ISBN 978-82-471-5854-8 (printed ver.)
ISBN 978-82-471-5868-5 (electronic ver.)
ISSN 1503-8181

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Theses at NTNU, 2008:6

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Science and Technology
Thesis for the degree of
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Printed by Tapir Uttrykk

# Lifetime profit modelling of ageing systems utilising information about technical condition

**by**

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# Abstract

This dissertation focuses on a framework to support decision making in the management of ageing (oil and gas) facilities. Within the oil and gas sector on the Norwegian Continental Shelf the topic has become important as the installations are approaching the end of their intended lifetime. The objective of the thesis is to provide decision support for managing ageing systems and equipment with respect to the inspection, overhaul and replacement strategy in a life cycle perspective. The replacement strategy also include evaluation of obsolescence i.e. changes in external requirements and thus functional demands that call for a replacement.

The thesis presents a model designed to investigate and seek optimal solutions when it is possible to classify the items present condition and predict future development based on previous condition monitoring results and future functional demands. The deterioration process is described by a Markov process, and the sequential decision problem is modelled as a discrete time Semi-Markov Decision Process. The transition probabilities of the controlled time-variant Markov process are described in a condition transition probability matrix. To account for the end-of-horizon effect and time dependent external parameters e.g. varying production profile, the optimal solution is found by use of the value iteration procedure (stochastic dynamic programming).

The model has been applied to a case study including inspection and repair of an offshore gas turbine. Two main degradation processes have been studied and added to the model. In the case study, an investigation has been made into the effect on the optimal plan due to forthcoming known turnarounds (planned shutdowns for inspection, maintenance and modifications), and knowledge about new technology.

The provision of methodologies that can support decision making for future maintenance and operation activities is challenging, but hopefully the thesis presents ideas that will improve understanding of the applicability of Markov Decision Processes in this matter.

# Acknowledgment

This dissertation is the result of my PhD study at the Norwegian University of Science and Technology, NTNU, Department of Marine Technology.
Professor Magnus Rasmussen, Department of Marine Technology, NTNU, has been supervisor during the work. I am thankful to Magnus for his helpful guidance and support.

I also wish to thank MARINTEK AS that has contributed with additional funding and given me the opportunity to conduct and finalize this work. Especially I would thank Trond M. Andersen for being a valuable discussion partner, Roar Bye and Geir Langli for their encouragement to finalize my work, and Andrew K. Gibson for his valuable correction and comments in my writing of this thesis.

Last, but not least I wish to express my gratitude to my wife Arnhild, for her support, patience and understanding during this work, and to my two dear children, Emil and Dora, my parents, rest of the family and friends for all their support.

# Contents

# Nomenclature

*Abbreviations*

| | |
|---|---|
| AHP | Analytic Hierarchy Process , page 20 |
| BPD | Barrels per day, page 13 |
| CBM | Condition Based Maintenance , page 48 |
| CM | Condition Monitoring, page 13 |
| CTPM | Condition Transition Probability Matrix , page 53 |
| DA | Decision Analysis, page 19 |
| DM | Decision Maker, page 20 |
| DP | Dynamic Programming, page 21 |
| DTM | Delay Time Model, page 35 |
| ETGT | Efficiency Thermal Gas Turbine [%] , page 129 |
| GPP | General Planning Problem , page 20 |
| HSRI | Hot Section Repair Interval , page 125 |
| IAEA | The International Atomic Energy Agency , page 1 |
| LCC | Life Cycle Costing, page 15 |
| LCP | Life Cycle Profit, page 7 |
| LMDP | Latent Markov Decision Process , page 47 |
| LP | Linear Programming, page 40 |
| MARR | Minimum Attractive Rate of Return , page 17 |
| MAUT | Multi-Attribute Utility Theory , page 20 |
| MCDM | Multi Criteria Decision Making , page 20 |
| MDP | Markov Decision Processes , page 28 |
| MRL | Mean Residual Life, page 13 |
| MTTF | Mean Time To Failure , page 100 |

| | |
|---|---|
| NCS | The Norwegian Continental Shelf , page 149 |
| NPP | Nuclear Power Production , page 2 |
| NPV | Net Present Value, page 16 |
| POMDP | Partially Observable Markov Decision Process , page 35 |
| RBI | Risk Based Inspection , page 150 |
| RCM | Reliability Centered Maintenance , page 8 |
| ROCOF | The Rate of OCcurrence Of Failure , page 35 |
| SMDP | Semi-Markov Decision Process , page 44 |
| SPPT | Shaft Power from Power Turbine $[MW]$ , page 129 |
| TCI | Technical Condition Index, where the minimum and maximum values are set to 0 and 100, respectively , page 11 |

*Greek letters*

| | |
|---|---|
| $\alpha$ | Smoothing constant , page 111 |
| $\beta$ | Discount factor , page 54 |
| $\beta^*$ | Annual discount factor $\beta^* = \frac{100+inflation}{100+interest\ rate}$ , page 16 |
| $\beta_i$ | Constants of a time series model , page 118 |
| $\delta$ | Small time increment , page 71 |
| $\Delta(T)$ | The mean absolute deviation , page 116 |
| $\Delta\tau$ | Number of stages (time intervals) to the next inspection , page 55 |
| $\Delta O(\cdot)$ | The benefit achieved when an unforeseen replacement has taken place and the item is in a better condition than before , page 65 |
| $\epsilon$ | Random error , page 110 |
| $\epsilon^*$ | Limit value of the error bound (small value) , page 66 |
| $\epsilon_t$ | Random component $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ , page 118 |
| $\eta$ | The characteristic life parameter for a three parametric Weibull distribution , page 73 |

| | |
|---|---|
| $\gamma$ | The location parameter for a three parametric Weibull distribution , page 73 |
| $\gamma^*$ | Discount factor, direct smoothing , page 114 |
| $\iota$ | The shape parameter for a three parametric Weibull distribution , page 73 |
| $\mu_n$ | Function mapping the decision at stage $n$ , page 57 |
| $\omega$ | Condition measurement - continuous , page 71 |
| $\Phi$ | Set of present and new technologies (state space of technology) , page 54 |
| $\phi$ | A specific technology, $\phi \in \Phi$ , page 54 |
| $\pi_n^*$ | Optimal policy at stage $n$ , page 54 |
| $\Psi_E(z)$ | Energy consumption profile, given condition $z$ , page 60 |
| $\Psi_S(z)$ | Salvage profile , page 63 |
| $\Psi_{cap}(z)$ | Capacity profile , page 58 |
| $\Psi_{PM}(z)$ | Maintenance cost profile — include all costs associated with the day-to-day maintenance (minimal repairs) , page 60 |
| $\Psi_{pp}(n)$ | Production profile , page 54 |
| $\Psi_{ps}(n)$ | Planned shutdown profile, $\Psi_{ps}(n) = MDT_{ps}(n)/\Delta t$ , page 58 |
| $\sigma^2$ | Variance, page 111 |
| $\sigma_e$ | The standard error of forecast , page 112 |
| $\sigma_\epsilon$ | The standard error of regression , page 110 |
| $\tau'$ | number of years between the first inspection of facility and start of the planning horizon , page 48 |
| $\xi(t)$ | Deterioration process , page 70 |
| $\xi^\delta(\omega)$ | The condition $\delta$ time units ahead given that present condition is $\omega$ , page 71 |

*Mathematical operators*

| | |
|---|---|
| $\otimes$ | Denotes any mathematical relationship, including addition, subtraction, multiplication and root operators , page 23 |
| $\Sigma$ | Denotes the summarization of , page 23 |

<u>*Roman letters*</u>

| | |
|---|---|
| $\mathbf{G'}$ | $\mathbf{G'} \equiv \sigma_\epsilon^{\mathbf{2}} \mathbf{V'^{-1}}$ , page 119 |
| $\mathbf{V}$ | Variance-covariance matrix, $\mathbf{V} \equiv \mathrm{Cov}(\hat{\mathbf{b}}) = \mathbf{G^{-1}} \sigma_\epsilon^{\mathbf{2}}$ , page 119 |
| $\mathbf{W}$ | State space of the deterioration processes , page 55 |
| $\bar{\mathbf{b}}'$ | $\bar{\mathbf{b}}' = \mathbf{E}(\mathbf{b})$ , page 119 |
| $\mathbf{b}$ | Matrix containing parameters describing the deterioration process, $\mathbf{b} = [\mathbf{b_1}, \mathbf{b_2}, \cdots, \mathbf{b_k^t}]$ , page 118 |
| $\mathbf{G}$ | $\mathbf{G} = \mathbf{Z^t Z}$ , page 119 |
| $\mathbf{P}$ | The transition probability matrix that contains all combinations of transition probabilities between different states (conditional on state and decision) , page 65 |
| $\mathbf{Z}$ | State space, page 54 |
| $\mathbf{z(t)}$ | Matrix containing independent variables describing the deterioration process, $\mathbf{z(t)} = [\mathbf{z_1(t)}, \mathbf{z_2}, \cdots, \mathbf{z_k}]$ , page 118 |
| $\hat{a}_i$ | Estimates of the model parameters, direct smoothing , page 114 |
| $\hat{b}_k$ | Estimate of the model parameter , page 110 |
| $\hat{p}_{i,j}(s)$ | The probability that the condition of the component is $j$ while the condition was $i$ ($0 \leq i \leq j \leq M+2$) $s$ time periods ago , page 73 |
| $\hat{x}_{T+t}(T)$ | Forecast for period $t$, given an observation at $T$ , page 111 |
| $\hat{z}_n$ | Measured condition at the start of stage $n$ , page 47 |
| $\overline{C_E}$ | The energy cost of operating on the baseline over one stage , page 60 |
| $\overline{c_n}$ | Upper error bound , page 66 |
| $\overline{C_I}$ | Baseline set-up cost (fixed costs) for an inspection , page 62 |
| $\overline{C_{OH}}$ | Baseline cost of restoring an item which has failed , page 63 |
| $\overline{C_{pen}}$ | Penalty factor that reduces the income if the demand is not fulfilled , page 60 |
| $\overline{C_{PM}}$ | Baseline maintenance cost (minimal repair) , page 60 |
| $\overline{I}$ | Baseline income per stage , page 60 |

| | |
|---|---|
| $\overline{S}$ | Baseline salvage value (normally set equal to the cost of purchase a new item) , page 63 |
| $\overline{W(N)}$ | Regular annual rent , page 31 |
| $b$ | A variance parameter given by the standard normal distribution, $b \geq 0$ , page 71 |
| $b_k$ | Model parameter, page 110 |
| $B_T$ | The smoothed value $\hat{b}_0(T)$, $(B_T \equiv \hat{b}_0(T))$ , page 111 |
| $C(t)$ | Cost per unit time over a single replacement cycle , page 30 |
| $c(t)$ | Operating cost per unit time , page 30 |
| $C_I(z,n)$ | Inspection costs at stage $n$, given condition $z$ , page 62 |
| $c_n$ | Lower error bound , page 66 |
| $C_{CAP}$ | Capital costs, page 30 |
| $C_{OH}(z,n,x_n)$ | Overhaul/replacement costs at stage $n$, given condition $z$ , page 62 |
| $C_{OP}(z)$ | Total total operational cost, given condition $z$ , page 61 |
| $C_{OP}(z,n)$ | Total operation cost at stage $n$, given condition $z$ , page 60 |
| $C_{UCM}(z,n)$ | Cost of an unforeseen failure at stage $n$, given condition $z$ , page 63 |
| $cp_w$ | The condition parameter describing the state of deterioration process $w \in W$ , page 55 |
| $E_i$ | Expectation operator, conditioned on $z = i$ , page 42 |
| $e_{T+t}(T)$ | $t$-period-ahead forecast error where $T$ is the point in time when the forecast was made , page 115 |
| $f(\omega)$ | is the density of $\xi^s(\omega)$, when $\xi^s(\omega) \in [L_{u,i}, L_{l,i}]$ , page 72 |
| $F(n)$ | the net profit at year $n$ , page 16 |
| $f(t)$ | The probability density function , page 73 |
| $g(\hat{\mathbf{b}})$ | The marginal distribution of $\hat{\mathbf{b}}$ , page 119 |
| $G(\cdot)$ | The net profit of operating the item between to consecutive inspections , page 64 |
| $g(t)$ | Drift function — a deterministic decay of the item condition , page 70 |

| | |
|---|---|
| $h(z)$ | Objective function conditioned on $z$ , page 42 |
| $h_0(b)$ | Prior distribution , page 119 |
| $H_n$ | Fuel lower heat value $[kJ/kg]$ , page 129 |
| $h_T(\mathbf{b}\hat{\mathbf{b}})$ | The posterior distribution of $\mathbf{b}$, given $\hat{\mathbf{b}}$ at time $T$ , page 120 |
| $H_\phi(n)$ | The profit generated if the technology $\phi$ is selected at stage $n$ , page 67 |
| $i$ | A single condition parameter, $i \in Z^*, Z^* = [1, M+1]$ , page 64 |
| $I(z,n)$ | Income at stage $n$, given condition $z$ , page 60 |
| $I_n$ | State space at decision epoch $n$, that includes the entire history of measured states up to $n$, and the decision made up to $n-1$ , page 45 |
| $J_n(z_n)$ | Maximum expected profit at stage $n$, given condition $z_n$ , page 65 |
| $k$ | Counter - integer, page 31 |
| $k$ | Deterioration rate (increase in fuel consumption per time unit) $[\%]$ , page 137 |
| $K(N)$ | Income at age $N$ , page 31 |
| $l$ | Counter - integer, page 31 |
| $L_i$ | Express time from beginning of $i$th cycle to the replacement of sub-fleet $i$,$i \in O$ , page 32 |
| $L_{L,i}$ | Lower limit of condition level $i$ , page 71 |
| $L_{u,i}$ | Upper limit of condition level $i$ , page 71 |
| $M$ | Number of possible states , page 41 |
| $M^*$ | Evident fault level , page 74 |
| $M_w+1$ | Disjunct condition intervals for deterioration process $w$ , page 55 |
| $M_w$ | Number of disjunct condition intervals for deterioration process $w$, where $0 \le cp_w \le 100$ , page 70 |
| $MDT_I$ | Mean Down Time caused by an inspection , page 62 |
| $MDT_{OH}$ | Mean Down Time caused by an overhaul/replacement , page 63 |
| $MDT_{UCM}$ | Mean Down Time caused by an unforeseen replacement , page 63 |
| $Mf$ | Fuel consumption $[kg/s]$ , page 129 |

$N$ Planning horizon, page 54

$n$ Discrete stage or counter, $n \in N$ , page 54

$O$ Number of operate-sell-and-buy cycles , page 32

$P^s_{i,j}(1)$ The one-step probability of operating in state $j$ after $s+1$ time periods, when the state $i$ in previous time unit $s$ is uncertain , page 76

$P^s_{i,j}(2)$ The probability of state $j$, when the previous state $i$ was known $s$ periods ago , page 76

$P^s_{i,M^*}(1)$ The one-step probability of having an unforeseen maintenance task in time interval [s, s+1], when the condition $i$ at time $s$ is uncertain , page 76

$P^{s=1}_{i,j=1}(3)$ The probability that the next state is $j = 1$ after a replacement or an overhaul , page 76

$p_\phi(n)$ The probability that technology $\phi \in \Phi = \{A, B, C, \ldots\}$ is available at stage $n$ , page 67

$p_{i,j}(s)$ The “natural transition probabilities” , page 72

$P_{z_n,z_{n+1}}(x_n)$ The transition probability from state $z_n$ to state $z_{n+1}$ given decision $x_n$ , page 54

$Q(T)$ Smoothed error, page 116

$q_1$ The probability to detect a failure without an inspection , page 73

$q_2$ The probability of a transition between the two fault conditions if $q_1 < 1$ , page 73

$R(t)$ The reliability (survivor) function , page 73

$r(z, x)$ Return/reward function dependent on the condition $z$ and the decision/action $x$ , page 42

$S$ Baseline salvage value , page 63

$s$ The number of time periods (age) since last replacement/overhaul , page 55

$S(t_p)$ Salvage value (the resale value) , page 30

$s^2_\epsilon$ Estimate of the variance , page 110

$t$ Time - continuous, page 71

| | |
|---|---|
| $T_N(i)$ | The maximum number of time units between two successive inspections when the condition is $i \in Z^* = [1, M^* - 1]$ , page 75 |
| $t_p^*$ | Optimum interval between preventive replacements, $t_p = t_r + T_r$ , page 29 |
| $T_r$ | Time required for a replacement , page 30 |
| $t_r$ | Operation time between replacement , page 30 |
| $U$ | $U \sim N(0,1)$, page 71 |
| $u$ | Index, page 32 |
| $v$ | Fleet size, page 32 |
| $w$ | A deterioration process, $w \in \mathbf{W}$ , page 55 |
| $W(N)$ | Regular annual rent , page 31 |
| $X$ | The action/decision state space , page 57 |
| $x$ | Action/decision, $x \in X$ , page 57 |
| $x_k$ | Predictor variables , page 110 |
| $y$ | Response variable, page 110 |
| $z_i(t)$ | Regressor variable , page 114 |
| $z_n$ | True condition at the start of stage $n$ , page 47 |

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# Chapter 1

# Introduction

## 1.1 Background

Oil-companies operating on the Norwegian shelf have considerable challenges as installations are facing the end of their design lifetime. Increased oil recovery has made it desirable to produce for a longer period than initially estimated. Extended production periods due to technology improvements are however in most cases too small to defend investment in totally new installations.

Ageing management has thus become important as the installations are approaching the end of their intended lifetimes. Within Nuclear Power Plant industry ageing management has been an important area over the past decades. The International Atomic Energy Agency (IAEA) defines ageing management as *engineering, operations and maintenance actions to control within acceptable limits ageing degradation systems, structures and components* [1]. Ageing management provides vital input to life cycle management.

According to IAEA, life cycle management can be described as all activities over the life of an industrial system in order to (1) maintain an acceptable level of performance, (2) optimize the operation, maintenance and service life of structures, systems and components, (3) maximize return on investment over the operational life and (4) take into account strategies for decommissioning [1]. Ageing management activities are primarily directed to ensure that operatability and availability are in accordance with the requirements for safety, environment and economy. Economic profit is a condition that has to be fulfilled, but in several cases this alone is not sufficient to justify further operation. Change in requirements with respect to safety, environment, and reduced availability of product support (spares and experts) are important factors leading to obsolescence.

In a broad context ageing management includes both the consideration of material degradation and technology obsolescence, as well as human and organizational aspects. Figure 1.1 presents the main topics in ageing management.

Physical impairment (material degradation) is a result of physical mechanisms inherent in system/equipment/component materials. The degradation is dependent on the design, assembly and functional characteristics, and it is influenced by the stresses from the environment and from the operation. Knowledge about the present condition is essential to select cost effective activities that also ensure safe operations. The lack of reliable information is a challenge, and substantial resources have been directed to research and development to quantify degradation mechanisms and to develop condition monitoring techniques.

Cost effective solutions may imply modifications of existing facilities or even a total change of technology. Over the production lifetime several main systems will normally need to be modified. The production facilities offshore have to adapt to the changing production flows from the reservoir. Typically the water treatment systems have to be upgraded to meet the increase in water from the wells. External factors such as new rules and regulations may also have significant impact on performance standards and require a modification.

The transfer of knowledge and information from experienced plant personnel e.g. due to retirement of individuals is also an issue in ageing management. The same applies to adequate training of operation and maintenance personnel in the use of new equipment. The organization of an ageing management program is demanding because the activities involved typically are distributed among several organizational units (integrity management, spare parts management, operation, purchasing etc.).

The offshore industry has not yet developed a similar guideline for ageing management as the IAEA has done for the Nuclear Power Production (NPP) community. Although such overall guidelines do not exists, the use of reliability methods in assigning proper inspection and maintenance programs is adopted by all operators. On the strategy level methods like Reliability Centered Maintenance, Risk Based Maintenance and Safety Integrity Level analysis are applied to define preventive maintenance programs and inspection programs for different types of equipment. These methodologies are transparent and effectively combine the probability of an event with the consequences of such an event. The degree of utilizing statistical data and estimating residual life do however vary. Risk based inspection analysis, which focuses on the integrity of structures, pipes and other static equipment, is in general more quantitative than reliability centered maintenance analysis which primarily is directed to machinery components. The ability to classify and describe the stresses

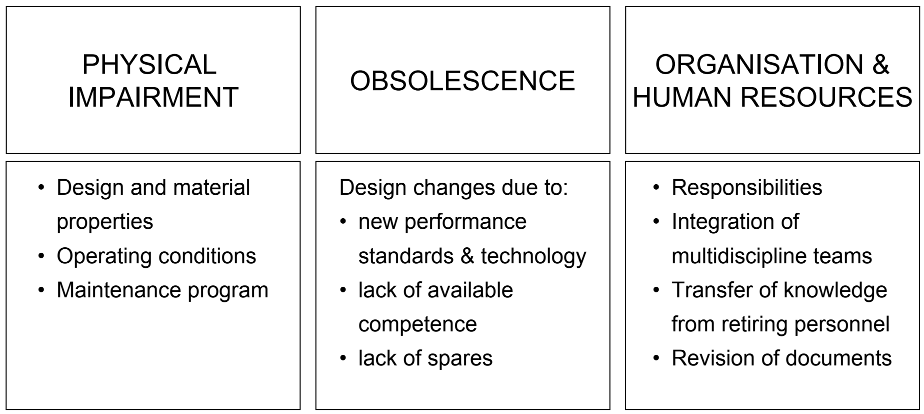


Figure 1.1: The main topics in ageing management.

and the deterioration process may be the main cause of this difference. Structures, pipes and other static equipment normally suffer from only a few dominating deterioration processes that are well known, which make it easier to monitor and state the condition and calculate the probability of an event.

Important aspects in ageing management decision support are the ability to handle several and sometimes competitive criteria, and to provide methodologies able to handle the uncertainty in the available information, both historical records and future prognosis. The use of condition monitoring and update of the estimates of residual life based on knowledge of present condition is a mean to reduce the uncertainty in predicting the residual life. For slowly deteriorating systems, equipment or components there may not exist any suitable statistical data to adapt, and condition monitoring may be the only practical approach.

In an operational setting, the costs of performing an inspection, replacement, an overhaul or a modification may depend on the accessability to the system in operation. If a production shutdown is necessary, the economic consequences will be substantial and may often dominate the overall costs of an action. In some cases actions may be rejected to defend against high production losses during execution. They may have to line up with other ongoing activities to share the losses. Planned production shutdowns or turnarounds can be viewed as opportunities reducing the overall cost of the replacement or modification.

The culture of applying more advanced mathematical models have been limited within optimization of maintenance, which often tends to be very pragmatic based on experience and *rule of thumb* guidance. In-service decision support methodologies for maintenance strategies have to provide capabilities to systemize and utilize information that is vital for the decision making process. The work in this thesis focus on optimization models for repair/replacement/renewal of equipment, utilizing the technical condition of the system/equipment as the basic information.

## 1.2 The challenge of maintenance decision support modelling

In addition to describing the present technical condition of equipment/system/facility under evaluation, the modelling approach should also include e.g. the knowledge of planned shutdowns, the remaining time to operate before closure (end-off horizon), the probability of having new and improved technology on the market in the near future. The effects of obsolescence and limited time to operate often have significant impact on the decision policy. An approach that combines the information about present technical condition, planned shutdowns, remaining time to operate, effect of new technology etc. should therefore represent a step forward to grasp the decision problem related to maintenance policies as part of managing ageing systems.

## 1.3 Objective of the thesis

The objective of the thesis is to develop a methodology that provides decision support for managing ageing systems and equipment with respect to the inspection, overhaul and replacement strategy in a life cycle perspective.

The purpose of the developed methodology is thus to:

- Provide decision support for maintenance or modification activities based on technical condition and system boundaries.
- Include the effect of operating a degraded system/equipment both with respect to integrity as well as with respect to economy.
- Include the effect of obsolescence.
- Include the effect of production prognosis and limited (fixed) production horizon.

## 1.4 Limitations

The thesis focuses on the technology aspects of physical impairment and obsolescence and to utilise condition assessments as the primary source to classify the technical condition. For the proposed model the following assumptions and limitations apply:

- a decision can only be made after the condition (state) has been determined
- the condition of the system is revealed without uncertainty at inspection
- each deterioration process is independent
- a fault is detected immediately with probability $q$
- an overhaul/replacement action returns the item back to “as good as new” or if modified “better than as previously installed”
- a new technology always has better properties than the existing one

The issues relating to organisation and human resources are not considered. These issues are quite comprehensive and each area may justify an in-depth study in itself.

## 1.5 Overview of the plan for the thesis

In order to give an overview of the thesis, a short outline is presented as follows supported by figure 1.2. Topics related to management of ageing systems are discussed in chapter 2. The chapter includes topics such as monitoring technical condition and methods applicable to support decisions in a stochastic environment with multiple goals. Chapter 3 presents an overview of different mathematical model approaches to support optimized maintenance actions based on an economic criterion of either maximum revenue or minimum costs.

Ageing management decision making can be characterized by sequentiality, stochastic environment, and several requirements to be fulfilled. The Markov decision process provides suitable capabilities for handling such a decision environment, and the topic is discussed in chapter 4.

A description of the proposed decision process and a mathematical formulation of the decision model is presented in chapter 5. A short description of the different input parameters is also included.

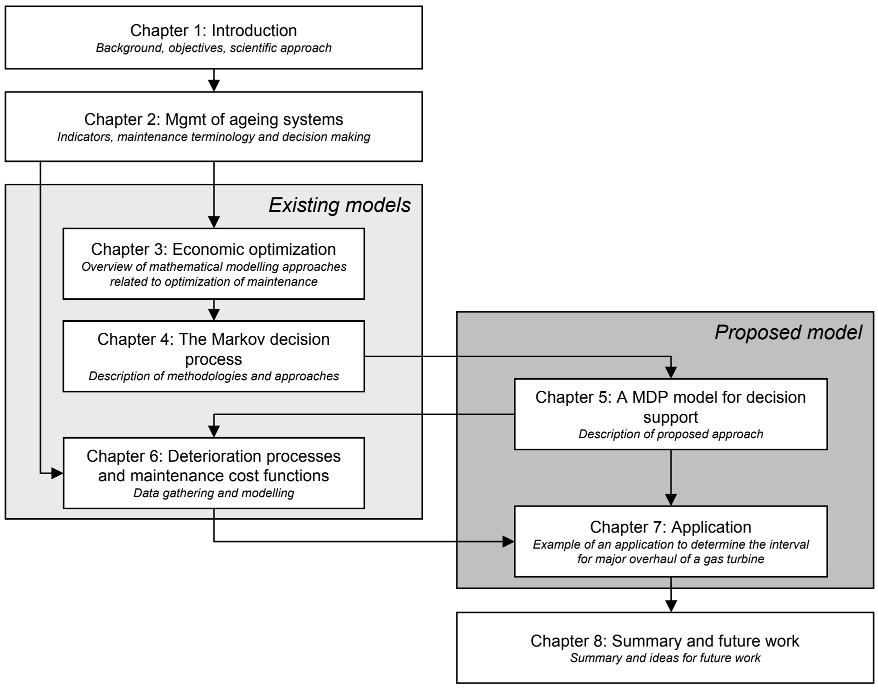


Figure 1.2: Illustration of the thesis structure.

Estimation of model parameters is the central issue of chapter 6. The chapter contains descriptions of possible approaches to establish deterioration processes and maintenance cost functions.

To illustrate the applicability of the methodology and model developed, chapter 7 presents the results from an optimisation of the major overhaul interval of a gas turbine installed on a platform in the North Sea.

Chapter 8 contains a summary and gives some ideas for further development of the proposed method.

.

# Chapter 2

# Management of ageing systems

## 2.1 Introduction

The primary objective of the chapter is to introduce important aspects that have to be considered in supporting management of ageing systems. The chapter also includes an introduction to methods and terms normally used in maintenance and maintenance decision making.

A central issue in management of ageing systems is to understand and monitor the deterioration processes. The degradation is due to physical mechanisms inherent in the equipment materials and linked to equipment design, assembly and functional characteristics. It is influenced by the stresses from the equipment environment and from the equipment operation. Stress factors originate from the manufacture, pre-service, or in-service operating conditions. Deterioration processes and stress factors are further discussed in chapter 6.

To ensure that the equipment complies with the performance requirements, condition monitoring must be in place. The condition monitoring should be dedicated to monitor the dominating degradation processes leading to an overhaul or a replacement. The challenge in management of ageing systems is further to select an appropriate action and schedule for the action that may balance between multiple and perhaps competitive goals. The main objective is in general to have the highest life cycle profit (LCP) without violating any safety, environmental or other sorts of requirements. An increase in safety level often results in increased cost thereby lowering the LCP. Suitable methods for such multi criteria decisions are discussed in this chapter.

## 2.2 Maintenance management

According to NS-EN13306 [2] maintenance management is defined as:

> *All activities of the management that determine the maintenance objectives, strategies, and responsibilities and implement them by means such as maintenance planning, maintenance control and supervision, improvement of methods in the organization including economical aspects.*

A common approach today is to write a document describing the philosophy with respect to life cycle management (operation & maintenance philosophy) to address the objectives, requirements and responsibilities. The document(s) should include a description of all aspects as mentioned above and relate the activities to the overall Business Management. The Business Management is the management of all activities influencing the business performance.

The maintenance objectives define the targets assigned and accepted for the maintenance activities which may be e.g. cost reduction, product quality, environment preservation and safety. The methods and tools applicable for maintenance management decision making may be classified in several ways. Pintelon and Gelders [3] have proposed a three level classification scheme ranging from the top level *strategical planning*, through *tactical planning*, down to *operational planning*. According to their classification scheme the strategic planning is concerned with provision of production resources to ensure company's competitive capabilities. The decisions involves e.g. consideration of capacity, technology and investment criteria to retain or increase availability and keep maintenance costs low.

Tactical planning is focusing on selecting the optimal maintenance strategies to ensure available and reliable equipment and a cost effective solution. The maintenance strategy specifies maintenance activity(ies), action interval and the need for resources. Different maintenance management approaches can be applied to work out a maintenance plan, connecting basic maintenance strategies to identified maintainable items. Reliability Centered Maintenance (RCM) is a methodology often applied to determine cost effective policies[4], [5],[6]. To some extent, use of maintenance optimization models provides a quantitative balancing of costs and benefits of alternative maintenance strategies.

Operational planning deals with day-to-day operational and scheduling decisions. Maintenance scheduling addresses the problem of arranging the sequence in which work orders will be executed to utilize available resources and minimize profit losses. It is necessary to consider job priorities and the availability of workers, spare parts,

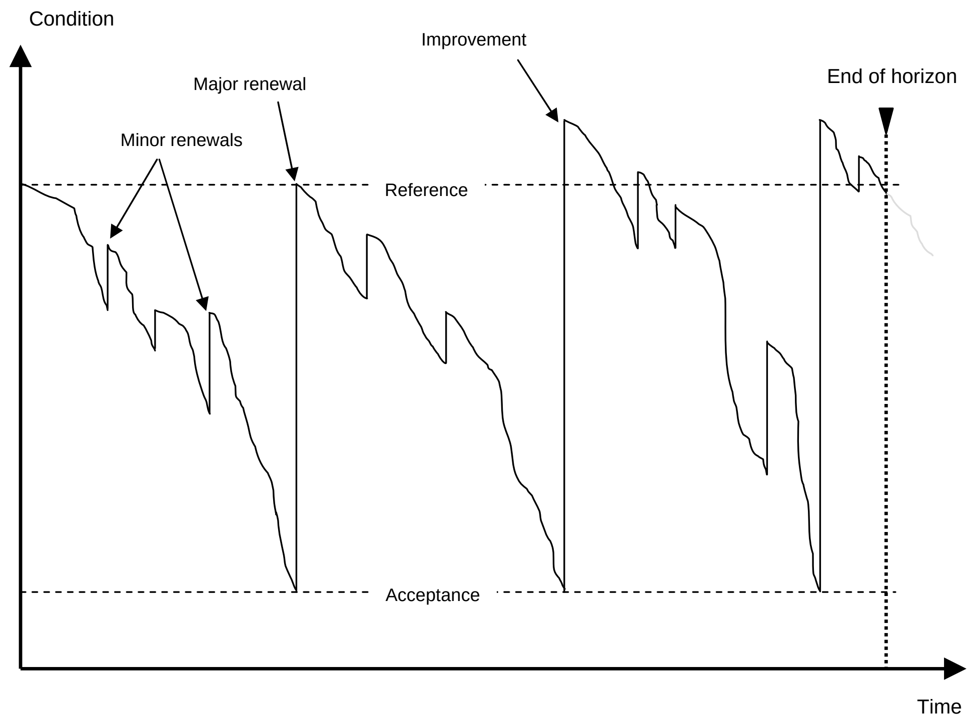


Figure 2.1: Short term and long term planning based on system condition.

tools and equipment to be maintained. The benefit of grouping activities to reduce down-time costs and logistic support is an important issue in this context [7], [8], [9], [10]. Figure 2.1 illustrate different approaches to maintenance planning. Short and medium term activities are often not sufficient to keep the condition of a system to an acceptable level, thus major overhaul or replacement has to take place from time to time.

Applying the classification scheme proposed by Dekker, Wildeman and Egmond [8], management of ageing systems is a vital part of strategic and tactical decisions. The ability to ensure cost efficient and safe operations over the entire lifetime is a key issue in ageing management. Over the past decades the industry has started to undertake more formal methods to determine the economic life of physical assets. Complex production facilities involving high overhaul and replacement costs force the companies to include criterion modelling in their decision processes. Although maximum profit is the main driver, other factors have also a significant influence on the decision process i.e issues regarding safety, environment, comfort, shine, etc. A study performed by Hsu [11] in late 80's, questioning 200 of the largest firms in the United States, revealed that $89\%$ of the firms had an equipment replacement policy. Their study indicated a relationship between use of formal replacement policies and the capital intensity. Use of formal methods tended to increase as the measure fixed-assets-to-employee ratio increased.

An overhaul or replacement is primarily triggered by the condition of the asset itself necessitating an action. If not, an action is released by external requirements or a combination of the condition and the external requirements. Change of external requirements can be exemplified by the introduction of $CO_2$ taxes. The new tax regime gave new incentives to reduce the emission to air calling for action to increase the performance. According to Park and Sharp-Bette [12] the main reasons for considering replacement of a physical asset are:

- Physical impairment — changes in the condition of the asset itself.
- Obsolescence — changes in the environment external to the asset.

**Physical impairment** may cause a decline in quality of products, speed losses, and increase in down-time, maintenance costs and other logistic support costs. The physical impairment can be measured in two different ways – directly by measuring physical deterioration on the object itself and indirectly by measuring the symptoms of the physical deterioration. The performance indicators which classify the physical impairment may also differ, dependent on the objectives. Several techniques are available to classify the physical impairment, ranging from specific physical measures of the deterioration to economic performance measures of profit and costs. Several terms have been used to describe the physical impairment of an equipment or system e.g. technical health and technical condition. The topics are discussed in the next section.

**Obsolescence** is caused by changes in the function which the physical asset is intended to fulfil. New and more efficient technology, regulations and an increase in capacity are typical causes which may force a modification and/or replacement.

Physical impairment and obsolescence often occur jointly, but they also occur independently with respect to a particular asset.

## 2.3 Technical condition and residual life

The ability to monitor the technical condition of systems and equipment is vital in managing ageing systems and the term technical condition is often used but without a common definition. The technical condition can be described by quantitative and/or qualitative measures of the system, by representative parameters from which it is possible to determine the soundness of the system. Reinertsen [13] describes technical health as "the soundness of a technical system".

It has been recognized that it is difficult to classify the technical condition by a single parameter. To illustrate this a car is used as an example. If the brakes of a new car fail to function due a brake hose rupture, the technical condition of the car with respect to safety may be classified as poor. From an economical perspective, the technical condition may on the contrary be classified as good due to the low cost of replacing the hose compared to the total value of the car.

The research project *Ageing Management* proposed the following definition of the technical condition [14]:

> *The technical condition is defined as the degree of degradation relative to the design condition. It may take values between a maximum and minimum value, where the maximum value describes the design condition and the minimum value describes the state of total degradation.*

It also introduced a technical condition index (TCI) where the minimum and maximum values are set to 0 and 100, respectively. The design condition is taken as a reference in order to make the technical condition independent of the demands of the system in question. The TCI is always related to a specific context e.g. safety, environment, production availability etc.

The TCI value on a plant or a system level is calculated based on an aggregation methodology. In short, it comprises the following four steps; (1) Establish a hierarchy of objects which represents the actual industrial system. (2) Assign a weight to each of the objects according to its criticality. (3) Assign relevant input variables, which characterize the objects technical condition (mainly at the bottom level) (4) Based on values of the input variables (e.g. maintenance statistics, process data, condition monitoring and inspection data/notifications), the TCI values are then aggregated upwards in the hierarchy. Figure 2.2 is a screenshot from TeCoMan, a software designed to calculate TCI values based on the above mention methodology [14],[15].

Application of the TCI methodology and the TeCoMan software has revealed difficulties in stating the consequences of a specific index level. This is due to the fact that a single TCI value on a specific level normally consists of several measurements, and is based on rules that have converted the measurements to TCI and aggregated these to the level of interest. The solution has been to focus on trends rather than the absolute figures, and to compare different TCI levels with the real situation on the plant. On a low level in the hierarchy, the TCI value is only based on a single or

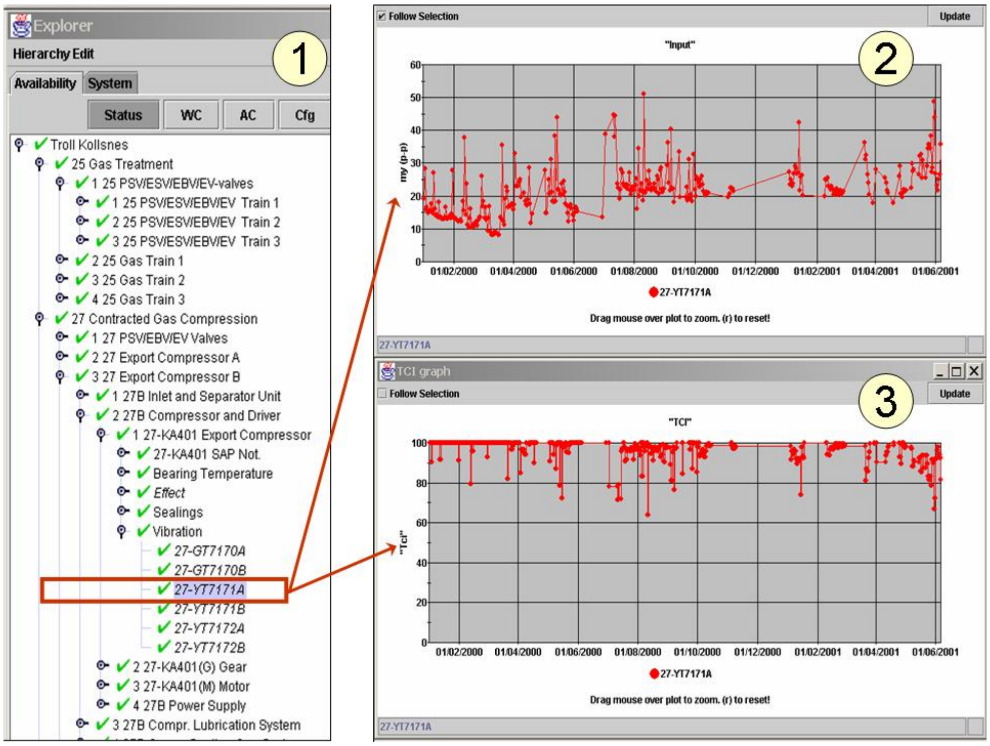


Figure 2.2: Aggregation of technical condition in TeCoMan. 1) The hierarchy, 2) Measurement values 3) TCI values calculated from input values.

few measurements, and therefore the connection between TCI and the consequences becomes more obvious.

In the literature the term "residual" life often appears. An important distinction should therefore be made between technical condition and the residual life. Technical condition can be viewed as a static value while the residual life is both dependent on usage and external demands as well as present technical condition. The operators may strongly influence the residual life by changing the operating conditions. By reducing or increasing the stresses, the residual life may increase or decrease respectively.

In the reliability community the term *Mean Residual Life* (MRL) has a different meaning from residual life as described above. The definition is limited to purely the technical ability to survive (strength) taking into account the knowledge that the component may have been operating for a time. The MRL of a non-repairable system, is given by [16]:

$$MRL(t_1) = \int_0^\infty R(t \mid t_1)dt \tag{2.1}$$

where $R(t \mid t_1)$ is the condition survival function of a component given survival up to time $t_1$.

## 2.4 Condition monitoring

Use of condition monitoring (CM) as a means to improve maintenance performance has been adopted to a great extent in industry. Especially in tactical and operational maintenance planning CM is a well established strategy. Development of new CM techniques, improved diagnostic methods and tools has made it more beneficial compared to periodic overhaul and replacements. Introduction of production philosophies such as e.g. "Just In Time", and the gradual increase in complexity and serial dependencies in production facilities have in general increased the importance of reducing equipment down time and unnecessary repairs by means of CM.

The CM data is collected using either on-line or off-line measurements. Selection of appropriative methods depends on available CM technology, investment costs and equipment criticality. Selecting off-line CM implies that inspections have to be performed to reveal the condition. To choose an appropriate CM interval, the benefit gained by an inspection has to be compared with the cost of performing the task, which may be significant if an inspection requires a production shutdown. On a typical offshore installation the production down time cost is typically $5 - 12$ MUSD/-day[1]. Thus there is a great incentive to keep the number of inspections to a minimum if a production shutdown is required.

Application of CM techniques requires a rational consideration of the investment costs and benefits. Quantitative methods to compare potential cost savings when considering CM are available today [17]. The potential benefits of condition monitoring can be summarized by:

- Reduced repair time and costs. A planned maintenance action reduces the costs with respect to acquiring necessary labour resource, spare parts and tools. Use of CM gives detailed knowledge of failures and repair requirements.

- Avoided revenue loss. An impending failure is detected well in advance, thus the availability can be increased by planning actions at convenient times with respect to known outage periods or periods with lower production requirements.

[1]On the Norwegian shelf, the production capacities typically vary between $80.000$ and $200.000$ BPD . $1$ barrel $\approx$ USD 60 (2006).

- Maintenance cost savings. Unnecessary maintenance work is avoided and savings can be achieved through reduction in maintenance induced failures, reduction in scheduled maintenance, reduced spares inventory and reduced planned outage.
- Increased equipment lifetime. The CM allows longer service time, because the life of each individual equipment item is utilized at a maximum level without increase in damage severity. An incipient failure is stopped.
- Higher efficiency. Performance monitoring is useful in scheduling maintenance actions such as e.g. cleaning of heat exchangers and washing of rotor blades of a gas turbine.
- Sound basis for continuous improvement. The CM suits procedures for an efficient evaluation to improve maintenance actions. By monitoring the condition both before and after a maintenance action, means of improvement can easily be detected.
- Improved safety assurance. Increased equipment knowledge reduces consequences for personnel and environment due to primary and secondary damage caused by machine failure.

Based on equipment criticality and the time from an incipient failure to a failure occurrence, different CM policies are eligible and have to be evaluated. Off-line surveillance generally requires less investment than on-line CM, and the reasonable method is often visual inspection without use of any measuring devices. A more objective approach is by use of condition measuring devices at regular intervals. Use of CM equipment generally increases the pre-warning period before a failure occurs. On-line CM is in general only beneficial for equipment where failures may have huge consequences either for safety or costs.

## 2.5 Economic criteria

In addition to ensuring safe operations, management of ageing systems includes economic evaluations. The ability to select cost-effective solution provides an important competitive advantage. Measures like availability and efficiency are readily converted to economic figures. Safety measures and risk reduction to achieve a specific safety level also have their merits and price, but are in general not converted to an economic value.

Economic analysis of overhaul and replacement decisions require a variety of data inputs, either to make individual decisions or to study a class of assets. Two types of

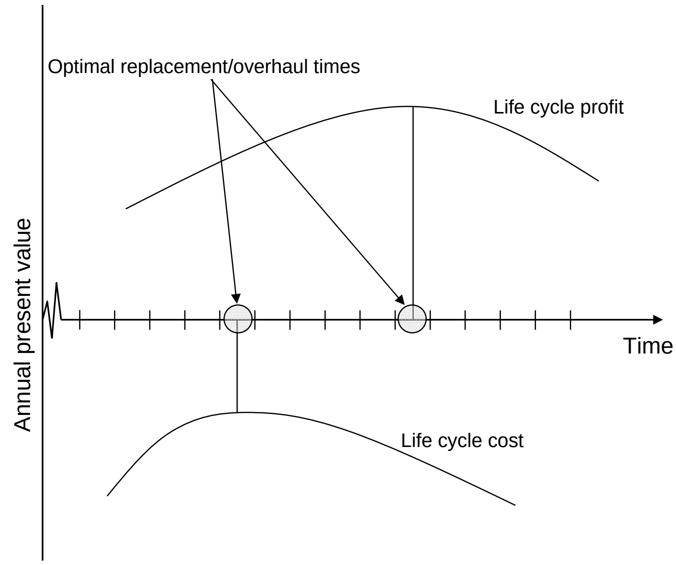


Figure 2.3: Optimal overhaul and replacement periods by using LCC and LCP calculations.

data are needed for the economical evaluation — general economic data and specific asset data (see 2.5.2).

### 2.5.1 Life Cycle Profit versus Life Cycle Costs

A common method to compare the economic features of alternatives is to apply Life Cycle Costing (LCC). The alternative which results in the lowest LCC is generally preferred. The benefit of applying the LCC concept is that the investment costs are only considered as a part of the total cost. The LCC involves all costs associated with the system life cycle i.e. it includes research and development, production and construction, operating and maintenance and system retirement and phase-out costs [18]. A weakness of the LCC methodology is the omission of the profit achieved by a selected alternative. The Life Cycle Profit method recognizes this limitation, and includes the income to evaluate the net profit instead of costs only, to classify the benefit of a specific alternative. The alternative with lowest LCC does not always equal the alternative which results in the highest LCP. In overhaul, replacement and modification decision problems, this difference may often have an impact on the optimum schedule for actions (see figure 2.3).

The cash flows identified by using the LCC and LCP method have to be compared on the same basis. A common basis is the net present value criterion, which is expressed as:

$$NPV(\beta^*, N) = \sum_{n=0}^{N} \beta^{*n} F(n) \tag{2.2}$$

where $\beta^*$ represent the annual discount rate [2], $F(n)$ the net profit at year $n$, and $N$ the expected number of years to operate.

### 2.5.2 Generic and specific asset data

The challenge using LCP and LCC criteria is to select and collect necessary input data. Some data are generic and independent of the asset to be evaluated, while other data largely depends on the selected asset. The preferred technology and present technical condition during operation are typical examples of asset properties in this context.

The amount of data needed for the analysis depends on the required level of detail. Looking at economic and financial aspects for a group of equipment items, it is normally sufficient to collect average data on purchase costs, operation and maintenance expenditures, overhaul costs and salvage value. The disadvantage of such an approach occurs when the inhomogeneity of the group, with respect to the technical condition, is large. While the mean net profit of the group is below acceptance criteria, some of the items may have a high profit and should continue to operate for a longer period. To consider the individual properties of each asset requires a much higher degree of detail and the analysis is therefore generally more cost exhaustive. On a detailed level, the individual properties which may/should be considered are:

- *Capacity* – depends on selected technology and the technical condition during operation.
- *Product quality* – depends on selected technology, and the technical condition during operation.
- *Energy consumption* – depends on the efficiency and may have a significant effect on pollution taxes. The efficiency is a result of selected technology and the technical condition during operation.
- *Maintenance expenditure* (including spares) –depends on selected technology, time in service and the technical condition during operation.
- *Overhaul costs* – depends on selected technology, time in service and technical condition.

[2] Annual discount rate, $\beta^* = \frac{100+inflation}{100+interest\ rate}$.

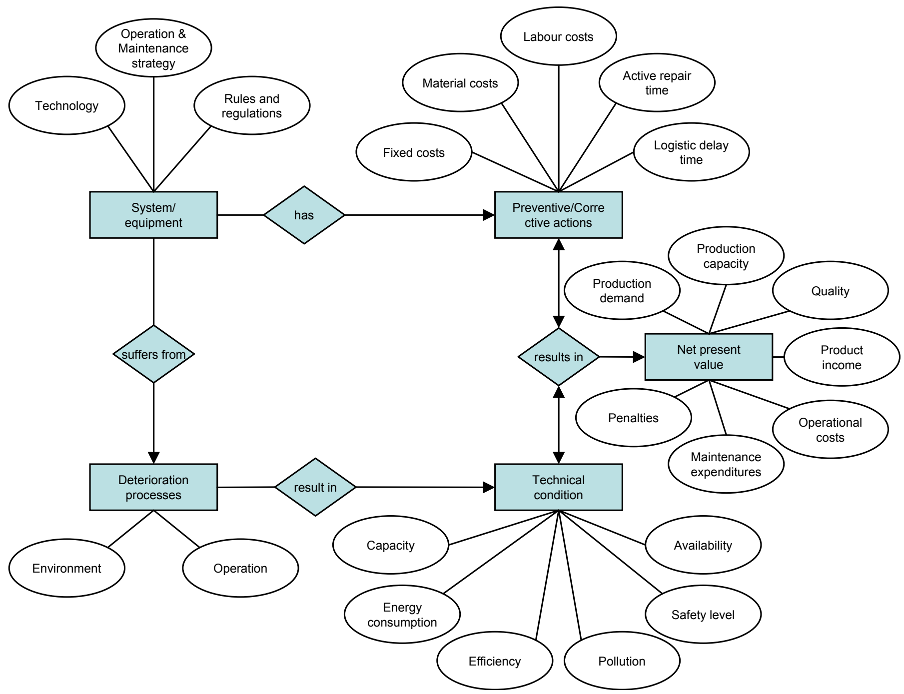


Figure 2.4: The effect of operation and maintenance on the technical condition, and on the resulting net present value (NPV).

- *Salvage value* – depends on the technology, time in service and the technical condition at replacement and market demands

A prediction of the future development for each of these properties has to be based on historical data combined with expert knowledge. Figure 2.4 presents an overview of different physical elements and objectives which may have an effect on the resulting net present value (NPV) during operation.

The generic data relates either to the company policy regarding e.g. minimum attractive rate of return (MARR) or to external condition such as e.g. authority requirements and market constraints. Internal condition such as budget constraints and production constraints may have a significant impact on the final decision. In addition to these constraints, the following properties may have an influence — depreciation schedules and tax allowances, property tax rates on assets, overhead rates on e.g. maintenance parts and labour, and old price indices [12].

### 2.5.3 Planning horizon

Technical systems are normally designed to operate for a specific number of years. During operation, several overhaul and replacement actions have to take place to maintain a specific standard. As the facilities reach the end of the operating horizons, the probable profit resulting from overhaul, replacement or modification often reduces. In the end, the decision may either be to overhaul or replace the equipment one or two times before the final shut down. This *end of horizon* effect may be significant and should not be ignored. Although the predicted operating horizon is highly uncertain in the early phases, the operating horizon becomes more predictable as the age of the facility gets closer to the final shutdown.

Looking at slowly deteriorating systems, it may be even important to consider *end of horizon* effect early in the life-cycle. Despite day-to-day maintenance on systems and major equipment, replacement or major overhaul may be the only option to facilitate continued operation. In general these actions are infrequent, and thus the historical records (mean time to failure) are often insufficient. Use of condition monitoring is therefore a necessity to improve the decision process.

## 2.6 Decision making

Ageing management can be described as engineering, operations and maintenance actions to control within acceptable limits ageing degradation of systems, structures and components.

Management of ageing systems involves decision processes to prioritize and select between different activities to ensure a safe and cost effective operation. Decisions relating to ageing management are often characterized by *uncertainties* and *multiple goals*.

### 2.6.1 Uncertainties

Uncertainties are introduced by the physical properties of the system or equipment item itself, change in environmental properties, imprecise measurements, lack of data and inaccuracy in the prescribed decision model.

Starting with the *physical properties* of the equipment, the ability to withstand influence from the environment depends on the quality of the equipment itself. The quality is a result of the production process and is in general assumed to vary randomly within given limits. During operation, the equipment is influenced by different modes of operation and external environmental factors causing the equipment to deteriorate. These *environmental properties* generally vary randomly, but they may also vary as functions of time due to the season and/or change in production plans etc. Other properties tied to economic figures do also come into play here such as e.g. discount rate, market and release of new technologies. A general problem in management of ageing systems is the lack of data and the quality of that data which is available. As a consequence, the *statistical models* of physical attributes becomes uncertain. Both mean and variance may be affected. The process to fit data to statistical distributions or stochastic processes should therefore be considered as a source of uncertainty. The statistical models are based on data collected through measurements. The *uncertainty introduced by measurements* may consist of several components, and the International Committee for Weights and Measures (CIPM) has divided these components into two categories, according to the method used to estimate their numerical values [19]:

- those which are evaluated by statistical methods,
- those which are evaluated by other means.

The first type of uncertainty is commonly classified as "random", and the second as "systematic". The "random" effects often arise from real world phenomena. The "systematic" errors may result from alack of precision and/or relevance in the information available. The last source of uncertainty is caused by the *mathematical models* of the decision process. The models are general a simplification of real world phenomena and decision processes, thereby introducing uncertainty (i.e. approximations, rounding errors,, etc.).

The "randomness" in the information can be managed through use of probalistic theories. By applying statistical methods, the historical records may be fitted to statistical distribution or statistical processes.

Decision analysis provides methods for decision-making under conditions of uncertainty and multiple objectives. These methods are therefore applicable for ageing management decision making.

### 2.6.2 Decision analysis

Decision analysis (DA) is a general term which applies to modern microeconomics and statistical theory for decision under uncertainty with multiple goals. DA provides methodologies for both structuring decision situations and making rational choices

[20]. DA analysis may be divided into two broad classes of problems. The first class concentrates on *one-time* decisions, where a group of alternatives must be compared on the basis of multiple competing goals. The second class involves *sequential decisions* where uncertainties and probalistic dependencies are very important.

The decision problems often involve a trade off between multiple goals. In addition to achieve a high profit, the production facility is generally required to keep a high standard to avoid safety and environmental hazards. The latter demands often do not coincide with economic goals, and the decision maker (DM) must therefore relax on some requirements. Applying specific methodologies and techniques such as hazard studies, fault-tree analysis and LCP analysis, the decision maker is capable of generating the input to an overall evaluation. To describe the overall planning process in formal terms, Massam [21] has defined the General Planning Problem (GPP) as:

> *Given a set of $U$ plans, and for each an evaluation on a set of $V$ criteria, for a set of $W$ interest groups, classify the $U$ plans in such a way as to identify their relative attractiveness so that agreement among interested groups is maximized.*

A class of methods which considers the GPP is the "multi criteria decision making" (MCDM) methods. The MCDM methods are concerned with the general class of problems that involve multiple attributes, objectives and goals [22]. Attributes refer to descriptors of the objective reality while objectives are closely identifiable with DM's needs and desires. An attribute becomes an objective when it is assigned a purpose, a direction of desirability or improvement. Criteria are measures, rules and standards that guide the decision making, and goals are fully identifiable with DM's needs and desires. Common methods within MCDM are the “multi-attribute utility theory” (MAUT) and the Analytic Hierarchy Process (AHP). Both are classified as additive MCDM models [21]. The additive models seek to reduce the plan evaluation and selection problem to one in which each of the alternate plans is classified using single score which represents attractiveness. They use a *multi-measure utility function* that allows an alternatives overall desirability to be computed based on how it performs on a set of evaluation measures. Both MAUT and AHP have been applied to evaluate system design and different reconditioning alternatives [23].

Sequential decisions are modelled using tools from stochastic optimization theory and simulation. Typical solution methods within stochastic optimization are decision trees, dynamic programming and scenario aggregation [24].

A decision tree consist of nodes and arcs. The nodes represents states, the arcs decisions. In a stochastic environment the decision tree is extended to incorporate circular nodes called chance nodes. Figure 2.5 presents an example of a decision tree includ-

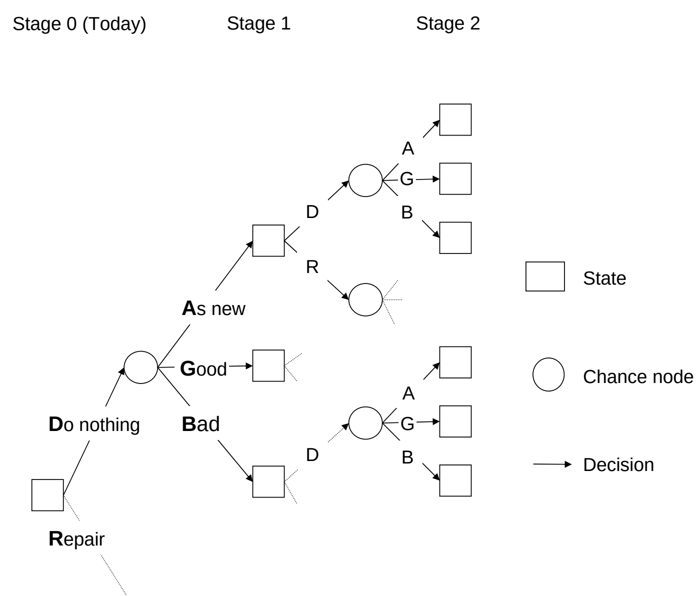


Figure 2.5: Basic set-up for a decision tree.

ing chance nodes. The chance nodes represent points in which chance and probability plays a dominant role and reflect alternatives over which the DM has (effectively) control. A third type of nodes, called terminal nodes, represents the ends of paths from left to right through the decision tree. Each branch in the tree is checked, and the best decision for each node is found by *folding back* from right to left. Due to the stochastic nature, it is only possible to determine the best possible first decision. The general approach is to chose a *wait and observe strategy* thus, a decision is only made after observing the first step.

The decision tree has its advantages when the input state is known. When the input state is known, the decision tree produce numbers in the leaves, which are not functions of initial states (which is the result when using stochastic dynamic programming (DP)). The folding back is therefore very simple and efficient. The strength of DP lies in the number of cases which need to be investigated compared to the decision tree [3]. Figure 2.6 presents an example of dynamic programming.

Important aspects in dynamic programming are the time horizon, state variables, de-

[3] In the case of a decision problem evolving over 10 periods, each period involving 2 possible decision, the decision tree method would require $2^{10} + 2^9 + 2^8 + ... + 1 = 2^{11} - 1 = 2047$ calculations, while DP only would require 20 calculations.

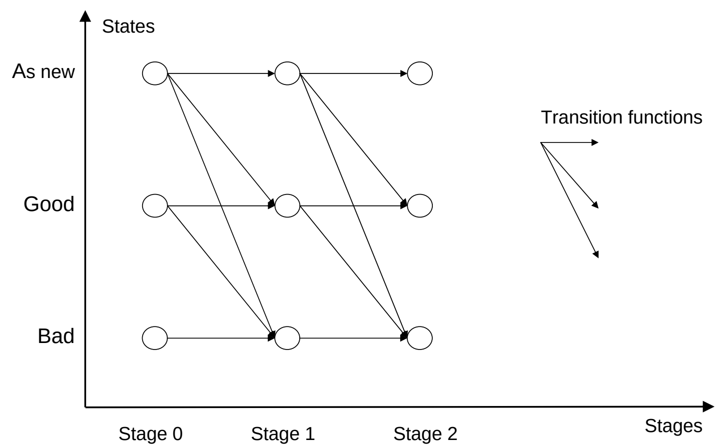


Figure 2.6: Basic set-up for a dynamic programming.

cision variables, return functions, accumulated return functions, optimal accumulated return function and transition functions. The *time horizon* refers to the number of *stages* (time periods) in the problem. The stage will be represented by the variable $n$, where $n = 1...N$. $N$ is the time horizon or in this context the planning horizon. The stage is the element in DP which transfers the problem from a multi-variable optimization problems, to a sequential series of mono-variable optimization problem. *State variables*, $z_n \in Z$, describe the state of the system, for example present technical condition related to safety. The set of all possible choices is reflected and/or governed by the state at each stage. *Decision variables*, $x_n \in X_n$ are the variables under one's control. They can represent the decision to modify a plant, replace or restore an equipment etc. The *transition function* shows how the state variable changes as a function of decisions, i.e the transition function dictates the state that will result from the combination of present state and the present decisions. The transition function can be expressed as [25][4]:

$$z_{n-1} \quad = \quad z_n \otimes x_n \tag{2.3}$$

[ output of stage $n$ ] = [ input to stage $n$ ] $\otimes$ [ decision made at stage $n$ ]

A *return function*, $r_n(z_n, x_n)$, shows the immediate returns (costs or profits) as a result of making a specific decision in a specific state. The sequential solution procedure makes it necessary to keep track of all the returns accumulated in the decision process as it proceeds from stage to stage. The accumulated returns calculated over $n$-stages, given a particular state variable $z_n$, are denoted by $J_n(z_n)$. The optimal accumulated returns show the value of making the optimal decision based on an accumulated return function, in other words, the best return that can be achieved from the present state until the end of the time horizon. A particular value of $z_n$ may give rise to many possible decisions, $x_n$, among which is a decision, $x_n^*$, which gives rise to an optimal $n$-stage accumulated return, $J_n^*(z_n)$. This situation can be expressed by the equation[25]:

$$J_n^*(z_n) = opt\{r_n(z_n, x_n) \otimes J_{n-1}^*(z_{n-1})\} \qquad , \forall\, x_n \tag{2.4}$$

Once the $N$-stage optimal policy has been discovered, the $N$-component decision vector can be recovered by tracing back through the $N$-stage transition function.

Scenario aggregation operates on an event tree, which is a tree that branches off for each possible value of the random variable, $P$, in each stage. DP is considered to be more flexible than scenario aggregation in terms of distribution of $P$, similar to scenario aggregation with respect to decision variable $x_n$ but much more restrictive with respect to state variable $z$ [24]. Figure 2.7 presents an example of scenario aggregation.

A methodology utilizing simulation is "influence diagrams", which are considered to substitute conventional decision trees in modelling and solving real world decision problems. It offers an effective representation of independencies between variables compared to decision trees. An influence diagram consists of nodes which represents uncertain quantities or random variables. Lines joining the nodes represent conditional dependencies/independencies between them. The problem is solved using Monte-Carlo methods, which combine the diagram with statistical variation into simulation models. Random number generation is used to represent statistical aspects of the system being modelled.

[4] Here the symbol $\otimes$ denotes any mathematical relationship between $z_n$ and $x_n$, including addition, subtraction, multiplication and root operators.

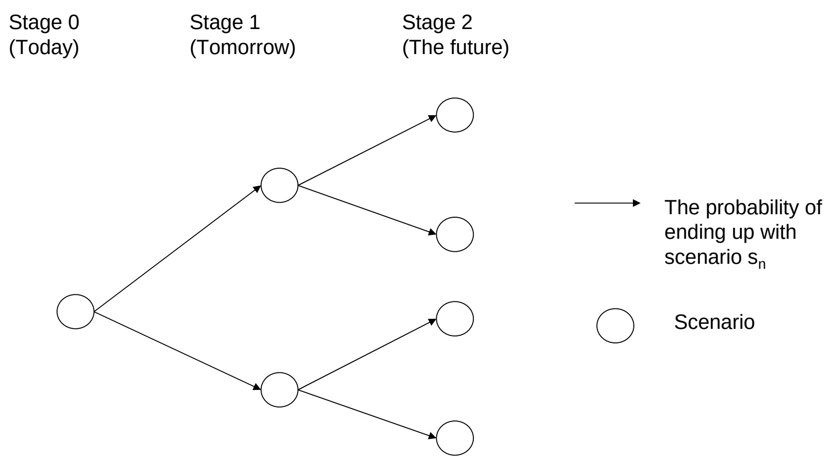


Figure 2.7: Example of an event tree.

## 2.7 Final remarks to the chapter

Management of ageing systems includes all technical activities ensuring safe and efficient operation through plant service life, including any extended life. It often requires balancing between multiple and in some cases competitive goals. A general challenge in management of ageing systems is lack of data and data quality i.e handling uncertainties. Managing the ageing process means predicting and/or detecting when a system or component within a system suffers from physical impairment or is obsolete. It also includes taking appropriate corrective or mitigatory actions. The sequentiality in the decision process, updating requirements and system knowledge based on surveillance and the uncertainties in available information call for formal methodologies provided by decision analysis. The process consists of:

1. definition of the objectives and criteria for which the systems are going to be evaluated (safety, environment, economy, etc.);

2. selection of important plant systems and/or components for which ageing should

be evaluated;

3. perform ageing management studies/evaluation for the prioritized systems

4. managing the ageing degradation in the selected systems by selecting proper surveillance, maintenance and operations.

Several studies have been conducted and reported within this area, especially within the Nuclear Power Plant Industry. The Nuclear Power Plant Industry and the International Atomic Energy Agency (IAEA) have published several technical documents concerning proper life cycle management and ageing management to ensure safety and sustainability [1], [26],[27], [28], [29].

Application of mathematical models in management of ageing systems is often directed to ensure safety and environmental issues such as methodologies within Probabilistic Safety Assessments focusing on the system reliability. Risk reduction may also be measured in profit or costs but is generally treated separately. Probalistic Safety Assessment analysis is thoroughly treated in several technical documents provided by IAEA e.g. [30],[31]. The topic is not discussed any further in this thesis. The same applies for the process of defining objectives and criteria, and methodologies to prioritize system importance. Within the oil and gas industry the criteria are directed to safety, environment, availability and costs. When it comes to maximize the profit, the availability normally overrules the costs due to the high income, until the reservoir becomes depleted, and the reduced production levels (or "tail end production") alter the economics substantially.

The focus of this thesis is to provide a methodology for decision support in an ageing system management context. These decisions and corresponding decision making processes are often related to rare and sometimes *one of a kind* events. The basis of the thesis is to utilise CM and description of system status as the key input elements for the decision support. CM information can be provided on a system or plant level utilising e.g. the approach of Technical Condition Index as described in section 2.3 or at equipment level by utilising standard CM analysis. The decision support includes a proposal for an inspection and replacement strategy. The extensions to traditional models with respect to life cycle profit/cost calculations are to include scenarios describing the possibility of obsolescence due to improved technology and to handle the effect of change in functional requirements (e.g. production demand) over the life cycle. The latter also includes an assumption of remaining time to operate i.e. the effect caused by defining a finite horizon.

Different types of economic models are discussed in chapter 3 and 4.

# Chapter 3

# Economic optimizations

## 3.1 Introduction

Maintenance modelling includes organizational issues, logistic support issues, and issues regarding the selection of maintenance strategies. A maintenance strategy includes both a description of activities and of the frequencies to carry out these activities. This chapter deals with existing mathematical models suitable to support maintenance engineers in managing ageing systems with respect to decisions concerning inspections, overhauls, replacements and modifications. According to Dekker [32], the subject of maintenance optimization models covers four aspects:

1. a description of a technical system, its function and its importance
2. modelling of the deterioration of the system with time and possible consequences for the system
3. a description of the available information about the system and actions available to management
4. an objective function and optimization technique

Dependent on the modelling approach, the models may support strategic or tactical decision problems (see section 2.2). Two distinct types of models have developed within mathematical modelling in this area:

- Capital replacement models, and
- Stochastic inspection, repair and replacement models

The capital replacement models reflect the managers point of view. Thinking of equipment as an asset value, the manager strives to maximize the profit by choosing the optimal strategy based on a balance between income and costs. Although technological and economic factors may be the principal drivers for equipment replacement, maintenance costs and unavailability are also important here. Developments of capital replacement models are briefly covered in section 3.2. An extensive amount of paper and theories has been published within the field of stochastic inspection, repair and replacement models. Instead of fitting income and cost data to functional forms, these models try to utilize information about the failure processes. Statistical distribution or stochastic processes are used to describe the physical deterioration of the equipment. Several authors, however, have claimed that several of these models have little impact upon the solution of real maintenance problems [33], [16]. The absence of sufficient data relating to maintenance problems of interest for a plausible model to be fitted and validated, and the complexity of the models that are often proposed, have been emphasized to support this statement. A brief presentation of different stochastic models is given in section 3.3.

Condition monitoring (CM) has in recent years been adopted to a great extent in the industry. Expensive equipment, high down-time costs, improved condition monitoring methods and diagnostic tools favour use of this policy compared to the traditional predetermined maintenance and corrective maintenance. Instead of looking at failure frequencies, the approach is to base the decision process on collecting data and updating information about the present condition, diagnosis and severity analysis. The nature of the decision process, including consideration of inspection tasks to reveal present condition, and the ongoing failure process, can nicely be modelled by methods developed within finite discrete sequential decision problems in a stochastic environment. These decision processes are often solved by applying techniques classified as Markov Decision Processes (MDP). Chapter 4 presents an overview of MDP and argues for the applicability of this modelling approach in an ageing management context. The MDP models are considered to belong to the class of stochastic inspection, repair and replacement models.

## 3.2 Capital replacement modelling

In general, capital replacement is often regarded as a part of strategic planning of capital expenditure. The strategy is to spend the capital in a reasonable manner with the objective of minimizing expenses and maximizing profit. Replacement policies consider all sorts of items and systems, from small components of a device to total production plants. There is, however, a major difference between the approach selected when dealing with a component compared to a whole plant. In the component replacement case, the objective is to minimize the expected running cost per unit time. Based on a predicted income and cost functions, the replacement models seek to find the optimum balance between preventive maintenance costs and the risk of incurred costs for corrective actions. For large expensive plant on the other hand, economic factors such as discount factor, rate of inflation, interest rate and tax parameters are considered, with purchase, operation, maintenance and resale costs also taken into account. The implication is that capital expenditure is planned over a certain specified period, the planning horizon. The planning horizon may have a finite or an infinite length.

There are mainly two different approaches to solve capital replacement problems. When the replacement policy aims at replacing equipment at fixed intervals due to cost minimization (or profit maximization), this is normally termed *economic life policy*. This is also the classical approach of modelling capital replacement problems. Scarf and Hashem [34] have divided the economic life models even further into three separate classes depending on the solution method applied. These classes are infinite, variable finite (with length of the horizon as a function of decision variables) and finite/fixed horizon methods (with variable number of replacement 'cycles'). A major weakness in this type of policy is that it does not allow the decision maker to take advantage of good equipment in the sense that it could continue to operate for a period longer than its 'economic life', nor will this policy dispose of bad equipment at an early date.

The second approach within capital replacement modelling is termed *cost limit models*. These models, as opposed to economic life models, consider conditional information available when a decision has to be made. They can be classified to be special cases of Markov Decision Processes (see sec. 4). Using information about operating cost on an individual basis, limits are put up to determine whether replacement of an existing item is optimal or not. The input data and policies influencing the decision process are described previously in chapter 2.

An extensive number of **economic life models** have been published in the literature. The objective is to determine the optimum interval, $t_p^*$ , to maximize the total discounted net benefits derived from operating the equipment over a long period of time. The modelling approach can be illustrated as shown in figure 3.1. Earlier economic life models (rent models) were basically all expressed as:

$$C(t_p) = \frac{1}{t_p}\left\{C_{CAP} + \int_0^{t_p} c(t)dt - S(t_p)\right\} \tag{3.1}$$

where $C_{CAP}$, $c(t)$ and $S(t_p)$ are respectively the capital cost, the operating cost per unit time and the resale value. Here C(t) represents the cost per unit time over a single replacement cycle. The optimum value of $t_p^*$ is the economic life which min-

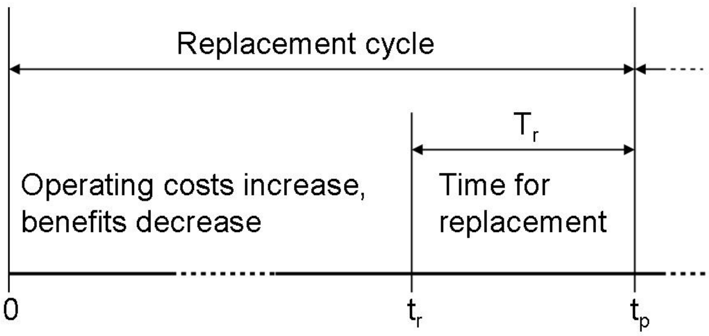


Figure 3.1: Preventive replacement policy for capital equipment. The system/equipment is put into operation at age $t = 0$. When its age, or accumulated operating time, $t$, reaches a pre-specified value $t_r$, it is replaced with a new identical one with replacement time $T_r$. The replacement cycle is $t_p = t_r + T_r$. $T_r$ is often small compared to $t_r$ such that $t_p = t_r$ when ($t_r \gg T_r$).

imizes equation 3.1. Another criterion well established and widely used is the total discounted cost over all times, expressed as:

$$C(t_p) = \frac{\left\{ C_{CAP} + \int_0^{t_p} c(t)\beta^t dt - S(t_p) \right\}}{(1 - \beta^{t_p})} \qquad \beta < 1 \tag{3.2}$$

where $\beta$ represents the annual discount rate ($t_p = t_r$). The criterion given by eq. 3.1 suffers from the assumption that costs do not change with time i.e. the discount rate equals one. Eq. 3.2 becomes invalid if $\beta \geq 1$, which in some cases makes the criterion inapplicable for use. Further, it has been emphasised that criterion 3.2 lacks credibility in an unsteady economy, because the optimum economic life is based on cost estimates for an indefinite number of replacements across future decision points [35]. Technological developments are nor considered in these criterion. However, due to their simplicity, such models have been used extensively by many authors.

The economic life models (or criterion) are often classified according to length of the planning horizon. The criteria above belong to the group of infinite horizon models. These types of models have also been named *stationary long-term models* by Dekker [36]. The infinite planning period is just a concept that allows performance of some convenient tricks in the modelling process. Even though their credibility may be discussed, the infinite horizon models give an indication of the optimal interval between replacements.

Due to the weaknesses of the former models, several criteria within the class of *finite* *horizon models* have been developed. Finite horizon models are the methods within economic life models that are desired and accepted for use in real world applications. The term “discounted rent criterion” was introduced in 1984 by Christer [37]. The replacement period is identified as the interval which optimises a regular annual rent, $\overline{W(N)}$. The total rent must equate to the total discounted cost/benefit at the end of the planning horizon. Looking at costs, we seek the solution to minimise $\overline{W(N)}$, where the regular annual rent is calculated as:

$$\overline{W(N)} = \frac{NPV(\beta, N)}{\sum_{i=1}^{N} \beta^i} \tag{3.3}$$

Here, the cost profile $NPV(\beta, N)$ is presented as discrete sums, payable at the end of a period. The NPV over a single cycle of integer length N is generally expressed as:

$$NPV(\beta, N) = \sum_{i=1}^{N} C_i \beta^i + K(N)\beta^N \tag{3.4}$$

where K(N) expresses an income at age $N$. An alternative criterion often applied is the net discounted present value per unit time given by [37]:

$$\overline{NPV(N)} = \frac{NPV(\beta, N)}{N} \tag{3.5}$$

The concept of two-cycle models was introduced by Christer and Goodbody [35] to determine optimum replacement age for fork lift trucks in an unsteady economy. The criterion function to be minimised was expressed as[1]:

$$\overline{NPV(n,k,l)} = \frac{\left\{ \int_0^k c(n+t)\beta^t dt + \beta^k \left( C_{CAP} + \int_0^l c(t)\beta^t dt + \beta^l C_{CAP} \right) \right\}}{(k+l)} \qquad \beta > 0 \tag{3.6}$$

which represents the total discounted cost per unit time of operating a plant currently $n$ years old for a further $k$ years, maintainig/upgrading it with possible different equipment models, and operating for a further $l$ years before again replacing equipment models of same type, $N = k + l$ (see figure 3.2). The advantage of using the two cycle model is primarily the relatively short term estimates of costs and discount rate needed, compared to the required assumptions using infinite horizon models. The formulation also had the advantages of coping with variable discount rate and the situation in which discount factor $\beta \geq 1$. The model was later extended to include the influence of tax parameters, but the inclusion showed very little influence on the replacement age mainly due to tax legislation prevailing at that time [38]. Scarf and Hashem [34] have classified this model as a variable finite horizon model, because the length of the horizon is a function of the decision variables. In

[1]The salvage value (resale value), $S(n)$, has not been explicitly incorporated into the criterion function, but the expression can readily be modified simply by subtracting the value from $C_{CAP}$,

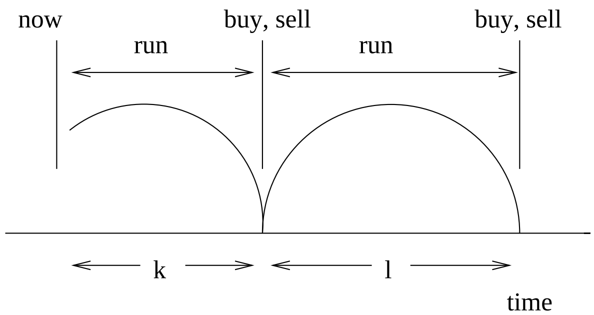


Figure 3.2: Two cycle replacement model [39].

some cases, the breakdown cost may be significant, causing a financial problem for the manager/owner. Scarf and Christer [40] extended the two-cycle model to include such penalties. They determined replacement policies for medical ventilator equipment for a range of values of the penalty cost. The inclusion of a penalty cost has also been used in the decision problem to select optimum replacement periods for a mixed fleet of buses [41]. The penalty cost was included when the number of failures exceeded the number of spares available, assuming that the failures occurred according to a nonhomogeneous Poisson process. Scarf [33] states that "the model proposed is flexible in that: the plant in a particular sub-fleet may themselves be of differing ages and specification; technological change is allowed for in that costs relating to replacement plant may be assigned as appropriate; the assignment of equipment to sub-fleets would be under the control of the fleet operator, along with some indication of the sub-fleets which are candidates for replacement." A weakness of the model proposed is that the optimum value of the horizon length $(k + l)$ depends on the choice of sub-fleets to be replaced. To avoid difficulties in selecting sub-fleets to be replaced first and second etc., a modified model by notionally fixing the length of horizon, $N$, was proposed by Scarf & Hashem [34]. They calculate optimum policies for a range of horizons, and select a horizon not too large, but large enough in order not to increase costs by imposing a poorly scheduled replacement. Within a horizon, they operate with a variable number of operate-sell-and-buy cycles, $O$. The model is also capable of incorporating technological changes which they term a “Non-like-with-like” replacement. In the model, it is possible to describe several sub-fleets (technologies), $\Phi$. The current sub-fleets (technologies) in use is indexed by $u = 1, \ldots, \phi$ and new replacement sub-fleets are indexed by $u = \phi + 1, \ldots, \phi + O$. The fleet size may be constant $(v_i = v_{\phi+i} \forall i)$ or variable, with sub-fleet sizes $v_{\phi+i}(i = 1, \ldots, O)$. They let $L_i(i = 1, \ldots, O)$ express time from beginning of $i$th cycle to the replacement of sub-fleet $i$, and calculate the total discounted cost for a given replacement schedule over the horizon, $N$, by:

$$NPV(\beta, O, L_1, \ldots, L_O; N) = \sum_{i=1}^{O} \beta^{m_i} \left\{ \sum_{t=m_{i-1}+1}^{m_i} \overline{c_i(t)} \beta^{t-m_i-\frac{1}{2}} + v_i C_{CAP\phi+i} - \overline{S_i(m_i)} \right\} \tag{3.7}$$

where $m_i = \sum_{j=0}^{i} L_i$. Here, $\overline{c_i(\cdot)}$ is the age related operating cost of the whole fleet in cycle $i$, and $\overline{S_i(\cdot)}$ is the salvage value of plant in sub-fleet $i$. The costs and salvage value are expressed as:

$$\begin{aligned} \overline{c_i(t)} &= \sum_{u=i}^{\phi+i-1} \sum_{j=1}^{v_u} c_u(\tau_{uj} + t), \qquad (i = 1, \ldots, O), \\ \overline{S_i(m_i)} &= \sum_{j=1}^{v_i} s_i(\tau_{ij} + m_i), \qquad (i = 1, \ldots, O), \end{aligned}$$

where $\tau_{ij}(i = 1, \ldots, \phi{+}O; j = 1, \ldots, v_i)$ is the current age of plant. $C_{CAP\phi+i}$ is the cost of each replacement (to buy a new) in sub-fleet $\phi + i$ ($i = 1, \ldots, O$). Using the rent criterion described in eq. 3.3, they studied replacement schedules for a fleet of buses. They concluded that it was difficult to determine optimal retirement policy for the fleet as a whole, because both the usage and maintenance level were uncertain for those sub-fleets which were partially retired. They further emphasised that the length of the planning horizon needs to be chosen with some care, due to discontinuities in the models first derivate. This model represents a state of the art of economic life modelling, capable of incorporating several sub-fleets (also new technologies), and is easily extendable to incorporate penalty costs and end-of-horizon effects which merit study. Bouamra [42] discusses variations and use of the above model and other capital replacement models.

## 3.3 Stochastic inspection, repair and replacement models

An extensive amount of paper has been published within maintenance optimisation of units or systems subjected to stochastic deterioration. The literature has frequently been reviewed. Based on the objective of the review, each has proposed a new classification scheme. Some of the most widely known reviews are (in chronological order):

- Sherif and Smith (1981) [524 references] [43]
- Thomas (1986) [46 references] [44]
- Valdez-Flores and Feldman (1989) [129 references] [45]

- Cho and Parlar (1991) [123 references] [46]
- Dekker (1996) [132 references] [32]

The survey by Cho and Parlar discusses different classification schemes based on previous reviews. They summarise possible criteria of classifying stochastic maintenance models, and present a list of eight bullets: (a) information availability, (b) single-unit or multi unit system, (c) time-event/action relationship, (d) state event/action relationship, (e) model types, (f) optimisation criterion, (g) methods of solution and (h) planning horizon.

Information availability addresses the decision makers (DM) knowledge of the deterioration process and system state upon a decision. Sherif and Smith [43] distinguish e.g. between stochastic models under risk, and under uncertainty. Under risk, the distribution of time to failure is assumed to be known, while under uncertainty this assumption is not imposed. Decker [32] states that the latter models have to be based on adaptive policies. The models can be subdivided even further based on information availability. Some models assume that the state of the system is known with certainty at all times, while others state that the condition is only available through inspection. The former models have been termed preventive models and the latter preparedness models [43]. In some cases, the deterioration process can only be partially observed due to no inspections or insufficient inspection techniques. The degradation of the system is thus only stochastically known. The theory of partially observed Markov Decision Processes (MDPs) is concerned with such problems. MDP is covered in chapter 4.

A frequently applied classification scheme is to distinguish between single-unit and multi-unit systems. While the review by e.g. Sherif and Smith (1981) [43] covers both areas, the review by Valdez-Flores and Feldman (1989) [45] applies only to single unit systems, and the review by Thomas (1986) [44] and Cho and Parlar (1991) [46] only concentrates on multi-unit systems.

**The single-unit models** seek the optimal strategy based on a deterioration model and economic values governing the unit. Using a model classification approach, the single-unit system models may be divided into several sub categories such as age replacement models, minimal repair models, shock models, inspection and inspection/repair/replacement models.

*Age (dependent) replacement models* seek the replacement interval which minimises the total operating cost per time unit by balancing the risk of failure forcing a corrective maintenance action (costly), and the probability of survival and subsequently a preventive maintenance action (less expensive) [16]. An extension to the original age-dependent policy is to include age-dependent costs, which reflects increased maintenance burden or reduced productivity as a function of the age since last renewal.

While age replacement models follow a renewal theory, assuming the system is set back to as-good-as new condition after repair, the *minimal repair* models assume that the rate of occurrence of failure (ROCOF) of the system remains as it was just before failure. Since the ROCOF increases with age, it would become increasingly expensive to maintain operation by minimal repairs. Therefore the minimal repair models seek a policy which determines the optimal time to replace (renew) the system instead of repairing it. Ascher and Feingold [16] and Valdez-Flores and Feldman [45] present an extensive overview of models based on the concept of minimal repair modelling.

An optimal solution for replacement of systems subjected to *random shocks* is often a *control-limit policy*. Each shock causes damage to the systems and the damage accumulates additively until replacement or failure. Both the time between shocks and the damage caused by a shock are random variables [45]. A usual approach is to apply a nondecreasing Markov process to describe the cumulative damage caused by the shocks.

The *standard inspection models* (or periodic inspections) usually assume that the state of the system is completely unknown unless an inspection is performed. The standard inspection models assume that a component is inspected at fixed intervals, with subsequent replacement when at inspections the component turns out to have failed. If a component fails before it is inspected, it stays inoperative until it is inspected. Some models also consider problems where the inspection does not reveal the true status of the system thus, the information obtained through inspection is not reliable. The Delay Time Model (DTM) and modelling by partially observable Markov decision process (POMDP) are capable of handling such decision problems. POMDP is discussed in section 4.3. The DTM tries to determine the distribution of the time from which a potential failure could be observed to that at which a failure occurs. The motivation for such an approach is that the uncertainty in estimating only a small part of the life cycle (the last part) should be less than the uncertainty in estimating the distribution of the total life cycle. The delay-time is similar to the P-F interval [2] in an RCM context [47]. Baker and Christer [48] provide an comprehensive overview of the delay-time model and its extensions.

The *repair and inspections models* (or condition based maintenance) can be considered as an extension to the standard inspection models. As the system deteriorates, following a nondecreasing stochastic process, the decision maker has to decide at

[2] P-F interval: The time from which a **P**otential failure can be detected to that at which the **F**ailure occurs.

which condition the system should be repaired and/or replaced, and to select a time for the next inspection to take place. A usual approach to solve the sequential decision problem is to apply a Markov decision process (MDP) method. The MDP methods also have the capability to model multi-unit systems[49].

**The multi-unit models** seek optimal maintenance strategy for components which have economic dependencies, structural dependencies, and/or stochastic (failure) dependencies. Economic dependence implies that cost can be reduced when several components are jointly maintained due to reduced down time cost, and/or other logistic support costs. Structural dependencies apply if maintenance of a failed component implies maintenance of other components as well. If the state of a component influences the lifetime distribution of other components or if there are causes outside the system which bring about simultaneous failures (common-cause failures), these are termed stochastic dependencies. Wildeman [7] has divided the multi-component models (grouping of single-unit maintenance activities) into two major classes — stationary grouping and dynamic grouping. The classification is according to their planning aspects.

The *stationary models* provide static rules for grouping of maintenance activities, but they provide no framework to take the short-term information into account. The stationary models are applicable in selecting interval for corrective maintenance activities, preventive maintenance activities and combinations of these. The latter is not necessarily planned in advance, but set-up savings can be obtained if maintenance of a component (both corrective and preventive) yields an opportunity for maintenance of other components. The disadvantage of so-called *opportunistic maintenance models* is that it is often not known in advance which actions will be taken, thus the benefit obtained by planning activities and work preparations diminishes. A simple multi-unit optimisation model is the block-replacement model presented by Barlow and Hunter in early 1960 [50]. Dekker et al [8] used a *penalty function* in what they called *joint replacement* in an operational planning phase. The penalty function reflected the cost of shifting a single unit replacement interval from its optimum to fit the optimal interval for the group of items.

According to Wildeman the *dynamic grouping* can be subdivided into two categories: finite horizon and rolling horizon models. The finite horizon models incorporate the residual value of the system at the end of the horizon whereas the rolling-horizon models consider a new horizon once a decision on the finite horizon is implemented (according to short-term circumstances). These *opportunity maintenance models* utilise the accumulated system knowledge and seek grouping alternatives that minimize costs. The area of multi-unit optimisation models has recently been reviewed by Dekker et al [51], Wildeman [7] and Andersen [9]. Dekker & Plasmeijer [52] have presented a case study applying a multi-component model. They compared a joint-to-joint maintenance strategy of grouping aged adjacent road segments to obtain economies of scale versus maintaining a whole road stretch in one operation. They also considered the merits in maintaining segments of different lanes.

Several methods exists to solve the stochastic inspection/repair /replacement models mentioned above. If it is impossible to derive a closed form solution, the model may be solved by simulation, stochastic programming or other operational research techniques e.g. linear programming, dynamic programming, mixed-integer programming, nonlinear-programming [43], [53],[54].

Most models assume an infinite horizon and seek solutions to minimize the long-term operational cost. The assumption of an infinite-horizon in general simplifies the solution methods. In some cases though, the assumptions of an infinite horizon can not be justified and methods to solve finite-horizon models have to be used. Typical objective functions are to minimize the accumulated long term costs or discounted costs. A common approach is also to seek the solution which minimises the cost per unit time.

## 3.4 Summary

The main objective of all models presented in this context is either to reduce cost or to maximize profit. Based on the objective of this thesis, presented in chapter 1.3, the solution provided by MDP is considered to be a practical approach. The ability to utilize condition monitoring data and to connect the observed or predicted state with different attributes such as operating costs, maintenance costs etc. are important properties of MDPs. Some solution techniques also provide the ability to include need for inspections and fixed horizon. The features and limitations of MDP, and present application of such models are discussed in the next chapter.

# Chapter 4

# The Markov decision process

## 4.1 Introduction

The Markov decision process (MDP) originates from the principles of dynamic programming introduced by Bellman in 1957 [55]. The new numerical method which he presented sought solution of *sequential decision problems*. The basic elements of the method are the *"Bellman principle of optimality"* and *"functional equations"*. The basic principles of the method can be illustrated as follows.

Consider a system being observed over a finite or infinite time horizon split into periods or *stages*. At each stage, the *state* of the system is observed and a decision has to be made. The decision influences the state to be observed at the next stage either deterministically or stochastically, and depending on the state and the decision to be made, an immediate *reward* is gained. The expected total rewards from the present stage until the end of the planning horizon are expressed as a *value function*. The relation between the value function at the present stage and the one at the following stage is expressed by the functional equation (recurrence relation). Optimal decision, depending on stage and state, is determined by backwards step by step maximizing the right hand side of the functional equation. This way of determining an optimal policy is based on the Bellman principle of optimality which says ([55], p. 83):

> *An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.*

The Markov Decision Process was introduced in 1960 by Howard [56], and combined the work of Bellman with the well established theory of *Markov chains*. Do to the nature of Markov chains, Howard also presented an optimisation technique which he named *policy iteration*. The technique was efficient in solving problems modelled as Markov decision processes with infinite horizons. The solution technique presented by Bellman was named *value iteration*, today also known as *stochastic dynamic programming* or *successive approximation*. While the policy iteration method is only applicable to infinite horizon problems, the value iteration method may also be used to solve finite horizon problems [57].

Selecting an appropriate method depends on the problem to be solved. The main features of stochastic dynamic programming versus policy iteration is presented in table 4.1. The infinite discounted MDP can also be solved by Linear Programming (LP). LP is a general technique and does not take advantage of the special structure of MDPs. To be specific, White and White (1989) [57] comment that policy iteration method is a simplex method with block pivoting. While the policy iteration method focuses on changing the plan until an optimal plan is attained, the value iteration focuses on changing the value function until it becomes sufficiently close to the stationary optimal strategy. Several extensions of the solution algorithms have been published to make them more efficient and to make them capable of solving specific problems. The principles are briefly discussed in e.g., [57].

In this thesis, the value iteration method has been selected. The main reasons for this are the ability to:

- optimize over a fixed/finite horizon that enables evaluation of *end of horizon effects*
- handle non-cyclic time dependent variation in demand such as production profile, forecast of turnarounds, forecast of new technologies
- incorporate a time variant and huge transition probability matrix (based on models of deterioration functions and results from condition monitoring)

In addition, the value iteration process provides a simple mathematical and intuitive formulation.

The following section will briefly present the basic Markov Decision processes. The limitations are discussed and some extensions (improvements) are presented. The contents are mainly based on the work of Bertsekas (1987)[58], Tijms (1986) [59], and the review of White & White (1989)[57].

Table 4.1: Comparison of the policy and value iteration methods.

| | Policy Iteration | Value Iteration (stochastic dynamic programming) |
|---|---|---|
| Mathematical formulation | More complex | Simple |
| State space | Small (few hundred) due to inversion of an $uxu$ matrix | Capable of handling thousands of states |
| Time (stage) dependent input/output | Possible only to incorporate cyclic variation | Easy to incorporate |
| Application | Infinite horizon | Both finite and infinite horizon |
| Efficiency | Convergence after few iteration | Convergence rather slow [a] |
| Results | Exact solution | Not exact (infinite solution) |

[a] Can be improved by use of error bounds

## 4.2 Description of Markov decision process

A Markov decision process is a controlled stochastic process satisfying the Markov property with cost assigned to state transitions [1]. A *Markov decision problem* is a Markov decision process together with a performance criterion. A solution to Markov decision problems is a policy ($\pi_t^i$), mapping states actions, that determines states transitions to maximise profit (or minimise cost) according to the performance criterion. A Markov decision process describes the dynamics of an agent interacting with a stochastic environment. Given an initial state or distribution over states and a sequence of actions, the Markov decision process describes the subsequent evolution of the system state over a finite or infinite sequence of times referred to as the *stages* of the process. A discrete Markov decision process is defined with the following properties [59]:

- system state $i \in Z$ ($Z$ is the set of $M$ possible states) is recorded only in discrete moments of time $n, n+1, \ldots$
- in each moment of time $(n, n+1, \ldots)$ a decision $x \in X(i)$ is made $X(i)$ is the set of possible decision when the system is in state $i$.

[1] The Markov property states that [60]: Given that a system is in a state $i$ at time $t$ $(s(t) < i)$, the future states $(s(t+v))$ do not depend on the previous states $s(u), u < t$.

- after the decision $x$ has been made in state $i$ a reward $r(i, x)$ is incurred
- after the decision $x$ has been made in state $i$ at time $n$ the system will transit to state $j$ at time $n+1$ with probability $p_{ij}(x) = p[z(n+1) = j|z(n) = i, x(n) = x]$

The dynamics of a MDP is described by the conditional transition probabilities, which has the property that:

$$\sum_{j \in Z} p_{ij}(x) = 1 \qquad \forall i \in Z, x \in X(i) \tag{4.1}$$

and

$$0 \leq p_{ij}(x) \leq 1 \qquad \forall i \in Z, j \in Z, x \in X(i) \tag{4.2}$$

The objective of the MDP is to find the optimal policy ($\pi^*$)according to a selected decision criterion. There are basically two alternative criteria:

- maximisation of the total (cumulative) expected discounted reward
- maximisation of the average reward per unit time

The criteria may alternatively be to minimise the total expected discounted cost or to minimise the average cost per unit time. The planning horizon may be finite, infinite or random. For the sake of simplicity, the mathematical expressions of the objective functions are only presented for finite horizon MDPs in the following text. Let $N < \infty$ be the number of decision epochs in a finite planning horizon problem. If the objective, $h$, is to maximise the total expected discounted reward over a finite horizon, $N$, it is mathematically expressed as:

$$\max h(z_0) = \max_{x_0,\ldots,x_{N-1}} \underset{z_1,\ldots,z_T}{E_i} \left\{ \sum_{n=0}^{N-1} \beta^{n-1} r(z_n, x_n) + \beta^N r(z_N) \right\} \tag{4.3}$$

where $E_i$ is the expectation operator, conditioned on $z_0 = i$. $\beta$ represents the discount rate, $z_0$ is the known initial state, and $r_(z_N)$ is the terminal reward associated with state of the system at the end of the last period. The objective function is subjected to the state transition probabilities, $p_{ij}$ where:

$$p_{ij} = p(z_{n+1} = j|z_n = i, x_n), \qquad i, j = 1, \ldots, M; n = 0, \ldots, N-1)$$

The solution to the objective function 4.3 is an optimal policy $\pi_n^*$, which is a function of stage $n$ and the initial condition $z_0$. The optimal policy only recommends the optimal *first* decision, $x_0^*$, due to the stochastic nature of the problem. In general, the result is a look-up table which determines the optimal decision $x_n^*$ based on present stage $n$ and condition $z_n$. When $n \to \infty$ and $\beta \geq 1$, the total expected discounted reward criterion may not be defined. However, the average expected reward per decision epoch MDP may be used as a criterion when $\beta \leq 1$.

In a finite-horizon MDP the only applicable solution method is the value-iteration algorithm. The mathematical formulation of this procedure is:

$$J_n(z_n) = \max_{x_n} \left\{ r_(z_n, x_n) + \beta \sum_{j=1}^{M} p_{ij} J_{n+1}(z_{n+1} = j) \right\} \tag{4.4}$$

where $z_n = 1, \ldots, M$ and $n = 1, \ldots, N-1$. A terminal cost $J_N(z_N)$ is incurred at the end-of-horizon, given that the system is in state $z_N$, that is:

$$J_N(z_N) = r_(z_N), \qquad z_n = 1, \ldots, M$$

The MDP is solved recursively as described in chapter 2.6. Starting at the end of horizon and tracing the optimum decisions, $x_n^*$, that are selected for every state, for all time periods (stages), the optimal policy is expressed as:

$$\pi_n^* = \{\mu_0(z_0), \ldots, \mu_{N-1}(z_{N-1})\}$$

where $\mu$ is a function mapping the decision $x_n$ based on the condition $z_n$, i.e. $x_n = \mu_n(z_n)$.

For the basic MDP problem there is a strong linkage between inspection and decisions (actions.) Since the state of the system at $n+1$ is only known probabilistically at $n$, the decision-maker can only select an activity after the state $z_n$ becomes known, i.e. in the beginning of stage $n+1$. Thus an inspection has to be performed *before* a decision can be made.

In addition to this assumption, the basic MDP also assumes that the condition can be revealed without uncertainty after each inspection. This implicitly means that the inspection technology is considered free of systematic and random errors.

To summarize, the following assumptions apply to a basic MDP:

1. system being modelled evolves over time according to a finite Markov chain.
2. at each stage the condition has to be revealed by an inspection *before* a decision can be made
3. the condition state of the system is revealed without uncertainty after each inspection

The next section discusses some improvements to relax the latter two assumptions concerning the information pattern i.e. what the decision maker knows at a decision epoch.

## 4.3 Applications and extension of the basic MDPs

Due to the limitations of the standard MDPs, several improvements have been proposed and adopted in real world problem solving. The main modifications have been done either to relax the assumption of inspection at each stage and each decision, or to handle the assumptions regarding uncertainty in the inspection output. Two methods which relax the assumption regarding required inspections are Semi-Markov problem formulation and state-augmentation. The techniques from state-augmentation are also utilised in Partially Observed Markov Decision processes (POMDP) to handle the uncertainty in the inspection output. In PODMP the transition probabilities $P_{ij}$ and return function $r$ are known for all states. The performed observations, however, only infer probabilities for the various states in $Z$. In the basic MDP, the components of the model such as transition probabilities and discount rate are assumed known. If these components depend upon a fixed, but unknown, parameter $\Theta$, a special structured POMDP occurs which is known as Adaptive Markov decision process.

### 4.3.1 Semi-Markov Decision process

Instead of requiring an inspection at each decision epoch, the Semi-Markov decision process seeks decision epochs where the information gained by the inspection is considered profitable compared to the cost of performing the inspection. An inspection still has to be performed before a decision can be made, but a decision does not have to be made at each stage. The general form of a Semi-Markov objective function is:

$$J_n(z_n) = \max_{x_n, \Delta\tau} \left\{ r(z_n, x_n) + \sum_{j=1}^{M} \left( \sum_{\tau=1}^{\Delta\tau} \beta^{\tau} P_{ij}^{\tau} \right) J_{n+1}(z_{n+\Delta\tau} = j) \right\} \tag{4.5}$$

where $\Delta\tau$ is the period to the next decision.

### 4.3.2 State-augmentation

The technique termed state-augmentation enables use of MDP methods when the state of condition at present decision epoch is unknown or uncertain. The basic principle of state-augmentation is to redefine the original state space $Z$, to a new state

Table 4.2: An example

| Do nothing $x_n = a$ | | $z_{n+1}$ 1 | 2 | 3 | Repair $x_n = b$ | | $z_{n+1}$ 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 0.8 | 0.2 | | | 1 | 1.0 | | |
| $z_n$ | 2 | | 0.5 | 0.5 | $z_n$ | 2 | 0.9 | 0.1 | |
| | 3 | | | 1.0 | | 3 | | 0.7 | 0.3 |

space $I$, which accounts for all the information available for the decision maker and relevant for future decisions. The new state space at decision epoch $n$, $I_n$, includes the entire history of measured states up to $n$, and the decision made up to $n-1$. As an example, assume that the condition is exactly revealed by an inspection, but that the decision maker is allowed to make a decision prior to an inspection. In the extreme case, where the condition is never measured throughout the planning horizon, the augmented state is:

$$\begin{aligned} I_n &= \{z_0, x_0, x_1, \ldots, x_{n-1}\}; \qquad n = 1, 2, \ldots, N \\ I_0 &= \{z_0\} \end{aligned} \tag{4.6}$$

Here, $z_0$ is the condition at the start of the planning horizon, which is assumed to be known. From these equations it can be seen that $I_n = \{I_{n-1}, x_{n-1}\}$. Before using the state information ($I_n$) in a dynamic program, the relationship between $I_n$ and the probability distribution of $z_n$ has to be established. These probabilities are calculated using Bayes' law:

$$\begin{aligned} p_n(z_n = j|I_n) &= \frac{prob(z_n = j, I_n)}{prob(I_n)} \\ &= \frac{\sum_i p(z_n = j|z_{n-1} = i, x_{n-1}) p_{n-1}(z_{n-1} = i|I_{n-1})}{\sum_j \sum_i p(z_n = j|z_{n-1} = i, x_{n-1}) p_{n-1}(z_{n-1} = i|I_{n-1})}, \\ & \qquad j = 1, \ldots, M \end{aligned} \tag{4.7}$$

To see how the new information state transition probabilities are calculated, a simple example incorporating two possible decisions $x_n = \{a, b\}$ at each decision epoch and three possible states $z_n = \{1, 2, 3\}$ is presented. The two possible decisions are; $a$= do nothing, $b$=repair. The states range from 1 denoting a new condition to 3 denoting a bad condition. The one-step transition probabilities are presented in table 4.2.

Figure 4.1 shows a decision tree for the decision problem. From table 4.2 it is easily seen that $p_1(z_1 = 1|I_1) = p_1(z_1|z_0 = 1, x_0 = a) = 0.8$, $p_1(z_1 = 2|I_1) = 0.2$ and $p_1(z_1 = 3|I_1) = 0$. Here, the denominator of eq. 4.7 equals 1. If the decision is *no action* in two consecutive decision epochs ($I_2 = \{z_0 = 1, x_0 = a, x_1 = a\}$), the distribution of the true condition state at $n = 2$ on $I_2$, $P_2|I_2$ is:

$$P_2|I_2 = \begin{bmatrix} p_2(z_2 = 1|I_2) \\ p_2(z_2 = 2|I_2) \\ p_2(z_2 = 3|I_2) \end{bmatrix} = \begin{bmatrix} 0.8 \cdot 0.8 & = 0.64 \\ 0.8 \cdot 0.2 + 0.5 \cdot 0.2 & = 0.26 \\ 0.5 \cdot 0.2 & = 0.10 \end{bmatrix}$$

Equation 4.7 is repeated for all possible combinations of state conditional on the state of information $I_n$, and for all decision epochs $n$, that is:

$$p_n(z_n|I_n) \qquad \forall z_n, \forall I_n, \forall n \tag{4.8}$$

Using the state of information, $I_n$, the dynamic programming formulation is given by:

$$\begin{aligned} J_n(I_n) = \quad & \max_{x_n} \Bigg\{ \sum_{i=1}^{M} p_n(z_n = i|I_n) r(z_n, x_n) \\ & +\beta J_{n+1}(I_{n+1}) \Bigg\} \quad \forall I_n, n = 0, \ldots, N-1 \end{aligned} \tag{4.9}$$

The disadvantage of the state space transformation is the increase in size of the problem. While the original MDP involved $M$ states per stage, the new state space $I_n$, involves $v^n$ information states, where $v$ is the number of possible activities which can be applied to the facility. Thus, having the option of two different activities over a period of $10$ decision epochs involves consideration of $2^{10} = 1024$ different information states.

Madanat [61] has used the state augmentation technique to develop a model where the inspections are no longer required to take place prior to a maintenance and repair decision. To allow for the option of an inspection at the beginning of every time period, the information state at stage $n$ is described by:

$$\begin{aligned} I_n &= \{z_0, x_0, \vartheta_1 \cdot z_1, x_1, \ldots, x_{n-1}, \vartheta_n \cdot z_n\}; \qquad t = 1, 2, \ldots, N \\ I_0 &= \{z_0\} \end{aligned} \tag{4.10}$$

where the variable $\vartheta_n = 1$ at an inspection and $\vartheta_n = 0$ otherwise. Through an empirical comparison on the minimum expected life cycle cost, he presents the benefit of the latter model compared to a standard MDP and a Semi-Markov model under varying standard error of forecasting. The disadvantage of the more complex decision model is however the increasing state space. Using eq. 4.10, the number of states to

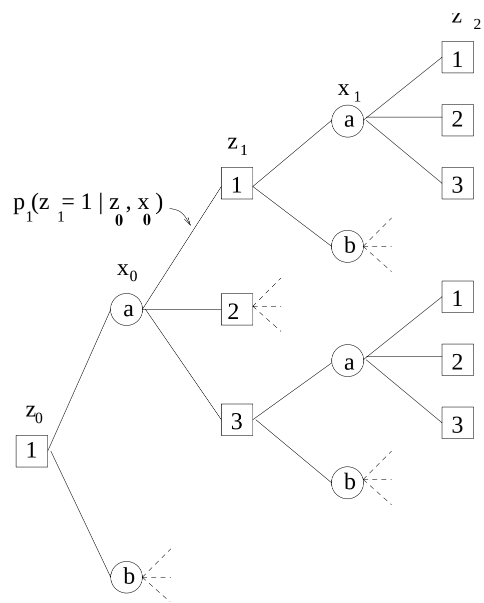


Figure 4.1: Decision tree showing 3 states and 2 possible options in each decision epoch.

which the recursion equation has to be applied at stage $n$ is equal to $(v \cdot (1 + M))^n$. For a problem of 3 condition levels, and 3 possible activities over a period of 10 years this equals $6.19 \cdot 10^{10}$ states.

### 4.3.3 Partially observed Markov decision processes

The state augmentation technique is also suitable for problems with imperfect inspections [57], [62]. These problems are generally termed Partially Observed Markov Decision Processes (POMDP). A POMDP method termed *The Latent Markov decision Process (LMDP)* has been presented to handle the presence of random errors in measurement of the condition of infrastructure facilities [62], [63], [64]. The result of an inspection, measuring the performance of the facility, is probabilistically related to the true performance through a known probability mass function $q(\hat{z}_n = k|z_n = j)$, where $\hat{z}_n$ and $z_n$ is the measured and true condition at the start of stage $n$ respectively . $k$ is indices of elements in the set of discrete condition states. The new state $I_n$ is expressed as:

$$
\begin{aligned}
I_n &= \{I_0, x_0, \hat{z}_1, x_1, \ldots, \hat{z}_{n-1}, \hat{x}_n, \hat{z}_n\} \qquad n = 1, \ldots, N \\
I_0 &= \{\hat{z}_{-\tau'}, \ldots, \hat{z}_{-1}, x_{-1}, \hat{z}_0\}
\end{aligned}
$$

where $\tau'$ denotes number of years between the first inspection of facility and start of the planning horizon. By this redefinition, the state space $I_n = \{I_{n-1}, x_{n-1}, \hat{z}_n\}$ evolves in a Markovian way. Because the basic LMDP model assumes the inspection schedule to be fixed, a model which relaxes on this requirement has been proposed [64]. The state space is extended to incorporate the inspection decision in a similar manner to that presented in eq. 4.10. The model operates on the information vector $P_t|I_t$ instead of $I_t$. The reason for this redefinition is to utilize the technique referred to as "sufficient statistics". While the states $I_n$ are impossible to compare, the information vector $P_n|I_n$ allows for a direct comparison of corresponding elements. When two states have similar or almost similar values they are combined into a single state, which reduces the number of replications of the objective function $J(P_n|I_n)$.

### 4.3.4 Adaptive Markov decision processes

If the transition probabilities $P_{ij}$, reward function $r$ and/or discount rate $\beta$ depend upon a fixed, but unknown parameter $\Theta$, a specially structured PODMP occurs termed *Adaptive Markov Decision Processes* [57]. The parameter $\Theta$ is assumed to take a known prior distribution, and is completely unobserved.

## 4.4 Application of MDP in ageing system management

Markov decision processes modelling comprises an efficient method for handling sequential decision problems, which typically are of concern in condition based maintenance. The MDP has been applied to a variety of CBM applications, especially within civil engineering concerning pavement and bridge maintenance management. The main reason is the huge amount of data already available based on inspection records, which is often the major problem in maintenance optimization. The literature within pavement management has been concerned with both development of new efficient optimisation procedures and handling of data records in a Markovian framework.

The Latent Markov Decision Process is a method developed to support decision problems in the area of pavement management [63],[62] and [64]. The LMDP explicitly recognizes the presence of random errors in the measurement of facility condition.

The LMDP algorithm has been developed to solve problems with an unconstrained inspection frequency. The solution method is based on POMDP applying the technique of state-augmentation, and an optimal solution is found by the value-iteration methodology. The LMDP is a single-unit, discrete optimization procedure and does not seek solution to problems on the network-level. Madanat and Ben-Akiva [64] suggests that the LMDP could be extended to a network-level including e.g. budget constraints through the use of "random policies". A randomized policy does not specify a single optimal activity for each state of the system, but it specificies optimal probabilities for different activities for each state of the system. This has been described by Smilowitz and Madenat [65]. They express the true condition state at $n$ conditional on $I$ as $P|I \forall I$ [2]. They reduce the number of information vectors $(P|I)_n$ by pair-wise comparison of corresponding elements. When two states are found to have equal, or almost equal, values of $(P|I)_n$, they are combined to a single state, which reduces the required computational effort. They further define a decision variable $W_{x(P|I)n}$, which denotes the fraction of facilities in information vector $P|I$ to which activity $x$ is applied in time $n$. The budget constraints are solved by linear programming, where $W_{x(P|I)n}$ is the decision variable. It should be noted here that the main limiting factors on the problem size are the number of condition states and the level of discretization, since the number of these two factors determines the size of the state space.

Ellis, Jiang and Corotis [66] have developed an exact POMDP implementation, to remove the possible effect of discretization on the optimal policy space-effects that continuous (exact) solution approaches do not produce. They comment on the computational burden by solving POMDPs (the main benefit of discretization is less computational resources compared to continuous solutions). In [67], Jiang, Corotis and Ellis present an application of their proposed model to highway bridges subjected to fatigue and corrosion.

Scherer and Glagola [68] have applied a MDP method for bridge maintenance management. They concentrate on the work to determine appropriate state variables and transition matrices. They propose a method to analyze bridge surveillance records. An inference analysis using chi-square statistics is formulated to test the significance of the Markovian assumption. Although not all of the possible state transition could be analyzed for Markovian compliance because of insufficient recorded samples, they concluded that the most frequently occurring state transitions were sufficient to establish the Markovian property as it applies to the entire deterioration model. They apply a simple frequency analysis to sequence of occurrence of a predefined three-state transition scheme for three conditions: past, present and future.

Thorstensen and Rasmussen [69] have presented an approach using MDP to develop

[2] The vector form $P|I \forall I$ equals $p(x|I) \forall x, \forall I$

a cost model for condition based overhaul/repair. Defining a finite number of condition levels of the system, the continuous-time deterioration process is described by a condition transition probability matrix. All input data are modelled as a function of time or system status. The model have also the flexibility to include cyclic variation as for example changes in production demand.

Wijnmalen and Hontelez [49] have proposed a method for coordinated condition-based repair strategies for components of multi component maintenance systems with discount. Their “random policy” method is based on an heuristic approach, which decompose the multi-unit problem into several single-unit Markov Decision problems. The "decomposition" is performed due to the dimensionality of the state space, which they argue would be computationally intractable if a MDP was applied to the multi-unit problem. The single-unit model applied originated from work performed by Tijms and Duyn Schouten [70], which proposed a special purpose algorithm to solve Semi-Markov MDPs over an infinite horizon. In their model, the condition at each inspection is considered to be revealed without uncertainty. The algorithm is a modification of the general policy-iteration technique and has later been modified by Wijnmalen and Hontelez to meet further specific requirements from practice [71]. Relevant information about each component is aggregated into steady-state repair probabilities.

Grall et al [72] have proposed a condition-based maintenance approach for a single-unit system considering a continuous state. The continuous state problem is solved by simulation and an optimal solution is found by a gradient based method. The major benefit is that the discretization of a continuous deterioration process is not necessary. The additional approximation errors imposed by the discretisation are therefore avoided.

Chen and Trivedi [73] presents a SMDP for the maintenance policy optimization of condition-based preventive maintenance problems. Under a special case when the optimization objective is steady-state availability and the deterioration rate at each failure stage is the same (and homogeneous), they present the results as thresholds determining the relationship between the inspection rate, the observed condition and the preferred action (major or minimal maintenance). The inspection rate is taken as an input to the SMDP model.

Sloan [74] presents an application of MDP within a manufacturing environment, where the objective is to choose simultaneously the equipment maintenance schedule as well as production quantity that minimizes the sum of expected production, back-order, and holding costs. They apply the policy improvement algorithm to solve the infinite horizon problem.

## 4.5 Remarks

There are a lot of examples of MDP and solutions to MDP in the literature and some of the references are mentioned above. The majority of these papers are written from a theoretical point of view, looking at means to improve methodologies for solving different types of MDP problems such as SMDP, POMDP, LMDP, infinite and finite horizon.

Many MDP's have been proposed as decision support tools to infrastructure management such as pavement and bridges, to optimize time between inspections and corrective maintenance actions.

The major part of the literature focusses on minimizing the long-term costs or maximizing the long-term profit in an *infinite horizon environment*. Their common objective is to seek an optimal decision policy given boundaries that are fixed in time such as capacity demand.

The usual approach to represent the degradation (and corresponding condition) is by a time-invariant deterioration process (time homogeneous process), such that the entire deterioration process is described by a single transition matrix. The single stage transition matrix holds the probabilities for a transition from one state to another (within the state space) for a single step ahead. Utilizing information from condition monitoring (or technical condition) will often imply a time-variant transition matrix, thus the transition probabilities from one stage to another will be dependent on the time.

In-service decision support methodologies for inspection and repair strategies have to provide capabilities to systemize and utilize information that is vital for the decision making process. Such information, in addition to describing the present condition of equipment/system/facility under evaluation, may also include e.g. the knowledge of planned shut downs, the remaining time to operate before closure (end-off horizon), the probability of having new and improved technology on the market in the near future. The effects of obsolescence and limited time to operate often have significant impact on the decision policy. Incorporating the effect of new technology on the market has mainly been considered in economic life models. The results achieved by combining the information given above should therefore represent a step forward to grasp the decision problem related to repair and inspection policies as part of ageing management decision making.

# Chapter 5

# A MDP model for decision support

## 5.1 Introduction

This chapter presents a model with the objective of relating ageing system management decision support to economy. An optimal plan includes selection of appropriate inspection intervals and overhaul/replacement decisions which consider both physical impairment and obsolescence. The system/equipment, hereafter referred to as the item, may either become inefficient, insecure or inappropriate to fulfill the intended function making a repair or replacement essential.

The model is designed to investigate and seek optimal solutions when it is possible to classify the item's present condition and predict future development based on previous condition monitoring results. The deterioration process is described by a Markov process, and the sequential decision problem is modelled as a discrete time Semi-Markov Decision Process (SMDP). The transition probabilities of the controlled time-variant Markov process are described in a condition transition probability matrix (CTPM). To account for the end-of-horizon effect and time dependent external parameters such as varying production profile, the optimal solution is found by use of the value iteration procedure (stochastic dynamic programming). The model is considered to have its strength in the in-service phase where new results from condition monitoring are utilised to optimise forthcoming inspections and maintenance actions. However, the model may also provide decision support to establish a maintenance program in the design phase. The main difference compared to the in-service phase is the information concerning stresses and degradation. In the design phase, the data will have to be based on the results from a similar item assuming that the data from the observed stresses and degradations are valid for the present case. In the in-service phase, the data quality can be improved by utilising condition monitoring (if applicable). Change in the external requirements (boundaries) may also have impact on the selected policy.

The chapter is structured as follows. First, the main objective, assumptions and limitations are described followed by a description of the state space, the decision space, modelling of income and cost figures, technology improvements etc. Having introduced the model mathematically, the chapter continues with a methodology to establish the CTPM. The last part of the chapter provides analysis and descriptions of the model features. The model is implemented in Fortran F90 and the source code is listed in appendix A and appendix B.

## 5.2 Formulation of the proposed planning model

The main objective is to define an optimal policy $\pi_n^*$ that maximises the total expected discounted cost over the planning horizon $N$ . The model consists of a state space $Z$ , a decision space $X(Z)$, a profit function $J$, a production profile $\Psi_{pp}(n)$ , several cost figures and a discount factor $\beta$ . Each deterioration process included is represented in a separate condition transition probability matrix. The transition probabilities are dependent on both present state and selected decision, thus if decision $x_n \in X(z_n)$ is made in state $z_n$ the item will transit to state $z_{n+1}$ with probability $P_{z_n,z_{n+1}}(x_n)$. .

In principle there are three parameters that effect the income and cost data:

- the stage (time)
- the present condition
- the decision (activity)

The different income and cost data is presented as functions of one or several of these three parameters. Inclusion of the stage makes it possible to make e.g. the required and achieved output dependent on the time. If the item does not satisfy the functional demands, technical obsolescence may result in a replacement with a technology, $\phi \in \Phi$ , available on the market which fulfills these requirements. The central assumption of Markov decision processes is that the “system” being modelled evolves over time according to a finite state Markov chain. For the proposed SMDP model the following assumptions also apply:

- a decision can only be made after the condition (state) has been determined

- the condition (state) of the system is revealed without uncertainty at inspection
- each deterioration process is independent
- a fault is detected immediately with probability $q$
- an overhaul/replacement action returns the item back to "as good as new" or if modified "better than as previously installed"
- a new technology always has better properties than the existing one

The first assumption implies that the decision maker has to verify the condition before a decision can be made, and follows from the theory of MDP. The algorithm seeks the optimal policy $\pi_n^*$, which consists of function $\mu_n^*(z_n) = (x_n, \Delta\tau)$ , thus the policy states both an action and the time to next inspection after the action has been performed. The optimal-policy is time-dependent.

### 5.2.1 A description of the state space

There are in principle two different ways to describe the condition of an item — by age or by measuring a physical condition. Which method to choose will depend on the ability to observe the deterioration process and the significance of being able to distinguish between different condition levels by inspections (in terms of costs, income and risk of unforeseen failures). An item may have several condition (state) variables in which each may have different criticality in respect to safety, environment, availability, costs etc. (see section 2.3 considering TCI and residual life). Therefore it may be insufficient to represent the condition by a single condition parameter. To be able to handle different independent deterioration processes within the same optimisation procedure, the state space has been designed to incorporate both information about the age since last overhaul/replacement, and condition measured by other means. The state space $Z$ is expressed as:

$$Z = \{\phi, s, cp_1, cp_2, \ldots\ cp_w\} \tag{5.1}$$

Here, $\phi$, represents the technology, $s$ the number of time periods (age) since last replacement/overhaul and $cp_w$ the condition parameter describing the state of deterioration process $w \in W$ . For each deterioration process it is possible to classify $M_w + 1$ disjunct condition intervals . For those failure modes which only result in two different conclusions based on an inspection (working or failed), $M_w = 1$. The notation $M_w^*$ is used for an evident fault level.

The number of disjunct condition intervals needed to approximate the continuous time and continuous state deterioration process may be high to get sufficient accuracy

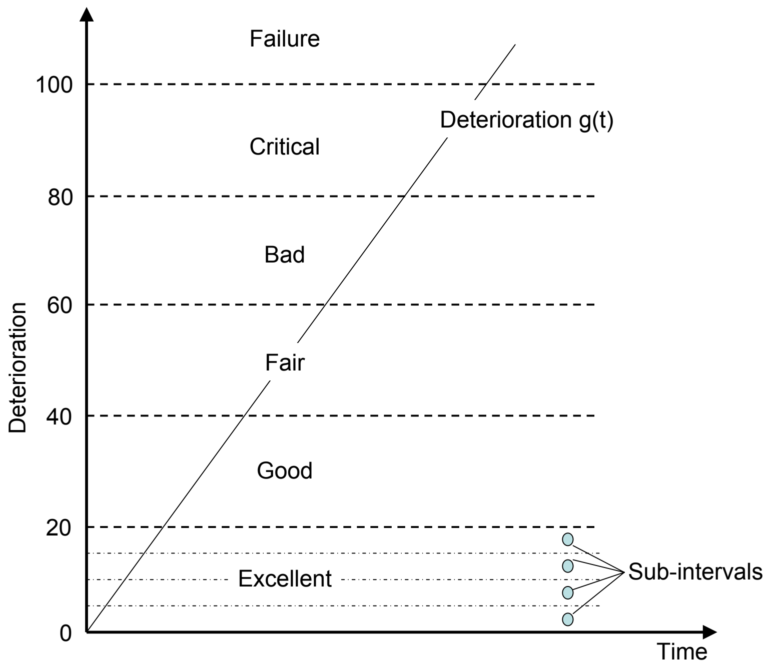


Figure 5.1: An example of a classification scheme which includes 6 main intervals, each divided into four sub-intervals.

(see sec. 5.4.1). However, a large number of condition intervals would require an extensive effort to specify all the item properties. From a profit/cost point of view, a number of five to ten level classification scheme seems to be practical in most cases [62], [75].

To meet both requirements, each condition interval determined from a profit/cost point of view is allocated into groups of discrete condition intervals. As an example, a six level interval scale ranging from *excellent* to *failure* is presented in table 5.1. If the deterioration after an inspection is classified to be in the interval $[0, 20 >$ the item is said to be in an excellent condition. If the deterioration is classified at or above $100$ the item is said to have failed. Each level of condition is further allocated into appropriate number of discrete intervals to meet the required accuracy such that e.g. each condition level $1, 2, \ldots$ are further divided in $4$ discrete sub-intervals (see figure 5.1).

The "failure limit" can also be defined based on safety and environmental requirements which include a "safety limit". Thus, depending on the definition of a failure mode the item may have failed according to the classification scheme but may still be able to operate from an economic point of view.

Table 5.1: An example of a condition classification scheme.

| Level | Classification | Description | Interval |
|---|---|---|---|
| 1 | Excellent | *As good as new* | $[0, 20 >$ |
| 2 | Good | *x below baseline* | $[20, 40 >$ |
| 3 | Fair | *y below baseline* | $[40, 60 >$ |
| 4 | Bad | *z below baseline* | $[60, 80 >$ |
| 5 | Critical | *Approaching lower acc. criterion* | $[80, 100 >$ |
| 6 | Failure | *Lower acc. criterion exceeded* | $\geq 100$ |

### 5.2.2 The decision space

At a given state $z$ and stage $n$ the model provides two recommendations — (1) keep or replace existing system/item and (2) the number of stages till the next inspection, $\Delta\tau$. The decision maker must at time of the decision know the exact condition of the system/item, thus an inspection is required *before* a new decision can be made.

Between two consecutive inspections and possible replacements there are no planned actions such that the system/item is left as it is. To represent this option and corresponding state transition probabilities the option is described as "No action (continue without an inspection)".

The possible actions in the decision space, $x \in X(z)$, are:

**[x=1** ] No action (continue without an inspection)

**[x=2** ] Inspect

**[x=3** ] Overhaul/replace by similar or new technology

A solution to the Markov decision process is thus a policy ($\pi_n^*$), mapping state actions, that determines state transitions to maximise profit according to the performance criterion. At a given state and stage the model $\mu$ is a function mapping the decision $x_n$ based on the condition $z_n$, i.e. $x_n = \mu_n(z_n)$ (ref. section 4.2).

### 5.2.3 Modelling of income and cost figures

All income and cost figures are modelled as a function of the state and/or the stage. Every condition parameter defined in the state space may have different consequences, hence each condition parameter has a separate set of income and cost figures. Table 5.2 presents an overview of all basic income and cost parameters and their principle relation to the state and stage variables.

Table 5.2: An overview of income and cost profile functions

| Mathematical expression | Description |
|---|---|
| $\Psi_{pp} \sim f(n)$ | Production profile |
| $\Psi_{ps} \sim f(n)$ | Planned production shutdowns (opportunities) |
| $\Psi_{cap} \sim f(z)$ | Capacity |
| $\Psi_{E} \sim f(z)$ | Energy cost profile |
| $\Psi_{PM} \sim f(z)$ | Maintenance cost profile |
| $\Psi_{OH} \sim f(z)$ | Overhaul and replacement cost profile |
| $\Psi_{S} \sim f(z)$ | Salvage value profile |

**Income,** $I(z, n)$

The income during operation is a function of both the present state and stage. Two main elements are assumed to affect the income at each stage — the production profile and the capacity of the item in production.

*The production profile,* $\Psi_{pp}(n)$, and known opportunities (production shutdown caused by other equipment)$\Psi_{ps}(n) \in [0, 1]$ [1] constitute the production demand. The production demand originates from the production strategy, which is based on system and external demands as well as on the resource constraints. As an example, the oil production facilities in the North Sea are scheduled for a major production shutdown every year or every second year to perform required overhauls and replacement. A reservoir is an example of resource constraint which will influence the maximum delivery from the producing facility, and therefore may influence the functional requirements of the item. As the pressure in the oil reservoir reduces, the production level will drop on some systems, while others may have an increase in the demands placed on them. The requirement for separation of water from oil typically increases during the life cycle. A principle drawing of a production profile is shown in figure 5.2. The production profile is specified in a deterministic manner, where each stage ($\Delta t$) is assigned a specific production demand. The production profile is given as a percentage of an initial demand.

*The capacity,* $\Psi_{cap}(z)$, is modelled as a function of the state. The capacity determines the maximum available throughput at a specific condition operating with an item of a specific technology. The capacity is given as a percentage of the initial design requirement. If an item in a new condition fulfills the initial production demand, the

[1] $\Psi_{ps}(n) = MDT_{ps}(n)/\Delta t$

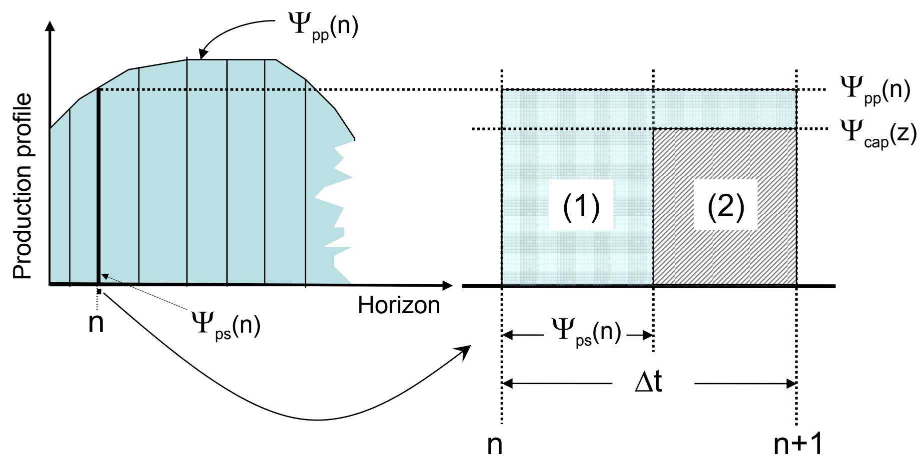


Figure 5.2: The production profile incorporating opportunities for overhauls and modifications (planned shutdowns). 1) Lost production (i.e production which is deferred) during the planned shutdown and 2)Income.

maximum capacity is set to $100\%$. A deterioration may reduce the maximum available capacity. It is assumed that an increase in capacity can only be achieved by a modification or replacement with a new technology.

At each stage the income $I(z, n)$ is calculated as:

$$\begin{aligned} I(z,n) = \; & \overline{I} \cdot \min[\Psi_{pp}(n), \Psi_{cap}(z)] \cdot (1 - \Psi_{ps}(n)) - \overline{C_{pen}} \Psi_{pp}(n) \cdot \Psi_{ps}(n) \\ & - \overline{C_{pen}} \max[0, \Psi_{pp}(n) - \Psi_{cap}(z)] \cdot (1 - \Psi_{ps}(n)) \end{aligned} \tag{5.2}$$

where, $\overline{I}$ expresses the baseline income per stage at full production $100\%$, $\Psi_{ps}(n) \in [0, 1]$ represents planned shutdown at stage $n$ and $\overline{C_{pen}}$ is a penalty factor that reduces the income if the demand is not fulfilled. The following assumptions have been made regarding planned shutdowns:

- Planned shutdown always starts at the beginning of a new period.
- If the inspection and repair action is performed within an opportunity, there are no downtime costs associated with the activities.

- The residual life is independent of shutdown periods (the opportunities) if no overhaul action takes place, thus it is assumed that the equipment/item continues to deteriorate similar to full operation. (Reasonable as the sum of hours for shutdowns is negligible compared to the total lifetime).

**Operational costs,** $C_{OP}(z, n)$

The operational costs in this context include both cost by use of energy and the cost of performing the day-to-day maintenance activities. If the efficiency reduces during the operational phase, two elements may be affected. The production capacity may be reduced and the energy consumption may increase. The effect of reduced capacity is already included in eq. 5.2 and has therefore not been included in the direct operational costs.

*The energy consumption profile,* $\Psi_E(z)$, describes the energy consumption as a function of the state. For some items, the energy costs of running may be very crucial to the overall profit, thus the energy consumption has to be considered. The efficiency may differ depending on the technology in use and the physical condition of the specific technology in operation. In addition to the direct increase in energy consumption, loss of efficiency may also cause an increase in pollution. On the Norwegian continental shelf, the authorities operate a $CO_2$ tax. Dependent on the energy consumption and related taxes, the cost may increase drastically if the efficiency reduces, e.g. for gas turbines producing electricity on the platform, the taxes constitute approximately $60\%$ of the total operating cost [76]. To establish the energy profile, the energy consumption at each condition level is compared to a baseline. The baseline is defined as the consumption of an item of a specific technology in an excellent condition. The cost factor $\overline{C_E}$ is defined as the energy cost of operating on the baseline over one stage . This formulation makes it easy to model the energy consumption of the new items with an improved technology. The value of $\Psi_E(z)$ is simply adjusted according to baseline figures.

*The maintenance cost profile,* $\Psi_{PM}(z)$, is used to model the maintenance costs. The maintenance costs include all costs associated with day-to-day maintenance (minimal repairs) to keep the items in an appropriate state . Major inspections and replacements / overhauls are treated separately and therefore not included. The maintenance costs are assumed to depend on the selected technology and the present physical condition of this technology. The modelling of maintenance cost is done in the same way as for the energy cost, by first defining a mean maintenance cost to a baseline $\overline{C_{PM}}$, and then defining the whole maintenance cost profile as a percentage of this value .

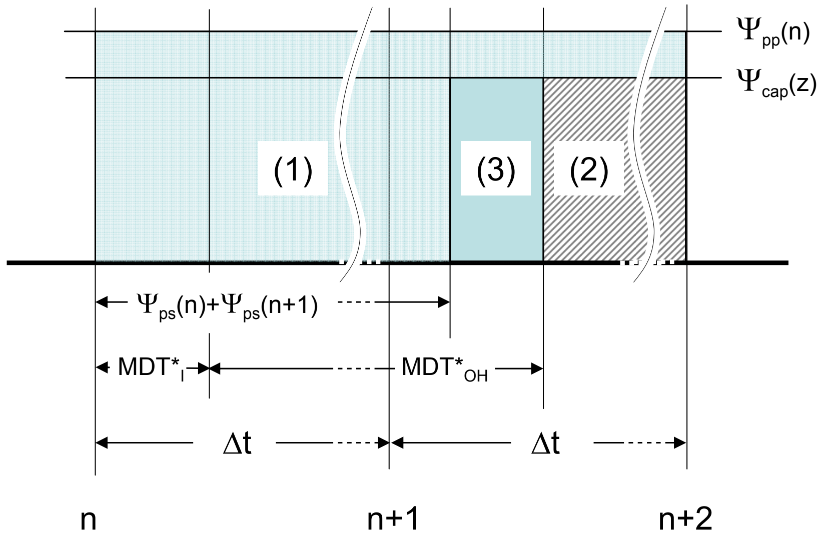


Figure 5.3: The production profile incorporating opportunities for overhauls and modifications (planned shutdowns). 1) Production deferred from the planned shutdown; 2) Income; 3) Lost production due to inspection and maintenance if the actions extend the planned shutdown.

It is assumed that the operating costs are zero during an opportunity i.e a production shutdown not caused by the equipment evaluated. The total operational costs at each stage are expressed as:

$$\begin{aligned} C_{OP}(z) =& \quad \overline{C_E} \cdot \Psi_E(z) + \overline{C_{PM}} \cdot \Psi_{PM}(z) \\ C_{OP}(z,n) =& \quad C_{OP}(z) \cdot (1 - \Psi_{ps}(n)) \end{aligned} \tag{5.3}$$

There will not be any operational costs during inspection and overhauls and the operational costs are therefore subtracted from the cost of such actions as described in the three following subsections.

**Inspection costs,** $C_I(z,n)$

Inspections are needed to reveal the condition. The inspection costs is divided in to separate cost items — the set-up cost and shutdown costs. The set-up costs, $\overline{C_I}$ may include scaffolding, hiring of experts, special equipment to perform inspections etc.. The shutdown cost depends on the extent of the shutdown (equipment, train, system or facility level), and the time required to fulfill the inspection. In many cases the shutdown cost may be more significant than the set-up costs. Area 3 in figure 5.3 represents the associated downtime costs due to an inspection. The length of the inspection, $MDT_I^*$ [2], is assumed to be short compared to the total length of each stage $\Delta t$ ($MDT_I^* << \Delta t$). This is also the usual case in real applications. and implies that the condition is revealed early in the stage if it is decided to do an inspection. The inspection costs are determined by:

$$\begin{aligned} C_I(z,n) = \quad & \overline{C_I} + max\left[0, MDT_I^* - \Psi_{ps}(n)\right] \\ & \cdot \left(\overline{I} \cdot \min[\Psi_{pp}(n), \Psi_{cap}(z)] - C_{OP}(z)\right) \end{aligned} \tag{5.4}$$

where $\overline{C_I}$ represents the set-up costs (fixed cost) for an inspection.

**Overhaul/replacement costs,** $C_{OH}(z,n,x_n)$

Overhaul and replacement costs are modelled as a function of state and stage and the selected action. The state is considered, because the cost of restoring an item may in some cases depend on the present condition. At replacement, a salvage value may be subtracted if the item can be reused in another application. Further, the time of action may have significant importance because unnecessary downtime may be avoided (opportunistic maintenance).

Area 3 in figure 5.3 represents the associated downtime costs due to an overhaul action.

The cost of replacement/overhaul is expressed as:

$$C_{OH}(z,n,x_n) = \begin{cases} C_{OH}(z,n) & x_n = 3 \\ 0 & x_n \neq 3 \end{cases}$$

[2] $MDT_I^* = MDT_I/\Delta t$

where

$$
\begin{aligned}
C_{OH}(z,n) = \quad & \overline{C_{OH}} \cdot \Psi_{OH}(z) - \overline{S} \cdot \Psi_S(z) \\
& + \Big( max \Big[ 0, MDT^*_{OH} + MDT^*_I - \textstyle\sum_{u=n}^{u^*_{OH}} \Psi_{ps}(u) \Big] \\
& - \; max \left[ 0, MDT^*_I - \Psi_{ps}(n) \right] \Big) \\
& \cdot \; \left( \overline{I} \cdot \min[\Psi_{pp}(n), \Psi_{cap}(z)] - C_{OP}(z) \right)
\end{aligned}
\tag{5.5}
$$

where $\overline{C_{OH}}$ represents the reference cost of restoring an item which has failed, and $\overline{S}$ is the salvage value reference, which is normally set equal to the cost of purchase of a new item. $MDT^*_{OH}$ represents the length of the overhaul action, and $u^*_{OH}$ [3] expresses the number of periods ahead to be considered.

### Cost of an unforeseen failure, $C_{UCM}(z,n)$

In addition to planned maintenance and replacement actions, there is always the risk of an unforeseen failure. An unforeseen failure may cause severe damage and secondary costs to equipment nearby. In addition to the secondary costs, the mean down time (MDT) normally increases compared with a planned action because no or minimal initial preparation has been done to have e.g. spare parts, personnel and tools available. It is assumed that the opportunity to modify with new technology when a replacement is forced by an unforeseen corrective maintenance is impossible. In addition, the downtime cost will depend on the cost of a downtime at that specific period of time. The cost of an unforeseen failure leading to a replacement is therefore described as:

$$
\begin{aligned}
C_{UCM}(z,n) = \quad & \overline{C_{OH}} \cdot \Psi_{OH}(z) - \overline{S} \cdot \Psi_S(z) \\
& + max \Big[ 0, MDT^*_{UCM} - \textstyle\sum_{u=n}^{u^*_{UCM}} \Psi_{ps}(u) \Big] \\
& \cdot \; \left( \overline{I} \cdot \min[\Psi_{pp}(n), \Psi_{cap}(z)] - C_{OP}(z) \right)
\end{aligned}
\tag{5.6}
$$

where $\overline{C_{OH}}$ represents the cost of restoring an item which has failed, $\overline{S}$ is a salvage value equal to the cost of purchase a new item and $MDT^*_{UCM}$ [4] is the Mean Down Time of an unforeseen replacement. $u^*_{UCM}$ [5].The salvage values are scaled by the discrete salvage profile $\Psi_S(z)$, in a similar manner as for the overhaul costs.

[3] $u^*_{OH} = n + Int^-(MDT^*_{OH} + MDT^*_I)$
[4] $MDT^*_{UCM} = MDT_{UCMn}/\Delta t$
[5] $u^*_{UCM} = n + Int^-(MDT^*_{UCM})$

The net benefit (savings) of having an item in a "'new" condition after an unforeseen replacement has to be included when the total costs of an unforeseen replacement are considered. This is thoroughly covered in sec. 5.2.4.

### 5.2.4 Optimisation

#### The profit function

The optimisation is performed by the value iteration method. To simplify the presentation, the state space is reduced and totally described by a single condition parameter i.e $i \in Z^*, Z^* = [1, M+1]$. Condition level 1 represents an item in an excellent condition (new). The fault level is defined as $M+1$. The maximum expected profit at stage $n$, when operating to the next decision interval at stage $n + \Delta\tau$ is given as:

$$J_n(i) = \frac{max}{x_n \in X_n, \Delta\tau} \left\{ G(i, n, \Delta\tau, x_n) + \beta^{\Delta\tau} \sum_{j=1}^{M} P_{i,j}^{\Delta\tau}(2) J_{n+\Delta\tau}(j) \right\} \quad (5.7)$$

also known as the optimality equation. Here, $G(\cdot)$ expresses the net profit of operating the item between two consecutive inspections and $P_{i,j}^{\Delta\tau}(\cdot)$ is the state transition probability, conditional on the state and the decision (see section 5.3.2). The state transition probabilities used are:

— $P_{i,j}^{s}(2)$ The probability of state $j$, when the previous state $i$ was known $s$ periods ago.

— $P_{i,j}^{s}(1)$ The one-step probability of operating in state $j$ after $s+1$ time periods, when the state $i$ in previous time unit $s$ is uncertain.

— $P_{i,M^*}^{s}(1)$ The one-step transition probability of having an unforeseen maintenance task in time interval [s, s+1], when the condition $i$ at time $s$ is uncertain.

The net profit of operating until next inspection, $G(\cdot)$ is expressed as:

$$\begin{aligned} G(i, n, \Delta\tau, x_n) = & -C_I(i, n) - C_{OH}(i, n, x_n) \\ & + \sum_{s=1}^{\Delta\tau-1} \sum_{j=i}^{M} \beta^s P_{i,j}^{s}(1)(I(j, n+s) - C_{OP}(j, n+s)) \quad (5.8) \\ & - \sum_{s=1}^{\Delta\tau-1} \beta^s P_{i,M+1}^{s}(1) C_{UCM}(M+1, n+s) + \Delta O(i, n, \Delta\tau) \end{aligned}$$

where $C_I(\cdot)$ represents the cost of an inspection (see eq. 5.4), $C_{OH}(\cdot)$ is the overhaul cost (see eq. 5.5), $I(\cdot)$ is the income (see eq. 5.2),$C_{OP}(\cdot)$ is the operational cost (see eq. 5.3), $C_{UCM}(\cdot)$ is the cost of an unforeseen corrective maintenance action (see eq. 5.6), and $\Delta O(\cdot)$ is the benefit achieved when an unforeseen replacement has taken place and the item is in a better condition than before. $s$ is the number of time periods ($\Delta t$) since last inspection. $\Delta O(\cdot)$ is given by:

$$\begin{aligned}\Delta O(i, n, \Delta\tau) &= \sum_{s=1}^{\Delta\tau-1} \beta^s P^s_{i,M+1}(1) \\ &\cdot \Big( I(1, n+s) - C_{OP}(1, n+s) + J_{n+s+1}(1) - \sum_{j=i}^{M} P^s_{i,j}(2) J_{n+s+1}(j) \Big)\end{aligned} \tag{5.9}$$

The net benefit after an unforeseen replacement is here found by subtracting the net accumulated profit at the next stage $n+s+1$ (assuming the item survives stage $n+s$) from the accumulated profit of an item in a "new" condition at stage $n+s+1$.

The optimality equation provides the mechanism for recursively determining the value of the objective function at the start of the planning horizon, beginning with the values at the boundary. Starting with the boundary conditions $J_N(j_N)$ at the end of the planning horizon, the objective value at stage $n = N-1$, $J_{N-1}(j_{N-1})$, is found using eq. 5.7. Then, with $n+1 = N-1$ in the equation, the objective value at stage $n = N-2$, $J_{N-2}(j_{N-2})$ is derived from $J_{N-1}(z_{N-1})$ determined from the previous step. The objective value at the start of the planning horizon $J_1(i)$ is thus eventually determined with $n = 1$. The set of optimal decisions for each state at each stage, forms the optimal policy for the problem.

The general formula of the optimisation procedure, including state space $Z$ and decision space $X$ is expressed as:

$$\begin{aligned}J_n(z_n) &= \frac{max}{x_n \in X_n, \Delta\tau} \Big\{ G(z_n, n, \Delta\tau, x_n) + \\ &\beta^{\Delta\tau} \sum_{z\in Z} \mathbf{P}^{\Delta\tau}_{z_n,z_{n+1}}(2) J_{n+\Delta\tau}(z_{n+1}) \Big\}\end{aligned} \tag{5.10}$$

where $\mathbf{P}$ contains all combinations of transition probabilities between different states. $\mathbf{P}$ is calculated based on the assumption that all deterioration processes are independent of each other. Here, $z^*_{n+1}$, represents a state where at least one failure has occurred. A thorough description to establish each CTPM matrix is presented in section 5.3.

**Infinite horizon and error bounds**

The solution to infinite horizon problems is found by an approximation over a finite but very large number of stages. The approximation in general results in a control-limit policy, which states the optimal stationary policy for different initial states ($\pi^* = \{\mu^*, \mu^*, \ldots\}$). According to Bertsekas (1987) [58], three main types of problem may arise:

**(a)** Discounted case with bounded profit (cost) per stage. $0 < \beta < 1$, $X_1 \leq r(z_n, x_n) \leq X_2$.

**(b)** Discounted case with unbounded profit (cost) per stage. $\beta > 0$, $X_1 \leq r(z_n, x_n) \leq X_2$.

**(c)** Average profit (cost) per stage. $\beta \geq 0$, $0 \leq r(z_n, x_n)$ or $r(z_n, x_n) \leq 0$.

$X_1$ and $X_2$ are arbitrary scalars. The solution to problems of type **(a)** may be found by applying the functional form of eq. 5.10. In problems involving finite-state Markov chains and no discounting, type **(b)**, the optimal total expected cost is either infinite for some initial states or finite for all initial states. From the assumption of problem type **(b)** the latter implies a special case of some "cost-free states" that are eventually entered and subsequently never left [58]. If the total expected cost is infinite ($J_\pi(z_0) = +\infty$), the problem has to be redefined into the average cost per stage framework. The optimality equation for the average cost per stage is expressed as:

$$\lim_{N \to \infty} \frac{1}{N} J_n(z_n) \tag{5.11}$$

To reduce the number of iterations in an infinite horizon domain, it is possible to establish *error bounds* to terminate the calculation when the value is sufficiently close to the optimal policy. Under the assumption of problem type *(a)*, the error bounds are calculated based on the following two equations:

$$\begin{aligned} \underline{c_n} &= \tfrac{\beta}{1-\beta} \min_{z \in Z} \Big[ J_n(z_n) - J_{n-1}(z_{n-1}) \Big] \\ \overline{c_n} &= \tfrac{\beta}{1-\beta} \max_{z \in Z} \Big[ J_n(z_n) - J_{n-1}(z_{n-1}) \Big] \end{aligned} \tag{5.12}$$

where $\underline{c_n}$ and $\overline{c_n}$ denote the lower and upper bound respectively. The value iteration is terminated when the difference $(\overline{c_n} - \underline{c_n})$ of the error bounds becomes sufficiently small ($< \epsilon^*$).

### 5.2.5 Technology improvements

Due to obsolescence, a modification may be necessary to fulfill the operational goals. The profit achieved by modifying versus replacement with an item of similar technology will depend on the extent of improvement in the different parameters such as capacity, efficiency, maintenance cost etc. The inclusion of technology improvements has mainly been considered in capital replacement models. Hopp and Nair (1992) [77] have proposed a model where the appearance of future technologies is non-stationary in time and cost and revenues of technologies are different but constant over time. Under the assumption that the costs and revenues are bounded and that the discount factor is less than one ($\beta < 1$), they calculate a forecast horizon (finite) sufficiently long to secure an optimal solution to a infinite horizon problem. The optimal solution is found by applying stochastic dynamic programming over the forecasted horizon, with transition probabilities given as the probability of appearance of a new technology.

To include the effect of future technological improvements on a decision at the present time, a similar approach is applied. In this work an heuristic approach is selected to include the effect of future technology improvements. It is assumed that the decision maker may only consider to replace existing technology with an improved one after having performed a planned inspection to reveal the present condition of the item of existing technology. An important assumption is that a modification to the most recent available technology is always the best choice, independent of the remaining time to operate.

At the beginning of the planning horizon, the decision maker is allowed to specify a separate set of properties for each technology (see fig. 5.4). For each included technology, the decision maker also has to make a forecast of the probability of release. As an example, assume the present technology (in operation) $A$ is challenged by two forthcoming technologies $B, C$. At the moment these technologies are not available. Let $p_\phi(n)$ denote the probability that technology $\phi \in \Phi = \{A, B, C, \ldots\}$ is available at stage (time period) $n$ (see fig. 5.5), and $H_\phi(n)$ expresses the profit generated if the technology $\phi$ is selected at stage $n$. Based on the assumption that a modification to the most recent available technology is always the best choice ( $H_A(n) < H_B(n) < H_C(n)$), then the expected profit $\overline{H(n)}$, at any stage $n$, is given by:

$$\begin{aligned} \overline{H(n)} &= H_A(n)(1 - p_B(n))(1 - p_C(n)) \\ &\quad + H_B(n)p_B(n)(1 - p_C(n)) + H_C(n)p_C(n) \end{aligned} \tag{5.13}$$

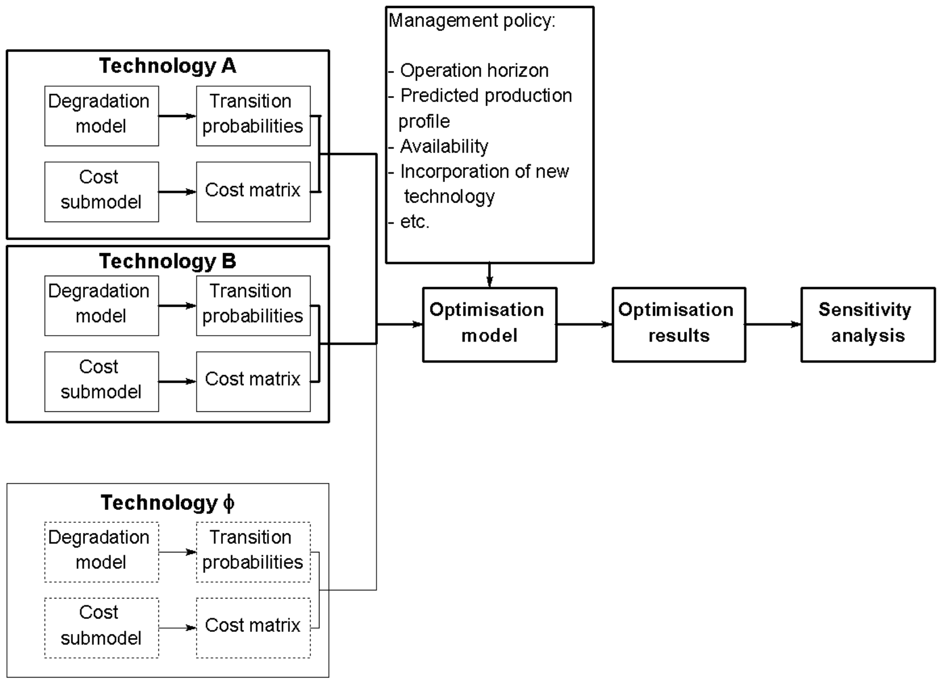


Figure 5.4: The data model.

## 5.3 The condition transition probability matrix

A critical part of utilising a Markovian probabilistic modelling approach is generating condition states that adhere to the Markovian property. The Markovian property states that the future probabilistic behaviour of the process depends only on the present state of the process and is not influenced by its past history [59]. This section deals with different approaches to represent inspection records within a condition transition probability matrix (CTPM), conditional on the state and the decision at a decision interval. The CTPMs are based on calculation of natural transition probabilities describing the stochastic deterioration process.

The natural transition probabilities may be established by several methods, depending on the nature of the deterioration process. In this section two basic approaches are presented. The first approach considers deterioration processes in which the performance of the item is observable and may be classified according to a discrete condition scheme based on surveillance records. The second approach considers the deterioration processes which are non-observable and thus, at each inspection, it can only be stated if the item is working or not. For the latter deterioration processes, the state of deterioration may be related to e.g. number of years in operation, stress

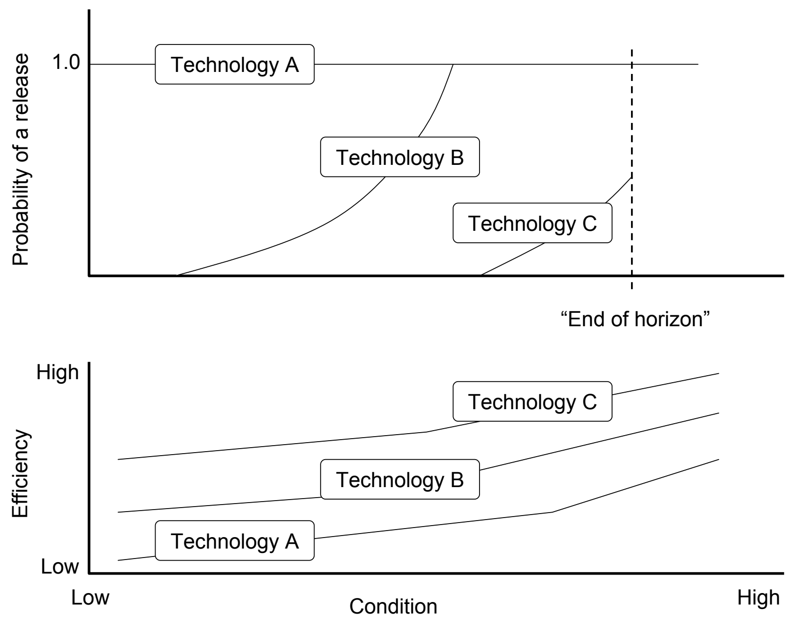


Figure 5.5: The probability of release and degree of technological improvements. Technology $A$ is the current one in use.

cycles etc.

### 5.3.1 "The natural transition probabilities"

"The natural transition probabilities" describe the nature of the stochastic process when the item is left undisturbed (without an action that restores or preserves it). In this context, a special method to calculate these natural transition probabilities based on trend-analysis and forecasting of the ongoing deterioration process is described. The method differs from the general transition probability modelling approach, where the $\Delta\tau$ step transition probability matrices are recursively determined from a one-step transition probability matrix e.g. [68], [61], [75]. In those modelling approaches, "the natural transition probabilities" over a number of time periods $\Delta\tau$ are entirely derived from a one-step matrix, $\mathbf{P}(x)$, given as:

$$\mathbf{P}(x=1) = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,M+1} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,M+1} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ p_{M+1,1} & p_{M+1,2} & \cdots & p_{M+1,M+1} \end{bmatrix} \tag{5.14}$$

where $p_{ij} = p(z_{n+1} = j | z_n = i, x_n = 1)$. The transition from state $i$ to state $j$ in $\Delta\tau$ time units is expressed as:.

$$P_{i,j}^{\Delta\tau}(x_n) = P^{\Delta\tau}(z_{n+\Delta\tau} = j | z_n = i, x_n) = (\mathbf{P}(x_n) * \mathbf{P}(x=1)^{\Delta\tau - 1})_{ij} \tag{5.15}$$

Here, each row represents the initial condition $i$, and each column the expected future condition $j$ during one stage transition.

This approach clearly has its weaknesses. To establish the transition probabilities, there has to be relevant and reliable data available for all possible state transitions. This may often not be the case, especially if the number of discrete condition levels is high and/or the deterioration process is slow (little data available). Second, the transition probabilities will entirely be based on previously observed deterioration processes, and do not explicit include knowledge of present deterioration processes. Third, this modelling approach is not capable of reflecting that uncertainty in the estimated (forecasted) deterioration, which often increases as a function of time.

Utilising condition monitoring information seems to be a more practical approach, where the information gained from surveillance makes it possible to treat each item independently. The condition development is often presented both as short-term and long-term trend graphs. An appealing approach is therefore to convert the information in the trend-graphs to a set of transition probabilities describing the ongoing deterioration processes.

Due to the weaknesses of the one-stage transition probability matrix and the exhaustive work of modelling, Hontelez et. al. [78] has proposed a method to derive the transition probabilities from a continuous deterioration function $\xi(t)$. In their approach a drift function $g(t)$ represents a deterministic decay of the items condition. To include the uncertainty, a stochastic part $\epsilon_t$ is added to $g(t)$, and the deterioration process $\xi(t)$ is expressed as:

$$\xi(t) = g(t) + \epsilon_t \tag{5.16}$$

The functional form of the deterioration process can be determined by several methods, and a thorough discussion of these methods is presented in chapter 6. In the following text it is assumed that the stochastic part of the deterioration process $\epsilon_t$ is given as:

$$\epsilon_t = b \cdot U \cdot \sqrt{t}, \qquad U \sim N(0,1) \tag{5.17}$$

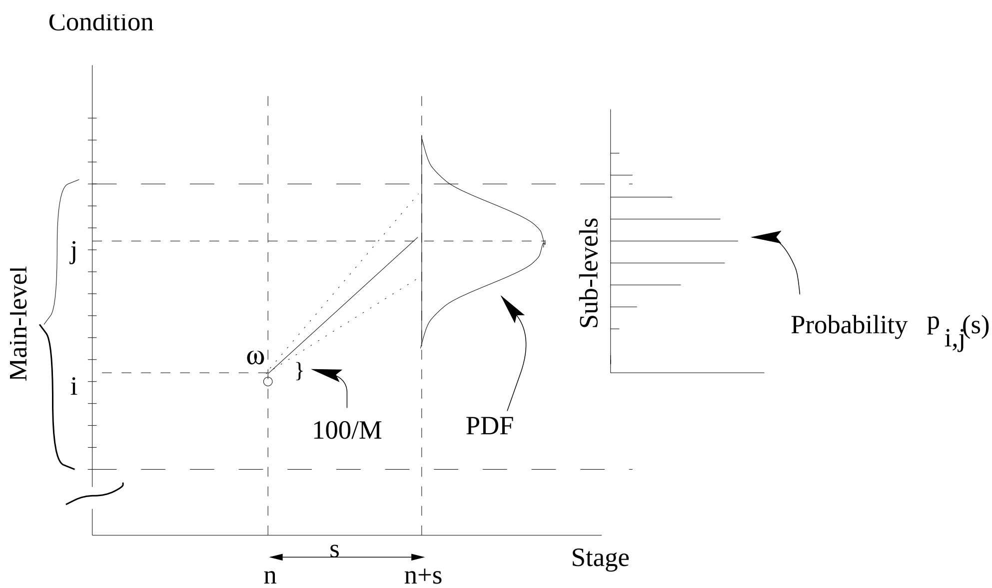


Figure 5.6: The natural transition probabilities $p_{ij}(s)$ generated from the functional form of $\xi(t)$. All sub-levels within the same main-level have the same economic properties. All state dependent cost and income values are constant within a specific main-level.

where N(0,1) stands for the standard normal distribution, and $b \geq 0$ is a variance parameter given by the standard normal distribution. To determine the discrete transition probabilities, the deterioration process is divided into $M+1$ discrete intervals (condition levels). Condition level 1 represents an item in a new condition, and condition level $M+1$ represents a failure, i.e. the component is in an inoperable condition. The upper limit of condition level $i$ is denoted as $L_{u,i}$ and the lower limit as $L_{l,i}$, i.e. considering the classification scheme presented in table 5.1 for $i = 2$, $L_{u,i} = 20$. Based on knowledge about the condition, $\omega$, at time $t$, the objective is to predict the probability of having condition $\xi^*$ at time $t + \delta$. Using the assumption that the stochastic part of the model is independent of the deterioration in previous time intervals, it is known that the distribution of the time increment $\delta$ will also be normally distributed [78]. Hence, the condition $\xi^\delta(\omega)$, when the condition was $\omega$ at the start of the increment, is:

$$\xi^\delta(\omega) = g(g^{-1}(\omega) + \delta) + b \cdot U \cdot \sqrt{\delta}$$

$\xi^\delta(\omega)$ is normally distributed, where:

$$P[\xi^\delta(\omega) \leq y] = \Phi\left[\frac{y - g(g^{-1}(\omega) + \delta)}{b\sqrt{\delta}}\right] \tag{5.18}$$

The "natural transition probabilities" $p_{i,j}(s)$ express the probability that the item deteriorates in $s$[6] time periods from condition $i$ to condition $j$ $(1 \leq i \leq j \leq M+1)$. Using eq. 5.18, the transition probabilities are calculated using the following equations:

$$
\begin{aligned}
p_{i,j}(s) &= \int_{L_{u,i}}^{L_{l,i}} P[L_{l,j} \leq \xi^s(\omega) \leq L_{u,j}] f(\omega) d\omega \quad (i < j) \\
p_{i,i}(s) &= \int_{L_{u,i}}^{L_{l,i}} P[\xi^s(\omega) \leq L_{l,i}] f(\omega) d\omega \\
p_{i,j}(s) &= 0 \qquad (i > j)
\end{aligned}
\tag{5.19}
$$

$f(\omega)$ is the density of $\xi^s(\omega)$, given that $\xi^s(\omega)$ is a part of the interval $[L_{u,i}, L_{l,i}]$:

$$
f(\omega) = \left| \frac{\partial(g^{-1}(\omega))}{\partial\omega} \right| f(g^{-1}(\omega)) \tag{5.20}
$$

After obtaining the transition probabilities by eq. 5.19, the values of $p_{i,j}(s)(i \leq j)$ may have to be normalised, such that $\sum_{j \geq i} p_{i,j}(s) = 1.0$, $(1 \leq i \leq M+1)$. The integrals of 5.19 are approximated by Simpson's rule [7]. Eq. 5.20 is approximated by:

$$
f(\omega) = \frac{\partial(g^{-1}(\omega))\partial\omega}{g^{-1}(L_{l,i}) - g^{-1}(L_{u,i})} \tag{5.21}
$$

where we assume that the time is uniformly distributed over the interval $[g^{-1}(L_{u,i})$ $, g^{-1}(L_{l,i})]$.

### Discretization of continuous distributions

The process previously presented is a multi-state deterioration process, however for several deterioration processes the state is only classified as working or failed. Such failure processes are also fitted to continuous distributions. The continuous distributions have to be discretisized if the process is to be included. As an example, a brief

[6] $t = s \cdot \Delta t$

[7] The estimation of $\int_a^b f(x)dx$ is performed by a numerical integration over two equal subintervals with partition points $a$, $a+h$, and $a+2h=b$. Using Taylor series the integral can be expressed as [79]:

$$
\int_a^{a+2h} f(x)dx \simeq \frac{h}{3}[f(a) + 4f(a+h) + f(a+2h)]
$$

presentation of e.g. the Weibull distribution and its modelling is given. The same framework will apply to other continuous distributions.

The probability density function $f(t)$ for a three parametric Weibull distribution is expressed as [60]:

$$f(t) = \frac{\iota(t-\gamma)^{\iota-1}}{\eta^{\iota}} \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\iota}\right]$$

where $\eta$ is the characteristic life parameter, $\iota$ is the shape parameter, and $\gamma$ represents the location parameter. Further, the reliability (survivor) function $R(t)$ is defined as:

$$R(t) = \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\iota}\right] \tag{5.22}$$

Based on the definition of the survivor function above, the transition probabilities are simply presented as a function of the age ($s$) e.g. number of time periods elapsed since last replacement. Thus:

$$\begin{aligned} p_{1,1}(s) &= R(s \cdot \Delta t) \\ p_{1,2}(s) &= F(t) = 1 - R(s \cdot \Delta t) \end{aligned}$$

**Fault detection**

In some cases a fault may only be detected by an inspection. As an example, a pipe may be corroded beyond a predefined failure level but still function. In such cases it is appropriate to split the failure condition into two separate condition levels $M+1$ and $M+2$. Condition level $M+1$ corresponds to a failure which can only be detected by an inspection, while condition level $M+2$ is immediately noticed at its occurrence. The probability of detecting a failure without an inspection is denoted as $q_1$. If the item should fail without being noticed, there is still a chance that the failure will be noticed later.

Because the natural transition probabilities $p_{i,j}(s)$ do not differentiate between these two failure conditions, $\hat{p}_{i,j}(s)$ is defined as the probability that the condition of the component is $j$ while the condition was $i$ ($0 \leq i \leq j \leq M+2$) $s$ time periods ago.

To be able to calculate transition probabilities $\hat{p}_{i,j}(s)$, the probability of detecting a failure without an inspection ($q_1$), and the transition probability $q_2$ between the two failure levels $M+1$ to $M+2$ has to be estimated. Having estimated these probabilities, the revised transition probabilities are determined by[78]:

$$\begin{array}{rcll}
\hat{p}_{i,j}(s) & = & p_{i,j}(s) & (1 \leq i \leq j \leq M+1) \\
\hat{p}_{i,M+1}(s) & = & (1-q_1)\Big(p_{i,M+1}(s) - p_{i,M+1}(s-1)\Big) & \\
 & & +(1-q_2)\hat{p}_{i,M+1}(s-1) & (1 \leq i \leq M+1) \\
\hat{p}_{i,M+2}(s) & = & p_{i,M+1}(s) - \hat{p}_{i,M+1}(s) & (1 \leq i \leq M+1) \\
\hat{p}_{M+2,M+2}(s) & = & 1 &
\end{array} \tag{5.23}$$

For a failure process described by a continuous distribution, these probabilities may be calculated in a similar manner. However here there are only three different condition levels instead of $M+2$. Condition level $1$ is defined as "still operating", level 2 as "a hidden failure", and level 3 as "an evident failure". Thus, the transition probabilities $\hat{p}_{i,j}(s)$ is determined by:

$$\begin{array}{rcll}
\hat{p}_{1,1}(s) & = & R\big(s \cdot \Delta t\big) & (0 \leq s \cdot \Delta t \leq T_N) \\
\hat{p}_{1,2}(s) & = & (1-q_1)\Big(R\big((s-1)\cdot \Delta t\big) - R\big(s \cdot \Delta t\big)\Big) & \\
 & & +(1-q_2)\hat{p}_{1,2}(s-1) & (0 < s \cdot \Delta t \leq T_N) \\
\hat{p}_{1,3}(s) & = & 1 - R\big(s \cdot \Delta t\big) - \hat{p}_{1,2}(s) & (0 \leq s \cdot \Delta t \leq T_N) \\
\hat{p}_{2,2}(s) & = & (1-q_1)(1-q_2)\hat{p}_{2,2}(s-1) & (0 < s \cdot \Delta t \leq T_N) \\
\hat{p}_{2,3}(s) & = & q_2 \cdot \hat{p}_{2,2}(s-1) & (0 < s \cdot \Delta t \leq T_N) \\
\hat{p}_{1,2}(s) & = & 0 & (s \cdot \Delta t = 0) \\
\hat{p}_{2,2}(s) & = & 0 & (s \cdot \Delta t = 0) \\
\hat{p}_{2,3}(s) & = & 0 & (s \cdot \Delta t = 0) \\
\hat{p}_{3,3}(s) & = & 1 &
\end{array} \tag{5.24}$$

$T_N$ is the expected time to end of horizon.

To simplify the formulation in the next sections, the evident fault level is denoted by $M^*$ and is given as:

$$M^* = \begin{cases} M+1 & q_1 = 1 \\ M+2 & 0 \leq q_1 < 1 \end{cases} \tag{5.25}$$

### 5.3.2 Decision dependent transition probabilities

Based on the natural probabilities, $\hat{p}_{i,j}(s)$, calculated according to the methods described in the previous section, the next step is to calculate the transition probabilities, conditional on state and on the decision. The state space is given as $i \in$

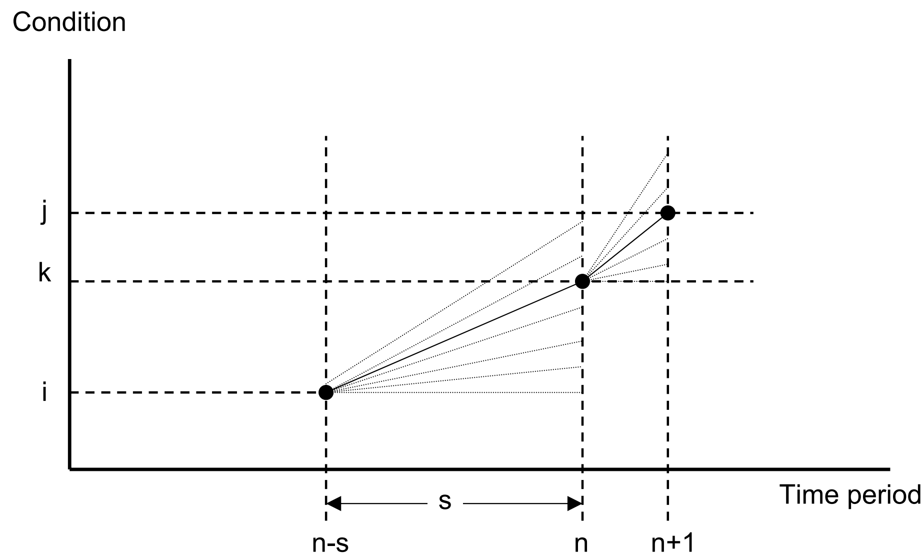


Figure 5.7: Schematic view of the transition probability to state $j$ from state $i$, when an inspection at stage $n-s$ has revealed condition $i$.

$Z^* = [1, M^*]$, where $M^*$ is the failed state. Further, $T_N(i)$ denotes the maximum number of time units between two successive inspections when the condition is $i \in Z^* = [1, M^* - 1]$. According to decision space described in sec. 5.2.2, the decision maker may choose between 3 possible actions, $X = \{x|x = 1, 2, 3\}$ — do nothing ($x = 1$), inspect ($x = 2$), restore or replace by a similar technology or modify (improve by a new technology) ($x = 3$).

Let $P_{i,j}(x)$ express the conditional transition probability from state $i$ to state $j$, where $x \in X$ and $(1 \le i \le j \le M^*)$. Next, consider the principle drawing of figure 5.7. Assume the condition at present stage $n$ is $k$. If an exact condition $i \in \{1, \ldots, M^* - 1\}$ was determined at stage $n-s$, the probability operating in condition $k$ at present stage $n$ (in absence of inspection and revisions) is expressed as:

$$\frac{\hat{p}_{i,k}(s)}{1 - \hat{p}_{i,M^*}(s)}$$

Let $\hat{p}_{k,j}(1)$ express the probability that the item is in condition $j \in \{1, \ldots, M^*\}$, at stage $n+1$. If the condition is uncertain at stage $n$, the probability that the condition is $j$, knowing the condition was $i$ $n-s$ time periods ago, is given as:

$$\frac{\sum_{k=i}^{M^*-1} \hat{p}_{i,k}(s)\hat{p}_{k,j}(1)}{1-\hat{p}_{i,M^*}(s)} = \frac{\hat{p}_{i,j}(s+1)}{1-\hat{p}_{i,M^*}(s)} \tag{5.26}$$

Using the equations above, the decision (action) dependent transition probabilities $P^s_{i,j}(x)$ may now be developed. For the Markov decision problem of eq. 5.10, the probability of five different situations has to be calculated:

**[1] The one-step probability of operating in state $j$ after $s+1$ time periods, when the state $i$ in previous time unit $s$ is uncertain.**

$$P^s_{i,j}(1) = \tfrac{\hat{p}_{i,j}(s+1)}{1-\hat{p}_{i,M^*}(s)} \quad 1 \leq i < M^*, 1 \leq j < M^*-1, 1 \leq s \leq T_N(i) \tag{5.27}$$

**[2] The one-step transition probability of having an unforeseen maintenance task in time interval [s, s+1], when the condition $i$ at time $s$ is uncertain.**

$$P^s_{i,M^*}(1) = \tfrac{\hat{p}_{i,M^*}(s+1)-\hat{p}_{i,M^*}(s)}{1-\hat{p}_{i,M^*}(s)} \quad 1 \leq i < M^*, 1 \leq s \leq T_N(i) \tag{5.28}$$

**[3] The probability of state $j$, when the previous state $i$ was known $s$ periods ago.**

$$P^s_{i,j}(2) = \tfrac{\hat{p}_{i,j}(s)}{1-\hat{p}_{i,M^*}(s)} \quad 1 \leq i < M^*, i \leq j < M^*, 1 \leq s \leq T_N(i) \tag{5.29}$$

**[4] The probability that the next state is $j=1$ after a replacement or an overhaul.**

$$P^{s=1}_{i,j=1}(3) = 1 \quad 1 \leq i \leq M^* \tag{5.30}$$

In addition to these conditional transition probabilities, the probability of being in state $j$ ($1 \leq j \leq M$) after $s$ time-units of operation when $q_1 > 0$ is given according to eq. 5.29.

### 5.3.3 Discussion

#### Discretisation

Due to the discretisation of the continuous function $\xi(t)$ describing the deterioration process, an uncertainty is introduced in the modelling. An important task is therefore to secure sufficient accuracy. Two parameters have an important effect on the accuracy achieved — the number of states $M$ and the length of each stage. Because a large number of stages and states increases the effort to find an "optimal" policy, the number of states and stages has to be kept at a minimum without violating the required accuracy. For this purpose, the following procedure is proposed:

1. Select an initial number of states $M$ and an initial stage length $\Delta t$.
2. Calculate all natural transition probabilities according to eq. 5.19.
3. Calculate the mean residual life ($MRL_1$) at different condition levels based on the deterioration function $g(t)$, such that [8]:
$$MRL_1(i) = g^{-1}(100) - g^{-1}((i-1)100/M)$$
4. Calculate the mean residual life at different condition levels from the natural transition probabilities, such that:
$$MRL_2(i) = \sum_{s=1}^{\infty} [p_{i,M+1}(s) - p_{i,M+1}(s-1)] \cdot s$$
5. Compare the values of $MRL_1(i)$ and $MRL_2(i)$. Let $\epsilon_{MRL}(i)$ denote the deviation in the estimate at condition level $i$ given as:
$$\epsilon_{MRL}(i) = MRL_1(i) - MRL_2(i)$$
If $\epsilon_{MRL}(i) < \epsilon_{max}\ \forall i$, a sufficient accuracy is achieved and the iteration terminates. Otherwise, the number of states has to be increased and/or the stage length has to be reduced, and the procedure of steps 2-5 is repeated.

[8] The continuous deterioration function is always scaled to a function $g(t)$ which ranges from 0 to 100, where $g(t) = 0$ represents an item in a new condition and $g(t) > 100$ represents an item which has failed. Condition level $i$ is condition level given as $[(i-1) \cdot 100/M, i \cdot 100/M >$. $M$ is the number of discrete condition levels between 0 an 100.

Table 5.3: The error in the approximation of the $MRL_2$ for different numbers of states ($M$), $\xi(t) = 2.0t + 2.0\sqrt{t}U$, $U \sim N(0,1)$

| M | $i = 1$ | $MRL_2$ | $\epsilon_{MRL}(i) = MRL_1 - MRL_2$ | | |
|---|---|---|---|---|---|
| 5 | [0,20> | 45.5000 | a: 4.5000 | a-b | 2.5000 |
| 10 | [0,10> | 48.0000 | b: 2.0000 | b-c | 1.2500 |
| 20 | [0, 5> | 49.2500 | c: 0.7500 | c-d | 0.6250 |
| 40 | [0,2.5> | 49.8750 | d: 0.1250 | d-f | 0.3125 |
| 50 | [0, 2> | 50.0000 | e: 0.0000 | e-g | 0.2500 |
| 80 | [0,1.25> | 50.1875 | f: -0.1815 | - | - |
| 100 | [0, 1> | 50.2500 | g: -0.2500 | - | - |

It should be noted that calculation of transition probabilities using the method presented assumes that the variable state number $i$ represents an interval within $[(i-1)\cdot 100/M, (i \cdot 100/M >$. If $M$ equals $5$, the initial state $i$ (at $s = 0$), is in the range $[0, 20 >$. Table 5.3 presents the accuracy for different values of $M$, for $i = 1$, and a deterministic function $g(t) = 2t$, and the variance $b = 2.0$. The mean residual life when the item is in a new condition is $50$ time units. By a comparison of the error at different levels of discretisation, it can be seen that the effect of increasing the number of states on the accuracy of the estimated residual life diminishes as the number of states increases. In this case, the reduction in the error is proportional to $M$ by $12.5 \cdot M^-1$, and by doubling the number of states from e.g. $5$ to $10$ the reduction in the error is twice as big as the reduction achieved by increasing the number of states from $10$ to $20$. From this it can be seen that the number of states increases drastically if the required accuracy is high. Fortunately, by comparing the results achieved by different deterioration functions and discretisation levels, an "optimal policy" can be found within a reasonable size of $M$. This will be further elaborated in section 5.4.1.

**An alternative method to generate "the natural transition probabilities"**

The transition probabilities may for some special cases be exactly derived from the inverse Gaussian distribution. When the deterministic part of the Wiener process, $g(t)$ (see eq. 5.16), is represented by a constant drift function ($g(t) = \eta t$, $\eta > 0$), the first passage time to the lower boundary condition $L_l$ may be shown to have an inverse Gaussian distribution $T = \inf_t\{t; \xi(t) > L_l\}$[80]. By a reparameterisation of the the Wiener process, the probability density function for a transition from condition $\omega$ to the lower boundary $L_l$ is:

$$f_T(t;\mu,\lambda) = \sqrt{\frac{\lambda}{2\pi\, t^3}} e^{-\left(\frac{\lambda}{2\mu^3}\right)\left(\frac{(t-\mu)^2}{t}\right)} \qquad \text{for } t>0, \mu>0 \text{ and } \lambda>0 \qquad (5.31)$$

where

$$\mu = \tfrac{L_l-\omega}{\eta} \quad \text{and} \quad \lambda = \tfrac{(L_l-\omega)^2}{\sigma^2} \qquad (5.32)$$

Here $\mu$ is the mean time to failure (MTTF), and $\frac{\mu^3}{\lambda}$ is the variance. The survival function is given by:

$$\begin{aligned} R(t) &= P(T>t) = [1-F_T(t)] \\ &= 1-\left(\Phi\Big[\sqrt{\tfrac{\lambda}{t}}\Big(\tfrac{t}{\mu}-1\Big)\Big] + e^{2\lambda/\mu}\Phi\Big[-\sqrt{\tfrac{\lambda}{t}}\Big(\tfrac{t}{\mu}+1\Big)\Big]\right) \end{aligned} \qquad (5.33)$$

where $\Phi[\cdot]$ denotes the cumulative distribution function of the standard normal distribution.

The transition probabilities may therefore be calculated by two different methods when $g(t)$ is represented by a constant drift function. As an example consider the case where $g(t) = 2t$ and $b = 2.0$. The resulting survival function when $L_l = 20$ for $M = 20$ and $M = 100$ is given figure 5.8. In the case of M=100, the approximation becomes close to the actual R(t), while the approximation when $M = 20$ is more inaccurate.

## 5.4 Examples

To investigate the MDP model features, three different inspection and replacement problems are employed. Each fabricated problem is made to visualise different model features.

- A single state variable and a single technology
- Multiple state variables for a single technology
- Multiple state variables and multiple technologies

The aim of the first problem is to study the effect of a selected number of states for each state variable, the functional form of the deterioration function, the uncertainty, the discounting and the effect of an end of horizon boundary. Several deterioration processes may in general affect the items ability to perform its function. The deterioration processes may be considered as competing, and the general approach is

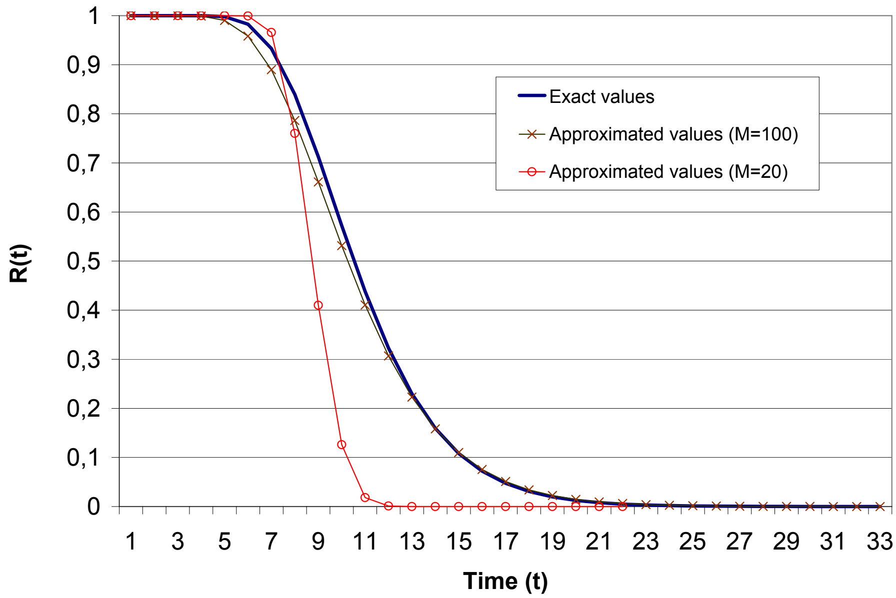


Figure 5.8: Approximation of $R(t)$ for $M = 20$ and $M = 100$ when $g(t) = 2t$, $b = 2.0$, $i = 1 \rightarrow \omega \in [0, 100/M >$ and $L_l - \omega = 100$.

to select the most dominant deterioration process. In some cases however, one deterioration process may affect the efficiency, while the other is only important with respect to the availability. In those cases it may be necessary to expand the state space to include several state variables. The goal of the second subsection is to exploit the model features including several state space variables having only a single technology available. In the last part of this session, the effect of incorporating multiple state variables and multiple technology is investigated.

In the problems to follow several deterioration functions, income and cost figures are applied. The economic figures of three different problems are presented in table 5.4. If not otherwise stated, the cost/income profiles $\Psi_E(z)$, $\Psi_{PM}(z)$, $\Psi_{OH}(z,x)$, $\Psi_S(z)$ equals $1.0$ for all possible states $z$ and decisions $x$. The same assumption applies to the production demand profile $\Psi pp(n)$ and the capacity profile $\Psi_{cap}(z)$ for all states and stages. Both the salvage value ($\bar{S} \cdot \Psi_s(z)$) and penalty cost ($\overline{C_{pen}}$) are neglected. The length of each time period $\Delta t$ is set to 1. The different drift functions and the variance parameters of the deterioration process are given as follows:

| | | | |
|---|---|---|---|
| Drift function: | 1: $g(t) = 2t$ | 2: $g(t) = 10\sqrt{2t}$ | 3: $g(t) = 0.02t^2 + t$ |
| Variance parameter: | 1: $b = 0.05$ | 2: $b = 2.0$ | $3 : b = 6.0$ |

Table 5.4: Income and cost parameters for three different cases

| Case | $\bar{I}$ $[USD]$ | $\overline{C_{OH}}$ $[USD]$ | $\overline{C_I}$ $[USD]$ | $\overline{C_E}$ $[USD]$ | $MDT_{OH}$ $[time]$ | $MDT_{UCM}$ $[time]$ | $MDT_I$ $[time]$ |
|---|---|---|---|---|---|---|---|
| $[A]$ | 8000 | 2000 | 100 | 0 | 0 | 0.25 | 0 |
| $[B]$ | 8000 | 2000 | 100 | 0 | 0.1 | 0.45 | 0.05 |
| $[C]$ | 8000 | 2000 | 100 | 100 | 0 | 0.25 | 0 |

### 5.4.1 A single state variable and a single technology

In this section it is assumed that the state of the item is entirely determined by a single state variable. In addition there are no competing technologies. The state of the item is therefore entirely determined by $z \in Z = \{cp_1 = i\}$. The aim is to present different model properties and to discuss:

- the effect of increasing the number of state variables upon the optimal solution.
- the effect of change in the drift function $g(t)$
- the effect of uncertainty, $\epsilon(t)$
- the effect of a fixed horizon

By way of introduction, a simple inspection and replacement problem is considered based on the economic properties of case [A] (ref. table 5.4). The cost of an inspection (fixed) is therefore assumed to equal USD $100$ and the cost of a planned and unplanned replacement is assumed to be USD $2000$ and USD $4000$ respectively. If not otherwise stated, the discount rate ($\beta$) is set to $1.0$, the number of condition levels ($M$) equals $20$ and the deterioration function and variance ($b$) are as given on page 80, then;

$$\xi(t) = 2.0 \cdot t + 2.0 \cdot \sqrt{t} \cdot U, \qquad U \sim N(0,1) \tag{5.34}$$

#### Discretisation of the deterioration function

As mentioned in section 5.3, the deterioration function $\xi(t)$ has to be discretisised for the MDP to be solved. An important issue when discretisising the continuous deterioration function is to select the appropriate number of condition levels. A high number of condition levels $M$ reduces the approximation error compared to the continuous function, but increases the effort required to solve the MDP. To investigate the effect by increasing the number of condition levels, the model has been run with five levels of discretisation. Figure 5.9 presents the recommended control-limit to perform an inspection, dependent on the initial condition levels for the four levels of discretisation when $N = 200$, i.e. when there are 200 time units left before end of horizon. From the figure it can be seen that the recommended inspection and replacement policy becomes slightly more conservative when the number of condition levels decreases. The average costs per time-unit are presented in figure 5.10. As the policy becomes more conservative the average costs per time-unit increase. Table 5.5 presents the average cost per time-unit for an item which initially is in a new condition ($i = 1, \omega \in [0, 100/M >$) at $N = 200$. The second column presents the recommended time-units to the next inspection if the condition at present decision interval ($N = 200$) equals $1$. If the condition ($i$) is better than the replacement control limit given in the right column of the table, the decision maker is recommend to continue and perform the next inspection according to the control limits of figure 5.9. From figure 5.10, the average cost per time unit for an item with a condition worse than the recommended replacement limit given in table 5.5 is [USD] $2000/200 = 10$ higher than the average costs for $i = 1$, due to the need for an additional replacement. By comparing the average costs for the different levels of discretisation, it can be seen that the benefit by doubling $M$ ( with respect to the achieved accuracy of the average cost estimate) increases as $M$ increases. However, the deviation between the calculated average cost for $M = 40$ and $M = 20$ is only $16\%$ of the deviation between the calculated average costs for $M = 10$ and $M = 5$. The same applies to the optimal policy, where there is only a minor difference between the optimal policies suggested for $M = 20$, $M = 40$ and $M = 100$ compared to $M = 5$ and $M = 10$.

From a practical point of view, a number between 10 and 20 condition levels seems to give an adequate approximation to the continuous deterioration function. Though the average cost per time-unit becomes more accurate as the number of condition levels increases, the control limit for the inspections and replacements is less dependent on $M$. As shown in table 5.5, the control limit for the replacement decision does only change slightly when the number of condition levels $M$ increases from 20 to 100 and the recommended time to the next inspection deviates with only a few time units.

**The effect of uncertainty**

To see how the uncertainty affects the control limit rules and the calculated costs, three variance parameters are specified for the same drift function $(1)$: $g(t) = 2t$. The results are compared with the case where the drift function is assumed to be known with certainty, thus the variance parameter equals $0$. All income and cost figures in the following example are based on case $[A]$.

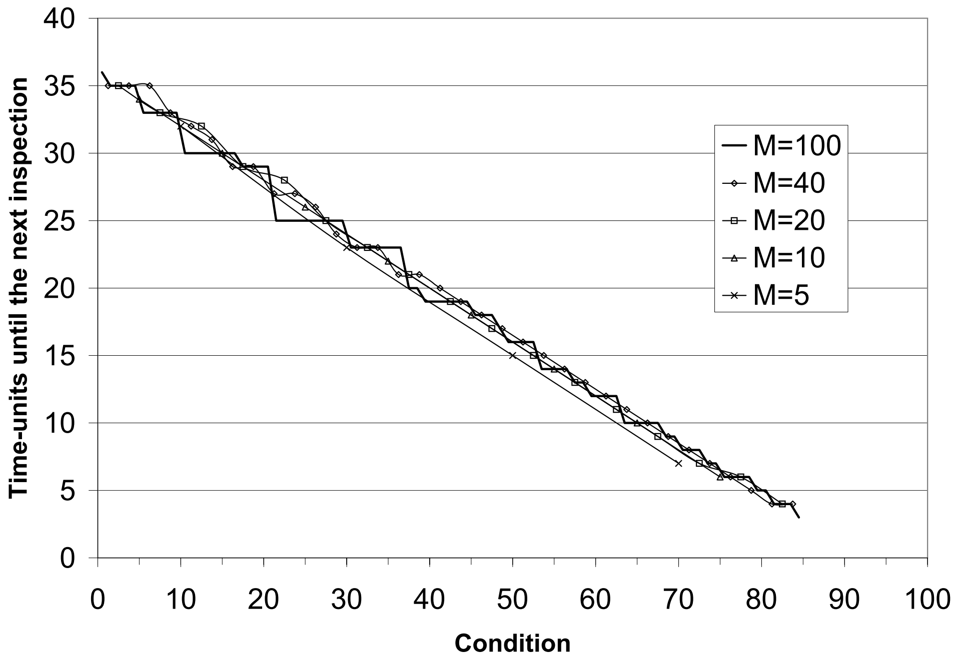


Figure 5.9: The inspection control limit rules for five different levels of state discretisation $M$. Case [A], $\xi(t) = 2t + 2\sqrt{t}U$, $U \sim N(0,1)$, N=200.

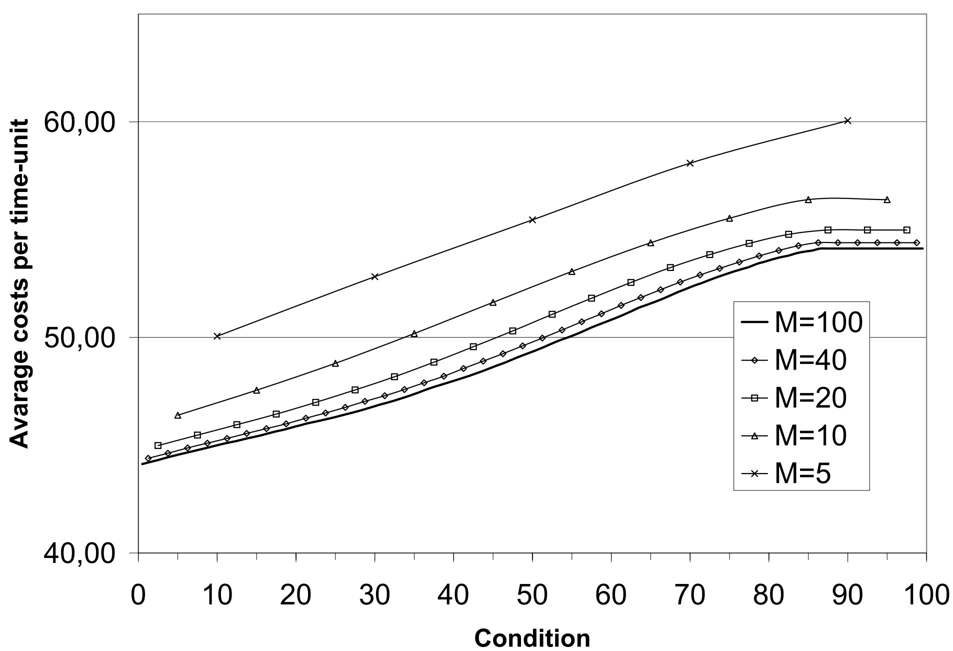


Figure 5.10: The average costs per time-unit for different number of state variables ($M$). Case [A], $\xi(t) = 2t + 2\sqrt{t}U$, $U \sim N(0,1)$, N=200.

Table 5.5: The influence on the level of discretisation (Case: [A], $\xi(t) = 2t + 2\sqrt{t}U$, $U \sim N(0,1)$, $N = 200$).

| M | Average cost per stage if $i = 1$[USD] | Time units to the next inspection if $i = 1$ | Control limit ("continue if") |
|---|---|---|---|
| 5 | 50.06 | 32 | Condition $\omega < 80$, $(i < 5)$ |
| 10 | 46.39 | 34 | Condition $\omega < 80$, $(i < 9)$ |
| 20 | 44.98 | 35 | Condition $\omega < 85$, $(i < 18)$ |
| 40 | 44.39 | 35 | Condition $\omega < 87.5$, $(i < 35)$ |
| 100 | 44.13 | 36 | Condition $\omega < 88$, $(i < 88)$ |

If it is assumed that the time left until end of horizon equals 400 time-units, the minimum number of replacements and average costs per stage is achieved when the variance parameter equals zero. In that case, the expected lifetime is $50$ time units, and the item is replaced and new at the start of the period. The number of replacements during 400 time units of operation is then $8$, and the minimum average cost per time-unit is thus USD $8 \cdot (2000 + 100)/400 = 42.00$.

From table 5.6 it can be seen that the average costs increase as the variance parameter increases. The time to next inspection for the case when $b = 6.0$, is almost half the time if the deterioration function is known with certainty. From figure 5.12 it can be seen that the average cost functions for the different levels of uncertainty vary as a function of time remaining to the end of horizon. The effect of the variance parameters on the proposed time-units to the next inspection is presented in figure 5.11. The effect on the proposed time until the next inspection increases when the item is in a good condition c.f. if the item is in a poor condition. The reason is simply that the inspection interval is larger for an item in a good condition and thus, the reduction in time due to uncertainty becomes more distinctive.

Table 5.6: The influence of uncertainty (Case: [A], $g(t) = 2t$, $N = 400$).

| b | Average cost per stage if $i = 1$[USD] | Time units to the next inspection if $i = 1$ | Control limit ("continue if") |
|---|---|---|---|
| 0 | 42.00 | 50 | Condition $\omega < 100$ |
| 0.5 | 42.71 | 44 | Condition $\omega < 85$, $(i < 18)$ |
| 2.0 | 48.06 | 35 | Condition $\omega < 90$, $(i < 19)$ |
| 6.0 | 58.93 | 27 | Condition $\omega < 75$, $(i < 16)$ |

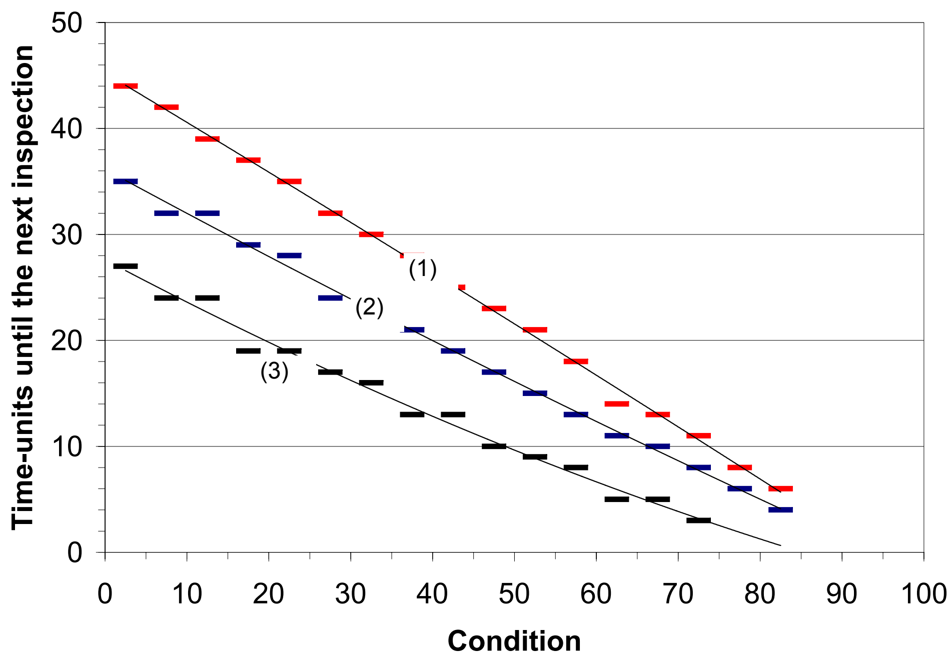


Figure 5.11: Control limit rule for different values of the variance parameter $b$ and condition. Case [A], $g(t) = 2t$. 1: b=0.5, 2: b=2.0, 3: b=6.0.

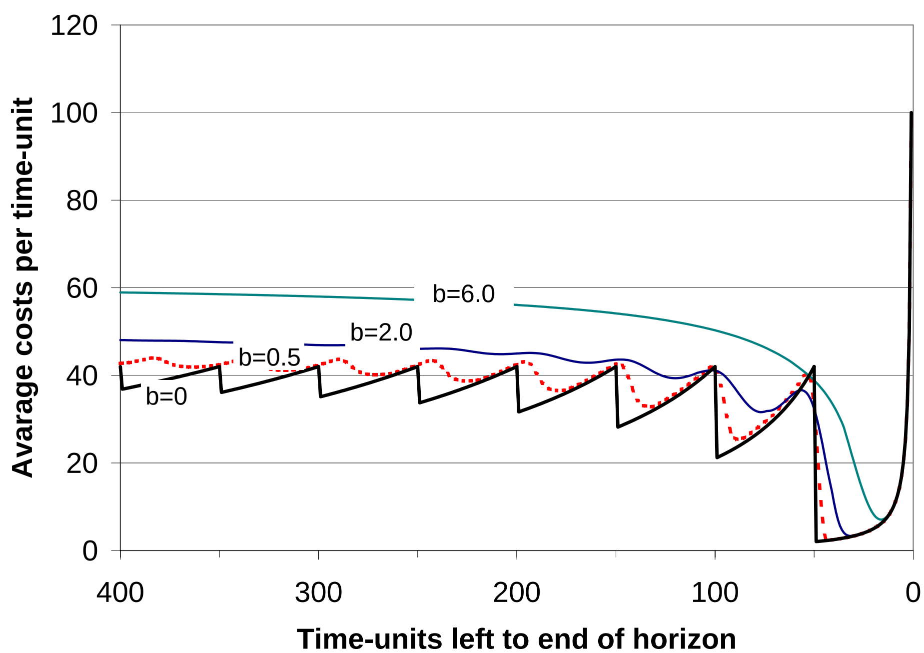


Figure 5.12: A comparison of the average cost per time-unit for different values of the variance parameter $b$ and condition $i = 1$. Case [A], $g(t) = 2t$.

**The drift function**

The drift function may take several functional forms. Until now the drift function has been modelled as linear. Here, the results of two other functional forms are compared with the linear case. All three drift functions reach the breakdown limit after 50 time units and the variance parameter of the deterioration process equals $2.0$ in all three cases. All income and cost figures are based on case $[A]$. The three different drift functions are:

| | | |
|---|---|---|
| 1: | Linear | $g(t) = 2t$ |
| 2: | Square root | $g(t) = 10\sqrt{2t}$ |
| 3: | Quadratic | $g(t) = 0.02t^2 + t$ |

Figure 5.13 presents the calculated time -units to the next inspection for the different drift functions ($N = 400$, $M = 20$). From the figure it can be seen that the replacement limit belonging to the square root drift function is higher $(< 90)$ than the replacement (control) limit belonging to the linear drift function $(< 85)$. The average costs per time-unit as a function of the initial condition are visualised in figure 5.14.

**The effect of a finite horizon**

A central part of the modelling approach selected is to be able to find the "optimal" policy when the horizon is finite. Depending on the problem at hand, the end of horizon will have a minor or major impact on the decision process. If e.g. the uncertainty in the deterioration function is low, the end of horizon will have a greater impact on the decision process than if the deterioration function is very uncertain (see fig. 5.12 - less variation in the avarage cost for high values of $b$). A decrease in the discount factor will in a similar manner affect the influence by a finite horizon. The influence of the discount factor is presented later.

As the distance between the present decision interval and the end of horizon increases, the control limits and average costs per time-unit reach the solution of an infinite horizon problem. Figure 5.15 presents the proposed time until the next inspection for five different condition levels. It can easily be seen that the recommended inspection periods move towards a steady state policy as the distance between the end of horizon and present decision interval increases.

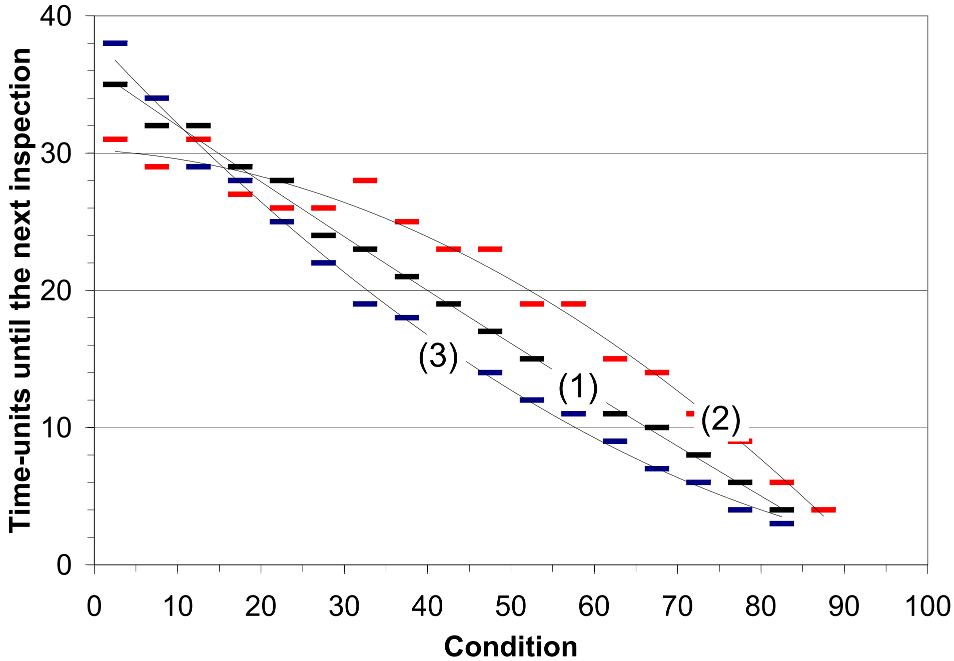


Figure 5.13: Different control limit rules dependent on the functional form of the drift function, 1: $g(t) = 2t$, 2: $g(t) = 10\sqrt{2t}$, 3: $g(t) = 0.02t^2 + t$. Case [A], $b = 2.0$, $N = 400$

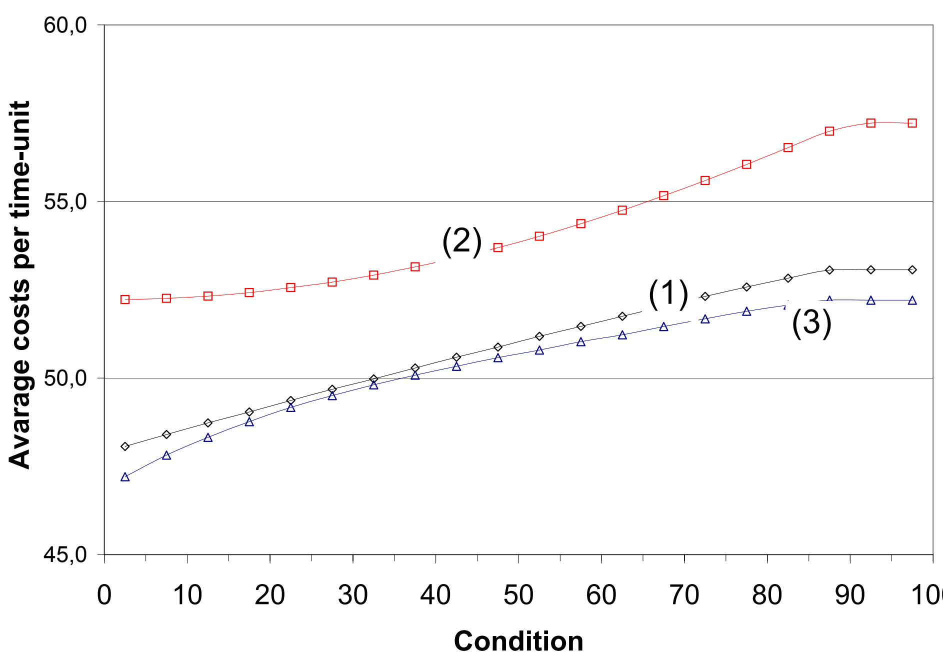


Figure 5.14: Average costs per time-unit for three different drift functions.($N = 400$, $M = 20$, $b = 2.0$). 1: $g(t) = 2t$, 2: $g(t) = 10\sqrt{2t}$, 3: $g(t) = 0.02t^2 + t$.

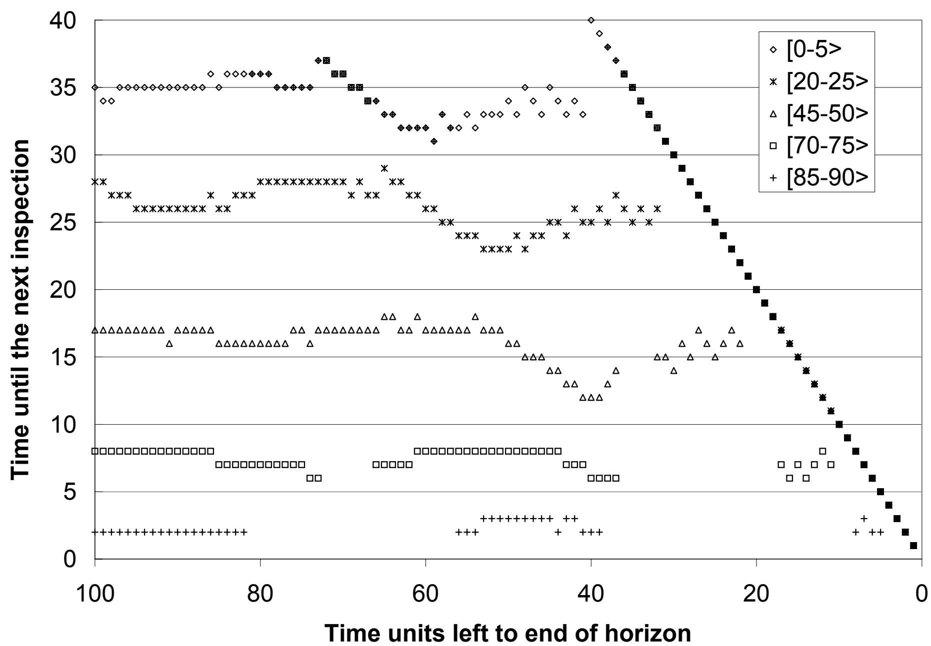


Figure 5.15: Control limit rules for five different condition classes as a function of the time-units left until end of horizon. Case [A], $\xi(t) = 2t + 2\sqrt{t}U$, $U \sim N(0, 1)$.

**The effect of the discount factor**

Change in the discount rate $\beta$ may have significant impact on the "optimal" policy. The discount rate is defined as $\beta = (100 + \text{inflation})/(100 + \text{internal rate of return})$. It may therefore have a value greater, equal to or less than $1.0$, although the latter is the most common situation. For an infinite horizon problem ($n \to \infty$), the objective function of total expected discounted reward can not be applied if $\beta \geq 1.0$ (see fig. 5.17). The average costs objective function may however converge and give an optimal solution. In a finite horizon problem however, an optimal solution may be found by both objective functions.

Intuitively it is expected that the decisions becomes less sensitive to future costs as the discount rate decreases. Figure 5.16 shows that the end of horizon effect only has a minor effect on the optimal solution for $\beta = 0.95$, while the optimal solution for $\beta = 1.0$ is strongly affected by a fixed horizon even for $N > 100$.

The number of iterations required to seek a solution in an infinite environment reduces as the discount rate decreases below 1.0. From fig. 5.17 it is seen that the total cost per time-unit approaches a fixed steady-state value as $n \to \infty$. The effect increases as $\beta$ decreases. From a policy point of view, the importance of considering the end of horizon effect reduces when the discount rate decreases. The number of iterations required to obtain an optimal policy for an infinite horizon problem also decreases. Using the error bounds of equation 5.12 ($\beta < 1$) makes it possible to terminate the optimisation procedure when a sufficient accuracy is achieved.

**The influence of forthcoming shutdowns**

From a maintenance planning perspective, the inspection and replacement policy may be bounded by other external situations in addition to "the end of horizon" boundary. In some cases, there may exist some predefined shutdown periods, where the item may be down for repair or replacement without causing production loss. These periods are in general caused by repair or replacement of other items or systems which demand a total shutdown or a shutdown of the system of which the item is a part. Figure 5.18 and 5.19 presents the results when there exists a predefined shutdown schedule with a duration of one time-unit at $160$, $120$, $80$ and $20$ time-units before end of horizon. The objective function is defined as maximising the profit over the life-cycle, and the income and cost figures are taken from case [B] (ref. table 5.4). In fig. 5.19 it can be seen that the predefined shutdown periods have a similar effect on the average costs and decision policies as the termination of the horizon, though not so significant. The impact of a predefined shutdown does however depend on the duration, and as the duration of a shutdown increases, the shutdown period becomes similar to an end of horizon.

### 5.4.2 Multiple state variables for a single technology

The example in the previous section is here extended to incorporate several deterioration processes. Each deterioration process may cause damage to the item, demanding an instant repair or replacement if it is recognized. Some deterioration processes may cause significant loss of efficiency or increase in maintenance effort to keep the item running. Even if these deteriorations do not violate any safety requirements, it may be considered as failed from an economic point of view, thus it is more profitable to replace than continue. An important assumption made is that the item is entirely renewed when a replacement takes place.

To investigate the feature of modelling multiple states, two deterioration processes are considered. In addition to the deterioration process ($cp_1$) given by eq. 5.34, a second deterioration process ($cp_2$) is included, and this process is modelled as a three parametric Weibull distribution with $\eta = 40$, $\iota = 0$, $\gamma = 3.5$. From the parameters of the distribution, the expected MTTF equals 36.0 time-units. The costs of an in-

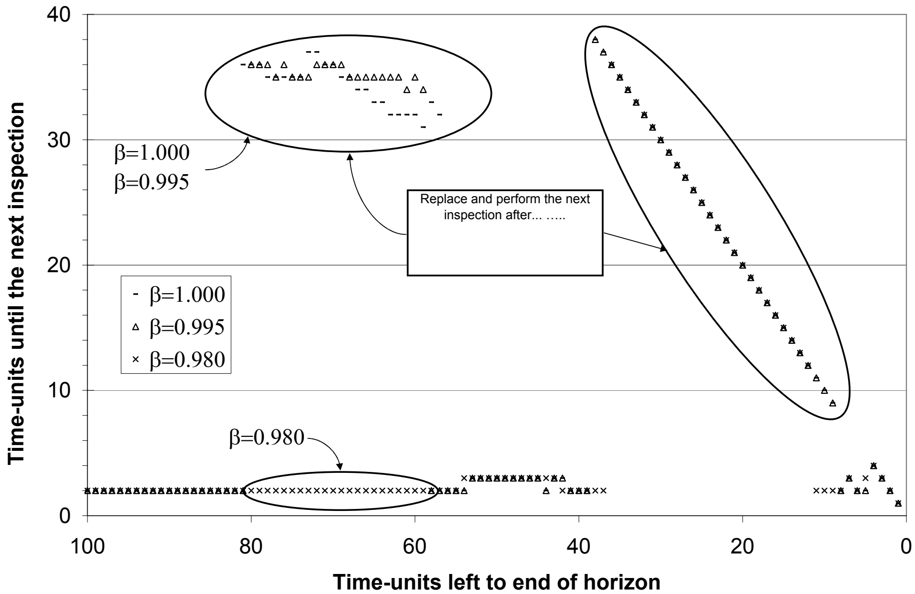


Figure 5.16: Time to next inspection as a function of time-units left to the end of horizon and three different values of the discount factor. The inspection policies are presented for $i = 18$ ($\omega \in [85, 90>$). Case [A], $\xi(t) = 2t + 2\sqrt{t}U$, $U \sim N(0, 1)$.

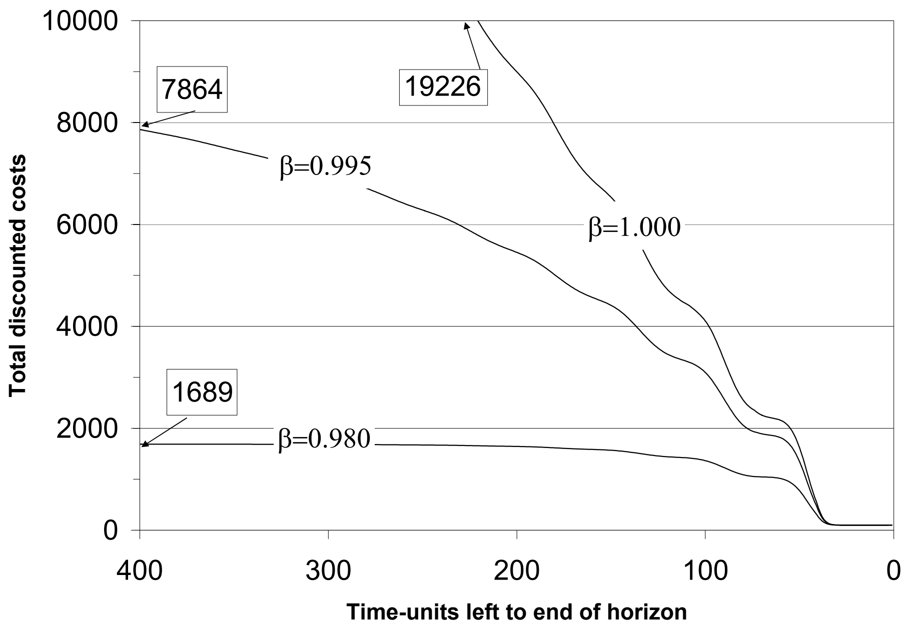


Figure 5.17: Total discounted costs for different discount factor values and $i = 1$ ($\omega \in [0, 5>$). Case [A], $\xi(t) = 2t + 2\sqrt{t}U$, $U \sim N(0, 1)$.

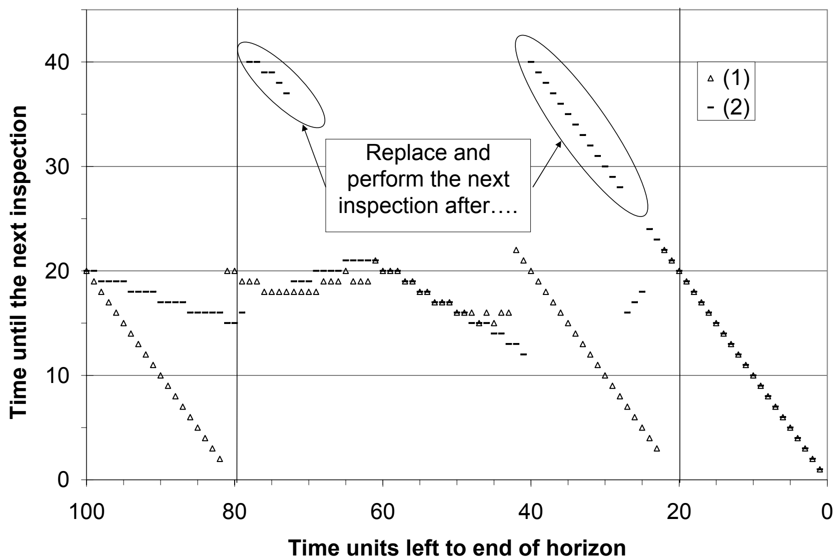


Figure 5.18: The "optimal" inspection policy with $(1)$ and without $(2)$ predefined shutdown periods of one time-unit at $n = 20$ and $n = 80$. The values are given for $i = 10$ ($\omega \in [45, 50 >$). Case [B], $\xi(t) = 2t + 2\sqrt{t}U$, $U \sim N(0, 1)$.

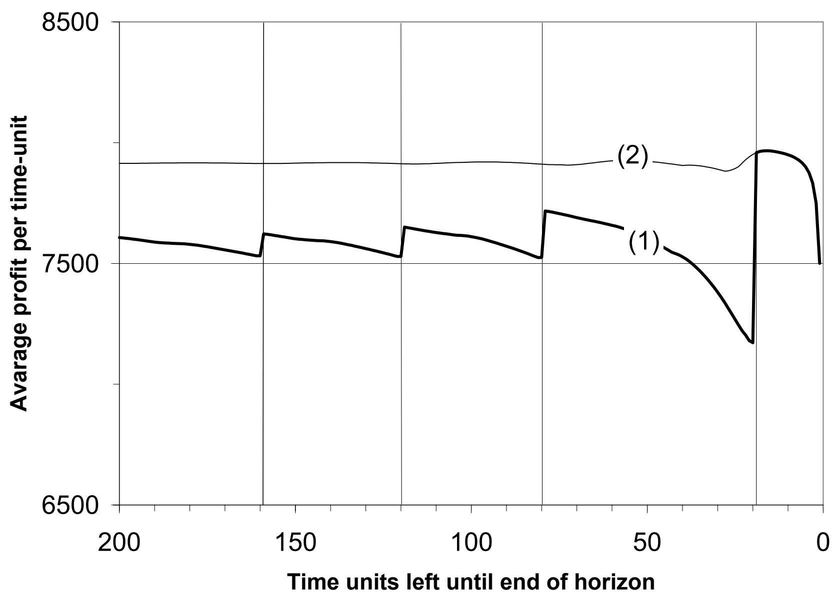


Figure 5.19: The expected average profit per time-unit with (1) and without (2) predefined shutdown periods of one time-unit at $n = 20$, $n = 80$, $n = 120$ and $n = 160$ time-units before end of horizon. The values are given for $i = 10$ ($\omega \in [45, 50 >$). Case [B], $\xi(t) = 2t + 2\sqrt{t}U$, $U \sim N(0, 1)$.

spection and replacement are given according to case [A] in table 5.4. The calculated avarage costs per time-unit due to this single deterioration process are presented in fig. 5.20, where the average costs for the remaing period to operate depends on the the age ($cp_2$) at present decision interval. When the state of the item is only classified as working or failed, there is no need for an inspection, unless the failure is hidden during normal operation. The state of the deterioration process is then entirely determined by the age and thus, it may be considered as an age replacement policy. The age-replacement model gives the optimal age replacement policy assuming an infinite horizon. The objective is to minimise the avarage cost per time-unit and it is expressed as [16],[81]:

$$C(s \cdot \Delta t) = \frac{\Big(C_{OH}(\cdot) + C_I(\cdot)\Big) R(s \cdot \Delta t) + C_{UCM} \cdot F(s \cdot \Delta t)}{\int_0^{s \cdot \Delta t} R(t) dt} \tag{5.35}$$

where $C_{OH}(\cdot)$, $C_I(\cdot)$ and $C_{UCM}(\cdot)$ are the costs of a planned overhaul, an inspection and the cost of an unforeseen overhaul action. The expressions for the cost terms are given in equations 5.4-5.6.

Inserting the values of case [A] gives average costs per time-unit of USD $96.70$, and a policy to replace at an age of $32$ time-units. From fig. 5.20 it can be seen that the average costs per time-unit using the MDP model approach this value as $n \to \infty$. At $n = 400$ time-units, the proposed policy is to replace at an age of $32$ time periods at an average cost per time-unit of USD $96.90$, which is only a small deviation compared to the infinite horizon age-replacement policy. The deviation between average cots per time-unit comparing $age = 0$ and $age = 32$ equals USD $(2000/n)$ (a single replacement). At $n = 200$ the deviation thus equal USD $10$ as shown in the fig.5.20.

Table 5.7 and fig. 5.21 present the “optimal” inspection policy and the average cost per time-unit if both deterioration processes are used to describe the overall state of the item. The proposed policy at $n = 200$, given $cp_1 = 1$ and $cp_2 = 0$ (age), is to perform the next inspection 31 time-units ahead. The results show that the inspection period is dominated by the second failure process ($cp_2$). The reason is that the mean time to failure (MTTF) is $36.0$ time-units, which is less than the MTTF for the deterioration process modelled by $cp_1$. The MTTF of $cp_1$ equals $50.0$ time-units.

In this example, the operational costs (energy, maintenance etc.) are assumed to be independent of the state of the system. If these costs are made state dependent and increase as the item deteriorates, the costs involved at an unforeseen failure may be less important for the overall optimisation compared to e.g. efficiency and energy consumption of operating in a poor condition. The “optimal” policy may be dominated by other properties than MTTF. The individual income and cost figures may easily be specified by the profile functions $\Psi_{OH}$, $\Psi_{PM}$ etc.

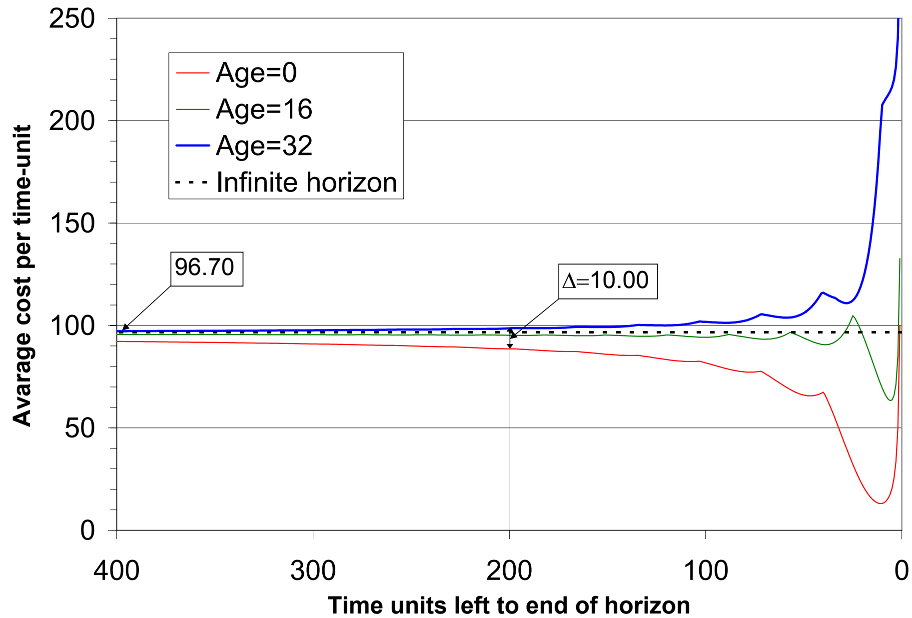


Figure 5.20: The expected average costs per time-unit as a function of time left until end of horizon if the age at decision interval is 0, 16 or 32 time-units. Case [A]. Single deterioration process, Weibull distributed with $\eta = 40$, $\iota = 0$, $\gamma = 3.5$

Table 5.7: The "optimal" inspection policy for different combinations of the state variable $cp_1$ ($M = 10$ levels) and $cp_2$ (age) at $n = 200$ (time units left until end of horizon). Case [A].

| CP1 | CP2 (Age) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0 | 4 | 8 | 12 | 16 | 20 | 24 | ≥26 |
| 1 | I-31 | I-27 | I-24 | I-20 | I-16 | I-12 | I-9 | R/I-31 |
| 2 | I-30 | I-27 | I-24 | I-20 | I-16 | I-12 | I-9 | R/I-31 |
| 3 | I-27 | I-26 | I-24 | I-20 | I-16 | I-12 | I-9 | R/I-31 |
| 4 | I-22 | I-23 | I-22 | I-20 | I-16 | I-12 | I-9 | R/I-31 |
| 5 | I-18 | I-18 | I-18 | I-18 | I-16 | I-12 | I-9 | R/I-31 |
| 6 | I-14 | I-14 | I-14 | I-14 | I-14 | I-12 | I-9 | R/I-31 |
| 7 | I-10 | I-10 | I-10 | I-10 | I-10 | I-9 | I-8 | R/I-31 |
| 8 | I-6 | I-6 | I-6 | I-6 | I-6 | I-6 | I-5 | R/I-31 |
| 9 | I-3 | I-3 | I-3 | I-3 | I-3 | I-3 | R/I-31 | R/I-31 |
| 10 | R/I-31 | R/I-31 | R/I-31 | R/I-31 | R/I-31 | R/I-31 | R/I-31 | R/I-31 |

R: Immediate Replacement, I-xx: Next inspection xx time units ahead.

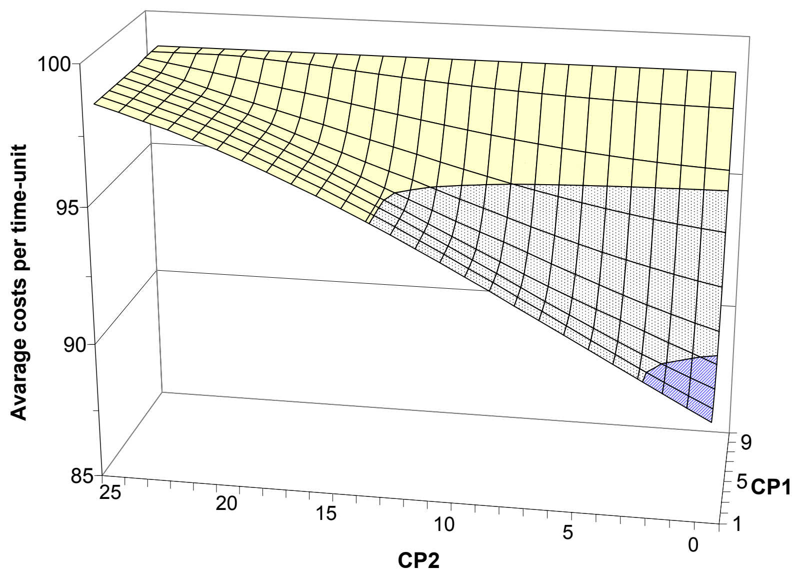


Figure 5.21: The expected average costs per time-unit for different combinations of the state variable $cp_1$ ($M = 10$ levels) and $cp_2$ (age) at $n = 200$ (time units left until end of horizon). Case [A].

### 5.4.3 Multiple state variables and multiple technologies

The aim of this subsection is to investigate how an assumption regarding the release of a new (future) technology will effect the "optimal" policy at present time. Two examples are presented. The first example involves two technologies, where the condition of each technology is determined by monitoring a single deterioration process. The second example is an extension of the first where the state description of each technology involves two deterioration processes.

**Two technologies ($\phi = 2$) and a single deterioration process**

The release date of a new technology is normally uncertain, and a date of release must therefore be predicted. To investigate how this prediction may effect the "optimal" inspection policy the results of three different scenarios are presented. The first scenario assumes an instant release of a new technology at a specific time and the second scenario involves a predicted probability of release within a time interval. The third scenario only involves the present technology, and is included to compare the results of the previous two scenarios.

Table 5.8: The energy cost profile and overhaul cost profile.

| Main condition levels ($i$): | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $[a]$: Present technology | $\Psi_E(i)$ | 1.0 | 1.0 | 1.1 | 1.2 | 1.4 |
| | $\Psi_{OH}(i, a-a)$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| | $\Psi_{OH}(i, a-b)$ | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 |
| $[b]$: New technology | $\Psi_E(i)$ | 0.8 | 0.8 | 0.88 | 0.96 | 1.12 |
| | $\Psi_{OH}(i, b-b)$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |

The deterioration process of the item in operation (of present technology), and the estimated deterioration process of an item having an improved technology are given as:

$$\begin{array}{ll} [a]\text{: Present technology:} & \xi(t) = 2.0t + 2.0\sqrt{t}U, U \sim N(0,1) \\ [b]\text{: New technology:} & \xi(t) = 1.5t + 2.0\sqrt{t}U, U \sim N(0,1) \end{array}$$

In addition to increased strength (slower degradation) it is assumed that the energy costs are reduced by utilising the new technology. The energy cost is set to be $80\%$ of the energy cost involved by operating the present technology. Further, the cost of an overhaul is assumed to be equal for both technologies, except for the modification cost of changing to the new technology. The cost of a modification is assumed to be $20\%$ higher compared to the normal overhaul costs. Case $[C]$ in table 5.4 presents the general cost figures that are used in this example. The income and cost levels (main condition levels) are divided into five separate levels. Each item of different technology has its specific cost figures. The energy profile, $\Psi_E(i)$ and the overhaul cost profile $\Psi_{OH}(i, x)$ are presented in table 5.8. A horizon of maximum $150$ time-units is calculated. In the first scenario, it is assumed that a new technology appears instantly at $n = 99$ (with probability $1.0$), while in the second scenario the probability of a release is described as normally distributed with mean 99 and standard deviation equal to 2.

Figure 5.22 presents the "optimal" inspection policy for the three different scenarios. Each inspection policy is only valid for an item in a new condition at the time of decision i.e. $i = 1$ $\omega \in [0, 10 >$. When a technology is known to occur at $n = 99$ (scenario [1], the decision at $n = 150$ is *to perform a new inspection at* $n = 126$ *(because time-units until next inspection equal* $24$. When the new technology is available ($n = 99$) as stated in the first scenario the decision is to replace the present with the new one immediately. It can be seen that the introduction of a new technology, deterministic or stochastic, has an *end-of-horizon* effect on the optimal decision policy.

Table 5.9 presents the "optimal" inspection policy at $n = 150$ for the same three

Table 5.9: Optimal inspection and replacement policy.

| Condition | Scenarios | | |
|---|---|---|---|
| | $[1]$ Instant release | $[2]$ Prob. of release N(99,2) | $[3]$ No release |
| 1 | I-24 | I-25 | I-26 |
| 2 | I-20 | I-24 | I-22 |
| 3 | I-20 | I-21 | I-20 |
| 4 | R/I-24 | R/I-25 | R/I-26 |
| 5 | I-4 | I-4 | I-5 |
| 6 | R/I-24 | R/I-25 | R/I-26 |
| 7 | I-4 | I-4 | I-4 |
| 8 | R/I-24 | R/I-25 | R/I-26 |
| 9 | R/I-24 | R/I-25 | R/I-26 |
| 10 | R/I-24 | R/I-25 | R/I-26 |

R: Immediate Replacement, I-xx: Next inspection xx time units ahead.

different scenarios. Due to the step function of the cost profiles (see table 5.8) the proposed policy may seem to be odd. At e.g. $i = 6$ ($\omega \in [50, 60 >$) the "optimal" policy is to replace, while at $i = 7$ ($\omega \in [60, 70 >$) the "optimal" policy is to continue for a further $4$ time-units. If a replacement is performed for $i = 6$, the replacement cost is $25\%$ less than if a replacement is performed for $i = 7$ (see the cost profile $\Psi(i, a - a)$).

### Two technologies ($\phi = 2$) and two deterioration processes

In addition to the deterioration process described for each technology in the previous text, a second deterioration process is incorporated, which together with the first one mentioned, fully describes the state of the item currently in use. The second deterioration process ($cp_2$) is assumed to be similar for both technologies, and the probability of a failure follows a Weibull distribution with parameters $\eta = 40$, $\iota = 0$ and $\gamma = 3.5$. The probability of a release is assumed to be similar to the previous example. The resulting replacement and inspection policy dependent on the initial condition at $n = 150$ is presented in table 5.10. Thus if $cp_1$ equals $1$ and $cp2$ equals $0$ (i.e. the item is in a new condition), the optimal decision is to inspect at $19$ time-units ahead. Because of the "end-of-horizon" effect caused by introducing a new technology, the inspection policy will have some odd steps (rapid change) for small changes in combinations of $cp_1$ and $cp_2$. Figure 5.23 presents the corresponding average cost profile. The difference between the minimum and maximum average cost for CP1 equals the average cost of one additional replacement over the horizon, i.e. USD $2000/150(13.33)$ per time-unit.

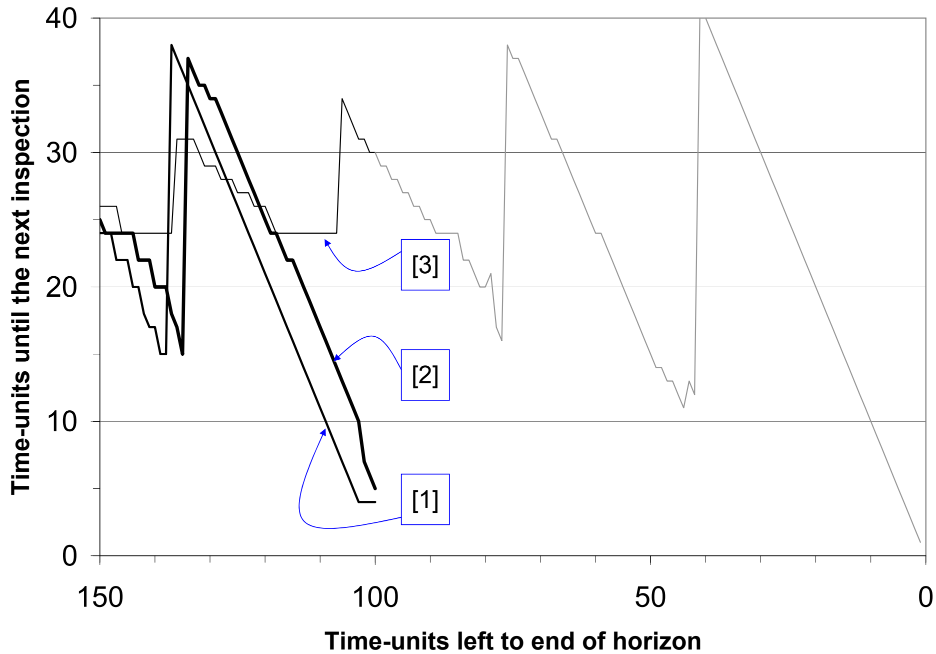


Figure 5.22: The effect of considering the release of a new technology with improved performance characteristics. The figure presents an item in new condition at the decision interval i.e. $i = 1$ ($\omega \in [0, 10 >$); Case [C]. The presented control limits are only valid for $n \in [150, 100]$. [1] Instant release of a new technology at N=99 i.e. 50 times units ahead. [2] The probability of a new technology is assumed to be normally distributed with mean 99 and standard deviation 2. [3] shows the control limit if the release of a new technology is neglected.

## 5.5 The decision process

The proposed MDP model may provide useful input to decision support for managing ageing systems and equipment. Although the MDP model has its strength in an operational phase where updated information about the ongoing deterioration processes is available, the model may also be utilised to define the optimal inspection interval dependent on observed condition and the condition boundary (control limit) requiring a replacement in an initial maintenance program. The application of the MDP model in these two different situations is discussed at the end of the chapter. The proposed steps to make use of the MDP model are:

- to determine the system boundary and define the requirements

Table 5.10: The optimal inspection and replacement policy at $n = 150$. Including two technologies each described with two condition parameters. The probability of a new technology is assumed to be normally distributed with mean 99 and standard deviation 2.

| Condition | Age (CP2) | | | | | |
|---|---|---|---|---|---|---|
| (CP1) | 0 | 4 | 8 | 12 | 16 | ≥19 |
| 1 | I-20 | I-16 | I-14 | I-5 | I-4 | R/I-20 |
| 2 | I-18 | I-17 | I-11 | R/I-20 | R/I-20 | R/I-20 |
| 3 | I-15 | I-14 | I-12 | I-4 | I-4 | R/I-20 |
| 4 | I-11 | R/I-20 | R/I-20 | R/I-20 | R/I-20 | R/I-20 |
| 5 | I-14 | I-5 | I-4 | I-4 | R/I-20 | R/I-20 |
| 6 | R/I-20 | R/I-20 | R/I-20 | R/I-20 | R/I-20 | R/I-20 |
| 7 | I-4 | I-3 | I-4 | R/I-20 | R/I-20 | R/I-20 |
| 8/9/10 | R/I-20 | R/I-20 | R/I-20 | R/I-20 | R/I-20 | R/I-20 |

R: Immediate Replacement, I-xx: Next inspection xx time units ahead.

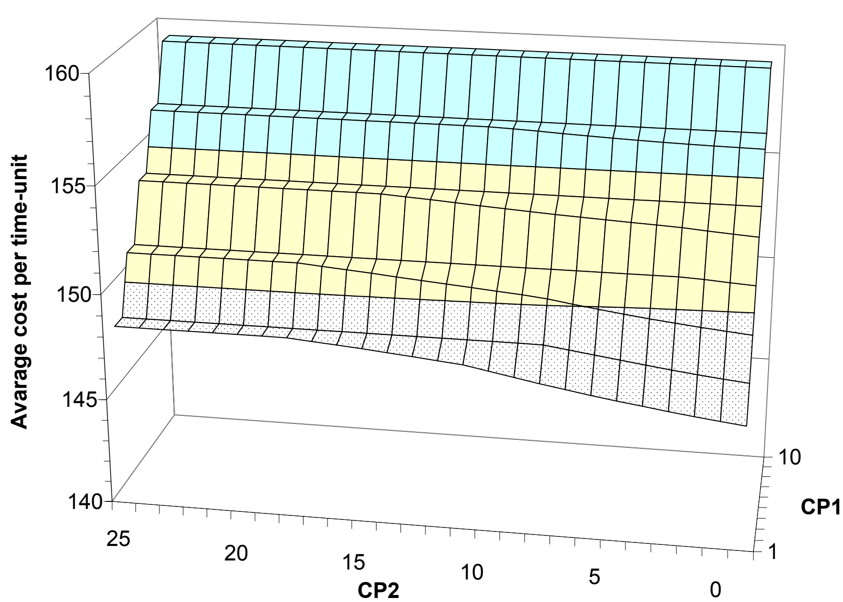


Figure 5.23: The average cost profile at $n = 150$ (time-units left to the end of horizon). The probability of a new technology is assumed to be normally distributed with mean 99 and standard deviation 2. Each technology is described by two deterioration processes. Case [C].

- to select and model the deterioration process(es) of interest
- to specify the income and cost parameters (determine the main condition levels)
- to investigate the results

In the following text, these four steps are outlined.

**Determine the system boundaries and define the requirements**

The initial preparation involves an approach to define the system requirements for the item which is going to be analysed. The main requirements to consider are:

- *Health, safety and environmental requirements*. The item should as a minimum satisfy acceptance criteria laid down by the authority and company (operator)
- *Regularity requirements*. The producing item should provide capability and availability to fulfill the required demand.
- *Cost efficient operations*. In order to contribute to maximum life cycle profit, the decision maker should consider acquisition & installation costs, operation & maintenance costs including durability of materials and technology.

The maintainability and required logistic support are obviously important aspects to be considered to achieve cost efficient operations. In addition, the decision maker must also consider the use of new or proven (standard) technology to meet the demands.

Table 5.11 presents the six model parameters that have to be specified considering the overall requirements stated above. If the horizon is assumed to be infinite, the value of $T_N$ has to be sufficiently large to reach a steady state solution (if possible).

**Select and quantify the deterioration process(es) of interest, $\xi(t)$**

When the system boundaries are defined and the requirements are determined, the next step is to identify and analyse the most significant long term deterioration process(es). Although there may exist several which are critical to the operation of the selected item, some deterioration processes are normally more important (dominating) and especially those leading to a major overhaul/replacement.

Table 5.11: General parameters.

| Parameter | Description [unit] |
|---|---|
| $T_N$ | Expected time to end of horizon [time] |
| $\Psi_{pp}(n)$ | Production profile [%] |
| $\Psi_{ps}(n)$ | Opportunity profile |
| $\overline{C_{pen}}$ | Cost of an inspection [USD] |
| $p_\phi(n)$ | The probability that techn. $\phi$ is available at $n$ |
| $\beta$ | The discount rate (dependent on the length of each stage) |

For the deterioration processes included, the first step is to establish a mathematical description of the deterioration as a function of time ($\xi(t)$). The mathematical description of the deterioration process consists of a drift function $g(t)$ with an uncertainty $\epsilon(t)$ (see eq. 5.16). In this context, the drift function $g(t)$ is defined to be $0$ at $t = 0$ and $100$ at $t = MTTF$ [9]. The condition monitoring data must therefore be transformed to fit within this scheme. Chapter 6 presents some procedures to establish a drift function.

When the mathematical description of the deterioration process has been determined, the next step is to establish the CTPM. In the process to calculate the CTPM values, the decision maker has to select both the number of condition levels $M$, and the length of each stage $\Delta t$. In addition to these two parameters, the decision maker also has to select the probability of detecting a failure without an inspection $q_1$, and the probability $q_2$ of a transition between the two fault conditions if $q_1 < 1$.

To include the knowledge of possible future technology(ies), experts have to make assumptions on the schedule of release(s) and degree of improvement compared with existing technology. There may be improvements in several areas such as capacity, efficiency and durability. If the new technology is assumed to be more resistant to the specific deterioration processes of interest, the mathematical description of the deterioration process $\xi(t)$ may have to be modified.

### Specify the income and cost parameters

Each income/cost value is modelled by a combination of a reference parameter and a profile function. The reference parameters are defined as the income/cost values obtained by operating the item of present technology just after a replacement, except for the overhaul and replacement parameter. The latter parameter is based on the predicted cost of performing an overhaul replacement action for an item which

[9] If a failure is critical with respect to safety and/or environment, the condition at which an item is said to have failed is stated by these requirements

Table 5.12: Reference parameters

| Parameter | Description [unit] |
|---|---|
| $\overline{I}$ | Income [USD/stage] |
| $\overline{C_{pen}}$ | Penalty cost [USD/$\%$ deviation] |
| $\overline{C_E}$ | Energy cost [USD/stage] |
| $\overline{C_{PM}}$ | Cost of the day-to-day maintenance [USD/stage] |
| $\overline{C_{OH}}$ | Cost of a planned overhaul action [USD] |
| $\overline{S}$ | Salvage value [USD] |
| $\overline{C_I}$ | Cost of an inspection [USD] |
| $MDT_{OH}$ | MDT due to a planned overhaul [stage] |
| $MDT_{UCM}$ | MDT due to an unforeseen corrective maint. action [stage] |
| $MDT_I$ | MDT due to an inspection |

has failed. For each reference parameter the corresponding profile function ($\Psi_{(\cdot)}(\cdot)$) describes the relationship between the condition of the item and the actual income/-cost value for that specific condition. Table 5.12 presents all the reference parameters used in the MDP model.

The first step is to select and determine the values of income and cost parameters of interest. When the parameters have been determined, the next step is to select an appropriate number of main condition levels. The number of main condition levels must be equal to or less than $M + 1$ and should be kept at a minimum to limit the necessary work to specify the profile functions. The choice will depend on the ability and benefit achieved by increasing or decreasing the number of main levels. Usually a number of 10 to 20 levels will be sufficient. The second step is to assign a value to each profile function for each defined level. The value reflects the deviation between the reference value and the actual income/cost value at that specific state. The process is repeated for all technologies and state variables of interest.

**Investigate the results**

Sensitivity analysis should always be conducted to reveal the robustness of the recommended policy. Sensitivity analysis can easily demonstrate the effect of change in e.g. length of horizon, discount rate and other parameters which are estimated or have been strictly judged by experts.

**Remarks**

The outlined MDP model is developed to provide decision support for managing ageing systems and equipment with respect to the inspection, overhaul and replacement strategy, where the effect of e.g. obsolescence and fixed horizon may be significant and therefore should be included. However, the MDP model may also be used to establish an initial maintenance policy, as well as to update the policy in an operational phase based on results from the condition assessments. In the design phase, when the initial maintenance policies have to be established, it is normal to assume an infinite horizon as a basis for optimization of the maintenance intervals. Under this assumption, the maintenance intervals may be determined by use of infinite horizon models. However, in some cases the assumption of an infinite horizon may be incorrect. This may be the case if the speed of deterioration is low and the horizon to operate is bounded and short compared to the normal operation time of the asset between two subsequent overhaul/replacements, which implies few replacements over the systems life-cycle. Each replacement will then have to be considered with respect to the time left to operate to secure a maximum life-cycle profit. Although the number of replacements may be low, the assumption of an infinite horizon may still hold. As the discount rate decreases below 1.0, the effect of future income/costs diminishes, and the consequences of a finite horizon are less important. A similar effect is observed if the variance $b$ of the deterioration process increases (see sec. 5.4.1). Because the item has to be overhauled at an earlier stage when the uncertainty of the deterioration process increases, the number of replacements also increases, which diminishes the effect of a fixed horizon.

Under the assumption of an infinite horizon and constant external parameters such as discount rate, production demand, the “optimal” policy will constitute a set of “steady state” control limits, regarding both time-units until the next inspection and the state (condition level) to perform a replacement.

In the operational phase, both physical impairment and obsolescence will effect the “optimal” policy. If the horizon is fixed and the time left to operate is decreasing, the end of horizon effect may be significant. Operational experience would also effect the “optimal decision”. Through operation it is possible to observe the actual speed of deterioration and the effect of the deterioration on the income and cost values. This information is useful and important to include in seeking the “optimal” policy to follow until the next decision interval. If obsolescence is the main cause of a reduced profit, a modification may be required. To evaluate a future modification, the decision maker will have to consider the possible improvements achieved by use of a new technology and make predictions or assumptions about the features of that specific technology. The MDP model may then suit as a decision support tool to estimate the benefits and the ”optimal” policy in the period until a modification is possible.

# Chapter 6

# Deterioration processes and maintenance cost functions

The decision process outlined in section 5.5 is based on information that has to be gathered from management systems, condition monitoring systems and/or expert judgements. In this chapter methods of estimation and forecasting of model parameters are presented. The selection of appropriate methods is limited to those which fit the typical decision process and sources of information. The techniques outlined are based on regression analysis, exponential and direct smoothing, and Bayesian methods in forecasting. They differ by the importance they give to the data and their complexity. Forecast based on time series analysis is based on a 3-step procedure — (1) select the model (increasing/decreasing/seasonal shape); (2) parameterize the model; (3) make a forecast and estimate confidence. The use of time series to represent a deterioration process has a major drawback. That is, the model does not contain knowledge about the ongoing physical failure process, it only represents the symptoms of the process that are measurable. Expert judgement may however compensate for the lack of knowledge in the model. If the regressor variables in the model are simple mathematical functions of time, techniques such as exponential smoothing and direct smoothing can be applied to update the model and forecast future values. These techniques are described in section 6.2.3 and 6.2.4. First, the nature of the decision problem is briefly discussed.

## 6.1 Features of the decision process

The basic idea of the proposed model is to utilize condition information to improve the maintenance function. The proposed MDP model presented in chapter 5 requires the decision maker to perform an inspection at proposed time intervals, evaluate the inspection results and modify the model parameters if necessary. If the model parameters have changed significantly, new calculations have to be performed to select the next *optimal* inspection time.

In order to find the model or value(s) that describe each distinct parameter, the characteristics of the input parameter have to be evaluated. The following parameters have to be treated in the light of the specific decision problem:

- Economic figures (Production profile, discount rate, energy price, man-hour and spare part costs)
- Deterioration processes $(\xi(t))$
- Technology development (The time of release and operational cost savings utilizing the new technology)

The quantity, accuracy, and timeliness of historical data are important in selecting the appropriate methods to estimate and forecast the model parameters. The nature of the decision process also plays an important part in this respect. Usually the number of observations is limited, they may be objective but often they are subjective, and they will in general be updated on a regular basis (determined by the decision process, the estimated deterioration rate and the uncertainty in the observations). Prior modelling processes, using information from similar applications to cover for lack of data, are considered to be important. If physical models of similar deterioration processes exist they should form a basis for the applied models and this topic is discussed in section 6.1.2. Typically the decision maker must handle a decision process which may have:

- Few observations available
- Prior model known from similar applications
- Information updated on a regular basis from new observations
- Subjective information often in combination with objective measurements
- Physical models of the deterioration processes

### 6.1.1 Economic figures

The economic figures may strongly depend on external conditions as well as internal requirements. The income is dependent on the price of the product produced by the equipment, system or plant of interest. The same applies to maintenance costs, energy costs and the cost of reduced availability (lost or deferred production). The discount rate is set based on the company policy and market conditions. As mentioned in section 2.5.2, two types of data are needed for economic evaluation — general economic data and specific asset data.

There are different methods for establishing the cost figures such as trend analysis to determine previous and predict future changes, or influence analysis to determine market situation. The methods of trend analysis and forecasting that are discussed in section 6.2 can be useful to estimate income and cost figures. The specific income and cost functions that are needed in the proposed model are outlined in section 5.2.3.

### 6.1.2 Deterioration processes

A major challenge in describing the deterioration process (physical impairment) is the low number of observations available. For a new specific item, the necessary information may not exist, while for an item that has been operating for some time there may be both qualitative and quantitative information available. The former may require a comparison with a similar type of item, either from internal company sources or from various external sources. A drawback however is that the influence of external environmental factors and previous preventive maintenance tasks, usually has a significant impact on those few data that are available. This applies also to those items for which specific operating conditions and maintenance records exist. Basically, the following qualitative and quantitative sources of information can be available for an item [9]:

- historical information related to operation and maintenance of the same type of item, either from internal company sources or from various external sources such as international standards, databanks, suppliers or other companies (having experience with the same type of item).

- item specific information collected during operation and maintenance of the item up until today, such as process parameters, technical condition parameters, minor repairs and follow ups, design changes, operating conditions, operation patterns and operating environments.

- current information such as the existence of potential failure types and their severity, that is, their consequences with regard to operation, cost, safety and environmental hazards.

- information about the future, such as expected operations strategies and influences from operation and environment.

There is a comprehensive number of papers concerning the assignment of lifetime distribution to complete and censored failure data [16]. When it comes to use of knowledge about the failure mechanisms, fewer papers exist.

A common method for generating deterioration models based on condition monitoring results is forecasting using historical records from time series and use of failure mechanism models. A physical understanding of the deterioration process is of great importance in selecting an appropriate time series model. Previous history can be used to suggest the appropriate form to be regressed. Section 6.2 discusses the use of regression analysis to determine such models.

Deterioration processes may be described by several statistical methods. In Rausand and Reinertsen [82], the use of stochastic models of life span for non-reparable items under the influence of dominant failure mechanisms like fatigue, corrosion and wear are treated. The authors concluded that the model selection must be based on a thorough knowledge of actual failure mechanisms and the associated time dependent deterioration. The methods discussed in this section are based on establishing time series models based on available condition monitoring information. A brief overview of typical models is given. Fitting models to the observed symptoms data is also discussed.

Hontelez, Burger and Wijnmalen [78] have described several deterioration processes for civil structures as a diffusion process with drift function $g(t)$. They give some examples of different models such as:

- Corrosion of steel not sufficiently protected by concrete or preservation, can be described as a linear process $g(t) = At, (A > 0, t > 0)$.
- Carbonation is a chemical deterioration process working on concrete by diffusion of $CO_2$. The thickness of the coat which is affected can be described by $g(t) = A\sqrt{t}, (A > 0, t > 0)$.
- Crack development is a deterioration process affecting steel and some times concrete. The size of the crack caused by wear can be described by $g(t) = (1/2[Ct + a_0])^2, (t > 0)$.
- Shrinkage occurs in structures made of concrete. It causes the distortions which increase as time goes on and can be described as $g(t) = A(1 - e^{B(t-t_0)}) +$

Table 6.1: Failure mechanisms and life distribution[82].

| | Life distribution | | | | |
|---|---|---|---|---|---|
| Mechanism | Weibull | Lognormal | Inverse Gauissian | Birnbaum-Saunders | Gumbel |
| Fatigue (cyclic) | | • | • | | |
| Fatigue (cum.) | | | • | • | |
| Corrosion (pitting) | • | | | | • |
| Wear | | | • | | |

$$C(t > t_0 > 0, g(t) = 0 | t \leq t_0).$$

Samdal [83] has modelled the deterioration of a pipeline located in the water injection system of an offshore installation as $g(t) = A + Bt, (t > 0)$. The uncertainty in the model estimates are described as $\sigma^2 = C + Dt^2$. Expressions are given for both pitting-corrosion, and uniform-corrosion, which were determined to be the main deterioration mechanisms. He also proposes a similar model for a charge pipe system in use at a plant producing FeSi. The general expression of the deterioration process models is similar to the expression used for the pipeline. The models cover uniform corrosion and abrasive wear of a charge pipe.

Kobbacy [84] presents a system which incorporates decision support for selection of a suitable model for preventive maintenance scheduling. By analysing PM records, the Hybrid Intelligent Maintenance Optimization System (HIMOS) is stated to be capable of assigning an appropriate mathematical model to more than 47% of the data current analysed.

Rausand and Reinertsen [82] have proposed a preliminary guidance for selection of life distributions for four different failure mechanisms. These failure mechanisms with corresponding distributions are presented in table 6.1. They emphasize that the effect of several failure mechanisms may be much higher than the sum of the effects of each individual mechanism. These synergy effects are difficult, if not impossible to model using the knowledge available today.

### 6.1.3 Technology development

Technological advances take place all the time - driven by need. The current trends in products lead to changes in the design environment such as a) larger, more integrated systems, b) increased complexity, c) higher levels of competition, and d) increased legal requirements.

Although the field of technology forecasting has existed for nearly 50 years, rarely has it been institutionalized in the strategic technology planning efforts of technology intensive organizations. Aside from a number of governmental initiatives, technology forecasting techniques have been studied by relatively few. A study performed by Technology/Engineering Management, Inc. [85] has revealed that most technology forecasting, where it has been done, has primarily relied on expert opinion or other judgmental approaches rather than trend analyses or other data driven techniques. The five most applied approaches were (sorted from top to bottom):

- Expert opinion
- Scenarios
- Patent analysis
- Delphi Techniques
- Technology trend analysis

Expert opinion was clearly the most widely applied approach in spite of known problems with the bias of individual experts.

The predominant time horizon was 2 to 5 years (61%). This was followed distantly by less than 2 years (18%), greater than 10 years (12%), and 5 to 10 years (9%). Thus, most of the companies have relatively short time horizon.

There are several possible tradeoffs in evaluation of technology in a life-cycle perspective. The importance of each of the areas will depend on the defined requirements. Figure 6.1 presents an example of a trade-off relationship diagram.

## 6.2 Forecasting and regression analysis

Regression analysis is a statistical technique for modelling and investigating the relationship between two or more variables. Regression models are used for several purposes, including the following: 1) Data description, 2) Parameter estimation, 3) Prediction and estimation and 4) Control [87]. A function model representing the deterioration process $\xi$ can be determined by applying regression analysis if the history of the speed of deterioration over a long period is known. The length of the period should be at least the same as the length of the period to be forecasted. If the forecast is based on the time-order sequence of the measured deterioration only, time series models can be applied. Forecasting methods that are based on *time series models* are called trending or extrapolation models. The other type of statistical forecasting models is the *causal models*, where the deterioration rate is explained by some external

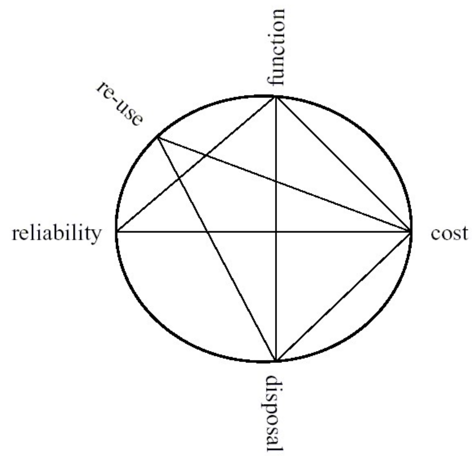


Figure 6.1: Trade-off relationships [86]

factors e.g. maintenance effort, load demand. Such forecast methods are also called multivariate methods. Both these types of models apply the regression analysis.

In addition to the the quantitative methods, where the logic is clearly stated and the operations are mathematical, qualitative procedures are often applied. The qualitative procedures involve subjective estimates through opinion of experts and they usually make use of formal procedures for obtaining predictions in this manner such as the Delphi method [88]. The methods of forecasting may thus be divided into two main categories depending upon the extent to which mathematical and statistical methods are used [89]— the *quantitative* methods and the *qualitative* methods.

The selection of appropriate forecasting methods is influenced by the following factors [89]:

- Form of forecast required
- Forecast horizon, period, and interval
- Data availability
- Accuracy required
- Behaviour of process being forecast (demand pattern)
- Cost of development, installation, and operation
- Ease of operation
- Management comprehension and cooperation

Usually the forecast will take one of the following forms: (1) an estimate of the expected value of the variable, plus an estimate of the standard deviation of forecast error, or (2) an interval that has a stated probability of containing the actual future value. The latter is called a prediction interval.

### 6.2.1 Regression analysis

Regression analysis is a widely used statistical technique for investigating and modelling the relationship between variables. The general form of a linear regression model is:

$$y = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_k x_k + \epsilon \tag{6.1}$$

where $y$ is the response variable, $x_1, \ldots, x_k$ predictor variables , $b_0, \ldots, b_k$ parameters of the model and $\epsilon$ the random error. The estimates for the parameters $b_0, \ldots, b_k$ are denoted by $\hat{b}_0, \ldots, \hat{b}_k$. They can be determined by the method of least squares. The resulting regression equation gives the predicted values for $y$ denoted by $\hat{y}$:

$$\hat{y} = \hat{b}_0 + \hat{b}_1 x_1 + \hat{b}_2 x_2 + \ldots + \hat{b}_k x_k + \epsilon \tag{6.2}$$

Considering a deterioration process, the response variable is the degree of deterioration at time $\xi(t)$. If the time $t$ is the only predictor variable the linear regression equation can be given in the form:

$$\xi(t) = \hat{b}_0 + \hat{b}_1 t + \epsilon_t \tag{6.3}$$

However, the speed of deterioration is seldom linear, and thus various non-linear models are generally preferred. As an example, a polynomial fit of order 3 is given below:

$$\xi(t) = \hat{b}_0 + \hat{b}_1 t + \hat{b}_2 t^2 + \hat{b}_3 t^3 + \epsilon_t \tag{6.4}$$

By linearization the parameters of the model can easily be estimated using standard methods of regression analysis. Such linearization can also be applied to other functional forms to estimate the model parameters.

However, the polynomial may give poor results outside the range of historical data, and use of such models should be handled with care.

In order to determine the prediction intervals for the forecast, the estimates for the variance of the random error have to be calculated:

$$\hat{\sigma}_\epsilon^2 = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-2} \tag{6.5}$$

Several deterioration processes can be described by a deterioration function $g(t)$. The TCI values, as described in chapter 2.3, may also be modelled using the methods within time-series analysis as described in the following text.

### 6.2.2 The Wiener Process

The Wiener process has some excellent features which make it very suitable for describing an item whose physical condition can be measured/observed as it deteriorates.

Let the deterioration function g(t) represent the deterministic decay of the items condition. The deterioration process will normally be influenced by several external conditions. To be able to describe the random nature of our process we have to add a stochastic part to the deterioration model, thus giving us the following general model of a deterioration process, $\xi(t)$, at time $t$ as:

$$\xi(t) = g(t) + b \cdot U\sqrt{t}, \qquad U \sim N(0,1) \tag{6.6}$$

where N(0,1) stands for the standard normal distribution. The stochastic part of the Wiener process is thus described by $b \cdot \sqrt{t}$, $t \geq 0$, where $b \geq 0$ is a parameter given by the standard normal distribution.

In situations where the independent variables are simple mathematical functions of time, very efficient estimation and forecasting techniques can be derived. Direct smoothing presented in section 6.2.4 is such a method.

### 6.2.3 Exponential smoothing methods

Exponential smoothing is one of the most widely used class procedures for smoothing discrete time series in order to forecast the immediate future. The main reason can be attributed to its simplicity, its computational efficiency, the ease of adjusting its responsiveness to changes in the process to be forecast, and its reasonable accuracy [89].

Consider a simple exponential smoothing for a constant process ($y_t = b_0 + \epsilon_t$), where $b_0$ is the expected demand in any period and $\epsilon_t$ is a random component having mean 0 and variance $\sigma^2$. To simplify the expression, the smoothed value $\hat{b}_0(T)$ is written as $B_T$ ($B_T \equiv \hat{b}_0(T)$). The new estimate for the expected value using simple exponential smoothing is then [89]:

$$B_T = \alpha x_T + (1 - \alpha)B_{T-1} \tag{6.7}$$

The fraction $\alpha$ is called the *smoothing constant*. The forecast ($\hat{x}_{T+t}(T)$) of a value in any future period $T + t$ for a constant process is simply $B_T$. Selecting an appropriate smoothing constant is generally performed by carrying out a sequence of trials on a set of actual historical data using different values of $\alpha$. The selection of an "optimal" smoothing constant can be performed by comparing e.g. minimum sum of squared errors. The smaller the value of $\alpha$, the slower the response. Larger values of $\alpha$ cause the smoothed values to be more sensitive to the latest measurements, which incorporate both real changes and random fluctuations. It can be shown that for a constant demand process and simple exponential smoothing, the forecast errors can be expressed as[89]:

$$\sigma_e^2 = \frac{2}{2-\alpha}\sigma_\epsilon^2 \tag{6.8}$$

where the standard deviation $\sigma_e$ for a constant process is found by computing the sample standard deviation of a sequence of actual demand realizations.

For a process where the values increase or decrease linearly with time, double exponential smoothing for a linear trend process can be applied. The forecast equation is expressed as [89]:

$$\hat{x}_{T+t}(T) = \left(2 + \frac{\alpha \cdot t}{1-\alpha}\right) B_T - \left(1 + \frac{\alpha \cdot t}{1-\alpha}\right) B_T^{[2]} \tag{6.9}$$

where $B_T^{[2]}$ implies double exponential smoothing, and equals:

$$B_T^{[2]} = \alpha B_T + (1-\alpha) B_{T-1}^{[2]} \tag{6.10}$$

Exponential smoothing can be used to estimate coefficients in polynomial models of any degree. For a quadratic model, a procedure called triple exponential smoothing can be applied. It is recommended that the smoothing constant, $\alpha_1$ for a constant process (single smoothing) should be between $0.01$ and $0.3$. The equivalent value of the smoothing constant for a model of $k$ parameters to be estimated, $\alpha_k$, would be [89]:

$$\alpha_k = (1-\alpha_1)^{1-k}$$

*Example*

To exemplify the applicability of double exponential smoothing, a fictitious case is given. Assume that a pipe inspection is performed each third month. A new pipe has a wall thickness of $792 \cdot 10^{-2}$ mm. After five year in operation the observed data is as presented in table 6.2. Using simple linear regression methods the initial deterioration model is (all wall thickness values given as $10^{-2}mm$):

$$g(t) = \hat{x}_t = 794.47 - 7.13t$$

Letting $\alpha = 0.1$, and using equation 6.8-6.10, the forecast for the subsequent periods can be calculated. Smoothing the results sequentially for the 20 periods of historical

Table 6.2: Observed wall thickness at each 3 month inspection over 20 periods (5 years). All measurements are given as $10^{-2}$ mm.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (1-10) | 773 | 797 | 765 | 758 | 783 | 752 | 785 | 669 | 727 | 716 |
| (11-20) | 710 | 696 | 688 | 675 | 643 | 709 | 656 | 657 | 613 | 649 |

data yields $B_{20} = 705.35$ and $B_{20}^{[2]} = 774.85$. The forecast for period $21$ given an observation at period $20$ is then:

$$\hat{x}_{20+1}(T = 20) = \left(2 + \frac{0.1 \cdot 1}{1 - 0.1}\right) 705.35 - \left(1 + \frac{0.1 \cdot 1}{1 - 0.1}\right) 774.85 \approx 628$$

After the inspection in period $21$ the wall thickness is measured to be $611 \cdot 10^{-2}$ mm. Figure 6.2 presents the one-period ahead forecast from period $21$ (i.e. the forecast includes the observation in the previous period).

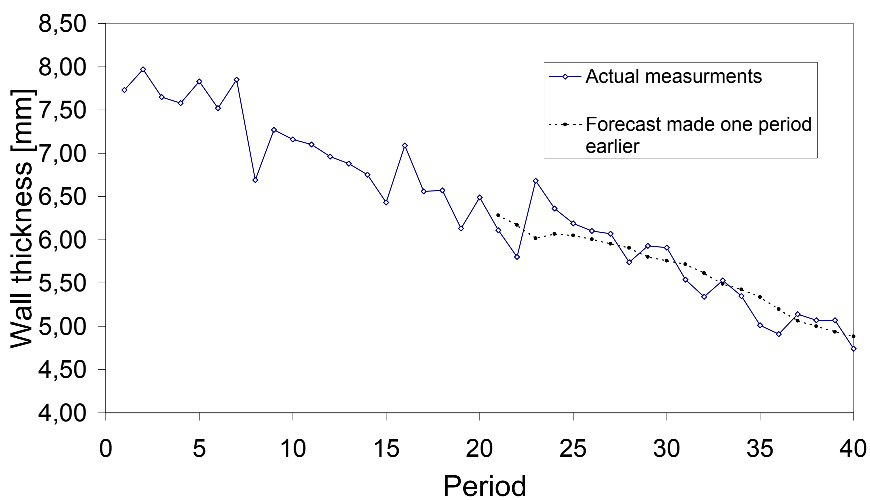


Figure 6.2: Pipe thickness measurements and one-period-ahead forecast using double exponential smoothing.

### 6.2.4 Direct smoothing

Direct smoothing (or adaptive smoothing) is a technique which can be applied if the regressor variables of the model are simple mathematical functions of time. The method smoothes the old coefficients with the current period's forecast error to obtain new coefficients. When the regressor variables are polynomial functions of time, direct smoothing is equivalent to exponential smoothing [89]. Direct smoothing applies the techniques of discounted least squares. It is assumed that the time series $\{x_t\}$ can be represented by the general model:

$$x_t = \sum_{i=1}^{k} b_i z_i(t) + \epsilon_t \qquad t = 1, 2, \ldots, T$$

where $b_i$ is the coefficient of the $i$th term in the model and the regressor variables $z_i(t), i = 1, 2, \ldots, k$, are appropriate mathematical functions of $t$. The $\{\epsilon_t\}$ are random errors such that $E(\epsilon_t) = 0, V(\epsilon_t) = \sigma_\epsilon^2$, and $E(\epsilon_t \cdot \epsilon_{t+u}) = 0$. Let $\hat{x}_T(T-1)$ be the forecast for period $T$, made in the end of period $T-1$. The forecast for any future time period $T+t$, of a linear trend model ($x_t = b_1 + b_2 t + \epsilon_t$) is determined by [89]:

$$\hat{x}_{T+t}(T) = \sum_{i=1}^{k} \hat{a}_i(T) z_i(t) \tag{6.11}$$

where the estimates of the model parameters $\hat{a}_i$ are found using the following equations:

$$\begin{aligned} &\hat{a}_1(T) = \hat{a}_1(T-1) + \hat{a}_2(T-1) + (1-\gamma^{*2}) e_1(T) \\ &\hat{a}_2(T) = \hat{a}_2(T-1) + (1-\gamma)^{*2} e_1(T) \\ &e_1(T) = x_T - \hat{x}_T(T-1) \end{aligned}$$

$\gamma^*$ is called the discount factor, and chosen so that $0 < \gamma^* < 1$. Direct smoothing of polynomial model of degree $k$ is equivalent to multiple smoothing of order $k+1$ with $\alpha = 1 - \gamma^*$. It is possible to represent seasonal time series with use of trigonometric functions, such as sine and cosine. Several examples are found in the literature [89].

*Example*

Recall the example of the previous section concerning the deterioration of a pipeline. By direct smoothing, the new estimated parameters and the forecast are calculated using equation 6.11. The discount factor is $\beta = 0.90$, and the results are presented in table 6.3. The revised parameters are shown in columns $3$ and $5$.

### 6.2.5 Forecasting errors

A time series model is of the general form:

$$x_t = g(t) + \epsilon_t$$

Table 6.3: Comparison of direct smoothing with double exponential smoothing

| Period T | $x_T$ | $\hat{a}_1(T)$ | $\hat{a}_2(T)$ | $\hat{x}_T(T-1)$, by direct smoothing | $S_T$ | $S_T^{[2]}$ | $\hat{x}_T(T-1)$, by double smoothing |
|---|---|---|---|---|---|---|---|
| 0 | | 794.47 | -7.13 | | 858.30 | 922.81 | |
| 1 | 773 | 784.62 | -7.27 | 787.34 | 849.77 | 915.51 | 786.62 |
| 2 | 797 | 781.08 | -7.08 | 777.34 | 844.49 | 908.40 | 776.73 |
| 3 | 765 | 772.29 | -7.17 | 774.00 | 836.54 | 901.22 | 773.48 |
| 4 | 758 | 763.77 | -7.24 | 765.12 | 828.69 | 893.97 | 764.68 |
| 5 | 783 | 761.56 | -6.97 | 756.53 | 824.12 | 886.98 | 756.16 |
| 6 | 752 | 754.10 | -7.00 | 754.59 | 816.91 | 879.97 | 754.28 |
| 7 | 785 | 754.30 | -6.62 | 747.10 | 813.72 | 873.35 | 746.84 |
| 8 | 669 | 732.73 | -7.41 | 747.68 | 799.25 | 865.94 | 747.46 |
| 9 | 727 | 725.64 | -7.39 | 725.32 | 792.02 | 858.55 | 725.14 |
| 10 | 716 | 717.82 | -7.41 | 718.25 | 784.42 | 851.13 | 718.10 |

where the expected value of $x_t$ is $g(t)$ and $\epsilon_t$ is a random component with mean $0$ and variance $\sigma_\epsilon^2$.

Let $e_{T+t}(T)$ be the $t$-period-ahead forecast error defined as [89]:

$$e_{T+t}(T) = x_{T+t} - \hat{x}_{T+t}(T) \tag{6.12}$$

where $T$ is the point in time when the forecast was made. The forecast error is the difference between two random variables, the demand process in period $T$ and the forecast made $t$ periods earlier. The forecast error variance is thus the sum of the variance of the measured process, $Var(x_{T+t})$, and the variance of the forecast:

$$Var[e_{T+t}(T)] = Var(x_{T+t}) + Var[\hat{x}_{T+t}(T)] \tag{6.13}$$

where $Var(x_{T+t}) = \sigma_\epsilon^2$. $Var[\hat{x}_{T+t}(T)]$ expresses the variance of $t$-period ahead forecast made at time $T$.

The variance of the forecast is a function of the variance and the covariance describing the uncertainty in using estimates of the model parameters in the forecasting procedure. The variance of error in the forecasting depends on the selected forecasting method, and must therefore be determined for each specific case. Table 6.4 presents the expressions for the variance of error in the forecasting for period for period $T+t$ at the end of period $T$, for two different cases.

Table 6.4: Two examples of the variance of error in forecasting of linear trend models [89].

| Forecasting methods | $Var[\hat{x}_{T+t}(T)]$ |
|---|---|
| Convent. least-squares analysis | $\frac{2}{T(T^2-1)}[(2T-1)(T-1)+6t(T+t-1)]\sigma_\epsilon^2$ |
| Direct smoothing | $\frac{\alpha}{(1+\beta)^3}[(1+4\beta+5\beta^2)+2\alpha(1+3\beta)t+2\alpha^2t^2]\sigma_\epsilon^2$ |

**Estimation of the variance of the observation $\sigma_\epsilon^2$**

A usual method to estimate the variance, is first to calculate the *mean absolute deviation* at time $T$, $\hat{\Delta}(T)$. To estimate $\hat{\Delta}(T)$, the following statistics may be used:

$$\hat{\Delta}(T) = E[|e - E(e)|] = \frac{\sum_{t=T-N+1}^{T}[e_1(t) - \bar{e}_1(T)]}{N} \tag{6.14}$$

where $\bar{e}_1(T)$ is the average of the last $N$ errors and is defined by:

$$\bar{e}_1(T) = \frac{1}{N}\sum_{t=T-N+1}^{T} e_1(t)$$

Another method to estimate $\hat{\Delta}(T)$ is to apply the exponential smoothing concept. $\hat{\Delta}(T)$ is then estimated using:

$$\hat{\Delta}(T) = \alpha|e_1(T) - Q(T)| + (1-\alpha)\hat{\Delta}(T-1) \tag{6.15}$$

where $0 < \alpha < 1$ and the smoothed error, $Q(T)$, is given by the following expression:

$$Q(T) = \alpha e_1(T) + (1-\alpha)Q(T-1)$$

If the forecast error is normally distributed, the estimate of variance is given as [89]:

$$\hat{\sigma}_e^2 = \sqrt{\frac{\Pi}{2}}\hat{\Delta}(T) \approx 1.25\hat{\Delta}(T) \tag{6.16}$$

For simple exponential smoothing, equation 6.8 presents the relation between the variance of the observation, $\sigma_\epsilon^2$, and $\sigma_e^2$. Similar expressions of relationship between $\sigma_\epsilon^2$ and $\sigma_e^2$ can be developed for other types of forecasting procedures. For direct smoothing procedures, the expressions will in general be of the form $\sigma_e^2 = c(\gamma^*, t) \cdot \sigma_\epsilon^2$, where $c(\gamma^*, t)$, can be calculated for different discount values and forecasting horizons.

**Prediction interval**

If the forecast is unbiased and the forecast errors are normally distributed, the $100(1-\gamma^*)$ percent prediction interval for $x_{T+t}$ is expressed as:

$$x_{T+t} = \hat{x}_{T+t}(T) \pm u_{\gamma^*/2,\nu} \cdot \sigma_{e_t} \tag{6.17}$$

where $u_{\gamma^*/2,\nu}$ represents the *t*-distribution with $\nu$ degrees of freedom.

### 6.2.6 Other forecasting models

The different smoothing techniques presented in previous sections are all derived from the least-square criterion. Several other methods exist such as the Winter's method (triple exponential smoothing) [89]. The major drawback of these methods is that they usually assume independent errors, i.e. independent observations.

A common method to represent time series is to define a linear combination of independent random variables, that are drawn from a stable probability distribution with mean $0$ and variance $\sigma_\epsilon^2$. Methods based on linear filter models define functions that transform a white noise process into a time series model. A white noise process is a sequence of uncorrelated random variables ($\epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \ldots$), each with zero mean and finite variance $\sigma_\epsilon^2 > 0$. The linear combination of the $\epsilon_i$ could be written as:

$$x_t = \mu + \psi_0\epsilon_t + \psi_1\epsilon_{t-1} + \psi_2\epsilon_{t-2} + \cdots \tag{6.18}$$

where $\psi_j (j = 0, 1, \ldots)$ are usually weights and $\mu$ is a constant that determines the level of the process. Equation 6.18 is usually called a linear filter.

Various different models exist which are based on such a time series model such as autoregressive processes, moving average processes, mixed autoregressive-moving average processes and nonstationary processes. The two latter, which are both nonstationary in the mean, can be represented by use of autoregressive integrated moving average process (abbreviated to ARIMA). However, there are two major disadvantages in applying these models, which are:

- Use of ARIMA models will require at least 50 to 100 observations. That amount of historical data would usually be unavailable, and ARIMA models will therefore be impossible to use.
- There is not a convenient way to modify or update the estimate of the model parameters when each new observation becomes available, such as there is in direct smoothing. One has periodically to refit the model completely.

In general, they are most suitable for time series where the sampling interval is very small since relatively long history can be obtained.

### 6.2.7 Bayesian methods of forecasting

The methods described in the previous sections can only be applied if historical data are available. Unfortunately, lack of historical data is often the case at the time the initial forecast is required. Thus the forecast has to be based largely on subjective information. As the time series becomes available the model may be adjusted in the light of actual data. Bayesian methods are often useful in statistical inference problems of this type.

In this section, it is assumed that the original subjective forecast can be translated into a subjective estimate of the parameters of the forecasting model. The methods outlined below are limited to parameter estimation and forecasting for time series models, that are linear in the unknown parameters $b_1, b_2, \ldots, b_k$ [1]. Suppose a deterioration process, $\xi(t)$, can be expressed as a time series process of the following general form:

$$\begin{aligned} \xi(t) = x_t &= b_1 z_1(t) + b_2 z_2(t) + \cdots + b_k z_k(t) + \epsilon_t \\ &= \textstyle\sum_{i=1}^{k} b_i z_i(t) + \epsilon_t \end{aligned} \tag{6.19}$$

where the $\{b_i\}$ are constants, the $\{z_i(t)\}$ are mathematical functions of $t$ and are the independent variables in the model, and $\epsilon_t$ is $N(0, \sigma_\epsilon^2)$. $\epsilon_t$ is assumed to be independent of $\epsilon_{t+j}$ for all $j$. It is convenient to write equation 6.19 with the following matrix notation:

$$\xi(t) = \mathbf{b}^t \mathbf{z}(t) + \epsilon_t \tag{6.20}$$

where $\mathbf{z(t)} = [\mathbf{z_1(t)}, \mathbf{z_2}, \cdots, \mathbf{z_k}]$ and $\mathbf{b} = [\mathbf{b_1}, \mathbf{b_2}, \cdots, \mathbf{b_k^t}]$. The probability distribution of $\xi(t)$, assuming the variance to be known, is:

$$f(\xi_t | \mathbf{b}, \sigma_\epsilon^{\mathbf{2}}) = \mathbf{N}(\mathbf{b^t z(t)}, \sigma_\epsilon^{\mathbf{2}}) \tag{6.21}$$

To establish a posterior distribution, the following procedure is followed:

- Select a prior distribution
- Based on observation, estimate the parameters $\mathbf{b}$
- Determine the posterior distribution

[1] The nonlinear models are omitted because they are difficult to estimate by least-square methods, and the sampling distribution in such models generally does not have a normal distribution.

**Prior distribution**

Prior to observing the time series, it is necessary to make an assumption to select the most appropriate *prior distribution*, $h_0(\mathbf{b})$. If the $\{b_i\}$ is assumed to be jointly normally distributed with $\mathrm{E}(b_i) = \bar{b}'_i$, $\mathrm{Var}(b_i) = v'_{ii}$, and $\mathrm{Cov}(b_i, b_j) = v'_{ij}$. The prior distribution of $\mathbf{b}'$ then follows a multivariate normal distribution:

$$h_0(\mathbf{b}) = (2\pi)^{-(1/2)k}|\mathbf{V}'^{-1}|^{1/2}exp\{-\frac{1}{2}[\mathbf{b} - \bar{\mathbf{b}}']^t\mathbf{V}'^{-1}[\mathbf{b} - \bar{\mathbf{b}}']\} \tag{6.22}$$

where $\bar{\mathbf{b}}' = E(\mathbf{b})$ and $\mathbf{V}'$ is the variance-covariance matrix. Further, a matrix $\mathbf{G}'$ is defined as:

$$\mathbf{G}' \equiv \sigma_\epsilon^2 \mathbf{V}'^{-1}$$

**Estimation of parameters**

Based on observations of the deterioration process, the prior distribution can be updated. Let $\mathbf{x}$ represent the observed data after $T$ periods, expressed as:

$$\mathbf{x} = [x_1, x_2, \dots, x_T]^t$$

and

$$\mathbf{Z} = \begin{bmatrix} \mathbf{z}'(1) \\ \mathbf{z}'(2) \\ \vdots \\ \mathbf{z}'(k) \end{bmatrix} = \begin{bmatrix} z_1(1) & z_2(1) & \cdots & z_k(1) \\ z_1(2) & z_2(2) & \cdots & z_k(2) \\ \cdots & \vdots & \cdots & \cdots \\ z_1(k) & z_2(k) & \cdots & z_k(k) \end{bmatrix}$$

$\mathbf{b}$ can be estimated from the actual time series by the least-squares estimators given as [89]:

$$\hat{\mathbf{b}} = \hat{\mathbf{b}}(T) = \mathbf{G}^{-1}\mathbf{Z}^t\mathbf{x} \tag{6.23}$$

which are unbiased ($\mathbf{E}(\hat{\mathbf{b}}|\mathbf{b}) = \mathbf{b}$), and have variance-covariance matrix:

$$\mathrm{Cov}(\hat{\mathbf{b}}) \equiv \mathbf{V} = \mathbf{G}^{-1}\sigma_\epsilon^2 \tag{6.24}$$

Here, $\mathbf{G} = \mathbf{Z^tZ}$. It can be shown that $\hat{\mathbf{b}}$ is sufficient to estimate $\mathbf{b}$ when $\sigma_\epsilon^2$ is known and that it follows the multivariate normal distribution [89]:

$$f(\hat{\mathbf{b}}|\mathbf{b}; \mathbf{Z}, \sigma_\epsilon^{\mathbf{2}}) = (2\pi)^{-(1/2)k}|\mathbf{V}^{-1}|exp\{\frac{1}{2}[\hat{\mathbf{b}} - \mathbf{b}]^t\mathbf{V}^{-1}[\hat{\mathbf{b}} - \mathbf{b}]\} \tag{6.25}$$

For fixed $\mathbf{Z}$ and $\sigma_\epsilon^{\mathbf{2}}$ the marginal distribution of $\hat{\mathbf{b}}$ is given by:

$$g(\hat{\mathbf{b}}) = \int_{-\infty}^{\infty} f(\hat{\mathbf{b}}|\mathbf{b})h_0(\mathbf{b})d\mathbf{b} \tag{6.26}$$

**Posterior distribution**

The posterior distribution of $\mathbf{b}$, given $\hat{\mathbf{b}}$, is computed at time $T$ as:

$$h_T(\mathbf{b}|\hat{\mathbf{b}}) = \frac{h_0(\mathbf{b})f(\hat{\mathbf{b}}|\mathbf{b})}{g(\hat{\mathbf{b}})} \tag{6.27}$$

It can be shown that the posterior distribution above equals [89]:

$$h_T(\mathbf{b}|\hat{\mathbf{b}}) = N(\bar{\mathbf{b}}'', \mathbf{V}'') \tag{6.28}$$

where the mean $\bar{\mathbf{b}}''$ and the variance-covariance matrix $\mathbf{V}''$ are determined from:

$$\mathbf{V}''^{-1} = \mathbf{V}'^{-1} + \mathbf{V}^{-1} \tag{6.29}$$

$$\bar{\mathbf{b}}'' = \mathbf{V}''(\mathbf{V}'^{-1}\bar{\mathbf{b}}' + \mathbf{V}^{-1}\hat{\mathbf{b}}) \tag{6.30}$$

*Example*

Recall the example in section 6.2.3. For a new installation, the operators may use knowledge from similar installed pipelines and environmental conditions to estimate the speed of deterioration. In the following hypothetical case, it is assumed the wall thickness at design varies between $787 \cdot 10^{-2}$mm and $797 \cdot 10^{-2}$mm. Based on similar installations the experts also estimate the speed of deterioration to be $1.5 - 3 \cdot 10^{-2}$ mm/month. The experts interpret this information to mean that a linear trend model $\xi(t) = x_t = b_1 + b_2 t + \epsilon_t$ is appropriate. Since the experts think the assumption that $\epsilon_t$ is $N(0, \sigma_\epsilon^2)$ is reasonable, a normal prior for $\mathbf{b}$ is applied. In addition, the experts decide the value for $\sigma_\epsilon^2$ to be $25^2 = 625$ based on forecasting of similar deterioration processes .

After $9$ months of operation (3 observations), the experts wish to update the deterioration process model and incorporate gained knowledge in the model. The three first observations are $\mathbf{x}^t = \{773, 797, 765\}$ $10^{-2}$mm (see table 6.2). Based on the initial assumption about the deterioration process, the forecast of the deterioration process at $T = 0$ is $\hat{x}_{0+t}(T = 0) = 792 - 6.5t + \epsilon_t$, where the $95$ percent Bayesian prediction interval for $\hat{x}_{T+t}(T)$, made at time $T = 0$, is given by $\hat{x}_{0+t}(T = 0) \pm 1.960\sqrt{2,7778 + 0,0625t^2 + 625}$. Using the information from the available inspection results after three periods, the updated posterior distribution (at $T = 3$) is established using equation 6.23-6.30. The least-square estimates of the parameters at $T = 3$ are:

$$\begin{aligned} \hat{\mathbf{b}} = \quad & \hat{\mathbf{b}}(T=3) = \mathbf{G}^{-1}\mathbf{Z}^t\mathbf{x} \\ = \quad & \begin{bmatrix} 2.333 & -1.000 \\ -1.000 & 0.500 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 773 \\ 797 \\ 765 \end{bmatrix} = \begin{bmatrix} 783.66 \\ -4.00 \end{bmatrix} \end{aligned}$$

Using equation 6.24, the corresponding sample variance-covariance matrix equals:

$$\mathrm{Cov}(\hat{\mathbf{b}}) \equiv \mathbf{V} = \mathbf{G}^{-1}\sigma_\epsilon^2 = \begin{bmatrix} 525.00 & -225.00 \\ -225.00 & 112.50 \end{bmatrix}$$

Using equation 6.29 and 6.30, the parameters of the posterior distribution are computed as:

$$\begin{aligned} \mathbf{V}''^{-1} &= \begin{bmatrix} 0.373 & 0.027 \\ 0.027 & 3.312 \end{bmatrix} \\ \bar{\mathbf{b}}'' &= \begin{bmatrix} 792.011 \\ -6.985 \end{bmatrix} \end{aligned}$$

From the results above, the updated forecast of the deterioration process in period $3+t$ is given by $\hat{x}_{3+t}(T=3) = 792.011 - 6.985t + \epsilon_t$. The 95 percent Bayesian prediction interval for $x_{3+t}(T=3)$ is thus $\hat{x}_{3+t}(T=3) \pm 1.960\sqrt{2.681 - 0.062t + 0.433t^2 + 625}$. A similar approach has been used for $\hat{x}_{16+t}(T=16)$, i.e. using measurements from the last 16 inspections. The updated forecast of the deterioration process at $16+t$ is $\hat{x}_{16+t}(T=16) = 791.949 - 7.508t + \epsilon_t$. The 95 percent Bayesian prediction interval for $\hat{x}_{16+t}(T=16)$ is calculated to be $\hat{x}_{16+t}(T=16) \pm 1.960\sqrt{2.56 - 0.348t + 0.124t^2 + 625}$.

Figure 6.3 presents the results of the forecast based on a) the initial assumption, b) Bayesian update after 3 periods and c) Bayesian update after 16 periods. It can easily be seen that the latter fits the data better than the two previous estimates.

**Comment:**

Nonlinear models are difficult to estimate by least-square methods, and the sampling distribution in such models generally does not have a normal distribution. This usually makes Bayesian methods impractical [89]. However, if revisions are to be made infrequently, the Bayesian approach may be reasonable.

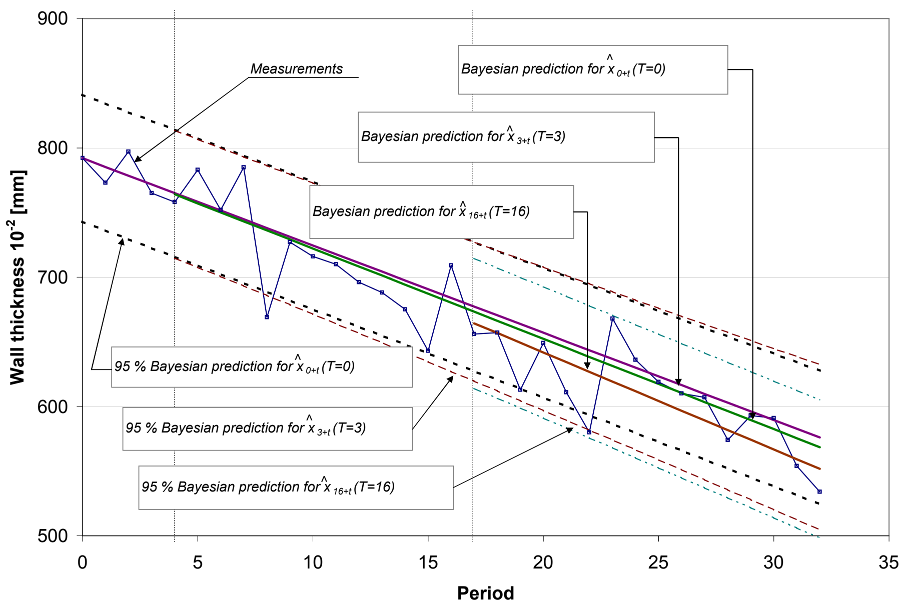


Figure 6.3: Comparison of the initial forecast at $T = 0$ and forecast based on information after $T = 3$ and $T = 16$.

# Chapter 7

# Application

The objective of this section is to present an application of the proposed MDP model. In addition to the specific result from the model itself, the example includes the decision process and the procedure to prepare the input data which are of importance for the specified maintenance decision problem.

In the Norwegian Sector of the North Sea, several offshore facilities have been producing oil and gas for over 20 years. Although new and improved technology has increased the oil recovery, several facilities are going to be shut-down in the forthcoming years as the oil reservoirs become less profitable to operate. The challenge is to maximize the profit for the remaining life-time without violating any regulatory,safety, health or environmental requirements. To maximize the profit, the maintenance effort has to be consistent with requirements of continued prudent operation until the planned sale or decommissioning of the asset.

In a short-term and medium term planning perspective, the use of an on-condition maintenance policy or condition-based maintenance has proven to be essential for cost-efficient operation. By using condition monitoring, the interval between overhaul actions may increase, and the efficiency with respect to maintenance may improve by directing repair and overhaul actions toward specific deficiencies. In a long-term planning perspective, physical impairment is not the only reason to consider a replacement or a modification. An important aspect here is to evaluate new technology with respect to efficiency, safety and environmental requirements. Hence, in addition to physical impairment, obsolescences may require a modification. Other important factors which also have to be considered are e.g. the investment costs, oil prices, discount rate and operation horizon.

The condition of equipment is normally affected by several deterioration processes. Some failures are easily repaired, while others may have severe effects and e.g. require an entire replacement of the equipment. On a system level, the latter deterioration processes may also affect the efficiency and the competitiveness of the facility if the equipment is expensive. There may therefore be totally different deterioration processes which are of interest in a short term perspective as opposed to a long term perspective.

As an example, consider large centrifugal pumps. Typical failures on centrifugal pumps are bearing, seal and wear-ring failures. In some cases both the impeller and casing may also have severe damage due to the operating condition causing e.g. corrosion or erosion. In a long-term perspective, the condition of costly components such as casing and impeller may be more important than the other typical failure modes.

## 7.1 Operation of a gas turbine

Gas turbines are the main utility equipment for electric power generation on most platforms in the Norwegian Sector of the North Sea. Because the weight is an important factor for these installations, aero gas turbine technology is often chosen. Operation and maintenance of the gas turbines is expensive, and even a slight improvement may therefore result in significant cost savings. High fuel prices and emission taxes[1] means that turbines must operate efficiently with minimum degradation. To secure a safe and cost efficient operation, the operators have to consider factors such as operating costs, machine deterioration, production deferment, maintenance cost, environmental protection, machine availability, etc. Amongst these factors, there are at least three major tradeoffs:

- The tradeoff between maintenance cost and machine performance.
- The tradeoff between the costs associated with preventive maintenance, fuel costs and emission taxes associated with operating the turbine when it is less efficient.
- The tradeoff between overhaul costs and increased turbine availability as well as reduced production deferment. The cost of overhauling the turbine is dependent on the extent and duration of the overhaul action.

To control and reduce the effect of component deterioration, on-line condition monitoring equipment systems are normally used. To cover all the most dominating dete-

[1] At the present time only for $CO_2$.

rioration processes, both periodic surveillance and minor time-based maintenance is performed offshore during operation.

Savings have been achieved through optimisation of washing procedures and inlet filter refit procedures. Both these actions are performed quite often compared to compressor and hot section major overhaul. During the compressor and hot-section life cycle, there will be a non-recoverable degradation due to thermal effects and mechanical wear. Today, the hot section repair interval (HSRI) is typically 20 000 - 25 000 running hours. The decision to overhaul is mainly based on calculated equivalent operating hours [2] and an evaluation of the thermal efficiency of the gas turbine, and the possibility to avoid production losses during the overhaul actions. A $5\%$ drop in the gas generator thermal efficiency may increase the operational cost by approximately $1.8\ [MUSD/year]$ (see sec. 7.2.2).

At an early stage in the platforms life-cycle, insufficient data is available concerning the actual deterioration process(es) for a specific turbine. The period between overhauls will therefore have to be based on the suppliers recommendations and on condition assessments to check that the turbine performance is within acceptable limits. At the end of the production life of a production platform, it is more complicated to select the proper action and time schedule. As an example the manager may have to decide whether the hot section turbine should be overhauled one or two times in the remaining production horizon.

### 7.1.1 Objective

The objective is to seek an "optimal" policy for the Hot Section Repair and Inspection (HSRI) utilizing on-line performance monitoring facilities combined with off-line inspection of internal parts. The focus is directed towards the non-recoverable deterioration processes, where the only method to recover the turbine's performance and reliability is by a replacement of deteriorated parts.

### 7.1.2 The system boundaries and requirements

The specific application of interest consists of two parallel gas turbine trains, each train capable of producing $23MW$ ($GE\ LM2500$). The gas turbines are the main source of electrical power on the oil-platform, which is producing $100\,000\ [barrels/day]$. The platform has been producing for 11 years and the residual life of the main reservoir is estimated to be 17 years (approximately 150 thousand hours). Once a year,

[2]The equivalent operating hours includes in addition to the real operating hours, the deterioration effects caused by start and stop sequences, trip signals and extreme loads.

Table 7.1: General parameters.

| Parameter | Description [unit] |
|---|---|
| $T_N = 150$ | Expected time to end of horizon [kHRS] |
| $\beta^*$=0.971 | The annual discount rate |
| $\Psi_{pp}(n)$(see fig. 7.1) | Production profile (power demand) [$\%$] |
| $\overline{C_{pen}}$ (see sec. 7.2.2) | Cost of an inspection [USD] |
| $p_\phi(n)$ (see sec. 7.3) | The probability that techn. $\phi$ is available at $n$ |

there is an total shutdown (turnaround) of the main systems for major overhaul/modification purposes. The elapsed time during a planned shutdown is $14$ days. If a shutdown (planned/unplanned) of a single train should occur, the production capacity is reduced by $40\%$ to 60 000 $[barrels/day]$. During the turnaround period, only a single gas generator unit is required and the other unit may be down for a major repair/replacement without causing any losses.

The demand for electric power is assumed to be constant over the entire life-cycle. Although a significant drop in the oil production is to be expected at the end of horizon, the demand for electric power is assumed to be constant. The need for power to keep the utility systems running such as the water injection system will increase at the end of the horizon.

The gas turbine parameters are monitored by a turbine thermal analysis system. A platform data acquisition system provides data (on-line logging) and selection of representative readings. The data are stored in a central data base on-shore. The sampling frequency is set to a 6 hours interval. The system is also capable of monitoring the vibration and number of start, stop and trip signals. In addition to on-line performance monitoring, scheduled inspection of internal parts has to be carried out, to determine the actual state of the turbine.

The decision to overhaul the turbine is based on the information obtained by the on-line monitoring and on an *à priori* model of the deterioration process. Because the cost of production deferment is significant, the turbine overhaul, if necessary, normally takes place during the yearly shutdown.

The yearly interest rate is set to $6\%$ while the inflation is $3\%$, which gives a discount rate of $0.971$ per year. The oil-price is set to $60$ $[USD/barrel]$.

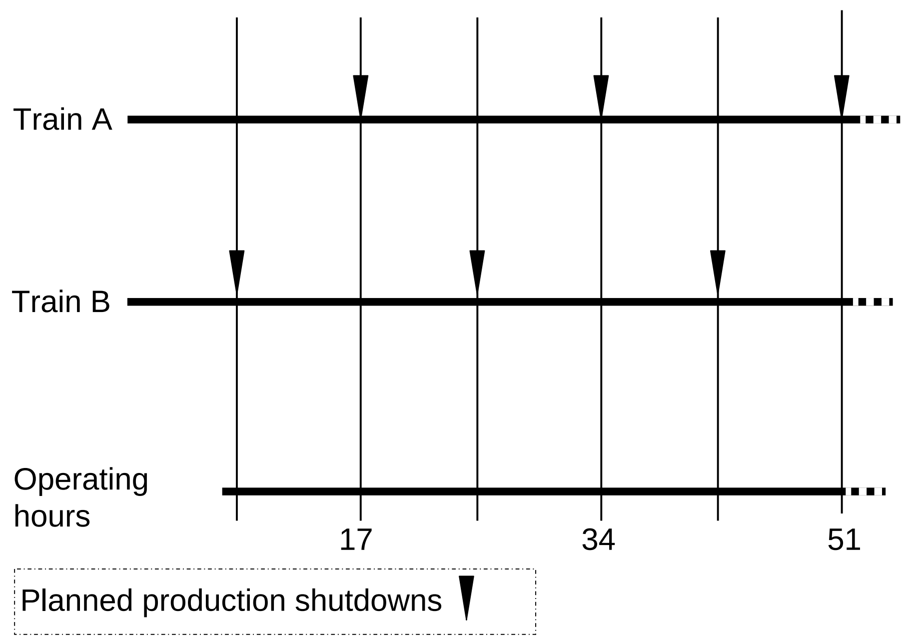


Figure 7.1: The planned shutdown periods for the two forthcoming years.

### 7.1.3 The decision process

The proposed inspection and overhaul strategy is affected by change in external parameters (boundaries), knowledge about the speed of the deterioration, and present condition. To ensure that the optimal strategy is selected, the decision process has to include an approach to modify input parameters on a regular basis. Both the economic values and the models of the deterioration process have to be adjusted.

The economic values are affected by change in e.g. taxes, oil prices, discount rate etc. The models of the ongoing deterioration processes have to be modified based on the experience gained during operation. Some deterioration processes can be studied without performing an inspection, while others have to be determined by inspections. Further, the results from an inspection also depend on the quality of the inspection itself.

The optimal strategy will depend on present system condition and change of external boundaries. Examples of situations which may require an update of strategy are:

- inspection of the gas turbine revealing the condition

- a shift in economic input figures beyond the limit which determines another strategy.
- new production plans incorporating planned shutdown times.
- knowledge about a significant improvement in technology

### 7.1.4 Forecasting of the deterioration process

A key issue is to identify and describe the most important deterioration processes affecting the replacement time. It is a significant challenge to model the deterioration processes and working conditions influencing the processes. The modelling procedure will depend on the quantity and quality of the information available.

Chapter 6 outlines methods that may be applied to define necessary processes that also include Bayesian methods in forecasting. The Bayesian method provides an approach to combine expert knowledge with measured values which can be very fruitful when the number of measured values is low.

## 7.2 Present technology - multiple deterioration processes

The following section describes the proposed approach to recommend a HSRI policy for each gas turbine when the only option is to replace with a gas turbine hot section of similar technology. In section 7.3, the decision support process also includes the option of replacing present technology with a new and improved technology, and the assumptions regarding the probability of release at specific times ahead.

First the deterioration processes of interest are covered. The income and cost figures are treated separately in section 7.2.2.

### 7.2.1 Deterioration processes

The performance degradation phenomena can be divided into recoverable degradation (typically fouling effects) and non-recoverable effects which include structural damage and wear. In addition to the deterioration processes which effect the performance, several other mechanical deterioration processes occurs such as bearing problems and thermal stress. Thermal stress may lead to failure (rupture) of internal parts which may cause severe secondary damage to the gas turbine.

The model of the deterioration process is based on the knowledge from existing and other similar gas turbines (à priori), the on-line performance monitoring and internal inspections.

**Performance degradation**

Due to the high complexity of the gas generator, it is vulnerable to component failures. Not every component is critical for operation, but several of these components may cause unforeseen shutdowns of the entire gas generator or severe performance losses if they are operated in a bad condition. It is not possible (or desirable) to model all these failures and failure processes. However, there are some dominating failure processes which normally cause loss of performance. The main causes for performance degradation are [90]:

- Fouling which is defined as performance loss caused by adherence of particulates to the airfoils and the annulus surfaces. The fouling mainly effects the compressor efficiency.
- Erosion which is defined as abrasive removal of material from the flow path by hard particles suspended in the gas stream
- Object damage by foreign object and/or domestic object.
- Seal wear which is caused by vibrations and temperature changes.
- Over-heating which may result in turbine blades (hot section) being burned down due to high temperature.

Other sources of performance deterioration are plugging of inlet filter, incorrect guide vane position, bleed valve leakage and increased mechanical losses (gearboxes, bearings and couplings). The source of the deterioration may be detected using diagnostic tools. The effects of different fault conditions on the typical engine performance parameters may be listed in the form of a fault matrix (see table 7.2).

In this context, the overall performance of the turbine is represented by the total thermal efficiency ($ETGT$), which is defined as $ETGT = SPPT/(Mf \cdot H_n)$. Here, SPPT is the abbreviation for the measured Shaft Power from the Power Turbine [kW], $Mf$ is the fuel consumption [kg/s], and $H_n$ is the fuel lower heat value [kJ/kg]. The main cause of efficiency losses is general fouling in the compressor, mainly due to air born particles such as dust and salt.

The condition monitoring parameters, both measured and derived will vary with the operating conditions. Off-design operation may often result in lower efficiencies. To

Table 7.2: Example of a qualitative fault matrix [90]

| | | Speed | Compressor pressure ratio | Mass flow (air) | Power | $GG_{EGT}$ (exhaust) |
|---|---|---|---|---|---|---|
| Fouling | Compr.* | ↓ | ↓ | ↓ | ↓ | ↑ |
| | HPT* | ↑ | ↑ | ↑ | ↓ | ↓↑ |
| | Pow. T. | ↓ | ↓ | ↓ | ↓ | |
| Erosion | | ↓ | ↓ | ↓ | ↓ | ↑ |
| Object damage | | ↓ | | ↓ | ↓ | ↑ |
| Seal wear | | ↓ | ↓ | ↓ | ↓ | ↑ |
| Burned HPT | | ↓ | ↓ | ↓ | ↓ | ↑ |

* The gas generator consists of compressor and high pressure turbine.

account for such variation, all data are normalized and transposed to a constant SPPT to be comparable [91].

Low quality in condition monitoring data is considered as a challenge to optimizing the HSRI policy. Specific parameters such as the fuel flow measurement and heat value requires high quality instruments for sufficient accuracy and the instrument has to be calibrated on a regular basis. Thorough analysis of existing condition monitoring data has revealed uncertainties in these measurements. It has therefore been difficult to establish quality data concerning the speed of deterioration, although it may easily be shown that there is a distinct efficiency reduction during operation. However, in future it is expected that more effort will be put into collecting the data with less uncertainties. If the cost of operating the turbine increases due to increase in fuel costs, the benefit of operating with higher thermal efficiency increases. To secure an efficient operation more effort has to be directed towards data quality for decision support purposes.

In the following optimization, the "long-term" reduction in performance is described as a function of the operating hours. Due to limited data quality as mentioned above, the given mathematical expression of the deterioration process must only been viewed as an example. The thermal efficiency (ETGT) is assumed to decrease according to the following equation:

$$ETGT(t) = ETGT_{t=0} - 0.17653 \cdot t + 5.1700 \cdot 10^{-4} \cdot t^2 \tag{7.1}$$

For a specific thermal efficiency, the fuel consumption (at $SPPT = 20\ [MW]$) can be found by the following equation:

$$Mf = \frac{20000 \cdot 3600}{ETGT \cdot H_n} \tag{7.2}$$

The corresponding deterioration process $\xi_{ETGT}(t)$ and drift function $g(t)_{ETGT}$ are found by normalizing equation 7.1. The drift function range from value $0$ at $t = 0$ to

$100$ which represents a condition where the turbine has failed. The normalization is performed according to the following equation:

$$g(t)_{ETGT} = \frac{ETGT_{t=0} - ETGT(t)}{ETGT_{failure}} * 100 = 2.94217 \cdot t - 8.6167 \cdot 10^{-3} \cdot t^2 \quad (7.3)$$

Further, the deterioration process is defined as:

$$\xi(t)_{ETGT} = g(t)_{ETGT} + b\sqrt{t} \cdot U \qquad U \sim N(0,1) \quad (7.4)$$

The value of the gas turbine thermal efficiency $(ETGT)$ has to be transferred to the *condition classification scheme* applied by the model. The classification scheme ranges from 0 (new condition) to 100 (unacceptable condition). Table 7.3 presents the applied classification scheme. In a new condition, the gas turbine performance ($ETGT$) at 20 MW is $37.15\%$. A decrease in the thermal efficiency of 1% corresponds to an increase in the fuel consumption of approximately $126$ $[kg/h]$ (or $2.9\%$)[3]. The turbine does not fail (technically) to operate even if the specific fuel consumption is very high. But from an economic point of view, there exists an efficiency limit which makes it more profitable to overhaul/replace than to continue. Because this limit is difficult to define, the "fault limit", according to the classification scheme, is selected at a very low "thermal efficiency". In fact, the defined "thermal efficiency" level would not be allowed to occur during normal circumstances. Figure 7.2 presents the predicted efficiency reduction due to deterioration.

The "failure level" in respect to a low thermal efficiency is defined as $29.13$ $\%$, i.e. a reduction of $8$ $\%$. A reduction of $8$ $\%$ in the thermal efficiency corresponds to an increase in the fuel consumption of approximately $1208$ $[kg/hr]$ e.g. an increase in the operating cost of approximately $3.11[MUSD/year]$ (see sec. 7.2.2).

Because the additional operating cost is extremely high at this condition, an overhaul or replacement will generally occur before reaching this condition.

### Damage to internal parts

Due to thermal stresses during operation, there is a risk of cracking/rupture (failure) of materials in blades, vanes and other internal parts. Damage to internal parts may cause an instant stop of the turbine and there is a risk of extensive secondary costs. Therefore, the equipment supplier operates limits regarding recommended maximum equivalent operating hours. Their recommendations are taken into account together with inspection of internal parts during operation, to make a decision on when to overhaul/replace. A simplified description of the deterioration process is applied in

[3] It is here assumed a lower heat value of 44194 MJ/kg

Table 7.3: The condition classification scheme.

| Class | | Reduction in ETGT [%] | Condition boundaries |
|---|---|---|---|
| 1 | Excellent | $\Delta ETGT < 1.0$ | $[0, 12.5 >$ |
| 2 | Very good | $1.0 \leq \Delta ETGT < 2.0$ | $[12.5, 25.0 >$ |
| 3 | Good | $2.0 \leq \Delta ETGT < 3.0$ | $[25.0, 37.5 >$ |
| 4 | Reduced | $3.0 \leq \Delta ETGT < 4.0$ | $[37.5, 50.0 >$ |
| 5 | Poor | $4.0 \leq \Delta ETGT < 5.0$ | $[50.0, 62.5 >$ |
| 6 | Bad | $5.0 \leq \Delta ETGT < 6.0$ | $[62.5, 75.0 >$ |
| 7 | Very bad | $6.0 \leq \Delta ETGT < 7.0$ | $[75.0, 87.5 >$ |
| 8 | Critical | $7.0 \leq \Delta ETGT < 8.0$ | $[87.5, 100.0 >$ |
| 9 | Failure | $\Delta ETGT \geq 8.0$ | $\geq 100$ |

this application to include the risk of damage to internal parts causing a failure. The estimated mean time to failure (MTTF) is set to 35 kUHR with variance parameter $b$ equal to 3:

$$\xi(t)_{internal\ parts} = \frac{10}{7}\,t + \frac{2}{49}\,t^2 + 3\,\sqrt{t}\,U \qquad U \sim (N(0,1) \tag{7.5}$$

The next step is to define the number of condition levels. First, the number of condition levels (states) should be sufficient to describe the current state with respect to discretization and estimation of residual life. Second, the number of states should also be sufficient and applicable to classify the condition from an expert point of view. The proposed approach to defined necessary number of states with respect to accuracy is outlined in chapter 5.3.3. Table 7.4 presents the results by comparing estimated and exact MRL for different numbers of condition levels. Based on the comparison, M=10 is selected as a sufficient number of condition levels.

Table 7.4: The error in the approximation of the $MRL_2$ for different numbers of states ($M$)

| M | $i = 1$ | $MRL_2$ | $\epsilon_{MRL}(i = 1) = MRL_1 - MRL_2$ |
|---|---|---|---|
| 5 | [0,20> | 29.748 | 5.252 |
| 10 | [0,10> | 32.107 | 2.893 |
| 20 | [0, 5> | 33.487 | 1.513 |

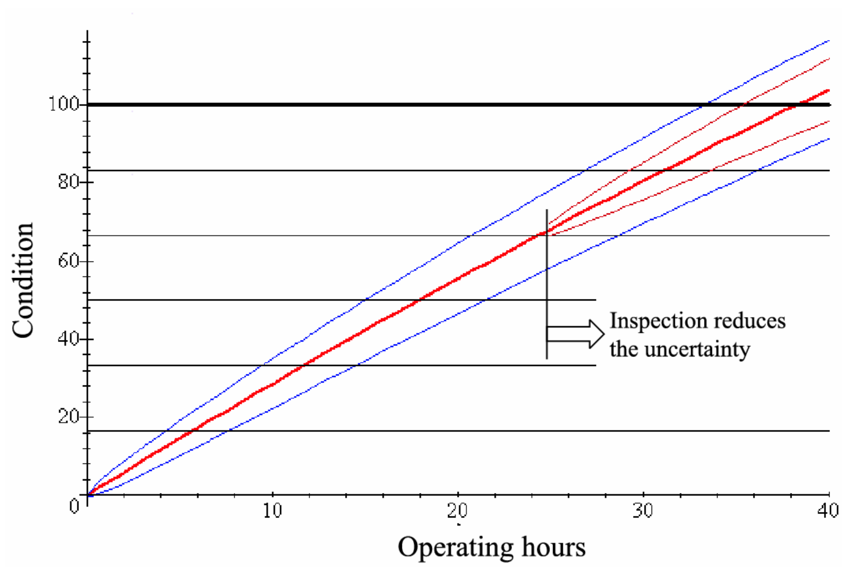


Figure 7.2: The predicted efficiency reduction due to deterioration. The condition is classified in 7 separate levels. An inspection reduces the uncertainty.

### 7.2.2 Income and cost data

#### General income and cost data

The general income is estimated to be $6$ $[MUSD/day]$ [4]. At present time, the fuel cost is assumed to be $0.167$ $[USD/Sm^3]$. The values are summarized in table 7.5. Table 7.6 presents an overview of the estimated reference values for each parameter which is included in the model. These values are discussed in the following text in addition to the profile functions which describe the influence on the costs to operate in a specific condition.

#### Fuel and emission-tax costs

A drop of $5\%$ in the gas turbine overall efficiency corresponds to an increase in the fuel consumption of approximately $15\%$. From table 7.7 it can be seen that an in-

[4] If the oil price is assumed to be 60.0 $[USD/barrel]$ and the production capacity equals 100000 $[barrels/day]$

Table 7.5: Production and cost data.

| Description | Value | Max/min |
|---|---|---|
| Fuel costs | 0.167 $[USD/Sm^3]$ | $[0, +20\%]$ |
| $CO_2$ tax | 0.130 $[USD/Sm^3]$ [5] | $[0, +20\%]$ |
| Production capacity | 100 000 [barrels/day] | - |
| Oil price | 60.0 [USD/barrel] | $+/-20\%$ |

Table 7.6: Gas turbine operation reference parameters.

| Parameter | Value | |
|---|---|---|
| $\overline{I}$ | $250$ | $[MUSD/kHR]$ |
| $\overline{C_{pen}}$ | $250$ | $[MUSD/kHR]$ |
| $\overline{C_E}$ | $1289$ | $[USD/HR]$ |
| $\overline{C_{PM}}$ | $20.0$ | $[USD/HR]$ |
| $\overline{C_{OH}}$ | $0.5$ | $[MUSD]$ |
| $\overline{S}$ | $9.5$ | $[MUSD]$ |
| $\overline{C_I}$ | $1250$ | $[USD]$ |
| $MDT_{OH}$ | $40$ | $[HR]$ |
| $MDT_{UCM}$ | $100$ | $[HR]$ |
| $MDT_I$ | $4$ | $[HR]$ |

crease in the fuel consumption of $15\%$ has dramatic consequences on the cost. A $15\%$ fuel consumption increase increases the cost by $193$ $[USD/hr]$ or approx. $1.7$ $[MUSD/year]$. The loss equals an increase of the SFC from $0.219$ $[kg/kJ]$ to $0.252$ $[kg/kJ]$. Based on the figures above, the eneregy cost reference value is set to $1289$ $[USD/HR]$, and the corresponding energy profile (with respect to efficiency) is as shown in table 7.8. At present time there is no tax associated with $NO_X$ emission. The extra cost of such a tax and the consequences have been neglected here. However, it may have a significant impact on the costs and efficiency demand, and can easily be incorporated.

**Inspection**

In addition to the sensor-based monitoring, manual inspection may be a reliable and efficient method to detect incipient failures. The most common technique for manual inspection of a gas turbine is to use a borescope. The gas turbine is equipped with inspection ports to give access for the borescope, so that internal parts may be inspected without opening the casing. The main parts which are inspected are compressor, turbine blades, combustion chamber and seals. The inspection may reveal incipient crack/failure, fouling and general wear of these parts. The man hours re-

Table 7.7: Engine energy and emission costs with and without performance losses

| Costs (SPPT=20MW) | New engine USD/hr | Combined deterioration (15% increase in fuel consump.) USD/hr |
|---|---|---|
| $CO_2$-tax | $564^{a)}$ | 649 |
| Fuel cost | $725^{b)}$ | 833 |
| Total | 1289 | 1482 |

a) $CO_2 - tax = 4385[kg/hr] \cdot (1.0106)^{-1}[Sm^3/kg] \cdot 0.130$ $[USD/Sm^3]$=564 $[USD/hr]$

b) $Fuel\ cost = 4385[kg/hr] \cdot (1.0106)^{-1}[Sm^3/kg] \cdot 0.167$ $[USD/Sm^3]$=725 $[USD/hr]$

Table 7.8: The gas generator energy and emission cost profile.

| $z$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\Psi_E(z)$ | 1.012 | 1.041 | 1.071 | 1.103 | 1.139 | 1.175 | 1.212 | 1.430 |

quired to do a boroscope inspection is estimated to 12 hours and the turbine has to be totally shutdown for 4 operation hours.

**Maintenance cost for utility equipment**

In addition to the maintenance cost associated with the main parts of the gas turbine, several subsystems and equipment within the gas-turbine module have to be maintained. The cost of maintaining this equipment is very small compared to the operating cost, and is approximately $20$ $[USD/HR]$. The cost of maintaining the gas generator between major overhauls is thus very small compared to the other costs involved, e.g. the maintenance costs are less than $3\%$ of the sum of energy and emission costs. Because the day-to-day maintenance cost parameter is so low, the maintenance costs are assumed to be constant and independent of the condition. Thus the maintenance cost profile ($\Psi_{PM}(\cdot)$) is set to $1.0$ for all conditions (state) levels.

**Major overhaul costs**

The major overhaul costs include the total costs of all activities and resources needed to restore the gas-generator to "as-good-as-new" condition. In addition to the fixed cost in connection with dismantling/re-assembling the gas generator and transportation to the onshore facilities, the overhaul cost is affected by two other factors. First, the condition of the gas generator to be overhauled may have significant impact on the total costs. Dependent on the degradation/damages, the cost may vary from MUSD 0.45 to MUSD 0.85. Second, the downtime costs will depend on the shutdown period

Table 7.9: The gas generator overhaul cost profile with respect to degree of internal crack/failure.

| $z$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\Psi_{OH}(z)$ | 0.90 | 1.10 | 1.20 | 1.30 | 1.35 | 1.40 | 1.45 | 1.50 | 1.55 | 1.70 |

in connection with the work on the platform. A significant cost reduction is achieved if the actions that require shutdown are shifted to fall within the planned platform shutdowns.

The change in the overhaul costs is described by the overhaul cost profile function presented in table 7.9. It is assumed that the overhaul cost profile function is independent of the condition parameter classifying the efficiency, but dependent on the condition parameter classifying the severity of internal crack/failure. The overhaul cost reference parameter, $\overline{C_{OH}}$, is set to $MUSD\ 0.5$. The overhaul cost profile with respect to efficiency is constant and equal 1.0. Remember that the overhaul cost at a specific condition is given as the general overhaul reference cost multiplied by the overhaul cost profile value at that specific condition level. If the turbine is classified to be at e.g. condition level $5$ with respect to damage to internal parts, the overhaul cost is calculated as MUSD $1.35 \cdot 0.5 = 0.675$.

**Salvage value**

The salvage value of the turbine depends on the condition and the second hand market. A future market price may also be affected by future environmental requirements regarding the emission of eg. $NO_x$ and $CO_2$.

A new gas generator costs approximately MUSD $9.5$. Today, a secondhand overhauled 20 year old gas generator of similar can be purchased at a cost starting at MUSD $3.0$. The salvage value will depend on the actual condition (and generation). In the calculation the salvage value is stated to be MUSD $5.0$ after a major overhaul and MUSD $4.5$ when the gas generator is in a bad condition as presented in table 7.10.

Table 7.10: The gas generator salvage value profile with respect to degree of internal crack/failure.

| $z$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\Psi_{OH}(z)$ | 1.000 | 0.989 | 0.978 | 0.967 | 0.956 | 0.944 | 0.933 | 0.922 | 0.911 | 0.900 |

### 7.2.3 Results

**Comparison of results with a simple economic life model**

To view the results of the proposed model a comparison is made with a simple economic life model. The following economic life model is applied (see sec. 3.2, eq. 3.1) :

$$C(t_p) = \frac{1}{t_p}\left\{C_{CAP} + \int_0^{t_p} c\,(1 + k\,t)\,dt)\right\} \tag{7.6}$$

where $C_{CAP}$, $c(\cdot)$ and $t_p$ represent the overhaul costs, the operating costs and the time period between two consecutive overhauls. Here the overhaul cost is independent of the condition, while the operating cost reflects only the fuel consumption. The parameter $k$ describes the increase in fuel consumption (deterioration rate). In this case, the fuel consumption is described by equation 7.1, where $ETGT_{t=0} = 37.15$.

Based on the simple economic life model, where the fuel and emission costs add up to a total of USD/URH $1289$ after a replacement, the optimal time between replacements is found to be $37$ kURH. The calculated results for different cost levels are presented in fig. 7.3. When the discount rate decreases the optimal replacement strategy moves towards infinity i.e. breakdown maintenance, as shown in figure 7.4. In a real situation other failure processes such as damage to internal parts due to material degradation will dominate and force a replacement at an earlier stage. The economic life model given in 7.6 optimizes the interval based on a deterministic description of the deterioration process.

The economic life model is easy to apply but is not capable of including the effect of uncertainty that always applies in describing a deterioration process. The effect of uncertainty can be viewed by applying the proposed MDP model. Table 7.11 presents the results of changing the variance coefficient $b$ from 0.5 to 6. An increase in the variance have significant impact on the proposed time to the next inspection. If the condition is classified within e.g. level one at present time, the TTNI decreases from $40$ kURH to $20$ kURH when the variance parameter increases from $0.5$ to $6$. At first, $40$ kURH may seem odd knowing that the effect of uncertainty decreases the TTNI and that the economic life model proposed an interval of 37 kURH. The results can be explained by the "end of horizon effect" which tries to minimize the total number of inspections and replacements before shutting down the entire process. The "end of horizon effect" also becomes more visible on the result as the variance coefficient decreases.

The difference in the cost between a gas turbine in a new condition and a gas turbine

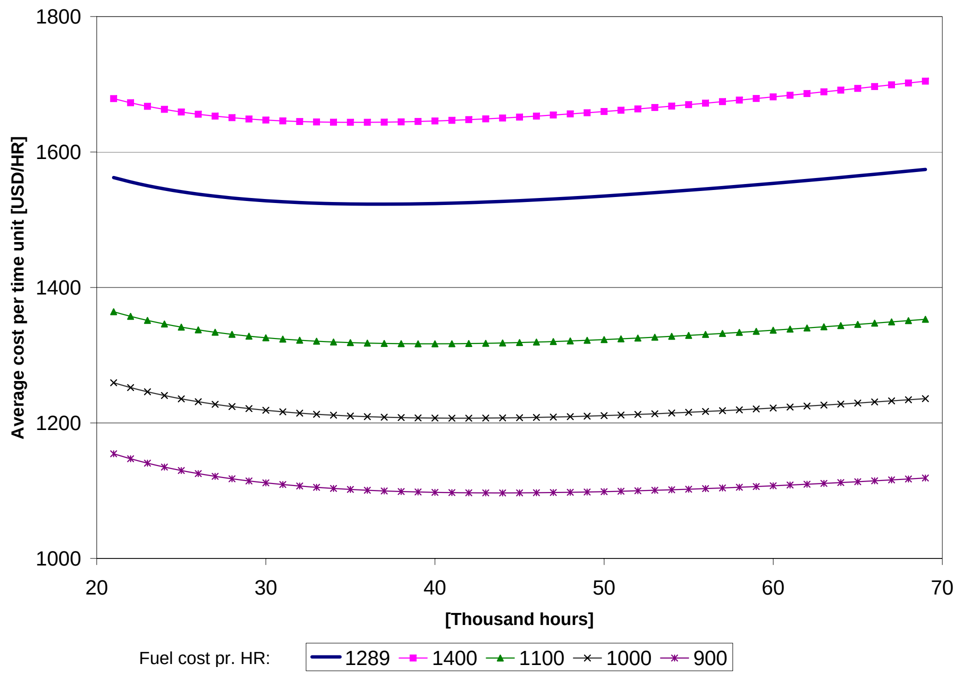


Figure 7.3: The influence of the fuel costs on the optimal overhaul/replacement time ($\beta = 1.0$).

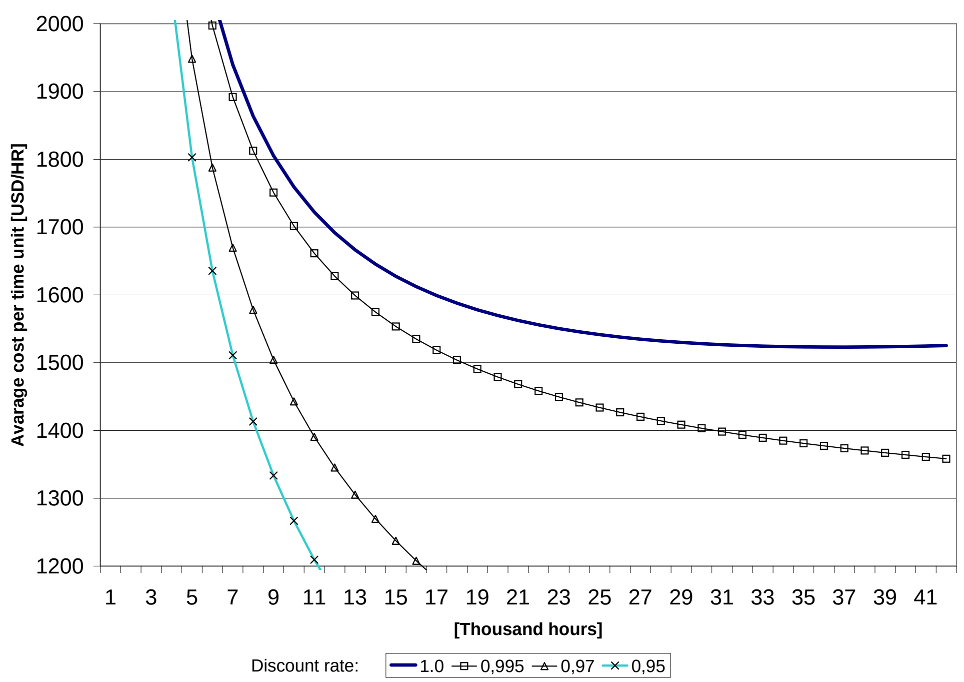


Figure 7.4: The influence of the discount rate on the optimal overhaul/replacement time.

with a condition equal to or less than level $8$ is USD $4425.5$ [6], which equals one additional replacement.

Another external factor which has an effect on the decision process is the discount rate $\beta$. A decrease in the discount rate causes an increase in the proposed time to the next inspection as shown in table 7.12.

Table 7.11: The optimal intervals to the next inspection (and average LCC [USD/UHR]) when there are 200 time units to end of horizon ($\beta = 1.0$).

| | Condition class (efficiency) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| The variance | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| b=0.5 | I-40 (1555) | I-28 (1563) | I-24 (1569) | I-18 (1573) | I-13 (1576) | R/I-40 (1578) | | |
| b=2.0 | I-32 (1559) | I-28 (1566) | I-18 (1572) | I-17 (1577) | I-11 (1580) | R/I-32 (1581) | | |
| b=3.0 | I-27 (1563) | I-24 (1570) | I-16 (1576) | I-13 (1581) | I-8 (1585) | R/I-27 (1586) | | |
| b=6.0 | I-20 (1573) | I-17 (1580) | I-12 (1587) | I-8 (1592) | R/I-20 (1595) | | | |

R: Immediate Replacement, I-xx: Next inspection xx time units ahead.

Table 7.12: The recommended intervals to the next inspection (and average LCC[USD/UHR]) when there are 200 time units left to end of horizon ($b = 3.0$).

| | Condition class (efficiency) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Discount | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $\beta = 1.0$ | I-27 (1563) | I-24 (1570) | I-16 (1576) | I-13 (1581) | I-8 (1585) | R/I-27 (1586) | | |
| $\beta = 0.995$ | I-28 (981) | I-24 (988) | I-18 (994) | I-15 (999) | I-8 (1002) | R/I-28 (1003) | | |
| $\beta = 0.960$ | I-30 (178) | I-26 (182) | I-21 (186) | I-16 (191) | I-11 (195) | I-6 (199) | R/I-30 (200) | |

R: Immediate Replacement, I-xx: Next inspection xx time units ahead.

### The risk of damage to internal parts and its effect on the LCC

Maintaining a high efficiency is important to reduce the cost, but the deterioration processes causing damage to internal parts (internal crack/failure) often dominate

[6] The difference in average cost per time unit is USD/kURH $1577.52 - 1555.39 = 22.13$ at $N = 200$ kURH, thus $22.13 \cdot 200 = 4425.5$ USD

in the timing of a replacement. The dominant factor or parameter depends on the balance between energy costs, power demand, the cost of downtime, secondary costs at breakdown and the repair cost. Table 7.13 presents the result by only looking at the risk of an internal crack/failure and neglecting the energy costs. As the uncertainty of the predicted rate of deterioration increases the time between inspections is reduced, and the cost increases rapidly.

Table 7.13: The optimal intervals to the next inspection (and average LCC [USD/URH]) when there are 200 time units to end of horizon ($\beta = 1.0$).

| Variance | Condition class (Damage to int. parts) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 10 |
| b=2.0 | I-24 | I-21 | I-18 | I-14 | I-10 | I-8 | I-6 | I-4 | R/I-24 |
| | (192) | (198) | (203) | (206) | (209) | (211) | (213) | ((215) | (216) |
| b=3.0 | I-24 | I-20 | I-16 | I-12 | I-9 | I-7 | I-5 | I-3 | R/I-24 |
| | (199) | (204) | (209) | (212) | (215) | (218) | (221) | (222) | (223) |
| b=6.0 | I-17 | I-17 | I-12 | I-9 | I-7 | I-6 | I-4 | R/I-17 | |
| | (217) | (222) | (227) | (231) | (234) | (237) | (240) | (240) | (241) |

R: Immediate Replacement, I-xx: Next inspection xx time units ahead.

### Multiple deterioration processes

Previous results determine the inspection and replacement decision when each of the deterioration models is treated separately. In a real situation, all significant deterioration processes have to be considered together. The following assumptions yield the result of this section:

- the deterioration processes are independent
- the discount rate is constant over the entire horizon
- the major shutdowns are known and deterministic

The first assumption implies that the deterioration processes causing an internal crack-/failure do not effect the efficiency and vice versa. Although major damage to the hot section may cause efficiency losses, the losses are usually caused by fouling on compressor parts. Increased pressure drop over the filter package in front of the compressor also results in a reduced efficiency. The ongoing deterioration processes due to thermal stresses are usually not observable without an internal inspection. The processes cause general wear and tear leading to cracks on the surface of blades and vanes. The combustion chambers and nozzles also suffer from thermal stresses and material degradation. The discount rate varies and is affected by internal as well as external premises. The time between each major shutdown is normally a part of the design basis and generally known from the beginning of the operational period. Due to weather condition these major shutdown periods are scheduled in the summer season, and within a short time window. The analyses of three different external parameters affecting the decision process are initially performed. These parameters are:

- Change of discount rate
- Change in fuel costs and $CO_2$ taxes
- Inclusion known shutdowns

The first analysis is performed assuming a situation with no planned shutdowns, a discount rate of $0.995$ and income and cost values as presented in table 7.6. If the time left to operate is $200$ kURH ($N = 200$), the optimal time to perform the next inspection is presented in table 7.14. If e.g. the condition is classified as $(1, 2)$ with respect to the efficiency and damage of internal parts respectively, the "optimal" TTNI is $20$ kURH. Further, it can be seen that a replacement should be performed immediate if an inspection reveals a condition class equal or less than 6, with respect to efficiency. With respect to damage to internal parts, the boundary value for a replacement action is less or equal to 9. The calculated LCC per time unit as a function of the condition at $N = 200$ is presented in fig. 7.5.

During the forthcoming time period, the income and cost figures would normally change, and thus it is important to investigate how robust the proposed results are with respect to external conditions. Two major income and cost elements are the market oil price and the fuel price to fire the gas turbine. Table 7.15 presents the results from the sensitivity analysis with respect to these two figures. An increase in the fuel cost by $20\%$ only leads to small deviation in TTNI. Deviations are mainly observed if the condition is in state 5, leading to a shorter time to the next inspection. If the condition is in state (5,7) the recommended policy is to perform a replacement followed by an inspection after 24 kURH. Changing the the oil price (income) from $-10\%$ up to $+10\%$ has even less impact on the results from the baseline case. The energy costs and deterioration with respect to efficiency dominate the optimal decision policy.

Planned shutdowns of systems are normally held during a turnaround each second summer as presented in figure 7.1. Table 7.16 presents the optimal inspection and replacement policy at $N = 200$ knowing there will be a shutdown each second year $(17kURH)$, thus the first planned turnaround will occur at $N = 184$. The shutdowns have a significant impact on the optimal policy and the policy is directed to

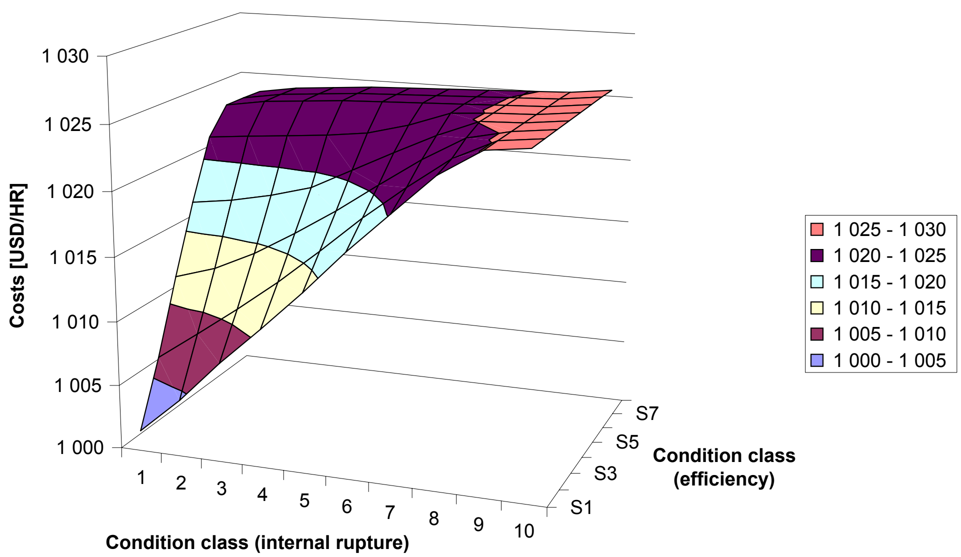


Figure 7.5: The calculated life-cycle costs dependent on the observed condition at N=200. The discount rate ($\beta$) is assumed to be $0.995$.

perform the next inspection (and replacement) at the planned turnaround. If the condition is too poor at $N = 200$, the recommended policy is to perform an immediate replacement and plan for a new inspection at the next turnaround (marked "Replace (16)").

Today the operators are challenging the interval between each turnaround and in many cases the time between each turnaround is extended by typically 1-2 years. Thus, if the time between planned turnarounds is extended from 2 to 4 years, the turbine would probably have to be overhauled between two consecutive turnarounds. Figure 7.6 presents the consequences with respect to average costs per time unit by changing the elapsed time between two consecutive turnarounds. The average costs are presented for the scenario of 2 years, 3 years and 4 years interval respectively for a turbine in as good-as-new condition at the time of decision. In general, the average cost per time unit of operating the gas turbine decreases as the interval between turnarounds is reduced, which seems intuitive. However, due to the end of horizon effect, the resulting costs from increasing the time between two consecutive turnarounds may in some time intervals give a lower cost per time unit for a scenario with a lower number of tunrarounds over the operating period (e.g. TTNI $\epsilon[184, 192]$).

Table 7.14: The "optimal" time to the next inspection, when combining the effect of the two deterioration processes.

| C.class | Condition class (Damage to internal parts) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| eff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | I-24 | I-20 | I-17 | I-12 | I-10 | I-7 | I-6 | I-3 | | |
| 2 | I-21 | I-20 | I-16 | I-12 | I-9 | I-7 | I-5 | I-3 | | |
| 3 | I-21 | I-18 | I-16 | I-12 | I-10 | I-7 | I-5 | I-3 | | |
| 4 | I-15 | I-14 | I-14 | I-12 | I-9 | I-7 | I-5 | | | |
| 5 | I-11 | I-9 | I-11 | I-11 | I-9 | I-6 | I-4 | | | |
| 6 | | | | | | | | | | |
| 7 | R/I-24 | | | | | | | | | |
| 8 | | | | | | | | | | |

R: Immediate Replacement, I-xx: Next inspection xx time units ahead.

Table 7.15: Influence of change in fuel cost and oil price (income) on the "optimal" time to the next inspection, compared to 7.14.

| C.class | Condition class (Damage to int. parts)[a)] | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| eff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | -2/0/0 | -2/0/0 | | | | | | | | |
| 4 | | | | | -1/-1/1 | | | | | |
| 5 | -3/-3/3 | -1/-1/0 | -2/0/0 | -2/-2/0 | -1/-1/0 | | 20/20/0 | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |

[a)] The values represent deviation from the basic values of table 7.14. The first value shows the result of an increase in fuel cost of +20%, and the second and third a change in oil price of -10% and +10% respectively.

## 7.3 Present and future technologies - multiple deterioration processes

The gas turbine technology is constantly developing, even though no revolutionary design changes have been presented over the last years. Based on recent improvements within material technology, it is expected that the performance characteristics of the gas turbine presented above may improved even further. The improvements are related to e.g. re-coating of turbine blades and vanes to reduce fouling and increase their strength against non-recoverable degradation. Research on new materials which may be exposed to higher temperature without deteriorating is a target area. Increased temperature resistance may increase not only the length between HSRI, but also the efficiency and the gas generator maximum output.

Table 7.16: The optimal time to the next inspection at N=200 when the turbine is in as good as new condition knowing there will be shutdowns (opportunities) each second year (17 kURH).

| C.class | Condition class (Damage to internal parts) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| eff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | I-16 | I-16 | I-16 | I-12 | I-10 | I-6 | I-4 | | | |
| 2 | I-16 | I-16 | I-16 | I-12 | I-8 | I-6 | | | | |
| 3 | I-16 | I-16 | I-16 | I-12 | I-8 | | | | | |
| 4 | I-16 | I-16 | I-16 | I-12 | | | | | | |
| 5 | I-16 | I-16 | I-16 | | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | R/I-16 | | | | | |
| 8 | | | | | | | | | | |

R: Immediate Replacement, I-xx: Next inspection xx time units ahead.

The objective of this section is to visualise the impact on the initial decision policy by describing a new scenario that incorporates a release of an improved gas turbine technology. The new technology is stated to have:

- improved efficiency by $10\%$ (see tbl. 7.17)
- increased life-time by $35$' kURS to $40$' kURS with respect to internal crack/-failure

Because it is difficult to determine the exact time of a release on the market, the time of release is described using the normal distribution (other distributions may also be used for this purpose). The probability of release is described as $p_B(t) \sim N(30', 2')$, thus after 30 kURS (3.4 years) the probability that a new technology will be released equals $0.5$.

Table 7.17: The energy and emission cost profile for an improved technology - B.

| $z$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\Psi_E(z)$ | 0.911 | 0.937 | 0.964 | 0.993 | 1.025 | 1.058 | 1.091 | 1.287 |

The estimated mean time to failure (MTTF) is set to 40' UHR with variance parameter $b$ equal to 3:

$$\xi(t)_{internalparts} = \frac{7}{6}\,t + \frac{1}{39}\,t^2 + 3\,\sqrt{t}\,U \qquad U \sim (N(0,1) \tag{7.7}$$

The number of states (condition classification scheme) is 8 with respect to efficiency and $10$ with respect to internal crack/failure, and thus fit the scheme applied to describe present available technology.

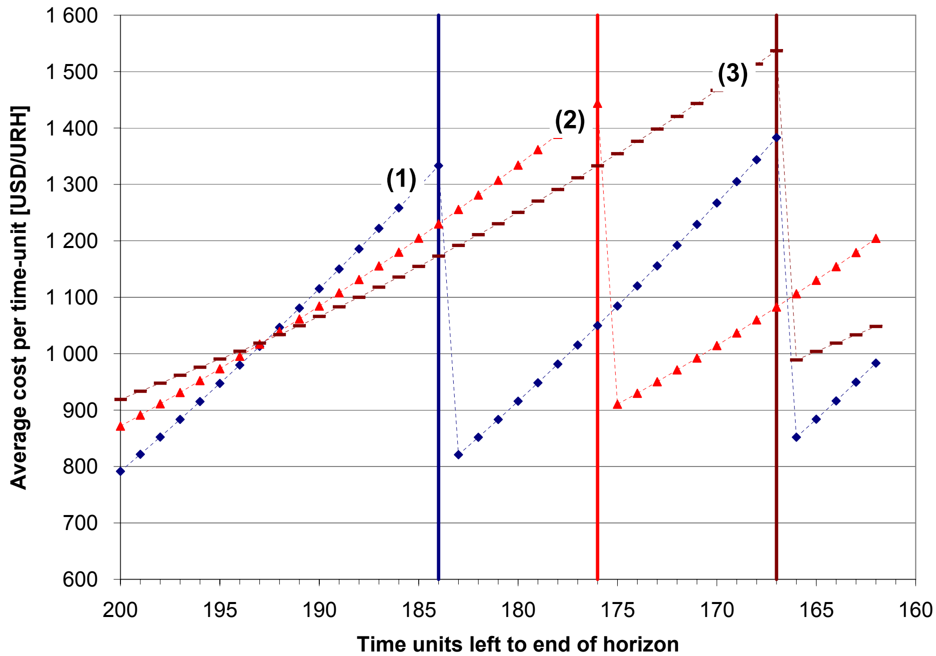


Figure 7.6: The figure presents three different scenarios with planned shutdown each (1) $2^{nd}$ year(17 URH), (2) $3^{rd}$ year (24 URH) (3) $4^{th}$ year (34 URH) and corresponding costs dependent on remaining time until end of horizon given as-good-as new condition at the point of decision.

The repair and replacement costs of a turbine based on the new technology are stated to be the same as for the present technology. However, there are extra costs associated with the modification by changing from present to the new technology. The investment costs are stated to be 9.5 MUSD which includes both procurement and installation of the new turbine. The salvage value of the existing turbine depends on the condition at replacement. The salvage value profile with respect to degree of internal crack/failure is given in table 7.10.

### 7.3.1 Results

The model has been applied for three different scenarios to investigate the effect of including an assumption regarding the forecast of a technology improvement.

The results show that the effect of including shutdowns overrides the effect of including technology improvements. The only deviation in the inspection policy compared to the results presented in table 7.16 (at $N = 200$) is observed when the gas generator is described to be in condition (1,7) leading to an increase in TTNI from 4 kURH to 5 kURH. This conclusion can also be supported by looking at the recommended inspection policy and varying the time units left to end of horizon. Figure 7.7 presents the result if the gas generator is stated to be in condition (1,1) with respect to efficiency and internal crack/failure at the decision interval. Three scenarios are presented:

1. New technology available at $N \sim (30', 2')$, neglecting possible planned shutdown activities (i.e. opportunities)
2. Single technology (present), including planned shutdown each $2^{nd}$ year.
3. New technology available at $N \sim (30', 2')$, also including planned shutdowns each $2^{nd}$ year.

The only deviation observed by comparing scenario [2] and scenario [3] for the specified condition in the presented time frame is at $N = 187$. At that decision interval, the optimal policy when including for the scenario [3] is to continue without inspection and replacement at 20 kURH, thus not to utilise the opportunity given by the next planned shutdown at $N = 184$. For scenario [2] the recommended policy is to perform an inspection and possible replacement at $N = 184$. Although the effect of planned shutdowns is the most important factor to define the optimal policy in the presented case, the effect of including possible improvements in technology can be of importance depending on the magnitude of costs associated with shutdowns. In table 7.14 the planned shutdowns and possible technology improvements have been neglected. To investigate the effect of scenario [2] (including technology improvements) on the proposed policy, the deviation with resect to TTNI by including and not including technology improvements is presented in table 7.18. The optimal policy at $N = 200$ recommends an inspection and possible replacement mid-way between today and the possible time of a release.

## 7.4 Summary

Use of the proposed methodology and tools may apply both to developing an initial plan as well as to support in-service inspection and maintenance planning. The section primarily focuses on in-service inspection and maintenance planning, which differs from an initial planning in at least two ways. First, more information becomes available about the deterioration processes and it is therefore necessary to adjust the forecast on the ongoing deterioration processes. Second, the economic values and external boundary conditions for the decision problem may have to be adjusted according to present knowledge. This implies that the decision maker has to incorporate

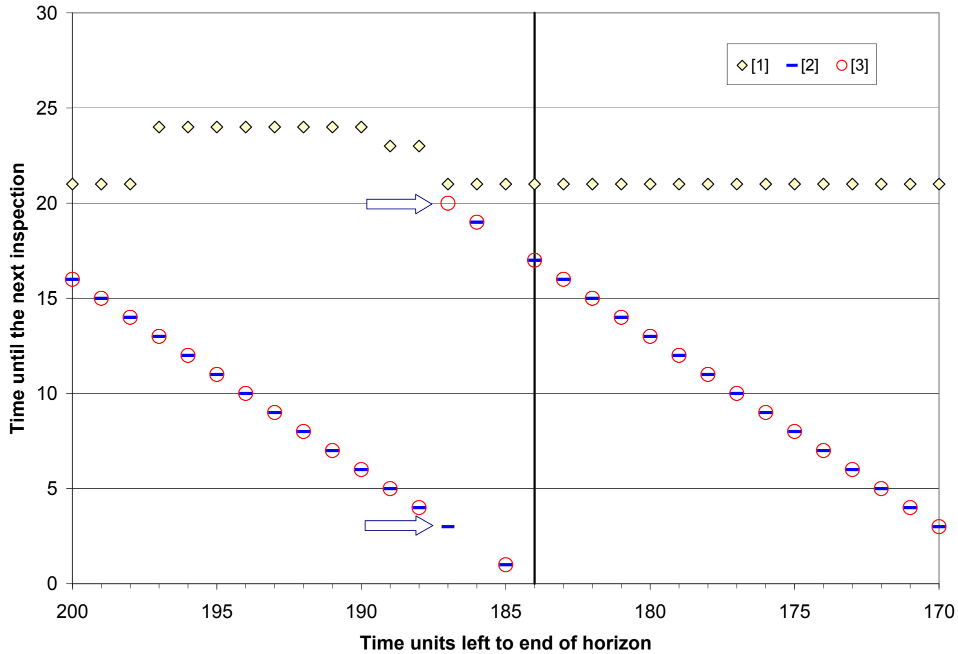


Figure 7.7: The inspection and replacement policy given the following three scenarios when the gas turbine is in condition (1,1) at the decision interval. [1]: New technology available $(N \sim (30', 2'))$, neglecting possible planned shutdowns; [2]: Single technology, including planned shutdown each $2^{nd}$ year; [3] Includes both the the effect of a new technology $(N \sim (30', 2'))$ and planned shutdowns.

updated strategy for planned shutdowns as well as new forecasts for income and cost values to encompass the present situation. Obsolescences must also be considered in in-service inspection and maintenance planning compared to initial planning.

The results presented in this chapter show the importance of incorporating knowledge of possible technology improvements and planned shutdowns. In the case presented here, incorporating planned shutdowns dominates the optimal decision policy due to the high downtime cost rate.

The same applies for the end of horizon effect. This will be even more important in those cases where the remaining time to operate is low.

The discount rate will affect the importance of future activities, thus as the discount rate decreases the effect of future planned shutdowns, future technology development and end of horizon reduces.

Table 7.18: Influence of including the effect of technology improvement presented as deviation from optimal time to next inspection, compared to 7.14.

| C.class | Condition class (Damage to int. parts)$^{a)}$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| eff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | -3 | | -3 | | | | | | -3 | -3 |
| 2 | | -4 | -3 | | 1 | | | | -3 | -3 |
| 3 | -5 | -2 | -2 | | | 1 | 1 | | -3 | -3 |
| 4 | -2 | -2 | -1 | | 1 | | | $-21^*$ | -3 | -3 |
| 5 | -1 | | -1 | -1 | | 1 | 1 | $-21^*$ | -3 | -3 |
| 6 | $-18^*$ | $-18^*$ | $-18^*$ | $-18^*$ | $-18^*$ | $-18^*$ | $-20^*$ | -3 | -3 | -3 |
| 7 | -3 | -3 | -3 | -3 | -3 | -3 | -3 | -3 | -3 | -3 |
| 8 | -3 | -3 | -3 | -3 | -3 | -3 | -3 | -3 | -3 | -3 |

$^{a)}$The values represent deviation from the basic values of table 7.14. In the scenario for including new technology the optimal TTNI in condition (1,1) is thus $24 - 3 = 21kURH$. For those values marked with $^*$, the optimal decision have changed from "replace" to "continue as-is" and perform the next inspection as described.

# Chapter 8

# Summary and future work

## 8.1 Summary

Managing ageing systems has been on the agenda within the Nuclear Power Plant industry over the past decades. The topic has now also come on the agenda in the oil and gas industry of the Norwegian Continental Shelf, which includes facilities that have been operating for more than two decades.

Annual costs related to operation and maintenance on the Norwegian Continental Shelf (NCS) are predicted to be in the range of 20-35 BNOK over the next ten years. At the same time, the unit costs are predicted to increase strongly. To meet this challenge, the authorities call for coordinated initiatives and new solutions in the renewal of the oil and gas industry [92]. The challenges have been addressed by the industry as well as the government. *Oil and Gas in the 21st Century* (OG21) is a Task Force established by the Ministry of Petroleum and Energy (MPE) of Norway in 2001 to help the petroleum industry to formulate a national technology strategy for added value and competitive advantage in the oil and gas industry. The objective is to develop a more co-ordinated and focused approach to research and development throughout the oil and gas industry.

An important aspect addressed by OG21 is to improve both technology and methods for condition monitoring and the ability to utilise the information for decision support. That is also a key issue in managing ageing systems as discussed in chapter 2.6.2.

This thesis presents one approach for decision support utilising condition monitoring data to manage ageing systems. The thesis focuses on the technology aspect of physical impairment and obsolescence. The issues relating to organisation and human resources are not considered but they are not seen as less important, on the contrary the area would itself justify comprehensive studies.

The thesis presents a model designed to investigate and seek optimal solutions when it is possible to classify the items present condition and predict future development based on previous condition monitoring results. The deterioration process is described by a Markov process, and the sequential decision problem is modelled as a discrete time Semi-Markov Decision Process (SMDP). The transition probabilities of the controlled time-variant Markov process are described in a condition transition probability matrix (CTPM). To account for the end-of-horizon effect and time dependent external parameters such as varying production profile, the optimal solution is found by use of the value iteration procedure (stochastic dynamic programming).

The input data has to be gathered from management systems, condition monitoring systems and/or expert judgements. The method for determining the deterioration process which is the basis for generating the CTPM is based on regression and time series analysis, and Bayesian methods in forecasting. Regression analysis is used to establish the proper time series models. However, the use of time series to represent a deterioration process has a major drawback. That is, the model does not contain knowledge about the ongoing physical failure process, it only represents the symptoms of the process that are measurable. Expert judgement may compensate however for the lack of knowledge in the model. Further, the thesis describes one approach to convert trend models to time dependent condition transition matrixes, making the results from condition monitoring applicable to models requiring a description of state transition.

The model provides capability to incorporate information about future planned shutdowns which provide opportunities for inspections and repairs. The opportunities may have significant impact on the optimal inspection and repair strategy.

The modelling approach is considered to be strongest when applied to planning of in-service inspection and repair, but the solution methods can also be applied in the design phase. The most important achievement is the development of a method for obtaining cost optimal inspection and repair strategy over the entire life cycle.

In decision problems concerning *strategic* maintenance planning, quantitative methods like Reliability-Centered Maintenance (RCM) and Risk Based Inspection (RBI) are often applied [93], [94]. These methods lend themselves to the initial selection of the appropriate maintenance strategies based on an evaluation of the criticality of each identified failure mode. A major challenge, however, is to specify the task interval. For those items assigned a condition-based maintenance policy, the proposed model may in some cases support the maintenance engineer in selecting an appropriate inspection interval and/or control limit to perform a replacement. From a strategic point of view, the capability to consider the impact of change in e.g. production demand, efficiency requirements and release of new improved technology in situations which require an evaluation of the benefit of continuation versus overhaul/replace is important. A combination of the methods supported by theories within decision analysis as presented in section 2.6.2 and the results achieved by applying the proposed model may constitute a powerful tool for strategic maintenance decision making.

## 8.2 Future work

The main challenge regarding maintenance optimization in general, and specifically application of the methodology as presented in this thesis, is implementation in real operations. Application will have impact on work processes and condition monitoring and diagnostic methodologies to measure and model deterioration processes as a basis for seeking the optimal policies. Real application will also require work on the graphical user interface both related to input of data and output and interpretation of the results.

When it comes to the solution methodologies and software routines there are several areas for improvement. These are:

1. To relax the assumption of knowing the true state of condition after an inspection.
2. Modelling of deterioration processes
3. Modelling of unforseen events that provide opportunities for maintenance action.
4. Development of mathematical models and solution techniques that enable faster solution algorithms.

(1) A limitation of the fully observable Markov decision processes stems from the assumption that the condition state of the system is revealed without uncertainty after inspection, which is equivalent to assuming that the inspection technology used is perfectly precise (free of random errors) and accurate (free of bias). An area of future work is to adopt the theory and solution methods provided by partial observable Markov decision processes as discussed in section 4.3. The POMDP approach relaxes the above assumption by recognizing that the true state of the components or systems is typically unknown. Different observations are obtained using various inspection methodologies, having different costs, and providing more or less accurate

depictions of true system state. The extension to accommodate partial observability does however significantly increase the computational demand.

(2) A major challenge in describing the deterioration process (physical impairment) is to compensate for the lack of data. For a new item, the necessary information for the specific item may not exist, while for an item that has been in operation for some time, there may both be qualitative and quantitative information available. This subject has been discussed in chapter 6.

(3) The methodologies presented in the thesis provide a good framework to include known (deterministic) events that utilise opportunities to perform actions (inspections, repair and or modifications) to limit production losses for individual (single) items. In real time operations, there are normally many opportunities which arise stochastically. This is a key issue within short-term maintenance planning [9]. Here, item dependencies and grouping of activities to reduce the loss of availability and minimize costs of a maintenance action are of key interest. The MDP approach has the capability of describing more than two states (operating versus failed), which in some cases may be vital to select the appropriate action.

(4) Application of Markov decision processes often requires high computational capability. In an operational setting the response time is vital in providing a user friendly solution. Development of mathematical models and solution techniques that enable faster solution algorithms is therefore of key interest. This will be even more important if the solution is to include uncertainty in the observation (results from inspections) as outlined within the area of POMDP.

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# Appendix A

# Fortran code for the MDP model

The Fortran (F90) source code for the SMDP.

```
PROGRAM MDP05

USE i_o_data


!    VARIABLES

IMPLICIT NONE
INTEGER*4                      :: i, j, k, l, m, n, o, s, y,count, delta_t, max_kj !, max_k
INTEGER*4                      :: delta_1_t, delta_2_t, delta_3_t, delta_4_t
INTEGER*4                      :: delta_1_add, delta_2_add, delta_3_add, delta_4_add
INTEGER*4                      :: CP1_levels,CP2_levels,CP3_levels,CP4_levels,sensitivity_check
INTEGER*4, DIMENSION(1:35)     :: caseoption !1:Horizon (time),2:Present technology,
            !3: Present age,   4: Present CP1 (Safety),   5: Present CP2 (env.),
            !6: Present CP3 (Availab.),7: Present CP4 (Costs),8: No.of cond. param. used
            !9: No. of avail. techn.   10: Discount variation,   11: Sensitivity Horizon,
            !12: Sensitivity age ,13: Sensitivity Technology, 14: Sensitivity CP1,
            !15: Sensitivity CP2, 16: Sensitivity CP3, 17: Sensitivity CP4
            !18: Timescale, 19: Max no. of ageclasses  20: Dummy, 21: No of CP1 levels,
            !22: No of CP2 levels, 23: No of CP3 levels, 24: No. of CP4 levels, 25: Dummy
            !26: CP1-Evident/Hidden, 27: CP2-Evident/Hidden, 28: CP3-Evident/Hidden,
            !29: CP4-Evident/Hidden,30: Dummy,31: Lower cond. bound-CP1 ,
            !32: Lower cond. bound-CP2, 33: Lower cond. bound-CP3,
            !34: Lower cond. bound-CP4  35: Dummy
REAL*8, DIMENSION(1:1,1:4,1:25,1:27,0:200)  :: CTPM
            ! 1:Technology, 2: Condition parameter 3: Present condition, 4: Next condition
            ! 5: Time elapsed since last inspection
REAL*8, DIMENSION(1:3,1:25,1:12,1:1,1:1)         :: capacity, energy_c
            ! 1: Technology, 2: CP1, 3: CP2, 4: CP3 ,5: CP4
REAL*8, DIMENSION(1:3,1:25,1:12,1:1,1:1,1:6)     :: maint_c
            ! 1: Technology, 2: CP1, 3: CP2, 4: CP3 ,5: CP4, 6: Age
REAL*8, DIMENSION(1:5,1:25,1:12,1:1,1:1,1:6)     :: restore_c, salvage
            ! 1: From technology x to technology y 2: CP1, 3: CP2, 4: CP3 ,5: CP4, 6: Age
REAL*8, DIMENSION(1:15)                          :: ec_value
            ! 1: Discount rate, 2: Income, 3: Energy costs, 4: Maintenance costs,
            ! 5: Restoration costs, 6: Penalty value, 7: Salvage value,
            ! 8: MDT_Planned, 9:MDT_Unplanned, 10: MDT_Inspect, 11: Inspection costs,
            !12: Epsilon (termination value if discount rate<1), 13-15:Dummy
REAL*8, DIMENSION(1:400)     :: demand, planned_shutdown
REAL*8, DIMENSION(1:3,1:400):: p_tech
            ! Probability of available technology at stage i; 1: Tech. A, 2: Techn B,
            ! 3: Tech.  C

INTEGER*4,DIMENSION(1:200,1:2,0:1,1:12,1:12,1:1,1:1)     :: decision, next_inspection
            ! 1: Horizon, 2: Technology, 3: Age , 4: CP1, 5: CP2, 6: CP3, 7:CP4
REAL*8, DIMENSION(1:200,1:2,0:1,1:25,1:1 ,1:1,1:1)       :: b, c, f, g
            ! 1: Horizon, 2: Technology, 3: Age , 4: CP1, 5: CP2, 6: CP3, 7:CP4
REAL*8, DIMENSION(1:250,1:4)     :: disc_sens
REAL*8, DIMENSION(1:3,1:4,1:25) :: res_life
```

```
            ! 1: Technology, 2: Condition parameter, 3:Residual life at condition i

INTEGER*4, DIMENSION(1:5)    :: imax, max_k
REAL*8                       :: keep, modify, modify_with_A, modify_with_B,
REAL*8                       :: modify_with_C, modify_with_D, modify_with_E
REAL*8                       :: beta, discount, stop_crit ,stop_sens, max_profit_keeping,
REAL*8                       :: max_profit_modifying
REAL*8                       :: c_upper, c_lower, gap, max_income


CALL input(caseoption, ec_value, demand, planned_shutdown, capacity, energy_c, &
           maint_c, restore_c, salvage, p_tech, CTPM, res_life)


! Initializaion of the discount rate sensitivity check method.
beta = ec_value(1)
discount=beta
stop_crit=ec_value(1)    ! stopping criteria and step size for the discount
                         ! sens.test - remains
stop_sens=10             ! unchanged if sensitivity_check.NE.1


sensitivity_check=caseoption(10) ! 1: equals discount rate sensitivity calculation
IF (caseoption(10).eq.1) THEN
    stop_crit=ec_value(1)+(ec_value(1)/100)
    beta=ec_value(1)-(ec_value(1)/100)
    stop_sens=MIN(((stop_crit-beta)/10),0.005)
    count=1
ENDIF


cp1_levels=caseoption(21)+caseoption(26)
cp2_levels=caseoption(22)+caseoption(27)
cp3_levels=caseoption(23)+caseoption(28)
cp4_levels=caseoption(24)+caseoption(29)

max_k=999
IF ((caseoption(21).eq.1).or.((caseoption(22).eq.1).and.(caseoption(8)>1)) &
    .or.((caseoption(23).eq.1).and.(caseoption(8)>2)).or.&
    ((caseoption(24).eq.1)).and.(caseoption(8)>3)) THEN
    DO j=1, caseoption(9)
        !max_kj=min(res_life(j,1,1),res_life(j,2,1),res_life(j,3,1),res_life(j,4,1))
        !max_k= min(max_kj,max_k)
        max_k(j)=min(res_life(j,1,1),res_life(j,2,1),res_life(j,3,1),res_life(j,4,1))
    END DO
ELSE
    max_k=0
END IF


!************************************************************
! ****                                                  ****
!   ****    RETURN FUNCTION MATRIX GENERATION         ****
! **** (ref.eq. 5.2 and 5.3)                            ****
!************************************************************

DO i=1, caseoption(1)       ! Horizon
    DO j=1, caseoption(9)   ! Options; 1= Use technology (A), 2= Use technology (B),
                            ! 3= Use technology (C), 4= Use technology (D),
                            ! 5= Use technology (E)
        DO k=0, max_k(j)                        ! Age
            DO l=1, CP1_levels                  ! CP1
                DO m=1, CP2_levels              ! CP2
                    DO n=1, CP3_levels          ! CP3
                        DO o=1, CP4_levels      ! CP4
                            g(i,j,k,l,m,n,o)=income(i,j,l,m,n,o) - &
                            op_costs(j,k,l,m,n,o)*(1-planned_shutdown(caseoption(1)+1-i))
                            f(i,j,k,l,m,n,o)=-99999
                        END DO
                    END DO
                END DO
            END DO
        END DO
    END DO
END DO

! ---END RETURN FUNCTION GENERATION---



!************************************************************
```

```
! ****                                                          ****
!   ****    RECURRENCE FUNCTION (OBJECTIVE)                       ****
! ****                                                          ****
!************************************************************

i=1 !Equals the stage at the End-Of-Horizon
print *, i
DO j=1, caseoption(9)   ! Options; 1= Use technology (A), 2= Use technology (B),
                        ! 3= Use technology (C), 4= Use technology (D), 5= Use technology (E)
     DO k= 0, max_k(j)                      ! Age       |
          DO l=1, CP1_levels                ! CP1       ||
               DO m=1, CP2_levels           ! CP2       ||| -> Present condition
                    DO n=1, CP3_levels      ! CP3       ||
                         DO o=1, CP4_levels ! CP4       |


               !Possible decisions at stage i:
               !d=1 :do nothing

               keep= - insp_costs(i,j,L,m,n,o)+puni(i,j,k,L,m,n,o,1)

               !d=2 :inspect
               !(an inspection has to be done to reveal the condition before a
               ! planned action)
               !d=3 :overhaul/replace by similar or new technology

               ! Technology E in operation
                 modify_with_E=  - oh_costs(i,j,k,caseoption(9),l,m,n,o) &
                      - insp_costs(i,j,L,m,n,o) + puni(i,caseoption(9),0,1,1,1,1,1)
                 modify=modify_with_E

               ! ASSUMPTION: p_A >= p_B >= p_C >= p_D >= p_E  for all stages ):
               !                 p_A=p_tech_(A,i)
               ! ASSUMPTION: p_A >= p_B >= p_C >= p_D >= p_E  for all stages ): p_A=p_tech_(A,i)

               IF (j<caseoption(9)) THEN ! Technology D in operation
                  modify_with_D=- oh_costs(i,j,k,(caseoption(9)-1),l,m,n,o) &
                  - insp_costs(i,j,L,m,n,o)+puni(i,(caseoption(9)-1),0,1,1,1,1,1)

                  modify= modify_with_D*(1-p_tech(caseoption(9),i)) + &
                          modify_with_E*p_tech(caseoption(9),i)

                 IF (j<caseoption(9)-1) THEN ! Technology C in operation
                    modify_with_C=- oh_costs(i,j,k,(caseoption(9)-2),l,m,n,o) &
                    - insp_costs(i,j,L,m,n,o)+puni(i,(caseoption(9)-2),0,1,1,1,1,1)

                    modify= modify_with_C*(1-p_tech((caseoption(9)-1),i))*(1-p_tech(caseoption(9),i))+ &
                            modify_with_D*p_tech((caseoption(9)-1),i)*(1-p_tech(caseoption(9),i)) + &
                            modify_with_E*p_tech(caseoption(9),i)


                    IF (j<caseoption(9)-2) THEN ! Technology B in operation
                      modify_with_B=- oh_costs(i,j,k,(caseoption(9)-3),l,m,n,o) - insp_costs(i,j,L,m,n,o) &
                      +puni(i,(caseoption(9)-3),0,1,1,1,1,1)

                      modify= modify_with_B*(1-p_tech((caseoption(9)-2),i))&
                           *(1-p_tech((caseoption(9)-1),i))*(1-p_tech(caseoption(9),i))&
                           + & modify_with_C*p_tech((caseoption(9)-2),i)*(1-p_tech((caseoption(9)-1),i))&
                           *(1-p_tech(caseoption(9),i))+ modify_with_D*p_tech((caseoption(9)-1),i)&
                           *(1-p_tech(caseoption(9),i)) + modify_with_E*p_tech(caseoption(9),i)

                       IF (j<caseoption(9)-3) THEN ! Technology A in operation (modeling 5 technologies)

                           modify_with_A=- oh_costs(i,j,k,(caseoption(9)-4),l,m,n,o) &
                                      - insp_costs(i,j,L,m,n,o)+puni(i,(caseoption(9)-4),0,1,1,1,1,1)
                                      modify= modify_with_A*(1-p_tech((caseoption(9)-3),i))&
                                      *(1-p_tech((caseoption(9)-2),i))*(1-p_tech((caseoption(9)-1),i))&
                                      *(1-p_tech(caseoption(9),i)) &
                                      + modify_with_B*p_tech((caseoption(9)-3),i)&
                                      *(1-p_tech((caseoption(9)-2),i))*(1-p_tech((caseoption(9)-1),i))&
                                      *(1-p_tech(caseoption(9),i))+ &
                                      modify_with_C*p_tech((caseoption(9)-2),i)*&
                                      (1-p_tech((caseoption(9)-1),i))*(1-p_tech(caseoption(9),i))+ &
                                      modify_with_D*p_tech((caseoption(9)-1),i)&
                                      *(1-p_tech(caseoption(9),i))+modify_with_E*p_tech(caseoption(9),i)
                         END IF
                      END IF
                    END IF
                  END IF
```

```
                        IF ((keep.ge.modify) &
                            .and.((L<caseoption(31)).and.(m<caseoption(32)).and.&
                            (n<caseoption(33)).and.(o<caseoption(34)))) THEN

                            decision(i,j,k,l,m,n,o)=1
                            next_inspection(i,j,k,l,m,n,o)=1
                            b(i,j,k,l,m,n,o)=keep-modify     !the benefit of keeping the equipment
                                                             !instead of modifying
                            c(i,j,k,l,m,n,o)=income(i,j,l,m,n,o)*beta-keep
                            f(i,j,k,l,m,n,o)=keep
                        ELSE
                            decision(i,j,k,l,m,n,o)=3
                            next_inspection(i,j,k,l,m,n,o)=1
                            b(i,j,k,l,m,n,o)=keep-modify
                            c(i,j,k,l,m,n,o)=income(i,j,l,m,n,o)*beta-modify
                            f(i,j,k,l,m,n,o)=modify
                        END IF
                    END DO  !CP4
                END DO  !CP3
            END DO  !CP2
        END DO  !CP1
    END DO  !Age
END DO  !Technology

! Second to n'th stage


delta_1_add=0
delta_2_add=0
delta_3_add=0
delta_4_add=0
IF (caseoption(21)>1) THEN    !Necessary variables used to switch between
                              !age or condition dependent state description
    delta_1_add=1
END IF
IF (caseoption(22)>1) THEN
    delta_2_add=1
END IF
IF (caseoption(23)>1) THEN
    delta_3_add=1
END IF
IF (caseoption(24)>1) THEN
    delta_4_add=1
END IF

700 DO WHILE (beta<(stop_crit+stop_sens)) !sensitivity tests
    DO i=2, caseoption(1)
        print *, i, (c_upper-c_lower)
        DO j=1, caseoption(9)         ! Options; 1= Use technology (A),
                    !2= Use technology (B), 3= Use technology (C),
                    !4= Use technology (D), 5= Use technology (E)
            DO k= 0, max_k(j)         ! Age
                DO L=1, CP1_levels        ! CP1
                    DO m=1, CP2_levels        ! CP2
                        DO n=1, CP3_levels        ! CP3
                            DO o=1, CP4_levels        ! CP4


!******************************************************************
!
!PARAGRAPH MOVED LEFT TO FIT PRINTABLE LAYOUT



DO delta_t=1, MIN(i,INT(res_life(j,1,L)-1)+delta_1_add,INT(res_life(j,2,m)-1)+ &
        delta_2_add, INT(res_life(j,3,n)-1)+delta_3_add,INT(res_life(j,4,o)-1)+&
        delta_4_add)

    ! If a failure mode is only described by two condition levels
    !--- working and failed --, present condition is only determined by the age.
    ! Thus age is used to describe the present condition instead of 'time elapsed
    ! since last inspection at a spesific condition level'.
    delta_1_t=delta_t
    delta_2_t=delta_t
    delta_3_t=delta_t
    delta_4_t=delta_t
    IF (caseoption(21).eq.1) THEN   !CP1
        delta_1_t=k+delta_t
    END IF
    IF ((caseoption(22).eq.1).and.(caseoption(8)>1)) THEN   !CP2
```

```
        delta_2_t=k+delta_t
    END IF
    IF ((caseoption(23).eq.1).and.(caseoption(8)>2)) THEN  !CP3
        delta_3_t=k+delta_t
    END IF
    IF ((caseoption(23).eq.1).and.(caseoption(8)>3)) THEN  !CP4
        delta_4_t=k+delta_t
    END IF

    !Possible decisions at stage i:

    ! ************************************************************
    ! * Nomenclature:                                            *
    ! *                                                          *
    ! * - puni(*) equals G(*) as described by eq. 5.8 and gives  *
    ! *       the net profit of operating to the next inspection *
    ! *                                                          *
    ! ************************************************************

    !d=1 :do nothing

    IF (delta_t.eq.i) THEN
       keep=puni(i,j,k,L,m,n,o,delta_t) - insp_costs(i,j,L,m,n,o)
    ELSE
       keep=puni(i,j,k,L,m,n,o,delta_t) - insp_costs(i,j,L,m,n,o) &
       +(beta**(delta_t))* profit(i-delta_t,j,k,L,m,n,o,delta_t)
    END IF
    IF (f(i,j,k,l,m,n,o).le.keep) THEN
        IF (beta.ne.1.0) THEN
            discount=beta*(1-beta**(i))/(1-beta)
        ELSE
            discount=i
        END IF
        decision(i,j,k,l,m,n,o)=1
        next_inspection(i,j,k,l,m,n,o)=delta_t
        max_profit_keeping=keep
        f(i,j,k,l,m,n,o)=keep
        max_income=0

        DO y=1, i
            max_income = max_income+income(y,j,1,1,1,1)*beta**y
        END DO
        c(i,j,k,l,m,n,o) = max_income- f(i,j,k,l,m,n,o)
    END IF
ENDDO   !delta_t


!d=3 :replace (replace with the best technology available)
 max_profit_modifying=-999999

DO delta_t=1, MIN(i,INT(res_life(j,1,1)-1)+&
        delta_1_add,INT(res_life(j,2,1)-1)+delta_2_add,  &
        INT(res_life(j,3,1)-1)+delta_3_add,INT(res_life(j,4,1)&
        -1)+delta_4_add)

! If a failure mode is only described by two condition levels
!--- working and failed --, present condition is only determined by the age.
! Thus age is used to describe the present condition instead of 'time elapsed
! since last inspection at a spesific condition level'.

    ! Technology E in operation.
    modify_with_E=-oh_costs(i,j,k,caseoption(9),L,m,n,o) &
                - insp_costs(i,j,L,m,n,o)+puni(i,caseoption(9),0,1,1,1,1,delta_t)
                !puni: Profit Until Next Inspection
    IF (delta_t<i) THEN
        modify_with_E=modify_with_E + (beta**(delta_t))&
                      *profit(i-delta_t,caseoption(9),0,1,1,1,1,delta_t)
    ENDIF
    modify=modify_with_E


    ! ASSUMPTION: p_A >= p_B >= p_C >= p_D >= p_E  for all stages ): p_A=p_tech_(A,i)

    IF (j<caseoption(9)) THEN ! Technology D in operation
       modify_with_D=-oh_costs(i,j,k,caseoption(9)-1,L,m,n,o) &
       - insp_costs(i,j,L,m,n,o)+ puni(i,caseoption(9)-1,0,1,1,1,1,delta_t)
                 !puni: Profit Until Next Inspection
       IF (delta_t<i) THEN
           modify_with_D=modify_with_D + (beta**(delta_t))&
                         *profit(i-delta_t,caseoption(9)-1,0,1,1,1,1,delta_t)
       ENDIF
```

```
        modify= modify_with_D*(1-p_tech(caseoption(9),i)) + &
                modify_with_E*p_tech(caseoption(9),i)

        IF (j<caseoption(9)-1) THEN ! Technology C in operation
           modify_with_C=-oh_costs(i,j,k,caseoption(9)-2,L,m,n,o) &
                  - insp_costs(i,j,L,m,n,o) + puni(i,caseoption(9)-2,0,1,1,1,1,delta_t)
                  !puni: Profit Until Next Inspection
        IF (delta_t<i) THEN
            modify_with_C=modify_with_C + (beta**(delta_t))&
                   *profit(i-delta_t,caseoption(9)-2,0,1,1,1,1,delta_t)
        ENDIF
        modify= modify_with_C*(1-p_tech((caseoption(9)-1),i))*(1-p_tech(caseoption(9),i))+ &
             modify_with_D*p_tech((caseoption(9)-1),i)*(1-p_tech(caseoption(9),i)) + &
             modify_with_E*p_tech(caseoption(9),i)
             IF (j<caseoption(9)-2) THEN ! Technology B in operation
                modify_with_B=-oh_costs(i,j,k,caseoption(9)-3,L,m,n,o) &
                - insp_costs(i,j,L,m,n,o) + puni(i,caseoption(9)-3,0,1,1,1,1,delta_t)
                !puni: Profit Until Next Inspection
                IF (delta_t<i) THEN
                    modify_with_B=modify_with_B + (beta**(delta_t))&
                    *profit(i-delta_t,caseoption(9)-3,0,1,1,1,1,delta_t)
                ENDIF
                modify= modify_with_B*(1-p_tech((caseoption(9)-2),i))&
                        *(1-p_tech((caseoption(9)-1),i))*(1-p_tech(caseoption(9),i))+ &
                         modify_with_C*p_tech((caseoption(9)-2),i)&
                         *(1-p_tech((caseoption(9)-1),i))*(1-p_tech(caseoption(9),i))+ &
                         modify_with_D*p_tech((caseoption(9)-1),i)&
                         *(1-p_tech(caseoption(9),i))+modify_with_E*p_tech(caseoption(9),i)

                    IF (j<caseoption(9)-3) THEN ! Technology A in operation (modeling 5 technologies)
                        modify_with_A= -oh_costs(i,j,k,caseoption(9)-4,L,m,n,o) &
                                     - insp_costs(i,j,L,m,n,o) + &
                                     puni(i,caseoption(9)-4,0,1,1,1,1,delta_t)
                                     !puni: Profit Until Next Inspection
                        IF (delta_t<i) THEN
                            modify_with_A=modify_with_A + (beta**(delta_t))&
                                          *profit(i-delta_t,caseoption(9)-4,0,1,1,1,1,delta_t)
                        ENDIF
                            modify= modify_with_A*(1-p_tech((caseoption(9)-3),i))&
                              *(1-p_tech((caseoption(9)-2),i))*(1-p_tech((caseoption(9)-1),i))&
                              *(1-p_tech(caseoption(9),i))+modify_with_B*p_tech((caseoption(9)-3),i)&
                              *(1-p_tech((caseoption(9)-2),i))*(1-p_tech((caseoption(9)-1),i))&
                              *(1-p_tech(caseoption(9),i))+ modify_with_C*p_tech((caseoption(9)-2),i)&
                              *(1-p_tech((caseoption(9)-1),i))*(1-p_tech(caseoption(9),i))+ &
                               modify_with_D*p_tech((caseoption(9)-1),i)*(1-p_tech(caseoption(9),i)) + &
                              modify_with_E*p_tech(caseoption(9),i)
                    END IF
                 END IF
             END IF
        END IF

     IF ((f(i,j,k,l,m,n,o).le.modify) &
         .or.((L.ge.caseoption(31)).or.(m.ge.caseoption(32)).or.(n.ge.caseoption(33)).or.&
         (o.ge.caseoption(34)))) THEN

         !IF (beta.ne.1.0) THEN
         !    discount=beta*(1-beta**(i))/(1-beta)
         !ELSE
         !    discount=i
         !END IF
         decision(i,j,k,l,m,n,o)=4
         next_inspection(i,j,k,l,m,n,o)=delta_t
         f(i,j,k,l,m,n,o)=modify
         max_income=0
         DO y=1, i
             max_income = max_income + income(y,j,1,1,1,1)*beta**y
         END DO
         c(i,j,k,l,m,n,o) = max_income- f(i,j,k,l,m,n,o)

     END IF
     IF (max_profit_modifying.le.modify) THEN
          max_profit_modifying=modify
     END IF
END DO !delta_t
b(i,j,k,l,m,n,o)=max_profit_keeping-max_profit_modifying
        !the benefit of keeping the equipment instead of modifying



!PARAGRAPH END
!******************************************************************
```

```
                        END DO   !CP4
                    END DO   !CP3
                END DO   !CP2
            END DO   !CP1
        END DO   !Technology
    END DO   !Age
        !Error Bounds
    ! Quit approximation when the error is belowe "epsilon"
    c_upper=0
    c_lower=9999999

    IF (beta<1) THEN
        DO k= 0, max_k(j)                                  ! Age
            DO L=1, CP1_levels                    ! CP1
                DO m=1, CP2_levels                ! CP2
                    DO n=1, CP3_levels            ! CP3
                        DO o=1, CP4_levels        ! CP4
                            gap=(beta/(1-beta))*(f(i,1,k,L,m,n,o)-&
                            f(i-1,1,k,L,m,n,o))
                            IF (c_lower>gap) THEN
                                c_lower=gap
                            END IF
                            IF (c_upper<gap) THEN
                                c_upper=gap
                            END IF
                        END DO
                    END DO
                END DO
            END DO
        END DO
        IF (ec_value(12)>(c_upper-c_lower)) THEN
                caseoption(1)=i
                GOTO 900
        END IF
    END IF !Error bounds

END DO   !Horizon
    !---Discount rate sensitivity tests---
    IF (sensitivity_check.EQ.1) THEN

    disc_sens(count,1)= beta
    disc_sens(count,2)= c(caseoption(1),caseoption(2),caseoption(3),&
    caseoption(4),caseoption(5),caseoption(6),caseoption(7))
    disc_sens(count,3)= f(caseoption(1),caseoption(2),caseoption(3),&
    caseoption(4),caseoption(5),caseoption(6),caseoption(7))
    disc_sens(count,4)= next_inspection(caseoption(1),caseoption(2),&
    caseoption(3),caseoption(4),caseoption(5),caseoption(6),caseoption(7))
    count=count+1
    beta=beta+stop_sens
ELSE
    beta=stop_crit+stop_sens
ENDIF
END DO ! DO WHILE LOOP


900 IF (sensitivity_check.EQ.1) THEN !Performs a new calculation of
                              !keep, modify and decision array
                              !for the initial discount rate. This is necessary because these
                              !arrays contain values based on maximum discount rate solutions.

    beta= ec_value(1)     ! discount rate
    stop_crit=beta        ! stopping criteria and step size for the discount
                          ! sens.test- remains
    stop_sens=10          ! unchanged if sensitivity_check.NE.1
    sensitivity_check=0 ! Turns off the discount rate sensitivity calculation
    GOTO 700

ENDIF

! --- END RECURRENCE FUNCTION---


CAll output(caseoption, ec_value, f, b, c, next_inspection, &
disc_sens, decision, res_life)
```

```
!************************************************************
! ****                                                  ****
CONTAINS
!    ****                                             ****
! ****                                                  ****
!************************************************************


SUBROUTINE condition_class(L,m,n,o, L_class, m_class, n_class, o_class, arg)

INTEGER ::L,m,n,o, L_class, m_class, n_class, o_class, arg


! SELECT ONE OF THE OPTIONS BELOW:

IF (arg .eq. 1) THEN
! OPTION 1:
!    Only to be used if the number of economic condition classes with respect to
!    capacity are less than the number of condition levels in use to classify
!    the condition.

    L_class=cp_class(1,L)
    m_class=1
    n_class=1
    o_class=1
    !Defining condition limits
    IF (caseoption(8) > 1) THEN
        m_class=cp_class(2,m)
        IF (caseoption(8) > 2) THEN
            n_class=cp_class(3,n)
            IF (caseoption(8) > 3) THEN
                o_class=cp_class(4,o)
            END IF
        END IF
    END IF
ELSE IF (arg .eq. 2) THEN
! OPTION 2:
!    Used when the number of condition levels in use to classify the condition
!    equals the number of economic classes with respect to overhaul costs (the
!    restore_c(.....) array).
    L_class=L
    m_class=m
    n_class=n
    o_class=o
END IF
END SUBROUTINE  condition_class

FUNCTION cp_class(t,u)
! The function classifies the present condition according to a predefined condition
! scheme. The vaule is used to select the correct income or cost at present
! condition. At present time 5 different condition classes are defined, where
! condition class 5 also is used to represent a breakdown situation.

INTEGER :: cp_class, t, u, cp_levels

cp_levels=caseoption(20+t)
cp_class=u

! This procedure has to be adjusted for the spesific case and number of
! condition levels
IF (cp_levels.ne.1) THEN
    IF (cp_levels.eq.10) THEN            ! Rupture  Application-Gas turbine

    cp_class=u
    !   IF(u<3) THEN
    !       cp_class=1                        ! New equipment
    !   ELSE IF ((u>2).and.(u<5)) THEN
    !       cp_class=2
    !   ELSE IF ((u>4).and.(u<7)) THEN
    !       cp_class=3
    !   ELSE IF ((u>6).and.(u<9)) THEN
    !       cp_class=4
    !   ELSE IF ((u>8).and.(u<11)) THEN
    !       cp_class=5   !   6              ! Breakdown
    !   END IF


    ELSEIF (cp_levels.eq.8) THEN            ! Efficiency - Application-Gas turbine, table 7.3
        IF(u<INT((1/8.0)*cp_levels)+1) THEN
            cp_class=1                        ! New equipment
        ELSE IF ((u>INT((1/8.0)*cp_levels)).and.(u<INT((2/8.0)*cp_levels)+1)) THEN
```

```
            cp_class=2
        ELSE IF ((u>INT((2/8.0)*cp_levels)).and.(u<INT((3/8.0)*cp_levels)+1)) THEN
            cp_class=3
        ELSE IF ((u>INT((3/8.0)*cp_levels)).and.(u<INT((4/8.0)*cp_levels)+1)) THEN
            cp_class=4
        ELSE IF ((u>INT((4/8.0)*cp_levels)).and.(u<INT((5/8.0)*cp_levels)+1)) THEN
            cp_class=5
        ELSE IF ((u>INT((5/8.0)*cp_levels)).and.(u<INT((6/8.0)*cp_levels)+1)) THEN
            cp_class=6
        ELSE IF ((u>INT((6/8.0)*cp_levels)).and.(u<INT((7/8.0)*cp_levels)+1)) THEN
            cp_class=7
        ELSE
            cp_class=8  !  8              ! Breakdown
        END IF


        ELSE   ! Efficiency — with 5 condition levels (Table 5.1)
        IF(u<INT((1/5.0)*cp_levels)+1) THEN
            cp_class=1          ! New equipment
        ELSE IF ((u>INT((1/5.0)*cp_levels)).and.(u<INT((2/5.0)*cp_levels)+1)) THEN
            cp_class=2
        ELSE IF ((u>INT((2/5.0)*cp_levels)).and.(u<INT((3/5.0)*cp_levels)+1)) THEN
            cp_class=3
        ELSE IF ((u>INT((3/5.0)*cp_levels)).and.(u<INT((4/5.0)*cp_levels)+1)) THEN
            cp_class=4
        ELSE
            cp_class=5  !  5              ! Breakdown
        END IF
    END IF
END IF


END FUNCTION cp_class


SUBROUTINE age_class(k,d)
  !Determines the position "d" (age class) in the cost matrixes based on the
  !age "k". The routine is applied to the maintenance cost, salvage value
  !and operational cost matrixes.

INTEGER :: k,d


IF (caseoption(18).eq.1) THEN   ! Timescale months
    IF (k<13) THEN                      !   necessary to move the pointer d rows in
        d=1                             !   the energy & maintenance cost matrix based on age j
    ELSE IF ((k>12).AND.(k<25))  THEN  ! 2nd year
        d=2
    ELSE IF ((k>24).AND.(k<37)) THEN    ! 3rd year
        d=3
    ELSE IF ((k>36).AND.(k<47)) THEN    ! 4th year
        d=4
        ELSE
        d=5
    END IF
ELSEIF (caseoption(18).eq.2) THEN  ! Timescale 1000 Hours
    IF (k<10) THEN                      !   necessary to move the pointer d rows in
        d=1                             !   the energy & maintenance cost matrix based on age j
    ELSE IF ((k>9).AND.(k<19))  THEN   ! 2nd year
        d=2
    ELSE IF ((k>18).AND.(k<28)) THEN   ! 3rd year
        d=3
    ELSE IF ((k>27).AND.(k<37)) THEN   ! 4th year
        d=4
        ELSE
        d=5
    END IF
ELSE   !Timescale years
    IF (k.le.1) THEN
         d=1
    ELSE IF ((1<k).and.(k<5)) THEN
         d=k
    ELSE
         d=5
    ENDIF
END IF

END SUBROUTINE age_class
```

```
          ! Stage
          ! | Options; 1= Use technology(A),2=Use technology(B),3=Use technology(C)...
          ! | | CP1
          ! | | | CP2
          ! | | | | CP3
          ! | | | | | CP4
          ! | | | | | |
FUNCTION income(i,j,L,m,n,o)

! This function calculates income for the next time period based on following
! information: present stage, age of facility since last major action,
! decision made at present time, CP1, CP2, CP3 and CP4. A penalty cost is
! calculated if production shortage exists.

INTEGER :: i, j, L, m, n, o, L_class, m_class, n_class, o_class
REAL    :: income

CALL condition_class(L,m,n,o, L_class, m_class, n_class, o_class, 1)


  !Income= production value - penalty due to two low production ec_value.
  !Note that the penalty is additional to loss of production.
  !(Equation 5.2)

income=   ec_value(2)*(MIN(demand(caseoption(1)+1-i), &
             capacity(j,L_class,m_class,n_class,o_class))) &
             * (1.0-planned_shutdown(caseoption(1)+1-i)) &
             - ec_value(6)*demand(caseoption(1)+1-i)&
             *planned_shutdown(caseoption(1)+1-i) &
             - ec_value(6)*(MAX(0.0, (demand(caseoption(1)+1-i)&
             -capacity(j,L_class,m_class,n_class,o_class))))&
             * (1.0-planned_shutdown(caseoption(1)+1-i))

END FUNCTION   income



FUNCTION sum_production_shutdown(i,action_downtime)

INTEGER    :: i, u, u_lower
REAL*8     :: action_downtime, production_shutdown, sum_production_shutdown

u_lower=1+INT(action_downtime-.5)
production_shutdown=0.0
DO u=1,u_lower
   IF ((u-i).le.(0)) THEN
           production_shutdown=production_shutdown+&
           planned_shutdown(caseoption(1)+u-i)
   ENDIF
END DO
  sum_production_shutdown=production_shutdown

END FUNCTION sum_production_shutdown


                    !  Stage
                    ! Age
                    ! | CP1
                    ! | | CP2
                    ! | | | CP3
                    ! | | | | CP4
                    ! | | | | |
FUNCTION insp_costs(i,j,l,m,n,o)

INTEGER :: i,j,l,m,n,o,L_class, m_class, n_class, o_class
REAL        :: insp_costs

CALL condition_class(L,m,n,o, L_class, m_class, n_class, o_class, 1)

  !(Equation 5.4)

insp_costs= ec_value(11)+ max(0.0, ec_value(10)-&
            sum_production_shutdown(i,ec_value(10))) &
            * (ec_value(2)*ec_value(13)*min(demand(caseoption(1)+1-i),&
            capacity(j,L_class,m_class,n_class,o_class)) &
               - op_costs(j,k,L,m,n,o))

END FUNCTION insp_costs
```

```
                 ! Stage     (The stage is included due to the necessity
                 !               to model change in salvage value due to age
                 ! |             since release. This is not included at present time)
                 ! | Present technology
                 ! | | Age
                 ! | | | Options; (1: -), 2= Replace with technology (A),
                                     !3= Replace technology (B)...
                 ! | | | | CP1
                 ! | | | | | CP2
                 ! | | | | | | CP3
                 ! | | | | | | | CP4
                 ! | | | | | | |     |
FUNCTION  oh_costs(i,j,k,e,l,m,n,o) ! Overhaul costs

INTEGER     :: d,e,i,j,k,l,m,n,o, L_class, m_class, n_class, o_class

REAL        :: oh_costs

d=i           ! only introduced due to supress warning FOR4269

CALL age_class(k,d)
CALL condition_class(L,m,n,o, L_class, m_class, n_class, o_class, 1)

  !(Equation 5.5)

oh_costs = ec_value(5)*restore_c(pos(j,e),L_class,m_class,n_class,o_class,d) &
           - ec_value(7)*salvage(pos(j,e),L_class,m_class,n_class,o_class,d) &
           + (max(0.0,ec_value(8)+ec_value(10)-&
           sum_production_shutdown(i,ec_value(8)+ec_value(10))) &
              -max(0.0, ec_value(10)-sum_production_shutdown(i,ec_value(10)))) &
           * (ec_value(2)*ec_value(13)*min(demand(caseoption(1)+1-i),&
           capacity(j,L_class,m_class,n_class,o_class)) &
               - op_costs(j,k,L,m,n,o))


END FUNCTION oh_costs

                 ! Stage     (The stage is included due to the necessity
                                     !to model change in salvage
                 ! |              value due to age since release.
                                 !This is not included at present time)
                 ! | Present technology
                 ! | | Age
                 ! | | | Options; (1: -), 2= Replace with technology (A),
                                     !3= Replace technology (B)...
                 ! | | | | CP1
                 ! | | | | | CP2
                 ! | | | | | | CP3
                 ! | | | | | | | CP4
                 ! | | | | | | |     |
FUNCTION  ucm_costs(i,j,k,e,l,m,n,o) ! Overhaul costs

INTEGER     :: d,e,i,j,k,l,m,n,o, L_class, m_class, n_class, o_class

REAL        :: ucm_costs

d=i           ! only introduced due to supress warning FOR4269


CALL age_class(k,d)
CALL condition_class(L,m,n,o, L_class, m_class, n_class, o_class, 1)

  !(Equation 5.6)
ucm_costs = ec_value(5)*restore_c(pos(j,e),L_class,m_class,n_class,o_class,d) &
            - ec_value(7)*salvage(pos(j,e),L_class,m_class,n_class,o_class,d) &
           + max(0.0,ec_value(9)-sum_production_shutdown(i,ec_value(9))) &
           * (ec_value(2)*ec_value(13)*min(demand(caseoption(1)+1-i),&
           capacity(j,L_class,m_class,n_class,o_class)) &
               - op_costs(j,k,L,m,n,o))


END FUNCTION ucm_costs



FUNCTION pos(present_tech,new_tech)

INTEGER:: pos, present_tech, new_tech
```

```
IF (caseoption(9).eq.1) THEN
        pos=1
    ELSE IF (caseoption(9).eq.2) THEN
        IF ((present_tech.eq.1).and.(new_tech.eq.1)) THEN
            pos=1     !(A-A)
        ELSE IF ((present_tech.eq.1).and.(new_tech.eq.2)) THEN
            pos=2     !(A-B)
        ELSE
            pos=3     !(B-B)
        END IF
    ELSE IF (caseoption(9).eq.3) THEN
        IF ((present_tech.eq.1.).and.(new_tech.eq.1)) THEN
            pos=1     !(A-A)
        ELSE IF ((present_tech.eq.1).and.(new_tech.eq.2)) THEN
            pos=2     !(A-B)
        ELSE IF ((present_tech.eq.1).and.(new_tech.eq.3)) THEN
            pos=3     !(A-C)
        ELSE IF ((present_tech.eq.2).and.(new_tech.eq.2)) THEN
            pos=4     !(B-B)
        ELSE IF ((present_tech.eq.2).and.(new_tech.eq.3)) THEN
            pos=5     !(B-C)
        ELSE
            pos=6     !(C-C)
        END IF

! Note: "else if" option for technology D and E not presented

    END IF
END FUNCTION pos




                !  Technology
                !  | Age
                !  | | CP1
                !  | | | CP2
                !  | | | | CP3
                !  | | | | | CP4
                !  | | | | | |
FUNCTION  op_costs(j,k,l,m,n,o)   ! Operational costs

INTEGER :: d,j,k,l,m,n,o, L_class, m_class, n_class, o_class

REAL         :: op_costs

d=0          ! only introduced due to supress warning FOR4269

CALL age_class(k,d)

CALL condition_class(L,m,n,o, L_class, m_class, n_class, o_class, 1)

  !(Equation 5.3)

    op_costs= ec_value(3)*energy_c(j,L_class,m_class,n_class,o_class) &
             + ec_value(4)*maint_c(j,L_class,m_class,n_class,o_class,d)

END FUNCTION op_costs


              ! Next stage
              ! |Technology 1= Use technology (A), 2= Use technology (B),
                            !3= Use technology (C)...
              ! | | Age
              ! | | | CP1
              ! | | | | CP2
              ! | | | | | CP3
              ! | | | | | | CP4
              ! | | | | | | | Time elapsed since last inspection
              ! | | | | | | | |
FUNCTION profit(i,j,k,L,m,n,o,s)

INTEGER :: age, i, j, k, L, m, n, o, s, s1, s2, s3, s4
INTEGER :: cp1_next, cp2_next, cp3_next, cp4_next
INTEGER :: cp1_next_max, cp2_next_max, cp3_next_max, cp4_next_max
REAL         :: profit, acc_profit

!     ---initialisation---
acc_profit=0
s1=s
```

```
s2=s
s3=s
s4=s
age=k
cp1_next_max=caseoption(21)+caseoption(26)
cp2_next_max=caseoption(22)+caseoption(27)
cp3_next_max=caseoption(23)+caseoption(28)
cp4_next_max=caseoption(24)+caseoption(29)


IF (caseoption(21).eq.1) THEN  !CP1
    s1=MIN(k+s,INT(res_life(j,1,1)))
    age=MIN(k+s,INT(res_life(j,1,1)))
END IF
IF ((caseoption(22).eq.1).and.caseoption(8)>1) THEN  !CP2
    s2=MIN(k+s,INT(res_life(j,2,1)))
    age=MIN(k+s,INT(res_life(j,2,1)))
END IF
IF ((caseoption(23).eq.1).and.caseoption(8)>2) THEN  !CP3
    s3=MIN(k+s,INT(res_life(j,3,1)))
    age=MIN(k+s,INT(res_life(j,3,1)))
END IF
    IF ((caseoption(24).eq.1).and.caseoption(8)>3) THEN  !CP4
s4=MIN(k+s,INT(res_life(j,4,1)))
    age=MIN(k+s,INT(res_life(j,4,1)))
END IF


DO cp1_next=L, cp1_next_max
    IF (caseoption(8)>1) THEN
        DO cp2_next=m, cp2_next_max
            IF (caseoption(8)>2) THEN
                DO cp3_next=n, cp3_next_max
                    IF (caseoption(8)>3) THEN
                        DO cp4_next=o, cp4_next_max
                            acc_profit=acc_profit+ &
                                (CTPM(j,1,L,cp1_next,s1)/(1-CTPM(j,1,L,(cp1_next_max+1),s1)) )* &
                                (CTPM(j,2,m,cp2_next,s2)/(1-CTPM(j,2,m,(cp2_next_max+1),s2)) )* &
                                (CTPM(j,3,n,cp3_next,s3)/(1-CTPM(j,3,n,(cp3_next_max+1),s3)) )* &
                                (CTPM(j,4,o,cp4_next,s4)/(1-CTPM(j,4,o,(cp4_next_max+1),s4)) )* &
                                f(i,j,age,cp1_next,cp2_next,cp3_next,cp4_next)
                        END DO
                    ELSE
                        acc_profit=acc_profit+ &
                               (CTPM(j,1,L,cp1_next,s1)/(1-CTPM(j,1,L,(cp1_next_max+1),s1)) )* &
                               (CTPM(j,2,m,cp2_next,s2)/(1-CTPM(j,2,m,(cp2_next_max+1),s2)) )* &
                               (CTPM(j,3,n,cp3_next,s3)/(1-CTPM(j,3,n,(cp3_next_max+1),s3)) )* &
                               f(i,j,age,cp1_next,cp2_next,cp3_next,1)
                    ENDIF
                END DO
            ELSE
                acc_profit=acc_profit+ &
                        (CTPM(j,1,L,cp1_next,s1)/(1-CTPM(j,1,L,(cp1_next_max+1),s1)) )* &
                        (CTPM(j,2,m,cp2_next,s2)/(1-CTPM(j,2,m,(cp2_next_max+1),s2)) )* &
                        f(i,j,age,cp1_next,cp2_next,1,1)
            ENDIF
        END DO
    ELSE
        acc_profit=acc_profit+(CTPM(j,1,L,cp1_next,s1)&
        /(1-CTPM(j,1,L,(cp1_next_max+1),s1)) )* &
                f(i,j,age,cp1_next,1,1,1)
    END IF

END DO
profit=acc_profit
END FUNCTION profit




                ! Present stage
                ! | Options; 1= Use technology (A), 2= Use technology (B),
                                                !3= Use technology (C)...
                ! | | Age
                ! | | | CP1
                ! | | | | CP2
                ! | | | | | CP3
                ! | | | | | | CP4
                ! | | | | | | | Time elapsed since last inspection
                                    !where the condition was known
                ! | | | | | | | |
```

```
FUNCTION    puni(i,j,k,l,m,n,o,delta_t) !Generates the Profit Until
                                         !Next Inspection (PUNI)

INTEGER :: i,j,k,l,m,n,o,s, s1, s2, s3, s4, delta_t, l_next, m_next, n_next
INTEGER :: o_next, age, min_age
INTEGER :: cp1_next_max, cp2_next_max, cp3_next_max, cp4_next_max
REAL        :: puni,op, ucmc, acc_op, acc_ucmc, f_survived, savings
acc_op=0
acc_ucmc=0

cp1_next_max=caseoption(21)+caseoption(26)
cp2_next_max=caseoption(22)+caseoption(27)
cp3_next_max=caseoption(23)+caseoption(28)
cp4_next_max=caseoption(24)+caseoption(29)

! Semi Markov loop

DO s=1,delta_t
    s1=s
    s2=s
    s3=s
    s4=s
    age=k
    min_age=k+s
    IF (caseoption(21).eq.1) THEN   !CP1
        s1=MIN(min_age,INT(res_life(j,1,1)))
        age=s1
    END IF
    IF ((caseoption(22).eq.1).and.(caseoption(8)>1)) THEN   !CP2
        s2=MIN(min_age,INT(res_life(j,2,1)))
        age=s2
    END IF
    IF ((caseoption(23).eq.1).and.(caseoption(8)>2)) THEN   !CP3
        s3=MIN(min_age,INT(res_life(j,3,1)))
        age=s3
    END IF
    IF ((caseoption(23).eq.1).and.(caseoption(8)>2)) THEN   !CP4
        s4=MIN(min_age,INT(res_life(j,4,1)))
        age=s4     !s4-1
    END IF

    op=0
    ucmc=0
    f_survived=0
    savings=0
    DO L_next= L, cp1_next_max
        IF (caseoption(8)>1) THEN
            DO m_next= m, cp2_next_max
                IF (caseoption(8)>2) THEN
                    DO n_next= n, cp3_next_max
                        IF (caseoption(8)>3) THEN
                            DO o_next= o, cp4_next_max
                                !Operational profit s stages ahead
                                op= op + g(i-s+1,j,age,l_next,m_next,n_next,o_next) &
                                        *CTPM(j,1,L,L_next,s1)/(1-CTPM(j,1,L,cp1_next_max+1,s1-1))&
                                        *CTPM(j,2,m,m_next,s2)/(1-CTPM(j,2,m,cp2_next_max+1,s2-1))&
                                        *CTPM(j,3,n,n_next,s3)/(1-CTPM(j,3,n,cp3_next_max+1,s3-1))&
                                        *CTPM(j,4,o,o_next,s4)/(1-CTPM(j,4,o,cp4_next_max+1,s4-1))

                                IF ((i-s).ne.0)THEN
                                    f_survived=f_survived+f(i-s,j,age,L_next,m_next,n_next,o_next) &
                                        *CTPM(j,1,L,L_next,s1)/(1-CTPM(j,1,L,cp1_next_max+1,s1)) &
                                        *CTPM(j,2,m,m_next,s2)/(1-CTPM(j,2,m,cp2_next_max+1,s2))      &
                                        *CTPM(j,3,n,n_next,s3)/(1-CTPM(j,3,n,cp3_next_max+1,s3))      &
                                        *CTPM(j,4,o,o_next,s4)/(1-CTPM(j,4,o,cp4_next_max+1,s4))
                                END IF
                            END DO !o_next
                        ELSE
                            op= op + g(i-s+1,j,age,L_next,m_next,n_next,1) &
                                    *CTPM(j,1,L,L_next,s1)/(1-CTPM(j,1,L,cp1_next_max+1,s1-1))&
                                    *CTPM(j,2,m,m_next,s2)/(1-CTPM(j,2,m,cp2_next_max+1,s2-1))&
                                    *CTPM(j,3,n,n_next,s3)/(1-CTPM(j,3,n,cp3_next_max+1,s3-1))
                            IF ((i-s).ne.0) THEN
                                f_survived=f_survived+f(i-s,j,age,L_next,m_next,n_next,1) &
                                *CTPM(j,1,L,L_next,s1)/(1-CTPM(j,1,L,cp1_next_max+1,s1)) &
                                *CTPM(j,2,m,m_next,s2)/(1-CTPM(j,2,m,cp2_next_max+1,s2))      &
                                *CTPM(j,3,n,n_next,s3)/(1-CTPM(j,3,n,cp3_next_max+1,s3))
                            END IF
                        END IF
                    END DO  !n_next
                ELSE
```

```
                    op= op + g(i-s+1,j,age,L_next,m_next,1,1) &
                                *CTPM(j,1,L,L_next,s1)/(1-CTPM(j,1,L,cp1_next_max+1,s1-1))&
                                *CTPM(j,2,m,m_next,s2)/(1-CTPM(j,2,m,cp2_next_max+1,s2-1))
                    IF ((i-s).ne.0) THEN
                            f_survived=f_survived+f(i-s,j,age,L_next,m_next,1,1) &
                                    *CTPM(j,1,L,L_next,s1)/(1-CTPM(j,1,L,cp1_next_max+1,s1)) &
                                    *CTPM(j,2,m,m_next,s2)/(1-CTPM(j,2,m,cp2_next_max+1,s2))
                    END IF
                END IF
            END DO !m_next
        ELSE
        op= op+ g(i-s+1,j,age,L_next,1,1,1) &
            *CTPM(j,1,L,L_next,s1)/(1-CTPM(j,1,L,cp1_next_max+1,s1-1))
            IF ((i-s).ne.0)THEN
                f_survived=f_survived+f(i-s,j,age,L_next,1,1,1)*CTPM(j,1,L,L_next,s1)&
                /(1-CTPM(j,1,L,cp1_next_max+1,s1))
            ENDIF
        ENDIF
    END DO !l_next
        acc_op=acc_op+(beta**s)*(op)
        !
    !Unforceen corrective maintenance cost (UCMC) at stage i=N. The cost due to an
    !unforceen actions is calculated as the sum of restoring the item back to
    !as-good-as new condition, and cost due to loss of production (downtimecost).
    !It is not possible to modify to a new technology. Savings achieved
    !(over the time to end of horizon) after an unforceen replacement.
    !The item is restored back to a new condition at a higher cost compared to a
    !planned action.
        IF (caseoption(8)>1) THEN
        IF  (caseoption(8)>2) THEN
            IF (caseoption(8)>3) THEN
                    ucmc=ucm_costs(i-s+1,j,age,j,L,m,n,o)   &!->Cost of a failure at stage i
                            *(1 - (1 - (CTPM(j,1,L,(cp1_next_max+1),s1)&
                            -CTPM(j,1,L,(cp1_next_max+1),s1-1))&
                             /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))) &
                            *(1 - (CTPM(j,2,m,(cp2_next_max+1),s2)&
                            -CTPM(j,2,m,(cp2_next_max+1),s2-1))&
                            /(1-CTPM(j,2,m,(cp2_next_max+1),s2-1))) &
                            *(1 - (CTPM(j,3,n,(cp3_next_max+1),s3)&
                            -CTPM(j,3,n,(cp3_next_max+1),s3-1))&
                            /(1-CTPM(j,3,n,(cp3_next_max+1),s3-1))) &
                            *(1 - (CTPM(j,4,o,(cp4_next_max+1),s4)&
                            -CTPM(j,4,o,(cp4_next_max+1),s4-1))&
                            /(1-CTPM(j,4,o,(cp4_next_max+1),s4-1))))

                    IF ((i-s).ne.0) THEN
                        savings=(g(i-s+1,j,0,1,1,1,1)+f(i-s,j,0,1,1,1,1)- f_survived ) &
                                *(1 - (1 - (CTPM(j,1,L,(cp1_next_max+1),s1)&
                                -CTPM(j,1,L,(cp1_next_max+1),s1-1))&
                                /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))) &
                                *(1 - (CTPM(j,2,m,(cp2_next_max+1),s2)&
                                -CTPM(j,2,m,(cp2_next_max+1),s2-1))&
                                /(1-CTPM(j,2,m,(cp2_next_max+1),s2-1))) &
                                *(1 - (CTPM(j,3,n,(cp3_next_max+1),s3)&
                                -CTPM(j,3,n,(cp3_next_max+1),s3-1))&
                                /(1-CTPM(j,3,n,(cp3_next_max+1),s3-1))) &
                                *(1 - (CTPM(j,4,o,(cp4_next_max+1),s4)&
                                -CTPM(j,4,o,(cp4_next_max+1),s4-1))&
                                /(1-CTPM(j,4,o,(cp4_next_max+1),s4-1))))
                    END IF
            ELSE
                ucmc=ucm_costs(i-s+1,j,age,j,L,m,n,1)   &!   ->Cost of a failure at stage i
                        *(1 - (1 - (CTPM(j,1,L,(cp1_next_max+1),s1)&
                        -CTPM(j,1,L,(cp1_next_max+1),s1-1))&
                        /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))) &
                        *(1 - (CTPM(j,2,m,(cp2_next_max+1),s2)&
                        -CTPM(j,2,m,(cp2_next_max+1),s2-1))&
                        /(1-CTPM(j,2,m,(cp2_next_max+1),s2-1))) &
                        *(1 - (CTPM(j,3,n,(cp3_next_max+1),s3)&
                        -CTPM(j,3,n,(cp3_next_max+1),s3-1))&
                        /(1-CTPM(j,3,n,(cp3_next_max+1),s3-1))))
                    IF ((i-s).ne.0) THEN
                        savings=(g(i-s+1,j,0,1,1,1,1)+f(i-s,j,0,1,1,1,1)- f_survived ) &
                                        *(1 - &
                            (1 - (CTPM(j,1,L,(cp1_next_max+1),s1)&
                            -CTPM(j,1,L,(cp1_next_max+1),s1-1))&
                            /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))) &
                            *(1 - (CTPM(j,2,m,(cp2_next_max+1),s2)&
                            -CTPM(j,2,m,(cp2_next_max+1),s2-1))&
                            /(1-CTPM(j,2,m,(cp2_next_max+1),s2-1))) &
                            *(1 - (CTPM(j,3,n,(cp3_next_max+1),s3)&
```

```
                        -CTPM(j,3,n,(cp3_next_max+1),s3-1))&
                        /(1-CTPM(j,3,n,(cp3_next_max+1),s3-1))))
              !ELSE
              ! The code is not printed out but follows
              ! the same structure as presented above.

              END IF
              END IF
        ELSE
           ucmc=ucm_costs(i-s+1,j,age,j,L,m,1,1)    &! ->Cost of a failure at stage i
                   *(1 -    (1-(CTPM(j,1,L,(cp1_next_max+1),s1)-&
                    CTPM(j,1,L,(cp1_next_max+1),s1-1))&
                    /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))) &
                   *(1-(CTPM(j,2,m,(cp2_next_max+1),s2)&
                   -CTPM(j,2,m,(cp2_next_max+1),s2-1))&
                   /(1-CTPM(j,2,m,(cp2_next_max+1),s2-1))))

              IF ((i-s).ne.0) THEN
                 savings=(g(i-s+1,j,0,1,1,1,1)+f(i-s,j,0,1,1,1,1)- f_survived ) &
                     *(1 - (1 - (CTPM(j,1,L,(cp1_next_max+1),s1)&
                      -CTPM(j,1,L,(cp1_next_max+1),s1-1))&
                      /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))) &
                     *(1 - (CTPM(j,2,m,(cp2_next_max+1),s2)&
                     -CTPM(j,2,m,(cp2_next_max+1),s2-1))&
                     /(1-CTPM(j,2,m,(cp2_next_max+1),s2-1))))
                        ELSE
               savings= g(i-s+1,j,0,1,1,1,1) &
                     *(1 - (1 - (CTPM(j,1,L,(cp1_next_max+1),s1)-&
                     CTPM(j,1,L,(cp1_next_max+1),s1-1))&
                     /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))) &
                     *(1 - (CTPM(j,2,m,(cp2_next_max+1),s2)-&
                     CTPM(j,2,m,(cp2_next_max+1),s2-1))&
                     /(1-CTPM(j,2,m,(cp2_next_max+1),s2-1))))
              END IF
        END IF
    ELSE                ! (CP1_levels+1)
        ucmc=ucm_costs(i-s+1,j,age,j,L,1,1,1)   &!    ->Cost of a failure at stage i
              *(CTPM(j,1,L,(cp1_next_max+1),s1)-&
              CTPM(j,1,L,(cp1_next_max+1),s1-1))&
              /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))
        IF ((i-s).ne.0) THEN
           savings= (g(i-s+1,j,0,1,1,1,1)+f(i-s,j,0,1,1,1,1)- f_survived ) &
                    *(CTPM(j,1,L,(cp1_next_max+1),s1)-&
                    CTPM(j,1,L,(cp1_next_max+1),s1-1))&
                    /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))
        ELSE
           savings= g(i-s+1,j,0,1,1,1,1) &
                    *(CTPM(j,1,L,(cp1_next_max+1),s1)-&
                    CTPM(j,1,L,(cp1_next_max+1),s1-1))&
                    /(1-CTPM(j,1,L,(cp1_next_max+1),s1-1))
        END IF
    END IF

    acc_ucmc=acc_ucmc+(beta**s)*(ucmc-savings)



    END DO   !s
        puni=acc_op-acc_ucmc
    END FUNCTION     puni


!**********************

!         END SUBROUTINE

END PROGRAM MDP05
```

# Appendix B

# Fortran code for the CTPM

The Fortran (F90) source code for used to generate the Condition Transition Probability Matrix.

In addition to the code given below the program also uses a library called DCDFLIB [95]. DCDFLIB is a FORTRAN90 library, using double precision arithmetic, for evaluating cumulative density functions.

DCDFLIB includes routines for evaluating the cumulative density functions of a variety of standard probability distributions. An unusual feature of this library is its ability to easily compute any one parameter of the CDF given the others. This means that a single routine can evaluate the CDF given the usual parameters, or determine the value of a parameter that produced a given CDF value.

```
PROGRAM CTPM

USE NumUtils

IMPLICIT NONE

REAL*8                              :: Li_lower, Li_upper, step, b, q1, q2 !(q2=q_M,M+1)
REAL*8                              :: delta, Lj_lower, Lj_upper, delta_p_ij
REAL*8,DIMENSION(1:105,1:400)       :: sum_norm
REAL*8,DIMENSION(1:105)             :: res_life, horizon  !Horizon included 5.7 2006
REAL*8,DIMENSION(1:105,1:106,1:400) :: q_ij, qn_ij, p_ij, p2
REAL*8,DIMENSION(1:105,1:2 ,1:400)  :: p1
INTEGER                             :: i, j, k,i_eq_j, T, t_s, M, M_q !M: Number of Cond. Levels
REAL*8,DIMENSION(0:105)             :: CB_value


!INPUT VALUES**************************************************************

M=10            ! number of condition levels (between 0-100)
T=200           ! number of timesteps
q1=1.0          ! The probability that a failure will be detected by an inspection q<=1
q2=0.0          ! The one step transition probability from condition M+1 to condition
                ! M+2 if q1<1.
b=4.0           ! standard deviation
step=0.01       ! time increment     (used in the numerical integration)


!NOTE:Remeber to change g(t), g_inv, g_inv_deriv and the condition boundaries as
```

```
!      described below.


CB_value(0)= 0.0   !(new condition)
DO i=1, M   !————!Note: (M=10,  10; M=20, 5;  M=40, 2.5; M=80, 1.25)
   CB_value(i)=CB_value(i−1)+ 10
END DO

!Gas turbine efficiency losses
  !CB_value(0)= 0           !(new condition) (M=8)
  !CB_value(1)= 50.0/4.0
  !CB_value(2)= 50.0/2.0
  !CB_value(3)= 150.0/4.0
  !CB_value(4)= 50.0
  !CB_value(5)= 125.0/2.0
  !CB_value(6)= 150.0/2.0
  !CB_value(7)= 100.0∗7.0/8.0
  !CB_value(8)= 100.0     !(Breakdown)

! END INPUT VALUES∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗



M_q = M+1
IF (q1.eq.1) THEN
    M_q=M
ENDIF

DO i=1,M
    DO j=1, M+1
        DO k=1,T
            q_ij(i,1,k)=0
        END DO
    END DO
END DO


DO i=1, M ! Number of working condition levels
    print ∗,  'CONDITION LEVEL', i
    Li_lower=CB_value(i−1)
    Li_upper=CB_value(i)
    DO k=1,T
        i_eq_j=1   ! True i.e. qii is calculated
        delta=k∗1.0
        DO  j= i, M
            Lj_lower=CB_value(j−1)
            Lj_upper=CB_value(j)

            q_ij(i,j,k)=Simp(Li_lower,Li_upper,step,delta,Lj_lower,Lj_upper,i_eq_j,b)
            i_eq_j=0
            sum_norm(i,k)=sum_norm(i,k)+q_ij(i,j,k)
        END DO
    END DO
END DO

! Correction when sum_norm>1.0 because of rounding errors.
DO i=1, M
    DO k=1,T
        IF  (sum_norm(i,k)>1.0) THEN
          !DO j=i,(M−1)
           DO j=i, M
               q_ij(i,j,k)=q_ij(i,j,k)/sum_norm(i,k)
           END DO
           sum_norm(i,k)=1.0
        END IF
    ENDDO
ENDDO

!Failure condition level
DO i=1, M
    DO k=1,T
        q_ij(i,M+1,k)=1−sum_norm(i,k)
    ENDDO
ENDDO

qn_ij=q_ij

IF (q1.eq.1) THEN
    p_ij=qn_ij
ELSE
    DO i=1, M_q
```

```
        DO j=i,M_q
            DO k=1, T
                IF (i.eq.M_q) THEN
                    qn_ij(i,j,k)=1.0
                END IF
                IF (j<M_q) THEN
                    p_ij(i,j,k)=qn_ij(i,j,k)
                ELSE
                    IF (k.eq.1) THEN
                     p_ij(i,j,k)=(1-q1)*qn_ij(i,j,k)
                    ELSE
                     p_ij(i,j,k)=(1-q1)*(qn_ij(i,j,k)-qn_ij(i,j,(k-1)))+(1-q2)*p_ij(i,j,(k-1))
                    ENDIF
                     p_ij(i,j+1,k)=qn_ij(i,j,k)-p_ij(i,j,k)
                ENDIF
            END DO
        END DO
    END DO
END IF

!Calculation of maximum residual life at each condition level
!SUM FROM k=1 TO T: [F(k)-F(k-1)]*k
DO i=1, M_q        !********
    res_life(i)=0
    horizon(i)=0
    DO k=1, T
        IF (k .eq. 1) THEN
            delta_p_ij=p_ij(i,M_q+1,k)
        ELSE
            delta_p_ij=p_ij(i,M_q+1,k)-p_ij(i,M_q+1,k-1)
        ENDIF
        IF (delta_p_ij>0) THEN   ! else, the equipment becomes better after use
            res_life(i)=res_life(i)+delta_p_ij*(k-0.5)
        ENDIF
        IF (p_ij(i,M_q+1,k)<0.99999)THEN
            horizon(i)=horizon(i)+1
        ENDIF
    ENDDO
ENDDO

!Ouput

OPEN(1, FILE='CTPMx.dat')        !Natural transition probabilities
OPEN(2, FILE='MRL_error.dat')    !MRL error due to discretization

!Natural transition probabilities
WRITE(1,26) (res_life(i), i=1,M_q)
DO i=1, M_q    !*******************
    DO j=1, (M_q+2-i)
        WRITE(1,25) (p_ij(i,i+j-1,k), k=1, T)
    END DO
END DO

!Mean residaul life error (MRL1-MRL2), ref. sec 5.3.3)
WRITE (2,*) 'MRL error due to discretization'
DO i=1, M_q    !*******************
    WRITE (2,22) i, ExactRes(CB_value(M))-ExactRes((CB_value(i-1)+&
            CB_value(i))/2.0),res_life(i),ExactRes(CB_value(M))- &
            ExactRes((CB_value(i-1)+CB_value(i))/2.0)-res_life(i)
ENDDO

CLOSE(1)
CLOSE(2)

PRINT *, 'The following files have been produced:'
PRINT *, '1. ctpmx.dat       : Natural transition probabilities'
PRINT *, '2. MRL_error.dat   : MRL error due to discritization'


10      FORMAT (I3,';',I3,';',F8.6,';......................
20      FORMAT (I3,';',I3,';',F8.6,';......................
21      FORMAT (I3,';',I3,';',F8.6,';......................
22      FORMAT ('Condition level: ;',......................
25      FORMAT (F8.6,',', F8.6,',', ......................
26      FORMAT (F8.3,',', F8.3,',', ......................


END PROGRAM CTPM
```

```fortran
Module NumUtils
!************************************
!** Use of the normal distribution **
!************************************

CONTAINS

FUNCTION Fung(Li_lower, Li_upper, omega, delta, Lj_lower, Lj_upper, i_eq_j, b)


EXTERNAL cdfnor, g_inv, g_inv_deriv, g_func

REAL*8  :: Fung, mean, Phi1, Phi2, q, b, sd, z1, z2, std
REAL*8  :: Li_upper, Li_lower, omega, Lj_lower, Lj_upper, delta
INTEGER :: status, bound, i_eq_j


! The equation which calculate the variables "b" and "mean" will
! depend on the function developed for the degradation process:
sd=b*sqrt(delta)
mean=g_func(omega, delta)
z1=Lj_lower
z2=Lj_upper

CALL cdfnor(1, Phi1, q, z1, mean, sd, status, bound)
CALL cdfnor(1, Phi2, q, z2, mean, sd, status, bound)

!  Fung depends on the g(t) function:


!ORIGINAL
IF (i_eq_j.eq.0) THEN
    Fung=((Phi2-Phi1)/(g_inv(Li_upper)-g_inv(Li_lower)))*g_inv_deriv(omega)
ELSE
    Fung=(Phi2/(g_inv(Li_upper)-g_inv(Li_lower)))*g_inv_deriv(omega)
ENDIF

END FUNCTION Fung

!Integration using Simpson algorithm

FUNCTION Simp(Li_lower,Li_upper,h,delta,Lj_lower,Lj_upper,i_eq_j, b)

REAL*8, INTENT(IN)  :: Li_upper, Li_lower, h, delta, Lj_upper, Lj_lower, b
REAL*8              :: Simp
INTEGER, INTENT(IN) :: i_eq_j
INTEGER             :: i, n



Simp = 0
n = NINT( (Li_upper-Li_lower) / (2 * h) )          ! 2N panels now
DO i = 1, n-1
    Simp = Simp + 2 * Fung( Li_lower, Li_upper, (Li_lower +  2 * i * h), delta, &
          Lj_lower, Lj_upper, i_eq_j,b)
END DO
DO I = 1, n
    Simp = Simp + 4 * Fung( Li_lower, Li_upper, (Li_lower +  (2 * i - 1) * h),delta, &
          Lj_lower, Lj_upper , i_eq_j, b)
END DO
    Simp = h / 3 * (Fung(Li_lower, Li_upper, Li_lower, delta,Lj_lower,Lj_upper, &
          i_eq_j, b) + Fung(Li_lower, Li_upper, Li_upper,delta,Lj_lower,Lj_upper, &
          i_eq_j, b) + Simp)

END FUNCTION Simp



FUNCTION ExactRes(CB)

REAL*8  :: CB
REAL*8  :: ExactRes

ExactRes=g_inv(CB)

END FUNCTION ExactRes
```

```
FUNCTION g_func(omega, delta)


EXTERNAL g_inv
REAL*8                :: g_func, t, omega, delta


t=g_inv(omega)+delta


!chapter 5
g_func=2*t !techn:A
! g_func=1.5*t !techn:B
! g_func=10*SQRT(2*t) ! square root
! g_func=0.02*(t)**2+t !quadratic

!chapter 7
! g_func=2.942166667*t-.00861667*t**2   ! Efficiency

! g_func= (10.0/7.0)*t+(2.0/49.0)*t**2 !cracking MTTF=35
! g_func= (7.0/6.0)*t+(1.0/30.0)*t**2  !cracking MTTF=40


END FUNCTION g_func


FUNCTION g_inv(omega)
! The inverse function of the detoriation process

REAL*8  :: g_inv, omega


!chapter 5
g_inv= 0.5* omega                              ! techn.:A
!  g_inv= (2.0/3.0)* omega                      ! techn: B
!  g_inv= (1.0/200.0)*(omega)**2                ! square root
!  g_inv=-25.0+5.0*sqrt(25.0+2*(omega))         ! quadratic


!chapter 7
! g_inv= 170.7253385-58.02707929*(8.6563447-&
        !.0344667*omega)**(0.5) !Efficiency


! g_inv=-(35.0/2.0)+3.5*(25.0+2*omega)**(0.5)  !cracking MTTF=35
! g_inv=-(35.0/2.0)+0.5*(1225.0+120.0*omega)**(0.5) !cracking MTTF=40


END FUNCTION g_inv



FUNCTION g_inv_deriv(omega)
! The inverse derived function of the detoriation process

REAL*8  :: g_inv_deriv, omega


!chapter 5
g_inv_deriv=0.5        !techn:A                 !linerar
! g_inv_deriv=2.0/3.0    !techn:B               !linear

! g_inv_deriv=(1.0/100.0)*(omega)               ! square root
! g_inv_deriv=5.0/sqrt(25.0+2.0*(omega))        ! quadratic


!chapter 7
! g_inv_deriv=1.0/(8.6563447-.0344667*omega)**(0.5)   !Efficiency

! g_inv_deriv=3.5/((25.0+2*omega)**(0.5))       ! cracking MTTF35
! g_inv_deriv=30.0/(1225.0+120*omega)**(0.5)    ! cracking MTTF40

END FUNCTION g_inv_deriv


END MODULE
```

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