MSc Thesis

Prediction of Design Response for North Sea

N T N U

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Abstract

The methods to predict long term extreme structure responses induced by environmental loads are reviewed. Due to the variability of both environmental condition and response in a given environmental condition, the prediction should be performed statistically. Generally, one should carry out a full long term response analysis, i.e. convoluting the variability of response in a given environmental condition with the variability of environmental characteristics. One may consider the response distributions during all sea states or storm peaks. However, full long term analysis can be quite time-consuming if the problem under consideration is of complicated non-linear nature. In this condition it is more convenient to perform short term response analysis. The most common method in connection with this is the environmental contour lines method. Extreme heave motion of a semi-submersible in the North Sea is evaluated as an example.

Key Words: Characteristic response, all sea states, peak over threshold, environmental contour lines.

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List of abbreviations

ULS	Ultimate Limit State
ALS	Accidental Limit State
ITTC	International Towing Tank Conference
ISSC	International Ship Structures Congress
FORM	First Order Reliability Method
IFORM	Inverse First Order Reliability Method

Nomenclature

Some of the most frequently used nomenclatures in this paper are listed as following. Other notations will be introduced when they first appear.

x_q	x_q the loads and/or responses corresponding to an annual probability of being exceeded, q
γ_E	safety factor for environmental loads
x_E	environmental loads
q	the annual probability for responses and loads being exceeded
H_s	the significant wave height
T_p	the spectrum peak period
Θ	wave direction
$F_{X_\Gamma}(x)$	the long term distribution of global loads and/or responses maxima
$F_{X_{\Gamma,3h}}(x)$	the long term distribution of 3-hour loads and/or responses maxima
$F_{X_\Gamma \| H_s T_p}(x \| h_s, t_p)$	the distribution of global loads and/or responses maxima in a given sea state
$F_{X_{\Gamma,3h} \| H_s T_p}(x \| h_s, t_p)$	the distribution of 3-hour loads and/or responses maxima in a given sea state
$f_{H_s T_p}(h_s, t_p)$	the joint distribution of H_s and T_p
$S_{\Xi\Xi}$	the wave spectrum
$S_{\Gamma\Gamma}$	the responses spectrum
$\tilde{x}$	the most probable responses
d	the duration of a stationary sea state

∇	Laplace operator
ϕ	velocity potential
u_i	transformed variable in U-space
Φ	standard normal distribution

Chapter 1

Introduction

In the process of designing offshore structures, one's major concern is to ensure that the structure is able to withstand all foreseen loads and/or responses with an adequate margin during its life time. Generally, one should account for permanent loads, variable functional loads and the environmental loads. Permanent loads are loads that are constant in magnitude, position and direction during the period under consideration, such as mass of structure, permanent ballast and equipment, etc. Variable functional loads are loads caused by functional units and people on board and may vary in magnitude, position or direction. Examples of variable functional loads are personnel, stored materials and crane operational loads. Environmental loads are loads induced by environmental conditions, such as hydrodynamic loads induced by waves and current, wind loads, etc.

According to NORSOK (2007), safety of offshore structures is ensured by requiring that:

$$\gamma_P x_P + \gamma_V x_V + \gamma_E x_E \leq \frac{\sigma}{\gamma_M} \quad (1.1)$$

where x_P, x_V and x_E are permanent loads, variable functional loads and environmental loads, respectively; σ is the nominal capacity of a structural component, which is usually taken to be the 5%-value of the elastic component capacity; γ_P, γ_V, γ_E and γ_M are safety factors.

In the present study, we will merely focus on environmental loads and/or responses. x_E in Equation (1.1) is generally taken to be the characteristic loads and/or responses, x_q,whose annual probability of being exceeded is q. We may also refer to x_q as the response corresponding to a return period of T years, or T-year response, where

$T = \frac{1}{q}$. Equation (1.1) can be simplified as:

$$\gamma_E x_q \le \frac{\sigma}{\gamma_M} \qquad (1.2)$$

In NORSOK (2007), offshore structures are controlled against overload failure on two levels: ultimate limit state control (ULS) and accidental limit state control (ALS). For ULS, $q = 1/100$, and typical values for γ_E and γ_M are 1.3 and 1.15, respectively. For ALS, $q = 1/10000$, and γ_E and γ_M are usually taken to be 1.

Usually, ULS control will govern the design. As long as the relation between loads and the corresponding annual exceedance probability does not change abruptly, ULS might be sufficiently safe. If this is the case, $\gamma_E x_{1/100}$ (γ_E is generally taken to be 1.3 for ULS control) approximately corresponds to the annual exceedance probability of 1/10000, which is typically the required probability level for accidental loads, see Haver and Winterstein (2008). However, if the loads-annual exceedance probability relation change abruptly towards the unfavorable side between the annual exceedance probability 1/100 and 1/10000, $\gamma_E x_{1/100}$ might correspond to an annual exceedance probability much larger than required level. From the perspective of safety, ALS should govern the design in this condition.

The environmental loads and/or responses acting on offshore structures are in principle functions of wave, wind and current. For illustrative purpose, we will primarily focus on wave-induced loads and/or responses throughout this paper.

When we model the wave, some mean characteristics like the significant wave height H_s and spectrum peak period T_p change slowly over time, and we may consider them to be constant in a short period of time, commonly 3 hours in the North Sea or 0.5 hours in the Gulf of Mexico. On the contrary, in a given sea state defined by significant wave height and spectrum peak period, the instantaneous sea surface elevation changes much faster, typically with period of order 10s. In short term, the structures are subject to loads and/or responses due to the instantaneous sea surface elevation, while in the long term, the significant wave height and spectrum peak period, which characterize instantaneous sea surface elevation, will vary slowly with time. These are two sources of randomness associated with extreme loads and/or responses. Therefore the characteristic loads and/or responses should be estimated in a statistical sense. Both of the two mentioned sources of randomness should be accounted for if proper characteristic loads and/or responses shall be obtained.

To obtain a consistent estimate of x_q, some sort of long term distribution of the loads and/or responses under consideration should be established. One may consider the long term distribution of various quantities, such as:

i) The global response maxima, i.e. the largest response maxima between two successive zero-up-crossings;

ii) The d-hour response maxima, where d is the duration of stationary sea state;

iii) The storm response maxima.

For example we may consider the long term distribution of 3-hour response maxima. The characteristic response, x_q, could be exceeded in a wide range of 3-hour sea states. Denote the distribution of 3-hour maxima in the sea state with significant wave height h_{si} and spectrum peak period t_{pj} by $F_{X_{\Gamma,3h}|H_sT_p}(x|h_{si},t_{pj})$ and the probability of the occurrence of this sea state by $p(h_{si},t_{pj})$, the long term distribution of 3-hour maxima reads:

$$F_{X_{\Gamma,3h}}(x)=\sum_i\sum_j F_{X_{\Gamma,3h}|H_sT_p}(x|h_{si},t_{pj})p(h_{si},t_{pj}) \tag{1.3}$$

or in a continuous sense:

$$F_{X_{\Gamma,3h}}(x)=\int_h\int_t F_{X_{\Gamma,3h}|H_sT_p}(x|h,t)f_{H_sT_p}(h,t)dhdt \tag{1.4}$$

The characteristic response can then be estimated by:

$$1-F_{X_{\Gamma,3h}}(x_q)=\frac{q}{n_{3h}} \tag{1.5}$$

where n_{3h} is the number of 3-hour sea states in one year.

It can be seen from the Equation (1.5) that the prediction of extreme loads and/or responses consists of a short term problem, i.e. the probability of the loads and/or responses exceeding a certain value within a stationary sea state and a long term problem, i.e. the probability of the occurrence of the particular sea state. Such long term analysis can be performed by considering all sea states, i.e. the ‘all sea states approach’, or only taking the storms whose significant wave heights are above a certain threshold into account, i.e. the “peak over threshold approach”.

If we are dealing with complicated non-linear response problem, we may instead of performing the full long term analysis consider only a limited number of sea states, i.e. the short term response analysis. The most frequently used approach in connection with this is the environmental contour lines approach. Using the environmental contour line method, one should first construct the contour lines for d-hour sea states corresponding to a certain annual exceedance probability and then decide a relatively narrow part on the contour lines where the most unfavorable sea state locates. Thereafter some few time domain simulations or model tests are performed for each sea state over the selected region. From these simulations or model tests one will be able to identify the worst sea state. As the worst sea state is selected, one should run more simulations or model tests to determine the response distribution for this sea state. The number of simulations or model tests should be many enough that the upper tail of the distribution could be reasonably well modeled. An estimate for x_q is then solved by:

$$F_{X_{\Gamma,3h}|H_sT_p}(x|h_q^{'},t_q^{'})=\alpha \tag{1.6}$$

where $h_q^{'}$ and $t_q^{'}$ are the characteristics of the worst sea state on the contour lines corresponding to annual exceedance probability of q. α is in the range of 0.85-0.95 for most practical cases, see by Haver (1998), Kleiven and Haver (2003). Sødahl (2006) suggest a lower value for some cases.

Both long term and short term response analysis will be illustrated in detail in this paper.

Chapter 2

Environmental modeling

Offshore structures in the sea will be subject to environmental loads caused by wave, wind, current, etc. In the present study, we are mainly interested in the wave-induced loads and/or motions. Wave-induced loads on structures are the consequences of the motion of water particles. Therefore, to better understand the wave-induced loads acting on offshore structures and consequent structure motions, it would be helpful to investigate the motion of water particles first. A detailed discussion on this issue can be found in Faltinsen (1993). We will briefly introduce the wave theory based on his discussion in the following.

2.1 Wave theory

Figure 2.1 shows a structure in the sea confronting a regular wave. A Cartesian coordinate system fixed in space is established to illustrate the wave condition. The wave is propagating in the x direction.

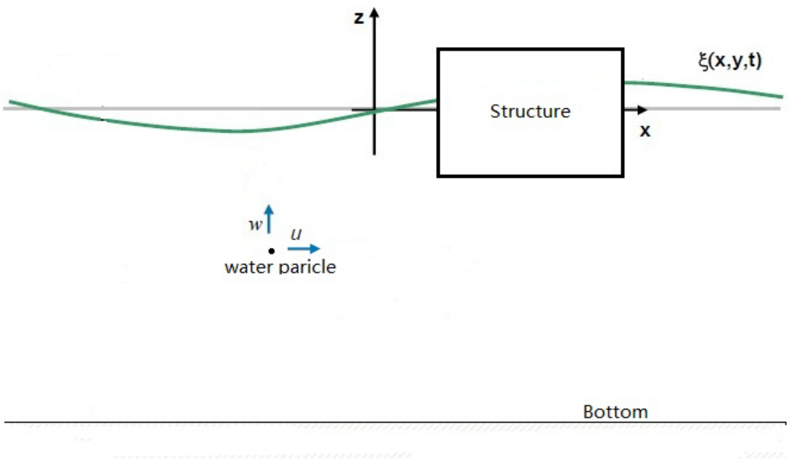

Figure 2.1 Offshore structure in regular wave

The real sea water is viscid, compressible. These features, however, have only minor impact on the general offshore structures. Therefore some basic assumptions can be made without introducing too much error in practice:

i) The sea water is assumed to be incompressible and inviscid;

ii) The fluid motion is irrigational, i.e.: $\nabla \times \vec{V} = 0$, where $\vec{V}(x, y, z, t) = (u, v, w)$ is the velocity vector of the water particle at time t at the point (x, y, z) and ∇ is the Laplace operator.

Based on the above assumptions, a velocity potential, ϕ, can be utilized to describe the velocity vector:

$$\vec{V} = \nabla \phi = \vec{i}\frac{\partial \phi}{\partial x} + \vec{j}\frac{\partial \phi}{\partial y} + \vec{k}\frac{\partial \phi}{\partial z} \tag{2.1}$$

Since the water is incompressible, i.e. $\nabla \cdot \vec{V} = 0$, it follows that the velocity potential satisfies the Laplace equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{2.2}$$

In addition to the Equation (2.2), the velocity potential should fulfill some boundary conditions:

Kinematic boundary conditions:

i) No water particles can penetrate the bottom, i.e. $\frac{\partial \phi}{\partial z} = 0$ on the bottom;

ii) The water particles on the free-surface will remain there, i.e. $\frac{\partial \phi}{\partial z} = \frac{D\xi}{Dt}$, where $\frac{D}{Dt}$ is the Eulerian derivative which reads:

$$\frac{D\xi}{Dt} = \frac{\partial \xi}{\partial t} + u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} + w\frac{\partial \xi}{\partial z}$$

Therefore,

$$\frac{\partial \phi}{\partial z} = \frac{\partial \xi}{\partial t} + u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} + w\frac{\partial \xi}{\partial z} \quad \text{on the free-surface} \tag{2.3}$$

If the wave amplitude is very small, Equation (2.3) can be linearized, i.e. the 2nd order terms can be neglected. The linearized formulation reads:

$$\frac{\partial \phi}{\partial z} = \frac{\partial \xi}{\partial t} \quad \text{on the free-surface} \tag{2.4}$$

Dynamic free-surface condition

Bernoulli equation states that the hydrodynamic pressure in the fluid domain will be constant. It reads:

$$p + \rho g z + \rho \frac{\partial \phi}{\partial t} + \frac{\rho}{2} \vec{V} \cdot \vec{V} = C(t) \tag{2.5}$$

where $C(t)$ is a constant which is only dependent on time.

The dynamic free-surface condition requires that the water pressure on the free-surface is identical to the atmospheric pressure, p_0:

$$p_0 + \rho g \xi + \rho \frac{\partial \phi}{\partial t} + \frac{\rho}{2} \left(\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right) = p_0 \quad \text{on the free-surface} \tag{2.6}$$

Under linear assumption, the 2nd order terms are neglected:

$$\xi = -\frac{1}{g} \left(\frac{\partial \phi}{\partial t} \right) \quad \text{on the free-surface} \tag{2.7}$$

The boundary conditions (2.4) and (2.7) can be combined into one equation:

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0 \tag{2.8}$$

Assuming water depth is infinite, a solution for ϕ fulfilling Equation (2.8) can be obtained, see Newman (1977):

$$\phi(x, z, t) = \phi_0 e^{kz} \cos(\omega t - kx) \tag{2.9}$$

where k is the wave number and $k = \frac{\omega^2}{g}$.

The corresponding wave profile on the surface is then found using Equation (2.7):

$$\xi(x, t) = \frac{\omega}{g} \phi_0 \sin(\omega t - kx) = \xi_0 \sin(\omega t - kx) \tag{2.10}$$

In reality, the wave process is of a random nature. If we assume that the sea surface is constructed by a series of long crested waves, and the waves are propagating in the same direction, the real sea surface can then be expressed by the linear superposition of a series of regular waves:

$$\xi = \sum_{i=1}^{N} \xi_{0i} \sin(\omega_i t - k_i x + \varepsilon_i) \tag{2.11}$$

where ε_i is the phase angle. ε_i is considered to be uniformly distributed between 0 and 2π.

It is commonly assumed that the wave elevation is Gaussian distributed with a mean value of 0 and variance of σ^2. And in a short period of time, The wave process is stationary, meaning that within a short period the mean value and variance of the process will be constant.

2.2 Wave spectrum

The wave spectrum, $S_{\Xi\Xi}(\omega)$, can be utilized to characterize the wave process. Wave spectrum is defined in such way that:

$$\frac{1}{2}\xi_{0i}^2 = S_{\Xi\Xi}(\omega_i)\Delta\omega \tag{2.12}$$

where ξ_{0i}^2 and ω_i is the amplitude and frequency of wave component i, $\Delta\omega$ is a constant difference between successive frequencies.

$S_{\Xi\Xi}(\omega)$ contains all the information about the statistical properties of $\xi(t)$, since

$$\sigma^2 = \int_0^\infty S_{\Xi\Xi}(\omega) d\omega \tag{2.13}$$

See Faltinsen (1993).

At present determination of wave spectrum directly from wind profile is out of reach due to the complicated mechanism how wind induces wave. Therefore, some semi-empirical spectrum models are developed for practice use. Generally, an analytical form is adopted to express the spectrum. Some definitions are first introduced before we present different kinds of spectrum models.

Wind sea: Wind sea refers to the sea state that is generated by a local wind field. Wind sea can be further divided into growing wind sea and fully developed wind sea. Fully developed wind sea refers to a sea condition where the wave process has reached equilibrium, meaning that the spectrum form will not change.

Swell sea: Swell sea is a sea condition which is not caused by a local wind field. It can be viewed as a decaying phase after the local wind is significantly reduced or the wind has moved out of the area.

Combined sea: Most sea states will be a combination of wind sea and swell sea. We refer to these sea conditions as combined sea.

Some of the most frequently used wave spectrums are introduced in the following:

Pierson-Moskowitz Spectrum

This spectrum is proposed by Pierson and Moskowitz (1964). It is derived based on data collected from North Atlantic and is applicable for fully developed sea states. Haver (2009, a) proposed that whether the fully developed sea condition was fulfilled could be assessed by checking if the relation between significant wave height and spectrum peak period satisfied the following equation:

$$t_p = 5\sqrt{h_s} \tag{2.14}$$

Pierson-Moskowitz spectrum has the shape:

$$S_{\Xi\Xi}(\omega) = \frac{A}{\omega^5}\exp(-\frac{B}{\omega^4})$$

By fitting the collected data, the parameters A and B are given by Pierson and Moskowitz (1964):

$A = 0.0081g^2, B = 0.74(\frac{g}{V})^4$, where V is the wind speed 19.5m above sea surface.

Therefore the Pierson-Moskowitz spectrum reads:

$$S_{\Xi\Xi}(\omega) = \frac{0.0081g^2}{\omega^5}\exp\left(-0.74\left(\frac{g}{V\omega}\right)^4\right) \tag{2.15}$$

ITTC and ISSC spectrum

These two spectrums are respectively proposed by ITTC (International Towing Tank Conference) and ISSC (International Ship Structures Congress). They are both applicable for fully developed sea states on open sea. They have the same shape as Pierson-Moskowitz spectrum:

$$S_{\Xi\Xi}(\omega) = \frac{A}{\omega^5}\exp(-\frac{B}{\omega^4})$$

In ITTC spectrum, see ITTC (1966):

$$A = 0.78, B = \frac{3.11}{h_s^2}$$

In ISSC spectrum, see ISSC (1964):

$$A = 173\frac{h_s^2}{T_1^4}, B = \frac{691}{T_1^4}, \text{ where } T_1 = \frac{2\pi m_0}{m_1} \text{ and } m_j = \int_0^\infty \omega^j S_{\Xi\Xi}(\omega)d\omega$$

JONSWAP Spectrum

This is presently the most common wave spectrum in North Sea. It is proposed by Hasselmann (1973). The spectrum reads:

$$s_{\Xi\Xi}(f) = 0.3125 h_s^2 t_p \left(\frac{f}{f_p}\right)^{-5} \exp\left\{-1.25\left(\frac{f}{f_p}\right)^{-4}\right\}(1 - 0.287\ln\gamma)\gamma^{\exp\left\{-0.5\left(\frac{f - f_p}{f_p\sigma}\right)^2\right\}} \tag{2.16}$$

where σ reads, Hasselmann (1973):

$$\sigma = \begin{cases} 0.07, & f \le f_p \\ 0.09, & f \ge f_p \end{cases} \tag{2.17}$$

The peak enhancement factor can be computed from Torsethaugen (2004):

$$\gamma = 42.2\left(\frac{2\pi h_s}{g t_p^2}\right)^{\frac{6}{7}} \tag{2.18}$$

Each spectrum has its own validity area. Haver (2008) provided a illustration diagram, see Figure 2.2. One should be aware of which sea states are most critical for the problem under consideration before deciding which spectrum model is to be utilized.

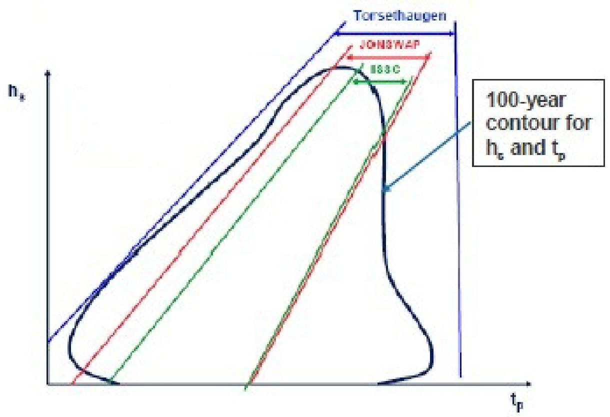

Figure 2.2 Validity of various wave spectrums

So far we have been assuming that the wave is long crested. The effect of short crested might be important and should be accounted for. For short crest waves, we shall include the wave directionality:

$$S_{\Xi\Xi}(\omega,\theta) = S_{\Xi\Xi}(\omega)\, f(\theta)$$

where θ is an angle measuring the wave propagation direction of wave components in the sea.

2.3 Long term analysis of sea state characteristics

For a short period of time, the wave process is stationary and can be completely characterized by the wave spectrum. The most commonly used JONSWAP in Norwegian Continental Shelf is defined by significant wave height and spectrum peak period. Therefore the long term variability of sea state characteristics can be modeled by the joint distribution of significant wave height and spectrum peak period. In addition, the direction of wave propagation should also be included.

Since the short term sea state is characterized by Θ , H_s and T_p, the long term distribution of wave conditions can be described by the joint density function can be conveniently written as:

$$f_{H_sT_p\Theta}(h,t,\theta) = f_{\Theta}(\theta)\, f_{H_sT_p|\Theta}(h,t|\theta) = f_{\Theta}(\theta)\, f_{H_s|\Theta}(h|\theta)\, f_{T_p|H_s\Theta}(t|h,\theta) \qquad (2.19)$$

2.3.1 Marginal distribution of Θ

The marginal distribution of Θ is difficult to model by some few parameters. In most practical cases, the distribution of Θ is obtained by dividing the circle into a number of sectors (a common choice is 12 sectors with width of $30°$), and assigning each sector its probability of occurrence. If the problem under consideration is very sensitive to direction, a finer resolution should be adopted. It should be kept in mind that the finer the resolution is, the fewer data there are to establish the join distribution of significant wave height and spectrum peak period. Therefore one should be careful if a too fine resolution is to be adopted.

In practice, the main direction of propagation is often assumed to be the wind direction. This is accurate for storm seas, but the accuracy is questionable for low and moderate sea states. Another option is to use the information provided by hindcast data. Comparisons between the measured wave directions and hindcast wave direction suggest these two shows a good agreement with each other, see Haver (2009, b). It is likely that the hindcast data will be used for obtaining direction information in the future.

2.3.2 Conditional distribution of H_s given Θ

As far as we consider all 3-hour sea states in the North Sea, a 3-parameter Weibull model is proper for the distribution of H_s given Θ, see Bury (1975). Haver and Nyhus (1986) proposed another model that gave very good fit:

$$f_{Hs}(h) = \begin{cases} \frac{1}{\sqrt{2\pi}\alpha h} \exp(-0.5(\ln h - \lambda)^2 / \alpha^2; h \le \eta \\ \beta \frac{h^{\beta-1}}{\rho^{\beta}} \exp(-(h/\rho)^{\beta}); h > \eta \end{cases} \tag{2.20}$$

λ and α^2 are the mean and variance of $\ln h$, respectively. ρ and β are estimated in such way that the model is continuous at $h = \eta$, i.e.:

$$\begin{aligned} f_{Hs}^{\log-normal}(\eta) &= f_{Hs}^{Weibull}(\eta) \\ F_{Hs}^{\log-normal}(\eta) &= F_{Hs}^{Weibull}(\eta) \end{aligned} \tag{2.21}$$

To obtain a good fit, various values of η are selected and corresponding ρ and β are calculated. For each value of η a test for the goodness test of the fit can be performed, such as Chi-square test or Kolmogorov test. The value of η is chosen to be the one minimizing the error.

2.3.3 Conditional distribution of T_p given Θ and H_s

Experience suggest that T_p will follow a log-normal distribution for given H_s and Θ. Haver (2009, c) proposed that a lower bound of T_p should be adopted:

$$f_{T_p|H_s\Theta}(t|h,\theta) = \frac{1}{\sqrt{2\pi}\sigma(h,\theta)\left[t-\varepsilon(h,\theta)\right]} \exp\left\{-\frac{1}{2}\left(\frac{\ln\left[t-\varepsilon(h,\theta)\right]-\mu(h,\theta)}{\sigma(h,\theta)}\right)^2\right\} \tag{2.22}$$

where $\mu = E\left[\ln(T_p - \varepsilon)\right]$, $\sigma^2 = Var\left[\ln(T_p - \varepsilon)\right]$

The reason for including the lower limit is to account for the consequences of wave breaking. Ochi and Tsai (1983) gave reasonable lower boundaries as far as individual waves were of concern. However the lower limit is often set to be zero in Norwegian Continental Shelf since the establishment of ε is not clear yet, although sometimes a better can be obtained adopting a low limit. In the Gulf of Mexico, an improved fitting model to the hindcast hurricane data can be obtained when a proper lower limit is introduced.

2.4 Environmental modeling of wind and current

In addition to the wave-induced loads, offshore structures will also be subject to environmental loads induced by wind, current, etc. These kinds of environmental loads will be briefly introduced in the following.

2.4.1 Wind

Wind force acting on structures is generally proportional to the square of wind speed. Wind speed consists of two parts: a slowly varying mean wind, $V_m(t)$, and a rapidly fluctuating wind component riding on the mean wind, $V_f(t)$. The mean wind speed is considered to be constant over a short period of time, say 3 hours, while the period of the fluctuating component will be around several seconds to a few minutes. An important quantity in connection with $V_m(t)$ and $V_f(t)$ is the turbulence intensity, I. The turbulence intensity is defined as the ratio between the standard deviation of the fluctuating component and the mean wind speed.

In practice, the mean wind speed is assumed to follow a Rayleigh distribution. The mean direction of wind is described by a probability density for directional sectors of constant degrees, typically 30 degrees per sector.

2.4.2 Current

The current is considered to be a slowly varying phenomenon, i.e. it can be assumed to be constant over a short period of time.

In practice, the current is modeled as a velocity field varying over depth. At present, the velocity of current is measured at various depths, and speed between different depths is assumed to be linearly distributed.

Current can be divided into several components:

i) Tidal current. Tidal current is constant through water column.

ii) Background current.

iii) Wind driven current. Wind current is approximately 1%-2% in magnitude of mean wind speed at the surface decaying linearly to zero at about 50m below surface.

iv) Meanders or vortex current.

If data are out of reach, current field may be taken to be the sum of tidal current and the wind driven current.

Chapter 3

Responses problem

In chapter 2, the environmental characteristics are reviewed. However, it is the environmental loads acting on structures and consequent structure responses that are of our interest. To estimate the long term extreme response, one has to establish the long term distribution of response by convoluting the short term distribution of response in a given sea state with the long term variability of sea state characteristics. The former one, i.e. the short term response problem, is often more challenging to solve. We will investigate the short term response problem and discuss how the short term response distribution can be established in this chapter.

3.1 Classification of structure responses problems

Based on Newton Second Law, the motion, x, of an offshore structure due to environmental loading can be solved by the equation:

$$m\ddot{x}(t)+c(x,\dot{x})\dot{x}(t)+k(x,\dot{x})x(t)=Q(t) \tag{3.1}$$

where m is the mass of the structure, $c(x,\dot{x})$ is the damping coefficient associated with the motion degree of freedom, $k(x,\dot{x})$ is the stiffness coefficient associated with the motion degree of freedom, $Q(t)$ is the external load acted on the structure in the direction of the motion degree of freedom. Generally, x will be a vector, containing all degrees of freedom of the structure.

A simple classification can be made in connection with the left side of equation (3.1). Generally speaking, the damping coefficient and stiffness coefficient are of a non-linear nature. In practice, however, results with enough accuracy can be obtained under linearization of these coefficients, i.e. the damping force and stiffness are linear functions of $\dot{x}(t)$ and x, respectively. The response problem can therefore be divided into two categories:

i) Linear mechanical system
ii) Non-linear mechanical system

linear mechanical system

Linear mechanical system refers to problems whose damping force and stiffness are linear functions of $\dot{x}(t)$ and x. It can be further divided into two subcategories:

i) linear response problem

If we further assume that the right hand side of Equation (3.1) is a linear function of the sea surface elevation process, the response is called a linear response problem. The estimation of response extreme for this kind of problem is rather simple and it can be solved in the frequency domain. Examples fall into this category are fixed large volume structures, slender structures subject to loads dominated by mass term.

ii) 2^{nd} and higher order of force

In the 2^{nd} order loading theory, we shall keep all terms in the velocity potential in Equation (2.3) and (2.6). This will result in mean forces, and forces oscillating with difference frequencies and sum frequencies in addition to the linear solution. For slender structures where the drag term is important, the loading is of order higher than 2 due to the integration to the exact surface. These forces are generally much smaller than the wave frequency force, but they might be important if their frequencies coincide with one of the nature frequencies of the structure.

Non-linear mechanical system

If the displacement is too large, the stiffness coefficient and damping coefficient should be updated with time in the analysis, i.e. $c(x,\dot{x})$ and $k(x,\dot{x})$ in Equation (3.1) will be a function of time. Examples of non-linear mechanical responses are surge motions of a moored vessel and motions of flexible risers.

3.2 Distribution of short term responses

The long term distribution of response extreme will be obtained by convoluting distribution of the short term response in a given sea state with the long term variability of environmental characteristics. In practice, it is often the short term response problems that are more challenging. Therefore it is necessary to discuss various methods to establish such short term response distribution. For simple linear responses, it is possible to solve the response in frequency domain. For non-linear problems, however, solutions in frequency domain are no longer possible, and the problems have to be solved in the time domain. If the problems are so complicated that time domain integrations are out of reach, model test is the only available method at the moment.

3.2.1 Linear responses

If we assume that:

i) The relationship between sea surface elevation and response amplitude is linear for a given frequency;

ii) The response is of the same frequency as the wave;

iii) The response from different frequency is independent and the total response is the linear superposition of response from each frequency.

The responses will also be a Gaussian process as the wave process is. A transfer function may be applied to obtain the response process:

$$x_0(\omega_i) = H(\omega_i)\xi_0(\omega_i) \tag{3.2}$$

Both sides of Equation are squared:

$$x_0^2(\omega_i) = H^2(\omega_i)\xi_0^2(\omega_i) \tag{3.3}$$

Inserting Equation (2.12), it follows that:

$$\frac{1}{2}x_0^2(\omega_i) = H^2(\omega_i)\frac{1}{2}\xi_0^2(\omega_i) = H^2(\omega_i)S_{\Xi\Xi}(\omega_i)\Delta\omega \tag{3.4}$$

We may define a response spectrum in a similar way as the wave spectrum:

$$\frac{1}{2}x_0^2(\omega_i) = S_{\Gamma\Gamma}(\omega_i)\Delta\omega$$

Therefore we have:

$$S_{\Gamma\Gamma}(\omega) = H^2(\omega_i)S_{\Xi\Xi}(\omega) \tag{3.5}$$

The variance, σ_Γ, and the zero-up-crossing frequency, $\upsilon^+_{\Gamma,0}$, can be found by

$$\sigma_\Gamma^2(h,t) = m_{\Gamma\Gamma}^{(0)}(h,t)$$
$$\upsilon^+_{\Gamma,0}(h,t) = \sqrt{\frac{m_{\Gamma\Gamma}^{(2)}(h,t)}{m_{\Gamma\Gamma}^{(0)}(h,t)}} \tag{3.6}$$

where the spectrum moments, $m_{\Gamma\Gamma}^{(i)}(h,t)$ are defined by:

$$m_{\Gamma\Gamma}^{(i)}(h,t) = \int_0^\infty \omega^i s_{\Gamma\Gamma}(\omega;h,t)d\omega \tag{3.7}$$

Under the linear assumption, global response maxima, i.e. the largest response maxima between two adjacent zero-up-crossings, can be modeled by the Rayleigh distribution as a conditional distribution given the sea state characteristics:

$$F_{X_\Gamma|H_sT_p}(x|h,t) = 1 - \exp\left\{-\frac{1}{2}\left[\frac{x}{\sigma_\Gamma(h,t)}\right]^2\right\} \tag{3.8}$$

Assuming all global responses maxima during the sea state are independent, the distribution of d-hour responses maxima can then be written as:

$$F_{X_{\Gamma,d}|H_sT_p}(x|h,t) = \left\{1 - \exp\left\{-\frac{1}{2}\left[\frac{x}{\sigma_\Gamma(h,t)}\right]^2\right\}\right\}^{n_\Gamma(h,t)} \tag{3.9}$$

where d is the duration of a stationary sea state, and $n_\Gamma(h,t) = 3600 \cdot d \cdot \upsilon^+_{\Gamma,0}$ is the expected number of global response maxima in this sea state.

As $n_d(h,t)$ increases, the Equation (3.9) will approach to Gumbel distribution:

$$F_{X_{\Gamma,d}|H_sT_p}(x|h,t) = \exp\left\{-\exp\left\{-\left[\frac{x-\gamma}{\beta}\right]\right\}\right\} \tag{3.10}$$

where γ is the most probable responses, and the parameters read, see Haver (2009):

$$\gamma = \sigma_\Gamma(h,t)\sqrt{2\ln n_\Gamma(h,t)}$$
$$\beta = \frac{\sigma_\Gamma(h,t)}{\sqrt{2\ln n_\Gamma(h,t)}} \tag{3.11}$$

Alternatively, we can adopt the α-percentile response instead of the most probable response:

$$x_\alpha = \gamma - \beta \ln(-\ln \alpha) \tag{3.12}$$

3.2.2 Time domain simulations

For non-linear response problems where frequency domain solutions are no longer available, a step-by-step solution in the time domain might be useful.

The method is briefly introduced as following. For illustrative purpose, we will consider the simplest situation, i.e. a one-degree-of-freedom response problem.

Consider the equation of motion:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = Q(t)$$

The relation between $\ddot{x}(t)$, $\dot{x}(t)$ and $x(t)$ is given by:

$$\ddot{x}(t) = \frac{d\dot{x}(t)}{dt} \tag{3.13}$$

$$\dot{x}(t) = \frac{dx(t)}{dt} \tag{3.14}$$

We will now look at the time interval between time t and $t+h$. Assuming that we already know the motion quantities at time t, it is possible to determine the motion quantities at time $t+h$ once we know the acceleration of motion over the time interval:

$$\dot{x}(t+h) = \dot{x}(t) + \int_0^h \ddot{x}(\tau)d\tau \tag{3.15}$$

$$x(t+h) = x(t) + \int_0^h \dot{x}(\tau)d\tau \tag{3.16}$$

A frequently used method here is Newmark-beta method, see Newmark (1959):

$$\dot{x}(t+h) = \dot{x}(t) + (1-\gamma)\ddot{x}(t) + \gamma\ddot{x}(t+h), \tag{3.17}$$

$$x(t+h) = x(t) + h\dot{x}(t) + \left(\frac{1}{2} - \beta\right)h^2\ddot{x}(t) + \beta h^2\ddot{x}(t+h) \tag{3.18}$$

A common simplification is to assume that the acceleration in a time interval is constant and equal to the average of acceleration at interval ends, i.e. $\gamma = \frac{1}{2}, \beta = \frac{1}{4}$:

$$\ddot{x}(\varepsilon) = \frac{1}{2}\left(\ddot{x}(t) + \ddot{x}(t+h)\right), \quad t \le \varepsilon \le t+h \tag{3.19}$$

Introducing Equation (3.19) into (3.15), we have:

$$\dot{x}(t+h) = \dot{x}(t) + \frac{1}{2}\left(\ddot{x}(t) + \ddot{x}(t+h)\right)\int_0^h d\tau = \dot{x}(t) + \frac{1}{2}h\left(\ddot{x}(t) + \ddot{x}(t+h)\right) \tag{3.20}$$

$$x(t+h) = x(t) + \int_0^h \left(\dot{x}(t) + \frac{1}{2}\varepsilon\left(\ddot{x}(t) + \ddot{x}(t+h)\right)\right)d\varepsilon = x(t) + \dot{x}(t)h + \frac{1}{4}h^2\left(\ddot{x}(t) + \ddot{x}(t+h)\right) \tag{3.21}$$

Denote the $\ddot{x}(t)$, $\dot{x}(t)$, $x(t)$, $\ddot{x}(t+h)$, $\dot{x}(t+h)$ and $x(t+h)$ by $\ddot{x}_i$, $\dot{x}_i$, x_i, $\ddot{x}_{i+1}$, $\dot{x}_{i+1}$ and x_{i+1}, respectively, we can obtain some sort of a step-by-step method:

$$\dot{x}_{i+1} = \dot{x}_i + \frac{1}{2}h\left(\ddot{x}_i + \ddot{x}_{i+1}\right) \tag{3.22}$$

$$x_{i+1} = x_i + \dot{x}_i h + \frac{1}{4}h^2\left(\ddot{x}_i + \ddot{x}_{i+1}\right) \tag{3.23}$$

Rewrite Equation (3.23):

$$\ddot{x}_{i+1} = \frac{4}{h^2}x_{i+1} - \frac{4}{h^2}x_i - \frac{4}{h}\dot{x}_i - \ddot{x}_i \tag{3.24}$$

Introduce Equation (3.24) to Equation (3.22):

$$\dot{x}_{i+1} = \dot{x}_i + \frac{1}{2}h\left(\ddot{x}_i + \left(\frac{4}{h^2}x_{i+1} - \frac{4}{h^2}x_i - \frac{4}{h}\dot{x}_i - \ddot{x}_i\right)\right) = \frac{2}{h}x_{i+1} - \frac{2}{h}x_i - \dot{x}_i \tag{3.25}$$

The motion equation at step i+1 is:

$$m\ddot{x}_{i+1} + c\dot{x}_{i+1} + kx_{i+1} = Q_{i+1} \tag{3.26}$$

By introducing Equation (3.24) and (3.25) into (3.26):

$$m\left(\frac{4}{h^2}x_{i+1} - \frac{4}{h^2}x_i - \frac{4}{h}\dot{x}_i - \ddot{x}_i\right) + c\left(\frac{2}{h}x_{i+1} - \frac{2}{h}x_i - \dot{x}_i\right) + kx_{i+1} = Q_{i+1} \tag{3.27}$$

Therefore,

$$x_{i+1} = \frac{Q_{i+1} + m\ddot{x}_i + (\frac{4m}{h} + c)\dot{x}_i + (\frac{4m}{h^2} + \frac{2c}{h})x_i}{(\frac{4m}{h^2} + \frac{2c}{h} + k)} \tag{3.28}$$

Once the displacement at time i+1 is obtained, the speed and acceleration at time step i+1 can be solved using Equation (3.24) and (3.25). The complete motion quantities are so far obtained using the motion quantities at the previous time step. Therefore, the response history of the structure can be calculated using time domain simulations if we know the initial condition of the structure.

To utilize time domain simulations to establish the short term response distribution, a d-hour realization of wave elevation should first be generated from the wave spectrum. The water particle speed and acceleration are then calculated. Loads on the various submerged structure members are then determined. Loads are given at each submerged nodal point of structure. The equation of motion is then solved step-by-step as discussed above, resulting in a time series of all nodal points' displacements. This is in general rather quick if the motion of the structure is small. From this series of displacements one can establish the distribution of global response maxima. Alternatively, one can identify the d-hour maxima. By repeating these procedures, say

20 times, we obtain 20 simulated d-hour extreme responses, and a proper distribution of d-hour extreme response can thereafter be established.

3.2.3 Model tests

For very complicated response problems, numerical analysis might be out of reach at the moment. Under this circumstance, model test is the only option.

The detailed procedures for performing model tests, including the scaling of the model, etc, will not be covered here. Instead we merely focus on how one can estimate extreme response based on model test results.

It is unrealistic to run model test for too many the sea states since it can be very time and effort consuming. In practice we will select some few d-hour sea states corresponding to a certain annual exceedance probability, say 1/100. A couple of tests are run for each selected sea state, and the one with the largest response is taken to be the worst sea state. Note that it might be difficult to identify the worst sea state based on such few tests. But this is not of too much of our concern. Since there is not much difference between the sea states under consideration, the error by taking the sea state next to the worst one is not too significant, so one should not be too concerned about this.

Once the worst sea state is decided, a series of model tests should be performed in order to establish the short term response distribution. Obviously enough tests should be run so that the upper tail of the distribution can be reasonably well fitted. In practice, 20-24 tests are performed. Apparently a lot of uncertainty is associated with only 20-24 tests. However significant improve in the accuracy require a lot more tests, resulting in considerably increase in time and effort. This is not efficient in the practical point of view. So performing 20-24 tests is considered to be reasonable compromise. Generally a high percentile value of the distribution will be adopted as the characteristic response, generally 85%-95% value for 100-year value and 90-95% value for 10000-year value, see Haver (1998).

Chapter 4

Methods to estimate characteristic responses

As mentioned before, we have to estimate the characteristic loads and/or responses corresponding to a certain annual probability of being exceeded in the design of offshore structures. There are several ways to do this, such as all sea states approach, peak over threshold method, environmental contour lines method, etc. Which method is to be utilized depends on both the nature of the problem under consideration and the environmental condition where the structure locates. Some frequently utilized methods will be introduced in this chapter.

4.1 All sea states approach

As discussed previously, the long term distribution of response is established by convoluting the short response variability in a given sea state with the long term variability of sea state characteristics. With all sea states approach, the distribution is established based on all sea states. This method was introduced to the field of naval architecture in the 1950s, see e.g. Jasper (1956). Major improvements of all sea states approach during the last three decades have been in the joint modeling of the environmental characteristics like significant wave height and spectrum peak period, i.e. $f_{H_sT_p}(h_s,t_p)$, see e.g. Haver and Nyhus (1986).

One may assume that duration for stationary sea state is constant. A common choice is 3 hours for North Sea conditions, while 0.5 hours may be more accurate for hurricane conditions like Gulf of Mexico. The response distribution in a given sea state can be established by considering either global maxima or d-hour maxima. For example, if the d-hour maxima distribution is considered, the long term distribution of d-hour maximum response can then be established by convoluting conditional distribution of d-hour response maxima in a given state with the joint distribution of significant wave height and spectrum peak period, i.e. $f_{H_sT_p}(h_s, t_p)$. Therefore, the long term distribution of responses in a random d-hour sea state can be expressed as:

$$F_{X_{\Gamma,d}}(x) = \int_h \int_t F_{X_{\Gamma,d}|H_sT_p}(x|h,t) f_{H_sT_p}(h,t) dt dh \quad (4.1)$$

The characteristic response, x_q, is given by:

$$1 - F_{X_{\Gamma,d}}(x_q) = \frac{q}{n_d} \quad (4.2)$$

where n_d is the number of d-hour sea state in per year.

All sea state approach is a slightly conservative approach. This is mainly due to the assumption that all sea states are statistically independent and the correlation between adjacent sea states is neglected. However, this assumption will yield conservative results and the extent of conservatism is only 3-5%, see e.g.Winterstein et al. (2001). Therefore, it is not an important limitation.

4.2 Peak over threshold method

On the contrary of taking all sea states into consideration, we may concentrate merely on the sea states whose significant wave heights are higher than a threshold. By doing this, we obtain a number of storms, as shown in Figure 4.1. A storm can be defined as a continuous period consisting of several adjacent d-hour sea states with significant wave height exceeding a certain threshold. It can generally be characterized by a development phase during which the significant wave height increases to a maximum and a subsequent decaying phase, as shown in Figure 4.2. With this approach, we treat storms as the independent events. The statistical independence between sea states is avoided, resulting in a less conservative extreme responses value.

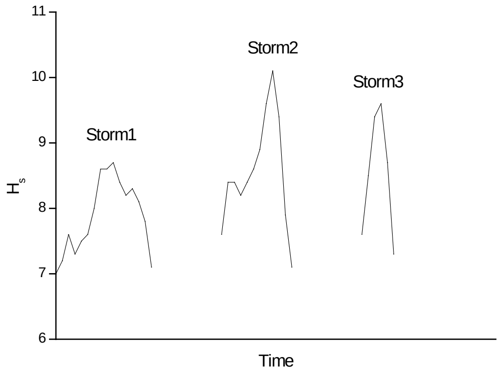

Figure 4.1 Typical storms in wave climate

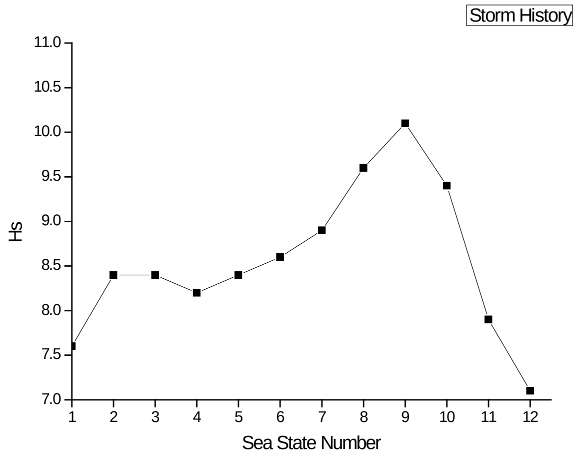

Figure 4.2 Typical history of a storm

The basic idea of peak over threshold approach is to establish the distribution of the largest responses during a random storm. These could be achieved in various ways. For example, Johns and Wheeler (1973) and Petrauskas and Aagaard (1971) merely consider the observed storms. It is assumed that all storms will occur with the same probabilities of these observed storms, and each observed storm has the same probability. Therefore,

$$F_{X|random\_storm}(x|random\_storm) = \frac{1}{n_{storm}} \sum_{j=1}^{n_{storm}} F_{X|storm_j}(x|storm_j) \tag{4.3}$$

where n_{storm} is the number of observed storms.

This method may yield reasonable results when the number of observed storms is large enough that they can represent the true long-term storm distribution. In reality, however, this is often not the case. Tromans and Vanderschuren (1995) recommended that we should take non-observed storms into account. According to their theory, the distribution of the largest response in a random storm can be obtained by convoluting conditional distribution of largest response in a storm given the storm's most probable largest response with the distribution of storm largest most probable response. This way the non-observed storms are accounted for and the result will be closer to that obtained by all sea states approach.

Peak over threshold method is more frequently used in hurricane-governed areas like Gulf of Mexico. This is because in these areas, the hurricanes, which represent the design conditions, are so scarce that a very long period of measurements would be required before a reliable joint model of the environmental characteristics could be established. Due to the availability of environmental data, peak over threshold approach is the dominant design approach in these areas instead of all sea state approach.

4.3 Reliability method

To estimate the extreme response with a very low annual exceedance probability, we are mainly interested in the very upper tail part of the long term response distribution. In this condition, reliability analysis is a very efficient tool.

Say we now intend to determine the probability for the response exceeding a given high value, x_c , and the structure will fail if x_c is exceeded. The distribution of d-hour response maxima is a function of significant wave height and spectrum peak period, and the d-hour response maxima itself, $X_{\Gamma,d}$, is a variable. A function $g()$ is proposed such that:

$$g\left(X_{\Gamma,d}, H_s, T_p; x_{\Gamma,d}\right) = x_c - X_{\Gamma,d}(H_s, T_p)$$

Obviously, the d-hour response maxima will exceed x_c when $g < 0$. Therefore, the probability x_c being exceeded is given by:

$$p(x_c) = \iiint_{g<0} f_{X_{\Gamma,d}|H_sT_p}(x|h,t)\, f_{H_sT_p}(h,t)\,dx\,dh\,dt \qquad (4.4)$$

This integration can be solved easily numerically. Another available method is First-Order-Reliability-Method (FORM). It avoids the explicit integration. FORM will be illustrated as following.

The variables in the physical space are first transformed to another space: U-space. The transformed variables in the U-space follow standard Gaussian distributed. This transformation is called Rosenblatt transformation, see Madsen (1986):

$$\Phi(u_1) = F_{H_s}(h)$$

$$\Phi(u_2) = F_{T_p|H_s}(t|h) \tag{4.5}$$

$$\Phi(u_3) = F_{X_{\Gamma,d}|H_s T_p}(x|h,t)$$

The points with identical probability density in the physical space will be transformed to a sphere in U-space since all the transformed variables are all standard normal distributed. Meanwhile, the fail surface in the physical space will also be transformed to U-space. In FORM, the transformed fail surface will turn into a plane. The design point in the U-space is taken to be the one closet to the origin point on the fail surface. The distance r between design point and origin reads:

$$r = \sqrt{\hat{u}_1^2 + \hat{u}_2^2 + \hat{u}_3^2} \tag{4.6}$$

where $\hat{u}_1$, $\hat{u}_2$ and $\hat{u}_3$ are coordinates of the design point.

The probability of x_c being exceeded then reads:

$$p(x_c) = 1 - \Phi(r) = \Phi(-r) \tag{4.7}$$

We need to search the fail surface in U-space to determine the point closet to origin. This might be time-consuming and computer codes shall be utilized for the searching, see Haver (1980).

If the FORM approach is utilized to determine the characteristic response, x_q , we shall try various values of x_c, until the corresponding annual exceedance probability, $p(x_c)$, equal to the desired level, q. Alternatively, we may establish the upper tail of response distribution by fitting various pairs of x_c and $p(x_c)$.

If we are mainly interested in estimating extreme response corresponding to a certain annual exceedance probability, we can use a slightly different method, called inverse-FORM, i.e. IFORM, see Winterstein (1993). Since we intend to find the design point with given annual exceedance probability, say q, we may establish a sphere in the U-space first. The radius of the sphere r reads:

$$r = -\Phi^{-1}(q) \tag{4.8}$$

In IFORM approach, the failure plane will be tangent to the sphere at a point somewhere on this sphere. Therefore we just need to find the worst sea state along the contour lines in the physical space corresponding to the sphere in U-space.

4.4 Environmental contour lines method

In some complicated cases, time domain simulations or model tests are required to solve the short term response problem. This could be so time-consuming that all sea state approach and peak over threshold method become unrealistic since many short term response analyses have to be performed. An approximate method called environmental contour lines method can be an effective tool under this circumstance.

In environmental contour lines method, the contour lines for the significant wave height and spectrum peak period are determined such that all d-hour stationary sea states on the contour lines are of the same annual exceedance probability.

The contour lines are established using FORM. First, the significant wave height and spectrum peak period are transformed from the physical space to U-space:

$$\Phi(u_1) = F_{H_s}(h)$$
$$\Phi(u_2) = F_{T_p|H_s}(t|h) \tag{4.9}$$

where Φ is standard normal distribution, and the transformed variables u_1 and u_2 are independent. Therefore the contour lines on the transformed space will be a circle:

$$u_1^2 + u_2^2 = r^2$$

Therefore :

$$\Phi(r) = 1 - q \Rightarrow r = -\Phi^{-1}(q) \tag{4.10}$$

where q is the probability of the sea states on the contour lines being exceeded.

The values of u_1 and u_2 on the circle are given by:

$$u_1 = \beta_C \cos(\theta) \tag{4.11}$$

$u_2 = \beta_C \sin(\theta)$

The corresponding value of H_s and T_p are given by:

$$\Phi(u_1) = F_{Hs}(h)$$

$$\Phi(u_2) = F_{Tp|Hs}(t|h) \quad (4.12)$$

After the contour lines are established, we shall select some sea states along the contour lines which must include the worst sea state. A few time domain simulations or model tests are performed for each selected sea state, and the one with the largest response is taken to be the worst sea state. As the worst sea state is chosen, more simulations or model tests should be run for the worst sea state to determine the response distribution.

If we assume the response distribution in a stationary sea state is extremely narrow, we can neglect the randomness of x and take the median, x_{median}, to be the extreme. In reality, however, the extreme distribution is not extremely narrow and there is inherent randomness associated with the extreme response in a given sea state. Neglecting this kind of randomness will result in an underestimation of about 10%-15%, see Haver (2009, d). Therefore, short term variability should be included to obtain an accurate estimation of extreme response. There are several ways to take this source of randomness into account:
i) Increase the duration of the sea state artificially.
ii) Increase the sea state level artificially, see Winterstein (1993).
iii) Increase both the sea state level and duration artificially, see Haver (1996).
iv) Adopt a higher percentage value of the extreme response distribution.
v) Multiply the most probable responses extreme by a factor, typically between 1.1-1.3, see Sagli (2000).

To establish the short term extreme value distribution for worst sea state along the contour line might be a great challenge in practice. Generally we have the following methods.

i) Run a number of 3-hour time domain simulations for the worst sea state along the target contour lines. Identify the simulated largest 3-hour response for each time series, and we get, say 20, simulated 3-hour response extremes. These 20 samples are utilized to establish the response distribution;

ii) Run a number of 3-hour model tests for the worst sea state along the contour line. From each test one may identify the 3-hour extreme. The results obtained from a series of model tests are used to establish the short term response distribution. In order to properly model the upper tail which we are mainly interested in, a number of tests should be performed, say 20-30 times.

4.5 Remarks regarding the choice of method

Generally, a full long term response analysis should be performed to get a consistent estimate of extreme response. In North Sea, this is commonly done using all sea states approach. If the response quantities under consideration are approximately linear to the wave process, the short term response distribution can be easily solved using transfer function in the frequency domain. In this condition, all sea states approach is the favorable method. Examples of such response problems are response of fixed large volume platforms, slender structures when the mass term of Morison equation dominates, etc. Further discussion regarding all sea states approach is made in Chapter 5.

In some cases, where a lot of weather characteristics should be included to establish a joint distribution function, peak over threshold is more attractive. Using peak over threshold method, the long term distribution of response can be established by convoluting the storm response maxima given storm characteristics, e.g. the storm most probable response maxima, with the distribution of storm characteristics. This way the establishment of joint distribution of weather characteristics is avoided. Peak over threshold is also convenient in hurricane governed areas like Gulf of Mexico. In such areas, we are primarily interested in response during storms. However, due to the limited number of data, it is difficult to establish a reliable joint distribution of sea characteristics. We may therefore merely focus on storms whose significant wave heights exceeding a certain threshold instead of all sea states. This method will be illustrated in Chapter 6.

Environmental contour lines method is convenient for complicated response problems where time domain simulations or model tests are required. Only short term analysis is to be performed and numerous simulations or tests can be avoided using this method. Environmental contour lines method could be applied to problems such as green water problems of ships, water-deck impacts of fixed and floating structures, etc. A detailed illustration in connection with this method will be made in Chapter 7.

Chapter 5

All sea states approach

Previously we have introduced how a consistent estimation of response extreme can be obtained using full long term analysis. All sea state approach is an option for such long term analysis. In this chapter we will illustrate the application of all sea states approach in detail. An example will be given.

5.1 Example description

To illustrate the concept and procedure of predicting design extreme responses, we will analyze extreme heave motion for a semi-submersible located in the North Sea. In this study we will assume linear responses mechanism applies, i.e. the heave motion is linearly related to the wave process by the transfer function shown in Figure 5.1:

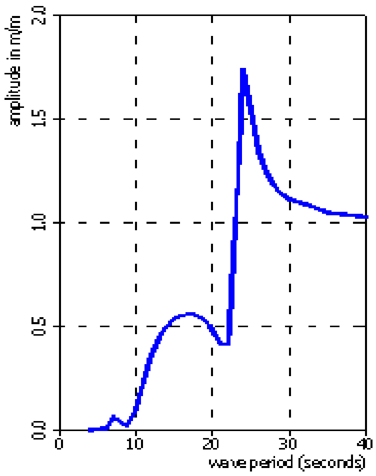

Fig 5.1 Transfer function for heave motion of aimed structure

A hindcast data series in the aimed area is available, which give significant wave height and spectrum peak period for every 3 hour from September 1957 until December 2007. Here it is assumed that the distribution function estimated from the hindcast data are representative for the 3-hour significant wave height, and all waves are propagating in the same direction.

5.2 Environmental description

The long term distribution of the wave climate can be described by the joint probability of H_s and T_p, $f_{HsTp}(h,t)$, which can be estimated from the joint frequency table based on hindcast data given in Table 5.1.

The joint distribution function for significant wave height and spectrum peak period, $f_{HsTp}(h,t)$, can be conveniently written as $f_{HsTp}(h,t) = f_{Hs}(h)\, f_{Tp|Hs}(t|h)$, where $f_{Hs}(h)$ and $f_{Tp|Hs}(t|h)$ are fitted to the observations separately.

Table 5.1 Joint Frequency Table from Hindcast Data

Hs\Tp	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	SUM
0-0.5	0	0	0	6	55	88	77	98	60	43	62	9	11	3	3	0	0	0	0	0	0	0	515
0.5-1	0	0	3	167	862	1778	3514	1955	1854	1600	986	488	300	124	57	0	35	19	0	8	0	0	13750
1-1.5	0	0	0	18	848	2792	4555	3806	4609	3719	2674	1982	968	510	231	0	122	49	0	17	0	6	26906
1.5-2	0	0	0	0	135	1682	3809	2537	3579	4136	3436	2577	1911	933	415	0	212	79	0	34	0	16	25491
2-2.5	0	0	0	0	5	372	2553	1925	2287	2929	3386	2573	1960	1282	580	0	222	85	0	32	0	8	20199
2.5-3	0	0	0	0	0	56	1134	1598	1785	2121	2633	2660	1944	1218	700	0	259	113	0	16	0	7	16244
3-3.5	0	0	0	0	0	3	310	869	1427	1655	1991	2138	1811	1110	704	0	286	104	0	27	0	9	12444
3.5-4	0	0	0	0	0	0	66	293	953	1391	1638	1668	1494	994	623	0	263	118	0	24	0	1	9526
4-4.5	0	0	0	0	0	0	7	62	421	824	1320	1380	1218	881	540	0	215	106	0	12	0	1	6987
4.5-5	0	0	0	0	0	0	0	12	113	433	971	1231	924	636	470	0	153	81	0	6	0	1	5031
5-5.5	0	0	0	0	0	0	0	1	26	159	581	1004	853	487	313	0	135	65	0	3	0	1	3628
5.5-6	0	0	0	0	0	0	0	0	3	73	243	701	761	449	268	0	125	53	0	4	0	1	2681
6-6.5	0	0	0	0	0	0	0	0	1	20	75	420	645	363	204	0	113	42	0	3	0	0	1886
6.5-7	0	0	0	0	0	0	0	0	0	1	30	214	569	343	141	0	63	44	0	3	0	1	1409
7-7.5	0	0	0	0	0	0	0	0	0	1	12	51	403	357	135	0	48	41	0	0	0	0	1048
7.5-8	0	0	0	0	0	0	0	0	0	0	3	20	196	314	121	0	31	38	0	1	0	0	724
8-8.5	0	0	0	0	0	0	0	0	0	0	1	5	93	271	114	0	20	23	0	1	0	0	528
8.5-9	0	0	0	0	0	0	0	0	0	0	0	0	28	171	115	0	14	16	0	1	0	0	345
9-9.5	0	0	0	0	0	0	0	0	0	0	0	0	9	99	85	0	25	13	0	1	0	0	232
9.5-10	0	0	0	0	0	0	0	0	0	0	0	0	1	55	85	0	25	9	0	0	0	0	175
10-10.5	0	0	0	0	0	0	0	0	0	0	0	0	0	22	51	0	17	12	0	0	0	0	102
10.5-11	0	0	0	0	0	0	0	0	0	0	0	0	0	5	33	0	10	9	0	0	0	0	57
11-11.5	0	0	0	0	0	0	0	0	0	0	0	0	0	5	19	0	5	4	0	0	0	0	33
11.5-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	13	0	8	2	0	0	0	0	23
12-12.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	7	2	0	0	0	0	13
12.5-13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	4	1	0	0	0	0	6
13-13.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	4	0	0	0	0	0	5
13.5-14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	4
14-14.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0	0	0	0	3
14.5-15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
15-15.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15.5-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16-16.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16.5-17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
SUM	0	0	3	191	1905	6771	16025	13156	17118	19105	20042	19121	16099	10632	6026	0	2425	1133	0	193	0	52	149997

Haver and Nyhus (1986) proposed that $f_{Hs}(h)$ can be reasonably well modeled by a log normal distribution for $Hs \le \eta$ and by a Weibull distribution for $Hs > \eta$, i.e.:

$$f_{Hs}(h) = \begin{cases} \frac{1}{\sqrt{2\pi}\alpha h} \exp(-0.5(\ln h - \lambda)^2 / \alpha^2; h \le \eta \\ \beta \frac{h^{\beta-1}}{\rho^{\beta}} \exp(-(h/\rho)^{\beta}); h > \eta \end{cases} \quad (5.1)$$

λ and α^2 are the mean and variance of $\ln h$, respectively. In the present case, they are found to be:

$\lambda = 0.8136$, $\alpha^2 = 0.3315$

ρ and β are estimated in such way that the model is continuous at $h = \eta$, i.e.:

$$\begin{aligned} f_{Hs}^{\log-normal}(\eta) &= f_{Hs}^{Weibull}(\eta) \\ F_{Hs}^{\log-normal}(\eta) &= F_{Hs}^{Weibull}(\eta) \end{aligned} \quad (5.2)$$

Various values of η are selected, and for each of these values the goodness for the corresponding fit is assessed using Chi-square test, as shown in Table 5.2:

Table 5.2 Chi-square test for different η

H_s	Number from data	Expected number of observations					Chi-square				
		4.4	4.5	4.6	4.7	4.8	4.4	4.5	4.6	4.7	4.8
5-6	6769	6200.4	6159.9	6116.5	6070.2	6024.9	52.1	60.2	69.6	80.4	91.9
6-7	3518	3236.2	3241.0	3241.2	3236.7	3232.8	24.5	23.7	23.6	24.4	25.2
7-8	1893	1611.8	1631.5	1647.3	1658.9	1670.8	49.1	41.9	36.7	33.0	29.6
8-9	935	770.3	790.1	807.1	821.2	835.7	35.2	26.6	20.3	15.8	11.8
9-10	445	354.8	369.5	382.7	394.2	406.0	22.9	15.4	10.1	6.6	3.7
10-11	173	158.0	167.4	176.1	183.9	192.1	1.4	0.2	0.1	0.7	1.9
11-12	63	68.2	73.7	78.9	83.7	88.7	0.4	1.5	3.2	5.1	7.5
12-13	19	28.6	31.6	34.4	37.2	40.1	3.2	5.0	6.9	8.9	11.1
13-14	10	11.7	13.2	14.7	16.1	17.7	0.2	0.8	1.5	2.3	3.4
14-15	5	4.6	5.4	6.1	6.9	7.7	0.0	0.0	0.2	0.5	0.9
15-16	0	1.8	2.1	2.5	2.9	3.3	1.8	2.1	2.5	2.9	3.3
16-17	1	0.7	0.8	1.0	1.2	1.4	0.1	0.0	0.0	0.0	0.1
$\sum$							191.2	177.5	174.7	180.6	190.3

It is seen from the table above that a good fit is obtained when

$\eta = 4.6$, $\rho = 2.5327$, $\beta = 1.3408$

The distribution of significant wave height is shown in Figure 5.2:

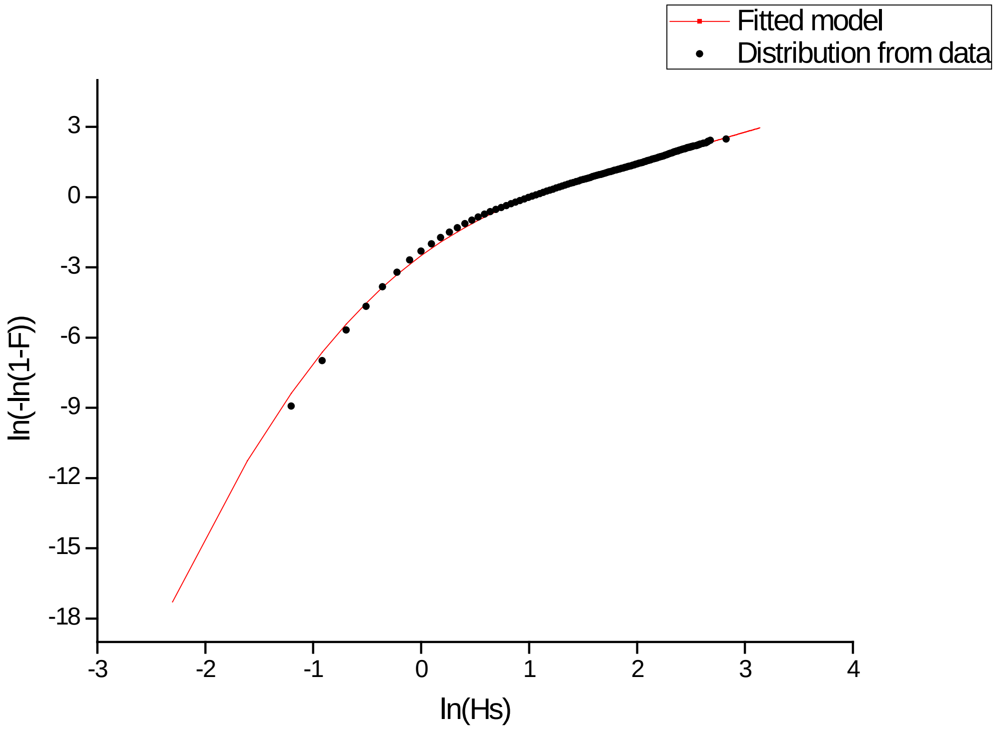

Figure 5.2 Distribution of H_s and fitted model

The significant wave height corresponding to return period of T years can be found that:

$$1 - F_{Hs}(h) = \frac{1}{T \cdot n_{3h}} \tag{5.3}$$

where $n_{3h} = 2920$ is the number of 3-hour stationary sea state per year.

➔

$$Hs^{100} = F_{Hs}^{-1}(1 - \frac{1}{292000}) = 16.74m$$

$$Hs^{10000} = F_{Hs}^{-1}(1 - \frac{1}{29200000}) = 21.13m$$

The conditional distribution of T_p given H_s is fitted by log normal distribution:

$$f_{Tp|Hs}(t|h) = \frac{1}{\sqrt{2\pi}\alpha h}\exp(-0.5(\ln h - \mu)^2/\phi^2 \qquad (5.4)$$

where $\mu = E(\ln Tp)$, $\phi^2 = Var(\ln Tp)$

μ and ϕ^2 are estimated from the sample of each class of significant wave height. Continuous functions are fitted to these samples in order to obtain estimates for the most extreme sea states:

$$\begin{aligned} \mu &= a_1 + a_2 H_s^{a_3} \\ \phi^2 &= b_1 + b_2 \exp(-b_3 H_s^{b_4}) \end{aligned} \qquad (5.5)$$

In the present case, a good fit using chi-square method is obtained as following:

$$\mu = 1.642 + 0.4617 Hs^{0.3597}$$

$$\phi^2 = 0.005 + 0.07373\exp(-0.08708 Hs^{1.658})$$

Note that b_1 in Equation (5.5) is set to be 0.005 artificially to ensure that ϕ^2 is always positive.

The fitted model is shown in Figure 5.3.

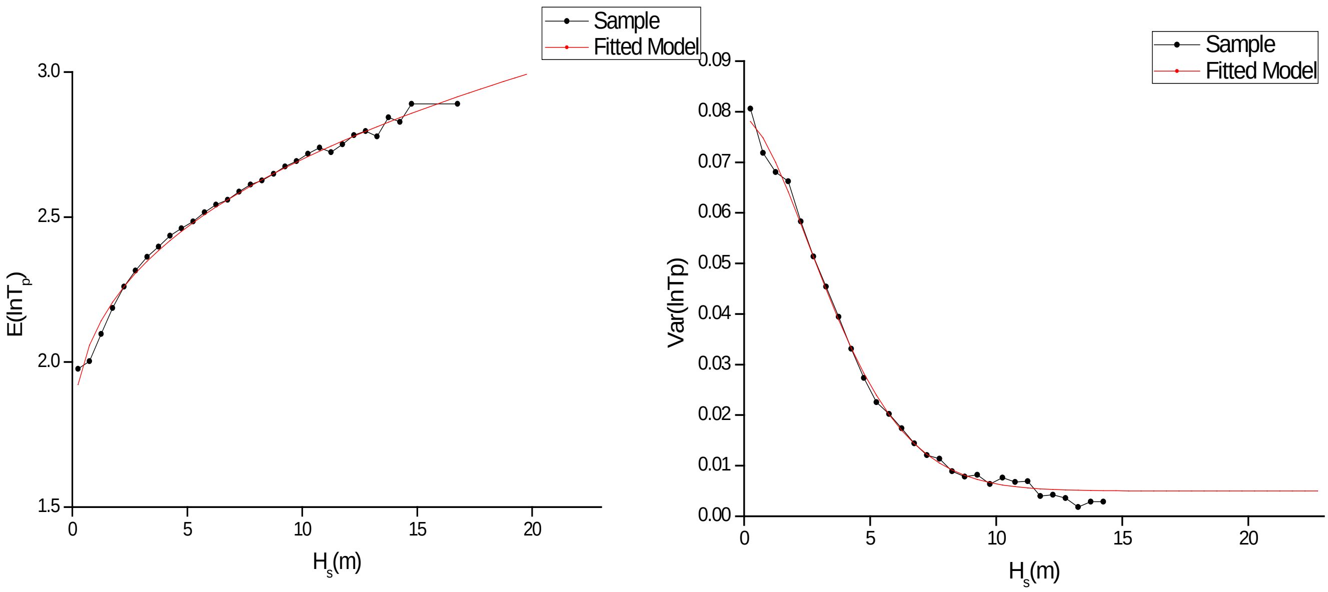

Table 5.3 Distribution of $E(\ln Tp)$ and $Var(\ln Tp)$ and fitted model

Mean value of Tp corresponding to given H_s can then be calculated as Moan and Hever (2005) suggested:

$$E(Tp) = (\mu + 0.5\phi^2) \quad (5.6)$$

The sea state characteristics corresponding to return period of 100 years and 10000 years are listed in Table 5.3.

Table 5.3 Estimation of 100-year and 10000-year Sea State Characteristics

Return Period (years)	Extreme sea states		
	$H_s(m)$	$T_p(s)$	90% range of T_p
100	16.74	18.5	16.4-20.7
10000	21.13	20.7	18.3-23.1

5.3 Extreme responses analysis

Distribution of global response maxima

To estimate the long term response extreme, we may establish the long term distribution of global response maxima:

$$F_{X_\Gamma}(x_q) = \frac{1}{\bar{\nu}^+_{\Gamma,0}} \int_h \int_t \nu^+_{\Gamma,0}(h,t) F_{X_\Gamma|H_s T_p}(x_q|h,t) f_{H_s T_p}(h,t) dt dh \quad (5.7)$$

where $\bar{\nu}^+_{\Gamma,0}$ is the mean zero-up-crossing frequency:

$$\bar{\nu}^+_{\Gamma,0} = \int_h \int_t \nu^+_{\Gamma,0}(h,t) f_{H_s T_p}(h,t) dt dh \quad (5.8)$$

The characteristic response is taken to be the value which will be exceeded only one zero-up-crossing during T years:

$$1 - F_{X_\Gamma}(x_q) = \frac{1}{n_T} \quad (5.9)$$

where $n_T = T \cdot 365 \cdot 24 \cdot 3600 \cdot \bar{\nu}^+_{\Gamma,0}$ is the number of zero-up-crossings in T years:

Distribution of 3-hour response maxima

Since we are mainly interested in predicting the response extreme, we may use a slightly different approach. The long term wave climate is assumed to be constant in 3 hours in the North Sea, so we may concentrate on the 3-hour largest response instead of all global response maxima. The distribution of largest response in a 3-hour sea state is given by Equation (3.10). Therefore, the long term distribution of 3-hour response maxima can be expressed as:

$$F_{X_{\Gamma,3h}}(x_q) = \int_h \int_t F_{X_{\Gamma,3h}|H_sT_p}(x_q|h,t) f_{H_sT_p}(h,t)dtdh \qquad (5.10)$$

We now divide significant wave height and spectrum peak period into classes. Denote the interval of each class of significant wave height in the table by Δh, the interval of each class of spectrum peak period by Δt. Denote the mid-value of significant wave height in class i by h_{si}, the mid-value of spectrum peak period in class j by t_{pj}, the probability density for sea state (h_{si}, t_{pj}) by $p(h_{si}, t_{pj})$, Equation (5.10) can be rewritten as:

$$F_{X_{\Gamma,3h}}(x) = \sum_i \sum_j F_{X_{\Gamma,3h}|H_sT_p}(x|h_{si}, t_{pj}) p(h_{si}, t_{pj}) \qquad (5.11)$$

This way the joint distribution of wave characteristics can be expressed in a joint frequency table. It will of course be different from Table 5.1 since it accounts for non-observed sea states. This joint frequency table can be established as following.

$p(h_{si}, t_{pj})$ can be written as:

$$p(h_{si}, t_{pj}) = p(h_{si}) p(t_{pj}|h_{si}) \qquad (5.12)$$

$p(h_{si})$ and $p(t_{pj}|h_{si})$ can be obtained from the joint distribution model of H_s and T_p in Equation (5.1) and (5.3):

$$p(t_{pj}|h_{si}) = \begin{cases} F_{Tp|Hs}(t_{pj} + \frac{\Delta h}{2}|h_{si}) - F_{Tp|Hs}(t_{pj} - \frac{\Delta h}{2}|h_{si}), \ j > 1 \\ F_{Tp|Hs}(t_{p1}|h_{si}), \ j = 1 \end{cases} \qquad (5.13)$$

$$p(h_{si}) = \begin{cases} F_{H_s}(h_{si} + \frac{\Delta h}{2}) - F_{H_s}(h_{si} - \frac{\Delta h}{2}), i > 1 \\ F_{H_s}(h_{s1}), i = 1 \end{cases} \qquad (5.14)$$

In the present case we adopt an interval of 0.5m for H_s and 1s for T_p. The joint frequency table is shown in Table 5.4.

Table 5.4 Join frequency table

Hs\f(Tp)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5
0.25	1E-14	2E-08	7E-06	0.0001	0.0005	0.0008	0.0009	0.0008	0.0006	0.0003	0.0002	1E-04	5E-05
0.75	2E-15	2E-08	2E-05	0.0005	0.0032	0.0085	0.0131	0.0141	0.0122	0.0089	0.0059	0.0036	0.002
1.25	4E-17	3E-09	6E-06	0.0003	0.0032	0.0113	0.0219	0.0284	0.0282	0.0233	0.0169	0.0111	0.0068
1.75	0	2E-10	1E-06	0.0001	0.0015	0.0074	0.018	0.0278	0.0316	0.0289	0.0227	0.0159	0.0102
2.25	0	5E-12	1E-07	2E-05	0.0005	0.0034	0.0107	0.0201	0.0262	0.0266	0.0225	0.0167	0.0111
2.75	0	6E-14	6E-09	3E-06	0.0001	0.0012	0.0053	0.0122	0.0186	0.0212	0.0195	0.0152	0.0105
3.25	0	3E-16	2E-10	2E-07	2E-05	0.0004	0.0022	0.0065	0.0119	0.0155	0.0156	0.0129	0.0093
3.75	0	0	2E-12	1E-08	3E-06	8E-05	0.0007	0.003	0.0069	0.0105	0.0118	0.0105	0.0079
4.25	0	0	1E-14	3E-10	2E-07	1E-05	0.0002	0.0012	0.0036	0.0067	0.0086	0.0084	0.0066
4.75	0	0	1E-17	4E-12	9E-09	1E-06	4E-05	0.0004	0.0017	0.004	0.006	0.0066	0.0055
5.25	0	0	0	2E-14	2E-10	1E-07	6E-06	0.0001	0.0007	0.0022	0.0041	0.0051	0.0047
5.75	0	0	0	2E-17	2E-12	4E-09	6E-07	2E-05	0.0002	0.001	0.0025	0.0038	0.0038
6.25	0	0	0	0	8E-15	7E-11	4E-08	3E-06	6E-05	0.0004	0.0014	0.0026	0.003
6.75	0	0	0	0	1E-17	7E-13	1E-09	3E-07	1E-05	0.0001	0.0007	0.0016	0.0023
7.25	0	0	0	0	0	2E-15	3E-11	2E-08	1E-06	3E-05	0.0003	0.0009	0.0016
7.75	0	0	0	0	0	4E-18	3E-13	6E-10	1E-07	7E-06	9E-05	0.0004	0.001
8.25	0	0	0	0	0	0	1E-15	1E-11	1E-08	1E-06	2E-05	0.0002	0.0006
8.75	0	0	0	0	0	0	5E-18	2E-13	5E-10	1E-07	5E-06	6E-05	0.0003
9.25	0	0	0	0	0	0	0	3E-15	2E-11	1E-08	1E-06	2E-05	0.0001
9.75	0	0	0	0	0	0	0	4E-17	8E-13	1E-09	2E-07	6E-06	5E-05
10.25	0	0	0	0	0	0	0	4E-19	3E-14	8E-11	3E-08	1E-06	2E-05
10.75	0	0	0	0	0	0	0	0	9E-16	6E-12	4E-09	3E-07	7E-06
11.25	0	0	0	0	0	0	0	0	3E-17	5E-13	5E-10	7E-08	2E-06
11.75	0	0	0	0	0	0	0	0	1E-18	4E-14	7E-11	1E-08	6E-07
12.25	0	0	0	0	0	0	0	0	7E-20	3E-15	9E-12	3E-09	2E-07
12.75	0	0	0	0	0	0	0	0	4E-21	3E-16	1E-12	6E-10	5E-08
13.25	0	0	0	0	0	0	0	0	0	3E-17	2E-13	1E-10	1E-08
13.75	0	0	0	0	0	0	0	0	0	3E-18	3E-14	2E-11	4E-09
14.25	0	0	0	0	0	0	0	0	0	3E-19	4E-15	5E-12	9E-10
14.75	0	0	0	0	0	0	0	0	0	3E-20	6E-16	9E-13	2E-10
15.25	0	0	0	0	0	0	0	0	0	3E-21	8E-17	2E-13	5E-11
15.75	0	0	0	0	0	0	0	0	0	5E-22	1E-17	3E-14	1E-11
16.25	0	0	0	0	0	0	0	0	0	0	2E-18	6E-15	3E-12
16.75	0	0	0	0	0	0	0	0	0	0	2E-19	1E-15	7E-13
17.25	0	0	0	0	0	0	0	0	0	0	4E-20	2E-16	2E-13
17.75	0	0	0	0	0	0	0	0	0	0	5E-21	4E-17	3E-14
18.25	0	0	0	0	0	0	0	0	0	0	7E-22	6E-18	7E-15
18.75	0	0	0	0	0	0	0	0	0	0	1E-22	1E-18	1E-15
19.25	0	0	0	0	0	0	0	0	0	0	2E-23	2E-19	3E-16
19.75	0	0	0	0	0	0	0	0	0	0	0	3E-20	6E-17
20.25	0	0	0	0	0	0	0	0	0	0	0	5E-21	1E-17
20.75	0	0	0	0	0	0	0	0	0	0	0	8E-22	2E-18
21.25	0	0	0	0	0	0	0	0	0	0	0	1E-22	5E-19
21.75	0	0	0	0	0	0	0	0	0	0	0	2E-23	9E-20
22.25	0	0	0	0	0	0	0	0	0	0	0	3E-24	2E-20
22.75	0	0	0	0	0	0	0	0	0	0	0	5E-25	3E-21
23.25	0	0	0	0	0	0	0	0	0	0	0	0	6E-22
23.75	0	0	0	0	0	0	0	0	0	0	0	0	1E-22
24.25	0	0	0	0	0	0	0	0	0	0	0	0	2E-23
24.75	0	0	0	0	0	0	0	0	0	0	0	0	8E-24

Hs\f(Tp)	13.5	14.5	15.5	16.5	17.5	18.5	19.5	20.5	21.5	22.5	23.5	24.5
0.25	2E-05	1E-05	6E-06	3E-06	1E-06	6E-07	3E-07	1E-07	7E-08	3E-08	2E-08	8E-09
0.75	0.0011	0.0006	0.0003	0.0002	8E-05	4E-05	2E-05	1E-05	6E-06	3E-06	1E-06	8E-07
1.25	0.004	0.0022	0.0012	0.0007	0.0003	0.0002	9E-05	5E-05	3E-05	1E-05	7E-06	3E-06
1.75	0.0062	0.0036	0.002	0.0011	0.0006	0.0003	0.0002	8E-05	4E-05	2E-05	1E-05	5E-06
2.25	0.0069	0.004	0.0023	0.0012	0.0006	0.0003	0.0002	9E-05	4E-05	2E-05	1E-05	5E-06
2.75	0.0066	0.0039	0.0022	0.0012	0.0006	0.0003	0.0001	7E-05	3E-05	2E-05	8E-06	4E-06
3.25	0.0059	0.0035	0.0019	0.001	0.0005	0.0002	0.0001	5E-05	2E-05	1E-05	5E-06	2E-06
3.75	0.0051	0.003	0.0016	0.0008	0.0004	0.0002	8E-05	4E-05	1E-05	6E-06	3E-06	1E-06
4.25	0.0044	0.0026	0.0014	0.0007	0.0003	0.0001	5E-05	2E-05	8E-06	3E-06	1E-06	4E-07
4.75	0.0038	0.0022	0.0011	0.0005	0.0002	9E-05	3E-05	1E-05	4E-06	2E-06	5E-07	2E-07
5.25	0.0033	0.002	0.001	0.0004	0.0002	6E-05	2E-05	7E-06	2E-06	7E-07	2E-07	5E-08
5.75	0.0029	0.0017	0.0008	0.0004	0.0001	4E-05	1E-05	4E-06	1E-06	3E-07	7E-08	2E-08
6.25	0.0025	0.0015	0.0007	0.0003	1E-04	3E-05	8E-06	2E-06	4E-07	1E-07	2E-08	4E-09
6.75	0.0021	0.0013	0.0006	0.0002	7E-05	2E-05	5E-06	1E-06	2E-07	3E-08	6E-09	9E-10
7.25	0.0016	0.0011	0.0005	0.0002	6E-05	1E-05	3E-06	5E-07	8E-08	1E-08	2E-09	2E-10
7.75	0.0012	0.0009	0.0005	0.0002	5E-05	1E-05	2E-06	3E-07	4E-08	4E-09	5E-10	5E-11
8.25	0.0009	0.0008	0.0004	0.0002	4E-05	8E-06	1E-06	2E-07	2E-08	2E-09	2E-10	1E-11
8.75	0.0006	0.0006	0.0004	0.0001	3E-05	7E-06	1E-06	1E-07	1E-08	9E-10	6E-11	4E-12
9.25	0.0004	0.0004	0.0003	0.0001	3E-05	6E-06	8E-07	9E-08	7E-09	5E-10	3E-11	2E-12
9.75	0.0002	0.0003	0.0002	0.0001	3E-05	6E-06	8E-07	8E-08	6E-09	4E-10	2E-11	9E-13
10.25	1E-04	0.0002	0.0002	1E-04	3E-05	6E-06	8E-07	8E-08	6E-09	3E-10	2E-11	7E-13
10.75	5E-05	0.0001	0.0001	8E-05	3E-05	6E-06	8E-07	8E-08	6E-09	4E-10	2E-11	7E-13
11.25	2E-05	6E-05	9E-05	7E-05	3E-05	6E-06	9E-07	1E-07	8E-09	5E-10	2E-11	9E-13
11.75	8E-06	3E-05	6E-05	5E-05	2E-05	6E-06	1E-06	1E-07	1E-08	7E-10	3E-11	1E-12
12.25	3E-06	2E-05	3E-05	4E-05	2E-05	6E-06	1E-06	2E-07	1E-08	1E-09	5E-11	2E-12
12.75	1E-06	7E-06	2E-05	2E-05	2E-05	6E-06	1E-06	2E-07	2E-08	1E-09	9E-11	4E-12
13.25	4E-07	3E-06	1E-05	2E-05	1E-05	5E-06	1E-06	2E-07	3E-08	2E-09	1E-10	7E-12
13.75	1E-07	1E-06	5E-06	9E-06	8E-06	4E-06	1E-06	2E-07	3E-08	3E-09	2E-10	1E-11
14.25	4E-08	5E-07	3E-06	5E-06	6E-06	3E-06	1E-06	3E-07	4E-08	4E-09	3E-10	2E-11
14.75	1E-08	2E-07	1E-06	3E-06	4E-06	3E-06	1E-06	3E-07	4E-08	5E-09	5E-10	4E-11
15.25	4E-09	7E-08	5E-07	2E-06	2E-06	2E-06	8E-07	2E-07	5E-08	7E-09	7E-10	6E-11
15.75	1E-09	3E-08	2E-07	8E-07	1E-06	1E-06	7E-07	2E-07	5E-08	8E-09	9E-10	8E-11
16.25	3E-10	9E-09	9E-08	4E-07	8E-07	8E-07	5E-07	2E-07	5E-08	9E-09	1E-09	1E-10
16.75	9E-11	3E-09	4E-08	2E-07	4E-07	5E-07	4E-07	2E-07	5E-08	9E-09	1E-09	2E-10
17.25	2E-11	1E-09	1E-08	8E-08	2E-07	3E-07	3E-07	1E-07	4E-08	9E-09	2E-09	2E-10
17.75	6E-12	3E-10	5E-09	3E-08	1E-07	2E-07	2E-07	9E-08	4E-08	9E-09	2E-09	2E-10
18.25	2E-12	9E-11	2E-09	1E-08	5E-08	1E-07	1E-07	7E-08	3E-08	8E-09	2E-09	3E-10
18.75	4E-13	3E-11	7E-10	6E-09	3E-08	5E-08	7E-08	5E-08	2E-08	7E-09	2E-09	3E-10
19.25	1E-13	8E-12	2E-10	2E-09	1E-08	3E-08	4E-08	3E-08	2E-08	6E-09	2E-09	3E-10
19.75	2E-14	2E-12	8E-11	9E-10	5E-09	1E-08	2E-08	2E-08	1E-08	5E-09	1E-09	3E-10
20.25	6E-15	7E-13	2E-11	3E-10	2E-09	7E-09	1E-08	1E-08	8E-09	4E-09	1E-09	3E-10
20.75	1E-15	2E-13	8E-12	1E-10	9E-10	3E-09	7E-09	8E-09	6E-09	3E-09	1E-09	3E-10
21.25	3E-16	5E-14	2E-12	5E-11	4E-10	2E-09	3E-09	4E-09	4E-09	2E-09	8E-10	2E-10
21.75	7E-17	1E-14	7E-13	2E-11	2E-10	7E-10	2E-09	3E-09	2E-09	1E-09	6E-10	2E-10
22.25	2E-17	3E-15	2E-13	5E-12	6E-11	3E-10	8E-10	1E-09	1E-09	1E-09	5E-10	2E-10
22.75	3E-18	9E-16	7E-14	2E-12	2E-11	1E-10	4E-10	7E-10	8E-10	6E-10	3E-10	1E-10
23.25	7E-19	2E-16	2E-14	6E-13	8E-12	5E-11	2E-10	4E-10	5E-10	4E-10	2E-10	9E-11
23.75	2E-19	5E-17	5E-15	2E-13	3E-12	2E-11	9E-11	2E-10	3E-10	2E-10	2E-10	7E-11
24.25	3E-20	1E-17	1E-15	6E-14	1E-12	9E-12	4E-11	9E-11	1E-10	1E-10	1E-10	5E-11
24.75	2E-20	7E-18	9E-16	4E-14	8E-13	8E-12	4E-11	1E-10	2E-10	2E-10	1E-10	8E-11

Hs\f(Tp)	25.5	26.5	27.5	28.5	29.5	30.5	31.5	32.5	33.5	34.5
0.25	4E-09	2E-09	1E-09	5E-10	2E-10	1E-10	7E-11	3E-11	2E-11	2E-11
0.75	4E-07	2E-07	1E-07	6E-08	3E-08	2E-08	8E-09	5E-09	2E-09	3E-09
1.25	2E-06	9E-07	5E-07	3E-07	1E-07	7E-08	4E-08	2E-08	1E-08	1E-08
1.75	3E-06	1E-06	7E-07	4E-07	2E-07	1E-07	5E-08	3E-08	1E-08	2E-08
2.25	3E-06	1E-06	6E-07	3E-07	2E-07	8E-08	4E-08	2E-08	1E-08	1E-08
2.75	2E-06	8E-07	4E-07	2E-07	9E-08	4E-08	2E-08	9E-09	5E-09	4E-09
3.25	1E-06	4E-07	2E-07	8E-08	4E-08	2E-08	7E-09	3E-09	1E-09	1E-09
3.75	4E-07	2E-07	7E-08	3E-08	1E-08	5E-09	2E-09	8E-10	3E-10	2E-10
4.25	2E-07	6E-08	2E-08	8E-09	3E-09	1E-09	4E-10	1E-10	5E-11	3E-11
4.75	5E-08	2E-08	6E-09	2E-09	6E-10	2E-10	5E-11	2E-11	5E-12	3E-12
5.25	2E-08	4E-09	1E-09	3E-10	8E-11	2E-11	6E-12	2E-12	4E-13	2E-13
5.75	4E-09	9E-10	2E-10	4E-11	1E-11	2E-12	5E-13	1E-13	2E-14	6E-15
6.25	8E-10	1E-10	3E-11	5E-12	9E-13	2E-13	3E-14	5E-15	8E-16	2E-16
6.75	1E-10	2E-11	3E-12	4E-13	6E-14	8E-15	1E-15	1E-16	3E-17	0
7.25	3E-11	3E-12	3E-13	4E-14	4E-15	4E-16	4E-17	0	0	0
7.75	5E-12	4E-13	3E-14	3E-15	2E-16	2E-17	0	0	0	0
8.25	9E-13	6E-14	4E-15	3E-16	2E-17	0	0	0	0	0
8.75	2E-13	1E-14	6E-16	3E-17	0	0	0	0	0	0
9.25	8E-14	3E-15	1E-16	6E-18	0	0	0	0	0	0
9.75	4E-14	1E-15	5E-17	2E-18	0	0	0	0	0	0
10.25	3E-14	9E-16	3E-17	0	0	0	0	0	0	0
10.75	3E-14	8E-16	2E-17	0	0	0	0	0	0	0
11.25	3E-14	9E-16	2E-17	6E-19	0	0	0	0	0	0
11.75	5E-14	1E-15	4E-17	9E-19	0	0	0	0	0	0
12.25	8E-14	3E-15	7E-17	2E-18	0	0	0	0	0	0
12.75	2E-13	5E-15	1E-16	4E-18	0	0	0	0	0	0
13.25	3E-13	1E-14	3E-16	9E-18	2E-19	0	0	0	0	0
13.75	6E-13	2E-14	7E-16	2E-17	6E-19	0	0	0	0	0
14.25	1E-12	5E-14	2E-15	5E-17	2E-18	4E-20	0	0	0	0
14.75	2E-12	1E-13	4E-15	1E-16	4E-18	1E-19	0	0	0	0
15.25	4E-12	2E-13	8E-15	3E-16	1E-17	3E-19	0	0	0	0
15.75	6E-12	3E-13	2E-14	7E-16	3E-17	8E-19	2E-20	0	0	0
16.25	9E-12	6E-13	3E-14	1E-15	6E-17	2E-18	7E-20	0	0	0
16.75	1E-11	1E-12	6E-14	3E-15	1E-16	5E-18	2E-19	5E-21	0	0
17.25	2E-11	2E-12	1E-13	6E-15	3E-16	1E-17	4E-19	2E-20	0	0
17.75	3E-11	2E-12	2E-13	1E-14	5E-16	2E-17	1E-18	4E-20	1E-21	0
18.25	3E-11	3E-12	2E-13	2E-14	1E-15	5E-17	2E-18	9E-20	3E-21	0
18.75	4E-11	4E-12	4E-13	3E-14	2E-15	9E-17	4E-18	2E-19	8E-21	0
19.25	4E-11	5E-12	5E-13	4E-14	3E-15	2E-16	9E-18	4E-19	2E-20	7E-22
19.75	5E-11	6E-12	7E-13	6E-14	4E-15	3E-16	2E-17	8E-19	4E-20	2E-21
20.25	5E-11	7E-12	8E-13	8E-14	6E-15	5E-16	3E-17	2E-18	8E-20	4E-21
20.75	5E-11	8E-12	1E-12	1E-13	9E-15	7E-16	5E-17	3E-18	2E-19	8E-21
21.25	5E-11	8E-12	1E-12	1E-13	1E-14	1E-15	7E-17	5E-18	3E-19	2E-20
21.75	4E-11	8E-12	1E-12	2E-13	2E-14	1E-15	1E-16	8E-18	5E-19	3E-20
22.25	4E-11	8E-12	1E-12	2E-13	2E-14	2E-15	2E-16	1E-17	8E-19	5E-20
22.75	3E-11	8E-12	1E-12	2E-13	2E-14	2E-15	2E-16	2E-17	1E-18	9E-20
23.25	3E-11	7E-12	1E-12	2E-13	3E-14	3E-15	3E-16	3E-17	2E-18	1E-19
23.75	2E-11	6E-12	1E-12	2E-13	3E-14	4E-15	4E-16	3E-17	3E-18	2E-19
24.25	2E-11	5E-12	1E-12	2E-13	3E-14	4E-15	5E-16	5E-17	4E-18	3E-19
24.75	3E-11	1E-11	2E-12	5E-13	8E-14	1E-14	1E-15	1E-16	1E-17	1E-18

Note that it is impossible and unnecessary to include unlimited number of H_s and T_p.

Therefore we artificially set $F_{H_s}(h_{s50})=1$ and $F_{T_p|H_s}(t_{j35}\,|h)=1$, meaning that there are no sea states with $H_s > 25m$ or $T_p > 35s$.

Since we are dealing a rather simple response problem where a transfer function is available, the short term response can be solved in the frequency domain, as discussed in Section 3.2.1. For each sea state, $F_{X_{\Gamma,3h}|H_sT_p}(x|h_{si},t_{pj})$ can be solved by:

$$F_{X_{3h}|H_sT_p}(x|h_{si},t_{pj}) = \exp\left\{-\exp\left\{-\left[\frac{x-\gamma(i,j)}{\beta(i,j)}\right]\right\}\right\} \tag{5.15}$$

where $\gamma(i,j)$ and $\beta(i,j)$ are obtained as Equation (3.11) suggested.

The long term distribution of responses extreme can then be expressed as:

$$F_{X_{\Gamma,3h}}(x) = \sum_i\sum_j \exp\left\{-\exp\left\{-\left[\frac{x-\gamma(i,j)}{\beta(i,j)}\right]\right\}\right\} p(h_{si},t_{pj}) \tag{5.16}$$

The response extreme corresponding to an annual exceedance probability of 1/100 and 1/10000 are determined by:

$$F_{X_{\Gamma,3h}}(x) = 1-\frac{q}{n_{3h}} \tag{5.17}$$

It can be easily obtained that the 100-year and 10000-year response extreme are 9.87m and 16.11m, respectively.

It is interesting to notice that the 10000-year significant wave height is about 30% larger than the 100-year significant wave height. Since we are dealing with a linear response problem, generally the magnification from 100-year response extreme to 10000-year extreme should also be around 1.3. However, in our case the magnification for the response is approximately 60%. This is because the period of estimated 10000-year sea state is close to the nature period of the structure, resulting in a significant increase in the response.

5.4 Uncertainties associated with all sea states approach

As stated before, we estimate the extreme response in a statistical sense. Therefore, the result will be associated with uncertainty, both aleatory uncertainty and epistemic uncertainty. As an illustration, the uncertainty associated with estimated 100-year significant wave height is investigated as following.

Aleatory Uncertainty

This kind of uncertainty is due to the random nature of 100-year significant wave height. Assuming the fitted model of H_s in Equation (5.1) is the true model, the distribution of largest H_s in 100 years reads:

$$F_{Hs^{100}}(h) = \begin{cases} (\phi(\frac{\ln h - 0.8136}{\sqrt{0.3315}}))^{292000}; h \le 4.6 \\ (1 - \exp(-(h/2.5327)^{1.3408}))^{292000}; h > 4.6 \end{cases}$$

The distribution function is shown in Figure 5.4, it is seen the 80% range for the variability of H_s is given by 16m-19m.

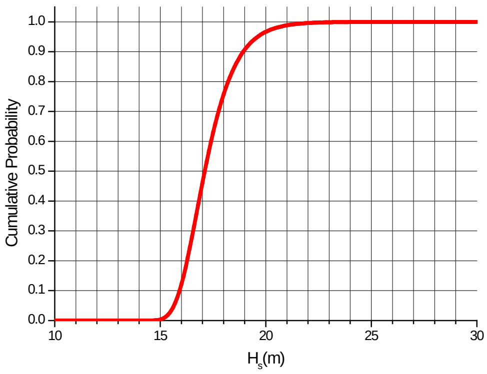

Table 5.4 Distribution of 100-year largest significant wave height

This kind of randomness cannot be removed since it is inherent to the problem.

Epistemic Uncertainty

Another source of uncertainty, i.e. epistemic uncertainty, is related to a limited amount of available data. This kind of uncertainty can be indicated by either classical bootstrapping, see e.g. Efron and Tibshirani (1994), or a parametric bootstrapping, see e.g. Haver and Bergsvik (2009). Here we will adopt the latter approach.

Assuming the model to fit the significant wave height is the true model, we can generate 10 series of significant wave height samples with size 10000 by Monte Carlo simulation. All these samples could as well have been our observations just as the one we did observe. A more proper size would be 149997, which is the exact number of observed sea states. However, a size as large as 149997 is unmanageable in the present study, and a size10000 is chosen as a compromise. One can still see the point with size of 10000.

The sample distributions are compared to the true distribution in Figure 5.5. It is seen that the sample distribution show an obvious variation relative to the true distribution. Since we are mainly interested in estimating the extreme significant wave height, the upper parts of sample distributions, i.e. the Weibull parts are shown in Figure 5.6.

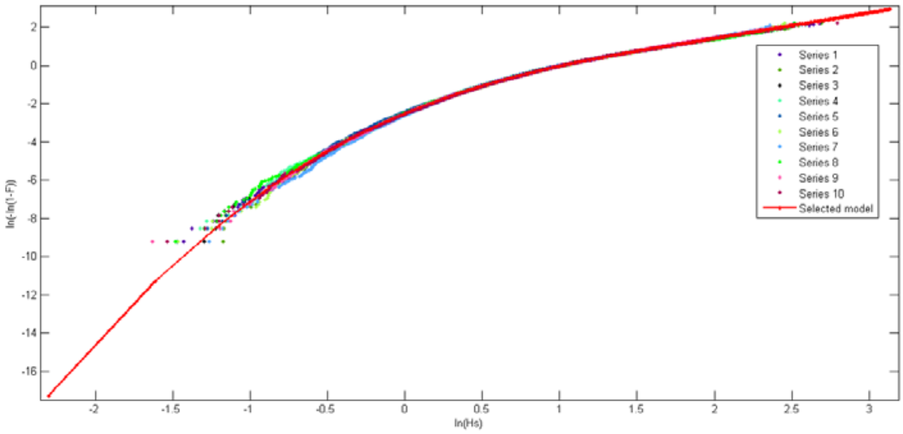

Figure 5.5 Distribution of simulated samples compared to underlying distribution

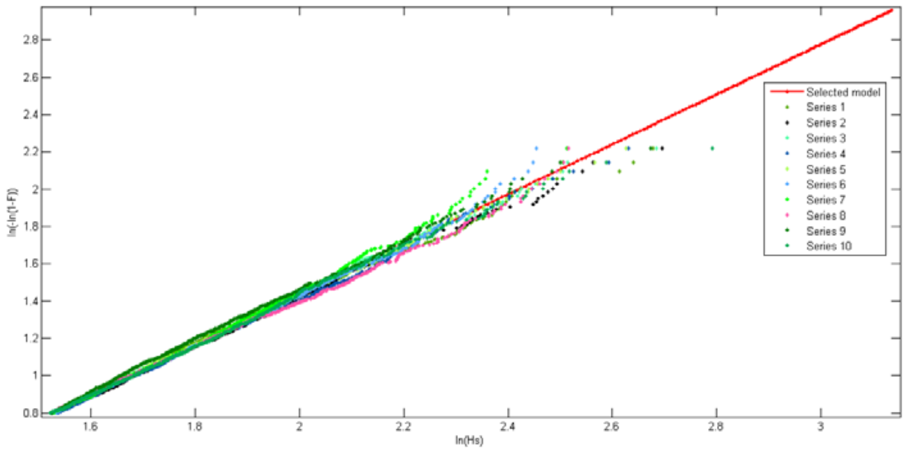

Figure 5.6 Upper tails of the sample distributions

We may use Weibull model to fit the 10 generated series of samples on the upper part, i.e. $H_s > 4.6$. Thereafter we will get 10 different estimations of 100-year significant wave height, as shown in Figure 5.7

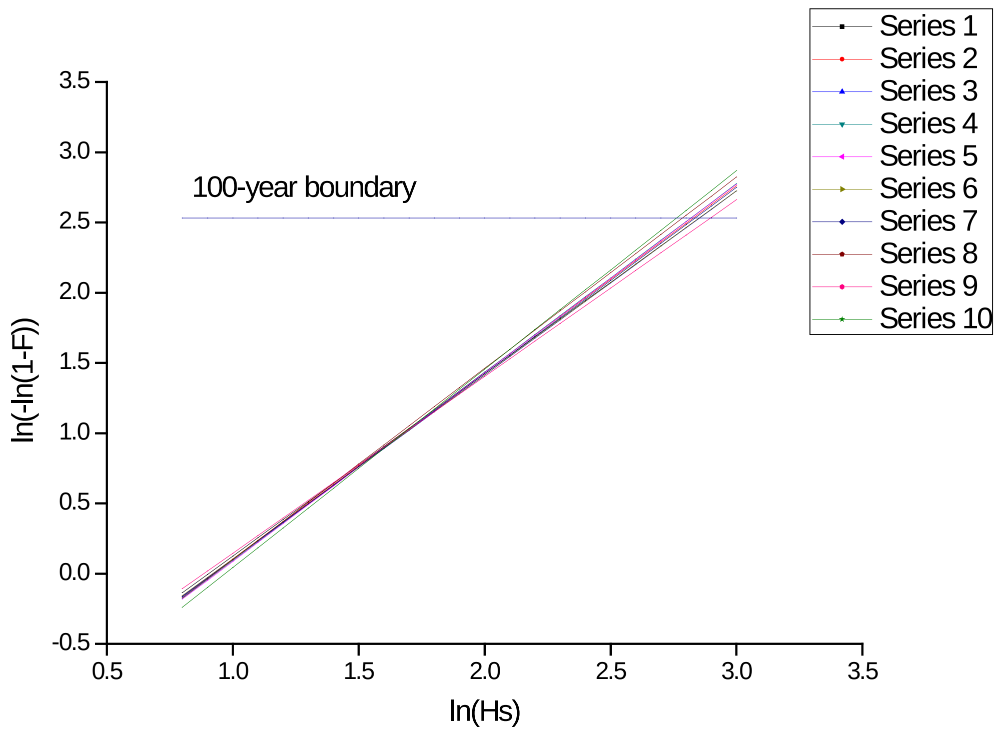

Figure 5.7 Fitting the simulated samples

It is seen that the 100-year significant wave height from the 10 sample series range from 15.8m to 18.1m. Gaussian model is used to fit the 10 estimations, see Figure 5.8.

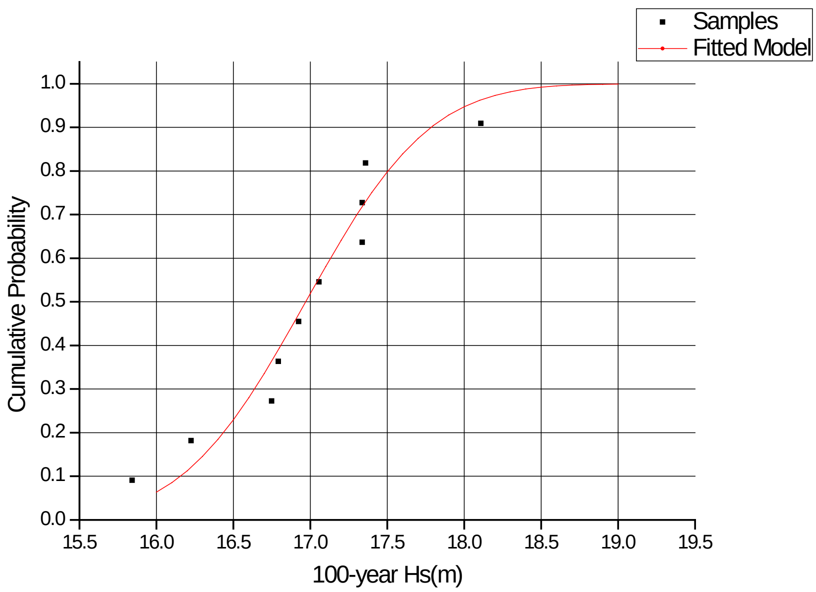

Figure 5.8 Distribution of estimation of 100-year H_s

Using moment method we obtain the distribution of 100-year largest significant wave height and we find that the 80% confidence band of 100-year largest significant wave height is 16.2m-17.8m. This is a little narrower than the range from aleatory uncertainty.

5.5 A conservative method

In all sea states approach, it is assumed that all sea states are independent. By neglecting the correlation between successive sea states, we will get a conservative result. But the extent of conservatism is not significant, and it will be on the safe side, so this is not a significant drawback in practice. There are several ways to avoid this kind of independence. One of these would be considering the annual extreme, i.e. we can consider the extreme significant wave height in one year as the independent event. Gumbel model is used to fit the distribution of annual largest significant wave height, see Figure 5.9.

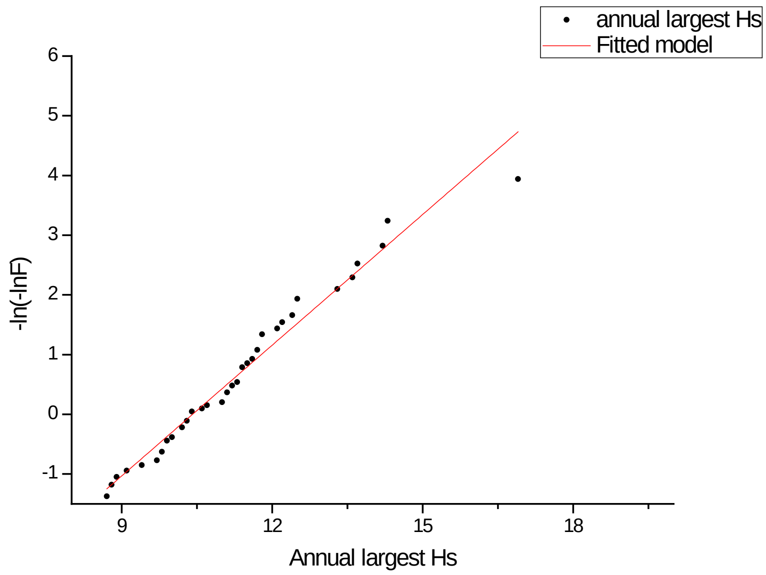

Figure 5.9 Distribution of annual largest significant wave height

Using moment method, the 100-year and 10000-year significant wave height are found to be 16.07m and 20.66m, respectively, which are lower than the values obtained when we consider all 3-hour response maxima.

Another method is the peak over threshold approach, which will be illustrated in the following chapter.

Chapter 6

Peak over threshold method

In cases where a lot of characteristics will affect the response, it will be time-consuming and difficult to establish a reliable joint distribution of these characteristics. In this circumstance, peak over threshold is an attractive method. The distribution of storm characteristics, e.g. the most probable storm response maxima can be established, and the response distribution given storm characteristics is estimated. A long term estimation of extreme response is then obtained by convoluting the distribution of short term response maxima given the storm characteristics with the distribution of the storm characteristics.

This method is more used is hurricane governed areas, e.g. Gulf of Mexico than in Norway. This is because the limited amount of available data in such areas. The hurricanes, which represent the design condition, are so rare that a very long time of observation is required to establish a reliable joint distribution model of environmental characteristics.

6.1 Environmental description

Establish Storm Data base

With peak over threshold method, we treat storm peaks as independent events. Therefore a reliable storm database is crucial for accurate estimation of responses extreme. This could be done conveniently using hindcast technique. In areas like Gulf of Mexico, where hurricanes dominate the design condition, one merely focused on generating reliable hindcast data for historical hurricanes where meteorological basis data are available. Detailed discussion on this issue is made by Bergsvik (2009), Cooper and Stear (2006). For the North Sea, however, a continuous hindcast data is available for the past 50 years. To establish the storm data base, we need to first identify all sea states whose significant wave heights are higher than a reasonable threshold, say 7m.Thereafter 712 storms are identified, in the past 51.25 years with available hindcast data, and each storm consist of several 3-hour stationary sea state.

Determine Storm Peaks Distribution

We define storm peak as the sea state in a storm that has the largest significant wave height. The distribution of H_s can be estimated by establishing the distribution of the storm peaks' significant wave heights from storm database. A 3-parameter Weibull distribution is a reasonable model for this distribution, see Haver and Bergsvik (2009):

$$F_{Hs}(h) = 1 - \exp\left\{ -\left(\frac{h - h_0}{\rho} \right)^{\beta} \right\} \tag{6.1}$$

where h_0 is the threshold.

In our case we have 712 storm peaks and by using moment method, fitted model for distribution of storm peak H_s is obtained:

$$F_{Hs_{peak}}(h) = 1 - \exp\left\{ -\left(\frac{h - 7}{1.38965} \right)^{1.06831} \right\} \tag{6.2}$$

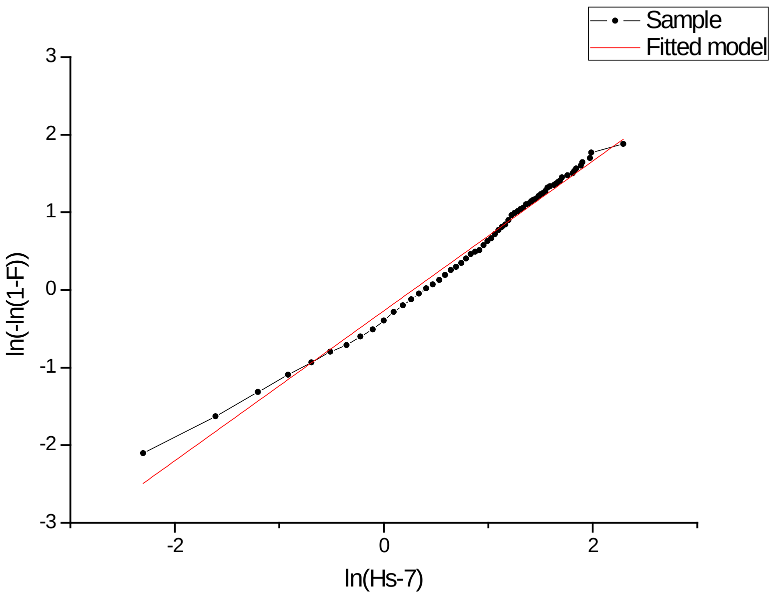

Figure 6.1 Distribution of storm peaks' significant wave heights

Significant wave height corresponding to return period of 100 years and 10000 years are found to be:

$$1 - F_{Hs_{peak}}(h) = \frac{1}{T \cdot n_{storm}}$$

where n_{storm} is the number of storms happen per year. Therefore, 100-year and 10000-year extreme significant wave heights are found to be:

$$Hs^{100} = F_{Hs}^{-1}\left(1 - \frac{1}{100 \times \frac{712}{51.25}}\right) = 15.86m$$

$$Hs^{10000} = F_{Hs}^{-1}\left(1 - \frac{1}{10000 \times \frac{712}{51.25}}\right) = 21.05m$$

Similar to all sea state approach, T_p of storm peaks also follow a log normal distribution and can be estimated for each class of storm peaks significant wave heights. Mean value and deviation of $\ln T_p$ can be fitted as following:

$$\begin{aligned} E(\ln Tp) &= a_1 + a_2 (h - h_0)^{a_3} \\ Var(\ln Tp) &= b_1 + b_2 \exp(-b_3 (h - h_0)) \end{aligned} \qquad (6.2)$$

A good fit using chi-square method for the present case can be found, as shown in Figure 6.2:

$$E(\ln Tp) = 2.52 + 0.08559 \times (h - 7)^{0.6458}$$

$$Var(\ln Tp) = 0.005 + 0.01737 \times \exp(-0.7821 \times (h - 7))$$

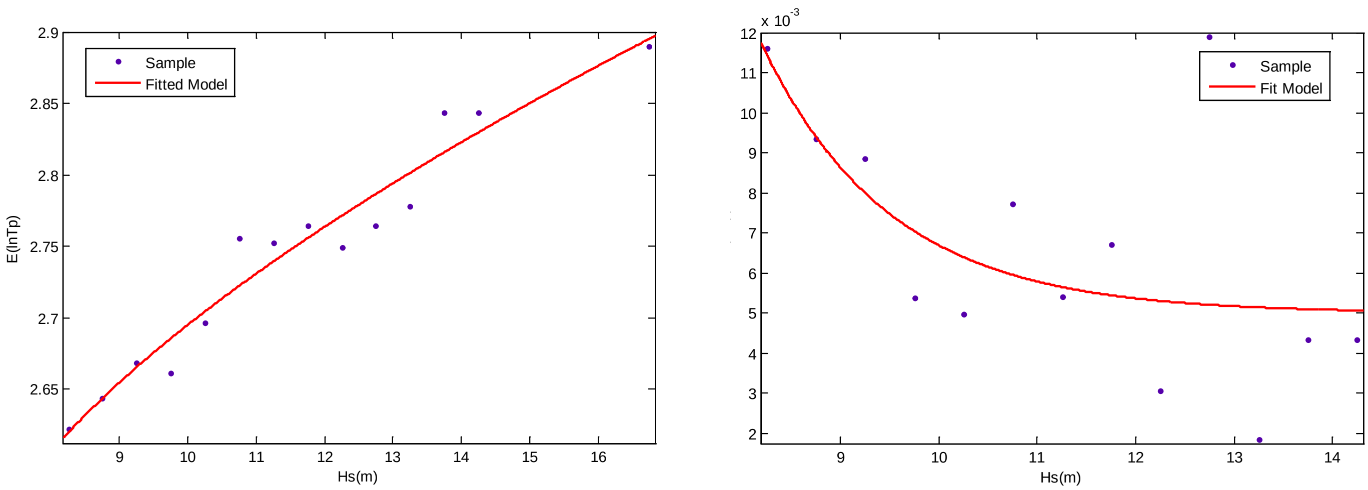

Figure 6.2 Distribution of $E(\ln Tp)$ and $Var(\ln Tp)$ and fitted model

Mean value and 90% confidence band of T_p for return period of 100 years and 10000 years are found to be 17.7s, 15.7s-19.9sand 20.0s, 17.7s-22.4s respectively.

A different threshold can also be applied. Estimation for sea states characteristics using storm database with different are also made. A summary is made in Table 6.1.

Table 6.1 Extreme sea state characteristics

threshold(m)	Return periods (years)	Extreme sea states		
		$H_s(m)$	$T_p(s)$	90% range of T_p
7	100	15.98	17.7	15.7-19.9
	10000	21.05	20.0	17.7-22.4
8	100	15.88	17.7	15.8-19.9
	10000	21.03	20.2	17.9-22.6
9	100	15.85	17.8	15.8-19.9
	10000	21.07	20.2	17.9-22.6

It is seen that reasonably stable results are obtained for different thresholds. Further discussion regarding the choice of threshold will be made in Section 6.2.

6.2 Extreme responses analysis

The analysis of extreme responses using peak over threshold method consists of a short term problem, i.e. the conditional distribution of storm response maxima given the most probable response maxima of this storm, and a long term problem, i.e. the long term distribution of the most probable responses maxima of a random storm.

Distribution of storm most probable response extreme

For a storm consisting of a number of stationary sea states, one should perform short term response analysis for each sea state. This is fairly easy in the present study since we are solving the response in frequency domain. Consequently we obtain a series of most probable responses, as shown in Figure 6.3. Denote the largest most probable 3-hour responses of storm number i by $\tilde{x}_i$.

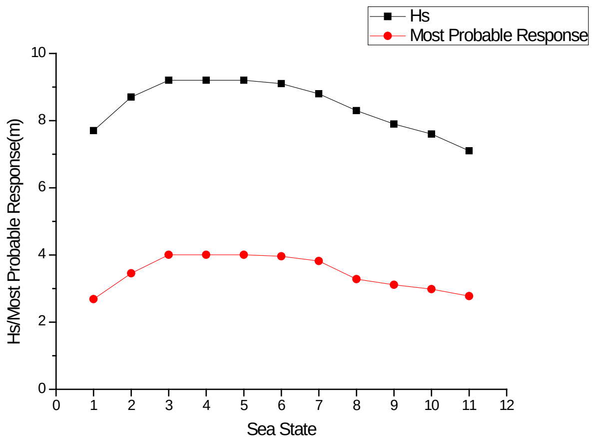

Figure 6.3 The significant wave heights and most probable responses in a storm

The distribution of $\tilde{x}$ from all storms can be reasonably well modeled by a 3-parameter Gumbel distribution:

$$F_{\tilde{X}}(\tilde{x}) = \exp\left\{-\exp\left(-\frac{\tilde{x}^{s} - \alpha}{\beta}\right)\right\} \tag{6.3}$$

A reasonable well fitted model for our case is found, as shown in Figure 6.4:

$$F_{\tilde{X}}(\tilde{x}) = \exp\left\{-\exp\left(-\frac{\tilde{x}^{0.7} - 1.9766}{0.3442}\right)\right\}$$

Here the s is chosen by judgement, to make the sample points approach a straight line on a Gumbel probability paper.

Conditional distribution of storm maximum response given storm's most probable maximum response

If we observe the structure's responses during a storm, we would find that actual response maxima scatter around the most probable largest response. To account for this randomness, we can generate a possible observation for each 3-hour stationary sea state for all storms by using Mont Carlo simulation. A random number $u_{i,j}$ between 0 and 1 is generated for the number j sea state in number i storm. Replace $F_{X_{\Gamma,d}|H_sT_p}(x|h,t)$ in Equation (3.10) with $u_{i,j}$ and solve the equation with respect to x. A realization of 3-hour responses maxima for this sea state can then be expressed as following:

$$x_{i,j} = \alpha_{i,j} - \beta_{i,j}\ln(-\ln(u_{i,j})) \tag{6.4}$$

Two set of simulations are shown in Figure 6.5 and Figure 6.6.

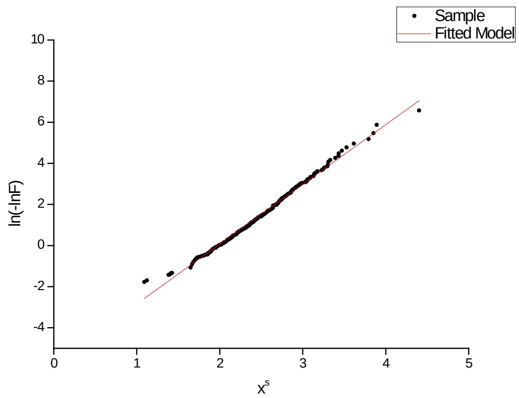

Figure 6.4 Distribution and fitted model of largest most probable response

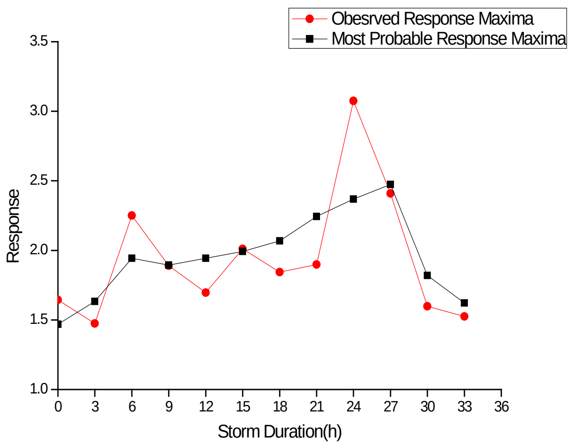

Figure 6.5 One set of simulation of responses in a storm

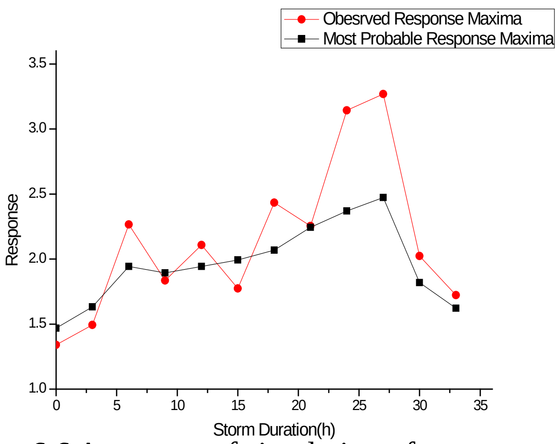

Figure 6.6 A new set of simulation of responses in a storm

Denote the largest observed 3-hour responses in storm number i by x_i. Tromans and Vanderschuren (1995) proposed a method to establish the conditional distribution of response extreme in a storm given the largest most probable response of that storm. Denote the ratio between x_i and $\tilde{x}_i$ by v_i, i.e. $v_i = \frac{x_i}{\tilde{x}_i}$. v is assumed to follow Gumbel distribution:

$$F_V(v) = \exp(-\exp(-\frac{v-\alpha_G}{\beta_G})) \tag{6.5}$$

Denote the mean value and standard deviation of v by $\bar{v}$ and s_V, respectively. Bury (1975) gives the Gumbel parameters using moment principle:

$$\hat{\beta}_G = 0.7797 s_V$$
$$\hat{\alpha}_G = \bar{v} - 0.57722\hat{\beta}_G \tag{6.6}$$

$\bar{v}$ and s_V are found to be 1.0799 and 0.094, respectively. The distribution of v is shown in Figure 6.7.

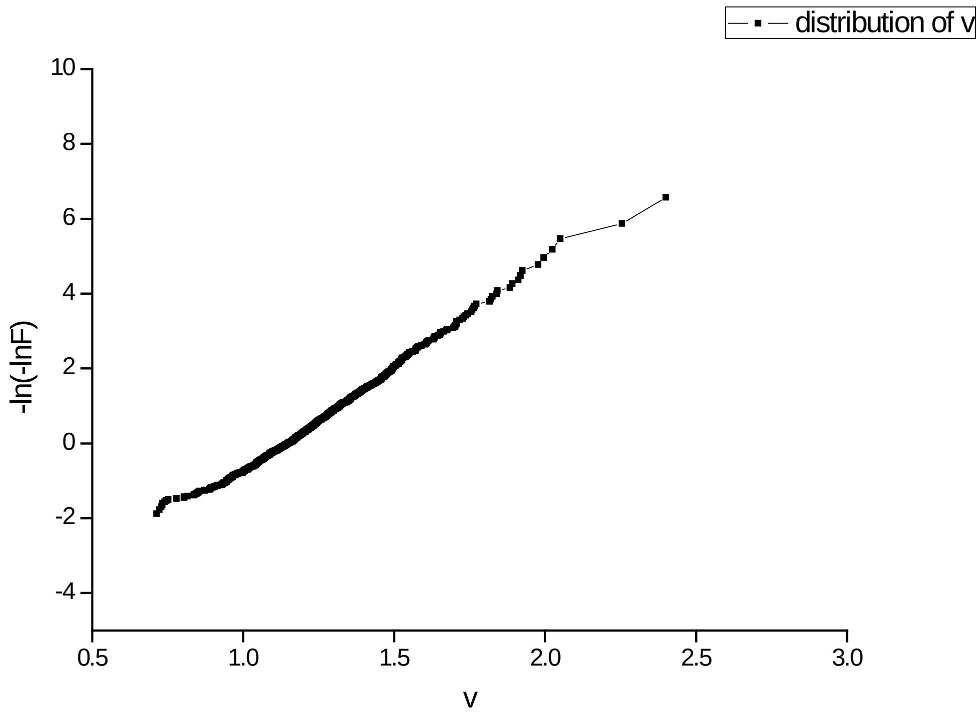

Figure 6.7 Distribution of v

By simple transformation

$$F_{X|\tilde{X}}(x|\tilde{x}) = P(X \le x | \tilde{X} = \tilde{x}) = P(V\tilde{x} \le x) = P(V \le x/\tilde{x}) = F_V(x/\tilde{x})$$

we get:

$$F_{X|\tilde{X}}(x|\tilde{x}) = \exp(-\exp(-\frac{x-\alpha_G \tilde{x}}{\beta_G \tilde{x}})) \tag{6.7}$$

Using Equation (6.6), the conditional distribution of largeste response in a storm given this storm's largest most probable response is found to be:

$$F_{x|\tilde{x}}(x|\tilde{x}) = \exp(-\exp(-\frac{x-1.037607\tilde{x}}{0.073295\tilde{x}})) \tag{6.8}$$

By convoluting conditional distribution of responses extreme given most probable responses extreme with the distribution of most probable responses extreme:

$$F_X(x) = \int_{\tilde{x}} \exp(-\exp(-\frac{x-1.037607\tilde{x}}{0.073295\tilde{x}}))\frac{0.7}{0.3442}\tilde{x}^{-0.3}\exp\left(-\frac{\tilde{x}^{0.7}-1.9766}{0.3442}\right)\exp\left\{-\exp\left(-\frac{\tilde{x}^{0.7}-1.9766}{0.3442}\right)\right\}d\tilde{x} \tag{6.9}$$

The characteristic response is found by:

$$1 - F_X(x) = \frac{q}{n_{storm}} \tag{6.10}$$

where $n_{storm} = \frac{712}{51.25}$ is the annual number of storms. The extreme response corresponding to return period of 100 years and 10000years are found to be 9.46m and 15.27m, respectively.

Similarly, results when adopting storm data base with different threshold can be obtained, as shown in Table 6.2.

Table 6.2 Estimated extreme responses using peak over threshold

Threshold(m)	Return periods (years)	Extreme Responses(m)
7	100	9.46
	10000	15.27
8	100	9.44
	10000	15.65
9	100	9.20
	10000	15.38

It can be seen that the estimates of extreme response using different threshold are reasonably stable. Generally, we should try to adopt a high threshold since we are mainly interested in fitting the upper tail part. But a problem with a high threshold is that we will have relatively few available data, resulting in a large extent of uncertainty in the estimates. A reasonable choice of threshold could be obtained as following:

i) A high threshold in the view of data is chosen and a model is used to fit the data. We shall obtain an estimate for the extreme storm response as discussed previously.

ii) We can generate a series of samples of the same size using Monte Carlo simulation. This series of samples could as well be our observation. A new estimate can be made based on the generated samples. By generating several, say 20, series of samples, we will be able to establish the distribution of the estimates and determine a certain confidence band of the estimates, say 90% confidence band.

iii) Adopt a lower threshold, and repeat step i) and ii). Generally, we will have a relatively stable result concerning the estimated of extreme value and a narrower 90% band as threshold decreases since we have more data. We shall keep lowering the threshold until the estimation of extreme response deviate from the originally stable value. This is when we enter another zone where the mechanism for the distribution changes.

iv) We will choose the threshold with least uncertainty, i.e. with the narrowest 90% confidence band, before the mechanism changes.

6.3 Uncertainties associated with peak over threshold approach

As for all sea state approach, there are uncertainties associated with peak over threshold method, both aleatory uncertainty and epistemic uncertainty.

Aleatory uncertainty

Assuming the model we used to fit the long term distribution of significant wave height is correct, i.e. $F_{Hs}(h) = 1 - \exp\left\{-\left(\frac{h-7}{1.38965}\right)^{1.06831}\right\}$. The largest significant wave height in 100 years will be the largest storm peaks among storms in 100 years. Expected number of storms in 100 years is $\frac{712 \times 100}{51.25} = 1389$. The formulation reads:

$$F_{Hs}(h) = \left\{1 - \exp\left\{-\left(\frac{h-7}{1.38965}\right)^{1.06831}\right\}\right\}^{1389}$$

The distribution of 100-year extreme is shown in Figure 6.8.

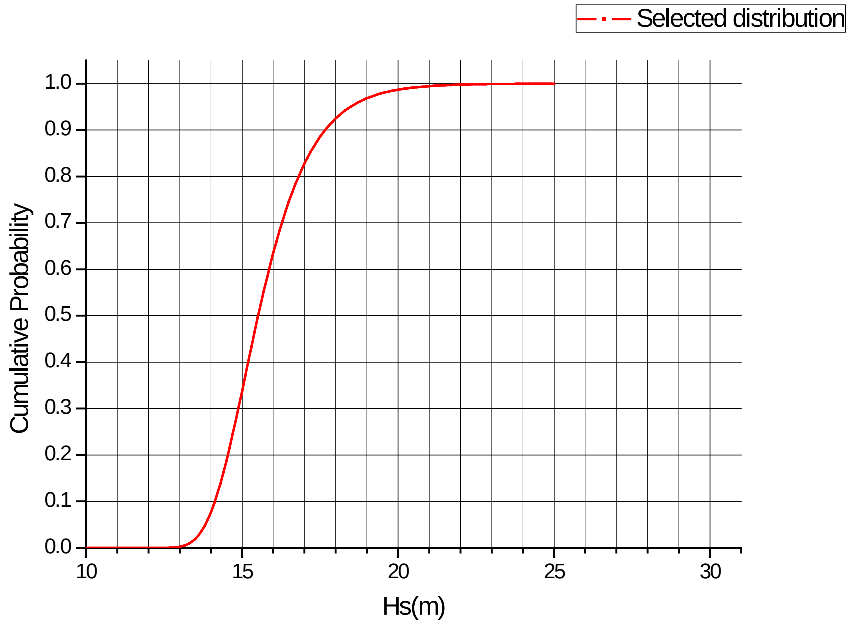

Figure 6.8 Distribution of 100-year significant wave height

It is seen that the 80% range for the 100-year largest significant wave height is given by 14.1m-17.6m.

Epistemic uncertainty

We still assume that the fitted model for significant wave height is true. We can generate other samples of storm peaks with size of 1389 using Monte Carlo simulation. See Figure 6.9. 10 samples of size 1389 are generated.

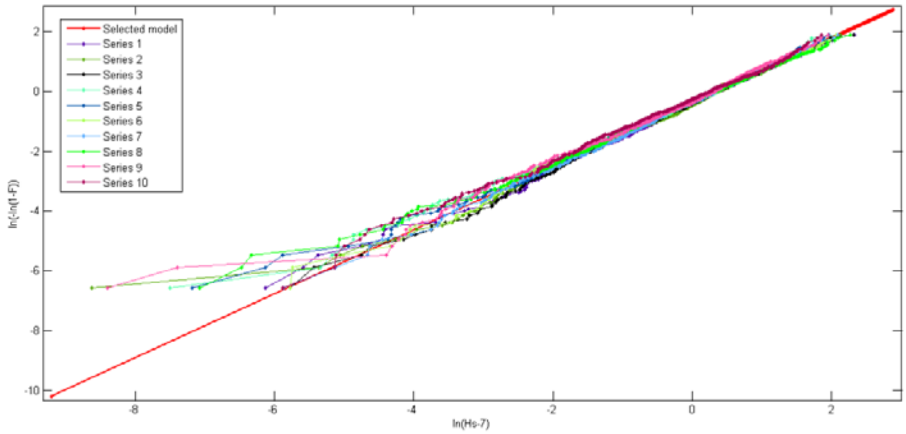

Figure 6.9 Distribution of simulated samples compared to underlying distribution

If we fit the generated samples, 10 corresponding 100-year extreme significant wave height estimates are obtained, see Figure 6.10. These estimates vary from 15.55m to 17.44m.

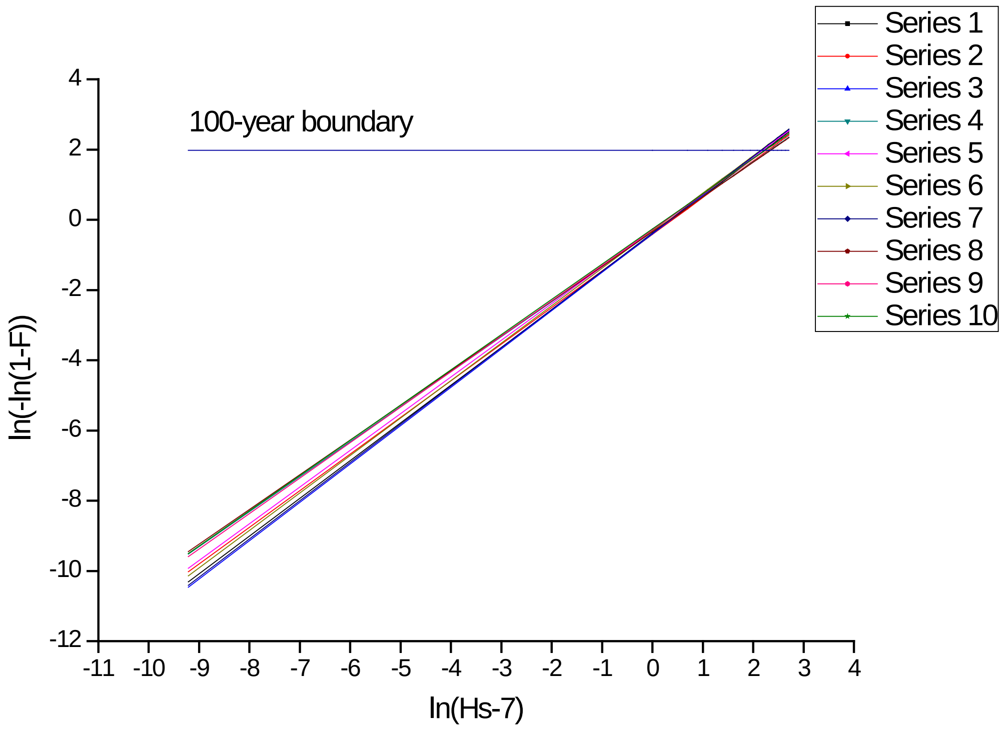

Figure 6.10 Fitting the generated samples

We can assume that these estimates follow Gaussian distribution. Using moment method, the distribution is found, as shown in Figure 6.11:

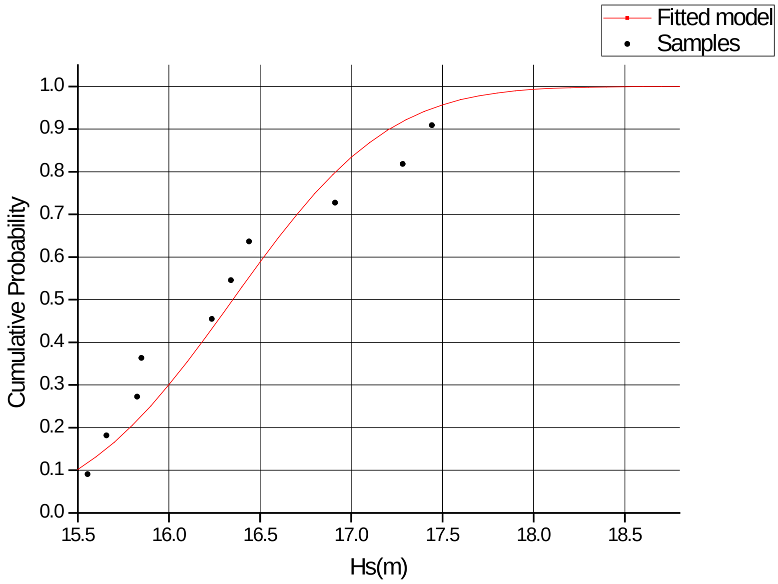

Figure 6.11 Distribution of estimates of 100-year H_s

The 80% band of extreme significant wave height reads 15.5m to 16.9m, which is narrower than the corresponding range due to aleatory uncertainty.

In addition to aleatory uncertainty and epistemic uncertainty, another source of randomness for peak over threshold approach is the variability of storm database.

So far we have been assuming that the annual number of storm is constant because we have only one storm database. But by utilizing hindcast technique, we are able to generate as many 50-year storm databases as we intend to. Haver and Bergsvik (2009) compare the storm numbers and 100-year extreme significant wave height obtained from 20 different storm databases. It was found that the 80% confidence band of 100-year significant wave height is slightly wider than that when we rule out storm database variability.

The variability in number of hurricanes per unit time can easily be accounted for as shown in Haring and Heideman (1978). Instead of focusing on the distribution of storm peak significant wave height, we may determine the distribution of the annual largest significant wave height. In this way, the effects of randomness in annual number of storms are accounted for.

6.4 Comparison of all sea states approach and peak over threshold method

All sea state approach and peak over threshold, as discussed above, are two efficient approaches to estimate responses extreme, but their target quantities are not the same. The all sea state approach estimates extreme response, x_q, that is to be exceeded for only one 3-hour sea state per 1/q years if we are interested in long term distribution of 3-hour response maxima, while the peak over threshold approach predicts the response that will be exceeded in only one storm in 1/q years.

It is interesting to compare the difference in severity between these two methods. From the results above it is observed that the difference is not too significant. Both the significant wave heights and extreme responses obtained from peak over threshold are slightly lower than the correspondent values using all sea states approach. The difference is only about 5%. Therefore too much emphasize should not be put on which method is better. The choice between these methods in designing offshore structures mainly depends on the aimed area. In North Sea for example, where continuous hindcast data are available, all sea state approach is the most frequently used method. On the contrary, in hurricane dominated areas like Gulf of Mexico, peak over threshold approach sometimes is the only available method due to the lack of available data. Peak over threshold method is also more convenient when the weather condition should include so many characteristics that a reliable joint distribution for these characteristics is difficult to establish.

Chapter 7

Environmental contour lines method

It is seen from the previous chapters that either all sea states approach or peak over threshold method is adopted, distributions for a lot of short term responses should be established in order to get a consistent estimation for the long term response extreme. This is no problem for simple response, e.g. response problems where a frequency domain solution is available. But the response mechanism is so complicated that time domain simulations or model tests have to be performed, it is no longer an easy job. This is when environmental contour lines method becomes attractive. This is an approximated method which makes estimating long term extreme response possible without carrying out a full long term analysis. We will illustrate this method in the following. Other discussions have been made by e.g. Winterstein (1993), Kleiven and Haver (2004).

7.1 Construction of contour lines

The contour lines method is used to predict loads and/or responses maxima with given annual exceedance probability without carrying out full long term analysis. Using contour lines method, the environmental and response analysis is decoupled. It means that we can establish contour lines of d-hour sea state using environmental characteristics and perform response analysis for a set of d-hour sea states on the contour lines. The design sea state is then the one along the contour lines that causes the largest response.

Initially, the contour lines are determined such that they follow curves of constant probability density, see Haver (1980). A more consistent approach is to determine the contour lines to correspond to a certain exceedance probability. With environmental contour lines method, the contour lines corresponding to constant exceedance probability can be established using the methods within the field of reliability analysis. Usually the aim of utilizing reliability analysis is to estimate the exceedance probability of a particular load or capacity. This can be efficiently and with sufficient accuracy done by the First-Order Reliability Method (FORM), which will be illustrated in the following.

The d-hour sea states along contour lines on the physical space with identical annual exceedance probability can be transformed to a U-space using so-called Rosenblatt transformation, see e.g. Madsen (1986). The physical variables, i.e. the significant wave height and the spectrum peak period, can be transformed by:

$$\Phi(u_1)\Phi(u_2) = F_{Hs}(h)F_{Tp|Hs}(t|h) \tag{7.1}$$

where Φ is standard normal distribution, and the transformed variables u_1 and u_2 are independent. Therefore the contour lines on the transformed space will be a circle with radius equal to r:

$$u_1^2 + u_2^2 = r^2$$

where $\Phi(-r) = q \Rightarrow r = -\Phi^{-1}(q)$

The values of u_1 and u_2 are given by:

$$u_1 = r\cos(\theta) \tag{7.2}$$

$$u_2 = r\sin(\theta)$$

The corresponding value of H_s and T_p are given by:

$$\Phi(u_1) = F_{Hs}(h)$$

$$\Phi(u_2) = F_{Tp|Hs}(t|h) \tag{7.3}$$

$F_{Hs}(h)$ and $F_{Tp|Hs}(t|h)$ are to be determined from joint distribution model of significant wave height and spectrum period, using either all sea states characteristics or storm peak characteristics.

The contour lines of 3-hour sea states in the present case can be established as following.

7.1.1 Determine contour lines using all sea state wave climate models.

Using all sea states characteristics, H_s is fitted by log-normal model when $h \le \eta$ and by Weibull model when $h > \eta$. Therefore,

When $h \leq \eta$

$$\Phi(u_1) = \Phi(\frac{\ln h - \mu_{\ln Hs}}{\sigma_{\ln Hs}}) \Rightarrow h = \exp(\mu_{\ln Hs} + \sigma_{\ln Hs} u_1) \qquad (7.4)$$

When $h > \eta$

$$\Phi(u_1) = 1 - \exp(-(h/\beta)^{\gamma}) \Rightarrow h = \beta(-\ln(1-\Phi(u_1)))^{\gamma} \qquad (7.5)$$

The distribution of is T_p fitted by log normal model:

$$\Phi(u_2) = \Phi(\frac{\ln t - \mu_{\ln Tp}}{\sigma_{\ln Tp}}) \Rightarrow t = \exp(\mu_{\ln Tp} + \sigma_{\ln Tp} u_2) \qquad (7.6)$$

The contour lines can then be determined, as shown in Figure 7.1.

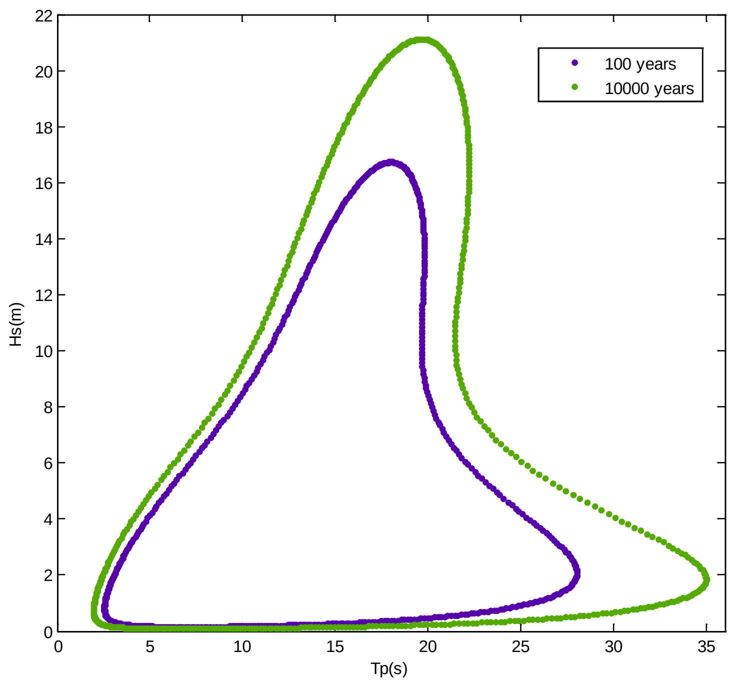

Figure 7.1 Contour lines using all sea states characteristics

7.1.2 Determine contour lines for storm peaks characteristics.

The joint distribution function for significant wave height using storm peak characteristics has been established in Equation (6.1) and (6.2).

From Equation (7.3), we have

$$\Phi(u_1) = 1 - \exp(-((h - h_0)/\beta)^\gamma) \Rightarrow h = \beta(-\ln(1 - \Phi(u_1)))^\gamma + h_0 \tag{7.7}$$

and the spectrum peak period is given by:

$$\Phi(u_2) = \Phi(\frac{\ln t - \mu_{\ln Tp}}{\sigma_{\ln Tp}}) \Rightarrow t = \exp(\mu_{\ln Tp} + \sigma_{\ln Tp} u_2) \tag{7.8}$$

The contour lines are shown in Figure 7.2.

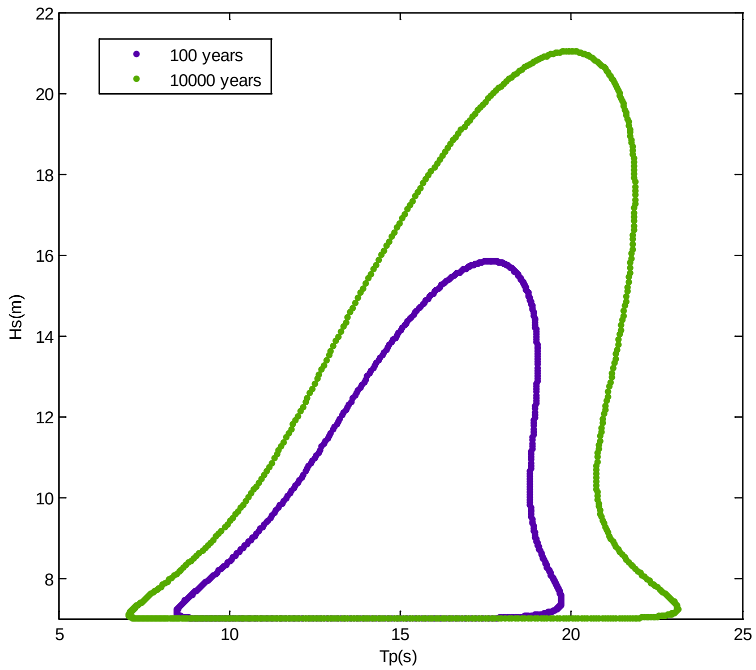

Figure 7.2 Contour lines using storm peak characteristics

7.2 Estimation of extreme response neglecting short term variability

Once the contour lines have been established, we will select a part of the contour lines where the most severe 3-hour sea state locates. The choice will be dependent on the problem under consideration. Generally it will be close to the sea state with largest significant wave height. The sea states whose spectrum peak periods are close to one of the structures nature periods should in principle also be included.

In general, the establishment of short term response distribution is a challenge. Once we select the critical area along the contour lines, several 3-hour time domain simulations or model tests should be performed for all the sea states in this area, thereafter we will be able to identify the worst 3-hour sea state. Note that with so few tests a lot of uncertainty is associated with the result, meaning that we might not be able identify the real worst sea state. However, this is not of too much of our concern, since the selected part is not too wide and the difference between those sea states is not too significant. Therefore not too much error is introduced if the sea state close to the worst one is selected.

For illustrative purpose, we will solve the response in frequency domain, meaning that a reliable response distribution can be obtained using transfer function and the effort to perform time domain simulations or model tests is avoided. The response distribution in the worst sea state can be solved as discussed in Section 3.2.1. The distribution will follow a Gumbel distribution.

If we assume the response distribution in the worst stationary sea state is extremely narrow, we can neglect the randomness of x and take the most probable response, x_{mp} as the extreme. This is a proximate assumption. In reality the extreme distribution is not extremely narrow and there is inherent randomness associated with the extreme response in a given sea state. The critical sea states are selected to be the ones close to the largest significant wave heights and close to the nature period of the structure. The most probable response of these sea states are calculated, see Table 7.1-7.4:

Table 7.1 Worst sea states-all sea states characteristics, 100 years

Selected 3-hour sea states	Significant wave height	Spectrum peak period	Most probable response
1	16.74	17.97	8.22
2	16.73	18.14	8.30
3	16.19	19.09	8.62
4	16.15	19.13	8.63
5	16.10	19.16	8.63
6	16.04	19.20	8.63
7	15.99	19.23	8.62

Table 7.2 Worst sea states-all sea states characteristics, 10000 years

Selected 3-hour sea states	Significant wave height	Spectrum peak period	Most probable response
1	21.13	19.61	11.91
2	21.12	19.83	12.10
3	21.11	19.90	12.16
4	19.75	21.55	13.29
5	19.66	21.59	13.29
6	19.57	21.64	13.30
7	19.48	21.68	13.30
8	19.39	21.71	13.29

Table 7.3 Worst sea states-storm peaks characteristics, 1000 years

Selected 3-hour sea states	Significant wave height	Spectrum peak period	Most probable response
1	15.86	17.64	7.62
2	15.85	17.80	7.69
3	15.56	18.47	7.90
4	15.53	18.49	7.90
5	15.50	18.52	7.90
6	15.47	18.54	7.90
7	15.44	18.57	7.90

Table 7.4 Worst sea states-storm peaks characteristics, 10000 years

Selected 3-hour sea states	Significant wave height	Spectrum peak period	Most probable response
1	21.05	19.92	11.91
2	21.04	20.10	12.10
3	21.03	20.16	12.16
4	19.96	21.42	13.22
5	19.89	21.45	13.22
6	19.82	21.48	13.22
7	19.75	21.50	13.22

The worst sea states can be identified for various cases as shown in Table 7.1-7.4. Neglecting the short term variability, the most probable response of the worst sea state is taken to be the characteristics response. The estimated characteristic responses are shown in Table 7.5.

Table 7.5 Estimated characteristic responses neglecting short term variability

Characteristic responses (m)	Return period	
	100	10000
All sea states characteristics	8.63	13.30
Storm peaks characteristics	7.90	13.22

It is obvious that the results significantly under estimate response extremes. Compared to full long term analysis, neglecting short term variability results in estimates 15%-20% lower. This clearly illustrates that short term variability should not be neglected.

7.3 Accounting for short term variability

Compared to the values obtained by all sea state approach and peak over threshold method, the largest most probable responses along contour lines above are apparently too low to be an effective estimation. It demonstrates that in practice short term variability cannot be neglected. There are several ways to account for the short term randomness. We will illustrate two most commonly used approaches as following.

Adopt a higher percentile response

One may instead of taking the most probable response, use a higher percentile value of the short term response distribution in the worst sea state as the characteristic response, as shown in Equation (3.12). Since the worst sea states have been determined in Table 7.1-7.4, a higher percentile, α, is selected to calculate the corresponding response. One shall iterate the percentile until the α-percentile response is equal to the corresponding response obtained using full long term analysis, see Table 7.6-7.9:

Table 7.6 α-Percentile value-all sea states characteristics, 100 years

	Characteristics response from full long term analysis	α -Percentile response		
		84%	85%	86%
Response	9.87	9.81	9.86	9.91

Table 7.7 α -Percentile value-all sea states characteristics, 100 years

	Characteristics response from full long term analysis	α -Percentile response		
		92%	93%	94%
Response	16.11	15.95	16.10	16.27

Table 7.8 α -Percentile value-storm peaks characteristics, 100 years

	Characteristics response from full long term analysis	α -Percentile response		
		92%	93%	94%
Response	9.46	9.38	9.47	9.81

Table 7.9 α -Percentile value-storm peaks characteristics, 10000 years

	Characteristics response from full long term analysis	α -Percentile response		
		86%	87%	88%
Response	15.27	15.23	15.31	15.41

The proper percentile to account for short term variability is then shown in Table 7.10:

Table 7.10 Proper percentile to account for short term variability

	Return period	
	100	10000
All sea states characteristics	85%	93%
Storm peaks characteristics	93%	87%

It is seen from Table 7.10 that the percentiles using all sea states characteristics are in agreement with Kleiven and Haver (2003) that a percentile of 85%-95% is reasonable to estimate the 100-year response, while 90%-95% is more suitable 10000-year response. A recommendation of 90% percentile is proposed when using storm peaks characteristics for both 100-year and 10000-year extreme response based on the result given in Table 7.10.

Further discussions regarding an adequate percentile for various problems can be found in Kleiven and Haver (2004).

Multiply by a factor
A simple way to account for the short term randomness is to multiply the largest most probable responses by a factor.

This factor could be estimated by calculating the ratio between estimated responses using all sea state approach or peak over threshold approach and corresponding largest most probable responses along contour line, as shown in Table 7.11 and 7.12:

Table 7.11 Ration between estimated extreme responses using all sea state approach and worst most probable responses along contour lines

Return period	Estimation by all sea state approach	Largest most probable responses along contour lines	Ration
100 years	9.87	8.63	1.14
10000 years	16.11	13.3	1.21

Table 7.12 Ration between estimated extreme responses using peak over threshold method and worst most probable responses along contour lines

Return period	Estimation by peak over threshold method	Largest most probable responses along contour lines	Ration
100 years	9.46	7.8	1.21
10000 years	15.27	13.22	1.16

As shown in Table 7.11 and 7.12, a correction factor 1.1-1.25 may be recommended, no matter the all sea states characteristics or storm peaks characteristics are utilized. Apparently the correction factor used will also dependent somewhat on the nature of the responses problem and aimed annual exceedance probability, but it will typically be a factor of the order of 1.1 - 1.3, see Sagli (2000).

Note that either adopting a high percentile response value or utilizing a correction factor is a pretty rough method. The percentile and correction factor may vary from case to case. Therefore, full long term analysis is recommended as long as condition permits.

Other methods to include the short term variability are: Increase the duration of the sea state artificially; Increase the sea state level artificially, see Winterstein (1993); Increase both the sea state level and duration artificially, see Haver (1996).

Chapter 8

Conclusion and Future work

In the present study, statistical methods to estimate offshore structures' extreme response are reviewed. As an example, extreme response estimation for a semi-submersible located in North Sea is performed. Some conclusions can be drawn:

i) All sea state approach is the most common full long term analysis used in North Sea. This is a slightly conservative method. Compared to the estimation using annual extremes or storm peaks, all sea states approach will give a result about 5% higher.

ii) Peak over threshold method is attractive in hurricane governed areas. The result is not sensitive to the choice of threshold as long as it is within a reasonable range.

iii) There are uncertainties associated with both all sea states approach and peak over threshold method, both aleatory uncertainty and epistemic uncertainty. In addition, the annual number of storms may vary when using peak over threshold method.

iv) Environmental contour lines method is an approximate method useful for complicated response problems. A consistent construction of contour lines can be established using FORM. Generally the short term variability cannot be neglected. To account for this randomness, 85%-95% percentile response value is recommended. Alternatively one may utilize a correction factor between 1.1-1.25. Of course these are very rough estimates. One should perform a full long term analysis if possible.

v) For complicated response problems, time domain simulations or model tests shall be performed to establish the short term response distribution. From a practical point of view, not too few, say 20, times of such simulations or tests should be performed to establish response distribution in a given sea state.

So far we have been using a transfer function and short term response problem is solved in the frequency domain. In the future more concern should be put in non-linear response problems, such as green water problems of ships, wave-deck impacts, ringing response, etc, since these problems are crucial to structures' safety in extreme weather conditions. More theoretical research on these issues is required.

To estimate extreme responses, we are mainly interested in the very upper tail of response distribution. Therefore, more attention should be given to how to better fit the upper tail distribution. Presently we have to use the data observed in time period as short as 50 years. It is likely that the upper tail of distribution cannot be reasonably well fitted because of the limited amount of data. A potential different mechanism in the very upper tail is also possible. More research on the upper tail part is suggested. This relies on the amount of available data; also more thorough understanding of response mechanism is required.

Due to the limit of time and data, the present study does not include extreme response estimations in areas other than the North Sea. In other areas with active offshore oil & gas industry, like Gulf of Mexico, South China Sea, the principles in predicting characteristic response are the same. But there are differences on weather severity as well as the nature of weather condition. More study of extreme responses in these areas in the future is recommended.

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Appendix

A. Problem description

M.Sc. thesis 2010

for

Stud.techn.

Wu Ruichao

Prediction of Design Response for Norwegian and Chinese Waters

(Estimering av karakteristisk design respons for norske og kinesiske farvann.)

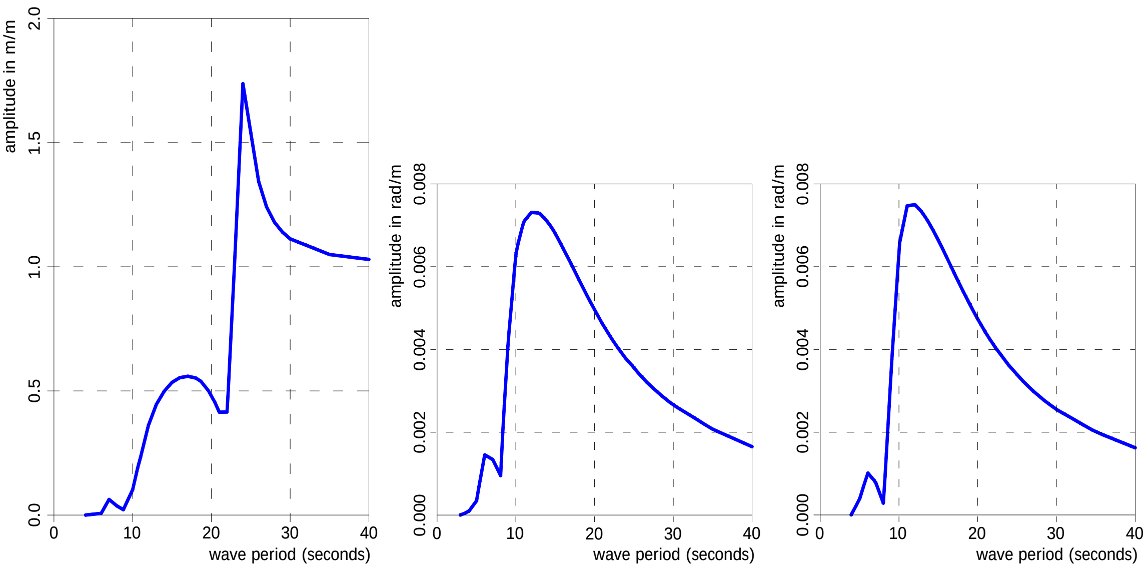
In most cases the aim of a design process is that the structure shall fulfil various limit states, SLS, FLS, ULS and ALS. Here we will focus on ULS and ALS. In connection with these limit states environmental loads and responses corresponding annual exceedance probabilities of 10^{-2} and 10^{-4}, respectively, are required. Various methods are available for predicting these quantities. The overall aim of the investigation is to present a review of various methods for estimating responses corresponding to a given annual exceedance probability. This should include a discussion of their advantages and disadvantages.

The title above is the title expected to be used for the master thesis in the spring semester 2010. The aim of the project for the autumn semester in 2009 is to get familiar with subjects that will represent important background knowledge for doing the master thesis. The difference between the estimates for the various methods should be demonstrated for a site at the Norwegian Continental Shelf. If data become available for an offshore site in Chinese waters that should also be included. If a sufficient amount of data from Chinese waters does not become available, one can apply the methods to a generic hindcast data base for Gulf of Mexico conditions, Bergsvik (2009).

As an example structure response quantity for this project we will adopt the heave and pitch motions of a semi-submersible. These motions are assumed to be linearly related to the wave process. The response characteristics for heave and pitch are given by the transfer functions given in Figs. 1a and 1b. If it is found convenient to illustrate difference between methods one may include an additional linear response case being sensitive a shorter spectral peak period band.

The work should be carried out in steps as follows:

As an extended introduction a brief review of the various methods being available for predicting response corresponding to a given annual exceedance probability. The following methods must be included: i) Long term analysis using the all sea state approach, ii) Long term analysis using a POT formulation, iii) Long term analysis using structural reliability analysis, iv) Approximate long term analysis using environmental contour lines, v) Long term analysis using level crossing formulations. The environmental information required by the various methods should be pointed out.



(a) (b) (c)

Fig. A.1 Transfer functions

Present an environmental description for the selected site on the Norwegian Continental Shelf based on available hindcast data. Use this long term wave climate model for carrying out a full long term analysis using the all sea state approach for the selected response quantities. Illustrate the part of the sample space of H_s and T_p that is of major importance regarding exceeding 10^{-2} – probability response and 10^{-4} – probability response. As far as possible uncertainties associated with these extremes shall be indicated.

Perform a long term analysis for the same offshore site using a POT formulation and the storm maximum response as the primary response variable. Uncertainties associated with the predicted 10^{-2} - and 10^{-4} – probability response shall be indicated. Compare and discuss the estimated extremes for the two approaches.

Determine 10^{-2}- and 10^{-4}- probability contour lines for H_s and T_p using all sea state wave climate models. Estimating extremes using Norsok recommendations regarding how to use environmental contour line method.

Determine a joint model for storm peak characteristics used in 3). Estimate 10^{-2}- and 10^{-4} – probability environmental contours for storm peaks. Assuming that the storm peak event has duration of 3 hours and estimate the percentile of the 3-hour extreme value distribution that will equal the long term result of 3). What would be the adequate percentile if duration of storm peak was artificially increased to 6 hours abd 9 hours.

At this time of the work one must review status regarding time left for the work. If a sufficient amount of data from a location in Chinese waters is available, one should repeat the steps above for this location. If not one may apply the methods to generic hurricane data from Gulf of Mexico.

Perform a full long term analysis using methods above for a site outside the Norwegian Continental Shelf. How extensive the investigation is for the second site must be decided in view of the experiences obtained for the first site.

If time permits one may also illustrate the applications of other methods using the environmental data available for the location on the Norwegian Continental Shelf.

Finally, the work is to be summarized. Findings shall be clearly pointed out. Recommendations for future work should also be included.

The work may show to be more extensive than anticipated. Some topics may therefore be left out after discussion with the supervisor without any negative influence on the grading.

The candidate should in his report give a personal contribution to the solution of the problem formulated in this text. All assumptions and conclusions must be supported by mathematical models and/or references to physical effects in a logical manner. The candidate should apply all available sources to find relevant literature and information on the actual problem.

The report should be well organised and give a clear presentation of the work and all conclusions. It is important that the text is well written and that tables and figures are used to support the verbal presentation. The report should be complete, but still as short as possible.

The final report must contain this text, an acknowledgement, summary, main body, conclusions, suggestion for further work, symbol list, references and appendices. All figures, tables and equations must be identified by numbers. References should be given by author and year in the text, and presented alphabetically in the reference list. The report must be submitted in two copies unless otherwise has been agreed with the supervisor.

The supervisor may require that the candidate should give a written plan that describes the progress of the work after having received this text. The plan may contain a table of content for the report and also assumed use of computer resources.

From the report it should be possible to identify the work carried out by the candidate and what has been found in the available literature. It is important to give references to the original source for theories and experimental results.

The report must be signed by the candidate, include this text, appear as a paperback, and - if needed - have a separate enclosure (binder, diskette or CD-ROM) with additional material.

Supervisor: Prof. II Sverre Haver, Senior Spesialist, Statoil ASA.

B. Hindcast data

See the file 'WAM10_6529N_0732E' in the attached CD-ROM.

C. Calculation sheets

See the file 'Prediction of design response for North Sea' in the attached CD-ROM.