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M.Sc.THESIS

Title: **Stochastic analysis of Workover Risers**	**Delivered:** June 14th 2010 **Availability:** Open
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Abstract:

Top tensioned risers are widely used in the search and extraction of oil and gas offshore around the world. Drilling and workover risers are held up by a tensioned system to keep the pipe stretched with an overpull regardless of the semisubmersible rigs motions. These systems must be analyzed for fatigue, accidental loads and ultimate strength when exposed to environmental loads such as waves, currents and rig motion.

To define the safe operation limitations for the use of top tensioned risers the designer must run a large set of analyses of different sea states, selected from long-term statistics. It is possible to use both regular and irregular waves to model the dynamic wave load. The common practice is to use regular waves with a wave height corresponding to the expected largest wave height during the short-term sea state. In reality, however, the sea is irregular, and the stochastic response of the system should be investigated.

But running long simulations of nonlinear time-domain analyses with irregular wave loading is extremely time consuming. The standards do not have clear guidelines with regards to all aspects of irregular analyses.

In this report there is an overview of dynamic analyses of slender marine structures, and wave modeling and statistics. The case study explores the properties of irregular and regular wave analyses to find one or more parameters that correlate strongly to the response. These parameters may be used to find a link between regular and irregular analyses, and be used to fine tune a method for using regular waves to find the operation limitations while still maintaining the extreme values that corresponds to irregular wave analyses.

The main results are:

- The extreme response of an irregular simulation does not seem to correlate with the largest wave height in the simulation.

- The bending moment seems to correlate weakly to the displacement and velocity of the rig.

- The bending moment seems to correlate well with the displacement of the riser pipe. More in surge and pitch than in heave
.
- The bending moment seems to correlate well with the velocity of the riser pipe. More in surge and pitch than in heave.

Keyword:

Dynamic riser analysis
Stochastic wave loads
Extreme value statistics

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Stochastic Analysis of Workover Risers

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Master of Science Thesis
performed at

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for
Stud.techn. Knut Johannessen

STOCHASTIC ANALYSIS OF WORKOVER RISERS

Work over risers are used during specific operations on subsea wells, and have to remain connected during the actual operation. It is therefore important to have rational criteria for sea state parameters that allow an operation to start and continue, but also for conditions that requires an unplanned disconnection.

The criterion must in principle be based on a probability level for a specific response parameter to exceed a critical level during a specific time period in a given sea state. Hence, if the probability of exceedence is too high, the operation can not be allowed in the actual sea state, and if the sea state parameters exceed a certain level one have to stop the operation and disconnect the riser. Stochastic analyses with irregular sea must be carried out in a set of sea states in order to estimate probability distributions and check if the probability to exceed the critical response level is acceptable. The purpose of this project is to investigate the feasibility of using results from regular wave analysis as basis for decisions regarding start of operations and emergency disconnect.

The work should be organised in steps as follows:

1. Describe the format and analysis procedures that represent common practice for verifying the safety of selected operations with work over risers.

2. Describe how stochastic analyses and regular wave analyses may formally be used in a consistent way to define sea state limitations for safe operations.

3. Carry out a case study for a given operation in an area with known long term wave statistics (scatter diagram), and demonstrate and verify the use of regular wave analyses to define operational limitations.

4. Propose a general way for tuning procedures based on regular waves for verification of safe operation in a specific sea state.

The work may show to be more extensive than anticipated. Some topics may therefore be left out after discussion with the supervisor without any negative influence on the grading.

The candidate should in his report give a personal contribution to the solution of the problem formulated in this text. All assumptions and conclusions must be supported by mathematical models and/or references to physical effects in a logical manner.

The candidate should apply all available sources to find relevant literature and information on the actual problem.

The report should be well organised and give a clear presentation of the work and all conclusions. It is important that the text is well written and that tables and figures are used to support the verbal presentation. The report should be complete, but still as short as possible.

The final report must contain this text, an acknowledgement, summary, main body, conclusions and suggestions for further work, symbol list, references and appendices. All figures, tables and equations must be identified by numbers. References should be given by author name and year in the text, and presented alphabetically by name in the reference list. The report must be submitted in two copies unless otherwise has been agreed with the supervisor.

The supervisor may require that the candidate should give a written plan that describes the progress of the work after having received this text. The plan may contain a table of content for the report and also assumed use of computer resources.

From the report it should be possible to identify the work carried out by the candidate and what has been found in the available literature. It is important to give references to the original source for theories and experimental results.

The report must be signed by the candidate, include this text, appear as a paperback, and - if needed - have a separate enclosure (binder, DVD/ CD) with additional material.

Supervisor at NTNU is Professor Carl M. Larsen
Supervisor at FMC is Per Thomas Moe

Trondheim, February 2010

Carl M. Larsen

Submitted: January 25, 2010
Deadline: June 14, 2010

Preface

This report is the result of my Master's thesis for 5^{th} year students at The Norwegian University of Science and Technology (NTNU) at the Department of Marine Technology the spring of 2010.

The motivation for me to cover the topics of this thesis is a result of discussions with colleagues during an internship at FMC Kongsberg Subsea on using irregular waves in structural analysis of top tensioned risers. The approach to describing the challenges, and the methods for trying to solve them has been outlined by my advisor at NTNU and myself.

This thesis has given me an opportunity to go deeper into the field of applied statistics, which is an area that I earlier had little experience in. With the literature study and the following case study I have learned more about probability distributions and applied extreme value statistics than I expected, and it has become a field that I would like to pursue further. The project has been done in cooperation and with support from FMC Kongsberg Subsea.

I would like to thank my advisor, Professor Carl M. Larsen at the Department of Marine Technology, NTNU for excellent guidance and deep insight on the topic. I would also like to thank my advisor at FMC Kongsberg Subsea, Per Thomas Moe and colleagues for great advice and support.

Trondheim, June 14, 2010.

Knut Johannessen

Summary

The goal of this thesis is to investigate the properties of dynamic analysis of slender, top tensioned risers when using both regular and irregular waves. The goal was to discover properties in the response that correlates to a parameter that can be found in both methods. The approach is to use a model of a riser in open water connected to the sea bed, and top tensioned by a semi submersible rig subjected to the dynamic load of waves and currents.

The theories that hold the basis of dynamic analyses using finite elements are outlined, and different methods of solving the dynamic equilibrium equation are discussed. Relevant wave theories and their statistical properties are investigated and outlined, followed by a clear methodology for performing the case study.

The data from the case study is based on a large number of irregular analyses, and one regular wave analysis for each sea state. The extreme values is extracted from the irregular simulations and fitted to a Gumbel extreme value distribution. The distributions are extrapolated to return periods of 1, 10 and 100 years, and compared to the extreme values from the regular wave analyses.

The main results from the case study are:

- The extreme response of an irregular simulation does not seem to correlate with the largest wave height in the simulation.
- The bending moment seems to correlate weakly to the displacement and velocity of the rig.
- The bending moment seems to correlate well with the displacement of the riser pipe. More in surge and pitch than in heave.
- The bending moment seems to correlate well with the velocity of the riser pipe. More in surge and pitch than in heave.

The results from the case study are used in a discussion on how to fine tune a regular wave analysis to be used in a consistent way to define the safe operation limitations for a top tensioned work over riser.

Notation

The Roman and Greek letters most frequently used throughout the thesis are given here. Bold types are used exclusively to denote vectors and matrices.

Roman

A	Cross-sectional area.
C_D	Drag coefficient.
$\mathbf{C}$	Damping matrix.
c	Damping.
D	Diameter or duration.
g	Acceleration of gravity.
H, H_{MAX}	Individual wave height, maximum wave height.
H_s, H_{m0}	Significant wave height, estimate for significant wave height based on spectrum.
H_M	Expected largest wave height during a sea state.
h	Current water depth.
$\mathbf{K}$	Stiffness matrix.
k	Stiffness or shape parameter.
L	Length of riser
$\mathbf{M}$	Mass matrix.
m	Mass.
P, P_i, P_e	Pressure, internal pressure, external pressure.
$S(\omega)$	Wave spectrum.
T, T_e	Tension or individual wave period, effective tension.
T_p	Spectral peak period.
T_z, T_{m0}	Zero-crossing period (measured), estimated mean zero-crossing period (from spectrum).
u_w	Fluid velocity
N_m	Number of occurrence of a sea state in m years.
p	Probability level.
$\mathbf{Q}$	External force matrix.
$r, \dot{r}, \ddot{r}$	Response displacement, velocity and acceleration.
$\mathbf{X}$	Complex load vector.
$\mathbf{Y}$	Normal coordinate matrix.

Greek

α	Shape parameter in *JONSWAP*.
γ	Peak enhancement factor in *JONSWAP*.
ϵ	Phase angle.
ζ, ζ_A	Free surface elevation, amplitude.
μ	Mean.
Φ	Velocity potential or eigenvector.
ρ, ρ_i	Density, internal fluid density.
σ, σ^2	Standard deviation, variance.
ω, ω_p	Wave frequency, peak frequency.

Abbreviations

API	American Petrolium Institute
CPU	Central Processing Unit
DNV	Det Norske Veritas
FE	Finite Element(s)
FFT	Fast Fourier Transform
GEV	General Extreme Value distribution
ISO	International Organization for Standardization
JONSWAP	Joint North Sea Wave Project
MSL	Mean Sea Level
PM	Pierson-Mozkowitz
PWM	Probability of weighted moments
RAO	Response Amplitude Operator
RP	Recommended Practice

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Chapter 1

Introduction

In the search for offshore oil and gas top tensioned risers are beeing used by semi-submersible rigs for drilling, well completion and workover tasks. The risers are tensioned by heave compensating systems to maintain a constant pull independent of rig movement. The riser and the connected equipment are subjected to environmental loads from waves, currents, rig movement and load effects due to internal fluid flow, temperature and pressure.

All equipment that is used in petroleum industry must follow rigorous requirements set by the governing laws of the country it is used in. These regulations usually require that certain standards and recommended practices set by organisations such as the International Organization for Standardization (ISO), The American Petroleum Institute (API), or Det Norske Veritas (DNV) are followed.

These standards prescribes that the operators and their subcontractors must perform a design analysis of all equipment before it is used with regards to ultimate strength, fatigue damage and accidental loads. The dynamic load from surface waves can be modelled as regular or irregular. The two different ways of modelling this load are different in many aspects, and these differences will be discussed in the thesis.

To determine limitations for the safe use of the equipment, several analyses with combinations of sea states and configurations must be performed. In the Norwegian petroleum industry today it is common to use regular waves in dynamic analysis. The use of regular waves has a huge advantage with respect to computational time. An structure subjected to regular waves will after only a few incoming wave periods achieve the maximum and minimum values of the response, and only one analysis needs to be run for each sea state.

But the reality is that the sea surface is not regular, it is irregular. The equipment used in the marine environment is submitted to stochastic loads, and using a regular load to replicate this is not a straightforward procedure. To use regular waves to represent the same response as from irregular waves one or more parameters must be identified that strongly correlates to the response, and can be used as a link between the response from regular and irregular waves.

The dangers of miscalculating the forces from waves can in the worst case lead to the equipment being used in operations that gives forces that exceeds the ultimate strength of the materials. This can cause equipment failure, and cause a risk to the health and safety of personnel, and damage the environment. On the Norwegian Continental Shelf the largest uncontrolled oil spill happened on the Bravo drilling platform in 1977 on the Ekofisk field. 12 700 cubic meters of raw oil leaked into the Norwegian Sea for a total of eight days. These kinds of accidents happen still today. During the final weeks of writing this thesis probably the largest oil spill in Americas history was caused by a blow-out during drilling in the Gulf of Mexico. The following explosion and fire caused the life of eleven workers, the rig to sink and an oil spill that was still uncontrolled when this thesis was completed.

This report will contain an overview of a theoretical background for dynamic analysis of slender structures, wave modelling and statistics based on relevant literature. The methodology chapter will include an overview of relevant standards, a literature review of studies made on the subject and the methodology for the case study. The case study will seek to discover the properties of the response of a riser subjected to both regular and irregular waves to find one or several parameters that links the methods together.

Chapter 2

Dynamic analysis of slender structures

This chapter outlines the basic methods for structural analysis of slender structures under dynamic loading from a marine environment. The methods can be applied to e.g. anchor lines, tension leg platform tendons, flexible risers or, as used in this thesis, for top tensioned work over risers.

2.1 Dynamic analysis using finite elements

A dynamic analysis using the finite element method will consist of:

1. **Discretization** the structure is divided into a finite number of elements connected with nodes. The element type, size and number of nodes vary.

2. **Element analysis** The stiffness matrix, the mass matrix and the loads for each element is found.

3. **System analysis** The entire structures‘ mass and stiffness-matrix and load vector is found from the element matrices.

4. **Solve the dynamic equilibrium equation**

The solution should converge toward the correct values when the finite element sizes are reduced. To reduce computational time the element size should not be reduced any further when convergence is reached. To ensure this a sensitivity study should be performed when creating a finite element model.

In the following sections different relevant methods for solving the dynamic equilibrium are discussed.

2.2 Dynamic Analysis

There are several methods for dynamic analysis of marine risers by using finite elements. A structure exposed to a dynamic load will have the following forces acting:

- Inertia forces
- Damping forces
- External, time-dependent forces

According to d'Alemberts principle the inertia forces can be added to the other force components. The equilibrium of these forces in a system with viscous damping and forced oscillations can be described by the equation of dynamic equilibrium:

$$M\ddot{r} + C\dot{r} + Kr = Q(t) \tag{2.1}$$

where:

$\boldsymbol{r}$	Matrix of nodal displacement vectors
$\dot{r}$	Matrix of nodal velocity vectors
$\ddot{r}$	Matrix of nodal acceleration vectors
$\boldsymbol{Q(t)}$	Matrix of external, time-dependent forces
$\boldsymbol{K}$	System stiffness matrix
$\boldsymbol{C}$	System damping matrix
$\boldsymbol{M}$	System mass matrix

There are several ways to solve the dynamic equilibrium equation, and a few relevant methods are briefly discussed in the following sections.

2.3 Mode superposition

This method is similar to the Rayleigh-Ritz method, as it describes the response as a linear combination of the eigenmodes. However, instead of basic shapes it uses known eigenmodes. To find the nodal displacements $\boldsymbol{r}$ in equation 2.1 the displacements are described by mode shapes found from the eigenvalue problem:

$$[\boldsymbol{K} - \omega^2 M]\phi = 0 \tag{2.2}$$

Here ϕ holds the linearly independent eigenvectors, and ω is the eigenfrequency. The eigenmodes can be written on matrix-form, and the orthogonal properties of the eigenmodes can be employed. From these formulations the modal mass, the modal stiffness and the modal forces can be found for the dynamic equation as generalized terms. The mass matrix and the stiffness matrix are diagonal matrices that give a set of uncoupled equations. The damping however does not always hold the same orthogonal properties, and can rather be found by modal damping or proportional damping (Larsen, 2007). Finally the response history is expressed by the normal coordinate transformation:

$$r(t) = \sum_{i=1}^{S} \phi_i y_i \tag{2.3}$$

Here $\mathbf{y}$ holds the normal coordinates. Mode superposition is a good method if the load distribution can be approximated with the inertia forces of the lowest mode shapes, and the load frequencies corresponds to the lowest eigenfrequencies.

However this way of solving the dynamic equilibrium equation for this type of riser is not suitable. The riser here will have a displacement and a response velocity that is relatively too large for the wave induced water particle motion that the drag force from the modified Morrison‘s equation will have to account for the relative speed. There will be a quadratic coupling between the wave induced velocity and the response velocity of the structure. This makes it impossible to split the drag force into an excitation term and a damping term. The force term in equation 2.1 will be coupled, and mode superposition is therefore not a suitable solution to the problem that is modeled here.

2.4 Frequency-response method

Equation 2.1 can be solved in the frequency domain:

$$M\ddot{r} + C\dot{r} + Kr = Xe^{i\omega t} \tag{2.4}$$

$\mathbf{X}$ is generally a complex number that describes the load amplitude in the frequency domain. Solving equation 2.1 in the frequency domain is mainly applicable to linear problems. Solving it directly can be very time consuming, but it can be dramatically reduced by using Fast Fourier Transform (FFT) (Langen and Sigbjörnsson, 1979). The method is well suited for stochastic analyses, because it gives a complete statistical description of the process (Larsen, 2007). The method is often used for fatigue analysis to find estimates for root-mean-square axial and bending stresses.

2.5 Time domain method

The dynamic equilibrium equation can be solved numerically by step-by-step integration in the time domain. The time period that is to be solved is divided into intervals, and the dynamic equilibrium is found for each time step. The displacement and the velocity are found by integrating the acceleration twice for each time step. There are different methods for describing how the acceleration varies over the interval. Examples are constant initial acceleration, constant average acceleration or linear acceleration.

One method is the Newarks $\beta - family$. The displacement and velocity for the next time step is found from Taylor expansions with parameters β and γ. In this case $\beta = \frac{1}{4}$ is used to give constant average acceleration, and is unconditionally stable. This is the trapeze method (Euler-Gauss) formulated for problems of second order. $\gamma = \frac{1}{2}$, ensures no artificial damping (Langen and Sigbjörnsson, 1979).

This method is very useful in analysing nonlinear systems, as it can give a quite accurate representation of nonlinear behavior. The mass, and therefore the inertia forces, are assumed to be constant. Elastic and damping forces can be nonlinear functions of respectively displacement and velocity. The elastic forces must be calculated from the stiffness state in the elements.

By linearisation of the damping matrix and the stiffness matrix the incremental matrices can be found. In the time integration the displacement and the velocities can only be found by iterations at each time step. If the stiffness and the damping matrix are found for each iteration the method is called Newton-Rhapson. This is time consuming, and by only updating them once or a few selected times, time is saved and the method is called modified Newton-Rhapson. Although the rate of convergence is slower, it is usually faster than true Newton-Rhapson (Langen and Sigbjörnsson, 1979).

2.6 Dynamic loads

The environmental dynamic load in the analysis of slender marine structures comes from waves, current and vessel motions. The drag forces acting on the riser is usually calculated from the Morison equation. By assuming that the wave induced motions are harmonic, and that the relative displacement response of the riser is large, the drag expression for time domain is:

$$m\ddot{r} + c\dot{r} + kr = Q(t) + \frac{1}{2}C_D DL(u_w - \dot{r})|u_w - \dot{r}| \tag{2.5}$$

where:

r	Response displacement
$\dot{r}$	Response velocity
$\ddot{r}$	Response acceleration
Q(t)	Time-dependent forces
k	Stiffness
c	Damping
m	Mass
C_D	Drag coefficient
D	Diameter of riser
L	Length of riser
u_w	Wave induced velocity

Equation 2.5 must be iterated for each time step. First assume $\dot{r}$ for the next time step, compute the drag from Equation 2.5, calculate displacement, velocity and acceleration and then compare computed and assumed velocity. Repeat the iteration until the difference is smaller that the convergence criterion. This will lead to a correct solution (Larsen, 2005).

In slender structures the quadratic term $(u_w - \dot{r})|u_w - \dot{r}|$ is known to be significant, and gives the response nonlinear properties.

Chapter 3

Wave theory and statistics

The surface waves, also known as gravity waves, that are discussed and used in this thesis are separated into two main categories; regular waves and irregular waves. There are several mathematical models available to create both regular and irregular waves to use in dynamic analysis of marine structures. The properties of the waves which are modelled and used in the case study will be discussed in the following sections.

3.1 Wave modelling

Mathematically, a general solution does not exist for gravity waves and approximations must be made for even simple waves. Due to simplifying assumptions the different wave theories have different validities (Mader, 2004). By introducing boundary conditions and assumptions models can be made to describe the waves. Generally it is assumed for all waves that the flow of sea water is incompressible and inviscid, and the velocity vector of the fluid can be described by a velocity potential. The flow is also assumed to be irrotational if the vorticity vector is zero, and it follows that the velocity potential must satisfy the Laplace equation.

The first boundary condition is that no water particles can cross the free surface. This is called the kinematic boundary condition, and means that a particle on the free surface stays on the free surface. The second is the dynamic free-surface condition that demands that the water pressure on the free surface is equal to the constant atmospheric pressure. Furthermore, no fluid particle can cross the solid boundary (Faltinsen, 1990).

3.1.1 Wave theories

Waves can be modelled using linear or nonlinear terms. In this thesis the linear wave theory will be referred to as Airy wave theory. There are several nonlinear wave theories, but the nonlinear waves used herein are 5^{th} order Stokes waves.

The Airy wave theory is an oscillatory wave where particle follows an elliptic path in a closed orbit, which results in no net mass transport. The free surface is sinusoidal, and higher order terms

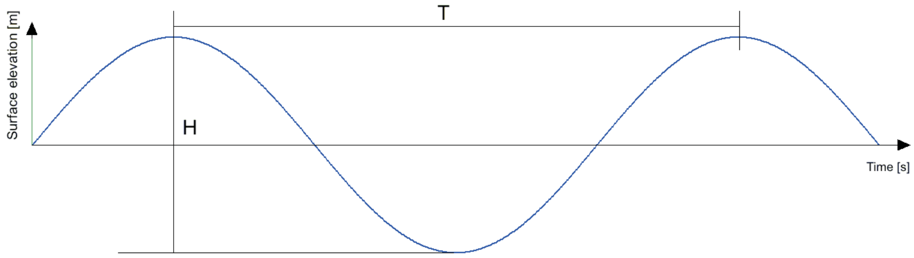

Figure 3.1: A regular wave.

are neglected (Mader, 2004). We assume a horizontal sea bottom and a free surface of infinite horizontal extent. Linear wave theory assumes the velocity potential and fluid velocity to be constant from the mean free surface to the free surface level (Faltinsen, 1990). All irregular waves used in the case study are based on linear Airy waves.

Linear theory can only approximate the conditions for the free surface by first order terms. This can be improved by using Stokes expansions of higher order. The velocity potential is represented by a higher number of terms from a series of small perturbations (McCormick, 1973). In that case the sum of five sinusoidal terms describes the wave. Terms higher than the fifth power are neglected, and the theory is only valid when the wave height is less than ten percent of the wave length. The wave profile is characterized by a more peaked wave crest and flatter troughs. Where as in the linear theory there is no mass transport, there will be a small mass transport in higher order Stokes theory, known as Stokes drift (Faltinsen, 1990). All regular waves used in the case study are based on nonlinear 5^{th} order Stokes waves.

3.1.2 Regular waves

Regular waves are uniquely described by two variable parameters; the wave period (T) and the wave height (H), as shown in figure 3.1. They are periodic and invariant, and the fact that they have these properties means that dynamic analyses with regular waves only need a few periods after transients have died out to find the response of the structure. This is an advantage over irregular waves regarding analysis time.

3.1.3 Irregular waves

In reality however, the surface elevation is irregular, and can be described as a stochastic process. The exact surface of the wave cannot be determined in advance, as can be done with regular waves. Irregular waves must be predicted using statistical methods. Mathematically the surface elevation of an irregular sea state can be described as a function of time (t) using the sum of a finite number of linear, regular waves with different amplitudes (ζ_{An}), frequencies (ω_n) and a random phase angle (ϵ_n) uniformly distributed between 0 and 2π.

$$\zeta(t) = \sum_{n=1}^{N} \zeta_{An} cos(\omega_n t + \epsilon_n) \tag{3.1}$$

When using irregular waves in dynamic analysis we must create realisations of the stochastic process of finite time lengths. Each new realisation is unique, and represents a sample of the process and its properties. For the realizations to be valid it is assumed that the process is stationary and ergodic, and that the surface elevation is Gaussian distributed (Myrhaug, 2006).

3.2 Wave spectra

A wave spectrum, or energy spectrum, holds the statistical properties of the surface elevation $\zeta(t)$. If the wave spectrum for the area of interest is not available, standardised wave spectra can be used. There are several different spectra with different approaches, and parameters. Since standardised spectra represent approximations of an average spectrum, they must be treated as such. For numerical analysis and model testing the *JONSWAP*-spectrum and the *Torsethaugen*-spectrum has been frequently used on the Norwegian Continental Shelf. Outside this area the *Ochi-Hubble*-spectrum is also adopted.

3.2.1 JONSWAP

The **Jo**int **N**orth **S**ea **Wa**ve **P**roject (*JONSWAP*) was a multi-national research programme. By measuring waves in the south-east parts of the North Sea in 1968 and 1969 it was found that the shape of the sharply peaked shape spectra remained fairly similar, so that it could be represented using only a few characteristic parameters (Komen et al., 1984). It is a one-peaked Pierson-Moskowitz type spectrum with three parameters; a peak frequency (ω_p), a peak enhancement parameter (γ), a shape-parameter (α). There are two σ-parameters, but they can be held constant. The expression for the spectrum, its parameters and valid area of use are found in appendix B.

3.2.2 Torsethaugen

Torsethaugen is a double-peaked spectrum that was originally based on two *JONSWAP* shaped models fitted to average measured spectra from the Norwegian Continental Shelf (Haltenbanken and Statfjord). It assumes that ocean waves at one location can be divided into wind generated sea and swell sea coming from another location. The significant wave height at wind dominated sea can be higher than what is expected for fully developed sea at a given spectral peak period (T_p). This means that swell sea must be present to account for the extra energy. A simplified version of the double-peaked spectrum has later been developed (Torsethaugen and Haver, 2004). The expression for the spectrum, its parameters and valid area of use are found in (Torsethaugen, 1996) from (Torsethaugen and Haver, 2004).

3.3 Wave statistics

The elevation of the water surface, the individual wave heights and a selection of the largest waves can be described using known statistical models and distributions. By utilising these properties the distributions can be used to model dynamic wave loads based on statistical parameters measured at the geographical area of interest. Combined with standardised wave spectra it is possible to replicate realisations of sea loads that are representative for that area, within certain limits of probability.

3.3.1 Surface elevation

The instantaneous wave elevation is a naturally occuring, random process. The Central Limit Theorem, formulated by Laplace and Gauss, states that if you look at random samples of size n from a distribution with mean μ and variance σ^2, then it will converge towards a normal (Gaussian) distribution for large n. This is true even if the samples come from a distribution that is not neccessarily normal (Milton and Arnold, 2003).

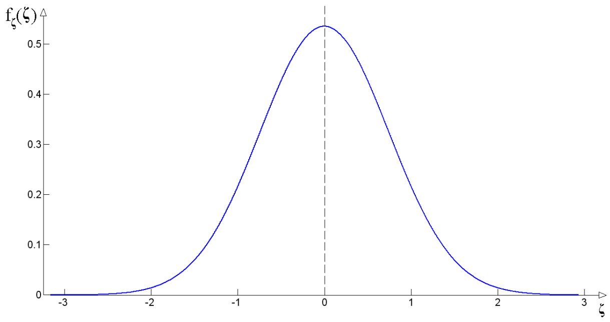

Figure 3.2: The surface elevation distribution.

This is illustrated in figure 3.2, that shows the Gaussian distribution of the surface elevation in a 6000 second realisation of a irregular sea state with significant wave height (Hs) of 3 meters, and a spectral peak period (Tp) of 6 seconds. The dotted vertical line represents the mean, which is $\mu \cong 0$.

3.3.2 Individual wave heights

The global maxima and minima of a Gaussian process will be Rayleigh distributed. Hence, the wave heights will be Rayleigh distributed since they represent the maxima and minima of the surface elevation process. The Rayleigh distribution of the individual wave heights are illustrated in figure 3.3, where the samples are taken from the same realisation as in figure 3.2.

These properties can be useful if the response of the structure is a linear transformation of the surface elevation. The linear transformation of a Gaussian process is also Gaussian distributed, and its global maxima and minima are also Rayleigh distributed. For slender marine structures the quadratic drag term in the Morison equation can be significant, resulting in a non-Gaussian hydrodynamic load. Therefore can the Gaussian process be used as a reference to measure the non-Gaussianness of the response (Passano, 1994).

The non-dimensional coefficient of skewness γ_1 and kurtosis γ_2 can be used to quantify non-linearities. This is done by comparing the coefficients with the known skewness and kurtosis for

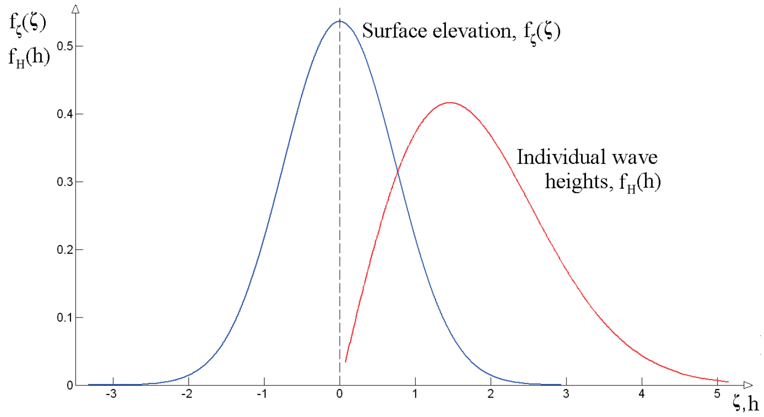

Figure 3.3: The surface elevation and wave height distribution.

Gaussian distributions of $\gamma_1 = 0$ and $\gamma_2 = 3$ (Sødahl, 1991). They are defined from the third and fourth central moments:

$$\gamma_1 = \frac{\mu_3(X)}{\sigma_x^3} \tag{3.2}$$

$$\gamma_2 = \frac{\mu_4(X)}{\sigma_x^4} \tag{3.3}$$

3.3.3 Distribution of the largest waves

When examining the properties of realisations of a stochastic process it is also interesting to look at the expected value and the distribution of the *largest* waves. Each realisation has N number of waves with $H_1, H_2, ..., H_N$ representing the wave heights, and H_{MAX} is the largest. By assuming, perhaps conservatively, that all wave heights are statistically independent the cumulative distribution ($F_{HMAX}(h)$) and the probability density function ($f_{HMAX}(h)$) of the largest wave can be expressed using the zeroth moment of the wave spectrum *(m_0)* and the individual wave height *(h)*. These distributions can be found in appendix A.

From these distributions it is possible to calculate the expected largest wave in a realisation. For large N it can be approximated by:

$$H_M \approx H_{m0}\sqrt{\frac{lnN}{2}} \tag{3.4}$$

Here H_{m0} is an estimate for H_s based on the wave spectrum. This H_M is the wave height that is expected to be exceeded only once during the N number of waves (Myrhaug, 2006). The expected *extreme* value during N number of waves is somewhat higher, and from (DNV, 2009) it is found to be:

$$H_EXT \approx H_{m0}\sqrt{\frac{lnN}{2}}(1 + \frac{0.577}{lnN}) \tag{3.5}$$

To investigate this further 1000 realisations with the same H_s and T_p were simulated from the *JONSWAP* spectrum and analysed. Each realisation was sampled with a different seed number so that each sample was unique, but with the same statistical parameters. From each realisation the highest wave was found, and all of the highest waves were empirically fitted to a Gumbel extreme value distribution.

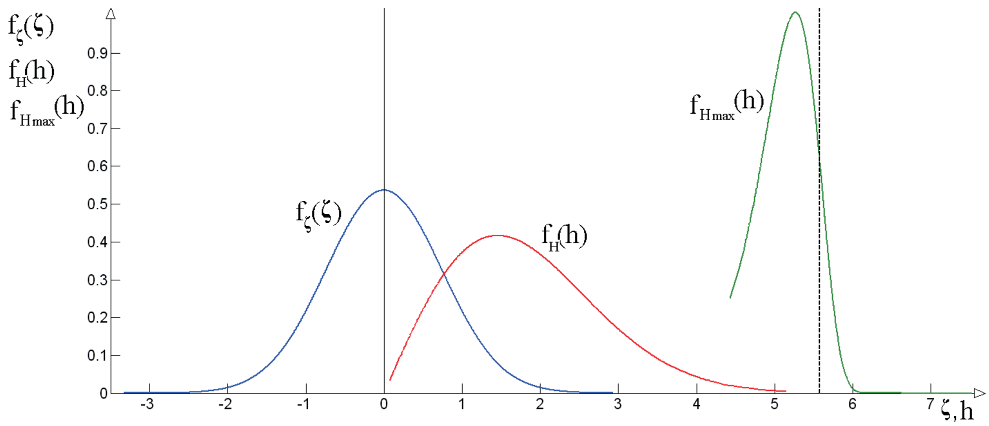

Figure 3.4: The surface elevation, the wave height and the highest waves distributions.

The probability density function of H_{MAX} was plotted in green, with the surface elevation distribution (blue) and individual wave height distributions (red) in figure 3.4. Note that the H_{MAX}-distribution is taken from 1000 individual realisations, and the surface elevation and individual wave height distributions are taken from a single realisation.

The dotted vertical line to the right represents the expected largest wave height as found by equation 3.4. It is exceeded 34.9 % of the times, and the largest of the largest waves was 6 % higher than the calculated H_M.

In dynamic analysis using *regular* waves the response increases with increasing wave height (H). When the frequency of a load is fixed, the response will increase with increasing amplitude of the load. However, in dynamic analysis using *irregular* waves the load frequency is variable, and therefore it is not necessarily true that the highest wave gives the highest response.

With regards to *extreme* waves, a study was done by Olagnon and Prevosto (2005) analysing wave measurements taken in the North Sea over 20 years to investigate the distribution of high waves with respect to conventional predictions. They found that extreme waves are not found more frequently than expected, or less frequently, than what is to be expected from conventional analyses. The H_{MAX} / H_s data fitted to a Gumbel extreme value distribution is found in Figure 3.5.

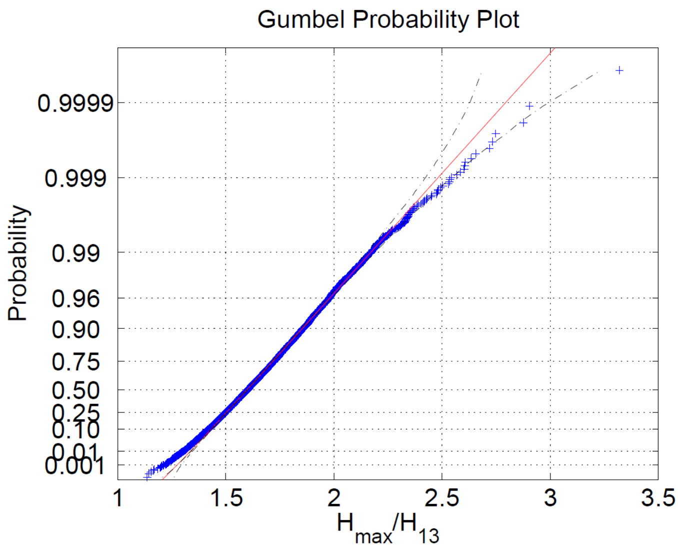

Figure 3.5: Gumble probability plot of H_{MAX} / H_s from North Sea study.

Chapter 4

Methodology

The aim of this chapter is to describe the motivation for examining the subjects of this thesis, and the methods used to investigate them. It will cover the standards and regulations that governs the common practice in the industry, and relevant studies performed on the topic.

4.1 Stating the problem

Laws and regulations set the premise for which standards to follow when analysing structures to be used in the industry. Still, numerous methods exist for analysing slender marine structures that are used in the industry and in educational institutions today. One of the problems is that the standards and regulations are not specific enough to regulate in detail the diversity of types of analysis that can be used.

A common way for designing a shallow water riser is to base the wave load on regular waves, where the wave height is normally set to the characteristic largest wave of the design sea state. This approach has an underlying assumption that the largest wave height will give the largest response. That assumption may not be always correct, at least for a stochastic analysis.

If maximum wave height of the design sea state shows not be the best parameter of determining the hydrodynamic load from incoming waves, other parameters will be explored to find a stronger correlation between regular and irregular wave analysis.

Dynamic analysis of permanent equipment using irregular waves in the time domain is also very time consuming with regards to computational time. Each design sea state must have a sufficient number of periods to account for randomness and a return period for wave heights. Also, each sea state must run several times to have a large enough data set for the statistical models to have stable values. If one or several more fitting parameters are discovered, the maxima / minima values of those parameters can be extracted to run more time-efficient analyses.

4.2 Standards

The government upholds laws and regulations for the equipment used in the industry in the respective country. In Norway, The Petroleum Safety Authority has the authority for the rules and regulations regarding planning, design, construction in all phases of the petroleum activity. Similar governmental agencies exists in other petroleum producing countries. The operators and their subcontractors must follow the regulations laid down by these agencies. The regulations usually calls to follow standards and recommended practices from organisations and foundations such as ISO, API and DNV. In the following sections only the relevant subjects for this thesis are discussed.

4.2.1 ISO

The International Organization for Standardization (*ISO*) is a non-governmental organisation, with a network of the national standards of 162 countries. For Petroleum and natural gas industries it has a standard for design and operation of subsea production systems (ISO 13628), where part 7 is for Completion/workover riser systems (ISO, 2006).

In part **6.2.4** it is stated that: *"The C/WO riser system design shall be based on calculations supplemented by necessary testing."* This calculations can be analytical-based equations or numerical analysis, e.g. finite element analysis or boundary element analysis. And for environmental loads: *"...for permanent operational conditions the most probable extreme combined load effect for a 100-year return period (10-2 annual exceedance probability) shall apply."*

With regards to the irregular wave loads versus regular wave loads, the standard allows the user to decide. In section **6.3.4.3** on extreme load conditions it states that *either* regular or irregular wave analysis in the time domain, irregular wave analysis in the frequency domain, or a combination of the three, can be applied. In any case a variation in wave period shall be performed to identify the most unfavorable loading condition within a 90 % confidence interval from the wave scatter diagram.

4.2.2 API

The American Petroleum Institute (API) is a national trade association in The United States of America, and has developed standards of petroleum and petrochemical equipment and operating standards. The recommended practice document RP2RD covers design of risers for floating production systems (ISO, 2009).

The document states in **6.3.1.6** that the most usual method is to use regular waves in design of shallow water risers, where the wave height is set to the maximum wave height of the design sea state. It also suggests to *consider* irregular wave time-domain simulation or spectral analysis in the frequency-domain, if the natural periods of the riser within the frequency range of the wave spectrum.

With regards to linear versus nonlinear waves it is stated in **6.3.1.3.5** that nonlinear aspects of steep waves should be considered for calculating the impact velocity on the riser, using third order

Stokes wave theory and the Green-Naghdi theory of fluid sheets. A short study performed with a simple top tensioned riser model suggested that using regular 5^{th} order Stokes waves can give significantly larger bending moments than regular linear (Airy) wave theory, at least for certain wave periods. Some of the results are shown in Figure 4.1.

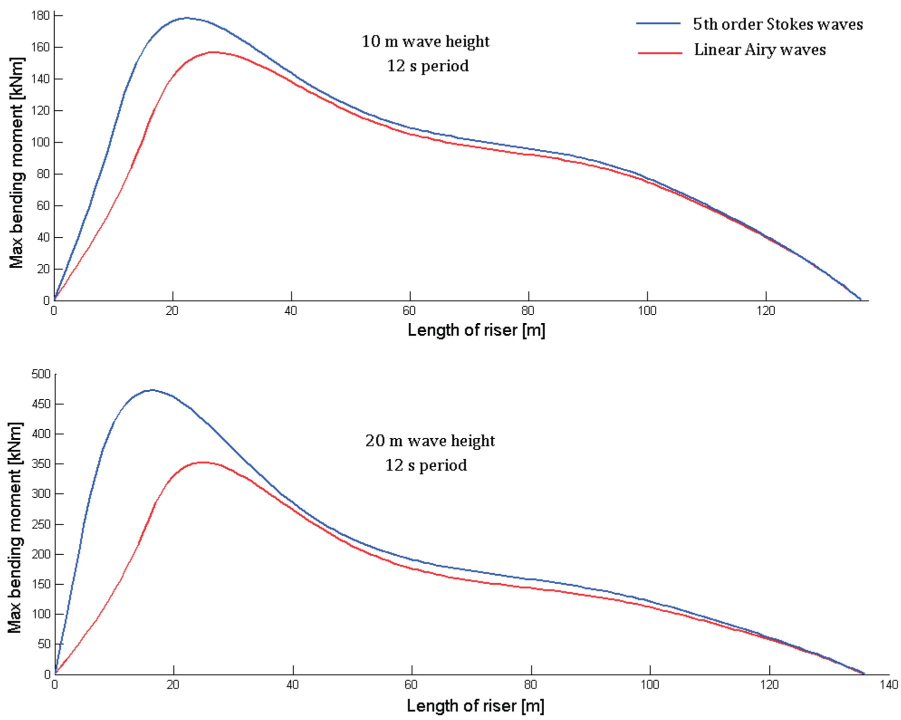

Figure 4.1: Linear Airy waves versus 5^{th} order Stokes.

In the figure, the value 0.0 at the x-axis is at the top of the riser 16 meters above the mean sea level, and the y-axis shows maximum bending moment. Only a limited number of wave periods and water depths were analysed, and further inquiries must be made to draw final conclusions. However, to use the most conservative method, 5^{th} order Stokes waves are used in regular waves in the case study of this thesis.

A far more thorough study on nonlinear, irregular waves was performed using a large number of 4.5 hour realisations for several sea states (Stansberg, 1998). Although not looking at the response, the study showed a 15 % increase in the extreme crest of a typical 100-year sea state, relative to the Rayleigh estimate, when using second-order waves.

4.2.3 DNV

Det Norske Veritas (*DNV*) is an independent foundation that creates standards and recommended practices for a large variety of industries. The different recommended practices have several ways of describing the recommended length of irregular wave analysis and the number of analyses needed to ensure a wide enough variation, depending on the type of operation. The Recommended Practice (RP) named *RP-H103 - Modelling and Analysis of Marine Operations* (2009) address many of the main topics in this thesis.

The Storm Factor (SFT) is the relationship H_{MAX}/H_S, and its recommended time periods of use is found in table 4.1 taken from (DNV, 2004). This recommendation is for operations concerning removal of offshore installations.

Table 4.1: DNV Recommended Practice - Storm Factor.

T_R - Operation Reference Period	Storm Factor
$T_R < 10$ min.	1.6
10 min. $\leq T_R < 30$ min.	1.7
30 min. $\leq T_R < 1$ hour	1.8
1 hour $\leq T_R < 3$ hours	1.9
3 hours $\leq T_R < 72$ hours	2.0

Concerning the probability limit for marine operation it is stated in *DNV Rules for planning and executing marine operations* in section **1.1.2.1** "Recommendations and guidance aims at a probability of structural failure equal to, or better than 1/10000 (10^{-4}) per operation."

4.3 Common practice

In the industry today the operators and the subcontractors are required to follow the standards and regulations imposed by the The Petroleum Safety Authority. In Norway this means that the ISO 13628-7 standard must be followed when analysing marine risers to be used in the petroleum industry, supported by recommended practices, such as DNV RP‘s.

4.3.1 Type of analyses

The ISO-standard does not demand stochastic analysis in the design of risers, but suggests that either irregular or regular wave analysis should be performed. An operator may demand that regular wave analysis should be confirmed by comparing it to an irregular wave analysis, but there are no clear guidelines as to how such an analysis should be executed. The standard do not specify the simulation length of such an analysis of permanent equipment, nor the number of analyses needed to ensure stable values with low standard errors. The standard do not specify what extreme value statistical models should be used, how the statistical data should be managed and the probability limits that applies in different configurations and operations.

As a result of this lack of guidelines, the common practice is that the operators only demand regular wave analysis for riser design, with a wave height based on the most probable highest wave calculated from DNV RP‘s, and a variation of T_p-values extracted from a scatter diagram of measured wave data for the given location. For normal sea states operators accepted the use of a storm factor of 1.9, as described in section 4.2.3.

4.3.2 Operation limitations

To determine the limitations for the safe use of equipment based on sea states, several analyses with combinations of sea states and configurations must be performed. As mentioned, it is common to use regular waves in dynamic analysis, and use the expected largest wave for the given sea state. The combinations of different sea states must be extracted from a 90 % confidence interval of the scatter diagram from long-term statistics measured at the area of interest. An example is shown in Figure 4.2, with the circles representing sea state combinations of H_s and T_p within the 90 % confidence interval.

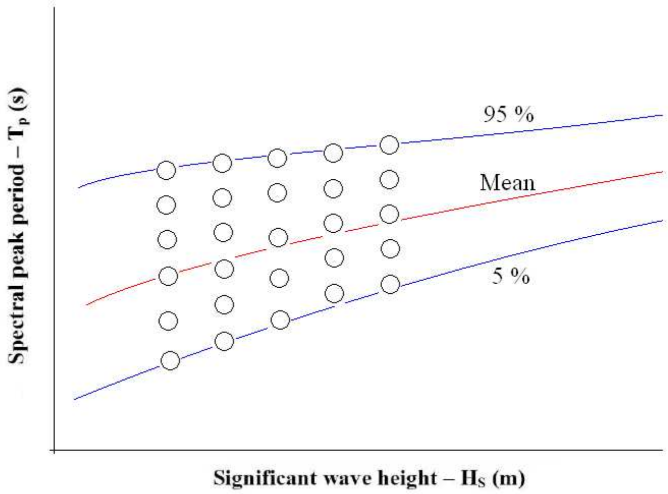

Figure 4.2: Selecting sea states for operation limitation analysis.

The operation limitations for a structure is usually only given in significant wave height, not peak period. It follows that for each limiting H_s must be valid for all T_ps. That means that if a structure is limited to e.g. $H_s = 5$ meters, then the ultimate strength of the structure is never exeeded for all possible sea state combinations within the 90 % confidence interval of the scatter diagram for $H_s = 5$ meters and lower. The procedure is repeted for each rig offset that is possible for the rig to obtain without being forced to disconnect.

If the same procedure should have been performed using irregular waves the methodology changes. For each sea state a large set of analyses is needed, and each analysis must run for a significant simulation time. Then the maximum response is extracted from each analysis and extrapolated to a given probability level or return period using an extreme value distribution. This must be repeated for all sea states, and all rig offsets. The details of simulation length, probability level and distributions are discussed in the following sections.

4.4 Literature review

There are many papers, studies and books on the subject of wave modelling, stochastic analysis of marine structures, reducing computational time of nonlinear structures and finding parameters that correlates strongly with the response. Here, a few relevant cases will be highlighted, in addition to the ones that are mentioned in the sections of the thesis where they are relevant.

Methods for stochastic analysis of slender marine structures has been investigated for many years. Sødahl (1991) found methods for using stochastic analysis and extreme value statistics for describing the long-term response of nonlinear structures such as flexible risers. This method calls for long and numerous simulations to achieve a large enough data set to reduce standard errors and fit the extreme value distributions well.

Several ways of reducing the computational time needed to run a stochastic analysis in the time domain are suggested by Passano (1994). One of the methods is to assume that the largest response come from the parts of the realisation that have the largest waves. These parts are extracted from the time series, and a long simulation can be shortened to a selection of the largest waves, as shown in Figure 4.3 from (Passano, 1994). The underlying assumption that the highest waves give the largest response may not be true for all systems, and must be investigated further.

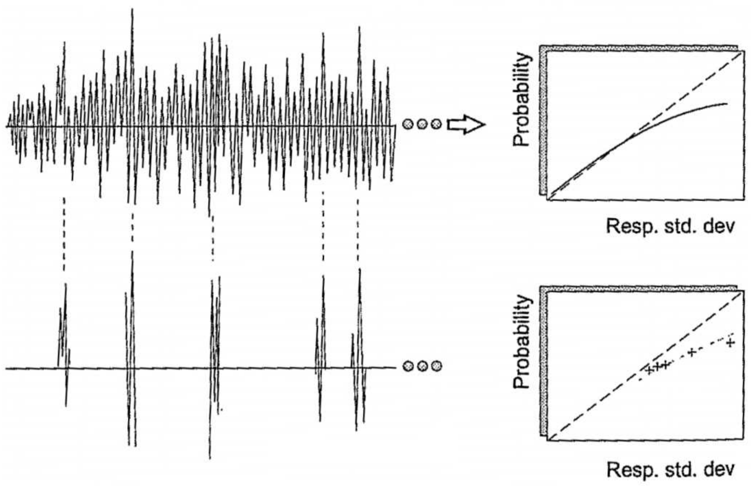

Figure 4.3: Simplified method of extracting wave maxima.

Passano and Larsen (2007) investigates other parameters than wave height to predict extreme response for a catenary riser. A clear trend was found between the axial velocity at the upper end of the riser and the response of the riser directly above the touch-down area. The study showed a method for estimation of the underlying response distributions that agreed with longer simulations, and the approach seems promising for reducing computational time drastically. It is improbable that the same relationship is to be found in top tensioned risers, as the axial velocity is close to zero. However, the study shows that in dynamic analysis other parameters can be identified to show strong correlation to the response.

4.5 Methodology for the Case Study

In this section the methodology for the case study will be presented. First, there are a few important variables with regards to irregular analysis that were not specified in the standards. The length of simulations, the number of simulations per sea state and lastly the probability level. When these variables have been decided, the procedures of the case study can be set.

4.5.1 Length of simulations

The duration of simulations when using irregular waves can be difficult to decide. The expected largest wave height increases for increasing number of waves, see Figure 4.4 and equation 3.4. Guidance from the standards are also not complete. For specific marine operations, such as towing operations or lifting equipment through the wave zone, it is recommended to perform 3 hour simulations in irregular sea states. And for running lifting equipment through the wave zone, at least 10 simulations for each sea state should be performed (DNV, 2009).

However, none of the governing standards or RP‘s prescribes the recommended length of a simulation of an irregular wave analysis for permanent equipment, or the number of analyses for each sea state. This is perhaps because it is not a straightforward topic to make absolute and clear regulations for. One could argue that it should be as long as the duration of the short-term sea state that is used. A short-term sea state is defined from Myrhaug (2006) from 20 minutes to 3 hours, and from DNV RP-H103 it states that a sea state is usually take to be 3 hours. Figure 4.5 is taken from (Myrhaug, 2006), where the duration of a sea state is plotted as a function of H_S. It is based on measured data from the winter season at Haltenbanken in the North Sea, and shows that sea states with high H_S have a short duration, and low H_S can have longer durations. E.g. an $H_S = 15$ meters lasts for typically 1 hour, and $H_S = 5$ can last for 8.1 hours. These variations can make it difficult to base simulation lengths on sea state durations.

Another way of determining the simulation length is to perform a sensitivity study, and determine how long the simulation must run for the response maxima to become stable. Of course, the longer a simulation runs, the higher the probable highest value gets. But the simulation should run until randomness of the rig movement and the riser response is ensured, and the values can be extrapolated without a large systematic error.

A third way of deciding simulation lengths is to base it on the number of waves N. If the mean zero crossing period *(T_z)* is known, then the duration D is:

$$D = NT_z \tag{4.1}$$

The issue here is that the *(T_z)* is determined from *measured* data, but it can also be estimated from the wave spectrum. Then it is denoted T_{m0}, and found from:

$$T_{m0} = 2\pi\sqrt{\frac{m_0}{m_2}} \tag{4.2}$$

where m_0 and m_2 is the zeroth and second moment of the spectrum. For spectra of the PM type, the relation between the spectral peak period *(T_p)* and the spectral mean zero crossing pe-

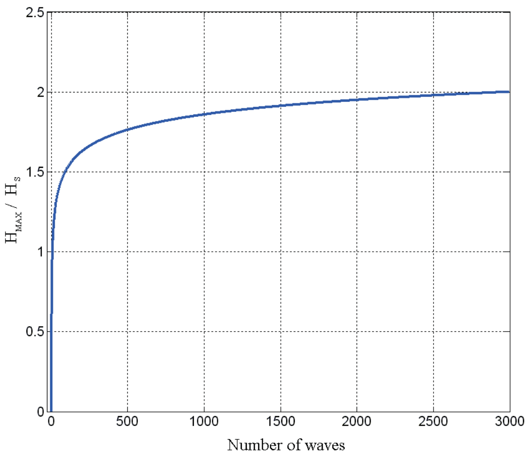

Figure 4.4: Plot of H_{MAX} / H_S as a function of N number of waves

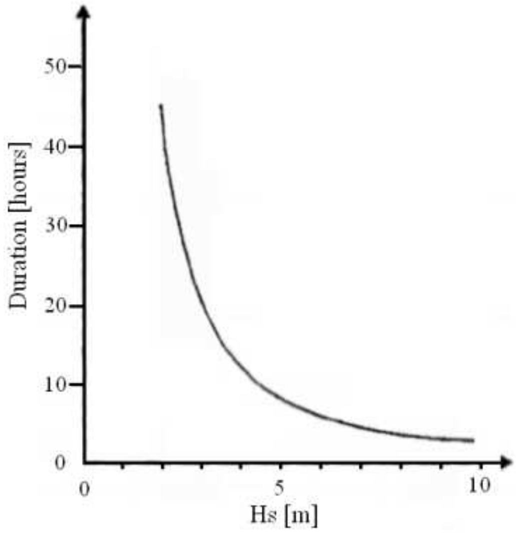

Figure 4.5: Duration of sea states as a functuion of H_s.

riod *(T_{m0})* is T_p / T_{m0} = 1.41. For the *JONSWAP*-spectrum the relation is related to the peak enhancement parameter (γ), and varies between 1.2 and 1.4 (Myrhaug, 2006).

This relationship will be exploited in determining the simulation lengths in the case study. The storm factor of H_{MAX} / H_S = 1.9 taken from section 4.2.3 is the most probable largest wave based on 1366.5 waves. If T_p is used in equation 4.1 instead of T_z, the duration would be N number of waves multiplied by the spectral peak period in the sea state.

$$D = NT_p \tag{4.3}$$

In the case study the number of waves is set to N = 1000 based on peak period (T_p), and therefore the duration of the simulations vary with the T_p. The advantage is that all simulations, regardless of sea state, can be measured against the regular wave analysis used in common practice of using H_{MAX} / H_S = 1.9, as shown in Table 4.2. Given the relationship between T_p and T_{m0}, the number of measured waves T_z will give a storm factor close to 1.9. It also makes sense to base the simulation length on the number of waves, since the statistics also use number of waves.

Table 4.2: Simulation length and properties.

T_p [s]	Simulation length [s]	Mean no. of waves	Standard deviation	Storm Factor (from mean)
8	8000	1208	8.5	1.88
10	10000	1317	11.2	1.90
12	12000	1382	10.1	1.90

4.5.2 The number of simulations

The utilisation was chosen as the parameter to be evaluated for sensitivity to the number of runs performed. The utilisation is the capacity of each riser joint with respect to tension and bending according to ISO (2006). The maximum utilisation for the entire riser was extracted from each run. The extrapolated value from the Gumbel distribution to probability levels of exceedance of every 100, 1000 and 10 000 times was plotted against the number of runs. The result is shown in Figure 4.6.

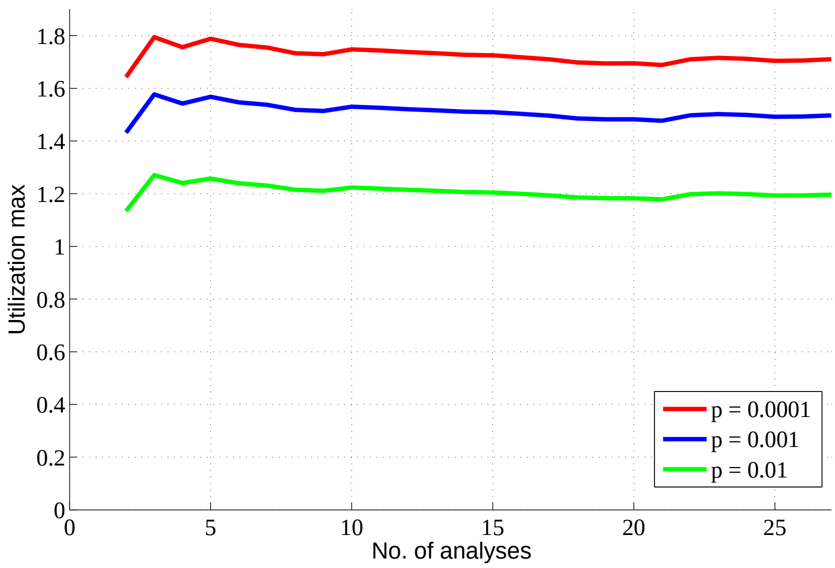

Figure 4.6: The no. of analyses to get stable values. Sea state: $H_s = 5$ m, $T_p = 8$ sec.

The graph shows stable results already after 10-15 runs, and to have a good margin at least 20 analyses will be performed for each sea state.

4.5.3 Selecting the probability level

After each sea state has bed simulated 20 times, the data is fitted to a extreme value distribution. Using the distribution the value for a given probability level or return period can be extrapolated. Long term wave statistics is used to find the return period of a given sea state. From scatter tables as shown in Figure 4.7 the occurrence of each sea state can be extracted, and calculated to represent a return period. The table is for sea states of 3 hours, and normalized to the number of sea states during approximately 34 years at a specific location in the North Sea.

The three sea states in the case study are marked with red. If these values are divided by 34, we find the expected occurrence per year. Then, the occurrence per year is multiplied by the

H_s	SPECTRAL PEAK PERIOD																			SUM
	0-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	<20	
0-1	55	426	1105	1561	1545	1228	849	536	319	182	101	55	30	16	9	5	3	1	2	8028
1-2	4	136	992	2957	5061	6057	5704	4553	3234	2113	1300	765	436	243	133	72	39	21	23	33843
2-3	0	5	104	678	2048	3703	4709	4670	3869	2808	1846	1127	650	359	192	100	51	26	26	26971
3-4	0	0	4	61	370	1108	2038	2639	2640	2178	1552	989	578	315	163	81	39	18	15	14788
4-5	0	0	0	3	37	208	613	1121	1438	1410	1125	765	459	249	125	59	26	11	8	7657
5-6	0	0	0	0	2	25	131	373	664	823	771	580	366	201	99	44	19	7	4	4109
6-7	0	0	0	0	0	1	17	83	225	380	444	387	267	153	75	32	13	5	2	2084
7-8	0	0	0	0	0	0	1	11	53	132	207	221	175	108	55	23	9	3	1	999
8-9	0	0	0	0	0	0	0	1	8	33	75	105	101	70	38	16	6	2	1	456
9-10	0	0	0	0	0	0	0	0	1	6	20	40	49	41	25	11	4	1	0	198
10-11	0	0	0	0	0	0	0	0	0	1	4	12	20	21	15	8	3	1	0	85
11-12	0	0	0	0	0	0	0	0	0	0	1	3	7	9	8	5	2	1	0	36
12-13	0	0	0	0	0	0	0	0	0	0	0	0	2	3	4	3	1	0	0	13
13-14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	4
14-15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
SUM	59	567	2205	5260	9063	12330	14062	13987	12451	10066	7446	5049	3140	1789	942	461	216	97	82	99272

Figure 4.7: Joint frequency table for sea state duration of 3 hours, normalized to the number of sea states during approximately 34 years.

return period of e.g. 1, 10 or 100 years. This gives N_m, which is the number of occurrence of sea state per m years. From this number the probability level p for each sea state for the given return period is found from Equation 4.4:

$$p = \frac{1}{N_m} \tag{4.4}$$

This probability level p can be used in the cumulative density function of the extreme value distribution to find the value that corresponds to that return period. All results found in this way will be presented with the results from using return periods of 1, 10 and 100 years. The probability levels for the sea states in the case study is shown in Table 4.3.

Table 4.3: Probability levels of return periods.

Sea	**State**	**No. of**	**Prob. level**	(p)	
H_s	T_p	**occurrence**	1 year	10 years	100 years
5	8	131	0.2595	0.0260	0.00260
5	10	664	0.0512	0.0051	0.00051
5	12	771	0.0441	0.0044	0.00044

4.5.4 Analysis procedure

For each sea state the same procedure must be performed. The procedure is different for regular and irregular analyses. The procedure for analyses using *irregular* waves is:

1. Run at least 20 analyses of T_p x 1000 seconds duration.

2. Post-process, and extract maximum response from all simulations.
3. Fit the data to an extreme value distribution.
4. Set the return period.
5. Extract the value from the cumulative density function.

And for analyses using *regular* waves:

1. Run one analysis for 80 seconds, using the expected largest wave.
2. Post-process, and extract the maximum response.

4.6 Selecting statistical models

Selecting a statistical model to fit the data is of crucial importance. Choosing the wrong model will lead to misrepresentation of data, and can lead to overestimation, or worse, underestimation of the response. The methods used here are based on fitting sampled data to the different distributions to estimate the extreme values. Then the distribution can be extrapolated to the desired duration, or probability level.

The extreme value distributions that are considered in this thesis are the General Extreme Value (GEV) distribution, the Gumbel distribution and the Weibull distribution. The functions and several other properties for all distributions are found in appendix A. When selecting the distribution that fits the data best, several tools can be used.

For fitting sampled data to distributions, and estimating the parameters some *Matlab*-routines was used. The WAFO toolbox (Brodtkorb et al., 2000) is a project by the University of Lund in Sweden that shares freely redistributable software for wave and response statistics. Other *Matlab*-routines have been written to supplement the WAFO programs to manage statistical data and results. All such programs are found in Appendix C.

4.6.1 Standard errors

If a distribution is selected that does not fit the tail of the sampled data, then a systematic error is introduced that will not be reduced by increasing the number of data points. The easiest way of quantifying the statistical uncertainty is by generating several sample realisations and estimating the mean value and standard deviation of the extreme values. Estimators for standard errors can also be used (Sødahl, 1991).

4.6.2 Residual analysis

When selecting distribution it is possible to plot the sampled data and observe if the data seems to fit or not. A more analytical method is to use a residual analysis. Ideally the plot of of the residuals should show random fluctuations around the value zero (Walpole et al., 2002). If the residuals appear to behave randomly, and not follow a systematic pattern on one side of the value

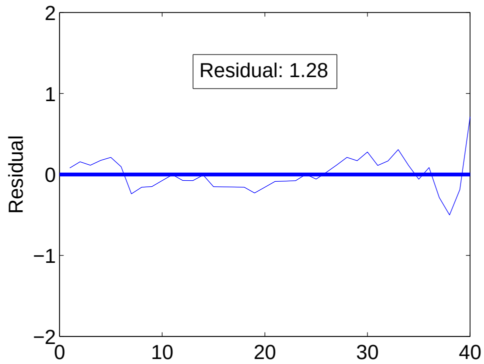

Figure 4.8: Residual for a good fit to data.

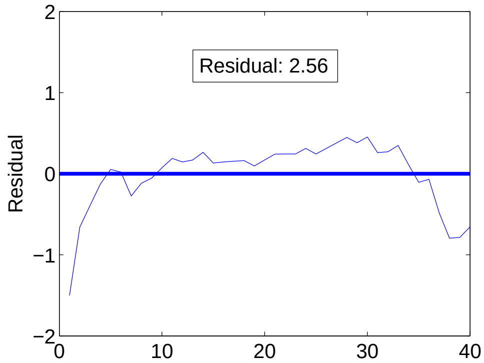

Figure 4.9: Residual for a poor fit to data.

zero, the model is a good fit for the data. An example is shown in Figure 4.8.

Other methods to quantify the goodness of a fit for a specific model are *the sum of squares due to error (SSE), R-square, Adjusted R-square*, and *Root mean squared error (RMSE)*.

4.6.3 Hypothesis testing of the Gumbel distribution

To ensure that the a distribution is fit to interpret the data, it is also possible to run a statistical hypothesis test. The Gumbel distribution is a General Extreme Value (GEV) distribution with a shape parameter $k = 0$. Therefore the data can be tested by using the Probability Weighted Moments (PWM) estimator of k. When estimating an unknown parameter, the estimator is said to be unbiased if the mean of the estimator (μ_E) is equal to the parameter estimated for finite samples (Walpole et al., 2002). In other words, the estimation will be correct, on average, without systematic errors.

To use the The null hypothesis is that the data has a Gumbel distribution, hence the parameter k = 0 for the GEV (Brodtkorb et al., 2000). The analysis of the sampled data is done with a probability factor for a Type I error (the error of rejecting a valid hypothesis) of 5 %, denoted by the greek letter alpha. This is called the significance level of the test, and common practice is to use $\alpha = 0.05$. The standard way of preforming such tests is:

1. **State the hypothesis:** The null hypothesis: "The sampled data has a Gumbel distribution." The alternative hypothesis: "The sampled data does not have a Gumbel distribution.
2. **Analysis plan:** To use the method of Probability Weighted Moments estimator for k.
3. **Analyse the sampled data:** Test all sets of data according to the analysis plan.
4. **Interpret results:** The null hypothesis is either rejected, or not rejected.

This method will be used to help choosing the correct distribution in the data analysis section of the case study.

Chapter 5

Case study

In the case study a work over riser is connected to the sea bed from a semi submersible rig, with no rig offset.The model is created with a realistic configuration and equipment, and the environmental loads are modeled from measured wave and current data from a field in the North Sea.

This chapter consists of description of the model, the environmental loads, the analysis plan and execution and lastly the results.

5.1 The model

The model is of a top tensioned work over riser. It is an open sea riser, which means it does not have a marine riser. It is connected to the sea floor directly under a semi submersible rig. The top of the riser is connected to the top drive in the derrick of the rig. The top drive is connected to a heave compensation system, and supports the weight of the tension frame and the riser down to a few meters below the drill floor. Here a tension ring is connected to two rucker wires that carries the rest of the weight of the submerged riser, its content, equipment and applies a stable over-pull at the bottom. The basic data is presented in table 5.1.

5.1.1 Sign conventions

The vessel is a semi submersible rig with a heading positive in the x-direction according to the global co-ordinate system given in figure 5.1. The the global co-ordinate system is defined with the origin at the mean sea level *(MSL)*, a positive x-direction to the right along the mean sea level, positive z-direction upwards and the y-direction is positive out of the paper. Only three degrees of freedom are used, since the model is two-dimensional. The definitions are:

- Surge is in x-direction, and positive to port.
- Heave is in z-direction, and positive upwards.
- Pitch is about y-axis, and is positive starboard side up.

Table 5.1: Configuration and properties for the model.

Configuration	
Water depth	380 m
Total riser length	405 m
Average element length	1.37 m
No. of elements	296
Properties	
Outer diameter	0.2191 m
Inner diameter	0.1746 m
Wall thickness	2.225 cm
Dry weight of pipe	197.20 kg/m
Contents fluid density	1024 kg/m^3
Top tension	
Top tension	386 kN
No. of rucker wires	2
Tension in each rucker wire	542.7 kN
Over-pull at sea bottom	200 kN

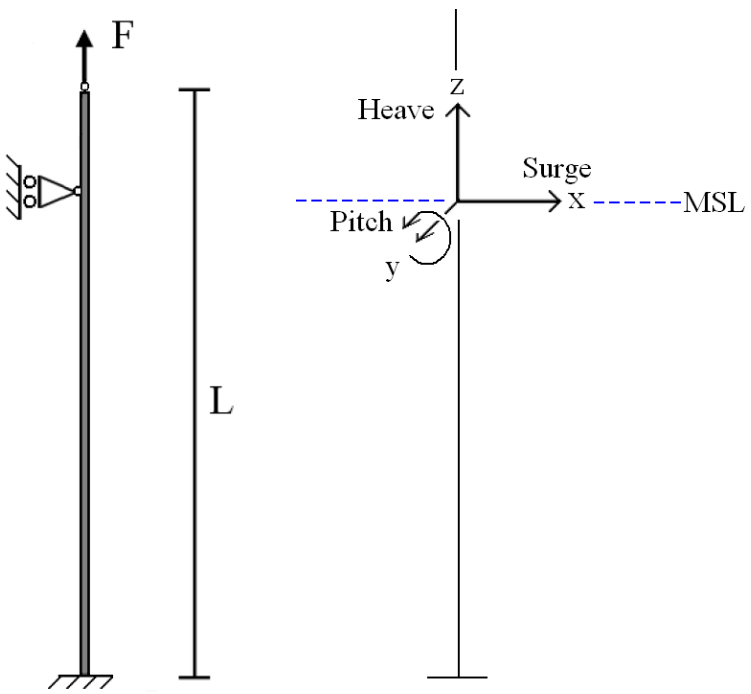

Figure 5.1: Boundary conditions, coordinate system and directions of motion.

Table 5.2: Coefficients used in the case study.

Coefficients	
Transverse drag coefficient	1.0
Axial (tangential) drag coefficient	0
Transverse fluid inertia coefficient	2.0
Transverse added mass coefficient	1.0

5.1.2 Boundary conditions

Topside the riser is fixed from all translations and rotations to the top drive. The top drive is fixed with respect to rotation around the z-axis and the x-axis, and fixed with respect to translation in the y-direction. At the drill floor the riser runs through a slick-joint. The riser slides up and down when the rig moves, and it has a continuous contact formulation through the slick-joint.

At the sea floor the riser is connected to a tapered stress joint. Below the stress joint is fixed with respect to all translations and rotations. The tapered stress joint is modeled as several smaller pipes with incrementally increasing width and stiffness. When looking at elements higher up the riser and topside this boundary condition is indistinguishable from a totally fixed condition.

5.1.3 Element type

The element type used to model the riser in Abaqus is linear, in-plane *PIPE21H*-elements. It is a type of Timosenko beam modelled as pipe, and it allows for transverse shear deformation. The elements are linear elastic with a fixed modulus of elasticity, and therefore independent of the response of the beam section to axial stretch and bending. The H stands for hybrid formulation, which makes the elements well suited for cases involving contact. This is useful in contact formulations where the riser moves up and down through the drill floor through a slick-joint with continuously changing contact points (Abaqus, 2008).

5.2 Environmental loads

Environmental loads are waves, currents and loads caused by environmentally induced vessel motions. The motion of the rig is generated in advance using a transfer function file with response amplitude operators *(RAO)* and phase angles. Loads from forces such as buoyancy, hydrostatic pressure, internal pressure, inertia forces and fluid mechanical calculations are handled by the *Abaqus/Aqua* program. Geographically specific loads such as earthquakes and ice will not be covered.

5.2.1 Waves

The sea states assessed this case study are from an area in the North Sea with known long term wave statistics. The sea states to be used are taken from the scatter diagram in figure 5.2. The reason for using $H_s = 5$ meters is that it is a commonly used limit for operation limitations for connected top tensioned riser in normal operations. The three T_ps that are used are selected to investigate the sensitivity of spectral peak frequency.

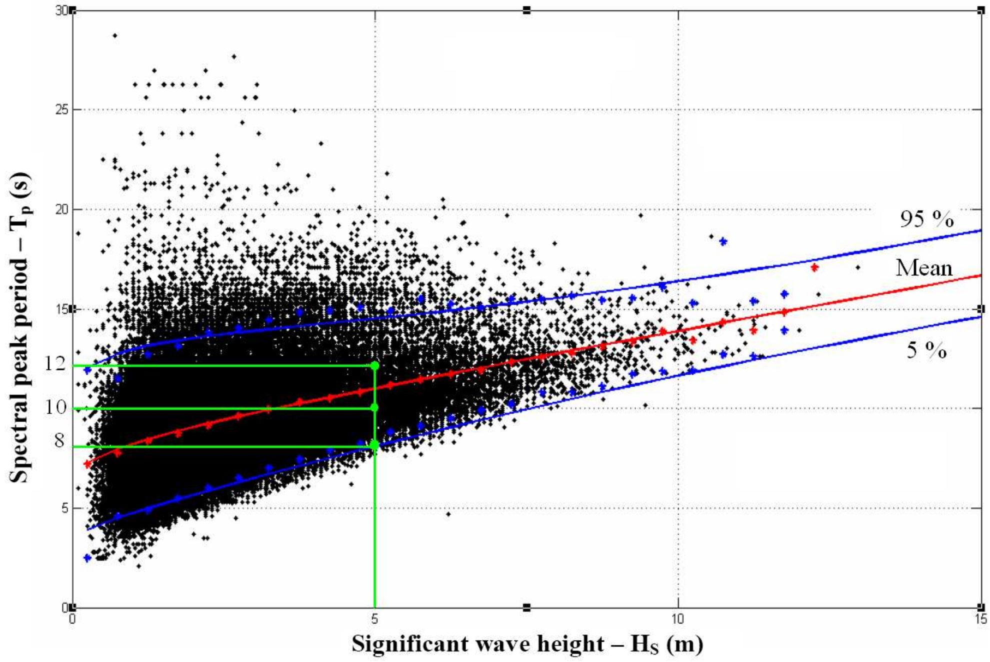

Figure 5.2: Scatter diagram showing the selected sea states.

All waves are uni-directional, and formulated as stated in section 3.1.1. They are incoming waves, and propagating in the positive x-direction of the global co-ordinate system. The sea states and the *JONSWAP* parameters are given in Table 5.3.

Table 5.3: Wave parameters.

Sign. wave height H_s	**Peak period** T_p	**Peakedness factor** γ
5	8	5.39
5	10	1.60
5	12	1.09

5.2.2 Current

The design current velocity, profile and direction shall be selected using recognized statistical methods. In the case study a current with a one-year return period is used, based on statistics from measured data in the North Sea. The current profile is shown in Figure 5.3.

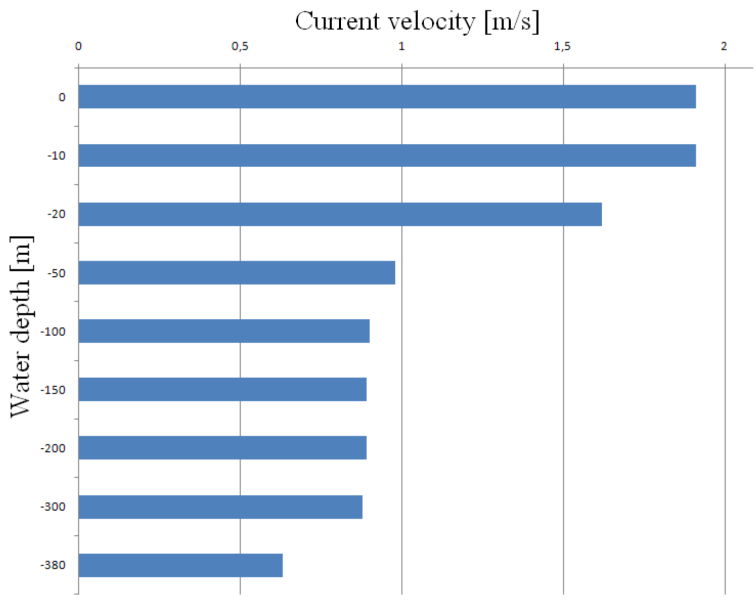

Figure 5.3: The current profile with one-year return period.

5.3 Analysis of the model

The procedure outlined in the Methodology chapter calls for at least 20 irregular wave analyses per sea state, and at least one regular wave analysis for each sea state for comparison. The analyses are nonlinear simulations in the time domain, with a time step of 0.1 seconds. *Abaqus* is the main program that runs all the analyses. Other specific programs generate the input-files, and simplifies the post-processing.

5.3.1 Irregular analysis

The irregular wave analyses were performed according to the procedure described in 4.5. Since the length of the simulations vary with the peak period, the computational time also varies. The analyses were run on HP XW8400 Work Stations, with the ability to run up to four analyses simultaneously. The approximate CPU-time for each sea state is shown in Table 5.4.

Table 5.4: Computational time for irregular analysis.

Sea	**State**	**Simulation time**	**CPU time**
H_s	T_p	[s]	[hours]
5	8	8000	12
5	10	10000	15
5	12	12000	18

This results in about 900 hours of computational time to achieve 20 analyses per sea state. By dividing the work load on 2 computers, and utilising the 4 cores in each computer the theoretical total time was less than 5 days. In reality, more than 20 analyses were run for each sea state. The first sea state was run to study the sensitivity of stable values to the number of runs. Also, some analyses failed, some analyses were discarded due to input errors or post-processing errors.

5.3.2 Regular analysis

The regular analyses were run for 80 seconds after start-up values had stabilized. For each sea state a regular wave analysis was performed with the common practice $H_{MAX}/H_s = 1.9$ factor to replicate the characteristic largest wave height of the simulation time used in the the irregular analyses. This results in 9.5 meter high regular waves (H), with the same periods (T) as the spectral peak periods in the irregular waves. The results from the regular analyses will be used to compare regular and irregular analyses. The analysis plan for regular waves are shown in Table 5.5.

Table 5.5: Computational time for regular analysis.

Sea	**State**	**Simulation time**	**CPU time**
H [m]	T [s]	[s]	[minutes]
9.5	8	80	6.5
9.5	10	80	6.5
9.5	12	80	6.5

This results in a huge benefit when it comes to computational time. In this case study a regular wave analysis is more than 2200 times faster than running 20 analyses of $T_p = 8$ seconds, or more than 3300 times faster than $T_p = 12$ seconds with irregular waves. This is computational time per sea state. In order to determine the operational limitations for a riser a large number of combinations of sea states must be performed.

5.3.3 Post processing

The post processing of the data extracts the parameters that are of interest. Below is a list of the output that will be extracted as a function of time:

- Bending moment
- Shear force
- Effective tension

and the utilisation of the element with the highest utilisation in the entire simulation.

For each simulation the unique surface elevation, the nodal displacement for the rig and the element nodes as a function of time can be found. The following are the parameters that will be explored for finding a correlation between regular and irregular analyses:

- Rig COG displacements in surge, heave and pitch.
- Each elements nodal displacements in surge, heave and pitch.
- Complete surface elevation.

From these the extreme values can be extracted, such as the maximum wave heights, or the largest displacements in the degrees of motion for both the rig and each node.

5.3.4 Selecting extreme value distribution

In order to select the best distribution, the methods described in Section 4.6 must be used. The data is fitted to the Gumbel and the Weibull distribution, and the residuals of the fit is plotted in Figure 5.4 and 5.5 for $H_s = 5$ m, $T_p = 8$ seconds.

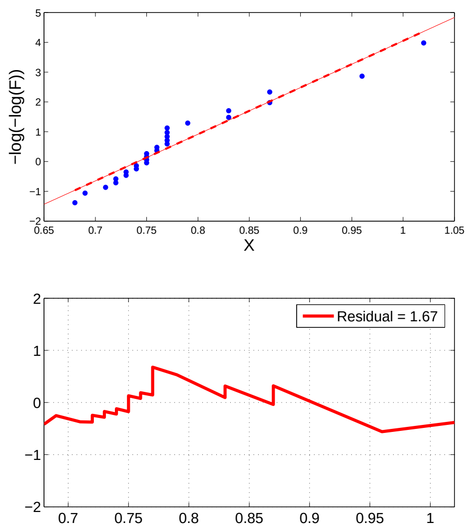

Figure 5.4: Fitting data to Gumbel distribution, and residuals.

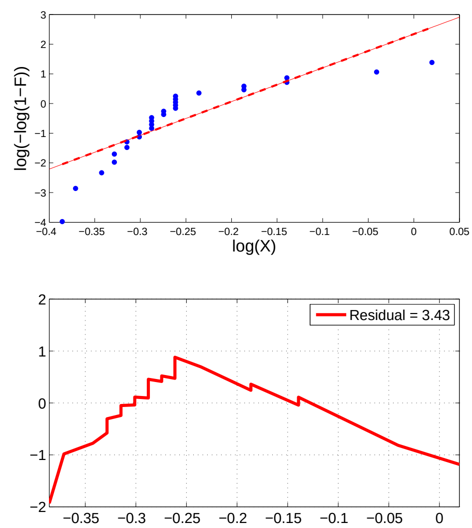

Figure 5.5: Fitting data to Weibull distribution, and residuals.

The plot seems to fit the Gumbel distribution well, but not perfectly. The residual value is 1.67, and seems to somewhat vary close to the value of zero. However, the data does not seem to fit the Weibull distribution as well. The tails of the plot is showing a systematic error, and the data seems to follow a nonlinear trend. The residual plot agrees with a residual of 3.43.

The data sets were fitted for the two other sea states, as shown in Figure 5.6 and 5.7. The fit for $T_p = 10$ seconds is also not perfect, but can still be used. However,when drawing conclusions from extrapolations of the data it must be kept in mind that it is not a perfect fit. For $T_p = 12$

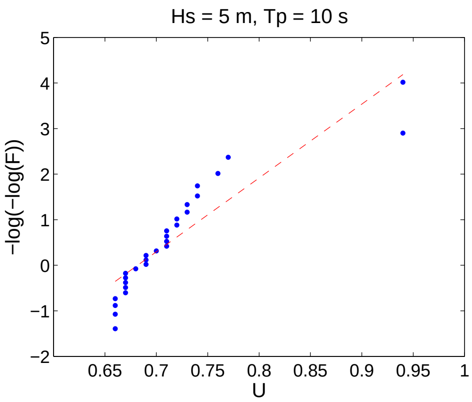

Figure 5.6: Fitting data to Gumbel distribution, $H_s = 5$ m, $T_p = 10$ sec.

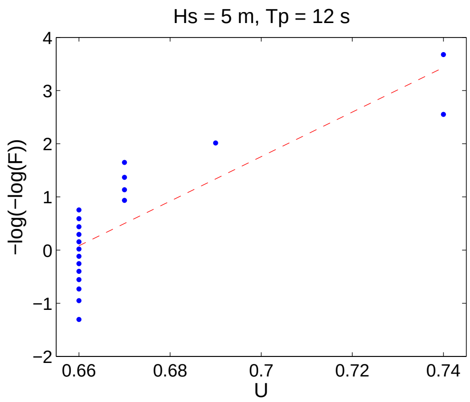

Figure 5.7: Fitting data to Gumbel distribution, $H_s = 5$ m, $T_p = 12$ sec .

seconds the distribution is not a good fit for the data, as it seems to group around specific values. The reason for this is a very low variance in the data, and it follows that extrapolations can not be drawn from this data set. The data from $T_p = 12$ seconds will therefore not be included in the results from extreme value statistics, but the simulations can still be used to investigate parameters that correlates with regular analysis.

The data was also subjected to the hypothesis test described in Section 4.6.3. The test rejected the null hypothesis that the data fits a Gumbel distribution for all three sea states, with a significance level of $\alpha = 0.05$. This suggests that the Gumbel distribution may not be the best distribution. The solution may be to formulate a tail-fitting technique, as done in (Sødahl, 1991), or to use another distribution. The generalised extreme value distribution may fit better, but since it has three parameters it is more sensitive to the number of runs to get sufficient confidence of the results. Therefore it can not be used with certainty on the data sets available in this case study. The Gumbel distribution is therefore preferred, but the goodness of the fit is questionable.

5.4 Results

The results from the case study are divided into two main sections. The first concerns the direct results from the regular and irregular analyses, and comparing the extreme value results. The second section investigates relevant properties of single simulations to see if some parameters correlates strongly with the response.

5.5 Extreme value results

As discussed in the Methodology chapter, the method for calculating the extreme value response is different for analyses using regular and irregular waves. The extreme values from *regular* waves were extracted from one 80 second simulation, using the largest expected wave height for the given sea state. Then the utilisation for the element with the largest utilisation was found after post-

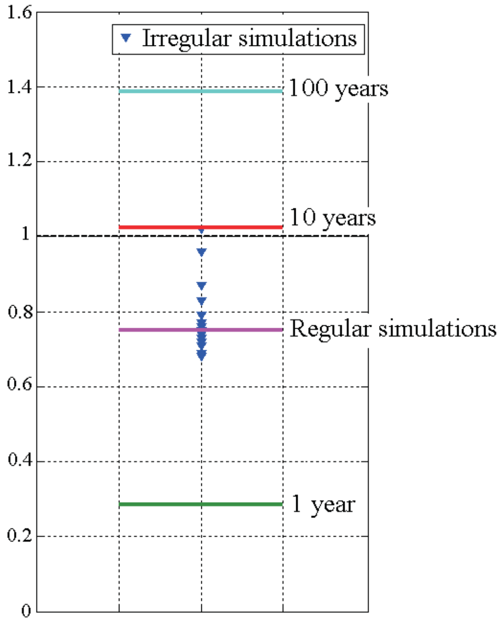

Figure 5.8: Extreme value results from Hs = 5 m, Tp = 8 sec.

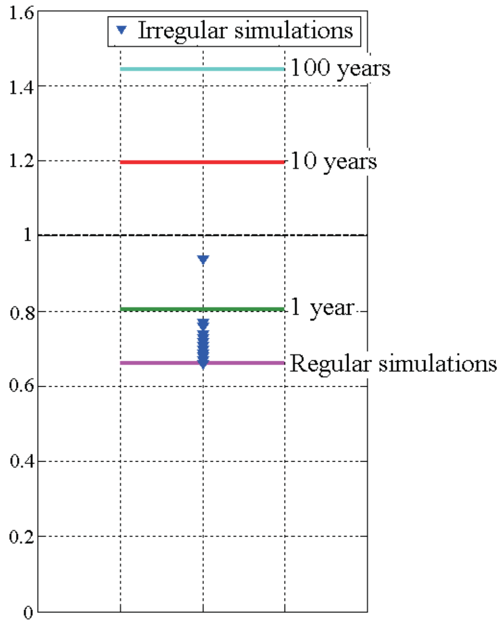

Figure 5.9: Extreme value results from Hs = 5 m, Tp = 10 sec.

processing.

The extreme values from *irregular* waves were calculated using the cumulative density function of a Gumbel extreme value distribution. It used the results from a large number of simulations, each running for T_p x 1000 seconds duration. The extreme value was calculated for a probability level corresponding to return periods of 1, 10 and 100 years. The results for T_p = 8 seconds and T_p = 10 seconds are shown in Figure 5.8 and 5.9.

However, it is not absolutely correct to compare a regular wave analysis based on the largest wave in *one* sea state duration to irregular wave results based on a return period of the same sea state during one, ten or a hundred years. The sea state (Hs=5, Tp=8) occurs on average almost four times per year, and almost 400 times in 100 years.

It is also interesting to observe that the extreme value from the regular wave is exceeded more than $\frac{2}{3}$ of the time in (Hs=5, Tp=8), and every time in (Hs=5, Tp=10) by the individual irregular analyses. This suggests that the stochastic response of the system based on an irregular load can be larger than if it is subjected to a regular load. The cause for this is not obvious, and might be one of the following reasons:

- As seen in section 3.3.3, the actual largest wave height can be higher than the expected largest wave. Higher waves can lead to larger response, and cause the difference in results.

- The stochastic behavior of the system subjected to irregular waves can lead to combinations of displacement of the rig that cause higher forces. It might be the combination of maximum displacement in two or all three degrees of freedom of the *rig* that leads to the maxima in the irregular wave response.

- It might be the combination of maximum displacement in two or all three degrees of freedom of *specific elements* that leads to the maxima in the irregular wave response.

- As seen in section 2.6, the relative horizontal velocity of the response is a significant part of the drag force. This response velocity of can be different in the regular waves and the irregular waves.

If one of these parameters can be identified as correlating in the two different analyses, then it might be possible to use regular waves to simulate the same response as in irregular waves, and save computational time. Another approach is to identify the parameters that give the largest response, and create an irregular wave realisation that corresponds to that repsonse. Using the transfer function and phase angles a realisation can be computed using Fast Fourier Transforms (FFT) to generate only the surface elevation that gives the largest response.

5.5.1 Correlation in wave height

Since the expected largest wave height can be exceeded, it is interesting to examine the effect that the largest waves have on the response with irregular wave loading. First the extreme values are examined. From each simulation with irregular waves the largest response is found. Then, for the same simulation, the surface elevation is analysed to find the largest wave height. From regular waves we could expect to find a correlation between maximum wave height and maximum response. The values from sea state (Hs=5, Tp=8) is shown in Figure 5.10.

The figure shows an apparently random scatter of values. If the two data sets correlated then the figure would show a systematic scatter that followed a linear pattern. The correlation can also be calculated numerically using Pearsons correlation factor p. For this sea state the correlation was -0.0535. That result is very close to zero, and can be read as no correlation at all. If the values correlate, then the Pearsons correlation factor would have been close to 1.0.

Since the parameters correlates so badly, it can be interesting to examine how the e.g. bending moment behaves close to the highest wave, and how the surface elevation behaves around the largest bending moment. This could give us more insight to how the irregular response is different from the regular. Figure 5.11 shows the surface elevation and bending moment at the largest wave, and Figure 5.12 shows the same at the largest bending moment.

The plots clearly show that the bending moment is much lower at the largest wave height, than it is at its maximum. This observation was consistent through all the individual simulations that were examined. It is somewhat counter-intuitive since at figure 5.11 shows that at the largest wave the three waves seem almost regular with a period close to the T_p.

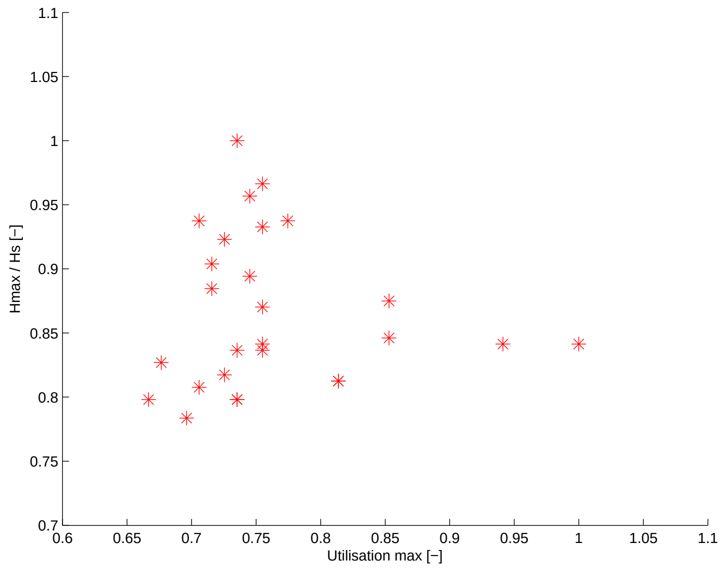

Figure 5.10: Scatter plot of the correlation between H_{MAX} and U_{MAX}.

Even if there is no correlation between the maximum bending moment and the maximum wave height in a long simulation, there still might be a correlation between the individual wave heights and the resulting bending moment within a simulation. But there is a phase angle between the incoming wave and the response, so the time series should be divided into a few seconds around each wave, and then look at the highest bending moment within that time window. The correlation will probably be larger, but the expected largest wave in a sea state is still not a good parameter to compare regular and irregular wave analyses.

5.5.2 Correlation in rig displacement

The displacement of the center of gravity (COG) as a function of time was extracted for a 8000 second analysis using irregular waves. First the 10 largest values of surge in the time series, and the largest bending moment within 10 seconds range of that value was extracted. All of the extracted moments were within 0 to 13 % of the maximum bending moment of the entire time series. The largest bending moment was found at the 10^{th} largest surge displacement.

A program analysed the bending moment and the surge displacement of the rig, and saved the largest value within 10 seconds for both. The values were calculated for correlation, and the Pearsons correlation factor was 0.54. This is still a weak correlation, but much stronger than the correlation between maximum wave height and bending moment. The same procedure was performed for heave and pitch, and the correlation was respectively 0.44 in heave and 0.55 in pitch.

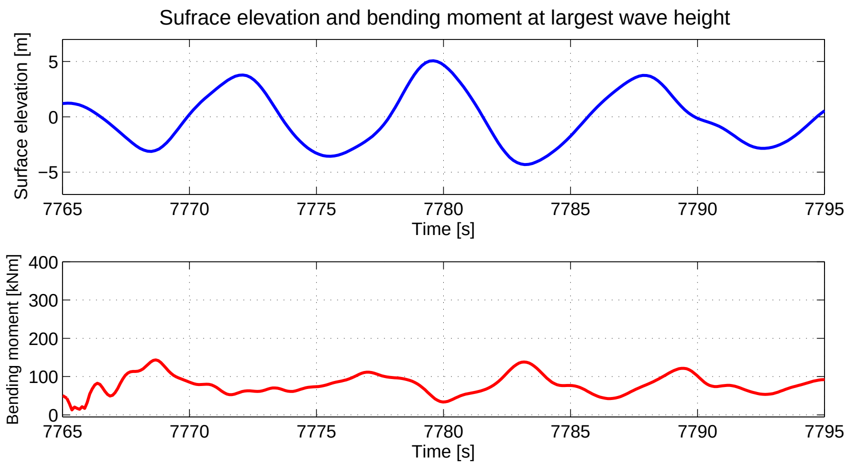

Figure 5.11: Surface elevation and bending moment at largest wave.

5.5.3 Correlation in rig velocity

The rig velocity can be calculated if the displacement as a function of time is known. The same procedure for finding the correlation used with the displacement was followed after the velocities were calculated. The correlation was 0.45 in heave, 0.49 in surge and 0.62 in pitch. This is somewhat higher than the correlation with rig displacement.

The heave displacement and velocity has the lowest correlation with the bending moment. This is probably because the riser is top tensioned and heave compensated. The rig moves up and down in heave, but the riser keeps the vertical position. The displacement and velocity in surge moves the riser from side to side, and the pitch bends the riser back and forth. These movements contribute more to the bending moments in the riser pipe. However, they do not correlate good enough to be used as parameters for comparison between regular and irregular wave analysis.

5.5.4 Correlation in riser displacement

The next natural step is to look at the riser displacement and velocity. The same procedure used for rig displacement and velocity can be applied to the riser pipe. The displacement in surge, heave and pitch for a single element in the riser some meters below the surface The velocity is calculated from the displacements, and the maximum values within a 10 second time frame are measured against the corresponding maximum values of the bending moment in the same time frame.

The results are a correlation of 0.90 in surge, 0.73 in heave and 0.87 in pitch. This suggests a much stronger correlation than the rig displacement and velocity. The riser is subjected to environmental dynamic forces both from the rig movements *and* the waves and current. This is mainly due to the fact that the contributions from the waves and current are significant for the displacement of the riser. The local displacement of the pipe appears to strongly correlate to the bending moment.

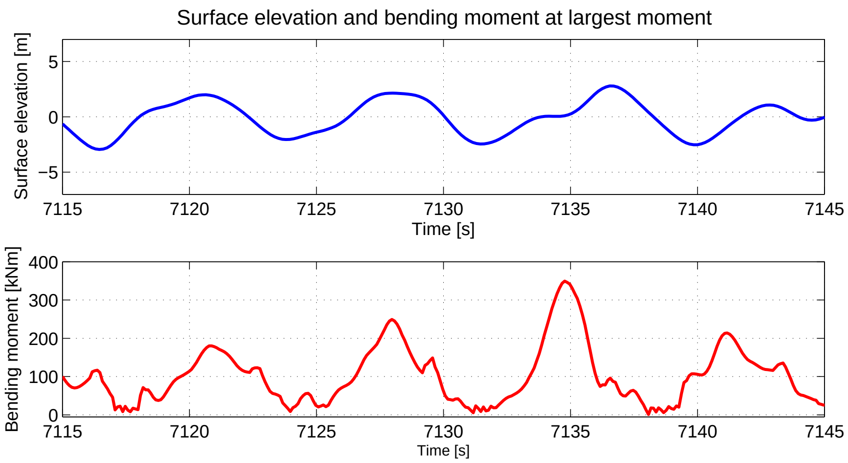

Figure 5.12: Surface elevation and bending moment at largest moment.

5.5.5 Correlation in riser velocity

Finally the velocity of the same element used in the previous section regarding the riser displacement is investigated. The velocity was calculated for each of the degrees of freedom, and then the method of using a 10 second time frame was applied. The resulting correlations was also much stronger than with the rig displacement and velocity. The correlation in surge was 0.80, in heave it was 0.70 and finally the correlation in pitch was as high as 0.93. The correlation in surge and pitch are very high, and suggests that they can be used as parameters for finding a link between regular and irregular analysis.

It is also interesting to look at *where* in the time series the maximum values occur, and if they occur at the same time. The maximum moment in this specific time series occurs at the same time as the maximum velocities in heave and pitch. At this time step the surge velocity is also large, with a velocity that is less than 6 % lower than the maximum. It appears that the highest bending moment occurs when a combination of the three degrees of freedom has its maximum velocities occurring at the same time.

In Figure 5.13 the velocities in the three degrees of motion are plotted together with the bending moment. The maximum values are marked with a red dot, except in surge where the red dot represent the local maximum, 5.8 % lower than the global maximum velocity. The heave motion is very small, since the pipe is heave compensated and in constant tension, but the maximum value still occurs at the same time as the maximum bending moment. The surge was expected to correspond somewhat with the bending moment, since the relative horizontal velocity gives a large contribution in the drag force, as seen in Equation 2.5

Both the displacement and the velocity of the riser suggest a close relationship to the bending moment. Surge and pitch seems to be stronger correlated than heave. This is possibly due to the fact that a top tensioned riser is heave compensated, and does not have a large displacement in

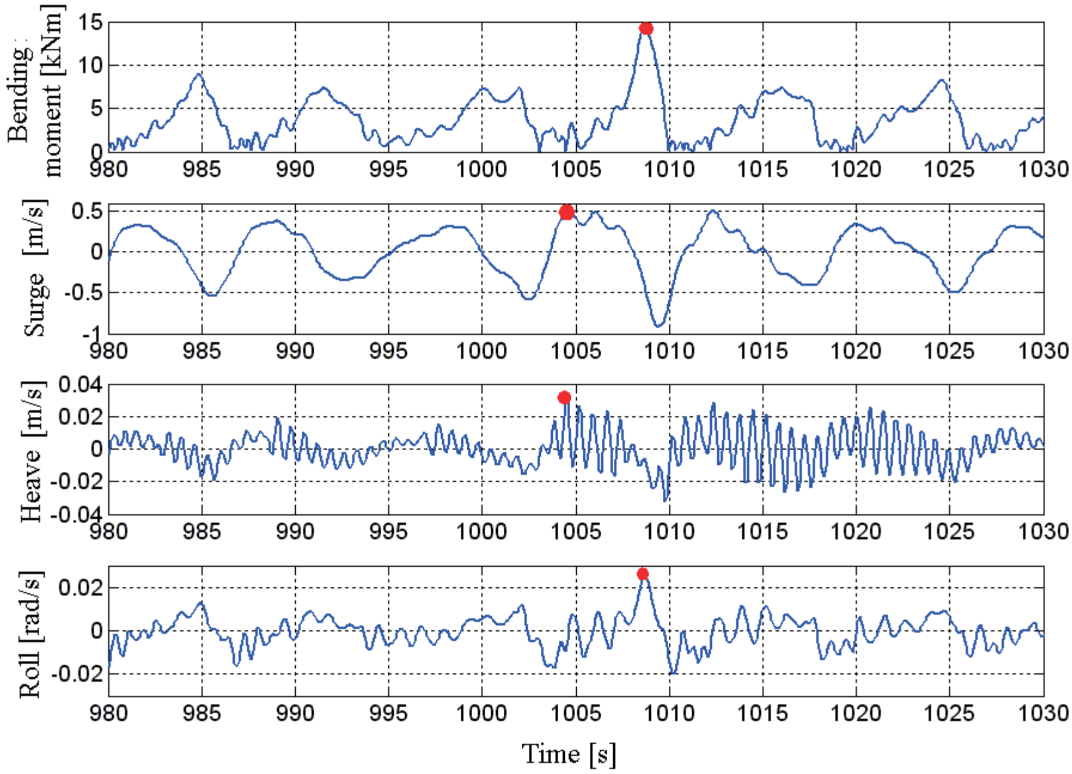

Figure 5.13: The velocities in surge, heave and pitch at the largest bending moment

the axial direction. Both the displacement and the velocity can be used as parameters to identify the response that gives extreme values. They can be used either to tune regular analyses to represent long irregular simulations in a short regular simulation, or they can be used to create shorter irregular realisations with only the desired response that gives the extreme values.

5.5.6 Comparing with regular wave analysis

In the results of irregular waves the displacement and velocities of the riser showed a strong correlation to the bending moment. To find a link to a regular wave analysis the same parameters were plotted for an analysis using regular waves. The results are found in Figure 5.14, and are quite similar to the maximum bending moment and velocities found with the irregular waves in figure 5.13.

These results suggests that the similarities exist between the maximum response of a long, stochastic analysis and a short regular one. The identified parameters that correlates to the bending moment can be used as numerical markers for tuning the response in regular waves.

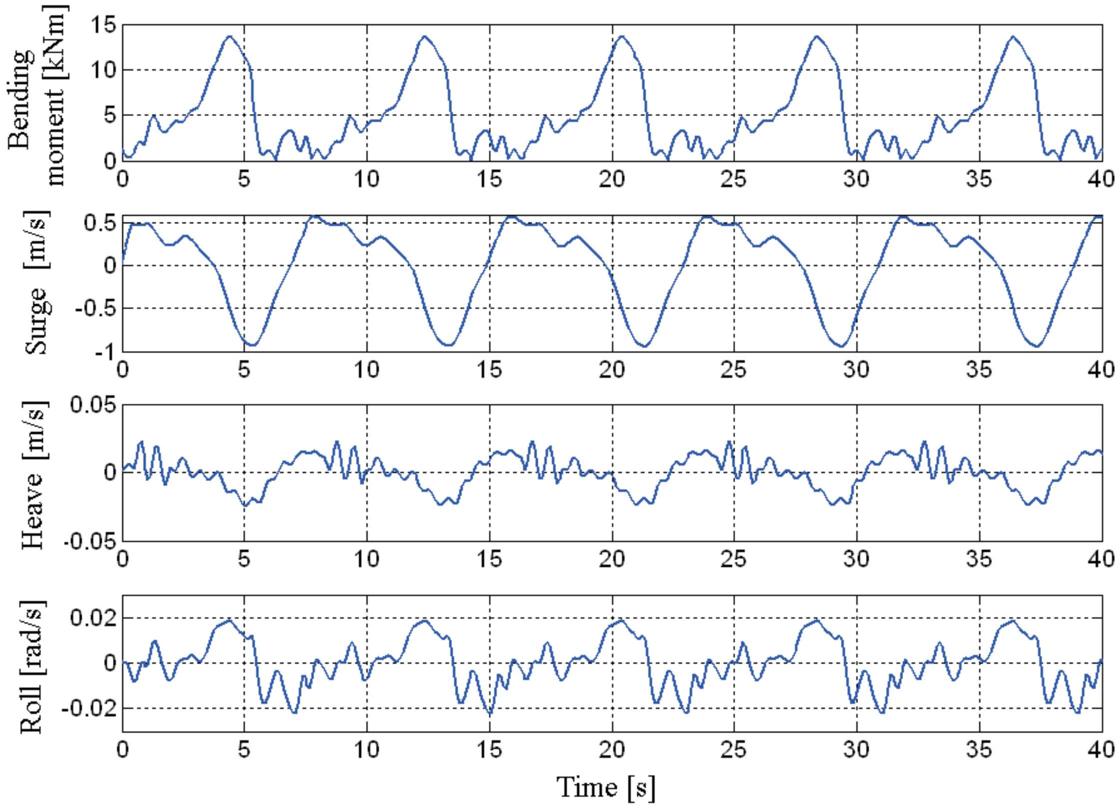

Figure 5.14: The velocities in surge, heave and pitch for a regular wave analysis

5.6 Operation limitations

A study to find the operation limitations is mandatory in the design analysis of a work over riser. The safe limits for sea states and rig offset must be set before an operation can be performed using the riser. ISO 13628-7 states that no part of the riser or the equipment that is connected to it can have a utilisation above 1.0. The utilisation factor is based on the capacities of the riser, its connectors or connected equipment in tension, bending and, in the case of connectors, separation. If the utilisation of a riser joint or connector exceeds 1.0 the structure will yield or separate. This means that for each time step the utilisation must be calculated for all parts of the model.

The common practice is to use regular waves based on the expected largest wave during the given sea state. The results from the case study suggests that this may not be the correct parameter to use. The case study identified parameters that corresponds more strongly with the response than wave height, and can rather be used to fine tune the regular waves to replicate the extreme values of long and time consuming irregular analyses. This tuning must be based on a large data set of irregular analyses for it to have any validity. Such procedures have been suggested in the case study.

Another way is to still use irregular waves, but exploit the correlation to the displacement and velocity of the riser to identify the response that will give a large bending moment, and then creating realisations that will give the response that generates extreme values. This procedure has the equal potential to save computational time, but still ensure a correct extreme response within certain limits.

Chapter 6

Discussion

The results from the case study are many, and I have tried to present and discuss them in a logical manner. First the extreme value results were presented. This chapter represents a huge workload, compressed into two figures. The third, large data set with $T_p = 12$ seconds had to be discarded due to lack of fit to the extreme value distribution. It is hard to compare regular and irregular wave analyses since they are so different in nature. The regular wave analysis is based on *one* single sea state, and its expected largest wave. The irregular analyses are based on many, long simulations fitted to an extreme value distribution, and finally the value is found by reversing the cumulative density function, and calculated using a probability level based on a return period.

The reality is, however, that the sea is irregular in nature. It is more correct to analyse a structure using a stochastic approach, but how should one perform such analyses, and how should we interpret the results? The current standards are lacking on this point, and in this thesis I have tried to use reasoning and theory to decide on parameters that are not decided by standards or recommended practices. Other parameters, such as return period, I have used three different values to show the different options.

In the search for a parameter that correlates to the response many different approaches were pursued. Perhaps other parameters exist that correlates almost one to one with the response. The correlation was based on values within a 10 second time frame, and a better way of finding the direct connection between the parameters that are examined exist. Also, the value of 10 seconds may not be the best value to correctly capture the cause of the correlation.

I have also written many small programs to manage the huge amount of data that is used in post-processing the results. These have not been checked by any other than the author, and may be a source of error. They have been written with care, and tested and checked thoroughly but programming errors are a possible source of error.

The use of a correlation factor also creates an opportunity for misinterpretation. The phrase *"Correlation does not imply causation"* means that a correlation between two parameters does not always mean that one parameter is the cause of the other. In this thesis Pearsons correlation factor is used in many different settings. In the last sections of the results it is used to find the parameters that correlates with the bending moment. These data are taken from one single simulation of 8000 seconds. To conclude that some parameters do in fact correlate a large number of simulations should be examined, and checked for sensitivity to different parameters.

For the fatigue analysis of a riser, and certain marine operations the effect of multi-directional waves should be accounted for. But in structural, extreme value analysis a directional short-crested wave spectrum can rather be expressed as a uni-directional spectrum (DNV, 2009). If directional information on waves is not available, wind direction may be used for wave direction (ISO, 2006). In the case study of this thesis waves are uni-directional, and moving in the positive x-direction of the global coordinate system described in section 5.2.2.

The use of a current profile with a 1 year return period is also a topic of discussion. In many cases the current profile for each month of the year is available. It may be too conservative to use the current with a one year return period. Even if it is better to err on the side of caution, it is still preferable to try to be as correct as possible.

The extreme values of the response from the irregular wave analyses did not fit the extreme value distributions perfectly. The Gumbel distribution seemed to fit the data best, but it did not fit the data set from the analyses with $T_p = 12$ seconds. This data set had very little variation, and the data seemed to group around certain values. This makes it difficult to draw statistical conclusions, and the fit to a Gumbel distribution is questionable. It did not pass the hypothesis test either, even if this test can be used in a conclusive way.

The standards and recommended practices proved insufficient to give a clear guidance with respect to analyses using irregular waves. To implement irregular wave analysis as a required part of design analysis of work over risers a change in standards must be implemented. In any case it should specify:

- The length of simulations (with respect to e.g. time or no. of waves).
- The statistical distributions to fit the data. (By stating the specific distributions, or a more general guideline: "select a distribution that fits the data sufficiently").
- The probability limit of exceedance (in 10^{-n} where n = 2, 3, 4 or by return period).

Chapter 7

Conclusion

The main objective of this thesis has been to investigate the properties of dynamic analysis using both regular and irregular waves. The goal was to discover parameters in the response that correlates to a parameter that can be found in both methods.

In this thesis there has been described a general description of dynamic analysis for slender marine structures, and wave modelling and statistics as a foundation to describe the methods used in the case study. A literature review has been performed, and the findings that came from that study was presented in a dedicated section in the Methodology chapter. Other relevant literature has been referred to in the text where it is relevant. The standards and recommended practices that are relevant to work over risers, marine operations and design analysis has been reviewed for sections and paragraphs that regulates the topics of this thesis.

A thorough description of the methodology used in the case study has been written. It contains the choices made to investigate the problems in the case study, and the reasons for selecting the different methods and parameters. The methodology chapter should have enough details for others to replicate the case study to test the results.

A case study of a top tensioned work over riser has been performed to investigate the properties of analyses using regular and irregular waves as a dynamic load. For irregular waves a large number of long simulations has been calculated to create enough data to draw reliable statistical conclusions. For each sea state one regular analysis has been calculated, and used as a reference to the irregular waves.

The simulations from the case study have been post-processed to extract parameters such as bending moments, utilisation factors and displacement. The displacement as a function of time has been calculated to find the velocity of the rig and the riser as a function of time. The maximum wave height, the displacement and velocity of the rig and the displacement and velocity of the riser has been investigated to find a correlation to the bending moment.

Finally the parameters that were found to correlate most strongly with the bending moment were compared to a sample from a regular analysis showing the response of the same parameters. The findings can be used to identify what gives the extreme response, and can serve as identifiable markers to tune the regular wave analyses to give the correct response. If this is successful, regular waves can be used in the design analysis to find the safe areas of operational limitations.

Chapter 8

Further work

In this thesis some parameters have been identified that seems to correlate well with the response. Still, further work should be performed to investigate the sensitivity of the correlation to other parameters e.g. riser length or configuration, wave period, riser pipe geometry, current profile, top tension etc. If the correlation is sensitive to any change in parameters such as these then the correlation is not general and must be used with caution.

Further investigations should also be conducted on the length of simulations for irregular analyses. This thesis suggests that the length is based on the number of waves, and is therefore dependent on the period. This may not be the best procedure over all, and it may prove hard to find a standard way. One suggestion is to define a reasonable limit of e.g. variance or standard deviation in the response that must be satisfied before a duration limit is set.

Also the probability level to be used in the extreme value distributions fitted to the data of irregular analyses should be looked at. There is no current standard that gives guidance to how this probability level is set. The method used in this case study with long term statistics and return periods may not be the correct way to set the probability levels.

A new way of setting operation limitations has been tested out in the North Sea the last year. By utilising a large database of load cases, real time measurement of current, waves and rig offset the real time operation limitations can be found. This method is not limited to only significant wave height and rig offset as limiting parameters. The measured peak period and weather forecasts makes planning and operations more predictable, accurate and hopefully more safe. As this thesis has suggested, other parameters than wave height can be used as limiting parameters. An instrumented riser can show the real time displacement, and databases of known load cases can be combined with measured data to give a more accurate estimate of the utlilsation of the riser. In the future other parameters may give more accurate estimates, and larger operation windows, instead of always having to assume the worst case scenario.

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Appendix A

Statistical distributions

Here the statistical distributions used in the thesis are presented.

A.1 The Gauss distribution

Also known as the normal distribution. The probability density function is:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{2\sigma^2}) \tag{A.1}$$

where:

σ^2 = variance.
μ = mean.

A.2 The Rayleigh distribution

The probability density function for Rayleigh distribution is:

$$f(x) = \frac{x}{\sigma^2}(\exp -\frac{x^2}{2\sigma^2}) \tag{A.2}$$

And cumulative distribution is:

$$F(x) = 1 - (\exp -\frac{x^2}{2\sigma^2}) \tag{A.3}$$

where:

Mean: $\mu = \sigma\sqrt{\frac{\pi}{2}}$

Variance: $s^2 = \frac{4-\pi}{2}\sigma^2$

A.3 The Gumbel distribution

Probability density function:

$$P(x) = \frac{1}{\beta} \exp(\frac{x-\alpha}{\beta} - \exp(\frac{x-\alpha}{\beta})) \tag{A.4}$$

and Cumulative density function:

$$D(x) = 1 - \exp(-\exp\frac{x-\alpha}{\beta}) \tag{A.5}$$

where:

α = location parameter.

β = scale parameter.

The location and scale parameters must be estimated from empirical data fitting.

(Walpole et al., 2002) (Brodtkorb et al., 2000)

Appendix B

The JONSWAP spectrum

The *JONSWAP* spectrum on frequency form:

$$S(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp(-\frac{5}{4}(\frac{f}{f_p})^{-4}) \gamma^{\exp(-\frac{(\frac{f}{f_p}-1)^2}{2\sigma^2})} \tag{B.1}$$

where:

f = frequency

f_p = frequency at the peak of the spectrum.

α = parameter that decides the shape of the high frequecy part of the spectrum.

γ = peakedness factor.

(Myrhaug, 2006)

Appendix C

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