xi = 0$. The "contour lines" show the boundaries between stable and unstable domains for different values of $\xi(\omega_n/\omega_e)$. The figure is from Faltinsen (2005).

height will contribute to make this restoring moment larger than the restoring moment for the same heel angle in calm water. By the time the ship passes the next wave crest, it will have heeled back one quarter of a roll period and the metacentric height is now at a minimum again. Due to conservation of energy the ship will continue to heel, and the story repeats itself for the next two quarter of roll periods. We see that for every wave trough the restoring moment reaches its maximum. This happens with a period corresponding to the natural period in roll, since there is half a roll period between each wave trough. The variation of the restoring moment will hence serve as a excitation and drive the roll motion.

5.2 Natural frequencies

As we saw in section 5.1.1, the ratio between the natural frequency in roll and the frequency of encounter is of importance. We will in this section calculate the undamped natural frequencies or periods for the vessel, both in coupled sway-roll-yaw and coupled heave-pitch.

5.2.1 Sway-roll-yaw

The natural frequency of a mass-spring system is the frequency the system will oscillate with when it is oscillating freely, i.e. without excitation. The damping will have little influence on the natural frequency, so we will disregard this here. Since we have a coupled system, we cannot calculate the natural frequency of one single mode of motion by itself, but we get the natural frequency of the total coupled motion. If we write equations (3.5) to (3.7) without damping and excitation and on matrix form we obtain

$$\begin{bmatrix} -\omega^2\left(M+A_{22}\right) & -\omega^2\left(-Mz_G+A_{24}\right) & -\omega^2 A_{26} \\ -\omega^2\left(-Mz_G+A_{42}\right) & -\omega^2\left(A_{44}+I_{44}\right)+C_{44} & -\omega^2\left(A_{46}-I_{46}\right) \\ -\omega^2 A_{62} & -\omega^2\left(A_{64}-I_{46}\right) & -\omega^2\left(A_{66}+I_{66}\right) \end{bmatrix} \begin{bmatrix} \bar{\eta}_2 \\ \bar{\eta}_4 \\ \bar{\eta}_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (5.6)$$

where we have used the assumption that the response is harmonic, as stated in equation (3.19). The only non-trivial solution of equation (5.6) is when the determinant of the coefficient matrix is zero, and that will give us the natural frequency. The determinant for zero forward speed is plotted in figure 5.3. The natural period in coupled sway-roll-yaw, T_{n246}, as function of forward speed at a wave frequency

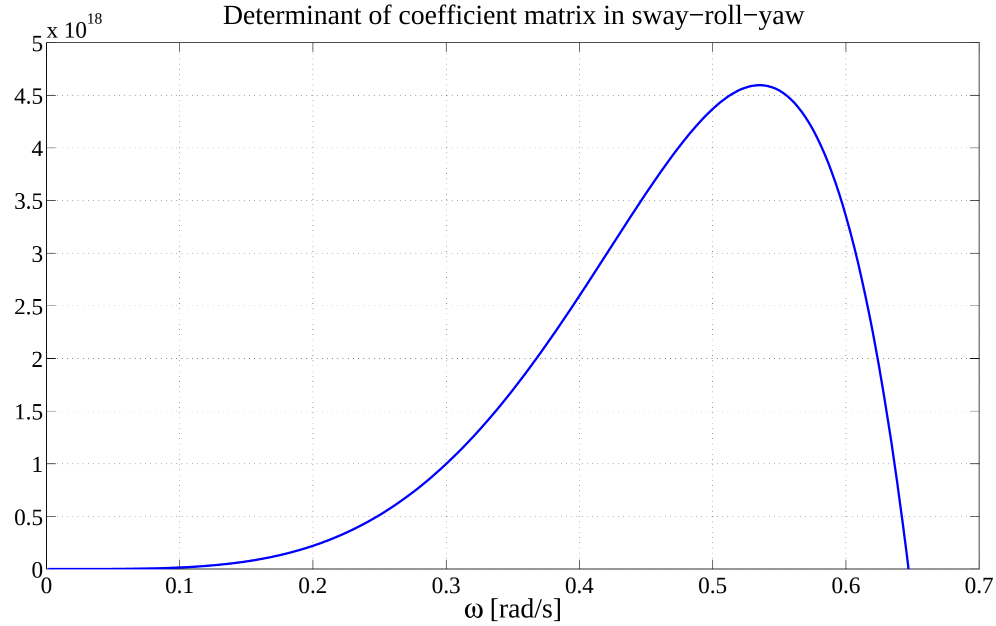


Figure 5.3: Determinant of the coefficient matrix in coupled sway-roll-yaw for zero forward speed. We find the natural frequency when this determinant is equal to zero.

of 7 s in head sea is shown in figure 5.4. We see that it is almost constant over the speed range for this particular sea state.

5.3 Heave-pitch

To find the natural frequencies in heave-pitch we proceed the same way as we did for sway-roll-yaw. The undamped, unexcited equations of motion on matrix form becomes

$$\begin{bmatrix} -\omega^2\left(M+A_{33}\right)+C_{33} & -\omega^2 A_{24}+C_{35} \\ -\omega^2 A_{53}+C_{53} & -\omega^2\left(I_{55}+A_{55}\right)+C_{55} \end{bmatrix} \begin{bmatrix} \bar{\eta}_3 \\ \bar{\eta}_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (5.7)$$

When we set the determinant of the coefficient matrix equal to zero we get an equation on the form

$$a\omega^4 + b\omega^2 + c = 0, \quad (5.8)$$

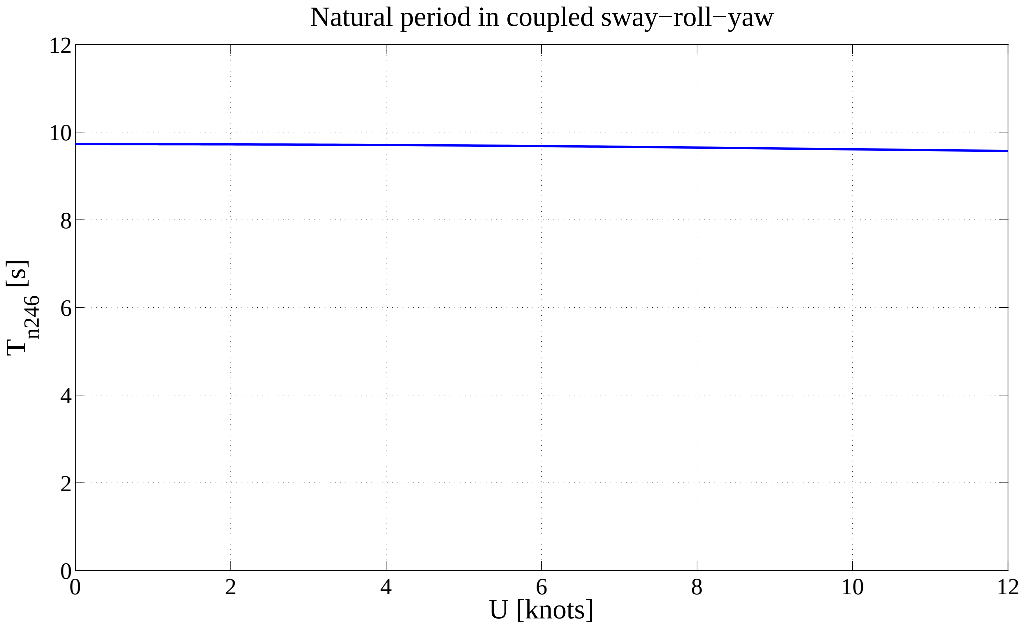


Figure 5.4: Natural period in coupled sway-roll-yaw.

where

$$\begin{aligned} a &= (M + A_{33})\,(I_{55} + A_{55}) - A_{35}A_{53}, \\ b &= -\,(M + A_{33})\,C_{55} - (I_{55} + A_{55})\,C_{33} + A_{35}C_{53} + A_{53}C_{35}, \\ c &= C_{33}C_{55} - C_{35}C_{53}. \end{aligned} \tag{5.9}$$

Equation (5.8) is in reality a second order equation in ω^2. By substituting $\omega^2 = u$, we can solve the equation by the quadratic formula

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{5.10}$$

This will give us two solutions, and thereby two natural frequencies, which makes sense since we have restoring forces both in heave and pitch. This is shown in figure 5.5, where we have plotted the determinant of the coefficient matrix in equation (5.7) at zero forward speed. The natural periods in coupled heave-pitch, T_{n35}, as function of forward speed at a wave frequency of 7 s in head sea is shown in figure 5.6, and we clearly see that they are not constant.

Since the variation in stability is caused by the ship's motions in waves, it is interesting to compare the natural periods in heave-pitch to the natural period in sway-roll-yaw. In figure 5.7 we have plotted the ratio T_{n35}/T_{n246} as function of forward speed at a wave period of 7 s in head sea. We see that both ratios are close to or equal to 0.5 over the whole speed range. This means that when we have resonance in heave-pith, and hence maximum vertical motions, the danger of parametric resonance is at its largest. In other words, this vessel is expected to be very vulnerable to parametric roll resonance.

5.4 Restoring moment

In section 4.2.1.1 we saw how we could calculate the non-linear restoring moment for calm water. In order to reach a steady state roll motion during parametric resonance, it is essential to use the non-linear

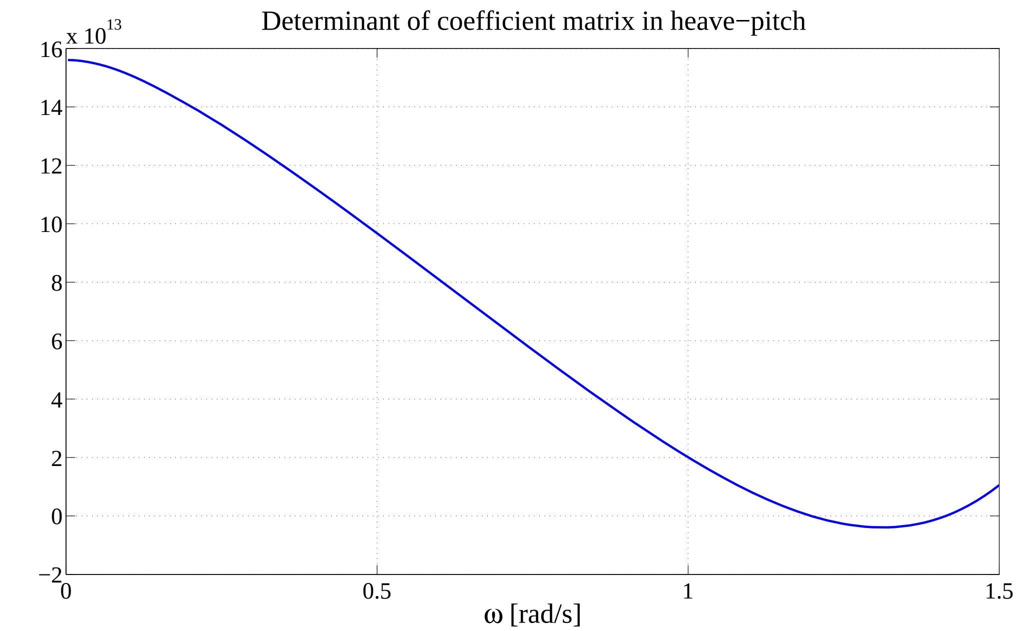


Figure 5.5: Determinant of the coefficient matrix in coupled heave-pitch for zero forward speed. We find the natural frequencies when this determinant is equal to zero.

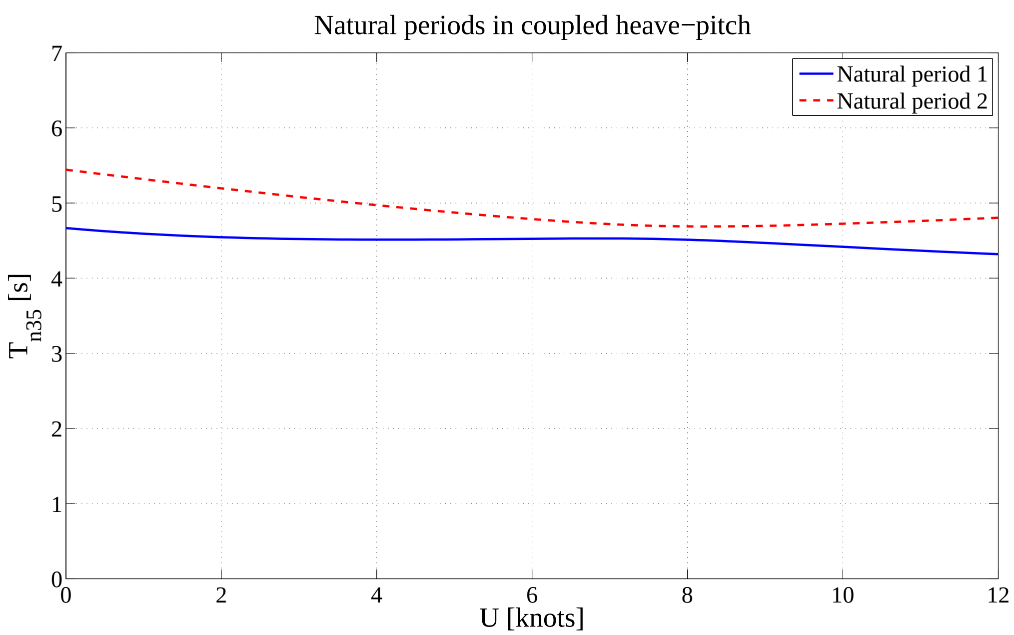


Figure 5.6: Natural periods in coupled heave-pitch.

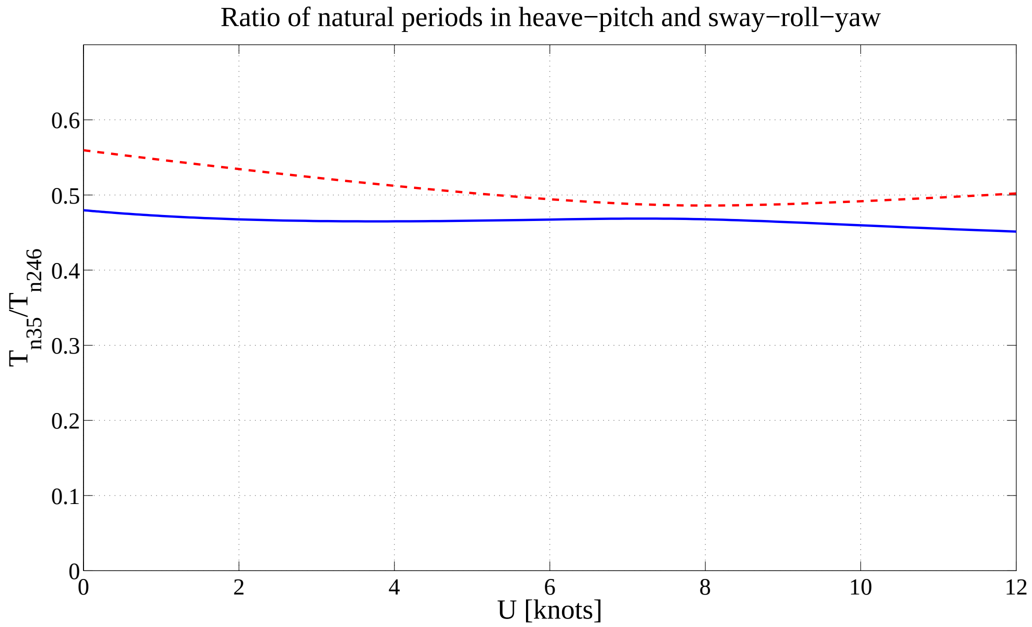


Figure 5.7: Ratios of the natural periods in heave-pitch and sway-roll-yaw. The ratios are close to 0.5 which will give the vessel maximum vertical motions when the danger of parametric resonance is largest.

restoring moment in roll in equation (3.9), otherwise the roll amplitude will go to infinity (Shin et al., 2004). We calculate the restoring moment for the ship moving in waves the same way we did it for the calm water case. We substitute the hydrostatic pressure in equation (4.15) with the total pressure given by equation (1.18)

$$M_R = -\iint_S p\,(\mathbf{r} \times \mathbf{n})\,\mathrm{d}S, \tag{5.11}$$

A ship section in waves is shown in figure 5.8. To find the instantaneous wetted surface, we need to find where each section cuts the free surface. We find the z-position from the relative position between the wave and the vertical position of the ship and use this to interpolate the y-position between the two offset points on either side of the this position. The motion of any point at the ship is given as (Faltinsen, 1990)

$$\mathbf{s} = \eta_1\mathbf{i} + \eta_2\mathbf{j} + \eta_3\mathbf{k} + \boldsymbol{\omega} \times \mathbf{r}. \tag{5.12}$$

Here $\boldsymbol{\omega}$ is the vector containing the rotational degrees of freedom and is written as

$$\boldsymbol{\omega} = \eta_4\mathbf{i} + \eta_5\mathbf{j} + \eta_6\mathbf{k}. \tag{5.13}$$

Further $\mathbf{r}$ is the position of the given point relative to the origin. Hence equation (5.12) can be written as

$$\mathbf{s} = (\eta_1 + z\eta_5 - y\eta_6)\,\mathbf{i} + (\eta_2 - z\eta_4 + x\eta_6)\,\mathbf{j} + (\eta_3 + y\eta_4 - x\eta_5)\,\mathbf{k}. \tag{5.14}$$

We are interested in the vertical component of the motion. The z-position we are looking for can then be written as

$$z = \zeta - (\eta_3 + y\eta_4 - x\eta_5) = \Re\left(-i\zeta_a e^{-ik(x\cos\beta + y\sin\beta)}e^{i\omega_e t}\right) - (\eta_3 + y\eta_4 - x\eta_5)\,. \tag{5.15}$$

This means that the non-linear restoring moment becomes a function of heave, roll, pitch and time, $M_R = M_R\,(\eta_3, \eta_4, \eta_5, t)$, and this is the key to be able to catch the phenomenon parametric resonance. We see that the z-position is dependent of the y-position, which we are going to find by interpolating the

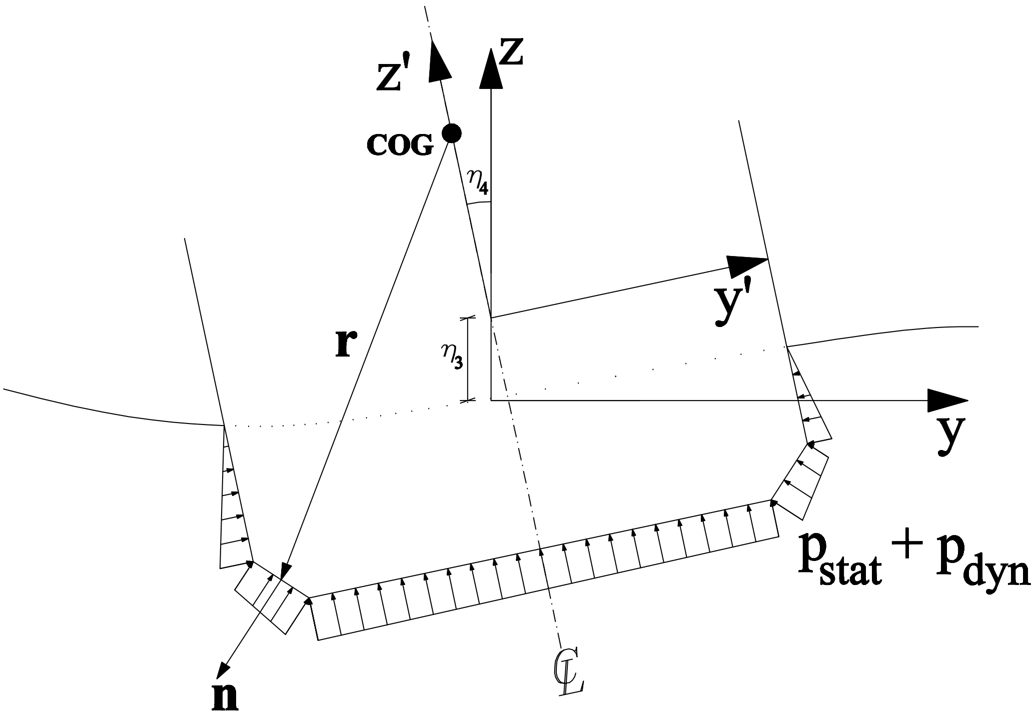


Figure 5.8: Calculation of restoring moment in waves. We measure the moment about the center of gravity. The restoring moment is a function of the vessel's position in waves.

z-position. This means that we need to iterate in order to find the right positions. However, this will slow down the program further, so we approximate the y-value in the wave elevation in equation (5.15) with the half-beam in the mean water line without the vessel moving, i.e. $\zeta = \eta_3 = \eta_4 = \eta_5 = 0$. This is not correct, but since the slope of the wave is not to large, we think this assumption to a certain extent can be justified, and we are still able to catch the variations of the shape of the submerged hull. From figure 5.8 we also see that the sideways motion, i.e. sway and yaw will affect the pressure distribution around the hull, and hence the restoring moment. We have made a simplification in this project and neglected this fact when calculating the restoring moment. This can also be seen from figure 5.8. The reason is the same as for using the mean breadth to find the wave elevation; the wave slope is small. Having found the y- and z-positions of where the hull cuts the free surface, we know the limits in equation (5.11). The coupled equation on roll now becomes

$$\begin{aligned}(-Mz_G + A_{42})\,\ddot{\eta}_2 + B42\dot{\eta}_2 + (A_{44} + I_{44})\,\ddot{\eta}_4 + B_{44}\dot{\eta}_4 + B_v|\dot{\eta}_4|\dot{\eta}_4 \\ + M_R(\eta_3, \eta_4, \eta_5, t) + (A_{46} - I_{46})\,\ddot{\eta}_6 + B_{46}\dot{\eta}_6 = F_4 e^{\omega_e t},\end{aligned} \tag{5.16}$$

Since the restoring moment is dependent of the time and the motion of the vessel, we need to find the wetted surface for every time step, i.e. apply equation (5.15) together with equation (5.11) for every time step and this makes the computation extremely time consuming.

5.5 Simulation of parametric roll resonance

In this section we will solve the equations of motion in the time domain using the built-in Matlab function *ode45*. We will do this for different forward speeds, wave headings and wave periods in regular waves. For each wave period, we will present the results in terms of a polar diagram showing where we get parametric resonance or not for different forward speeds and wave headings. It is believed that this will give a good illustration of the dangerous areas and this way of presenting the results could serve as an aid for the crew on board the ship in order to avoid resonance. Due to time consuming calculations we only do the analyses for one wave height and loading condition, although both full load, partial load and ballast condition at different wave heights are of concern and hence very relevant. We use the loading condition as given in table 1.1. The wave height and wave periods used are shown in table 5.1

Table 5.1: Sea states used in the simulations

Wave period, T_0 [s]	Wave height, $2\zeta_a$ [m]
7.0	2
7.5	2
8.0	2

5.5.1 Wave period 7 s, wave height 2 m

We choose a sea state with a wave height of 2 m and a wave period of 7 s. This corresponds to a wave length of about 76 m which is approximately three times the ship length. We run simulations for many forward speeds and wave directions in order to distinguish between where we get parametric resonance or not. The result is shown in figure 5.9. We see that the area where we get parametric resonance is quite wide. We get parametric resonance at wave headings up to 70°. The span of forward speeds is also quite large, we get parametric resonance between approximately 4.5 and 12 knots in head sea. The ratio T_e/T_n as function of forward speed for wave headings up to 70° is shown in figure 5.10. Here we see that the range of T_e/T_n spans from approximately 0.62 to .47, and that this range increases with increasing wave heading. We hence see that there is danger of parametric resonance even if T_e/T_n is not exactly 0.5. The reason why we stop the forward speed at 12 knots is that this corresponds to a Froude number of 0.4, which is the fastest a displacement ship can go. When the Froude number is 0.4 the wave generated at the bow has the same length as the ship. Since the wave length is dependent of the wave velocity and hence the forward speed of the ship, a faster moving ship would cause the ship to make a longer wave than its own length. The ship will then feel like it is sailing uphill, and that is the reason why a Froude number of 0.4 is the maximum. The Froude number is given as

$$Fn = \frac{U}{\sqrt{gL_{PP}}}. \tag{5.17}$$

We also see in figure 5.9 that we get parametric resonance for following and quartering sea. Or at least resonance, because this area corresponds to T_e/T_n around 1, so this might as well be "normal" resonance. It is difficult to say what causes the large roll motions here, if it is the wave excitation or the variation of the restoring moment or a combination of these. Anyway, the area is dangerous and should be avoided in the same way as for head or bow sea. The range of T_e/T_n for the following and quartering waves is shown in figure 5.11. From this figure we can see that the span of T_e/T_n is somewhat larger than for the head or bow sea, spanning from 0.92 to 1.34. However, the speed range is lower, with the range spanning from 5.5 to 9.25 knots for following waves.

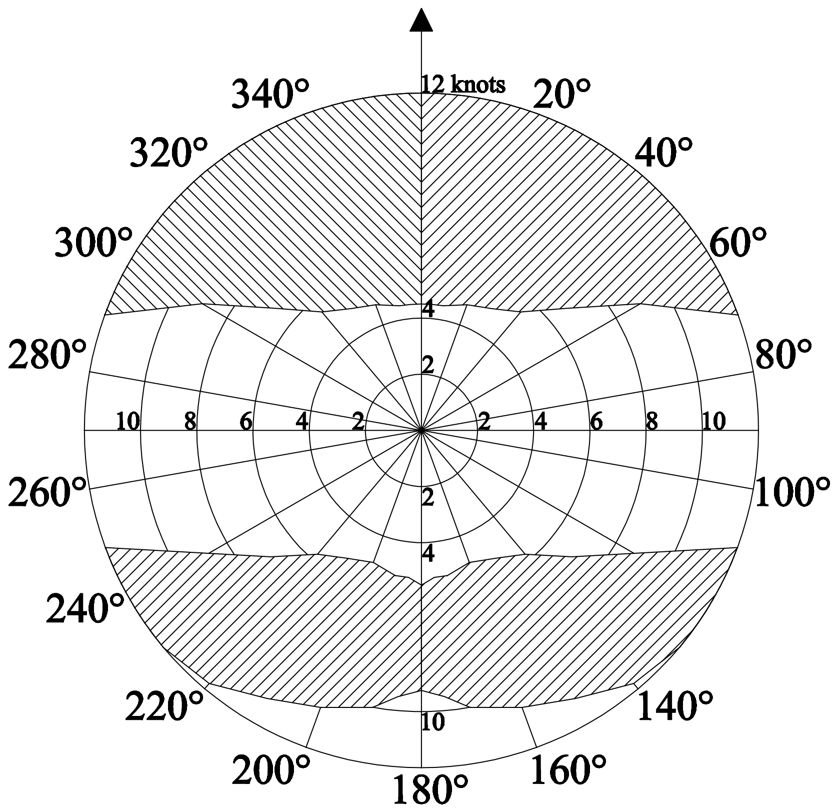


Figure 5.9: Shaded area shows which forward speeds and wave headings that are dangerous with respect to parametric roll resonance. Wave height of $2\ \mathrm{m}$ and wave period of $7\ \mathrm{s}$.

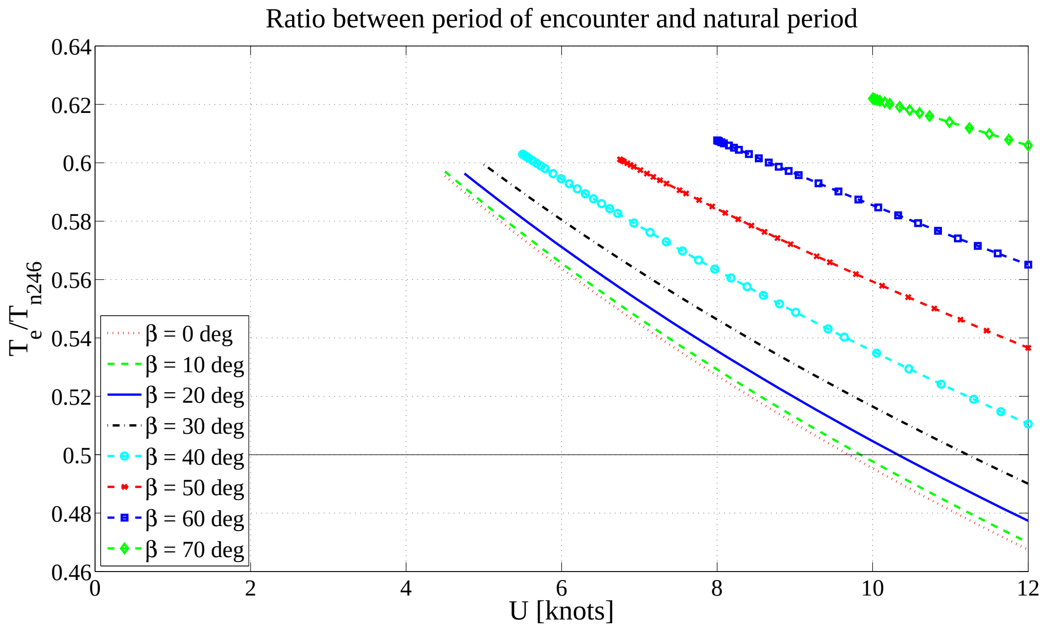


Figure 5.10: The ratio T_e/T_n as function of forward speed for wave headings up to $70°$. The wave period is $7\ \mathrm{s}$ and the wave height is $2\ \mathrm{m}$.

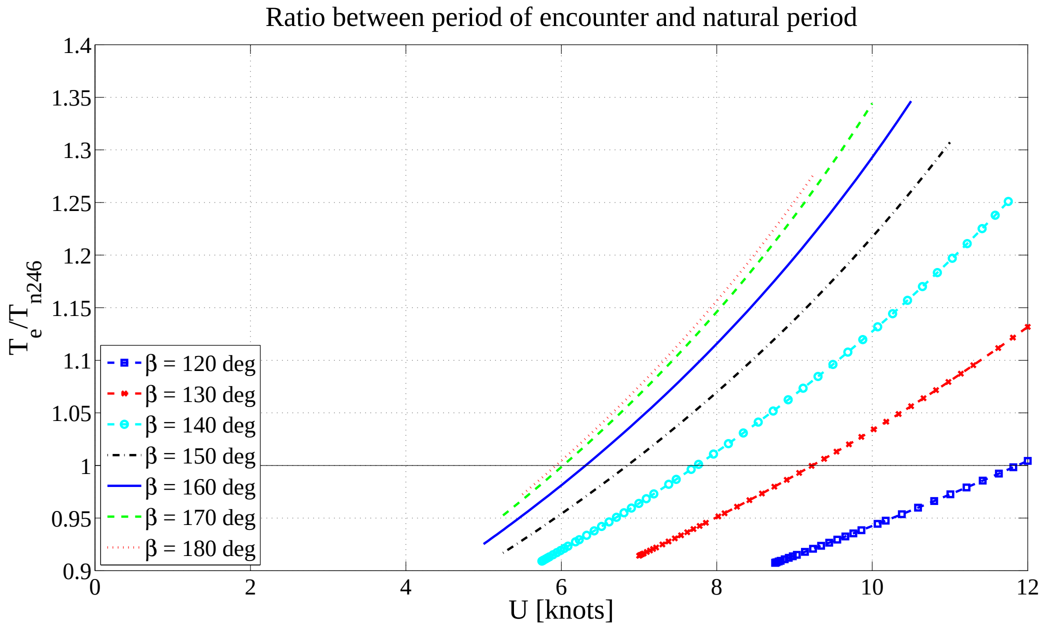


Figure 5.11: The ratio T_e/T_n as function of forward speed for wave headings between 110 and $250°$. The wave period is 7 s and the wave height is 2 m.

It is interesting to study the roll motion during parametric resonance closer. In figure 5.12 we have plotted the roll and pitch motion for a forward speed of 10 knots and a wave heading of $20°$. We see that the pitch motion is oscillating with a period of approximately 4.85 s or 1.29 rad/s, which corresponds to the frequency of encounter, see equation (1.13). The roll motion oscillates with the natural frequency, even though the excitation frequency in roll is the same as the frequency of encounter. This is because it is the variation of the restoring moment that is the driving force for the motion when we have triggered parametric resonance in roll. In figure 5.13 we have plotted the time series for all modes of motion for a forward speed of 10 knots and a wave heading of $20°$. We see that the sway and yaw motions both begin to oscillate with the frequency of encounter. When the resonance occurs, the yaw motion starts to oscillate with the same frequency as the roll motion. The sway motion continues to oscillate with the frequency of encounter as the zero-crossing frequency, but the maximum amplitude has the same frequency as the roll motion. The sway motion is therefore more influenced by the wave excitation than yaw. We also see that the sway and yaw motions are larger against one side than the other. This makes sense, because we have oblique waves. A selection of time series are shown in appendix F.

If we look at the roll amplitude during parametric resonance, we see that this increases with increasing forward speed. This is plotted in figure 5.14, where we have shown a time series for forward speeds ranging from 7 to 11 knots in head sea. We also see from these time series that the amplitude increases rapidly when the forward speed exceeds a certain level, in this case somewhere between 8 and 9 knots. Before this level the roll amplitude is small, even though it is resonance. This means that close to the lower speed limits in figure 5.9 the danger is not that large, but also that just a little increase in the forward speed may cause the vessel to roll violent. But if we see it the other way around, when large roll amplitudes has occurred, an efficient way to stop it is to slow down. We also from figure 5.9 that increasing the heading angle is an effective way to escape parametric resonance, but for a ship captain it may feel a little weird that bearing away should improve the roll motion. However, a fishing vessel of this kind often has a lot of engine power and is therefore highly maneuverable, so it should not be too hard to escape a situation where parametric resonance has occurred if the crew reacts the right way in time. On the other hand, then the vessel is rolling like it is shown in figure 5.14e and worse, it may

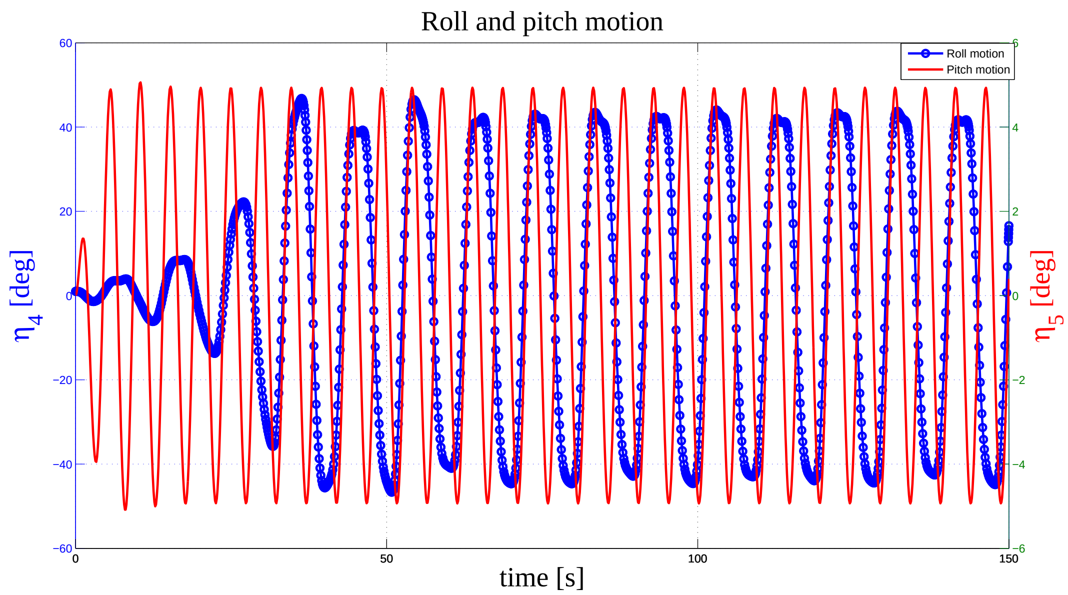
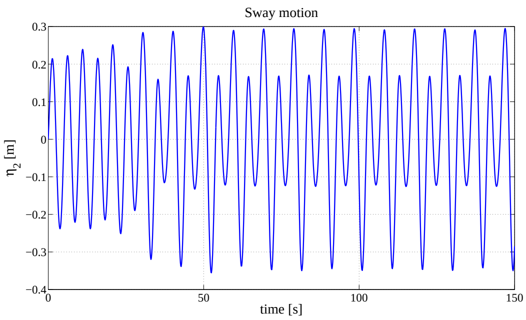
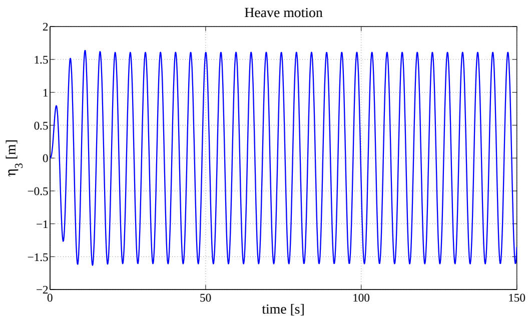


Figure 5.12: Time series of roll and pitch during parametric resonance. The pitch motion oscillates with the frequency of encounter, while the roll motion oscillates with the natural frequency. The forward speed is $10\ \mathrm{knots}$ and the wave heading 20°.

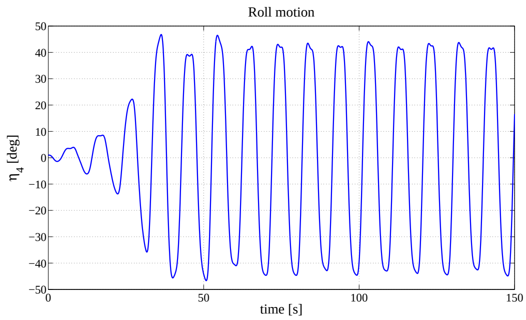
capsize. Parametric roll resonance is therefore a very dangerous situation and should by all means be avoided.



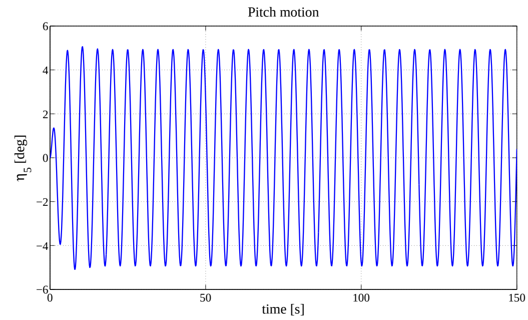
(a) Time series in sway.



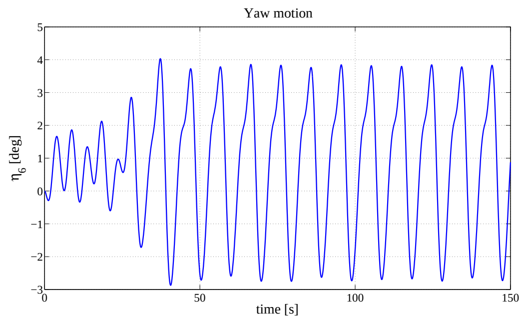
(b) Time series in heave.



(c) Time series in roll.

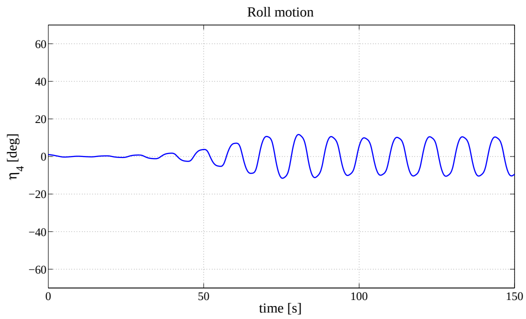


(d) Time series in pitch.

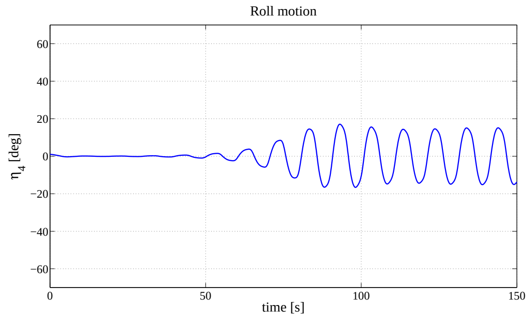


(e) Time series in yaw.

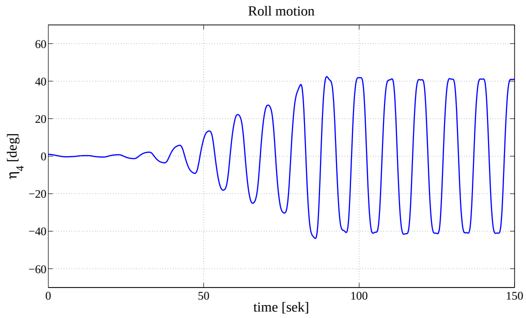
Figure 5.13: Time series of all modes of motion for a forward speed of 10 knots and a wave heading of $20°$. The mean sway and yaw amplitudes are not zero, due to oblique waves.



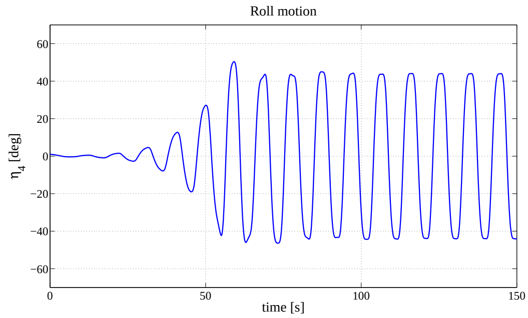
(a) $U = 7$ knots, $T_e/T_{n246} = 0.545$



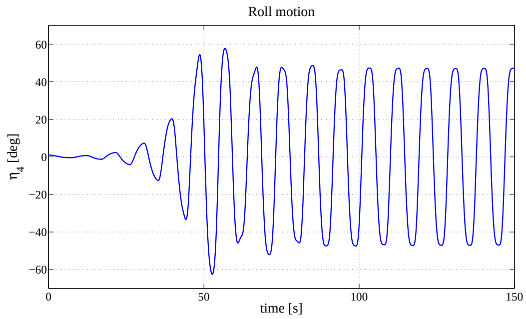
(b) $U = 8$ knots, $T_e/T_{n246} = 0.527$



(c) $U = 9$ knots, $T_e/T_{n246} = 0.511$



(d) $U = 10$ knots, $T_e/T_{n246} = 0.495$



(e) $U = 11$ knots, $T_e/T_{n246} = 0.481$

Figure 5.14: The roll amplitude during parametric resonance increases as the forward speed increases.

5.5.2 Wave period 7.5 s, wave height 2 m

We increase the wave period by 0.5 s and do the same as in the previous section. The result is shown in figure 5.15. We can see that both the speed range and the maximum wave heading in head or bow

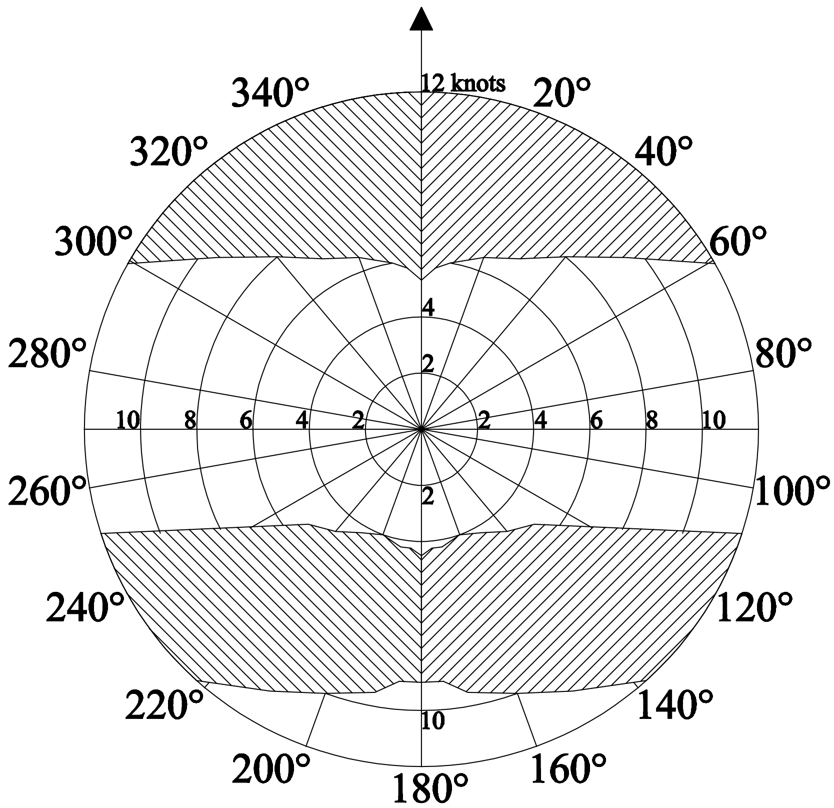


Figure 5.15: Shaded area shows which forward speeds and wave headings that are dangerous with respect to parametric roll resonance. Wave height of 2 m and wave period of 7.5 s.

sea are smaller than for the case where the wave period is 7 s. This can also be seen from figure 5.16 where the ratio T_e/T_n is plotted as a function of forward speed and wave headings. When it comes to following and quartering sea, the speed range is larger than for head and bow sea. It spans from 4.5 to 9 knots at an heading angle of $180°$, and the ratio t_e/T_n is plotted in figure 5.17.

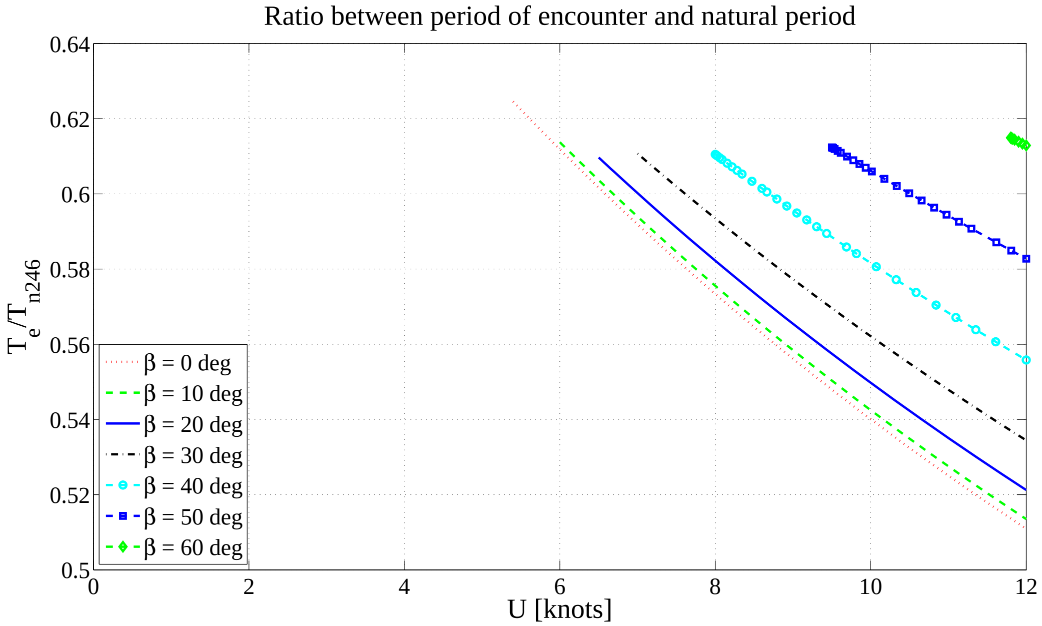


Figure 5.16: The ratio T_e/T_n as function of forward speed for different wave headings.

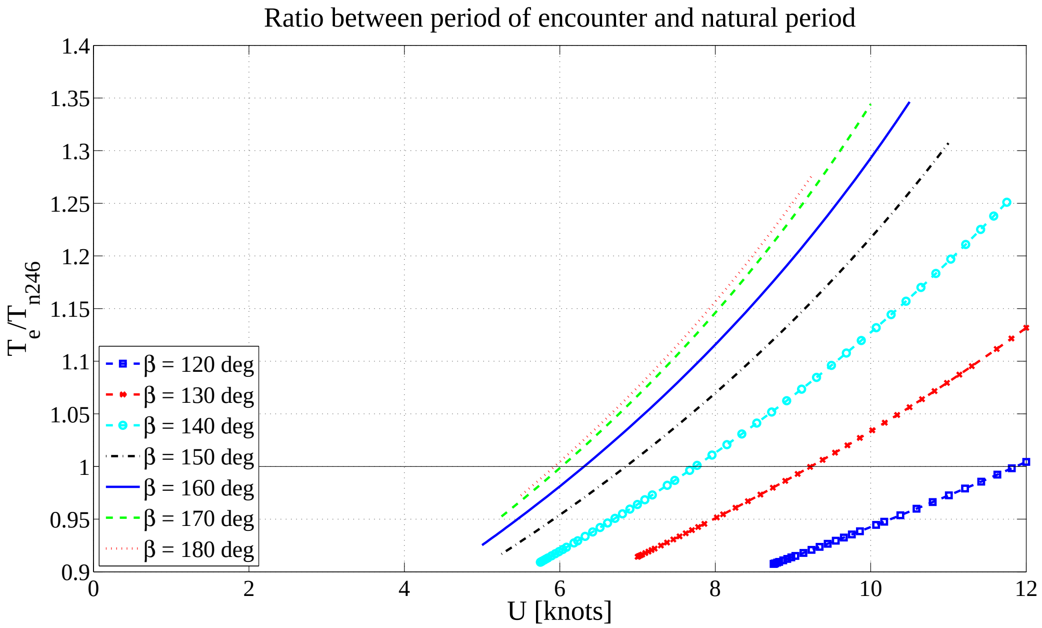


Figure 5.17: The ratio T_e/T_n as function of forward speed for wave headings between 110 and $250°$. The wave period is 7 s and the wave height is 2 m.

5.5.3 Wave period 8 s, wave height 2 m

The polar diagram when the forward speed is 12 knots and the wave period is 8 s is shown in figure 5.18. We see that the trend continues from the two previous cases; the speed range for head and bow sea

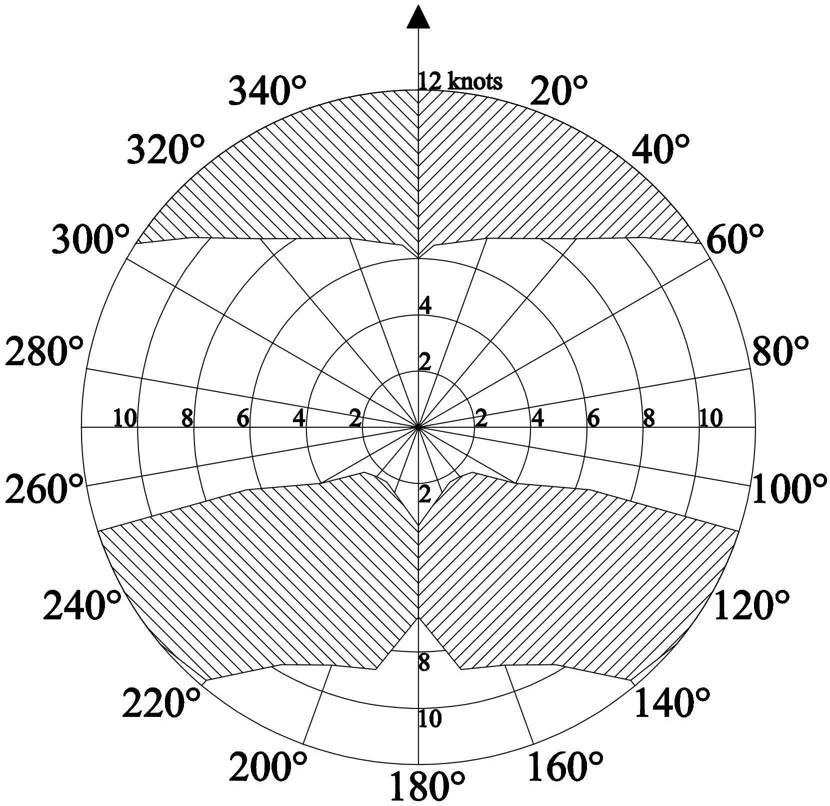


Figure 5.18: Shaded area shows which forward speeds and wave headings that are dangerous with respect to parametric roll resonance. Wave height of 2 m and wave period of 8 s.

decreases, while it increases for following and quartering sea.

As we mentioned in section 4.2.1.1, the vessel complies with all the intact stability rules. However, we have seen that this is not enough in order to avoid parametric roll resonance.

Chapter 6

Conclusions and Further Work

6.1 Conclusions

We have in this study made a mathematical model that can calculate the coupled linear ship motions for any wave heading in regular waves. This model is based on a strip theory approach, where the two dimensional added mass and damping coefficients for the ship sections are calculated beforehand by a separate program. These coefficients are used as input to the model. The linear motions are presented in terms of transfer functions. In order to catch the phenomenon parametric roll resonance, we have modified the restoring term in roll. This has been made non-linear and time varying by integrating the pressure over the instantaneous wetted hull surface. The imbalance in trim moment and vertical force when the ship is heeling is not taken into account and that is a simplification. Nor is the sideways motion's influence on the vessel's position in waves accounted for.

The model is used to simulate parametric roll resonance at different forward speeds and wave headings for wave periods of 7, 7.5 and $8\ \mathrm{s}$. The wave height is $2\ \mathrm{m}$ in all cases. We get parametric resonance both for head and bow sea, as well as following and quartering sea. A trend seems to be that the roll amplitude increases with increasing forward speed. This increase is at first slowly, but becomes rapid as we approach $T_e/T_n \approx 0.51-0.52$ for head and bow sea. The resonance here starts around $T_e/T_n \approx 0.6$. The speed range, and hence the area where we get resonance decreases with increasing wave period for head or bow sea and increases for following or quartering sea.

The roll motion oscillates with the natural frequency in coupled sway-roll-yaw during parametric resonance, even if the wave excitation moment in roll oscillates with the frequency of encounter.

Another important aspect, that concerns this particular vessel, is the ratio between the natural periods in coupled heave-pitch or pitch-heave and coupled sway-roll-yaw. It this ratio is close to or equal to 0.5, the vessel will have its maximum vertical motions, and hence a large change of metacentric height when the risk of parametric resonance is highest. This is the case for this vessel. Even if the vessel complies with the intact stability rules, this is not sufficient to avoid parametric roll resonance.

If parametric resonance has occurred, an effective way to escape it is to slow down and increase the heading relative to the waves.

6.2 Further work

Parametric roll resonance is a strong non-linear phenomenon that involves large roll amplitudes. We have assumed that the added mass, excitation forces, damping and restoring forces for all modes of motion except the restoring moment in roll to be linear. An improvement would might be to calculate all these non-linear quantities. The added mass and damping coefficients used here, were calculated beforehand by a separate program and then used as input into the current code. By implementing these calculation into the current program would improve the flexibility of the code. However, since the code is written in Matlab, it is very slow. It might hence be required to translate it into another, and faster programming language, such as Fortran or C.

When studying the roll motion, the roll damping is very important. In this text we have only accounted for the damping due bilge keels in addition to the potential damping. One should study the different damping components mentioned in section 2.1.2.1 further, to find out how much difference this would make. One should also quantify the importance of the initial stability.

Another very important source of roll damping that is not mentioned here is the passive free surface roll damping tank. The effect of this should be studied in detail. The use of this kind of tank, the hydrodynamics related to sloshing and fluid motions in tanks and the coupling with ship motions is extensively discussed by Faltinsen & Timokha (2009).

The restoring moment and hence the $\overline{GZ}$-curve is calculated by rotating the ship about a fixed axis. In reality the ship will both change its trim and sinkage when heeling. This should been taken into account, and that can be done by means of an iteration procedure with respect to sinkage and trim. The ship will also have sideways motions, i.e. sway and yaw and this will affect the ship's position in the waves, which again will affect the pressure distribution and hence the restoring moment. This should be accounted for in a future study.

We have also seen that we can get parametric resonance in oblique waves. The frequency of the motion is then governed by the restoring moment, and hence the natural frequency, even though we have an excitation moment with another frequency. It would have been interesting to compare the restoring moment in roll with the excitation moment to see if it is possible to find a connection of when we get parametric resonance or not.

When using strip theory, as done in this project, we miss out some three dimensional effects, specially towards the ends of the vessel. Fishing vessels often have a large bulbous bow and a very pronounced skeg at the stern. The three dimensional flow in these areas should be accounted for.

The way the polar diagrams are presented in this text, we only distinguish between parametric resonance and not parametric resonance. A better approach would be to include the roll amplitude in this diagram by shade of colors or similar, since we have small amplitudes towards the lower speed limit.

Parametric roll resonance may ultimately lead to capsizing. A closer study of the governing mechanisms related to capsizing should be performed. This may require an own project by itself.

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Appendix A

Roll damping due to bilge keels

The damping from bilge keels is divided into two contributions. One contribution from the normal force on the bilge keels, and one contribution from the pressure created by the bilge keels. In this appendix we will state the empirical formulas used to calculate this damping. The formulas below are taken from Ikeda & Tanaka (1976) and Ikeda et al. (1977).

A.1 Damping due to normal force on the bilge keels

Damping due to normal force on the bilge keels is given as.

$$B_{BK_N} = \frac{8}{3\pi}\rho r l \omega f^2 \left(22.5\frac{b^2 r}{\pi} + 22.40 b r^2 \eta_{4a}\right) \tag{A.1}$$

Here b is the breadth of the bilge keel, l is the length of the bilge keel and r is the distance from the roll axis to the bilge keel. The roll axis is in Ikeda & Tanaka (1976) taken to be the axis corresponding to the y-axis in the Cartesian coordinate system, even though it is not generally true that a ship rolls around a fixed axis, see Faltinsen (2005). Further f is a factor correcting for the velocity at the bilge keel, and this is given as

$$f = 1 + 0.3e^{-160(1-C_S)}. \tag{A.2}$$

Here C_S is the sectional area coefficient and is given by equation (2.65).

We also see that the last term in equation (A.1) is dependent on the roll amplitude. This means that we need an iterative procedure in order to solve the equation of motion, which makes this computational demanding.

A.2 Damping due to hull surface pressure created by the bilge keels

This contribution is given as

$$\frac{8}{3\pi}\rho r \eta_{4a} l \omega f^2 \frac{T^2}{2}\left(-AC_p^- + BC_p^+\right) \tag{A.3}$$

Here T is the mean draught of the ship. The pressure coefficients C_p^+ and C_p^- are given as

$$C_p^+ = 1.2, \tag{A.4}$$

$$C_p^- = -22.5\frac{b}{\pi f r \eta_{4a}} + 2.6. \tag{A.5}$$

Further, the coefficients A and B are given by

$$A = (m_3 + m_4)\, m_8 - m_7^2, \tag{A.6}$$

$$B = \frac{m_4^3}{3\,(H_0 - 0.215m_1)} + \frac{(1-m_1)^2\,(2m_3 - m_2)}{6\,(1 - 0.215m_1)} + m_1\,(m_3 m_5 + m_4 m_6)\,, \tag{A.7}$$

where

$$m_1 = \frac{R}{T}, \tag{A.8}$$

$$m_2 = \frac{z_G}{T}, \tag{A.9}$$

$$m_3 = 1 - m_1 - m_2 \tag{A.10}$$

$$m_4 = H_0 - m_1 \tag{A.11}$$

$$m_5 = \frac{0.414H_0 + 0.0651m_1^2 - (0.382H_0 + 0.0106)\,m_1}{(H_0 - 0.215m_1)\,(1 - 0.215m_1)}, \tag{A.12}$$

$$m_6 = \frac{0.414H_0 + 0.0651m_1^2 - (0.382 + 0.0106H_0)\,m_1}{(H_0 - 0.215m_1)\,(1 - 0.215m_1)}, \tag{A.13}$$

$$m_7 = \begin{cases} \frac{S_0}{d} - 0.25\pi m_1 & \text{if } S_0 > 0.25\pi R \\ 0 & \text{if } S_0 \leq 0.25\pi R, \end{cases} \tag{A.14}$$

$$m_8 = \begin{cases} m_7 + 0.414m_1 & \text{if } S_0 > 0.25\pi R \\ m_7 + \sqrt{2}\,(1 - \cos(S_0/R))\,m_1 & \text{if } S_0 \leq 0.25\pi R. \end{cases} \tag{A.15}$$

Here R is the bilge radius, in our case equal to 0, and H_0 is half the beam-to-draught ratio. Further S_0 is the constant pressure distribution length, given as

$$S_0 = 0.3\pi f r \eta_{4a} + 1.95b. \tag{A.16}$$

Hence the total viscous damping due to bilge keels can be written as

$$B_{v_{BK}} = \frac{8}{3\pi}\rho r^2 \omega \eta_{4a} f^2 l \left[rbC_D + \frac{T^2}{2}\left(-AC_p^- + BC_p^+\right)\right], \tag{A.17}$$

where C_D is given as

$$C_D = 22.5\frac{b}{\pi f r \eta_{4a}} + 2.4. \tag{A.18}$$

Appendix B

Offset points input file

B.1 Explanation of the input file

Here we will briefly explain the different lines in the offset points input file.

The first line is the name of the vessel. Further line 5 shows the L_{PP}, and line 6 the frame number. Line 7 shows the longitudinal position of the frame relative to $L_{PP}/2$. Negative value means forward at the vessel. Line 8 shows how many offset points there are in the half frame, and the following lines, $9-20$, contain the y- and z-coordinates of the half frame. Line 21 shows the frame number of the next frame and the story repeats itself all the way to the bottom of the input file. The input with the two first frames is shown in section B.2.

B.2 Excerpt from the input file

```
1  Trønderhav 90’
2
3
4
5   24.00
6              1
7     -13.50000
8             12
9       1.419000        8.023001
10      1.370000        7.975000
11      1.046000        7.682000
12      0.755000        7.465000
13      0.620000        7.381000
14      0.364000        7.260000
15      0.241000        7.222000
16      0.120000        7.199000
17      0.000000        7.192000
18      0.000000        2.166000
19      0.009000        2.071000
20      0.000000        1.972000
21             2
22    -13.45000
23            24
24      1.461917        7.956706
25      1.462724        7.938760
```

```
1.237625    7.727132
1.067602    7.586263
0.791918    7.365288
0.643677    7.268975
0.528932    7.202433
0.489525    7.182488
0.367658    7.129994
0.000000    7.108000
0.000000    2.553000
0.065754    2.518095
0.083023    2.478553
0.102256    2.392863
0.112502    2.305553
0.121304    2.173993
0.123761    2.086126
0.124000    1.998219
0.116466    1.866587
0.103278    1.779677
0.093480    1.736827
0.078204    1.695568
0.044295    1.671595
0.000000    1.670766
```

Appendix C

Matlab codes

Here we will show the Matlab code and describe briefly the output from each function. In order to run the program, type *main* in the command window. The electronic version is found an the CD in appendix G.

C.1 variables.m

Here we describe all the global variables used in the program.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This file contains the description of the variables used in the program %
% The "variables" witout any further explanation are structures containing%
% variables below
%
% Name.................Unit..........................Description........
% inp
%   nFr................[-]..............................Number of frames
%   Lpp................[m].................Length between perpendiculars
%   xFr................[m]...........................x-position of frame
%   nPktSp.............[-]..................Number of points per section
%   Offsets............[m].....................Vector with offset points
% const
%   rho..............[kg/m^3]..................Mass density of sea water
%   g................[m/s^2].....................Acceleration of gravity
%   T..................[s]..........................Incident wave period
%   w0...............[rad/s].....................Incident wave frequency
%   U.................[m/s]................................Forward speed
%   betta............[rad]..................................Wave heading
%   we...............[rad/s]......................Frequency of encounter
% h....................[m]...................................Wave height
% zg...................[m]....................Vertical center of gravity
% d....................[m]..................................Mean draught
%   k.................[1/m]..................................Wave number
%   bolgehev...........[m]................................Wave elevation
%   pdyn...............[Pa].............................Dynamic pressure
%   ptot...............[Pa]...............................Total pressure
%   v.................[m/s]....................Horizontal fluid velocity
%   w.................[m/s]......................Vertical fluid velocity
%   ay...............[m/s^2]...............Horizontal fluid acceleration
```

```
%   az...............[m/s^2].................Vertical fluid acceleration
% dim
%   Beam...............[m]................................Beam of vessel
%   maxDraught.........[m]...............................Maximum draught
%   maxFreeboard.......[m].............................Maximum freeboard
%   B..................[m]..........................Beam of each section
%   Bwl................[m]............................Beam in water line
%   Vol...............[m^3]..........................Volume displacement
%   LCB................[m]...............Longitudinal center of buoyancy
%   zm.................[m].......Vertical center of area of each section
%   zm.................[m]...................Vertical center of buoyancy
%   LCF................[m]..............Longitudinal center of flotation
%   sac...............[m^2]...............................Sectional area
% frame
%   dx.................[m].................................Frame spacing
%   nPktSp.............[-]..................Number of points per section
%   bodypl.............[-].....................................Body plan
%   wbodypl............[-]...........................Submerged body plan
%   xFr................[m]..........................x-position of frames
%   SpSb...............[m]........................Starboard half-section
%   SpPs...............[m]........................Port side half-section
%   SpSbsort...........[m]................Starboard half-section, sorted
%   wFr................[m]................Submerged part of each section
% els
%   nEls...............[-]................Number of elements per section
%   tanVec.............[m]................Tangent vector of each element
%   lengde.............[m]........................Length of each element
%   midpoint...........[m]......................Midpoint of each element
%   n2.................[-].........Horizontal part of unit normal vector
%   n3.................[-]...........Vertical part of unit normal vector
% inert
%   M..................[kg]....................................Ship mass
%   I44..............[kgm^2]...................Moment of inertia in roll
%   I46...............[kgm]......Product of inertia in coupled roll-yaw
%   I55..............[kgm^2]..................Moment of inertia in pitch
%   I66..............[kgm^2]....................Moment of inertia in yaw
%   Mass.........[kg,kgm,kgm^2]..............................Mass matrix
% twoD
%   a22..............[kg/m]........................2D added mass in sway
%   a24...............[kg]............2D added mass in coupled sway-roll
%   a33..............[kg/m].......................2D added mass in heave
%   a33...............[kgm]........................2D added mass in roll
%   b22............. [kg/ms]..........................2D damping in sway
%   b24..............[kg/s]..............2D damping in coupled sway-roll
%   b33..............[kg/ms].........................2D damping in heave
%   b44..............[kg/s]...........................2D damping in roll
% amass
%   A22...............[kg].........................3D added mass in sway
%   A24...............[kgm]...........3D added mass in coupled sway-roll
%   A26...............[kgm]............3D added mass in coupled sway-yaw
%   A33...............[kg]........................3D added mass in heave
%   A35...............[kgm].........3D added mass in coupled heave-pitch
%   A42...............[kgm]...........3D added mass in coupled roll-sway
%   A44..............[kgm^2].......................3D added mass in roll
%   A46...............[kgm]............3D added mass in coupled roll-yaw
%   A53...............[kgm].........3D added mass in coupled pitch-heave
%   A62...............[kgm]............3D added mass in coupled yaw-sway
%   A64..............[kgm^2]...........3D added mass in coupled yaw-roll
%   A66..............[kgm^2]........................3D added mass in yaw
%   A............[kg,kgm,kgm^2]........................Added mass matrix
% gz
```

```
%   GZ.................[m]..................................Righting arm
%   GM.................[m].................Transverse metacentric height
% GMl..................[m]...............Longitudinal metacentric height
% damp
%   B22..............[kg/s]...........................3D damping in sway
%   B24..............[kgm/s].............3D damping in coupled sway-roll
%   B26..............[kgm/s]..............3D damping in coupled sway-yaw
%   B33..............[kg/s]..........................3D damping in heave
%   B35..............[kgm/s]...........3D damping in coupled heave-pitch
%   B42..............[kgm/s].............3D damping in coupled roll-sway
%   B44.............[kgm^2/s].........................3D damping in roll
%   Bv442...........[kgm^2/s]....................Viscous damping in roll
%   B46..............[kgm/s]..............3D damping in coupled roll-yaw
%   B53..............[kgm/s]...........3D damping in coupled pitch-heave
%   B62..............[kgm/s]..............3D damping in coupled yaw-sway
%   B64.............[kgm^2/s].............3D damping in coupled yaw-roll
%   B66.............[kgm^2/s]..........................3D damping in yaw
%   B..........[kg/s,kgm/s,kgm^2/s].......................damping matrix
% rest
%   C33...............[N/m]...............Restoring coefficient in heave
%   C35................[N].......Restoring coeff. in coupled heave-pitch
%   C44...............[Nm]...................Non-linear restoring moment
%   C44lin............[Nm]..........Linear restoring coefficient in roll
%   C53...............[Nm].......Restoring coeff. in coupled pitch-heave
%   C..............[N/m,N,Nm]...............Restoring coefficient matrix
%   wn3..............[rad/s]........Natural freq. in coupled heave-pitch
%   wn4..............[rad/s].........Nat. freq. in coupled sway-roll-yaw
%   wn5..............[rad/s]........Natural freq. in coupled pitch-heave
% excit
%   F2.................[N].................Wave excitation force in sway
%   F2fk...............[N]..Froude-kriloff wave excitation force in sway
%   F2d................[N].....Diffraction wave excitation force in sway
%   F3.................[N]................Wave excitation force in heave
%   F3fk...............[N].Froude-Kriloff wave excitation force in heave
%   F3d................[N]....Diffraction wave excitation force in heave
%   F4................[Nm]................Wave excitation moment in roll
%   F4fk..............[Nm].Froude-Kriloff wave excitation moment in roll
%   F4d...............[Nm]....Diffraction wave excitation moment in roll
%   F5................[Nm]...............Wave excitation moment in pitch
%   F5fk..............[Nm]Froude-Kriloff wave excitation moment in pitch
%   F5d...............[Nm]...Diffraction wave excitation moment in pitch
%   F6................[Nm].................Wave excitation moment in yaw
%   F6fk..............[Nm]..Froude-Kriloff wave excitation moment in yaw
%   F6d...............[Nm]......Difraction wave excitation moment in yaw
%   Fa...............[N,Nm].............Wave excitation amplitude matrix
% trans
%   H2.................[-].....................Transfer function in sway
%   H3.................[-]....................Transfer function in heave
%   H4.................[-].....................Transfer function in roll
%   H5.................[-]....................Transfer function in pitch
%   H6.................[-]......................Transfer function in yaw
% resp
%   eta2...............[m]...................................Sway motion
%   eta3...............[m]..................................Heave motion
%   eta4..............[rad]..................................Roll motion
%   eta5..............[rad].................................Pitch motion
%   eta6..............[rad]...................................Yaw motion
% t....................[s]..........................................Time
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

C.2 Main.m

This is the main file of the program. This file calls every function or subroutine. The output from each function is a structure, i.e. a variable containing many variables.

```
clear all %Clear all variables
clc %Clear the screen
tic
set(0,'RecursionLimit',1500)
%This is the main file of the program. This file calls every function or
%subroutine. The output from each function is a structure, i.e. a variable
%containing many variables.

%% Read the input files
[inp] = ReadInput();

%% Some constants
[const h] = constants();

%% Ship data
[zg d] = shipdata();

%% Wave potential
[wav] = wavepot(const,h);

%% Geometrical properties of the ship
[dim frame els] = geometry(inp,const,wav,h,d);

%% Moment of inertia
[inert] = mominertia(inp,const,dim,zg);

%% 2-Dimensional added mass and damping coefficients
[twoD] = coeff2d(inp);

%% 3-Dimensional added mass coefficients
[amass] = addedmass(inp,const,frame,twoD);

%% GZ curve
[gz] = gzcurve(inp,const,wav,dim,frame,zg);

%% Hydrostatic properies
[GMl] = hydrostatic(inp,dim,frame,zg,d);

%% 3-Dimensional damping coefficients
[damp] = damping(inp,const,dim,frame,inert,twoD,amass,gz,zg);

%% Restoring coefficients and natural frequencies
[rest] = restoring(inp,const,wav,dim,frame,inert,amass,gz,GMl,zg,h);

%% Exitation forces
[excit] = excitation(inp,const,wav,dim,frame,els,twoD,h);

%% Transfer function
[trans] = transfer(inp,const,inert,amass,rest,damp,excit,h);

%% Solving the equation of motion in the time domain
[resp t] = eqmotion(const,inert,amass,rest,damp,excit,zg);

%% Print relevant information to the screen
```

```
skjerm(inp,const,dim,frame,gz,rest,zg,amass,inert,excit,trans,resp,t,d,h);

toc
```

C.3 ReadInput.m

This reads the input file containing the offset points as given by ShipX. The format of the file is .mgf, and an excerpt is found in appendix B. We have to specify the number of frames.

```
function [inp] = ReadInput()
% This function reads the input file containing the offset points

inp = struct; % Structure containing the variables in the input file

inp.nFr = 38; % Number of frames in the input file

Input = 'Input/Trhav/offsets.MGF'; % Open the file containing the results
fid = fopen(Input); % File identifier
a = textscan(fid,'%f',1,'headerlines',4);% Read first 4 lines of input file
inp.Lpp = a{1}(1); % Length of vessel

inp.xFr = zeros(inp.nFr,1); % Vector with x-positions of each frame
inp.nPktSp = zeros(inp.nFr,1); % Vector with number of points per frame
inp.Offsets = cell(inp.nFr,1); % Cell array with offset points

for aa = 1:inp.nFr % Loop going through all frames
    b = textscan(fid,'%f',3); % Reading the three next lines
    inp.xFr(aa,1) = b{1}(2); % Content of line 2
    inp.nPktSp(aa,1) = b{1}(3); % Content of line 3
    c = textscan(fid,'%f %f',inp.nPktSp(aa,1)); % Reading the offset points
    inp.Offsets{aa}(:,1) = c{1}(:); % y offset
    inp.Offsets{aa}(:,2) = c{2}(:); % z offset
end

fclose(fid); % Close the input file

end
```

C.4 constants.m

Here we define the constants mass density of water, acceleration of gravity, wave height, wave period, ship speed and ship heading. In addition we calculate the frequency of encounter.

```
function [const h] = constants()
%Defining constants

%% Structure containing all the constants
const = struct;

%% mass density of sea water [kg/m3]
const.rho = 1025;
```

```

%% acceleration of gravity [m/s2]
const.g = 9.81;

%% wave height [m]
h = 2;

%% wave period [s]
const.T = 7;

%% wave angular frequency [rad/s]
const.w0 = 2*pi/const.T;

%% Velocity [knots]
u = 10;

%% Velocity [m/s]
const.U = u*1.852/3.6;

%% Heading relative to the waves [rad}
beta = 20;
const.betta = beta*pi/180;

%% Frequency of encounter [rad/s]
const.we = const.w0 + ((const.w0^2)/const.g)*const.U*cos(const.betta);

end
```

C.5 shipdata.m

In this subroutine we can specify different properties of the vessel like the center of gravity and mean draught.

```
function [zg d] = shipdata()
%% Defines constants related to the ship

%% Vertical centre of gravity (relative to the mean surface) [m]
zg = -0.105;

%% Draught of vessel [m]
d = 3.989;

end
```

C.6 wavepot.m

Here we calculate the properties of the waves, such as surface elevation, the dynamic and total pressure and the y- and z-components of the velocity and acceleration of the fluid.

```
function [wav] = wavepot(const,h)
%Caculate the properties of the waves
```

```
%% Structure containing the wave properties
wav = struct;

%% Wave heading
B = const.betta;

%% Wave number [1/m]
wav.k = @(w0) w0*w0/const.g;

%% Surface elevation
wav.bolgehev = @(h,x,y,w0,we,t) -1i*0.5*h*exp(-1i*wav.k(w0)*(x*cos(B) + y*sin(B)))*exp(1i*we*t);

%% Dynamic pressure
wav.pdyn = @(x,y,z,w0,we,h,t) -1i*const.rho*const.g*0.5*h*exp(wav.k(w0)*z)*exp(-1i*wav.k(w0)*(x*cos(B) + y*sin(B)))*exp(1i*we*t);

%% Total pressure
wav.ptot = @(x,y,z,w0,we,h,t) totpres(const,wav.bolgehev,wav.pdyn,h,x,y,z,w0,we,t);

%% Horizontal velocity component, y-direction
wav.v = @(x,y,z,w0,we,t) -1i*sin(B)*w0*0.5*h*exp(wav.k(w0)*z)*exp(-1i*wav.k(w0)*(x*cos(B) + y*sin(B)))*exp(1i*we*t);

%% Vertical velocity component
wav.w = @(x,y,z,w0,we,t) w0*0.5*h*exp(wav.k(w0)*z)*exp(-1i*wav.k(w0)*(x*cos(B) + y*sin(B)))*exp(1i*we*t);

%% Horizontal acceleration component, y-direction
wav.ay = @(x,y,z,w0,we,t) sin(B)*we*w0*0.5*h*exp(wav.k(w0)*z)*exp(-1i*wav.k(w0)*(x*cos(B) + y*sin(B)))*exp(1i*we*t);

%% Vertical acceleration component
wav.az = @(x,y,z,w0,we,t) 1i*we*w0*0.5*h*exp(wav.k(w0)*z)*exp(-1i*wav.k(w0)*(x*cos(B) + y*sin(B)))*exp(1i*we*t);

end
```

C.7 totpres.m

Here we calculate the total linear pressure. The pressure under the wave crest is assumed to be hydrostatic. See figure 1.2 and equation (1.18) for definitions.

```
function [ptot] = totpres(const,bolgehev,pdyn,h,x,y,z,w0,we,t)
% Calculate the total linear pressure under the waves. Includes linear
% dynamic pressure and hydrostatic pressure.

if z > 0
    ptot = const.rho*const.g*(bolgehev(h,x,y,w0,we,t) - z);
else
    ptot = -const.rho*const.g*z + pdyn(x,y,z,w0,we,h,t);
end

end
```

C.8 geometry.m

In this subroutine we calculate all the geometrical properties of the vessel. The subroutine itself is subdivided into many smaller subroutines.

```
function [dim frame els] = geometry(inp,const,wav,h,d)
%Geometric properties of the ship

%% Structure containing the variables related to the dimensions of the ship
dim = struct;
%% Structure containing information of each frame of the ship
frame = struct;
%% Structure containing information of each element of the frames
els = struct;

%% Length of each strip
[frame.dx] = stripLength(inp);

%% Half section when draught and x-coordinates are accounted for
[SpSb SpPs SpSbSort] = newVertCoord(inp,d);

%% Extreme breadth, draught and freeboard
[dim.Beam dim.maxDraught dim.maxFreeboard dim.maxdraught] = extreme(inp,SpSb);

%% Include a deck at each section
[SpSb SpPs SpSbSort frame.nPktSp] = deck(SpSb,SpPs,SpSbSort,inp);

%% Beam in mean water line of each section
[dim.B dim.Bwl] = halfBeam(inp,SpSb,SpPs);

%% Under water part of total section
[wFr els.nEls wSp nPktwSp] = wetFrame(inp,const,wav,SpSb,SpPs,SpSbSort,frame.nPktSp,inp.xFr,dim.B,0,0);

%% Tangent vector of each element in each section
[els.tanVec] = tangentVec(wFr,els.nEls,inp);

%% Length of each element in each section
[els.lengde] = elLength(inp,els.nEls,els.tanVec);

%% Midpoint of each element in each section
[els.midpoint] = midPoint(inp,els.nEls,wFr);

%% Normal vector of each element in each section
[els.n2 els.n3] = normalVec(inp,els.nEls,els.tanVec,els.lengde);

%% Body plan
[frame.bodypl frame.wbodypl] = bodyPlan(inp,SpSb,wSp);

%% Sectional area and volume displacement
[sac dim.Vol dim.LCB] = sectionalArea(inp,frame.dx,wFr);

%% centre of volume of each section and the global centre of buoyancy
[dim.zm dim.zb] = centreOfVolume(inp,nPktwSp,wFr,sac,frame.dx,dim.Vol);

%% Water plane area
[dim.Aw dim.LCF] = wlarea(inp,frame.dx,dim.B);

%% New origin in x-direction
```

```
[frame.xFr frame.SpSb frame.SpPs frame.SpSbSort frame.wFr dim.sac] = newxCoord(
    inp,dim.LCB,SpSb,SpPs,SpSbSort,wFr,sac);

end
```

C.9 stripLength.m

This subroutine calculates the distance between each frame of the vessel, which corresponds to the length of each strip.

```
function [dx] = stripLength(inp)
%Calculate the length of each strip, i.e. the frame spacing

%% Length of each strip
dx = zeros((inp.nFr - 1),1);

for aa = 1:(inp.nFr - 1) % Loop going through all strips
    dx(aa+1,1) = inp.xFr(aa+1) - inp.xFr(aa,1);
end

%% Length of the first strip, assumed to be equal to the second strip
dx(1,1) = dx(2,1);
%% Length of the last strip, assumed to be equal to the second last strip
dx(inp.nFr,1) = dx(inp.nFr-1,1);

end
```

C.10 newVertCoord.m

Since the input file has its vertical origin at the base line, we need to transform the offsets points in the vertical direction such that the origin is in the mean water line.

```
function [SpSb SpPs SpSbSort] = newVertCoord(inp,d)
%Calculate the half section when accounting for the draught. The vertical
%origin is now in the mean water plane. Also include the x-coordinate of
%the section.

%% Half section at starboard side when draught is accounted for
SpSb = cell(inp.nFr,1);

%% Half section at port side when draught is accounted for
SpPs = cell(inp.nFr,1);

%% Half section at port side when draught is accounted for and offset
%% points sorted
SpSbSort = cell(inp.nFr,1);

% Loop going through each frame
for aa = 1:inp.nFr
    % Loop going through each point of each frame
    for bb = 1:inp.nPktSp(aa,1)
```

```
            %% x-coordinate of starboard side half section
            SpSb{aa}(bb,1) = inp.xFr(aa);
            %% y-coordinate of starboard side half section
            SpSb{aa}(bb,2) = inp.Offsets{aa}(bb,1);
            %% z-coordinate of starboard side half section
            SpSb{aa}(bb,3) = inp.Offsets{aa}(bb,2) - d;
            %% x-coordinate of port side half section
            SpPs{aa}(bb,1) = SpSb{aa}(bb,1);
            %% y-coordinate of port side half section
            SpPs{aa}(bb,2) = -SpSb{aa}(bb,2);
            %% z-coordinate of port side half section
            SpPs{aa}(bb,3) = SpSb{aa}(bb,3);
        end
    end

    %% Loop sorting the points at port side half section into same direction as
    %% the starboard side half section.
    for aa = 1:inp.nFr
        for bb = 1:inp.nPktSp(aa,1)
            SpSbSort{aa}(bb,1) = SpSb{aa}(inp.nPktSp(aa,1) - bb + 1,1);
            SpSbSort{aa}(bb,2) = SpSb{aa}(inp.nPktSp(aa,1) - bb + 1,2);
            SpSbSort{aa}(bb,3) = SpSb{aa}(inp.nPktSp(aa,1) - bb + 1,3);
        end
    end

    end
```

C.11 extreme.m

Here we calculate the maximum dimensions of the vessel, such as maximum beam, maximum draught and so on.

```
function [Beam maxDraught maxFreeboard maxdraught] = extreme(inp,SpSb)
%Calculate extreme breadth, extreme draught and extreme freeboard

beam = zeros(inp.nFr,1); %% Max beam of each section
maxdraught = zeros(inp.nFr,1); %% Max draught of each section
maxfreeboard = zeros(inp.nFr,1); %% Max freeboard of each section

%% Finding maximum values of each half frame
for aa = 1:inp.nFr
    beam(aa,1) = 2*max(SpSb{aa}(:,2));
    maxdraught(aa,1) = -min(SpSb{aa}(:,3));
    maxfreeboard(aa,1) = max(SpSb{aa}(:,3));
end

Beam = max(beam(:,1)); %% Max beam of ship
maxDraught = max(maxdraught(:,1)); %% Max draught of ship
maxFreeboard = max(maxfreeboard(:,1)); %% Max freeboard of ship

end
```

C.12 deck.m

In order to be able to integrate the pressure for all heel angles wee need a closed section. Here we close each section by adding some horizontal elements on each section that represents the deck.

```
function [SpSb SpPs SpSbSort nPktSp] = deck(SpSb,SpPs,SpSbSort,inp)
% Provides each section with a horisontal deck in order to be able to
% calculate restoring moment with deck immersion

%% Array containing deck elements on starboard half section
deckSb = cell(inp.nFr,1);
%% Array containing deck elements in port side half section
deckPs = cell(inp.nFr,1);
deckSbSort = cell(inp.nFr,1);
%% Number of points per section
nPktSp = zeros(inp.nFr,1);

for aa = 1:inp.nFr
    %% Checks if the section is fully submerged. In that case, no deck
    if max(SpSb{aa}(:,3)) < 0
        SpSb{aa} = SpSb{aa};
        SpPs{aa} = SpPs{aa};
    else
        %% Creates horizontal elements on top of each section
        deckSb{aa}(:,1) = linspace(SpSb{aa}(1,1),SpSb{aa}(1,1),5);
        deckSb{aa}(:,2) = linspace(0,(SpSb{aa}(1,2) - SpSb{aa}(1,2)/5),5);
        deckSb{aa}(:,3) = linspace(SpSb{aa}(1,3),SpSb{aa}(1,3),5);
        deckPs{aa}(:,1) = linspace(SpPs{aa}(1,1),SpPs{aa}(1,1),5);
        deckPs{aa}(:,2) = linspace(0,(SpPs{aa}(1,2) - SpPs{aa}(1,2)/5),5);
        deckPs{aa}(:,3) = linspace(SpPs{aa}(1,3),SpPs{aa}(1,3),5);
        ind = length(SpPs{aa}(:,1));
        deckSbSort{aa}(:,1) = linspace(SpSbSort{aa}(ind,1),SpSbSort{aa}(ind,1),5);
        deckSbSort{aa}(:,2) = linspace((SpSbSort{aa}(ind,2) - SpSbSort{aa}(ind,2)/5),0,5);
        deckSbSort{aa}(:,3) = linspace(SpSbSort{aa}(ind,3),SpSbSort{aa}(ind,3),5);
        %% Assembles the deck to the rest of the section
        SpSb{aa} = [deckSb{aa}; SpSb{aa}];
        SpPs{aa} = [deckPs{aa}; SpPs{aa}];
        SpSbSort{aa} = [SpSbSort{aa}; deckSbSort{aa}];

        %% Each section now have a new number of points
        nPktSp(aa,1) = length(SpSb{aa}(:,3));

    end
end

end
```

C.13 halfBeam.m

This subroutine calculates the half beam in the mean water line.

```
function [B Bwl] = halfBeam(inp,SpSb,SpPs)
%Calculate the half beam and the beam of each section in the mean water line.

%% Beam in mean water line
B = zeros(inp.nFr,1);

for aa = 1:inp.nFr
    %% Half beam in mean water line
    bSB = abs(wlbredde(SpSb{aa},0));
    bPS = abs(wlbredde(SpPs{aa},0));
    %% Beam in mean water line
    B(aa,1) =  bSB + bPS;
end
%% Max beam in water line
Bwl = max(B(:,1));

end
```

C.14 wlbredde.m

This is the interpolation function used in the subroutine halfBeam.m.

```
function [svar] = wlbredde(spant,bolge)
% Find the half-breadth at a sepcified z-position in a body-fixed
% coordinate system by linear interpolation between the points on either
% side.

%% Given z-position
z = bolge;

%% Find the index of the nearest point above the given z-position
upper = find(spant(:,3) > z);
%% Find the index of the nearest point below the given z-position
lower = find(spant(:,3) < z);

%% Checks if the section is totally submerged
if isempty(upper);
    svar = 0;
elseif isempty(lower);
    svar = 0;
else
    %% Linear interpolation between the two points
    index1 = upper(length(upper));
    index2 = lower(1);

    y1 = spant(index1,2);
    z1 = spant(index1,3);

    y2 = spant(index2,2);
    z2 = spant(index2,3);

    %% The seeked half beam
    svar = y1 + ((y2 - y1)/(z2 - z1))*(z - z1);
end

end
```

C.15 wetFrame.m

This subroutine adds an offset point in the free surface and removes all offset points above. It also gives the number of points and elements of each section.

```
function [wFr nEls wFrPS nPktwSp] = wetFrame(inp,const,wav,SpSb,SpPs,SpSbSort,
    nPktSp,xFr,B,h,tid)
%Define the under water part of each section. In addition number of points
%and elements of each section.

%% Under water part of total section
wFr = cell(inp.nFr,1);
wFrSB = cell(inp.nFr,1);
wFrPS = cell(inp.nFr,1);

%% Number of points in each under water section (wFr)
nPkt = zeros(inp.nFr,1);

%% Number of points in each under water half section (wSp)
nPktwSp = zeros(inp.nFr,1);

%% Number of elements in each under water section (wFr)
nEls = zeros(inp.nFr,1);

%% Determine the underwater part
for aa = 1:inp.nFr
    %% Wave elevation
    bolgeSB = @(tid) real(wav.bolgehev(h,xFr(aa),B(aa,1),const.w0,const.we,tid))
        ;
    bolgePS = @(tid) real(wav.bolgehev(h,xFr(aa),-B(aa,1),const.w0,const.we,tid)
        );

     for bb = 1:nPktSp(aa,1)
        wFrSB{aa}(bb,1) = SpSbSort{aa}(bb,1);
        wFrSB{aa}(bb,2) = SpSbSort{aa}(bb,2);
        wFrSB{aa}(bb,3) = SpSbSort{aa}(bb,3);
        wFrPS{aa}(bb,1) = SpPs{aa}(bb,1);
        wFrPS{aa}(bb,2) = SpPs{aa}(bb,2);
        wFrPS{aa}(bb,3) = SpPs{aa}(bb,3);
     end

        %% Determine the z-position where the section cuts the free surface
        wFrSB{aa}(1,3) = bolgeSB(tid);
        wFrPS{aa}(1,3) = bolgePS(tid);
        %% Determine the y-position where the section cuts the free surface
        wFrSB{aa}(1,2) = wlbredde(SpSb{aa},wFrSB{aa}(1,3));
        wFrPS{aa}(1,2) = wlbredde(SpPs{aa},wFrPS{aa}(1,3));


        %% Delete points above the free surface
        wetSB = wFrSB{aa}(:,3) > bolgeSB(tid);
        wFrSB{aa}(wetSB,:) = [];
        wetPS = wFrPS{aa}(:,3) > bolgePS(tid);
        wFrPS{aa}(wetPS,:) = [];

        if wFrPS{aa}(1,2) == 0
            wFrPS{aa}(1,:) = [];
            wFrSB{aa}(1,:) = [];
        else
```

```
            wFrSB{aa}((length(wFrSB{aa}(:,1)) + 1),:) = wFrSB{aa}(1,:);
            wFrSB{aa}(1,:) = [];
        end

        %% Assembles the two wet half section to one section
        wFr{aa} = [wFrPS{aa}; wFrSB{aa}];

%% Number of points in each section
nPkt(aa,1) = length(wFr{aa}(:,1));

%% Number of elements in each section
nEls(aa,1) = nPkt(aa,1) - 1;

%% Number of elements in each half section
nPktwSp(aa,1) = length(wFrSB{aa}(:,1));

end

end
```

C.16 tangentVec.m

This subroutine calculates the vector of each element of each section.

```
function [tanVec] = tangentVec(wFr,nEls,inp)
%% Calculate the tangent vector of each element in each under water section

%% Tangent vector of each element
tanVec = cell(inp.nFr,1);

for aa = 1:inp.nFr
    for dd = 1:nEls(aa,1)
        tanVec{aa}(dd,1) = wFr{aa}(dd+1,2) - wFr{aa}(dd,2);
        tanVec{aa}(dd,2) = wFr{aa}(dd+1,3) - wFr{aa}(dd,3);
    end
end

end
```

C.17 elLength.m

Here we calculate the length of each element of each section.

```
function [lengde] = elLength(inp,nEls,tanVec)
%% Calculate the length of each element in each under water section

%% Length of each element
lengde = cell(inp.nFr,1);

for aa = 1:inp.nFr
    for bb = 1:nEls(aa,1)
        lengde{aa}(bb,1) = sqrt(tanVec{aa}(bb,1)^2 + tanVec{aa}(bb,2)^2);
```

```
    end
end

end
```

C.18 midPoint.m

When integrating the pressure over the wetted surface we evaluate the pressure at the midpoint of each element. This subroutine calculates that midpoint.

```
function [midpoint] = midPoint(inp,nEls,wFr)
%% Calculate the midpoint of each element in each section

%% Midpoint of each element
midpoint = cell(inp.nFr,1);

for aa = 1:inp.nFr
    for bb = 1:nEls(aa,1)
        midpoint{aa}(bb,1) = (wFr{aa}(bb + 1,2) + wFr{aa}(bb,2))/2;
        midpoint{aa}(bb,2) = (wFr{aa}(bb + 1,3) + wFr{aa}(bb,3))/2;
    end
end

end
```

C.19 normalVec.m

This subroutine calculates the normal vector of each element of each section. This is done by rotating the tangent vector 90°.

```
function [n2 n3] = normalVec(inp,nEls,tanVec,lengde)
%% Calculate the unit normal vectors of each element in each section

%% Normal vector of each element
normVec = cell(inp.nFr,1);

%% Horizontal component of unit normal vetor
n2 = cell(inp.nFr,1);

%% Vertical component of unit normal vetor
n3 = cell(inp.nFr,1);

%% Determine the normal vectors
for aa = 1:inp.nFr
    for bb = 1:nEls(aa,1)
        normVec{aa}(bb,1) = tanVec{aa}(bb,2);
        normVec{aa}(bb,2) = -tanVec{aa}(bb,1);
        n2{aa}(bb,1) = normVec{aa}(bb,1)/lengde{aa}(bb,1);
        n3{aa}(bb,1) = normVec{aa}(bb,2)/lengde{aa}(bb,1);
    end
end
```

C.20 bodyPlan.m

The body plan is best shown when the forward part of the vesses is at one side and the aft part at the other.

```
function [bodypl wbodypl] = bodyPlan(inp,SpSb,wSp)
%Determine the body plan

%% Half sections organized to be plotted in body plan
bodypl = cell(inp.nFr,1);

%% Under water half section to be plotted in body plan
wbodypl = cell(inp.nFr,1);

%% Foreship on the right hand side and aftship on the left hand side
for aa = 1:inp.nFr

    if aa < inp.nFr/2
        bodypl{aa}(:,1) = SpSb{aa}(:,2);
        bodypl{aa}(:,2) = SpSb{aa}(:,3);
        wbodypl{aa}(:,1) = -wSp{aa}(:,2);
        wbodypl{aa}(:,2) = wSp{aa}(:,3);
    else
        bodypl{aa}(:,1) = -SpSb{aa}(:,2);
        bodypl{aa}(:,2) = SpSb{aa}(:,3);
        wbodypl{aa}(:,1) = wSp{aa}(:,2);
        wbodypl{aa}(:,2) = wSp{aa}(:,3);
    end
end

end
```

C.21 sectionalArea.m

This subroutine calculates the submerged area of each section. Integrating this along the length gives the volume displacement, and the horizontal volume moment divided by the volume displacement gives the longitudinal center of buoyancy, which corresponds to the longitudinal center of gravity when the vessel is at its static position.

```
function [sac Vol LCB] = sectionalArea(inp,dx,wFr)
%% Calculate the area of each section and the volume displacement

%% Sectional area of each under water section
sac = zeros((inp.nFr + 4),1);

for aa = 1:inp.nFr
    sac(1,1) = inp.xFr(1,1) - 0.5*dx(1,1) - eps;
    sac(1,2) = 0;
    sac(2,1) = inp.xFr(1,1) - 0.5*dx(1,1);
    sac(2,2) = abs(trapz(wFr{1}(:,2),wFr{1}(:,3)));
    sac((aa + 2),1) = inp.xFr(aa,1);
    sac((aa + 2),2) = abs(trapz(wFr{aa}(:,2),wFr{aa}(:,3)));
    sac((inp.nFr + 3),1) = inp.xFr(inp.nFr,1) + 0.5*dx(inp.nFr,1);
    sac((inp.nFr + 3),2) = abs(trapz(wFr{aa}(:,2),wFr{aa}(:,3)));
```

```
    sac((inp.nFr + 4),1) = inp.xFr(inp.nFr,1) + 0.5*dx(inp.nFr,1) + eps;
    sac((inp.nFr + 4),2) = 0;
end

%% Horizontal volume moment
xmom = 0;
Vol = 0;
for aa = 1:inp.nFr
    %% Volume displacement
    Vol = abs(trapz(wFr{aa}(:,2),wFr{aa}(:,3)))*dx(aa) + Vol;
    xmom = abs(trapz(wFr{aa}(:,2),wFr{aa}(:,3)))*dx(aa)*(inp.xFr(aa)) + xmom;
end

%% Longitudinal centre of buoyancy
LCB = xmom/Vol;

end
```

C.22 centreOfVolume.m

Here we calculate the vertical centre of volume of each section, and the global vertical centre of buoyancy.

```
function [zm zb] = centreOfVolume(inp,nPktwSp,wFr,sac,dx,Vol)
%Calculate the centre of volume of each section and the centre of buoyancy
%of the entire hull

%% Centre of area of each under water section
zm = zeros(inp.nFr,1);

zbmom = 0;

for aa = 1:inp.nFr
    amom = 0;
    htot = 0;
    %% Area moment of each section
    for bb = 1:nPktwSp(aa,1)-1
        a = -wFr{aa}(bb,2);
        b = -wFr{aa}((bb + 1),2);
        a1 = b;
        a2 = a - b;
        h = wFr{aa}(bb,3) - wFr{aa}((bb + 1),3);
        amom = -2*(a1*h*(0.5*h + htot) + 0.5*a2*h*((1/3)*h + htot)) + amom;
        htot = h + htot;
    end
    %% Centre of area of each section
    zm(aa,1) = amom/sac((aa + 2),2);

    %% Volume moment of all sections
    zbmom = amom*dx(aa,1) + zbmom;

end

%% Centre of buoyancy of the hull
zb = zbmom/Vol;

```

```
end
```

C.23 wlarea.m

Here we calculate the mean water plane area.

```
function [Aw LCF] = wlarea(inp,dx,B)
%Calculate the water plane area and the longitudinal center of flotation

Aw0 = 0;
Awmom = 0;
for aa = 1:inp.nFr
    %% Water plane area
    Aw0 = B(aa)*dx(aa) + Aw0;
    %% Longitudinal area moment of water plane
    Awmom = B(aa)*dx(aa)*inp.xFr(aa) + Awmom;
end
Aw = Aw0;

%% Longitudinal center of flotation
LCF = Awmom/Aw;

end
```

C.24 newxCoord.m

The coordinate system used in the calculations has its horizontal origin, $x = 0$, at the longitudinal center of gravity, while the coordinate system used in the input file has its horizontal at $L_{pp}/2$. This subroutine transforms the x-coordinates of the sections according to LCB.

```
function [xFr SpSb SpPs SpSbSort wFr sac] = newxCoord(inp,LCB,SpSb,SpPs,SpSbSort
    ,wFr,sac)
% Moves x = 0 from Lpp/2 to the longitudinal centre of gravity

xFr = inp.xFr - LCB;

for aa = 1:inp.nFr
    SpSb{aa}(:,1) = SpSb{aa}(:,1) - LCB;
    SpPs{aa}(:,1) = SpPs{aa}(:,1) - LCB;
    SpSbSort{aa}(:,1) = SpSbSort{aa}(:,1) - LCB;
    wFr{aa}(:,1) = wFr{aa}(:,1) - LCB;
end

sac(:,1) = sac(:,1) - LCB;

end
```

C.25 mominertia.m

Here we calculate the ship mass and its moment of inertia in roll, pitch pitch and yaw. In addition we calculate the total mass matrix.

```
function [inert] = mominertia(inp,const,dim,zg)
%Calculate the mass and moments of inertia of the vessel

inert = struct;

%% Ship mass, M [kg]
inert.M = const.rho*dim.Vol;

%% Moment of inertia in roll [kgm^2]
[inert.I44 inert.I46] = momin4(dim,inert.M);

%% Moment of inertia in pitch [kgm^2]
[inert.I55] = momin5(inp,inert.M);

%% Moment of inertia in yaw [kgm^2]
[inert.I66] = momin6(inp,inert.M);

%% Mass matrix
inert.Mass = [inert.M     0           0        0            inert.M*zg 0;...
              0           inert.M     0        -inert.M*zg  0          0;...
              0           0           inert.M  0            0          0;...
              0           -inert.M*zg 0        inert.I44    0          -inert.I46;
                  ...
              inert.M*zg  0           0        0            inert.I55  0;...
              0           0           0        -inert.I46   0          inert.I66];

end
```

C.26 momin4.m

Here we calculate the moment of inertia in roll. The radius of gyration is given as a fraction of the ship beam.

```
function [I44 I46] = momin4(dim,M)
%Calculate the moment of inertia in roll

%% Radius of gyration in roll [m] (assumed value)
%It is common for a fishing vessel to define the radius of gyration in roll
%as a fraction of the beam of the vessel.
r44 = 0.36*dim.Bwl;

%% Moment of inertia in roll due to roll motion,I44 [kgm^2]
I44 = M*r44^2;

%% Product of inertia in coupled roll-yaw
I46 = 0;
end
```

C.27 momin5.m

Here we calculate the moment of inertia in pitch. The radius of gyration is given as a fraction of the ship length.

```
function [I55] = momin5(inp,M)
%Calculate the moment of inertia in pitch

%% Radius of gyration in pitch [m] (assumed value)
%It is common for a fishing vessel to define the radius of gyration in
%pitch as a fraction of the length of the vessel. A typical value may be
%0.33 - 0.35
r55 = 0.34*inp.Lpp;

%% Moment of inertia in pitch, I55 [kgm^2]
I55 = M*r55^2;

end
```

C.28 momin6.m

Here we calculate the moment of inertia in yaw. The radius of gyration is given as a fraction of the ship length.

```
function [I66] = momin6(inp,M)
%Calculate the moment of inertia in yaw

%% Radius of gyration in yaw [m] (assumed value)
r66 = 0.25*inp.Lpp;

%% Moment of inertia in roll due to roll motion, I66 [kgm^2]
I66 = M*r66^2;

end
```

C.29 coeff2d.m

In this subroutine we calculate the two dimensional added mass and damping coefficients in sway, sway-roll, heave and roll.

```
function [twoD] = coeff2d(inp)
%% 2-Dimensional added mass and damping coefficients

twoD = struct;

%% 2-Dimensional added mass coefficients
[twoD.a22 twoD.a24 twoD.a33 twoD.a44] = amass2d(inp);

%% 2-Dimensional damping coefficients
```

```
[twoD.b22 twoD.b24 twoD.b33 twoD.b44] = damp2d(inp);

end
```

C.30 amass2d.m

Here we read in the two dimensional added mass coefficients in sway, sway-roll, heave and roll from input files.

```
function [a22 a24 a33 a44] = amass2d(inp)
%% 2-Dimensional added mass coefficients
amass22 = cell(inp.nFr,1); % added mass in sway
amass24 = cell(inp.nFr,1); % added mass in coupled sway-roll
amass33 = cell(inp.nFr,1); % added mass in heave
amass44 = cell(inp.nFr,1); % added mass in roll

a22 = cell(inp.nFr,1);
a24 = cell(inp.nFr,1);
a33 = cell(inp.nFr,1);
a44 = cell(inp.nFr,1);

%% Open the files containing the coefficients
Amass22 = fopen('Input/Trhav/2D/a22.dat');
Amass24 = fopen('Input/Trhav/2D/a24.dat');
Amass33 = fopen('Input/Trhav/2D/a33.dat');
Amass44 = fopen('Input/Trhav/2D/a44.dat');

for aa = 1:inp.nFr
    %% Read the files
    a = textscan(Amass22,'%f',2);
    npkt = a{1}(2);
    b = textscan(Amass22,'%f %f',npkt);
    amass22{aa}(:,1) = b{1}(:);
    amass22{aa}(:,2) = b{2}(:);

    a = textscan(Amass24,'%f',2);
    npkt = a{1}(2);
    b = textscan(Amass24,'%f %f',npkt);
    amass24{aa}(:,1) = b{1}(:);
    amass24{aa}(:,2) = b{2}(:);

    a = textscan(Amass33,'%f',2);
    npkt = a{1}(2);
    b = textscan(Amass33,'%f %f',npkt);
    amass33{aa}(:,1) = b{1}(:);
    amass33{aa}(:,2) = b{2}(:);

    a = textscan(Amass44,'%f',2);
    npkt = a{1}(2);
    b = textscan(Amass44,'%f %f',npkt);
    amass44{aa}(:,1) = b{1}(:);
    amass44{aa}(:,2) = b{2}(:);

    %% Two dimensional added mass in sway
    a22{aa} = @(w) interp1(amass22{aa}(:,1),amass22{aa}(:,2),w);

```

```
    %% Two dimensional added mass in coupled sway-roll
    a24{aa} = @(w) interp1(amass24{aa}(:,1),amass24{aa}(:,2),w);

    %% Two dimensional added mass in heave
    a33{aa} = @(w) interp1(amass33{aa}(:,1),amass33{aa}(:,2),w);

    %% Two dimensional added mass in roll
    a44{aa} = @(w) interp1(amass44{aa}(:,1),amass44{aa}(:,2),w);

end

%% Close the files
fclose(Amass22);
fclose(Amass33);
fclose(Amass44);
fclose(Amass24);

end
```

C.31 damp2d.m

Here we read in the two dimensional damping coefficients in sway, sway-roll, heave and roll from input files.

```
function [b22 b24 b33 b44] = damp2d(inp)
%% 2-Dimensional damping coefficients
damp22 = cell(inp.nFr,1); % damping in sway
damp24 = cell(inp.nFr,1); % damping in coupled sway-roll
damp33 = cell(inp.nFr,1); % damping in heave
damp44 = cell(inp.nFr,1); % damping in roll

b22 = cell(inp.nFr,1);
b24 = cell(inp.nFr,1);
b33 = cell(inp.nFr,1);
b44 = cell(inp.nFr,1);

%% Open the files containing the coefficients
Damp22 = fopen('Input/Trhav/2D/b22.dat');
Damp24 = fopen('Input/Trhav/2D/b24.dat');
Damp33 = fopen('Input/Trhav/2D/b33.dat');
Damp44 = fopen('Input/Trhav/2D/b44.dat');

for aa = 1:inp.nFr
    %% Read the files
    a = textscan(Damp22,'%f',2);
    npkt = a{1}(2);
    b = textscan(Damp22,'%f %f',npkt);
    damp22{aa}(:,1) = b{1}(:);
    damp22{aa}(:,2) = b{2}(:);

    a = textscan(Damp24,'%f',2);
    npkt = a{1}(2);
    b = textscan(Damp24,'%f %f',npkt);
    damp24{aa}(:,1) = b{1}(:);
    damp24{aa}(:,2) = b{2}(:);

```

```
    a = textscan(Damp33,'%f',2);
    npkt = a{1}(2);
    b = textscan(Damp33,'%f %f',npkt);
    damp33{aa}(:,1) = b{1}(:);
    damp33{aa}(:,2) = b{2}(:);

    a = textscan(Damp44,'%f',2);
    npkt = a{1}(2);
    b = textscan(Damp44,'%f %f',npkt);
    damp44{aa}(:,1) = b{1}(:);
    damp44{aa}(:,2) = b{2}(:);

%% Two dimensional damping in sway
    b22{aa} = @(w) interp1(damp22{aa}(:,1),damp22{aa}(:,2),w);

%% Two dimensional damping in coupled sway-roll
    b24{aa} = @(w) interp1(damp24{aa}(:,1),damp24{aa}(:,2),w);

%% Two dimensional damping in heave
    b33{aa} = @(w) interp1(damp33{aa}(:,1),damp33{aa}(:,2),w);

%% Two dimensional damping in roll
    b44{aa} = @(w) interp1(damp44{aa}(:,1),damp44{aa}(:,2),w);

end

%% Close the files
fclose(Damp22);
fclose(Damp33);
fclose(Damp44);
fclose(Damp24);

end
```

C.32 addedmass.m

Here we calculate the three dimensional added mass coefficients.

```
function [amass] = addedmass(inp,const,frame,twoD)
%This function calculates the vessel's added mass

amass = struct;

%% Added mass coefficients in sway
[amass.A22 amass.A24 amass.A26] = amass2(inp,const,frame,twoD);

%% Added mass coefficients in heave
[amass.A33 amass.A35] = amass3(inp,const,frame,twoD);

%% Added mass coefficients in roll
[amass.A42 amass.A44 amass.A46] = amass4(inp,const,frame,twoD);

%% Added mass coefficients in pitch
[amass.A53 amass.A55] = amass5(inp,const,frame,twoD);

%% Added mass coefficients in pitch
```

```
[amass.A62 amass.A64 amass.A66] = amass6(inp,const,frame,twoD);

%% Total added mass matrix
amass.A = @(we)  [0  0           0            0            0            0; ...
                  0  amass.A22(we)  0            amass.A24(we)  0            amass.A26(we); ...
                  0  0           amass.A33(we)  0            amass.A35(we)  0; ...
                  0  amass.A42(we)  0            amass.A44(we)  0            amass.A46(we); ...
                  0  0           amass.A53(we)  0            amass.A55(we)  0; ...
                  0  amass.A62(we)  0            amass.A64(we)  0            amass.A66(we)];

end
```

C.33 amass2.m

Calculate the added mass coefficients in sway.

```
function [A22 A24 A26] = amass2(inp,const,frame,twoD)
%Calculate added mass coefficients in sway

A22 = @(we) 0;
A24 = @(we) 0;
A26 = @(we) 0;
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Added mass in sway due to sway acceleration, A22 [kg]
    A22 = @(we) twoD.a22{aa}(we)*frame.dx(aa) + A22(we);
    %% Added mass in sway due to roll accelration, A24 [kgm]
    A24 = @(we) twoD.a24{aa}(we)*frame.dx(aa) + A24(we);
    %% Added mass in sway due to yaw accelration, A26 [kgm]
    A26 = @(we) frame.xFr(aa)*twoD.a22{aa}(we)*frame.dx(aa) - (U/(we^2))*twoD.b22{aa}(we)*frame.dx(aa) + A26(we);
end

A22 = @(we) A22(we) - (U/(we^2))*twoD.b22{inp.nFr}(we);
A24 = @(we) A24(we) - (U/(we^2))*twoD.b24{inp.nFr}(we);
A26 = @(we) A26(we) - (U/(we^2))*xT*twoD.b22{inp.nFr}(we) - ((U/we)^2)*twoD.a22{inp.nFr}(we);

end
```

C.34 amass3.m

Calculate the added mass coefficients in heave.

```
function [A33 A35] = amass3(inp,const,frame,twoD)
%Calculate added mass coefficients in heave

A33 = @(we) 0;
A35 = @(we) 0;
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Added mass in heave due to heave acceleration, A33 [kg]
    A33 = @(we) twoD.a33{aa}(we)*frame.dx(aa) + A33(we);
    %% Added mass in heave due to pitch acceleration, A35 [kgm]
    A35 = @(we) -frame.xFr(aa)*twoD.a33{aa}(we)*frame.dx(aa) + (U/(we^2))*
        twoD.b33{aa}(we)*frame.dx(aa) + A35(we);
end

A33 = @(we) A33(we) - (U/(we^2))*twoD.b33{inp.nFr}(we);
A35 = @(we) A35(we) + (U/(we^2))*xT*twoD.b33{inp.nFr}(we) + ((U/we)^2)*twoD.a33{
    inp.nFr}(we);

end
```

C.35 amass4.m

Calculate the added mass coefficients in roll.

```
function [A42 A44 A46] = amass4(inp,const,frame,twoD)
%Calculate added mass coefficients in roll

A42 = @(we) 0;
A44 = @(we) 0;
A46 = @(we) 0;
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Added mass in roll due to sway acceleration, A42 [kgm]
    A42 = @(we) twoD.a24{aa}(we)*frame.dx(aa) + A42(we);
    %% Added mass in roll due to roll acceleration, A44 [kgm^2]
    A44 = @(we) twoD.a44{aa}(we)*frame.dx(aa) + A44(we);
    %% Added mass in roll due to yaw acceleration, A46 [kgm]
    A46 = @(we) frame.xFr(aa)*twoD.a24{aa}(we)*frame.dx(aa) - (U/(we^2))*
        twoD.b24{aa}(we)*frame.dx(aa) + A46(we);
end

A42 = @(we) A42(we) - (U/(we^2))*twoD.b24{inp.nFr}(we);
A44 = @(we) A44(we) - (U/(we^2))*twoD.b44{inp.nFr}(we);
A46 = @(we) A46(we) - (U/(we^2))*xT*twoD.b24{inp.nFr}(we) - ((U/we)^2)*twoD.a24{
    inp.nFr}(we);

end
```

C.36 amass5.m

Calculate the added mass coefficiens in pitch.

```
function [A53 A55] = amass5(inp,const,frame,twoD)
%Calculate added mass coefficients in pitch

A53 = @(we) 0;
A55 = @(we) 0;
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Added mass in pitch due to heave acceleration, A53 [kgm]
    A53 = @(we) -frame.xFr(aa)*twoD.a33{aa}(we)*frame.dx(aa) - (U/(we^2))*twoD.b33{aa}(we)*frame.dx(aa) + A53(we);
    %% Added mass in pitch due to pitch acceleration, A55 [kgm^2]
    A55 = @(we) frame.xFr(aa)*frame.xFr(aa)*twoD.a33{aa}(we)*frame.dx(aa) + ((U/we)^2)*twoD.a33{aa}(we)*frame.dx(aa) + A55(we);
end

A53 = @(we) A53(we) + (U/(we^2))*xT*twoD.b33{inp.nFr}(we);
A55 = @(we) A55(we) - (U/(we^2))*(xT^2)*twoD.b33{inp.nFr}(we) - ((U/we)^2)*xT*twoD.a33{inp.nFr}(we);

end
```

C.37 amass6.m

Calculate the added mass coefficiens in yaw.

```
function [A62 A64 A66] = amass6(inp,const,frame,twoD)
%Calculate added mass coefficients in roll

A62 = @(we) 0;
A64 = @(we) 0;
A66 = @(we) 0;
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Added mass in yaw due to sway acceleration, A62 [kgm]
    A62 = @(we) frame.xFr(aa)*twoD.a22{aa}(we)*frame.dx(aa) + (U/(we^2))*twoD.b22{aa}(we)*frame.dx(aa) + A62(we);
    %% Added mass in yaw due to roll acceleration, A64 [kgm]
    A64 = @(we) frame.xFr(aa)*twoD.a24{aa}(we)*frame.dx(aa) + (U/(we^2))*twoD.b24{aa}(we)*frame.dx(aa) + A64(we);
    %% Added mass in yaw due to yaw acceleration, A66 [kgm^2]
    A66 = @(we) frame.xFr(aa)*frame.xFr(aa)*twoD.a22{aa}(we)*frame.dx(aa) + ((U/we)^2)*twoD.a22{aa}(we)*frame.dx(aa) + A66(we);
end

A62 = @(we) A62(we) - (U/(we^2))*xT*twoD.b22{inp.nFr}(we);
```

```
A64 = @(we) A64(we) - (U/(we^2))*xT*twoD.b24{inp.nFr}(we);
A66 = @(we) A66(we) - (U/(we^2))*xT*xT*twoD.b22{inp.nFr}(we) - ((U/we)^2)*xT*
    twoD.a22{inp.nFr}(we);

end
```

C.38 gzcurve.m

Here we calculate the restoring moment and hence $\overline{GZ}$ for different heel angles in calm water in order to get the calm water $\overline{GZ}$-curve.

```
function [gz] = gzcurve(inp,const,wav,dim,frame,zg)
%Calculate the restoring moment

gz = struct;

eta4max = pi/2; % Max heel angle
number = 50; % Number of angles

restMom = zeros(number,1);
vinkel = zeros(number,1);

for aa = 1:number

rull = eta4max*(aa - 1)/(number - 1);
vinkel(aa) = rull*180/pi;

%% Transform the sections according to the motions
[rotSpSb rotSpPs rotSpSbSort] = transformation(inp,frame,0,rull,0);

%% Determine the rotated wet frames
[rotwFr rotnEls] = wetFrame(inp,const,wav,rotSpSb,rotSpPs,rotSpSbSort,
    frame.nPktSp,frame.xFr,dim.B,0,0);

%% Tangent vector of each element in each rotated section
[rottanVec] = tangentVec(rotwFr,rotnEls,inp);

%% Length of each element in each rotated section
[rotlengde] = elLength(inp,rotnEls,rottanVec);

%% Midpoint of each element in each rotated section
[rotmidpoint] = midPoint(inp,rotnEls,rotwFr);

%% Normal vector of each element in each section
[rotn2 rotn3] = normalVec(inp,rotnEls,rottanVec,rotlengde);

%% Restoring moment
[restMom(aa)] = restoringMoment(inp,const,wav,frame,rotnEls,rotmidpoint,rotn2,
    rotn3,zg,rotlengde,0,rull,0);

end

gz.GZ = restMom/(const.rho*const.g*dim.Vol);

gz.GM = ((gz.GZ(2,1) - gz.GZ(1,1))/(vinkel(2,1) - vinkel(1,1)))*180/pi;

```

```
end
```

C.39 transformation.m

This subroutine transforms the coordinates of each frame according to the rigid body motion heave, roll and pitch.

```
function [rotSpSb rotSpPs rotSpSbSort] = transformation(inp,frame,hiv,rull,stamp
    )
%Transforms the sections according to heave, roll and pitch motions

rotSpSb = cell(inp.nFr,1);
rotSpPs = cell(inp.nFr,1);
rotSpSbSort = cell(inp.nFr,1);

%Transformation matrix
Transform = @(rull) [1 0 0;0 cos(rull) -sin(rull);0 sin(rull) cos(rull)];

for aa = 1:inp.nFr
    %% Rotate the sections
    rotSpSb{aa} = frame.SpSb{aa}*Transform(rull);
    rotSpPs{aa} = frame.SpPs{aa}*Transform(rull);
    rotSpSbSort{aa} = frame.SpSbSort{aa}*Transform(rull);
    %% Taking heave and pitch motion into account
    rotSpSb{aa}(:,3) = rotSpSb{aa}(:,3) + hiv - frame.xFr(aa,1)*stamp;
    rotSpPs{aa}(:,3) = rotSpPs{aa}(:,3) + hiv - frame.xFr(aa,1)*stamp;
    rotSpSbSort{aa}(:,3) = rotSpSbSort{aa}(:,3) + hiv - frame.xFr(aa,1)*stamp;
end

end
```

C.40 restoringMoment.m

Here we integrate the pressure over the instantaneous wetted surface in order to get the restoring moment about the center of gravity.

```
function [restMom] = restoringMoment(inp,const,wav,frame,rotnEls,rotmidpoint,
    rotn2,rotn3,zg,rotlengde,h,rull,tid)
% Calculate the restoring moment at a given roll angle

moment = zeros(inp.nFr,1);

for aa = 1:inp.nFr

    y = zeros(rotnEls(aa),1);
    z = zeros(rotnEls(aa),1);
    ybar = zeros(rotnEls(aa),1); % Body-fixed y-coordinate
    zbar = zeros(rotnEls(aa),1); % Body-fixed z-coordinate
    n2 = zeros(rotnEls(aa),1);
    n3 = zeros(rotnEls(aa),1);
    n2bar = zeros(rotnEls(aa),1); % Body-fixed normal vector
```

```
    n3bar = zeros(rotnEls(aa),1); % Body-fixed normal vector
    elmoment = zeros(rotnEls(aa),1);

    for bb = 1:rotnEls(aa)
        y(bb,1) = rotmidpoint{aa}(bb,1);
        z(bb,1) = rotmidpoint{aa}(bb,2);
        ybar(bb,1) = y(bb,1)*cos(rull) + z(bb,1)*sin(rull);
        zbar(bb,1) = -y(bb,1)*sin(rull) + z(bb,1)*cos(rull);
        zbar(bb,1) = zbar(bb,1) + zg;
        n2(bb,1) = rotn2{aa}(bb,1);
        n3(bb,1) = rotn3{aa}(bb,1);
        n2bar(bb,1) = n2(bb,1)*cos(rull) + n3(bb,1)*sin(rull);
        n3bar(bb,1) = -n2(bb,1)*sin(rull) + n3(bb,1)*cos(rull);

        Z = -rotmidpoint{aa}(bb,2);
        p = real(wav.ptot(frame.xFr(aa),y(bb,1),Z,const.w0,const.we,h,tid));

        elmoment(bb,:) = p*frame.dx(aa)*rotlengde{aa}(bb,1)*(ybar(bb,1)*n3bar(bb
            ,1) - zbar(bb,1)*n2bar(bb,1));

    end
    moment(aa,1) = sum(elmoment(:,1));
end

restMom = sum(moment(:,1));

end
```

C.41 hydrostatic.m

Here we calculate the longitudinal metacentric height.

```
function [GMl] = hydrostatic(inp,dim,frame,zg,d)
%Calculate the instantaneous transverse metacentric height and the mean
%longitudinal metacentric height.

%% Vertical center of buoyancy
KB = dim.zb + d;

%% Transverse second moment of inertia
Iwl = 0;
for aa = 1:inp.nFr
   %% Longitudinal second moment of inertia
   Iwl = (1/12)*dim.B(aa)*frame.dx(aa)^3 + dim.B(aa)*frame.dx(aa)*frame.xFr(aa)
       ^2 + Iwl;
end

%% Longitudinal metacentric radius
BMl = Iwl/dim.Vol;

%% Vertical center of gravity
KG = d + zg;

%% Longitudinal initial metacentric height
GMl = KB + BMl - KG;

```

```
end
```

C.42 damping.m

Here we calculate the three dimensional damping coefficients and the viscous damping due to bilge keels.

```
function [damp] = damping(inp,const,dim,frame,inert,twoD,amass,gz,zg)
%Calculate the linear and nonlinear damping coefficients in the
%different degrees of freedom.

damp = struct;

%% Linear damping coefficients in sway
[damp.B22 damp.B24 damp.B26] = damp2(inp,const,frame,twoD);

%% Linear damping coefficients in heave
[damp.B33 damp.B35] = damp3(inp,const,frame,twoD);

%% Linear and viscous damping coefficients in roll
[damp.B42 damp.B44 damp.B46] = damp4(inp,const,dim,frame,twoD,inert,amass,gz);
[damp.Bv442] = bilgekeel(inp,const,frame,dim,zg);

%% Linear damping coefficients in pitch
[damp.B53 damp.B55 damp.Bv55] = damp5(inp,const,frame,twoD);

%% Linear damping coefficients in yaw
[damp.B62 damp.B64 damp.B66] = damp6(inp,const,frame,twoD);

%% Total damping matrix
damp.B = @(we) [0 0 0 0 0 0; ...
                0 damp.B22(we) 0 damp.B24(we) 0 damp.B26(we); ...
                0 0 damp.B33(we) 0 damp.B35(we) 0; ...
                0 damp.B42(we) 0 (damp.B44(we)+(3*pi/8)*damp.Bv442(we,(20*pi/180))) 0 damp.B46(we); ...
                0 0 damp.B53(we) 0 damp.B55(we) 0; ...
                0 damp.B62(we) 0 damp.B64(we) 0 damp.B66(we)];

end
```

C.43 damp2.m

Calculate the damping coefficients in sway.

```
function [B22 B24 B26] = damp2(inp,const,frame,twoD)
%Calculate the damping coefficients in roll

B22 = @(we) 0;
B24 = @(we) 0;
B26 = @(we) 0;
```

```
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Linear damping coefficient in sway due to sway velocity, B22 [kg/s]
    B22 = @(we) twoD.b22{aa}(we)*frame.dx(aa) + B22(we);
    %% Linear damping coefficient in sway due to roll velocity, B24 [kgm/s]
    B24 = @(we) twoD.b24{aa}(we)*frame.dx(aa) + B24(we);
    %% Linear damping coefficient in sway due to yaw velocity, B26 [kgm/s]
    B26 = @(we) frame.xFr(aa)*twoD.b22{aa}(we)*frame.dx(aa) + U*twoD.a22{aa}(we)*frame.dx(aa) + B26(we);
end

B22 = @(we) B22(we) + U*twoD.a22{inp.nFr}(we);
B24 = @(we) B24(we) + U*twoD.a24{inp.nFr}(we);
B26 = @(we) B26(we) + U*xT*twoD.a22{inp.nFr}(we) - ((U/we)^2)*twoD.b22{inp.nFr}(we);

end
```

C.44 damp3.m

Calculate the damping coefficients in heave.

```
function [B33 B35] = damp3(inp,const,frame,twoD)
%Calculate damping coefficients in heave

B33 = @(we) 0;
B35 = @(we) 0;
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Damping coefficient in heave due to heave velocity, B33 [kg/s]
    B33 = @(we) twoD.b33{aa}(we)*frame.dx(aa) + B33(we);
    %% Damping coefficient in heave due to pitch velocity, B35 [kgm/s]
    B35 = @(we) -frame.xFr(aa)*twoD.b33{aa}(we)*frame.dx(aa) - U*twoD.a33{aa}(we)*frame.dx(aa) + B35(we);
end

B33 = @(we) B33(we) + U*twoD.a33{inp.nFr}(we);
B35 = @(we) B35(we) - U*xT*twoD.a33{inp.nFr}(we) + ((U/we)^2)*twoD.b33{inp.nFr}(we);

end
```

C.45 damp4.m

Calculate the linear damping coefficients in roll.

```
function [B42 B44 B46] = damp4(inp,const,dim,frame,twoD,inert,amass,gz)
%Calculate the damping coefficients in roll

% %% Critical damping in roll [kgm^2/s]
% B4cr = 2*sqrt((inert.I44 + amass.A44(const.we))*const.rho*const.g*dim.Vol*gz.GMmean);

B42 = @(we) 0;
B44 = @(we) 0;
B46 = @(we) 0;
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Damping coefficient in roll due to sway velocity, B42 [kgm/s]
    B42 = @(we) twoD.b24{aa}(we)*frame.dx(aa) + B42(we);
    %% Damping coefficient in roll due to roll velocity, B44 [kgm^2/s]
    B44 = @(we) twoD.b44{aa}(we)*frame.dx(aa) + B44(we);
    %% Damping coefficient in roll due to yaw velocity, B46 [kgm/s]
    B46 = @(we) frame.xFr(aa)*twoD.b24{aa}(we)*frame.dx(aa) + U*twoD.a24{aa}(we)*frame.dx(aa) + B46(we);
end

B42 = @(we) B42(we) + U*twoD.a24{inp.nFr}(we);
B44 = @(we) B44(we) + U*twoD.a44{inp.nFr}(we);
B46 = @(we) B46(we) + U*xT*twoD.a24{inp.nFr}(we) - ((U/we)^2)*twoD.b24{inp.nFr}(we);

end
```

C.46 damp5.m

Calculate the damping coefficients in pitch.

```
function [B53 B55 Bv55] = damp5(inp,const,frame,twoD)
%Calculate the damping coefficients in pitch

B53 = @(we) 0;
B55 = @(we) 0;
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Damping coefficient in pitch due to heave velocity, B53 [kgm/s]
    B53 = @(we) -frame.xFr(aa)*twoD.b33{aa}(we)*frame.dx(aa) + U*twoD.a33{aa}(we)*frame.dx(aa) + B53(we);
    %% Damping coefficient in pitch due to pitch velocity, B55 [kgm^2/s]
    B55 = @(we) frame.xFr(aa)*frame.dx(aa)*twoD.b33{aa}(we)*frame.xFr(aa) + ((U/we)^2)*twoD.b33{aa}(we)*frame.dx(aa) + B55(we);
end

B53 = @(we) B53(we) - U*xT*twoD.a33{inp.nFr}(we);
B55 = @(we) B55(we) + U*(xT^2)*twoD.a33{inp.nFr}(we) - ((U^2)/(we^2))*xT*twoD.b33{inp.nFr}(we);
```

```

%% Viscous damping coefficient in pitch, Bv55 [kgm^2/s^2]
Bv55 = 0*15*10^9;


end
```

C.47 damp6.m

Calculate the damping coefficients in yaw.

```
function [B62 B64 B66] = damp6(inp,const,frame,twoD)
%Calculate the damping coefficients in roll

B62 = @(we) 0;
B64 = @(we) 0;
B66 = @(we) 0;
%% x-position of transom stern [m]
xT = frame.xFr(inp.nFr);
%% Forward speed [m/s]
U = const.U;
for aa = 1:inp.nFr
    %% Linear damping coefficient in sway due to sway velocity, B22 [kg/s]
    B62 = @(we) frame.xFr(aa)*twoD.b22{aa}(we)*frame.dx(aa) - U*twoD.a22{aa}(we)
        *frame.dx(aa) + B62(we);
    %% Linear damping coefficient in sway due to roll velocity, B24 [kgm/s]
    B64 = @(we) frame.xFr(aa)*twoD.b24{aa}(we)*frame.dx(aa) - U*twoD.a24{aa}(we)
        *frame.dx(aa) + B64(we);
    %% Linear damping coefficient in sway due to yaw velocity, B26 [kgm/s]
    B66 = @(we) frame.xFr(aa)*frame.xFr(aa)*twoD.b22{aa}(we)*frame.dx(aa) + ((U/
        we)^2)*twoD.b22{aa}(we)*frame.dx(aa) + B66(we);
end

B62 = @(we) B62(we) + U*xT*twoD.a22{inp.nFr}(we);
B64 = @(we) B64(we) + U*xT*twoD.a24{inp.nFr}(we);
B66 = @(we) B66(we) + U*xT*xT*twoD.a22{inp.nFr}(we) - ((U/we)^2)*xT*twoD.b22{
    inp.nFr}(we);

end
```

C.48 bilgekeel.m

Calculate the viscous damping coefficient in roll due to bilge keel. This is based on empirical formulas found in Ikeda et al. (1977) and Ikeda & Tanaka (1976).

```
function [Bv442bk] = bilgekeel(inp,const,frame,dim,zg)
%Calculate the damping coefficients in roll

ca = zeros(inp.nFr,1);
f = zeros(inp.nFr,1);
Bv44bk = @(we,rull) 0;

```

```
fid = fopen('Input/Trhav/Bilgekeel.txt');

a = textscan(fid,'%f %f %f %f',inp.nFr);
fclose(fid);
ybk(:,1) = a{2}(:);
zbk(:,1) = a{3}(:);
bbk(:,1) = a{4}(:);

for aa = 1:inp.nFr
    B = 2*max(frame.wFr{inp.nFr - aa + 1}(:,2));
    d = -1*min(frame.wFr{inp.nFr - aa + 1}(:,3));
    ca(aa) = dim.sac((inp.nFr - aa + 3),2)/(B*d);
    f(aa) = 1 + 0.3*exp(-160*(1 - ca(aa)));
    if bbk(aa,1) == 0;
        l = 0;
        rbk = 1;
    else
        l = frame.dx(inp.nFr - aa + 1);
        rbk = sqrt((ybk(aa,1))^2 + (zbk(aa,1))^2);
    end

    H0 = B/(2*d);
    R = 0;
    S0 = @(rull) 0.3*(pi*f(aa)*rbk)*rull + 1.95*bbk(aa,1);
    m1 = R/d;
    m2 = zg/d;
    m3 = 1 - m1 - m2;
    m4 = H0 - m1;
    m5 = (0.414*H0 + (0.0651*m1^2) - (0.382*H0 + 0.0106)*m1)/((H0 - 0.215*m1)*(1
         - 0.215*m1));
    m6 = (0.414*H0 + (0.0651*m1^2) - (0.382 + 0.0106*H0)*m1)/((H0 - 0.215*m1)*(1
         - 0.215*m1));
    m7 = @(rull) (S0(rull)/d) + 0.25*pi*m1;
    m8 = @(rull) m7(rull) + 0.414*m1;
    A = @(rull) (m3 + m4)*m8(rull) - m7(rull)*m7(rull);
    B = ((m4^3)/(3*(H0 - 0.215*m1))) + ((1 - m1)*(1 - m1)*(2*m3 - m2)/(6*(1 - 0
        .215*m1))) + m1*(m3*m5 + m4*m6);
    Cd = @(rull) 22.5*(bbk(aa,1)/(pi*f(aa)*rbk*(rull + eps))) + 2.4;
    Cppluss = 1.2;
    Cpminus = @(rull) -22.5*(bbk(aa,1)/(pi*rbk*f(aa)*(rull + eps))) - 1.2;

    Bv44bk = @(we,rull) (8/(3*pi))*const.rho*we*rull*f(aa)*f(aa)*l*(rbk*bbk(aa
        ,1)*Cd(rull) + 0.5*d*d*(-A(rull)*Cpminus(rull) + B*Cppluss)) + Bv44bk(we,
        rull);

end

Bv442bk = @(we,rull) Bv44bk(we,rull);



end
```

C.49 restoring.m

In this subroutine we calculate the restoring coefficients and the natural frequencies of each rigid body mode.

```
function [rest] = restoring(inp,const,wav,dim,frame,inert,amass,gz,GMl,zg,h)
%Calculating the restoring coefficients and undamped natural
%frequencies for the different degrees of freedom.

rest = struct;

%% Restoring coefficients in heave
[rest.C33 rest.C35] = restor3(inp,const,dim,frame);

%% Restoring coefficients in roll
[rest.C44 rest.C44lin] = restor4(inp,const,wav,dim,frame,gz,zg,h);

%% Restoring coefficients in pitch
[rest.C53 rest.C55] = restor5(inp,const,dim,frame,GMl);

%% Undamped natural frequencies
[rest.wn3 rest.wn4 rest.wn5 rest.a rest.b rest.c] = natfreq(inert,amass,rest.C33
    ,rest.C35,rest.C44lin,rest.C53,rest.C55,zg);

%% Restoring matrix
rest.C = [0 0 0        0           0        0;...
          0 0 0        0           0        0;...
          0 0 rest.C33 0           rest.C35 0;...
          0 0 0        rest.C44lin 0        0;...
          0 0 rest.C53 0           rest.C55 0;...
          0 0 0        0           0        0];

end
```

C.50 restor3.m

Here we calculate the restoring coefficient in heave.

```
function [C33 C35] = restor3(inp,const,dim,frame)
%Calculate restoring coefficients in heave

%% Restoring coefficient in heave due to displacement in heave, C33 [N/m]
C33 = const.rho*const.g*dim.Aw;

%% Restoring coefficient in heave due to a pitch angle, C35 [N]
C35 = 0;
for aa = 1:inp.nFr
    C35 = -const.rho*const.g*frame.xFr(aa)*dim.B(aa)*frame.dx(aa) + C35;
end

end
```

C.51 restor4.m

Here we calculate the restoring moment i roll as a function of heave, roll, pitch and time in addition to the linear restoring coefficient in roll.

```
function [C44 C44lin] = restor4(inp,const,wav,dim,frame,gz,zg,h)
%Calculate resoring coefficients in roll

%% Restoring coefficient in roll due to roll angle, C44 [Nm]
C44 = @(hiv,rull,stamp,tid) restoring4(inp,const,wav,frame,dim,zg,h,hiv,rull,
    stamp,tid);

%% Linear restoring coefficient in roll
C44lin = const.rho*const.g*dim.Vol*gz.GM;

end
```

C.52 restoring4.m

This subroutine calculates the restoring moment in roll from prescribed values of heave, roll, pitch and wave elevation (time).

```
function [restMom rotn2 rotn3] = restoring4(inp,const,wav,frame,dim,zg,h,hiv,
    rull,stamp,tid)
%Calculate the restoring moment

%% Rotate the sections
[rotSpSb rotSpPs rotSpSbSort] = transformation(inp,frame,hiv,rull,stamp);

%% Determine the rotated wet frames
[rotwFr rotnEls] = wetFrame(inp,const,wav,rotSpSb,rotSpPs,rotSpSbSort,
    frame.nPktSp,frame.xFr,dim.B,h,tid);

%% Tangent vector of each element in each rotated section
[rottanVec] = tangentVec(rotwFr,rotnEls,inp);

%% Length of each element in each rotated section
[rotlengde] = elLength(inp,rotnEls,rottanVec);

%% Midpoint of each element in each rotated section
[rotmidpoint] = midPoint(inp,rotnEls,rotwFr);

%% Normal vector of each element in each section
[rotn2 rotn3] = normalVec(inp,rotnEls,rottanVec,rotlengde);

%% Restoring moment
[restMom] = restoringMoment(inp,const,wav,frame,rotnEls,rotmidpoint,rotn2,rotn3,
    zg,rotlengde,h,rull,tid);

end
```

C.53 restor5.m

Here we calculate the restoring coefficient in pitch.

```
function [C53 C55] = restor5(inp,const,dim,frame,GMl)
```

```
%Calculate resoring coefficients in pitch

%% Restoring coefficient in pitch due to heave displacement, C53 [N]
C53 = 0;
for aa = 1:inp.nFr
    C53 = -const.rho*const.g*frame.xFr(aa)*dim.B(aa)*frame.dx(aa) + C53;
end

%% Restoring coefficient in pitch due to a pitch angle, C55 [Nm]
C55 = const.rho*const.g*dim.Vol*GMl;

end
```

C.54 natfreq.m

Here we calculate the undamped natural frequencies in coupled heave-pitch and coupled sway-roll-yaw.

```
function [wn3 wn4 wn5 a b c] = natfreq(inert,amass,C33,C35,C44lin,C53,C55,zg)
%Calculate the undamped natural frequencies in coupled heave-pitch,
%pitch-heave and sway-roll-yaw.

%% Sway-roll-yaw
nevnar = @(we) (inert.M+amass.A22(we))*(amass.A44(we)+inert.I44)*(amass.A66(we)+
    inert.I66)-(inert.M+amass.A22(we))*(amass.A64(we)-inert.I46)*(amass.A46(we)-
    inert.I46)-(-inert.M*zg+amass.A24(we))*(-inert.M*zg+amass.A42(we))*(amass.A66
    (we)+inert.I66)+(-inert.M*zg+amass.A24(we))*(amass.A46(we)-inert.I46)*
    amass.A62(we)+amass.A26(we)*(-inert.M*zg+amass.A42(we))*(amass.A64(we)-
    inert.I46)-amass.A26(we)*(amass.A44(we)+inert.I44)*amass.A62(we);
teljar = @(we) (inert.M+amass.A22(we))*(amass.A66(we)+inert.I66)*C44lin -
    amass.A26(we)*amass.A62(we)*C44lin;
wn4 = @(we) sqrt(teljar(we)/nevnar(we));

%% Heave-pitch
a = @(we) (inert.M + amass.A33(we))*(inert.I55 + amass.A55(we)) - amass.A35(we)*
    amass.A53(we);
b = @(we) -(inert.M + amass.A33(we))*C55 - (inert.I55 + amass.A55(we))*C33 +
    amass.A53(we)*C35 + amass.A35(we)*C53;
c = C33*C55 - C35*C53;

wn3sq = @(we) (-b(we) + sqrt(b(we)*b(we) - 4*a(we)*c))/(2*a(we));
wn3 = @(we) sqrt(wn3sq(we));

wn5sq = @(we) (-b(we) - sqrt(b(we)*b(we) - 4*a(we)*c))/(2*a(we));
wn5 = @(we) sqrt(wn5sq(we));


end
```

C.55 excitation.m

This subroutine calculates the excitation forces and moments in heave and pitch.

```
function [excit] = excitation(inp,const,wav,dim,frame,els,twoD,h)
%Calculate the exitation forces in each degree of freedom

excit = struct;

%% Sway force
[excit.F2 fk2 fd2 excit.F2fk excit.F2d] = force2(inp,const,els,frame,wav,h,twoD,
    dim);

%% Heave force
[excit.F3 fk3 fd3 excit.F3fk excit.F3d] = force3(inp,const,els,frame,wav,h,twoD,
    dim);

%% Roll moment
[excit.F4 excit.F4fk excit.F4d] = force4(inp,const,els,frame,wav,h,twoD,dim);

%% Pitch moment
[excit.F5 excit.F5fk excit.F5d] = force5(inp,const,frame,dim,twoD,wav,fk3,fd3);

%% Yaw moment
[excit.F6 excit.F6fk excit.F6d] = force6(inp,const,frame,dim,twoD,wav,fk2,fd2);

%% Total force vector
excit.Fa = @(w0,we) [0; excit.F2(w0,we,0); excit.F3(w0,we,0); excit.F4(w0,we,0);
     excit.F5(w0,we,0); excit.F6(w0,we,0)];

end
```

C.56 force2.m

Here we calculate the Froude-Kriloff and diffraction forces in sway.

```
function [F3 fk3 fd3 F3fk F3d] = force3(inp,const,els,frame,wav,h,twoD,dim)
%Calculate the wave excitation force in heave

%% Sectional Froude-Kriloff force in heave
fk3 = cell(inp.nFr,1);
%% Sectional diffrction force in heave
fd3 = cell(inp.nFr,1);

F3fk = @(w0,we,t) 0;
F3d = @(w0,we,t) 0;
F3 = @(w0,we,t) 0;

%% Sectional Froude-Kriloff force in heave
for aa = 1:inp.nFr
    fk3{aa} = @(w0,we,t) 0;
    x = frame.xFr(aa);
    d = dim.maxdraught(aa);
    s = dim.sac((aa+2),2)/(dim.maxdraught(aa)*2*max(frame.wFr{aa}(:,2)));
    ds = d*s;

    %% Sectional Froude-Kriloff force in heave
    for bb = 1:els.nEls(aa)
        y = els.midpoint{aa}(bb,1);
        z = els.midpoint{aa}(bb,2);
```

```
        n3 = els.n3{aa}(bb);
        fk3{aa} = @(w0,we,t) -wav.pdyn(x,y,z,w0,we,h,t)*n3*els.lengde{aa}(bb) +
            fk3{aa}(w0,we,t);

    end

    %% Sectional diffrction force in heave
    fd3{aa} = @(w0,we,t) (twoD.a33{aa}(we)*(wav.az(x,0,-ds,w0,we,t)) + twoD.b33{
        aa}(we)*(wav.w(x,0,-ds,w0,we,t)));

    %% Total Froude-Kriloff force in heave
    F3fk = @(w0,we,t) fk3{aa}(w0,we,t)*frame.dx(aa) + F3fk(w0,we,t);
    %% Total diffraction force in heave
    F3d = @(w0,we,t) fd3{aa}(w0,we,t)*frame.dx(aa) + F3d(w0,we,t);

    F3 = @(w0,we,t) (fk3{aa}(w0,we,t) + fd3{aa}(w0,we,t))*frame.dx(aa) + F3(w0,
        we,t);
end

%% Total wave excitation force in heave
F3 = @(w0,we,t) F3(w0,we,t) + const.U*twoD.a33{inp.nFr}(we)*wav.w(x,0,-ds,w0,we,
    t);

end
```

C.57 force3.m

Here we calculate the Froude-Kriloff and diffraction forces in heave.

```
function [F3 fk3 fd3 F3fk F3d] = force3(inp,const,els,frame,wav,h,twoD,dim)
%Calculate the wave excitation force in heave

%% Sectional Froude-Kriloff force in heave
fk3 = cell(inp.nFr,1);
%% Sectional diffrction force in heave
fd3 = cell(inp.nFr,1);

F3fk = @(w0,we,t) 0;
F3d = @(w0,we,t) 0;
F3 = @(w0,we,t) 0;

%% Sectional Froude-Kriloff force in heave
for aa = 1:inp.nFr
    fk3{aa} = @(w0,we,t) 0;
    x = frame.xFr(aa);
    d = dim.maxdraught(aa);
    s = dim.sac((aa+2),2)/(dim.maxdraught(aa)*2*max(frame.wFr{aa}(:,2)));
    ds = d*s;

    %% Sectional Froude-Kriloff force in heave
    for bb = 1:els.nEls(aa)
        y = els.midpoint{aa}(bb,1);
        z = els.midpoint{aa}(bb,2);
        n3 = els.n3{aa}(bb);
        fk3{aa} = @(w0,we,t) -wav.pdyn(x,y,z,w0,we,h,t)*n3*els.lengde{aa}(bb) +
            fk3{aa}(w0,we,t);
```

```

    end

    %% Sectional diffrction force in heave
    fd3{aa} = @(w0,we,t) (twoD.a33{aa}(we)*(wav.az(x,0,-ds,w0,we,t)) + twoD.b33{aa}(we)*(wav.w(x,0,-ds,w0,we,t)));

    %% Total Froude-Kriloff force in heave
    F3fk = @(w0,we,t) fk3{aa}(w0,we,t)*frame.dx(aa) + F3fk(w0,we,t);
    %% Total diffraction force in heave
    F3d = @(w0,we,t) fd3{aa}(w0,we,t)*frame.dx(aa) + F3d(w0,we,t);

    F3 = @(w0,we,t) (fk3{aa}(w0,we,t) + fd3{aa}(w0,we,t))*frame.dx(aa) + F3(w0,we,t);
end

%% Total wave excitation force in heave
F3 = @(w0,we,t) F3(w0,we,t) + const.U*twoD.a33{inp.nFr}(we)*wav.w(x,0,-ds,w0,we,t);

end
```

C.58 force4.m

Here we calculate the Froude-Kriloff and diffraction moments in roll.

```
function [F4 F4fk F4d] = force4(inp,const,els,frame,wav,h,twoD,dim)
%Calculate the wave excitation moment in roll

%% Sectional Froude-Kriloff moment in roll
fk4 = cell(inp.nFr,1);
%% Sectional diffrction moment in roll
fd4 = cell(inp.nFr,1);

F4fk = @(w0,we,t) 0;
F4d = @(w0,we,t) 0;
F4 = @(w0,we,t) 0;

%% Sectional Froude-Kriloff moment in roll
for aa = 1:inp.nFr
    fk4{aa} = @(w0,we,t) 0;
    x = frame.xFr(aa);
    d = dim.maxdraught(aa);
    s = dim.sac((aa+2),2)/(dim.maxdraught(aa)*2*max(frame.wFr{aa}(:,2)));
    ds = d*s;

    %% Sectional Froude-Kriloff moment in roll
    for bb = 1:els.nEls(aa)
        y = els.midpoint{aa}(bb,1);
        z = els.midpoint{aa}(bb,2);
        n2 = els.n2{aa}(bb);
        n3 = els.n3{aa}(bb);
        n4 = y*n3 - z*n2;
        fk4{aa} = @(w0,we,t) -wav.pdyn(x,y,z,w0,w0,h,t)*n4*els.lengde{aa}(bb) + fk4{aa}(w0,we,t);
    end
```

```
    %% Sectional diffrction moment in roll
    fd4{aa} = @(w0,we,t) (twoD.a24{aa}(we)*wav.ay(x,0,-ds,w0,we,t) + twoD.b24{aa
        }(we)*wav.v(x,0,-ds,w0,we,t));

    %% Total Froude-Kriloff moment in roll
    F4fk = @(w0,we,t) fk4{aa}(w0,we,t)*frame.dx(aa) + F4fk(w0,we,t);
    %% Total diffraction moment in roll
    F4d = @(w0,we,t) fd4{aa}(w0,we,t)*frame.dx(aa) + F4d(w0,we,t);

    F4 = @(w0,we,t) (fk4{aa}(w0,we,t) + fd4{aa}(w0,we,t))*frame.dx(aa) + F4(w0,
        we,t);

end

%% Total wave excitation moment in roll
F4 = @(w0,we,t) F4(w0,we,t) + const.U*twoD.a24{inp.nFr}(we)*wav.v(x,0,-ds,w0,we,
    t);

end
```

C.59 force5.m

Here we calculate the Froude-Kriloff and diffraction moments in pitch.

```
function [F5 F5fk F5d] = force5(inp,const,frame,dim,twoD,wav,fk3,fd3)
%Calculate the wave excitation moment in pitch

%% Sectional Froude-Kriloff moment in pitch
fk5 = cell(inp.nFr,1);
%% Sectional diffrction moment in pitch
fd5 = cell(inp.nFr,1);

F5fk = @(w0,we,t) 0;
F5d = @(w0,we,t) 0;
F5 = @(w0,we,t) 0;

for aa = 1:inp.nFr
    x = frame.xFr(aa);
    d = dim.maxdraught(aa);
    s = dim.sac((aa+2),2)/(dim.maxdraught(aa)*2*max(frame.wFr{aa}(:,2)));
    ds = d*s;

    %% Sectional Froude-Kriloff moment in pitch
    fk5{aa} = @(w0,we,t) -x*fk3{aa}(w0,we,t);
    %% Sectional diffrction moment in pitch
    fd5{aa} = @(w0,we,t) -x*fd3{aa}(w0,we,t);

    %% Total Froude-Kriloff moment in pitch
    F5fk = @(w0,we,t) fk5{aa}(w0,we,t)*frame.dx(aa) + F5fk(w0,we,t);
    %% Total diffraction moment in pitch
    F5d = @(w0,we,t) fd5{aa}(w0,we,t)*frame.dx(aa) + F5d(w0,we,t);
    F5 = @(w0,we,t) -x*(fk3{aa}(w0,we,t) + fd3{aa}(w0,we,t))*frame.dx(aa) +
        const.U*twoD.a33{aa}(we)*wav.w(x,0,-ds,w0,we,t)*frame.dx(aa) + F5(w0,we,t
        );

```

```
end

%% Total wave excitation moment in pitch
F5 = @(w0,we,t) F5(w0,we,t) - const.U*frame.xFr(inp.nFr)*twoD.a33{inp.nFr}(we)*
    wav.w(x,0,-ds,w0,we,t);

end
```

C.60 force6.m

Here we calculate the Froude-Kriloff and diffraction moments in yaw.

```
function [F6 F6fk F6d] = force6(inp,const,frame,dim,twoD,wav,fk2,fd2)
%Calculate the wave excitation moment in yaw

%% Sectional Froude-Kriloff moment in yaw
fk6 = cell(inp.nFr,1);
%% Sectional diffrction moment in yaw
fd6 = cell(inp.nFr,1);

F6fk = @(w0,we,t) 0;
F6d = @(w0,we,t) 0;
F6 = @(w0,we,t) 0;

for aa = 1:inp.nFr
    x = frame.xFr(aa);
    d = dim.maxdraught(aa);
    s = dim.sac((aa+2),2)/(dim.maxdraught(aa)*2*max(frame.wFr{aa}(:,2)));
    ds = d*s;

    %% Sectional Froude-Kriloff moment in yaw
    fk6{aa} = @(w0,we,t) x*fk2{aa}(w0,we,t);
    %% Sectional diffrction moment in yaw
    fd6{aa} = @(w0,we,t) x*fd2{aa}(w0,we,t);

    %% Total Froude-Kriloff moment in yaw
    F6fk = @(w0,we,t) fk6{aa}(w0,we,t)*frame.dx(aa) + F6fk(w0,we,t);
    %% Total diffraction moment in yaw
    F6d = @(w0,we,t) fd6{aa}(w0,we,t)*frame.dx(aa) + F6d(w0,we,t);

    F6 = @(w0,we,t) x*(fk2{aa}(w0,we,t) + fd2{aa}(w0,we,t))*frame.dx(aa) -
        const.U*twoD.a22{aa}(we)*wav.v(x,0,-ds,w0,we,t)*frame.dx(aa) + F6(w0,we,t
        );
end

%% Total wave excitation moment in yaw
F6 = @(w0,we,t) F6(w0,we,t) + const.U*frame.xFr(inp.nFr)*twoD.a22{inp.nFr}(we)*
    wav.v(x,0,-ds,w0,we,t)*frame.dx(aa);

end
```

C.61 transfer.m

In this subroutine we solve the coupled equations of motion in the frequency domain. We use the frequency-response method in order to do this, and the results are presented in terms of transfer functions.

```
function [trans] = transfer(inp,const,inert,amass,rest,damp,excit,h)
%% Calculating the transfer functions in sway, heave, roll, pitch and yaw

nedr = 0.167; %% Lower frequency limit
ovr = 3; %% Upper frequency limit
ant = 100; %% Number of frequencies

w = zeros(ant,1); %% Wave frequency
T = zeros(ant,1); %% Wave period
lambda = zeros(ant,1); %% Wave length
LL = zeros(ant,1); %% Non-dimensional wave length
eta = zeros(ant,6); %% Reponse vector
eta2a = zeros(ant,1); %% Sway amplitude
eta3a = zeros(ant,1); %% Heave amplitude
eta4a = zeros(ant,1); %% Roll amplitude
eta5a = zeros(ant,1); %% Pitch amplitude
eta6a = zeros(ant,1); %% Yaw amplitude
trans.H2 = zeros(ant,1); %% Transfer function in sway
trans.H3 = zeros(ant,1); %% Transfer function in heave
trans.H4 = zeros(ant,1); %% Transfer function in roll
trans.H5 = zeros(ant,1); %% Transfer function in pitch
trans.H6 = zeros(ant,1); %% Transfer function in yaw

for aa = 1:ant
    w(aa,1) = nedr + (ovr-nedr)*(aa-0.5)/(ant-0.5);
    T(aa,1) = 2*pi/w(aa,1);
    lambda(aa,1) = 2*pi*const.g/(w(aa,1)*w(aa,1));
    LL(aa,1) = lambda(aa,1)/inp.Lpp;
    k = w(aa,1)*w(aa,1)/const.g;
    we = w(aa,1) + k*const.U*const.betta;

    H = -(we*we)*(inert.Mass + amass.A(we)) + (1i*we)*damp.B(we) + rest.C;
    F = excit.Fa(w(aa,1),we);
    eta(aa,:) = H\F;

    eta2a(aa,1) = abs(eta(aa,2));
    eta3a(aa,1) = abs(eta(aa,3));
    eta4a(aa,1) = abs(eta(aa,4));
    eta5a(aa,1) = abs(eta(aa,5));
    eta6a(aa,1) = abs(eta(aa,6));

    trans.H2(aa,1) = eta2a(aa,1)/(h/2);
    trans.H3(aa,1) = eta3a(aa,1)/(h/2);
    trans.H4(aa,1) = eta4a(aa,1)/((h/2)*k);
    trans.H5(aa,1) = eta5a(aa,1)/((h/2)*k);
    trans.H6(aa,1) = eta6a(aa,1)/((h/2));

end

trans.omega = w(:,1);
trans.lambdaL = LL(:,1);

```

```
end
```

C.62 eqmotion.m

In this subroutine we solve the coupled equations of motion in the time domain. We use the built-in function *ode45* in order to do this, and the results are presented in terms of time series.

```
function [resp t] = eqmotion(const,inert,amass,rest,damp,excit,zg)
%eqmotion: Solving the equations of motion by use of the built-in function
%ode45.

%time interval
tmin = 0;
tmax = 0.01;
tspan = [tmin, tmax];

M = inert.M; %% Vessel mass
I44 = inert.I44; %% Moment of inertia in roll
I55 = inert.I55; %% Moment of inertia in pitch
I46 = inert.I46; %% Product of inertia in coupled roll-yaw
I66 = inert.I66; %% Moment of inertia in yaw
A22 = amass.A22; %% Added mass in sway
A24 = amass.A24; %% Added mass in coupled sway-roll
A26 = amass.A26; %% Added mass in coupled sway-yaw
A33 = amass.A33; %% Added mass in heave
A35 = amass.A35; %% Added mass in coupled heave-pitch
A42 = amass.A42; %% Added mass in coupled roll-sway
A44 = amass.A44; %% Added mass in roll
A46 = amass.A46; %% Added mass in coupled roll-yaw
A53 = amass.A53; %% Added mass in coupled pitch-heave
A55 = amass.A55; %% Added mass in pitch
A62 = amass.A62; %% Added mass in coupled yaw-sway
A64 = amass.A64; %% Added mass in coupled yaw-roll
A66 = amass.A66; %% Added mass in yaw
B22 = damp.B22; %% Damping in sway
B24 = damp.B24; %% Damping in coupled sway-roll
B26 = damp.B26; %% Damping in coupled sway-yaw
B33 = damp.B33; %% Damping in heave
B35 = damp.B35; %% Damping in coupled heave-pitch
B42 = damp.B42; %% Damping in coupled roll-sway
B44 = damp.B44; %% Damping in roll
Bv442 = damp.Bv442; %% Viscous damping in roll
B46 = damp.B46; %% Damping in coupled roll-yaw
B53 = damp.B53; %% Damping in coupled pitch-heave
B55 = damp.B55; %% Damping in pitch
B62 = damp.B62; %% Damping in coupled yaw-sway
B64 = damp.B64; %% Damping in coupled yaw-roll
B66 = damp.B66; %% Damping in yaw
Bv55 = damp.Bv55; %% Viscous damping in pitch
C33 = rest.C33; %% Restoring coefficient in heave
C35 = rest.C35; %% Restoring coefficient in coupled heave-pitch
C44 = rest.C44; %% Restoring moment in roll
C53 = rest.C53; %% Restoring coefficient in coupled pitch-heave
C55 = rest.C55; %% Restoring coefficient in pitch
F2 = excit.F2; %% Excitation force in sway
F3 = excit.F3; %% Excitation force in heave
```

```
F4 = excit.F4; %% Excitation force in roll
F5 = excit.F5; %% Excitation force in pitch
F6 = excit.F6; %% Excitation force in sway


%% initial conditions:
eta2start = 0; %% initial sway displacement in metres.
eta21start = 0; %% initial sway velocity in metres per second.
eta3start = 0; %% initial heave displacement in metres.
eta31start = 0; %% initial heave velocity in metres per second.
eta4start = 1; %% initial roll angle in degrees.
eta41start = 0; %% initial roll velocity in degrees per second.
eta5start = 0; %% initial pitch angle in degrees.
eta51start = 0; %% initial pitch velocity in degrees per second.
eta6start = 0; %% initial yaw angle in degrees.
eta61start = 0; %% initial yaw velocity in degrees per second.

%% defines the initial conditions in radians insted of degrees:
y0 = [eta2start; eta21start;...
      eta3start; eta31start;...
      eta4start*pi/180; eta41start*pi/180;...
      eta5start*pi/180; eta51start*pi/180;...
      eta6start*pi/180; eta61start*pi/180];

%% Constants used in the equations of motions
A = @(we) ((A24(we)*(A46(we) - I46)/(I44 + A44(we))) + zg*M - A26(we));
B = @(we) ((A42(we)*(A64(we) - I46)/(I44 + A44(we))) + zg*M - A62(we));
C = @(we) (M + A22(we) - ((A24(we)*A42(we))/(I44 + A44(we))));
D = @(we) (I66 + A66(we) - ((A64(we) - I46)*(A46(we) - I46)/(I44 + A44(we))));
K1 = @(we) (1/(I44 + A44(we)));
K2 = @(we) ((A64(we) - I46)/(I44 + A44(we)));
K3 = @(we) (A24(we)/(I44 + A44(we)));
K4 = @(we) ((A46(we) - I46)/(I44 + A44(we)));
K5 = @(we) (A42(we)/(I44 + A44(we)));
B1 = @(we) (D(we)/(C(we)*D(we) - A(we)*B(we)));
B2 = @(we) (B(we)/(C(we)*D(we) - A(we)*B(we)));
C1 = @(we) -B1(we);
C2 = @(we) B1(we)*((A(we)/D(we))*K2(we) + K3(we));
C3 = @(we) -B1(we)*A(we)/D(we);
C4 = @(we) K5(we)*B1(we) + K4(we)*B2(we);
C5 = @(we) -K5(we)*K2(we)*(A(we)/D(we))*B1(we) - K5(we)*K3(we)*B1(we) - K4(we)*
    K2(we)*(A(we)/D(we))*B2(we) - K4(we)*K3(we)*B2(we) + K4(we)*K2(we)*(1/D(we))
    - K1(we);
C6 = @(we) K5(we)*(A(we)/D(we))*B1(we) + K4(we)*(A(we)/D(we))*B2(we);
C7 = @(we) -B2(we);
C8 = @(we) K2(we)*(A(we)/D(we))*B2(we) + K3(we)*B2(we) + K2(we)*(1/D(we));
C9 = @(we) -B2(we)*(A(we)/D(we)) - (1/D(we));
a = -zg*M;
b = -zg*M;
c = M;
d = @(we) I55 + A55(we) - ((A53(we)*A35(we))/(M + A33(we)));
k1 = @(we) (a*b/(c*d(we) - a*b));
c1 = @(we) ((-k1(we) + 1)*(((A53(we))^2)/(d(we)*(M + A33(we)))) - 1)*(1/(M + A33
    (we)));
c2 = @(we) -(k1(we) + 1)*A53(we)/(d(we)*(M + A33(we)));
c3 = @(we) -c2(we);
c4 = @(we) -(k1(we) + 1)*1/d(we);

%% coupled equation of motion in sway-roll-yaw. Written as a system of first
    order differential equations.
svai = @(t,y) (B22(const.we)*y(2) + B24(const.we)*y(6) + B26(const.we)*y(10) -
```

```
real(F2(const.w0,const.we,t)));
rull = @(t,y) (B42(const.we)*y(2) + (B44(const.we) + Bv442(const.we,abs(max(y(5))))*abs(y(6)))*y(6) + B46(const.we)*y(10) + C44(y(3),y(5),y(7),t) - real(F4(const.w0,const.we,t)));
gir = @(t,y) (B62(const.we)*y(2) + B64(const.we)*y(6) + B66(const.we)*y(10) - real(F6(const.w0,const.we,t)));

%% coupled equation of motion in heave-pitch. Written as a system of first order differential equations.
hiv = @(t,y) (B33(const.we)*y(4) + B35(const.we)*y(8) + C33*y(3) + C35*y(7) - real(F3(const.w0,const.we,t)));
stamp = @(t,y) (B53(const.we)*y(4) + (B55(const.we) + Bv55*abs(y(8)))*y(8) + C53*y(3) + C55*y(7) - real(F5(const.w0,const.we,t)));

%% Total equation system
shp = @(t,y) [y(2);...
              C1(const.we)*svai(t,y) + C2(const.we)*rull(t,y) + C3(const.we)*gir(t,y);...
              y(4);...
              c1(const.we)*hiv(t,y) + c2(const.we)*stamp(t,y);...
              y(6);...
              C4(const.we)*svai(t,y) + C5(const.we)*rull(t,y) + C6(const.we)*gir(t,y);...
              y(8);...
              c3(const.we)*hiv(t,y) + c4(const.we)*stamp(t,y);...
              y(10);...
              C7(const.we)*svai(t,y) + C8(const.we)*rull(t,y) + C9(const.we)*gir(t,y)];

%% Solving the equation of motion in heave and pitch
[t,y] = ode45(shp, tspan, y0);

resp.eta2 = y(:,2); %% Final response in sway
resp.eta3 = y(:,3); %% Final response in heave
resp.eta4 = y(:,5); %% Final response in roll
resp.eta5 = y(:,7); %% Final response in pitch
resp.eta6 = y(:,9); %% Final response in yaw

end
```

C.63 skjerm.m

Write relevant information to the screen and plot the results.

```
function skjerm(inp,const,dim,frame,gz,rest,zg,amass,inert,excit,trans,resp,t,d,h)
%Write relevant information to the screen

fprintf('SHIP MAIN PARTICULARS: \n \n')
fprintf(['Length between perpendiculars, inp.Lpp = ' num2str(inp.Lpp) ' [m] \n'])
fprintf(['Beam, B = ' num2str(max(dim.B)) ' [m] \n'])
fprintf(['Mean draught, d = ' num2str(d) ' [m] \n'])
fprintf(['Block coefficient, Cb = ' num2str(dim.Vol/(inp.Lpp*max(dim.B)*d)) ' [-] \n'])
fprintf(['Number of frames, inp.nFr = ' num2str(inp.nFr) ' [-] \n \n'])
```

```
fprintf('SHIP HYDROSTATICS: \n \n')
fprintf(['Transverse metacentric height, GM = ' num2str(gz.GM) ' [m] \n'])
fprintf(['Vertical centre of buoyancy, KB = ' num2str(d + dim.zb) ' [m] \n'])
fprintf(['Vertical centre of gravity, KG = ' num2str(d + zg) ' [m] \n'])
fprintf(['Volume displacement, Vol = ' num2str(dim.Vol) ' [m^3] \n'])
fprintf(['Water plane area, Aw = ' num2str(dim.Aw) ' [m^2] \n'])
fprintf(['Longitudinal centre of buoyancy (positive forward of inp.Lpp/2), LCB =
    ' num2str(-dim.LCB) ' [m] \n \n'])

fprintf('DYNAMIC PROPERTIES: \n \n')
fprintf(['Ship velocity, U = ' num2str(const.U*3.6/1.852) ' [knots] \n'])
fprintf(['Froude number, Fn = ' num2str(const.U/sqrt(const.g*inp.Lpp)) ' [-] \n'
    ])
fprintf(['Wave period, T = ' num2str(2*pi/const.w0) ' [s] \n'])
fprintf(['Wave height, h = ' num2str(h) ' [m] \n'])
fprintf(['Heading, Betta = ' num2str(const.betta*180/pi) ' [-] \n'])
fprintf(['Period of encounter, Te = ' num2str(2*pi/const.we) ' [s] \n'])
fprintf(['Frequency of encounter, we= ' num2str(const.we) ' [rad/s] \n \n'])

fprintf('NATURAL FREQUENCIES AND PERIODS: \n \n')
fprintf(['Natural period in heave, Tn3 = ' num2str(2*pi/rest.wn3(const.we)) ' [s
    ] \n'])
fprintf(['Natural period in roll, Tn4 = ' num2str(2*pi/rest.wn4(const.we)) ' [s]
     \n'])
fprintf(['Natural period in pitch, Tn5 = ' num2str(2*pi/rest.wn5(const.we)) ' [s
    ] \n \n'])

fprintf('PARAMETRIC ROLLING: \n \n')
fprintf(['Period ratio, Te/Tn = ' num2str(rest.wn4(const.we)/const.we) ' [-] \n'
    ])
fprintf(['Frequency ratio squared, (wn/we)^2 = ' num2str((rest.wn4(const.we)/
    const.we)^2) ' [-] \n \n'])

fprintf('ADDED MASS RATIOS: \n \n')
fprintf(['Heave, A33/M = ' num2str(amass.A33(const.we)/inert.M) ' [-] \n'])
fprintf(['Roll, A44/I44 = ' num2str(amass.A44(const.we)/inert.I44) ' [-] \n'])
fprintf(['Pitch, A55/I55 = ' num2str(amass.A55(const.we)/inert.I55) ' [-] \n \n'
    ])



%% Plot the submerged part of the body plan
for aa = 1:inp.nFr
    figure(1)
    plot(frame.wbodypl{aa}(:,1),frame.wbodypl{aa}(:,2),'.-')
    hold on
    title('Under water hull')
    axis([-5 5 -3 2])
    axis equal
    grid on
end

%% Plot the body plan
for aa = 1:inp.nFr
    figure(2)
    plot(frame.bodypl{aa}(:,1),frame.bodypl{aa}(:,2))
    hold on
    title('Body plan')
    axis([-5 5 -3 2])
    axis equal
```

```
    grid on
end

%% Plot the 3-dimensional hull
for aa = 1:inp.nFr
    figure(3)
    plot3(frame.SpSb{aa}(:,1),frame.SpSb{aa}(:,2),frame.SpSb{aa}(:,3))
    hold on
      plot3(frame.SpPs{aa}(:,1),frame.SpPs{aa}(:,2),frame.SpPs{aa}(:,3))
      hold on
    title('3D model entire hull')
    axis equal
    axis([(-0.5*inp.Lpp - 5) (0.5*inp.Lpp + 5) (-0.5*dim.Beam - 1) (0.5*dim.Beam
         + 1) (-dim.maxDraught - 1) (dim.maxFreeboard + 1)])
    grid on
end

%% Plot the submerged 3-dimensional hull
for aa = 1:inp.nFr
    figure(4)
    plot3(frame.wFr{aa}(:,1),frame.wFr{aa}(:,2),frame.wFr{aa}(:,3))
    hold on
    title('3D model under water hull')
    axis equal
    axis([(-0.5*inp.Lpp - 5) (0.5*inp.Lpp + 5) (-0.5*dim.Beam - 1) (0.5*dim.Beam
         + 1) (-dim.maxDraught - 1) (dim.maxFreeboard + 1)])
    grid on
end

%% plot the sway displacement in metres.
figure(5)
plot(t,resp.eta2)
xlabel('time [s]')
ylabel('\eta_2 [m]')
title('Sway motion')
grid on

%% plot the heave displacement in metres.
figure(6)
plot(t,resp.eta3)
xlabel('time [s]')
ylabel('\eta_3 [m]')
title('Heave motion')
grid on

%% plot the roll angle in degrees.
figure(7)
plot(t,180/pi*resp.eta4)
xlabel('time [s]')
ylabel('\eta_4 [deg]')
title('Roll motion')
grid on

%% plot the pitch angle in degrees.
figure(8)
plot(t,180/pi*resp.eta5)
xlabel('time [s]')
ylabel('\eta_5 [deg]')
title('Pitch motion')
grid on

```

```
%% plot the yaw angle in degrees.
figure(9)
plot(t,180/pi*resp.eta6)
xlabel('time [s]')
ylabel('\eta_6 [deg]')
title('Yaw motion')
grid on

%% Plotting the transfer function in sway
figure(10)
plot(trans.omega,trans.H2,'b')
xlabel('\omega [rad/s]')
ylabel('|\eta_2/\zeta_a|')
title('Transfer function in sway')
ylim([0 1.5])
grid on

figure(11)
plot(trans.lambdaL,trans.H2,'b')
xlabel('\lambda/L_{PP}')
ylabel('|\eta_2/\zeta_a|')
title('Transfer function in sway')
ylim([0 1.5])
xlim([0 10])
grid on

%% Plotting the transfer function in heave
figure(12)
plot(trans.omega,trans.H3,'b')
xlabel('\omega [rad/s]')
ylabel('|\eta_3/\zeta_a|')
title('Transfer function in heave')
ylim([0 1.5])
grid on

figure(13)
plot(trans.lambdaL,trans.H3,'b')
xlabel('\lambda/L_{PP}')
ylabel('|\eta_3/\zeta_a|')
title('Transfer function in heave')
ylim([0 1.5])
xlim([0 10])
grid on

%% Plotting the transfer function in roll
figure(14)
plot(trans.omega,trans.H4,'b')
xlabel('\omega [rad/s]')
ylabel('|\eta_4/\zeta_ak|')
title('Transfer function in roll')
ylim([0 5])
grid on

figure(15)
plot(trans.lambdaL,trans.H4,'b')
xlabel('\lambda/L_{PP}')
ylabel('|\eta_4/\zeta_ak|')
title('Transfer function in roll')
ylim([0 5])
xlim([0 10])
grid on
```

```
%% Plotting the transfer function in pitch
figure(16)
plot(trans.omega,trans.H5,'b')
xlabel('\omega [rad/s]')
ylabel('|\eta_5/\zeta_ak|')
title('Transfer function in pitch')
ylim([0 1.55])
grid on

figure(17)
plot(trans.lambdaL,trans.H5,'b')
xlabel('\lambda/L_{PP}')
ylabel('|\eta_5/\zeta_ak|')
title('Transfer function in pitch')
ylim([0 1.55])
xlim([0 10])
grid on

%% Plotting the transfer function in yaw
figure(18)
plot(trans.omega,trans.H6,'b')
xlabel('\omega [rad/s]')
ylabel('|\eta_6/\zeta_a|')
title('Transfer function in yaw')
ylim([0 0.05])
grid on

figure(19)
plot(trans.lambdaL,trans.H6,'b')
xlabel('\lambda/L_{PP}')
ylabel('|\eta_6/\zeta_ak|')
title('Transfer function in yaw')
ylim([0 0.05])
xlim([0 10])
grid on

%% Plotting the wave excitation load amplitudes
F2tot = @(w0) abs(excit.F2(w0,const.we,0));
F2fk = @(w0) abs(excit.F2fk(w0,const.we,0));
F2d = @(w0) abs(excit.F2d(w0,const.we,0));
F3tot = @(w0) abs(excit.F3(w0,const.we,0));
F3fk = @(w0) abs(excit.F3fk(w0,const.we,0));
F3d = @(w0) abs(excit.F3d(w0,const.we,0));
F4tot = @(w0) abs(excit.F4(w0,const.we,0));
F4fk = @(w0) abs(excit.F4fk(w0,const.we,0));
F4d = @(w0) abs(excit.F4d(w0,const.we,0));
F5tot = @(w0) abs(excit.F5(w0,const.we,0));
F5fk = @(w0) abs(excit.F5fk(w0,const.we,0));
F5d = @(w0) abs(excit.F5d(w0,const.we,0));
F6tot = @(w0) abs(excit.F6(w0,const.we,0));
F6fk = @(w0) abs(excit.F6fk(w0,const.we,0));
F6d = @(w0) abs(excit.F6d(w0,const.we,0));


%% Sway force
figure(20)
fplot(F2tot, [0 3],'r')
hold on
fplot(F2fk, [0 3],'b')
hold on
```

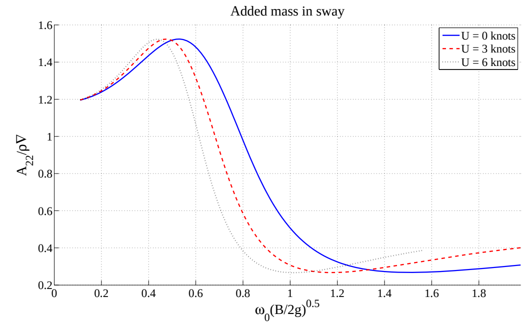
```
fplot(F2d, [0 3],'k')
grid on
xlabel('\omega_0 [rad/s]')
ylabel('|F_2(\omega)|, [N]')
title('Absolute value of wave excitation force in sway')
legend('Total sway force','Froude-krloff force in sway','Diffraction force in
    sway')

%% Heave force
figure(21)
fplot(F3tot, [0 3],'r')
hold on
fplot(F3fk, [0 3],'b')
hold on
fplot(F3d, [0 3],'k')
grid on
xlabel('\omega_0 [rad/s]')
ylabel('|F_3(\omega)|, [N]')
title('Absolute value of wave excitation force in heave')
legend('Total heave force','Froude-krloff force in heave','Diffraction force in
    heave')


%% Roll moment
figure(22)
fplot(F4tot, [0 3],'r')
hold on
fplot(F4fk, [0 3],'b')
hold on
fplot(F4d, [0 3],'k')
grid on
xlabel('\omega_0 [rad/s]')
ylabel('|F_4(\omega)|, [Nm]')
title('Absolute value of wave excitation moment in roll')
legend('Total roll moment','Froude-krloff moment in roll','Diffraction moment in
     roll')


%% Pitch moment
figure(23)
fplot(F5tot, [0 3],'r')
hold on
fplot(F5fk, [0 3],'b')
hold on
fplot(F5d, [0 3],'k')
grid on
xlabel('\omega_0 [rad/s]')
ylabel('|F_5(\omega)|, [Nm]')
title('Absolute value of wave excitation moment in pitch')
legend('Total pitch moment','Froude-krloff moment in pitch','Diffraction moment
    in pitch')


%% Yaw moment
figure(24)
fplot(F6tot, [0 3],'r')
hold on
fplot(F6fk, [0 3],'b')
hold on
fplot(F6d, [0 3],'k')
grid on
```

```
xlabel('\omega_0 [rad/s]')
ylabel('|F_6(\omega)|, [Nm]')
title('Absolute value of wave excitation moment in yaw')
legend('Total yaw moment','Froude-krloff moment in yaw','Diffraction moment in
    yaw')



end
```

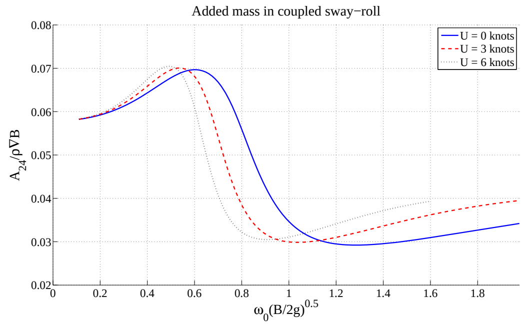
Appendix D

Added mass coefficients

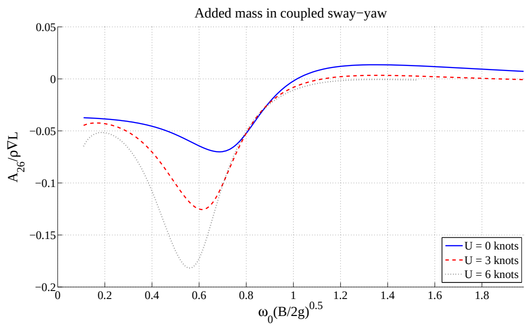
Here we will plot all the non-dimensional $3D$ added mass coefficients as function of frequency in head sea.



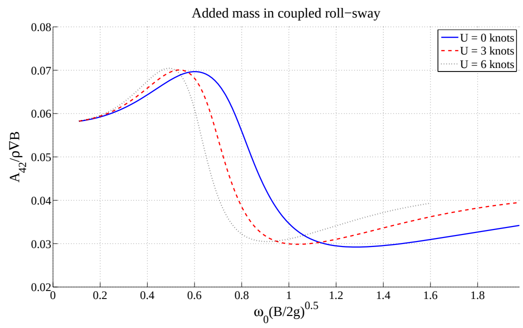
(a) A_{22}



(b) A_{24}

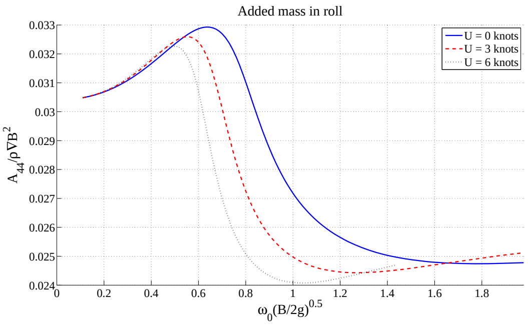


(c) A_{26}

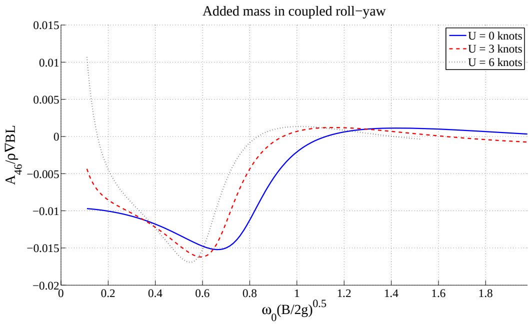


(d) A_{42}

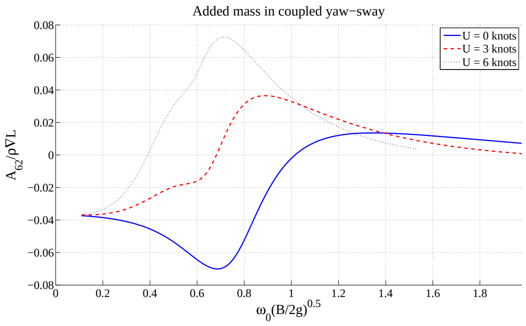
Figure D.1: Non-dimensional $3D$ added mass coefficients.



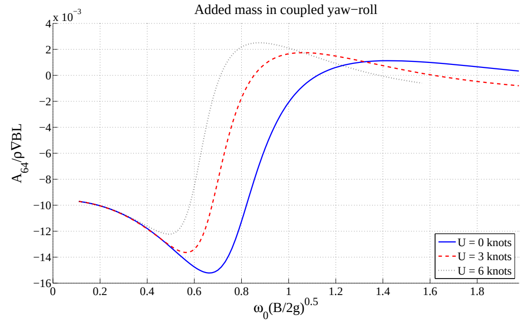
(a) A_{44}



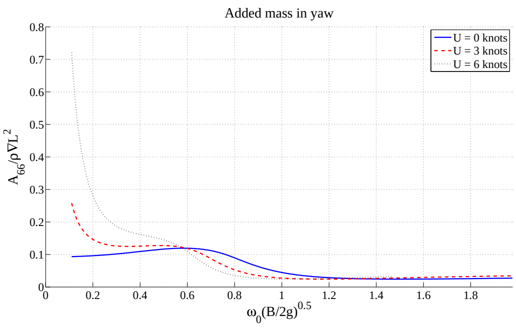
(b) A_{46}



(c) A_{62}

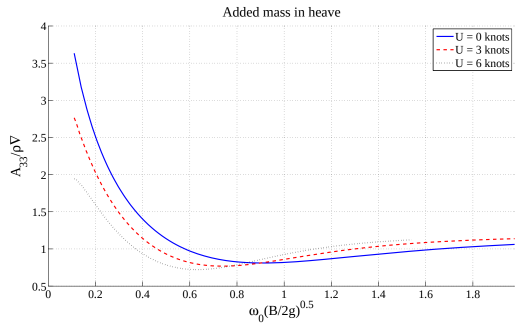


(d) A_{64}

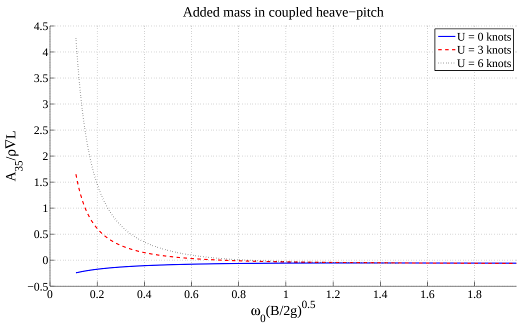


(e) A_{66}

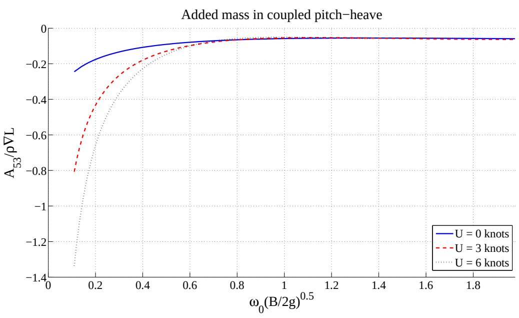
Figure D.2: Non-dimensional $3D$ added mass coefficients.



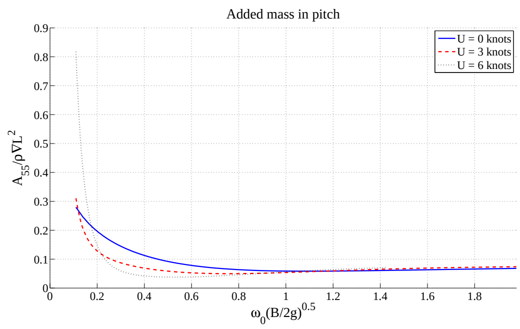
(a) A_{33}



(b) A_{35}



(c) A_{53}



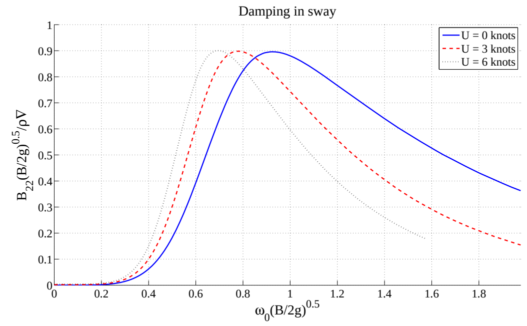
(d) A_{55}

Figure D.3: Non-dimensional $3D$ added mass coefficients.

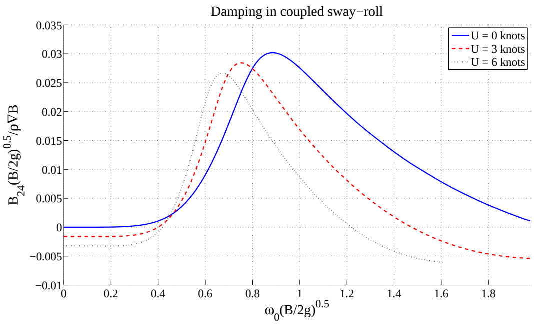
Appendix E

Damping coefficients

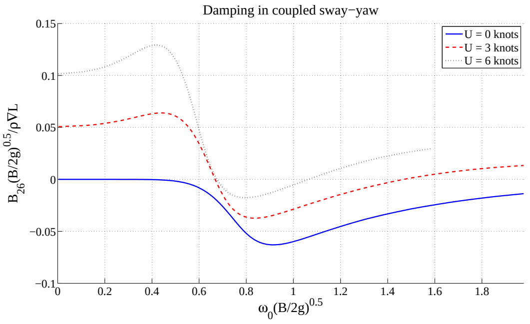
Here we will plot all the non-dimensional $3D$ damping coefficients as function of frequency in head sea.



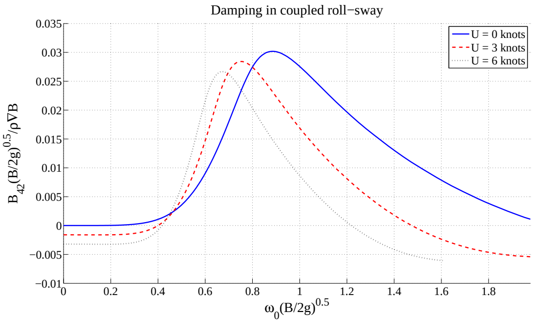
(a) B_{22}



(b) B_{24}

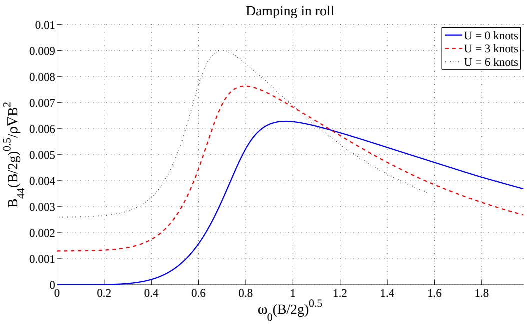


(c) B_{26}

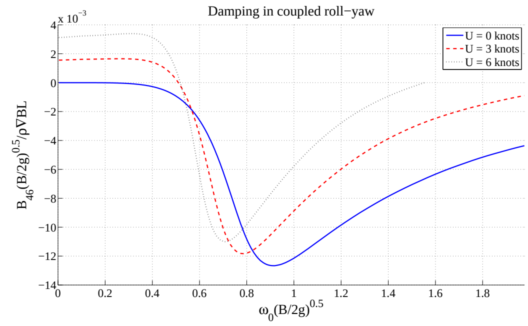


(d) B_{42}

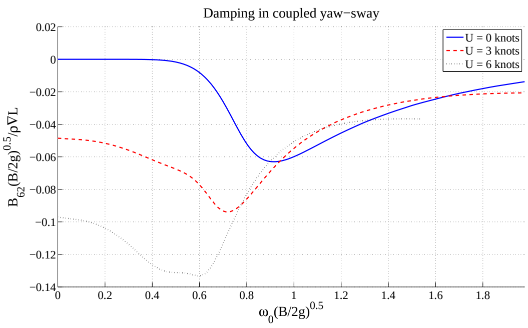
Figure E.1: Non-dimensional $3D$ damping coefficients.



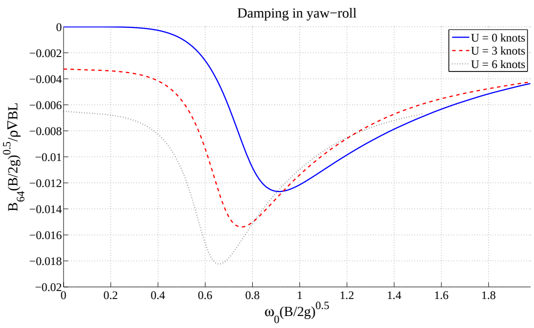
(a) B_{44}



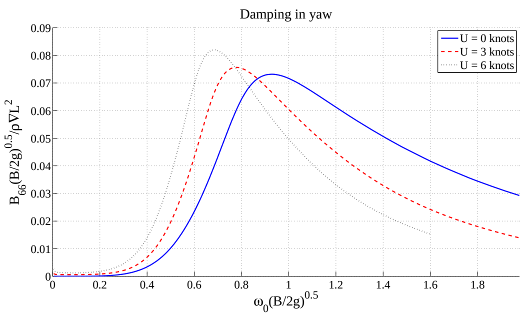
(b) B_{46}



(c) B_{62}

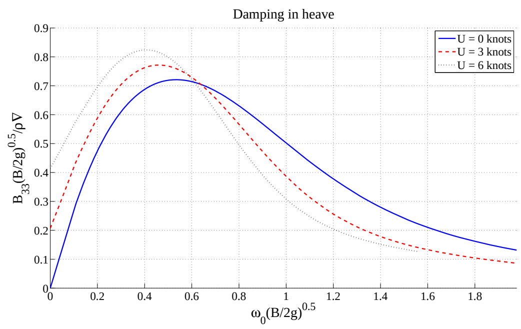


(d) B_{64}

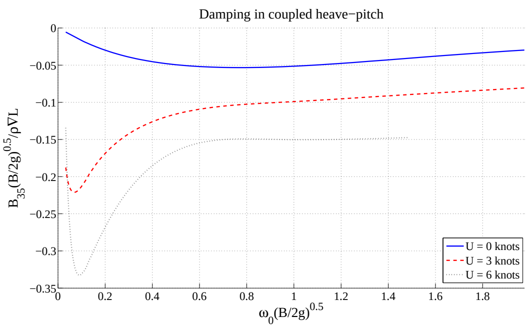


(e) B_{66}

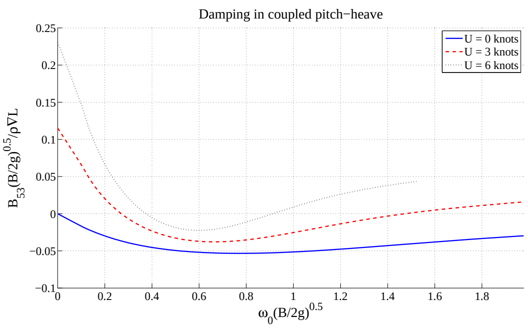
Figure E.2: Non-dimensional $3D$ damping coefficients.



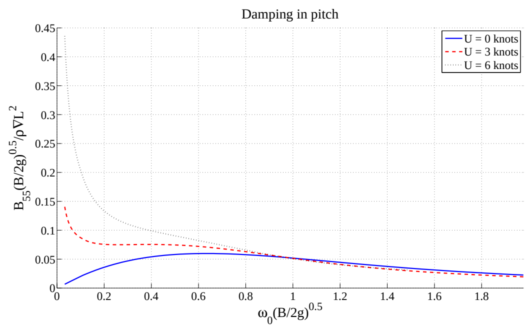
(a) B_{33}



(b) B_{35}



(c) B_{53}



(d) B_{55}

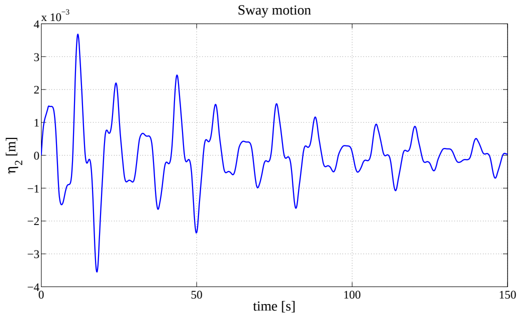
Figure E.3: Non-dimensional $3D$ added mass coefficients.

Appendix F

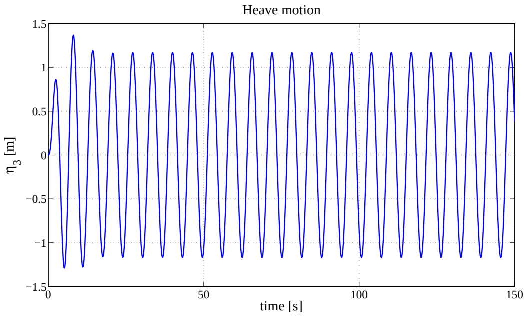
Time series

Here we will show a selection of time series.

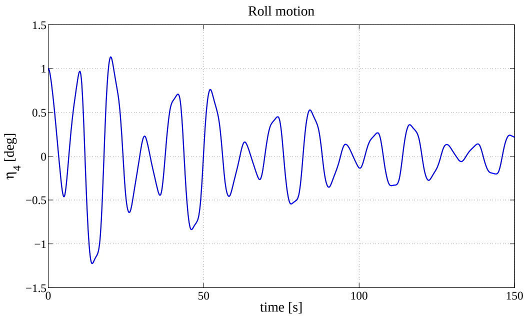
F.1 $U = 2$ knots, $\beta = 0°$, $T_0 = 7$ s



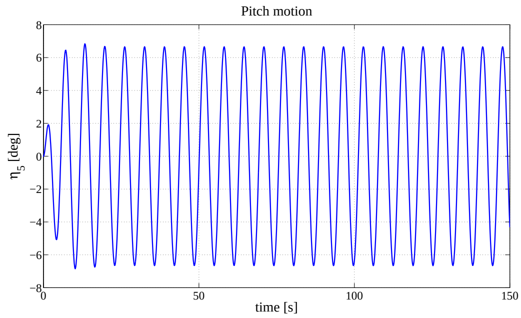
(a) Time series in sway.



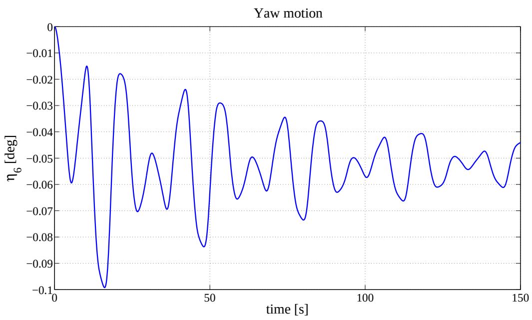
(b) Time series in heave.



(c) Time series in roll.



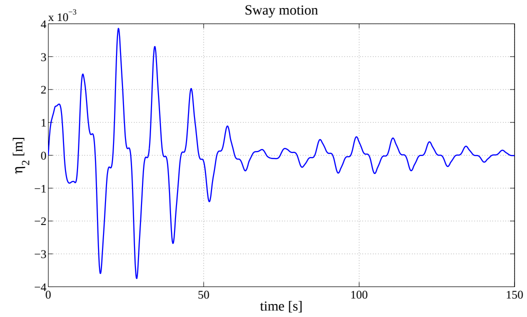
(d) Time series in pitch.



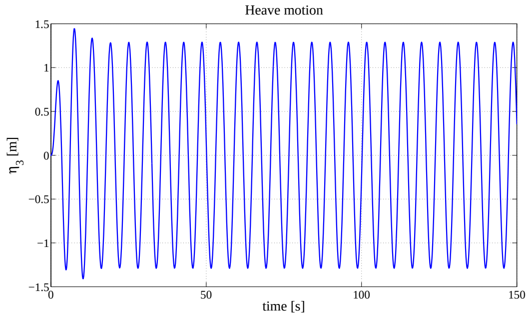
(e) Time series in yaw.

Figure F.1: Time series of all modes of motion for a forward speed of 2 knots and a wave heading of $0°$. Wave period of 7 s. No resonance.

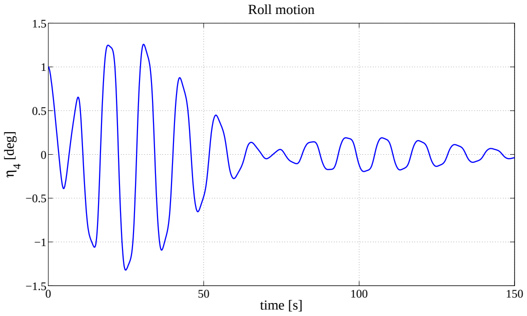
F.2 $U = 4$ knots, $\beta = 0°$, $T_0 = 7$ s



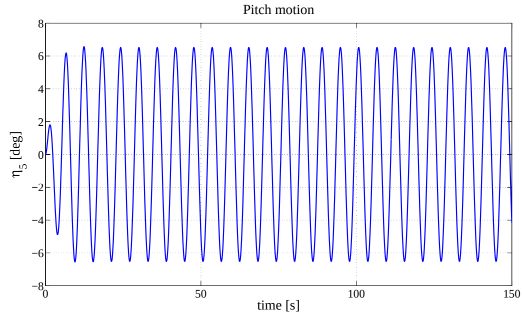
(a) Time series in sway.



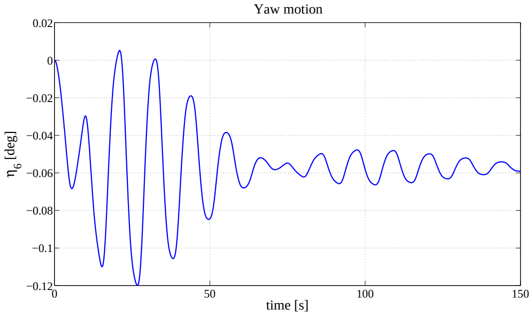
(b) Time series in heave.



(c) Time series in roll.



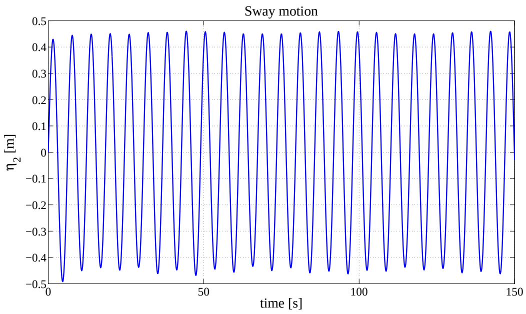
(d) Time series in pitch.



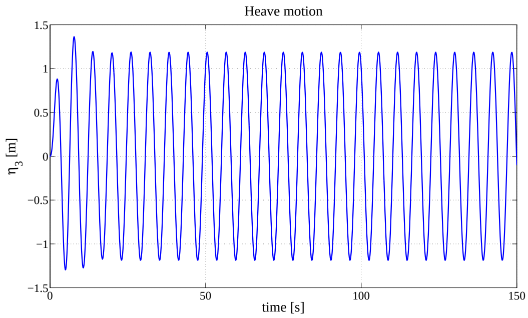
(e) Time series in yaw.

Figure F.2: Time series of all modes of motion for a forward speed of 4 knots and a wave heading of $0°$. Wave period of 7 s. No resonance.

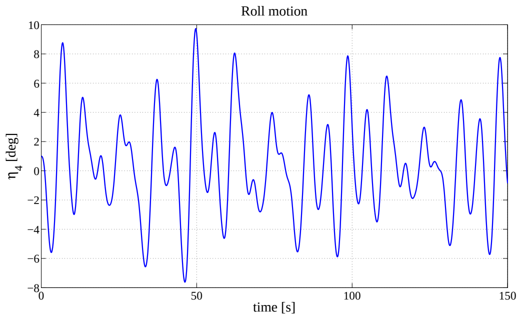
F.3 $U = 4$ knots, $\beta = 40°$, $T_0 = 7$ s



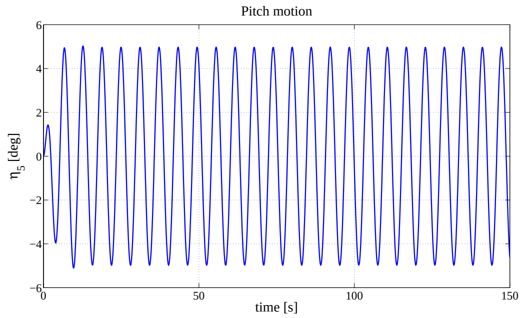
(a) Time series in sway.



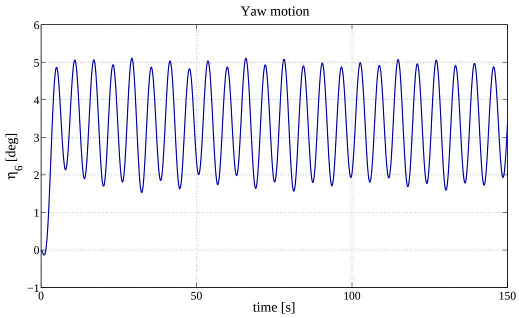
(b) Time series in heave.



(c) Time series in roll.



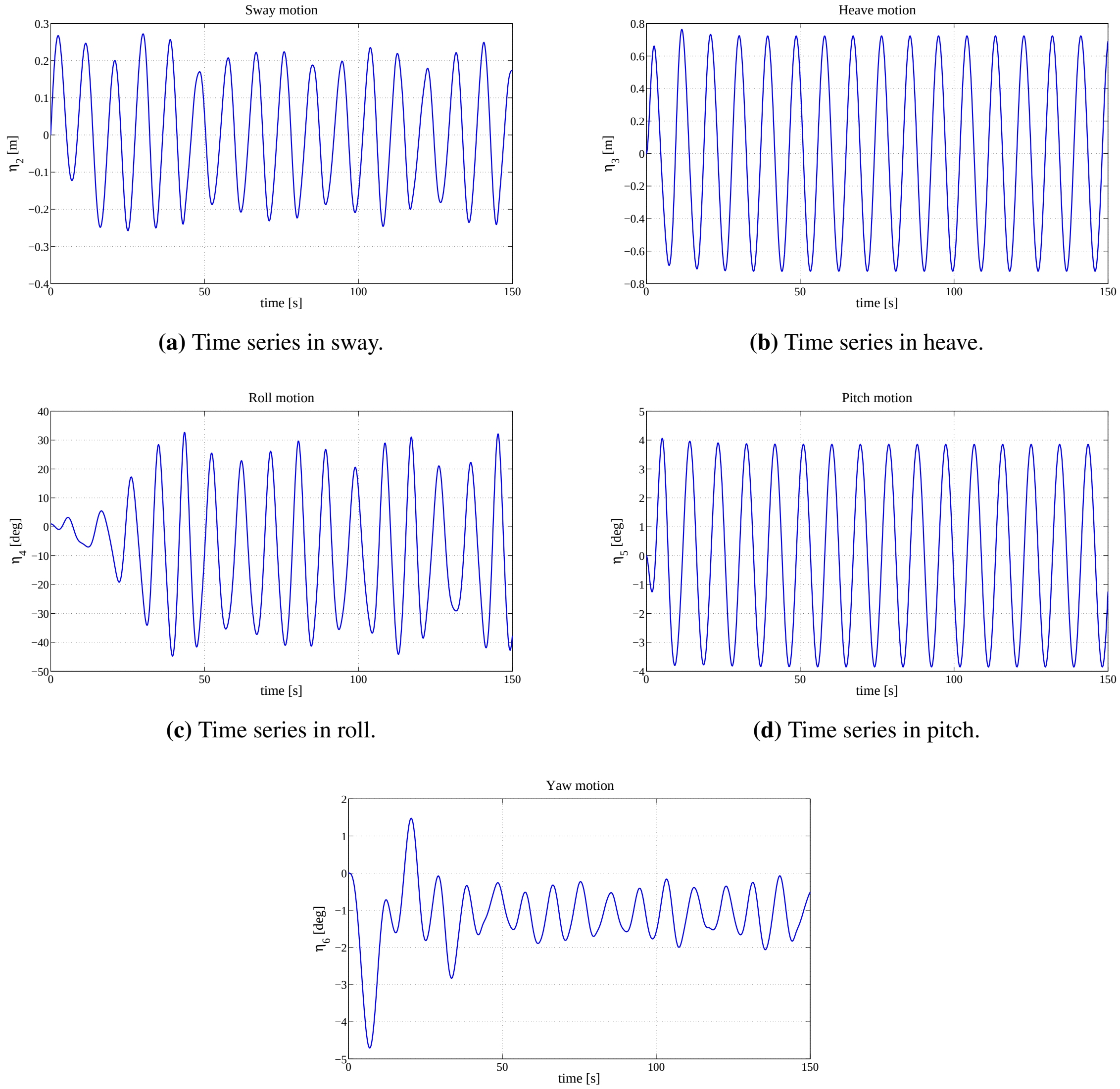
(d) Time series in pitch.



(e) Time series in yaw.

Figure F.3: Time series of all modes of motion for a forward speed of 4 knots and a wave heading of $40°$. Wave period of 7 s. No resonance.

F.4 $U = 5.5$ knots, $\beta = 160°$, $T_0 = 7$ s



(a) Time series in sway.

(b) Time series in heave.

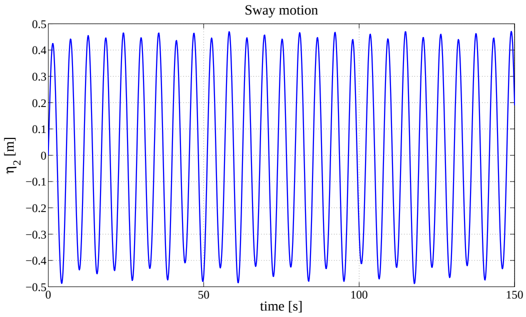
(c) Time series in roll.

(d) Time series in pitch.

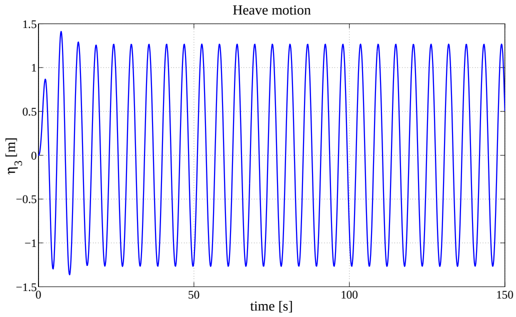
(e) Time series in yaw.

Figure F.4: Time series of all modes of motion for a forward speed of 5.5 knots and a wave heading of 160°. Wave period of 7 s. Resonance.

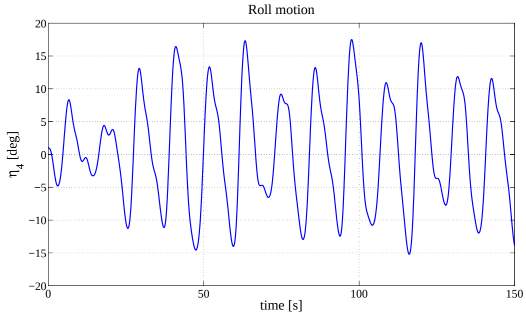
F.5 $U = 6.5$ knots, $\beta = 40°, T_0 = 7$ s



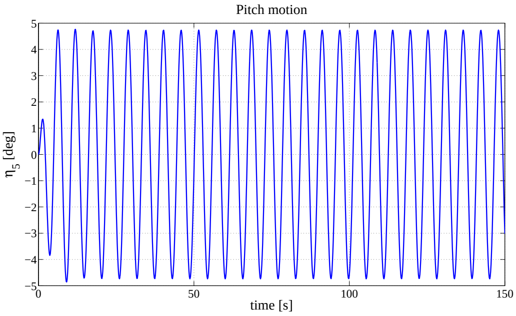
(a) Time series in sway.



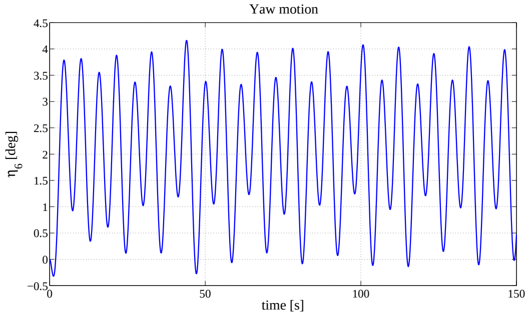
(b) Time series in heave.



(c) Time series in roll.



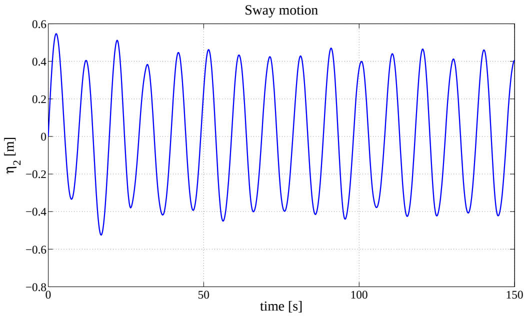
(d) Time series in pitch.



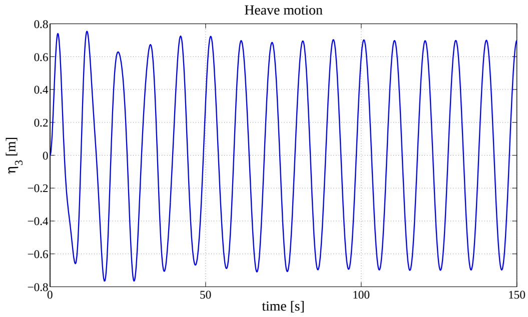
(e) Time series in yaw.

Figure F.5: Time series of all modes of motion for a forward speed of 6.5 knots and a wave heading of $40°$. Wave period of 7 s. Resonance.

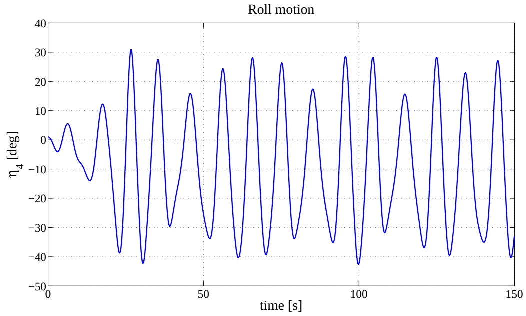
F.6 $U = 8$ knots, $\beta = 140°$, $T_0 = 7$ s



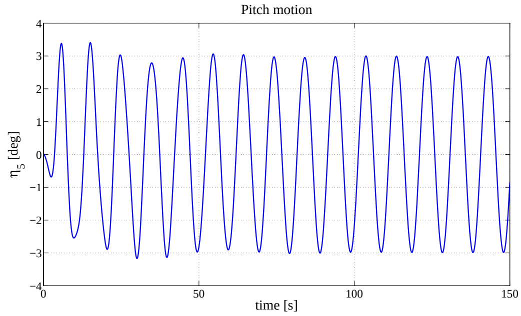
(a) Time series in sway.



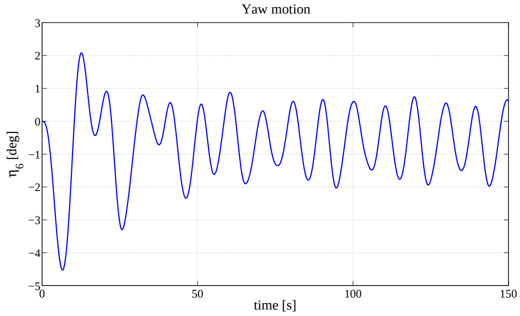
(b) Time series in heave.



(c) Time series in roll.



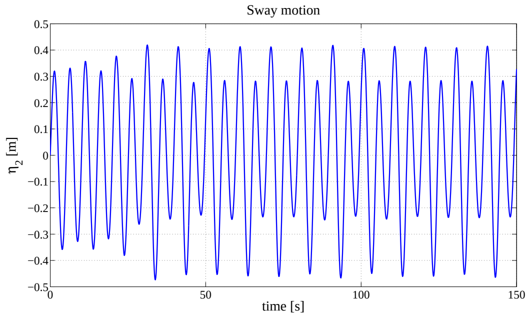
(d) Time series in pitch.



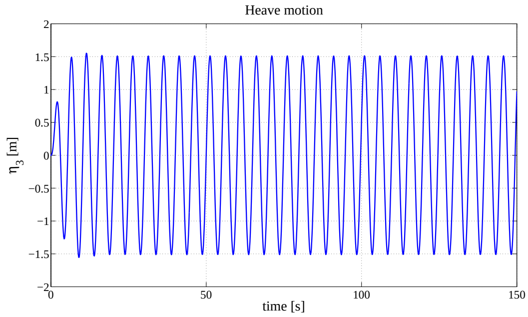
(e) Time series in yaw.

Figure F.6: Time series of all modes of motion for a forward speed of 8 knots and a wave heading of $140°$. Wave period of 7 s. Resonance.

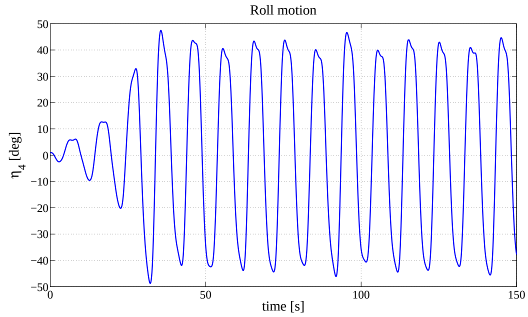
F.7 $U = 10$ knots, $\beta = 30°$, $T_0 = 7$ s



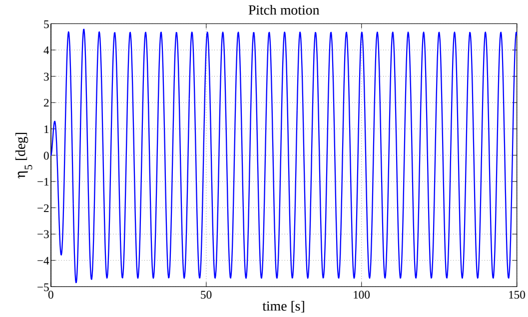
(a) Time series in sway.



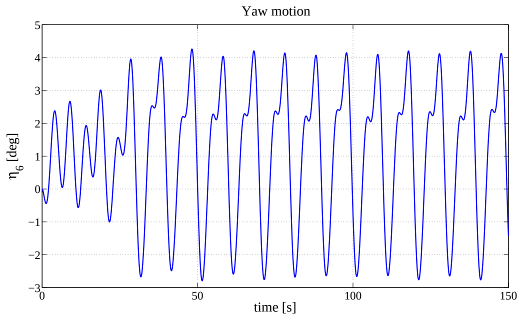
(b) Time series in heave.



(c) Time series in roll.



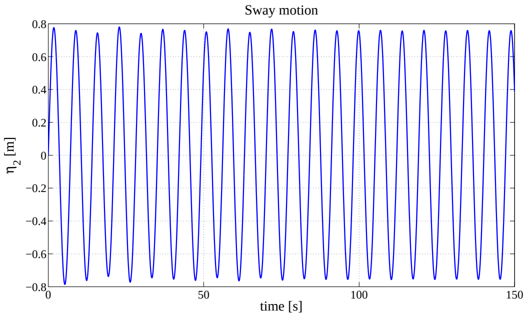
(d) Time series in pitch.



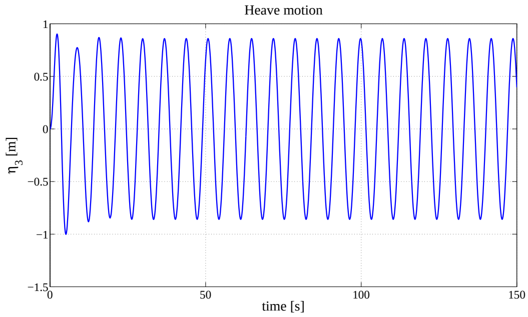
(e) Time series in yaw.

Figure F.7: Time series of all modes of motion for a forward speed of 10 knots and a wave heading of 30°. Wave period of 7 s. Resonance.

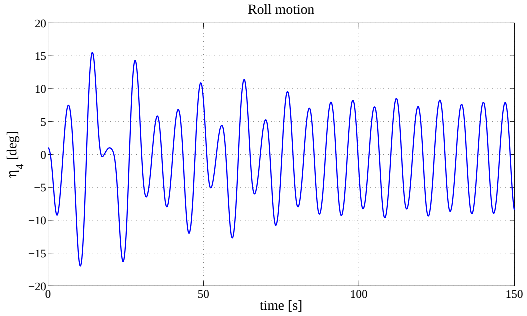
F.8 $U = 10$ knots, $\beta = 90°$, $T_0 = 7$ s



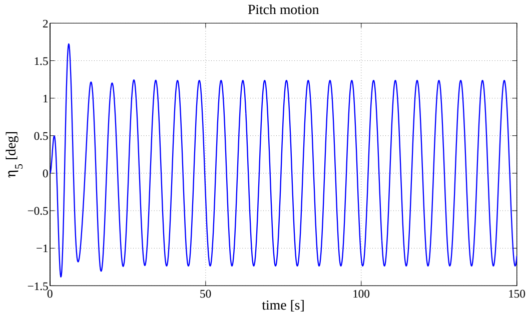
(a) Time series in sway.



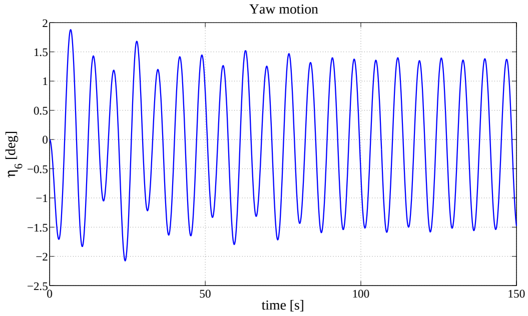
(b) Time series in heave.



(c) Time series in roll.



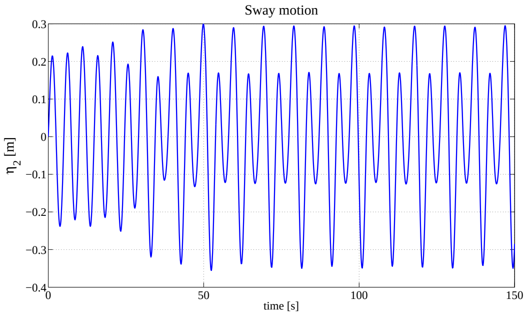
(d) Time series in pitch.



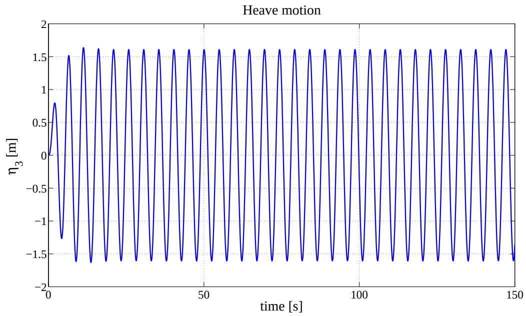
(e) Time series in yaw.

Figure F.8: Time series of all modes of motion for a forward speed of 10 knots and a wave heading of 90°. Wave period of 7 s. No resonance.

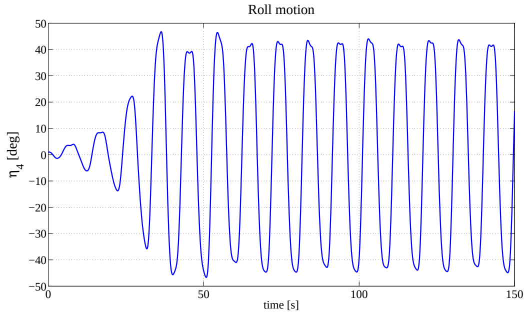
F.9 $U = 12$ knots, $\beta = 20°$, $T_0 = 7$ s



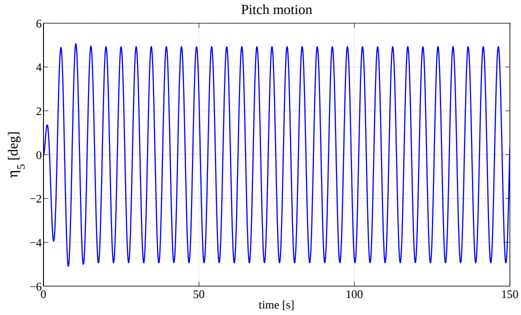
(a) Time series in sway.



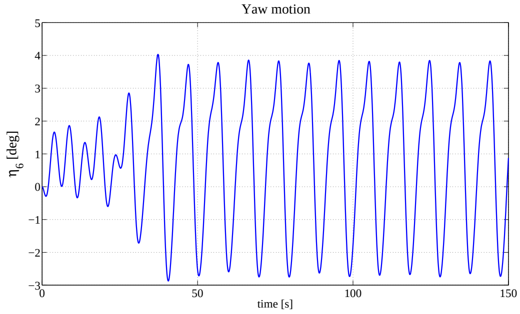
(b) Time series in heave.



(c) Time series in roll.



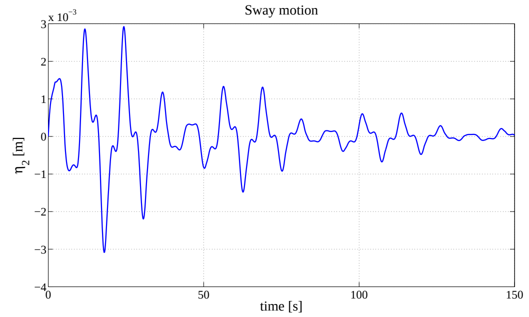
(d) Time series in pitch.



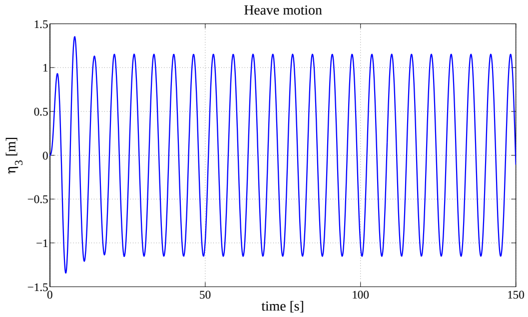
(e) Time series in yaw.

Figure F.9: Time series of all modes of motion for a forward speed of 12 knots and a wave heading of 20°. Wave period of 7 s. Resonance.

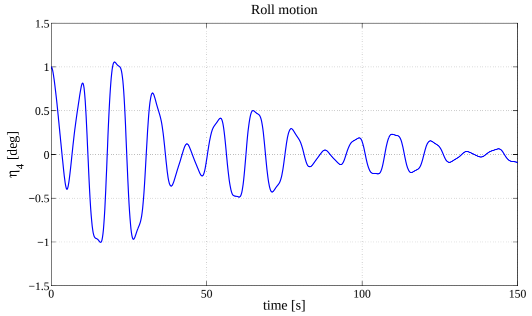
F.10 $U = 4$ knots, $\beta = 0°$, $T_0 = 7.5$ s



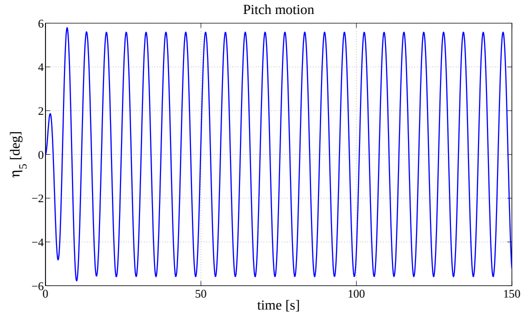
(a) Time series in sway.



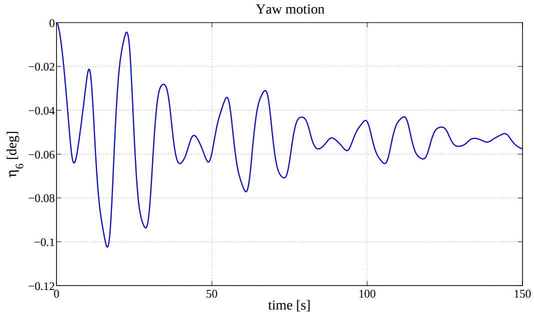
(b) Time series in heave.



(c) Time series in roll.



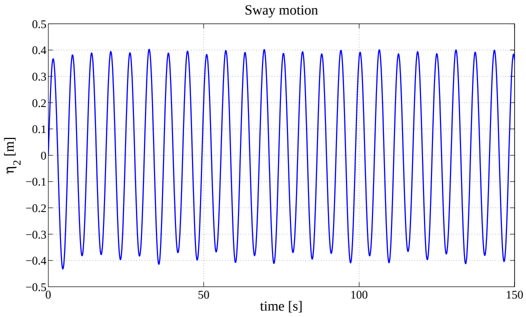
(d) Time series in pitch.



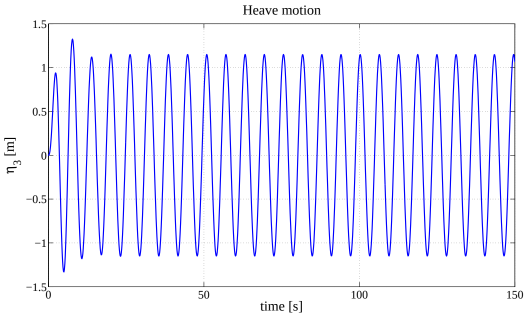
(e) Time series in yaw.

Figure F.10: Time series of all modes of motion for a forward speed of 4 knots and a wave heading of $0°$. Wave period of 7.5 s. No resonance.

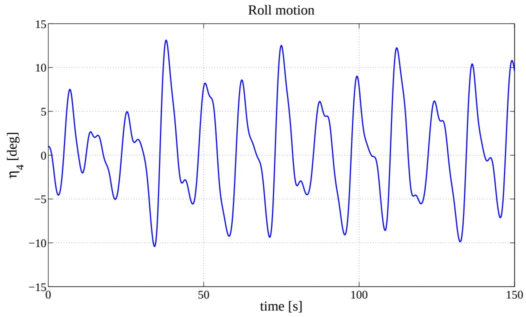
F.11 $U = 6$ knots, $\beta = 35°$, $T_0 = 7.5$ s



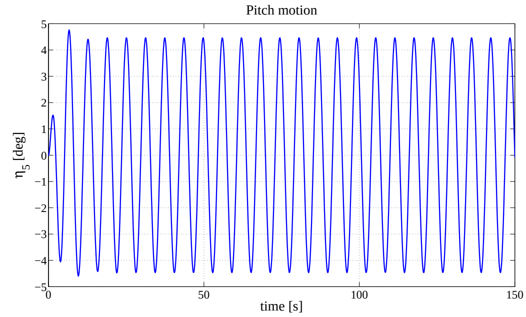
(a) Time series in sway.



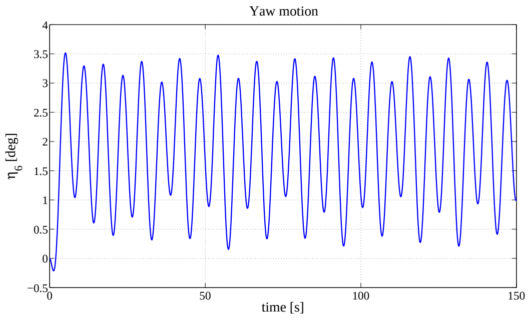
(b) Time series in heave.



(c) Time series in roll.



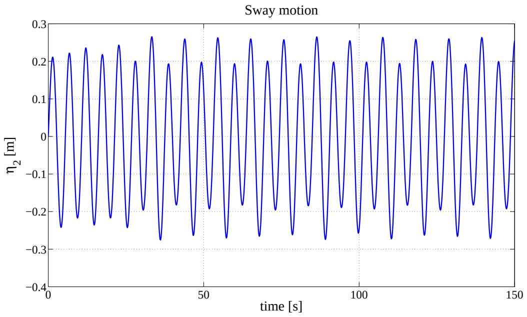
(d) Time series in pitch.



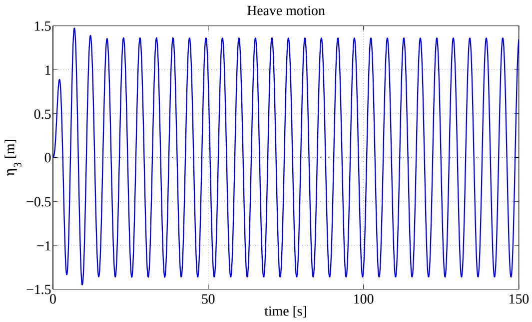
(e) Time series in yaw.

Figure F.11: Time series of all modes of motion for a forward speed of 6 knots and a wave heading of 35°. Wave period of 7.5 s. No resonance.

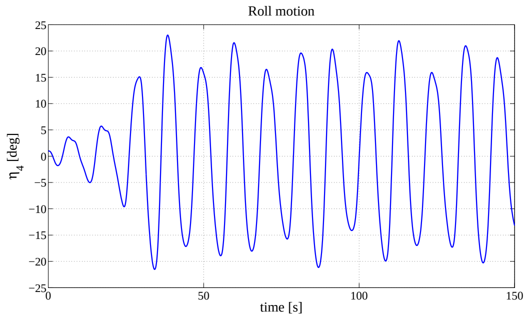
F.12 $U = 10$ knots, $\beta = 20°$, $T_0 = 7.5$ s



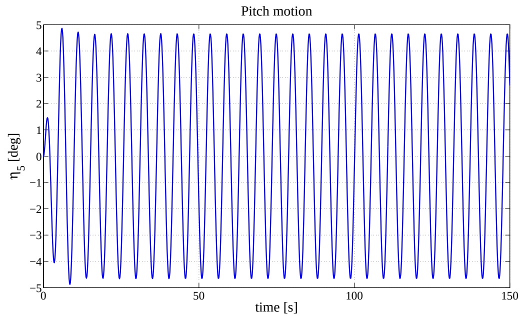
(a) Time series in sway.



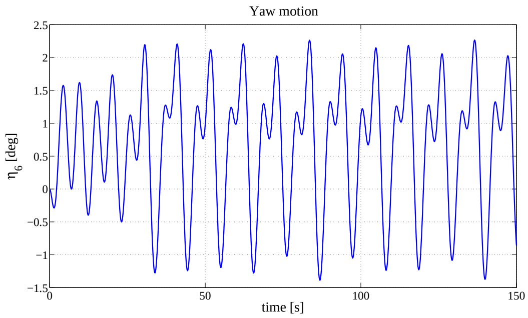
(b) Time series in heave.



(c) Time series in roll.



(d) Time series in pitch.



(e) Time series in yaw.

Figure F.12: Time series of all modes of motion for a forward speed of 10 knots and a wave heading of 20°. Wave period of 7.5 s. No resonance.

Appendix G

CD with contents

This is the contents of the appended CD:

- Report in pdf format
- Matlab code
- Figures found in the report in original format